TARGET 80

ತ್ರಿಕೋನಮಿತಿ ಅನ್ವಯ ಸಮಸ್ಯೆಗಳು

(2 ಅಥವಾ 3 ಅಂಕಗಳಿಗಾಗಿ)

1. $\sec^4\theta - \sec^2\theta = \tan^2\theta + \tan^4\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \sec^4\theta - \sec^2\theta$

$= \sec^2\theta(\sec^2\theta - 1)$

$= (1 + \tan^2\theta)\tan^2\theta$

$= \tan^2\theta + \tan^4\theta$

$\therefore LHS = RHS$

2. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\mathrm{cosec}\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2 \times 1 \times \cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos)}{(1+\cos)\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\mathrm{cosec}\theta$

$\therefore LHS = RHS$

3. $\frac{\cos(90-\theta)}{1+\sin(90-\theta)} + \frac{1+\sin(90-\theta)}{\cos(90-\theta)} = 2\mathrm{cosec}\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\cos(90-\theta)}{1+\sin(90-\theta)} + \frac{1+\sin(90-\theta)}{\cos(90-\theta)}$

$= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2 \times 1 \times \cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos)}{(1+\cos\theta)\sin\theta}$

$= \frac{2}{\sin\theta}$

$= 2\mathrm{cosec}\theta$

$\therefore LHS = RHS$

4. $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$

$= \frac{\mathrm{COS}^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$

$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2 \times 1 \times \sin\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{1 + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{2 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{2(1+\sin)}{(1+\sin\theta)\cos\theta}$

$= \frac{2}{\cos\theta}$

$= 2\sec\theta$

$\therefore LHS = RHS$

5. $\frac{\sin(90-\theta)}{1+\cos(90-\theta)} + \frac{1+\cos(90-\theta)}{\sin(90-\theta)} = 2\sec\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\sin(90-\theta)}{1+\cos(90-\theta)} + \frac{1+\cos(90-\theta)}{\sin(90-\theta)}$

$= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$

$= \frac{\mathrm{COS}^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$

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$$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2 \times 1 \times \sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{1 + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{2 + 2\sin\theta}{(1+\sin\theta)\cos\theta}$$

$$= \frac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}\cos\theta}$$

$$= \frac{2}{\cos\theta}$$

$$= 2\sec\theta$$

$\therefore LHS = RHS$

6. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \cos\theta + \sin\theta$ ಎಂದು ಸಾಧಿಸಿ.(***MQP*.1 − 2021, 3 *marks***)

ಪರಿಹಾರ :

$$LHS = \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$$

$$= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}} + \frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$

$$= \frac{(\cos\theta+\sin\theta)\cancel{(\cos\theta-\sin\theta)}}{\cancel{\cos\theta-\sin\theta}}$$

$$= \cos\theta + \sin\theta$$

$\therefore LHS = RHS$

7. $\frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta} = 2\cot\theta$ ಎಂದು ಸಾಧಿಸಿ.

(Preperatory − 2020, 3 marks)

ಪರಿಹಾರ :

$$LHS = \frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)^2 - \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1^2 + \cos^2\theta + 2 \times 1 \times \cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 + \cos^2\theta + 2\cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 - \sin^2\theta + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\cos^2\theta + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cos\theta\cancel{(\cos\theta+1)}}{\sin\theta\cancel{(1+\cos\theta)}}$$

$$= 2\cot\theta$$

$\therefore LHS = RHS$

8. $\frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2 - \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1^2 + \sin^2\theta + 2 \times 1 \times \sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1 + \sin^2\theta + 2\sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1 - \cos^2\theta + \sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\sin^2\theta + \sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2\sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2\sin\theta\cancel{(\sin\theta+1)}}{\cos\theta\cancel{(1+\sin\ )}}$$

$$= 2\tan\theta$$

$\therefore LHS = RHS$

9. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta.\csc\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

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$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta.\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\cancel{(\sin\theta - \cos\theta)}(\sin^2\theta + \cos^2\theta + \sin\ .\cos\theta}{\cos\theta.\sin\theta\cancel{(\sin\theta - \cos\theta)}}$$

$$= \frac{1 + \sin\theta.\cos\theta}{\cos\theta.\sin\theta}$$

$$= \frac{1}{\cos\theta.\sin\theta} + \frac{\cancel{\sin\theta.\cos\theta}}{\cancel{\cos\theta.\sin\theta}}$$

$$= \frac{1}{\cos\theta.\sin\theta} + 1$$

$$= \sec\theta.\mathrm{cosec}\theta + 1$$

$$= 1 + \sec\theta.\mathrm{cosec}\theta$$

$$\therefore LHS = RHS$$

10. $\sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A}}$ **= secA + tanA ಎಂದು ಸಾಧಿಸಿ.**

ಪರಿಹಾರ :

$$\mathrm{LHS} = \sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A}}$$

$$= \sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A} \times \frac{1+\mathrm{Sin}A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1^2-(\sin A)^2}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \sqrt{\left(\frac{1+\mathrm{Sin}A}{\cos A}\right)^{\cancel{2}}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$\therefore LHS = RHS$$

11. $\sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A}} + \sqrt{\frac{1-\mathrm{Sin}A}{1+\sin A}}$ **= 2secA ಎಂದು ಸಾಧಿಸಿ.**

ಪರಿಹಾರ :

$$\mathrm{LHS} = \sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A}} + \sqrt{\frac{1-\mathrm{Sin}A}{1+\sin A}}$$

$$= \sqrt{\frac{1+\mathrm{Sin}A}{1-\sin A} \times \frac{1+\mathrm{Sin}A}{1+\sin A}} + \sqrt{\frac{1-\mathrm{Sin}A}{1+\sin A} \times \frac{1-\mathrm{Sin}A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1^2-(\sin A)^2}} + \sqrt{\frac{(1-\sin A)^2}{1^2-(\sin A)^2}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} + \sqrt{\frac{(1-\sin A)^2}{1^2-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} + \sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$$

$$= \sqrt{\left(\frac{1+\mathrm{Sin}A}{\cos A}\right)^{\cancel{2}}} + \sqrt{\left(\frac{1-\mathrm{Sin}}{\cos A}\right)^{\cancel{2}}}$$

$$= \frac{1+\sin A}{\cos A} + \frac{1-\sin A}{\cos A}$$

$$= \frac{1+\sin A+1-\sin A}{\cos A}$$

$$= \frac{2}{\cos A}$$

$$= 2\sec A$$

$$\therefore LHS = RHS$$

12. $\sqrt{\frac{1+\cos A}{1-\cos A}}$ **= cosecA + cotA ಎಂದು ಸಾಧಿಸಿ.**

ಪರಿಹಾರ :

$$\mathrm{LHS} = \sqrt{\frac{1+\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1^2-(\cos A)^2}}$$

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$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$$

$$= \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2}$$

$$= \frac{1+cosA}{sinA}$$

$$= \frac{1}{sinA} + \frac{cosA}{sinA}$$

$$= cosecA + cotA$$

$$\therefore LHS = RHS$$

13. $\sqrt{\frac{1+\cos A}{1-\cos A}} + \sqrt{\frac{1-\cos A}{1+\cos A}} = 2\mathrm{cosec}A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\text{LHS} = \sqrt{\frac{1+\cos A}{1-\cos A}} + \sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}} + \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1^2-(\cos A)^2}} + \sqrt{\frac{(1-\cos A)^2}{1^2-(\cos A)^2}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}} + \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}$$

$$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2} + \sqrt{\left(\frac{1-\cos}{\sin A}\right)^2}$$

$$= \frac{1+cosA}{sinA} + \frac{1-cosA}{sinA}$$

$$= \frac{1+cosA+1-cosA}{sinA}$$

$$= \frac{2}{sinA}$$

$$= 2cosecA$$

$$\therefore LHS = RHS$$

14. $\frac{1+\cos A}{1-\cos A} = (\mathrm{cosec}A + \cot A)^2$ ಎಂದು ಸಾಧಿಸಿ.

(**March – 2019, 2marks**)

ಪರಿಹಾರ :

$$\text{LHS} = \frac{1+\cos A}{1-\cos A}$$

$$= \frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$$

$$= \frac{(1+\cos A)^2}{1^2-(\cos A)^2}$$

$$= \frac{(1+\cos A)^2}{1-\cos^2 A}$$

$$= \frac{(1+\cos A)^2}{\sin^2 A}$$

$$= \left(\frac{1+\cos A}{\sin A}\right)^2$$

$$= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right)^2$$

$$= (\mathrm{cosec}A + \cot A)^2$$

$$\therefore LHS = RHS$$

15. $\frac{1+\sin A}{1-\sin A} = (\sec A + \tan A)^2$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\text{LHS} = \frac{1+\sin A}{1-\sin A}$$

$$= \frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}$$

$$= \frac{(1+\sin A)^2}{1^2-(\sin A)^2}$$

$$= \frac{(1+\sin A)^2}{1-\sin^2 A}$$

$$= \frac{(1+\sin A)^2}{\cos^2 A}$$

$$= \left(\frac{1+\sin A}{\cos A}\right)^2$$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2$$

$$= (\sec A + \tan A)^2$$

$$\therefore LHS = RHS$$

16. $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A} = 4\sec A.\tan A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

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$$LHS = \frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A}$$
$$= \frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A} - \frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}$$
$$= \frac{(1+\sin A)^2}{1^2-(\sin A)^2} - \frac{(1-\sin A)^2}{1^2-(\sin A)^2}$$
$$= \frac{(1+\sin A)^2}{1-\sin^2 A} - \frac{(1-\sin A)^2}{1-\sin^2 A}$$
$$= \frac{(1+\sin A)^2}{\cos^2 A} - \frac{(1-\sin A)^2}{\cos^2 A}$$
$$= \left(\frac{1+\sin A}{\cos A}\right)^2 - \left(\frac{1-\sin A}{\cos A}\right)^2$$
$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)^2 - \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$
$$= (\sec A + \tan)^2 - (\sec A - \tan)^2$$
$$= sec^2A + tan^2A + 2secA.tanA - (sec^2A + tan^2A - 2secA.tanA)$$
$$= sec^2A + tan^2A + 2secA.tanA - sec^2A - tan^2A + 2secA.tanA$$
$$= 4secA.tanA$$
$\therefore LHS = RHS$

17. $\frac{1+\cos A}{1-\cos A} - \frac{1-\cos A}{1+\cos A} = 4\operatorname{cosec}A.\cot A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{1+\cos A}{1-\cos A} - \frac{1-\cos A}{1+\cos A}$$
$$= \frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A} - \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$$
$$= \frac{(1+\cos A)^2}{1^2-(\cos A)^2} - \frac{(1-\cos A)^2}{1^2-(\cos A)^2}$$
$$= \frac{(1+\cos A)^2}{1-\cos^2 A} - \frac{(1-\cos A)^2}{1-\cos^2 A}$$
$$= \frac{(1+\cos A)^2}{\sin^2 A} - \frac{(1-\cos A)^2}{\sin^2 A}$$
$$= \left(\frac{1+\cos A}{\sin A}\right)^2 - \left(\frac{1-\cos A}{\sin A}\right)^2$$
$$= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right)^2 - \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$
$$= (\operatorname{cosec}A + \cot)^2 - (\operatorname{cosec}A - \cot)^2$$
$$= cosec^2A + cot^2A + 2cosecA.c - (cosec^2A + cot^2A - 2cosecA.cotA)$$
$$= cosec^2A + cot^2A + 2cosecA.tanA - cosec^2A - cot^2A + 2cosecA.cotA$$
$$= 4cosecA.cotA$$
$\therefore LHS = RHS$

18. $\frac{1-\tan^2 A}{1+\tan^2 A} = 1 - 2\sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{1-\tan^2 A}{1+\tan^2 A}$$
$$= \frac{1-\tan^2 A}{\sec^2 A}$$
$$= \frac{1-\frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}}$$
$$= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}}$$
$$= \cos^2 A - \sin^2 A$$
$$= 1 - \sin^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A$$
$\therefore LHS = RHS$

19. $(\sin A + \operatorname{cosec}A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ ಎಂದು ಸಾಧಿಸಿ.

(*April* – **2022, 3** ***marks***)

ಪರಿಹಾರ :

$$LHS = (\sin A + \operatorname{cosec}A)^2 + (\cos A + \sec A)^2$$
$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A.\operatorname{cosec}A + \cos^2 A + \sec^2 A + 2\cos A.\sec A$$
$$= \sin^2 A + \cos^2 A + \sec^2 A + \operatorname{cosec}^2 A + 2\sin A.\operatorname{cosec}A + 2\cos A.\sec A$$
$$= 1 + 1 + \tan^2 A + 1 + \cot^2 + 2\sin A.\frac{1}{\sin A} + 2\cos A.\frac{1}{\cos A}$$
$$= 3 + \tan^2 A + \cot^2 + 2 + 2$$
$$= 7 + \tan^2 A + \cot^2 A$$
$\therefore LHS = RHS$

20. $\frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{1}{cosecA - cotA} - \frac{1}{sinA}$$
$$= \frac{1}{cosecA - cotA} - cosecA$$
$$= \frac{1 - cosecA(cosecA - cotA)}{cosecA - cotA}$$
$$= \frac{1 - cosec^2A + cosecA.cotA}{cosecA - cotA}$$
$$= \frac{-cot^2A + cosecA.cotA}{cosecA - cotA}$$
$$= \frac{cotA(-cotA + cosecA)}{cosecA - cotA}$$
$$= cotA$$
$$RHS = \frac{1}{sinA} - \frac{1}{cosecA + cotA}$$
$$= cosecA - \frac{1}{cosecA + cotA}$$
$$= \frac{cosecA(cosecA + cotA) - 1}{cosecA + cotA}$$
$$= \frac{cosec^2A + cosecA.cotA - 1}{cosecA + cotA}$$
$$= \frac{cosec^2A - 1 + cosecA.cotA}{cosecA + cotA}$$
$$= \frac{cot^2A + cosecA.cotA}{cosecA + cotA}$$
$$= \frac{cotA(cotA + cosec\ )}{cosecA + cotA}$$
$$= cotA$$
$\therefore LHS = RHS$

21. $\frac{tanA - sinA}{tanA + sinA} = \frac{secA - 1}{secA + 1}$ ಎಂದು ಸಾಧಿಸಿ.

(***September* − 2020, 2 *marks***)

ಪರಿಹಾರ :

$$LHS = \frac{tanA - sinA}{tanA + sinA}$$
$$= \frac{\frac{sinA}{cosA} - sinA}{\frac{sinA}{cosA} + sinA}$$
$$= \frac{sinA(\frac{1}{cosA} - 1)}{sinA(\frac{1}{cosA} + 1)}$$
$$= \frac{secA - 1}{secA + 1}$$
$\therefore LHS = RHS$

22. $\frac{cosA - sinA + 1}{cosA + sinA - 1} = cosecA + cotA$ ಎಂದು ಸಾಧಿಸಿ

ಪರಿಹಾರ :

$$LHS = \frac{cosA - sinA + 1}{cosA + sinA - 1}$$
ಅಂಶ ಮತ್ತು ಛೇದಗಳೆರಡನ್ನು sinA ದಿಂದ ಭಾಗಿಸಿದಾಗ
$$= \frac{\frac{cosA}{sinA} - \frac{sinA}{sinA} + \frac{1}{sinA}}{\frac{cosA}{sinA} + \frac{sinA}{sinA} - \frac{1}{sinA}}$$
$$= \frac{cotA - 1 + cosecA}{cotA + 1 - cosecA}$$
$$= \frac{cotA + cosecA - 1}{cotA - cosecA + 1}$$
$$= \frac{cotA + cosecA - (cosec^2A - cot^2A)}{cotA - cosecA + 1}$$
$$= \frac{cotA + cosecA - (cosecA + cotA)(cosecA - cotA)}{cotA - cosecA + 1}$$
$$= \frac{cotA + cosecA[1 - (cosecA - cotA)]}{cotA - cosecA + 1}$$
$$= \frac{cotA + cosecA[1 - cosecA + cotA]}{cotA - cosecA + 1}$$
$$= \frac{cotA + cosec\ [cotA - cosecA + 1]}{cotA - cosecA + 1}$$
$$= cosecA + cotA$$
$\therefore LHS = RHS$

23. $\frac{sinA - cosA + 1}{sinA + cosA - 1} = \frac{1}{secA - tanA}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{sinA - cosA + 1}{sinA + cosA - 1}$$
ಅಂಶ ಮತ್ತು ಛೇದಗಳೆರಡನ್ನು cosA ದಿಂದ ಭಾಗಿಸಿದಾಗ
$$= \frac{\frac{sinA}{cosA} - \frac{cosA}{cosA} + \frac{1}{cosA}}{\frac{sinA}{cosA} + \frac{cosA}{cosA} - \frac{1}{cosA}}$$
$$= \frac{tanA - 1 + secA}{tanA + 1 - secA}$$
$$= \frac{tanA + secA - 1}{tanA - secA + 1}$$
$$= \frac{tanA + secA - (sec^2A - tan^2A)}{tanA - secA + 1}$$
$$= \frac{tanA + secA - (secA + tanA)(secA - tanA)}{tanA - secA + 1}$$
$$= \frac{tanA + secA(1 - (secA - tanA)}{tanA - secA + 1}$$

$$= \frac{\tan A + \sec A(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec \cancel{(\tan A - \sec A + 1)}}{\cancel{\tan A - \sec A + 1}}$$

$$LHS = \tan A + \sec A$$

$$\text{RHS} = \frac{1}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{\sec A + \tan A}{\sec^2 A - \tan^2 A}$$

$$= \frac{\sec A + \tan A}{1}$$

$$\text{RHS} = \tan A + \sec A$$

$$\therefore LHS = RHS$$

24. $\frac{\cos A - \sin A + 1}{\cos A + \sin A + 1} = \frac{1}{\tan A + \sec A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{\cos A - \sin A + 1}{\cos A + \sin A + 1}$$

ಅಂಶ ಮತ್ತು ಛೇದಗಳೆರಡನ್ನು cosA ದಿಂದ ಭಾಗಿಸಿದಾಗ

$$= \frac{\frac{\cos A}{\cos A} - \frac{\sin A}{\cos A} + \frac{1}{\cos A}}{\frac{\cos A}{\cos A} + \frac{\sin A}{\cos A} + \frac{1}{\cos A}}$$

$$= \frac{1 - \tan A + \sec A}{1 + \tan A + \sec A}$$

$$= \frac{\sec A - \tan A + 1}{\sec A + \tan A + 1}$$

$$= \frac{\sec A - \tan A + (\sec^2 A - \tan^2 A)}{\sec A + \tan A + 1}$$

$$= \frac{\sec A - \tan A + (\sec A + \tan A)(\sec A - \tan A)}{\sec A + \tan A + 1}$$

$$= \frac{\sec A - \tan A\cancel{(1 + \sec A + \tan A)}}{\cancel{\sec A + \tan A + 1}}$$

$$= \sec A - \tan A$$

$$RHS = \frac{1}{\tan A + \sec A}$$

$$= \frac{1}{\tan A + \sec A} \times \frac{\tan A - \sec A}{\tan A - \sec A}$$

$$= \frac{\tan A - \sec A}{\tan^2 A - \sec^2 A}$$

$$= \frac{-(\sec A - \tan A)}{-(\sec^2 A - \tan^2 A)}$$

$$= \frac{\sec A - \tan A}{1}$$

$$= \sec A - \tan A$$

$$\therefore LHS = RHS$$

25. $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{\cot A - \cos A}{\cot A + \cos A}$$

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

$$= \frac{\cancel{\cos A}\left(\frac{1}{\sin A} - 1\right)}{\cancel{\cos A}\left(\frac{1}{\sin A} + 1\right)}$$

$$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$$

$$\therefore LHS = RHS$$

26. $\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A[1 - (\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A\cancel{[\cot A - \operatorname{cosec} A + 1]}}{\cancel{\cot A - \operatorname{cosec} A + 1}}$$

$$= \operatorname{cosec} A + \cot A$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$$= \frac{1 + \cos A}{\sin A}$$

$$\therefore LHS = RHS$$

27. $(\sec A + \tan A)^2 = \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = (\sec A + \tan A)^2$$

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$$= \left(\frac{1}{cosA} + \frac{sinA}{cosA}\right)^2$$

$$= \left(\frac{1 + sinA}{cosA}\right)^2$$

$$= \frac{(1 + sinA)^2}{cos^2A}$$

$$= \frac{(1 + sinA)^2}{1 - sin^2A}$$

$$= \frac{\cancel{(1+si\quad)}(1+sinA)}{\cancel{(1+sin\quad)}(1-sinA)}$$

$$= \frac{1 + sinA}{1 - sinA}$$

$$= \frac{1 + \frac{1}{cosecA}}{1 - \frac{1}{cosecA}}$$

$$= \frac{\frac{cosecA + 1}{cosecA}}{\frac{cosecA - 1}{cosecA}}$$

$$= \frac{cosecA + 1}{cosecA - 1}$$

$\therefore LHS = RHS$

28. $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta.\,cosec^2\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \tan^2\theta + \cot^2\theta + 2$

$= \sec^2\theta - 1 + cosec^2\theta - 1 + 2$

$= \sec^2\theta + cosec^2\theta$

$$= \frac{1}{cos^2\theta} + \frac{1}{sin^2\theta}$$

$$= \frac{sin^2\theta + cos^2\theta}{cos^2\theta.sin^2\theta}$$

$$= \frac{1}{cos^2\theta.sin^2\theta}$$

$= \sec^2\theta.\,cosec^2\theta$

$\therefore LHS = RHS$

29. $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$LHS = \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= tanA\,\frac{1 - 2(1 - \cos^2 A)}{2\cos^2 A - 1}$$

$$= tanA\,\frac{1 - 2 + 2\cos^2 A}{2\cos^2 A - 1}$$

$$= tanA\,\frac{\cancel{2\cos^2 A - 1}}{\cancel{2\cos^2 A - 1}}$$

$= \tan A$

$\therefore LHS = RHS$

30. $(\mathrm{cosec}A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = (\mathrm{cosec}A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{sinA} - sinA\right)\left(\frac{1}{cosA} - cosA\right)$$

$$= \left(\frac{1 - \sin A^2}{sinA}\right)\left(\frac{1 - \cos^2 A}{cosA}\right)$$

$$= \left(\frac{\cos^2 A}{sinA}\right)\left(\frac{\sin^2 A}{cosA}\right)$$

$= cosA.sinA$

$$RHS = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A.\sin A}}$$

$$= \frac{cosA.sinA}{\sin^2 A + \cos^2 A}$$

$$= \frac{cosA.sinA}{1}$$

$= cosA.sinA$

$\therefore LHS = RHS$

31. $(\sin A + \sec A)^2 + (\cos A + \mathrm{cosec}A)^2 = (1 + \sec A.\mathrm{cosec}A)^2$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = (\sin A + \sec A)^2 + (\cos A + \mathrm{cosec}A)^2$

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$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$

$= \left(\frac{\sin A.\cos A + 1}{\cos A}\right)^2 + \left(\frac{\cos A.\sin A + 1}{\sin A}\right)^2$

$= \frac{(\sin A.\cos A + 1)^2}{cosA^2} + \frac{(\sin A.\cos A + 1)^2}{sin^2A}$

$= \frac{(\sin A.\cos A + 1)^2 sin^2A + cos^2A(\sin A.\cos A + 1)^2}{cosA^2.sin^2A}$

$= \frac{(\sin A.\cos A + 1)^2(sin^2A + cos^2A)}{cosA^2.sin^2A}$

$= \frac{(\sin A.\cos A + 1)^2.1}{cosA^2.sin^2A}$

$= \left(\frac{\sin A.\cos A + 1}{cosA.sinA}\right)^2$

$= \left(\frac{\sin A.\cos A}{cosA.sinA} + \frac{1}{cosA.sinA}\right)^2$

$= \left(1 + \frac{1}{cosA.sinA}\right)^2$

$= (1 + \sec A.\mathrm{cosec}A)^2$

$\therefore LHS = RHS$

32. $\tan^2 A - \sin^2 A = \tan^2 A.\sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \tan^2 A - \sin^2 A$

$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$

$= \frac{\sin^2 A - \sin^2 A.\cos^2 A}{\cos^2 A}$

$= \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A}.\sin^2 A$

$= \tan^2 A.\sin^2 A$

$\therefore LHS = RHS$

33. $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \cos^4 A - \sin^4 A$

$= (cosA^2)^2 - (sinA^2)^2$

$= (cos^2A + sin^2A)\,(cos^2A - sin^2A)$

$= 1(cos^2A - sin^2A)$

$= 1 - sin^2A - sin^2A$

$= 1 - 2sin^2A$

$\therefore LHS = RHS$

34. $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A.\sec^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \sec^6 A - \tan^6 A$

$= (sec^2A)^3 - \tan^6 A$

$= (1 + tan^2A)^3 - \tan^6 A$

$= 1^3 + (tan^2A)^3 + 3 \times 1 \times tan^2A(1 + tan^2A) - \tan^6 A$

$= 1 + \tan^6 A + 3tan^2A.\sec^2 A - \tan^6 A$

$= 1 + 3\tan^2 A.\sec^2 A$

$\therefore LHS = RHS$

35. $\sqrt{\sec^2 A + \mathrm{cosec}^2 A} = \tan A + \cot A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \sqrt{\sec^2 A + \mathrm{cosec}^2 A}$

$= \sqrt{1 + \tan^2 A + 1 + \cot^2 A}$

$= \sqrt{\tan^2 A + \cot^2 A + 2}$

$= \sqrt{(tanA + cotA)^2}$

$= \tan A + \cot A$

$\therefore LHS = RHS$

36. $\frac{\cos A}{\sec A - \tan A} = 1 + \sin A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\cos A}{\sec A - \tan A}$

$= \frac{\cos A}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}$

$= \frac{\cos A}{\frac{1 - \sin A}{\cos A}}$

$= cosA \times \frac{cosA}{1 - sinA}$

$= \frac{cos^2A}{1 - sinA}$

$= \frac{1 - sin^2A}{1 - sinA}$

$= \frac{(1 + sinA)(1 - sinA)}{1 - sinA}$

$= 1 + sinA$

$\therefore LHS = RHS$

37. $\frac{\sin^2 A}{1 - \cos A} = \frac{1 + \sec A}{\sec A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

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$LHS = \frac{\sin^2 A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$

$= \frac{(1+cosA)(1-cos)}{1-cos}$

$= 1 + cosA$

$= 1 + \frac{1}{secA}$

$= \frac{secA + 1}{secA}$

$\therefore LHS = RHS$

38. $\frac{\mathrm{cosec}^2 A+1}{\mathrm{cosec}^2 A-1} = \sec^2 A + \tan^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\mathrm{cosec}^2 A + 1}{\mathrm{cosec}^2 A - 1}$

$= \frac{\frac{1}{\sin^2 A} + 1}{\frac{1}{\sin^2 A} - 1}$

$= \frac{\frac{1+\sin^2 A}{\sin^2 A}}{\frac{1-\sin^2 A}{\sin^2 A}}$

$= \frac{1+\sin^2 A}{1-\sin^2 A}$

$= \frac{1+\sin^2 A}{\cos^2 A}$

$= \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A}$

$= \sec^2 A + \tan^2 A$

$\therefore LHS = RHS$

39. $\frac{\sin A}{1-\cos A} = \mathrm{cosec}A + \cot A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = \frac{\sin A}{1-\cos A}$

$= \frac{\sin A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$

$= \frac{\sin A(1+\cos A)}{1^2 - \cos^2 A}$

$= \frac{\sin A(1+\cos A)}{\sin^2 A}$

$= \frac{1+\cos A}{\sin A}$

$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$

$= cosecA + cotA$

$\therefore LHS = RHS$

40. $(\tan A \times \sin A) + \cos A = \sec A$ ಎಂದು ಸಾಧಿಸಿ. ($june - 2019, 2\ marks$)

ಪರಿಹಾರ :

$LHS = (\tan A \times \sin A) + \cos A$

$= \left(\frac{\sin A}{\cos A} \times \sin A\right) + \cos A$

$= \frac{\sin^2 A}{\cos A} + \cos A$

$= \frac{\sin^2 A + \cos^2 A}{\cos A}$

$= \frac{1}{\cos A}$

$= \sec A$

$\therefore LHS = RHS$

41. $x = p\tan\theta + q\sec\theta$ ಮತ್ತು $x = p\sec\theta + q\tan\theta$ ಆದರೆ $x^2 - y^2 = q^2 - p^2$ ಎಂದು ಸಾಧಿಸಿ.

($june - 2020, 3\ marks$)

ಪರಿಹಾರ :

$LHS = x^2 - y^2$

$= (p\tan\theta + q\sec\theta)^2 - (p\sec\theta + q\tan\theta)^2$

$= (p\ tan\theta)^2 + (q\ sec\theta)^2 - 2p\ tan\theta.\text{p}\sec\theta - [(p\ sec\theta)^2 + (q\ tan\theta)^2 - 2p\ sec\theta.\text{p}\tan\theta]$

$= p^2tan^2\theta + q^2sec^2\theta - 2pq\ tan\theta.\sec\theta - [p^2sec^2\theta + q^2tan^2\theta - 2pq\ sec\theta.\tan\theta]$

$= p^2tan^2\theta + q^2sec^2\theta - 2pq\ tan\theta.sec\theta - p^2sec^2\theta - q^2tan^2\theta + 2pq\ sec\theta.tan\theta$

$= p^2tan^2\theta + q^2sec^2\theta - p^2sec^2\theta - q^2tan^2\theta$

$= p^2tan^2\theta - p^2sec^2\theta + q^2sec^2\theta - q^2tan^2\theta$

$= p^2(tan^2\theta - sec^2\theta) + q^2(sec^2\theta - tan^2\theta)$

$= p^2(-1) + q^2(1)$

$= q^2 - p^2$

$\therefore LHS = RHS$

42. $\frac{\cot^2(90°-\theta)}{\tan^2\theta - 1} + \frac{\mathrm{cosec}^2\theta}{\sec^2\theta - \mathrm{cosec}^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$ ಎಂದು ಸಾಧಿಸಿ. ($june - 2020, 3\ marks$)

ಪರಿಹಾರ :

$LHS = \frac{\cot^2(90° - \theta)}{\tan^2\theta - 1} + \frac{\mathrm{cosec}^2\theta}{\sec^2\theta - \mathrm{cosec}^2\theta}$

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$$= \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\text{cosec}^2\theta}{\sec^2\theta - \text{cosec}^2\theta}$$

$$= \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\text{cosec}^2\theta}{\sec^2\theta - \text{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - 1} + \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}} + \frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta.\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{\sin^2\theta - \cos^2\theta} + \frac{1}{\cancel{\sin^2\theta}} \times \frac{\cos^2\theta.\cancel{\sin^2\theta}}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1}{\sin^2\theta - \cos^2\theta}$$

$\therefore LHS = RHS$

43. $\frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta - \sin\theta} = \cot\theta$ **ಎಂದು ಸಾಧಿಸಿ.**

(MQP.1 – 2021, 3 marks)

ಪರಿಹಾರ :

$$LHS = \frac{\cos\theta - 2\cos^3\theta}{2\sin^3\theta - \sin\theta}$$

$$= \frac{\cos\theta(1 - 2\cos^2\theta)}{\sin\theta(2\sin^2\theta - 1)}$$

$$= \cot\theta \frac{1 - 2\cos^2\theta}{2(1 - \cos^2\theta) - 1}$$

$$= \cot\theta \frac{1 - 2\cos^2\theta}{2 - 2\cos^2\theta - 1}$$

$$= \cot\theta \frac{\cancel{1 - 2\cos^2\theta}}{\cancel{1 - 2\cos^2\theta}}$$

$$= \cot\theta$$

$\therefore LHS = RHS$

44. $\frac{\sin\theta}{1 - \cot\theta} + \frac{\cos\theta}{1 - \tan\theta} = \sin\theta + \cos\theta$ **ಎಂದು ಸಾಧಿಸಿ.**

(MQP.1 – 2021, Preperatory – 2020, 3 marks)

ಪರಿಹಾರ :

$$LHS = \frac{\sin\theta}{1 - \cot\theta} + \frac{\cos\theta}{1 - \tan\theta}$$

$$= \frac{\sin\theta}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1 - \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\cos\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \sin\theta \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \cos\theta \times \frac{\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} - \frac{\cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)\cancel{(\sin\theta - \cos\theta)}}{\cancel{\sin\theta - \cos\theta}}$$

$$= \sin\theta + \cos\theta$$

$\therefore LHS = RHS$

46. $\{\text{Cosec}(90 - \theta) - \sin(90 - \theta)\}\{(\text{Cosec}\theta - \sin\theta\tan\theta + \cot\theta\} = 1$ **ಎಂದು ಸಾಧಿಸಿ.** *(MQP.2–2021, 3 marks)*

ಪರಿಹಾರ :

$$LHS = \{\text{Cosec}(90 - \theta) - \sin(90 - \theta)\}\{(\text{Cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)\}$$

$$= \{\sec\theta - \cos\theta\}\{(\text{Cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)\}$$

$$= \left\{\frac{1}{\cos\theta} - \cos\theta\right\}\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\tan\theta + \frac{1}{\tan\theta}\right)\right\}$$

$$= \left\{\frac{1 - \cos^2\theta}{\cos\theta}\right\}\left\{\left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 + \tan^2\theta}{\tan\theta}\right)\right\}$$

$$= \left\{\frac{\sin^2\theta}{\cos\theta}\right\}\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\sec^2\theta}{\tan\theta}\right)\right\}$$

$$= \left\{\frac{\sin^2\theta}{\cos\theta}\right\}\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)\left(\frac{\frac{1}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta}}\right)\right\}$$

$$= \frac{\sin^2\theta}{\cancel{\cos\theta}} \times \frac{\cancel{\cos^2\theta}}{\sin\theta} \times \frac{1}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta}$$

$$= \frac{\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$$

$$= 1$$

$\therefore LHS = RHS$

47. $\frac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \frac{(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)} = \frac{2}{(2\sin^2\theta - 1)}$ **ಎಂದು ಸಾಧಿಸಿ.** *(MQP.2 – 2021, 3 marks)*

ಪರಿಹಾರ :

$$LHS = \frac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \frac{(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^2\theta + \cos^2\theta - \cancel{2\sin\theta.\cos\theta} + \sin^2\theta + \cos^2\theta + \cancel{2\sin\theta.\cos\theta}}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{1+1}{\sin^2\theta - (1-\sin^2\theta)}$$
$$= \frac{2}{\sin^2\theta - 1 + \sin^2\theta}$$
$$= \frac{2}{2\sin^2\theta - 1}$$
$\therefore LHS = RHS$

48. $\mathrm{CosecA}(1-\cos A)(\mathrm{CosecA}+\cot A) = 1$ ಎಂದು ಸಾಧಿಸಿ.

(Gadag dist level exam – 2021, 3 marks)

ಪರಿಹಾರ :

$LHS = \mathrm{CosecA}(1-\cos A)(\mathrm{CosecA}+\cot A)$
$$= \frac{1}{\sin A}(1-\cos A)\left(\frac{1}{\sin A}+\frac{\cos A}{\sin A}\right)$$
$$= \frac{1}{\sin A}(1-\cos A)\left(\frac{1+\cos A}{\sin A}\right)$$
$$= \frac{1-\cos^2A}{\sin^2A}$$
$$= \frac{\cancel{\sin^2A}}{\cancel{\sin^2A}}$$
$= 1$

$\therefore LHS = RHS$

49. $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta) = 1$ ಎಂದು ಸಾಧಿಸಿ. *(April – 2022, 3 marks)*

ಪರಿಹಾರ :

$LHS = \sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$
$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$$
$$= \frac{1}{\cos\theta}(1-\sin\theta)\left(\frac{1+\sin\theta}{\cos\theta}\right)$$
$$= \frac{1-\sin^2\theta}{\cos^2\theta}$$
$$= \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}}$$
$= 1$

$\therefore LHS = RHS$

50. $(\cot A \times \cos A) + \sin A = \mathrm{cosecA}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$LHS = (\cot A \times \cos A) + \sin A$
$$= \left(\frac{\cos A}{\sin A} \times \cos A\right) + \sin A$$
$$= \frac{\cos^2A}{\sin A} + \sin A$$
$$= \frac{\cos^2A + \sin^2A}{\sin A}$$
$$= \frac{1}{\sin A}$$
$= cosecA$

$\therefore LHS = RHS$

51. $\frac{2\cos(90°-30°)+\tan45°-\sqrt{3}.\mathrm{cosec}60°}{\sqrt{3}.\sec30°+2\cos60°+\cot45°}$ ರ ಬೆಲೆ ಕಂಡುಹಿಡಿಯಿರಿ. *(September – 2022, 3 marks)*

ಪರಿಹಾರ :

$$\frac{2\cos(90°-30°)+\tan45°-\sqrt{3}.\mathrm{cosec}60°}{\sqrt{3}.\sec30°+2\cos60°+\cot45°}$$
$$= \frac{2\cos60°+\tan45°-\sqrt{3}.\mathrm{cosec}60°}{\sqrt{3}.\sec30°+2\cos60°+\cot45°}$$
$$= \frac{2\times\frac{1}{2}+1-\sqrt{3}\times\frac{2}{\sqrt{3}}}{\sqrt{3}\times\frac{2}{\sqrt{3}}+2\times\frac{1}{2}+1}$$
$$= \frac{1+1-2}{2+1+1}$$
$$= \frac{0}{4}$$
$= 0$

$$\therefore \frac{2\cos(90°-30°)+\tan45°-\sqrt{3}.\mathrm{cosec}60°}{\sqrt{3}.\sec30°+2\cos60°+\cot45°} = 0$$

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