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(KEY ANSWERS BY SHIVA.T)
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CLASS: 10
Subject code: 81E

SUBJECT: MATHEMATICS
Date of exam: 29.02.2024
I.

1. (C) 2
2. (B) ${ }_{3}^{4} \pi r^{3}$ cubic units.
3. (D) $\mathrm{b}^{2}-4 \mathrm{ac}>0$
4. (A) $\frac{\sqrt{3}}{2}$
5. (A) 24
6. (C) Infinite solutions
7. (B) 13,7
8. (A) 8 cm
9. We know that $\mathrm{HCFxLCM}=\mathrm{axb}$

HCFx $72=24 \times 36$
$\mathrm{HCFx} 72=24 \times 36$
$\mathrm{HCF}=12$
10. $(x-1)=0$ and $(x+3)=0$
$x=1 \& x=-3$
11. Volume of the frustum of a cone $\mathrm{V}=\frac{1}{3} \pi h\left(r_{1}^{2}+r_{2}^{2}+r_{2} r_{1}\right)$
12. We know that the sum of the zeroes of the quadratic polynomial is $\alpha+\beta=-\frac{b}{a}$

$$
=-\frac{7}{1}=-7
$$

13. In the given figure,
$\frac{\operatorname{Sin} R}{\operatorname{Cos} R}=\frac{\frac{12}{13}}{\frac{5}{13}}=\frac{12}{5}$
14. $\mathrm{P}(\mathrm{E})=\frac{n(E)}{n(S)}=\frac{0}{4}=0$
15. Area of $\triangle \mathrm{PQR}=81 \mathrm{~cm}^{2}$.

16. $\operatorname{Sin}(90-A)=\cos 60$
$\cos A=\operatorname{Cos} 60$ $A=60$
III.
17. Prove that $7+\sqrt{5}$ is an irrational number.

Solution: Let us assume that $7+\sqrt{3}$ is a rational number with p and q as co-prime integer and $\mathrm{q} \neq 0$
$\Rightarrow \sqrt{5}=\mathrm{p} / \mathrm{q}$
$\Rightarrow \sqrt{5}=7-\mathrm{p} / \mathrm{q}$
$\Rightarrow \sqrt{5}=(7 \mathrm{q}-\mathrm{p}) / 5 \mathrm{q}$
$\Rightarrow(7 \mathrm{q}-\mathrm{p}) / 5 \mathrm{q}$ is a rational number
However, $\sqrt{5}$ is in irrational number
This leads to a contradiction that $7+\sqrt{5}$ is a rational number.
18. Find the roots of the quadratic equation $3 x^{2}-6 x+2=0$ using quadratic formula.

Solution: here $a=3, b=-6 \& c=2$.
Quadratic formula, $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
Put all the values of $\mathrm{a}, \mathrm{b} \& \mathrm{c}$ in above formula.

$$
\mathrm{x}=\frac{6 \pm \sqrt{6^{2}-4 \times 3 \times 2}}{2 \times 3}
$$

$$
\begin{aligned}
& =\frac{6 \pm \sqrt{36-24}}{6} \\
& =\frac{6 \pm \sqrt{12}}{2 x 1} \\
& =\frac{6 \pm 2 \sqrt{3}}{6} \\
& \qquad x=\frac{6+2 \sqrt{3}}{6} \text { or } x=\frac{6-2 \sqrt{3}}{6} \\
& \quad x=\quad \text { or } x=
\end{aligned}
$$

19. Solve the equation by elimination method,

Solution: let two equations be $4 x+y=15$ $\qquad$
\& $\quad x+y=6$---------(2)
By elimination method,
Multiple 4 to equation (2) \& 1 to equation (1)
We get $4 x+y=15$

$$
\frac{4 x+4 y=24}{-3 y=-9}
$$

$$
\mathrm{Y}=3
$$

Put this $y$ value in any one equation we get $x$ value.
Equation (2) becomes

$$
\begin{array}{r}
x+y=6 \\
X+3=6 \\
X=6-3
\end{array}
$$

20. Find the sum of first 20 terms of the A.P $4,9,14, \ldots \ldots \ldots$.

Solution: Given data here $a=4 d=9-4$
$\mathrm{n}=20$
$\mathrm{d}=5$
We are going to find $\mathrm{S}_{20}$
We know $\mathrm{S}_{\mathrm{n}}=\frac{n}{2}\{2 a+(n-1) d\}$

$$
\begin{aligned}
\mathrm{S}_{20} & =\frac{20}{2}\{2 x 4+(20-1) 5\} \\
& =10\{8+(19) 5\} \\
& =10 \times 103 \\
& =1030 .
\end{aligned}
$$

Find the sum of first 40 positive integers which are divisible by 6
Solution: the number which are divisible by 6 are $6,12,18$, $\qquad$
Given data here $a=6 d=12-6$
$n=40$
$\mathrm{d}=6$
We are going to find $\mathrm{S}_{40}$
We know $\mathrm{S}_{\mathrm{n}}=\frac{n}{2}\{2 a+(n-1) d\}$

$$
\begin{aligned}
\mathrm{S}_{40} & =\frac{40}{2}\{2 x 6+(40-1) 6\} \\
& =20\{12+(39) 6\} \\
& =20 \times 246 \\
& =4920 .
\end{aligned}
$$

21. Find the coordinates of the point which divides the line segment joining the points $(1,5)$ and $(-4,0)$ in the ratio of $2: 3$.
Solution: Here, $x_{1}=1, y_{1}=5, x_{2}=-4$ and $y_{2}=0$ and m:n=2:3
w.k.t, by section formula formula $(\mathrm{x}, \mathrm{y})=\left(\frac{m x_{2}+n x_{1}}{m+n}, \frac{m y_{2}+n y_{1}}{m+n}\right)$

$$
=\left(\frac{-8+3}{5}, \frac{0+15}{5}\right)
$$

$$
=\left(\frac{-5^{1}}{5_{1}}, \frac{15^{3}}{5_{1}}\right)=(x, y)=(-1,3)
$$

22. If $P(A)=\frac{3}{4}$ then, show that $P\left(A_{-}\right) \neq \frac{1}{2}$

Solution: given that $\mathrm{P}(\mathrm{A})=\frac{3}{4}$ and we know $\mathrm{P}(\mathrm{A})+\mathrm{P}\left(\mathrm{A}_{-}\right)=1$

$$
\frac{3}{4}+\mathrm{P}\left(\mathrm{~A}_{-}\right)=1, \mathrm{P}\left(\mathrm{~A}_{-}\right)=1-\frac{3}{4}=\frac{1}{4} \text { Hence the proof }
$$

23. Draw a circle of radius 3.5 cm , then construct a pair of tangents to the circle which are inclined to each other at an angle of $70^{\circ}$.
Solution:

24. Find the value of $: \frac{\cos 45 . \sin 45}{\sec 30-\cot 60}$

Solution: we have $\frac{\cos 45 \cdot \sin 45}{\sec 30-\cot 60}=\frac{\frac{1}{\sqrt{2}} x \frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{3}}}=\frac{\frac{1}{2}}{\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}}{2}$

> OR

Prove that $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\operatorname{Cos} A}$
Solution: $\frac{\cos 2 A+(1+\sin A) 2}{(1+\sin A) \cos A}=\frac{1+1+2 \sin A}{\cos A(1+\sin A)}=\frac{2(1+\sin A)}{\cos A(1+\sin A)}=2 \mathrm{Sec} A$
III.
25. Given that $p(x)=3 x^{3}+4 x^{2}-11 x+35$ by $g(x)=x^{2}-2 x+3$

$$
\begin{gathered}
\left.\mathrm{x}^{2}-2 \mathrm{x}+3\right) 3 \mathrm{x}^{3}+4 \mathrm{x}^{2}-11 \mathrm{x}+35(3 \mathrm{x}+10 \\
\frac{3 \mathrm{x}^{3}-6 \mathrm{x}^{2}+9 \mathrm{x}}{10 \mathrm{x}^{2}-20 \mathrm{x}+35} \\
\frac{10 \mathrm{x}^{2}-20 \mathrm{x}+30}{5}
\end{gathered}
$$

Therefore $\mathrm{q}(\mathrm{x})=3 \mathrm{x}+10$ and $\mathrm{r}(\mathrm{x})=5$
OR
Given that sum $=-3$ and product $=2$
We know that $x^{2}-($ sum $) x+$ product

$$
\begin{aligned}
& x^{2}-(-3) x+2 \\
& x^{2}+3 x+2
\end{aligned}
$$

by factorization method we have $x^{2}+2 x+1 x+2$

$$
\begin{aligned}
& x(x+2) x+1(x+2) \\
& (x+2)(x+1)
\end{aligned}
$$

Therefore zeroes of the polynomial are -2 and -1 .
26.

Given: PT and PS are tangents from an external point P to the circle with centre O .
To prove: PT = PS
Construction: Join O to $\mathrm{P}, \mathrm{T}$ and S .


Proof: In $\triangle \mathrm{OTP}$ and $\Delta \mathrm{OSP}$.
$\mathrm{OT}=\mathrm{OS} \ldots$ [radii of the same circle]
$\mathrm{OP}=\mathrm{OP} \ldots$ [common]
$\angle \mathrm{OTP}=\angle \mathrm{OSP} \ldots$ each $90^{\circ}$ ]
$\Delta \mathrm{OTP}=\Delta \mathrm{OSP} \ldots$ [R.H.S.]
$\mathrm{PT}=\mathrm{PS} \ldots$ [c.p.c.t.]
27.

Solution:

28. Find the mean for the following data by direct method.

| c.i | F | X | fx |
| :--- | :--- | :--- | :--- |
| $2-6$ | 4 | 4 | 16 |
| $6-10$ | 8 | 8 | 64 |
| $10-14$ | 2 | 12 | 24 |
| $14-18$ | 1 | 16 | 16 |
| $18-22$ | 5 | 20 | 100 |
|  | $\mathrm{~N}=20$ |  | $\sum f x=220$ |

Mean by direct method, $\mathrm{x}=\frac{\sum f x}{n}=\frac{220}{20}=11$

|  |  | OR |
| :--- | :--- | :--- |
| c.i | F |  |
| $5-15$ | 4 | $\mathrm{f}_{0}$ |
| $15-25$ | 8 | $\mathrm{f}_{1}$ |
| $25-35$ | 2 | $\mathrm{f}_{2}$ |
| $35-45$ | 5 |  |
| $45-55$ | 1 |  |
|  |  |  |

Here LRL=15, $\mathrm{f}_{1}=8, \mathrm{f}_{0}=4, \mathrm{f}_{2}=2$ and $\mathrm{h}=10$
Mode $=$ LRL $+\left\{\frac{f 1-f 0}{2 f 1-f 0-f 2}\right\} \times \mathrm{h}$

$$
\begin{aligned}
= & 15+\left\{\frac{8-4}{16-4-2}\right\} \times 10 \\
& =15+4
\end{aligned}
$$

## Mode=19

29. O-give graph

Solution:
More than type O-give
scale, along $x$-axis $1 \mathrm{~cm}=10$ units and $y$-axis $1 \mathrm{~cm}=10$ unis

30. Find the area of the triangle whose vertices are $\mathrm{A}(-5,-1), \mathrm{B}(3,-5) \& \mathrm{C}(5,2)$.

Solution: $(x 1, y 1)=(-5,-1),(x 2, y 2)=(3,-5)$ and $(x 3, y 3)=(5,2)$

We have Area of $\Delta^{l e}=\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$

$$
\text { Area of } \triangle \mathrm{PQR}=\frac{1}{2}[-5(-5-2)+3(2+1)+5(-1+5)]
$$

$$
\begin{aligned}
& =\frac{1}{2}[35+9+20] \\
& =\frac{1}{2}[64]=\frac{1}{z_{1}} \times 64^{32}
\end{aligned}
$$

## $\therefore$ Area $\Delta=32$ sq.units

31. Solution:


Just assume that


Area of triangle is $49 \sqrt{3} \mathrm{~cm}^{2}$
$\frac{\sqrt{3}}{4} a^{2}=49 \sqrt{3} \Rightarrow \mathrm{a}=14 \mathrm{~cm}$
The area of 3 circles is $=\frac{\theta}{360} \mathrm{x} \pi \mathrm{r}^{2}$.

$$
\begin{aligned}
& =\frac{60}{360} \times \frac{22}{7} \times 7 \times 7 \\
& =77 \mathrm{~cm}^{2} .
\end{aligned}
$$

Area of remaining part is
$49 \sqrt{3}-77=84.868-77=7.868 \mathrm{~cm}^{2}$.
Perimeter of 1 arc $=\frac{\theta}{360} \times 2 \pi r$

$$
=\frac{60}{360} 2 \times \frac{22}{7} x 7=\frac{22}{3} \mathrm{~cm} .
$$

Such 3 arcs $=3 x \frac{22}{3} \mathrm{~cm} .=22 \mathrm{~cm}$.
32. Solution:

Let speed of the stream be $\mathrm{xkm} / \mathrm{hr}$
Speed of the boat upstream = speed of the boat in still water - speed of the stream
Speed of the boat upstream $=(11-\mathrm{x}) \mathrm{km} / \mathrm{hr}$
Speed of the boat downstream $=$ speed of the boat in still water + speed of the stream
Speed of the boat downstream $=(11+\mathrm{x}) \mathrm{km} / \mathrm{hr}$
It is mentioned that the boat can go 12 km upstream and return downstream to its original point in 2 hr 45 min .
$\Rightarrow$ One-wayDistance traveled by boat (d) $=12 \mathrm{~km}$
$\Rightarrow \mathrm{T}_{\text {upstream }}+\mathrm{T}_{\text {downstream }}=2 \mathrm{hr} 45 \mathrm{~min}=(2+3 / 4) \mathrm{hr}=11 / 4 \mathrm{hr}$
$\Rightarrow$ [distance / upstream speed $]+[$ distance $/$ downstream speed $]=11 / 4$
$\Rightarrow[12 /(11-x)]+[12 /(11+x)]=11 / 4$
$\Rightarrow 12[1 /(11-x)+1 /(11+x)]=11 / 4$
$\Rightarrow 12\left[\{11-\mathrm{x}+11+\mathrm{x}\} /\left\{121-\mathrm{x}^{2}\right\}\right]=11 / 4$
$\Rightarrow 12\left[\{22\} /\left\{121-x^{2}\right\}\right]=11 / 4$
$\Rightarrow 12\left[2 /\left\{121-x^{2}\right\}\right]=1 / 4$
$\Rightarrow 24 /\left\{121-x^{2}\right\}=1 / 4$
$\Rightarrow 24(4)=\left\{121-x^{2}\right\}$
$\Rightarrow 96=121-x^{2}$
$\Rightarrow x^{2}=121-96$
$\Rightarrow \mathrm{x}^{2}=25$
$\Rightarrow \mathrm{x}=+5$ or -5
As speed to stream can never be negative, we consider the speed of the stream(x) as $5 \mathrm{~km} / \mathrm{hr}$.

## OR

Let the present age of Person be $x$.
Three years ago, Person age $=x-3$
Five years hence, Person age $=x+5$
It is given that, sum of the reciprocals of Person ages 3 years ago and 5 years from now is 13 .
$\Rightarrow 1 \mathrm{x}-3+1 \mathrm{x}+5=1 / 3$
$\Rightarrow \mathrm{x}+5+\mathrm{x}-3(\mathrm{x}-3)(\mathrm{x}+5)=1 / 3$
$\Rightarrow 2 \mathrm{x}+2(\mathrm{x}-3)(\mathrm{x}+5)=1 / 3$
$\Rightarrow 3(2 \mathrm{x}+2)=(\mathrm{x}-3)(\mathrm{x}+5)$
$\Rightarrow 6 \mathrm{x}+6=\mathrm{x}^{2}+2 \mathrm{x}-15$
$\Rightarrow \mathrm{x}^{2}+2 \mathrm{x}-6 \mathrm{x}-15-6=0$
$\Rightarrow \mathrm{x}^{2}-4 \mathrm{x}-21=0$
$\Rightarrow x^{2}-7 x+3 x-21=0$
$\Rightarrow(\mathrm{x}-7)(\mathrm{x}+3)=0$
$\Rightarrow \mathrm{x}-7=0$ or $\mathrm{x}+3=0$
$\Rightarrow x=7$ or $x=-3$
So, $x$ can be 7 or -3 .
Since, the age of a person cannot be negative.
Hence, Person present age is 7 years.
33. Given:


In triangle APQ and ABC ,
$\mathrm{AP}=2.5 \mathrm{~cm}, \mathrm{AB}=5 \mathrm{~cm}$ and $\mathrm{PQ}=2 \mathrm{~cm}$.
$\mathrm{A}=\mathrm{A}$ common
$\angle A B C=\angle A P Q$
$\lfloor A Q P=\lfloor A C B$ (corresponding sides)
By AA similarity
$\frac{A P}{A B}=\frac{P Q}{B C}$
$\frac{2.5}{5}=\frac{2}{B C}$ hence $B C=4 \mathrm{~cm}$
By theorem, $\frac{\text { ar of } \triangle A P Q}{\text { ar of } \triangle A B C}=\frac{P Q^{2}}{A B^{2}}=\frac{2^{2}}{4^{2}}=\frac{1}{2}$ Hence the proof
34. Given:
$\mathrm{n}=51, \mathrm{~d}=5$ and $20^{\text {th }}$ term from the last term is $(51+1)-20=32^{\text {nd }}$ term.
$\mathrm{a}_{32}=157$
$a+31 d=157$
$a+31 \times 5=157$
$a=157-155$
$a=2$
this A.P is a, a+d, $\mathrm{a}+2 \mathrm{~d}$
$2,2+5,2+10$
$2,7,12, \ldots \ldots \ldots$.

Given: $a_{2}+a_{4}=22 \quad S_{11}=253$
$a+d+a+3 d=22$
$2 \mathrm{a}+4 \mathrm{~d}=22 \rightarrow---\rightarrow(1)$

$$
\begin{aligned}
& \mathrm{S}_{11}=\frac{11}{2}\{2 a+(11-1) d\}=253 \\
& \frac{11}{2}\{2 a+10 d\}=253 \\
& 2 \mathrm{a}+10 \mathrm{~d}=46----\rightarrow(1)
\end{aligned}
$$

Subtract above two equations we get $6 d=24$
$\mathrm{d}=4$
hence common difference is 4 and $a=3$
A.P is $3,7,11$.

If last term is 67 , an =67 we have to find $n$ here.

$$
\begin{gathered}
a+(n-1) d=67 \\
3+4 n-4=67 \\
4 n=68 \\
n=17
\end{gathered}
$$

therefore number of terms is 17
35. Given equations are $2 x+y=8$ and $x+y=5$

We have to make table for this
$2 x+y=8$

| $x$ | 0 | 4 |
| :--- | :--- | :--- |
| $y$ | 8 | 0 |

$x+y=5$

| $x+y=5$ | 0 | 5 |
| :--- | :--- | :--- |
| $y$ | 5 | 0 |

,

Construction: Mark points G and H on the side AB and AC such that
$\mathrm{AG}=\mathrm{DE}, \mathrm{AH}=\mathrm{DF}$
proof: in triangle AGH and DEF
AG=DE.....by construction
$\mathrm{AH}=\mathrm{DF}$..... by contsruction
$\angle$ GAH= $\angle$ EDF...Given
therefore,
$\triangle A G H \cong \triangle F E D$ by SAS congruency thus
$\angle A G H=\angle D E F ~ . . .$. by CPCT
but $\angle \mathrm{ABC}=\angle \mathrm{DEF}$
$\angle A G H=\angle A B C$
Thus GH \|BC
Now, In triangle ABC
$\frac{A B}{A G}=\frac{B C}{G H}=\frac{C A}{H A}$
Hence, $\frac{A B}{D E}=\frac{B C}{E F}=\frac{C A}{F D}$
hence proved.
37.
in the given figure,
let h be the height of the tower $\mathrm{CE}=\mathrm{h}$
then $\mathrm{DE}=\mathrm{h}-10 \sqrt{3} \mathrm{~m}$
in triangle $\mathrm{ADE}, \tan 30=\frac{\mathrm{h}-10 \sqrt{3}}{30}$

$$
\begin{aligned}
& \frac{1}{\sqrt{3}}=\frac{h-10 \sqrt{3}}{30} \\
& \sqrt{3} \mathrm{~h}-10=30 \Rightarrow h=\frac{40}{\sqrt{3}}=23.12 \mathrm{~m}
\end{aligned}
$$



We have to find angle of depression formed from the top of the light house to the ship, it means at A. In triangle $\mathrm{ABF}, \tan \Theta=\frac{10}{10 \sqrt{3}}=\frac{1}{\sqrt{3}}$ hence $\Theta=60^{\circ}$.
38. Given:

Cylinder: $\mathrm{r}=8 \mathrm{~cm}$ \& height $\mathrm{h}=45-15=30 \mathrm{~cm}$
Cone: $\mathrm{r}=8 \mathrm{~cm}$ and $\mathrm{h}=15 \mathrm{~cm}$ we have to find $\mathrm{l}=\sqrt{h^{2}+r^{2}}$

$$
\begin{aligned}
& =\sqrt{15^{2}+8^{2}} \\
& =\sqrt{225+64} \\
& =\sqrt{289}
\end{aligned}
$$

$$
\mathrm{l}=17 \mathrm{~cm}
$$

Total surface area of wooden solid $=$ C.Surface area of cone + C.surface area of cylinder+area of base.

$$
\begin{aligned}
& =\pi \mathrm{rl}+2 \pi \mathrm{rh}+\pi \mathrm{r}^{2} . \\
& =\pi \mathrm{r}(1+2 \mathrm{~h}+\mathrm{r}) \\
& =\frac{22}{7} \times 8(17+60+8) \\
& =\frac{22}{7} \times 8 \times 85 \\
& =2137.14 \mathrm{~cm}^{2} .
\end{aligned}
$$

Volume of the wooden solid $=$ volume of cone+volume of cylinder.

$$
\begin{aligned}
& =\frac{1}{3} \pi r^{2} \mathrm{~h}+\pi \mathrm{r}^{2} \mathrm{H} \\
& =\pi \mathrm{r}^{2}\left(\frac{1}{3} \mathrm{~h}+\mathrm{H}\right) \\
& =\frac{22}{7} \times 8 \times 8(15+30) \\
& =\frac{1}{3} \times \frac{22}{7} \times 8 \times 8 \times 45 \\
& =7039.99 \mathrm{~cm}^{3} .
\end{aligned}
$$

