ಬೆಂಗಳೂರು ನಗರ ಜಿಲ್ಲಾ ಪಂಚಾಯತ್

(ಸಾರ್ವಜನಿಕ ಶಿಕ್ಷಣ ಇಲಾಖೆ)

ಉಪನಿರ್ದೇಶಕರ ಕಚೇರಿ, ಸಾರ್ವಜನಿಕ ಶಿಕ್ಷಣ ಇಲಾಖೆ
ಬೆಂಗಳೂರು ದಕ್ಷಿಣ ಜಿಲ್ಲೆ, ಕಲಾಸಿಪಾಳ್ಯ, ಬೆಂಗಳೂರು-2

2021-22 ನೇ ಸಾಲಿನ ***S.S.L.C,***ಫಲಿತಾಂಶ ಉತ್ತಮ ಪಡಿಸಲು ಗಣಿತ ವಿಷಯದ

"ಪರೀಕ್ಷಾ ಮಾರ್ಗದರ್ಶಿ"

ಸಲಹೆ ಮತ್ತು ಮಾರ್ಗದರ್ಶನ

ಶ್ರೀ. ಪಿ. ಬೈಲಾಂಜನಪ್ಪ
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ಶ್ರೀಮತಿ ಗಿರಿಜಮ್ಮ. *H.N.*
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ಉಪನಿರ್ದೇಶಕರ ಕಚೇರಿ ,ಸಾರ್ವಜನಿಕ ಶಿಕ್ಷಣ ಇಲಾಖೆಬೆಂಗಳೂರು ದಕ್ಷಿಣ ಜಿಲ್ಲೆ.

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ರಚನೆ

ಚರಣ್ ಸಿಂಗ್. ಸಿ.
ಸಹಶಿಕ್ಷಕರು, ಸರ್ಕಾರಿ ಪ್ರೌಢಶಾಲೆ.
ರಾಮಮೂರ್ತಿನಗರ ,ಬೆಂಗಳೂರು ದಕ್ಷಿಣ ವಲಯ *4.*

Board model paper chapter wise analysis

Topic	mcq	1m	2m	3m	4m	5m	marks
Arithmetic progression	1	1	2	1	-	-	9
Triangles	1	1	-	-	-	1	7
Circles	-	-	-	1	-	-	3
Pair of linear equations	1	1	1	-	1	-	8
Areas related to circles	-	1	-	1	-	-	4
constructions	-	-	1	1	1	-	9
Co-ordinate geometry	1	1	1	1	-	-	7
Quadratic equations	1	-	2	-	1	-	9
Trigonometry	1	1	1	1	-	-	7
Applications of trigonometry	-	-	-	-	1	-	4
statistics	1	-	-	2	-	-	7
Surface areas and volumes	1	2	-	1	-	-	6
Total	8	8	8	9	4	1	80

Unit: <u>Arithmetic progression</u>

……

<u>Concept 1 :</u> Finding a particular term of an AP.

1.If n^{th} term of an AP is 2n+3 then Find the value of 10^{th} term.

Solution: a_n***=2n+3***

n=10

a_{10}***=2(10)+3***

=20+3

a_{10} ***=23.***

2..If n^{th} term of an AP is n^2+2 then Find the value of 6^{th} term.

Solution: a_n***=*** n^2***+2***

n=6

a_6=6^2***+2***

=36+2

a_6***=38.***

<u>Problems for practice</u>

1. ***If n^{th} term of an AP is 2n-3 then Find the value of 8^{th} term.***
1. ***If n^{th} term of an AP is $2n^2$+2 then Find the value of 9^{th} term.***
2. ***If n^{th} term of an AP is 3n+4 then Find the value of 4^{th} term.***
3. ***.If n^{th} term of an AP is n^2-2 then Find the value of 5^{th} term.***

<u>Concept 2.</u> Finding the first four terms of an AP.

1. ***If the n^{th} term of an AP is 2n-1 then Find the first four terms of this AP.***

   Solution; a_n***=2n-3***

   n=1, then a_1***=2(1)-1=2-1=1***

   n=2, then a_2***=2(2)-1=4-1=3.***

   n=3, then a_3***=2(3)-1=6-1=5***

   n=4, then a_4***=2(4)-1=8-1=7.***

   The first four terms of the given AP are 1,3,5,7.

2. ***If n^{th} term of an AP is n^2+2 then,then Find the first four terms of this AP.***

   Solution; a_n=$2n^2$***+2,*** ***n=1, then*** a_1=$(1)^2$***+2=1+2=3.*** ***n=2, then*** a_2=$(2)^2$***+2=4+2=6.***

   n=3, then a_3=$(3)^2$***+2=9+2=11.*** ***n=4, then*** a_4=$(4)^2$***+2=16+2=18.***

   The first four terms of the given AP are 3,6,18.

Problems for practice; **1.If n^{th} term of an AP is 2n+1 then First four terms of this AP.**

2. If n^{th} term of an AP is n^2+1 then Find four terms of this AP.

Concept 3: *Finding the term n^{th} term of an AP.*

1. 2,5,8,......Find the 5^{th} term Of this AP.

Solution; Here a=2, d=3, n=5, a_5=

a_n=a+(n-1)d

a_5= 2+(5-1)3

= 2+4x3

= 2+12

a_5 = 14.

2. 100,90,80,.......Find the 25^{th} term Of this AP.

Solution; Here a=100, d = -10 , n=25, a_{25}= ?

a_n=a+(n-1)d

a_{25}= 100+(25-1)(-10)

= 100+24x(-10)

= 100-240

a_{25} = - 140.

Problems for practice

1. 1,4,7,......Find the 10^{th} term Of this AP.
2. 2,6,10,......Find the 50^{th} term Of this AP.
3. 20,16,12,......Find the 20^{th} term Of this AP.
4. 50,45,40,......Find the 10^{th} term Of this AP.

Concept 4 : *Finding Sum of first n terms of an AP.*

1. **2,6,10,...... Find the sum of the first 10 terms of this AP.**

Solution; Here a=2, d = 4 , n=10, S_{10}= ?

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(2) + (10-1)4]$

$= 5\ [4 + (9)4]$

$= 5\ [4 + 36]$

= 5 x40

S_{10} = 200.

2. **1,4,7,10,...... Find the sum of the first 25 terms of this AP.**

Solution; Here a=1, d = 3 , n=25, S_{25}= ?

$S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{25} = \frac{25}{2}[2(1) + (25-1)3]$

$= \frac{25}{2}[2 + (24)3]$

$= \frac{25}{2}[2 + 72] = \frac{25}{2}[74]$

= 25 X 37

=925 .

Problems for practice

1. 2,5,8,...... Find the sum of the first 20 terms of this AP.
2. 3,6,9,,...... Find the sum of the first 35 terms of this AP.

Unit; Statistics.

....

Concept 1. Finding the Mean by direct method.

Class interval	5-15	15-25	25-35	35-45	45-55
frequency	4	3	6	5	2

CI	f	x	fx	
5-15	4	10	40	
15-25	3	20	60	
25-35	6	30	180	
35-45	5	40	200	
45-55	2	50	100	
	Σf =20		Σfx=580	

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{580}{20}$$

Mean =29

Problems for practice :

1. *Find the Mean by direct method*

Class interval	5-15	15-25	25-35	35-45	45-55
frequency	4	3	6	5	2

2.Find the Mean by direct method

Class interval	1-5	6-10	11-15	16-20	21-25
frequency	5	6	4	3	2

Concept 2. Mode

Find the mode for the following data.

Class interval	*1-3*	*3-5*	*5-7*	*7-9*	*9-11*
frequency	*7*	*8*	*2*	*2*	*1*

CI	*f*	
1-3	*7*	f_o
3-5	***8***	f_1 *Modal class*
5-7	*2*	f_2
7-9	*2*	
9-11	*1*	

Here $l=3$, $f_o=7$, $f_1=8$, $f_2=2$, $h=2$.

Mode $= l + \left[\frac{f_1 - f_o}{2f_1 - f_o - f_2}\right] \times h$

$$= 3 + \left[\frac{8-7}{2(8)-7-2}\right] \times 2$$

$$= 3 + \left[\frac{1}{16-7-2}\right] \times 2 = 3 + \left[\frac{1}{16-9}\right] \times 2 = 3 + \left[\frac{1}{7}\right] \times 2 = 3 + \frac{2}{7}$$

$$= 3.286$$

Mode= 3.286

Problems for practice

Class interval	*5-15*	*15-25*	*25-35*	*35-45*	*45-55*	*55-65*
frequency	*6*	*11*	*21*	*23*	*14*	*5*

Concept 3. Median

Find the median for the following class interval.

CI	10-25	25-40	40-55	55-70	70-85	85-100
f	2	3	7	6	6	6

CI	f	Cf
10-25	2	2
25-40	3	2+3=5
40-55	7	5+7=12, cf=12
l=55-70	6=f	12+6=18 $\frac{n}{2}$=15
70-85	6	18+6=24
85-100	6	24+6=30
	n=30	

Here $\frac{n}{2}=\frac{30}{2}=15$, $h=15$

$$\text{Median}= l+\left[\frac{\frac{n}{2}-cf}{f}\right] xh.$$

$$=55+\left[\frac{15-12}{6}\right] x15 =55+\left[\frac{3}{6}\right]x15 =55+\frac{45}{6}= 55+7.5=62.5$$

Median =62.5

Problems for practice

1. Find the median for the following data.

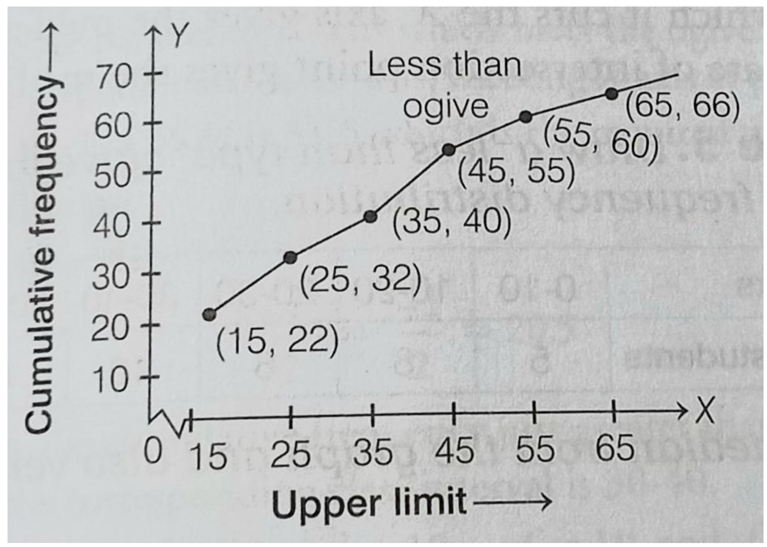
CI	65-85	85-105	105-125	125-145	145-165	165-185	185-205
F	4	5	13	20	14	8	4

Concept 4. Less than type ogive

Prepare a less than type ogive graph for the following data.

CI	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**
f	**22**	**10**	**8**	**15**	**5**	**6**

CI	f	cf
5-15	22	22
15-25	10	22+10=32
25-35	8	32+8=40
35-45	15	40+15=55
45-55	5	55+5=60
55-65	6	60+6=66



Problems for practice

Prepare a less than type ogive graph for the following data

Class interval	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**
frequency	**6**	**11**	**21**	**23**	**14**	**5**

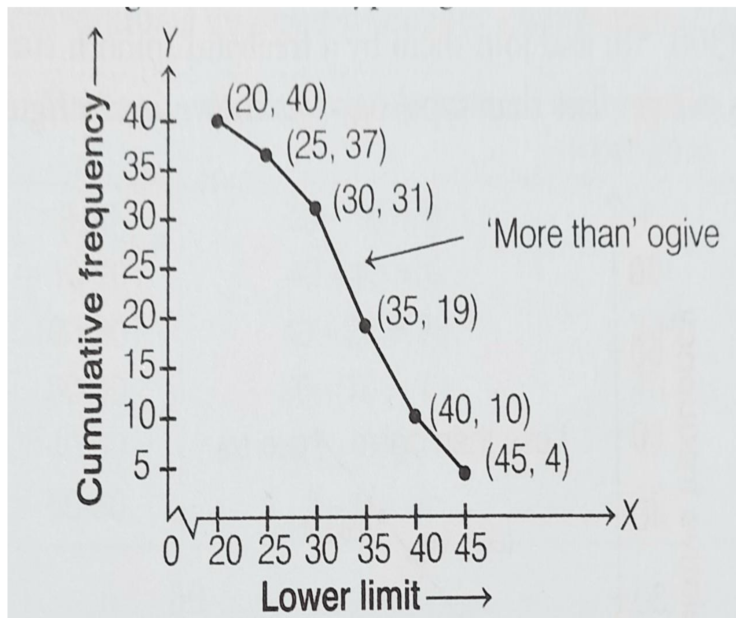
CI	**65-85**	**85-105**	**105-125**	**125-145**	**145-165**	**165-185**	**185-205**
F	**4**	**5**	**13**	**20**	**14**	**8**	**4**

Concept 5. ***More than type ogive***

Prepare a more than type ogive graph for the following data.

CI	***20-25***	***25-30***	***30-35***	***35-40***	***40-45***	***45-50***
f	***3***	***6***	***12***	***9***	***6***	***4***

CI	***f***	***Cf***
20-25	***3***	***40***
25-30	***6***	***40-3=37***
30-35	***12***	***37-6=31***
35-40	***9***	***31-12=19***
40-45	***6***	***19-9=10***
45-50	***4***	***10-6=4***
	Σf ***=40***	



Problems for practice

Prepare a more than type ogive graph for the following data

CI	***10-25***	***25-40***	***40-55***	***55-70***	***70-85***	***85-100***
f	***2***	***3***	***7***	***6***	***6***	***6***

CI	***5-15***	***15-25***	***25-35***	***35-45***	***45-55***	***55-65***
f	***22***	***10***	***8***	***15***	***5***	***6***

Unit; Quadratic equation

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The general form of quadratic equation is $ax^2+bx+c=0$

The discriminant of of quadratic equation is $\Delta = b^2-4ac$

Quadratic formula is $x = \frac{-(b)\pm\sqrt{b^2-4ac}}{2a}$

Nature of roots :

Discriminant	*Nature*
$b^2-4ac>0$	*Roots are real and distinct*
$b^2-4ac=0$	*The roots are real and equal*
$b^2-4ac<0$	*There is no real roots* *(Imaginary roots)*

Concept one ; Solving quadratic equation by factorization method

Solve $x^2-5x+6=0$ ***By factorization method.***

$x^2-5x+6=0$

$x^2-3x-2x+2(3)=0$

$x(x-3)-2(x-3)=0$

$(x-3)(x-2)=0$

$x-3=0 \quad x-2=0$

$x=3 \quad x=2$

similarly

$x^2+5x+6=0$

$x^2+3x+2x+2(3)=0$

$x(x+3)+2(x+3)=0$

$(x+3)(x+2)=0$

$x+3=0 \quad x+2=0$

$x= -3 \quad x= -2$

Problems for practice

Solve; $x^2-7x+10=0$

$x^2+7x+12=0$

$x^2-5x+6=0$

Concept 2 ***Solving quadratic equation by formula method***

Solve x^2-5x+6=0 by formula method
Solution;

x^2***-5x+6=0***

Here a=1, b= -5 , c=6

$$x = \frac{-(b)\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$$

$$= \frac{5\pm\sqrt{25-24}}{2}$$

$$= \frac{5\pm\sqrt{1}}{2}$$

$$= \frac{5+1}{2} \qquad x = \frac{5-1}{2}$$

$$= \frac{6}{2} \qquad = \frac{4}{2}$$

$$x=3 \qquad x=2$$

x^2***-3x-10=0 by formula method***

Here a=1, b=-3, c= -10.

$$x = \frac{-(b)\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-10)}}{2(1)}$$

$$= \frac{3\pm\sqrt{9+40}}{2}$$

$$= \frac{3\pm\sqrt{49}}{2}$$

$$= \frac{3\pm 7}{2}$$

$$= \frac{3+7}{2} \qquad = \frac{3-7}{2}$$

$$= \frac{10}{2} \qquad = \frac{-4}{2}$$

$$x = 5 \qquad x = -2$$

Problems for practice:

x^2***-7x+12=0 by formula method***

$2x^2$***-6x+8=0 by formula method***

Concept 3 ***Nature of roots***

Discriminant of quadratic equation $\Delta = b^2 - 4ac$

Discriminant	**Nature**
$b^2-4ac>0$	***Roots are real and distinct***
$b^2-4ac=0$	***The roots are real and equal***
$b^2-4ac<0$	***There is no real roots (Imaginary roots)***

Find the nature of the roots of the equation $x^2-4x+4=0$

Solution: here a =1, b=-4, c=1

$$\Delta = b^2-4ac$$
$$= (-4)^2-4(1)(4)$$
$$= 16-16$$
$$= 0 = 0$$

$\Delta = 0$ ***,Roots are real and equal***

Find the nature of the roots of the equation $3x^2+5x-2=0$

Solution: here a =3, b=-5, c= -2

$$\Delta = b^2-4ac$$
$$=(5)^2- 4(3)(-2)$$
$$= 25+24$$
$$= 49 > 0$$

$\Delta > 0$ ***,Roots are real and distinct***

Find the nature of the roots of the equation $7x^2-4x+5=0$

Solution: here a =7, b=-4, c= 5

$$\Delta = b^2-4ac$$
$$=(-4)^2- 4(7)(5)$$
$$= 16-140$$
$$= -124 < 0$$

$\Delta < 0$, ***There is no real roots (Imaginary roots)***

Problems for practice **:** ***Find the nature of the roots of the equation***

$4x^2$***+12x+9=0***

x^2***-8x+15=0***

Find the value of k for which the equation $9x^2$***+3kx+4=0 has equal roots***

Solution: here a =9, b=-3k, c= 4

b^2***-4ac=0***

$(3k)^2$***-4(9)(4)=0***

$9k^2$***-144=0***

$9k^2$***=144***

$k^2 = \frac{144}{9} = 16$

k=$\sqrt{16}$

k=4

Problems for practice **:**

Find the value of k for which the equation has equal

Roots (i) $3x^2$***+kx+3=0***

(ii) x^2***+4x+k=0***

Unit; Co-ordinate geometry

…………………………………………………………………………

Concept 1; Distance formula

(a). Distance between origin and a given point.

d= $\sqrt{x^2 + y^2}$

Find the distance of the point A(3,4) from the origin.

Solution; A(3,4) $\Rightarrow$ ***x=3, y=4***

d= $\sqrt{x^2 + y^2}$

d= $\sqrt{3^2 + 4^2}$

d= $\sqrt{9 + 16}$

d= $\sqrt{25}$

d=5.

Problems for practice;

Find the distance of the point A(3,-4) from the origin.
Find the distance of the point A(-6,-8) from the origin.

(b) Distance between two points

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between the points (6,7) and (1,5).
Solution: (6,7) and (1,5).

x_1**=6 ,** x_2**=1**

y_1**=7 ,** y_2**=5**

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\Rightarrow$ **d=** $\sqrt{(1 - 6)^2 + (5 - 7)^2}$

d= $\sqrt{(-5)^2 + (-2)^2}$

d= $\sqrt{25 + 4}$

d= $\sqrt{29}$

Problems for practice

Find the distance between the points (7,3) and (5,11).
Find the distance between the points (7,1) and (3,5).

Concept 2 ***Section formula ;***

$$p(x,y)= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Find the coordinates of the point divide the line segment joining the points (4,-3) and (9, 7) Internally in the ratio 3:2.

Solution; x_1=4, x_2=9

y_1=3 , y_2=7 ,

m=3, n=2

$$p(x,y)= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$p(x,y)= \left(\frac{3(9)+2(4)}{3+2}, \frac{3(7)+2(-3)}{3+2}\right)$$

$$p(x,y)= \left(\frac{27+8}{5}, \frac{21-6}{5}\right)$$

$$p(x,y)= \left(\frac{35}{5}, \frac{15}{5}\right)$$

p(x,y)= (7,3).

Problems for practice

1.Find the coordinates of the point divide the line segment joining the points (3,4) and (-5,-7) Internally in the ratio 4:5.

2.Find the coordinates of the point divide the line segment joining the points (3,-1) and (2,6) Internally in the ratio 2:3.

Concept 3 ***Midpoint formula***

$$p(x,y)= \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$$

Find the coordinates of the midpoint of the line segment joining the points (7,4) and (1,6) .

Solution; x_1=7, x_2=1, y_1= 4 , y_2=6

$$p(x,y)= \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right) = \left(\frac{1+7}{2}, \frac{6+4}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4,5).$$

Problems for practice;

Find the coordinates of the midpoint of the line segment joining the points (4,6) and (8,4) .
Find the coordinates of the midpoint of the line segment joining the points (5,3) and (3,1) .

Concept 4 : Area of triangle

A= $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$.

Find the area of the triangle with vertices (3,0), (7,0) and (8,4).
Solution:

A= $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

A= $\frac{1}{2}[3(0 - 4) + 7(4 - 0) + 8(0 - 0)]$

A= $\frac{1}{2}[3(-4) + 7(4) + 8(0)]$

A= $\frac{1}{2}[-12 + 28 + 0]$

A= $\frac{1}{2}[16]$

A= 8 sq units.

Problems for practice;

Find the area of the triangle with vertices (5,7), (4,5) and (1,6).
Find the area of the triangle with vertices (7,-3), (12,2) and (7,21).

Note : ***Prove that the points (2,-2), (-3,8) and (-1,4) are collinear***

Solution; **A=** $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

A= $\frac{1}{2}[2(8 - 4) + (-3)(4 - (-2)) + (-1)(-2 - 8)]$

A= $\frac{1}{2}[2(4) - 3(6) - 1(-10)]$

A= $\frac{1}{2}[8 - 18 + 10]$

A= $\frac{1}{2}[0]$ ***=0*** ***Since the area of triangle is zero, therefore all the points are collinear.***

Unit ;   Pair of linear equations in two variables

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Nature of lines and Consistency

The nature of lines and consistency corresponding to linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, is shown in the table given below:

Compare the ratios	Graphical representation	Algebraic interpretation	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)	System is consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent (dependent)
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

Concept 1 Elimination method ;

solve 6x+y=20 and 3x-y=7 .

Solution ;

6x+y=20
3x-y = 7
……………
9x =27

$x=\frac{27}{9}$

x=3

6x+y=20
6(3)+y=20
18+y=20
y=20-18
y= 2

Solve 6x+2y=28 and 2x+y=10

Solution ; 6x+2y =28 ……(1)

2x+y =10 …….(2) x2 ⇒ 4x+2y=20……(3)

(1)-(3)⇒ 6x+2y =28
4x+2y=20
(-) (-) (-)
…………………….

2x =8 ⇒ $x=\frac{8}{2}$ ***⇒ x=4***

6x+2y=28
6(4)+2y=28 ⇒ 24+2y=28
2y=28-24 ⇒ 2y=4 ⇒ y=2.

Problems for practice;

Solve i. $8x+5y=11$ and $x+y=4$.

ii . $x-y=3$ and $3x-2y=10$.

Concept 2 Graphical method of solving linear pair of equation

Solve $2x+y=10$ and $x+y=6$ graphically.

$2x+y=10$

x	0	5
y	10	0

$x=0,\ 2(0)+y=10$

$y=10$

$y=o,\ 2x+0=10$

$2x=10$

$x=5$

………………………………..

$x+y=6$

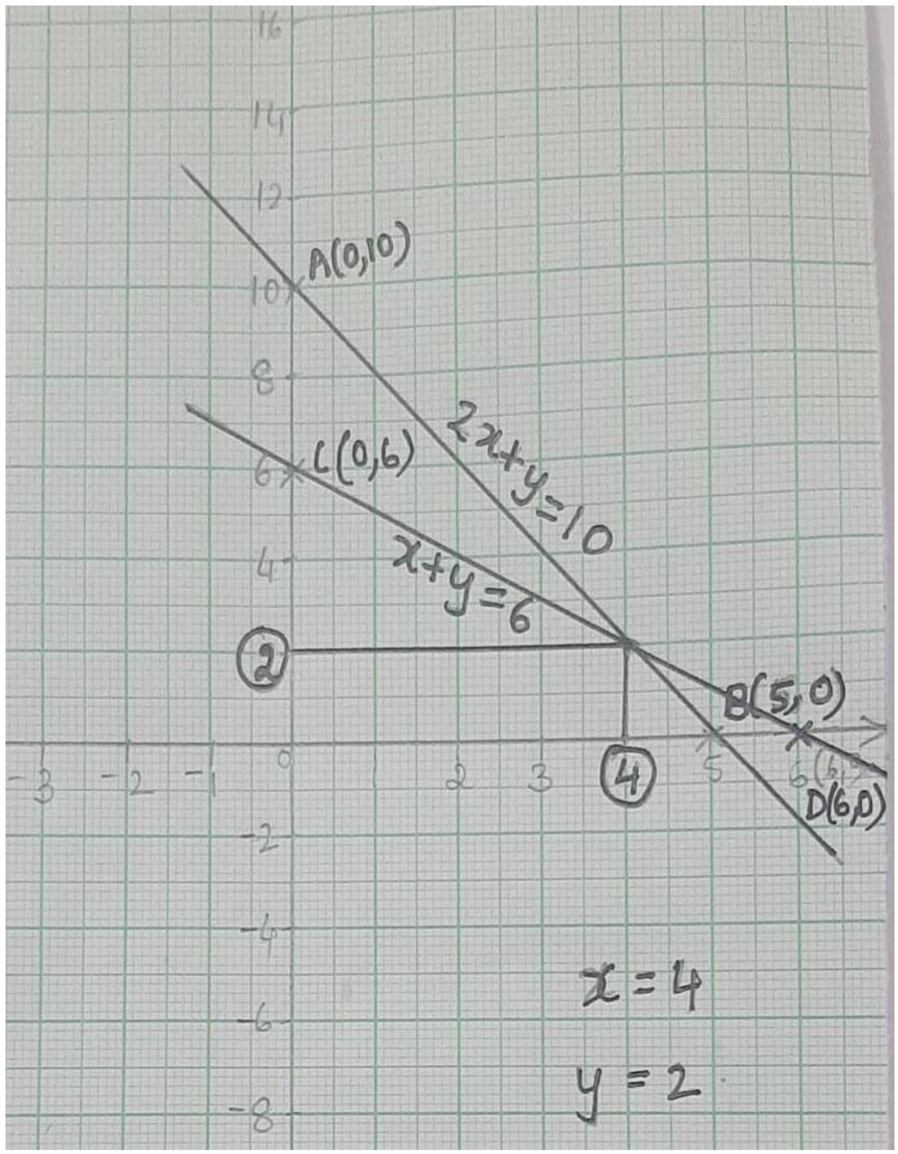
x	0	6
y	6	0

$x=0,\ 0+y=6$

$y=6$

$y=0,\ x+0=6$

$x=6$



Solve x+y=5 and x-y=1 graphically

x+y=5

x	*0*	*5*
y	*5*	*0*

x=0, 0+y=5
y=5

y=o, x+0=5
x=5

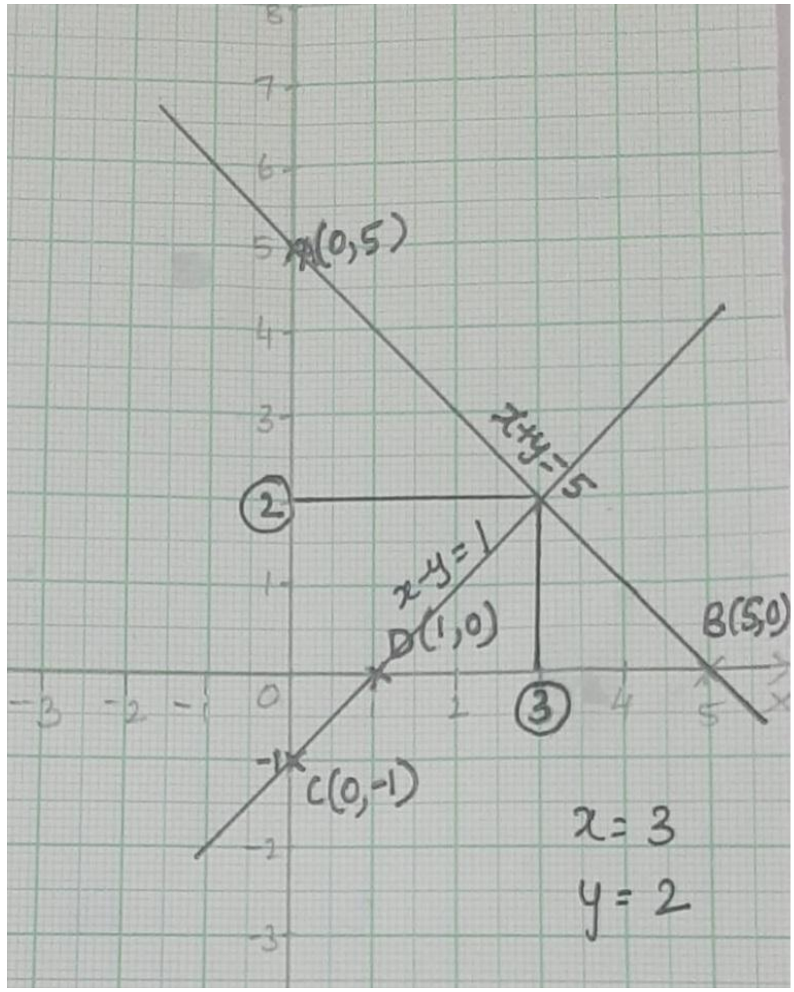
……………………………………

x-y=1

x	*0*	*1*
y	*-1*	*0*

x=0, 0-y=1
-y= 1
y=-1

y=0, x-0=1
x=1



<u>Problems for practice</u>

Solve ***i. 2x+y=8 and x-y=1***
Ii. 2x-y=2 and 4x-y=4
Iii. x+y=7 and 3x-y=1
Iv. 2x+y=8 and x-y=1

Unit : Trigonometry

……………………………………………………………………………………

TRIGONOMETRIC RATIOS

1. **sin**Θ = $\frac{opposite\ side}{hypotenuse}$
2. **cos**Θ = $\frac{adjacent\ side}{hypotenuse}$
3. **tan**Θ = $\frac{opposite\ side}{adjacent\ side}$
4. **cosec**Θ = $\frac{hypotenuse}{opposite\ side}$
5. **sec**Θ = $\frac{hypotenuse}{adjacent\ \ side}$
6. **cot**Θ = $\frac{adjacent\ side}{opposite\ side}$

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES.

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\text{cosec}\ \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TRIGONOMETRIC RATIOS OF C0MPLEMENTARY ANGLES.

sin(90^0**-A) =cosA** **cos(**90^0**-A) =sinA**

tan(90^0**-A) =cotA** **cot(**90^0**-A) =tanA**

sec(90^0**-A) =cosecA** **cosec(**90^0**-A) =secA**

TRIGONOMETRIC IDENTITIES:

$\sin^2 A + \cos^2 A = 1$

$\sec^2 A - \tan^2 A = 1$

$\mathrm{cosec}^2 A - 1 = \cot^2 A$

RECIPROCAL RATIOS :

sinA= $\frac{1}{\mathrm{cosec}A}$	**cosecA=** $\frac{1}{\sin A}$
cosA= $\frac{1}{\sec A}$	**secA=** $\frac{1}{\cos A}$
tanA= $\frac{1}{\cot A}$	**cotA=** $\frac{1}{\tan A}$
tanA= $\frac{\sin A}{\cos A}$	**cotA=** $\frac{\cos A}{\sin A}$

Given tanA= $\frac{4}{3}$ ***then find the other trigonometric ratios of the angle A.***

Solution ;

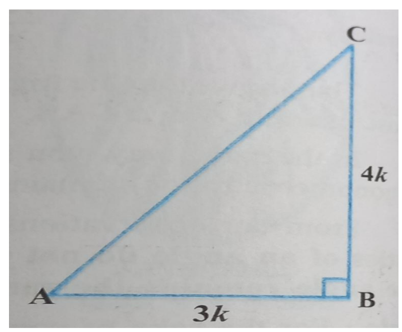
In the fig tanA= $\frac{4}{3} = \frac{opp}{adj}$

Using pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 3^2 + 4^2 = 9 + 16 = 25$

AC= $\sqrt{25} = 5$



sinA= $\frac{opp}{hyp} = \frac{4}{5} \Rightarrow$ ***cosecA=*** $\frac{5}{4}$

cosA= $\frac{adj}{hyp} = \frac{3}{5} \Rightarrow$ ***secA=*** $\frac{5}{3}$

tanA= $\frac{opp}{adj} = \frac{4}{3} \Rightarrow$ ***cotA=*** $\frac{3}{4}$

Find the value of $\frac{\sin 65^0}{\mathrm{cosec}25^0} \Rightarrow \frac{\sin 65^0}{\mathrm{cosec}25^0} = \frac{\sin 65^0}{\sin(90-65^0)} = \frac{\sin 65^0}{\sin 65^0} = 1$

Problems for practice:

If sinA= $\frac{3}{5}$ ***then find all other trigonometric ratios***

If secA= $\frac{10}{8}$ ***then find all other trigonometric ratios***

Find the value of $\frac{tan65^0}{cot25^0}$

Find the value of $\frac{sec50^0}{cos40^0}$

Find the value of cot 60^0 ***- tan*** 30^0

Unit: surface areas and volumes

SOLIDS	LSA	TSA	VOLUME
CUBE	$4a^2$	$6a^2$	a^3
CUBOID	$2h(l + b)$	$2(lb + bh + hl)$	$l \times b \times h$
CYLINDER	$2\pi rh$	$2\pi r(h + r)$	$\pi r^2 h$
CONE	πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
SPHERE	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
HEMISPHERE	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
FRUSTUM of a CONE	$\pi l(r_1 + r_2)$	$\pi l(r_1 + r_2) + \pi(r_1^2 + r_2^2)$	$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

AREAS RELATED TO CIRCLES

Length of the circumference of a circle of radius $r = 2\pi r$

Length of the circumference of a circle of diameter $d = \pi d$

Area of a circle of radius $r = \pi r^2$ sq.units

Area of the quadrant of a circle of radius $r = \frac{1}{4}\pi r^2$ sq.units

Area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$ sq.units

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$ units

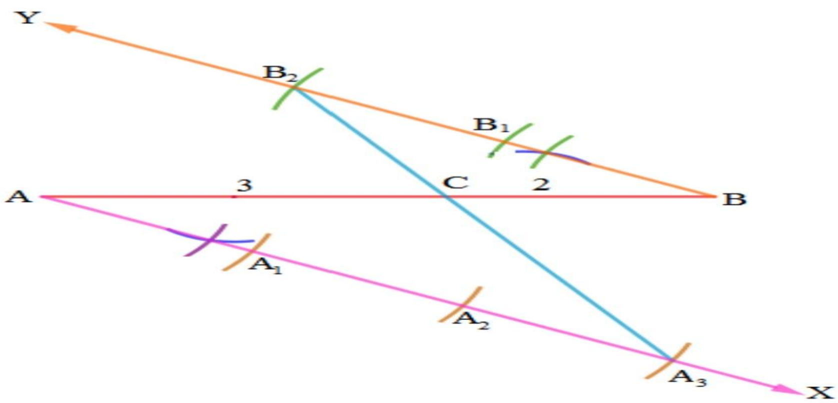
Area of the segment of a circle = (Area of the corresponding sector – Area of the corresponding triangle).

<u>Unit : constructions</u>

...

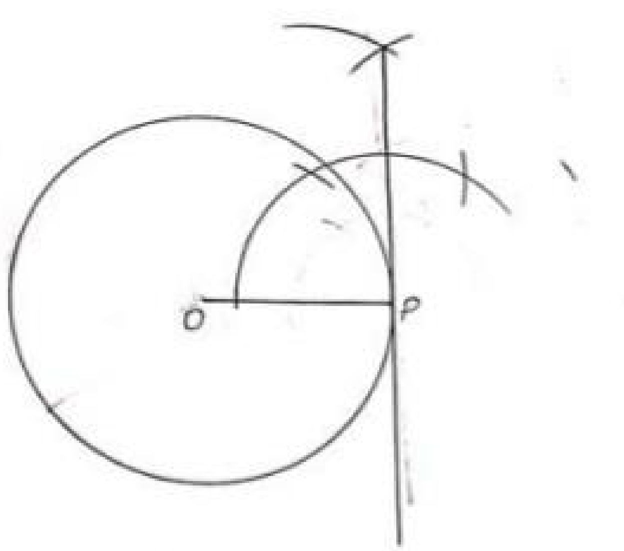
<u>Concept</u> 1. Division of line segment.

Divide the line segment AB=10cm in the ratio 3:2 internally

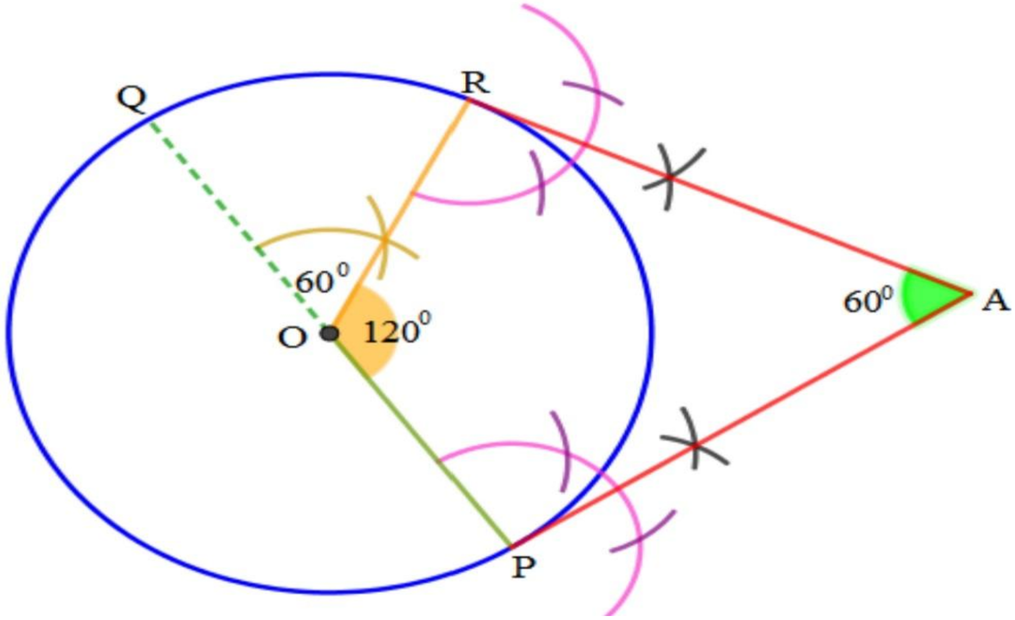


Concept 2. Construction of tangent on the circle

Construct a tangent to a circle of radius 4cm at any point p on it.

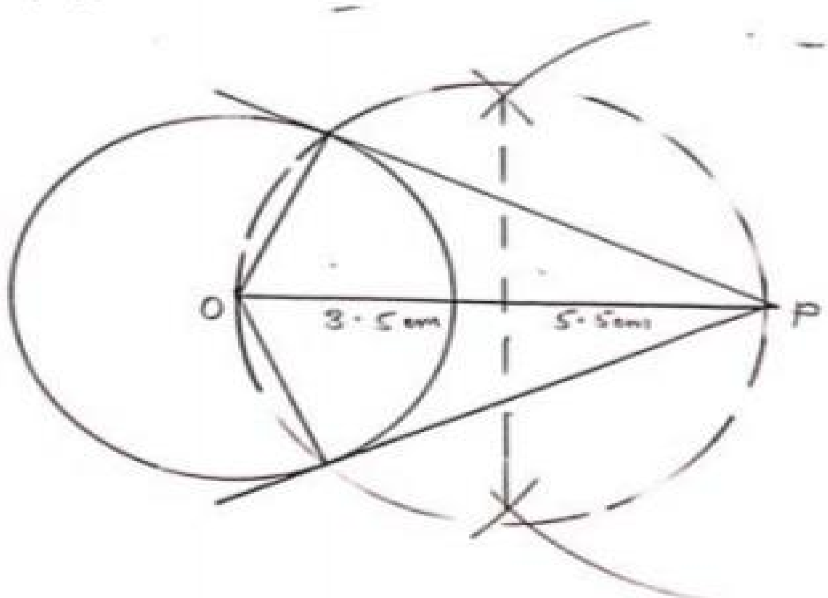


Construct a pair of tangents to a circle of radius 4cm where the angle between the radii is 120^{0}



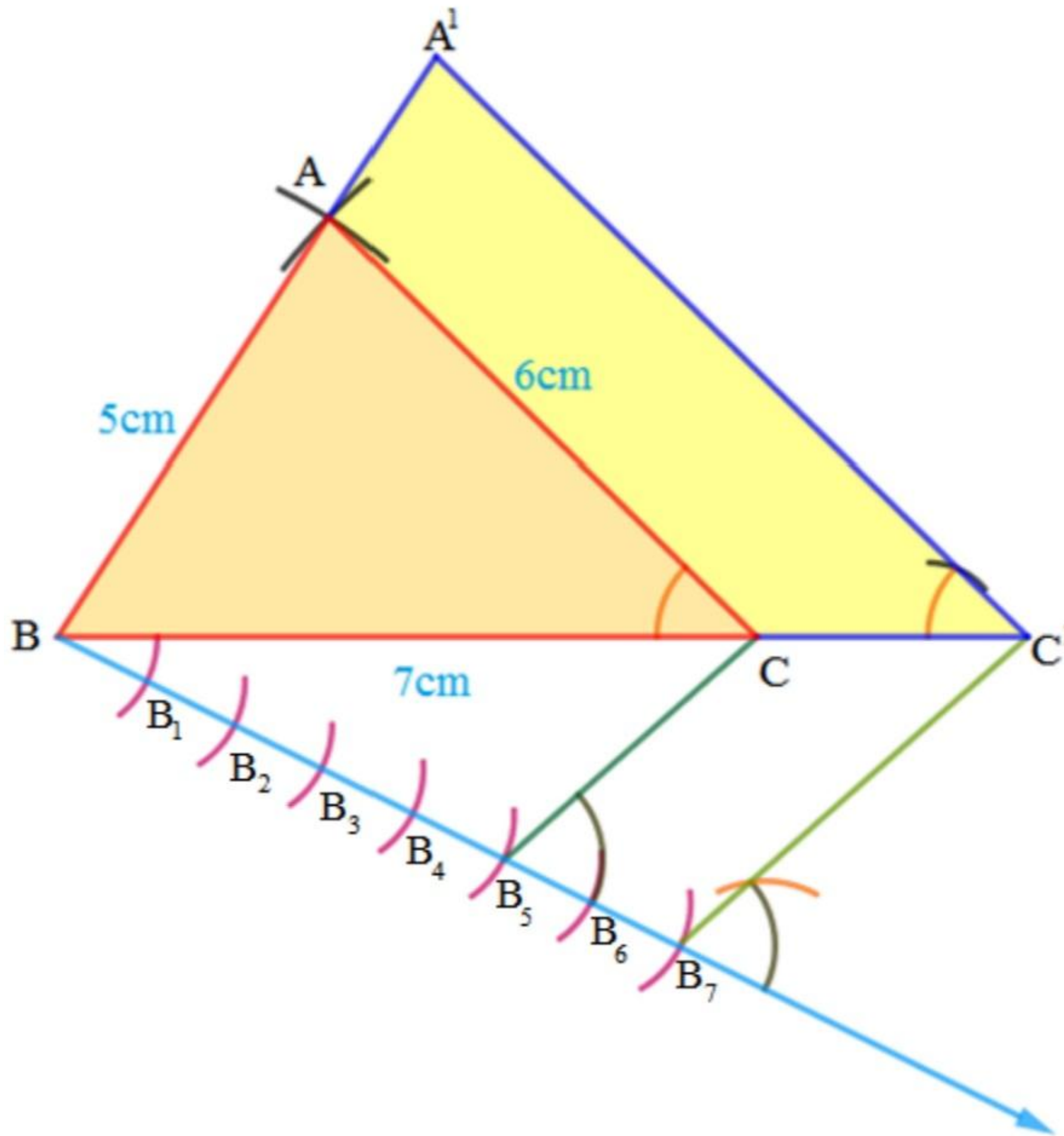
Concept 3.. construction of tangents from an external point.

Construct a pair of tangents to a circle of radius 3.5cm from an external point 9cm away from the centre.



***Concept* 4. Construction of similar triangles.**

Construct a triangle with sides 5cm ,6cm and 7cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle



<u>Problems for practice</u> :

Divide the line segment AB=8cm in the ratio 4:3 internally

Construct a tangent to a circle of radius 5 cm at any point p on it.

Construct a pair of tangents to a circle of radius 5cm where the angle between the radii is 110^{0}

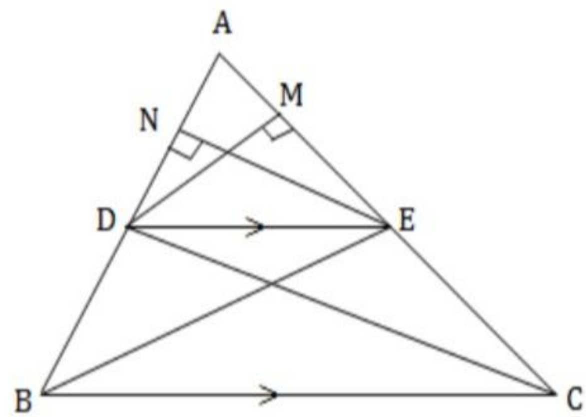
Construct a pair of tangents to a circle of radius 4cm from an external point 8cm away from the centre.

Construct a triangle with sides 6cm, 5cm and 4cm and then another triangle whose sides are $\frac{4}{3}$ ***of the corresponding sides of first triangle***

Theorems

39) State and prove Thales theorem (Basic Proportionality Theorem) .

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points , the other two sides are divided in the same ratio.



Data : In ΔABC $DE \parallel BC$.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw $DM \perp AC$ and $EN \perp AB$. Join BE and CD .

Proof :

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad \text{---------->} (1)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} \quad (\because \text{Area of } \Delta = \frac{1}{2} \times \text{base} \times \text{height})$$

$$\frac{ar(\Delta ADE)}{ar(\Delta CED)} = \frac{AE}{EC} \quad \text{---------->} (2)$$

But ΔBDE and ΔCED are standing on the same base DE and between $DE \parallel BC$.

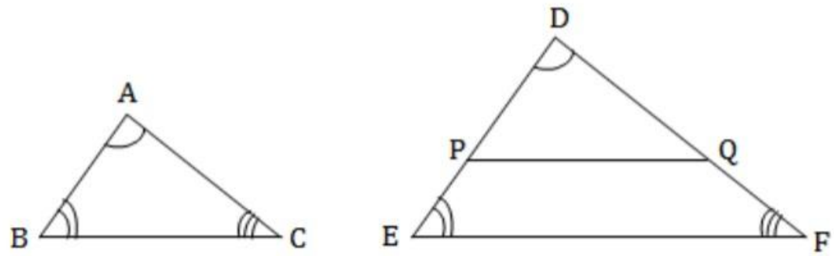
$ar(\Delta BDE) = ar(\Delta CED)$ ----------> (3)

$\therefore$ from equations (1), (2) and (3)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

40) AAA Similarity Criterion :

If in two triangles, corresponding angles are equal , then their corresponding sides are in the same ratio (or proportional). Hence prove that the two triangles are similar.



Data : In ΔABC and ΔDEF

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

To Prove : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction : Mark points P and Q on DE and DF such that $DP = AB$ and $DQ = AC$. Join PQ .

Proof : In ΔABC and ΔDPQ

$\angle A = \angle D$ ($\because$Data)

$AB = DP$ ($\because$Construction)

$AC = DQ$ ($\because$Construction)

$\Delta ABC \cong \Delta DPQ$ ($\because$ SAS Postulate)

$\therefore BC = PQ$ ($\because$CPCT) ----------> (1)

$\left.\begin{array}{l} \angle B = \angle P \quad (\because \text{CPCT}) \\ \angle B = \angle E \quad (\because \text{Data}) \end{array}\right\}$

$\therefore \angle P = \angle E$ ($\because$Axiom - 1)

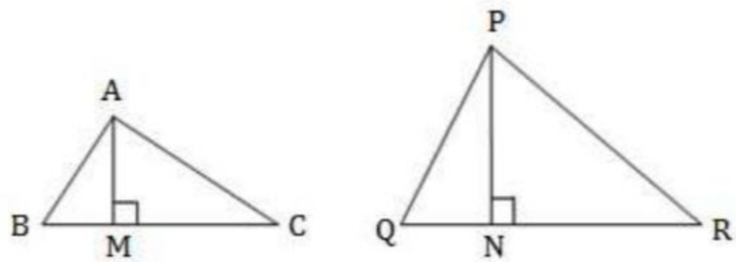
$\Rightarrow PQ \parallel EF$

$\frac{DP}{DE} = \frac{PQ}{EF} = \frac{DQ}{DF}$ ($\because$ Corollary of Thales theorem)

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [$\because$ from eq. (1) and construction]

41) **Areas of Similar Triangles:**

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides .



Data : $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

To Prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

Construction : Draw $AM \perp BC$ and $PN \perp QR$.

Proof : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ ($\because$ Area of $\Delta = \frac{1}{2} \times$ base $\times$ height)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC \times AM}{QR \times PN}$ ----------> (1)

In ΔABM and ΔPQN

$\angle B = \angle Q$ ($\because \Delta ABC \sim \Delta PQR$)

$\angle M = \angle N = 90^\circ$ ($\because$Construction)

$\therefore \Delta ABM \sim \Delta PQN$ ($\because$AA Similarity criterion)

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ --------> (2)

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ------> (3) ($\because$Data)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB}{PQ} \times \frac{AB}{PQ}$ ($\because$substituting eqs.(2) and (3) in (1))

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

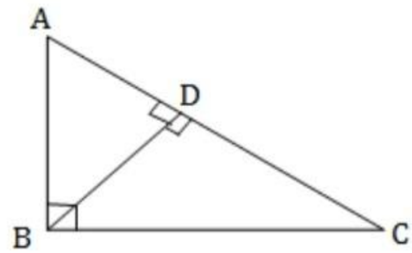
Now from eq.(3)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

Hence the proof.

42) **State and and Prove Pythagoras Theorem .**

" In a right angled triangle , the square on the hypotenuse is equal to the sum of the squares on other two sides ".



Data : ΔABC is a right triangle and $\angle B = 90^\circ$

To Prove : $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : In ΔADB and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle A = \angle A$ ($\because$ Common angle)

$\Delta ADB \sim \Delta ABC$ ($\because$ AAA Similarity Criterion)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ ($\because$ Proportional sides)

$AC. AD = AB^2$ ------> (1)

Similarly

In ΔBDC and ΔABC

$\angle D = \angle B = 90^\circ$ ($\because$ Data and Construction)

$\angle C = \angle C$ ($\because$ Common angle)

$\Delta BDC \sim \Delta ABC$ ($\because$AAA Similarity Criterion)

$\therefore \frac{DC}{BC} = \frac{BC}{AC}$ ($\because$ Proportional sides)

$AC. DC = BC^2$ ------> (2)

$AC . AD + AC. DC = AB^2 + BC^2$ [$\because$ By adding (1) and (2)]

$AC (AD + DC) = AB^2 + BC^2$

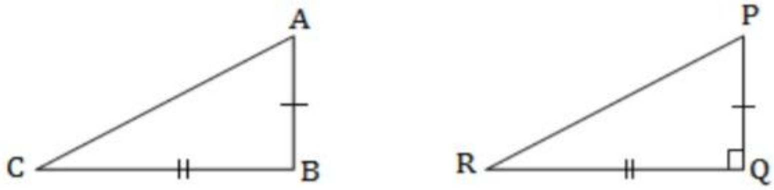
$AC \times AC = AB^2 + BC^2$ ($\because$ from fig. $AD + DC = AC$)

$AC^2 = AB^2 + BC^2$

Hence the proof.

43) **State and and Prove the converse of Pythagoras Theorem .**

" In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle " .



Data : In Δ ABC, $AC^2 = AB^2 + BC^2$

To Prove : $\angle B = 90^{\circ}$

Construction : construct another ΔPQR right angled at Q such that $PQ = AB$ and $QR = BC$

Proof : In ΔPQR

$PR^2 = PQ^2 + QR^2$ ($\because \angle Q = 90^{\circ}$, Pythagoras theorem)

$PR^2 = AB^2 + BC^2$ ---->(1) ($\because$ By Construction)

$AC^2 = AB^2 + BC^2$ ---->(2) ($\because$ Data)

$\therefore AC = PR$ ---->(3) [$\because$ From (1) and (2)]

In Δ ABC and Δ PQR

$AB = PQ$ ($\because$ By Construction)

$BC = QR$ ($\because$By Construction)

$AC = PR$ [$\because$ From (3)]

$\therefore \Delta ABC \cong \Delta PQR$ ($\because$ SSS congruence)

$\therefore \angle B = \angle Q$ ($\because$ CPCT)

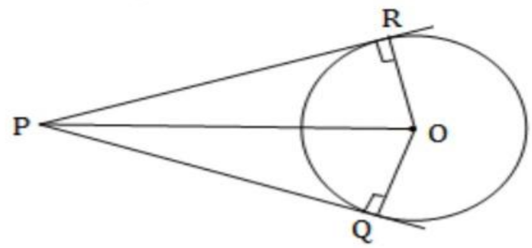
But, $\angle Q = 90^{\circ}$ ($\because$ By Construction)

$\therefore \angle B = 90^{\circ}$

Hence the proof.

18) **Theorem :- "The tangents drawn to a circle from an external point are equal" prove this.**

Data: A circle with centre O and P is an external point.
PQ and PR are the tangents drawn from an external point P.



To prove: $PQ = PR$

Construction: Draw OP, OQ and OR.

Proof: In ΔOQP and ΔORP

$OQ = OR$ ($\because$Radii of same circle)

$OP = OP$ ($\because$Common side)

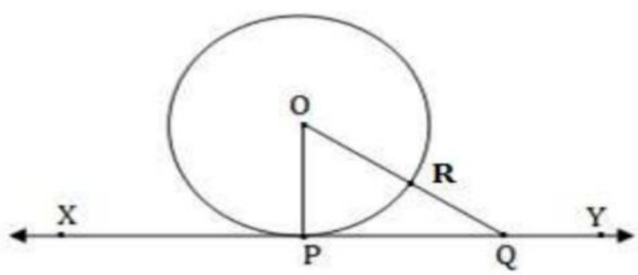
$\angle Q = \angle R = 90^{\circ}$ ($\because$tangent $\perp$radius)

$\Delta OQP \cong \Delta ORP$ ($\because$RHS criteria)

$PQ = PR$ ($\because$CSCT)

$\therefore$ Hence the proof.

17) **Theorem:- "The tangent at any point of a circle is perpendicular to the radius through the point of contact". Prove this.**



Data: A circle with centre O and XY is a tangent to the circle at point O.
OP is the radius through the Point P.

To prove: $OP \perp XY$

Construction: Take a point Q other than P on XY and join OQ.
Let OQ intersct the circle at R.

Proof: $OQ > OR$ ($\because$ from the figure)

But $OP = OR$ ($\because$ radii of the same circle)

$\therefore$ $OQ >$ radius OP

Since this happens for every point on the line XY cxcept the point P.

$\therefore$ OP is the shortest distance between the point O and tangent XY.

$\therefore$ $OP \perp XY$

Student's self assessment chart

sl.no	Concept	Marks	Remark
1	General form and formulas on AP	1	
2	Problems on nth term of AP	2	
3	Problems on sum of n terms of AP	2	
4	Theorems on triangles	5	
5	Theorems on circles	3	
6	Elimination method of solving linear pair	2	
7	Graphical method of solving linear pair	4	
8	Table 3.4 of Linear pair	1	
9	Constructions	9	
10	Distance formula	2	
11	Area of triangle	3	
12	mean median and mode	3	
13	Ogive graphs	3	
14	Formula method of solving QE	2	
15	Nature of roots of QE	2	
16	Formulas of solids	2	
17	Simple problems on trigonometric ratios	2	
	Total marks	48	

THANK YOU