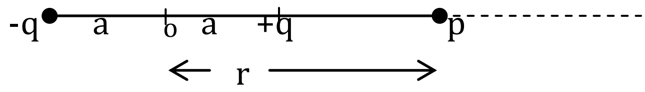
SCIENCE ACADEMY P U COLLEGE

DAVANAGERE

5 MARK IMPORTANT QUESTIONS WITH ANSWERS

1. Derive an Expression for Electric field at a point on the axis of a dipole (E_A).



Electric – field at 'P' due to +ve charge is $E_1 = \frac{1}{4\pi\mathcal{E}_0} x \frac{q}{(r-a)^2}$ along OP.

Electric – field at 'P' due to -ve charge is $E_2 = \frac{1}{4\pi\mathcal{E}_0} x \frac{q}{(r+a)^2}$ along PO.

Electric – field at 'P' due to the dipole = $E_A = E_1 - E_2$.

$$\therefore E_A = \frac{1}{4\pi\mathcal{E}_0} q \left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right)$$

$$E_A = \frac{1}{4\pi\mathcal{E}_0} x\, q \left(\frac{(r^2+a^2+2ra)-(r^2+a^2-2ra)}{(r^2-a^2)^2} \right)$$

$$E_A = \frac{1}{4\pi\mathcal{E}_0} x \frac{q \text{ x } 4ra}{(r^2-a^2)^2} = \frac{1}{4\pi\mathcal{E}_0} x \frac{q \text{ x } 2a \text{ x } 2r}{(r^2-a^2)^2}$$

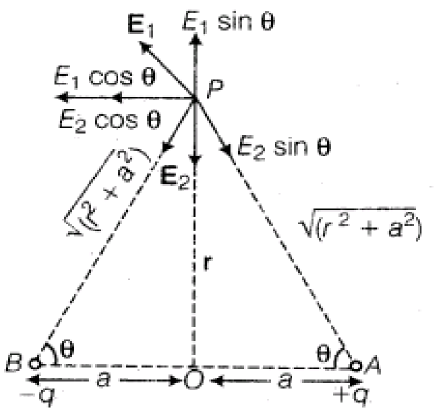
$$E_A = \frac{1}{4\pi\mathcal{E}_0} x \frac{2pr}{(r^2-a^2)^2} \qquad (\text{But, } q \text{ x } 2a = P)$$

.when a < < r, a^2 can be neglected.

$$E_A = \frac{1}{4\pi\mathcal{E}_0} x \frac{2pr}{r^4} = \frac{1}{4\pi\mathcal{E}_0} x \frac{2p}{r^3}.$$

$$\boxed{\vec{E_A} = \frac{1}{4\pi\mathcal{E}_0} x \frac{2\vec{p}}{r^3}}$$

2. Derive an Expression for Electric field at a point on the bisector line of dipole



In Δ OPA

$PA^2 = OP^2 + OA^2 = r^2 + a^2$

$PA = (r^2 + a^2)^{½}$

$$\text{Cos } \theta = \frac{OA}{PA} = \frac{a}{(r^2+a^2)^{1/2}}$$

Field at P due to +ve charge, $E_1 = \frac{1}{4\pi\mathcal{E}_0} x \frac{q}{(r^2+a^2)}$ along AP

Field at P due to -ve charge, $E_2 = \frac{1}{4\pi\mathcal{E}_0} x \frac{q}{(r^2+a^2)}$ along PB

$|E_1| = |E_2|$

∴ two equal and opposite components E sinθ cancel each other.

Field at 'P' due to the dipole is

$$E_B = E\cos\theta + E\cos\theta = 2E\cos\theta$$

$$E_B = 2 \times \frac{1}{4\pi\varepsilon_0} \times \frac{q}{(r^2+a^2)} \times \cos\theta$$

$$E_B = \frac{1}{4\pi\varepsilon_0} \times \frac{2q}{(r^2+a^2)} \times \frac{a}{(r^2+a^2)^{1/2}}$$

$$\cos\theta = \frac{a}{(r^2+a^2)^{1/2}}$$

$$q \times 2a = P$$

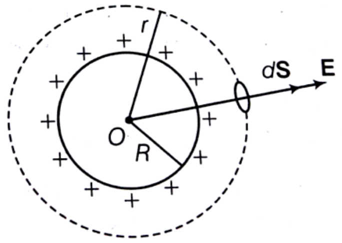
$$E_B = \frac{1}{4\pi\varepsilon_0} \times \frac{P}{(r^2+a^2)^{3/2}}$$

When a << r, a^2 can be neglected.

$$\boxed{\mathbf{E_B = \frac{1}{4\pi\varepsilon_0} \times \frac{P}{r^3}}}$$

3. Derive an expression for Electric field due to a uniformly charged spherical shell.

a) **At a point outside the shell:**



Electric flux through this sphere is given by

$$\Phi = \oint E\cos\theta \times ds$$

$$\Phi = \oint E \times ds \qquad (\because \theta = 0, \cos 0 = 1)$$

$$\Phi = E\oint ds$$

$$\Phi = E \times 4\pi r^2 \text{ ----- (1)} \qquad (\int ds = 4\pi r^2)$$

According to gauss theorem,

$$\Phi = \frac{1}{\varepsilon_0} \times q \qquad \text{------ (2)}$$

From equations (1) and (2) we get

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \times q$$

$$\boxed{\mathbf{E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}}}$$

b) At a point on the surface (r = R) :

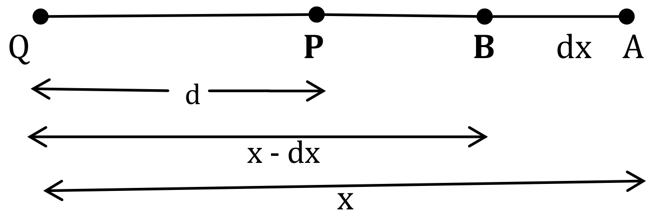
The electric intensity E for the points on the surface of charged spherical shell is given by

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{R^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{\sigma\, 4\pi R^2}{R^2} \qquad r = R \qquad (\text{By definition, } \sigma = \frac{q}{4\pi R^2}\ , \therefore q = \sigma 4\pi R^2)$$

$$\boxed{\mathbf{E = \frac{\sigma}{\varepsilon_0}}}$$

4. **Expression for Electric potential at a point due to a point charge**



Force acting on unit +ve charge (Q_0) at A due to charge 'Q' is given by

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{QQ_0}{x^2} \qquad Q_0 = 1\ C$$

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{x^2}$$

Work done in moving a unit +ve charge from A to B against the field direction is given by, $dw = -F \times dx$

$$dw = -\frac{1}{4\pi\varepsilon_0} \times \frac{Q}{x^2}\ dx$$

|||ly Work done in bringing a unit +ve charge from ∞ to the point ' P ' against the field direction is given by

$$\int dw = -\int \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{x^2}\ dx$$

$$w = -\frac{1}{4\pi\varepsilon_0} \times Q \int_{\infty}^{d} \frac{1}{x^2}\ dx$$

$$w = -\frac{1}{4\pi\varepsilon_0} \times Q \left[-\frac{1}{x}\right]_{\infty}^{d}$$

$$W = -\frac{1}{4\pi\varepsilon_0} \times Q \left[-\frac{1}{d} + \frac{1}{\infty}\right]$$

$$W = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{d}$$

By def W = V, Potential

∴

$$\boxed{\mathbf{V = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{d}}}$$

5. **State and Deduce of Ohm's Law**

Consider,

$$E = \frac{V}{l}$$

Current flowing through conductor $I = n\,e\,A\,v_d$ ----------------(1)

But, $a = F / m = e\,E / m = e\,V / m\,l$

The drift velocity of the electrons is given by, $v_d = a \times \tau = (eV / m\,l) \cdot \tau$

Substituting the v_d in (1) : $I = n\,e\,A \times (eV / ml) \times \tau$

$$I = n\,e^2 A\,\tau\,V / m\,l$$

$I = (n\, e^2\, \tau / m)\, (A / l)\, .\, V$

$V = (m / n\, e^2\, \tau)\, (l / A)\, .\, I$

$\therefore V = \rho\, \frac{l}{A} I$ where, $\rho = m / n\, e^2\, \tau$

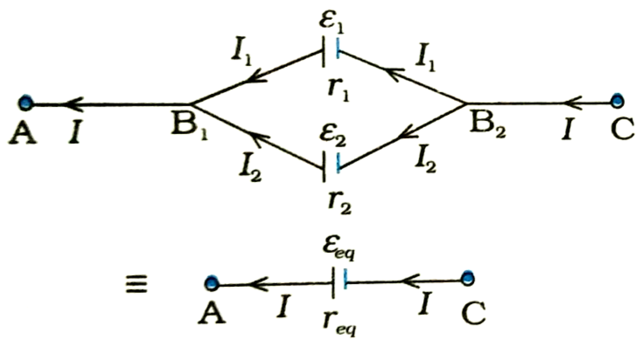
$V = R\, I$

$I = (1/R)\, V$

$\therefore \boxed{I \propto V}$

Thus current flowing through conductor is directly proportional to potential difference between the ends of the conductor provided temperature and other physical conditions remains constant. This is called 'Ohm's law'.

6. Obtain an expression for equivalent emf and internal resistance of two Cells in parallel.



Potential difference across cell E_1 is $V_1 = E_1 - I_1 r_1$

Potential difference across cell E_2 is $V_2 = E_2 - I_2 r_2$

$$I_1 = \frac{E_1 - V_1}{r_1} = \frac{E_1 - V}{r_1} \qquad (\because V_1 = V_2 = V)$$

|||ly $$I_2 = \frac{E_2 - V}{r_2}$$

Main current in the circuit is given by

$$I = I_1 + I_2$$

$$I = \frac{E_1}{r_1} - \frac{V}{r_1} + \frac{E_2}{r_2} - \frac{V}{r_2}$$

$$I = \frac{E_1}{r_1} + \frac{E_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - I$$

$$V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I\left[\frac{r_1 r_2}{r_1 + r_2}\right] \text{ ----- } (1)$$

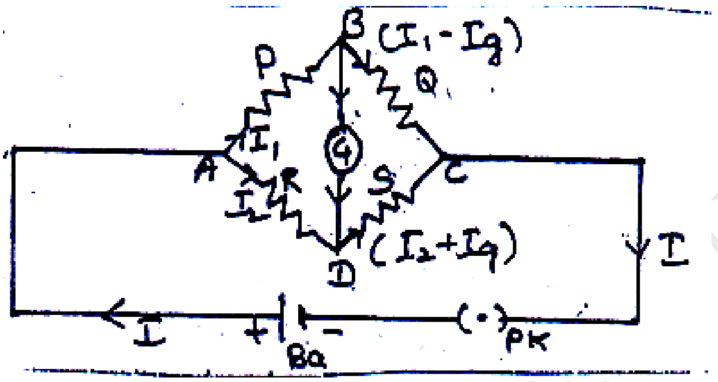
For the combination we have

$$V = E_{eff} - I\, r_{eff}. \quad \text{------} \quad (2)$$

Comparing equations (1) and (2) we get

$$\boxed{E_{eff} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \text{ and } r_{eff} = \frac{r_1 r_2}{r_1 + r_2}}$$

7. Deduce the condition for Wheat stone's Bridge network using Kirchhoff's laws.



Currents flowing through different branches of the bridge will be as shown in the figure.

Applying K.C.L to the junction A, $I = I_1 + I_2$ --------------(1)

Applying K.V.L to the mesh ABDA , $I_1P + I_gG - I_2R = 0$ -------(2)

Applying K.V.L to the mesh BCDB , $(I_1 - I_g)Q - (I_2 + I_g)S - I_gG = 0$

$I_1Q - I_gQ - I_2S - I_gS - I_gG = 0$ -------(3)

<u>**Condition for balance**</u>: current through galvanometer is zero ($I_g = 0$).

equations (2) and (3) becomes, $I_1P - I_2R = 0$

$I_1P = I_2R$ -----------(4)

And $I_1Q - I_2S = 0$

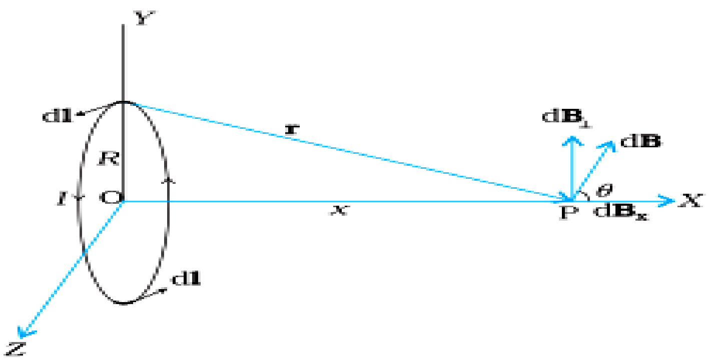
$I_1Q = I_2S$ -----------(5)

Divide (4) by (5) , we get

$I_1P / I_1Q = I_2R / I_2S$

$\therefore$ **P / Q = R / S**

8. Obtain an expression for Magnetic field at a point along the axis of circular coil carrying current.



$dB_x = dB \cos\theta$

$dB_\perp = dB \sin\theta$

From Biot Savart's law

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2} \qquad \theta = 90^0,\ \sin\theta = 1$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

Magnetic field produced at P due to current in the full loop is given by

$$\sum dB = \sum dB\cos\theta$$

$$B = \sum \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\cos\theta = \frac{\mu_0}{4\pi}\frac{I}{r^2}\cos\theta\sum dl \qquad (\sum dl = 2\pi R)$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\cos\theta \text{ x } 2\pi R$$

In Δ OAP, $\cos\theta = R/r$

$r^2 = R^2 + x^2 \quad \Rightarrow \quad r = (R^2 + x^2)^{1/2}$

$\therefore r^3 = (R^2 + x^2)^{3/2}$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r^2} \text{ x } \frac{R}{r} \text{ x } 2\pi R$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r^3}2\pi R^2$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I}{(R^2+x^2)^{\frac{3}{2}}}\,2\,\pi\,R^2$$ along the axis towards the observer

For n turns of the coil

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{2\,\pi\,R^2\,n\,I}{(R^2+x^2)^{\frac{3}{2}}}}$$

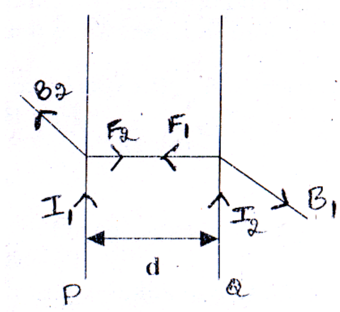
For field **at the centre of the loop**, $x = 0$

$$\boxed{\mathbf{B} = \frac{\mu_0}{2}\frac{n\,I}{R}}$$

9. Obtain an expression for Force between two parallel current carrying conductor.

Magnetic field produced by current I_1 on the conductor Q is given by

$$\mathbf{B_1} = \frac{\mu_0}{4\pi}\cdot\frac{2\,I_1}{d}$$



The conductor Q carrying current I_2 in the magnetic field B_1 experiences a mechanical force F_1 is given by

$$F_1 = B_1 I_2 L\sin\theta \qquad \text{If } \theta = 90^0$$

$$F_1 = B_1 I_2 L$$

$$\mathbf{F_1} = \frac{\mu_0}{4\pi}\cdot\frac{2\,I_1}{d}\cdot I_2\,L \quad \text{------------------(1}$$

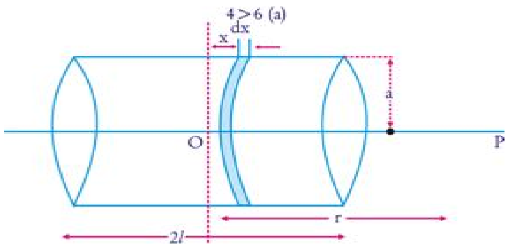
$|||^{ly}$, conductor P experiences mechanical force F_2 given by

$$F_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\,I_2}{d}\cdot I_1\,L \quad \text{------------------(2)}$$

From equations (1) and (2)

It is found that F_1 and F_2 are equal and opposite. The conductors attract each other when they carry current in same direction. They repel each other when they carry current in opposite direction.

10. Show that Bar magnet as an equivalent solenoid.



Number of turns in the element = n x dx

The magnetic field produced at 'P' due to the element is given by

$$dB = \frac{\mu_0}{4\pi} \text{ x } \frac{2\pi a^2 I\,(n \text{ x } dx)}{((r-x)^2 + a^2)^{3/2}} \qquad \text{along the axis.}$$

When $r >>>> a$, a^2 can be neglected compared with r^2

$$dB = \frac{\mu_0}{4\pi} \text{ x } \frac{2\pi a^2 I\, n \text{ x } dx}{r^3}$$

Total Magnetic field at 'P' due current in the solenoid in given by

$$\int dB = \int_{-l}^{l} \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 In\, dx}{r^3}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 In}{r^3} \int_{-l}^{+l} dx = \frac{2\pi a^2 In}{r^3} \cdot [x]_{-l}^{+l}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 In}{r^3} [l + l) = \frac{\mu_0}{4\pi} \text{ x } \frac{2\pi a^2 In x 2l}{r^3}$$

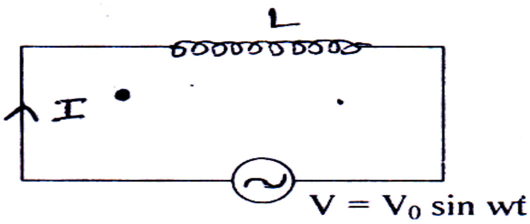
$N = n\,2\,l$

$\Pi a^2 = A$

$M = N\,I\,A$

$$\boxed{B = \frac{\mu_0}{4\pi} \text{x} \frac{2M}{r^3}}$$

11. Obtain an expression for current in an AC circuit containing pure inductance:



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Let, $V = V_0 \sin wt$ ------------ (1)

From Lenz's law

$$V^1 = -L \cdot \frac{dI}{dt}$$

Applying KVL for the circuit , we get

$$V + V^1 = 0 \qquad (\because R = 0,\ IR = 0)$$

$$V = L \cdot \frac{dI}{dt}$$

$$V_0 \sin \omega t = L \cdot \frac{dI}{dt}$$

$$dI = V_0 \sin \omega t \cdot \frac{dt}{L}$$

Integrating we get, $\int dI = \int (V_0 / L) \sin \omega t \cdot dt$

$$I = \frac{V_0}{L} \int \sin \omega t \, . \, dt$$

$$I = \frac{V_0}{L} \left[- \cos \omega t / w\right]$$

$$I = \frac{V_0}{\omega L} . \sin (\omega t - \pi/2) \qquad - \cos \omega t = \sin (\omega t - \pi/2)$$

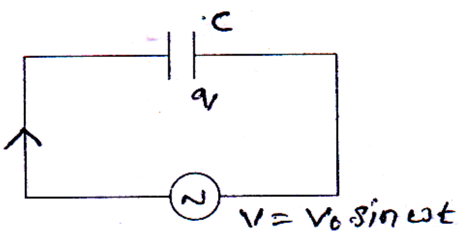
Current is maximum when $\sin (wt - \pi/2) = \pm 1$,

$I_0 = V_0 / \omega L$ when $\sin (wt - \pi/2) = \pm 1$

$$\mathbf{I = I_0 \sin (\omega t - \pi/2)} \text{ -------------(2)}$$

This is the expression for current in inductance circuit.
In inductive circuit voltage leads the current by 90^0 or $\pi/2$.

12. Obtain an expression for current in An AC circuit consisting of pure capacitance:



Let $V = V_0 \sin \omega t$ ----------(1) is the applied alternating voltage.

$I = dq / dt$ But, $q = CV$

$I = d (CV) / dt$

$$I = C . \frac{d}{dt} (V_0 \sin \omega t)$$

$$I = C V_0 \frac{d}{dt} (\sin \omega t)$$

$I = C V_0 \omega \cos \omega t$

$$I = \frac{V_0}{(1/\omega C)} . \sin (\omega t + \pi/2) \qquad \because \cos wt = \sin (\omega t + \pi/2)$$

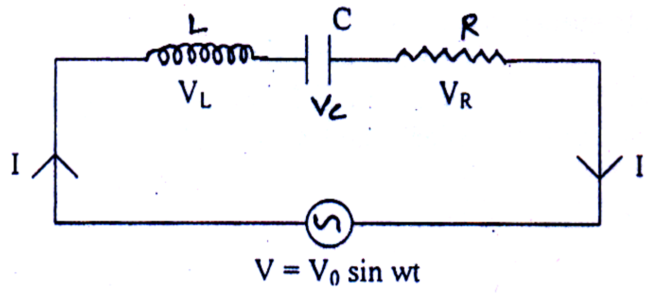
Current is maximum when $\sin (\omega t + \pi/2) = \pm 1$,

$I_0 = \frac{V_0}{(1/\omega C)}$ when $\sin (\omega t + \pi/2) = \pm 1$

$$\boxed{\mathbf{I = I_0 \sin (\omega t + \pi/2)}} \text{ ----------- (2)}$$

This is the expression for alternating current in a capacitor circuit.
In capacitor circuit voltage lags behind the current by 90^0 or $\pi/2$.

13. Obtain an Expression for Impedance, current and phase angle using phasor diagram.



Let $V = V_0 \sin wt$ is the applied voltage.

The voltage across each of them is given by,

$V_L = I X_L$ leads the current by $\pi/2$

$V_C = I X_C$ lags behind current by $\pi/2$

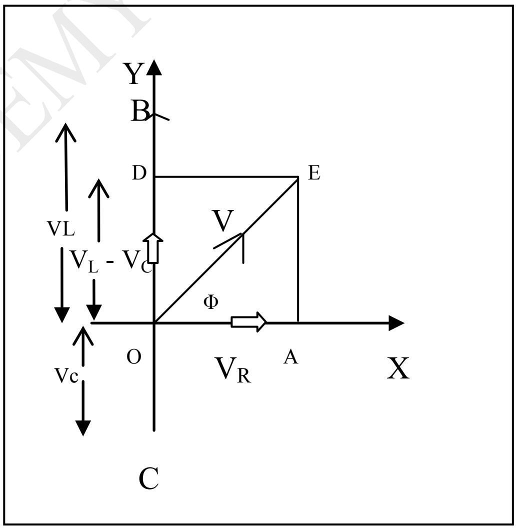
$V_R = I R$ in phase with current.

In Δ^{le} OAE,

$$OE^2 = OA^2 + AE^2$$
$$V^2 = V_R^2 + (V_L - V_C)^2$$
$$V^2 = (IR)^2 + (I X_L - I X_C)^2$$
$$V^2 = I^2 [R^2 + (X_L - X_C)^2]$$
$$V^2 / I^2 = R^2 + (X_L - X_C)^2$$
$$Z^2 = R^2 + (X_L - X_C)^2$$

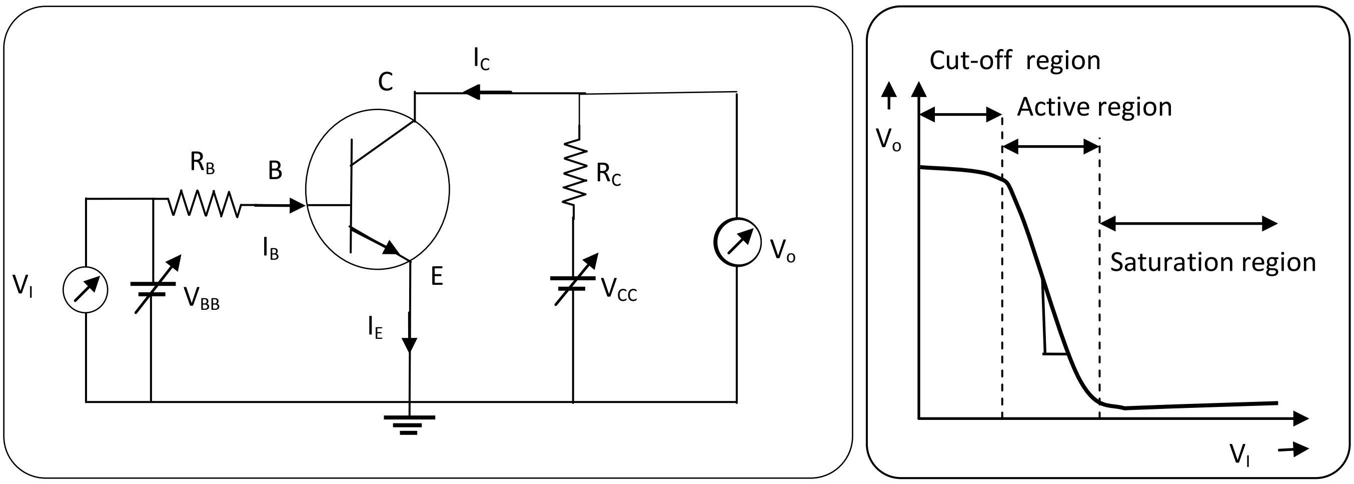
Where, Z = V / I , impedance of the circuit

$$\therefore \boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$



This is the expression for impedance for LCR circuit.

14. Explain the action of Transistor as a Switch:

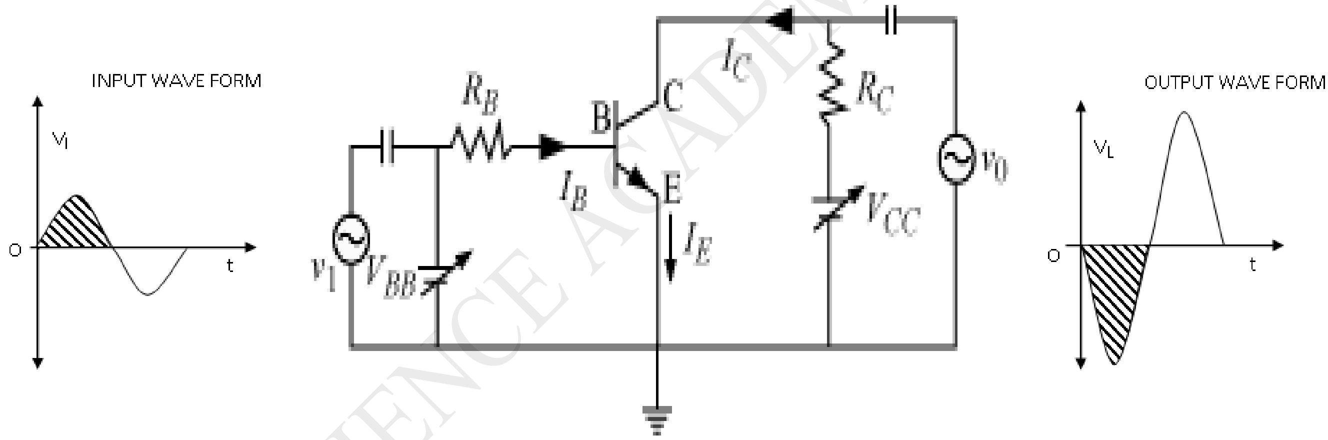


Let us study the working of Transistor as a switch in the following steps:

1. A small Positive voltage at Base, Positive voltage to collector and Negative voltage to emitter is being applied.
2. The voltage at Base is slightly positive than that of Emitter and the voltage at Collector is more positive than that of Base region.
3. Due to this arrangement the base region attracts electrons from the emitter region and the collector attracts the electrons from the base region. So, the electrons flow from emitter to collector and as we know that the flow of electrons is opposite to that flow of current and hence the current flows from Collector to Emitter.
4. From this we can conclude that the transistor acts as switch facilitating the flow of current from the Collector to Emitter.
5. Now take the case that the Base region is not connected to voltage supply, so the there would not be any charge flow from Collector to Emitter and hence the circuit acts as switch blocking the flow of current from the Collector to Emitter.

Thus in both the cases Transistor acts as a Switch.

15. Action of transistor as an amplifier in C-E mode with n-p-n transistor:



- The circuit of a simple N-P-N transistor in C-E configuration is as shown in the figure. The emitter base junction is forward biased and base collector junction of the transistor is reverse biased.
- The alternating signal to be amplified is fed to the input circuit [(E-B) junction]. Because of alternating voltage over and above the forward biasing voltage V_{BB}, the base current changes in the input circuit.
- As the base region is very thin most of the conduction electrons in the emitter are attracted to the collector through the base. A small change in base voltage due to input signal causes a large change in the voltage across the load resistor R_L. Thus a transistor amplifies a small signal into a large one. There is a voltage amplification as well as power amplification.
- Output is out of phase with the input by 180^0.

16. Explain the Einstein's explanation of photoelectric effect.

Einstein explained photoelectric effect on the basis of '***Quantum theory of radiation***'.

$$\boxed{\tfrac{1}{2} mv^2 = h\upsilon - W} \quad \text{----------------} (1)$$

This equation is called **Einstein's photoelectric equation**.

At threshold frequency, $\upsilon = \upsilon_0$, $\tfrac{1}{2} mv^2 = 0$

Therefore equation (1) becomes

$\therefore$ $\boxed{\phi_0 = h\upsilon_0}$ or

$$\boxed{K.E_{max} = \tfrac{1}{2} mv^2 = h(\upsilon - \upsilon_0)}$$

Verification of Laws of Photoelectric Emission based on Einstein's Photoelectric Equation

1) Photoelectric effect is instantaneous effect and takes place due to elastic collision between photon and electron inside the metal.
2) If $\nu < \nu_0$, then $\tfrac{1}{2} mv^2_{max}$ is negative, which is not possible. Therefore, for photoelectric emission to take place $\nu > \nu_0$.
3) Since one photon emits one electron, so the number photoelectrons emitted per second is directly proportional to the intensity of incident light.
4) It is clear that $\mathbf{\tfrac{1}{2} mv^2_{max} \propto \nu}$ as h and ν_0 are constant. This shows that K.E. of the photoelectrons is directly proportional to the frequency of the incident light.

17. Derive an Expression for energy of electron in an nth orbit of hydrogen atom.

Thus, for a dynamically stable orbit in a hydrogen atom

$$F_e = F_C$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \qquad \text{[For hydrogen, Z = 1]}$$

Thus, the relation between the orbit radius and the electron velocity is

$$mv^2 = \frac{e^2}{4\pi\epsilon_0 r} \text{------------------} (1)$$

The K.E (K) and electrostatic potential energy (U) of the electron in hydrogen atom are

$$K = \tfrac{1}{2} mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \qquad [\text{ from (1) }]$$

$$\text{And} \quad U = -\frac{e^2}{4\pi\epsilon_0 r} \qquad (U = -2 \times K)$$

Thus, the total mechanical energy E of the electron in a hydrogen atom is

$$E = K + U$$

$$E = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$E = -\frac{e^2}{8\pi\epsilon_0 r} \qquad \text{but } \therefore r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$$

$$E = -\frac{e^2}{8\pi\epsilon_0 \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}}$$

$$E_n = -\frac{m Z^2 e^4}{8 \epsilon_0^2 n^2 h^2}$$

for hydrogen Z = 1 , $\boxed{E_n = -\frac{m e^4}{8 \epsilon_0^2 n^2 h^2}}$

this is the expression for energy of electron is n^{th} hydrogen orbit.

18. State and Prove Law of radioactive decay or disintegration law.

This law states that ***"the rate of disintegration of a radioactive element is directly proportional to number of atoms present at that instant of time"***.

Let 'N' be the number of atoms of a radioactive element present at any instant of time 't'

Let 'dN' be the number of atoms disintegrates in a small interval of time 'dt',

Then According to law, $-(dN / dt) \propto N$

$$\mathbf{(dN / dt) = -\lambda N.}$$

Where λ –**decay constant**

$$dN / N = -\lambda .dt$$

Integrating both sides

$$\int dN / N = -\lambda \int dt$$

$$\log_e N = -\lambda t + k \quad \ldots\ldots\ldots(1)$$

where k – is integration constant

When t = 0, $N = N_0$ (Initial number of atoms)

then (1) becomes,

$$\log_e N_0 = -\lambda (0) + k$$

$$\therefore \boxed{\log_e N_0 = k}$$

substituting the value of k in the equation (1) , we get

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

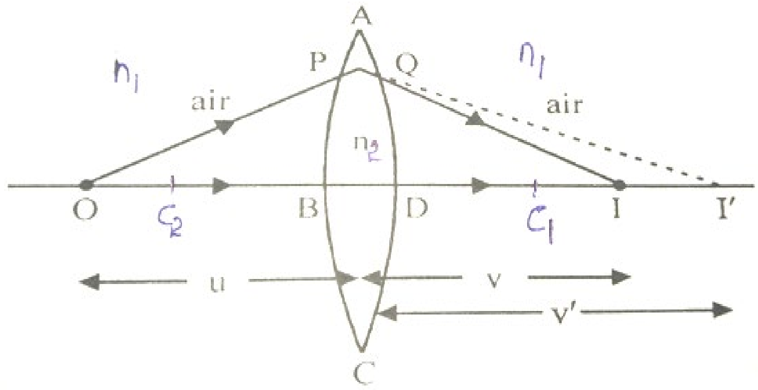
$$\log_e (N/N_o) = -\lambda t$$

Expressing in index form we get

$$N / N_0 = e^{-\lambda t}$$

$$\therefore \boxed{\mathbf{N = N_0 e^{-\lambda t}}}$$

19. Derive Lens makers formula.



From the figure

For refraction at the face ABC

$$\frac{n_1}{-u} + \frac{n_2}{v^1} = \frac{n_2 - n_1}{R_1} \quad \text{..........(i)}$$

For refraction at the face ADC of the lens,

$$\frac{n_2}{-v^1} + \frac{n_1}{v} = \frac{n_2 - n_1}{-R_2} \quad \text{.............(ii)}$$

Adding (i) and (ii), we get

$$\frac{n_1}{-u} + \frac{n_2}{v^1} + \frac{n_2}{-v^1} + \frac{n_1}{v} = [n_2 - n_1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

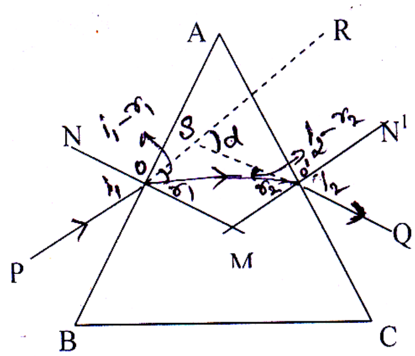
$$n_1 \left[-\frac{1}{u} + \frac{1}{v}\right] = [n_2 - n_1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{.............(iii)}$$

By definition (Principal focus) , when u = ∞, v = f and u = f , v = ∞

Equation (iii) becomes : $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\boxed{\frac{1}{f} = \left({}_1n_2 - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

20. **Derive** $\boxed{n = \frac{\sin (A + D) / 2}{\sin A / 2}}$ **for refraction through a prism.**



In quadrilateral $AOMO^1$

$\angle A + \angle M = 180^0$ ------------- (1)

In Δ^{le} OMO^1

$\angle r_1 + \angle r_2 + \angle M = 180^0$ ---------- (2)

From (1) and (2)

$\angle A + \angle M = \angle r_1 + \angle r_2 + \angle M$

$\angle A = \angle r_1 + \angle r_2$ ------------ (3)

In $\Delta\ SOO^1$,

$\angle d = (i_1 - r_1) + (i_2 - r_2)$

$= (i_1 + i_2) - (r_1 + r_2)$

$= (i_1 + i_2) - (A)$

$\mathbf{A + d = i_1 + i_2}$ -------------- (4)

At minimum, deviation position , $d = D$ and $i_1 = i_2 = i$ and $r_1 = r_2 = r$

Equations (3) and (4) becomes

$A = \angle r_1 + \angle r_2 = r + r = 2r$

$\mathbf{r = A / 2}$

$A + D = i_1 + i_2 = i + i = 2i$

$$\mathbf{i = \frac{A + D}{2}}$$

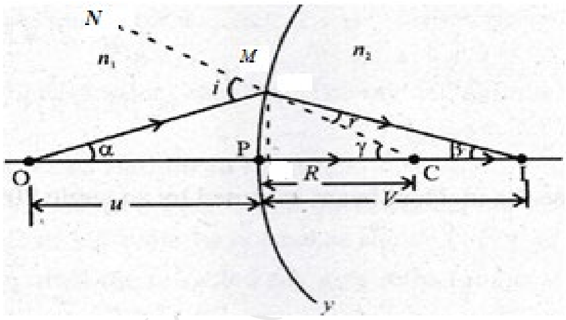
Substituting the values of i and r in the Snell's law equation. We get

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin (A + D) / 2}{\sin A / 2}$$

$$\boxed{n = \frac{\sin (A + D) / 2}{\sin A / 2}}$$

21. Obtain the relation between n, u, v and R. (Refraction at spherical surface)



In triangle PMO: $\tan MOP = \tan \alpha = \frac{PM}{PO}$

In triangle PMI: $\tan MIP = \tan \beta = \frac{PM}{PI}$

In triangle MCP: $\tan MCP = \tan \gamma = \frac{PM}{PC}$

In triangle OMC, $i = \alpha + \gamma = \frac{PM}{PO} + \frac{PM}{PC}$, (Exterior angle is equal to sum of interior angles)

In triangle MCI, $\gamma = r + \beta$ (Exterior angle is equal to sum of interior angles)

$$r = \gamma - \beta = \frac{PM}{PC} - \frac{PM}{PI}$$

By snell's law,

$$n_1 \sin i = n_2 \sin r$$ (as angles are small, sin i ≈ i & sin r ≈ r)

$$n_1 i = n_2 r$$

$$n_1 \left[\frac{PM}{PO} + \frac{PM}{PC}\right] = n_2 \left[\frac{PM}{PC} - \frac{PM}{PI}\right]$$

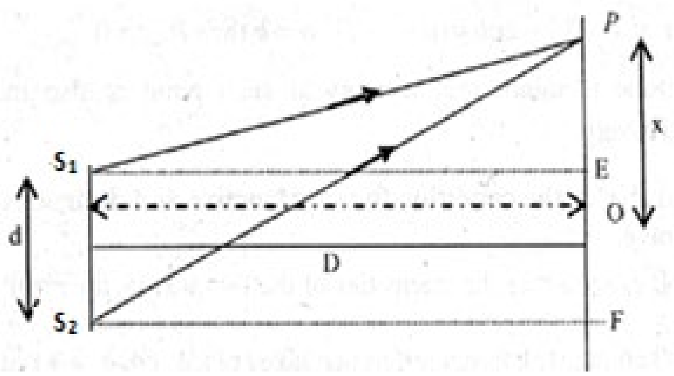
$$\frac{n_1}{PO} + \frac{n_1}{PC} = \frac{n_2}{PC} - \frac{n_2}{PI}$$

From figure: PO = -u, PC = R & PI = v

$$-\frac{n_1}{u} + \frac{n_1}{R} = \frac{n_2}{R} - \frac{n_2}{v}$$

$$\boxed{\therefore \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}$$

22. Expression for bandwidth of interference bands



Let P be a point on the screen at a distance x from O. Depending on the path difference S_2P - S_1P of light waves 'P' will be the position of dark band or bright band.

when $S_2P - S_1P = n\lambda$ ----- (1) P be the position of n^{th} bright band

$$S_2P^2 - S_1P^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right].$$

$$S_2P^2 - S_1P^2 = 2xd$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P} = \frac{2xd}{2D} = \frac{xd}{D} - -(2) \text{ [As } d \ll D,\ S_1P + S_2P \approx 2D]$$

From (1) and (2)

$$n\lambda = \frac{xd}{D}$$

$$x_n = \frac{n\lambda D}{d}$$

$$x_{n+1} = (n+1)\frac{\lambda D}{d}$$

$$\omega = x_{n+1} - x_n = \frac{\lambda D}{d}(n+1) - n$$

$$\boxed{\boldsymbol{\omega = \frac{\lambda D}{d}}}$$

in Δ, S_2FP

$$S_2P^2 = S_2F^2 + FP^2$$

$$= D^2 + \left(x + \frac{d}{2}\right)^2$$

in Δ, S_1EP

$$S_1P^2 = S_1E^2 + EP^2$$

$$= D^2 + \left(x - \frac{d}{2}\right)^2$$