

II PUC STATISTICS
ONE MARK TWO MARKS QUESTIONS

1. State merits and demerits of moving average method.

Ans. Merits: a) it is simple to understand.

b) it is more flexible than any other methods.

Demerits: a) no trend values in the beginning and in the end.

b) choice of period of moving average is difficult.

2. Define Interpolation and extrapolation.

Ans. Interpolation is the technique of estimating the value of the depending variable (y) for any intermediate value of the independent variable (x)

Extrapolation is the technique of estimating the value of the depending variable (y) for any value of the independent variable (x) which is outside the range of the given series.

3. What is discrete probability distribution and continuous probability distribution?

Ans: The probability distribution of a discrete random variable is known as discrete probability distribution.

The probability distribution of a continuous random variable is known as continuous probability distribution.

4. Define Bernoulli distribution.

Ans: A random variable X is said to follow Bernoulli distribution if its p.m.f is given by

$$P(x) = p^x q^{1-x}, x=0,1.$$

5. Define Binomial distribution.

Ans: A random variable X is said to follow Bernoulli distribution if its p.m.f is given by

$$p(x) = {}^n C_x p^x q^{n-x}, x=0,1,\dots,n.$$

6. Define Poisson distribution.

Ans: A random variable X is said to follow Poisson distribution if its p.m.f is given by

7. Define Hyper Geometric Distribution.

Ans: A random variable X is said to follow Hyper Geometric distribution if its p.m.f is given by

8. Define Normal distribution.

Ans: A random variable X is said to follow Normal distribution if its p.d.f is given by

9. Mention the features of Bernoulli distribution.

Ans: a) the parameter is 'p'

b) mean = p and variance = pq.

c) mean is greater than variance

10. Mention the features of Bernoulli distribution.

Ans: a) the parameters are 'n' and 'p'.

- b) mean= np and variance= npq .
- c) mean is greater than variance.

11. Mention the features of Poisson distribution.

- Ans: a) the parameter is λ .
- b) mean= λ and variance= λ .
 - c) mean and variance are equal.

12. Mention the features of normal distribution.

- Ans: a) the normal curve is bell shaped.
- b) mean=median=mode= μ .
 - c) the distribution is mesokurtic.
 - d) Q.D= $2\sigma/3$ M.D= $4\sigma/5$
 - e) the total area under the curve is one.
 - f) the normal distribution has only one mode.

13. Mention the features of Hyper Geometric Distribution.

- Ans: a) the parameters are a,b and n.
- b) mean = $na/(a+b)$ variance= $nab(a+b-n)/(a+b)^2 (a+b-1)$.

14. Mention the features of chi square distribution.

- Ans: a) the parameter is n.
- b) mean= n variance= $2n$.
 - c) mode= $n-2$, $n > 2$.

15. Mention the features of t distribution.

- Ans: a) variance= $n/n-2$, $n > 2$.
- b) it is symmetrical i.e mean=median=mode=0.
 - c) the distribution is leptokurtic.

16. Define population.

Ans: The total of all the units under study is called population.

17. Define parameter.

Ans: A statistical constant of the population is called parameter.

18. Define statistic.

Ans: A function of the sample values is called statistic.

19. Define sampling distribution.

Ans: The distribution of the values of a statistic for different samples of same size is called sampling distribution.

20. Define standard error.

Ans: the standard deviation of the sampling distribution is known as standard error.

21. Mention two branches of statistical inference.

Ans: a)estimation b)testing of hypothesis.

22. What is estimation?

Ans: Method of obtaining the most likely value of the population using statistic is called estimation.

23. Define estimator.

Ans: Any statistic which is used to estimate an unknown parameter is called an estimator.

24. What is an estimate?

Ans: An estimate is the most likely value of the unknown parameter.

25. Differentiate between point estimation and interval estimation.

Ans:

<u>Point estimation</u>	<u>interval estimation</u>
A single value is proposed as an estimate of the unknown parameter	an interval is proposed as an estimate of the unknown parameter.
Eg :Mean marks of 60 students is 70.	Eg:The mean marks of 60 of students lies between (60-70)

26. Define confidence interval.

Ans: an interval which is proposed to contain the unknown parameter is called confidence interval

27. Define confidence coefficient.

Ans: The probability that the confidence interval contains the unknown parameter is called confidence coefficient.

28. Define confidence limits.

Ans: The boundary values of confidence interval are confidence limits.

29. Define statistical hypothesis.

Ans: a statistical hypothesis is a statement regarding the parameters of the population.

30. Differentiate between null and alternative hypothesis.

Ans:

<u>Null hypothesis</u>	<u>Alternative hypothesis</u>
it is the hypothesis which is being tested for a possible rejection under the assumption that it is true.	it is the hypothesis which is accepted when null hypothesis is rejected
It is denoted by H_0	It is denoted by H_1

31. Differentiate between type I error and type II error.

Type I error	Type II error
The error that occurs by rejecting the null hypothesis when it is actually true	The error that occurs by accepting the null hypothesis when it is actually not true
It is also known as first kind error	It is also known as second kind error.

32. Define level of significance.

Ans: maximize size of the test is called level of significance.

33. What is size of the test?

Ans: the probability of rejecting H_0 , when it is true is called size of the test.

34. What is power of the test?

Ans: the probability of rejecting the H_0 , when it is not true is called power of a test.

35. What is critical region and accepting region.

Ans: critical region is the set of those values of the test statistic, which leads to the rejection of null hypothesis.

Accepting region is the set of those values of the test statistic, which leads to the acceptance of the null hypothesis.

36. Define statistical quality control.

Ans: It is the method of collecting the quality of the products using statistical techniques.

37. What is acceptance sampling or product control?

Ans: The manufactured lot will be accepted after inspection and confirmation of the quality standards. Two types are a) single sampling plan b) double sampling plan.

38. Differentiate between chance causes and assignable causes.

Ans:

Chance causes	Assignable causes
Variation for which no specific Cause can be attributed is called Chance causes.	Variation for which causes can be Precisely identified is called assignable causes.
This cannot be eliminated	This can be eliminated

39. What is variable and attribute?

Ans: A measurable quality characteristic which varies from unit to unit is called a variable.

A qualitative characteristic which cannot be measured and can only be identified by its presence or absence is called an attribute.

40. Differentiate between defect and defective.

Ans:

defects	defectives
It is a quality characteristics which Does not conform to specifications.	An item having one or more defects Is a defective item.
Misprint or damage in weaving of a Cloth is a defect	Cloth having one or more defects is Called defectives.

41. What is SSP and DSP?

Ans: In a single sampling plan , the decision about accepting or rejecting the lot is based on one sample only.

In a double sampling plan the decision about accepting or rejecting the lot is based on two samples.

42. Define feasible solution.

Ans: Any solution to a general LPP which satisfies the non negativity restrictions is called a feasible solution to the LPP.

43. What is unbounded solution?

Ans: In case of maximization problem, when the LPP do not posses a finite optimum that is value of z can be increased indefinitely then the LPP is said to have unbounded solution.

44. What is no solution?

Ans: when an LPP has no feasible region, it is said to have no solution.

45. What is multiple optimal solution?

Ans: When there exists more than one feasible solution having the same optimal value for the objective function then the LPP is said to have multiple or alternative optimal solution.

46. Define degenerate and non degenerate solution.

Ans: When the number of positive allocations in any basic feasible solution is less than $(m+n-1)$, then the solution is said to be a degenerate solution. When the number of positive allocations in any basic feasible solution is equal to $(m+n-1)$, then the solution is said to be a non degenerate solution.

47. When is a transportation problem said to be balanced?

Ans : when the sum of supply is equal to the sum of the requirements, then the transportation problem is said to be balanced.

48. State the properties of competitive .

Ans: a) There are finite number of competitors called players .

b) Each player has a finite number of courses of action .

c) A game is said to be played when each player chooses one of his courses of action available to him. These choices are assumed to be simultaneously.

49. Define n-person game and two person game.

Ans: A game with n participants is called n-person game.

A game involving two participants is called two person game.

50. Define strategy and mention its types.

Ans: The strategy of a player is the pre determined rule by which a player determines his course of action. There are two types of strategy. a) pure strategy b) mixed strategy.

51. What is zero sum game and two person zero sum game?

Ans: A game in which sum of the payoffs of the players is zero is called zero sum game.

A game with two players in which the gain of one player is the loss of other player is called two person zero sum game.

52. Define saddle point and a fair game.

Ans: The position in the payoff matrix where maximin coincides with minimax is called saddle point.

A game is said to be a fair game if the value of the game is zero.

53. What is value of the game?

Ans: The value at the saddle point is called the value of the game.

54. what is replacement theory? State the need of replacement theory.

Ans: Replacement theory deals with the problem of deciding the age at which an item which deteriorates with time has to be replaced by a new one. The need for replacement are 1) the item that has become inefficient with time. 2) it requires more maintenance cost. 3) Some items suddenly fails 4) A better or a more efficient design

55. What is inventory?

Ans: Inventory is the physical stock of goods for future use.

56. Define stock replenishment and lead time?

Ans. The quantity of goods acquired in one replenishment in order to maintain a certain level of inventory is known as stock replenishment.

The time gap between placing of order and arrival of goods at the inventory is the lead time.

57. Define the two variables of Inventory problem?

Ans: The variables of Inventory problem are

1) Controlled Variable- the variables that may be controlled like frequency of replenishment, quantity of goods required etc are known as controlled variables.

2) Uncontrolled variables – the variables that may not be controlled in an inventory problem like demand, lead time is known as Uncontrolled Variables.

58. What is vital statistics?

Ans: Vital statistics are the numerical records, analysis and interpretation of numerical data of vital events such as birth, death etc.

59. Explain the methods of obtaining vital statistics.

Ans: Different methods of obtaining vital statistics are

- a) registration method-here vital events such as births, deaths etc are continuously recorded. The government etc authorities like gram panchayats, municipalities etc record the data of vital events. This method is successful when registration becomes compulsory.
- b) census enumeration method-in most of the countries census enumeration is conducted once in ten years. Here data is collected regarding religion, educational status etc. information is obtained only for census year.

60. State the formula to measure the population at time t.

Ans: $P_t = P_0 + (B-D) + (I-E)$.

61. State the uses of vital statistics.

- Ans: a) They are useful in medical research.
b) They are essential in demographic research.

62. Define fertility and fecundity.

Ans: Fertility refers to the births occurring to women of child bearing age (15-49 years).
Fecundity refers to the capacity of a woman to bear children.

63. Define Mortality.

Ans: Mortality refers to deaths occurring in the population due to sickness, accident etc.

64. What is Infant mortality rate?

Ans: Infant mortality rate is defined as average number of infant deaths per thousand live births in a year or children die before they attain age of one year.

65. What is Neo natal mortality rate?

Ans: Neo natal mortality rate is defined average number of neo natal deaths (death before the child attains 1 month of age) per 1000 live births in a year.

66. What is maternal mortality rate?

Ans: Maternal mortality rate is defined as average number of deaths among mother due to child birth per 1000 births in a year.

67. Define Cohort.

Ans: Cohort is a group of individuals who are born at the same time & who experience the same mortality conditions.

68. What is Radix?

Ans: Radix is the size of the cohort (Generally 100000).

69. What is longevity?

Ans: Longevity is the expected number of years that a new born baby would live.

70. Define Index Number.

Ans: Index number are statistical devices designed to measure the relative changes in the

level of a phenomenon with respect to time, income etc.

71. Index number act as economic barometer.

Ans: Index numbers measures the pulse of the economy and act as a barometer to find the variations economic condition of the country. Hence index number acts as a economic barometer.

72. What is price relative?

Ans: Price relative is the price of the current year expressed as the percentage of the price in the base year. $P = (p_1/p_0)*100$.

73. What is quantity relative?

Ans: Quantity relative is the quantity of the current year expressed as the percentage of the quantity in the base year. $Q=(q_1/q_0)*100$.

74. What is value relative?

Ans: Value relative is the value of the current year expressed as the percentage of the value in the base year. $P=(v_1/v_0)*100$.

75. State the characteristics of index number.

Ans: a) Index numbers are specialized type of averages.
c) it facilitates comparison.

76. Mention the uses of index number.

Ans: a) index numbers are used in studying trend and tendencies.
b) it simplifies the data and hence facilitates comparison.

77. Mention the steps used in the construction of index number.

Ans: a) defining the purpose of the index number.
b) selection of the base period.
c) selection of commodities.
d) obtaining price quotations
e) choice of an average.
f) selection of weights.
g) selection of suitable formula.

78. Name the index number that satisfies TRT.

Ans: Marshall Edgeworth index number, Fisher's index number and Kelly's index number satisfies TRT.

79. Name the index number that satisfies that FRT.

Ans: Fisher's index number

80. What is consumer price index number?

Ans: CPI is the index number of the cost met by a specified class of consumers in buying a 'basket of goods and services'.

81. What is time series?

Ans: The chronological arrangement of numerical data is called time series.

Note: a graphic representation of time series is called histogram.

82. Explain the components of time series.

Ans: The components of time series are

Secular trend- the general tendency of the time series data to increase or decrease or remain constant over a long period of time is called secular trend. ex - increase in the price of gold in the past many years.

Seasonal variation- the regular, periodic and short term variations in a time series is called seasonal variation. Usually the period of seasonal variation will be less than one year. ex- umbrellas are sold more in rainy season.

Cyclical variation(business cycle)- the periodic up and down movements in economic and business time series is called cyclical variation. Its four stages are prosperity, decline, depression and recovery.

Irregular variation(random or erratic variation)- it is the irregular movements of the data over a period of time. These are sudden unexpected variations due to war, natural calamities etc. ex- sudden increase in death rate in a locality due to tsunami.

FIVE MARKS QUESTIONS

1. From the following data calculate TFR, GFR and ASFR.

Age(years)	Women Population	No.of births to women
15-19	84790	343
20-24	70010	14541
25-29	72660	16736
30-34	75920	12218
35-39	75100	756
40-44	71620	82
45-49	66660	45

Solution :

Age(years)	Women Population	No.of births to women	ASFR
15-19	84790	343	4.5
20-24	70010	14541	207.7
25-29	72660	16736	230.33
30-34	75920	12218	160.93
35-39	75100	756	10.07
40-44	71620	82	1.15
45-49	66660	45	0.68
TOTAL	516760	44721	614.91

$$\text{GFR} = \frac{\text{No. Of live births occurring in the year} \times 1000}{\text{Average population of women of child bearing age}}$$

$$= \frac{44721}{516760} \times 100 = 86.54$$

TFR = $5 \times \sum \text{ASFR} = 5 \times 614.91 = 3074.55$ per thousand women.

2. Calculate Net Reproduction Rate from the following data.

Age	Women Population	Survival rate	No of female birth
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15-19	4907	0.956	101
20-24	4817	0.947	431
25-29	4441	0.937	316
30-34	3911	0.929	161
35-39	3684	0.917	68
40-44	3371	0.905	26
45-49	2911	0.890	9

Solution :

Age	Women Population	Survival rate	No of female Birth	WSFR	S X WSFR
15-19	4907	0.956	101	20.6	19.69
20-24	4817	0.947	431	89.5	84.76
25-29	4441	0.937	316	71.2	66.86
30-34	3911	0.929	161	41.2	38.27
35-39	3684	0.917	68	18.4	16.87
40-44	3371	0.905	26	7.7	6.97
45-49	2911	0.890	9	3.1	2.76
TOTAL	2842				236.18

$$NRR = i \times \sum Sx WSFR$$

$$= 5 \times 236.18$$

$$= 1180.9$$

$$NRR \text{ per women} = \frac{1180.9}{1000} = 1.1809$$

Since the value is greater than 1 the population is increasing.

3. Calculate Gross Reproduction Rate from the following data.

Age	Women Population	No of female Birth
15-19	54000	2371
20-24	52000	8935
25-29	49000	8414
30-34	46000	4072
35-39	41000	1846
40-44	36000	437
45-49	31000	184

Solution:

Age	Women Population	No of female Birth	WSFR
15-19	54000	2371	43.91
20-24	52000	8935	171.83
25-29	49000	8414	171.71
30-34	46000	4072	88.52
35-39	41000	1846	45.02
40-44	36000	437	12.14
45-49	31000	184	5.94
TOTAL	309000	26259	539.07

$$GRR = i \times \Sigma WSFR$$

$$= 5 \times 539.07 = 2695.35$$

$$GRR \text{ per women} = \frac{2695.35}{1000} = 2.695$$

Since this number is greater than 1 population is increasing.

4. Calculate cost of living index number using Family Budget method from the following data.

Items	Weight	Price in Base year	Price in Current year
Food	10	150	225
House Rent	5	50	150
Clothing	2	30	60
Fuel	3	30	75
Others	5	50	75

Solution :

Items	W = p ₀ q ₀	p ₀	p ₁	P=p ₁ /p ₀ X100	PW
Food	10	150	225	150	1500
House Rent	5	50	150	300	1500
Clothing	2	30	60	200	400
Fuel	3	30	75	250	750
Others	5	50	75	150	750
Total	25				4900

$$CPI = \frac{\sum WP}{\sum W} = \frac{4900}{25} = 196$$

5. Explain briefly the steps in the construction of consumer price index number.

Step 1 : Defining Scope and coverage

At the outset it is necessary to decide the class of consumers for which the index number is required. The class may be that of bank employees, government employees, merchants, farmers etc. In any case, the geographical coverage should also be decided. That is the locality, city or town where the class dwells should be mentioned. Any how the consumer in the class should have almost the same pattern of consumption.

Step 2: Conducting Family Budget enquiry and selecting the weights

The next step is to conduct a sample survey of consumer families regarding their budget on various items. The survey should cover a reasonably good number of representative families. It should be conducted during a period of economic stability. In the survey information commodities consumed by the families, their quality, and the respective budget are collected. The items included in the index number are generally classified under the heads (i) Food (ii) clothing (iii) Fuel and lighting and (iv) others. Sufficiently large number of representative items are included under each head.

Step 3 : Obtaining Price Quotations

The quotations of retail prices of different commodities are collected from local market. The quotations are collected from different agencies and from different places. Then they are averaged and these averages are made use in the construction of index numbers. The price quotations of the current period and that of the base period should be collected.

Step 4 : Computing the index number.

a) Aggregative expenditure method

$$CPI = \frac{\sum p_{1q_0}}{\sum p_{0q_0}} \times 100$$

b) Family Budget Method

$$CPI = \frac{\sum PW}{\sum W}$$

6. Calculate 3 yearly and 5 yearly moving averages for the following data.

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Profit	110	104	98	105	109	120	115	110	114	122	130	127

Solution :

Year	Profit	3 yearly Moving sum	Trend values	5 yearly Moving sum	Trend values
1991	110	_____	_____	_____	_____
1992	104	312	312/3 = 104	_____	_____
1993	98	307	102.33	526	526/5 =105.2
1994	105	312	104	536	107.2
1995	109	334	111.33	547	109.4
1996	120	344	114.67	559	111.8
1997	115	345	115	568	113.6
1998	110	339	113	581	116.2
1999	114	346	115.33	591	118.2
2000	122	366	122	_____	_____
2001	130	379	126.33	_____	_____

2002	127	_____	_____	_____	_____
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In 3 yearly moving trend: it is an oscillatory trend

In 5 yearly moving trend: it is upward trend.

7. Calculate 4 yearly moving average for the moving data.

Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Value	103	104	107	101	102	104	105	99	100

Solution:

Year	Value	4 yearly Moving sum	2 yearly moving sum	Trend (previous col/8)
1998	103			
1999	104			
		415		
2000	107		829	103.6
		414		
2001	101		828	103.5
		414		
2002	102		826	103.3
		412		
2003	104		822	102.8
		410		
2004	105		818	102.3
		408		

2005	99			
2006	10			

It is a downward trend.

8. The following are the figures of production (in thousand quintals) of a sugar factory.

Year	1992	1994	1996	1998	2000	2002	2004
Production	77	81	88	94	94	96	98

- (i) Fit a straight line trend to the data.
- (ii) Estimate the production in the year 2006.

Solution:

Year	y	x	x ²	xy	Trend values (thousand quintals)
1992	77	-3	9	-231	78.87
1994	81	-2	4	-162	82.48
1996	88	-1	1	-88	86.09
1998	94	0	0	0	89.7
2000	94	1	1	94	93.31
2002	96	2	4	192	96.92
2004	98	3	9	296	100.53
Total	628	0	28	101	627.9 = 628

Thus,

$$a = \frac{\Sigma y}{n} = \frac{628}{7} = 89.7$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{101}{28} = 3.61$$

Thus the trend equation is $y = 89.7 + 3.61 x$

The value of x corresponding to the year 2006 is 4. Therefore the estimate of production for the year 2006 is : $y = 89.7 + 3.61 x 4 = 104.14$ thousand quintals.

9. Team A has probability $\frac{2}{3}$ of winning a game. If it plays 4 games, find the probability that

it wins (i) 2 games (ii) atleast one game.

Solution : Given $n = 4$, $p = 2/3$, $q = 1-p = 1/3$

$$\begin{aligned} \text{(i) } P(\text{ team A wins 2 games}) &= P(X = 2) = {}^n C_x p^x q^{n-x} \\ &= {}^4 C_2 (2/3)^2 (1/3)^2 = 0.2963 \end{aligned}$$

$$\begin{aligned} \text{(ii) } P(\text{ A wins atleast one game}) &= P(X \geq 1) = 1 - P(X=0) \\ &= 1 - {}^4 C_0 (2/3)^0 (1/3)^4 = 1 - 0.0123 \\ &= 0.9877 \end{aligned}$$

10. On an average a typist makes 3 mistakes while typing one page. What is the probability that a randomly observed page is free of mistakes? Among 200 pages, in how many pages would you expect mistakes ?

Solution: Given $\lambda = 3$

$$P(\text{ page is free of mistakes}) = e^{-\lambda} \lambda^x / x! = 0.0498$$

$$P(\text{ page has mistakes}) = P(X \geq 1) = 1 - P(\text{ has no mistakes}) = 1 - 0.0498 = 0.9502$$

$$\begin{aligned} \text{Among 200 pages, the expected number of pages containing mistakes} &= 200 \times 0.9502 \\ &= 190 \end{aligned}$$

11. Write down the properties of Normal Distribution.

1. The curve is bell shaped.

a. It is symmetrical

b. The mean , median and Mode are equal

2. The curve is asymptotic to the X axis. That is the curve touches the X axis at $-\infty$ to $+\infty$

3. The curve has points of inflexion at $\mu - \sigma$ and $\mu + \sigma$

4. For the distribution a) Standard deviation = σ

$$\text{b) Quartile deviation} = \frac{2}{3} \sigma$$

$$\text{c) Mean deviation} = \frac{4}{5} \sigma$$

5. The distribution is mesokurtic.

6. The total area under the curve is unity.

7. The Quartiles are $Q1 = \mu - 0.6745 \sigma$ and $Q3 = \mu + 0.6745 \sigma$

12. There are 12 girls and 4 boys in a class. 4 Students are selected randomly for the award of

a prize. Find the probability that the selection does not have girls.

Solution : Given $a = 12, b = 4, n = 4$

$$\begin{aligned} P(\text{the selection does not have girls}) &= \frac{{}^a C_x \times {}^b C_{n-x}}{{}^{a+b} C_n} \\ &= \frac{{}^{12} C_0 \times {}^4 C_0}{{}^{16} C_4} \\ &= 0.0005495 \end{aligned}$$

13. It is required to test the hypothesis that, on an average, Punjabis are taller than 180 cms. For this, 50 Punjabis are randomly selected and their heights are measured. If the mean height is 181.1 cms and the standard deviation is 3.3 cms, What is your conclusion ?

Solution : $\mu = 180$ cms, $n = 50$, $\bar{x} = 181.1$ cms, $s = 3.3$ cms

H_0 : Mean height of Punjabis = 180 cms

H_1 : Mean height of Punjabis is greater than 180 cms

Test Statistic is

$$\begin{aligned} Z &= \frac{\bar{x} - \mu}{s/\sqrt{n}} \\ &= \frac{181.1 - 180}{3.3/\sqrt{50}} = 2.36 \end{aligned}$$

Test is upper tailed

At 1% level of significance, the critical value is 2.33

Z_{cal} 2.36 is less than greater than 2.33

H_0 is rejected

Conclusion : On an average, Punjabis are taller than 180 cms

14. A random sample of 1000 apples from an orchard has mean weight 187 gms and standard deviation 8 gms. A random sample of 800 apples from another orchard has mean weight 188.4 gms and standard deviation 10 gms. Test the hypothesis that the mean weight of apples of the two orchards are the same.

Solution: Given I orchard : $n_1 = 1000$, $\bar{x}_1 = 187$, $s_1 = 8$

II orchard : $n_2 = 800$, $\bar{x}_2 = 188.4$, $s_2 = 10$

H_0 : Mean height of two orchards are equal

H_1 : Mean height of two orchards are not equal s_1^2 s_2^2

Test Statistic is

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{187 - 188.4}{\sqrt{\frac{8^2}{1000} + \frac{10^2}{800}}} = -3.22$$

Test is two tailed

At 5 % level of significance table value = (-1.96,+1.96)

Since $Z_{cal} = -3.22$ is a value outside the interval (-1.96,1.96)

H_0 is rejected

Conclusion : Mean weight of apples of two orchards are equal

15. The manufacturers of Brand R pens contend that the proportion of college students of Bangalore who use Brand R pens is more than 0.3. In order to test this contention, 40 college students were randomly picked and questioned in this regard. Among these 40 students, 10 were found to use Brand R pens. At 5% level of significance, test whether the manufacturers' contention is acceptable.

Solution : Given $P = 0.3$, $n = 40$, $p = 0.25$

H_0 : The proportion of users of Brand R pens is 0.3

H_1 : The proportion of users of Brand R pens is not equal to 0.3

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$
$$= \frac{0.25 - 0.3}{\sqrt{\frac{0.3 \times 0.7}{40}}}$$
$$= -0.69$$

Test is upper tailed

At 5 % i.o.s. table value = 1.645

$Z_{cal} = -0.69$ is less than 1.645, H_0 is accepted

Conclusion : The proportion of Brand R pens is 0.3

16. From the following data, test whether the difference between the proportion in the two samples is significant.

	Size	Proportion
Sample I	1000	0.02
Sample II	1200	0.01

Solution :

$$P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{1000 \times 0.02 + 1200 \times 0.01}{1000 + 1200}$$

$$= 0.0146$$

H_0 : The proportion of size of two samples are equal

H_1 : The proportion of size of two samples are not equal

$$Z = \frac{p_1 - p_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{0.2 - .01}{\sqrt{0.0146 \times 0.9854 \left(\frac{1}{1000} + \frac{1}{1200} \right)}}$$

$$= 1.9471$$

Test is two tailed

At 5 % table value = (-1.96,+1.96)

$Z_{cal} = 1.9471$ lies within the interval (-1.96,+1.96)

H_0 is accepted

Conclusion : The proportion of size of two samples are equal

17. The cost of a scooter is Rs.36,000. Its resale value and maintenance cost at different age are given below. When the scooter has to be replaced?

Year of service	1	2	3	4	5	6
Maintenance cost	800	1300	1900	2700	3900	5400
Resale value	28000	22000	20000	18000	17000	16000

Solution : $P = \text{Rs } 36,000$

Year	P	S_n	$P - S_n$	C_i	ΣC_i	$(P - S_n) + \Sigma C_i$	$A(n) = (P - S_n) + \Sigma C_i / n$
1	36000	28000	8000	800	800	8800	8800
2	36000	22000	14000	1300	2100	16100	8050
3	36000	20000	16000	1900	4000	20000	6667
4	36000	18000	18000	2700	6700	24700	6175
5	36000	17000	19000	3900	10600	29600	5980 (min)
6	36000	16000	20000	5400	16000	36000	6000

Annual average cost is minimum for $n = 5$. The scooter should be replaced after using it for 5 years.

18. Solve the following game using Dominance Principle.

	B1	B2	B3	B4
A1	20	15	12	35
A2	25	14	8	10
A3	-5	4	11	0

Solution : A3 dominated A1 delete A3

	B1	B2	B3	B4
A1	20	15	12	35
A2	25	14	8	10

B2 dominates B3 ,delete B3

	B1	B3	B4
A2	20	12	35
A3	25	8	10

B4 dominates B3 , delete B4

	B1	B3
A2	20	12
A3	25	8

B1 dominates B3 , delete B3

	B3
A2	12
A3	8

A3 dominates A2, delete A2

	B3
A3	8

Solution: (i) The game has a saddle point

(ii) Optimum strategy for A is A2 and optimum strategy for B is B3

(iii) Value of the game is $v = 12$

(iv) Game is not fair since $v \neq 0$

19. A firm has to supply 80 electric motors every week to customers. Production cost is instantaneous and the setup cost is Rs 280. For immediate delivery of motors to the customers, the company maintains an inventory where the inventory carrying cost is Rs 15 per unit per year, suggest an inventory policy which is most economical.

Solution : $R = \text{demand} = 80 \text{ units per week}$

$$= 80 \times 52 = 4160 \text{ units per year}$$

$C1 = \text{carrying cost} = \text{Rs } 15 \text{ per year}$

$C3 = \text{set up cost} = \text{Rs } 280$

$$\text{EOQ} = \sqrt{\frac{2C3R}{C1}} = \sqrt{\frac{2 \times 280 \times 4160}{15}} = 394 \text{ units}$$

$$t0 = \frac{Q}{R} = \frac{394}{4160} = 0.0947 \text{ years} = 4.93 \text{ weeks}$$

$$n0 = \frac{1}{t} = \frac{1}{0.0947} = 10.6 \text{ per year}$$

$$C(Q0) = \sqrt{2C1C3R} = \sqrt{2 \times 15 \times 280 \times 80} = \text{Rs } 5911.35$$

20. The population of a city in different census years are as below. Find by interpolation/extrapolation the populations of the city in the years 1981 and 2021.

Census year	1971	1981	1991	2001	2011
Population	5	?	12	19	27

Solution :

Census year	1971	1981	1991	2001	2011	2021
Population	5	?	12	19	27	?
	y_0	y_1	y_2	y_3	y_4	y_5

By Binomial Expansion Method

$$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$1 \quad 1$$

$$27 - 4 \times 19 + 6 \times 12 - 4y_1 + 5 = 0$$

$$1 \quad 2 \quad 1$$

$$4y_1 = 28$$

$$y_1 = 7$$

$$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$$

$$y_5 - 4x^{27} + 6x^{19} - 4x^{12} + 7 = 0$$

$$y_5 - 35 = 0$$

$$y_5 = 35$$

$$\begin{array}{cccccc}
 & & & 1 & & 3 & & 3 & & 1 \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 \\
 & & & y_4 & & y_3 & & y_2 & & y_1 & & y_0
 \end{array}$$

21. The annual premium rates for an insurance policy of Rs 1000/for person of different age are below. Calculate the annual premium rate at age 23.

Age (years)	20	25	30	35	40
Premium(Rs)	23	26	30	35	42

Solution :

The difference table is as below.

Age	Premium	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
20	23				
		3			
25	26		1		
		4		0	
30	30		1		1
		5		1	
35	35		2		
		7			
40	42				

$$h = \frac{x - x_0}{h} = \frac{23 - 20}{5} = 0.6$$

$$\begin{aligned}
 y &= y_0 + h \Delta y + \frac{h(h-1)}{2} \Delta^2 y + \frac{h(h-1)(h-2)}{3!} \Delta^3 y + \frac{h(h-1)(h-2)(h-3)}{4!} \Delta^4 y \\
 &= 23 + 0.6 \times 3 + \frac{0.6(0.6-1) \times 1}{2} + \frac{0.6(0.6-1)(0.6-2) \times 0}{6} + \frac{0.6(0.6-1)(0.6-2)(0.6-3) \times 1}{24} \\
 &= 23 + 1.8 - 0.12 - 0.0336 \\
 &= \text{Rs } 24.65
 \end{aligned}$$

22. For the following Transportation problem find the initial basic feasible solution by North West corner rule.

		A	B	C	available
From	I	50	30	220	10
	II	90	45	170	30
	III	270	200	50	40
Requirement		40	20	20	

Solution ;

Total requirement = Total availability = 80

	A	B	C	Bj
I	10 50	30	220	10
II	30 90	45	170	30
III	270	20 200	20 50	40
ai	40	20	20	80

Initial Basic Feasible solution is : $x_{IA} = 10$; $x_{IIA} = 30$; $x_{IIIB} = 20$; $x_{IIIC} = 20$

Transportation cost = $10 \times 50 + 30 \times 90 + 20 \times 200 + 20 \times 50 = \text{Rs } 8200$

$$m+n-1 = 3+3-1 = 5$$

no of allocations = 4

The solution is non degenerate.

23. For the following Transportation problem find the initial basic feasible solution by Matrix

Minima Method.

		Destination			
		A	B	C	available
Source O1	10	9	8	8	

O2	12	7	10	7
O3	11	9	7	9
O4	12	14	10	4
Required	10	10	8	28

Solution: Total availability = Total Requirement = 28

	A	B	C	Ai
O1	8 10	9	8	8
O2	2 12	5 7	10	7
O3	11	5 9	4 7	9
O4	12	14	4 10	4
bj	10	10	8	28

Transportation cost = $8 \times 10 + 2 \times 12 + 5 \times 7 + 5 \times 9 + 4 \times 7 + 4 \times 10$

= Rs 252

$$m+n-1 = 3+4-1 = 6$$

no of allocations = 6

solution is nondegenerate.

24. The standard deviation of weight of 18 new born babies is 0.32 kgs. Test at 1% level of significance that standard deviation of weight of new born babies is less than 0.35 kgs.

Solution ;

H0 : Population standard deviation = 0.35 kgs

H1 : Population standard deviation is less than 0.35 kgs

$$X^2 = \frac{ns^2}{\sigma^2}$$

$$= \frac{18 \times (0.32)^2}{(0.35)^2}$$

$$= 15.05$$

At 1 % i.o.s. table value = 6.41

Test is lower tailed

$$\chi^2 = 15.05 \text{ is greater than } 6.41$$

H_0 is accepted

Conclusion : Population standard deviation = 0.35 kgs

25. There are two candidates A and B contesting an election. A pre-election survey of 80 men and 120 women gave the following results.

	Voted for A	Voted for B	Total
Men	27	53	80
Women	64	56	120

Apply chi-square test to see whether voting pattern is the same among men and women.

Solution : H_0 : Voting pattern is the same among men and women.

H_1 : Voting pattern is not the same among men and women.

$$\chi^2 = \frac{N (ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

$$= \frac{200 (27 \times 56 - 53 \times 64)^2}{80 \times 120 \times 91 \times 109}$$

$$= 7.42$$

The degree of freedom is 1

At 5% i.o.s. critical value = 3.84

$$\chi^2 = 7.42 > 3.84, H_0 \text{ is rejected}$$

Conclusion : Voting pattern is not the same among men and women.

26. There is a coaching class for CET. 10 randomly selected students were given a test before coaching and they also were given a test after coaching. The test scores are as follows.

Before coaching	35	39	47	53	27	19	36	46	8	17
After coaching	41	37	45	56	31	21	47	41	5	12

Can we conclude that the coaching is effective?

Solution :

x	y	d = x - y	d ²
35	41	-6	36
39	37	2	4
47	45	2	4
53	56	-3	9
27	31	-4	16
19	21	-2	4
36	47	-11	121
46	41	5	25
8	5	3	9
17	12	5	25
		-9	253

$$\bar{d} = \frac{\sum d}{n} = \frac{-9}{10} = -0.9$$

$$s_d = \sqrt{\frac{\sum d^2}{n} - (\bar{d})^2} = 4.95$$

H₀: Marks before coaching = Marks after coaching

H₁ : Marks before coaching < Marks after coaching

$$t = \frac{\bar{d}}{\frac{s_d}{\sqrt{n-1}}} = \frac{-0.9}{\frac{4.95}{\sqrt{10-1}}} = -0.55$$

At 5 % l.o.s. table value = -1.83. Test is upper tailed

T = -0.55 is greater than -1.83

H₀ is accepted

Solution: Coaching is not effective .

27. 70 accidents that have occurred in a state in a week are tabulated as follows. Test whether

accident occurs uniformly throughout the week.

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
Accidents	7	8	11	12	5	13	14

Solution : H₀ : Accidents occur uniformly through out the year

H₁ : Accidents occur uniformly through out the year

Under H₀, E_i = 70/7 = 10

O _i	E _i	O _i - E _i	(O _i - E _i) ²	(O _i - E _i) ² / E _i
7	10	-3	9	0.9
8	10	-2	4	0.4
11	10	1	1	0.1
12	10	2	4	0.4
5	10	-5	25	2.5
13	10	3	9	0.9
14	10	4	16	1.6
				6.8

$$X^2 = \frac{\sum (O_i - E_i)^2}{E_i} = 6.8$$

test is upper tailed. d.f. = 7- 1=6

At 5 % l.o.s. table value = 16.81

$X^2 = 6.8$ is less than 3.84

H₀ is accepted

Conclusion : Accidents occur uniformly throughout the year.

28. The management of a factory contends that the mean sound intensity is less than 120 decibels. 23 random measurement have 117 decibels and standard deviation 8 decibels. Test at 1 % level of significance whether the contention of the management is acceptable.

Solution : n = 23 μ = 120 cms, $\bar{x} = 117$, s = 8

H₀ : Mean sound intensity = 120

H_1 : Mean sound intensity is less than 120

Test Statistic is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{117 - 120}{\frac{8}{\sqrt{23-1}}} = -1.76$$

Test is lower tailed

At 1% level of significance, the critical value is -2.51

$t = -1.76$ is less than greater than 2.33

H_0 is accepted

Conclusion : Mean sound intensity = 120

29. In a textile mill, at regular intervals, cloth is inspected for knitting defects. Draw c chart and analyze.

Sample number	1	2	3	4	5	6	7	8	9	10
Defects / square metre	0	1	0	2	0	1	1	4	3	1

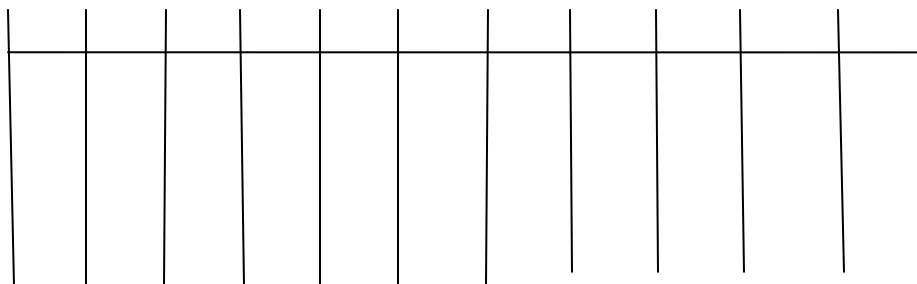
Solution : $\bar{c} = \frac{\sum c}{n} = \frac{13}{10} = 1.3$

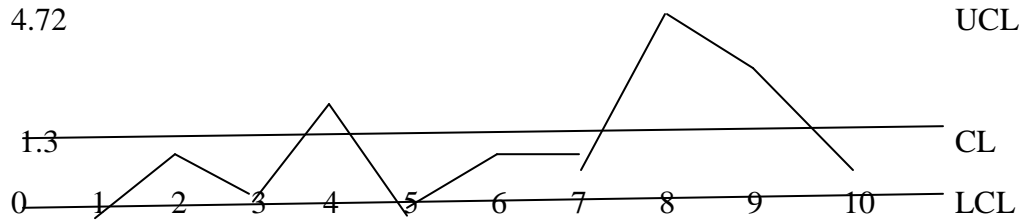
The control limits for c – chart are

Control limit = $\bar{c} = 1.3$

Lower control limit = $\bar{c} - 3\sqrt{\bar{c}} = 1.3 - 3 \times \sqrt{1.3} = -2.12 = 0$

Upper control limit = $\bar{c} + 3\sqrt{\bar{c}} = 1.3 + 3 \times \sqrt{1.3} = 4.72$





Interpretation : All the points are within the control limits. And so the process is in statistical control.

30. A drilling machine is to set holes with a mean diameter of 15 mms and a standard deviation of 0.2 mms. Find the control limits for mean and range, for a given sample of size 5.

Solution : Standards are known : $\bar{X} = 15$ and $\sigma = 0.2$, $n = 5$

The control limits for \bar{X} chart are:

$$\text{Control limit} = \bar{X} = 15$$

$$\text{Lower control limit} = \bar{X} - A \sigma = 15 - 1.342 \times 0.2 = 14.4316$$

$$\text{Upper control limit} = \bar{X} + A \sigma = 15 + 1.342 \times 0.2 = 15.2684$$

The control limits for R chart are :

$$\text{Control limit} = d_2 \sigma = 2.326 \times 0.2 = 0.4652$$

$$\text{Lower control limit} = D_1 \sigma = 0.0 \times 0.2 = 0$$

$$\text{Upper control limit} = D_2 \sigma = 4.918 \times 0.2 = 0.9836$$

TEN MARKS QUESTIONS

1. For the following data compute standardised death rates in two towns A and B. Which town is healthier ?

Age group (in years)	Town A		Town B		Standard Population
	Population	Deaths	Population	Deaths	
0 – 10	4000	36	3000	30	2000
10 -25	12000	48	20000	100	3000
25 – 60	6000	60	4000	48	6000
60 and above	8000	152	3000	60	4000

Solution :

For Town A

Age group (in years)	Town A		ASDR [A]	Standard Population [P]	PA
	Population	Deaths			
0 – 10	4000	36	9	2000	18000
10 -25	12000	48	4	3000	12000
25 – 60	6000	60	10	6000	60000
60 and above	8000	152	19	4000	76000
				15000	166000

$$\text{SDR} = \frac{\sum PA}{\sum P} = \frac{166000}{15000} = 11.06$$

For Town B

Age group (in years)	Town B		ASDR [B]	Standard Population [P]	PB
	Population	Deaths			
0 – 10	3000	30	10	2000	20000
10 -25	20000	100	5	3000	15000
25 – 60	4000	48	12	6000	72000
60 and above	3000	60	20	4000	80000
				15000	187000

$$\text{SDR} = \frac{\sum PB}{\sum P} = \frac{187000}{15000} = 12.46$$

Comment : Town A is more healthier as death rate is low.

2. Compute Standardised death rate for the towns A and B by taking town A as the standard population. Find which city is more healthy.

Age group (years)	Town A		Town B	
	Population	Deaths	Population	Deaths
Below 5	35000	360	80000	1000

5 – 30	40000	400	100000	1040
Above 30	20000	280	16000	240

Solution :

Town A

Age group (years)	Town A		ASDR A	PA
	Population	Deaths		
Below 5	35000	360	10.2857	360000
5 – 30	40000	400	10	400000
Above 30	20000	280	14	280000
	95000			1040000

$$SDR = \frac{\sum PA}{\sum P} = \frac{1040000}{95000} = 10.95$$

For Town B

Age group (years)	Town B		ASDR A	Standard Population [P]	PB
	Population	Deaths			
Below 5	80000	1000	12.5	35000	437500
5 – 30	100000	1040	10.4	40000	416000
Above 30	16000	240	15	20000	300000
				95000	1153500

$$SDR = \frac{\sum PB}{\sum P} = \frac{1153500}{95000} = 12.14$$

Conclusion : Town A is more healthier.

3. From the following data calculate standardised deathrate and hence find out which Town is healthier.(take Town B as standard)

Age group (years)	Town A		Town B	
	Population	Death per 1000	Population	Death per 1000

0-9	400	40	600	30
10-19	1500	4	1000	5
20-59	2400	10	3000	8
60 & above	700	30	400	50

Solution :

Here Deaths per 1000 means ASDR.

Age group (years)	Town A			Town B	
	P (St.popln)	A=ASDR	PA	B = ASDR	PB
0-9	600	40	24000	30	18000
10-19	1000	4	4000	5	5000
20-59	3000	10	30000	8	24000
60 & above	400	30	12000	50	20000
	5000		70000		67000

$$\text{SDR (for town A)} = \frac{\sum PA}{\sum P} = \frac{70000}{5000} = 14$$

$$\text{SDR (for town B)} = \frac{\sum PB}{\sum P} = \frac{67000}{5000} = 13.4$$

Town B is healthier.

4. Compute Laspeyre's , Paasche's, Marshall- Edgeworth, Dorbish – Bowley, and Fisher's Index numbers for 2000 from the following data. Show that Fisher's index numbers satisfies TRT and FRT.

Items	1995		2000	
	Price	quantity	Price	Quantity
A	6	50	10	56
B	2	100	2	120
C	4	60	6	60
D	10	30	12	24
E	8	40	12	36

Solution :

Items	p0	q0	p1	q1	p0q0	p1q0	p0q1	p1q1
A	6	50	10	56	300	500	336	560
B	2	100	2	120	200	200	240	240
C	4	60	6	60	240	360	240	360
D	10	30	12	24	300	360	240	288
E	8	40	12	36	320	480	288	432
Total					1360	1900	1344	1880

Laspeyers Price Index Number is

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100 = \frac{1900}{1360} \times 100 = 139.71$$

Paasche's Price Index Number is

$$P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100 = \frac{1880}{1344} \times 100 = 139.88$$

Marshall Edge worth Price index number is

$$P_{01} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100 = \frac{1900+1880}{1360+1344} \times 100 = 139.79$$

Dorbish Bowley Price Index Number is

$$P_{01} = \frac{L+P}{2} = \frac{139.71+139.88}{2} = 139.795$$

Fisher's Price index number is

$$P_{01} = \sqrt{L \times P} = \sqrt{139.71 \times 139.88} = 139.7949$$

Time reversal Test (TRT)

$$P_{01} \times P_{10} = 1$$

$$\begin{aligned} \text{L.H.S.} = P_{01} \times P_{10} &= \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1 \times \sum p_0 q_1 \times \sum p_0 q_0}{\sum p_0 q_0 \times \sum p_0 q_1 \times \sum p_1 q_1 \times \sum p_1 q_0}} = \sqrt{\frac{1900 \times 1880 \times 1344 \times 1360}{1360 \times 1344 \times 1880 \times 1900}} \\ &= 1 = \text{R.H.S.} \end{aligned}$$

TRT is verified

Factor reversal Test (FRT)

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

$$\text{L.H.S.} = P_{01} \times Q_{01} = \sqrt{\frac{\sum p_1 q_0 \times \sum p_1 q_1 \times \sum q_1 p_0 \times \sum q_1 p_1}{\sum p_0 q_0 \times \sum p_0 q_1 \times \sum q_0 p_0 \times \sum q_0 p_1}} = \sqrt{\frac{1900 \times 1880 \times 1344 \times 1880}{1360 \times 1344 \times 1360 \times 1900}}$$

$$= \frac{1880}{1360} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = = \text{R.H.S.}$$

FRT is verified.

5. Compute Laspeyre's , Paasche's, Marshall- Edgeworth, Dorbish – Bowley, and Fisher's Index numbers for 2000 from the following data.

Items	Base Year		Current Year	
	Price	Expenditure	Price	Expenditure
A	50	100	60	180
B	40	120	40	200
C	100	100	120	12
D	20	80	25	100

Solution :

Expenditure = price x quantity

Items	p ₀	Expenditure= p ₀ q ₀	p ₁	Expenditure = p ₁ q ₁	q ₀	q ₁	p ₀ q ₁	p ₁ q ₀
A	50	100	60	180	2	3	150	120
B	40	120	40	200	3	5	200	120
C	100	100	120	12	1	1	100	120
D	20	80	25	100	4	4	80	100
Total		400		600			530	460

Laspeyers Price Index Number is

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100 = \frac{460}{400} \times 100 = 115$$

Paasche's Price Index Number is

$$P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100 = \frac{600}{530} \times 100 = 113.21$$

MarshallEdge worth Price index number is

$$P_{01} = \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100 = \frac{460 + 600}{400 + 530} \times 100 = 113.97$$

Dorbish Bowley Price Index Number is

$$P_{01} = \frac{L+P}{2} = \frac{115+113.21}{2} = 114.105$$

Fisher's Price index number is

$$P_{01} = \sqrt{L \times P} = \sqrt{115 \times 113.21} = 114.1$$

6. Compute Laspeyre's, Paasche's, Marshall-Edgeworth, Dorbish – Bowley, and Fisher's Quantity Index numbers for 2000 from the following data.

Items	Price		Quantity	
	Base year	Current year	Base year	Current year
A	400	85	100	120
B	320	690	20	60
C	720	1600	10	10
D	720	2100	10	20

Solution :

Items	p ₀	p ₁	q ₀	q ₁	p ₀ q ₀	p ₁ q ₀	p ₀ q ₁	p ₁ q ₁
A	400	85	100	120	40000	85000	48000	102000
B	320	690	20	60	6400	13800	19200	41400
C	720	1600	10	10	7200	16000	7200	16000
D	720	2100	10	20	7200	21000	14400	42000
Total					60800	135800	88800	201400

Laspeyres quantity Index Number is

$$P_{01} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100 = \frac{88800}{60800} \times 100 = 146.05$$

Paasche's quantity Index Number is

$$P_{01} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100 = \frac{201400}{135800} \times 100 = 148.31$$

Marshall-Edgeworth quantity index number is

$$P_{01} = \frac{\sum q_1 p_0 + \sum q_1 p_1}{\sum q_0 p_0 + \sum q_0 p_1} \times 100 = \frac{88800 + 201400}{60800 + 135800} \times 100 = 147.61$$

Dorbish-Bowley quantity Index Number is

$$P_{01} = \frac{L+P}{2} = \frac{146.051+148.31}{2} = 147.18$$

Fisher's quantity index number is

$$P_{01} = \sqrt{L \times P} = \sqrt{146.51 \times 148.31} = 147.18$$

7. The sales of a company in lakhs of rupees for the year 2005 to 2011 are given below. Estimate the sales figure for the year 2012 using an equation of the form $y = a b^x$

Year	2005	2006	2007	2008	2009	2010	2011
Sales	32	47	65	92	132	190	275

Solution :

Year	Sales	x	X ²	Log y	X log y
2005	32	-3	9	1.5051	-4.5153
2006	47	-2	4	1.6721	-3.3442
2007	65	-1	1	1.8129	-1.8129
2008	92	0	0	1.9638	0
2009	132	1	1	2.1206	2.1206
2010	190	2	4	2.2788	4.5576
2011	275	3	9	2.4398	7.3119
		0	28	13.7926	4.3237

If $\sum x = 0$,

$$\text{Log } a = \frac{\sum \log y}{n} = \frac{13.7926}{7} = 1.9704$$

$$a = \text{antilog} (1.9704) = 93.4114$$

$$\text{Log } b = \frac{\sum x \log y}{\sum x^2} = \frac{4.3237}{28} = 0.1544$$

$$b = \text{antilog} (0.1544) = 1.4269$$

The exponential curve is $y = a b^x$

$$Y = 93.4114 (1.4269)^x$$

For the year 2012, $x = 4$

$$Y = 93.4114 (1.4269)^4 = \text{Rs.} 387.23 \text{ lakhs}$$

8. Fit an exponential trend curve $y = a b^x$ to the following data.

Year	2005	2006	2007	2008	2009	2010
Production (crores)	7	1	12	14	17	24

Solution :

Year	Sales	X	X ²	Log y	X log y
2005	7	-5	25	0.8451	-4.2255
2006	1	-3	9	1	-3
2007	12	-1	1	1.0792	-1.0792
2008	14	1	1	1.1461	1.1461
2009	17	3	9	1.2304	3.6912
2010	24	5	25	1.3802	6.901
		0	70	6.6810	3.4336

If $\Sigma x = 0$,

$$\text{Log } a = \frac{\Sigma \log y}{n} = \frac{6.6810}{6} = 1.1135$$

$$a = \text{antilog} (1.1135) = 12.98$$

$$\text{Log } b = \frac{\Sigma x \log y}{\Sigma x^2} = \frac{3.4336}{70} = 0.0491$$

$$b = \text{antilog} (0.0491) = 1.119$$

The exponential curve is $y = a b^x$

$$Y = 12.98 (1.119)^x$$

9. The following data related to the number of mistakes per page of a book containing 180 pages. Test whether Poisson distribution is a good fit to this observed distribution.

No. of mistakes	0	1	2	3	4	5
Per page						
Number of pages	130	32	15	2	1	0

Solution :

H₀ : Poisson distribution is a good fit.

H₁ : Poisson distribution is not a good fit.

x	f	Fx
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0	130	0
1	32	32
2	15	30
3	2	6
4	1	4
5	0	0
	180	72

$$\lambda = \frac{\sum fx}{\sum f} = \frac{72}{180} = 0.4$$

$$T_x = \frac{N e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$$

$$T_0 = \frac{180 e^{-0.4} 0.4^0}{0!} = 121$$

Similarly,

$$T_1 = 48, T_2 = 10, T_3 = 1, T_4 = 0, T_5 = 0$$

X	Observed frequency O _i	Theoretical frequency E _i	(O _i - E _i) ²	(O _i - E _i) ² / E _i
0	130	121	81	0.6694
1	32	48	256	5.3333
2	15	10		
3	2	1	49	4.4545
4	1	0		
5	0	0		
Total	180	180		10.4572

Here, the last three theoretical frequencies are less than 5. Therefore, they are pooled with the adjacent ones such that, finally all the frequencies are 5 or more.

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= 10.4572$$

test is upper tailed. d.f. = 3 - 2 = 1

At 5 % l.o.s. table value = 3.84

$\chi^2 = 10.4572$ is greater than 3.84

H_0 is rejected.

Conclusion : Poisson distribution is not a good fit.

10. A survey of 64 families with 3 children each is conducted and the number of male children in each family is noted. The results are tabulated as follows:

Male children	0	1	2	3	Total
Families	6	19	29	10	64

Apply Chi –square test of goodness of fit to test whether male and female children are equiprobable.

Solution :

H_0 : Male and female children are equiprobable.

H_1 : Male and female children are not equiprobable.

Given $n = 3, p = 0.5, q = 1 - p = 0.5$

$$T_x = N \times nC_x \times p^x \times q^{n-x}$$

$$T_0 = 64 \times {}^3C_0 \times (0.5)^0 \times (0.5)^{3-0} = 8$$

Similarly, $T_1 = 24, T_2 = 24, T_3 = 8$

X	Observed frequency O_i	Theoretical frequency E_i	$(O_i - E_i)^2$	$(O_i - E_i)^2 / E_i$
0	6	8	4	0.5
1	19	24	25	1.042
2	29	24	25	1.042
3	10	8	4	0.5
Total	64	64		3.084

$$\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= 3.084$$

test is upper tailed. d.f. = 4- 1 = 3

At 5 % l.o.s. table value = 7.81

$\chi^2 = 3.084$ is less than 7.81

H0 is accepted.

Conclusion : Male and female children are equi -probable.