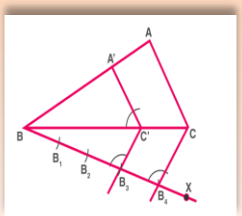
Government of Karnataka

Department of School Education and Literacy

OFFICE OF THE DEPUTY DIRECTOR, KOLAR DISTRICT, KOLAR



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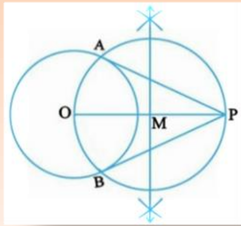
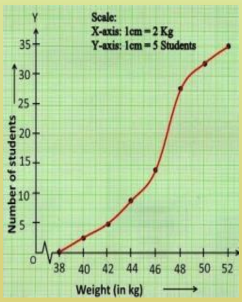


PRERANE

MATHEMATICS MADE EASY



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1. ARITHMETIC PROGRESSIONS

Example	Marks:-2
1. Find the 12^{th} term of an A.P, 2, 5, 8, 11, . . using formula a=2, d=5–2=3, n=12 a_n=a+(n-1)d a_{12}=2+(12-1)3 = 2+33 = 35 $\therefore \mathbf{a_{12} = 35}$	2. Find the 20^{th} term A.P, 61, 58, 55, ... using formula. a=61 d= 58 – 61= -3 , n=20 a_n=a + (n-1)d a_{20}=61 + (20-1)(-3) = 61 + 19(-3) = 61 – 57 = 4 $\therefore \mathbf{a_{20} = 4}$

Problems to practice	
1) Find the 30^{th} term of the Arithmetic Progression 3, 7, 11, - - - - - - - - - - - - - 3) Find the 20^{th} term of the A.P given 72, 68, 64, - - - - - - - - - -- - - - -	2) Find the 18^{th} term of the A.P given 5, 8, 11, 14, - - - - - - - - - - - - - - - - 4) Find the 12^{th} term of the A.P given -2, -7, -12, -17, ...

Example	Problems to solve practice
1. Find the sum of first 20 terms of an arithmetic series 2+7+12+……………… using the formula. a =2, d=5, n=20 $S_n = \frac{n}{2}[\ 2a+ (n-1)d]$ $S_{20} = \frac{20}{2}[\ 2(2)+ (20-1)(5)] = 10[4+95]$ = 10[99] = 990 $\therefore \mathbf{S_{20} = 990}$	1) Find the sum of an arithmetic series upto 30 terms 2 + 7 + 12 + ... ……. 2) Find the sum of first 15 terms of the A.P 3, 7, 11, 15, …………………… 3) Find the sum of first 10 terms of an A.P 5, 8, 11, 14, …………………

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Examples	Marks:-2
1. Solve the following pair of linear equations by elimination method. $x + y = 8$ *and* $2x - y = 7$ $x + \cancel{y} = 8$ $2x - \cancel{y} = 7$ $3x = 15$ (addition) $x = 5$ Substituting the value of x in $x + y = 8$ $5 + y = 8$ $y = 3$ $\therefore \boldsymbol{x = 5 \text{ and } y = 3}$	2. Solve by elimination method: $x + y = 5$ and $2x + 3y = 12$ $x + y = 5$ ----- (1) $2x + 3y = 12$ ----- (2) Multiplying the equation (1) by 2 we get $2x + 2y = 10$ − − − − − (3) Solving equation (2) and (3) $\cancel{2x} + 3y = 12$ $(-)\cancel{2x} + (-)2y = (-)10$ $y = 2$ (subtraction) Substitute the value of y in $x + y = 5$, we get $x = 3$ $\therefore \boldsymbol{x = 3 \text{ and } y = 2}$

Problems to practice

1) Solve the following pair of linear equations by elimination method

$$2x + y = 5 \quad and \quad x + y = 4$$

3) Solve the following pair of linear equations by elimination method

$$3x + 2y = 11 \; and \; 5x - 2y = 13$$

5) Solve the following pair of linear equations by elimination method

$$x + y = 5 \; and \; 2x - 3y = 5$$

2) Solve the following pair of linear equations by elimination method

2x + y = 8 and x – y = 1

4) Solve the following pair of linear equations by elimination method

$$2x + y = 11 \; and \; x + y = 8$$

6) Solve the following pair of linear equations by elimination method

$$3x + 2y = 15 \; and \; 2x - 3y = \; -4$$

Example Marks:-4

1) Solve graphically:

$2x + y = 8 \quad and \quad x - y = 1.$

$2x + y = 8$

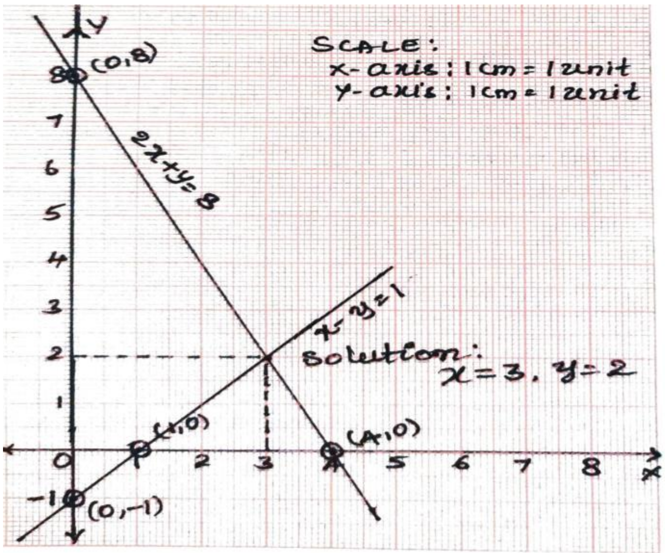
$y = 8 - 2x$

x	0	4
y	8	0

$x - y = 1$

$y = x - 1$

x	0	1
y	-1	0



$\therefore$ **x = 3 and y = 2**

8) Solve graphically:

$x + y = 5 \; and \; x + 2y = 6$

$x + y = 5$

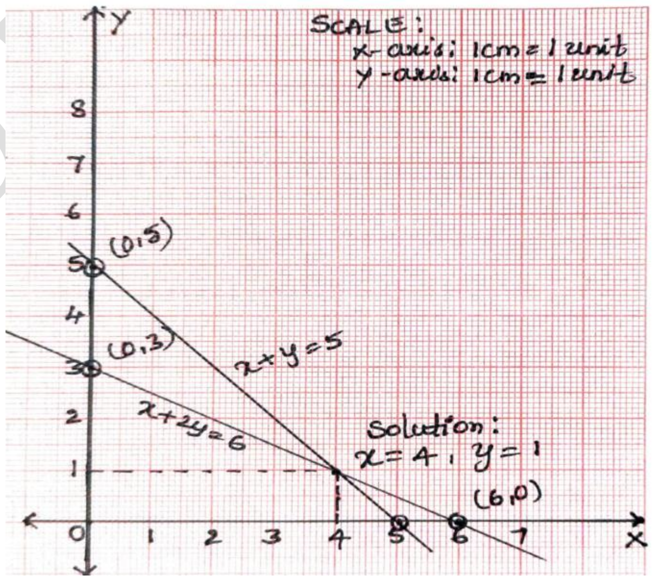
$y = 5 - x$

x	0	5
y	5	0

$x + 2y = 6$

$2y = 6 - x$

x	0	6
y	3	0



$\therefore$ **x = 4 and y = 1**

1) Solve graphically x + 3y = 6
2x – y = 5

2) Solve graphically 3x – y = 1
x + y = 7

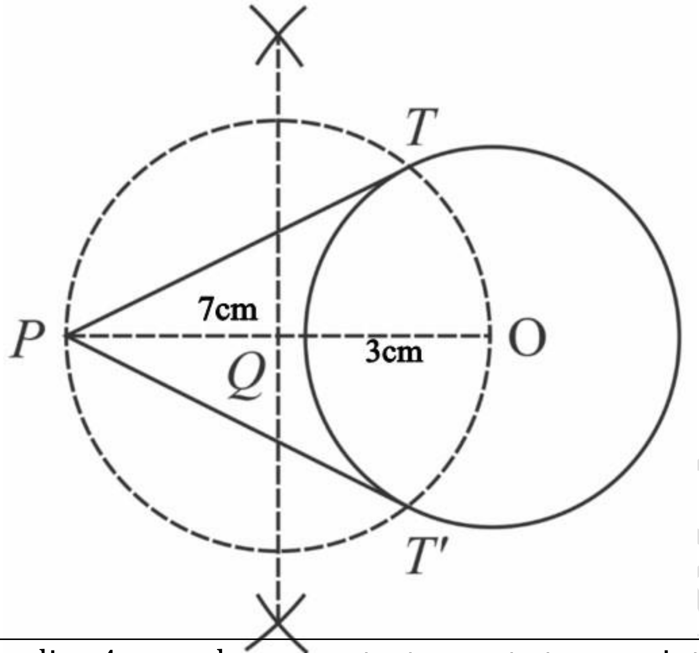
3) Solve graphically 3x + 2y = 12
x – y = -1

4) Solve graphically 2x + y = 6
2x – y = 2

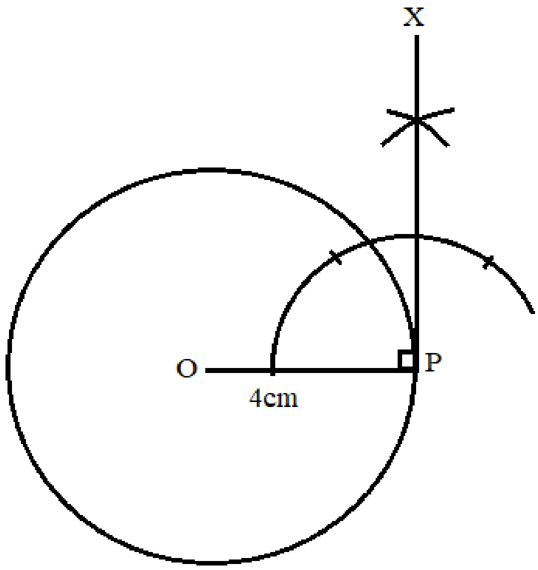
3. CONSTRUCTIONS

Example	Marks:-2

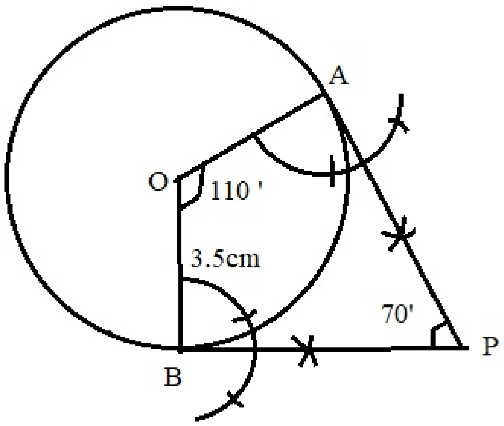
1) Draw a circle of radius 3cm and construct two tangents to the circle from a point 7cm away from the centre.

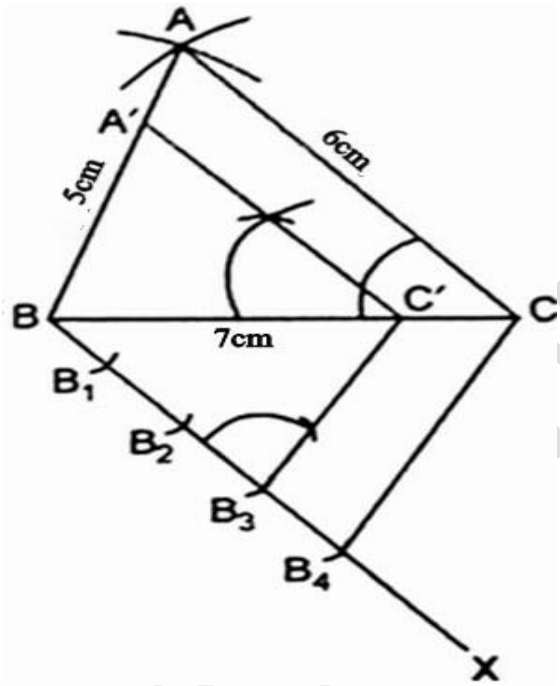
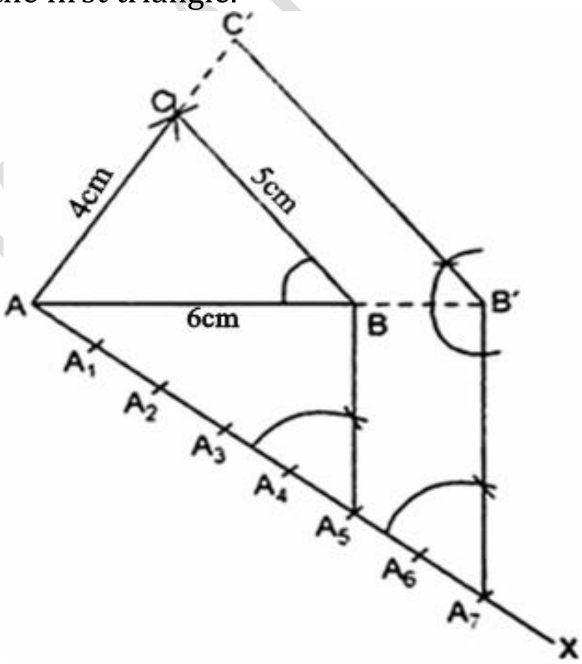


2) Draw a circle of radius 4cm and construct a tangent at any point 'P' on the circle.



3) Draw a circle of radius 3.5cm and construct two tangents to it such that the angle between the tangents is 70^0



Problems to construct	
1) Draw a circle of radius 4.5cm and construct two tangents to the circle from a point 8cm away from the centre. 3) Draw a circle of radius 4cm and construct two tangents to the circle from a point 9cm away from the centre. 5) Draw a circle of radius 5cm and construct a tangent at any point 'P' on the circle.	2) Draw a circle of radius 4cm and construct two tangents such that the angle between the tangents is 60^0 4) Draw a circle of radius 4.5cm and construct two tangents such that the angle between the tangents is 80^0
Example	Marks:-3
1. Construct a triangle with sides 5cm, 6cm and 7cm, then construct a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.  	2. Construct a triangle of sides 4cm, 5cm and 6cm and then construct a triangle similar to it whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. 
Problems for practice	
1) Construct a triangle similar to triangle ABC in which AB =4 cm, ∠ABC=60^o and BC= 6 cm such that each side of the new triangle is $\frac{3}{4}$ of the corresponding sides of the triangle ABC. 3) Construct a triangle similar to triangle ABC in which BC=3cm, AB =6 cm, and AC=4.5cm such that each side of the new triangle is $\frac{3}{5}$ of the corresponding sides of the triangle ABC.	2) Draw a triangle DEF with EF=7 cm, ∠DEF=60^0 and DE=6 cm then construct a triangle whose sides are $\frac{4}{3}$ of the corresponding sides of the triangle DEF. 4) Construct the triangle with sides 4cm, 6cm and 8cm, then construct the similar triangle which is $\frac{5}{3}$ of the given triangle.

4. COORDINATE GEOMETRY

Example Marks:-2

1) Find the distance between the points (3,2) and (-5,8).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-5-3)^2 + (8-2)^2}$$

$$= \sqrt{(-8)^2 + (6)^2}$$

$$= \sqrt{(64+36)}$$

$$= \sqrt{100}$$

$$\therefore \boldsymbol{d = 10 \text{ units}}$$

2) If the distance between the points $(4, p)$ and $(1, 0)$ is 5 units, find the value of 'p'

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$5 = \sqrt{(1-4)^2 + (0-p)^2}$$ [Squaring on both sides]

$$25 = (-3)^2 + p^2$$

$$25 = 9 + p^2$$

$$25 - 9 = p^2$$

$$16 = p^2$$

$$\therefore \boldsymbol{p = \pm 4}$$

Problems to practice

1) Find the distance between A(8,3) and B(2,11) using distance formula.	2) Find the distance between (-5,7) and (-1,3) using distance formula.
3) Find the distance between (3,4) and (4,7) using distance formula.	4) Find the distance between P(1,2) and Q(7,10) using distance formula.

Example

1. Find the co-ordinates of the midpoint of the line segment joining the points (0, 8) and (4, 0).

$$P(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$P(x, y) = \left(\frac{0+4}{2}, \frac{8+0}{2}\right)$$

$$P(x, y) = \left(\frac{4}{2}, \frac{8}{2}\right)$$

$$\boldsymbol{P(x, y) = (2, 4)}$$

Problems to practice

1) Find the co-ordinates of the midpoint of the line segment joining the points (5, 4) and (3, 6).

2) Find the co-ordinates of the midpoint of the line segment joining the points M(-2, 5) and N(6, -3).

5. QUADRATIC EQUATIONS

Example Marks:-2

1. Solve $2x^2 - 5x + 3 = 0$ by using the quadratic formula.

$$a = 2, \quad b = -5, \quad c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Solve $x^2 + 2x + 4 = 0$ by using the quadratic formula.

$$a = 1, \quad b = 2, \quad c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{4}$$

$$x = \frac{5 \pm 1}{4}$$

$$x = \frac{5+1}{4} \text{ or } \frac{5-1}{4}$$

$$\boldsymbol{x = \frac{3}{2} \; or \; x = 1}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm \sqrt{4(-3)}}{2}$$

$$x = \frac{2(-1 \pm \sqrt{-3})}{2}$$

$\boldsymbol{x} = (\boldsymbol{-1} + \sqrt{\boldsymbol{-3}})$ or $\boldsymbol{x} = (\boldsymbol{-1} - \sqrt{\boldsymbol{-3}})$

Problems for practice

1) Find the roots of the quadratic equation $x^2 + 3x - 5 = 0$ using quadratic formula.
3) Find the roots of the quadratic equation $x^2 + 2x - 15 = 0$ using quadratic formula.

2) Find the roots of the quadratic equation $4x^2 - 12x + 9 = 0$ using quadratic formula.
4) Find the roots of the quadratic equation $2x^2 - 3x + 5 = 0$ using quadratic formula.

Example

1. Find the discriminant of the quadratic equation $4x^2 - 12x + 9 = 0$ and write the nature of roots.

$a = 4, \quad b = -12, \quad c = 9$

$b^2 - 4ac = (-12)^2 - 4(4)(9)$

$= 144 - 144$

$b^2 - 4ac = 0$

∴ **Roots are Real and Equal**

2. Find the discriminant of the quadratic equation $x^2 + 2x - 15 = 0$ and write the nature of roots.

$a = 1, \quad b = 2, \quad c = -15$

$b^2 - 4ac = (2)^2 - 4(1)(-15)$

$= 4 + 60$

$= 64$

Here $b^2 - 4ac > 0$

∴ **Roots are Real and Distinct**

3. Find the discriminant of the quadratic equation $x^2 - x + 12 = 0$ and write the nature of roots.

$a = 1, \quad b = -1, \quad c = 12$

$b^2 - 4ac = (-1)^2 - 4(1)(12)$

$= 1 - 48$

$= -47$

Here $b^2 - 4ac < 0$

∴ **The equation has no real roots.**

1) Find the discriminant of the quadratic equation $x^2 + 4x + 4 = 0$ and write the nature of roots.

3) Find the discriminant of the quadratic equation $x^2 + x - 6 = 0$ and write the nature of roots.

2) Find the discriminant of the quadratic equation $x^2 + 2x + 1 = 0$ and write the nature of roots.

4) Find the discriminant of the quadratic equation $2x^2 - 15x + 18 = 0$ and write the nature of roots.

6. PROBABILITY

Examples — Marks:-2

1. A die, numbered from 1 to 6 on its each face is rolled once. Find the probability of getting an odd number.

Number of all possible outcomes,

$n(S) = 6$

Let A be the event of getting an odd number.

$\therefore n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} \qquad = \frac{3}{6}$$

$$\therefore \boldsymbol{P(A) = \frac{1}{2}}$$

2. A box contains 4 red marbles, 8 green marbles and 5 white marbles. One marble is taken out at random. Find the probability of the marble taken out to be red.

Number of all possible outcomes,

$n(S) = 4 + 5 + 8 = 17$

Let A be the event of taking out the red marble.

$\therefore n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\therefore \boldsymbol{P(A) = \frac{4}{17}}$$

Problems for practice

1) A cubical die whose faces are marked from 1 to 6 is rolled once, find the probability of getting a perfect square number.

2) Two identical coins are tossed simultaneously, find the probability of getting at least one head.

3) 12 defective pens got mixed with 132 good ones. One pen is taken randomly from the lot. Find the probability of getting a defective pen.

4) Two dice, numbered from 1 to 6 on their each face are together rolled once. Find the probability of getting the numbers whose sum is greater than 8.

7. REAL NUMBERS

Example — Marks:-2

1. Prove that $5 + \sqrt{2}$ is an irrational number.

Proof : Let $5 + \sqrt{2}$ is a rational number.

$\Rightarrow 5 + \sqrt{2} = \frac{p}{q}$ where $q \neq 0,\ p,\ q \in Z$

$\sqrt{2} = \frac{p}{q} - 5$

$\sqrt{2} = \frac{p-5q}{q}$

Irrational number ≠ Rational number

LHS ≠ RHS

Our assumption is wrong.

$\therefore 5 + \sqrt{2}$ is an irrational number.

2. Prove that $2 - \sqrt{3}$ is an irrational number.

Proof : Let $2 - \sqrt{3}$ is a rational number.

$\Rightarrow 2 - \sqrt{3} = \frac{p}{q}$ where $q \neq 0,\ p,\ q \in Z$

$2 - \frac{p}{q} = \sqrt{3}$

$\frac{2q-p}{q} = \sqrt{3}$

Rational number ≠ Irrational number

LHS ≠ RHS

Our assumption is wrong.

$\therefore 2 - \sqrt{3}$ is an irrational number.

Problems for practice	
1) Prove that $3+\sqrt{5}$ is an irrational number.	2) Prove that $5-\sqrt{7}$ is an irrational number.

Example Marks:-3

1. Prove that $\sqrt{2}$ is an irrational number.

Proof: Let $\sqrt{2}$ is a rational number.

$\sqrt{2} = \frac{p}{q}$ where $q \neq 0$, p *and* q are co-prime

$\sqrt{2}\, q = p$ squaring on both sides

$2q^2 = p^2$ ....(1)

$\therefore$ p^2 is divisible by 2 and also p is divisible by 2 ....(2)

=> $p = 2r$ .... (3)

Substitute equation (3) in (1)

$2q^2 = (2r)^2$

$2q^2 = (2r)^2$

$2q^2 = 4r^2$

$q^2 = 2r$

$\therefore$ q^2 is divisible by 2 and also q is divisible by 2 ....(4)

(2) and (4) => Both p and q have a common factor.

$\therefore$ both p and q are not co-primes. It is contradictory to our assumption.

$\therefore$ $\sqrt{2}$ is an irrational number.

1) Prove that $\sqrt{3}$ is an irrational number.	2) Prove that $\sqrt{5}$ is an irrational number.

8. INTRODUCTION TO TRIGONOMETRY

Example Marks:-2

1. If 5sinθ = 3 then find cosθ and tanθ.

sinθ = $\frac{3}{5}$

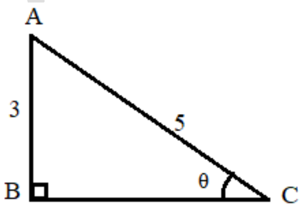
$AC^2=AB^2+BC^2$

$BC^2 = 5^2 - 3^2$

$BC^2= 25-9$

$BC^2=16$

$BC = \sqrt{16} = 4$



cosθ = $\frac{4}{5}$ tanθ = $\frac{3}{4}$

2. Evaluate:

$sin18^o - cos72^o - cos18^o + sin72^o$.

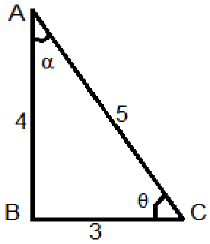
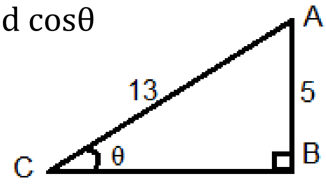
$= sin18^o - cos72^o - cos18^o + sin72^o$

$= sin(90^o - 72^0) - cos72^o - cos(90^o - 72^0) + sin72^o$

$= cos72^o - cos72^o - sin72^o + sin72^o$

$= 0$

Problems for practice

1) From the figure find the value of sinθ and tanα 	2) If $\sin\theta = \frac{5}{13}$ then find $\cos\theta$ 
3) Find the value of $\sin 30^0 + \cos 60^0$	4) Evaluate: $\tan 28^0 . \cot 62^0 - \sin 72^0 . \cos 18^0$

9. POLYNOMIALS

Example — Marks:-2

1. Find the quadratic polynomial having zeroes 5 and 3.

Let $\alpha = 5$ and $\beta = 3$

$\alpha + \beta = 8$ {sum of the roots}

$\alpha\beta = 15$ {product of the roots}

The quadratic polynomial is of the form

$x^2 - (\alpha + \beta)x + \alpha\beta$

$\therefore$ *The polynomial is* $x^2 - 8x + 15$

Problems for practice

1. Frame a quadratic polynomial given sum of the zeros 4 and product of the zeros as 1	2. Find the quadratic polynomial if, sum and product of its zeroes are -5 and 4 respectively.

Example — Marks:-3

1. Find the quotient and the remainder when $3x^3 + x^2 + 2x + 5$ is divided by $x^2 + 2x + 1$.

$$
\begin{array}{r|l}
 & 3x - 5 \\
\hline
x^2 + 2x + 1 & 3x^3 + x^2 + 2x + 5 \\
 & 3x^3 + 6x^2 + 3x \\
 & (-) \quad (-) \quad (-) \\
\hline
 & -5x^2 - x + 5 \\
 & -5x^2 - 10x - 5 \\
 & (+) \quad (+) \quad (+) \\
\hline
 & 9x + 10
\end{array}
$$

$\therefore$ *Quotient* $= 3x - 5$ *and*

Remainder $= 9x + 10$

2. Check whether the polynomial $(t - 3)$ is a factor of $t^2 - 6t + 9$ by suitable method.

$$
\begin{array}{r|l}
 & t - 3 \\
\hline
t - 3 & t^2 - 6t + 9 \\
 & t^2 - 3t \\
 & (-) \quad (+) \\
\hline
 & -3t + 9 \\
 & -3t + 9 \\
 & (+) \quad (-) \\
\hline
 & 0
\end{array}
$$

Here the remainder =0

$\therefore (t - 3)$ is a factor of $t^2 - 6t + 9$.

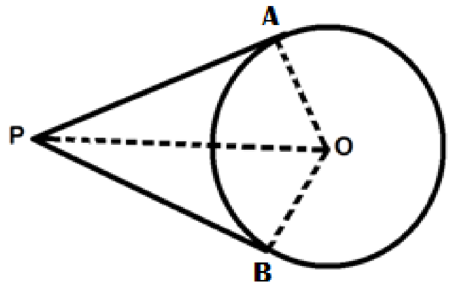
Problems for practice	
1) Divide $p(x) = x^3 - 3x^2 + 3x - 5$ by $g(x) = x^2 - x + 1$ and find the quotient and remainder.	2) Divide $p(x) = x^3 + 3x^2 + 3x + 1$ by $g(x) = x + 1$ and find the quotient and remainder.

10. CIRCLES

Theorems Marks:-3

A) Prove that "the lengths of tangents drawn from an external point to a circle are equal."

Given : 'O' is the centre of the circle, 'P' is an external point. AP and BP are the tangents



To Prove : **A**P = BP

Construction: Join OA, OB and OP.

Proof :

In ΔOQP and ΔORP

∠OAP = ∠OBP = 90^0 [Theorem 4.1]

OP = OP [Common side]

OA = OB [Radii of same circle]

$\Delta OAP \cong \Delta OBP$ [RHS Postulate]

AP=BP [CPCT]

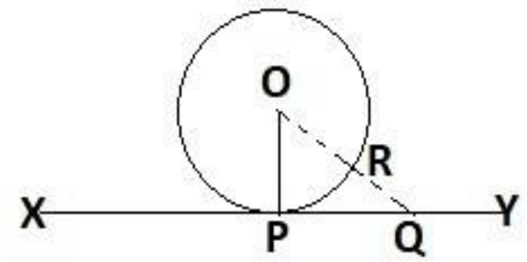
Hence proved.

B) Prove that " the tangent at any point of a circle is perpendicular to the radius through the point of contact."

Given : XY is the tangent at P to the circle with centre

To Prove : OP _|_ XY

Construction : Mark Any point 'Q' on XY, join OQ and it cuts the circle at R



Proof : OR < OQ

OR = OP (Radii of the same circle)

∴ OP < OQ

This holds good for all the points on XY

∴ OP is the least distance

=> OP _|_ XY

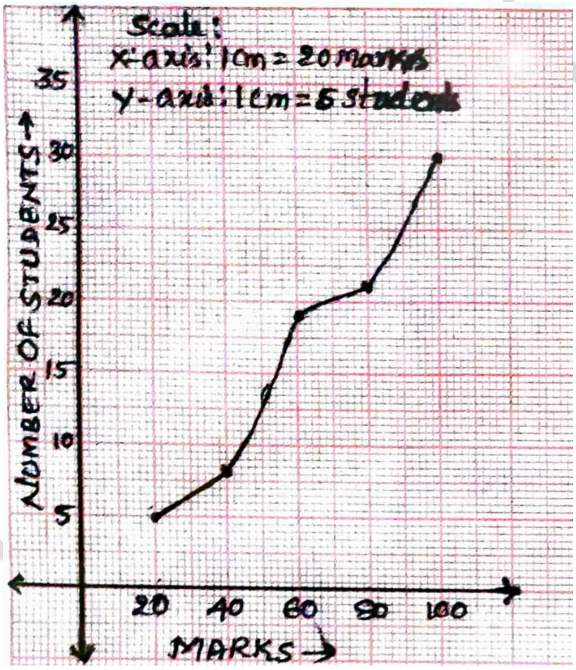
11. STATISTICS

Example	Marks:-3

1. The marks scored by 30 Students of class X, in the Mathematics are given below. Draw a less than type ogive.

Marks	0-20	20-40	40-60	60-80	80-100
Number of students	5	3	11	2	9

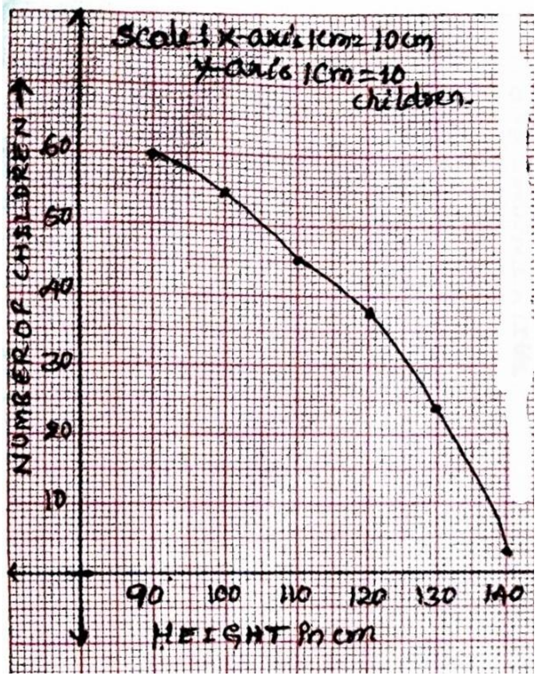
Marks	Number of students
Less than 20	5
Less than 40	5+3=8
Less than 60	8+11=19
Less than 80	19+2=21
Less than 100	21+9=30



6. Heights of 60 children are given below. Draw a more than type Ogive.

Height(in cm)	90-100	100-110	110-120	120-130	130-140	140-150
Number of children	5	10	7	24	11	3

Height(in cm)	Number of children
More than or equal to 90	60
More than or equal to 100	60 − 5 = 55
More than or equal to 110	55 − 10 = 45
More than or equal to 120	45 − 7 = 38
More than or equal to 130	38 − 24 = 14
More than or equal to 140	14 − 11 = 3



Problems for practice

1) During the medical check-up of 35 students of a class, their weights were recorded as follows. Draw a less than type Ogive for the given data.

Weights (in kg)	Number of students
Less than 40	3
Less than 45	5
Less than 50	9
Less than 55	14
Less than 60	28
Less than 65	32
Less than 70	35

2) Details of daily income of 50 workers in a food industry are given below. Draw a more than type Ogive for the following data.

Daily Income (in Rs.)	Number of workers
More than or equal to 80	50
More than or equal to100	38
More than or equal to 120	24
More than or equal to 140	16
More than or equal to 160	10
More than or equal to 180	0

3) The marks obtained by 40 students in Mathematics examination held in the school is given. Draw the less than type of Ogive.

Marks obtained	No of students
Less than 10	2
Less than 20	5
Less than 30	8
Less than 40	12
Less than 50	15
Less than 60	25
Less than 70	35
Less than 80	40

4) The yield of rice in the fields in a village is given. Draw more than type of Ogive to the data.

Yield (tons)	Number of farmers
>8	80
>12	60
>16	55
>20	45
>24	40
>28	35
>32	20
>36	15
>40	10

Example Marks:-3

1) Find mean for the following frequency distribution.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	3	5	9	5	3

Class Interval	**Frequency**	$\boldsymbol{x}$	$\boldsymbol{fx}$
0-10	3	5	15
10-20	5	15	75
20-30	9	25	225
30-40	5	35	175
40-50	3	45	135
	$\boldsymbol{\Sigma f = 25}$		$\boldsymbol{\Sigma fx = 625}$

$$Mean = \frac{\Sigma fx}{\Sigma f} = \frac{625}{25}$$

$\therefore$ **Mean = 25**

2) Find the Median of the following frequency distribution.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	7	13	9	3

Class Interval	Frequency	Cumulative Frequency
0-10	4	4
10-20	7	4+7=**11**
20-30	**13**	11+13=24
30-40	9	24+9=33
40-50	3	33+3=36

$n = 36, \quad \frac{n}{2} = 18, \quad f = 13, \quad cf = 11,$

$h = 10, \quad l = 20$

$$Median = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$Median = 20 + \left[\frac{18 - 11}{13}\right] \times 10$$

$$Median = 20 + 5.38$$

$$\therefore \boldsymbol{Median = 25.38}$$

Problems for practice

1) Find the average or mean of the given data.

Class Interval	Frequency
10-20	2
20-30	3
30-40	7
40-50	8
50-60	5
	Σf =25

2) Find the median for the following data.

Class Interval	Frequency
50 - 60	12
60 - 70	14
70 - 80	8
80 - 90	6
90 - 100	10

3) Find the average or mean of the given data.

Class Interval	Frequency
1 - 5	2
6 - 10	3
11 - 15	5
16 - 20	3
21 - 25	2
	Σf =15

4) Find the median for the following data.

Class Interval	Frequency
20 - 40	7
40 - 60	15
60 - 80	20
80 - 100	8

3. Find the mode of the following frequency distribution.

Class interval	*Frequency*
30-40	4
40-50	7
50-60	9
60-70	11
70-80	6
80-90	2

$f_1 = 11, \ f_0 = 9, \ f_2 = 6, \ l = 60, \ h = 10$

$$Mode = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$Mode = 60 + \left[\frac{11-9}{2(11)-9-6}\right] \times 10$$

$$Mode = 60 + 2.86$$

$$\therefore \boldsymbol{Mode = 62.86}$$

1) Find the mode of the following data.

Class interval	*Frequency*
1 – 3	6
3 – 5	9
5 – 7	15
7 – 9	9
9 - 11	1

2) Find the mode of the following data.

Class interval	*Frequency*
10 – 25	2
25 – 40	3
40 – 55	7
55 – 70	6
70 – 85	6
85 – 100	6

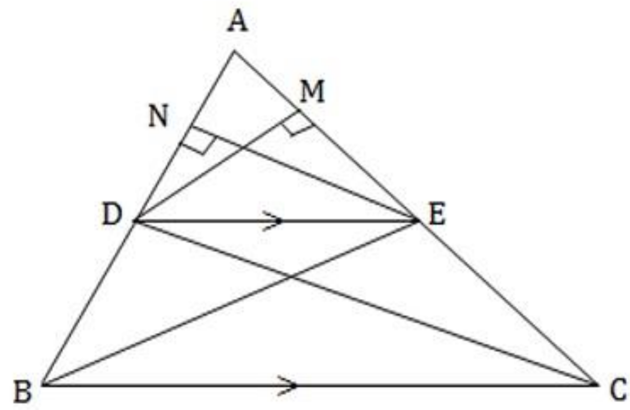
3) Find the mode of the following data.

Class interval	*Frequency*
1 – 3	7
3 – 5	8
5 – 7	2
7 – 9	2
9 - 11	1

13. THEOREMS

1. State and prove the Basic proportionality (Thales') theorem.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points , the other two sides are divided in the same ratio.



Data : In ΔABC $DE \parallel BC$.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw $DM \perp AC$ and $EN \perp AB$. Join BE and CD .

Proof :

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad \text{---------->} (1)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{AE}{EC} \quad \text{---------->} (2)$$

But ΔBDE and ΔCED are standing on the same base DE and between $DE \parallel BC$.

$ar(\Delta BDE) = ar(\Delta CED)$ ----------> (3)

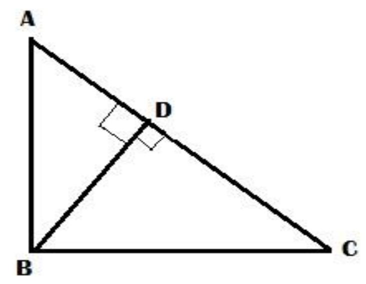
$\therefore$ from equations (1), (2) and (3)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence the proof.

2. State and prove the Pythagoras theorem.

" In a right angled triangle , the square on the hypotenuse is equal to the sum of the squares on other two sides " .



Data : ΔABC is a right triangle and $\angle B = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In ΔADB and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle A = \angle A$ ($\because$ Common angle)

$\Delta ADB \sim \Delta ABC$ ($\because$ AAA Similarity Criterion)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ ($\because$ Proportional sides)

$AC. AD = AB^2$ ------> (1)

Similarly

In ΔBDC and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle C = \angle C$ ($\because$ Common angle)

$\Delta BDC \sim \Delta ABC$ ($\because$ AAA Similarity Criterion)

$\therefore \frac{DC}{BC} = \frac{BC}{AC}$ ($\because$ Proportional sides)

$AC. DC = BC^2$ ------> (2)

$AC . AD + AC. DC = AB^2 + BC^2$ [$\because$ By adding (1) and (2)]

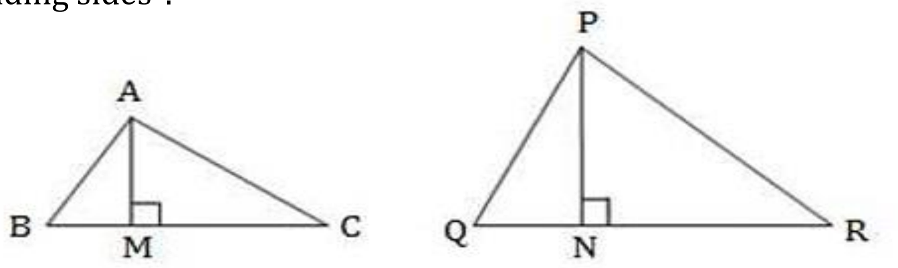
$AC (AD + DC) = AB^2 + BC^2$

$AC \times AC = AB^2 + BC^2$ ($\because$ from fig. $AD + DC = AC$)

$AC^2 = AB^2 + BC^2$

Hence the proof.

3. Prove that "The ratio of the areas of two similar triangles is equal to the ratio of their corresponding sides".



Data : $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction : Draw $AM \perp BC$ and $PN \perp QR$.

Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ $(\because$ Area of $\Delta = \frac{1}{2} \times$ base $\times$ height $)$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AM}{QR \times PN} \text{ ---------->} (1)$$

In ΔABM and ΔPQN

$\angle B = \angle Q$ $(\because \Delta ABC \sim \Delta PQR)$

$\angle M = \angle N = 90^\circ$ $(\because$Construction$)$

$\therefore \Delta ABM \sim \Delta PQN$ $(\because$AA Similarity criterion$)$

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ -------->(2)

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ------> (3) $(\because$Data$)$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ $(\because$substituting eqs.(2) and (3) in (1) $)$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

Now from eq.(3)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Hence the proof.

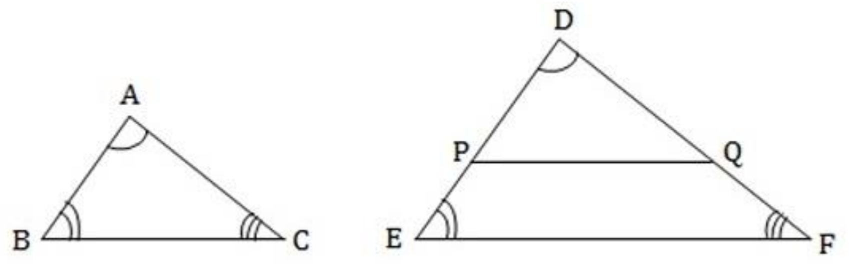
4. Prove that "If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (proportion) and hence the two triangles are similar".

Data: $In\ \Delta ABC\ and\ \Delta DEF$

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$



To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction: Mark points P and Q on DE and DF such that DP=AB and DQ=AC. Join PQ.

Proof: $In\ \Delta ABC\ and\ \Delta DPQ$

$\angle A = \angle D$ [Data]

AB=DP [Construction]

AC=DQ [Construction]

$\therefore \Delta ABC \cong \Delta DPQ$ [SAS postulate]

BC=PQ [By CPCT] -----(1)

$\angle B = \angle P$ [By CPCT]

$\angle B = \angle E$ [Data]

$\angle P = \angle E$ [Axiom 1]

$PQ || EF$

$\frac{DP}{DE} = \frac{PQ}{EF} = \frac{DQ}{DF}$ [Corollary of BPT]

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [From (1) and construction]

$\therefore$ **Hence the proof.**

IMPARTANT DEFINITIONS, STATEMENTS & FORMULAS

1) State 'Basic proportionality theorem' (Thale's Theorem)

The line drawn parallel to any one side of the triangle divides the other two sides proportionally.

2) State 'Pythagoras theorem'.

In right angled triangle the square of the hypotenuse is equal to sum of the squares of the other two sides.

3) State Euclid's Division Lemma.

Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r,\ 0 \leq r < b$.

4) State ‘Fundamental theorem of arithmetic.

Every composite number can be expressed as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

5) Define Arithmetic Progression.

Arithmetic Progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term, except the first term.

6) Define tangent of a circle.

The straight line that intersects the circle at only one point is called a **tangent**.

7) Define secant of a circle.

The straight line that intersects the circle at two points is said to be **secant** to the circle.

PROBABILITY	
Probability of occurrence of an event ‘A’	$P(A) = \frac{Number\ of\ outcomes\ favourable\ to\ A}{Number\ of\ all\ possible\ outcomes}$ i.e., $P(A) = \frac{n(A)}{n(S)}$
1) Probability of a **Sure Event** is **One**	2) Probability of an **Impossible Event** is **Zero**
3) Sum of the probabilities of all primary events of an experiment is **One**	4) $P(E) + P(\bar{E}) = 1$

POLYNOMIALS

Sl. No	Polynomials	Standard form	Degree
1	Linear polynomial	ax + b	1
2	Quadratic polynomial	$ax^2 + bx + c = 0$	2
3	Cubic polynomial	$ax^3 + bx^2 + cx + d = 0$	3

TRIGONOMETRY

Values of Trigonometric ratios of Standard Angles.						**Trigonometric Ratios of Complementary Angles**
Angles → Ratios ↓	**0^o**	**30^o**	**45^o**	**60^o**	**90^o**	
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$sin\ (90^O - A) = cos\ A$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$cos\ (90^O - A) = sin\ A$
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	$tan\ (90^O - A) = cot\ A$
cosec	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$cosec\ (90^O - A) = sec\ A$
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	$sec\ (90^O - A) = cosec\ A$
cot	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$cot\ (90^O - A) = tan\ A$

STATISTICS	
Direct method	$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ OR $\bar{x} = \frac{\sum fx}{N}$
Assumed Mean method	$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$
Step-deviation method	$\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$
Mode of the Grouped data	$l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \text{x } h$
Median of the Grouped data	$l + \left(\frac{\frac{n}{2} - cf}{f}\right) \text{x } h$
Empirical relationship between the three measures of central tendency	3 Median = Mode + 2 Mean

SURFACE AREA AND VOLUME OF SOLIDS

Name of the Solid	**C.S.A**	**T.S.A**	**Volume**
Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Cone	πrl	$\pi r(r+l)$ $l = \sqrt{r^2 + h^2}$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone	$\pi(r_1 + r_2)l$	$\pi[l(r_1 + r_2) + r_1^2 + r_2^2]$ $l = \sqrt{(r_1 - r_2)^2 + h^2}$	$\frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$