YK

Revised 2024–25

Mathematics

ENGLISH

10 Standard

Part –1

Version 1.1

Yakub S Koyyur, GHS Kayarthadka, Belthangady Taluk, D.K.-574216

1 Real Numbers

1.1 The Fundamental Theorem of Arithmetic

The Fundamental Theorem of Arithmetic says that every composite number can be factorised as a product of primes. Actually it says more. It says that given any composite number it can be factorised as a product of prime numbers in a'unique' way, except for the order in which the primes occur. That is, given any composite number there is one and only one way to write it as a product of primes, as long as we are not particular about the order in which the primes occur. So, for example, we regard $2 \times 3 \times 5 \times 7$ as the same as $3 \times 5 \times 7 \times 2$, or any other possible order in which these primes are written.

Theorem: 1.2
(Fundamental Theorem of Arithmetic):
Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime the prime factors occur

Example1: Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution: If the number 4^n, for any n, were to end with the digit zero,
then it would be divisible by 5.

That is, the prime factorisation of 4^n would contain the prime 5.

This is not possible because $4^n = (2)^{2n}$;

so the only prime in the factorisation of 4^n is 2.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 4^n.

So, there is no natural number n for which 4^n ends with the digit zero.

Example2: Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution: $6 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

HCF (6,20) = 2

LCM (6, 20) = 2 x 2 x 3 x 5 = 60

We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

For Any two positive integers a and b,
HCF (a, b) × LCM (a, b) = a × b.

Example 3 : Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Solution: We can write the prime factors of 96 and 404 are as follows

$96 = 2^5 \times 3$; $404 = 2^2 \times 101$

$HCF(96,404) = 2^2 = 4$

$\therefore$ LCM (96, 404) $= \frac{96 \times 404}{4}$

= 9696

Example 8 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

$6 = 2 \times 3$; $72 = 2^3 \times 3^2$; $120 = 2^3 \times 3 \times 5$

∴ HCF (6, 72, 120) $= 2^1 \times 3^1$

$= 2 \times 3 = 6$

∴ LCM (6, 72, 120) $= 2^3 \times 3^2 \times 5^1$

$= 8 \times 9 \times 5 = 360$

Exercise 1.2

1. Express each number as a product of its prime factors:
(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

(i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$
(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$ (iv) 5005
$= 5 \times 7 \times 11 \times 13$
(v) $7429 = 17 \times 19 \times 23$

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers. (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54.

(i) $26 = 2 \times 13$; $91 = 7 \times 13$
HCF = 13;
LCM $= 2 \times 7 \times 13 = 182$
Product of two numbers $= 26 \times 91 = 2366$;
LCM x HCF $= 13 \times 182 = 2366$

∴ LCM x HCF = Product of two numbers

(ii) $510 = 2 \times 3 \times 5 \times 17$; $92 = 2 \times 2 \times 23$
HCF = 2;
LCM $= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
Product of two numbers $= 510 \times 92 = 46920$;
LCM x HCF $= 2 \times 23460 = 46920$

∴ LCM x HCF = Product of two numbers

(iii) $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$; $54 = 2 \times 3 \times 3 \times 3$
HCF $= 2 \times 3 = 6$;
LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$
Product of two numbers $= 336 \times 54 = 18144$;
LCM x HCF $= 6 \times 3024 = 18144$

∴ LCM x HCF = Product of two numbers

3. Find the LCM and HCF of the following integers by applying the prime factorisation method (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

(i) $12 = 2 \times 2 \times 3$; $15 = 3 \times 5$;
$21 = 3 \times 7$
HCF = 3
LCM $= 2 \times 2 \times 3 \times 5 \times 7 = 420$
(ii) $17 = 1 \times 17$
$23 = 1 \times 23$

$29 = 1 \times 29$
HCF = 1
LCM = $1 \times 17 \times 19 \times 23 = 11339$

(iii) $8 = 1 \times 2 \times 2 \times 2$;
$9 = 1 \times 3 \times 3$
$25 = 1 \times 5 \times 5$
HCF = 1
LCM = $1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

LCM x HCF = Product of two numbers

$\therefore$ LCM (306, 657) = $\frac{306 \times 657}{9}$
= 22338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Here, n is a natural number.

If the number 6^n, for any n, were to end with the digit zero,
Then it would be divisible by 5.

That is, the prime factorisation of 4^n would contain the prime 5.
This is not possible because the prime factors of 6 are 2 and 3.
Therefore 5 is not a factor of 6.
$\Rightarrow 6^n = (2 \text{ x } 3)^n$

So, there is no natural number n for which 6^n ends with the digit zero.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers

$7 \times 11 \times 13 + 13 = 13 (7\text{x}11 + 1) = 13(77 + 1) = 13 (78) = 13\text{x}2\text{x}3\text{x}13$
The product of two or more than two prime numbers is a composite number.
Therefore 7 x 11 x 13 + 13 is a composite number.
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5(1008 + 1)$
$= 5(1009)$
The product of two or more than two prime numbers is a composite number.
Therefore $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

To find the time they meet again in the same point, we have to find the LCM of time
$18 = 2 \times 3 \times 3$
$12 = 2 \times 2 \times 3$
LCM $= 2 \times 2 \times 3 \times 3 = 36$
Therefore after 36 minutes they meet again at the starting point.

1.3 Revisiting Irrational Numbers

A number which can not be expressed in the form of $\frac{p}{q}$ is called irrational number.Here, $p, q \in Z, q \neq 0$

Theorem 1.2:

Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Theorm 1.3: $\sqrt{2}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{2}$ is rational.

$\Rightarrow \sqrt{2} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2}q = p$ Squaring on both sides we get,

$(\sqrt{2}q)^2 = p^2$

$\Rightarrow 2q^2 = p^2$ ------- (1)

$\Rightarrow$ 2 divides p^2

$\Rightarrow$ 2, divides p [By theorem]

∴ Let p = 2m,

(1) $\Rightarrow 2q^2 = (2m)^2$

$\Rightarrow q^2 = 2m^2$

$\Rightarrow$2,divides q^2

$\Rightarrow$ 2, divides q [By theorem]

∴ 2 is the common factor for both p and q

This contradicts that there is no common factor of p and q

Therefore our assumption is wrong. So, $\sqrt{2}$ is a an irrational number.

Example 5: Prove that $\sqrt{3}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{3}$ is rational.

$\Rightarrow \sqrt{3} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{3} = \frac{p}{q} \Rightarrow \sqrt{3}q = p$ Squaring on both sides we get,

$(\sqrt{3}q)^2 = p^2$

$\Rightarrow 3q^2 = p^2$ -------- (1)

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p [By theorem]

∴ Let p = 3m,

(1) $\Rightarrow 3q^2 = (3m)^2$

$\Rightarrow q^2 = 3m^2 \Rightarrow$3 divides q^2

$\Rightarrow$ 3 divides q [By theorem]

∴ 3 is the common factor for both p and q , This is not possible.

Therefore our assumption is wrong. So, $\sqrt{3}$ is a an irrational number.

- **The sum or difference of a rational and an irrational number is irrational and**
- **The product and quotient of a non-zero rational and irrational number is irrational.**

Example 6 : Show that $5 - \sqrt{3}$ is irrational

Proof: Assume that $5 - \sqrt{3}$ is a rational number.

$\Rightarrow 5 - \sqrt{3} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3}$

$\Rightarrow \frac{5q-p}{q} = \sqrt{3}$

Here, $\frac{5q-p}{q}$ is a rational number but $\sqrt{3}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $5 - \sqrt{3}$ is an irrational number.

Example 7 : Show that $3\sqrt{2}$ is irrational.

Proof: Assume that $3\sqrt{2}$ is a rational number.

$\Rightarrow 3\sqrt{2} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{2} = \frac{p}{3q}$

Here, $\frac{p}{3q}$ s a rational number but $\sqrt{2}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $3\sqrt{2}$ is an irrational number

Exercise 1.3

1. Prove that $\sqrt{5}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{5}$ is rational.

$\Rightarrow \sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{5} = \frac{p}{q}$

$\Rightarrow \sqrt{5}q = p$, squaring on both sides we get,

$(\sqrt{5}q)^2 = p^2 \Rightarrow 5q^2 = p^2$ --------- (1)

$\Rightarrow$ 5 divides p^2

$\Rightarrow$ 5 divide s p [By theorem]

∴ Let p = 5m,

(1) $\Rightarrow 5q^2 = (3m)^2$

$\Rightarrow q^2 = 5m^2 \Rightarrow 5$ divides q^2

$\Rightarrow$ 5 divides q [By theorem]

∴ 5 is the common factor for both p and q ; this is not possible

Therefore our assumption is wrong. So, $\sqrt{5}$ is a an irrational number.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Proof: Assume that $3 + 2\sqrt{5}$ is a rational number.

$\Rightarrow 3 + 2\sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow 2\sqrt{5} = \frac{p}{q} - 3 \Rightarrow \sqrt{5} = \frac{p-3q}{2q}$

Here, $\frac{p-3q}{2q}$ is a rational number but $\sqrt{5}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $3 + 2\sqrt{5}$ is an irrational number.

3. Prove that the following are irrationals: (i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

(i) Proof: Assume that $\frac{1}{\sqrt{2}}$ is a rational number.

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{p}{q}$

$\Rightarrow \sqrt{2} = \frac{2p}{q}$

Here, $\frac{2p}{q}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

Therefore our assumption is wrong.

$\therefore \frac{1}{\sqrt{2}}$ is an irrational number.

(ii) Proof: Assume that $7\sqrt{5}$ is a rational number.

$7\sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{5} = \frac{p}{7q}$

Here, $\frac{p}{7q}$ is a rational number, but $\sqrt{5}$ is an irrational. This is impossible.

Therefore our assumption is wrong.

$\therefore 7\sqrt{5}$ is an irrational number.

(iii) Proof: Assume that $6 + \sqrt{2}$ is a rational number

$\Rightarrow 6 + \sqrt{2} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{2} = \frac{p}{q} - 6$

$\Rightarrow \sqrt{2} = \frac{p-6q}{2}$

Here, $\frac{p-6q}{2}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

Therefore our assumption is wrong.

$\therefore 6 + \sqrt{2}$ is an irrational number.

Summery:

- **The Fundamental Theorem of Arithmetic :**
  Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
- If p is a prime and p divides a^2, then p divides a, where a is a positive integer.

2 Polynomials

Degree of the polynomial:

p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x)

$4x + 2$	$2y^2 - 3y + 4$	$5x^3 - 4x^2 + 2 - \sqrt{2}$
A polynomial in the variable x of degree 1	A polynomial in the variable y of degree 2	A polynomial in the variable x of degree 3
Linear polynomial	**Quadratic polynomial**	**Cubic polynomial**
Standard form **ax+b**	**Standard form** $ax^2 + bx + c$	**Standard form** $ax^3 + bx^2 + cx + d$

$a \neq 0$, a, b,c are real numbers

What is the value of p(x) = $x^2 - 3x - 4$ whenx = - 1 ?

$p(-1) = (-1)^2 - 3(-1) - 4 = 0$
Similarlly, p(4) = (4)2 - 3(4) - 4 = 0
As p(-1) = 0 and p(4) = 0;
-1 and 4 are called the zeros of the polynomial $x^2 - 3x - 4$

If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

If k is a real number such that p(k) = 0 then k is called the Zeros of the polynomial p(x)

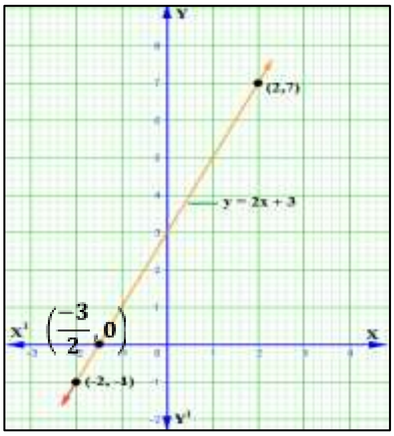
The zero of the lenear equation ax + b is $-\frac{b}{a}$

9.2 Geometrical Meaning of the Zeroes of a Polynomial

(i) Linear Polynomial

Example y = 2x + 3

x	-2	2
y	-1	7



The graph of y = 2x + 3 is a straight line passing through the points (– 2, –1) and (2, 7).

The graph of y = 2x + 3 intersects X – axis at the points $\left(\frac{-3}{2}, 0\right)$, Thus the zero of the polynomial 2x+3 is the x-coordinate of the point where the graph of y= 2x + 3 intersects the axis

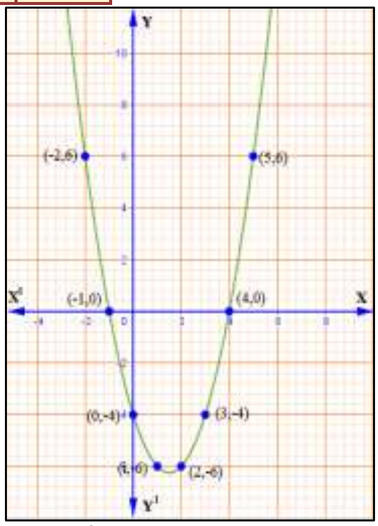
$\therefore \frac{-3}{2}$ is the zero of the linear polynomial y = 2x + 3

∴The linear polynomial $ax + b$ $(a \neq 0)$ has exactly one zero, namely the x-coordinate of the point where the graph of $y = ax + b$ intersects the axis

(i) Quadratic Polynomials:

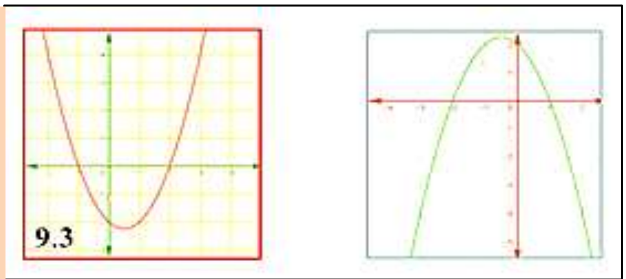
Example: $y = x^2 - 3x - 4$

x	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
y	6	0	-4	-6	-6	-4	0	6

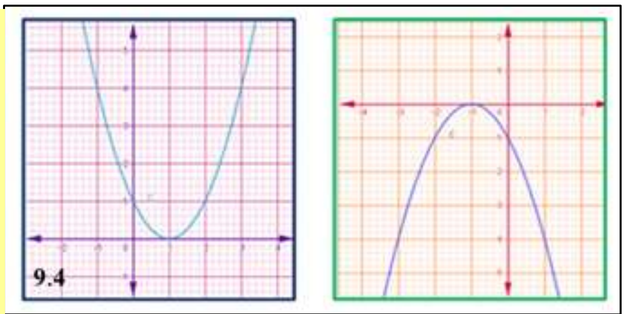


If we locate the points listed above on a graph paper and draw the graph, it will actually look like the one given in Fig. In fact, for any quadratic polynomial $ax^2 + bx + c$, a 0, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards or open downwards depending on whether $a > 0$ or $a < 0$. (Thesecurves are called parabolas.) –1 and 4 are the x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis. Thus, the zeroes of the quadratic polynomial $x^2 - 3x - 4$ are x-coordinates of the points where the graph of $y = x^2 - 3x - 4$ intersects the x-axis. This fact is true for any quadratic polynomial, i.e., the zeroes of a quadratic polynomial $ax^2 + bx + c$, a 0, are precisely the x-coordinates of the points where the parabola representing $y = ax^2 + bx + c$ intersects the x-axis

Case (i) : Here, the graph cuts x-axis at two distinct points A and A[1]. The x-coordinates of A and A[1] are the two zeroes of the quadratic polynomial $x^2 + bx + c$ in this case (see Fig.)

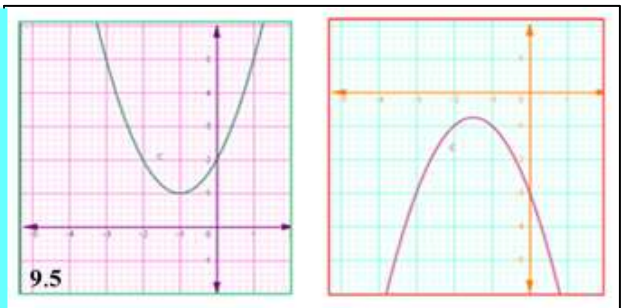


Case (ii) : Here, the graph cuts the x-axis at exactly one point, i.e., at two coincident points. So, the two points A and A[1] of Case (i) coincide here to become one point A (see Fig. 9.4). The x –coordinate of A is the only zero for the quadratic polynomial $ax^2 + bx + c$ in this case.



Case (iii) : Here, the graph is either completely above the x-axis or completely below the x-axis. So, it does not cut the x-axis at any point (see Fig. 9.5).

So, the quadratic polynomial $ax^2 + bx + c$ has no zero in this case.

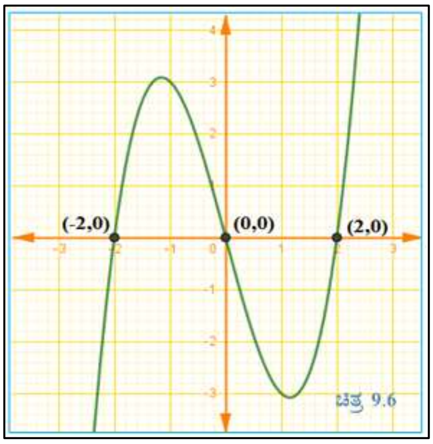


So, you can see geometrically that a quadratic polynomial can have either two distinct zeroes or two equal zeroes (i.e., one zero), or no zero. This also means that a polynomial of degree 2 has atmost two zeroes.

Cubic Polynomials:

Example: $y = x^3 - 4x$

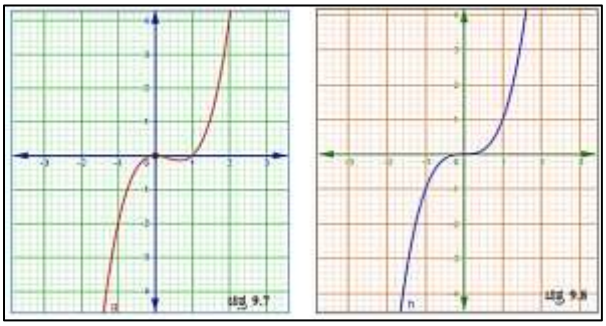
x	-2	-1	0	1	2
y	0	3	0	-3	0



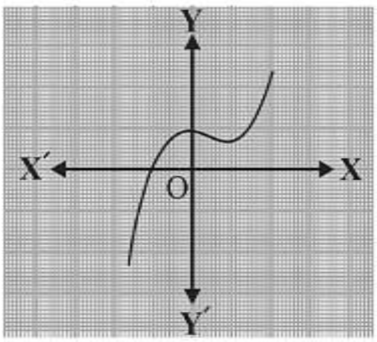
Locating the points of the table on a graph paper and drawing the graph, we see that the graph of $y = x^3 - 4x$ actually looks like the one given in fig

We see from the table above that – 2, 0 and 2 re zeroes of the cubic polynomial $x^3 - 4x$. Observe that – 2, 0 and 2 are, in fact, the x-coordinates of the only points where the graph of $y = x^3 - 4x$ intersects the x - axis. Since the curve meets the x - axis in onlythese 3 points, their x-coordinates are the only zeroes of the polynomial

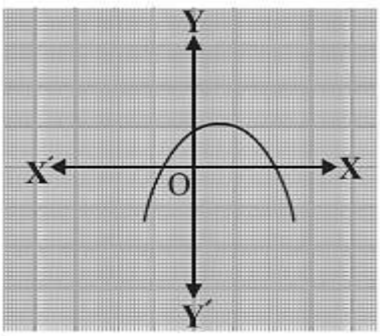
Let us take a few more examples. Consider the cubic polynomials x^3 and $x^3 - x^2$. We draw the graphs of $y = x^3$ and $y = x^3 - x^2$ in Fig. 9.7 and Fig. 9.8 respectively. Note that 0 is the only zero of the polynomial x^3. Also, from Fig. 9.7, you can see that 0 is the x-coordinate of the only point where the graph of $y = x^3$ intersects the x - axis. Similarly, since $x^3 - x^2 = x^2 (x - 1)$, 0 and 1 are the only zeroes of the polynomial $x^3 - x^2$. Also, from Fig. these values are the x-coordinates of the only points where the graph of $y = x^3 - x^2$ intersects the x-axisFrom the examples above, we see that there are at most 3 zeroes for any cubic polynomial. In other words, any polynomial of degree 3 can have at most three zeroes



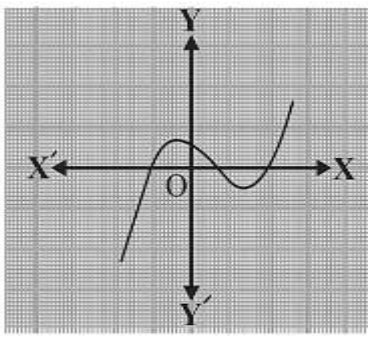
Example 1 : Look at the graphs in Fig. 9.9 given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).



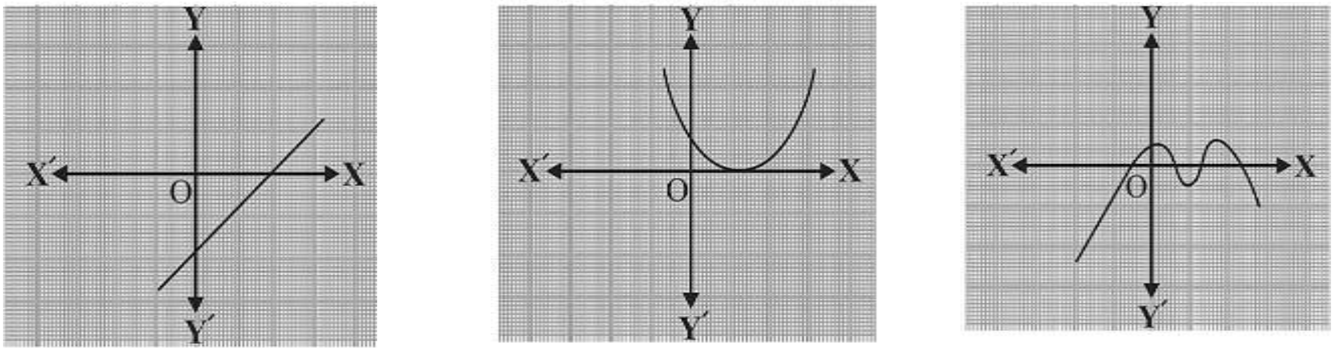
(i)



(ii)



(iii)



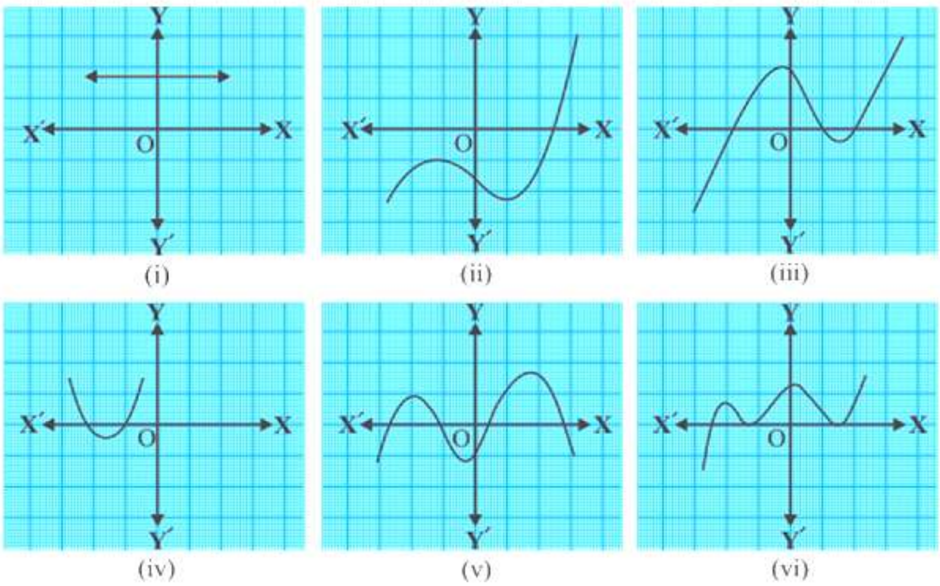
(iv) (v) (vi)

Solution :

(i) The number of zeroes is 1 as the graph intersects the x-axis at one point only.
(ii) The number of zeroes is 2 as the graph intersects the x-axis at two points.
(iii) The number of zeroes is 3 as the graph intersects the x-axis at three points
(iv) The number of zeroes is 1 as the graph intersects the x-axis at one point only.
(v) The number of zeroes is 1. as the graph intersects the x-axis at one point only.
(vi) The number of zeroes is 4. as the graph intersects the x-axis at four points

Exercise 2.1

1. The graphs of y = p(x) are given in Fig 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



Answer

(i) Zeroes 0 v (ii) Zeroes 1 (iii) Zeores 3 (iv) Zeores 2 (v) Zeores 4 (vi) Zeores 3

2.3 Relationship between Zeroes and Coefficients of a Polynomial

α and β are the zeros of the polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$

Sum of Zeros $\alpha + \beta = \frac{-b}{a}$

Product of Zeros $\alpha\beta = \frac{c}{a}$

Example 2: Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients

Solution:

$x^2 + 7x + 10$

$= x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 2)(x + 5)$

$\therefore$ The value of $x^2 + 7x + 10$ is zero when $x = -2$ or $x = -5$

$\therefore$ -2 and -5 are the zeros of $x^2 + 7x + 10$

Sum of the zeros $= (-2) + (-5) = -7$

$= \frac{-7}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (-2) \times (-5) = 10$

$= \frac{10}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients

Solution: $a^2 - b^2 = (a - b)(a + b)$

$\therefore x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

$\therefore \sqrt{3}$ and $-\sqrt{3}$ are the zeros of $x^2 - 3$

Sum of the zeros $= \sqrt{3} + -\sqrt{3} = 0 = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

Solution: Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\therefore \alpha + \beta = -3 = \frac{-b}{a}$ and $\alpha\beta = 2 = \frac{c}{a}$

$\Rightarrow$ If $a = 1$ then $b = 3$ and $c = 2$

$\therefore$ Quadratic polynomial $= x^2 + 3x + 2$

The relation between the zeros and the coefficients of Cubic polynomials:

If α, β, γ are the zeros of the cubic polynomia $ax^3 + bx^2 + cx + d$ then

$\alpha + \beta + \gamma = \frac{-b}{a}$; $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$; $\alpha\beta\gamma = \frac{-d}{a}$

Exercise 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s - 1$ (iii) $6x^2 - 3 - 7x$ (iv) $4u^2 - 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

(i) $x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= (x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

$\Rightarrow x = 4$ and $x = -2$ are the zeros of polynomial $x^2 - 2x - 8$

Sum of the zeros $= 4 + (-2) = 2$

$= \frac{-(-2)}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (4))(-2) = -8$

$= \frac{-8}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(ii) $4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

$\Rightarrow s = \frac{1}{2}$ and $s = \frac{1}{2}$ are the zeros of the polynomial $4s^2 - 4s + 1$

Sum of the zeros $= \frac{1}{2} + \frac{1}{2} = 1$

$= \frac{-(-4)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{1}{2} \text{ x } \frac{1}{2}$

$= \frac{1}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(iii) $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (3x+1)(2x-3)$

$\Rightarrow x = -\frac{1}{3}$ and $x = \frac{3}{2}$ are the zeros of the polynomial $6x^2 - 3 - 7x$

Sum of the zeros $= -\frac{1}{3} + \frac{3}{2} = \frac{-2+9}{6} = \frac{7}{6}$

$= \frac{-(-7)}{6} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= -\frac{1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u$

$= 4u^2 + 8u + 0$

$= 4u(u + 2)$

$\Rightarrow u = 0$ and $u = -2$ are the zeros of the polynomial $4u^2 + 8u$

Sum of the zeros $= 0 + (-2) = -2$

$= \frac{-(8)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (0 \times -2) = 0$

$= \frac{0}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(v) $t^2 - 15$

$= t^2 - 0.t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

$\Rightarrow t = \sqrt{15}$ and $t = -\sqrt{15}$ are the zeros of the polynomial $t^2 - 15$

Sum of the zeros $= \sqrt{15} + (-\sqrt{15}) = 0$

$= \frac{0}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \sqrt{15} \times (-\sqrt{15}) = -15$

$= \frac{-15}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

(vi) $3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

$\Rightarrow x = \frac{4}{3}$ and $x = -1$ are the zeros of the polynomial $3x^2 - x - 4$

Sum of the zeros $= \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3}$

$= \frac{-(-1)}{3} \quad \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{4}{3} + (-1) = -\frac{4}{3}$

$= \frac{-4}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

1. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively. (i) $\frac{1}{4}$, -1 (ii) $\sqrt{2}$, $\frac{1}{3}$ (iii) 0, $\sqrt{5}$ (iv) 1 , 1 (v) $-\frac{1}{4}$, $\frac{1}{4}$ (vi) 4 , 1

(i) $\frac{1}{4}$, -1 -

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = \frac{1}{4} = \frac{-(-1)}{4} = \frac{-b}{a}$ and $\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

$\Rightarrow a = 4, b = -1$ and $c = -4$

$\therefore$ The required polynomial is $4x^2 - x - 4$

(ii) $\sqrt{2}$, $\frac{1}{3}$ -

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = \sqrt{2} = \frac{-(3\sqrt{2})}{3} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$

$\Rightarrow a = 3, b = -3\sqrt{2}$ and $c = 1$

$\therefore$ The required polynomial is $3x^2 - 3\sqrt{2} + 1$

(iii) 0, $\sqrt{5}$

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$ and $\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = 0$ and $c = \sqrt{5}$

$\therefore$ The required polynomial is $x^2 + \sqrt{5}$

(iv) 1, 1

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = -1$ and $c = 1$

$\therefore$ The required polynomial is $x^2 - x + 1$

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β.

$\alpha + \beta = -\frac{1}{4} = \frac{-1}{4} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{4} = \frac{1}{4} = \frac{c}{a}$

$\Rightarrow$ a = 4, b = 1 and c = 1

$\therefore$ The required polynomial is $4x^2 + x + 1$

(vi) 4,1

Let the required polynomial be $ax^2 + bx + c$ and its zeros are α and β

$\alpha + \beta = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow$ a = 1, b = -4 and c = 1

$\therefore$ The required polynomial is $x^2 - 4x + 1$

Summary:

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.

2. A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with a 0

3. The zeroes of a polynomial p(x) are precisely the x - coordinates of the points, where the graph of y = p(x) intersects the x - axis

4. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

5. If α and β are the zeros of polynomial $ax^2 + bx + c$ then, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

6. If α, β and γ are the zeros of $ax^3 + bx^2 + cx + d$ then,

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

2

Pair of Linear Equations in two Variables

Linear equation with one variable:

The algebraic equation of the type $ax + b = 0$ ($a \neq 0$ and b are real numbers, x – variable) is called linear equation of one variable. These type of equations having only one solution.

Example:

$2x + 5 = 0$

$\Rightarrow 2x = -5 \Rightarrow x = \frac{-5}{2}$

3.2 Pair of Linear Equations in Two Variables

An equation which can be put in the form ax + by + c = 0, where a, b and c are real numbers, and both a and b are not zero, is called a linear equation in two variables x and y. A solution of such an equation is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.

In fact, this is true for any linear equation, that is, each solution (x, y) of a linear equation in two variables, ax + by + c = 0, corresponds to a point on the line representing the equation, and vice versa.

2x + 3y=5; x - 2y - 3 = 0 These two linear equations are in the same two variables x and y. Equations like these are called a **pair of linear equations in two variables.**

Example:

$2x + 3y = 5;$

$5x = y$

$x - 2y + 1 = 0;$

$17 = y$

$x + 0.y - 2 = 0;$

$-7x+2y+3=0;$

$9x - 2y + 8 = 0$

The general form for a pair of linear equations in two variables x and y is,

$$a_1x + b_1x + c_1 = 0; \quad a_2x + b_2x + c_2 = 0$$

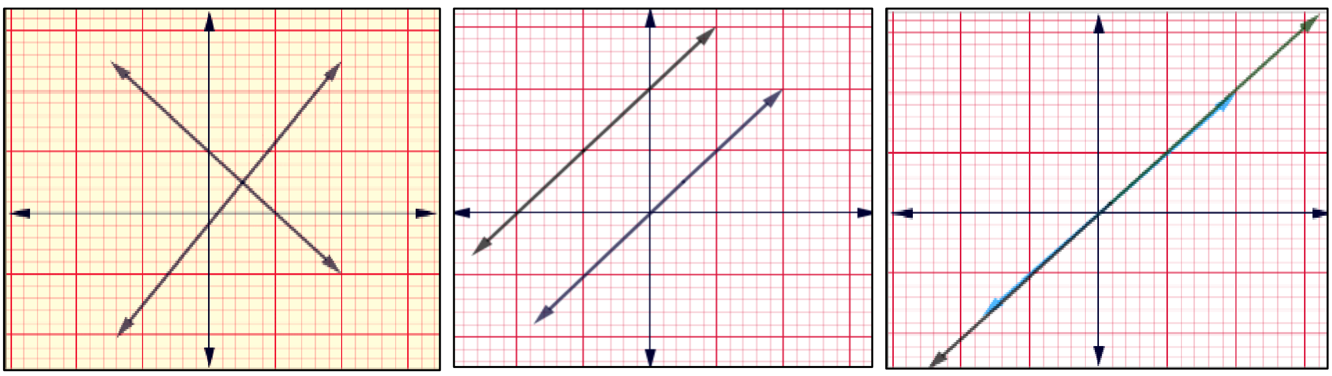
Here, $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers

Two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines will intersect at one point.

(ii) The two lines are parallel.

(iii) The two lines will be coincident.

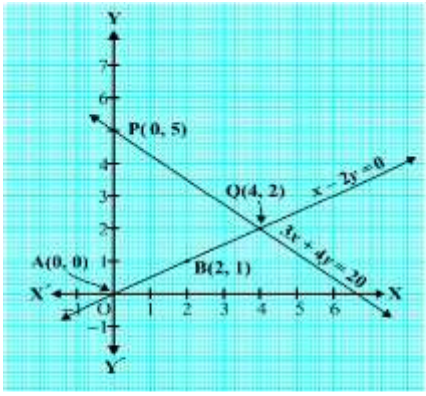


Akhila goes to a fair with Rs 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically).

The pair of equations formedis : $y = \frac{1}{2}x \Rightarrow 2y = x$

$\Rightarrow x - 2y = 0$ -------(1) and $3x + 4y = 20$ ----- (2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.



x	0	2
$y = \frac{x}{2}$	2	1

x	0	4	8
$y = \frac{20-3x}{4}$	5	2	-1

3.2 Graphical Method of Solution of a Pair of Linear Equations

Consistent pair: A pair of linear equations in two variables, which has a solution, is called a consistent pair of linear equations.

Dependent pair: A pair of linear equations which are equivalent has infinitely many distinct common solutions. Such a pair is called a dependent pair of linear equations in two variables.

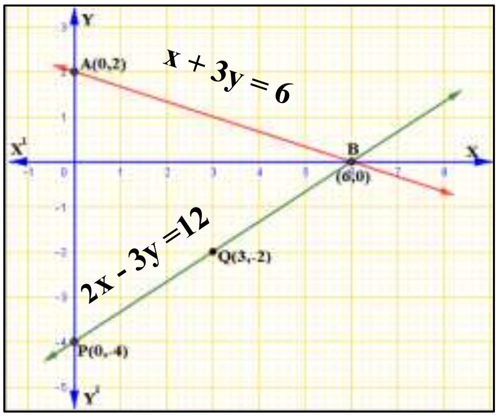
Inconsistent pair A pair of linear equations which has no solution, is called an inconsistent pair of linear equations.

For the linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Comparing the ratios	Representing on graph	Algebraic solution	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting each other	Unique solution	Consistant
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident Lines.	Infinite solutions	Dependent
$\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel Lines	No Solutions	Inconsistent

Example 1 : Check graphically whether the pair of equations
1) $x + 3y = 6$---(1);
2) $2x - 3y = 12$---(2)
is consistent. If so, solve them graphically.

$x + 3y = 6$



$\Rightarrow 3y = 6 - x$
$\Rightarrow y = \frac{6-x}{3}$

x	0	6
y	2	0

$2x - 3y = 12$
$\Rightarrow 3y = 2x - 12$
$\Rightarrow y = \frac{2x-12}{3}$

x	0	3
y	-4	-2

Both lines are intersecting at the point (6,0).
∴ The solution of the equation is $x = 6$ and $y = 0$
⇒ The equations are consistant pair.

Example 2 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:
$\mathbf{5x - 8y + 1 = 0}$ **--------(1) ;**
$\mathbf{3x - \frac{24}{5}y + \frac{3}{5} = 0}$ **--------(2)**

Multiplying equation (2) by $\frac{5}{3}$
$\Rightarrow 3\left(\frac{5}{3}\right)x - \frac{24}{5}\left(\frac{5}{3}\right)y + \frac{3}{5}\left(\frac{5}{3}\right) = 0$
$\Rightarrow 5x - 8x + 1 = 0$

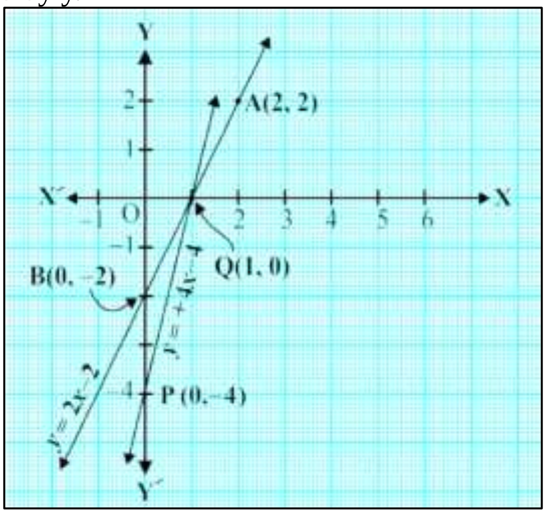
But, this is the same as Equation (1).

Hence the lines represented by Equations (1)and (2) are coincident.

Therefore, Equations (1) and (2) have infinitely many solutions.

Example 3 : Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Let us denote the number of pants by x and the number of skirts by y.
Then the equations are:
$y = 2x - 2$ ---(1);



x	2	1	0
$y = 2x - 2$	2	0	-2

$y = 4x - 4$ ---(2)

x	0	1
$y = 4x - 4$	-4	0

The two lines intersect at the point (1, 0).
So, x = 1, y = 0 is the required solution of the pair of linear equations, i.e., the number of pants she purchased is 1 and she did not buy any skirt.

Exercise 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen

(i) Let the number of girls be x number of boys be y

$x + y = 10$ --- (1) ; $x - y = 4$--- (2)

(1) $x + y = 10$

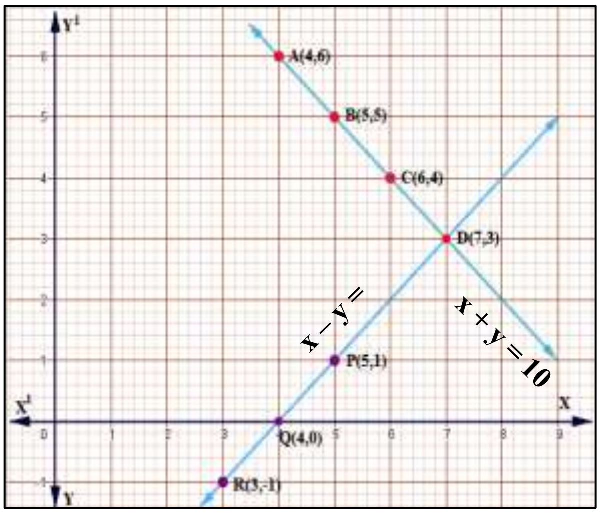
$\Rightarrow y = 10 - x$

x	5	4	6
$y = 10 - x$	5	6	4

(2)$\Rightarrow x - y = 4$

$\Rightarrow y = x - 4$

x	5	4	3
$y = x - 4$	1	0	-1



Two lines are intersecting at the point (7, 3) ∴ The solution is: $x = 7$, $y = 3$

⇒No. of Girs = 7, No. of boys = 3

(ii) Let the cost of a pencil be Rs x, and the cost of a pen is Rs y Then the equation are:

$5x + 7y = 50$ and $7x + 5y = 46$

$5x + 7y = 50$

$\Rightarrow y = \frac{50-5x}{7}$

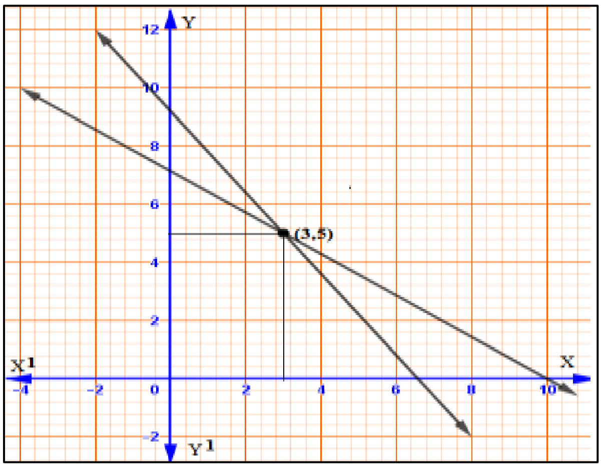
x	3	10	-4
$y = \frac{50-5x}{7}$	5	0	10

$7x + 5y = 46$

$\Rightarrow 5y = 46 - 7x$

$\Rightarrow y = \frac{46-7x}{5}$

x	8	3	-2
$y = \frac{300-4x}{2}$	-2	5	12



Two lines are intersecting at the point(3,5).

∴ The solution is: $x = 3$, $y = 5$

∴ **The cost of pencil = Rs 3; The cost of pen = Rs 5**

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, $\frac{c_1}{c_2}$ find out whether the lines representing the following pair of linear equations intersect at a point, parallel or coincident.

(i) $5x - 4y + 8 = 0$; $7x + 6y - 9 = 0$ **(ii)** $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$ **(iii)** $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

(i) $-4y + 8 = 0$; $7x + 6y - 9 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get

$a_1 = 5$, $b_1 = -4$, $c_1 = 8$ and $a_2 = 7$, $b_2 = 6$, $c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ **The pair of lines intersect at a point and have unique solution.**

(ii) $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get

$a_1 = 9, \ b_1 = 3, \ c_1 = 12$ and $a_2 = 18, \ b_2 = 6, \ c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2},$

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ **The pair of lines are coincident.and have infinite solution.**

(iii) $6x - 3y + 10 = 0$; $2x - y + 9 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get

$a_1 = 6, \ b_1 = -3, \ c_1 = 10$ and $a_2 = 2, \ b_2 = -1, \ c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = 3,$

$\frac{b_1}{b_2} = \frac{-3}{-1} = 3,$

$\frac{c_1}{c_2} = \frac{10}{9}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ **The pair of lines are parallel and have no solution.**

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing the following pair of linear equations are consistent or inconsistent?

(i) $3x + 2y = 5$; $2x - 3y = 7$ (ii) $2x - 3y = 8$; $4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$ (iv) $5x - 3y = 11$; $-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

(i) $3x + 2y = 5$; $2x - 3y = 7$

$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{2}{-3}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

⇒This pair of linear equations are consistent.

(ii) $2x - 3y = 8$; $4x - 6y = 9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{8}{9}$

$\therefore \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

⇒The equations are inconsistent

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7; \quad 9x - 10y = 14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6},$

$\frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{3} \times \frac{1}{-10} = -\frac{1}{6}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

⇒This pair of linear equations are consistent.

(iv) $5x - 3y = 11; \; -10x + 6y = -22$

$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \; \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

⇒This pair of linear equations are consistent and have infinite solutions.

(v) $\frac{4}{3}x + 2y = 8\,; \; 2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

⇒This pair of linear equations are consistent and have infinite solutions.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5; 2x + 2y = 10$ **(ii)** $x - y = 8; 3x - 3y = 16$

(iii) $2x + y - 6 = 0; 4x - 2y - 4 = 0$ **(iv)** $2x - 2y - 2 = 0; 4x - 3y - 5 = 0$

(i) $x + y = 5; \quad 2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2} \; \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ These are coincident lines and have infinite solutions. Hence the pair is consistent.

$x + y = 5$

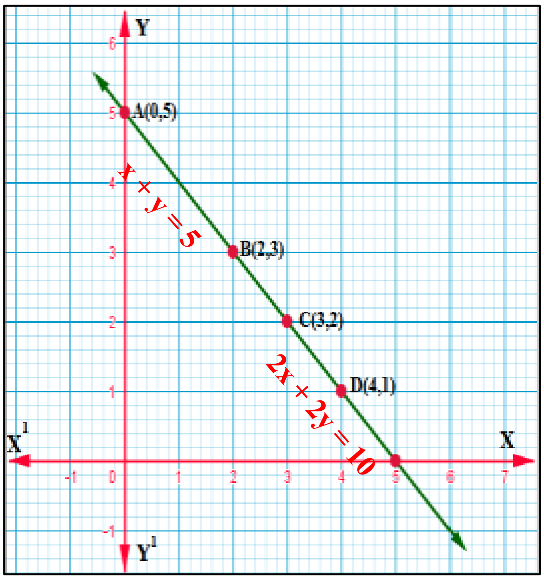
$\Longrightarrow y = 5 - x$

x	2	3	4
$y = 5 - x$	3	2	1

$2x + 2y = 10$

$\Longrightarrow y = \frac{10-2x}{2}$

x	2	3	4
$y = \frac{10-2x}{2}$	3	2	1



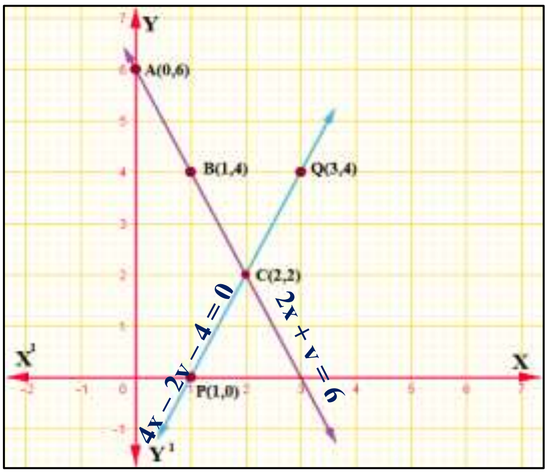
(ii) $x - y = 8; \quad 3x - 3y = 16$

$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$

$\therefore \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ These are parallel lines and have no solutions.Hence the pair is inconsistent.

(iii) $2x + y - 6 = 0$; $4x - 2y - 4 = 0$



$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴these lines are consistent and intersect each other. These lines have unique solution (2.2).

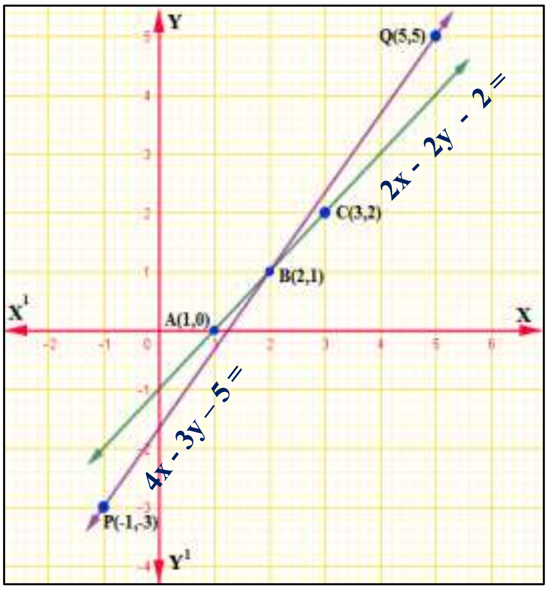
$2x + y - 6 = 0 \Rightarrow y = 6 - 2x$

x	0	1	2
$y = 6 - 2x$	6	4	2

$4x - 2y - 4 = 0 \Rightarrow y = \frac{4x-4}{2}$

x	1	2	3
$y = \frac{4x-4}{2}$	0	2	4

(iv) $2x - 2y - 2 = 0$; $4x - 3y - 5 = 0$



$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-2}{-3} = \frac{2}{3}$

$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ these lines are consistent and intersect each other. These lines have unique solution.

$2x - 2y - 2 = 0$

$\Rightarrow y = \frac{2x-2}{2}$

x	1	2	3
$y = \frac{2x-2}{2}$	0	1	2

$4x - 3y - 5 = 0$

$\Rightarrow y = \frac{4x-5}{3}$

x	2	5	-1
$y == \frac{4x-5}{3}$	1	5	-3

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Let the width of the Garden $= x$ and Length $= y$ but given that $y = x + 4$

x	0	8	16
$y = x + 4$	4	12	20

Half the perimeter: $\frac{2x+2y}{2} = 36$

$\Rightarrow x + y = 36 \Rightarrow y = 36 - x$

x	0	16	36
$y = 36 - x$	36	20	0

∴ These lines are consistent and intersect each other& have unique solution (16,20)

⇒Width = 16m Length = 20m

6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines (ii) Parallel lines (iii) Coinsident lines

(i) Given equation is $2x + 3y - 8 = 0$

If the lines are intersecting then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefor the second equation is $2x + 4y - 6 = 0$

$\frac{a_1}{a_2} = \frac{2}{2} = 1$, $\frac{b_1}{b_2} = \frac{3}{4}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) If the lines are parallel then $\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefor the the second line is $4x + 6y - 8 = 0$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-6}{-8} = \frac{3}{4}$

$\Rightarrow \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

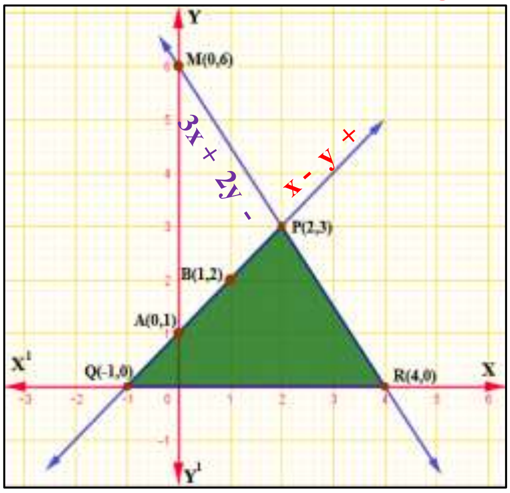
(iii) If the lines are coincident then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therfor the second line is $6x + 9y - 24 = 0$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

$\Rightarrow \frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.



$x - y + 1 = 0$

$\Longrightarrow y = x + 1$

x	0	1	2
$y = x + 1$	1	2	3

$3x + 2y - 12 = 0$

$\Longrightarrow y = \frac{12-3x}{2}$

x	0	2	4
$y = \frac{12-3x}{2}$	6	3	0

The coordinates of the vertices of the triangle are $(2,3)$, $(-1,0)$ $(4,0)$

3.3 Algebraic Methods of Solving a Pair of Linear Equations

In the previous section, we discussed how to solve a pair of linear equations graphically. The graphical method is not convenient in cases when the point representing the solution of the linear equations has non-integral coordinates

3.3.1 Substitution Method:

We have substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why the method is known as the substitution method.

Example 4: Solve the following pair of equations by substitution method:.

$7x - 15y = 2$ -----------(1); $x + 2y = 3$ ----------(2)

Equation (2) $\Longrightarrow x + 2y = 3$

$\Longrightarrow x = 3 - 2y$-----------(3)

Substitute the value of x in equation (1) we get,

$7(3 - 2y) - 15y = 2$
$\Rightarrow 21 - 14y - 15y = 2$
$-29y = 2 - 21$
$\Rightarrow y = \frac{-19}{-29} = \frac{19}{29}$
Substitute $y = \frac{19}{29}$ in equation (3),
$x = 3 - 2\left(\frac{19}{29}\right) = 3 - \frac{38}{29} = \frac{87-38}{29} = \frac{49}{29}$
∴The solution is, $x = \frac{49}{29}$, $y = \frac{19}{29}$

Example 5: Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then.Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Let the present age of Aftab be x Years, The present age of his daughter be y' years
The age of Aftab before 7 Years $= x - 7$ Years
The age of his daughter before 7 years $= y - 7$ yeares
$x - 7 = 7(y - 7)$
$\Longrightarrow x - 7y + 42 = 0$-----(1)
After 3 years, his age $= x + 3$ years and his daughter's age = y +3 years
$x + 3 = 3(y + 3)$
$\Longrightarrow x - 3y = 6$ ------- (2)
Equation (2) $\Longrightarrow x = 3y + 6$ --------------(3)
Substituting the value of x in (1) we get,
$3y + 6 - 7y + 42 = 0$
$\Rightarrow -4y + 48 = 0$
$\Rightarrow 4y = 48$
$\Rightarrow y = 12$
Substituting the value of y in equation we get,
$x = 3(12) + 6 = 36 + 6 = 42$
Therfor the age of Aftab and his daughter is 42 and 12 respectively.

Example 6 : The cost of 2 pencils and3 erasers is Rs 9 and the cost of 4 pencils and 6 erasers is Rs 18. Find the cost of each pencil and each eraser.

Let the cost of pencil be Rs x and the cost of rubber be Rs y , the the equations are
$2x + 3y = 9$ ---------(1)
$4x + 6y = 18$--------(2)
Equation (1) $\Longrightarrow 2x = 9 - 3y$
$\Longrightarrow x = \frac{9-3y}{2}$-----(3)
Substituting x in equation (2) we get,
$4\left(\frac{9-3y}{2}\right) + 6y = 18$
$\Rightarrow 18 - 6y + 6y = 18$
$\Rightarrow 18 = 18$
This statement is true for all values of y. However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x. This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

Example:7 Two rails are represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0 Represent this situation geometrically. Will the rails cross each other?

$x + 2y - 4 = 0$--------(1);

$2x + 4y - 12 = 0$------(2)

Equation (1) $\Longrightarrow x = 4 - 2y$ -----(3)

Substituting x in equation (2) we get,

$2(4 - 2y) + 4y - 12 = 0$
$\Rightarrow 8 - 4y + 4y - 12 = 0$
$\Rightarrow 8 - 12 = 0 \Rightarrow -4 = 0$

which is a false statement. Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

Exercise 3.2

1. Solve the following pair of linear equations by substitution method.

(i) $x + y = 14$; $x - y = 4$ (ii) $x + y = 14$; $x - y = 4$

(iii) $s - t = 3$; $\frac{s}{3} + \frac{t}{2} = 6$ (iv) $0.2x + 0.3y = 1.3$; $0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$; $\sqrt{3}x - \sqrt{8}y = 0$ (vi) $\frac{3x}{2} - \frac{5y}{2} = -2$; $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

(i) $x + y = 14$ ----------(1)

$x - y = 4$ ----------(2)

Equation (1) $\Longrightarrow x = 14 - y$ ----(3)

Substituting x in equation (2) we get,

$14 - y - y = 4$
$\Rightarrow 14 - 2y = 4$
$\Rightarrow -2y = 4 - 14$
$\Rightarrow -2y = -10$
$\Rightarrow y = \frac{-10}{-2} = 5$

Substituting $y = 5$ in equation (3)

$x = 14 - y = 14 - 5$
$\Rightarrow x = 9$
$\therefore x = 9,\ y = 5$

(ii) $s - t = 3$--- (1)

$\frac{s}{3} + \frac{t}{2} = 6$ --- (2)

Equation (1) $\Longrightarrow$ s = 3 + t ---(3)

Substituting s in equation (2) we get,

$\frac{3+t}{3} + \frac{t}{2} = 6$
$\Rightarrow \frac{6+2t+3t}{6} = 6$
$\Rightarrow 6 + 5t = 36$
$\Rightarrow 5t = 36 - 6$
$\Rightarrow t = \frac{30}{5}$

Substituting $t = 6$ in equation (3)

$s = 3 + t$
$\Rightarrow s = 3 + 6$
$\Rightarrow s = 9$
$\therefore s = 9,\ t = 6$

(iii) $3x - y = 3$ ------- (1)

$9x - 3y = 9$ -----(2)

Equation (1) $\Longrightarrow y = 3x - 3$ Substituting y in equation (2) we get,

$9x - 3(3x - 3) = 9$

$9x - 9x + 9 = 9$

$\Rightarrow 9 = 9$

This statement is true for all values of y. However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x. This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

(iv) $0.2x + 0.3y = 1.3$ ------(1)

$0.4x + 0.5y = 2.3$ -------(2)

$(1) \times 10 \Rightarrow 2x + 3y = 13$-------(3)

$(2) \times 10 \Rightarrow 4x + 5y = 23$-------(4)

Equation (3) $\Longrightarrow 2x = 13 - 3y$

$\Longrightarrow x = \frac{13-3y}{2}$ ---------(5)

Substituting x in equation (4) we get,

$4\left(\frac{13-3y}{2}\right) + 5y = 23$

$26 - 6y + 5y = 23 \Rightarrow 26 - 23 = y$

$\Rightarrow$y= 3;

Substituting $y = 3$ in equation (5)

$x = \frac{13-3(3)}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$

$\therefore x = 2, \quad y = 3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$ -----(1)

$\sqrt{3}x - \sqrt{8}y = 0$ ---------(2)

Equation (1)$\Longrightarrow \sqrt{2}x = -\sqrt{3}y$

$\Longrightarrow x = -\frac{\sqrt{3}\,y}{\sqrt{2}}$------(3)

Substituting $'x'$ in equation (2) we get,

$\sqrt{3}\left(-\frac{\sqrt{3}\,y}{\sqrt{2}}\right) - \sqrt{8}y = 0$

$\Rightarrow -\frac{3y}{\sqrt{2}} - \sqrt{4 \times 2}\,y = 0$

$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}\,y = 0$

$\Rightarrow y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

$y = 0$, Substituting $y = 0$ in equation (3) ,

$x = -\frac{\sqrt{3}\,(0)}{\sqrt{2}} = 0$

$\therefore x = 0, \quad y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{2} = -2$ -------(1)

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ ----------(2)

Equation (1) $\times 2 \Longrightarrow 3x - 5y = -4$----(3)

Equation $(2)\times 6 \Longrightarrow 2x+3y=13$-----(4)

$\text{Eq}^n(3) \Longrightarrow 3x=5y-4 \Longrightarrow x=\frac{5y-4}{3}$ ---(5)

Substituting x in equation (4) we get,

$2\left(\frac{5y-4}{3}\right)+3y=13$

$\Rightarrow \frac{10y-8+9y}{3}=13$

$19y-8=39$

$\Rightarrow 19y=39+8$

$\Rightarrow 19y=47 \Rightarrow y=\frac{47}{19}$

Substituting $y=\frac{47}{19}$ in eq^n (5)

$x=\frac{5\left(\frac{47}{19}\right)-4}{3}=\frac{235-76}{19}\times\frac{1}{3}$

$\Rightarrow x=\frac{159}{19}\times\frac{1}{3}\Rightarrow x=\frac{53}{19}$

$\therefore x=\frac{53}{19}; y=\frac{47}{19}$

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

$2x+3y=11$ ----(1); $2x-4y=-24$ -------(2)

Equation (2) $\Longrightarrow 2x=4y-24$

$\Longrightarrow x=2y-12$ -----(3)

Substituting x in equation (1)

we get, $2(2y-12)+3y=11$

$4y-24+3y=11$

$\Rightarrow 7y=11+24$

$\Rightarrow 7y=35$

$\Rightarrow y=5$

substituting $y=5$ in equation (3)

$x=2x5-12=10-12=-2$

$\therefore x=-2,\quad y=5$

$y=mx+3$

$\Rightarrow 5=m(-2)+3$

$\Rightarrow 5-3=-2m$

$\Rightarrow -2m=2 \Rightarrow m=\frac{2}{-2}$

$\Rightarrow \boldsymbol{m=-1}$

3. Form the pair of linear equations for the following problems and find their solution by substitution method

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Let the first number be x, the second number be y, then $y>x$.

By the given condition the equations are

$y-x=26$ ---(1)

$y=3x$ ------(2)

Substituting the value of y in equation (1) we get,

$3x-x=26$

$\Rightarrow 2x=26$

$x=13$, Substitute $x=13$ in equation (2)

$y=3(13)=39$

$\therefore \boldsymbol{x=13},\quad \boldsymbol{y=39}$

(ii)The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Let the larger angle be , the smaller angle be y. If the angles are supplementary then sum of two angles is 180^0 , By the given conditions the equations are,

$x + y = 180^0$ ----(1); $x = y + 18^0$ -------(2)

$y + 18^0 + y = 180^0$
$\Rightarrow 2y = 162^0$
$\Rightarrow y = 81^0$
Substitute $y = 81^0$ in equation (2)
$x = 81^0 + 18^0 = 99^0$
$\therefore$ $\boldsymbol{x = 99^0}$, $\boldsymbol{y = 81^0}$

(ii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Let the cost of a bat = Rs x, the cost of a ball = Rs y. the equations are

$7x + 6y = 3800$ -----(1); $3x + 5y = 1750$ -----(2)

Equation (1) $\Longrightarrow 7x = 3800 - 6y$

$\Longrightarrow x = \frac{3800-6y}{7}$ ------(3)

Substituting the value of x in equation (2)we get,

$3\left(\frac{3800-6y}{7}\right) + 5y = 1750$
$\Rightarrow \frac{11400-18y+35y}{7} = 1750$
$\Rightarrow 11400 + 17y = 12250$
$\Rightarrow 17y = 12250 - 11400$
$\Rightarrow 17y = 850$
$\Rightarrow y = \frac{850}{17} = 50$;
Substitute $y = 50$ in equation (3) we get,
$x = \frac{3800-6(50)}{7}$
$= \frac{3800-300}{7}$
$= \frac{3500}{7}$
$= 500$
$\therefore$ The cost of a bat = Rs 500 , The cost of a ball = Rs 50

(ii) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25km?

Let the fixed charges be Rs x, the charges/km be Rs y. Then the equations are,
$x + 10y = 105$ ------(1); $x + 15y = 155$ -------(2)

Equation (1) $\Longrightarrow x = 105 - 10y$-------(3) Substituting the value of x in equation (2)we get

$105 - 10y + 15y = 155$
$\Rightarrow 105 + 5y = 155$
$\Rightarrow 5y = 155 - 105$
$\Rightarrow y = \frac{50}{5} = 10$
Substitute $y = 10$ in equation (3) we get, $x = 105 - 10(10) = 105 - 100 = 5$
$\therefore$ the fixed charges is Rs 5 and charges per km is Rs 10
The total charges to travel 25km is $x + 25y = 5 + 25(10) = 5 + 250 =$ Rs 255

(iii) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$ Find the fraction.

Let the fraction be $\frac{x}{y}$. By the given condition,

$$\frac{x+2}{y+2} = \frac{9}{11}$$
$\Longrightarrow 11x + 22 = 9y + 18$
$\Longrightarrow 11x - 9y = 18 - 22$
$\Longrightarrow 11x - 9y = -4$---(1)
$\frac{x+3}{y+3} = \frac{5}{6}$
$\Longrightarrow 6x + 18 = 5y + 15$
$\Longrightarrow 6x - 5y = 15 - 18$
$\Longrightarrow 6x - 5y = -3$---(2)

Eq^n (1) $\Longrightarrow 11x = -4 + 9y$
$\Longrightarrow x = \frac{-4+9y}{11}$ ---(3)

Substituting the value of x in equation (2)we get

$6\left(\frac{-4+9y}{11}\right) - 5y = -3$
$\Rightarrow \frac{-24+54y-55y}{11} = -3$
$\Rightarrow -24 - y = -33$
$\Rightarrow -y = -33 + 24$
$\Rightarrow -y = -9$

Subtitute $y = 9$ in eq^n (3)

$x = \frac{-4+9(9)}{11}$
$= \frac{-4+81}{11} = \frac{77}{11} = 7$

The fraction $\frac{x}{y} = \frac{7}{9}$

(iv) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Let age of Jocob $= x$, and age of his son $= y$ By the given condition the equations are

After 5 years $x + 5 = 3(y + 5)$
$\Longrightarrow x + 5 = 3y + 15$
$\Longrightarrow x - 3y = 10$ ---(1)

Before 5 years $x - 5 = 7(y - 5)$
$\Longrightarrow x - 5 = 7y - 35$
$\Longrightarrow x - 7y = -30$---(2)

Equation (1) $\Longrightarrow x = 10 + 3y$----- (3)

Substituting the value of x in eq^n (2) we get,

$10 + 3y - 7y = -30$
$\Rightarrow 10 - 4y = -30$
$\Rightarrow -4y = -30 - 10$
$\Rightarrow -4y = -40$
$\Rightarrow y = \frac{-40}{-4} = 10$

Substitute $y = 10$ in equation (1),

$x = 10 + 3(10) = 10 + 30 = 40$

$\therefore$ Age of Jocob $= 40$ and age of his son $= 10$

3.3.2 Elimination Method

Example 8 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ` 2000 per month, find their monthly incomes

Solution: Let the incomes of two persons be Rs 9x and Rs 7x respectively.The expenditure be Rs 4y and Rs 3y, then we get the equations,

$9x – 4y = 2000$ ---------- (1); $7x – 3y = 2000$ --------- (2)

Multiply Equation (1) by 3 and Equation (2) by 4 to make the coefficients of y equal.

$9x – 4y = 2000$ ------(1) x 3

$7x – 3y = 2000$ ------(2) x 4

$27x – 12y = 6000$	----------------- (3)
$28x – 12y = 8000$	----------------- (4)
$-x = -2000$	

⇒ x = 2000

Substitute x = 2000 in equation (1), we get

$9(2000) – 4y = 2000$

$\Rightarrow 18000 – 2000 = 4y$

$\Rightarrow 4y = 16000 \Rightarrow y = 4000$

∴ **The monthly incomes of two persons are = Rs 18000 and Rs 14000**

The method used in solving the example above is called the elimination method, because we eliminate one variable first, to get a linear equation in one variable. In the example above, we eliminated y. We could also have eliminated x. Try doing it that way.

Example 12 : Use elimination method to find all possible solutions of the following pair of linear equations

$2x + 3y = 8$ -------(1); $4x + 6y = 7$ --------- (2)

Multiply Equation (1) by 2 to make the coefficients of x equal.

$2x + 3y = 8$ ------(1) x 2

$4x + 6y = 16$ ------(3)

(3) $\Rightarrow 4x + 6y = 16$
(2) $\Rightarrow 4x + 6y = 7$
0 = 9

Substracting (2) from (1) , we get 0 = 9, which is a false statement.

Therefore, the pair of equations has no solution

Example 13 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Let the two digits number = $10x + y$

Number after reversing the digits = $10y + x$

$\therefore 10x + y + 10y + x = 66$

$\Rightarrow 11x + 11y = 66$

$\Rightarrow x + y = 6$ ------(1)

We are also given that the digits differ by 2,

$\therefore x – y = 2$ -------(2)

substract (2) from (1)

(1) $\Rightarrow x + y = 6$
(2) $\Rightarrow x – y = 2$

$2y = 4 \Rightarrow y = 2$

Substitute y = 2 in equation (1)
$x + 2 = 6 \Rightarrow x = 4$
∴ he number is $10x + y = 10 \times 4 + 2 = 42$
⇒Two numbers are 42 and 24

Exercise 3.4

1.Solve the following pair of linear equations by the elimination method and the substitution method :
(i) $x + y = 5$ and $2x - 3y = 4$ (ii) $3x + 4y = 10$ and $2x - 2y = 2$
(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ (iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Solution: (i) $x + y = 5$ and $2x - 3y = 4$
Eliminating method: $x + y = 5$ ------ (1); $2x - 3y = 4$ -------(2)
Multiply Eq^n (1) by 2 to make the coefficients of x equal We get, $2x + 2y = 10$ --- (3) Substracting (2) from (1),

$2x + 2y = 10$	(3)
$2x - 3y = 4$	(2)
$5y = 6 \Rightarrow y = \frac{6}{5}$	

Substitute $y = \frac{6}{5}$ in equation (1) we get,
$x + \frac{6}{5} = 5 \Rightarrow 5x + 6 = 25$
$\Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$
$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

Substituting Method:
$x + y = 5$ --- (1); $2x - 3y = 4$ ---- (2)
(1) ⇒ $y = 5 - x$; Substitute $y = 5 - x$ in (2)
$\Rightarrow 2x - 3(5 - x) = 4$ ------(3)
$\Rightarrow 2x - 15 + 3x = 4$
$\Rightarrow 5x = 19$
$\Rightarrow x = \frac{19}{5}$
Substitute $x = \frac{19}{5}$ in (1)
$\Rightarrow \frac{19}{5} + y = 5$
$\Rightarrow 19 + 5y = 25$
$\Rightarrow 5y = 25 - 19$
$\Rightarrow y = \frac{6}{5}$
$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$
Eliminating Method: $3x + 4y = 10$ --------(1); $2x - 2y = 2$ ------(2)
Multiply Equation (2) by 2
to make the coefficients of y equal.
$2x - 2y = 2$ ---(2) x 2
$\Rightarrow 4x - 4y = 4$ --- (3)
Adding equation (1) and (3)
(1) ⇒ $3x + 4y = 10$

(3) ⇒	$4x - 4y = 4$
(1) +(3)⇒	$7x = 14$

⇒ $x = 2$
Substitute $x = 2$ in (1)
$3(2) + 4y = 10 \Rightarrow 6 + 4y = 10$
$\Rightarrow 4y = 10 - 6 \Rightarrow 4y = 4 \Rightarrow y = 1$
$\therefore x = 2, y = 1$

Substituting Method:
$3x - 5y - 4 = 0 \Rightarrow 3x - 5y = 4$ -----(1)
$9x = 2y + 7 \Rightarrow 9x - 2y = 7$ ------ (2)
(1) $\Rightarrow -5y = 4 - 3x \Rightarrow 5y = 3x - 4$
$\Rightarrow y = \frac{3x-4}{5}$ -------(3)
Substitute $y = \frac{3x-4}{5}$ in (2)
$9x - 2\left(\frac{3x-4}{5}\right) = 7 \Rightarrow 9x - \left(\frac{6x-8}{5}\right) = 7$
$\Rightarrow 45x - 6x + 8 = 35 \Rightarrow 39x = 27$
$\Rightarrow x = \frac{27}{39} = \frac{9}{13}$; Substitute $x = \frac{9}{13}$ in (1),
$3\left(\frac{9}{13}\right) - 5y = 4 \Rightarrow 27 - 65y = 52$
$\Rightarrow -65y = 52 - 27 \Rightarrow y = -\frac{25}{65}$
$\Rightarrow y = -\frac{5}{13}$
$\therefore x = \frac{9}{13}$ **and** $y = -\frac{5}{13}$

(iii) 3x - 5y - 4 = 0 and 9x = 2y + 7
liminating method: $3x - 5y - 4 = 0 \Rightarrow 3x - 5y = 4$ --- (1);
$9x = 2y + 7 \Rightarrow 9x - 2y = 7$ --- (2)
Multiply Equation (1) by 3
to make the coefficients of x equal.
$9x - 15y = 12$ -------(3)
Substracting (2) from (3)

(3) ⇒	$9x - 15y = 12$
(2) ⇒	$9x - 2y = 7$
(3) – (2) ⇒	$-13y = 5$

$-13y = 5 \Rightarrow y = -\frac{5}{13}$; Substitute $y = -\frac{5}{13}$ in (1)
$3x - 5\left(-\frac{5}{13}\right) = 4$
$\Rightarrow 3x + \frac{25}{13} = 4$
$\Rightarrow 39x + 25 = 52$
$\Rightarrow 39x = 27$
$\Rightarrow x = \frac{27}{39} = \frac{9}{13}$
$\therefore x = \frac{9}{13}$ and $y = -\frac{5}{13}$

Substituting Method:
$3x + 4y = 10$ --------(1)
$2x - 2y = 2$ ----------(2)
(2) $\Rightarrow -2y = -2x + 2$

⇒ y = x - 1
Substitute y = x – 1in equation (1)
3x + 4(x – 1) = 10
⇒ 3x + 4x - 4 = 10
⇒ 7x = 10 + 4⇒ 7x = 14 ⇒x = 2
Substitute x = 2 in (1)
2(2) – 2y = 2
⇒ 4 - 2y = 2⇒ -2y = 2 – 4
⇒ -2y = -2 ⇒ y = 1
∴ x = 2, y = 1

(i) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Eliminating Method: $\frac{x}{2} + \frac{2y}{3} = -1$ ⇒ 3x + 4y = -6 --- (1);

$x - \frac{y}{3} = 3$ ⇒ 3x - y = 9 --- (2)

Substracting (1) from (2)

(1) ⇒	3x + 4y =	-6
(2) ⇒	3x - y =	9
(2) - (1)⇒	+5y = - 15 ⇒ y = - 3	

Substitute y = - 3 in (1)
3x + 4(- 3) = -6
⇒3x - 12 = -6
⇒3x = 6
⇒x = 2
∴ $x = 2$ and $y = -3$

Substituting Method:

3x + 4y = -6 --- (1); 3x - y = 9 --- (2)
(2) ⇒ -y = 9 – 3x ⇒ y = 3x – 9--- (3)
Substitute y = 3x – 9 in (1)
3x + 4(3x – 9) = -6
⇒3x + 12x - 36 = -6
⇒ 15x = 30 ⇒ x = 2
Substitute x = 2 in (3)
y = 3(2) – 9 ⇒ y = 6 – 9 ⇒ y = -3
∴ $x = 2$ and $y = -3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Let the fraction = $\frac{x}{y}$ According to qn , $\frac{x+1}{y-1} = 1$
⇒ x + 1 = y – 1
⇒ x – y = -2 ----------(1)
and $\frac{x}{y+1} = \frac{1}{2}$
⇒ 2x = y + 1

⇒ 2x – y = 1 ----------(2)

Substract (1) from (2)

(1) ⇒	x – y = -2	
(2) ⇒	2x – y = 1	
(2)-(1)⇒	**-x**	**= -3** ⇒ x = 3

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Let the age of Nuri = *x* and that of Sonu = y .According to question

(x - 5) = 3(y - 5)

⇒ x - 3y = -10 -----(1) and

(x + 10y) = 2(y + 10)

⇒x - 2y = 10----- (2)

Substract (1) from (2)

(1)⇒	x - 3y	= -10
(2)⇒	x - 2y	= 10
(2)-(1)⇒	- y	= -20⇒*y* = 20

Substitute *y* = 20 in (1)

x - 60 = -10 ⇒ *x* = 50

∴ The age of Nuri = 50 years and

the age of Sonu = 20years

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Let the two digits number = xy ,

According to question,

x + y = 9 ---------- (1)

2(10y + x) = 9(10x + y)

20y + 2x = 90x + 9y

88x – 11y = 0 ⇒ 8x – y = 0 ------ ---(2)

By adding (1) and (2)

(1)⇒	x + y =	9
(2)⇒	8x – y =	0
(1)+(2) ⇒ 9x	=	9 ⇒ x = 1

⇒ x = 1; Substitute x = 1 in (1)

1 + y = 9 ⇒ y = 8

Therefor the number is xy = 18

(iv) Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received

Let the number of Rs 50 notes = x and

the number of Rs 100 notes = y.

According to qn

x + y = 25 ---------- (1) and

50*x* + 100*y* = 2000

⇒ x + 2y = 40-------(2)

Substract (1) from (2)

x + 2y = 40	(2)
x + y = 25	(1)
y = 15	

Substitute y = 15 in (1)

x + 15 = 25 ⇒ x = 25 – 15 ⇒ x = 10

Therefor the number of Rs 50 notes = 10 and the number of Rs 100 notes = 15

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Let the fixed charges for first 3 days = Rs 'x'

The additional charges for remaing days = Rs '*y*'

According to question

$x + 4y = 27$ ------- (1); $x + 2y = 21$--------(2)

Substract (1) from (2)

$x + 2y = 21$	(2)
$x + 4y = 27$	(1)
$-2y = -6$	

$\Rightarrow y = 3$

Substitute y = 3 in (1)

$x + 4\times3 = 27 \Rightarrow x + 12 = 27$

$\Rightarrow x = 27 - 12 \Rightarrow x = 15$

The fixed charges = Rs 15 and the additional charges = Rs 3

Summery:

1. A pair of linear equations in two variables can be represented, and solved, by the:
   (i) graphical method (ii) algebraic method
2. Graphical Method :
   The graph of a pair of linear equations in two variables is represented by two lines.
   (i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is consistent.
   (ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is dependent (consistent).
   (iii)If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent
3. Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :
   (i) Substitution Method
   (ii) Elimination Method
4. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the the following situations can arise :
   $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ In this case, the pair of linear equations is consistent
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ In this case, the pair of linear equations is inconsistent
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ In this case, the pair of linear equations is dependent and consistent
5. There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations

10 Quadratic Equations

When we equate quadratic polynomial to zero, we get a quadratic equation.

Standard form of quadratic equations:

$ax^2 + bx + c = 0$, Where $a \neq 0$

Any equation of the form p(x) = 0, where p(x) is a polynomial of degree 2, is a quadratic equation.

The features of quadratic equations:

- The quadratic equations has one variable
- The hieghest power of the variable is 2
- Standard form of quadratic equation: $ax^2 + bx + c = 0$,

Adfected quadratic equations

In a quadratic equation

$ax^2 + bx + c = 0, \; a \neq 0, \; b \neq 0$ then it is called adfected quadratic equations.

Example:

$x^2 - 3x - 5 = 0, x^2 + 5x + 6 = 0$,

$x + \frac{1}{x} = 5$, $(2x - 5)^2 = 81$

Pure Quadratic equations

The quadrtic equations where $a \neq 0$, $b = 0$ is called pure quadratic equations.

The standard form of pure quadratic equation: $ax^2 + c = 0$ $[a \neq 0]$

Example:

$x^2 = 9, \; x^2 + 7 = 43$, $x = \frac{1}{x}$

Example 1 : Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. We would like to find out the number of toys produced on that day.

(i) Let the number of marbles with Jhon be 'x'

Then the number of marbles with Jivanthi = 45 – x [∵Total number of marbles 45]

The number of marbles left with John, when he lost 5 marbles = x – 5

The number of marbles left with Jivanti, when she lost 5 marbles = 45 – x – 5 = 40 – x

∴ Their products = 124

$(x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124$

$\Rightarrow -x^2 + 45x - 200 = 124$

$\Rightarrow -x^2 + 45x - 324 = 0$

$\Rightarrow x^2 - 45x + 324 = 0$

Therefore, the number of marbles John had, satisfies the quadratic equation

$\mathbf{x^2 - 45x + 324 = 0}$ which is the required representation of the problem mathematically.

(ii) Let the number of toys produced on that day be x.

Therefore, the cost of production (in rupees) of each toy that day = 55 – x

So, the total cost of production (in rupees) that day = x (55 – x)

∴ $x(55 - x) = 750$

$\Rightarrow 55x - x^2 = 750$

$\Rightarrow -x^2 + 55x - 750 = 0$

$\Rightarrow x^2 - 55x + 750 = 0$

∴ The number of toys produced that day satisfies the quadratic equation

$\mathbf{x^2 - 55x - 750 = 0}$ which is the required representation of the problem mathematically.

Example 2 : Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$ (ii) $x(x+1)+8=(x+2)(x-2)$

(iii) $x(2x+3)=x^2+1$ (iv) $(x+2)^3=x^3-4$

(i) $(x-2)^2+1=2x-3$

$x^2-4x+4+1=2x-3$

$\Rightarrow\ x^2-4x-2x+5+3=0$

$\Rightarrow\ x^2-6x+8=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(ii) $x(x+1)+8=(x+2)(x-2)$

$x^2+x+8=x^2-4$

$\Rightarrow x^2-x^2+x+8+4=0$

$\Rightarrow x+12=0$

This is not in the form of $ax^2+bx+c=0$

Therefore the given equation is not a quadratic equation.

(iii) $x(2x+3)=x^2+1$

$2x^2+3x=x^2+1$

$\Rightarrow 2x^2-x^2+3x-1=0$

$\Rightarrow x^2+3x-1=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

(iv) $(x+2)^3=x^3-4$

$x^3+2^3+3(x)(2)^2+3x^2(2)=x^3-4$

$x^3+8+12x+6x^2=x^3-4$

$\Rightarrow x^3-x^3+6x^2+12x+8+4=0$

$\Rightarrow 6x^2+12x+12=0\div 6$

$\Rightarrow x^2+2x+2=0$

This is in the form of $ax^2+bx+c=0$

Therefore the given equation is quadratic equation.

Exercise 4.1

1. Check whether the following are quadratic equations:

(i) $(x+1)^2=2(x-3)$ (ii) $x^2-2x=(-2)(3-x)$ (iii) $(x-2)(x+1)=(x-1)(x+3)$

(iv) $(x-3)(2x+1)=x(x+5)$ (v) $(2x-1)(x-3)=(x+5)(x-1)$ (vi) $x^2+3x+1=(x-2)^2$

(vii) $(x+2)^3=2x(x^2-1)$ (viii) $x^3-4x^2-x+1=(x-2)^3$

(i) $(x+1)^2=2(x-3)$

$x^2+2x+1=2x-6$

$\Rightarrow x^2+2x-2x+1+6=0$

$\Rightarrow x^2+7=0$

This is in the form of $ax^2+bx+c=0$

$\therefore$ The given equation is quadratic equation.

(ii) $x^2-2x=(-2)(3-x)$

$x^2-2x=-6+2x\ \Rightarrow\ x^2-2x-2x+6=0$

$\Rightarrow\ x^2 - 4x + 6 = 0$

This is in the form of $ax^2 + bx + c = 0$

∴ The given equation is quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3)$

$x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow\ x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow\ x^2 - x^2 - x - 2x - 2 + 3 = 0$

$\Rightarrow\ -3x + 3 = 0 \times -1$

$\Rightarrow\ 3x - 1 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5)$

$2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow\ 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow\ 2x^2 - x^2 - 5x - 5x - 3 = 0$

$\Rightarrow\ x^2 - 10x - 3 = 0$

This is in the form of $ax^2 + bx + c = 0$

∴ The given equation is quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1)$

$2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow\ 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - x^2 - 7x - 4x + 3 + 5 = 0$

$\Rightarrow\ x^2 - 11x + 8 = 0$

This is in the form of $ax^2 + bx + c = 0$

∴ The given equation is quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2$

$x^2 + 3x + 1 = x^2 - 2(x)(2) + 2^2$

$\Rightarrow\ x^2 - x^2 + 3x + 4x + 1 - 4 = 0$

$7x - 3 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1)$

$x^3 + 2^3 + 3(x)(2)^2 + 3x^2(2) = 2x^3 - 2x$

$\Rightarrow\ x^3 + 8 + 12x + 6x^2 = 2x^3 - 2x$

$\Rightarrow\ x^3 - 2x^3 + 6x^2 + 12x + 2x + 8 = 0$

$\Rightarrow\ -x^3 + 6x^2 + 14x + 8 = 0 \times -1$

$\Rightarrow\ x^3 - 6x^2 - 14x - 8 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

$x^3 - 4x^2 - x + 1 = x^3 - 2^3 + 3(x)(2)^2 - 3x^2(2)$

$\Rightarrow\ x^3 - 4x^2 - x + 1 = x^3 - 8 + 12x - 6x^2$

$\Rightarrow x^3 - x^3 - 4x^2 + 6x^2 - x - 12x + 1 + 8 = 0$

$\Rightarrow\ 2x^2 - 13x + 9 = 0$

This is in the form of $ax^2 + bx + c = 0$

∴ The given equation is quadratic equation.

2. Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Let breadth b $= x$ m
$\Rightarrow$ Length $l = (2x+1)$m
Area of the rectangle $= l \times b$
$\Rightarrow 528 = x(2x+1)$
$\Rightarrow 528 = 2x^2 + x$
$\Rightarrow 2x^2 + x - 528 = 0$

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

Let two consecutive integers be x and $(x+1)$;
Their products $= 306$
$\Rightarrow x(x+1) = 306$
$\Rightarrow x^2 + x - 306 = 0$

(iii)Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Let the present age of Rohan $= x$;
The present age of his mother $= x + 26$
After 3 Rohan's age $= x + 3$
After 3 years his mothers age $= x + 26 + 3 = x + 29$
Product of their ages after 3 years $= 360$
$\therefore (x+3)(x+29) = 360$
$\Rightarrow x^2 + 29x + 3x + 87 = 360$
$\Rightarrow x^2 + 32x - 273 = 0$

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Let the speed of the train $=$ x km/h;
The time taken to travel 480 km $= \frac{480}{x}\ hrs$
Reducing speed by 8 km/h, the speed of the train $= (x-8)km/h$
Therefore the time taken to travel 480 km $= \left(\frac{480}{x-8}\right) hrs$
$\Rightarrow \frac{480}{x} + 3 = \frac{480}{x-8}$
$\Rightarrow 480(x-8) + 3x(x-8) = 480x$
$\Rightarrow 480x - 3840 + 3x^2 - 24x = 480x$
$\Rightarrow 3840 + 3x^2 - 24x = 0$
$\Rightarrow 3x^2 - 24x + 3840 = 0$
$\Rightarrow x^2 - 8x + 1280 = 0$

4.3 Solution of a Quadratic Equation by Factorisation

Note: The zeros of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation are the same.

Example 3 : Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorization.

$2x^2 - 5x + 3 = 0$
$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$
$\Rightarrow 2x(x-1) - 3(x-1) =$
$\Rightarrow (x-1)(2x-3) = 0$
$\Rightarrow x - 1 = 0, \quad 2x - 3 = 0$
$\Rightarrow x = 1, \ 2x = 3$
$x = 1, \ x = \frac{3}{2}$

> First term $= 2x^2$, Last term $= +3$
> Their product $= +6x^2$
> The middle term $= -5x$
> Divide middle term such that product $= +6x^2$ and their sum is $-5x$
> $\Rightarrow -5x = -2x - 3x$

Example 4: Find the roots the equation $6x^2 - x - 2 = 0$

$6x^2 - x - 2 = 0$
$6x^2 - 4x + 3x - 2 = 0$
$2x(3x - 2) + 1(3x - 2) = 0$
$(2x + 1)(3x - 2) = 0$
$2x + 1 = 0, 3x - 2 =$
$2x = -1, 3x = 2$
$\Rightarrow x = \frac{-1}{2}, x = \frac{2}{3}$

First term $= 6x^2$, Last term $= -2$
Their product $= -12x^2$
The middle term $= -x$
Divide middle term such that product $= -12x^2$ and sum $-x$
$\Rightarrow -x = -4x + 3x$

Example 5:Find the roots the equation $3x^2 - 2\sqrt{6}\,x + 2 = 0$

$3x^2 - 2\sqrt{6}\,x + 2 = 0$
$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$
$(\sqrt{3})^2 x^2 - \sqrt{2}.\sqrt{3}\,x - \sqrt{2}.\sqrt{3}\,x + (\sqrt{2})^2 = 0$
$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$
$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$
$(\sqrt{3}\,x - \sqrt{2}) = 0, \ (\sqrt{3}\,x - \sqrt{2}) = 0$
$\sqrt{3}\,x = \sqrt{2}, \ \sqrt{3}\,x = \sqrt{2}$
$\Rightarrow x = \sqrt{\frac{2}{3}}, \ x = \sqrt{\frac{2}{3}}$

First term $= 3x^2$, Last term $= +2$
Their product $= 6x^2$
The middle term $= -2\sqrt{6}\,x$
Divide middle term such that product $= 6x^2$ and sum $-2\sqrt{6}\,x$
$\Rightarrow -2\sqrt{6}\,x = \sqrt{6}\,x - \sqrt{6}\,x$

Example 6 : Find the dimensions of the prayer hall discussed in Section 4.1.

$2x^2 + x - 300 = 0$
$2x^2 - 24x + 25x - 300 = 0$
$2x(x - 12) + 25(x - 12) = 0$
$(x - 12)(2x + 25) =$
$x - 12 = 0, 2x + 25 = 0$
$x = 12$ or $2x = -25$
$\Rightarrow x = \frac{-25}{2} = \sout{-12.5}$
Breadth $= x = 12\,m$
Length $= 2x + 1 = 24 + 1 = 25\,m$

First term $= 2x^2$, Last term $= -300$
Their produ= $-600x^2$
The middle term $= +x$
Divide middle term such that product $= -600x^2$ and sum x
$\Rightarrow +x = -24x + 25x$

Exercise 10.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$ (ii) $2x^2 + x - 6 = 0$ (iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + \frac{1}{8} = 0$ (v) $100x^2 - 20x + 1 = 0$

(i) $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$
$\Rightarrow x(x - 5) + 2(x - 5) = 0$
$\Rightarrow (x - 5)(x + 2) = 0$
$\Rightarrow (x - 5) = 0, (x + 2) = 0$
$\Rightarrow \mathbf{x = 5, x = -2}$

(ii) $2x^2 + x - 6 = 0$

$2x^2 + x - 6 = 0$
$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$
$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$
$\Rightarrow (x + 2)(2x - 3) = 0$
$\Rightarrow x + 2 = 0, 2x - 3 = 0$
$\Rightarrow x = -2, 2x = 3 \Rightarrow x = -2, x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \sqrt{2}x + 5 = 0\,,\ x + \sqrt{2} = 0$

$\Rightarrow \sqrt{2}x = -5,\ x = -\sqrt{2}$

$\Rightarrow\ x = \frac{-5}{\sqrt{2}},\ x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow (4x - 1)(4x - 1) = 0$

$\Rightarrow 4x - 1 = 0, 4x - 1 = 0$

$\Rightarrow 4x = 1, 4x = 1$

$\Rightarrow x = \frac{1}{4},\ x = \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

$100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow 10x - 1 = 0, 10x - 1 = 0$

$\Rightarrow 10x = 1, 10x = 1$

$\Rightarrow\ x = \frac{1}{10},\ x = \frac{1}{10}$

2. Solve the problems given in Example 1.

In example 1 we got the equations:

(i) $x^2 - 45x + 324 = 0$ and (ii) $x^2 - 55x + 750 = 0$

(i) $x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

$\Rightarrow (x - 36) = 0,\ \ (x - 9) = 0$

$\Rightarrow x = 36,\ \ x = 9$

The marbles with John $= x = 36$ and the marbles with Jevan $= 45 - x = 45 - 36 = 9$

Or The marbles with John $x = 9$ and the marbles with Jevan: $45 - x = 45 - 9 = 36$

(ii) $x^2 - 55x + 750 = 0$

$x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

$\Rightarrow (x - 25) = 0,\ \ (x - 30) = 0$

$\Rightarrow x = 25,\ \ \ x = 30$

The number of toys are 25 or 30

3. Find two numbers whose sum is 27 and product is 182.

Let the first number $= x$ then second number $= 27 - x$

Their product $= 182$

$\therefore x(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$-x^2 + 27x - 182 = 0 \times -1$
$\Rightarrow \boldsymbol{x^2 - 27x + 182 = 0}$
$x^2 - 13x - 14x + 182 = 0$
$\Rightarrow x(x - 13) - 14(x - 13) = 0$
$\Rightarrow (x - 13)(x - 14) = 0$
$\Rightarrow (x - 13) = 0, (x - 14) = 0$
$\Rightarrow x = 13, x = 14$
If first number $= 13$ then the second number $= 27 - 13 = 14$
If first number $= 14$ then the second number$= 27 - 14 = 13$
Therefore the numbers are 13 and 14

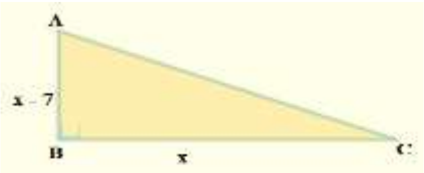
4. Find two consecutive positive integers, sum of whose squares is 365.

Let the positive numbsr be x and the concecutiveinteger $= x + 1$
$x^2 + (x + 1)^2 = 365$
$\Rightarrow x^2 + x^2 + 2x + 1 = 35$
$\Rightarrow 2x^2 + 2x + 1 - 365 = 0$
$\Rightarrow 2x^2 + 2x - 364 = 0 \div 2$
$\Rightarrow \boldsymbol{x^2 + x - 182 = 0}$
$x^2 + 14x - 13x - 182 = 0$
$\Rightarrow x(x + 14) - 13(x + 14) = 0$
$\Rightarrow (x + 14)(x - 13) = 0$
$\Rightarrow (x + 14) = 0, (x - 13) = 0$
$\Rightarrow x = -14 , x = 13$
$\therefore x + 1 = 13 + 1 = 14$
$\Rightarrow$ Two consecutive positive integers are $13, 14$.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Let the base of the right angle triangle $BC = x\ cm$;Height $AB = (x - 7)cm$

According to Pythagoras theorem, $BC^2 + AB^2 = AC^2$



$x^2 + (x - 7)^2 = 13^2$
$\Rightarrow x^2 + x^2 + 7^2 - 2(x)(7) = 169$
$2x^2 - 14x + 49 - 169 = 0$
$2x^2 - 14x - 120 = 0 \div 2$
$\Rightarrow \boldsymbol{x^2 - 7x - 60 = 0}$
$x^2 - 12x + 5x - 60 = 0$
$\Rightarrow x(x - 12) + 5(x - 12) = 0$
$\Rightarrow (x - 12)(x + 5) = 0$
$\Rightarrow (x - 12) = 0, \quad (x + 5) = 0$
$\Rightarrow x = 12, \ x = -5$
The base of $BC = x = 12\ cm$; Height $AB = (x - 7) = 12 - 7 = 5cm$

2. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ` 90, find the number of articles produced and the cost of each article.

Let the number of pots $= x$; The cost $= Rs(2x + 3)$
$\therefore x(2x + 3) = 90$
$\Rightarrow \boldsymbol{2x^2 + 3x - 90 = 0}$
$2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow x(2x+15)-6(2x+15)=0$
$\Rightarrow (2x+15)(x-6)=0$
$\Rightarrow (2x+15)=0, (x-6)=0$
$\Rightarrow 2x=-15,\ \ x=6$
$\Rightarrow x=\frac{-15}{2}, x=6$

Therefore number of pots $=x=6$;

Cost $=(2x+3)=2(6)+3=s12+3=R15$

4.5 Nature of Roots

The value of b^2 - 4ac decides the roots of quadratic equation $ax^2+bx+c=0$ has real or not, therefore

b^2 - 4ac is called the discriminant of this quadratic equation.and denoted by Δ [delta]

So, the quadratic equation $ax^2+bx+c=0$ has

Discriminant	Nature of the roots
$\Delta=0$	Two equal real roots
$\Delta>0$	Two distinct real roots
$\Delta<0$	No real roots

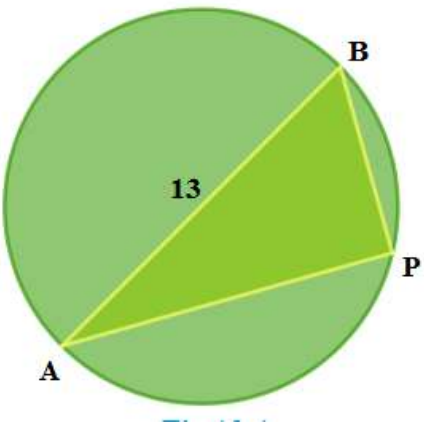
Example 8 : Find the discriminant of the quadratic equation $2x^2-4x+3=0$, and hence find the nature of its roots

Solution: $a=2,\ b=-4,\ \ c=3$

$\Delta=b^2-4ac$
$\Rightarrow \Delta=(-4)^2-4(2)(3)$
$\Rightarrow \Delta=16-24$
$\Rightarrow \Delta=-8<0$ Roots are imaginary

Example 9: A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

$BP=x$ m ;$AP=(x+7)$m ; Diameter $AB=13$ m
$\angle APB=90^0$



$\Rightarrow AP^2+PB^2=AB^2$
$\Rightarrow x^2+(x+7)^2=13^2$
$\Rightarrow x^2+x^2+7^2+2(x)(7)=169$
$\Rightarrow 2x^2+14x+49-169=0$
$\Rightarrow 2x^2+14x-120=0$ ---(1)

$(1)\div 2 \Rightarrow x^2+7x-60=0$
$\Rightarrow x^2+12x-5x-60=0$
$\Rightarrow x(x+12)-5(x+12)=0$
$\Rightarrow (x+12)(x-5)=0$
$\Rightarrow x+12=0,\ \ x-5=0$
$\Rightarrow x=-12,\ \ \ x=5$
$\Rightarrow$ x= -12 is not possible. Therefore $BP=x$ m $=5$m
$AP=(x+7)=5+7=12$ m

Example10: Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of the roots. Find them if they are real.

Solution: $a = 3, \quad b = -2, \quad c = \frac{1}{3}$

$b^2 - 4ac = (-2)^2 - 4(3)\left(\frac{1}{3}\right) = 4 - 4 = 0$

$b^2 - 4ac = 0$ Roots are real and equal

Roots are: $\frac{-b}{2a}$ or $\frac{-b}{2a}$

$= \frac{-(-2)}{2(3)}$ or $\frac{-(-2)}{2(3)}$

$= \frac{2}{6}$ or $\frac{2}{6} = \frac{1}{3}$ or $\frac{1}{3}$

Exercise: 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:.(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$

(i) $2x^2 - 3x + 5 = 0$ Here, $a = 2, \ b = -3, \quad c = 5$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4(2)(5)$

$\Rightarrow \Delta = 9 - 40 \Rightarrow \Delta = -31$

$\Rightarrow \Delta < 0 \Rightarrow$ Roots are imaginary

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3, b = -4\sqrt{3}, c = 4$

$\Delta = b^2 - 4ac$

$\Delta = (-4\sqrt{3})^2 - 4(3)(4)$

$\Rightarrow \Delta = 48 - 48$

$\Delta = 0 \Rightarrow$ Roots are real and equal

The roots are: $\frac{-b}{2a}$ or $\frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2(3)}$ or $\frac{-(-4\sqrt{3})}{2(3)}$

$= \frac{4\sqrt{3}}{6}$ or $\frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$ or $\frac{2\sqrt{3}}{3}$

$\Rightarrow \frac{2}{\sqrt{3}}$, or $\frac{2}{\sqrt{3}}$

(iii) $2x^2 - 6x + 3 = 0$

Here, $a = 2, b = -6, c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-6)^2 - 4(2)(3)$

$\Rightarrow \Delta = 36 - 24$

$\Rightarrow \Delta = 12 \Rightarrow \Delta > 0$

$\Rightarrow$ Roots are real and distinct

The roots $= \frac{-b+\sqrt{\Delta}}{2a}$ or $\frac{-b-\sqrt{\Delta}}{2a}$

$= \frac{-(-6)+\sqrt{12}}{2(2)}$ or $\frac{-(-6)-\sqrt{12}}{2(2)}$

$= \frac{6+\sqrt{12}}{4}$ or $\frac{6-\sqrt{12}}{4}$

$= \frac{6+2\sqrt{3}}{4}$ or $\frac{6-2\sqrt{3}}{4}$

$= \frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots (i) $2x^2 + kx + 3 = 0$ (ii) $kx(k - 2) + 6 = 0$

(i) $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$, $c = 3$
$b^2 - 4ac = 0$
$(k)^2 - 4(2)(3) = 0$
$\Rightarrow k^2 - 24 = 0$
$\Rightarrow k^2 = 24$
$\Rightarrow k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$
$\Rightarrow a = k$, $b = -2k$, $c = 6$
$b^2 - 4ac = 0$
$\Rightarrow (-2k)^2 - 4(k)(6) = 0$
$\Rightarrow 4k^2 - 24k = 0$
$\Rightarrow 4k(k - 6) = 0$
$\Rightarrow 4k = 0, k - 6 = 0$
$\Rightarrow k = 0, \quad k = 6$

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth

The breadth of the mango grove $= l$;The length $= 2l$

The area of the grove = Length x breadth

$\Rightarrow (l)(2l) = 800$
$\Rightarrow 2l^2 = 800$
$\Rightarrow l^2 = \frac{800}{2} = 400$
$\Rightarrow l = \pm\sqrt{400} = \pm 20$
$\therefore$ The breadth of the mango grove $= l = 20$ m
$\therefore$ The breadth of the mango grove $= 2l = 2 \times 20 = 40$ m

4. Is the following situation possible? If so, determine their present ages.The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Let the age of A friend $= x$ Years; The age of B friend $= (20 - x)$ years

The age of friend A before 4 $= (x - 4)$

The age of B friend before 4 years $= (20 - x - 4) = 16 - x$

According to question, $(x - 4)(16 - x) = 48$
$\Rightarrow 16x - x^2 - 64 + 4x = 48$
$\Rightarrow -x^2 + 20x - 64 - 48 = 0$
$\Rightarrow x^2 - 20x + 112 = 0$
Here, $a = 1, b = -20, c = 112$
$b^2 - 4ac = (-20)^2 - 4(1)(112)$
$= 400 - 448 = -48$
The equation has no real roots. Therefore this situation is not possible

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Let the length and breadth of the rectangle be l& b; The perimeter $= 2(l + b) = 80$

$l + b = \frac{80}{2} = 40 \Rightarrow l = 40 - b$
Area $l \times b = 400$
$\Rightarrow l(40 - l) = 400$
$\Rightarrow 40l - l^2 = 400$

$\Rightarrow l^2 - 40l + 400 = 0$
This is a quadratic equation with $a = 1, b = -40,\ c = 400$
$b^2 - 4ac = (-40)^2 - 4(1)(400)$
$= 1600 - 1600 = 0$
$b^2 - 4ac = 0$ Roots are rea and equal
Roots are: $\frac{-b}{2a}$ and $\frac{-b}{2a}$
$= \frac{-(-40)}{2(1)} and \frac{-(-40)}{2(1)}$
$= \frac{40}{2} and \frac{40}{26}$
$= 20\ and\ 20$
Length $= 20\ m$; Breadth $b = 40 - l = 40 - 20 = 20m$

Summary:

1. A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$
2. A real number is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a^2 + b + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
3. If we can factorise $ax^2 + bx + c$, a 0, into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
4. Roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where b2 - 4ac ≥ 0
5. For the quadratic equation $ax^2 + bx + c = 0$,
 (i) If $b^2 - 4ac > 0$ then roots are real and distinct
 (ii) If $b^2 - 4ac = 0$ roots are real and equal
 (iii) If $b^2 - 4ac < 0$ no real roots

5 Arithmetic progression

1.2 Arithmetic Progressions:

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the common difference of the AP. Remember that it can be positive, negative or zero.

i)	**1, 2, 3, 4................**	Each term is 1 more than the term preceding it.
ii)	**100, 70, 40, 10........**	Each term is 30 less than the term preceding it.
iii)	**-3, -2, -1, 0.............**	Each term is obtained by adding 1 to the term preceding it.
iv)	**3, 3, 3, 3...............**	All the terms in the list are 3, i.e., each term is obtained by adding (or subtracting) 0 to the term preceding it.
v)	**-1, -1.5,-2.0,-2.5.......**	Each term is obtained by adding – 0.5 to (i.e., subtracting 0.5 from) the term preceding it.

The fixed number is called common difference, and denoted by 'd'

Let us denote the first term of an AP by a_1, second term by a_2, . ., n^{th} term by a_n and the common difference by d. Then the AP becomes a_1, a_2, a_3 . . . a_n

So, $a_2 - a_1 = a_3 - a_2 = . . . = a_n - a_{n-1} = d$

a, a + d, a + 2d, a + 3d, . . .

Represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an AP.

Finite AP.:

In an AP there are only a finite number of terms. Such an AP is called a finite AP. Each of these Arithmetic Progressions (APs) has a last term.

a) The heights (in cm) of some students of a school standing in a queue in the morning assembly are 147 , 148, 149, . . ., 157.

b) The balance money (in Rs) after paying 5 % of the total loan of Rs 1000 every month is 950, 900, 850, 800, . . ., 50.

c) The total savings (in Rs) after every month for 10 months when Rs50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500.

Infinite AP.:

In an AP there are infinite number of terms. Such an AP is called a infinite AP. Each of these Arithmetic Progressions (APs) do not have last term.

a) 3, 7, 11 . . . . (b)1, 4, 7, 10, . . . (c) -10, -15, -20 . . . .

Note: You will If we know the first term a' and the common difference d' then we can write an AP.

Example 1: $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$...... **write the first term 'a' and the common difference 'd'.**

Here, $a_1 = \frac{3}{2}$ **d** = $a_2 - a_1 = \frac{1}{2} - \frac{3}{2} = -1$ $a_3 - a_2 = -\frac{1}{2} - \frac{1}{2} = -1$

Example 2: Which of the following list of numbers form an AP? If they form an AP, write the next two terms

i) 4, 10, 16, 22... (ii) 1,-1,-3,-5... (iii) -2, 2, -2, 2... (iv) 1, 1, 1, 2, 2, 2, 3, 3, 3...

Solution:

i) 4, 10, 16, 22........

$a_2 - a_1 = 10 - 4 = 6$; $a_3 - a_2 = 16 - 10 = 6$; $a_3 - a_2 = 22 - 16 = 6$

So, the given list of numbers forms an AP with the common difference d = 6.

The next two terms are: 22 + 6 = 28 and 28 + 6 = 34.

ii) 1, −1, −3, −5........

$a_2 - a_1 = -1 - 1 = -2$; $a_3 - a_2 = -3 - (-1) = -2$; $a_3 - a_2 = -5 - (-3) = -2$

So, the given list of numbers forms an AP with the common difference d = -2.

The next two terms are: – 5 + (– 2) = – 7 and – 7 + (– 2) = – 9

iii) -2, 2, -2, 2....

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$; $a_3 - a_2 = -2 - 2 = -4$; $a_3 - a_2 = 2 - (-2) = 2 + 2 = 4$

Here, $a_{k+1} \neq a_k$ So, the given list of numbers does not form an AP.

iv) 1, 1, 1, 2, 2, 2, 3, 3, 3..........

$a_2 - a_1 = 1 - 1 = 0$; $a_3 - a_2 = 1 - 1 = 0$; $a_3 - a_2 = 2 - 1 = 1$

$a_2 - a_1 = a_3 - a_2 \neq a_3 - a_2$ So, the given list of numbers does not form an AP.

EXERCISE 5.1

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

i). The taxi fare after each km when the fare is Rs15 for the first km and Rs 8 for each additional km

Solution: The first term $a_1 = 15$, $a_2 = 15 + 8 = 23$, $a_3 = 23 + 8 = 31$ ....

Here, each term is obtained by adding a common difference = 8, except first term.

ii).The amount of air present in a cylinder when a vacuum removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Solution: Let the initial volume of the air present in the cyclinder be V.

The remaining air in the cylinder after using vacuum pump first time V - $\frac{1}{4} = \frac{3V}{4}$,

Remaining air in the cylinder after using vacuum pump second time

$\frac{3V}{4} - \frac{3V}{4} \times \frac{1}{4} = \frac{3V}{4} - \frac{3V}{16} = \frac{9V}{16}$ and so on.; Here, the terms are V, $\frac{3V}{4}$, $\frac{9V}{16}$ .. …..

$a_2 - a_1 = \frac{3V}{4} - V = -\frac{V}{4}$; $a_3 - a_2 = \frac{9V}{16} - \frac{3V}{4} = \frac{9V}{16} - \frac{12V}{16} = -\frac{3V}{16}$

$\therefore a_2 - a_1 \neq a_3 - a_2$

Hence, it does not form an AP

iii). The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

Solution: The cost of digging for the first meter = Rs 150

Cost of digging for the second meter = 150+50 = Rs 200

Cost of digging for the third meter = 200+50 = Rs 250
Cost of digging for the fourth meter = 250+50 = Rs 300
Thus the list of numbers is 150, 200, 250, 300........

Here, we can find the common difference = 50; **So it forms an AP**.

iv). The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8 % per annum.

Solution: We know that amount $A = P\left(1 + \frac{r}{100}\right)^n$; Here, P =10,000; r = 8%, n = 1,2,3…

Amount in first year = $10000\left(1 + \frac{8}{100}\right)^1 = 10000\text{x}\frac{108}{100}$ = 100x108 = Rs 10800

Amount in second year = $10000\left(1 + \frac{8}{100}\right)^2 = 10000\text{x}\frac{108}{100}\text{x}\frac{108}{100}$ = 108 x 108 = Rs 11664

Thus the list of numbers is 10000, 10800, 11664 .. …..
$a_2 - a_1 = 10800 - 10000 = 800$; $a_3 - a_2 = 11664 - 10800 = 864$

There for $a_2 - a_1 \neq a_3 - a_2$; **Hence it does not form an AP**.

2. Write first four terms of the AP, when the first term 'a' and the common difference 'd' are given as follows:

i) (i) a = 10, d = 10 (ii) a = -2, d = 0 (iii) a = -2, d = 0 (iv) a = -1, d = $\frac{1}{2}$ (v) a = 10, d = 10

(i) a = 10, d = 10
$a_1 = 10$;
$a_2 = a_1 + d = 10 + 10 = 20$
$a_3 = a_2 + d = 20 + 10 = 30$;
$a_4 = a_3 + d = 30 + 10 = 40$
Thus the first four terms of an AP are 10, 20, 30, 40

ii) a = -2, d = 0
$a_1 = -2$;
$a_2 = a_1 + d = -2 + 0 = -2$
$a_3 = a_2 + d = -2 + 0 = -2$;
$a_4 = a_3 + d = -2 + 0 = -2$
Thus the first four terms of an AP are -2, -2, -2, -2,

(iii) a = 4, d = -3
$a_1 = 4$;
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$;
$a_4 = a_3 + d = -2 - 3 = -5$
Thus the first four terms of an AP are 4, 1, -2, -5

(iv) a = -1, d = $\frac{1}{2}$
$a_1 = -1$;
$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$
$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$; $a_4 = \frac{1}{2}$
Thus the first four terms of an AP are -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$

(v) a = -1.25, d = -0.25
$a_1 = -1.25$; $a_2 = a_1 + d = -1.25 - 0.25 = -1.50$
$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$; $a_4 = a_3 + d = -1.75 + 0.25 = -2.00$
Thus the first four terms of an AP are –1.25, -1.50, -1.75, -2.00

3. For the following APs, write the first term and the common difference:

i) 3, 1, - 1, - 3... ii) -5, -1, 3, 7... iii) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}...$ iv) 0.6, 1.7, 2.8, 3.9, ...

Solution:

i) 3, 1, - 1, - 3...

The first term a = 3,

Common difference

d = a_2 - a_1 = 1 – 3 = -2

ii) -5, -1, 3, 7.......

The first term a = 5,

Common difference

d = a_2 - a_1 = -1 – (-5) = -1 + 5 = 4

iii) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}...$

The first term a = $\frac{1}{2}$,

Common difference

d = a_2 - a_1 = $\frac{1}{2} - \frac{1}{2}$ = 0

iv) 0.6, 1.7, 2.8, 3.9, ...

The first term a = 0.6,

Common difference

d = a_2 - a_1 = 1.7 – 0.6 = 1.1

4. Which of the following are APs ? If they form an AP, find the common difference 'd' and write three more terms

i) 2, 4, 8, 16... (ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ ... (iii) -1.2, -3.2, -5.2, -7.2 ... (iv) -10, -6, -2, 2 ...

(v) 3, 3+$\sqrt{2}$, 3 +2$\sqrt{2}$, 3 + 3$\sqrt{2}$... (vi) 0.2, 0.22, 0.222, 0.2222 ...

(vii) 0, -4, -8, -12 ... (viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}...$ (ix) 1, 3, 9, 27 ... (x) a, 2a, 3a, 4a ... (xi) a, a^2, a^3, a^4...

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$... (xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$...

(xiv) 1^1, 3^2, 5^2, 7^2... (xv) 1^1, 5^2, 7^2, 73...

(i) 2, 4, 8, 16...

a_2 - a_1 = 4 – 2 = 2

a_3 - a_2 = 8 – 4 = 4

Here, a_2 - a_1 ≠ a_3 - a_2

∴The given list of numbers does not form an AP.

ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ ....

a_2 - a_1 = $\frac{5}{2}$ – 2 = $\frac{1}{2}$; a_3 - a_2 = 3 – $\frac{5}{2}$ = $\frac{1}{2}$

Here, a_2 - a_1 = a_3 - a_2 = a_4 - a_3

∴ Therefore the given list of numbers forms an AP with common difference

$\frac{7}{2} + \frac{1}{2} = 4$; $4 + \frac{1}{2} = \frac{9}{2}$; $\frac{9}{2} + \frac{1}{2} = 5$

iii) -1.2, -3.2, -5.2, -7.2 .....

a_2 - a_1 = -3.2 – (-1.2) = -3.2 + 1.2 = -2

a_3 - a_2 = -5.2 – (-3.2) = -5.2 + 3.2 = -2

a_4 - a_3 = -7.2 – (-5.2) = -7.2 + 5.2 = -2

Here, a_2 - a_1 = a_3 - a_2 = a_4 - a_3

∴ The given list of numbers forms an AP with common difference $d = -2$

Next 3 terms are, **-9.2,-11.2,-13.2**

iv) -10, -6, -2, 2 ......

$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ The given list of numbers forms an AP with common difference 'd' = 4

The next 3 terms are **6,10,14**

v) 3, 3+ $\sqrt{2}$, 3 + 2$\sqrt{2}$, 3 + 3$\sqrt{2}$, …

$a_2 - a_1 = 3+ \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3+ \sqrt{2}= \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ The given list of numbers forms an AP with common difference $d = \sqrt{2}$

The next 3 terms are **3+ 4$\sqrt{2}$, 3+5$\sqrt{2}$,**
3+6$\sqrt{2}$

(vii) 0, -4, -8, -12 ...

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ This is an AP with d = -4

The next 3 terms are **-16,-20,-24**

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ This is an AP with 'd' = 0

The Next 3 terms are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$

(ix) 1, 3, 9, 27 ....

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

(x) a, 2a, 3a, 4a ....

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Here,$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ This is an AP with 'd' = a

The next 3 terms are **5a**, **6a**, **7a**

(xi) a, a^2, a^3, a^4.......

$a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$........

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

∴ This is an AP with 'd' = $\sqrt{2}$

The next 3 terms are $\sqrt{\mathbf{50}}, \sqrt{\mathbf{72}}, \sqrt{\mathbf{98}}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$.........

$a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

(xiv) $1^1, 3^2, 5^2, 7^2$, ...

$a_2 - a_1 = 3^2 - 1^1 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

(xv) $1^1, 5^2, 7^2, 73$, ...

$a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$

$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 7^2 = 73 - 49 = 24$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference d = 24

The next 3 terms of this AP are 73 + 24 = 97, 97 + 24 = 121, 121 + 24 = 145

1.3 n^{th} Term of an AP

The n^{th} term is $a_n = a + (n-1)d$ [a – first term, d – c.d.]

n^{th} term from the last n: $l - (n-1)d$ [l-last term , d – c. d.]

Example 3 : Find the 10th term of the AP :2, 7, 12 . . .

Solution :

a = 2, d = 7 - 2 = 5and n = 10

$a_n = a + (n-1)d$

$a_{10} = 2 + (10-1)5$

$a_{10} = 2 + (9)5$

$a_{10} = 2 + 45$

$\boldsymbol{a_{10} = 47}$

Example 4 :Which term of the AP : 21,18,15 . . . is – 81? Also, is any term 0? Give reason for your answer.

Solution: a = 21, d = 18 - 21 = -3 and a_n = -81.

Now we have to find 'n'

$a_n = a + (n-1)d$

$-81 = 21 + (n-1)(-3) = 21 - 3n + 3$

$\Rightarrow -81 = 24 - 3n$

$\Rightarrow 3n = 24 + 81 = 105$

$\Rightarrow n = 35$

which term is Zero?

$0 = 21 + (n-1)(-3)$

$= 21 - 3n + 3$

$\Rightarrow 3n = 24$

$\Rightarrow n = 8$

∴ 8th term is Zero

Example 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solutin: $a + (n-1)d = a_n$

$a + (3-1)d = 5$

$a + 2d = 5$ ------- (1)

$a + (7-1)d = 9$

$a + 6d = 9$ --------(2)

a	+	**2d**	=	**5**
a	+	**6d**	=	**9**
		–4d	=	**- 4**

⇒ d = 1

$\Rightarrow a + 2(1) = 5$

⇒ a + 2 = 5

⇒ a = 5 – 2 = 3

⇒ a = 3

∴ AP: 3, 4, 5, 6, - - -

Alternate Method:

$d = \frac{a_p - a_q}{p-q}$

$a_p = a_7$; $a_q = a_3$

$d = \frac{a_7 - a_3}{7-3}$

$= \frac{9-5}{7-3} = \frac{4}{4} = 1$

∴ **d = 1**

a = $a_p + (p-1)d$ a

a = $a_7 + (7-1)1$

a = $9 + (7-1)1$

a = $9 + 6 = 3$

⇒ a = 3

∴ AP: 3, 4, 5, 6, - - -

Example 6: Check whether 301 is a term of the list of numbers 5, 11, 17, 23 . . .

Solution:

a = 5, d = 11 – 5 = 6
$a + (n-1)d = a_n$
$5 + (n-1)6 = 301$
$5 + 6n - 6 = 301$
$6n - 1 = 301$
$6n = 301 + 1$
$6n = 302$
$\Rightarrow n = \frac{302}{6}$
$= \frac{151}{3}$
Here n is not an integer
There fore 301 is not a term of the list of numbers 5, 11, 17, 13 …

Example 7 : How many two-digit numbers are divisible by 3?

Solution: 12, 15, 18 99

a = 12, d = 3, a_n = 99
$a + (n-1)d = a_n$
$12 + (n-1)3 = 99$
$12 + 3n - 3 = 99 \Rightarrow 3n + 9 = 99$
$\Rightarrow 3n = 99 - 9$
$\Rightarrow 3n = 90$
$\Rightarrow n = 30$
∴30,2- digit numbers are divisible by 3.

Example 8 : Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Solution: a = 10, d = 7 - 10 = -3, l = -62

$l = a + (n-1)d$
n^{th} term from the last $= l - (n-1)d$
$= -62 - (11-1)(-3)$
$= -62 + 33 - 3$
$= -62 + 30$
$= -32$

Example 9 : A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

Solution: Simple intrest $\mathbf{I} = \frac{\mathbf{PRT}}{\mathbf{100}}$

The interest at the end of the 1st year = $\frac{\mathbf{1000x8x1}}{\mathbf{100}}$ = **Rs 80**

The interest at the end of the 2nd year = $\frac{\mathbf{1000x8x2}}{\mathbf{100}}$ = **Rs 160**

the interest at the end of the 3rd year = $\frac{\mathbf{1000x8x3}}{\mathbf{100}}$ = **Rs 240**

∴ the terms are $80, 160, 240 - - -$

Here $a_2 - a_1 = a_3 - a_2$ = d = 80
It is an AP with d = 80,
The interest at the end of 30 years = $\mathbf{a_{30}}$;

a = 80, d = 80, n = 30
$a_n = a + (n-1)d$
$a_{30} = 80 + (30-1)80$
$a_{30} = 80 + 29x80$
$a_{30} = 80 + 2320$
$\mathbf{a_{30} = Rs\ 2400}$

Example 10 : In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution: The number of rose plants in the 1st, 2nd, 3rd... rows are :23, 21, 19...

Here $a_2 - a_1 = a_3 - a_2 = -2$

∴ it is an AP. a = 23, d = -2, $a_n = 5$, n = ?

$a + (n-1)d = a_n$

$23 + (n-1)(-2) = 5$

$23 - 2n + 2 = 5$

$-2n + 25 = 5$

$-2n = 5 - 25$

$-2n = -20$

$n = 10$

So,there are10 rows in the flower bed **=10.**

EXERCISE 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a the nth term of the AP: N

	a	d	n	a_n
(i)	7	3	8	-
(ii)	− 18	-	10	0
(iii)	-	− 3	18	− 5
(iv)	− 18.9	2.5	-	3.6
(v)	3.5	0	105	-

a_n
28
2
46
10
3.5

$\mathbf{a_n = a + (n-1)d}$

(i) $\mathbf{a_8 = 7 + (8-1)3}$

$= 7 + 7x3$

$= 7 + 21$

$= 28$

(ii) $\mathbf{0 = -18 + (10-1)d}$

$= -18 + 9d$

$\Rightarrow 9d = 18$

$\Rightarrow d = 2$

(iii) $\mathbf{-5 = a + (18-1)(-3)}$

$-5 = a - 17x3$

$\Rightarrow -5 = a - 51$

$\Rightarrow a = 46$

(iv) $\mathbf{3.6 = -18.9 + (n-1)(2.5)}$

$\Rightarrow 3.6 = -18.9 + 2.5n - 2.5$

$\Rightarrow 3.6 = -21.4 + 2.5n$

$\Rightarrow 2.5n = 3.6 + 21.4$

$\Rightarrow n \;= \frac{25}{2.5} = \frac{250}{25}$

$\Rightarrow n \;= 10$

(v) $\mathbf{a_n = 3.5 + (105 - 1)(0)}$

$= 3.5 + 104 \text{x} 0$

$\Rightarrow a_n = 3.5$

2. Choose the correct choice in the following and justify :

(i) 30th term of the AP: 10, 7, 4, . . . , is

(A) 97 (B) 77 (C) -77 (D) -87

$a_n \;= a + (n-1)d;$ d = a_2 - a_1 = 7 – 10 = -3

$a_{30} \;= 10 + (30-1)(-3)$

$= 10 + (29)(-3)$

$= 10 - 87$

$\mathbf{\Rightarrow a_{30} \;= -77}$

(ii) 11th term of an AP: -3, $-\frac{1}{2}$, 2, ... is

(A) 28 (B) 22 (C) -38 (D) $-48\frac{1}{2}$

$a_n = a + (n-1)d;$ d = a_2 - a_1 = $-\frac{1}{2} - (-3)$ = $-\frac{1}{2} + 3 = \frac{5}{2}$

$a_{11} \;= -3 + (11-1)\left[\frac{5}{2}\right]$

$= -3 + (10)\left[\frac{5}{2}\right]$

$= -3 + 25$

$\mathbf{\Rightarrow a_{11} \;= 22}$

3. In the following APs, find the missing terms in the boxes :

i)	2, □ 26	**Ans: 14**
ii)	□ 13 □ 3	**Ans: 18, 8**
iii)	5, □ □ $9\frac{1}{2}$	**Ans: $6\frac{1}{2}$, 8**
iv)	-4, □ □ □ □ 6	**Ans: −2, 0, 2, 4**
v)	□ 38, □ □ □ −22	**Ans: 53, 23, 8, -7**

4. Which term of the AP 3, 8, 13, 18 . . . is 78?

Solution: $a_n \;= a + (n-1)d$

d = 5; a = 3; a_n = 78; n = ?

$78 = 3 + (n-1)5$

$= 3 + 5n - 5$

$\Rightarrow 78 = 5n - 2$

$\Rightarrow 5n = 78 + 2$

$\Rightarrow 5n \;= 80$

$\Rightarrow n = 16$

5. Find the number of terms in each of the following APs :

i) 7, 13, 19 .... 205 (ii) 18, $15\frac{1}{2}$, 13 .... -4

i) 7, 13, 19 205

$a_n \;= a + (n-1)d$

d = 6; a = 7; a_n = 205; n = ?

$205 = 7 + (n-1)6$

$205 = 7 + 6n - 6$

$205 = 6n + 1$

$6n = 205 - 1$
$6n = 204$
$\Rightarrow n = \frac{204}{6}$
$\Rightarrow \mathbf{n = 34}$

(ii) 18, $15\frac{1}{2}$, 13 .... -47

$a_n = a + (n-1)d$
$d = -\frac{5}{2}$; $a = 18$; $a_n = -47$; $n = ?$
$-47 = 18 + (n-1)\left[-\frac{5}{2}\right]$
$-47 = 18 - \frac{5}{2}n + \frac{5}{2} = \frac{36-5n+5}{2}$
$\Rightarrow -47 = \frac{41-5n}{2}$
$\Rightarrow -94 = 41 - 5n$
$-5n = -94 - 41 = -135$
$\Rightarrow \mathbf{n = 27}$

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Solution: $a_n = a + (n-1)d$
$d = a_2 - a_1 = -3$; $a = 11$; $a_n = 150$; $n = ?$
$-150 = 11 + (n-1)(-3)$
$-150 = 11 - 3n + 3$
$-150 = 14 - 3n$
$-3n = -150 - 14 = -164$
$n = \frac{164}{3}$
'n' is not an integer. So, -150 is not a term of the AP: 11, 8, 5, 2, ..

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Solution: $a_n = a + (n-1)d$
$a_{11} = 38$, $a_{16} = 73$, $a_{31} = ?$
$a + (11-1)d = 38$
$a + 10d = 38$ -------------(1)
$a + (16-1)d = 73$
$a + 15d = 73$ --------------(2)
from (1) and (2)

$\mathbf{a + 10d = 38}$
$\mathbf{a + 15d = 73}$
-5d = -35

$d = \frac{-45}{-3} = 7$
(1) $\Rightarrow a + 10x7 = 38$
$\Rightarrow a + 70 = 38$
$\Rightarrow a = 38 - 70$
$\Rightarrow a = -32$
$a_{31} = -32 + (31-1)7$
$a_{31} = -32 + (30)7$
$a_{31} = -32 + 210$
$\mathbf{a_{31} = 178}$

Alternate Method:

$d = \frac{a_p - a_q}{p-q}$
$a_p = a_{16}$; $a_q = a_{11}$

$d = \frac{a_{16} - a_{11}}{16-11} = \frac{73-38}{5} = \frac{35}{5} = 7$

$a_n = a_p + (n - p)d$ a

$a_{31} = a_{16} + (31 - 16)7$

$a_{31} = 73 + (15)7$

$a_{31} = 73 + 105$

= **178**

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.50

$a + (n - 1)d = a_n$

$n = 50, a_3 = 12, a_n = 106$

$a + (50 - 1)d = 106$

$a + 49d = 106$ ----- (1)

$a + 2d = 12$ -------(2)

a + 49d = 106
a + 2d = 12
47d = 94

$\Rightarrow d = 2$

eqn (2)$\Rightarrow a + 2(2) = 12$

$a + 4 = 12$

$a = 12 - 4 = 8$

$a_{29} = 8 + (29 - 1)2$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56$

$a_{29} = 64$

Alternate Method:

$d = \frac{a_p - a_q}{p - q}$; $a_p = a_{50}$; $a_q = a_3$

$d = \frac{a_{50} - a_3}{50-3} = \frac{106-12}{47} = \frac{94}{47} = 2$

$a_n = a_p + (n - p)d$ a

$a_{29} = a_3 + (29 - 3)2$

$a_{29} = 12 + (26)2$

$a_{29} = 12 + 52$

$a_{29} = 64$

9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Solution: $a_3 = 4$, $a_9 = -8$

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$4 = a + 2d$ ------------(i)

$a_9 = a + (9 - 1)d$

$-8 = a + 8d$ ---------- (ii)

(i) - (ii),we get

$-12 = 6d$

$\Rightarrow d = -2$

$4 = a - 4$

$a = 8$

If $a_n = 0$,

$a_n = a + (n - 1)d$

$0 = 8 + (n - 1)(-2)$

$0 = 8 - 2n + 2$

$2n = 10$

n = 5

So, the 5^{th} term is 0

Alternate method:

d = $\frac{a_p - a_q}{p-q}$

$a_p = a_9$; $a_q = a_3$

d = $\frac{a_9 - a_3}{9-3}$ = $\frac{-8-4}{6}$ = $\frac{-12}{6}$ = -2

$a = a_p + (p-1)d$

$a = a_9 - (9-1)(-2)$

$a = -8 - (8)(-2)$

$a = -8 + 16 = 8$

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution:

$a_n = a + (n-1)d$

$a_{17} = a + (17-1)d$

$a_{17} = a + 16d$

Similarlly, $a_{10} = a + 9d$

But , $a_{17} - a_{10} = 7$

(a + 16d) − (a + 9d) = 7

7d = 7

d = 1

11.Which term of the AP: 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Solution:AP:3,15,27,39…

a = 3,d = 12

$a_{54} = a + (54-1)d$

$a_{54} = 3 + (53)(12)$

$a_{54} = 3 + 636 = 639$

132 + 639 = 771

Now ,a_n = 771.

$a_n = a + (n-1)d$

771 = 3 + (n − 1) 12

768 = (n − 1) 12

(n − 1) = 64

n = 65

∴ 65^{th} term is 132 more than54^{th} term.

Or

n'th term is 132 more than 54^{th} term.

n = 54 + $\frac{132}{12}$

= 54 + 11 = 65^{th} term.

12.Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Solution: Let the first terms of an AP's be a' and b'. Common difference – d

For the first AP,

$a_{100} = a + (100-1)d$

$a_{100} = a + 99d$

a_{1000} = a + (1000 − 1) d
a_{1000} = a + 999d
For 2nd AP,

a_{100} = b + (100 − 1) d
a_{100} = b + 99d
a_{1000} = b + (1000 − 1) d
a_{1000} = b+ 999d
The difference of 100th terms is 100

There for (a + 99d) − (b + 99d) = 100
a − b = 100 ------------------- (i)
The difference of 1000th terms is ?

(a + 999d) − (b + 999d) = a − b
From equation (i) ,

$a_1 - a_2 = 100$
So, the difference of 1000th terms is 100.

13. How many three-digit numbers are divisible by 7?

Solution: The first 3 digit number which is divisible by 7 is (a)= 105 and d = 7
The last 3 digit number which is divisible by 7 is (a_n) = 994

There for AP:105, 112, 119 …994
a_n = a + (n − 1) d
994 = 105 + (n − 1) 7⇒889 = (n − 1) 7
⇒(n − 1) = 127⇒ n = 128
There for 128 three digit numbers are divisible by 7.

Or

The 3-digit numbers which are divisible by 7are 105,112,119 994
These numbers are in AP:
a = 105 and d = 7, a_n = 994
⇒ a + (n - 1) d = 994
⇒ 105 + (n - 1) × 7 = 994
⇒7(n - 1) = 889
⇒ n - 1 = 127
⇒ n = 128

14. How many multiples of 4 lie between 10 and 250?

Solution: Multiples of 4 lie between 10 and 250 are
12, 16, 20, 24, …248
a = 12, d = 4, a_n = 248
a_n = a + (n - 1) d
248 = 12 + (n - 1) × 4
248 = 12 + 4n – 4 = 8 + 4n
⇒ 4n = 248 - 8 = 240

⇒ n =60
Hence, ther are 60 multiples of 4 lie between 10 and 250.

15. For what value of n, are the nth terms of two APs: 63, 65, 67 , . . . and 3, 10, 17, . . . equal?

Solution: a = 63, d = $a_2 - a_2$ = 65 − 63 = 2
a_n = a + (n − 1) d
a_n = 63 + (n − 1) 2 = 63 + 2n − 2
a_n = 61 + 2n ----------- (i)
3, 10, 17, … [a = 3, d = $a_2 - a_2$ = 10 − 3 = 7]
a_n = 3 + (n − 1) 7 = 3 + 7n – 7

$a_n = 7n - 4$ ------------ (ii)
According to qn , n^{th} term of both AP'^{s} are equal.
$\Rightarrow 61 + 2n = 7n - 4$
$\Rightarrow 61 + 4 = 5n$
$\Rightarrow 5n = 65$
$\Rightarrow n = 13$
Hence, the 13^{th} the two given AP'^{s} are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

$a_3 = 16 \Rightarrow a + (3 - 1)\,d = 16$
$a + 2d = 16$ ---------- (i)
$a_7 - a_5 = 12$
$[a + (7 - 1)\,d] - [a + (5 - 1)\,d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12 \Rightarrow$ **$d = 6$:** From equation (i),
$a + 2\,(6) = 16$
$\Rightarrow a + 12 = 16$
$\Rightarrow a = 4$
Then the AP is 4, 10, 16, 22, …

17. Find the 20th term from the last term of the AP: 3, 8, 13, . . ., 253.

Given AP: 3, 8, 13, …, 253
n^{th} term from the last = $l - (n - 1)d$
$l = 253,\ a = 3,\ d = 5$
n^{th} term from the last
$= 253 - (20 - 1)5$
$= 253 - (19)5$
$= 253 - 95$
$= 253 - 95$
$= 158$

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

$a_n = a + (n - 1)\,d$
$a_4 = a + (4 - 1)\,d$
$a_4 = a + 3d$
Similarlly,
$a_8 = a + 7d$; $a_6 = a + 5d$; $a_{10} = a + 9d$
But , $a_4 + a_8 = 24$
$a + 3d + a + 7d = 24$
$2a + 10d = 24$
$a + 5d = 12$ ------------(i)
$a_6 + a_{10} = 44$
$a + 5d + a + 910d = 44$
$2a + 14d = 44$
$a + 7d = 22$ -----------(ii)
By substracting (ii) from **(i)**,
$2d = 22 - 12 = 10$

d = 5
Substituting d = 5 in equation (i) ,

a + 5d = 12
a + 5 (5) = 12
a + 25 = 12
a = −13
a_2 = a + d = − 13 + 5 = −8
a_3 = a_2 + d = − 8 + 5 = −3
Hence the first three terms are
−13, −8, and −3.

19. Subbia Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

The annual salary received by Subba Rao in the years 1995 onwards are
5000, 5200, 5400,----7000
Hence, these numbers forms an AP.
a = 5000, d = 200, a_n = 7000.
a_n = a + (n − 1) d
7000 = 5000 + (n − 1) 200
200(n − 1) = 2000
(n − 1) = 10
n = 11
Thus the 11th years of his service or in 2005, Subba Rao received an annual salary of Rs 7000.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Solution:

a = 5, d = 1.75, a_n = 20.75, n = ?
a_n = a + (n − 1) d
20.75 = 5 + (n - 1) × 1.75
15.75 = (n - 1) × 1.75
15.75 = 1.75n – 1.75
1.75n = 15.75 + 1.75
1.75n = 17.50
$n = \frac{17.50}{1.75} = \frac{1750}{175}$
n = 10

1.4 Sum of First n Terms of an AP

$S = \frac{n}{2}[2a + (n - 1) d]$	**[First term – a, Common difference – d]**
$S = \frac{n}{2}[a + l]$	**[First term – a, last term - *l*]**

Example 11 : Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution:

Here a = 8, d = -5, n = 22.

$S = \frac{n}{2}[2a + (n - 1)d]$

$S = \frac{22}{2}[2x8 + (22 - 1)(-5)]$

S = 11[16 + 21(-5)]

S = 11[16 - 105]

S = 11x-89 **= -979**

Example 12 : If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Solution: Here,

S_{14} =1050,n = 14,a = 10

$S = \frac{n}{2}[2a + (n - 1)d]$

$1050 = \frac{14}{2}[2x10+(14-1)d]$

1050 = 7[20 + 13d]

1050 = 140 + 91d

91d = 1050 − 140

91d = 910

$d = \frac{910}{91} = 10$

$a_n = a + (n - 1) d$

a_{20} = 10 + (20 – 1)10

a_{20} = 10 + 19x10

a_{20} = 10 + 190

a_{20} = 200

Example 13 : How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Solution:a = 24, d = 21-24 = -3, Sn = 78 , We have to find 'n'

$S = \frac{n}{2}[2a + (n - 1)d]$

$78 = \frac{n}{2}[2x24 + (n - 1)(-3)]$

$78 = \frac{n}{2}[48 - 3n + 3]$

$156 = n[48 - 3n + 3] = 51n - 3n^2$

$52 = 17n - n^2$

$n^2 – 17n + 52 = 0$

$n^2 – 13n – 4n + 52 = 0$

n(n – 13) – 4(n – 13) = 0

(n – 13) (n – 4) = 0

n = 13 OR n = 4

Example 14 : Find the sum of :

(i) the first 1000 positive integers (ii) the first n positive integers

Solution:

(i) **Let S = 1 + 2 + 3 +...+ 1000**

$S = \frac{n}{2}[2a + (n - 1)d]$

S = 500[2 + 999]

S = 500[1001]

S = 500500

(ii) Let S = 1 + 2 + 3 +...... + n

S = $\frac{n}{2}$[2a + (n - 1)d]

S = $\frac{n}{2}$[2x1 + (n - 1)1]

S = $\frac{n}{2}$[2 + n - 1]

S = $\frac{n}{2}$[n + 1]

Example 15 : Find the sum of first 24 terms of the list of numbers whose nth term is given by a_n = 3 + 2n.

Solution:

a_n = 3 + 2n

a_1 = 3 + 2x1 = 3 + 2 = 5

a_2 = 3 + 2x2 = 3 + 4 = 7

a_3 = 3 + 2x3 = 3 + 6 = 9

There for AP is : 5, 7, 9, – – –

a = 5, d = 2, n = 24

S = $\frac{n}{2}$[2a + (n - 1)d]

S = $\frac{24}{2}$[2x5 + (24 - 1)2]

S = 12[10 + 23x2]

S = 12[10 + 46]

S = 12x56

S = 672

Example 16 : A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years

Solution:i) Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1st, 2nd, 3rd . . . years will form an AP.

Let us denote the number of TV sets manufactured in the n^{th} year by a_n

a_3 = 600, a_7 = 700,

a + 2d = 600

a + 6d = 700

By solving the equation we get,

d = 25 and a = 550

Therefore, production

(i) of TV sets in the first year is = 550

(ii) Production of TV sets in the 10th year is : a_{10} = a + 9d

a_{10} = 550 + 9x25 = 550 + 225 = **775**

(iii) The total production of TV sets in first 7 years is

S = $\frac{n}{2}$[2a + (n - 1)d]

S = $\frac{7}{2}$[2x550 + (7 - 1)25]

S = $\frac{7}{2}$[1100 + 6x25]

S = $\frac{7}{2}$[1100 + 150]

S = $\frac{7}{2}$[1250]
S = 7x625
S = 4375

Exercise 5.3

1. Find the sum of the following APs:

i) 2,7,12 ... to 10 terms ii) -37, -33,-29... to 12 terms iii) 0.6, 1.7, 2.5 ... to 100 terms iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ ---------- to 11 terms

i) 2,7,12 ... to 10 terms

a = 2, d = $a_2 - a_1$ = 7 − 2 = 5, n = 10
$S_n = \frac{n}{2}$ [2a + (n - 1) d]
$S_{10} = \frac{10}{2}$ [2(2) + (10 - 1) × 5]
S_{10} = 5[4 + (9) × (5)]
S_{10} = 5[4 + 45]
S_{10} = 5 × 49 = 245
S_{10} = 245

ii) -37, -33, -29 ...... to 12 terms

a = −37; d = 4; n = 12
$S_n = \frac{n}{2}$ [2a + (n - 1) d]
$S_{12} = \frac{12}{2}$ [2(-37) + (12 - 1) × 4]
S_{12} = 6[-74 + 11 × 4]
S_{12} = 6[-74 + 44]
S_{12} = 6(-30) **= -180**

iii) 0.6, 1.7, 2.5 ..... to 100 terms

a = 0.6; d = 1.1; n = 100
$S_n = \frac{n}{2}$ [2a + (n - 1) d]
$S_{100} = \frac{100}{2}$ [1. + (99) × 1.1]
S_{100} = 50[1.2 + 108.9]
S_{100} = 50[110.1] 2
S_{100} = 5505

iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ --------- to 11 terms

a = $\frac{1}{15}$; d = $\frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$; n = 11
$S_n = \frac{n}{2}$ [2a + (n - 1) d]
$S_{11} = \frac{11}{2}$ [2x$\frac{1}{15}$ + (11 – 1) × $\frac{1}{60}$]
$S_{11} = \frac{11}{2}$ [$\frac{2}{15}$+ $\frac{10}{60}$]
$S_{11} = \frac{11}{2}$ [$\frac{8+10}{60}$]
$S_{11} = \frac{11}{2}$ [$\frac{18}{60}$]
$S_{11} = \frac{11}{2}$ [$\frac{3}{10}$]

$S_{11} = \frac{33}{20}$

2. Find the sums given below :

i) $7 + 10\frac{1}{2} + 14 + \text{----} + 84$ (ii) $34 + 32 + 30 + \text{---} + 10$ (iii) $-5 + (-8) + (-11) + \text{----} + (-230)$

i) $7 + 10\frac{1}{2} + 14 + \text{--} + 84$

$a = 7;\ l = 84;\ d = \frac{7}{2}$

$l = a + (n - 1)d$

$84 = 7 + (n - 1) \times \frac{7}{2}$

$77 = (n - 1) \times \frac{7}{2}$

$154 = 7n - 7$

$7n = 161$

$n = 23$

$S_n = \frac{n}{2}(a + l)$

$S_{23} = \frac{23}{2}(7 + 84)$

$= \frac{23}{2} \times 91 = \frac{2093}{2}$

$= 1046\frac{1}{2}$

ii) $34 + 32 + 30 + \text{---} + 10$

$a = 34,\ d = -2,\ l = 10$

$l = a + (n - 1)d$

$10 = 34 + (n - 1)(-2)$

$-24 = (n - 1)(-2)$

$12 = n - 1$

$n = 13$

$S_n = \frac{n}{2}(a + l)$

$S_{13} = \frac{13}{2}(34 + 10)$

$S_{13} = \frac{13}{2} \times 44 = 13 \times 22$

$S_{13} = 286$

iii) $-5 + (-8) + (-11) + \dots + (-230)$

$a = -5,\ l = -230,$

$d = a_2 - a_1 = -3$

$l = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

$S_n = \frac{n}{2}(a + l)$

$S_{76} = \frac{76}{2}[(-5) + (-230)]$

$S_{76} = 38(-235)$

$S_{76} = -8930$

3. In an AP:

i) Given $a = 5$, $d = 3$, $a_n = 50$ find n and Sn

ii) Given $a = 7$, $a_{13} = 35$ find d and S_{13}

iii) Given $a_{12} = 37$, $d = 3$ find É a and S_{12}

iv) Given $a_3 = 15$, $S_{10} = 125$ find d and a_{10}

v) Given $d = 5$, $S_9 = 75$ find a and a_9

vi) Given a = 2, d = 8, Sn = 90 find n and a_n
vii) Given a = 8, an = 62, Sn = 210 find n and d
viii)Given an = 4, d = 2, Sn = -14 findÉ n and a
ix) Given a = 3, n = 8, S = 192 find d
x) Given l = 28, S = 144 and there are 9 terms. Find the value of a

Solution:

i) Given a = 5, d = 3, a_n = 50 find n and Sn

a = 5, d = 3, a_n = 50
$a_n = a + (n - 1)d$,
⇒ 50 = 5 + (n - 1) × 3
⇒ 3(n - 1) = 45
⇒ n - 1 = 15
⇒ n = 16
$S_n = \frac{n}{2}(a + a_n)$
$S_{16} = \frac{16}{2}(5 + 50) = 440$
$S_{16} = 8\ (55)$
$S_{16} = 440$

ii) Given a = 7 , a_{13} = 35 find d and S_{13}

a = 7, a_{13} = 35
$a_n = a + (n - 1)d$,
⇒ 35 = 7 + (13 - 1)d
⇒ 12d = 28
⇒ d = 28/12 = 2.33
$S_n = \frac{n}{2}(a + a_n)$
$S_{13} = \frac{13}{2}(7 + 35)$
$S_{13} = \frac{13}{2}(42) = 13x21$
$S_{13} = 273$

iii) Given a_{12} = 37, d = 3 find a and S_{12}

a_{12} = 37, d = 3
$a_n = a + (n - 1)d$,
⇒ $a_{12} = a + (12 - 1)3$
⇒ 37 = a + 33
⇒ a = 4
$S_n = \frac{n}{2}(a + a_n)$)
$S_{12} = \frac{12}{2}(4 + 37)$
$S_{12} = 6\ (41)$
$S_{12} = 246$

iv) Given a_3 = 15, S_{10} = 125 find d and a_{10}

$a_3 = 15,\ S_{10} = 125$
$a_n = a + (n - 1)d$,
$a_3 = a + (3 - 1)d$
15 = a + 2d ----------- (i)
$S_n = \frac{n}{2}[2a + (n - 1)\ d]$
$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$
125 = 5(2a + 9d)

$25 = 2a + 9d$ -------- (ii)

(i) x 2

$30 = 2a + 4d$ ------- (iii)

(iii) – (ii)

$-5 = 5d$

$d = -1$

From equation **(i)**,

$15 = a + 2(-1) = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9$

$a_{10} = 8$

v) Given $d = 5, S_9 = 75$ find a and a_9

$d = 5, S_9 = 75$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$75 = \frac{9}{2}[2a + (9 - 1)5]$

$75 = \frac{9}{2}(2a + 40) = 9(a + 20)$

$75 = 9a + 180 \Rightarrow 9a = 75 - 180$

$9a = \frac{-105}{3} \Rightarrow a = \frac{-35}{3}$

$a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)5 = \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$

vi) Given a = 2, d = 8, Sn = 90 find n and a_n

$a = 2, d = 8, S_n = 90$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$90 = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 180 = n(4 + 8n - 8)$

$\Rightarrow 180 = n(8n - 4)$

$\Rightarrow 180 = 8n^2 - 4n$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$

$\Rightarrow (2n - 9)(2n + 9) = 0$

n = 5 (Positive number)

There for $a_5 = 8 + 5 \times 4 = 34$

vii) Given a = 8, an = 62, Sn = 210 find n and d

$a = 8, a_n = 62, S_n = 210$

$S_n = \frac{n}{2}(a + a_n)$

$210 = \frac{n}{2}(8 + 62)$

$\Rightarrow 35n = 210$

$\Rightarrow n = \frac{210}{35} = 6$

a_n = a + (n – 1)d
62 = 8 + 5d
⇒ 5d = 62 - 8 = 54
⇒ d = $\frac{54}{5}$
⇒ d = 10.8

viii) Given an = 4, d = 2, Sn = -14 find n and a

$a_n = 4$, d = 2, $S_n = -14$
a_n = a + (n − 1)d
4 = a + (n − 1)2
4 = a + 2n − 2
a + 2n = 6
a = 6 − 2n ---------- **(i)**
$S_n = \frac{n}{2}(a + a_n)$
-14 = $\frac{n}{2}$(a + 4)
−28 = n (a + 4)
−28 = n (6 − 2n + 4)From **(i)**
−28 = n (− 2n + 10)
$-28 = -2n^2 + 10n$
$2n^2 - 10n - 28 = 0$
$n^2 - 5n - 14 = 0$
$n^2 - 7n + 2n - 14 = 0$
n (n − 7) + 2(n − 7) = 0
(n − 7) (n + 2) = 0
n − 7 = 0 or n + 2 = 0
n = 7 or ~~n = −2~~
From equation **(i)** ,
a = 6 − 2n
a = 6 − 2(7)
a = 6 − 14
a = −8

ix) Given a = 3, n = 8, S = 192 find d

a = 3, n = 8, S = 192
$S_n = \frac{n}{2}[2a + (n - 1)d]$
192 = $\frac{8}{2}$[2 × 3 + (8 - 1)d]
192 = 4 [6 + 7d]
48 = 6 + 7d
42 = 7d
d = 6

x) Given l = 28, S = 144 and there are 9 terms. Find the value of a

l = 28, S = 144, n = 9
$S_n = \frac{n}{2}(a + l)$
144 = $\frac{9}{2}$(a + 28)
(16) × (2) = a + 28
32 = a + 28
a = 4

4. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

$a = 9$; $d = a_2 - a_1 = 17 - 9 = 8$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$636 = \frac{n}{2}[2 \times a + (8 - 1) \times 8]$
$636 = \frac{n}{2}[18 + (n - 1) \times 8]$
$636 = n[9 + 4n - 4]$
$636 = n(4n + 5)$
$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
$n(4n + 53) - 12(4n + 53) = 0$
$(4n + 53)(n - 12) = 0$
$4n + 53 = 0$ or $n - 12 = 0$
$n =$ ~~(-53/4)~~ or $n = 12$
$\Rightarrow n = 12$

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

$a = 5$, $l = 45$, $S_n = 400$
$S_n = \frac{n}{2}(a + l)$
$400 = \frac{n}{2}(5 + 45)$
$400 = \frac{n}{2}(50)$
$25n = 400$
$n = 16$
$l = a + (n - 1)d$
$45 = 5 + (16 - 1)d$
$40 = 15d$
$d = \frac{40}{15}$
$d = \frac{8}{3}$

6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

$a = 17$, $l = 350$, $d = 9$
$l = a + (n - 1)d$
$350 = 17 + (n - 1)9$
$333 = (n - 1)9$
$(n - 1) = 37 \Rightarrow n = 38$
$S_n = \frac{n}{2}(a + l)$
$S_{38} = \frac{38}{2}(17 + 350)$
$S_{38} = 19 \times 367$
$\mathbf{S_{38} = 6973}$

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

$d = 7$, $a_{22} = 149$, $S_{22} = ?$
$a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$
$149 = a + 21 \times 7$
$149 = a + 147$
$a = 2$

$S_n = \frac{n}{2}(a + a_n)$

$S_{22} = \frac{22}{2}(2 + 149)$

$S_{22} = 11 \times 151$

$S_{22} = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

$a_2 = 14$, $a_3 = 18$, $d = 4$

$a_2 = a + d$

$14 = a + 4$

$\Rightarrow a = 10$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{51} = \frac{51}{2}[2\times10 + (51-1)4]$

$= \frac{51}{2}[20 + (50) \times 4]$

$= \frac{51}{2}[20 + 200] = \frac{51}{2}[220]$

$= 51 \times 110$

= 5610

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

$S_7 = 49$, $S_{17} = 289$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_7 = \frac{7}{2}[2a + (n - 1)d]$

$S_7 = \frac{7}{2}[2a + (7 - 1)d]$

$49 = \frac{7}{2}[2a + 6d]$

$7 = (a + 3d)$

$a + 3d = 7$ ----------- **(i)**

Similarlly,

$S_{17} = \frac{17}{2}[2a + (17 - 1)d]$

$289 = \frac{17}{2}(2a + 16d)$

$17 = (a + 8d)$

$a + 8d = 17$ --------- **(ii)**

Substract equation (ii) from (i)

$5d = 10$

$\Rightarrow d = 2$

From equation **(i)**

$a + 3(2) = 7$

$a + 6 = 7$

$\Rightarrow a = 1$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_n = \frac{n}{2}[2(1) + (n - 1) \times 2]$

$S_n = \frac{n}{2}(2 + 2n - 2)$

$S_n = \frac{n}{2}(2n)$

$S_n = n^2$

10. Show that $a_1, a_2, a_3 \ldots a_n \ldots$ form an AP where a_n is defined as below

(i) a_n=3+4n (ii) a_n= 9–5n Also, find the sum of the first 15 terms in each case.

(i) a_n= 3 + 4n

$a_1 = 3 + 4(1) = 7$

$a_2 = 11; a_3 = 15; a_4 = 19$

$\Rightarrow a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4; a_4 - a_3 = 19 - 15 = 4$

So, the given sequence forms an AP with a =7 and d = 4

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{15} = \frac{15}{2}[2(7) + (15-1)4]$

$= \frac{15}{2}[(14)+56]$

$= \frac{15}{2}(70)$

$= 15 \times 35$

$= 525$

(ii) a_n= 9 – 5n

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

ll^y a3 = $-6, a_4 = -11$

$\Rightarrow a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5; a_4 - a_3 = -5$

So, the given sequence forms an AP with a = 4 and d = -5

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{15} = \frac{15}{2}[2(4)+(15-1)(-5)]$

$S_{15} = \frac{15}{2}[8 + 14(-5)]$

$S_{15} = \frac{15}{2}(8 - 70)$

$S_{15} = \frac{15}{2}(-62)$

$S_{15} = 15(-31)$

$S_{15} = -465$

11.If the sum of the first n terms of an AP is 4n – n^2, what is the first term (that is S)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n^{th} terms.

$S_n = 4n - n^2$

$a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

$a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

n^{th} term $a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

So, the third term $a_3 = 5 - 2(3)$

$= 5 - 6 = -1$

10^{th} term $a_{10} = 5 - 2(10)$

$= 5 - 20$

$= -15$

12. Find the sum of the first 40 positive integers divisible by 6.

The positive integers divisible by 6 are 6, 12, 18, 24 …
This is an AP with d = 6 and a = 6
S_{40} = ?
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{40} = \frac{40}{2}[2(6) + (40 - 1) 6]$
= 20[12 + (39) (6)]
= 20(12 + 234) = 20 × 246
= 4920

13. Find the sum of the first 15 multiples of 8.

The numbers multiples of 8 are
8, 16, 24, 32…
These form an AP with d = 8 and a =8
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{15} = \frac{15}{2}[2(8) + (15 - 1)8]$
$= \frac{15}{2}[6 + (14) (8)]$
$= \frac{15}{2}[16 + 112]$
$= \frac{15}{2}(128)$
= 15 × 64
= 960

14. Find the sum of the odd numbers between 0 and 50.

The odd numbers between 0 and 50
1, 3, 5, 7, 9 … 49
This is an AP with a = 1 and d = 2
a = 1, d = 2, l = 49
$l = a + (n - 1) d$
49 = 1 + (n − 1)2
48 = 2(n − 1)
n − 1 = 24
⇒ n = 25
$S_n = \frac{n}{2}(a + l)$
$\Rightarrow S_{25} = \frac{25}{2}(1 + 49)$
$S_n = \frac{25}{2}(50)$
= (25)(25)
= **625**

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

This is an AP with common difference 50 and the first term 200 [a = 200, d = 50]
The penalty payable for the delay of 30 days = S_{30}
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_{30} = \frac{30}{2}[2(200) + (30 - 1) 50]$
$\Rightarrow S_{30}$ = 15 [400 + 1450]
= 15 (1850)
= **Rs 27750**

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Let the first prize = a

The amount of 2^{nd} prize = a − 20

The amount of 3^{rd} prize = a − 40

This is an AP with d = -20 and a = a

d = −20, S_7 = 700

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\frac{7}{2}[2a + (7 - 1)d] = 700$

$\frac{7}{2}[2a + 6d] = 700$

7 [a + 3d] = 700

a + 3d = 100

a + 3(−20) = 100

a − 60 = 100

a = 160

So, the values of prizes Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution: 1, 2, 3, 4, 5………………..12

This is an AP with common difference 1 and the first term 1

a = 1, d = 2 − 1 = 1

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{12} = \frac{12}{2}[2(1) + (12 - 1)(1)]$

= 6 (2 + 11)

= 6 (13)

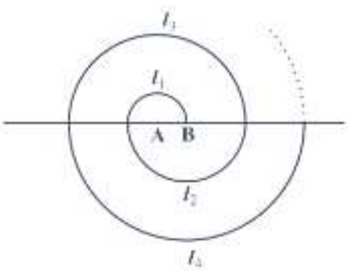
= 78

Hence, the trees planted by the students of each section = 78

There for the trees planted by the students of 3 sections = 78 x 3 = 234

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 1.4. What is the total length of such a spiral made up of thirteen consecutive semi circles (Take $\pi = \frac{22}{7}$)

[Hint:length of successive semi circles is l_1, l_2, l_3, l_4...with centers as A, B,A,B ...l_1, l_2, l_3, l_4...]



The length of the semi circles = πr

$l_1 = \pi(0.5) = \frac{\pi}{2}$ cm; $l_2 = \pi(1) = \pi$ cm; $l_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

∴ The lengths of semicircles are $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π, ….

$d = l_2 - l_1 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$; $a = \frac{\pi}{2}$cm

$S_n = \frac{n}{2}[2a + (n - 1)d]$

The total length of such a spiral made up of thirteen consecutive semi circles = S_{13}

$S_{13} = \frac{13}{2}[2x\frac{\pi}{2} + (13 - 1)\frac{\pi}{2}]$

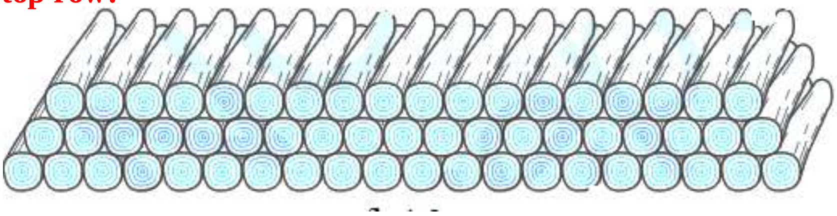
$= \frac{13}{2}[\pi + 6\pi]$

$= \frac{13}{2}(7\pi)$

$= \frac{13}{2} \times 7 \times \frac{22}{2}$

= 143 cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 1.5). In how many rows are the 200 logs placed and how many logs are in the top row?



The logs are in an AP: 20, 19, 18…

with a = 20, $d = a_2 - a_1 = 19 - 20 = -1$

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$200 = \frac{n}{2}[2(20) + (n - 1)(-1)]$

$200 = \frac{n}{2}[40 - n + 1]$

$\Rightarrow 400 = n(40 - n + 1)$

$\Rightarrow 400 = n(41 - n)$

$\Rightarrow 400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

$(n - 16) = 0$ or $n - 25 = 0$

n = 16 or n = 25

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16} = 20 - 15 = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1) = 20 - 24$

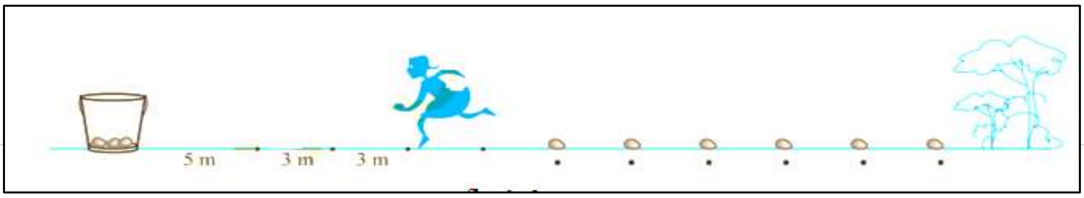
$= -4$ (negetive number is not possible)

Hence the number of rows is 16 and the number of logs in the top row is 5

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 5.6).

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 × 5 + 2 × (5 + 3)]



The distances from the bucket to potatos 5, 8, 11, 14…
They have to run twice, then the distacnes run by the competiror 10, 16, 22, 28, 34,……….
a = 10, d = 16 – 10 = 6, S_{10} =?
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{10} = \frac{10}{2}[2(10) + (10 - 1)(6)]$
= 5[20 + 54]
= 5 (74)
= 370
Hence, the distance the competitor has to run is 370km

Summery:

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference
- The general form of an AP : a, a + d, a + 2d, a + 3d......
- In an AP if there are only a finite number of terms. Such an AP is called a finite AP. Such AP has a last term.
- The AP has infinite number of terms is called infinite Arithmetic Progression. Such APs do not have a last term.
- The first term – a and the common difference is d then the nth term of an AP
  $a_n = a + (n - 1)d$
- The nth term from the last [last term – l , common difference – d]
  $l - (n - 1)d$
- a is the first term, d is the common difference then sum to nth term
  $S = \frac{n}{2}[2a + (n - 1)d]$
- If common difference is unknown then the sum to nth term
  $S = \frac{n}{2}[a + l]$ { l – the last term }

6 Triangles

2.2 Similar Figures

Two polygons of the same number of sides are similar, if

All the corresponding angles are equal and

All the corresponding sides are in the same ratio (or proportion).

EXERCISE 6.1

1. Fill in the blanks using the correct word given in brackets

i) All circles are ______________ (congruent, similar)

ii) All squares are _____________(similar, congruent)

iii) All_____________triangles are similar. (isosceles, equilateral)

iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are____________and (b) their corresponding sides are_____________(equal, proportional)

Ans: i) Similar (ii) Similar (iii) equilateral (iv) equal , proportional

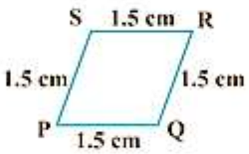
2. Give two different examples of pair of

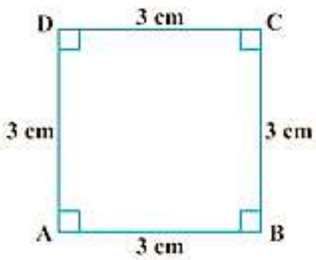
(i) similar figures. (ii) non-similar figures.

Ans: (i) Pair of circles; Pair of squares

(ii) A triangle and a square; A rectangle and a Quadrilateral

3. State whether the following quadrilaterals are similar or not:





Ans: The corresponding angles are not equal. Hence, they are not similar

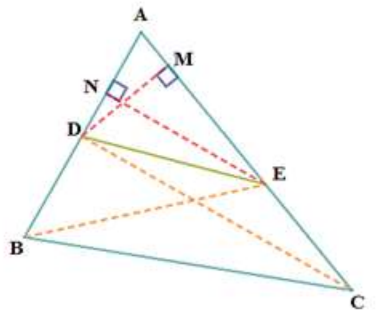
2.3 Similarity of Triangles [Basic proportionality theorem[Thales theorem]

Thale's theorem

Theorem 6.1

Basic proportionality theorem[Thales theorem

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio



Data: In ΔABC, the line drawn parallel to BC intersects AB and AC at D and E .

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD. Draw DM⊥AC and EN⊥AB.

Proof :

$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ --- (1)[∵Area of triangle = $\frac{1}{2}$ x Base x Height]

$\frac{Area(\Delta ADE)}{Area\ (\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ --- (2)[∵Area of triangle = $\frac{1}{2}$ x Base x Height]

ΔBDE and ΔDEC stand on the same base DE and in between BC∥DE

∴ Area (ΔBDE) = Area (ΔDEC) --- (3)

∴ From (1), (2) and (3),

$\frac{AD}{DB} = \frac{AE}{EC}$

Theorem 6.2

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Example 1 :If a line intersects sides AB and AC of a ABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$ (See fig 6.13)

Solution: DE∥BC (Data)

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ (Theorem 2.1)

Or $\frac{DB}{AD} = \frac{EC}{AE}$

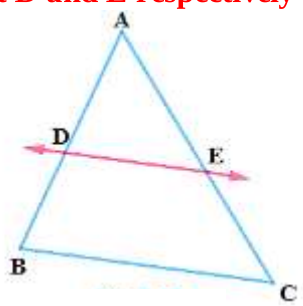


Fig. 6.13

$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$\frac{AB}{AD} = \frac{AC}{AE}$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ ($\because$ Taken reciprocals)

Example 2 :ABCD is a trapezium with AB || DC.E and F are points on non parallel sides AD and BC respectively such that EF is parallel to (fig 6.14) Show that $\frac{AE}{ED} = \frac{BF}{FC}$

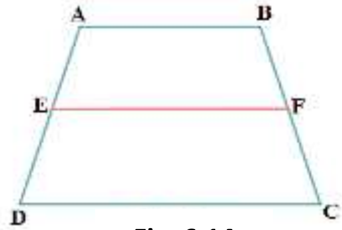


Fig. 6.14

solution: Join AC to intersect EF at G. (See fig 6.15)

AB∥DC and EF∥AB (Given)

So, EF ∥ DC

(Lines parallel to the same line are parallel to each other)

Now in ΔADC, EG∥DC ($\because$EF∥DC)

$\therefore \frac{AE}{ED} = \frac{AG}{GC}$ (Theorem 2.1) -----(1)

Similarly, from ΔCAB,

$\frac{CG}{AG} = \frac{CF}{BF}$

i.e., $\frac{AG}{GC} = \frac{BF}{FC}$ -----(2)

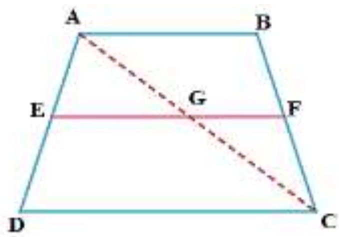


Fig. 6.15

$\therefore$ From (1), (2) axiom (1), $\frac{AE}{ED} = \frac{BF}{FC}$

Example 3 : In Fig. 2.16, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ prove that ΔPQR is an isosceles triangle.

Solution: Given that, $\frac{PS}{SQ} = \frac{PT}{TR}$

$\therefore$ ST∥QR ($\because$Theorem 2.2)

⇒∠PST = ∠PQR ($\because$Corresponding angles) ----(1)

But, ∠PST = ∠PRQ ($\because$ Given) -------(2)

So, ∠PRQ = ∠PQR [from (1), (2) and axiom (1)]

Therefore, PQ = PR ($\because$Sides opposite the equal angles)

i.e., PQR is an isosceles triangle.

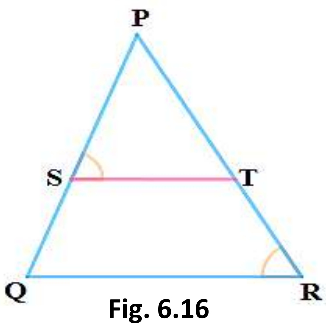


Fig. 6.16

EXERCISE 6.2

1. In Fig. 2.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i) In triangle△ABC, DE∥BC (Given)

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Thales theorem]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow EC = \frac{3x1}{1.5} = \frac{30}{15}$

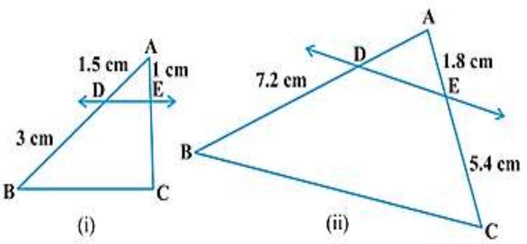


Fig. 6.17

⇒ EC = 2 cm.

(ii) In △ABC, DE∥BC (Given)

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [by BPT]

$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow AD = \frac{1.8x7.2}{5.4}$

$\Rightarrow AD = \frac{18x7.2}{54}$

⇒ AD = **2.4 cm.**

2. E and F are points on the sides PQ and PR respectively of a PQR. For each of the following cases, state whether EF || QR :

(i) PE = 3.9cm EQ = 3cm PF = 3.6cm FR = 2.4cm

(ii) PE = 4cm QE = 4.5cm PF = 8cm RF = 9cm

(iii) PQ = 1.28cm PR =2.56cm PE = 0.18cm PF = 0.36cm

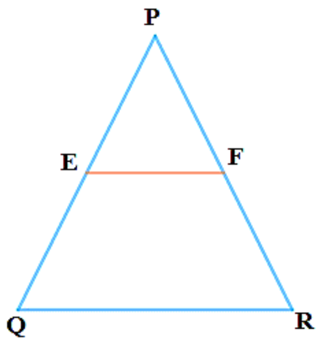
Solution:

(i) PE = 3.9 cm, EQ = 3 cm ,PF = 3.6 cm, FR = 2,4 cm

$\therefore \frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{3}$ = 1.3 [Thales theorem]

And $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{36}{24} = 1.5$

Therefore , $\frac{PE}{EQ} \neq \frac{PF}{FR}$



Hence, EF is not parallel to QR

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8cm, RF = 9cm

$\therefore \frac{PE}{QE} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$ [Thales theorem]

And, $\frac{PF}{RF} = \frac{8}{9}$

Therefore , $\frac{PE}{QE} = \frac{PF}{RF}$

Hence, EF∥QR

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

Here, EQ = PQ - PE = 1.28 - 0.18 = 1.10 cm

And, FR = PR - PF = 2.56 - 0.36 = 2.20 cm

$\therefore \frac{PE}{QE} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$ ... (i)

and, $\frac{PE}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ ... (ii)

$\therefore \frac{PE}{QE} = \frac{PE}{FR}$

There fore, EF∥QR

3. In Fig. 6.18, if LM || CB and LN || CD, prove that

$\frac{AM}{AB} = \frac{AN}{AD}$

In the given fig, LM || CB,

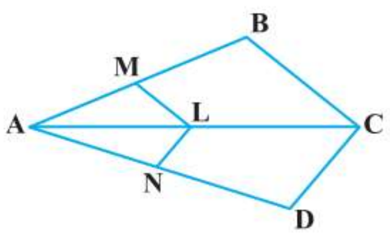


Fig. 6.18

$\frac{AM}{AB} = \frac{AL}{AC}$ ---- (i) [corollary of BPT]

Similarlly, LN || CD,

$\therefore \frac{AN}{AD} = \frac{AL}{AC}$---- (ii) [corollary of BPT]

From (i) and (ii) , $\frac{AM}{MB} = \frac{AN}{AD}$

4. In Fig. 6.19, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$

In ΔABC, DE || AC (Given)

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ --------(1) [Thales theorem]

In ΔABC, DF || AE (Given)

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ -----------(2) [Thales theorem]

From equation **(i)** and **(ii)**

$\frac{BE}{EC} = \frac{BF}{FE}$

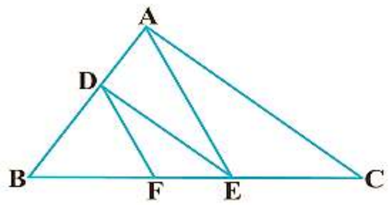


Fig. 6.19

5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR

In ΔPQO, DE || OQ (Given)

$\therefore \frac{PD}{DO} = \frac{PE}{EQ}$ -------------- (1) [Thales theorem]

In ΔPOR, DF || OR (Given)

$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ ---------------(2) [Thales theorem]

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$ [From equation **(1)** and **(2)**]

∴ In ΔPQR, EF || QR. [Converse of BPT]

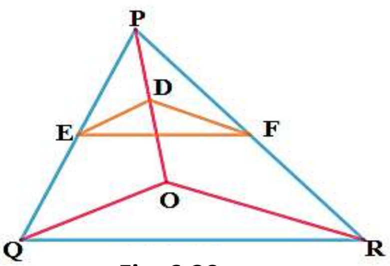


Fig. 6.20

6. In Fig. 2.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

In ΔOPQ, AB || PQ (Given)

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ -------------------(1) [Thales theorem]

In ΔOPR, AC || PR (Given)

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ -------------------(2) [Thales theorem]

$\frac{OB}{BQ} = \frac{OC}{CR}$ [From equation **(1)** and **(2)**]

∴ ΔOQR ನಲ್ಲಿ, BC || QR. [Converse of BPT]

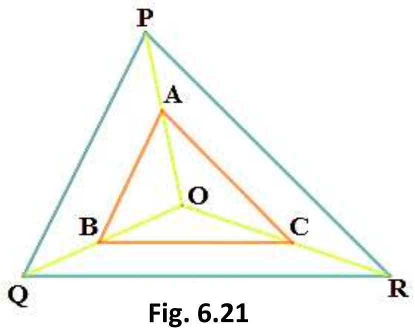


Fig. 6.21

7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Data: In ΔABC, D is the mid-point of AB⇒ AD=DB.

The parallel line DE to BC drawn from D intersects AC at E

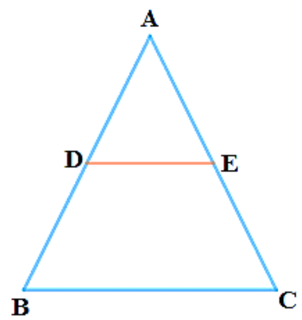
To prove: E is the mid-point of AC.

Proof: D is the mid-point of AB.

$\therefore$ AD = DB $\Rightarrow \frac{AD}{BD} = 1$ ----------------(1)

In ΔABC, DE || BC,

$\therefore \frac{AD}{BD} = \frac{AE}{EC}$ [Thales theorem]



$\Rightarrow 1 = \frac{AE}{EC}$ [From equation (1)]

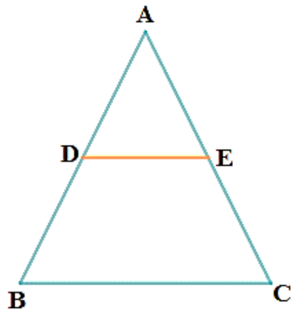
$\therefore$ AE =EC

$\Rightarrow$ E is the mid-point of AC

8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Data: In ΔABC, D and E are the mid-points of AB and AC

$\Rightarrow$AD=BD and AE=EC.



To prove: DE || BC

Proof: D is the mid-point of AB (Given)

$\therefore$ AD = DB $\Rightarrow \frac{AD}{BD} = 1$ ------------------------ (1)

E is the mid-point of AC (Given)

$\therefore$ AE=EC

$\Rightarrow \frac{AE}{EC} = 1$ --------------------------(2)

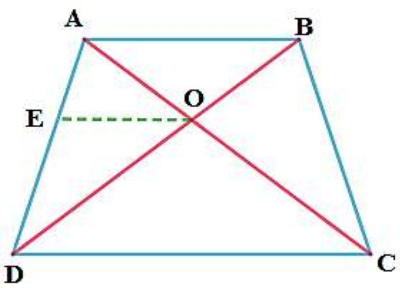
$\frac{AD}{BD} = \frac{AE}{EC}$ [From equation **(1)** and **(2)]**

$\therefore$ DE || BC [By BPT]

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{CO} = \frac{CO}{DO}$

Data: In trapezium ABCD, AB∥BC, and AC and BD intersects each other at O.

To Prove: $\frac{AO}{CO} = \frac{CO}{DO}$



Construction: Draw EO from O such that EO || DC || AB

Proof: In ΔADC, OE || DC (Construction)

$\frac{AE}{ED} = \frac{AO}{CO}$ -------------------------(1) [By BPT]

In ΔABD, OE || AB (Construction)

$\frac{ED}{AE} = \frac{DO}{BO}$ [By BPT]

$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO}$ ----------------------(2) [Taken reciprocals]

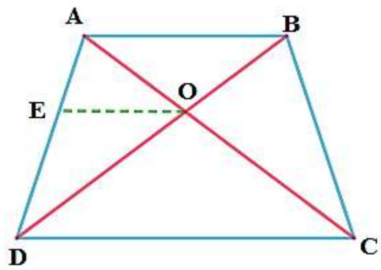
$\frac{AO}{CO} = \frac{BO}{DO}$ [From equation (1) and (2)]

$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium

Given: In ABCD, AC and BD intersects at O

Such that $\frac{AO}{BO} = \frac{CO}{DO}$



To prove: ABCD is a trapezium

Construction: Draw EO through O such that EO || AB which Intersects AD at E

Proof: In ΔDAB, EO || AB

$\therefore \frac{ED}{AE} = \frac{DO}{BO}$ [BPT]

$\Rightarrow \frac{AE}{\mathbf{ED}} = \frac{BO}{DO}$ ------------------------ (1) [taken reciprocals]

Similarlly, $\frac{AO}{\mathbf{BO}} = \frac{CO}{DO}$ (Given)

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$ ------------(2)

$\therefore \frac{AE}{\mathbf{ED}} = \frac{AO}{CO}$ [From equation **(1)** and **(2)]**

EO || DC and EO || AB [converse of BPT]

⇒ AB || DC. ∴ABCD is a trapezium

2.4 Criteria for Similarity of Triangles

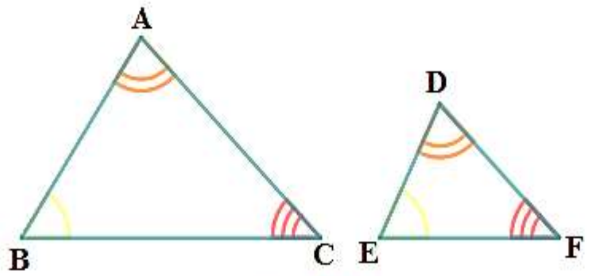
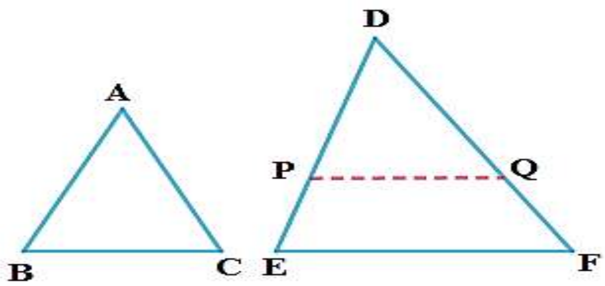


Fig. 6.22

It must be noted that as done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF of Fig. 6.22,
we cannot write ABC ~ EDF or ABC ~ FED. However, we can write BAC ~ EDF

Theorem 6.3 AAA

If in two triangles, corresponding angles are equal, then their Corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.



Data: In ΔABC and ΔDEF,

∠A = ∠D, ∠B = ∠E and ∠C = ∠F

To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (<1) and ΔABC~ ΔDEF

Construction: Cut DP = ABfrom DE &

DQ = AC from DF and join PQ

Proof: In ΔABC and ΔDPQ,

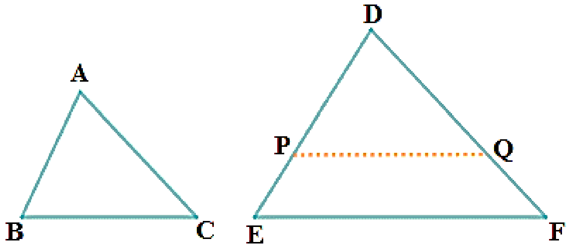
AB = DP [Construction]
AC = DQ [Construction]
∠A = ∠D [data]

∴ΔABC ≅ ΔDPQ [SAS Congruency rule]
⇒BC = PQ ---------(1) and
⇒ ∠B = ∠P [CPCT] But ∠B = ∠E [Given]
∴ ∠P = ∠E
∴**PQ∥EF** [corresponding angles are equal]
∴ $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$ [by Corolary of BPT]
⇒$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [By construction and (1)]
∴ΔABC~ ΔDEF

Theorem 6.4 SSS

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.



Data: In ΔABC and ΔDEF,
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ (<1) ----------(1)
To Prove: ∠A = ∠D, ∠B = ∠E and ∠C = ∠F
And ΔABC ≅ Δ DEF
Construction: Cut DP = AB from DE and
DQ = AC from DF.Join PQ
Proof : $\frac{AB}{DE} = \frac{AC}{DF}$ [Given]
⇒ $\frac{DP}{DE} = \frac{DQ}{DF}$ [∵ DP = AB, DQ = AC]
∴ PQ∥EF [corollary of Converse of BPT in Δ DEF]
⇒ ∠P = ∠E and ∠Q = ∠F
∴ ΔDPQ ~ΔDEF [AA Similarity criteria]
∴ $\frac{DP}{DE} = \frac{PQ}{EF}$
[Corresponding sides of similar triangles]
⇒ $\frac{AB}{DE} = \frac{PQ}{EF}$ ---(1) [AB = DP Construction]
But, $\frac{AB}{DE} = \frac{BC}{EF}$---(2) [Given]

$\Rightarrow \frac{PQ}{EF} = \frac{BC}{EF}$ [∵ From (1) and (2)]

⇒BC = PQ

In ΔABC and ΔDPQ ,

BC = PQ [Proved]

AB = DP [Construction]

AC = DQ [Construction]

∴ΔABC ≅ ΔDPQ [SSS Congruency rule]

Hence,

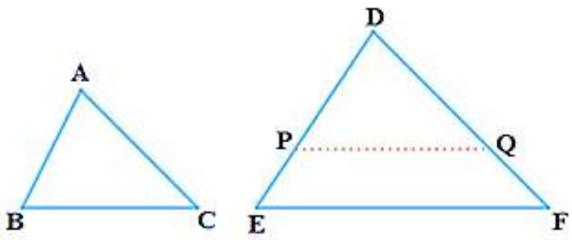
∠A = ∠D, ∠B = ∠P and ∠C = ∠Q

⇒∠A = ∠D, ∠B = ∠E and∠C = ∠F

and ΔABC ≅ Δ DEF

Theorem 6.5

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar



Given: In ΔABC and ΔDEF, ∠A = ∠D and

$\frac{AB}{DE} = \frac{AC}{DF}$ (< 1) ------------ (1)

To Prove: ΔABC ≅ ΔDEF

Construction: Cut DP = AB from DE

and DQ = AC from DF.Join PQ

Proof: In ΔABC and ΔDPQ,

AB = PQ [By Construction]

AC = DF [By Construction]

∠A = ∠D [Given]

ΔABC ≅ ΔDPQ [By SAS Congruency rule]----------(2)

From eqn (1) we get,

$\frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [AB = DP and AC = DQ]

⇒ PQ||EF [By converse of corollary of BPT]

⇒∠P = ∠E , ∠Q = ∠F [Corresponding angles]

∴ ΔDPQ ~ ΔDEF [by AA similarity ctriteria] --------(3)

⇒ ΔABC ≅ ΔDEF [From equation (2) and (3)]

Example 4 : In Fig. 6.29, if PQ ∥ RS, prove that ΔPOQ ~ ΔSOR

Solution: PQ||RS [given]

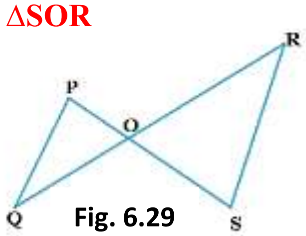


Fig. 6.29

∴ ∠P = ∠S [Alternate angles]

∠Q = ∠R [Alternate angles]

And ∠POQ = ∠SOR [vertically opposite angles]

∴ΔPOQ ~ ΔSOR [AAA similarity criteria]

Example 5 : Observe Fig. 2.30 and then find P.

Solution: In ΔABC and ΔPQR,

$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}$,

$\frac{BC}{CQ} = \frac{6}{12} = \frac{1}{2}$

And $\frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$

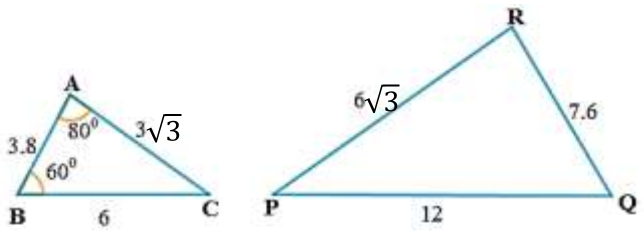


Fig. 6.30

$\Rightarrow \frac{AB}{RQ} = \frac{BC}{CQ} = \frac{CA}{PR}$

∴ ΔABC ~ ΔRQP [SSS similarity criteria]

∠C = ∠P [Corresponding angles of similar triangles]

But ∠C = 180 - ∠A - ∠B [The sum of interior angles of a triangle is 180^0]

$=180^0 – 80^0 - 60^0 = 40^0$

$\Rightarrow$ ∠P = 40^0

Example 6 : In Fig. 2.31, OA.OB = OC.OD. Show that ∠A = ∠C and ∠B = ∠D.

Solution: OA.OB = OC.OD [Given]

$\Rightarrow \frac{OA}{OC} = \frac{OD}{OB}$ ------- (1)

∠AOD = ∠COB [Vertically opposite angles] ------(2)

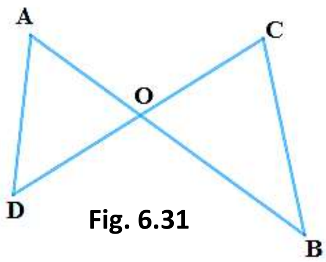


Fig. 6.31

From equation (1) and (2),

ΔAOD ~ ΔCOB [SAS similarity criteria]

∴ ∠A = ∠C and ∠D = ∠B [Corresponding angles of similar triangles]

Example 7 : A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution: AB is a Lamp post. CD i the hs hight of the girl, DE is the length of the shadow of the girl.

Let DE = *x*' mtr,

Now, BD = 1.2m × 4 = 4.8m

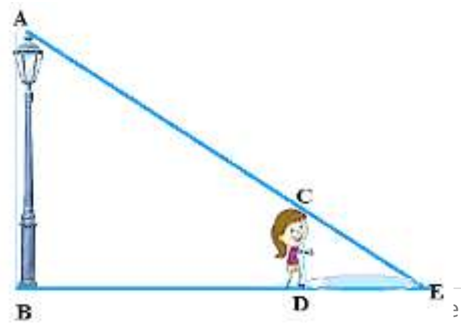
In ΔABE and ΔCDE, ∠B = ∠D = 90^0

And ∠E = ∠E [Common angle]

∴ ΔABE ~ ΔCDE [AA similarity criteria]

∴ $\frac{BE}{DE} = \frac{AB}{CD}$

$\Rightarrow \frac{4.8+x}{x} = \frac{3.6}{0.9}$ (∵90cm = $\frac{90}{100}$m = 0.9m)



$\Rightarrow$ 4.8+x = 4x

$\Rightarrow$ 3x = 4.8

$\Rightarrow$x = 1.6

Hence, the length of her shadow after 4 seconds is 1.6m

Example 8 : In Fig. 6.33, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, prove that :

(i) $\Delta AMC \sim \Delta PNR$ (ii) $\frac{CM}{RN} = \frac{AB}{PQ}$ iii) $\Delta CMB \sim \Delta RNQ$

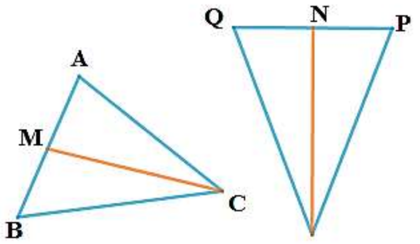


Fig. 6.33

Solution: i) $\Delta ABC \sim \Delta PQR$

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$--------(1)

and $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ ------ (2)

But, AB = 2AM and

PQ = 2PN [CM and RN are the medians]

$\Rightarrow \frac{2AM}{2PN} = \frac{CA}{RP}$

$\Rightarrow \frac{AM}{PN} = \frac{CA}{RP}$ ---(3)

But, $\angle A = \angle P$ [From (2)] -----(4)

From (3) and (4),

$\Delta AMC \sim \Delta PNR$ [by SAS] -------(5)

ii) From (5) $\frac{CM}{RN} = \frac{CA}{RP}$ ----(6)

But, $\frac{CA}{RP} = \frac{AB}{PQ}$ [From (1)] -----(7)

$\therefore \frac{\mathbf{CM}}{\mathbf{RN}} = \frac{\mathbf{AB}}{\mathbf{PQ}}$ --------(8)

(iii) $\frac{AB}{PQ} = \frac{BC}{QR}$ ------- (9)

$\Rightarrow \frac{CM}{RN} = \frac{BC}{QR}$ [From (8)]

$\Rightarrow \frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$ [CM and RN are the medians]

$\Rightarrow \frac{CM}{RN} = \frac{BM}{QN}$ -------- (10)

$\Rightarrow \frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$ [From (9) and (10)]

$\therefore \Delta CMB \sim \Delta RNQ$ [SSS similarity criteria]

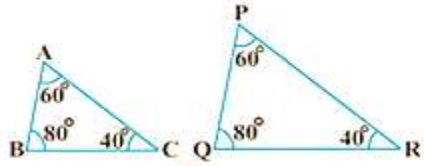
[Note: you can solve (ii) and (iii) using same method as solved for (i)]

Exercise 2.3

1. State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :.

(i) In ΔABC and ΔPQR,

∠A = ∠P = 60° [Given] ;
∠B = ∠Q = 80° [Given];
∠C = ∠R = 40° [Given]
∴ ΔABC ~ ΔPQR [AAA similarity criteria]

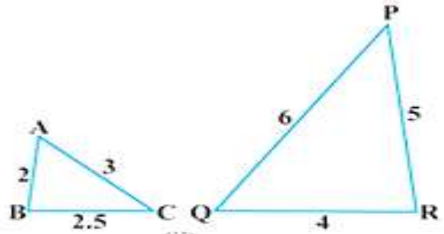


(i)

(ii) In ΔABC and ΔPQR ,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

∴ ΔABC ~ ΔQRP [SSS similarity criteria]



(ii)

(iii)In ΔLMP and ΔDEF,

LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

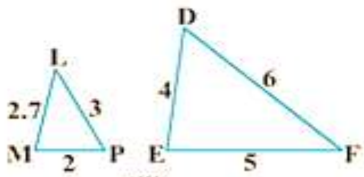
$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$$

Here, $\frac{MP}{DE} = \frac{PL}{DF} \neq \frac{LM}{EF}$

∴ ΔLMP and ΔDEF are not similar

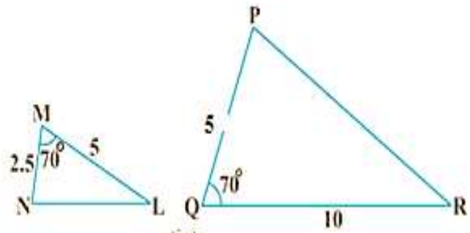


(iii)

(iv) In ΔMNL and ΔQPR,

$$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$$

∠M = ∠Q = 70°
∴ ΔMNL ~ ΔQPR [SAS similarity criteria]



(iv)

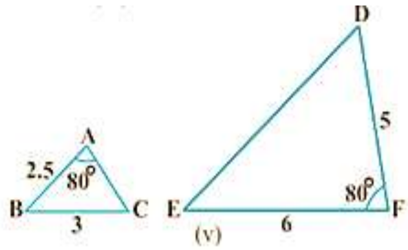
(v) In ΔABC and ΔDEF,

AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°

$$\Rightarrow \frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$$

and, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

⇒ ∠B ≠ ∠F
⇒ ΔABC and ΔDEF are not similar



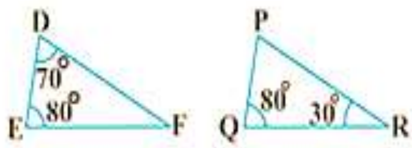
(v)

(vi) In ΔDEF,

∠D + ∠E + ∠F = 180°
⇒ 70° + 80° + ∠F = 180°
⇒ ∠F = 180° - 70° - 80°

In ΔPQR,

∠P + ∠Q + ∠R = 180



(vi)

$\Rightarrow \angle P + 80° + 30° = 180°$
$\Rightarrow \angle P = 180° - 80° - 30°$
$\Rightarrow \angle P = 70°$
$\Rightarrow \angle F = 30°$
In ΔDEF and ΔPQR ,

$\angle D = \angle P = 70°$; $\angle F = \angle Q = 80°$; $\angle F = \angle R = 30°$
$\Rightarrow \Delta DEF \sim \Delta PQR$ [AAA similarity criteria]

2. In Fig. 6.35, $\Delta OBA \sim \Delta ODC$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$

Solution: DOB is a straight line

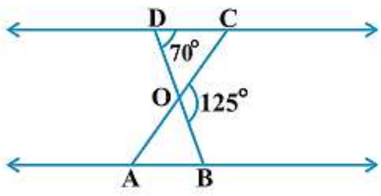
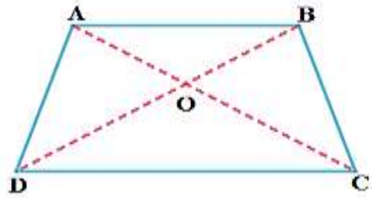


Fig. 6.35

$\therefore \angle DOC + \angle COB = 180°$
$\Rightarrow \angle DOC = 180° - 125° = 55°$
In ΔDOC,
$\angle DCO + \angle CDO + \angle DOC = 180°$
$\Rightarrow \angle DCO + 70° + 55° = 180°$
$\Rightarrow \angle DCO = 180^0 - 125° = 55^0$
$\Delta ODC \sim \Delta OBA$. [Given]
$\therefore \angle OAB = \angle OCD$ [CPCT]
$\Rightarrow \angle OAB = 55°$

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles. Show that $\frac{OA}{OC} = \frac{OB}{OD}$

In ΔBOA and ΔDOC,



$\angle ABO = \angle CDO$ [AB || CD, Alternate angles]
$\angle BAO = \angle DCO$ [AB || CD, Alternate angles]
$\angle BOA = \angle DOC$ [Vertically opposite angles]
$\therefore \Delta BOA \sim \Delta DOC$ [AAA Similarity criteria]
$\therefore \frac{OC}{OA} = \frac{OD}{OB}$ [By CPCT]
$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ [Taken on reciprocals]

4. In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ Show that $\Delta PQS \sim \Delta TQR$

In ΔPQR, $\angle 1 = \angle 2$ [Given]

$\therefore PQ = PR$ --------------(1)

But, $\frac{QR}{QS} = \frac{QT}{PR}$ [Given]

$\frac{QR}{QS} = \frac{QT}{PQ}$ [From (1)] ------------(2)

In ΔPQS and ΔTQR,

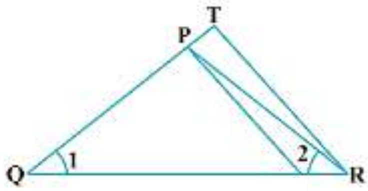
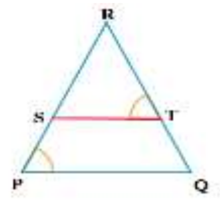


Fig. 6.36

$\frac{QR}{QS} = \frac{QT}{PQ}$ [From eqn (2)] ;

∠Q = ∠Q [Common angle]
∴ ΔPQS ~ ΔTQR [SAS similarity criteria]

5. S and T are points on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS



Solution: In ΔRPQ and ΔRST,

∠RTS = ∠QPS [Given]
∠R = ∠R [Common angle]
∴ ΔRPQ ~ ΔRTS [AA similarity criteria]

6. In Fig. 6.37, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC

Solution: ΔABE ≅ ΔACD [Given]

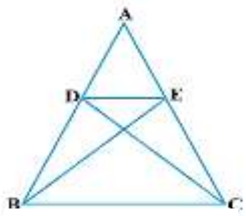


Fig. 6.37

∴ AB = AC -----------(1) [By CPCT]
and AD = AE -----------(2) [By CPCT]
In ΔADE and ΔABC,

Dividing (2) by (1)

$$\frac{AD}{AB} = \frac{AE}{AC}$$

∴ ΔADE ~ ΔABC [SAS Similarity criteria]

7. In Fig. 6.38, altitudes AD and CE of ABC intersect each other at the point P. Show that i) ΔAEP ~ ΔCDP ii) ΔABD ~ ΔCBE iii) ΔAEP ~ ΔADB iv) ΔPDC ~ ΔBEC

(i) In ΔAEP and ΔCDP,
∠AEP = ∠CDP = 90°
∠APE = ∠CPD [Vertically opposite angles]
∴ ΔAEP ~ ΔCDP[AA similarity criteria]

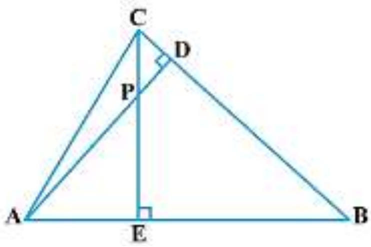
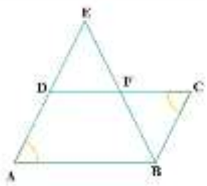


Fig. 6.38

(ii) In ΔABD and ΔCBE,
∠ADB = ∠CEB = 90°
∠ABD = ∠CBE [Common angle]
∴ ΔABD ~ ΔCBE [AA similarity criteria]
(iii) In ΔAEP and ΔADB,
∠AEP = ∠ADB = 90^0
∠PAE = ∠DAB [Common angle]
∴ ΔAEP ~ ΔADB [AA similarity criteria]
(iv) In ΔPDC and ΔBEC,
∠PDC = ∠BEC = 90°
∠PCD = ∠BCE [Common angle]
∴ ΔPDC ~ ΔBEC [AA similarity criteria]

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB



Solution: In ΔABE and ΔCFB,

∠A = ∠C [Opposite angles of a parallelogram]
∠AEB = ∠CBF [AE || BC,Alternate angles]
∴ΔABE ~ ΔCFB [AA similarity criteria]

9. In Fig. 6.39, ΔABC and ΔAMP are two right triangles, right angled at B and M respectively. Prove that: i) ΔABC ~ ΔAMP ii) $\frac{CA}{PA} = \frac{BC}{MP}$

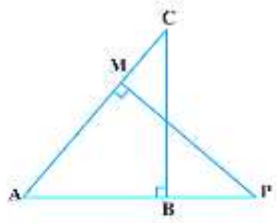


Fig. 6.39

(i) In **ΔABC and ΔAMP ,**

∠A = ∠A [Common angle]

∠ABC = ∠AMP = 90°

∴ ΔABC ~ ΔAMP [AA similarity criteria]

(ii) ΔABC ~ ΔAMP [Proved in(i)]

⇒ $\frac{CA}{PA} = \frac{BC}{MP}$ [the corresponding sides of similar triangles]

10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔEFG, show that: i) $\frac{CD}{GH} = \frac{AC}{FG}$ ii) ΔDCB ~ ΔHGE iii) ΔDCA ~ ΔHGF

(i) **ΔABC ~ ΔFEG [Given]**

∴ ∠A = ∠F, ∠ACB = ∠FGE [Corresponding angles of similar triangles]

CD is the bisector of ∠ACB, GH is the bisector of ∠FGE

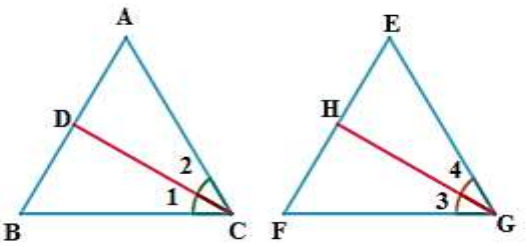
∴ ∠ACD = ∠FGH

In ΔACD and ΔFGH, ∠A = ∠F

∠ACD = ∠FGH

∴ΔACD ~ ΔFGH [AA similarity criteria]

⇒ $\frac{CD}{GH} = \frac{AC}{FG}$



(ii) ΔABC ~ ΔFEG [Given]

∴ ∠B = ∠E, and ∠ACB = ∠FGE [Corresponding angles of similar triangles]

CD is the bisector of ∠ACB, GH is the bisector of ∠FGE

∴∠DCB = ∠HGE

Now, In ΔDCB and ΔHGE,

∠DCB = ∠HGE , ∠B = ∠E

∴ ΔDCB ~ ΔHGE [AA similarity criteria]

(iii) ΔABC ~ ΔFEG [Given]

∴ ∠A = ∠F, ∠ACB = ∠FGE [Corresponding angles of similar triangles]

CD is the bisector of ∠ACB, GH is the bisector of ∠FGE

∴ ∠ACD = ∠FGH

In ΔDCA and ΔHGF,∠ACD = ∠FGH , ∠A = ∠F

∴ ΔDCA ~ ΔHGF [AA similarity criteria]

11. In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD⊥BC and EF⊥AC , prove that ΔABD ~ ΔECF

ABC is an isosceles triangle

AB = AC

⇒∠B = ∠C [Opposite sides of equal angles]

⇒ ∠ABD = ∠ECF ------ (1)

In ΔABD and ΔECF,

∠ADB = ∠EFC =90^0 [AD⊥BC, EF⊥AC]

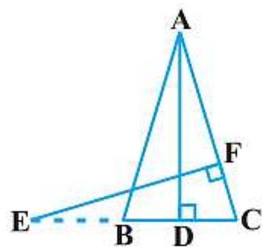


Fig. 6.40

⇒ ∠ABD = ∠ECF [From (1)]

∴ ΔABD ~ ΔECF [AA similarity criteria]

12. Sides AB and BC and median AD of a triangle ABC are respectively propor- tional to sides PQand QRand median PM of PQR (Fig.6.41).Show thatΔABC ~ ΔPQR

Given: In ΔABC and ΔPQR

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

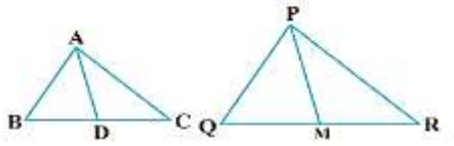


Fig. 6.41

To prove: ΔABC ~ ΔPQR

Proof: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$ [D and M are the mid-points of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

⇒ ΔABD ~ ΔPQM [SSS similarity criteria]

∴ ∠ABD = ∠PQM [Corresponding angles of similar triangles]

⇒ ∠ABC = ∠PQR -----------(1)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

∠ABC = ∠PQR [From (1)]

∴ ΔABC ~ ΔPQR [SAS similarity criteria]

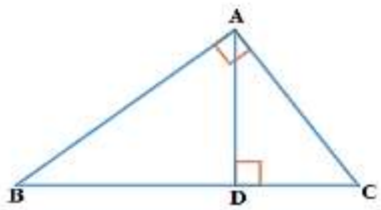
13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC . Show that CA^2 = CB.CD.

Solution: In ΔADC and ΔBAC,

∠ADC = ∠BAC [Given]

∠ACD = ∠BCA [Common angle]

∴ ΔADC ~ ΔBAC [AA similarity criteria]



∴ $\frac{CA}{CB} = \frac{CD}{CA}$ [The corresponding sides of the similar triangles are proportional]

⇒ CA^2 = CB.CD.

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR

Given: In ΔABC and ΔPQR, AD and PM are the medians drawn to BC and QR respectively. and $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

To prove: ΔABC ~ ΔPQR

Construction: Produce AD to E such that AD = DE, join CE and produce PM to N such that PM = MN, join RN

Proof: In ΔABD and ΔCDE,

AD = DE [Construction]

BD = DC [AD is Median]

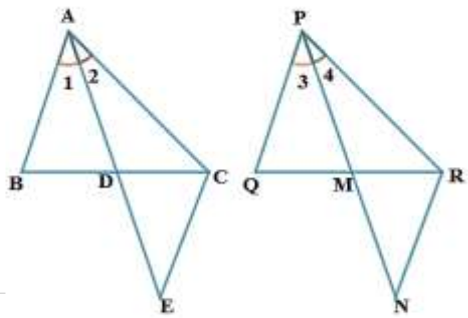
∠ADB = ∠CDE [Vertically opposite angles]

∴ ΔABD ≅ ΔCDE [SAS Congruency rule]

⇒ AB = CE [By CPCT] -----------(i)

Similarlly, In ΔPQM and ΔMNR,

⇒ PQ = RN [By CPCT]-----------(ii)



But, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ [Given]

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$ [From (i) and (ii)]$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [$\because$ 2AD = AE &2PM = PN]

$\therefore$ ΔACE ~ ΔPRN [SSS similarity criteria]

$\therefore$ ∠2 = ∠4

Similarlly, ∠1 = ∠3

$\therefore$ ∠1 + ∠2 = ∠3 + ∠4

$\Rightarrow$ ∠A = ∠P -------------(iii)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ [Given]

∠A = ∠P [From (iii)]

$\therefore$ ΔABC ~ ΔPQR [SAS similarity criteria]

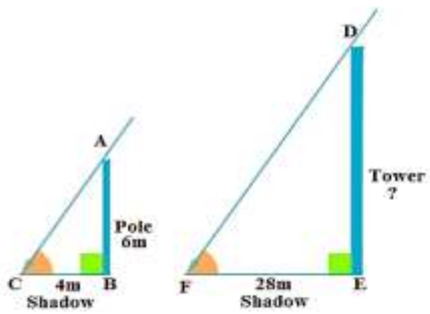
15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Length of the vertical Pole = AB = 6m

Length of the shadow by the Pole = BC = 4 m

Length of the shadow by the Tower = EF = 28 m

Let the height of the tower = DE = *h* ‘m



In ΔABC and ΔDEF,

∠C = ∠F [The angles make by sun at same time]

∠B = ∠E = 90°

$\therefore$ ΔABC ~ ΔDEF [AA similarity criteria]

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ [corresponding sides of the similar triangles]

$\therefore \frac{6}{h} = \frac{4}{28}$

$\Rightarrow h = 6\times\frac{6x28}{4} = 6 \times 7$

$\Rightarrow h = 42$ m

$\therefore$ Height of the tower = 42 m.

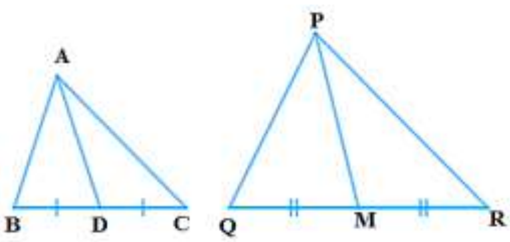
16. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

ΔABC ~ ΔPQR [Given]

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ -------------- (i)

and ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R --------- **(ii)**

AD and PM are the Medians



$\therefore$ BD = $\frac{BC}{2}$ and QM = $\frac{QR}{2}$ ----------**(iii)**

From equations **(i)** and **(iii)**, we get

$\therefore \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}$ ----------(iv)

In ΔABD and ΔPQM, ∠B = ∠Q [From (ii)] ;

$\frac{AB}{PQ} = \frac{BD}{QM}$ [From (iv)]

∴ ΔABD ~ ΔPQM [SAS similarity criteria]

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

2.7 Summary

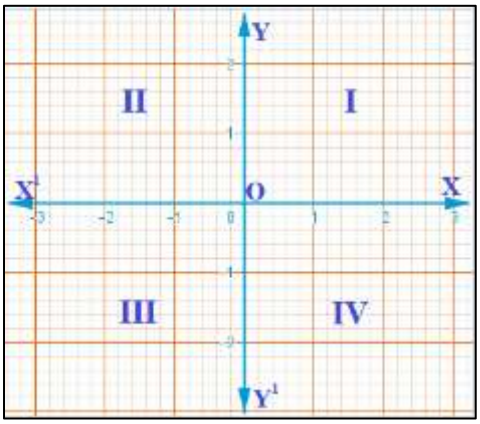
1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true
3. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equaland (ii) their corresponding sides are in the same ratio (i.e., proportion).
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
6. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
7. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
9. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).

7 Coordinate Geometry

Coordinate axes: **A set of a pair of perpendicular axes $X^{|}OX$ and $YOY^{|}$**

Horizontal line
- X^1OX
- X - Axis

Vertical line
- YOY^1
- Y - Axis



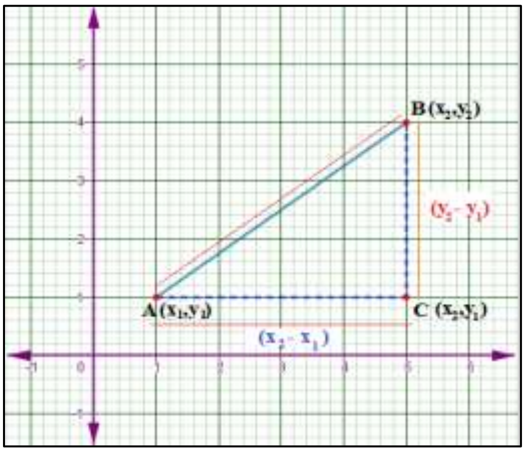
The intersection point of X and Y axes is called the Orgin 'O'

The distance of a point from the y-axis is called its x-coordinate, or abscissa. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The coordinates of a point on the x-axis are of the form (x, 0), and of a point on the y-axis are of the form (0, y).

The Coordinate axes divides the plane in to four parts. They are called quadrants.

The coordinaes of the orgin is $(0,0)$

7.2 Distance Formula



The distance between two points on X-axis or on the straight line parallel to X-axis is

$$\textbf{Distance} = x_2 - x_1$$

The distance between two points on Y-axis or on the straight line paralle to Y-axis is

$$\textbf{Distance} = y_2 - y_1$$

$AB^2 = AC^2 + BC^2$

The distance between two points which are neither on X or Y axis nor on the line parallel to X orY axis

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance between the point P(x,y) and the orgin

$$d = \sqrt{x^2 + y^2}$$

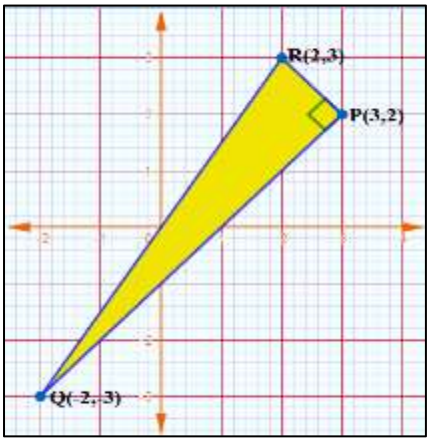
Example 1:Do the points (3, 2), (–2, –3) and (2, 3) form a triangle? If so, name the type of triangle formed.

$P(3,2),\ \ Q(-2,-3),\ \ R(2,3)$

Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(3-(-2))^2 + (2-(-3))^2}$

$= \sqrt{(3+2)^2 + (2+3)^2}$

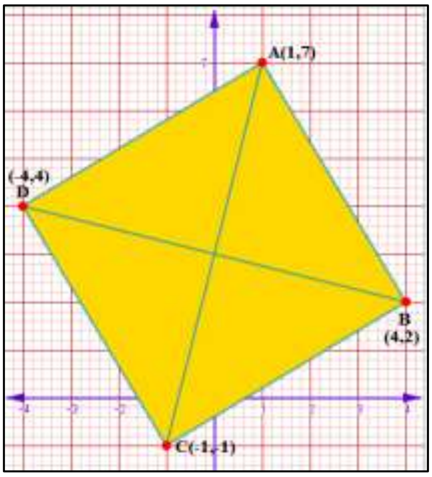


$= \sqrt{(5)^2+(5)^2}$
$= \sqrt{25+25}$
$= \sqrt{50}$
$= 7.07$
$QR = \sqrt{(-2-2)^2+(-3-3)^2}$
$= \sqrt{(-4)^2+(-6)^2}$
$= \sqrt{16+36}$
$= \sqrt{52}$
$= 7.21$
$PR = \sqrt{(3-2)^2+(2-3)^2}$
$= \sqrt{(1)^2+(-1)^2}$
$= \sqrt{1+1} = \sqrt{2} = 1.41$

Since the sum of any two of these distances is greater than the third distance, therefore the point P,Q and R form a triangle.
Also, $PQ^2 + PR^2 = QR^2$ by the converse of Pythagoras theorem $\angle P = 90^O$ we have
Therefore, PQR is a right triangle.

Example2: Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

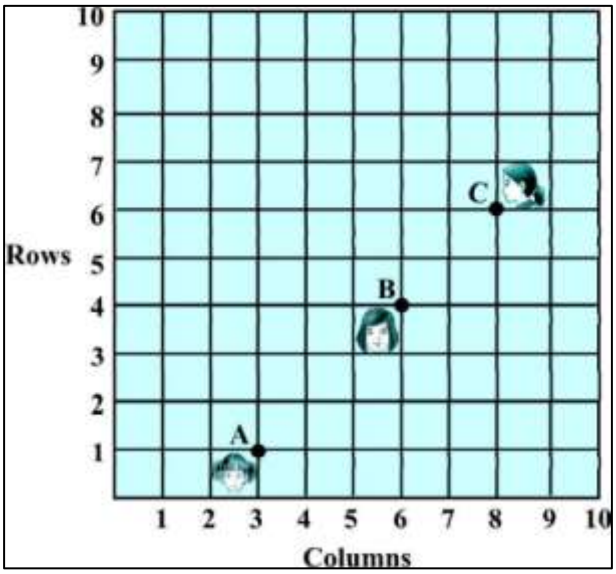
Solution: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)



$AB = \sqrt{(4-1)^2+(2-7)^2}$
$= \sqrt{(3)^2+(-5)^2} = \sqrt{9+25}$
$= \sqrt{34}$
$BC = \sqrt{(-1-4)^2+(-1-2)^2}$
$= \sqrt{(-5)^2+(-3)^2} = \sqrt{25+9}$
$= \sqrt{34}$
$CD = \sqrt{(-4-(-1))^2+(4-(-1))^2}$
$= \sqrt{(-4+1)^2+(4+1)^2}$
$= \sqrt{(-3)^2+(5)^2} = \sqrt{9+25}$
$= \sqrt{34}$
$DA = \sqrt{(1-(-4))^2+(7-4)^2}$
$= \sqrt{(1+4)^2+(3)^2}$
$= \sqrt{(5)^2+(3)^2} = \sqrt{25+9}$
$= \sqrt{34}$
$AC = \sqrt{(-1-1)^2+(-1-7)^2}$
$= \sqrt{(-2)^2+(-8)^2}$
$= \sqrt{4+64} = \sqrt{68}$
$BD = \sqrt{(-4-4)^2+(4-2)^2}$
$= \sqrt{(-8)^2+(2)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Thereore, ABCD is a square

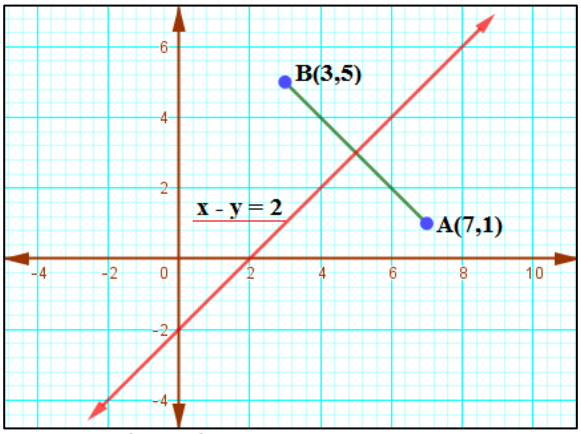
Example 3 : Fig. 7.6 shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at A (3, 1), B(6, 4) and C(8, 6) respectively. Do you think they are seated in a line? Give reasons for your answer.



$AB = \sqrt{(6-3)^2+(4-1)^2}$
$= \sqrt{(3)^2+(3)^2}$
$= \sqrt{9+9} = \sqrt{18}$
$= \sqrt{9\times 2}$
$= \mathbf{3\sqrt{2}}$
$BC = \sqrt{(8-6)^2+(6-4)^2}$
$= \sqrt{(2)^2+(2)^2}$
$= \sqrt{4+4} = \sqrt{8}$
$= \sqrt{4\times 2}$
$= \mathbf{2\sqrt{2}}$
$AC = \sqrt{(8-3)^2+(6-1)^2}$
$= \sqrt{(5)^2+(5)^2}$
$= \sqrt{25+25} = \sqrt{50}$
$= \sqrt{25\times 2}$
$= \mathbf{5\sqrt{2}}$
$AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$
Since,, **AB + BC = AC** we can say that the points A, B and C are collinear.
Therefore, they are seated in a line

Example 4 : Find a relation between x and y such that the point (x , y) is equidistant from the points (7, 1) and (3, 5).

Let the point P (x, y) is equi distance from the points A (7, 1) and B (3, 5)



PA = PB
$\Rightarrow PA^2 = PB^2$
$PA = \sqrt{(x-7)^2+(y-1)^2}$
$PB = \sqrt{(x-3)^2+(y-5)^2}$
$AP^2 = BP^2$
$\Rightarrow\left(\sqrt{(x-7)^2+(y-1)^2}\right)^2$
$= \left(\sqrt{(x-3)^2+(y-5)^2}\right)^2$
$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$
$x^2 + 7^2 - 2(x)(7) + y^2 + 1^2 - 2(y)(1) = x^2 + 3^2 - 2(x)(3) + y^2 + 5^2 - 2(y)(5)$
$x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$
$x^2 - x^2 - 14x + 6x + y^2 - y^2 - 2y + 10y = 34 - 50$
$-8x + 8y = -16 \quad \div -8$
$\Rightarrow x - y = 2$ Which is the require relation.

Remark : Note that the graph of the equation x – y = 2 is a line. From your earlier studies, you know that a point which is equidistant from A and B lies on the perpendicular bisector of AB. Therefore, the graph of x – y = 2 is the perpendicular bisector of AB

Example 5 : Find a point on the y-axis which is equidistant from the points A(6, 5) and B(– 4, 3).

We know that a point on the $y-$ axis is of the form $(0, y)$. P $(0, y)$ So, let the point P(0, y) be equidistant from A and B. Then PA = PB
$(6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$
$36 + 5^2 + y^2 - 2(5)(y) = 16 + 3^2 + y^2 - 2(3)(y)$
$36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y$

$y^2 - y^2 - 10y + 6y = 25 - 61$
$\Rightarrow\ -4y = -36$
$y = \frac{-36}{-4} = 9$
Therefore the required point is $(\mathbf{0}, \mathbf{9})$
$PA = \sqrt{(6 -\ 0)^2 + (5 - 9)^2}$
$= \sqrt{(6)^2 + (-4)^2} = \sqrt{36 + 16}$
$= \sqrt{52}$
$PB = \sqrt{(-4 -\ 0)^2 + (3 - 9)^2}$
$= \sqrt{(-4)^2 + (-6)^2} = \sqrt{16 + 36}$
$= \sqrt{52}$

Exercise 7.1

1. Find the distance between the following pairs of points :
i) (2, 3), (4, 1) ii) (-5, 7), (-1, 3) iii) (a, b), (-a, -b)

i) $(\mathbf{x_1}, \mathbf{y_1}) = (\mathbf{2}, \mathbf{3})$, $(\mathbf{x_2}, \mathbf{y_2}) = (\mathbf{4}, \mathbf{1})$

x_1	y_1	x_2	y_2
2	3	4	1

Formula $\mathbf{d} = \sqrt{(\mathbf{x_2} - \mathbf{x_1})^2 + (\mathbf{y_2} - \mathbf{y_1})^2}$
$d = \sqrt{(4 - 2)^2 + (1 - 3)^2}$
$= \sqrt{(2)^2 + (-2)^2}$
$d = \sqrt{4 + 4} = \sqrt{2 \times 4}$
$= 2\sqrt{2}$ Units

ii) $(\mathbf{x_1}, \mathbf{y_1}) = (-\mathbf{5}, \mathbf{7})$, $(\mathbf{x_2}, \mathbf{y_2}) = (-\mathbf{1}, \mathbf{3})$

x_1	y_1	x_2	y_2
-5	7	-1	3

$d = \sqrt{(-1 - [-5)^2 + (3 - 7)^2}$
$= \sqrt{(4)^2 + (-4)^2}$
$d = \sqrt{16 + 16}$
$= \sqrt{2 \times 16}$
$= 4\sqrt{2}$ Units

iii) $(\mathbf{x_1}, \mathbf{y_1}) = (\mathbf{a}, \mathbf{b})$, $(\mathbf{x_2}, \mathbf{y_2}) = (-\mathbf{a}, -\mathbf{b})$

x_1	y_1	x_2	y_2
a	b	-a	-b

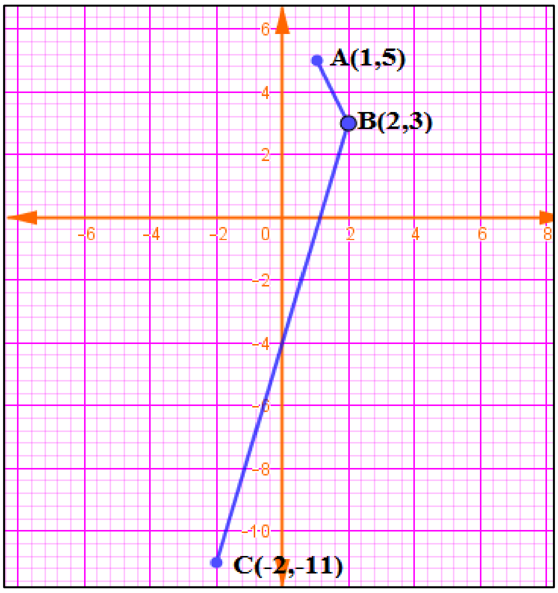
$d = \sqrt{(-a - a)^2 + (-b - b)^2}$
$= \sqrt{(-2a)^2 + (-2b)^2}$
$d = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2 + b^2)}$
$= 2\sqrt{a^2 + b^2}$ Units

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

$(x, y) = (36, 15)$
$d = \sqrt{x^2 + y^2}$
$= \sqrt{36^2 + 15^2}$
$= \sqrt{1296 + 225}$
$= \sqrt{1521}$
$= \mathbf{39}$ **Units**

We can find the distance between the two towns A and B.Suppose town A is at the Orgin, then the town has to be at (36,15). The distance between these two towns is 39km **(1, 5),**

3. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.



A (1, 5), B (2, 3) and C (-2, -11)

$AB = \sqrt{(2-1)^2+(3-5)^2}$

$= \sqrt{(1)^2+(-2)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$BC = \sqrt{(-2-2)^2+(-11-3)^2}$

$= \sqrt{(-4)^2+(-14)^2}$

$= \sqrt{16+196}$

$= \sqrt{212}$

$AC = \sqrt{(-2-1)^2+(-11-5)^2}$

$= \sqrt{(-3)^2+(-16)^2}$

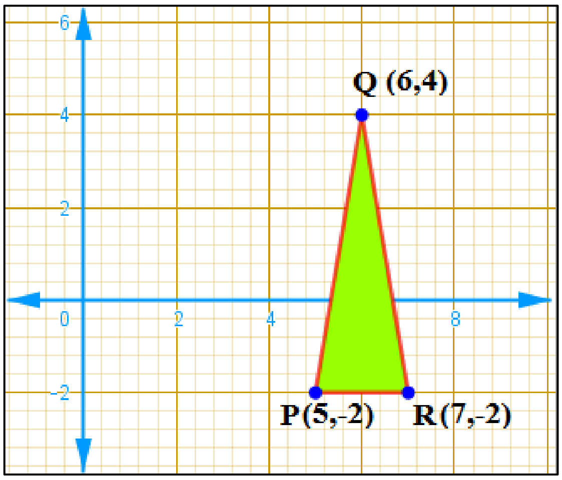
$= \sqrt{9+256}$

$= \sqrt{265}$

$AB + BC \neq AC$

$\therefore$ **These are non-collinear points**

3. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.



Formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ = \sqrt{(6-5)^2+(4-(-2))^2}$

$= \sqrt{(1)^2+(6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$ ------- (i)

$QR = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{(1)^2+(-6)^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$ -------- (ii)

$PR = \sqrt{(7-5)^2+(-2-[-2])^2}$

$= \sqrt{(2)^2+(0)^2}$

$= \sqrt{4} = 2$ ------ (iii)

(i), (ii), (iii) $\Rightarrow PQ = QR$,

Since,Two sides of the tringle are equal.

Hence, ΔPQR is an isosceles triangle.

4. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, “Don’t you think ABCD is a square?” Chameli disagrees. Using distance formula, find which of them is correct.

The coordinates of the points A,B,C and D are,

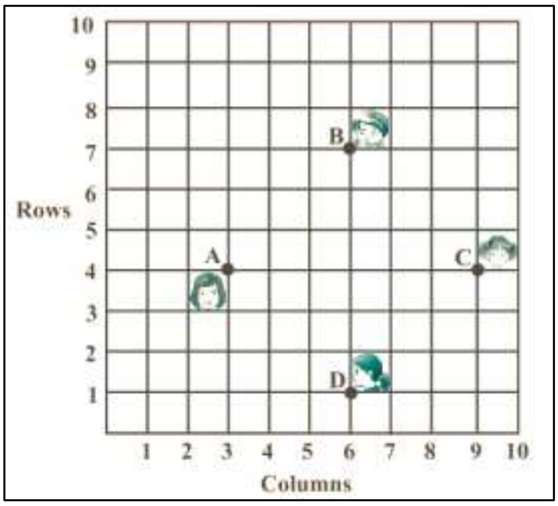
A(3,4) , B(6, 7) , C(9,4), D(6,1)

$AB = \sqrt{(6-3)^2+(7-4)^2}$

$= \sqrt{(3)^2+(3)^2}$

$= \sqrt{9+8} = \sqrt{18}$

$= 3\sqrt{2}$ ------------------**(i)**



$BC = \sqrt{(9-6)^2+(4-7)^2}$

$= \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ ------ **(ii)**

$CD = \sqrt{(6-9)^2+(1-4)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ ---------**(iii)**

$DA = \sqrt{(6-3)^2+(1-4)^2}$

$= \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ --------**(iv)**

$AB = BC = CD = DA$

Diagonal $AC = \sqrt{(9-3)^2+(4-4)^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36} = 6$ -----------------**(v)**

Diagonal $BD = \sqrt{(6-6])^2+(7-1)^2} = \sqrt{(0)^2+(6)^2} = \sqrt{36} = 6$ ----------------**(vi)**

$AC = BD$

Thus, $AB = BC = CD = DA$, diagonals: $AC = DB$

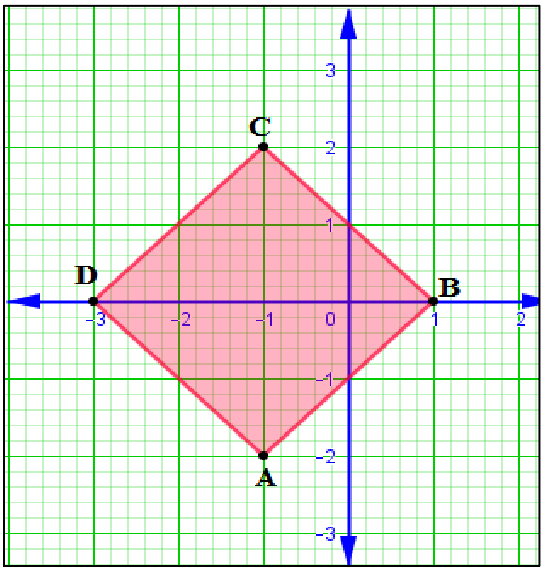
Since all the four sides and diagonals are equal.

Hence, ABCD is a square. So, Champa is correct.

5. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

i) (-1, -2), (1, 0), (-1, 2), (-3, 0) ii) (-3, 5), (3, 1), (0, 3), (-1, -4) iii) (4, 5), (7, 6), (4, 3), (1, 2)

i) $A(-1,-2)$, $B(1,0)$, $C(-1,2)$, $D(-3,0)$



$AB = \sqrt{(1-(-1))^2+(0-(-2))^2}$

$= \sqrt{(1+1)^2+(0+2)^2}$

$= \sqrt{(2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$= \sqrt{4\times 2} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2+(2-0)^2}$

$= \sqrt{(-2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$= \sqrt{4\times 2} = 2\sqrt{2}$

$CD = \sqrt{(-3-(-1))^2+(0-2)^2}$

$= \sqrt{(-3+1)^2+(-2)^2}$

$= \sqrt{(-2)^2+(-2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$= \sqrt{4\times 2} = 2\sqrt{2}$

$DA = \sqrt{(-3-(-1))^2+(0-(-2))^2}$

$= \sqrt{(-3+1)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8}$

$= \sqrt{4\times 2} = 2\sqrt{2}$

$AB = BC = CD = DA$

$AC = \sqrt{(-1-(-1))^2+(2-(-2))^2}$

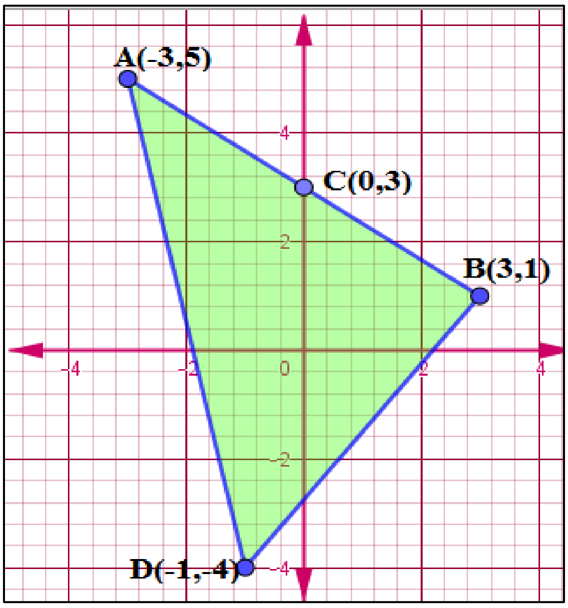
$= \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{(0)^2+(4)^2} = \sqrt{16} = 4$

$BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16} = 4$

Thus, $AC = BD$

Since, the four sides AB, BC, CD and DA are equal and the diagonals $AC = DB$ are equal. So the quadrilateral ABCD is a square.

ii) A(-3, 5), B(3, 1), C(0, 3), D(-1, -4)



$AB = \sqrt{(3-(-3))^2 + (1-(-3))^2}$

$= \sqrt{(3+3)^2 + (1+3)^2}$

$= \sqrt{(6)^2 + (4)^2}$

$= \sqrt{36+16} = \sqrt{52}$

$BC = \sqrt{(0-3)^2 + (3-1)^2}$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(-1-0)^2 + (-4-3)^2}$

$= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1+49} = \sqrt{50}$

$= \sqrt{25 \times 2} = 5\sqrt{2}$

$DA = \sqrt{(-3-(-1))^2 + (-4-5)^2}$

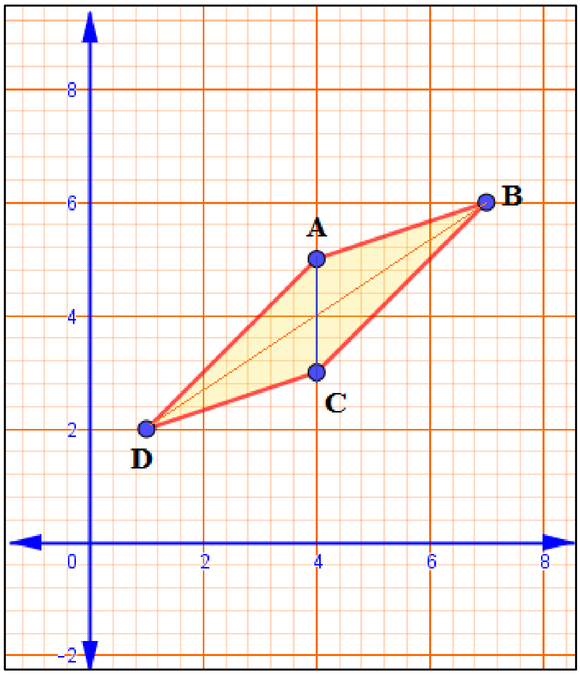
$= \sqrt{(-3+1)^2 + (-9)^2}$

$= \sqrt{(-2)^2 + (-9)^2}$

$= \sqrt{4+81} = \sqrt{85}$

$AB \neq BC \neq CD \neq DA$ Since, the four sides AB, BC, CD and DA are not equal.Hence these poist does not form a quadrilateral.

iii) A(4, 5), B(7, 6), C(4, 3), D(1, 2)



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(7-4)^2 + (6-5)^2}$

$= \sqrt{(3)^2 + (1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$BC = \sqrt{(4-7)^2 + (3-6)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9 \times 2}$

$= 3\sqrt{2}$

$CD = \sqrt{(1-4)^2 + (2-3)^2}$

$= \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$DA = \sqrt{(1-4)^2 + (2-5)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9 \times 2}$

$= 3\sqrt{2}$

$AB = CD, \; BC = DA$

$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{(0)^2 + (-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

AC ≠ DB

Thus opposite sides are equal. AB = CD , & BC = DA

But diagonals are not equal. AC ≠ DB

∴ The given points are forming a parallelogram.

6. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).

We know that a point on the X – axis is of the form (x, 0).

Let the point P (x, 0) is equi distance from the points A(2, -5) and B(-2, 9)

AP = BP

$(x - 2)^2 + (0 - (-5))^2 = (x - (-2))^2 + (0 - 9)^2$

$(x - 2)^2 + 5^2 = (x + 2)^2 + (-9)^2$

$x^2 + 2^2 - 2(x)(2) + 25 = x^2 + 2^2 + 2(x)(2) + 81$

$\Rightarrow -4x + 25 = 4x + 81$

$-4x - 4x = 81 - 25$

$\Rightarrow -8x = 56$

$\Rightarrow x = \frac{56}{-8} = -7$

Thus, the required point is $(-7, 0)$

7. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.

$(x_1, y_1) = (2, -3)$, $(x_2, y_2) = (10, y)$, $d = 10$

x_1	y_1	x_2	y_2
2	-3	10	y

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$10 = \sqrt{(10-2)^2 + (y - (-3))^2} = \sqrt{(8)^2 + (y+3)^2}$

$10^2 = 64 + (y+3)^2$

$\Rightarrow 100 - 64 = (y+3)^2$

$\Rightarrow (y+3)^2 = 36$

$\Rightarrow y + 3 = \pm\sqrt{36}$

$\Rightarrow y + 3 = \pm 6$

$\Rightarrow y = 6 - 3 = 3$ or $x = -6 - 3 = -9$

8. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

The point Q (0, 1) is equi distance from the points P (5, -3) and R (x, 6)

PQ = QR $\Rightarrow$ $PQ^2 = PR^2$

$PQ = \sqrt{(5 - 0)^2 + (-3 - 1)^2}$

$= \sqrt{(5)^2 + (-4)^2}$

$= \sqrt{25 + 16} = \sqrt{41}$

$QR = \sqrt{(x - 0)^2 + (6 - 1)^2}$

$= \sqrt{(x)^2 + (5)^2}$

$= \sqrt{x^2 + 25}$

$PQ^2 = PR^2 \Rightarrow \left(\sqrt{x^2 + 25}\right)^2 = \left(\sqrt{41}\right)^2$

$x^2 + 25 = 41$

$\Rightarrow x^2 = 41 - 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm\sqrt{16}$

$\Rightarrow x = \pm 4$

The coordinate of the point R is $(4,6)$ or $(-4,6)$

If the coordinates of R is $(4,6)$ then,

$QR = \sqrt{(4 - 0)^2 + (6 - 1)^2}$

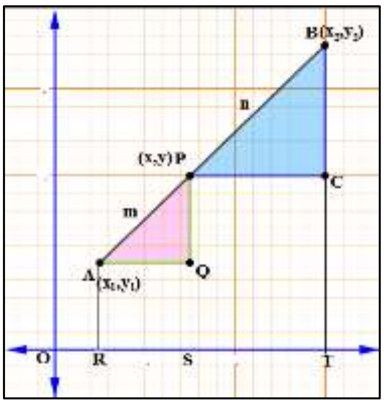
$= \sqrt{(4)^2+(5)^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$
$PR = \sqrt{(4-5)^2+(6-(-3))^2}$
$= \sqrt{(-1)^2+(6+3)^2}$
$= \sqrt{1+81}$
$= \sqrt{82}$
If the coordinates of R is $(-4,6)$ then,
$QR = \sqrt{(-4\ -0)^2+(6-1)^2}$
$= \sqrt{(-4)^2+(5)^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$
$PR = \sqrt{(-4-5)^2+(6-(-3))^2}$
$= \sqrt{(-9)^2+(6+3)^2}$
$= \sqrt{81+81}$
$= 9\sqrt{2}$

9. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

The point P (*x*, y) is equidistance from the points A (3, 6) and B (-3, 4).
$PA = PB \Rightarrow PA^2 = PB^2$
$PA = \sqrt{(x\ -\ 3)^2+(y-6)^2}$; $PB = \sqrt{(x\ -(-3))^2+(y-4)^2}$
$AP^2 = BP^2$
$\Rightarrow\left(\sqrt{(x\ -\ 3)^2+(y-6)^2}\right)^2 = \left(\sqrt{(x\ -(-3))^2+(y-4)^2}\right)^2$
$(x\ -\ 3)^2 + (y\ -\ 6)^2 = (x+\ 3)^2 + (y\ -\ 4)^2$
$x^2+3^2-2(x)(3)+y^2+6^2-2(y)(6) = x^2+3^2+2(x)(3)+y^2+4^2-2(y)(4)$
$x^2+9-6x+y^2+36-12y = x^2+9+6x+y^2+16-8y$
$x^2-x^2-6x-6x+y^2-y^2-12y+8y = 25-45$
$-12x-4y = -20 \quad \div -4$
$3x+y-5=0$
This is the required relation and it is representing a straight line
Thus the point equidistance from the point A and B on the ⊥ bisector of AB

7.3 Section Formula

The coordinates of the point P(x, y) which divides the line segment joining points $\mathbf{A}(\mathbf{x_1},\mathbf{y_1})$ and $\mathbf{B}(\mathbf{x_2},\ \mathbf{y_2})$, internally, in the ratio $m_1 : m_2$ are



$$\mathbf{P}(\mathbf{x},\mathbf{y}) = \left(\frac{\mathbf{m_1x_2 + m_2\ x_1}}{\mathbf{m_1+m_2}}\ ,\ \frac{\mathbf{m_1y_2 + m_2\ y_1}}{\mathbf{m_1+m_2}}\right)$$

The mid-point of a line segment divides the line segment in the ratio 1: 1. Then the coordinates of the midpoint of the line segment,

$$\mathbf{P(x,y) = \left(\frac{x_2 + x_1}{2} \ , \ \frac{y_2 + y_1}{2}\right)}$$

Example 6 : Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

$(x_1, y_1) = (4, -3), \ (x_2, y_2) = (8,5), m_1 : m_2 = 3:1$

x_1	y_1	x_2	y_2
4	-3	8	5

$x = \frac{m_1x_2 + m_2 \ x_1}{m_1+m_2}$

$= \frac{3(8)+1(4)}{3+1}$

$= \frac{24+4}{4}$

$= \frac{28}{4} = 7$

$y = \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}$

$= \frac{3(5)+1(-3)}{3+1}$

$= \frac{15-3}{4}$

$= \frac{12}{4} = 3$

Therefore the required point is (7,3)

Example 7 : In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

$P(x, y) = (-4, 6), \ A(x_1, \ y_1) = (-6, 10) \ , B(x_2, \ y_2) = (3, -8), m_1 = ?, m_2 = ?$

$(x, y) = \left(\frac{m_1x_2 + m_2 \ x_1}{m_1+m_2} \ , \ \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$(-4,6) = \left(\frac{m_1(3) + m_2 \ (-6)}{m_1+m_2} \ , \ \frac{m_1(-8) + m_2 \ (10)}{m_1+m_2}\right)$

$\Rightarrow -4 = \frac{3m_1 - 6 \ m_2}{m_1+m_2}$ Or $6 = \frac{-8m_1 + 10 \ m_2}{m_1+m_2}$

$\Rightarrow -4m_1 - 4m_2 = 3m_1 - 6 \ m_2$

$\Rightarrow \ -4m_1 - 3m_1 = -6 \ m_2 + 4m_2$

$\Rightarrow \ -7m_1 = -2m_2$

$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7}$

$\Rightarrow \ m_1 : m_2 = 2:7$

We should verify that the ratio satisfies the y-coordinate also.

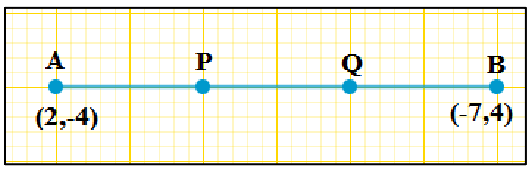
$\frac{-8m_1+10 \ m_2}{m_1+m_2} = \frac{-8(2)+10 \ (7)}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7

Example: Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).

Let P and Q be the trisection points of AB. ⇒ AP = PQ = QB

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula,



$A(x_1, y_1) = (2, -2) \ , B(x_2, y_2) = (-7, 4)$

$m_1 = 1, \ \ m_2 = 2$

$P(x, y) = \left(\frac{m_1x_2 + m_2 \ x_1}{m_1+m_2} \ , \ \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$= \left(\frac{1(-7)+2(2)}{1+2}, \ \frac{1(4)+2(-2)}{1+2}\right)$

$= \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$

$= \left(\frac{-3}{3}, \frac{0}{3}\right)$

$= (-1, 0)$

Now, Q also divides AB internally in the ratio 2 : 1. So, the coordinates of Q are

$A(x_1, y_1) = (2, -2)$, $B(x_2, y_2) = (-7, 4)$

$m_1 = 2, \quad m_2 = 1$

$Q(x, y) = \left(\frac{m_1 x_2 + m_2\ x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2\ y_1}{m_1 + m_2}\right)$

$= \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$

$= \left(\frac{-14+2}{3}, \frac{8-2}{3}\right)$

$= \left(\frac{-12}{3}, \frac{6}{3}\right)$

$= (-4, \quad 2)$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are (–1, 0) and (– 4, 2).

Example 9 : Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.

We know that a point on the $Y-$axis is of the form $(0, y)$. Let the ratio be k : 1

$A(x_1, y_1) = (5, -6)$, $B(x_2, y_2) = (-1, -4)$, $m_1 = k, \quad m_2 = 1$

$(x, y) = \left(\frac{m_1 x_2 + m_2\ x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2\ y_1}{m_1 + m_2}\right)$;

$(0, y) = \left(\frac{k(-1) + 1(5)}{k+1}, \frac{k(-4) + 1\ (-6)}{k+1}\right)$

$\Rightarrow 0 = \frac{-k+5}{k+1}$

$\Rightarrow \quad -k + 5 = 0$

$\Rightarrow \ k = 5$

$\Rightarrow$ The ratio is $5:1$

$y = \frac{5(-4)+1\ (-6)}{5+1}$

$= \frac{-20-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$

$\therefore$ The coordinates of the point of intersection $\left(0, \frac{-13}{3}\right)$

Example 10 : If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Solution: We know that diagonals of a parallelogram bisect each other.

So, the coordinates of the mid-point of AC = coordinates of the mid-point of BD

The coordinates of the Midpoint $= \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$\left(\frac{9+6}{2}, \frac{4+1}{2}\right) = \left(\frac{p+8}{2}, \frac{3+2}{2}\right)$

$\Rightarrow \frac{15}{2} = \frac{p+8}{2}$

$30 = 2p + 16$

$\Rightarrow \ 2p = 30 - 16$

$\Rightarrow \ p = \frac{14}{2}$

$\Rightarrow \ p = 7$

Exercise 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

x_1	y_1	x_2	y_2
-1	7	4	-3

Let the Coordinates of the Points be(x,y)

$m_1 : m_2 = 2:3, (x_1, y_1) = (-1, 7), \ (x_2, \ y_2) = (4, -3)$,

$(x, y) = \left(\frac{m_1x_2 + m_2 \ x_1}{m_1+m_2}, \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$= \left(\frac{2(4)+3(-1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right)$

$= \left(\frac{8-3}{5}, \frac{-6+21}{5}\right)$

$= \left(\frac{5}{5}, \ \frac{15}{5}\right)$

$\Rightarrow (x, y) = \ (1 \ , 3)$

2. Find the coordinates of the poin ts of trisection of the line segment join ing (4, –1) and (–2, –3).

x_1	y_1	x_2	y_2
-1	7	4	-3

Let P and Q are the trisection points of AB

$\Rightarrow$ AP = PQ = QB

$\therefore$ The point P divides AB internally in the ratio 1 : 2

$A(x_1, y_1) = (4, -1)$, $B(x_2, y_2) = (-2, -3)$,

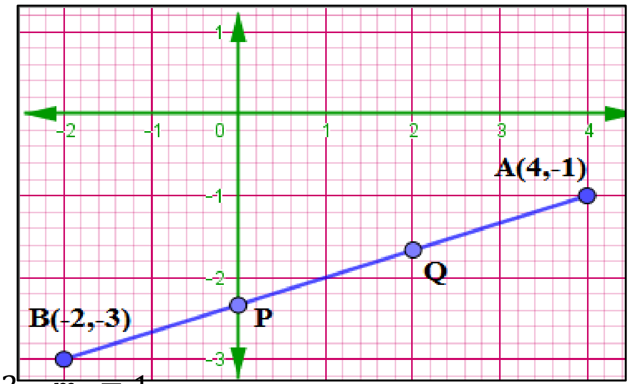
$m_1 = 1, \ \ m_2 = 2$ $\therefore$ The coordinates of P is,

$P(x, y) = \left(\frac{m_1x_2 + m_2 \ x_1}{m_1+m_2} \ , \ \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$= \left(\frac{1(-2)+2(4)}{1+2}, \ \frac{1(-3)+2(-1)}{1+2}\right)$

$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right) = \left(\frac{6}{3}, \ \frac{-5}{3}\right)$

$= \left(2, \ \frac{-5}{3}\right)$



The point Q divides AB internally in the ratio 2 : 1

$A(x_1, y_1) = (4, -1)$, $B(x_2, y_2) = (-2, -3)$; $m_1 = 2, \ \ m_2 = 1$

$Q(x, y) = \left(\frac{m_1x_2 + m_2 \ x_1}{m_1+m_2} \ , \ \frac{m_1y_2 + m_2 \ y_1}{m_1+m_2}\right)$ [Using section formula]

$= \left(\frac{2(-2)+1(4)}{2+1}, \ \frac{2(-3)+1(-1)}{2+1}\right)$

$= \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right)$

$= \left(\frac{0}{3}, \ \frac{-7}{3}\right)$

$= \left(0, \ \frac{-7}{3}\right)$

3. To conduct Sports day activities in your rectangular shaped school groun ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in fig 7.12. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eight line and posts a red flag. What is the distance between both the flags? If Rashmi has post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

Solution: The distance of green flagposted by Niharika on the 2nd line

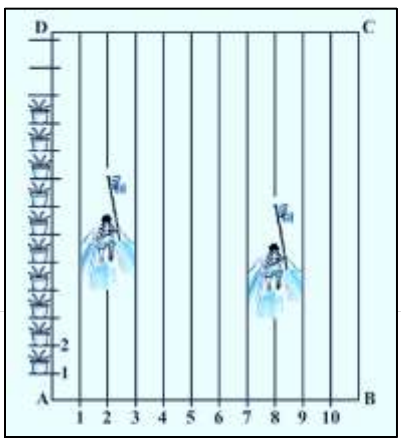
$= \frac{1}{4} \times AD = \frac{1}{4} \times 100 = 25$ m

The distance of red flag posted by Preet on the 8th line

$= \frac{1}{5} \times AD = \frac{1}{5} \times 100 = 20$ m

Coordinates of Green flag $= (2,25) = (x_1, y_1)$

Coordinates of red flag $= (8,20) = (x_2, y_2)$



The distance between flags

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(8-2)^2 + (20-25)^2}$

$= \sqrt{(6)^2 + (-5)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$ m

The coordinates of blue flag, if Rashmi post in between these two flags be

$(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$= \left(\frac{8+2}{2}, \frac{20+25}{2}\right)$

$= \left(\frac{10}{2}, \frac{45}{2}\right)$

$= (5, 22.5)$

4. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).

$P(x, y) = (-1, 6)$, $A(x_1, y_1) = (-3, 10)$, $B(x_2, y_2) = (6, -8)$, $m_1 = ?$, $m_2 = ?$

x_1	y_1	x_2	y_2
-3	10	6	-8

$(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$(-1, 6) = \left(\frac{m_1 6) + m_2 (-3)}{m_1 + m_2}, \frac{m_1(-8) + m_2 (10)}{m_1 + m_2}\right)$

$-1 = \frac{6m_1 - 3m_2}{m_1 + m_2}$ Or $6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$

$-m_1 - m_2 = 6m_1 - 3m_2$

$\Rightarrow -m_1 - 6m_1 = -3m_2 + m_2$

$\Rightarrow -7m_1 = -2m_2$

$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7}$ $m_1 : m_2 = 2 : 7$

We should verify that the ratio satisfies the y-coordinate also

$\frac{-8m_1 + 10m_2}{m_1 + m_2} = \frac{-8(2) + 10(7)}{2+7}$

$= \frac{-16+70}{9} = \frac{54}{9} = 6$

∴ the point (– 1, 6) divides the line segment joining the points A(– 3, 10) and B(6, – 8) in the ratio 2 : 7

5. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by thex-axis. Also find the coordinates of the point of division.

We know that a point on the X – axis is of the form $(x, 0)$ Let the ratio be $k : 1$

$A(x_1, y_1) = (1, -5)$, $B(x_2, y_2) = (-4, 5)$ $m_1 = k$, $m_2 = 1$

$(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$

$(x, 0) = \left(\frac{k(-4) + 1(1)}{k+1}, \frac{k(5) + 1(-5)}{k+1}\right)$

$0 = \frac{5k - 5}{k+1}$

$\Rightarrow 5k - 5 = 0$

$\Rightarrow 5k = 5$

$\Rightarrow k = 1$, the ratio is $1 : 1$

$x = \frac{1(-4) + 1(1)}{1+1} = \frac{-4+1}{2} = \frac{-3}{2}$

∴ The coordinates of the point of division = $\left(\frac{-3}{2}, 0\right)$

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution: Let A(1,2), B(4,y), C(x,6) and D(3,5) are the vertices of the parallelogram.

Since ABCD is a parallelogram

Therefore diagonals AC and BD bisects each other.

So, the coordinates of both AC and BD are same.

$\therefore$ Mid point of AC = Mid point of BD = $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$\left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \left(\frac{x+1}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

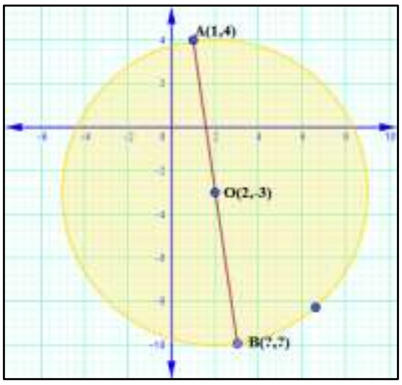
$\Rightarrow \frac{x+1}{2} = \frac{7}{2}$ & $\frac{5+y}{2} = \frac{8}{2}$

$\Rightarrow x+1 = 7$ and $5+y = 8$

$\Rightarrow x = 7-1$ and $y = 8-5$

$\Rightarrow x = 6, \quad y = 3$

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).



The center of the Circle is the mid-point of the diameter

$\therefore (x,y) = (2,-3),\ A(x_1,y_1) = ?\ , B(x_2,y_2) = (1,4)$

$(x,y) = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$(2,-3) = \left(\frac{1+x_1}{2}, \frac{4+y_1}{2}\right)$

$\Rightarrow \frac{1+x_1}{2} = 2, \frac{4+y_1}{2} = -3$

$\Rightarrow 1 + x_1 = 4,\ 4 + y_1 = -6$

$\Rightarrow x_1 = 4-1,\ \ y_1 = -6-4$

$\Rightarrow x_1 = 3,\ y_1 = -10$

$\therefore$ The coordinates of a point A is $(3,-10)$

8. If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB

x_1	y_1	x_2	y_2
-2	-2	2	-4

Given AP = $\frac{3}{7}$ AB

P divides AB in the ratio 3:4

$\Rightarrow$ AP: PB = 3: 4

$Q(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\frac{3(2)+4(-2)}{3+4}, \frac{3(-4)+4(-2)}{3+4}\right)$

$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

9. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts

x_1	y_1	x_2	y_2
-2	2	2	8

The point X divides AB in the ratio 1: 3

The coordinates of X is,

$(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$= \left(\frac{1(2)+3(-2)}{1+3}, \frac{1(8)+3(2)}{1+3}\right)$

$= \left(\frac{2-6}{4}, \frac{8+6}{4}\right)$

$= \left(\frac{-4}{4}, \frac{14}{4}\right)$

$= \left(-1, \frac{7}{2}\right)$

The point Y is the mid-point of AB. The coordinates of Y

$(x,y) = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$= \left(\frac{2-2}{2}, \frac{8+2}{2}\right)$

$= \left(\frac{0}{2}, \frac{10}{2}\right) = (0,\ 5)$

The point Z divides AB in the ratio 3: 1, The coordinates of Z is,

$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

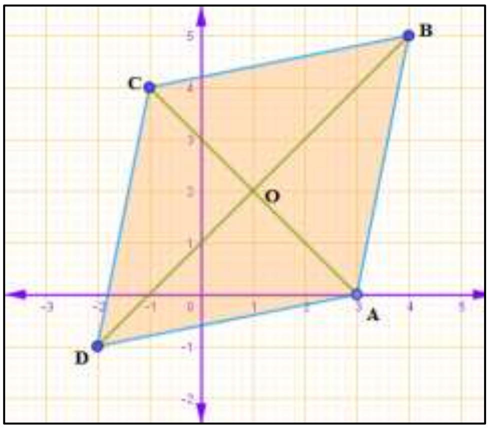
$= \left(\frac{3(2)+1(-2)}{3+1}, \frac{3(8)+1(2)}{3+1}\right)$

$= \left(\frac{6-2}{4}, \frac{24+2}{4}\right)$

$= \left(\frac{4}{4}, \frac{26}{4}\right)$

$= \left(1,\ \frac{13}{2}\right)$

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order[Hint: Area of rhombus = $\frac{1}{2}$(product of its diagonals)]



$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{(-4)^2+(4)^2}$

$= \sqrt{16+16} = \sqrt{16\times 2} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{36+36} = \sqrt{36\times 2} = 6\sqrt{2}$

The area of the rhombus $= \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$

$= \frac{24(\sqrt{2})^2}{2} = 12(2) = 24$ square units.

7.5 Summary

1. The distance between two given points $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2. The distance from the orgin to the given points $d = \sqrt{x^2+y^2}$
3. Section formula :P is the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2,\ y_2)$
   If the point P divides the line in the ratio m: n then the coordinates of P

$P(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

4. If P is the midpoint of AB, it divides in the ratio 1:1
   $P(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$