YK

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Mathematics

ENGLISH

10 Standard

Part –2

Version 1.1

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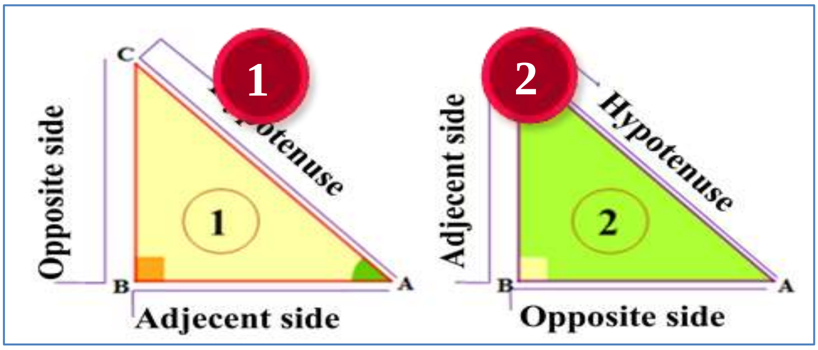
8 ITRODUCTION TO TRIGONOMETRY

Trigonometry is the study of relationships between the sides and angles of a triangle.

11.2 Trigonometric Ratios

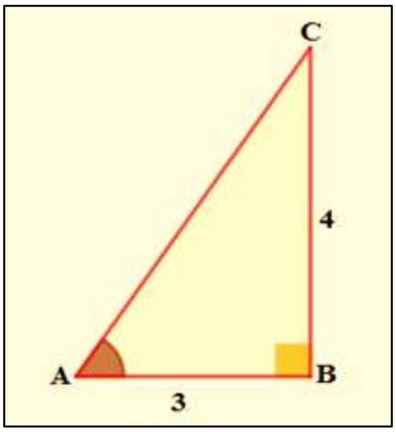
To know the trigonometric ratio we have to consider right angle triangle.



Trigonometric ratios		1	2
SinA	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{BC}{AC}$	$\frac{AB}{AC}$
CosA	$\frac{\text{Adjecent}}{\text{Hypotenuse}}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$
Tan A	$\frac{\text{Opposite}}{\text{Adjecent}}$	$\frac{BC}{AB}$	$\frac{AB}{BC}$
CosecA	$\frac{\text{Hypotenuse}}{\text{Opposite}}$	$\frac{AC}{BC}$	$\frac{AC}{AB}$
SecA	$\frac{\text{Hypotenuse}}{\text{Adjecent}}$	$\frac{AC}{AB}$	$\frac{AC}{BC}$
CotA	$\frac{\text{Adjecent}}{\text{Opposite}}$	$\frac{AB}{BC}$	$\frac{BC}{AB}$

There are six trigonometric ratios:

Example 1: tan A = $\frac{4}{3}$ find the other trigonometric ratios of the angle A



In ΔABC, $\angle ABC = 90^0$

$\therefore$ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

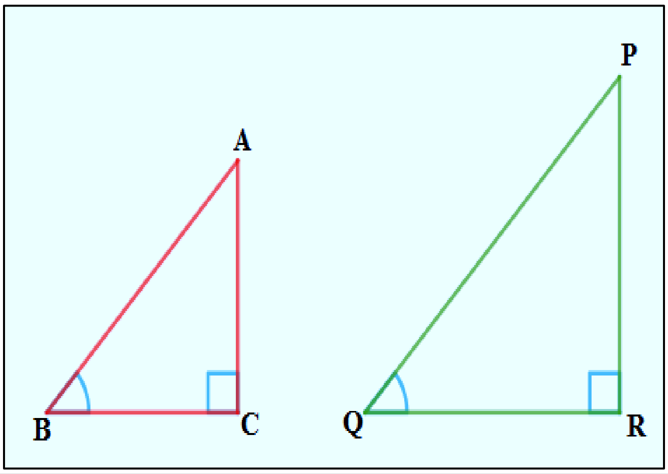
$\Rightarrow AC^2 = 4^2 + 3^3 = 16 + 9 = 25$

$\Rightarrow AC = 5$

$\text{Sin}A = \frac{BC}{AC} = \frac{4}{5}$; $\text{Cos }A = \frac{AB}{AC} = \frac{3}{5}$; $\text{Tan}A = \frac{BC}{AB} = \frac{4}{3}$

$\text{Cosec}A = \frac{AC}{BC} = \frac{5}{4}$; $\text{Sec}A = \frac{AC}{AB} = \frac{5}{3}$; $\text{Cot}A = \frac{AB}{BC} = \frac{3}{4}$

Example 2 :If B and Q are acute angles such that sin B = sin Q, then prove that $\angle B = \angle Q$.



Sin B = SinQ

$\Rightarrow \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \frac{AC}{PR} = \frac{AB}{PQ} = k$ ----- (1)

$BC = \sqrt{AB^2 - AC^2}$ [By Pythagoras theorem]

$\Rightarrow \sqrt{k^2PQ^2 - k^2PR^2}$

$\Rightarrow k.\sqrt{PQ^2 - PR^2}$ [from (1)]

$QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \frac{BC}{QR} = \frac{k.\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ------(2)

From (1) and (2),

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \Delta ABC \sim \Delta PQR$

$\therefore$ **$\angle B = \angle Q$**

Example 3 : Consider ΔACB, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$ (see Fig. 8.10). Determine the values of $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$

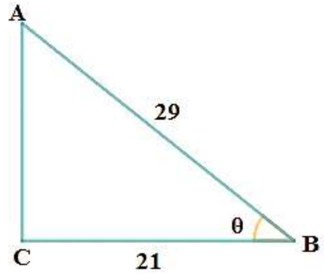


Fig. 8.10

In right angle triangle ACB, $\angle ACB = 90^0$

Therefore $AC = \sqrt{AB^2 - BC^2}$

$\Rightarrow AC = \sqrt{29^2 - 21^2}$

$\Rightarrow AC = \sqrt{841 - 441} = \sqrt{400} = 20$

(i) $\cos^2\theta + \sin^2\theta = \frac{21^2}{29^2} + \frac{20^2}{29^2} = \frac{441+400}{841} = \frac{841}{841} = 1$

(ii) $\cos^2\theta - \sin^2\theta = \frac{21^2}{29^2} - \frac{20^2}{29^2} = \frac{441-400}{841} = \frac{41}{841} = 1$

Example 4 : In a right triangle ABC, right-angled at B, if tan A = 1, then verify that 2sin A cos A = 1

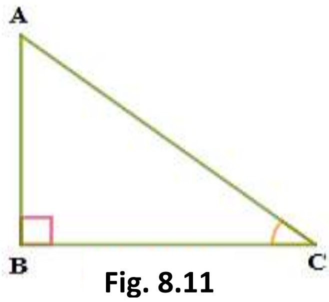


Fig. 8.11

In right angle triangle ACB ,

$\tan A = 1 \Rightarrow \frac{AB}{BC} = 1$

$\Rightarrow AB = BC$

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$\Rightarrow AC^2 = 2AB^2$ -------- (1)

Now, $2\sin A \cos A = 2.\frac{AB}{AC}.\frac{BC}{AC}$

$= 2.\frac{AB^2}{AC^2}$

$= 2.\frac{AB^2}{2AB^2} = 1$

Example 5 : In ΔOPQ right-angled at P, OP = 7 cm and OQ – PQ = 1 cm (see Fig. 11.12). Determine the values of sin Q and cos Q.

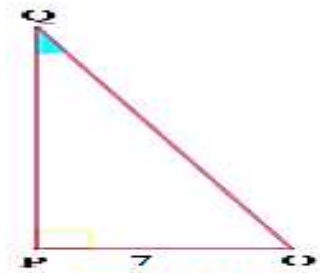


Fig. 8.12

In ΔOPQ, $OQ^2 = PQ^2 + OP^2$ [By Pythagoras theorem]

$\Rightarrow (1 + PQ)^2 = PQ^2 + 7^2$

$\Rightarrow 1 + PQ^2 + 2PQ = PQ^2 + 49$

$\Rightarrow 1 + 2PQ = 49$

$\Rightarrow 2PQ = 49 – 1 = 48$

$\Rightarrow$ PQ = 24cm

$\Rightarrow OQ = 1 + PQ = 1 + 24$

$\Rightarrow OQ = 25$

$\therefore$ **$\sin Q = \frac{7}{25}$ and $\cos Q = \frac{24}{25}$**

Inverse of trigonometric values

$\frac{1}{\sin A}$ $\frac{\text{Hypotenuse}}{\text{Opposite}}$ **CosecA**

$\frac{1}{\cos A}$ $\frac{\text{Hypotenuse}}{\text{Adjecent}}$ **SecA**

$\frac{1}{\tan A}$ $\frac{\text{Adjecent}}{\text{Opposite}}$ **CotA**

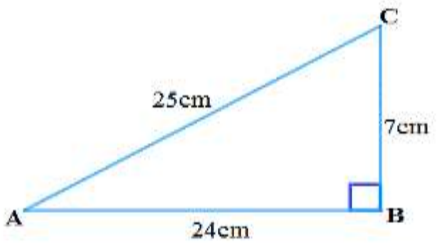
Inverse of trigonometric values		
$\frac{1}{\text{CosecA}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	**SinA**
$\frac{1}{\text{SecA}}$	$\frac{\text{Adjecent}}{\text{Hypotenuse}}$	**SecA**
$\frac{1}{\text{CotA}}$	$\frac{\text{Opposite}}{\text{Adjecent}}$	**CotA**

Exercise 8.1

[for solving problems, the value of constant k is taken as 1]

1. In ΔABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine

i) sin A, cos A (ii) sin C, cos C



In Δ ABC, $\angle B = 90^o$ $\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + 7^2$

$= (576+49)\ cm^2 = 625\ cm^2$

$\Rightarrow AC = 25$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$; $\cos C = \frac{BC}{AC} = \frac{7}{25}$

2. In Fig. 8.13, find tan P – cot R.

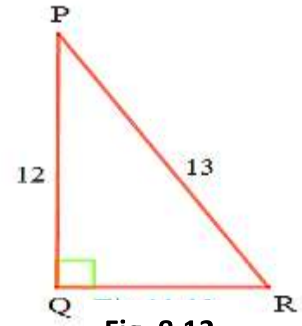


Fig. 8.13

In ΔPQR , By Pythagoras theorem,

$QR^2 = PR^2 - PQ^2 = (13)^2 - (12)^2 = 169 - 144$

$\Rightarrow QR^2 = 25$

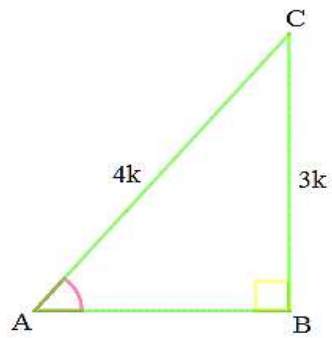
$\Rightarrow QR = 5$ cm

Now, $\tan P = \frac{QR}{PQ} = \frac{5}{12}$

$\cot R = \frac{QR}{PQ} = \frac{5}{12}$;

$\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

3. If $\sin A = \frac{3}{4}$ calculate the value of cos A and tan A



In ΔABC, $\angle B = 90^0$ According to question,

$\sin A = \frac{3}{4} = \frac{BC}{AC}$

$\Rightarrow AC = 4k, BC = 3k$ [Here, Let k = 1]

$\therefore$ By Pythagoras theorem,

$AB^2 = AC^2 - BC^2$

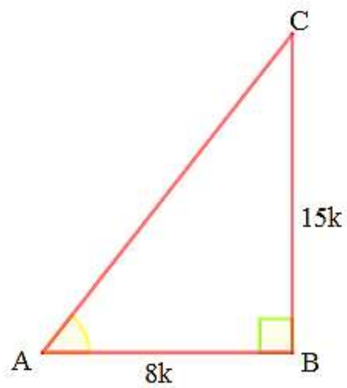
$\Rightarrow AB^2 = AC^2 - BC^2$

$AB^2 = 4^2 - 3^2 = 16 - 9 = 7$

$\Rightarrow AB = \sqrt{7}$

$\therefore \cos A = \frac{AB}{AC} = \frac{\sqrt{7}}{4}$; $\tan A = \frac{Bc}{AB} = \frac{3}{\sqrt{7}}$

4. Given 15 cot A = 8 find sin A and sec A



According to question, $\cot A = \frac{8}{15} = \frac{BC}{AB}$

In ΔABC , $\angle B = 90^0$

BC = 15k, AB = 8k [Here, Let k = 1]

$\therefore$ By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

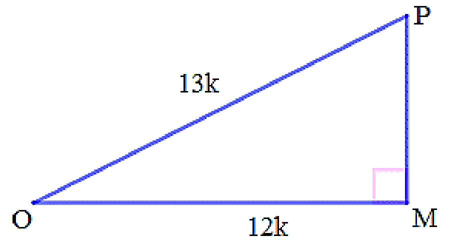
$AC^2 = 8^2 + 15^2 = 64 + 225 = 289$

$\Rightarrow AC = 17$

$\Rightarrow \sin A = \frac{BC}{AC} = \frac{15}{17}$;

$\sec A = \frac{AC}{AB} = \frac{17}{8}$

5. Given $\sec\theta = \frac{13}{12}$ calculate all other trigonometric values.



According to question, $\sec\theta = \frac{13}{12} = \frac{OP}{OM}$

In ΔPMO, $\angle M = 90^0$

OM = 12k, OP = 13k [Here, Let k = 1]

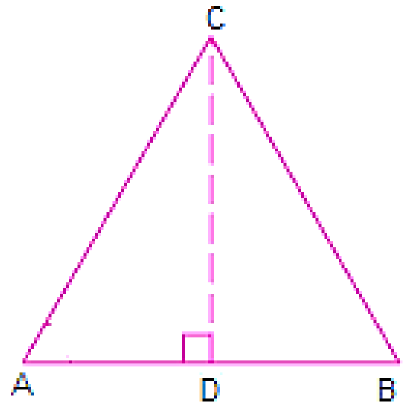
$\therefore$ By Pythagoras theorem, $PM^2 = OP^2 + OM^2$

$PM^2 = 13^2 + 12^2 = 169 - 144 = 25$

$\Rightarrow AC = 5$

$\sin\theta = \frac{MP}{OP} = \frac{5}{13}$; $\cos\theta = \frac{OM}{OP} = \frac{12}{13}$; $\tan\theta = \frac{MP}{OM} = \frac{5}{12}$ $\cot\theta = \frac{OM}{MP} = \frac{12}{5}$; $\text{cosec}\,\theta = \frac{OP}{MP} = \frac{13}{5}$

6. If A and B are acute angles such that cos A = cos B, then show that $\angle A = \angle B$



In ΔABC, $CD \perp AB$.

According to question, cos A = cos B

$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$

$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow AD = kBD$----(1)and

$\Rightarrow AC = kBC$----(2)

In ΔCAD and ΔCBD,

By Pythagoras theorem,

$CD^2 = AC^2 - AD^2$ ------(3) and $CD^2 = BC^2 - BD^2$ ------(4)

From (3) and (4), $AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (kBC)^2 - (k\,BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$; Substitute k = 1 in (2),

AC = BC

$\Rightarrow \angle A = \angle B$ [Angle opposite to the equal sides are equal]

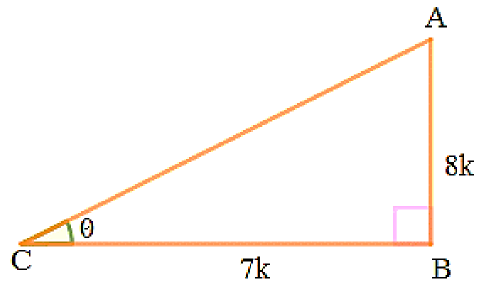
7. If $\cot\theta = \frac{7}{8}$ then find the value of i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ ii) $\cot^2\theta$

In Δ ABC, $\angle B = 90^\circ$ and $\angle C = \theta$ According to question,

$\cot\theta = \frac{BC}{AB} = \frac{7}{8}$

$\Rightarrow$ AB = 8 and BC = 7 [If k = 1]

In Δ ABC By Pythagoras theorem,



$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 8^2 + 7^2$

$\Rightarrow AC^2 = 64 + 49$

$\Rightarrow AC^2 = 113$

$\Rightarrow AC = \sqrt{113}$

$\sin\theta = \frac{AB}{AC} = \frac{8}{\sqrt{113}}$ and $\cos\theta = \frac{BC}{AC} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{\frac{113-64}{113}}{\frac{113-49}{113}} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$;

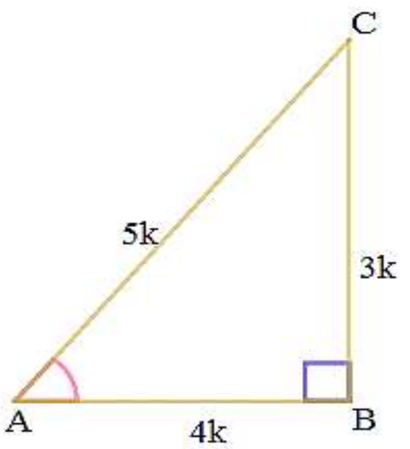
(ii) $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

8. If 3 cot A = 4 Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not?

In Δ ABC , ∠B = 90° Given,

$\cot A = \frac{AB}{AC} = \frac{4}{3}$

⇒AB = 4 ಮತ್ತು BC = 3, [taken k = 1]



In Δ ABC by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 4^2 + 3^2$

$\Rightarrow AC^2 = 16 + 9$

$\Rightarrow AC^2 = 25$

$\Rightarrow AC = 5$

$\tan A = \frac{BC}{AB} = \frac{3}{4}$; $\sin A = \frac{BC}{AC} = \frac{3}{5}$; $\cos A = \frac{AB}{AC} = \frac{4}{5}$

LHS $= \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$

$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$

R.H.S. $= \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{4}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

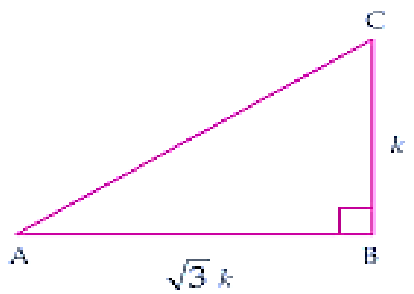
⇒R.H.S. = L.H.S.

$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

9. In triangle ΔABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$ then find the value of i) sin A cos C + cos A sin C ii) cos A cos C - sin A sin C

In Δ ABC , ∠B = 90° Given, $\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let AB = $\sqrt{3}$ and BC = 1,



In Δ ABC By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = (\sqrt{3})^2 + (1)^2$

$\Rightarrow AC^2 = 3 + 1$

$\Rightarrow AC^2 = 4$

$\Rightarrow AC = 2$

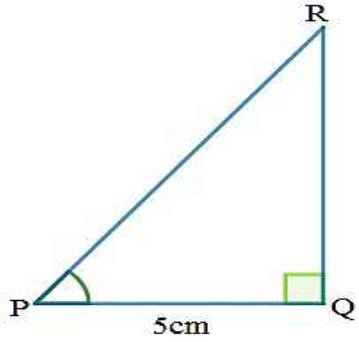
$\sin A = \frac{BC}{AC} = \frac{1}{2}$; $\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$; $\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$; $\cos C = \frac{BC}{AC} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C = \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

10. In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Given PR + QR = 25 , PQ = 5; Let PR = x.
∴ QR = 25 - x
By Pythagoras theorem, $PR^2 = PQ^2 + QR^2$
$x^2 = (5)^2 + (25 - x)^2$
$\Rightarrow x^2 = 25 + 625 + x^2 - 50x$
$\Rightarrow 50x = 650$
$\Rightarrow x = 13$
∴ PR = 13 cm
⇒ QR = (25 - 13) cm = 12 cm



$\sin P = \frac{QR}{PR} = \frac{12}{13}$; $\cos P = \frac{PQ}{PR} = \frac{5}{13}$; $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

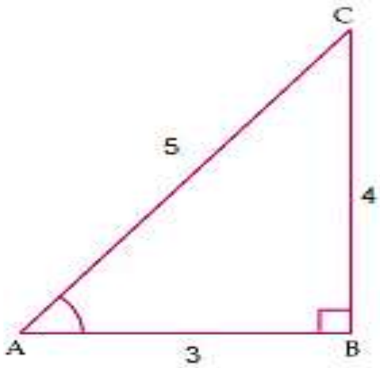
i) The value of tan A is always less than 1.

ii) sec A $= \frac{12}{5}$ for some values of A

iii) cos A is the abbreviation used for the cosecant of angle A

iv) cot A is the product of cot and A

v) sin θ $= \frac{4}{3}$ for some values of θ



(i) False

In ΔABC, $\angle B = 90^0$, Let AB = 3, BC = 4 and AC = 5 $\Rightarrow \tan A = \frac{4}{3} > 1$

(ii) True

In ΔABC , $\angle B = 90^0$, AC =13k and AB = 5k [k Positive real number]
⇒ AC =12, BC=5 and AB = 5 [If k = 1]
$BC^2 = AC^2 - AB^2$ [By Pythagoras theorem]
$\Rightarrow BC^2 = 12^2 - 5^2 \Rightarrow BC^2 = 144 – 25 \Rightarrow BC^2 = 119 \Rightarrow \sec A = \frac{12}{5}$

(iii) False

cosecA is a abbreviation of CosecantA and CosA is the abbreviation of CosineA.

(iv) False, cot A is not a product of cot and A . it is just a symbol

(v) False

$\sin\theta = \frac{\text{Opposite}}{Hypotenuse}$ In any triangle hypotenuse is the larger side.
∴ sin θ is always $\leq 1 \Rightarrow \sin\theta = \frac{4}{3}$, it is not possible for any value of θ

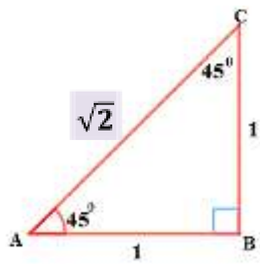
11.3 Trigonometric Ratios of Some Specific Angles:

Trigonometric ratio of 45^0

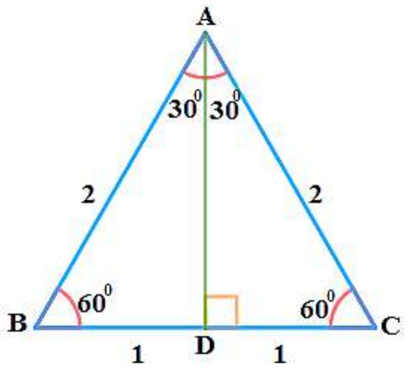
In ΔABC, $\angle B = 90^0, \angle A = 45^0 \Rightarrow \angle C = 45^0$ [Sum of interior angles is 180^0]

⇒ Let AB = BC = 1 , By Pythagoras theorem,
∴ $AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = 1^2 + 1^2 = 1+1 = 2 \Rightarrow AC = \sqrt{2}$

$Sin45^0$	$\frac{1}{\sqrt{2}}$	$Cosec45^0$	$\sqrt{2}$
$Cos45^0$	$\frac{1}{\sqrt{2}}$	$Sec45^0$	$\sqrt{2}$
$Tan45^0$	1	$Cot45^0$	1



Trigonometric Ratios of 30^0 and 60^0

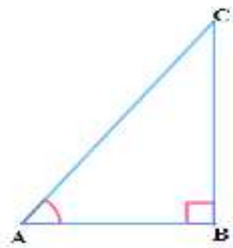


In equilateral triangle, the angles are equal.
$\Rightarrow \angle A = \angle B = \angle C = 60^0$
Draw $AD \perp BC$ $\Rightarrow BD = CD$
[In an equilateral triangle the perpendicular from the vertex bisects the base]
$\Rightarrow \angle BAD = \angle CAD = 30^0$
Let AB = BC = CA = 2,
$\Rightarrow BD = CD = 1$
In ΔABD , By Pythagoras theorem,
$AD^2 = AB^2 - BD^2$
$\Rightarrow AD^2 = 2^2 - 1^2 = 4 - 1 = 3$
$\Rightarrow AD = \sqrt{3}$

$Sin60^0$	$\frac{\sqrt{3}}{2}$	$Cosec60^0$	$\frac{2}{\sqrt{3}}$
$Cos60^0$	$\frac{1}{2}$	$Sec60^0$	2
$Tan60^0$	$\sqrt{3}$	$Cot60^0$	$\frac{1}{\sqrt{3}}$

$Sin30^0$	$\frac{1}{2}$	$Cosec30^0$	2
$Cos30^0$	$\frac{\sqrt{3}}{2}$	$Sec30^0$	$\frac{2}{\sqrt{3}}$
$Tan30^0$	$\frac{1}{\sqrt{3}}$	$Cot30^0$	$\sqrt{3}$

Trigonometric ratios of 0^0 and 90^0



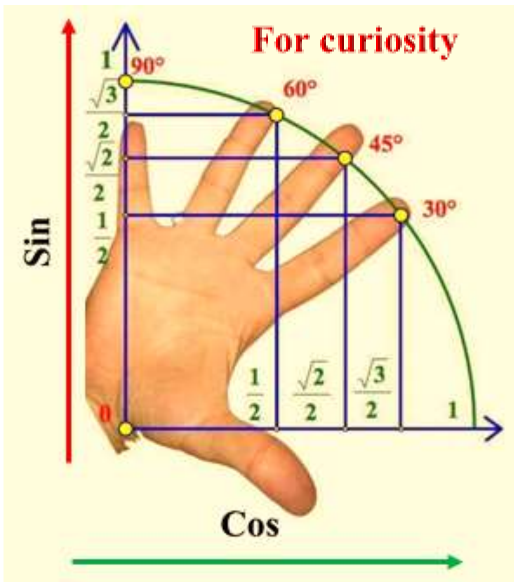
If $\angle A$ Closer to 0^0 then the length of BC closer to 0 and almost AB = AC,
If $\angle A$ closer to 90^0
Then the length of AB closer to 0 and almost AC = AC
Let AB = AC = 1 and BC = 0

$Sin0^0$	0	$Cosec0^0$	ND
$Cos0^0$	1	$Sec0^0$	1
$Tan0^0$	0	$Cot0^0$	ND

$Sin90^0$	0	$Cosec90^0$	ND
$Cos90^0$	1	$Sec90^0$	1
$Tan90^0$	0	$Cot90^0$	ND

Table 8.1

$\angle A$	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
Cosec	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
Cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Example 6 : In ΔABC, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ (see Fig. 8.19). Determine the lengths of the sides BC and AC.

$\text{Tan}30^0 = \frac{5}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC}$

$\Rightarrow 5\sqrt{3}\text{cm}$

$\text{Sin}30^0 = \frac{5}{AC}$

$\Rightarrow \frac{1}{2} = \frac{5}{AC}$

$\Rightarrow AC = 10\text{cm}$

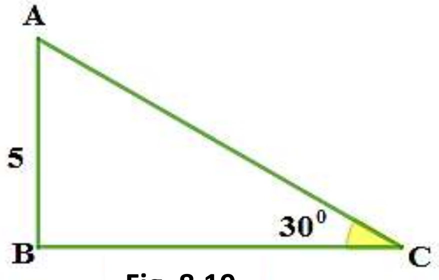


Fig. 8.19

Example 7 : In ΔPQR, right - angled at Q (see Fig. 11.20), PQ = 3 cm and PR = 6 cm. Determine $\angle QPR$ and $\angle PRQ$.

$\text{Sin}R = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \angle R = 30^0$

$\Rightarrow \angle PRQ = 30^0$

$\therefore \angle QPR = 60^0$

Fig. 8.20

Example 8: If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, $0 < A + B \le 90$, $A > B$ find A and B

If $\sin(A - B) = \frac{1}{2}$ then $\sin 30^0 = \frac{1}{2}$

$\Rightarrow A - B = 30^0$ ------------- (1)

If $\cos(A + B) = \frac{1}{2}$ then $\cos 60^0 = \frac{1}{2}$

$\Rightarrow A + B = 60^0$ ------------- (2)

(1) + (2) $= 2A = 90^0 \Rightarrow A = 45^0$;

(2) From (2) $\Rightarrow 45^0 - B = 30^0 \Rightarrow B = 15^0$

Exercise 8.2

1. Evaluate the following:

i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$ ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

iii) $\frac{\cos 45^0}{\sec 30^0 + \text{cosec}\, 30^0}$ iv) $\frac{\text{Sin}30^0 + \tan 45^0 - \text{Cosec}45^0}{\text{Sec}30^0 + \cos 60^0 + \cot 45^0}$ iv) $\frac{5\text{Cos}^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\text{Sin}^2 30^0 + \cos^2 30^0}$

i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4} = 1$

ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

iii) $\frac{\cos 45^0}{\text{Sec}\, 30^0 + \text{cosec}\, 30^0}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$=\frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}$

$=\frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$

$=\frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$

$=\frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2}$

$=\frac{2\sqrt{6}-6\sqrt{2}}{4\text{x}2-4\text{x}6}$

$=\frac{2(\sqrt{6}-3\sqrt{2})}{8-24}$

$=\frac{2(\sqrt{6}-3\sqrt{2})}{-16}$

$=\frac{\sqrt{6}-3\sqrt{2}}{-8}$

$=\frac{3\sqrt{2}-\sqrt{6}}{8}$

iv) $\frac{\sin 30^0+\tan 45^0-\text{Cosec}60^0}{\text{Sec}30^0+\cos 60^0+\cot 45^0}$

$=\frac{\left(\frac{1}{2}\right)+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$

$=\frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$

$=\frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}}$

$=\frac{3\sqrt{3}-4}{4+3\sqrt{3}}$

$\Rightarrow \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \frac{4-3\sqrt{3}}{4-3\sqrt{3}}$

$=\frac{12\sqrt{3}-16-9\sqrt{9}+12\sqrt{3}}{(4)^2-(3\sqrt{3})^2}$

$=\frac{12\sqrt{3}-16-27+12\sqrt{3}}{16-27}$

$=\frac{24\sqrt{3}-43}{-11} = \frac{43-24\sqrt{3}}{11}$

iv) $\frac{5\cos^2 60^0+4\sec^2 30^0-\tan^2 45^0}{\sin^2 30^0+\cos^2 30^0}$

$=\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-1}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}}$

$=\frac{\frac{15+64-12}{12}}{1}=\frac{67}{12}$

2. Choose the correct option and justify your choice:

i) $\frac{2\tan 30^0}{1+\tan^2 30^0}$

A) $\sin 60^0$ B) $\cos 60^0$ C) $\tan 60^0$ D) $\sin 30^0$

$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}=\frac{\sqrt{3}}{2}$

Ans: A) $\sin 60^0$

ii) $\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$

A) $\tan 90^0$ B) 1 C) $\sin 45^0$ D) 0

$\frac{1-1}{1+1} = \frac{0}{2} = 0$

Ans: D) 0

iii) sin 2A = 2 sin A is true when A =

A) 0 B) 30 C) 45 D) 60

sin 2x0 = 2 sin 0

⇒ sin 0 = 2 sin0 ⇒ 0 = 0

Ans: A) 0

iv) $\frac{2\tan 30^0}{1-\tan^2 30^0}$

A) $\cos 60^0$ B) $\sin 60^0$ C) $\tan 60^0$ D) $\sin 30^0$

$\frac{2x\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}}$

$= \sqrt{3}$

Ans: C) $\tan 60^0$

3. If tan (A + B) = $\sqrt{3}$ and tan (A - B) = $\frac{1}{\sqrt{3}}$, 0< A + B ≤ 90 ; A > B find A and B

tan (A + B) = $\sqrt{3}$

A + B = 60^0 -----(1)

tan (A - B) = $\frac{1}{\sqrt{3}}$

⇒ A – B = 30^0 -----(2)

(3) – (1) ⇒ 2B = 30^0

(4) ⇒ **B = 15^0**

(5) ⇒From (1) **A = 60 – 15 = 45^0**

4. State whether the following are true or false. Justify your answer.

i) sin (A + B) = sin A + sin B

Let A = 30^0 and B = 90^0

sin (30^0 + 60^0) = sin 90^0 = 1

⇒ sin 30^0 + sin 60^0 = $\frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

∴ sin (A + B) ≠ sin A + sin B

∴ The statement is false

ii) The value of sin θ increases as θ increases

sin 0^0 = 0 , sin 90^0 = 1

∴ The statement is true

iii)The value of cos θ increases as θ increases.

Cos 0^0 = 1 , Cos 90^0 = 0 Here,

we observe that as θ increases the value of cos θ dicreases

∴ The statement is false

iv)sinθ = cosθ for all values of θ

sin 30^0 = $\frac{1}{2}$; cos 30^0 = $\frac{\sqrt{3}}{2}$

⇒ sin θ ≠ cos θ for all values of θ

∴ The statement is false

v) cot A is not defined for A = 0^0

The statement is true

8.5 Trigonometric Identities:

An equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angle(s) involved.

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 + 1 = \sec^2 A$$

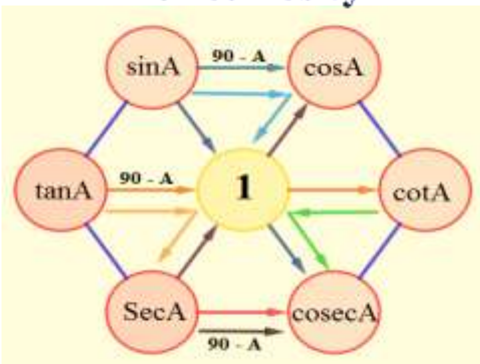
$$1 + \cot^2 A = \mathrm{cosec}^2 A$$

Note:

$\frac{\sin A}{\cos A} = \tan A$;

$\frac{\cos A}{\sin A} = \cot A$

For curiosity



Example 9 : Express the ratios cos A, tan A and sec A in terms of sin A.

$\cos^2 A + \sin^2 A = 1$

$\Rightarrow \cos^2 A = 1 - \sin^2 A$

$\Rightarrow \cos A = \sqrt{1 - \sin 2A}$

$\therefore \tan A = \frac{\sin A}{\cos A}$

$\Rightarrow \frac{\sin A}{\sqrt{1 - \sin 2A}}$

$\Rightarrow \sec A = \frac{1}{\cos A}$

$\Rightarrow \frac{1}{\sqrt{1 - \sin 2A}}$

Example 10 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.

LHS $= \sec A (1 - \sin A)(\sec A + \tan A)$

$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \left(\frac{1 - \sin A}{\cos A}\right)\left(\frac{1 + \sin A}{\cos A}\right)$

$= \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$

Example 11: Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\mathrm{cosec}\, A - 1}{\mathrm{cosec}\, A + 1}$

$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)}$

$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\mathrm{cosec}\, A - 1}{\mathrm{cosec}\, A + 1}$

Example 12:Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using the identity $\sec^2\theta = 1 + \tan^2\theta$

$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$

$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \times \frac{\tan\theta - \sec\theta}{\tan\theta - \sec\theta}$

$= \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{-1 - \tan\theta + \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{-1}{(\tan\theta - \sec\theta)}$

$= \frac{1}{\sec\theta - \tan\theta}$

Exercise 8.3

1. Express the trigonometric ratios sinA, secA and tanA in terms of cotA

Let $\text{cosec}^2A = 1 + \cot^2A$

$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2A$

$\Rightarrow \sin^2A = \frac{1}{1+\cot^2 A}$

$\Rightarrow \mathbf{\sin A} = \frac{\pm 1}{\sqrt{1+\cot^2 A}}$

Again, $\sin^2A = \frac{1}{1+\cot^2 A}$

$\Rightarrow 1 - \cos^2A = \frac{1}{1+\cot^2 A}$

$\Rightarrow \cos^2A = 1 - \frac{1}{1+\cot^2 A}$

$\Rightarrow \cos^2A = \frac{1+\cot^2 A - 1}{1+\cot^2 A}$

$\Rightarrow \frac{1}{\sec^2 A} = \frac{\cot^2 A}{1+\cot^2 A}$

$\Rightarrow \sec^2A = \frac{1+\cot^2 A}{\cot^2 A}$

$\Rightarrow \sec A = \frac{\pm\sqrt{1+\cot^2 A}}{\cot A}$

$\Rightarrow \tan A = \frac{1}{\cot A}$

2. Write all the trigonometric ratios ∠A in terms of sec A

$\sec A = \frac{1}{\cos A}$

$\Rightarrow \mathbf{\cos A} = \frac{1}{\sec A}$

Let $\sin^2A = 1 - \cos^2A$

$\Rightarrow \sin^2A = 1 - \frac{1}{\sec^2 A}$

$\Rightarrow \sin^2A = \frac{\sec^2 A - 1}{\sec^2 A}$

$\Rightarrow \mathbf{\sin A} = \frac{\pm\sqrt{\sec^2 A - 1}}{\sec A}$

$\sin A = \frac{1}{\text{cosec } A}$

$\Rightarrow \text{cosec } A = \frac{1}{\sin A}$

$\Rightarrow \mathbf{cosec\ A} = \frac{\pm \sec A}{\sqrt{\sec^2 A - 1}}$

Let $\tan^2A = \sec^2A + 1$

$\Rightarrow \mathbf{\tan A} = \sqrt{\sec^2 A + 1}$

$\tan A = \frac{1}{\cot A}$

$\Rightarrow \cot A = \frac{1}{\tan A}$

$\Rightarrow \mathbf{\cot A = \frac{1}{\sqrt{\sec^2 A + 1}}}$

3. Evaluate: i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$ ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

(i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$

$= \frac{\sin^2(90-27^0) + \sin^2 27^0}{\cos^2(90-73^0) + \cos^2 73^0}$

$= \frac{\cos^2 27^0 + \sin^2 27^0}{\sin^2 73^0 + \cos^2 73^0} = \frac{1}{1} = 1$

(ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

$= \sin(90°-25°) \cos 65° + \cos(90°-65°) \sin 65°$

$= \cos 65° \cos 65° + \sin 65° \sin 65°$

$= \cos^2 65° + \sin^2 65° = 1$

4. Choose the correct option and justifyyour choice

i) $9 \sec^2 A - 9 \tan^2 A$

A) 1 B) 9 C) 8 D) 0

$9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$ [$\because \sec^2 A - \tan^2 A = 1$]

Ans: B) 9

ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

A) 0 B) 1 C) 2 D) -1

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)$

$= \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \times \frac{\sin\theta + \cos\theta - 1}{\sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\cos\theta . \sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta}$

$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta}$

$= \frac{2\cos\theta\sin\theta}{\cos\theta . \sin\theta} = \mathbf{2}$

Ans C) 2

(iii) (secA + tanA) (1 - sinA) =

A) secA B) sinA C) cosecA D) cosA

(secA + tanA)(1 - sinA)

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin A)$

$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin A)$

$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$

$= \cos A$

Ans: D) cosA

iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$

A) $\sec^2 A$ B) -1 C) $\cot^2 A$ D) $\tan^2 A$

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$= \frac{1+\frac{1}{\cot^2 A}}{1+\cot^2 A}$

$= \frac{\cot^2 A+1}{\cot^2 A}$ x $\frac{1}{1+\cot^2 A}$

$= \frac{1}{\cot^2 A} = \tan^2 A$

Ans: D) $\tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

i) $(\text{cosec}\ \theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

LHS. $= (\text{cosec}\ \theta - \cot\theta)^2$

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec}\ \theta \cot\theta)$

$= \left(\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}\right)$

$= \left(\frac{1+\cos^2\theta-2\cos\theta}{1-\cos^2\theta}\right)$

$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$ = RHS

ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

LHS. $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

$= \frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A}$

$= \frac{2+2\sin A}{(1+\sin A)\cos A}$

$= \frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \frac{2}{\cos A}$

= 2 sec A = RHS.

iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta.\cos\theta$

[Hint : Write the expression in terms of sin and cos]

L.H.S. $= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{\sin^3\theta - \cos^3\theta}{\cos\theta.\sin\theta}\right]$

$= \frac{1}{(\sin\theta - \cos\theta)}\left[\frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\cos\theta.\sin\theta}\right]$

$= \left[\frac{(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\cos\theta.\sin\theta}\right]$

$= \left[\frac{1 + \sin\theta\cos\theta}{\cos\theta.\sin\theta}\right]$

$= \left[\frac{1}{\cos\theta.\sin\theta} + 1\right]$

$= 1 + \sec\theta \operatorname{cosec}\theta =$ R.H.S.

iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A} = 2\sec A$

[Hint: simplify LHS and RHS separately]

L.H.S. $= \frac{1+\sec A}{\sec A}$

$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \text{x} \frac{\cos A}{1}$

$= \cos A + 1$

R.H.S. $= \frac{\sin^2 A}{1-\cos A}$

$= \frac{(1+\cos A)(1-\cos A)}{1-\cos A}$

$= \cos A + 1$

$\Rightarrow$ L.H.S. = R.H.S.

v) Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$ using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

L.H.S. $= \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$= \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$

$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$ [Divide both denominator and numerator by sin A]

$= \frac{\cot A - \operatorname{cosec}^2 A + \cot^2 A + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$ (using $\operatorname{cosec}^2 A - \cot^2 A = 1$)

$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$

$= \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A - \cot A)}{1 - \operatorname{cosec} A + \cot A}$

$= \cot A + \operatorname{cosec} A =$ R.H.S.

vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$= \sqrt{\frac{1+\sin A}{1-\sin A} \text{x} \frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

$= \frac{1+\sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$

= secA + tanA=RHS

vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

L.H.S. = $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[1-2+2\cos^2\theta]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta[2\cos^2\theta - 1]}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta}{\cos\theta}$ = tan θ = R.H.S.

viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

LHS. = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right) + 1 + \tan^2 A + 1 + \cot^2 A$

$= 1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A$

$= 7 + \tan^2 A + \cot^2 A$ = RHS.

ix) (cosec A - sin A)(sec A - cos A) = $\frac{1}{\tan A + \cot A}$ [Hint: simplify LHS and RHS separately]

LHS. = (cosec A – sin A)(sec A – cos A)

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

= cos A sin A

RHS. = $\frac{1}{\tan A + \cot A}$

$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A . \sin A}}$

$= \frac{1}{\frac{1}{\cos A . \sin A}}$

$= \cos A . \sin A$

⇒ L.H.S. = R.H.S.

x) $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

LHS = $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$

$= \frac{1+\tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$

$= \frac{1+\tan^2 A}{\frac{1+\tan^2 A}{\tan^2 A}}$

$= \tan^2 A$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2$

$= \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)^2$

$= (-\tan A)^2$

$= \tan^2 A$

Summery:

1. In right angle triangle ABC, $\angle B = 90^0$

SinA	$\frac{\textbf{Opposite side}}{\textbf{Hypotenuse}}$
CosA	$\frac{\text{Adjacent side}}{\text{Hypotenuse}}$
Tan A	$\frac{\text{Opposite side}}{\text{Adjacent}}$

2.

$\frac{\mathbf{1}}{\textbf{SinA}}$	$\frac{\textbf{Hypotenuse}}{\textbf{oppsite side}}$	**CosecA**
$\frac{\mathbf{1}}{\textbf{CosA}}$	$\frac{\text{Hypotenuse}}{\text{Adjacent side}}$	**SecA**
$\frac{\mathbf{1}}{\textbf{Tan A}}$	$\frac{\text{Adjacent side}}{\text{Opposite side}}$	**CotA**

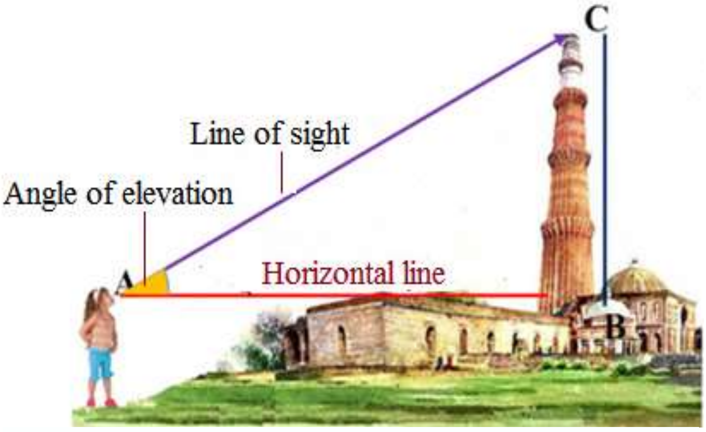
3. If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.
4. The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.
5. $\sin (90° – A) = \cos A$, $\cos (90° – A) = \sin A$;
   $\tan (90° – A) = \cot A$, $\cot (90° – A) = \tan A$;
   $\sec (90° – A) = \text{cosec } A$, $\text{cosec } (90° – A) = \sec A$
6. $\sec^2 A - \tan^2 A = 1, 0^0 \leq A < 90^0$
   $\text{cosec}^2 A = 1 + \cot^2 A, 0^0 \leq A < 90^0$
   $\sin^2 A + \cos^2 A = 1,$

9 Some Applications of Trigonometry

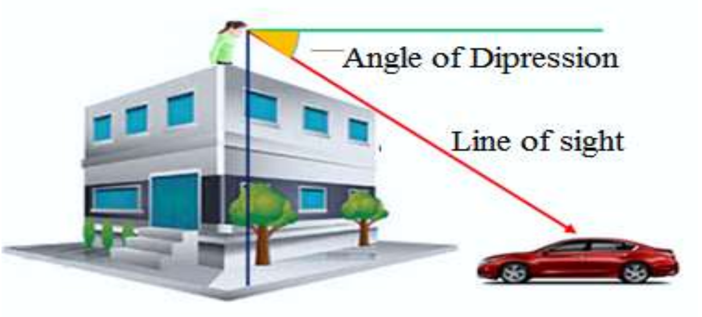
9.2 Height and distance:

Trigonometry is one of the most ancient subjects studied by scholars all over the world. Trigonometry was invented because its need arose in astronomy. Since then the astronomers have used it, for instance, to calculate distances from the Earth to the planets and stars. Trigonometry is also used in geography and in navigation. The knowledge of trigonometry is used to construct maps, determine the position of an island in relation to the longitudes and latitudes. Surveyors have used trigonometry for centuries. One such large surveying project of the nineteenth century was the 'Great Trigonometric Survey' of British India for which the two largest-ever theodolites were built. During the survey in 1852, the highest mountain in the world was discovered. From a distance of over160 km, the peak was observed from six different stations. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant theodolites (see the figure alongside). The theodolites are now on display in the Museum of the Survey of India in Dehradun.

The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer. **The angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object



Thus, **the angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



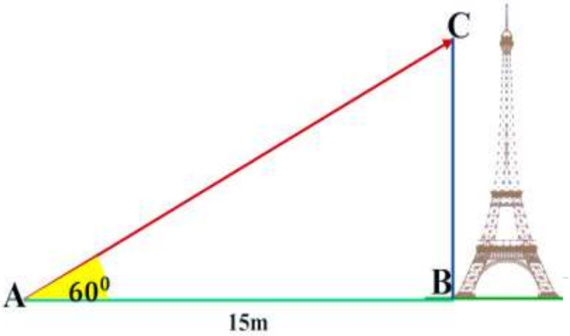
Example1: A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

Let Height of the tower = BC; AB = 15m

$\tan 60^0 = \frac{BC}{AB}$

$\Rightarrow \sqrt{3} = \frac{BC}{15}$

$\Rightarrow BC = 15\sqrt{3}$ m



Example 2 : An electrician has to repair an electric fault on a pole of height 5 m. She needs to reach a point 1.3m below the top of the pole to undertake the repair work (see Fig. 9.5). What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable her to reach the required position? Also, how far from the foot of the pole should she place the foot of the ladder? (You may take $\sqrt{3}$ = 1.73)

Height of the PoleAD = 5m;

The height in which repair work to be done

BD = 5 – 1.3 = 3.7m

Height of the Ladder BC = ?.

Distance between the foot of the pole and the foot of the ladder CD=?

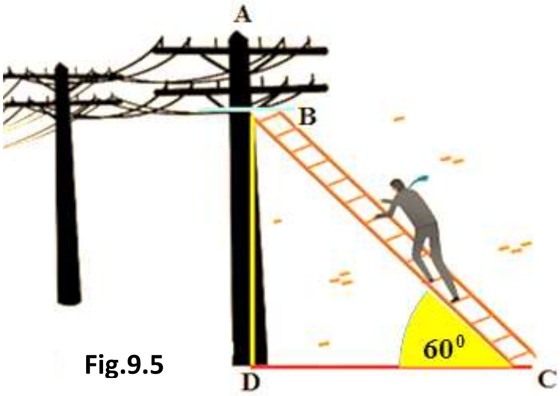


Fig.9.5

$\text{Sin } 60^0 = \frac{BD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$

$\Rightarrow BC = \frac{3.7x2}{\sqrt{3}}$

$= \frac{7.4}{\sqrt{3}} \text{ m} \approx \frac{740}{173} = 4.28\text{m}$

$\text{Tan}60^0 = \frac{BD}{CD}$

$\Rightarrow \sqrt{3} = \frac{3.7}{CD}$

$= \frac{3.7}{\sqrt{3}} \text{ m} \approx 2.14\text{m}$

∴ Height of the Ladder BC = 4.28m and

Distance between the foot of the pole and the foot of the ladder CD = 2.14m

Example 3 : An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

Height of the observer CD = BE = 1.5m,

Distance from the chimney to the observer

DE = CB = 28.5m;

Height of the chimney AB = ?

$\tan45^0 = \frac{AE}{DE}$

$\Rightarrow 1 = \frac{AE}{28.5}$

$\Rightarrow AE = 28.5\text{m}$

∴ Height of the chimney AB = AE + BE

= 28.5 + 1.5 = 30m

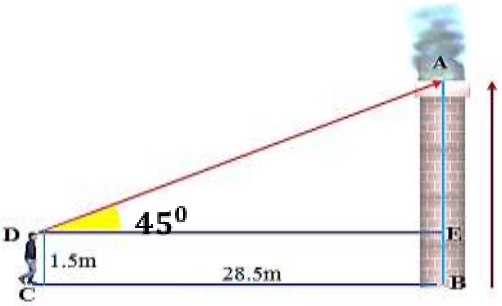


Fig.9.6

Example 4 : From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (you take $\sqrt{3}$ = 1.732)

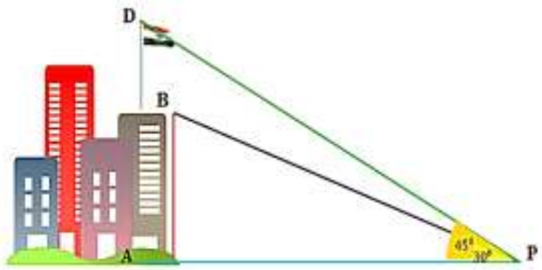
Height of the building AB = 10m

$\text{Tan}30^0 = \frac{AB}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow AP = 10\sqrt{3} = 10x1.732 = 17.32\text{m}$

$\text{Tan}45^0 = \frac{AD}{AP}$



$\Rightarrow 1 = \frac{AD}{17.32}$

$\Rightarrow$ AD = 17.32m

$\therefore$ Length of the flagstaff = AD – AB = 17.32 – 10 = 7.32m

Example5 : The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

Let length of the shadow when Sun's altitude 60^0

BC = x m

$\therefore$ length of the shadow when Sun's altitude 30^0

BD = (40 + x)m

$\tan 60^0 = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{AB}{x}$

$\Rightarrow AB = \sqrt{3}x$ ----- (1)

$\tan 30^0 = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{40+x}$

$\Rightarrow 40+x = \sqrt{3}AB$

$\Rightarrow 40 + x = \sqrt{3} . \sqrt{3}x$

$\Rightarrow 40 + x = 3x$

$\Rightarrow 2x = 40$

$\Rightarrow x = 20$m

$\therefore$ (1) $\Rightarrow AB = \sqrt{3}x$

$\Rightarrow AB = 20\sqrt{3}$ m

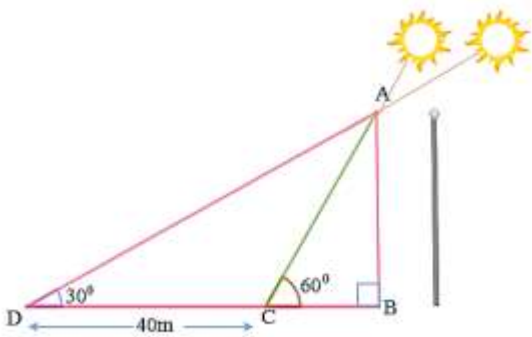


Fig.9.8

Example 6 : The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Height of the building = AB = 8m

Height of the multi-storeyed building

PC=PD+CD = PD+AB=PD+8m ------- (1)

Distance between the buildings=AC = BD

PQ∥BD,$\therefore \angle BPQ = \angle PBD$ [Alternate angles]

$\therefore \tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{PD}{BD}$

$\Rightarrow BD = \sqrt{3}PD$ --- (2)

PQ∥AC,$\therefore \angle APQ = \angle PAC$ [Alternate angles]

$\therefore \tan 45^0 = \frac{PC}{AC}$

$\Rightarrow 1 = \frac{PD+8}{\sqrt{3}PD}$ [From (1) and (2)]

$\Rightarrow \sqrt{3}PD = PD + 8$

$\Rightarrow PD(\sqrt{3} - 1) = 8$

$\Rightarrow PD = \frac{8}{\sqrt{3}-1} = \frac{8(\sqrt{3}+1)}{2} = 4(\sqrt{3}+1)$

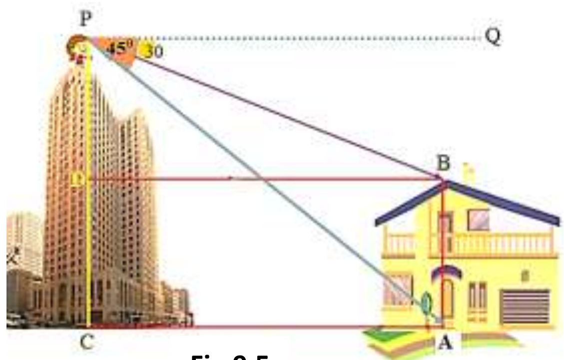


Fig.9.5

$\therefore$ Height of the multi-storeyed building PC = PD + 8m $=4(\sqrt{3}+1) + 8$

$= 4\sqrt{3} + 12 = 4(3+\sqrt{3})m$

$\therefore$ Distance between the buildings = Distance between the buildings $=4(3+\sqrt{3})m$

[Distance between the buildings = AC = BD $\Rightarrow$ BD

$= 4\sqrt{3}(\sqrt{3}+1)$ [(2) ರಿಂದ]

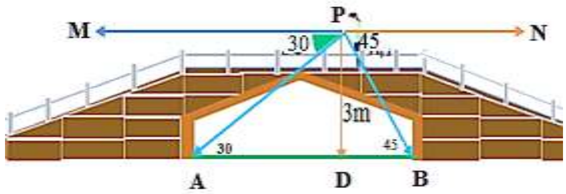
$\Rightarrow$ BD $=4(3+\sqrt{3})$m]

Example 7: From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Width of the river = AD + BD

MN∥AB $\Rightarrow \angle MPA = \angle A = 30^0$ and

$\angle NPD = \angle B = 45^0$ [Alternate angles]



$\tan 30^0 = \frac{PD}{AD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

$\Rightarrow AD = 3\sqrt{3}$ m ---(1)

$\text{Tan}45^0 = \frac{PD}{BD}$

$\Rightarrow 1 = \frac{3}{BD}$

$\Rightarrow BD = 3$m --------(2)

From (1) and (2)

Width of the river = AD + BD $= 3\sqrt{3} + 3$

$= 3(\sqrt{3}+1)$m

Exercise 12.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°

Height of the pole BC

$\text{Sin}30^0 = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{BC}{20}$

$\Rightarrow BC = 10$m

$\therefore$ Height BC = 10m

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Let BC is the broken part of the tree

$\therefore$ Total height of the tree = AB+BC

$\cos 30° = \frac{AC}{BC}$

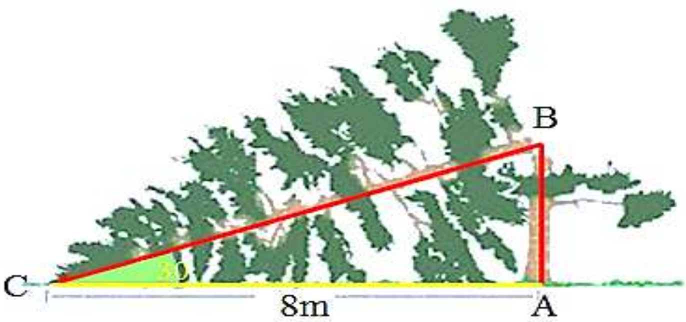
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{BC}$

$\Rightarrow BC = \frac{16}{\sqrt{3}}$;

$\tan 30° = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$

$\Rightarrow AB = \frac{8}{\sqrt{3}}$ m

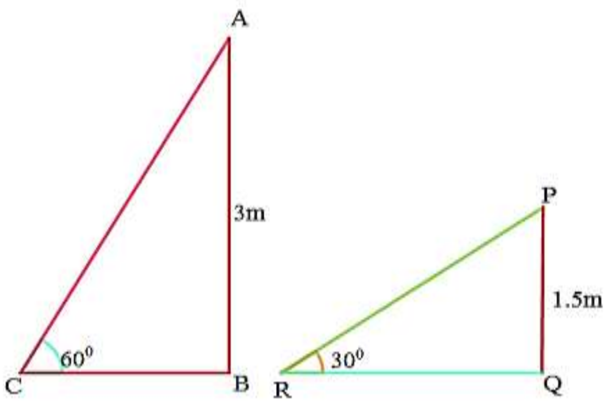


$\therefore$ Height of the tree = AB+ BC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$

$= \frac{24}{\sqrt{3}}$m

3.A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?



Let the length of the side making inclination 60^0 = AC and

Length of the slide making inclination 30° = PR

According to question,

In right angle Δ ABC,sin 30° = $\frac{PQ}{PR}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{PR}$

$\Rightarrow$ PR = 3m

In right angle Δ PQR, sin 60° = $\frac{AB}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{AC}$

$\Rightarrow AC = \frac{6}{\sqrt{3}}$ m = $2\sqrt{3}$ m

∴ Length of the slides 3m and $2\sqrt{3}$ m .

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower

Let height of the tower = AB

Distance from the foot of the tower to the point

BC = 30m

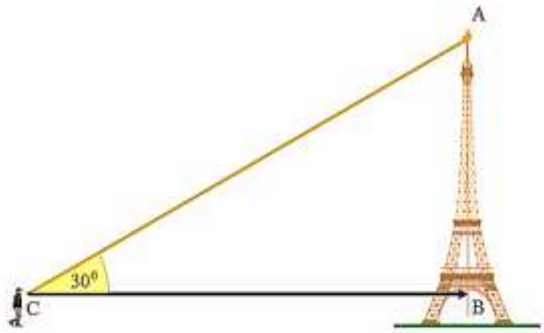
In right angle triangle ΔABC ,

$\tan 30^0 = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

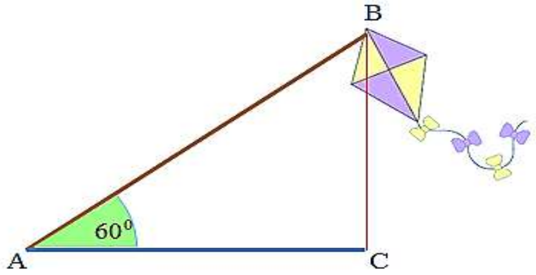
$\Rightarrow AB = \frac{30}{\sqrt{3}}$

$= 10\sqrt{3}$m



5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

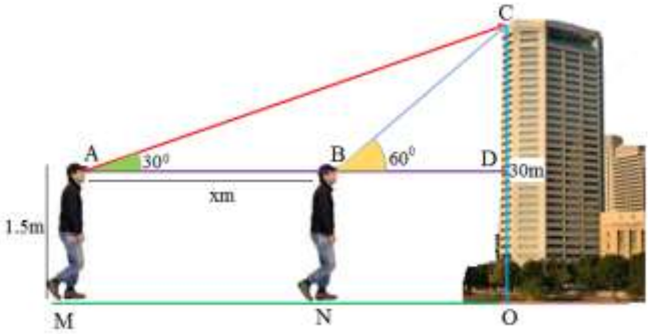
Height of the kite BC = 60m
Length of the tread = AB,
In right angle triangle ΔABC ,



$\sin 60^0 = \frac{BC}{AB}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AB}$
$\Rightarrow AB = \frac{120}{\sqrt{3}}$
$= 40\sqrt{3}$m

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building?

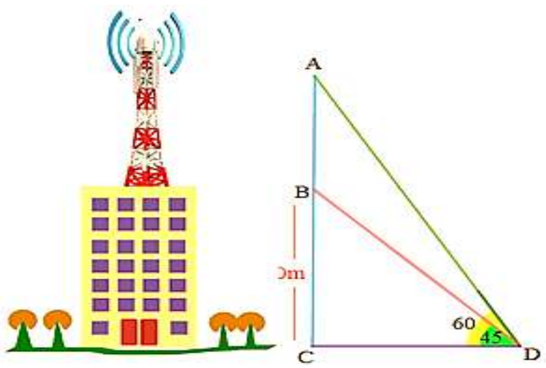
Angle of elevation when the boy is at M = 30^0 After walking x meter, the angle of elevation is 60^0 at N.
∴ MN = AB = x.
Height of the building = OC = 30 m
CD = OC - OD = (30 - 1.5) = 28.5 m,
According to question



In right angle ΔADC, $\tan 30° = \frac{CD}{AD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{AD}$
$\Rightarrow AD = 28.5\sqrt{3}$ m
In right angle ΔCBD, $\tan 60° = \frac{CD}{BD}$
$\Rightarrow \sqrt{3} = \frac{28.5}{BD}$
$\Rightarrow BD = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3}$ m
∴ MN = AB = $x = (28.5\sqrt{3} - 9.5\sqrt{3}) = 19\sqrt{3}$ m
∴The distance he walked towards the building = $19\sqrt{3}$ m

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower

Height of the building = BC= 20 m
A point on the ground where the angle of elevation measured is D
Height of the tansmission tower AB = AC - BC
According to question,

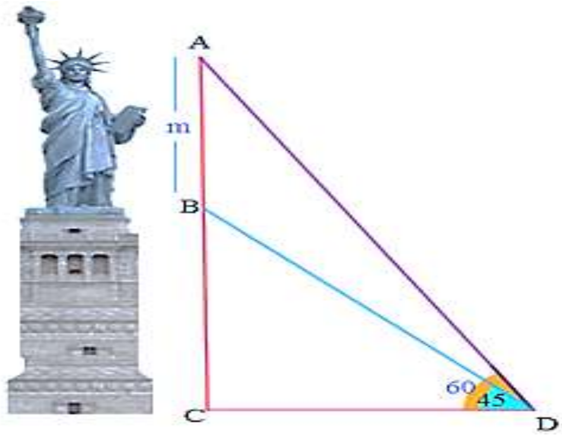


In right angle triangle ΔBCD,
$\tan 45° = \frac{BC}{CD}$
$\Rightarrow 1 = \frac{20}{CD}$
$\Rightarrow CD = 20$ m
In right angle triangle ΔACD,
$\tan 60° = \frac{AC}{CD}$
$\sqrt{3} = \frac{AC}{20}$
$\Rightarrow AC = 20\sqrt{3}$ m
Height of the tansmission tower AB = AC - BC = $(20\sqrt{3} - 20)$ m = $20(\sqrt{3} - 1)$ m.

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Let the height of the statue = AB

The point where the angle of elevation measured is D



Height of the pedestal BC = AC – AB

By question, In right angle ΔBCD ,

$\tan 45° = \frac{BC}{CD} \Rightarrow 1 = \frac{BC}{CD}$

⇒ BC = CD.

In right angle ΔACD,

$\tan 60° = \frac{AC}{CD}$

$\Rightarrow \sqrt{3}CD = 1.6\text{ m} + BC$

$\Rightarrow \sqrt{3}BC = 1.6\text{ m} + BC$

$\Rightarrow \sqrt{3}BC - BC = 1.6\text{ m}$

$\Rightarrow BC(\sqrt{3} - 1) = 1.6\text{ m}$

$\Rightarrow BC(\sqrt{3} - 1) = \frac{1.6}{\sqrt{3}-1}$

$\Rightarrow BC = 0.8(\sqrt{3} + 1)\text{ m}$

∴Height of the pedestal BC = 0.8 $(\sqrt{3}+1)$ m.

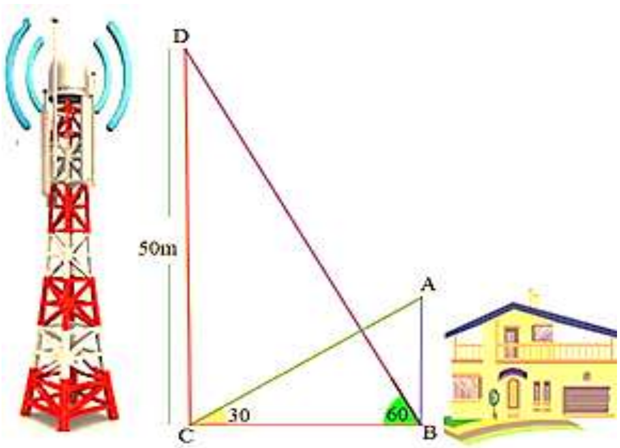
9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Given: the height of the tower CD = 50 m

Let the height of the building = AB

Distance from the foot of the building to the tower = BC

According to question In right angle triangle ΔBCD,



$\tan 60° = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{50}{BC}$

$\Rightarrow BC = \frac{50}{\sqrt{3}}$

In right angle triangle ΔABC,

$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC}$

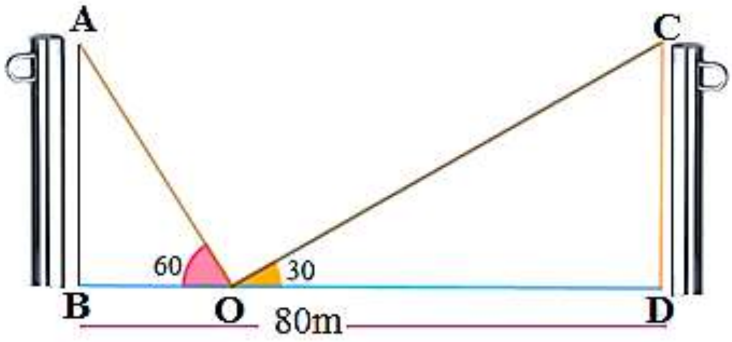
$\Rightarrow AB = \frac{BC}{\sqrt{3}}$

$\Rightarrow AB = \frac{50}{3}\text{m}$

∴ height of the building = $\frac{50}{3}$ m = $16\frac{2}{3}$ m

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

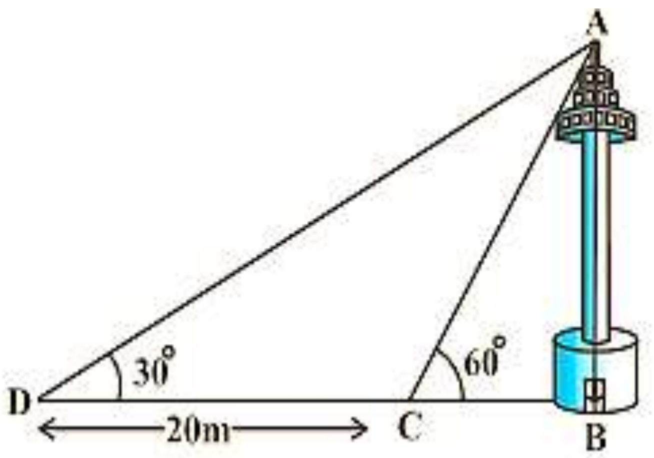
AB and CD are the two poles of equal height
The point on the ground where the angle of elevation is measured is O.
The distance between the poles = BD
According to question,
AB = CD, OB + OD = 80 m
In right angle triangle ΔCDO,
$\tan 30° = \frac{CD}{OD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{OD}$
$\Rightarrow CD = \frac{OD}{\sqrt{3}}$ --------------- (1)



In right angle triangle ΔABO, $\tan 60° = \frac{AB}{OB}$
$\Rightarrow \sqrt{3} = \frac{AB}{80-OD}$
$\Rightarrow AB = \sqrt{3}\,(80\text{-}OD)$
AB = CD [Given]
$\Rightarrow \sqrt{3}\,(80 - OD) = \frac{OD}{\sqrt{3}}$
$\Rightarrow 3(80\text{-}OD) = OD$
$\Rightarrow 240 - 3\,OD = OD$
$\Rightarrow 4\,OD = 240 \Rightarrow OD = 60$
Substitute OD = 60 in (1) we get, $CD = \frac{60}{\sqrt{3}}$
$\Rightarrow CD = 20\sqrt{3}$ m
OB + OD = 80 m
$\Rightarrow OB = (80\text{-}60)$ m = 20 m
Therefore the height of the poles = $20\sqrt{3}$ m and the distance from the point of elevation to the poles = 60m and 20m

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30^0 (see Fig. 12.12). Find the height of the tower and width of the canal.

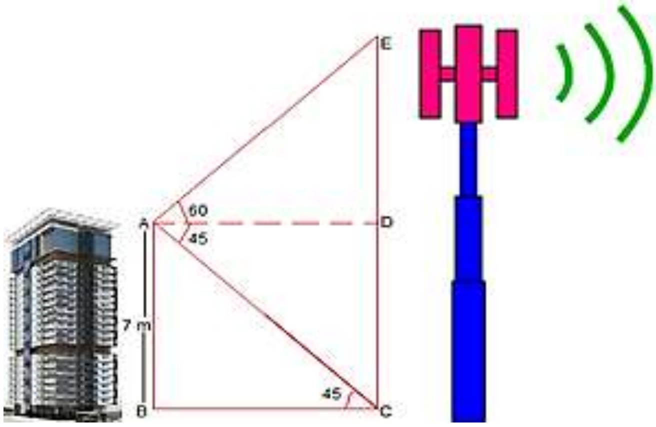
Let Height of the TV tower = AB; CD = 20 m [Given]
According to question, In triangle ΔABD,
$\tan 30° = \frac{AB}{BD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{CD+BC}$
$\Rightarrow AB = \frac{(20+BC)}{\sqrt{3}}$ -------- (1)



In right angle triangle ΔABC, $\tan 60° = \frac{AB}{BC}$
$\Rightarrow \sqrt{3} = \frac{AB}{BC}$
$\Rightarrow AB = \sqrt{3}\,BC$ ------- (2)
From equation (1) and (2)
$\frac{(20+BC)}{\sqrt{3}} = \sqrt{3}\,BC$
$\Rightarrow 3\,BC = 20 + BC$
$\Rightarrow 2\,BC = 20$
$\Rightarrow BC = 10$ m
Substitute BC =10 in equation (2) $AB = 10\sqrt{3}$ m
The height of the tower = $10\sqrt{3}$ m and width of the canal = 10 m.

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower

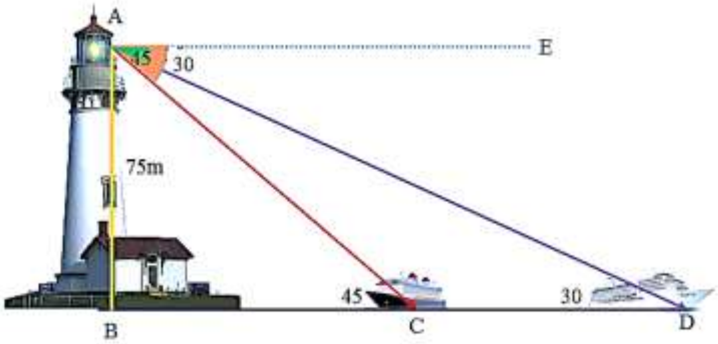
Height of the building AB = 7 m ;
Height of the tower = EC



A is the point of elevation
EC = DE + CD
CD = AB = 7 m ಮತ್ತು BC = AD
By question, In right angle triangle ΔABC,
$\tan 45° = \frac{AB}{BC} \Rightarrow 1 = \frac{7}{BC}$
$\Rightarrow BC = 7$ m = AD
In right angle triangle ΔADE, $\tan 60° = \frac{DE}{AD}$
$\Rightarrow \sqrt{3} = \frac{DE}{7}$
$\Rightarrow DE = 7\sqrt{3}$ m
Height of the tower = EC = DE + CD = $(7\sqrt{3} + 7)$ m = $7(\sqrt{3}+1)$ m.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships

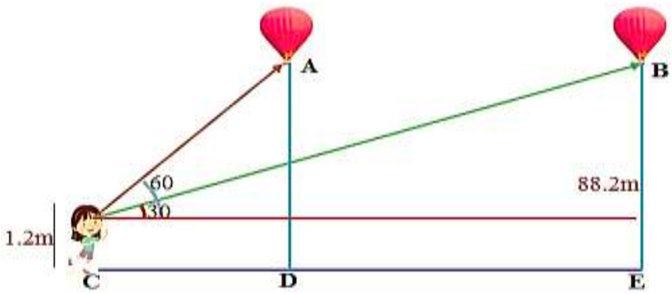
Height of the lighthouse AB = 75 m.
Let the positons of the ships C and D
According to question,



In right angle triangle ΔABC, $\tan 45° = \frac{AB}{BC}$
$\Rightarrow 1 = \frac{75}{BC}$
$\Rightarrow BC = 75$ m
In right angle triangle ΔABD, $\tan 30° = \frac{AB}{BD}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$
$\Rightarrow BD = 75\sqrt{3}$m
∴ the distance between the ships = CD = BD - BC
= $(75\sqrt{3} - 75)$m
= $75(\sqrt{3}-1)$m.

14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig.12.13). Find the distance travelled by the balloon during the interval.

Let the initial position of the balloon and the later position be A and B respectively
The height of the ballon = 88.2 m - 1.2 m = 87 m



The distance travelled by the bolloon
= DE = CE - CD
According to question in ΔBEC,
$\tan 30° = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{87}{CE}$
$\Rightarrow CE = 87\sqrt{3}$ m
In right triangle ΔADC, $\tan 60° = \frac{AD}{CD}$
$\Rightarrow \sqrt{3} = \frac{87}{CD}$

$\Rightarrow$ CD = $\frac{87}{\sqrt{3}}$ m = $29\sqrt{3}$ m

∴ The distance travelled by the bolloon DE = CE - CD

= $(87\sqrt{3} - 29\sqrt{3})$ m

= $58\sqrt{3}$ m.

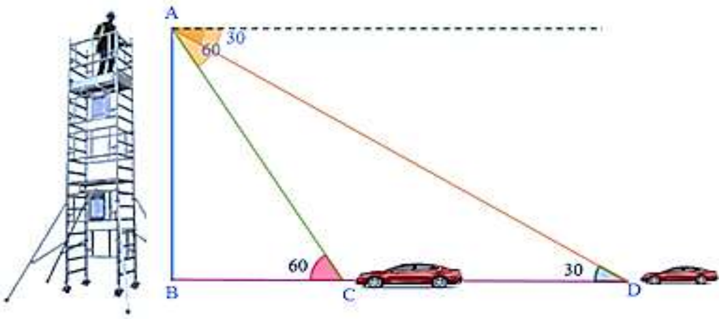
15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Let the height of the tower = AB

D is the initial position of the car and C is the later position

BC is the distance from the car to the tower

According to question, In right triangle ΔABC, $\tan 60° = \frac{AB}{BC}$



$\Rightarrow \sqrt{3} = \frac{AB}{BC}$

$\Rightarrow BC = \frac{AB}{\sqrt{3}}$ m

In right triangle ΔADB, $\tan 30° = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow AB\sqrt{3} = \frac{AB}{\sqrt{3}} + CD$

$\Rightarrow CD = AB\sqrt{3} - \frac{AB}{\sqrt{3}}$

$\Rightarrow CD = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$\Rightarrow CD = AB\left(\frac{3-1}{\sqrt{3}}\right)$

$\Rightarrow CD = \frac{2AB}{\sqrt{3}}$ m

Here, distance BC is half the distance CD

∴ time taken to move BC is half the time taken to move CD

Given that the time taken by the car to move distance CD = 6 sec.

∴ The time taken to move the distance BC = 6/2 = 3 sec.

Summary:

1. (i) The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

(ii) The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

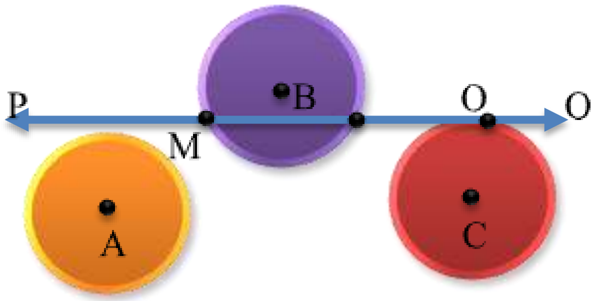
(iii) The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

2. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

10 Circles

Non-intersecting Line: The line and the circle have no common point. In this case, PQ is called a non-intersecting line to the circle of center A

Secant: There are two common points that the line and the circle have. In this case, we call the line PQ a secant of the circle of center B.



10.2 Tangent to a Circle

There is only one point which is common to the line and the circle. In this case, the line PQ is called a tangent to the circle of center C

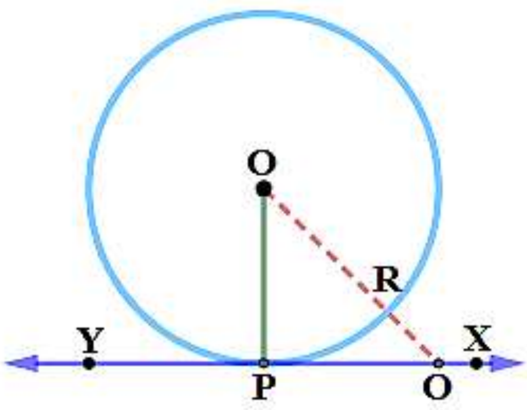
Tangent to a circle is a line that intersects the circle at only one point. There is only one tangent to a circle at a point. The common point of the tangent and the circle is called the point of contact.

Theorem 10.1 **The tangent at any point of a circle is perpendicular to the radius through the point of contact.**

Given: A circle with centre O and tangent XY at a point P.

To Prove: OP⊥XY

Consruction: Take any point Q, other than P on the tangent XY and join OQ



Proof: Hence, Q is a point on the tangent XY, other than the point of contact P. So Q lies outside the circle..
[∵There is only one point of contact to a tangent]
Let OQ intersect the circle at R
∴ OP = OR [∵ Radius of the same circle]
Now, OQ = OR + RQ
⇒OQ > OR
⇒OQ> OP [∵OP = OR]
Therefore, OP is the shortest distance to the tangent from the center O
∴ OP⊥XY [∵ Perpendicular distance is always the shortest distance]

Remarks

1. By theorem above, we can also conclude that at any point on a circle there can be one and only one tangent.
2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

Exercise 10.1

1. How many tangents can a circle have? [Answer: Infinite]

2. Fill in the blanks :

i) A tangent to a circle intersects it in __________ point (s). [Answer: One]

(ii) A line intersecting a circle in two points is called a __________ [Answer: Secant]

iii) A circle can have __________ parallel tangents at the most.

Answer: Two

[Note: we can draw only two(pair) parallel tangents each other. But we can draw infinite parallel pair of tangents]

iv) The common point of a tangent to a circle and the circle is called _________ .

[Answer: Point of cantact]

3.A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

a) 12 cm b) 13 cm c) 8.5 cm d) $\sqrt{119}$ cm

Answer:

The line drawn from the point of contact to the center of the circle is perpendicular to the tangent. $\Rightarrow OP \perp PQ$

In ΔOPQ, $OQ^2 = OP^2 + PQ^2$[Pythagoras Theorem]

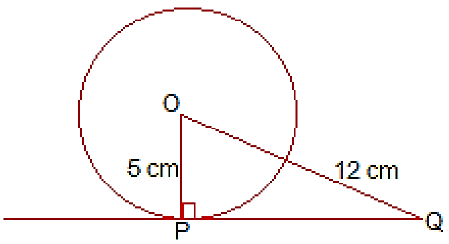
$\Rightarrow (12)^2 = 5^2 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25$

$\Rightarrow PQ^2 = 119$

$\Rightarrow PQ = \sqrt{119}$ cm

(d) $\sqrt{119}$ cm

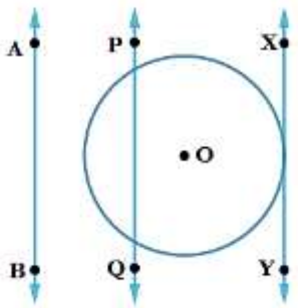


4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

AB – A line

PQ – A secant

XY – A tangent



10.3 Number of Tangents from a Point on a Circle

Case1: There is no tangent to a circle passing through a point lying inside the circle.

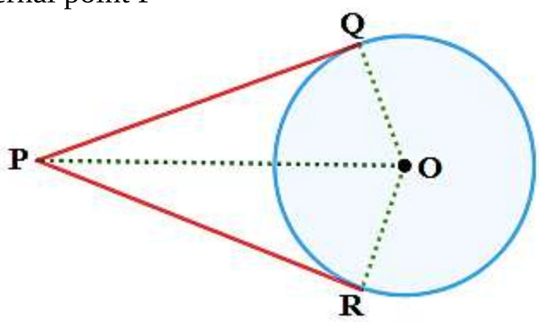
Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.

Case3: There are exactly two tangents to a circle through a point lying outside the circle.

Theorem 10.2

The lengths of tangents drawn from an external point to a circle are equal.

Given: PQ and PR are the two tangents drawn from an external point P to a circle of center O. JoinOP, OQ, OR



T Prove: PQ = PR

Proof: In right angle triangle OQP and ORP,

OQ = OR [Radius of the same circle]

OP = OP [Common side]

$\therefore$, $\Delta OQP \cong \Delta\ ORP$ [RHS]

$\therefore$, PQ = PR [CPCT]

Example 1 : Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

We are given two concentric circles C_1 and C_2 with centre O and a chord AB of the larger circle C_1 which touches the smaller circle C_2 at the point P (see Fig. 10.8). We need to prove that AP = BP.

Let us join OP. Then,

AB is a tangent to C_2 at P and OP is its radius.

Therefore, by Theorem 10.1,

Therefore $OP \perp AB$ [From theorem 10.1]

Now AB is a chord of the circle C_1 and $OP \perp AB$

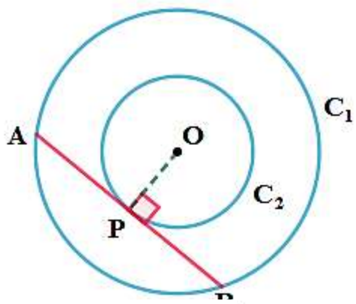


Fig. 10.8

Therefore, OP is the bisector of the chord AB, as the perpendicular from the centre bisects the chord,

$\Rightarrow$ AP = BP

Example 2 : Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that PTQ = 2 OPQ.

Solution: TP and TQ are the two tangents drawn from an external point T to the circle with centre O

To Prove: $\angle PTQ = 2\angle OPQ$

Let $\angle PTQ = \theta$-------(1)

TP = TQ [$\because$Theorem 4.2]

Therefore TPQ is an isosceles triangle.

$\angle TPQ = \angle TQP = \frac{1}{2}[180 - \theta]$

$\Rightarrow \angle TPQ = \angle TQP = 90^0 - \frac{1}{2}\theta$ ------(2)

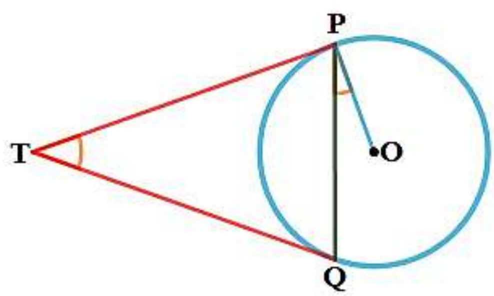


Fig. 10.9

$\angle OPT = 90^0$ -------- (3)

$\angle OPQ = \angle OPT - \angle TPQ$

$\Rightarrow \angle OPQ = 90^0 - \left(90^0 - \frac{1}{2}\theta\right)$ [$\because$ from(2)and(3)]

$\Rightarrow \angle OPQ = \frac{1}{2}\theta$

$\Rightarrow \angle PTQ = 2\angle OPQ$ [∵From (1)]

Example 3 : PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig. 4.10). Find the length TP.

Solution: Join OT Let it intersect PQ at the point R

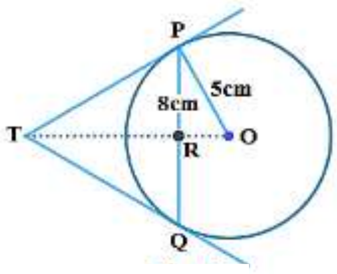


Fig. 10.10

Then ΔTPQ is isosceles and

TO is the angle bisector of $\angle PTQ$.

∴ $OT \perp PQ$ therefore OT bisects PQ

$\Rightarrow$ PR = RQ = 4 cm.

∴ $RO = \sqrt{5^2 - 4^2}$

$\Rightarrow RO = \sqrt{25 - 16}$

$\Rightarrow RO = \sqrt{9} \Rightarrow RO = 3cm$

$\angle OPR + \angle TPR = 90^0$ ---(1) [∵In ΔPRO, $\angle PRO = 90^0$]

$\angle PTR + \angle TPR = 90^0$ ---(2) [∵In ΔPTR, $\angle PRT = 90^0$]

From (1) and (2), $\angle OPR = \angle PTR$--- (3)

∴ ΔPRO and ΔPTR right triangles are similar [AA similarity criteria]

$\Rightarrow \frac{PT}{OP} = \frac{PR}{OR} \Rightarrow \frac{PT}{5} = \frac{4}{3}$

$\Rightarrow PT = \frac{4x5}{3} = \frac{20}{3}$

EXERCISE 10.2

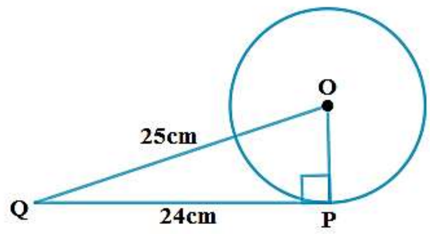
In Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

$OP \perp PQ$ and ΔOPQ is a right angle triangle.

OQ = 25 cm and PQ = 24 cm

In ΔOPQ, By Pythagoras theorem,



$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576$

$\Rightarrow OP^2 = 49$

$\Rightarrow$OP = 7 cm

Ans (A) 7 cm.

A) 7 cm B) 12 cm C) 15 cm D) 24.5 cm

2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that POQ = 110°, then PTQ is equal to

TP and TQ are the tangents to a circle at P and Q OP and OQ are radius of the circle at point of contacts P and Q

∴ $OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = \angle OQT = 90°$

In Quadrilateral POQT,

$\angle PTQ + \angle OPT + \angle POQ + \angle OQT = 360°$

$\Rightarrow \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \angle PTQ = 70°$

⇒Ans (B) 70°.

A) 60 **B) 70** C) 80 D) 90

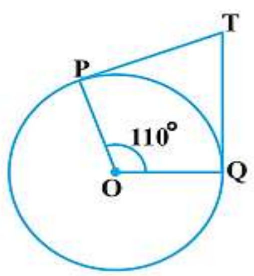
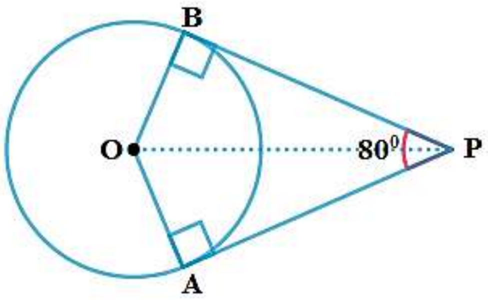


Fig. 10.11

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then POA is equal to

OA and OB are the radius drawn at the point of contact of the tangents BP and BQ

∴ OA ⊥ PA and OB ⊥ PB; ∠OBP = ∠OAP = 90°



In Quadrilateral AOBP,

∠AOB + ∠OBP + ∠OAP + ∠APB = 360°

⇒ ∠AOB + 90° + 90° + 80° = 360°

⇒ ∠AOB = 100°

Now, In ΔOPB and ΔOPA,

AP = BP [Tangents drawn from an external point]

OA = OB [Radius of the same circle]

OP = OP [Common]

∴ ΔOPB ≅ ΔOPA [SSS congruence rule]

⇒∠POB = ∠POA

∠AOB = ∠POB + ∠POA

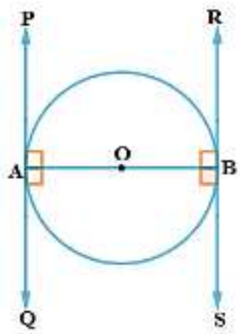
⇒ 2 ∠POA = ∠AOB

⇒ ∠POA = 50°

⇒ **(A) 50°**

A) 50^0 B) 60^0 C) 70^0 D) 80^0

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



AB is a diameter.

PQ and RS are the tangents drawn to the circle at point A and B

OA and OB are the radius drawn at point of contact.

∴ OA ⊥ PQ and OB ⊥ RS

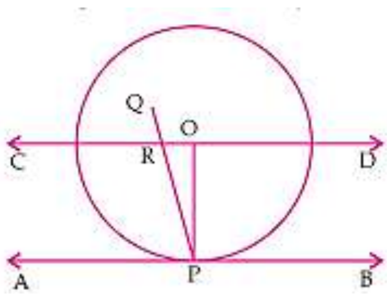
∠OAP = ∠OAQ = ∠OBR = ∠OBS = 90º

In the fig, ∠OBR = ∠OAQ [Alternate angles]

∠OBS = ∠OAP [Alternate angles] ⇒ PQ‖ RS

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

AB is the tangent drawn to the circle with radius O



To Prove:The perpendicular at P passes through the center O.

If possible,let the perpendicular passing through some other Point say Q , Join QP and OP

OP is the radius at point of contact AB is the tangent

∴ OP⊥AB

⇒ ∠OPA = 90^0 But, ∠RPA = 90° (PQ ⊥ AB)

⇒ Which is possible only when points P and Q coinside.

∴ the perpendicular at the point of contact to the tangent to a circle passes through the centre.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle

AB is the tangent to the circle at point B. OB is the radius at point of contact

$\therefore OB \perp AB$

OA = 5cm and AB = 4 cm [Given]

In ΔABO,

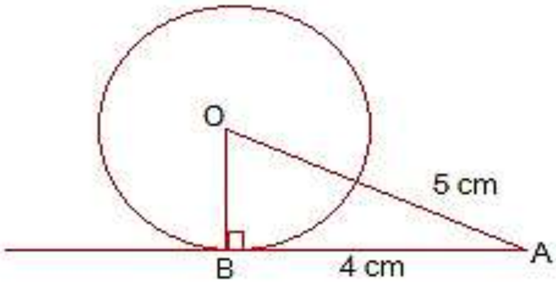
$OA^2 = AB^2 + BO^2$ [Pythagoras theorem]

$\Rightarrow 5^2 = 4^2 + BO^2$

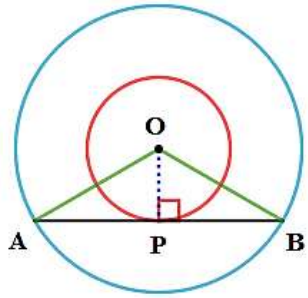
$\Rightarrow BO^2 = 25 - 16$

$\Rightarrow BO^2 = 9 \Rightarrow BO = 3$

$\therefore$ Radius = 3 cm.



7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Two concentric circles of radius 5cm and 3cm drawn with common center O

AB is the chord of circle with radius 5cm such that it touches the circle of radius 3cm at P

$\therefore$ AB is the tangent to the smaller circle at P

$\Rightarrow OP \perp AB$

$\therefore$ AP = PB [The perpendicular drawn from the center to the chord bisects the chord]

$OA^2 = AP^2 + OP^2$ [By Pythagoras theorem]

$\Rightarrow 5^2 = AP^2 + 3^2$

$\Rightarrow AP^2 = 25 - 9 \Rightarrow AP = 4$,

$AB = 2AP = 2 \times 4 = 8$ cm

$\therefore$ The length of the chord = 8 cm.

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that AB + CD = AD + BC

From the figure,

DR = DS [Tangents from the external point D] ---(1)

AP = AS [Tangents from the external point A] ---(2)

BP = BQ [Tangents from the external point B] ---(3)

CR = CQ [Tangents from the external point C] ---(4)

(1) + (2) + (3) + (4)

DR + AP + BP + CR = DS + AS + BQ + CQ

$\Rightarrow$ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

$\Rightarrow$AB + CD = AD + BC

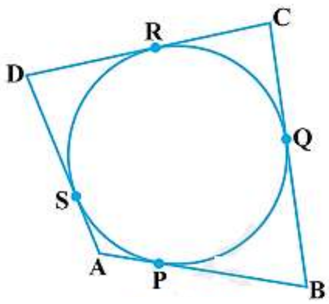


Fig. 10.12

9. In Fig. 10.13, XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B. Prove that $\angle AOB = 90°$.

Let the tangent AB touches the circle at C. Join OC.

In ΔOPA and ΔOCA, OP = OC [radius of the same circle]

AP = AC [The tangents from an external point A]

AO = AO [Common]

$\therefore \Delta OPA \cong \Delta OCA$ [SSS congruence rule]

$\Rightarrow \angle POA = \angle COA$ ---- (1)

Similarly, $\Delta OQB \cong \Delta OCB$

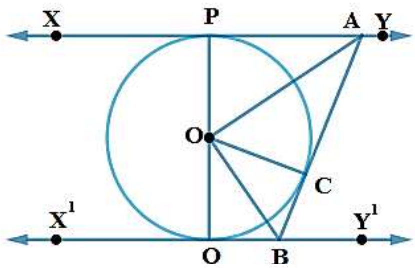


Fig. 10.13

$\angle QOB = \angle COB$ ----- (2)

POQ is the diameter

$\therefore \angle POQ = 180^0$

$\Rightarrow \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

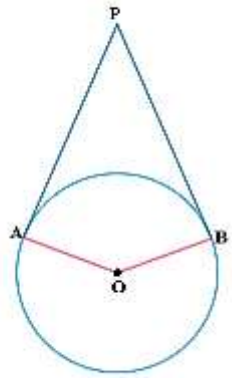
From (1) and (2) we get, $2\angle COA + 2\angle COB = 180^\circ$

$\Rightarrow \angle COA + \angle COB = 90^\circ$

$\Rightarrow \angle AOB = 90^\circ$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre

Let PA and PB are the tangents drawn from an external point to the circle with center O



Join OA and OB

To Prove: $\angle APB + \angle BOA = 180^\circ$

Proof: $OA \perp PA$ [Radius at point of contact to the circle]

$\therefore \angle OAP = 90^\circ$

Similarlly, $OB \perp PB$

$\therefore \angle OBP = 90^\circ$

In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$[Sum of interior agles]

$\Rightarrow 90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$

$\Rightarrow \angle APB + \angle BOA = 180^\circ$

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Given: ABCD is a parallelogram circumscribing a circle.

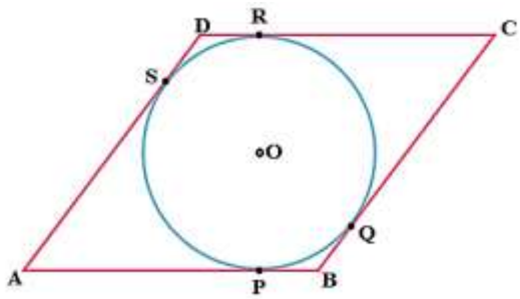
To prove: AB = BC = CD = DA

Proof: ABCD is a parallelogram

$\therefore$ AB = CD ------(1) and BC = AD -----(2)

We know that the tangent drawn from an external point to the circles are equal

$\therefore$ DR = DS, AP = AS, BP = BQ, and CR = CQ



Adding all these, we get

DR + CR + BP + AP = DS + CQ + BQ + AS

$\Rightarrow$ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

$\Rightarrow$ AB + CD = AD + BC ---------------(3)

Substituting (1) and (2) in (3),

2AB = 2BC

$\Rightarrow$ AB = BC ---- (4)

From equation (1), (2) and (4),

AB = BC = CD = DA $\therefore$ ABCD is a Rhombus

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

In ΔABC,

CF = CD = 6cm [Tangents from an external point]

BE = BD = 8cm [Tangents from an external point]

AE = AF = x [Tangents from an external point]

$\Rightarrow a = AB = AE + EB = x + 8$

$b = BC = BD + DC = 8 + 6 = 14$; $c = CA = CF + FA = 6 + x$

$S = \frac{AB+BC+CA}{2} = \frac{x+8+14+6+x}{2} = \frac{2x+28}{2}$

$\Rightarrow$ **S = 14 + x**

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{14+x\,[14+x-(x+8)](14+x-14)[14+x-(6+x)]}$

$= \sqrt{(14+x)\,[14+x-x-8](14+x-14)[14+x-6-x]}$

$= \sqrt{(14+x)\,(6)(x)(8)} = \sqrt{(14+x)\,48x}\ \text{cm}^2$ --------(1)

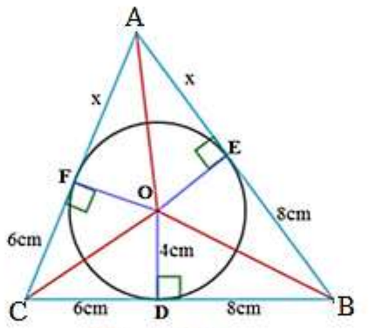


Fig. 10.14

Similarlly,

Area of ΔABC= Area of ΔOCB+ Area of ΔOBA+ Area of ΔOAC

$= \frac{1}{2}BC.OD + \frac{1}{2}AB.OE + \frac{1}{2}AC.OF$

$= \frac{1}{2}(14\text{x}4) + \frac{1}{2}(8+x)4 + \frac{1}{2}(6+x)4 = 28 + 16 + 2x + 12 + 2x = (56 + 4x)\text{cm}^2$ ------(2)

From (1) and (2), $\sqrt{(14+x)\,48x} = 56 + 4x$

$48x\,(14 + x) = (56 + 4x)^2$ [Squaring on both sides]

$\Rightarrow 48x = \frac{[4(14+x)]^2}{14+x}$

$\Rightarrow 48x = 16\,(14 + x)$

$\Rightarrow 48x = 224 + 16x$

$\Rightarrow 32x = 224$

$\Rightarrow x = 7$ cm

Therefore, AB = $x + 8 = 7 + 8 = 15$ cm;

CA = $6 + x = 6 + 7 = 13$ cm

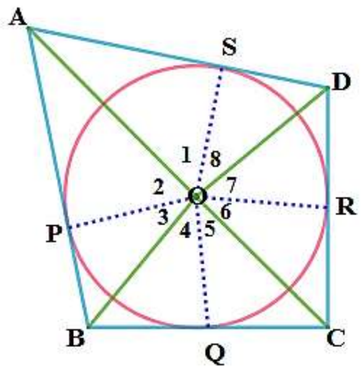
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Given: ABCD is a quadrilateral circumscribing a circle with center O.

Let the circle touches the quadrilateral at points P,Q,R and S

To Prove: $\angle AOB + \angle COD = 180^0$ and $\angle AOD + \angle BOC = 180^0$

Construction: Join OP, OQ, OR and OS.



Proof: The tangents drawn from an external point to the circle substend equal angle at the center.

$\Rightarrow \angle 1 = \angle 2;\ \angle 3 = \angle 4;\ \angle 5 = \angle 6;\ \angle 7 = \angle 8$ But,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + (\angle 5 + \angle 6) + (\angle 7 + \angle 8) = 360^\circ$

$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^\circ$

$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$

$\Rightarrow \angle AOB + \angle COD = 180^0$

Similarlly, $\angle AOD + \angle BOC = 180^0$

$\therefore$ **Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

Summary

1. **The tangent to a circle is perpendicular to the radius through the point of contact.**

2. **The length of the tangents from an external point to the circle are equal.**

11 Area Related to circles

Perimeter and Area of a Circle — A Review

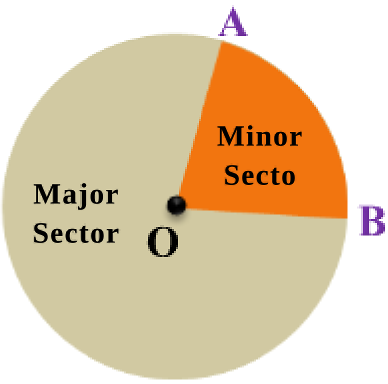
The distance covered by travelling once around a circle is its perimeter, usually called its circumference. Circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi').

$$\pi \rightarrow \frac{\text{circumference}}{\text{diameter}} \rightarrow \frac{2\pi r}{d} = \frac{22}{7} = 3.14$$

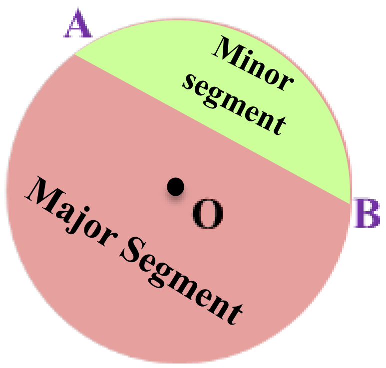
Area of Circle = πr^2

5.3 Areas of Sector and Segment of a Circle

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle and the portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.



The area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$



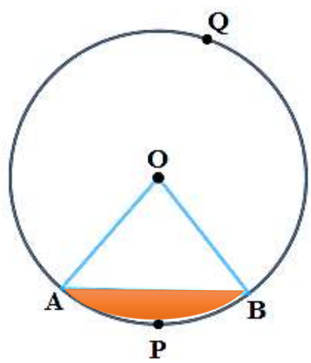
The length of the segment of angle $\theta = \frac{\theta}{360} \times 2\pi r$

Some formule related to calculate Areas of circle:

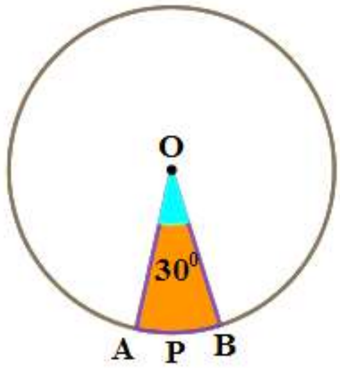
Areaof Segment APB = Area of OAPB Sector – Area of ΔOAB

Area of OAQB Major sector = πr^2 - Area of Minor sectorOAPB

Area of AQB Major Segment = πr^2 – Area of APB Minor segment



Example 1 : Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use = 3.14)



Solution: Given sector is OAPB.

Area of the sector OAPB = $\frac{\theta}{360}$ x πr^2

$\Rightarrow \frac{30}{360}$ x 3.14 x 4 x 4

= $\frac{12.56}{3}$ ≈ 4.19 cm^2

Area of the corresponding major sector

= πr^2 - Area of sector OAPB

= (3.14 x 16 - 4.19) cm^2 ≈ 46.1 cm^2

Alternate Method:

Area of the corresponding major sector

= $\frac{360-\theta}{360}$ x πr^2

= $\frac{360-30}{360}$ x 3.14 x 4 x 4

= 46.05 ≈ 46.1 cm^2

Example 2 : Find the area of the segment AYB shown in Fig. 11.9, if radius of the circle is 21 cm and ∠AOB = 120^0 (Use $\pi = \frac{22}{7}$)

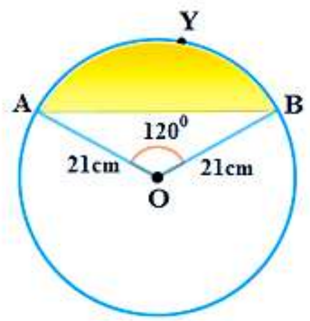


Fig.11.9

Solution: Area of the segment

= Area of sector OAYB – Area of ΔOAB -------- (1)

Area of the sector = $\frac{\theta}{360}$ x πr^2

= $\frac{120}{360}$ x $\frac{22}{7}$ x 21 x 21 = 462cm^2 -------- (2)

To find the area of ΔOAB, draw OM⊥AB as shown in fig.

Note that OA = OB

Therefore, by RHS congruence ΔAMO ≅ ΔBMO

So, M is the mid-point of AB and ∠AOM = ∠BOM = 60^0

In ΔOAM, $\frac{OM}{OA}$ = Cos60^0

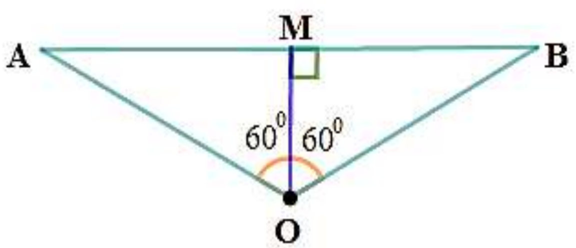
$\Rightarrow \frac{OM}{21} = \frac{1}{2}$

$\Rightarrow$ OM = $\frac{21}{2}$ cm

In ΔOAM, $\frac{AM}{OA}$ = Sin60^0

$\Rightarrow \frac{AM}{21} = \frac{\sqrt{3}}{2}$

$\Rightarrow$ AM = $\frac{21\sqrt{3}}{2}$ cm



$\Rightarrow$AB = 2AM

$\Rightarrow 21\sqrt{3}$cm

∴ Area of ΔOAB = $\frac{1}{2}$ x AB x OM

= $\frac{1}{2}$ x $21\sqrt{3}$ x $\frac{21}{2}$

= $\frac{441\sqrt{3}}{4}$$cm^2$ -------- (3)

Are of the segment = 462 - $\frac{441\sqrt{3}}{4}$

= $\frac{462x4-441\sqrt{3}}{4}$

= $\frac{21}{4}$(88 - $21\sqrt{3}$)cm^2

Exercise 11.1

[Unless stated, otherwise use $\pi = \frac{22}{7}$]

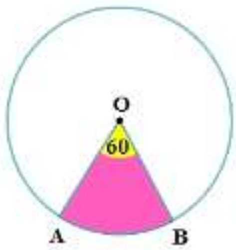
1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of the sector of angle $60^0 = \frac{60}{360°} \times \pi r^2 \text{ cm}^2$

$= \frac{1}{6} \times 6 \text{x} 6 \text{ x } \frac{22}{7}$

$= \frac{132}{7} \text{ cm}^2$



2. Find the area of a quadrant of a circle whose circumference is 22 cm

Quadrant of a circle = Angle of sector 90^0

Circumference $C = 2\pi r = 22$ cm

Radius $r = \frac{22}{2\pi}$ cm

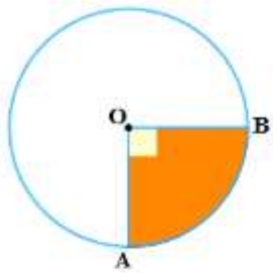
$= \frac{22 \text{x} 7}{2 \text{ x } 22} = \frac{7}{2}$ cm

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of the sector of angle $90° = \frac{90}{360°} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \text{x} \frac{7}{2} \text{x} \frac{7}{2}$

$= \frac{77}{8} \text{ cm}^2$



3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

The minute hand is the radius of the circle.

$\Rightarrow$ Radius(r) = 14 cm

The angle of rotation formed by minute hand in 1 hour = 360°

$\therefore$ The angle of rotation in 5 minutes $= \frac{360°}{60} \times 5 = 30°$

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

$\therefore$ Area of the sector of angle $30^0 = \frac{30}{360°} \times \frac{22}{7} \text{ x } 14 \text{ x } 14$

$= \frac{1}{12} \times \frac{22}{7} \text{ x } 14 \text{ x } 14$

$= \frac{1}{3} \times 22 \text{ x } 7$

$= \frac{154}{3} \text{ cm}^2$

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use = 3.14)

Radius of the circle = 10 cm

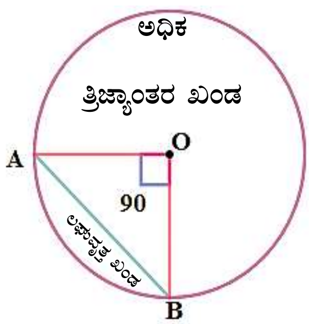
The angle of the Major sector = 360° - 90° = 270°

Area of the major sector $= \frac{270}{360°} \times \pi r^2 \text{ cm}^2$

$= \frac{3}{4} \times 3.14 \text{ x} 10 \text{ x } 10$

$= 75 \text{ x } 3.14 \text{ cm}^2$

$= 235.5 \text{ cm}^2$



In right angle ΔAOB , OA = 10 cm, OB = 10 cm

Area of $\Delta AOB = \frac{1}{2} \times OA \times OB$

$= \frac{1}{2} \times 10 \times 10$

$= 50\ cm^2$ --- (1)

The angle of the Minor sector = 90°

Area of the minor sector $= \frac{90}{360°} \times \pi r^2\ cm^2$

$= \frac{1}{4} \times 3.14\ x 10 x 10$

$= 25\ x\ 3.14\ cm^2$

$= 25 \times 3.14\ cm^2$

$= 78.5\ cm^2$ -------- (2)

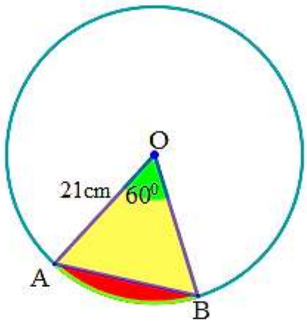
Area of minor segment = (2) – (1)

$= 78.5\ cm^2 - 50\ cm^2$

$= 28.5\ cm^2$

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc (ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord

Radius of the circle = 21 cm



(i) The length of the Arc AB $= \frac{\theta}{360°} \times 2\pi r$

Arc AB $= \frac{60}{360°} \times 2\ x\ \frac{22}{7}\ x\ 21$

$= \frac{1}{6} \times 2\ x\ 22\ x\ 3$

= **22cm**

(ii) The angle formed by arc AB = 60°

Area of the sector of angle $60^0 = \frac{60}{360°} \times \pi r^2\ cm^2$

$= \frac{60}{360°}\ x\ \frac{22}{7}\ x\ 21x21\ cm^2$

$= \frac{1}{6}\ x\ 22x\ 3\ x21\ cm^2$

$= \frac{1}{2}\ x\ 22x21\ cm^2$

$= 11x21\ cm^2$

= $\mathbf{231\ cm^2}$

(iii) The area of the equilateral $\Delta AOB = \frac{\sqrt{3}}{4}\ x\ (OA)^2$

$= \frac{\sqrt{3}}{4}\ x\ (21)^2$

$= \frac{441\sqrt{3}}{4}\ cm^2$

Hence the required area = Area of the sector formed by the Arc – area of ΔAOB

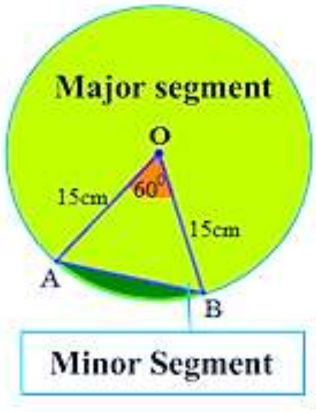
$= \mathbf{\left(231\ -\frac{441\sqrt{3}}{4}\right) cm^2}$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use = $\sqrt{3}$ = 1.73)

Radius of the circle = 15 cm

In triangle ΔAOB, $\angle AOB$ and $\angle A = \angle B = 60^0$ [$\because OA = OB = 15cm$]

$\therefore \Delta AOB$ is an equilateral triangle.



The area of $\Delta AOB = \frac{\sqrt{3}}{4} \text{ x } (OA)^2$

$= \frac{\sqrt{3}}{4} \text{ x } (15)^2$

$= \frac{225\sqrt{3}}{4} \text{ cm}^2$

$= \frac{225 \text{x } 1.73}{4} \text{ cm}^2$ **= 97.3 cm²**

The angle formed by the arc AB = 60°

$\therefore$ The area of the sector formed by the arc AB

$= \frac{60}{360°} \times \pi r^2 \text{ cm}^2 = \frac{60}{360°} \text{ x } (3.14) \text{x } 15\text{x}15 \text{ cm}^2$

$= \frac{1}{2} \text{ x } 3.14 \text{ x } 5\text{x}15 \text{ cm}^2$

$= 1.57 \text{ x } 75 \text{ cm}^2$ **= 117.75 cm²**

Area of the minor segment

= Area of the sector formed by the arc AB – Area of ΔAOB

= 117.75 - 97.3 **= 20.4 cm²**

Area of the major segment = Area of the circle – Area of minor segment

$= \pi r^2 - 20.4 \text{ cm}^2$

= 3.14 x 15x15 - 20.4

= 3.14 x 225 – 20.4

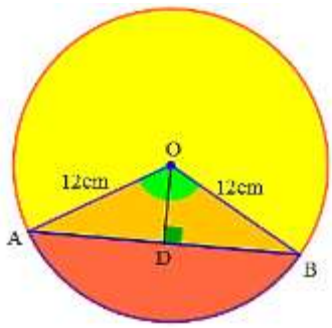
= 706.5 – 20.4 **= 686.1cm²**

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73).

Radius of the circle(r) = 12 cm

Draw $AB \perp OD \Rightarrow OD$ bisects AB

$\Rightarrow \angle A = 180° - (90° + 60°) = 30°$



$\cos 30° = \frac{AD}{OA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{12}$

$\Rightarrow AD = 6\sqrt{3}$ cm

$\Rightarrow AB = 2 \times AD = 12\sqrt{3}$ cm

$\sin 30° = \frac{OD}{OA}$

$\Rightarrow \frac{1}{2} = \frac{OD}{12}$

$\Rightarrow OD = 6$ cm

The area of $\Delta AOB = \frac{1}{2} \times AB \times OD$

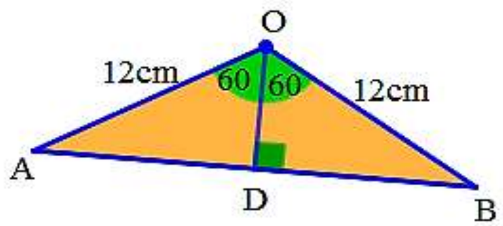
$= \frac{1}{2} \text{ x } 12\sqrt{3} \text{ x } 6 \text{ cm}^2$

$= 36\sqrt{3}$ cm

$= 36 \times 1.73$ **= 62.28 cm²**

The angle of minor sector = 120°

$\therefore$ Area of the minor sector $= \frac{120°}{360°} \times \pi r^2 \text{ cm}^2$



$= \frac{120°}{360°} \text{ x } 3.14 \text{ x } 12 \text{ x } 12 \text{ cm}^2$

$= \frac{1}{3} \text{ x } 3.14 \text{ x } 12 \text{ x } 12 \text{ cm}^2$

$= 3.14 \text{ x } 4 \text{ x } 12 \text{ cm}^2$

$= 3.14 \times 48\ cm^2 = \mathbf{150.72\ cm^2}$

$\therefore$ Area of the minor segment = Area of the minor sector – Area of ΔAOB

$= 150.72\ cm^2 - 62.28\ cm^2 = \mathbf{88.44\ cm^2}$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 11.11). Find

(i) The area of that part of the field in which the horse can graze

(ii) The increase in the grazing area if the rope were10 m long instead of 5 m. (Use = 3.14)

Solution: Given, the side of the square = 15 m

The length of the rope [Radius of the arc(r)] = 5 m

The radius of the field in which the horse can graze = 5 m.

Fig.11.11

(i) Area of the field graze by the horse

[Horse is tied at the corner of the square.So, it graze only quadrant of the circle of radius 5m]

$= \frac{\pi r^2}{4} = \frac{3.14 \times 5^2}{4}$

$= \frac{78.5}{4} = \mathbf{19.625\ m^2}$

(ii) The length of the rope is 10m then, the area graze

by the horse $= \frac{\pi r^2}{4} = \frac{3.14 \times 10^2}{4}$

$= \frac{314}{4} = \mathbf{78.5\ m^2}$

Therefore increase in grazing area

$= 78.5\ m^2 - 19.625\ m^2 = \mathbf{58.875\ m^2}$

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 11.12. Find

(i) the total length of the silver wire required

(ii) the area of each sector of the brooch.

Number of diameters = 5; Legnth of the diameter = 35 mm

$\therefore$ Radius (r) = 35/2 mm

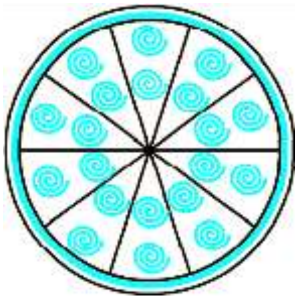


Fig.11.12

(i) The total lenth of wire required

= Perimeter of the brooch + length of 5 diameter

$= 2\pi r + (5 \times 35)$ mm

$= (2 \times \frac{22}{7} \times \frac{35}{2}) + 175$ mm

$= 110 + 175$ mm $= \mathbf{285\ mm}$

(ii) Number of sectors = 10

Therefore area of each sector $= \frac{\pi r^2}{10} = \frac{\frac{22}{7} \times \left(\frac{35}{2}\right)^2}{10}$

$= \frac{\frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}}{10}$

$= \frac{\frac{3850}{4}}{10}$

$= \mathbf{\frac{385}{4}\ mm^2}$

10. An umbrella has 8 ribs which are equally spaced (see Fig. 11.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Total ribs in the umbrella = 8

The radius of the umbrella

when it to be flat = 45 cm
The area between the two
consecutive ribs = $\frac{\text{Total Area}}{\text{number of ribs}}$

$= \frac{\pi r^2}{8} = \frac{\frac{22}{7} \text{x } 45^2}{8}$

$= \frac{44550}{56}$

$= \frac{22275}{28}$ cm^2

= 795.5 cm^2

Fig 11.13

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

The angle of the sector formed by the wiper = 115°

Radius of the sector = length of the wiper = 25 cm

Area of the sector formed by the wiper $= \frac{115°}{360°} \times \pi r^2$ cm^2

$= \frac{115°}{360°} \times \frac{22}{7}$ x 25 x 25 cm^2

$= \frac{23}{72} \times \frac{22}{7}$ x 625 cm^2

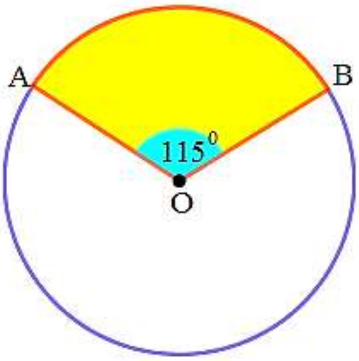
$= \frac{23}{36} \times \frac{11}{7}$ x 625 cm^2

$= \frac{158125}{252}$ cm^2

The total area coveed by blades of two wipers

$= 2 \times \frac{158125}{252}$ cm^2

$= \frac{158125}{126}$ **= 1254.96 cm^2**



12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use = 3.14)

Let the lighthouse be at O

Radius of the sector = length of the beam r = 16.5 km

Angle of the sector formed by the beam = 80°

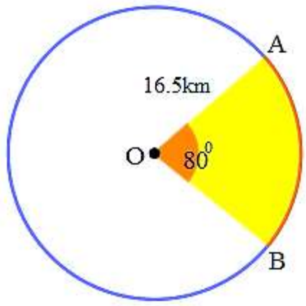
The area of the sector which light spreads

= Area of the sector $= \frac{80°}{360°} \times \pi r^2$ km^2

$= \frac{2}{9} \times 3.14$ x 16.5x16.5km^2

$= \frac{2}{9} \times 3.14$ x 272.25km^2

= 189.97 km^2



13. A round table cover has six equal designs as shown in Fig. 11.14. If the radius of the cover is 28 cm, find the cost of making the designs at th e rate of Rs 0.35 per cm^2 0.35 per cm^2 (Use $\sqrt{3}$ = 1.7).

The number of equal designs = 6;

The radius of the cover = 28 cm

Making Cost of the design = ರೂ 0.35 / cm^2

The angle of each sector $= \frac{360°}{6} = 60°$

ΔAOB ಯಲ್ಲಿ OA = OB [Radius of the same circle]

$\therefore \angle A = \angle B = 60^0$

∴Area of the equilateral ΔAOB $= \frac{\sqrt{3}}{4}$x (OA)^2

$= \frac{\sqrt{3}}{4}$x (28)^2

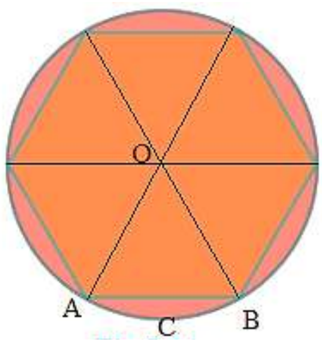


Fig.11.14

$= 1.7 \times 7 \times 28$ **= 333.2 cm²**

Area of the sector OACB = $\frac{60°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{6} \times \frac{22}{7} x\ 28^2\ cm^2$

$= \frac{1}{6} \times 22x\ 4x28\ cm^2$

$= \frac{1}{3} \times 22x\ 2x28\ cm^2 = 410.67 cm^2$

Area of the design = Area of the sector OACB - Area of the ΔAOB

$= 410.67\ cm^2 - 333.2\ cm^2 = 77.47\ cm^2$

∴ The total area of 6 designs = $6 \times 77.47\ cm^2$

$= 464.82\ cm^2$

∴ Total cost of making designs = $464.76\ cm^2 \times$ ರೂ $0.35\ /cm^2$

= Rs 162.68

14. Tick the correct answer in the following :

Area of a sector of angle p (in degrees) of a circle with radius R is

A) $\frac{P}{180}$ x $2\pi r$ B) $\frac{P}{180}$ x $2\pi r^2$ C) $\frac{P}{360}$ x $2\pi R$ D) $\frac{P}{720}$ x $2\pi R^2$

The area of the sector of angle p = $\frac{p°}{360°} \times \pi R^2\ cm^2$

$= \frac{p}{360°} \times \pi R^2 \times \frac{2}{2}$

$= \frac{P}{720} \times 2\pi R^2$

Answer **(D)** $\mathbf{\frac{P}{720} \times 2\pi R^2}$

Summary:

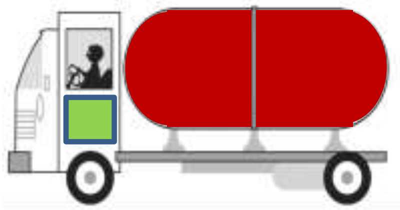
1. The radius of the circle r the angle measure with θ
   Then the Length of the Arc of the sector = $\frac{\theta}{360}$ x $2\pi r$
2. The radius of the circle r the angle measure with θ
   Then the area of the sector = $\frac{\theta}{360}$ x πr^2
3. Area of segment of a circle = Area of the corresponding sector - Area of the corresponding triangle.

12 Surface Area and Volumes

You must have seen a truck with a container fitted on its back (see Fig), carrying oil or water from one place to another. Is it in the shape of any of the four basic solids mentioned above? You may guess that it is made of a cylinder with two hemispheres as its ends. A test tube, is also a combination of a cylinder and a hemisphere. Similarly, while travelling, you may have seen some big and beautiful buildings or monuments made up of a combination of solids mentioned above.

15.2 Surface Area of a Combination of Solids

To find the surface area or the volume of a container or test tube we have to break it up two or more known solids.



For example,

Area of the container

= Area of the cylinder + 2 x Area of the hemisphere

Example 1 : Rasheed got a playing top (lattu) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig 12.6). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$)

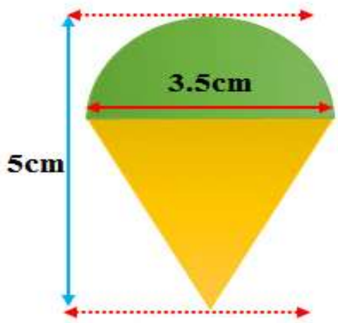


Fig.12.6

TSA of the toy = CSA of hemisphere + CSA of cone

CSA of hemisphere = $2\pi r^2$

$= \left(2x\ \frac{22}{7}\ x\ \frac{3.5}{2}\ x\ \frac{3.5}{2}\right) cm^2$

Height (cone)=Height(top)–Radius (hemisphere)

= 5 – 1.75

= **3.25cm**

Slant height of cone (l) = $\sqrt{r^2 + h^2}$

$=\sqrt{(1.75)^2 + (3.25)^2}$

≈ **3.7cm**

∴ CSA of cone = πrl

$= \left(\frac{22}{7}\ x\ \frac{3.5}{2}\ x\ 3.7\right) cm^2$

∴ TSA of the toy = $\left(2x\ \frac{22}{7}\ x\ \frac{3.5}{2}\ x\ \frac{3.5}{2}\right) + \left(\frac{22}{7}\ x\ \frac{3.5}{2}\ x\ 3.7\right)$

$=\frac{22}{7}\ x\ \frac{3.5}{2}(3.5 + 3.7)$

$=11\ x\ 0.5(3.5 + 3.7)$

= 5.5 x 7.2

= **39.6cm^2**

Example 2: The decorative block shown in Fig. 12.7 is made of two solids - a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block (Take $\pi = \frac{22}{7}$)

TSA of cube = $6a^2$

= 6 × (5×5) = 150 cm^2

The surface area of the block

= TSA(cube) –Base area(hemisphere) + CSA(hemisphere)

= $(150 - \pi r^2 + 2\pi r^2)cm^2$

= $(150 + \pi r^2)cm^2$

= $(150 + \frac{22}{7} x\ 2.1\ x\ 2.1\ )cm^2$

= $(150 + 13.86\ )cm^2$

= **163.86cm^2**

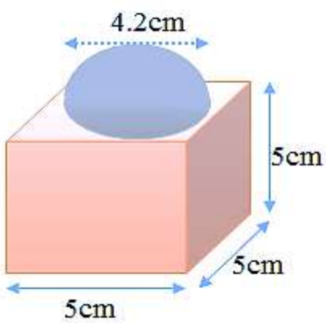


Fig.12.7

Example 3 : A wooden toy rocket is in the shape of a cone mounted on a cylinder, as shown in Fig. 12.8. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take = 3.14)

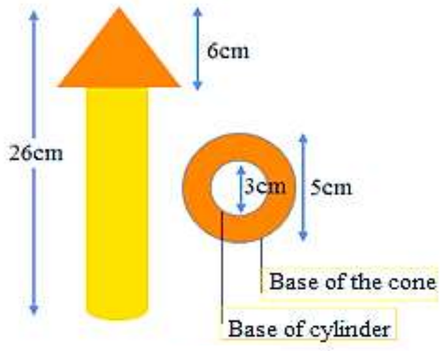
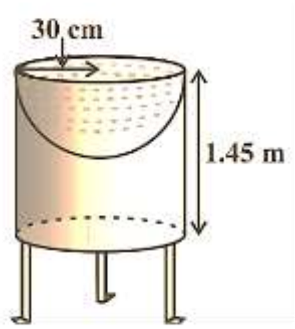


Fig.12.8

Let the the radius(Cone) = r, the slant height = l;
The height(Cone) = h ,
radius(cylinder) = r^1, and height of the cylinder = h^1
Then 'r' = 2.5 cm, h = 6 cm, r^1 = 1.5 cm, h^1 = 26 - 6 = 20 cm
and $l = \sqrt{r^2 + h^2}$
$= l = \sqrt{2.5^2 + 6^2} = 6.5$cm
Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted. The area to be painted orange
= CSA of the cone + base area of the cone– base area of the cylinder
$= \pi r l + \pi r^2 - \pi (r^1)^2$
$= \pi [(2.5 \times 6.5) + (2.5)^2 - (1.5)^2]$ cm^2
$= \pi [20.25]$ cm^2 = 3.14 × 20.25 cm^2 = **63.585 cm^2**
Now, the area to be painted yellow = CSA(cylinder)+ Area of one base of the cylinder
$= 2\pi r^1 h^1 + \pi (r^1)^2 = \pi r^1 (2h^1 + r^1)$
= (3.14×1.5) (2×20 +1.5) cm^2
= 4.71× 41.5 cm^2 = **195.465 cm^2**

Example 4 : Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (see Fig). The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird bath (Take $\pi = \frac{22}{7}$)

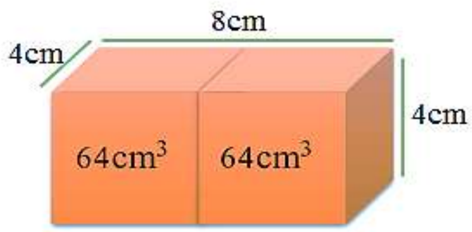


Let 'h' be height of the cylinder, and 'r' the common radius of the cylinder and hemisphere. Then,
The total surface area of the bird-bath
= CSA of cylinder + CSA of hemisphere
$= 2\pi rh + 2\pi r^2$
$= 2\pi r (h + r)$
$= 2 \times \frac{22}{7} \times 30 (145 + 30)$ cm^2
= 33000 cm^2 = **3.3m^2**

Exercise 12.1

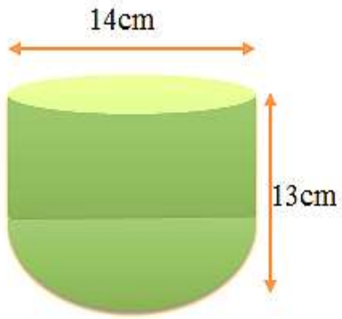
(Unless stated otherwise, take $\pi = \frac{22}{7}$)

1. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.



The volume of the cube = 64cm^3,
Therefore length of the side = 4cm
∴ The length of the cuboid = 4+4 = 8 cm
Breadth b = 4cm and height h = 4cm
The surface area of the cuboid = 2(lb+bh+hl)
= 2(8x4+4x4+4x8)= 2(32+16+32)= 2(80) = **160 cm^2**

2. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.



The inner surface area of the vessel =

Inner surface area of the cylinder+ Inner surface area of the hemisphere

$= 2\pi rh + 2\pi r^2$

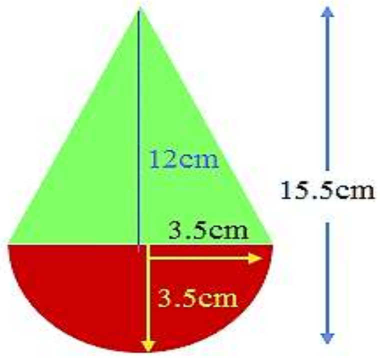
$\pi = \frac{22}{7}$; r = $\frac{14}{2}$ = 7cm; height of the cylinder h = 13 – 7 = 6cm

$= 2x\ \frac{22}{7} x\ 7x\ 6 + 2x \frac{22}{7}\ x\ 7x\ 7$

$= 2x\ 22x\ \ 6 + 2x22\ x\ 7$

$= 264 + 308 =$ **572cm^2**

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius.The total height of the toy is 15.5 cm. Find the total surface area of the toy.



TSA of the Toy = CSA of cone + CSA of hemisphere $= \pi rl + 2\pi r^2$

$\pi = \frac{22}{7}$; r = 3.5 ; h = 15.5 – 3.5 = 12cm

$l = \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + 3.5^2} = \sqrt{144 + 12.25}$

$l = \sqrt{156.25}$ = 12.5cm

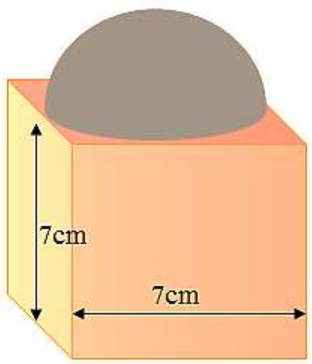
TSA of the Toy = $\frac{22}{7}$ x 3.5 x 12.5 + 2x $\frac{22}{7}$x3.5^2

= 22 x 0.5 x 12.5 + 2x 22x0.5x3.5

= 11 x 12.5 + 11x7

= 11 x 19.5 =**214.5cm^2**

4. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid



The greatest diameter of the hemisphere=Side of the square=7cm

Surface area(solid) =Surface area(cube)+CSA(hemisphere)

– The area of the circular base (hemisphere)

$= 6a^2 + 2\pi r^2 - \pi r^2$

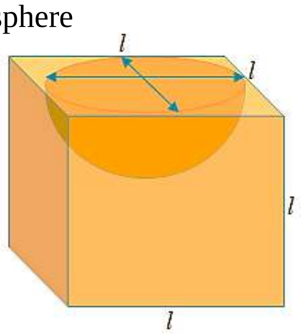
$= 6x7^2 + 2\ x\ \frac{22}{7} x \left(\frac{7}{2}\right)^2 - \frac{22}{7}\ x \left(\frac{7}{2}\right)^2$

$= 6x49 + 11x7 - 11\ x \frac{7}{2}$

$= 294 + 77 - 11\ x \frac{7}{2}$

= 371 – 38.5= **332.5cm^2**

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid



The surface area of the solid = Surface area of cube + Surface area of hemisphere

– Area of the circular base of the hemisphere

$= 6l^2 + 2\ \pi r^2 - \pi r^2$

$= 6l^2 + 2\ \pi \left(\frac{l}{2}\right)^2 - \pi \left(\frac{l}{2}\right)^2$

$= 6l^2 + 2\ \pi \left(\frac{l}{2}\right)^2 - \pi \left(\frac{l}{2}\right)^2$

$= 6l^2 + \pi \left(\frac{l}{2}\right)^2 = \frac{l^2}{4}(24 + \pi)$

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig. 12.10). The length of the entre capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.

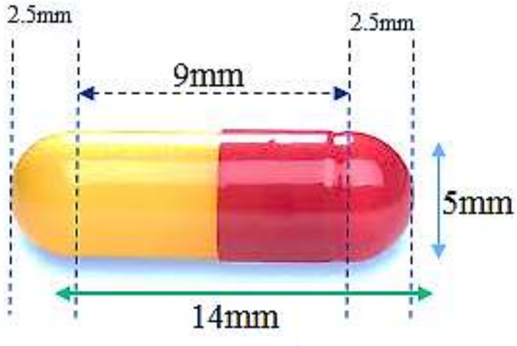


Fig.12.8

Surface area of the capsule =

2CSA of hemisphere CSA of cylinder = $2(2\pi r^2)+2\pi rh$

$\pi = \frac{22}{7}$; r = 2.5mm ; h = 9mm

$= 2(2\pi r^2) + 2\pi rh$

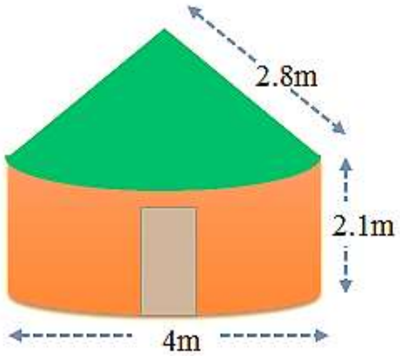
$= 2\pi r(2r + h)$

$= 2\text{x} \frac{22}{7} \text{ x } \frac{5}{2}(2\text{x}2.5 + 9)$

$= \frac{110}{7}(14)$

= 110 x 2 = **220mm^2**

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)



The area of the tent = CSA(cylinder) + CSA (cone)

$= 2\pi rh + \pi rl = \pi r(2h + l)$

$\pi = \frac{22}{7}$; r = 2m ; h = 2.1m ; l = 2.8m

$= \frac{22}{7}\text{x } 2(2\text{x}2.1 + 2.8)$

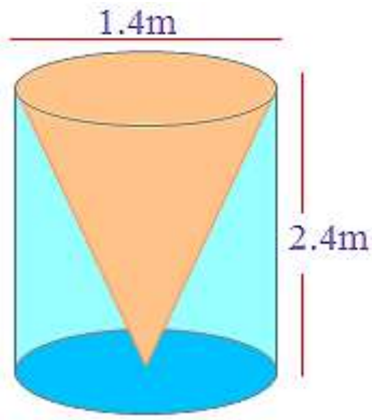
$= \frac{44}{7}\text{x } 7\ (2\text{x}0.3 + 0.4)$

= 44 (0.6 + 0.4) = **44m^2**

The total cost of the canvas at the rate of Rs 500/cm^2

= 44x500 = **Rs 22000**

8. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2



Surface area of the solid = TSA of cylinder + Inner CSA of cone

– Area of one the circular face of the cylinder

$= 2\pi r(r + h) + \pi rl - \pi r^2$

$\pi = \frac{22}{7}$; r = 0.7m ; h = 2.4m

$l = \sqrt{h^2 + r^2} = \sqrt{2.4^2 + 0.7^2}$

$= \sqrt{5.76 + 0.49}$

$= \sqrt{6.25} = 2.5\text{m}$

$= 2\text{x}\frac{22}{7}\text{x}0.7(0.7 + 2.4) + \frac{22}{7}\text{x } 0.7\text{x}2.5 - \frac{22}{7}\text{x}0.7\text{x}0.7$

$= 2\text{x}22\text{x}0.1(3.1) + 22\text{x } 0.1\text{x}2.5 - 22\text{x}0.1\text{x}0.7$

$= 4.4(3.1) + 2.2\text{x}2.5 - 2.2\text{x}0.7$

$= 13.64 + 5.5 - 1.54$

$= 13.64 + 5.5 - 1.54$

= **17.6m^2** ≈ **18m^2**

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 12.11. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.

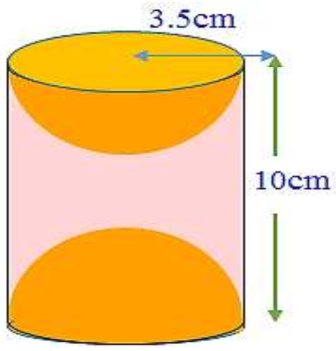


Fig.12.11

TSA (Article) = CSA (cylinder) + 2x inner CSA of hemisphere

$= 2\pi rh + 2x2\pi r^2$ $[\pi = \frac{22}{7}$; $r = 3.5cm$; $h = 10m]$

$= 2\pi r(h + 2r)$

$=2x \frac{22}{7} x\ 3.5\ (10 + 2x3.5)$

$=2x\ 22\ x\ 0.5\ (10 + 7)$

$= 22(17)$

$= 22\ (17)$ = **374cm^2**

12.3 Volume of a Combination of Solids

Example 5: Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder (see Fig.12.12). If the base of the shed is of dimension 7 m × 15 m, and the height of the cuboidal portion is 8 m, find the volume of air that the shed can hold. Further, suppose the ma chinery in the shed occupies a total space of 300 m^3, and there are 20 workers, each of whom occu py about 0.08 m^3 space on an average. Then, how much air is in the sshed (Take $\pi = \frac{22}{7}$)

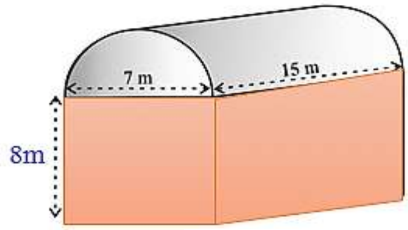


Fig.12.12

The volume of air inside the shed (when there are no people or machinery) is given by the volume of air inside the cuboid and inside the half cylinder, taken together.

Now, the length, breadth and height of the cuboid are 15 m, 7 m and 8 m, respectively. Also, the diameter of the half cylinder is 7 m and its height is 15 m

So, the required volume

= volume of cuboid + $\frac{1}{2}$ volume of cylinder

$= \left[15\ x\ 7\ x\ 8\ + \frac{1}{2}\ x\ \frac{22}{7}\ x\ \frac{7}{2}\ x\ \frac{7}{2}\ x\ 15\right] m^3$ = **1128.75 m^3**

Next, the total space occupied by the machinery = 300 m^3

And the total space occupied by the workers = 20 × 0.08 m^3 = 1.6m^3

∴ The volume of the air, when there are machinery and workers

= 1128.86 - (300.00 + 1.60) = **827.15 m^3**

Example 6: A juice seller was serving his customers using glasses as shown in Fig. 12.13. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14)

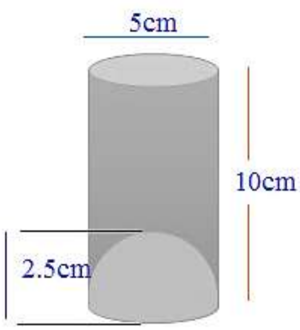


Fig.12.13

Since the inner diameter of the glass = 5 cm and

Height = 10 cm, the apparent capacity of the glass = $\pi r^2 h$

= 3.14 × 2.5 × 2.5 × 10 cm^3 = **196.25 cm^3**

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5$

= **32.71 cm^3**

So, the actual capacity of the glass

= apparent capacity of glass – volume of the hemisphere

= (196.25 – 32.71) **cm³**

= 164.08 cm³

Example 7: A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take π = 3.14).

Height of the cylinder (h)

= radius (hemisphere) + height (cone) = 2+2= **4cm**

Radius of the hemisphere

= radius of the cylinder = radius of the cone = **2cm**

The volume of the cylinder circumsubscried the toy

= $\pi r^2 h$ = 3.14 x 2 x 2 x 4

= 3.14 x 16

= 50.24cm³

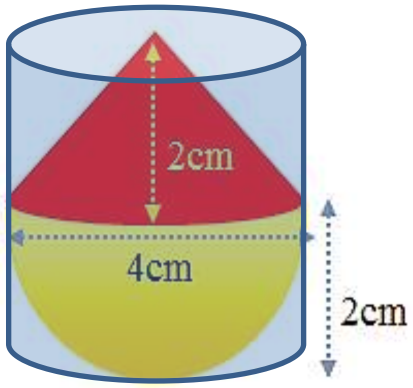


Fig.12.14

The volume of the toy = $\frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^2 h$

= $\frac{1}{3}\pi r^2[2r + h]$

= $\frac{1}{3}$x 3.14x $2^2[4 + 2]$

= $\frac{1}{3}$x 3.14x 4[6]= (196.25 – 32.71) **cm³**

= $\frac{1}{3}$x 3.14x 4[6] = 25.12cm³

Hence, the required difference of the two volumes = 50.24 - 25.12 cm³

= **25.12cm³**

Exercise 12.2

[Unless stated otherwise, take $\pi = \frac{22}{7}$]

1. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π

Volume of the solid = Volume of the cone + volume of the hemisphere

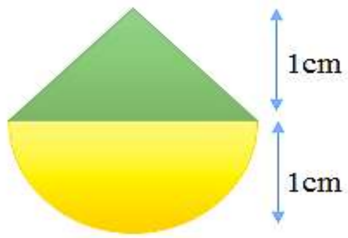
Given: $\pi = \frac{22}{7}$; h = 1cm; r = 1cm

Volume of the solid =$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

=$\frac{1}{3}\pi$x1x1 + $\frac{2}{3}$ π x1x1x1

=$\frac{2}{3}\pi + \frac{2}{3}\pi = \frac{3}{3}\pi$

= π**cm³**



2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly same]

Volume of the air contained in the model = 2xVolume of the cone +Volume of the cylinder ⇒ V = 2x$\frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$

Given:r = 1.5cm; h_1 = 2cm; h_2 = 8cm

V = 2x$\frac{1}{3}$x $\frac{22}{7}$ x $(1.5)^2$x 2 + $\frac{22}{7}$x$(1.5)^2$8

$= \frac{44}{21}$ x 2.25x2 $+ \frac{22}{7}$x 2.25x8

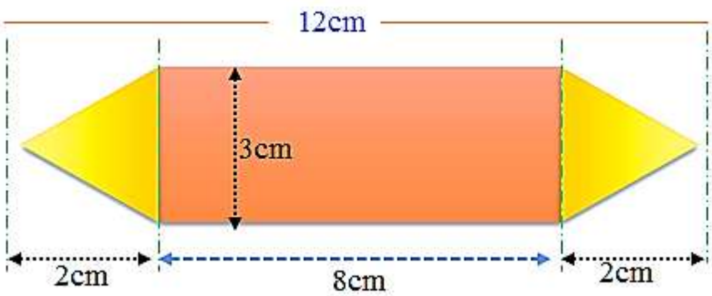
$= \frac{44}{21}$ x 4.5 $+ \frac{22}{7}$x 18

$= \frac{198}{21} + \frac{396}{7}$

$= \frac{198}{21} + \frac{1188}{21}$

$= \frac{1386}{21}$

= 66cm³



3. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig. 12.15).

Volume of the Jamun

= 2 x Volume of two hemisphere + Volume of the cylinder

$= 2\text{x} \frac{2}{3}\pi r^3 + \pi r^2 h$; Given: $\pi = \frac{22}{7}$; r = 1.4cm; h = 2.2cm;

$= 2\text{x} \frac{2}{3}\text{x}\frac{22}{7}$x 1.4 x 1.4 x 1.4 $+ \frac{22}{7}$ x 1.4 x1.4 x 2.2

$= 4\text{x}\frac{22}{3}$x 0.2 x 1.4x1.4 + 22 x 0.2 x1.4 x 2.2

$= \frac{34.496}{3} + 13.552$

= 11.5 + 13.552 = **25.05 cm³**

∴ The amount of sugar contained = 25.05 x $\frac{30}{100}$

= **7.515cm³**

∴ The total amount of sugar contained in 45 jamun = 7.515x 45

= 338.175cm³ ≈ **338cm³**

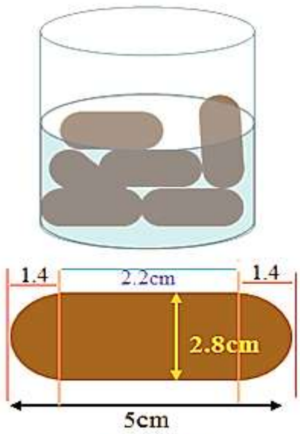


Fig.12.15

4. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig. 12.16)

The radius of the conical deoressions r = 0.5cm, the depth h_1 = 1.4 cm

Length of the cuboid shape l = 15cm, breadth b = 10cm height h = 3.5cm

4 (Volume of the conical depressions) = $4\left(\frac{1}{3}\pi r^2 h_1\right)$

$= 4\left(\frac{1}{3}\text{x}\frac{22}{7}\text{x } 0.5 \text{ x}0.5 \text{ x}1.4\right) = 4\left(\frac{1}{3}\text{x}22\text{x } 0.5 \text{ x}0.5 \text{ x } 0.2\right)$

$= 4\left(\frac{1}{3}\text{x}22\text{x } 0.5 \text{ x}0.5 \text{ x } 0.2\right)$

= 1.47cm³

Volume of the wood in the pen stand

= [Volume of the cuboid shape

– 4 (Volume of the conical depressions)]

= 15x10x3.5 – 1.47 = 525 – 1.47

= **523.53cm³**

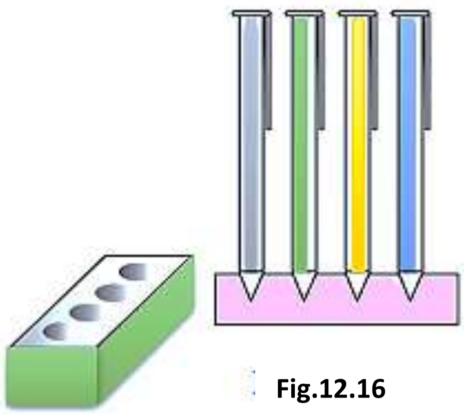


Fig.12.16

5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

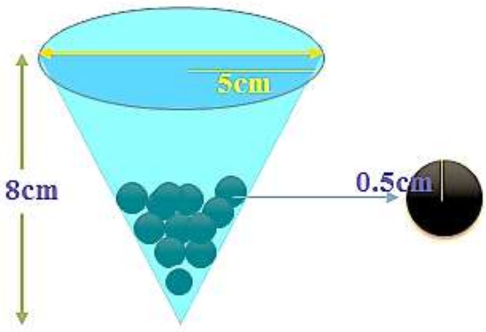
The Volume of the lead shots = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \text{ x } \frac{22}{7} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{11}{21}$ cm³

The volume of the water in the vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 8$

$= \frac{4400}{21}$ cm^3



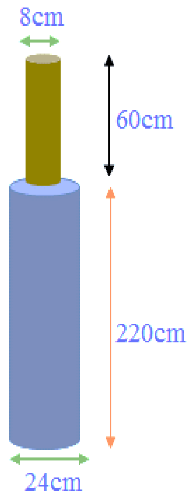
The volume of the water flows out $= \frac{4400}{21} \times \frac{1}{4} = \frac{1100}{21}$cm^3

$\therefore$ Number of lead shots $= \frac{\text{Amount of water folws out}}{\text{Volume of the lead shot}}$

$\therefore$ Number of lead shots $= \frac{\frac{1100}{21}}{\frac{11}{21}}$

= 100 shots

6. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use = 3.14)



$r_1 = 8$cm; $r_2 = \frac{24}{2} = 12$cm; $h_1 = 60$cm; $h_2 = 220$cm

Volume of the pole

= Volume of the first cylinder + Volume of the second cylinder

$= \pi r_2^2 h_2 + \pi r_1^2 h_1$

$= 3.14 \times 12 \times 12 \times 220 + 3.14 \times 8 \times 8 \times 60$

$= 99475.2 + 12057.6$

$= 111532.8$cm^3

The mass of the iron / 1 cm^3 = 8g

$\therefore$ Mass of the iron pole = 111532.8 x 8

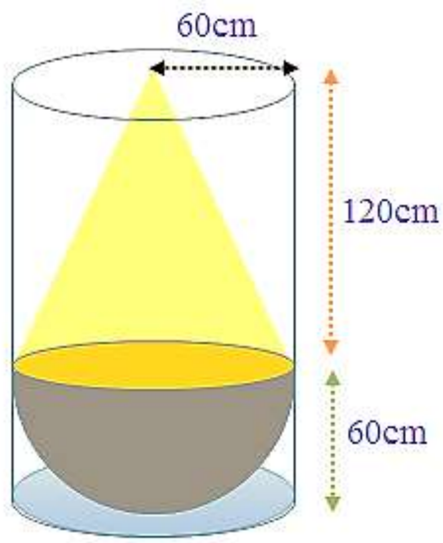
= 892262.4 g

= 892.26kg

7. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Radius of the cylinder r = 60cm; height h = 180cm

Height of the cone $h_1 = 120cm$



Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 60 \times 60 \times 180$

= 2036571.43cm^3

Volume of the cone $= \frac{1}{3}\pi r^2 h_1$

$= \frac{22}{7} \times 20 \times 60 \times 120$

= 452571. 43cm^3

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 60$

= 4525571.43cm^3

$\therefore$ The volume of the water left in the cylinder

= 2036571.43 – (452571.43 + 452571.43)

= 2036571.43 – 905142.86

= 1131428.57cm^3 = **1.131m^3**

Alternate method:

The volume of the water left in the cylinder

$= \left[\pi r^2 h - \frac{1}{3}\pi r^2 h_1 + \frac{2}{3}\pi r^3\right]$

$= \pi r^2 \left[h - \frac{1}{3}h_1 + \frac{2}{3}r\right]$

$= \frac{22}{7}$x 60x60 $\left[180 - \frac{1}{3}\text{x}120 + \frac{2}{3}\text{x}60\right]$

$= \frac{22}{7}$x 60x60$[180 - (40 + 40)]$

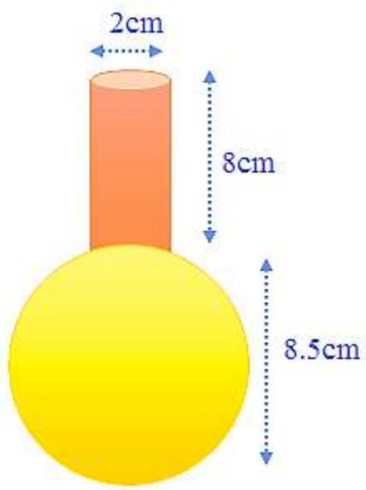
$= \frac{22}{7}$x 60x60x 100

= 1131428.57cm^3

= 1.131m^3

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Height of the cylinder h = 8cm; Radius $r_1 = \frac{2}{2} = 1$cm



Radius of the sphere $r_2 = \frac{8.5}{2}$ cm

Volume of the Vessel

= Volume of the cylinder + Volume of the sphere

$= \pi {r_1}^2 h + \frac{4}{3}\pi r_2^3$

$= 3.14\text{x}1^2\text{x}\,8 + \frac{4}{3}\text{x}3.14\text{x}\left(\frac{8.5}{2}\right)^3$

$= 25.12 + \frac{11}{21}$x 8.5x8.5x8.5

$= 25.12 + 321.39$

= **346.51cm^3**

So, there is little difference in her measurment

Summary:

1. To determine the surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
2. To find the volume of objects formed by combining any two of a cuboid, cone, cylinder, sphere and hemisphere.

13 Statistics

13.2 Mean of Grouped data:

Average: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ **[i = 1 to n]**

Example 1 : The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students

x	10	20	36	40	50	56	60	70	72	80	88	92	95
y	1	1	3	4	3	2	4	4	1	1	2	3	1

Solutions:

x	10	20	36	40	50	56	60	70	72	80	88	92	95
y	1	1	3	4	3	2	4	4	1	1	2	3	1
$x_i f_i$	10	20	108	160	150	112	240	280	72	80	176	276	96

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1779}{30} = 59.53$

Different methods to find average:

Example: Find the average of the following

C.I.	10-25	25-40	40-55	55-70	70-85	85-100
No.of students	2	3	7	6	6	6

Direct Method

C.I.	f_i	x_i	$f_i x_i$
10–25	2	17.5	35.0
25–40	3	32.5	97.5
40–55	7	47.5	332.5
55–70	6	62.5	375.0
70–85	6	77.5	465.0
85–100	6	92.5	555.0
$\sum f_i = 30$		$\sum f_i x_i = 1860.0$	

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{1860}{30} = 62$

Assumed Mean Method

C.I.	f_i	x_i	d_i	$f_i d_i$
10–25	2	17.5	–30	–60
25–40	3	32.5	–15	–45
40–55	7	47.5	0	0
55–70	6	62.5	15	90
70–85	6	77.5	30	182
85–100	6	92.5	45	270
$\sum f_i = 30$			$\sum f_i d_i = 435$	

$d_i = x_i$ – a [Here, a = 47.5]

Average $\bar{x}$ = a + $\frac{\sum f_i d_i}{\sum f_i}$

$= 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62$

Step Deviation Method

C.I.	(f_i)	(x_i)	$u_i = \frac{x_i - 47.5}{15}$	$f_i u_i$
10–25	2	17.5	–2	–4
25–40	3	32.5	–1	–3
40–55	7	47.5	0	0
55–70	6	62.5	1	6
70–85	6	77.5	2	12
85–100	6	92.5	3	18
$\sum f_i = 30$			$\sum f_i u_i = 29$	

$d_i = x_i - 47.5$ and h = 15

Note: If all di have common multiple then step deviation method is the best method
We get the same average in all three methods.
Assumed Mean and step deviation methods are the simplified form of Direct

Average $\bar{x}$= a + $\frac{\sum f_i u_i}{\sum f_i}$ x h

=47.5 + $\frac{29}{30}$ x 15

= 47.5 + $\frac{29}{2}$

= 47.5 + 14.5

= 62

Example 2 : The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by all the three methods discussed

Percentage of Female teachers	15-25	25-35	35-45	45-55	55-65	65-75	75-85
No.of states and union territories (U.T.)	6	11	7	4	4	2	1

C.I.	f_i	x_i	d_i	u_i	$f_i x_i$	$f_i d_i$	$f_i u_i$
15-25	6	20	-30	-3	120	-180	-18
25-35	11	30	-20	-2	330	-220	-22
35-45	7	40	-10	-1	280	-70	-7
45-55	4	50	0	0	200	0	0
55-65	4	60	10	1	240	40	4
65-75	2	70	20	2	140	40	4
75-85	1	80	30	3	80	30	3
$\sum f_i$ = 35					**1390**	**-360**	**-36**

a = 50, h = 10; $u_i = \frac{x_i - 20}{10}$

$\sum f_i = 35,\ \sum f_i x_i = 1390,\ \sum f_i d_i = -360,\ \sum f_i u_i = -36$

Direct Method

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{1390}{35}$

$= \mathbf{39.71}$

Assumed Mean Method

$\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i}$

$=50 - \frac{360}{35}$

$= 50 - 10.29 =$ **39.71**

Step Deviation Method

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$=50 - \frac{36}{35} \times 10$

$=50 - 10.29$

= 39.71

Example 3: The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

No.of Wickets	20 - 60	60 -100	100-150	150 - 250	250 - 350	350 - 450
No.of Bowlers	**7**	**5**	**16**	**12**	**2**	**3**

C.I.	f_i	x_i	$f_i x_i$
20 –60	7	40	280
60 –100	5	80	400
100 –150	16	125	2000
150 –250	12	200	2400
250 –350	2	300	600
350 –450	3	400	1200
	45		**6880**

Average by direct method.

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$= \frac{6880}{45} =$ **152.89**

Exercise – 13.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house

Which method did you use for finding the mean, and why?

No.of Plants	0–2	2–4	4–6	6–8	8–10	10–12	12–14
No.of Houses	1	2	1	5	6	2	3

C.I.	f_i	x_i	$f_i x_i$
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39

	$\sum f_i$ =20	$\sum x_i f_i$ =162

a = 7 ,h = 2

$\sum f_i = 35, \sum f_i x_i = 162$

Average from Direct Method

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{1620}{20} = 8.1$

[**You can use any method. Because of simple tabulation we can use direct method here**]

2. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method

Daily wages (Rs)	**100–120**	**120–140**	**140–160**	**160–180**	**180–200**
No.of workers	**12**	**14**	**8**	**6**	**10**

C.I.	f_i	x_i	$u_i = \frac{x_i - 150}{20}$	$f_i u_i$
100-120	12	110	-2	-24
120-140	14	130	-1	-14
140-160	8	150	0	0
160-180	6	170	1	6
180-200	10	190	2	20
$\sum f_i$ =50			$\sum x_i f_i$ = -12	

a = 150, h = 20

step deviation method

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 150 + \frac{-12}{50} \times 20$

$= 150 - 4.8 = $ **145.2**

[**Can use any method. But Assumed mean method is more suitable here**]

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ` 18. Find the missing frequency f

Daily Pocket allowences(Rs)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
No.of Children	7	6	9	13	f	5	4

C.I.	f_i	x_i	$f_i x_i$
11-13	7	12	84
13-15	6	14	84
15-17	9	16	144
17-19	13	18	234
19-21	f	20	20f
21-23	5	22	110
23-25	4	24	96
$\sum f_i$ =44+ f			**752+20f**

a = 18 , h = 2

Average from Direct Method

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{752+20f}{44+f}$

$\Rightarrow 18(44+f) = 752+20f$

$\Rightarrow 792+18f = 752+20f$

$\Rightarrow 40 = 2f \Rightarrow f = 20$

[Wecan use any method here]

4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method

No.of Heart beats/Minute	No.of women
65-68	2
68-71	4
71-74	3
74-77	8
77-80	7
80-83	4
83-86	2

C.I.	f_i	x_i	d_i	$f_i d_i$
65–68	2	66.5	–9	–18
68–71	4	69.5	–6	–24
71–74	3	72.5	–3	–9
74–77	**8**	**75.5**	**0**	**0**
77–80	7	78.5	3	21
80–83	4	81.5	6	24
83–86	2	84.5	9	18
	$\sum f_i = 30$			$\sum f_i d_i = 12$

a = 75.5 , h = 3; By Average from assumed Mean Method

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} = 75.5 + \frac{12}{30}$

= 75.5 + 0.4

= 75.9 [Direct method is not suitable here]

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

No.of Mangoes	50–52	53–55	56–58	59–61	62–64
No.of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

C. I.	f_i	x_i	$u_i = \frac{x_i - 57}{3}$	$f_i u_i$
50–52	15	51	–2	–30
53–55	110	54	–1	–110
56–58	135	57	0	0
59–61	115	60	1	115
62–64	25	63	2	50
	$\sum f_i = 400$			$\sum f_i u_i = 25$

a = 57 , h = 3

Average from step deviation method

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 57 + \frac{25}{400} \times 3$

= 57 + 0.1875

= 57.1875 ≈ 57.19

Here, Assumed mean method is more suitable

6. The table below shows the daily expenditure on food of 25 households in a locality

Daily expenditure(Rs)	100–150	150–200	200–250	250–300	300–350
No.of households	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

C.I.	f_i	x_i	$u_i = \frac{d-225}{50}$	f_iu_i
100-150	4	125	-2	-8
150-200	5	175	-1	-5
200-250	12	225	0	0
250-300	2	275	1	2
300-350	2	325	2	4
	$\sum f_i$ =25			$\sum f_iu_i$ =-7

a = 225 , h = 50

By step deviation method

$\bar{x}$ = a + $\frac{\sum f_iu_i}{\sum f_i}$ x h

= 225+$\frac{-7}{25}$ x 50

= 225- 14 = 211

For this problem step deviation method is more suitable

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below. Find the mean concentration of SO_2 in the air.

Concentration of SO_2	Freequency
0.00 – 0.04	4
0.04 – 0.08	9
0.08 – 0.12	9
0.12 – 0.16	2
0.16 – 0.20	4
0.20 – 0.24	2

C.I.	f_i	x_i	f_ix_i
0.00 – 0.04	4	0.02	0.08
0.04 – 0.08	9	0.06	0.54
0.08 – 0.12	9	0.10	0.90
0.12 – 0.16	2	0.14	0.28
0.16 – 0.20	4	0.18	0.72
0.20 – 0.24	2	0.22	0.44
	$\sum f_i$ =30		$\sum f_ix_i$ =2.96

By Direct Method

$\bar{x}$ = $\frac{\sum f_ix_i}{\sum f_i}$

= $\frac{2.96}{30}$

= 0.099

The mean concentration of SO_2 in the air

= 0.099ppm

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

No.of days	0 - 6	6 - 10	10 - 14	14 - 20	20 - 28	28 - 38	38 - 40
No.of students	11	10	7	4	4	3	1

C.I.	f_i	x_i	f_ix_i
0-6	11	3	33
6-10	10	8	80
10-14	7	12	84
14-20	4	17	68

20-28	4	24	96
28-38	3	33	99
38-40	1	39	39
	$\sum f_i$ =40		$\sum \boldsymbol{f_i x_i}$ = 499

From the above table

$\sum f_i$ = 40,

$\sum f_i\, x_i$ = 499,

Average from Direct Method

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{499}{40} = 12.475$

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate(%)	45-55	55-65	65-75	75-85	85-95
No.of cities	3	10	11	8	3

C.I.	$\boldsymbol{f_i}$	$\boldsymbol{x_i}$	$\boldsymbol{d_i = x_i - 70}$	$\boldsymbol{f_i d_i}$
45-55	3	50	-20	-60
55-65	10	60	-10	-100
65-75	11	70	0	0
75-85	8	80	10	80
85-95	3	90	20	60
	$\sum f_i$ =35			**$\sum f_i d_i$ =-20**

From the above table

$\sum f_i$ = 35, $\sum \boldsymbol{f_i d_i}$ **=-20**

Average by Assumed mean Method

$\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i}$

= 70 + $\frac{-20}{35}$ = **60.43**

13.3 Mode for the grouped data

A mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency

Example: 4 The wickets taken by a bowler in 10 cricket matches are as follows:

2 6 4 5 0 2 1 3 2 3

Find the mode of the data

No.of wickets	0	1	2	3	4	5	6
No.of matches	1	1	3	2	1	1	1

Clearly, 2 is the number of wickets taken by the bowler in the maximum number (i.e., 3) of matches. So, the mode of this data is 2

f_0 = frequency of the class preceding the modal class.

l = lower limit of the modal class

h = size of the class interval

$$\textbf{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

f_1 = frequency of the modal class.

f_2 = frequency of the class succeeding the modal class.

Example 5 : A survey ... resulted in the following frequency table for the number of family members in a household. Find the mode of this data

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
No.of families	7	8	2	2	1

Here the maximum class frequency is 8, and the class interval corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5

modal class = 3 – 5, l = 3, h = 2 , f_1 = 8, f_0 = 7, f_2 = 2

Now substitute the values in the formula:

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

$= 3 + \left[\frac{8-7}{2(8)-7-2}\right] \text{x } 2$

$= 3 + \left[\frac{1}{16-9}\right] \text{x } 2$

$= 3 + \frac{2}{7} = 3.286$

∴ Therefore, the mode of the data above is 3.286.

Example 6 : The marks distribution of 30 students in a mathematics examination are given in Table 13.3 of Example 1. Find the mode of this data. Also compare and interpret the mode and the mean.

Class Intervals	10-25	25-40	40-55	55-70	70-85	85-100
No.of students	2	3	7	6	6	6

Maximum students are in the class interval 40-45, it is the modal class,

∴ l = 40, h = 15, f_1 = 7 , f_0 = 3, f_2 = 6

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

Mode = $40 + \left[\frac{7-3}{2(7)-3-6}\right] \text{x } 15$

$= 40 + \left[\frac{4}{14-9}\right] \text{x } 15$

$= 40 + \frac{4}{5} \text{x } 15$

$= 40 + 12$

∴ The mode of the given data is 52

Exercise 13.2

1. The following table shows the ages of the patients admitted in a hospital during a year: Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Age(in years)	5 - 15	15-25	25-35	35-45	45-55	55-65
No.of patients	6	11	21	23	14	5

Maximum number of patients =23 ∴ 35-45 is the modal class interval

∴ $l = 35$, h = 10, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $35 + \left[\frac{23 - 21}{2(23)-21-14}\right] \times 10$

= $35 + \left[\frac{2}{46-35}\right] \times 10$

= $35 + \frac{2}{11} \times 10$

= 35 + 1.81

∴ The mode of the above data is 36.81

C.I.	f_i	x_i	$u_i = \frac{x_i - 30}{10}$	$f_i u_i$
5–15	6	10	–2	–12
15–25	11	20	–1	–11
25–35	21	30	0	0
35–45	23	40	1	23
45–55	14	50	2	28
55–65	5	60	3	15
	$\sum f_i = 80$		$\sum f_i u_i = 43$	

a = 30, h = 10

$\bar{x}$ = $a + \frac{\sum f_i u_i}{\sum f_i} \times h$

= $30 + \frac{43}{80} \times 10$

= 30 + 5.375

= **35.375**

So, we conclude that maximum number of patients admitted in the hospital are of the age 36.81years (Approx) whereas the average age of the patient admitted in the hospital is 35.375years

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components

Life time(in hours)	0 – 20	20–40	40–60	60–80	80–100	100–120
Freequency	10	35	52	61	38	29

Determine the modal lifetimes of the components

Maximum frequency =61. So, 60 - 80 is the modal class interval.

∴ $l = 60$, h = 20, $f_1 = 61$, $f_0 = 52$, $f_2 = 38$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

= $60 + \left[\frac{61 - 52}{2(61)-52-38}\right] \times 20$

= $60 + \left[\frac{9}{122-90}\right] \times 20$

= $60 + \frac{9}{32} \times 20$

= 60 + 5.625

= 65.625

∴ The mode of the above given data = **65.625**

4. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in Rs)	No.of families
1000 – 1500	24
1500 – 2000	40
2000 – 2500	33
2500 – 3000	28
3000 – 3500	30
3500 – 4000	22
4000 – 4500	16
4500 – 5000	7

The modal class interval is (1500 – 2000)

∴ l = 1500, h = 500, f_1 = 40, f_0 = 24, f_2 = 33

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

Mode = $1500 + \left[\frac{40-24}{2(40)-24-33}\right] \text{x } 500$

= $1500 + \left[\frac{16}{80-57}\right] \text{x } 500$

= $1500 + \frac{16}{23} \text{ x } 500 = 1500 + 347.83 = 1847.83$

∴ The mode of the given data = 1847.83

C.I.	f_i	x_i	$u_i = \frac{x_i - 2750}{500}$	$f_i u_i$
1000 – 1500	24	1250	–3	–72
1500 – 2000	40	1750	–2	–80
2000 – 2500	33	2250	–1	–33
2500 – 3000	28	2750	0	0
3000 – 3500	30	3250	1	30
3500 – 4000	22	3750	2	44
4000 – 4500	16	4250	3	48
4500 – 5000	7	4750	4	28
	$\sum f_i$ =200			$\sum f_i u_i$ =-35

By step deviation method Average

a = 2750, h = 500

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \text{ x h}$

= $2750 + \frac{-35}{200} \text{ x } 500$

= 2750 - 87.5

= 2662.5

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures

No.of students per teacher	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
No.of state/U.Ts	3	8	9	10	3	0	0	2

The modal class interval 30 – 35 is the modal class interval

∴ l = 30, h = 5, f_1 = 10, f_0 = 9, f_2 = 3

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

= $30 + \left[\frac{10-9}{2(10)-9-3}\right] \text{x } 5$

= $30 + \left[\frac{1}{20-12}\right] \text{x } 5$

$= 30 + \frac{1}{8} \times 5$

$= 30 + 0.625$

$= 30.625$

$\therefore$ The mode of the above data is **30.625**

C.I.	f_i	x_i	$u_i = \frac{x_i - 32.5}{5}$	$f_i u_i$
15 – 20	3	17.5	–3	–9
20 – 25	8	22.5	–2	–16
25 – 30	9	27.5	–1	–9
30 – 35	10	32.5	0	0
35 – 40	3	37.5	1	3
40 – 45	0	42.5	2	0
45 – 50	0	47.5	3	0
50 – 55	2	52.5	4	8
	$\sum f_i = 35$			$\sum f_i u_i = -23$

By step deviation Method

a=32.5, h = 5

$\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 32.5 + \frac{-23}{35} \times 5$

$= 32.5 - 3.29$

$= 29.21$

The students – teacher ratio is 30.625 and average ratio is 29.21

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches
Find the mode of the data.

Runs scored	No.of Batsman
3000 - 4000	4
4000 - 5000	18
5000 - 6000	9
6000 - 7000	7
7000 - 8000	6
8000 - 9000	3
9000 - 10000	1
10000 - 11000	1

The modal class interval is 4000 -5000

$\therefore$ $l = 4000$, h = 1000, $f_1 = 18$, $f_0 = 4$, $f_2 = 9$

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode $= 4000 + \left[\frac{18-4}{2(18)-4-9}\right] \times 1000$

$= 4000 + \left[\frac{14}{36-13}\right] \times 1000$

$= 4000 + \frac{14}{23} \times 1000 = 4000 + 608.7 = 4608.7$

$\therefore$ The mode of the above data is 4608.7

1. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data

No.of cars	0 – 10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Freequency	7	14	13	12	20	11	15	8

Maximum freequency = 20. It is in the class interval 40 - 50

The modal class interval is 40 – 50 ∴ l = 40, h = 10, f_1 = 20, f_0 = 12, f_2 = 11

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h = 40 + \left[\frac{20-12}{2(20)-12-11}\right] \times 10 = 40 + \left[\frac{8}{40-23}\right] \times 10$

$= 40 + \frac{8}{17} \times 10 = 40 + 4.71 = 44.71$

∴ **Mode of the given data 44.71**

13.4 Median of Grouped Data

Median: $\boldsymbol{l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h}$

l = lower limit of median class,

n = number of observations

cf = cf of class preceding the median class.

f = frequency of median class.

h = class size

The median is a measure of central tendency which gives the value of the middle-most observation in the data. we first arrange the data values of the observations in ascending order, then, if n is odd, then the meadian is $\left(\frac{n+1}{2}\right)$th observation and if n is an even, then the dedian is the average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2}+1\right)$ th observation.After finding the median class, we use the for calculating the median.

Example 7 : A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained. Find the median height.

Heights (in cm)	No.of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

C.I.	f	cf
< 140	4	4
140 - 145	7	11
145 - 150	18	29
150 - 155	11	40
155 - 160	6	46
160 - 165	5	51

Now, n = 51, ∴ $\frac{n}{2} = 25.5$ It is in the class interval 145 – 150

∴ l (lower limit) = 145, cf = 11. f = 18, h = 5 ;

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 145 + \left[\frac{25.5 - 11}{18}\right] \times 5$

$= 145 + \left[\frac{72.5}{18}\right] = 149.03.$

∴ **median = 149.03**

Example 8 : The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Freequency
0 -100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

C.I.	f	cf
0 -100	2	2
100-200	5	7
200-300	x	7+x
300-400	12	19+x
400-500	17	36+x
500-600	20	56+x
600-700	y	56+x+y
700-800	9	65+x+y
800-900	7	72+x+y
900-1000	4	76+x+y

Here, n = 100 ∴ 76 + x + y = 100
So, x + y = 24 .... (1)

Median is 525, which is lies in the class interval 500 – 600
∴ l = 500, f = 20, cf = 36 + x, h = 100;

Median= $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$525 = 500 + \left[\frac{50 - 36 - x}{20}\right] \times 100$

⇒525 = 500 + [14 – x] x 5
25 = 70 – 5x
⇒ 5x = 70 – 25
⇒ 5x = 45
∴ x = 9
From equation (1) 9 + y = 24
⇒ y = 15

Remarks: There is a empirical relationship between the three measures of central tendency

3 Median = Mode + 2 average

Exercise 13.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption	consumers	cf
65 – 85	4	4
85 – 105	5	9
105 – 125	13	22
125 – 145	20	42
145 – 165	14	56
165 – 185	8	64
185 – 205	4	68

Now, n = 68, ∴ $\frac{n}{2}$ = 34 It is in the class interval 125 - 145.
∴ l = 125, cf = 22, f = 20, h = 20

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

median = $125 + \left[\frac{34 - 22}{20}\right] \times 20$

= $125 + \left[\frac{12}{20}\right] \times 20 = 125 + 12$
= 137units
Therefore median is 137units

Average:

C.I.	f_i	x_i	$d_i = x_i - 135$	$u_i = \frac{x_i - 135}{20}$	$f_i d_i$
65 – 85	4	75	–60	–3	–12
85 – 105	5	95	–40	–2	–10
105 – 125	13	115	-20	-1	-13
125 – 145	20	135	0	0	0
145 – 165	14	155	20	1	14
165 – 185	8	175	40	2	16
185 – 205	4	195	60	3	12
	$\sum f_i$ =68				7

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

= $135 + \frac{7}{68} \times 20$
= 135 + 2.1
= 137.05

Mode: maximum freequency = 20, which lies in the class interval 125 – 145.

Therefore 125-145 is the modal class interval

∴ $l = 125$, h = 20, $f_1 = 20$, $f_0 = 13$, $f_2 = 14$

Mode= $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 125 + \left[\frac{20 - 13}{2(20) - 13 - 14}\right] \times 20$

$= 125 + \left[\frac{7}{40 - 27}\right] \times 20$

$= 125 + \frac{7}{13} \times 20$

$= 125 + 10.77$

= 135.77

∴ Therefore mode of the given data is **135.77**

So, we conclude that three measures are approximately same.

2. If the median of the distribution given below is 28.5, find the values of x and y

Total freequency = 45 + x + y ⇒ 60 = 45 + x + y

⇒x + y = 15 -------------------------- (1)

Now, n = 60,

∴ $\frac{n}{2} = 30$ this is in the class interval 20 – 30

∴ $l = 20$.$cf = 5 + x$, $f = 20$, h = 10

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10$

$8.5 \times 20 = (30 - 5 - x)10$

⇒ 170 =250 – 10x

⇒ 10x = 80 ⇒ x = 8

Substitute x = 8 in equation (1),

⇒8 + y = 15

⇒y = 7

Therefore x = 8 and y = 7

Class interval	Freequency	*cf*
0 - 10	5	5
10 - 20	x	5+x
20 - 30	20	25+x
30 - 40	15	40+x
40 - 50	y	40+x+y
50 - 60	5	45+x+y
Total	60	

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age(in years)	C. freequency
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

C.I.	f	cf
15-20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

Totol frequency = 100

Now, n = 100, ∴ $\frac{n}{2} = 50$ This is in the class interval 35 - 40

So, $l = 35$, cf $= 45$, $f = 33$, h $= 5$

Median $= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 35 + \left[\frac{50 - 45}{33}\right] \times 5$

$= 35 + \left[\frac{5}{33}\right] \times 5$

$= 35 + \frac{25}{33}$

$= 35 + 0.76$

⇒ **Median $=$ 35.76**

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table. Find the median length of the leaves.

(Hint : The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, ..., 171.5 - 180.5.]

Length(in mm)	No.of Leaves
118 - 126	3
127 - 135	5
136 - 144	9
145 - 153	12
154 - 162	5
163 - 171	4
172 - 180	2

C.I.	*f*	*cf*
117.5 – 126.5	3	3
126.5 – 135.5	5	8
135.5 – 144.5	9	17
144.5 – 153.5	12	29
153.5 – 162.5	5	34
162.5 – 171.5	4	38
171.5 – 180.5	2	40

Now, n = 40, $\therefore \frac{n}{2} = 20$ This is in the class interval 144.5 – 153.5

So, $l = 144.5$, cf $= 17$, $f = 12$, h $= 9$

Median $= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 144.5 + \left[\frac{20 - 17}{12}\right] \times 9$

$= 144.5 + \left[\frac{3}{12}\right] \times 9$

$= 144.5 + \frac{27}{12}$

$= 144.5 + 2.25$

=146.75mm

5. The following table gives the distribution of the life time of 400 neon lamps . Find the median life time of a lamp.

Life time in hours	No.of Lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Total freequencies $= 400$

C.I.	*f*	*cf*
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400

Now, n = 400, $\therefore \frac{n}{2} = 200$ this is in the class interval 3000 – 3500

Now, l = 3000, cf = 130, f = 86, h = 500

Median= $l + \left[\frac{\frac{n}{2} - cf}{f}\right]$ x h

Median= 3000 + $\left[\frac{200 - 130}{86}\right]$ x500=3000 + $\left[\frac{70}{86}\right]$ x 500

=3000 + 406.98 = **3406.98**

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as followsDetermine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames

No.of Letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
No.of surnames	6	30	40	16	4	4

Total freequencies = 100

C.I.	f	cf
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100

Now, n = 100, $\therefore \frac{n}{2} = 50$, this is in the C.I. 7 – 10

So, l =7, cf =36, f =40, h =3. Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right]$ x h

= 7 + $\left[\frac{50 - 36}{40}\right]$ x 3 =7 + $\left[\frac{14}{40}\right]$ x 3

=7 + 1.05 = **8.05**

To find the average:

C.I.	f_i	x_i	$u_i = \frac{x_i - 8.5}{3}$	$f_i u_i$
1-4	6	2.5	-2	-12
4-7	30	5.5	-1	-30
7-10	40	8.5	0	0
10-13	16	11.5	1	16
13-16	4	14.5	2	8
16-19	4	17.5	3	12
	$\sum f_i$ =100			$\sum f_i u_i$ =-6

[a = 8.5, h = 3]

By step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h

= 8.5 + $\frac{-6}{100}$ x 3

= 8.5 - 0.18

= **8.32**

To find the mode:

The modal class interval is 7 - 10 $\therefore l = 7$, h = 3, $f_1 = 30$, $f_0 = 30$, $f_2 = 16$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

= $7 + \left[\frac{40 - 30}{2(40) - 30 - 16}\right]$ x 3

= $7 + \left[\frac{10}{80 - 46}\right]$ x 3

= $7 + \frac{10}{34}$ x 3

= 7 + 0.88 = 7.88

$\therefore$ The mode of the given data is **7.88**

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students

Weight in Kgs	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No.of students	2	3	8	6	6	3	2

Total freequencies = 30, Now, n = 30, ∴ $\frac{n}{2} = 15$
which is in the class interval 55 – 60
So, l = 55, cf = 13, f = 6, h = 5

Meadian = $l + \left[\frac{\frac{n}{2} - cf}{f}\right]$ x h

= 55 + $\left[\frac{15 - 13}{6}\right]$ x 5

= 7 + $\left[\frac{2}{6}\right]$ x 5

Median= 55 + 1.67

= 56.67kg

C.I.	f	cf
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30

Summary:

1. The mean for grouped data can be found by :

   Direct Method $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

   Assumed mean method: $\bar{x}$ =a + $\frac{\Sigma f_i d_i}{\Sigma f_i}$

   Step deviation method: $\bar{x}$ = a + $\frac{\Sigma f_i u_i}{\Sigma f_i}$ x h

   with the assumption that the frequency of a class is centred at its mid-point, called its class mark

2. The mode for grouped data can be found by using the formula:

   Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right]$x h

   Where symbols have the meanings

3. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class

4. The median for grouped data is formed by using the formula:.

   meadian = $l + \left[\frac{\frac{n}{2} - cf}{f}\right]$ x h

   Where symbols have the meanings

14 Probabilty

14.2 Probability — A Theoretical Approach

Suppose a coin is tossed at random,the coin can only land in one of two possible ways — either head up or tail up.Suppose we throw a die once. For us, a die will always mean a fair die. They are 1, 2, 3, 4, 5, 6, Each number has the same possibility of showing up.

The experimental or empirical probability P(E) of an event E as

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trails}}$$

The theoretical probability (also called classical probability) of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{No of outcomes favarable to E}}{\text{No.of all possible outcomes of the experiment}}$$

When we speak of a coin, we assume it to be **'fair'**, that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being **'unbiased'**. By the phrase **'random experiment'**, we mean that the coin is allowed to fall freely without any bias or interference

Example 1 : Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Random experiment: Tossing a coin once

S - {H, T} [Here, H – Head T – Tail]

$\Rightarrow n(S) = 2$

Event A - {Getting Head} = {H} $\Rightarrow$ n (A) = 1

P (A) $= \frac{n(A)}{n(S)} = \frac{1}{2}$

Event B - {Getting tail} = {T} $\Rightarrow$ n (B) = 1

P (B) $= \frac{n(B)}{n(S)} = \frac{1}{2}$

Example 2 : A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the(i) Yellow ball (ii) Red ball (iii) Blue ball

S – {Total balls in a bag } $\Rightarrow n(S) = 3$

A – { Krthika picking up yellow ball}

$\Rightarrow n(A) = 1$; $P(A) = \frac{n(A)}{n(S)} = \frac{1}{3}$

B – { Krthika picking up red ball} – n(B) = 1;

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{3}$

C – { Krthika picking up blue ball} – n(C) = 1;

$P(C) = \frac{n(C)}{n(S)} = \frac{1}{3}$

Observe that the sum of the probabilities of all the elementary events of an experiment is 1

Example 3: Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

S – {Throwing a dice once} – {1, 2, 3, 4, 5, 6}

⇒ n(S) = 6

A – {Getting number more than 4} – {5, 6}

⇒ n (A) = 2;

P (A) = $\frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

B – {Getting a number equal or <4} – {1, 2, 3, 4}

⇒ n (B) = 4;

P(B) = $\frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

P (A) = 1 – P ($\bar{A}$) where A is an event and $\bar{A}$ is complement of an event A. 0 ≤ P(E) ≤ 1

Impossible Event

The probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**

Example: There are only six possible outcomes in a single throw of a die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 8, so there is no outcome favourable to 8, i.e., the number of such outcomes is zero. In other words,getting 8 in a single throw of a die, is impossible

Sure Event (Certain event)

The probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a certain event.

Example: Since every face of a die is marked with a number less than 7, it is sure that we will always get a number less than 7 when it is thrown once. So, the number of favourable outcomes is the same as the number of all possible outcomes, which is 6.

Now, let us take an example related to playing cards. Have you seen a deck of playing cards? It consists of 52 cards which are divided into 4 suits of 13 cards each— spades, hearts, diamonds and clubs. Clubs and spades are of black colour, while hearts and diamonds are of red colour. The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, queens and jacks are called face cards

Example 4 : One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will

(i) Be an aace (ii) Not be an ace

(i) S – {Picking a card from a deck of 52}

⇒ n(S) = 52

E – {The picked card is an ace}

⇒ n(E) = 4

P (A) = $\frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

(ii) F – {The card picked is not an ace}

⇒ n (F) = 48

⇒ P(F) = $\frac{n(F)}{n(S)} = \frac{48}{52} = \frac{11}{13}$

or P(F) = P($\bar{E}$) = 1 – p(E) = 1 - $\frac{1}{13} = \frac{11}{13}$

Example 5 : Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

The probability that Savith wins the match = P (A) = 0.62

The probability that Reshma wins the match

P ($\bar{A}$) = 1 – P (A) = 1 – 0.62 = 0.38

Example 6 : Savita and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year)

(i) Favarable days that Savitha and Hamida have different birthdays 365-1 = 364

Probabilty of having different birthdays P (A) $= \frac{364}{365}$

Probabilty of having same birthday P ($\bar{A}$) $= \frac{1}{365}$ [P ($\bar{A}$)= 1 – P (A)]

Example 7 : There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?

Total number of students: n(S) = 40; No. of Girls:n (A) = 25, No. of boys:n(B) = 15

The probability of drawn card with the name of a Girl P(A) $= \frac{n(A)}{n(S)} = \frac{25}{40} = \frac{5}{8}$

The probability of drawn card with the name of a BoyP(B) $= \frac{n(B)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

P(B) = 1 - P(A)
P(B) = 1 - $\frac{5}{8} = \frac{3}{8}$

Example 8 : A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white (ii) blue (iii) red

The number of marbles in a box = n (S) = 9

White – W, Blue – B, Red – R ;

Then, P (W) = $\frac{2}{9}$;

P (B) = $\frac{3}{9}$;

P (R) = $\frac{4}{9}$

Example 9 : Harpreet tosses two different coins simultaneously (say, one is of ` 1 and other of 2). What is the probability that she gets at least one head?

The two different coins are tossed, the outcomes are
S = { HH, HT, TH, TT }
⇒ n(S) = 4
The favorable outcomes to get atleast one head = {HT, TH, TT}

Therefore the probability of getting atleast one head = $\frac{3}{4}$

[Example 10 and 11 are not solved because they are optional]

Example 12: A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?

Total number of shirts = n(S) = 100; The number of good shirts = 88
(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88
Therefore, P (shirt is acceptable to Jimmy) = $\frac{88}{100}$ = 0.88
(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96
So, P (shirt is acceptable to Sujatha) = $\frac{96}{100}$ = 0.96

Example 13: Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12

The total number of outcomes when two dice are thrown at the same time
(1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

⇒ n(S) = 6x6 = 36

(i) A – The sum of two numbers be 8

A – { (2,6), (3,5), (4,4), (5,3), (6,2)}

⇒ n(A) = 5

∴ The probability of getting the sum of two numbers be 8 = $\frac{5}{36}$

(ii) B - The sum of two numbers be 13

⇒ n(B) = 0

∴ The probability of getting the sum of two numbers be 13 = $\frac{0}{36} = 0$

(iii) C -The sum of two numbers be equal or less than 12

∴ The probability of getting the sum of two numbers be equal or less than = $\frac{36}{36} = 1$

Exercise 14.1

1. Complete the following statements

(i) Probability of an event E + Probability of the event 'not E' = ____________

(ii) The probability of an event that cannot happen is ____Such an event is called_____

(iii)The probability of an event that is certain to happen is ____Such an event is called___

(iv)The sum of the probabilities of all the elementary events of an experiment is____

(v) The probability of an event is greater than or equal to __and less than or equal to___

Ansewers: (i) 1 (ii) 0, impossible event (iii) 1, Sure (iv) 1 (v) 0 , 1

2. Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong

(iv) A baby is born. It is a boy or a girl.

Answer

(i) It does not have equally likely outcomes as it depends on various reasons like mechanical problems, fuels etc.

(ii) It does not have equally likely outcomes as it depends on the player how he/she shoots.

(iii) It has equally likely outcomes.

(iv) It has equally likely outcomes.

3. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Yes, tossing of a coin is a fair way of deciding which team should get the ball at the beginning of a football game because it has only two outcomes either head or tail. A coin is always unbiased

4. Which of the following cannot be the probability of an event?

A) $\frac{2}{3}$ B) -1.5 C) 15% D) 0.7 3

(B) -1.5 cannot be the probability of an event because $\mathbf{0 \leq P(A) \leq 1}$

5. If P (E) = 0.05, what is the probability of 'not E'?

The probability of 'not E'=1 – P (E)

= 1 – 0.05

= 0.95

6. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out
(i) an orange flavoured candy? (ii) a lemon flavoured candy?

Answer (i) Since the bag contains only lemon flavoured.
Therefore, No. of orange flavoured candies = 0

Probability of taking out orange flavoured candies = $\frac{0}{1}$ = 0

(ii) The bag only have lemon flavoured candies.

Probability of taking out lemon flavoured candies = $\frac{1}{1}$ = 1

7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Answer: Let E be the event of having the same birthday. P(E) = 0.992
$\Rightarrow$ P(E) + P(not E) = 1
$\Rightarrow$ P(not E) = 1 – P(E)
$\Rightarrow$ 1 - 0.992 = 0.008
The probability that the 2 students have the same birthday is 0.008

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

Total number of balls in a bag = n(S) = 3 + 5 = 8

(i) Number of red balls = n(A) = 3;

Probabilty of drawing red balls $P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

(ii) Probabilty of drawing 'not red ball' $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white? (iii) not green?

Total number of marbles in abox = n(S) = 5 + 8 + 4 = 17

(i) Number of red marbles = n(A) = 5

$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{17}$

(ii) Number of white marbles = n(B) = 8

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{8}{17}$

(iii) Number of green marbles = n(C) = 4

$\Rightarrow P(C) = \frac{n(C)}{n(S)} = \frac{4}{17}$

$\therefore$ Probabilty of 'not green' marbles $P(C^1) = 1 - \frac{n(C)}{n(S)} = 1 - \frac{4}{17} = \frac{13}{17}$

10. A piggy bank contains hundred 50p coins, fifty Rs1 coins, twenty Rs2 coins and ten Rs5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ?(ii) will not be a Rs 5 coin?

Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180
Total number of 50 p coins = n(A) = 100;

Number of Rs 5 coins = n(B) = 10

(i) Probabilty of getting Rs 5 coins $P(A) = \frac{n(A)}{n(S)} = \frac{100}{180} = \frac{5}{9}$

(ii)Probabilty of it will not be a Rs 5 coin $1 - P(B)$

$= 1 - \frac{n(B)}{n(S)} = 1 - \frac{10}{180} = \frac{17}{18}$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 14.4). What is the probability that the fish taken out is a male fish?

Total number of fish in the tank

n(S) = 5+8 = 13

Number of male fish in the tank

n(A) = 5

The probability of taking out the male fish

P(A) = $\frac{n(A)}{n(S)}$

= $\frac{5}{13}$

Fig.14.4

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 14.5), and these are equally likely outcomes. What is the probability that it will point at (i) 8 (ii) an odd number (iii) A number greater than 2 (iv) A number less than 9

Possible number of events = 8

(i) A - Possible chances that an arrow pointing number 8 = 1;

P (A) = $\frac{1}{8}$

(ii) B - Chances of pointing an odd number (1, 3, 5 and 7) = 4;

P (B) = $\frac{1}{2}$

(iii) C - Chances of pointing a number >2 (i,e.3, 4, 5, 6, 7 and 8) = 6;

P (C) = $\frac{6}{8}$= $\frac{4}{4}$

(iv) D - Chances of pointing less than 9 (i.e, 1, 2, 3, 4, 5, 6, 7, 8) = 8

P (D) = $\frac{8}{8}$ = 1

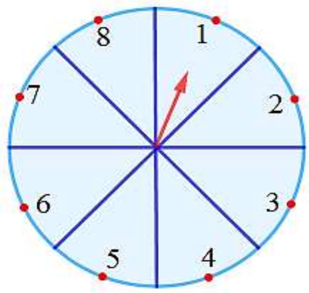


Fig.14.5

13. A die is thrown once. Find the probability of getting (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.

Possible numbers of events on throwing a dice n(S) = 6

(i) Event A-Prime numbers = 2,3 &5; n(A) = 3;

P (A) = $\frac{n(A)}{n(S)}\frac{3}{6} = \frac{1}{2}$

(ii) Event B - Numbers lying between 2 and 6 = 3,4 & 5

⇒ n(B) = 3; P (B)

$=\frac{n(B)}{n(S)}\frac{3}{6} = \frac{1}{2}$

(iii) Event C - Odd numbers = 1,3 & 5

⇒ n (C) = 3;

p (C) = $\frac{3}{6} = \frac{1}{2}$

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

Solution: n (S) = 52

(i) Event A: Numbers of king of red colour ⇒n (A)= 2;

P (A) = $\frac{2}{52} = \frac{1}{26}$

(ii) Event B: Numbers of face cards ⇒ n(B) = 12;

P(B) = $\frac{12}{52}$= $\frac{3}{13}$

(iii)Event C: Numbers of red face cards ⇒ n (C) = 6;

P (C) = $\frac{6}{52}$= $\frac{3}{26}$

(iv) Event D:Numbers of jack of hearts ⇒ n (D) =1;

P (D) = $\frac{1}{52}$

(v) Event E:Numbers of king of spade ⇒ n (E) = 13;

P (E) = $\frac{13}{52}$ = $\frac{1}{4}$

(vi) Event F: Numbers of queen of diamonds ⇒ n (F) = 1;

P (F) = $\frac{1}{52}$

15. Five cards the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Solution: Total numbers of cards n (S) = 5

(i) Numbers of queen = 1; Probability of picking a queen = $\frac{1}{5}$

(ii) When queen is drawn and put aside then total numbers of cards left is 4

(a) Numbers of ace = 1; Probability of picking an ace = $\frac{1}{4}$

(a) Numbers of queen = 0; Probability of picking a queen = $\frac{0}{4}$ = 0

16. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Solution: Numbers of defective pens = 12; Numbers of good pens = 132

Total numbers of pen n (S) = 132 + 12 = 144 pens

Probability of getting a good pen = $\frac{132}{144}$ = $\frac{11}{12}$

17. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

(i) Total numbers of bulbs n (S) = 20; Numbers of defective bulbs = 4

Probability of getting a defective bulb = $\frac{4}{20}$ = $\frac{1}{5}$

(ii) One non defective bulb is drawn in (i) then the total numbers of bulb left is 19

Total numbers of events = 19; Favourable numbers of events = 19 - 4 = 15

Probability that the bulb is not defective = $\frac{15}{19}$

18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5

Total numbers of discs = 90

(i) Event A: {Getting two digits no.} ⇒ n (A) = 81;

P (A) = $\frac{81}{90}$= $\frac{9}{10}$

(ii) Event B: { Getting Perfect square numbers} = 1, 4, 9, 16, 25, 36, 49, 64 and 81

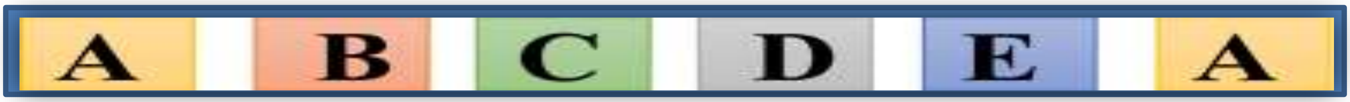
⇒ n (B) = 9;

Probability of getting a perfect square number = $\frac{9}{90}$ = $\frac{1}{10}$

(iii)Event C: { Numbers which are divisible by 5} = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90

$\Rightarrow$n (C) = 18; P (C) = $\frac{18}{90} = \frac{1}{5}$

19. A child has a die whose six faces show the letters as given below: The die is thrown once. What is the probability of getting (i) A? (ii) D?



Total numbers of events = 6

(i) Total numbers of faces having A on it = 2;

Probability of getting A = $\frac{2}{6} = \frac{1}{3}$

(ii) Total numbers of faces having D on it = 1 ;

Probability of getting A = $\frac{1}{6}$

20. Suppose you drop a die at random on the rectangular region shown in Fig. 14.6. What is the probability that it will land inside the circle with diameter 1m?[Not for examination]

Area of the rectangle = (3×2) m^2

= 6m^2

Area of the circle = πr^2

$= \pi\left(\frac{1}{2}\right)^2$

$= \frac{\pi}{4}$ m^2

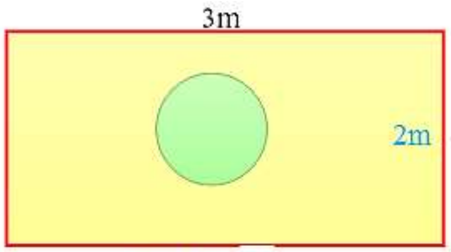


Fig.14.6

Probability that die will land inside the circle $= \frac{\frac{\pi}{4}}{6}$

$= \frac{\pi}{24}$

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that(i) She will buy it? (ii) She will not buy it?

Total numbers of pens = 144; Numbers of defective pens = 20

Numbers of non defective pens = 144 - 20 = 124

(i) Numbers of favourable events = 124 ;

Probability that she will buy it = $\frac{124}{144} = \frac{31}{36}$

(ii) Numbers of favourable events = 20;

Probability that she will not buy it = $\frac{20}{144} = \frac{5}{36}$

22. Refer to Example 13. (i) Complete the following table

Event Sum on two dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii)A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Events that can happen on throwing two dices are (1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

⇒ n(S) = 6x6 = 36

(i) To get sum as 2, possible outcomes = (1,1)

To get sum as 3, possible outcomes = (1,2) and (2,1)

To get sum as 4, possible outcomes = (1,3); (3,1); and (2,2)

To get sum as 5, possible outcomes = (1,4); (4,1); (2,3); and (3,2)

To get sum as 6, possible outcomes = (1,5); (5,1); (2,4); (4,2); and (3,3)

To get sum as 7, possible outcomes = (1,6); (6,1); (5,2); (2,5); (4,3); and (3,4)

To get sum as 8, possible outcomes = (2,6); (6,2); (3,5); (5,3); and (4,4)

To get sum as 9, possible outcomes = (3,6); (6,3); (4,5); and (5,4)

To get sum as 10, possible outcomes = (4,6); (6,4) and (5,5)

To get sum as 11, possible outcomes = (5,6) and (6,5)

To get sum as 12, possible outcomes = (6,6)

Event Sum on two dice	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) No, i don't agree with the argument. It is already justified in (i).

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Events that can happen in tossing 3 coins

={ HHH, HHT, HTH, THH, TTH, HTT, THT, TTT }; Total number of events = 8

Hanif will lose the game if he gets {HHT, HTH, THH, TTH, HTT, THT}

Favourable number of elementary events = 6; Probability of losing the game = $\frac{6}{8}$ = $\frac{3}{4}$

24. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?

[Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]

(i) Total number of possibilities = 6x6 =36

Possible outcomes: (1,1), (1,2), (1,3), (1,4),(1,6), (2,1), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,4), (3,6), (4,1), (4,2), (4,3), (4,4),(4,6), (6,1), (6,2), (6,3), (6,4), (6,6)

The possibility of 5 will not come either time = 25;

Required probability = $\frac{25}{36}$

(ii) Number of events when 5 comes at least once = 11;

Probability = $\frac{11}{36}$

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$

(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$

(i) **The statement is incorrect;** Possible events = (H,H); (H,T); (T,H) and (T,T)

Probability of getting two heads = $\frac{1}{4}$;Probability of getting one of the each = $\frac{2}{4} = \frac{1}{2}$

(ii) **Correct.** The two outcomes considered are equally likely.

Summary:

1. The difference between experimental probability and theoretical probability.
2. The theoretical (classical) probability of an event E, written as P(E), is defined as

   $$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

   where we assume that the outcomes of the experiment are equally likely.
3. The probability of a sure event (or certain event) is 1.
4. The probability of an impossible event is 0
5. The probability of an event E is a number P(E) such that
   $0 \leq P(E) \leq 1$
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is
7. For any event E, $P(E) + P(\overline{E}) = 1$ where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.