

ಎಸ್ ಹರ್ಷ & ಹೆಚ್.ಜಿ. ದೀಪಶ್ರೀ

ತ್ರಿಕೋನಮಿತಿ-ಅನ್ವಯಿಕ ಪ್ರಶ್ನೋತ್ತರಗಳು

ಸರ್ಕಾರಿ ಪ್ರೌಢಶಾಲೆ, ಹೆಡತಲೆ, ನಂಜನಗೂಡು, ಮೈಸೂರು

$$1) \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta}} = 2 \sec \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \sqrt{\frac{(1+\sin \theta) \times (1+\sin \theta)}{(1-\sin \theta) \times (1+\sin \theta)}} + \sqrt{\frac{(1-\sin \theta) \times (1-\sin \theta)}{(1+\sin \theta) \times (1-\sin \theta)}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{1-\sin^2 \theta}} + \sqrt{\frac{(1-\sin \theta)^2}{1-\sin^2 \theta}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}} + \sqrt{\frac{(1-\sin \theta)^2}{\cos^2 \theta}}$$

$$= \frac{1+\sin \theta}{\cos \theta} + \frac{1-\sin \theta}{\cos \theta}$$

$$= \frac{1+\sin \theta + 1-\sin \theta}{\cos \theta}$$

$$= \frac{2}{\cos \theta}$$

$$= 2 \sec \theta = \text{RHS}$$

$$2) \frac{1+\sin \theta}{1-\sin \theta} - \frac{1-\sin \theta}{1+\sin \theta} = 4 \sec \theta \tan \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{1+\sin \theta}{1-\sin \theta} - \frac{1-\sin \theta}{1+\sin \theta}$$

$$= \frac{1+\sin \theta}{1-\sin \theta} \times \frac{1+\sin \theta}{1+\sin \theta} - \frac{1-\sin \theta}{1+\sin \theta} \times \frac{1-\sin \theta}{1-\sin \theta}$$

$$= \frac{(1+\sin \theta)^2}{1-\sin^2 \theta} - \frac{(1-\sin \theta)^2}{1-\sin^2 \theta}$$

$$= \frac{(1+\sin \theta)^2 - (1-\sin \theta)^2}{1-\sin^2 \theta}$$

$$= \frac{1+\sin^2 \theta + 2 \sin \theta - 1 - \sin^2 \theta + 2 \sin \theta}{\cos^2 \theta}$$

$$= \frac{4 \sin \theta}{\cos^2 \theta}$$

$$= 4 \times \frac{\sin \theta}{\cos \theta} \times \frac{1}{\cos \theta}$$

$$= 4 \sec \theta \tan \theta$$

$$= \text{RHS}$$

3) $\sqrt{\frac{1-\sin \theta}{1+\sin \theta}} = \sec \theta - \tan \theta$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \sqrt{\frac{(1-\sin \theta) \times (1-\sin \theta)}{(1+\sin \theta) \times (1-\sin \theta)}}$$

$$= \sqrt{\frac{(1-\sin \theta)^2}{1-\sin^2 \theta}}$$

$$= \sqrt{\frac{(1-\sin \theta)^2}{\cos^2 \theta}}$$

$$= \frac{1-\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$$

$$= \sec \theta - \tan \theta$$

$$= \text{RHS}$$

$$4) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta \quad \text{ಎಂದು ಸಾಧಿಸಿ.}$$

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$$LHS = \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}}$$

$$= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} + \frac{\cos^2 \theta}{\sin \theta (\cos \theta - \sin \theta)}$$

$$= \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta (\sin \theta - \cos \theta)}$$

$$= \frac{(\sin \theta - \cos \theta) (\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta (\sin \theta - \cos \theta)}$$

$$= \frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta} + \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} + 1$$

$$= 1 + \tan \theta + \cot \theta = RHS$$

5) $\tan^2 \theta + \cot^2 \theta + 2 = \sec^2 \theta \operatorname{cosec}^2 \theta$ ಎಂದು ಸಾಧಿಸಿ.

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$$LHS = \tan^2 \theta + \cot^2 \theta + 2$$

$$= \sec^2 \theta - 1 + \operatorname{cosec}^2 \theta - 1 + 2$$

$$= \sec^2 \theta + \operatorname{cosec}^2 \theta$$

$$= \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$= \frac{1}{\cos^2 \theta \sin^2 \theta}$$

$$= \frac{1}{\cos^2 \theta} \times \frac{1}{\sin^2 \theta}$$

$$= \sec^2 \theta \operatorname{cosec}^2 \theta$$

$$= RHS$$

$$6) \frac{\sin \theta}{1 - \cos \theta} = \operatorname{cosec} \theta + \cot \theta \text{ ಎಂದು ಸಾಧಿಸಿ}$$

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta}{1 - \cos \theta} \\ &= \frac{\sin \theta}{1 - \cos \theta} \times \frac{1 + \cos \theta}{1 + \cos \theta} \\ &= \frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} \\ &= \frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta} \\ &= \frac{(1 + \cos \theta)}{\sin \theta} \\ &= \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = \operatorname{cosec} \theta + \cot \theta = \text{RHS} \end{aligned}$$

7) $\frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta} = \frac{\sec \theta + 1}{\sec \theta - 1}$ ಎಂದು ಸಾಧಿಸಿ.

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$$\text{LHS} = \frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} + \sin \theta}{\frac{\sin \theta}{\cos \theta} - \sin \theta}$$

$$= \frac{\frac{\sin \theta + \sin \theta \cos \theta}{\cos \theta}}{\frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta}}$$

$$= \frac{\sin \theta}{\sin \theta} \left[\frac{1 + \cos \theta}{1 - \cos \theta} \right]$$

$$= \frac{1 + \frac{1}{\sec \theta}}{1 - \frac{1}{\sec \theta}}$$

$$= \frac{\sec \theta + 1}{\sec \theta - 1}$$

$$= \text{RHS}$$

$$8) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$= \tan \theta \left[\frac{(\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta)}{2 \cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)} \right]$$

$$= \tan \theta \frac{[\cos^2 \theta - \sin^2 \theta]}{[\cos^2 \theta - \sin^2 \theta]}$$

$$= \tan \theta = \text{RHS}$$

$$9) \sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}}$$

$$= \sqrt{\frac{(\sec \theta - 1)(\sec \theta - 1)}{(\sec \theta + 1)(\sec \theta - 1)}} + \sqrt{\frac{(\sec \theta + 1)(\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)}}$$

$$= \sqrt{\frac{(\sec \theta - 1)^2}{\sec^2 \theta - 1}} + \sqrt{\frac{(\sec \theta + 1)^2}{\sec^2 \theta - 1}}$$

$$= \sqrt{\frac{(\sec \theta - 1)^2}{\tan^2 \theta}} + \sqrt{\frac{(\sec \theta + 1)^2}{\tan^2 \theta}}$$

$$= \frac{\sec \theta - 1}{\tan \theta} + \frac{\sec \theta + 1}{\tan \theta}$$

$$= \frac{\sec \theta - 1 + \sec \theta + 1}{\tan \theta}$$

$$= \frac{2 \sec \theta}{\tan \theta}$$

$$= \frac{2 \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}}$$

$$= \frac{2}{\sin \theta}$$

$$= 2 \operatorname{cosec} \theta$$

$$= \text{RHS}$$

10) $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2$ ಎಂದು ಸಾಧಿಸಿ.

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$$\text{LHS} = (1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$$

$$= \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right)$$

$$= \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right)$$

$$= \left[\frac{\{(\sin \theta + \cos \theta) - 1\} \{(\sin \theta + \cos \theta) + 1\}}{\sin \theta \cos \theta}\right]$$

$$= \left[\frac{(\sin \theta + \cos \theta)^2 - 1^2}{\sin \theta \cos \theta}\right]$$

$$= \frac{(\sin \theta)^2 + (\cos \theta)^2 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{1 + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= \frac{2 \sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1}{\sin \theta \cos \theta}$$

$$= 2$$

$$= \text{RHS}$$

$$11) \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \frac{1 + \cos A}{\sin A} \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A) - \{(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)\}}{\cot A - \operatorname{cosec} A + 1}$$

$$= \frac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{(\cot A - \operatorname{cosec} A + 1)}$$

$$= (\cot A + \operatorname{cosec} A)$$

$$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$

$$= \frac{1 + \cos A}{\sin A}$$

$$= \text{RHS}$$

$$12) \frac{\cos A - \sin A + 1}{\cos A + \sin A + 1} = \frac{1}{\tan A + \sec A} \quad \text{ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A + 1}$$

$$= \frac{\frac{\cos A}{\cos A} - \frac{\sin A}{\cos A} + \frac{1}{\cos A}}{\frac{\cos A}{\cos A} + \frac{\sin A}{\cos A} + \frac{1}{\cos A}}$$

$$= \frac{1 - \tan A + \sec A}{1 + \tan A + \sec A}$$

$$= \frac{(1 - \tan A + \sec A)}{(\sec^2 A - \tan^2 A) + (\tan A + \sec A)}$$

$$= \frac{(1 - \tan A + \sec A)}{(\sec A + \tan A)(\sec A - \tan A) + (\tan A + \sec A)}$$

$$= \frac{(1 - \tan A + \sec A)}{(\sec A + \tan A)(\sec A - \tan A + 1)}$$

$$= \frac{1}{\tan A + \sec A}$$

$$= \text{RHS}$$

13) $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta) = \frac{1}{\tan \theta + \cot \theta}$ ಎಂದು ಸಾಧಿಸಿ.

$$\begin{aligned}
 \text{LHS} &= (\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta) \\
 &= \left(\frac{1}{\sin \theta} - \sin \theta \right) \left(\frac{1}{\cos \theta} - \cos \theta \right) \\
 &= \left(\frac{1 - \sin^2 \theta}{\sin \theta} \right) \left(\frac{1 - \cos^2 \theta}{\cos \theta} \right) \\
 &= \frac{\cos^2 \theta \times \sin^2 \theta}{\sin \theta \cos \theta} \\
 &= \sin \theta \cos \theta \text{ -----} \rightarrow \textcircled{1}
 \end{aligned}$$

$$\begin{aligned}
 \text{RHS} &= \frac{1}{\tan \theta + \cot \theta} \\
 &= \frac{1}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \\
 &= \frac{1}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}} \\
 &= \frac{\cos \theta \sin \theta}{1} \\
 &= \sin \theta \cos \theta \text{ -----} \rightarrow \textcircled{2}
 \end{aligned}$$

① ಮತ್ತು ② ರಿಂದ LHS = RHS

$$14) \left(1 + \frac{1}{\tan^2 \theta}\right) \left(1 + \frac{1}{\cot^2 \theta}\right) = \frac{1}{\sin^2 \theta - \sin^4 \theta} \quad \text{ಎಂದು ಸಾಧಿಸಿ.}$$

$$\begin{aligned} \text{LHS} &= \left(1 + \frac{1}{\tan^2 \theta}\right) \left(1 + \frac{1}{\cot^2 \theta}\right) \\ &= \left(1 + \frac{1}{\frac{\sin^2 \theta}{\cos^2 \theta}}\right) \left(1 + \frac{1}{\frac{\cos^2 \theta}{\sin^2 \theta}}\right) \\ &= \left(1 + \frac{\cos^2 \theta}{\sin^2 \theta}\right) \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right) \\ &= \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta}\right) \left(\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}\right) \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sin^2 \theta \times \cos^2 \theta} \\ &= \frac{1}{\sin^2 \theta (1 - \sin^2 \theta)} \\ &= \frac{1}{\sin^2 \theta - \sin^4 \theta} \\ &= \text{RHS} \end{aligned}$$

$$15) \sec^2 \theta - \frac{\sin^2 \theta - 2\sin^4 \theta}{2\cos^4 \theta - \cos^2 \theta} = 1 \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \sec^2 \theta - \frac{\sin^2 \theta - 2\sin^4 \theta}{2\cos^4 \theta - \cos^2 \theta}$$

$$= \sec^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \left(\frac{1 - 2\sin^2 \theta}{2\cos^2 \theta - 1} \right)$$

$$= \sec^2 \theta - \tan^2 \theta \left(\frac{\sin^2 \theta + \cos^2 \theta - 2\sin^2 \theta}{2\cos^2 \theta - \sin^2 \theta - \cos^2 \theta} \right)$$

$$= \sec^2 \theta - \tan^2 \theta \left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta} \right)$$

$$= \sec^2 \theta - \tan^2 \theta$$

$$= 1$$

$$= \text{RHS}$$

16) $(1 + \cot A + \tan A)(\sin A - \cos A) = \sin A \tan A - \cot A \cos A$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{RHS} = \sin A \tan A - \cot A \cos A$$

$$= \sin A \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A} \cos A$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cos A)}{\sin A \cos A}$$

$$= (\sin A - \cos A) \left(\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A} + \frac{\sin A \cos A}{\sin A \cos A} \right)$$

$$= (\sin A - \cos A)(\tan A + \cot A + 1)$$

$$= \text{LHS}$$

17) $\frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta} = 2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta}$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta}$$

$$= \frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta} \times \frac{\operatorname{cosec} \theta - \cot \theta}{\operatorname{cosec} \theta - \cot \theta}$$

$$= \frac{\sin \theta \operatorname{cosec} \theta - \sin \theta \cot \theta}{\operatorname{cosec}^2 \theta - \cot^2 \theta}$$

$$= \frac{\sin \theta \times \frac{1}{\sin \theta} - \sin \theta \times \frac{\cos \theta}{\sin \theta}}{1}$$

$$= 1 - \cos \theta \text{ ----- } \rightarrow \textcircled{1}$$

① ಮತ್ತು ② ರಿಂದ LHS=RHS

$$\text{RHS} = 2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta}$$

$$= \frac{2 \cot \theta - 2 \operatorname{cosec} \theta + \sin \theta}{\cot \theta - \operatorname{cosec} \theta}$$

$$= \frac{2 \frac{\cos \theta}{\sin \theta} - 2 \frac{1}{\sin \theta} + \sin \theta}{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}}$$

$$= \frac{2 \cos \theta - 2 + \sin^2 \theta}{\frac{\sin \theta}{\cos \theta - 1}}$$

$$= \frac{2 \cos \theta - 2 + 1 - \cos^2 \theta}{\cos \theta - 1}$$

$$= \frac{2 \cos \theta - 1 - \cos^2 \theta}{\cos \theta - 1}$$

$$= \left[\frac{-\{\cos \theta - 1\}^2}{\cos \theta - 1} \right]$$

$$= -(\cos \theta - 1) = 1 - \cos \theta \text{ ----- } \rightarrow \textcircled{2}$$

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$$18) \frac{\sin \theta}{1 - \cos \theta} + \frac{\tan \theta}{1 + \cos \theta} = \cot \theta + \sec \theta \operatorname{cosec} \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta}{1 - \cos \theta} + \frac{\tan \theta}{1 + \cos \theta} \\ &= \frac{\sin \theta (1 + \cos \theta) + \tan \theta (1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)} \\ &= \frac{\sin \theta + \sin \theta \cos \theta + \tan \theta - \tan \theta \cos \theta}{1 - \cos^2 \theta} \\ &= \frac{\sin \theta + \sin \theta \cos \theta + \frac{\sin \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \cos \theta}{\sin^2 \theta} \\ &= \frac{\sin \theta + \sin \theta \cos \theta + \frac{\sin \theta - \sin \theta \cos \theta}{\cos \theta}}{\sin^2 \theta} \end{aligned}$$

$$\begin{aligned} &= \frac{\sin \theta \cos \theta + \sin \theta \cos^2 \theta + \sin \theta - \sin \theta \cos \theta}{\cos \theta \sin^2 \theta} \\ &= \frac{\sin \theta \cos^2 \theta}{\cos \theta \sin^2 \theta} + \frac{\sin \theta}{\cos \theta \sin^2 \theta} \\ &= \frac{\cos \theta}{\sin \theta} + \frac{1}{\cos \theta \sin \theta} \\ &= \cot \theta + \sec \theta \operatorname{cosec} \theta = \text{RHS} \end{aligned}$$

19) $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ ಮತ್ತು $x \cos \theta = y \cos \theta$ ಆದಾಗ $x^2 + y^2 = 1$ ಎಂದು ಸಾಧಿಸಿ.

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$$x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$$

$$x \sin \theta \sin^2 \theta + y \cos \theta \cos^2 \theta = \sin \theta \cos \theta$$

$$y \cos \theta \sin^2 \theta + y \cos \theta \cos^2 \theta = \sin \theta \cos \theta$$

$$y \cos \theta (\sin^2 \theta + \cos^2 \theta) = \sin \theta \cos \theta$$

$$y \cos \theta (1) = \sin \theta \cos \theta$$

$$y = \sin \theta$$

ಇದೇ ರೀತಿಯಲ್ಲಿ $x = \cos \theta$

$$x^2 + y^2 = \cos^2 \theta + \sin^2 \theta$$

$$x^2 + y^2 = 1$$

20) $\frac{1+\cos A}{1-\cos A} = (\operatorname{cosec} A + \cot A)^2$ ಎಂದು ಸಾಧಿಸಿ.

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$$\text{RHS} = \frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$$

$$= \frac{(1+\cos A)^2}{1-\cos^2 A}$$

$$= \frac{1 + \cos^2 A + 2 \cos A}{\sin^2 A}$$

$$= \frac{1}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} + \frac{2 \cos A}{\sin^2 A}$$

$$= \operatorname{cosec}^2 A + \cot^2 A + 2 \cot A \operatorname{cosec} A$$

$$= (\operatorname{cosec} A + \cot A)^2 = \text{RHS}$$

21) $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} = \sec \theta + \tan \theta$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \sqrt{\frac{(1+\sin \theta) \times (1+\sin \theta)}{(1-\sin \theta) \times (1+\sin \theta)}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{1-\sin^2 \theta}}$$

$$= \sqrt{\frac{(1+\sin \theta)^2}{\cos^2 \theta}}$$

$$= \frac{1+\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \sec \theta + \tan \theta$$

$$= \text{RHS}$$

$$22) \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A} \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A) - \{(\sec A + \tan A)(\sec A - \tan A)\}}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{(\tan A - \sec A + 1)}$$

$$= (\tan A + \sec A)$$

$$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$$

$$= \frac{\sin A + 1}{\cos A}$$

$$= \text{RHS}$$

$$23) \frac{\sin(90-A)}{1-\tan A} + \frac{\cos(90-A)}{1-\cot A} = \cos A + \sin A \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\begin{aligned} \text{LHS} &= \frac{\sin(90-A)}{1-\tan A} + \frac{\cos(90-A)}{1-\cot A} \\ &= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}} \\ &= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\ &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\ &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \end{aligned}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

$$= \text{RHS}$$

$$24) \frac{\cos \theta}{\sec \theta - \tan \theta} = 1 + \sin \theta \text{ ಎಂದು ಸಾಧಿಸಿ}$$

24

$$\begin{aligned} \text{LHS} &= \frac{\cos \theta}{\sec \theta - \tan \theta} \\ &= \frac{\cos \theta}{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\cos^2 \theta}{1 - \sin \theta} \\ &= \frac{1 - \sin^2 \theta}{1 - \sin \theta} \\ &= \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 - \sin \theta)} \\ &= 1 + \sin \theta = \text{RHS} \end{aligned}$$

$$25) \frac{\operatorname{cosec}A+1}{\operatorname{cosec}A-1} = (\sec A + \tan A)^2 \text{ ಎಂದು ಸಾಧಿಸಿ .}$$

25

$$\begin{aligned} LHS &= \frac{\operatorname{cosec}A+1}{\operatorname{cosec}A-1} \times \frac{\operatorname{cosec}A+1}{\operatorname{cosec}A+1} \\ &= \frac{\cos \theta}{\frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta}} \\ &= \frac{(\operatorname{cosec}A+1)^2}{\operatorname{cosec}^2A-1} \\ &= \frac{\operatorname{cosec}^2A+1+2 \operatorname{cosec}A}{\cot^2A} \\ &= \frac{\operatorname{cosec}^2A}{\cot^2A} + \frac{1}{\cot^2A} + \frac{2 \operatorname{cosec}A}{\cot^2A} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\frac{\sin^2A}{\cos^2A}} + \frac{1}{\cot^2A} + \frac{2 \frac{1}{\sin A}}{\frac{\cos A \cos A}{\sin A \sin A}} \\ &= \frac{1}{\cos^2A} + \tan^2A + \frac{2 \sin A}{\cos A \cos A} \\ &= \sec^2A + \tan^2A + 2 \sec A \tan A \\ &= (\sec A + \tan A)^2 \\ &= \text{RHS} \end{aligned}$$

26) $\sec^4 A - \sec^2 A = \tan^4 A + \tan^2 A$ ಎಂದು ಸಾಧಿಸಿ.

26

$$LHS = \sec^4 A - \sec^2 A$$

$$= \sec^2 A \sec^2 A - \sec^2 A$$

$$= \sec^2 A(\sec^2 A - 1)$$

$$= (\tan^2 A + 1)\tan^2 A$$

$$= \tan^4 A + \tan^2 A$$

$$= RHS$$

27) $\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

27

$$\text{LHS} = \tan^2 A - \sin^2 A$$

$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$$

$$= \frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}$$

$$= \frac{\sin^2 A (1 - \cos^2 A)}{\cos^2 A}$$

$$= \frac{\sin^2 A \sin^2 A}{\cos^2 A}$$

$$= \tan^2 A \sin^2 A$$

$$= \text{RHS}$$

$$28) \frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2} = \frac{1-\cos\theta}{1+\cos\theta} \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta+\cos\theta)^2}$$

$$= \frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+\sin^2\theta+\cos^2\theta+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1+1+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2+2\sin\theta+2\sin\theta\cos\theta+2\cos\theta}$$

$$= \frac{2(1+\sin\theta-\sin\theta\cos\theta-\cos\theta)}{2(1+\sin\theta+\sin\theta\cos\theta+\cos\theta)}$$

$$= \frac{(1-\cos\theta+\sin\theta-\sin\theta\cos\theta)}{(1+\cos\theta+\sin\theta+\sin\theta\cos\theta)}$$

$$= \frac{(1-\cos\theta)+\sin\theta(1-\cos\theta)}{(1+\cos\theta)+\sin\theta(1+\cos\theta)}$$

$$= \frac{(1-\cos\theta)(1+\sin\theta)}{(1+\cos\theta)(1+\sin\theta)}$$

$$= \frac{(1-\cos\theta)}{(1+\cos\theta)}$$

$$= \text{RHS}$$

29) $\frac{\sin^2 \theta}{1-\cos \theta} = \frac{1+\sec \theta}{\sec \theta}$ ಎಂದು ಸಾಧಿಸಿ.

$$\begin{aligned}\text{LHS} &= \frac{\sin^2 \theta}{1-\cos \theta} \\ &= \frac{1-\cos^2 \theta}{1-\cos \theta} \\ &= \frac{(1+\cos \theta)(1-\cos \theta)}{(1-\cos \theta)} \\ &= 1+\cos \theta \\ &= 1+\frac{1}{\sec \theta} \\ &= \frac{\sec \theta+1}{\sec \theta} \\ &= \text{RHS}\end{aligned}$$

$$30) \frac{\sin A}{(\sec A + \tan A - 1)} + \frac{\cos A}{(\operatorname{cosec} A + \cot A - 1)} = 1 \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$LHS = \frac{\sin A}{(\sec A + \tan A - 1)} + \frac{\cos A}{(\operatorname{cosec} A + \cot A - 1)}$$

$$= \frac{\sin A}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1\right)} + \frac{\cos A}{\left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1\right)}$$

$$= \frac{\sin A}{\left(\frac{1 + \sin A - \cos A}{\cos A}\right)} + \frac{\cos A}{\left(\frac{1 + \cos A - \sin A}{\sin A}\right)}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\cos A \sin A}{1 + \cos A - \sin A}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\cos A \sin A}{1 - (\sin A - \cos A)}$$

$$= \frac{\sin A \cos A \{1 - (\sin A - \cos A)\} + \cos A \sin A \{1 + \sin A - \cos A\}}{\{1 + \sin A - \cos A\} \{1 - (\sin A - \cos A)\}}$$

$$= \frac{\sin A \cos A - \sin^2 A \cos A + \sin A \cos^2 A + \sin A \cos A + \sin^2 A \cos A - \sin A \cos^2 A}{1 - (\sin^2 A + \cos^2 A - 2 \sin A \cos A)}$$

$$= \frac{2 \sin A \cos A}{1 - (1 - 2 \sin A \cos A)}$$

$$= \frac{2 \sin A \cos A}{1 - 1 + 2 \sin A \cos A}$$

$$= \frac{2 \sin A \cos A}{2 \sin A \cos A}$$

$$= 1$$

$$= \text{RHS}$$

31) $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$ ಎಂದು ಸಾಧಿಸಿ.

31

$$\text{LHS} = (\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2$$

$$= \left(\sin \theta + \frac{1}{\cos \theta}\right)^2 + \left(\cos \theta + \frac{1}{\sin \theta}\right)^2$$

$$= \left(\frac{\sin \theta \cos \theta + 1}{\cos \theta}\right)^2 + \left(\frac{\cos \theta \sin \theta + 1}{\sin \theta}\right)^2$$

$$= \frac{(\sin \theta \cos \theta + 1)^2}{\cos^2 \theta} + \frac{(\sin \theta \cos \theta + 1)^2}{\sin^2 \theta}$$

$$= \frac{(\sin \theta \cos \theta + 1)^2 \sin^2 \theta + (\sin \theta \cos \theta + 1)^2 \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}$$

$$= \frac{(\sin \theta \cos \theta + 1)^2 (\sin^2 \theta + \cos^2 \theta)}{(\sin \theta \cos \theta)^2}$$

$$= \left(\frac{\sin \theta \cos \theta + 1}{\sin \theta \cos \theta}\right)^2$$

$$= \left(\frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} + \frac{1}{\sin \theta \cos \theta}\right)^2$$

$$= (1 + \sec \theta \operatorname{cosec} \theta)^2$$

$$= \text{RHS}$$

$$32) \frac{1}{(\operatorname{cosec}A - \cot A)} - \frac{1}{(\sin A)} = \frac{1}{(\sin A)} - \frac{1}{(\operatorname{cosec}A + \cot A)} \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{(\operatorname{cosec}A - \cot A)} - \frac{1}{(\sin A)} \\ &= \frac{1}{(\operatorname{cosec}A - \cot A)} \times \frac{(\operatorname{cosec}A + \cot A)}{(\operatorname{cosec}A + \cot A)} - \frac{1}{(\sin A)} \\ &= \frac{(\operatorname{cosec}A + \cot A)}{(\operatorname{cosec}^2 A - \cot^2 A)} - \frac{1}{(\sin A)} \\ &= \frac{(\operatorname{cosec}A + \cot A)}{1} - \operatorname{cosec}A \\ &= \operatorname{cosec}A + \cot A - \operatorname{cosec}A \\ &= \cot A \text{ -----} \rightarrow \textcircled{1} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{(\sin A)} - \frac{1}{(\operatorname{cosec}A + \cot A)} \\ &= \frac{1}{(\sin A)} - \frac{1}{(\operatorname{cosec}A + \cot A)} \times \frac{(\operatorname{cosec}A - \cot A)}{(\operatorname{cosec}A - \cot A)} \\ &= \operatorname{cosec}A - \frac{(\operatorname{cosec}A - \cot A)}{(\operatorname{cosec}^2 A - \cot^2 A)} \\ &= \operatorname{cosec}A - \frac{(\operatorname{cosec}A - \cot A)}{1} \\ &= \operatorname{cosec}A - \operatorname{cosec}A + \cot A \\ &= \cot A \text{ -----} \rightarrow \textcircled{2} \end{aligned}$$

① ಮತ್ತು ② ರಿಂದ LHS = RHS

$$33) \frac{\operatorname{cosec}^2 A + 1}{\operatorname{cosec}^2 A - 1} = \sec^2 A + \tan^2 A \text{ ಎಂದು ಸಾಧಿಸಿ .}$$

33

$$\text{LHS} = \frac{\operatorname{cosec}^2 A + 1}{\operatorname{cosec}^2 A - 1}$$

$$= \frac{\frac{1}{\sin^2 A} + 1}{\frac{1}{\sin^2 A} - 1}$$

$$= \frac{\frac{1 + \sin^2 A}{\sin^2 A}}{\frac{1 - \sin^2 A}{\sin^2 A}}$$

$$= \frac{1 + \sin^2 A}{1 - \sin^2 A}$$

$$= \frac{1 + \sin^2 A}{\cos^2 A}$$

$$= \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A}$$

$$= \sec^2 A + \tan^2 A$$

$$= \text{RHS}$$

34) $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}$$

$$= \sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}$$

$$= \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}}$$

$$= \sqrt{\frac{1}{\cos^2 \theta \sin^2 \theta}}$$

$$= \sqrt{\sec^2 \theta \operatorname{cosec}^2 \theta}$$

$$= \sec \theta \operatorname{cosec} \theta$$

$$\text{RHS} = \tan \theta + \cot \theta$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$= \sec \theta \operatorname{cosec} \theta$$

$$= \text{RHS}$$

36) $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$= 2 \operatorname{cosec} \theta$$

$$= \text{RHS}$$

36) $(\sin \theta + \cos \theta)(\tan \theta + \cot \theta) = \sec \theta + \operatorname{cosec} \theta$ ಎಂದು ಸಾಧಿಸಿ.

36

$$\text{LHS} = (\sin \theta + \cos \theta)(\tan \theta + \cot \theta)$$

$$= (\sin \theta + \cos \theta) \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= (\sin \theta + \cos \theta) \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right)$$

$$= (\sin \theta + \cos \theta) \left(\frac{1}{\cos \theta \sin \theta} \right)$$

$$= \left(\frac{\sin \theta}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} \right)$$

$$= \left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta} \right)$$

$$= \sec \theta + \operatorname{cosec} \theta$$

$$= \text{RHS}$$

$$37) 1 + \frac{\cot^2 A}{1 + \operatorname{cosec} A} = \operatorname{cosec} A \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\begin{aligned} \text{LHS} &= 1 + \frac{\cot^2 A}{1 + \operatorname{cosec} A} \\ &= 1 + \frac{\operatorname{cosec}^2 A - 1}{1 + \operatorname{cosec} A} \\ &= 1 + \frac{\operatorname{cosec}^2 A - 1}{1 + \operatorname{cosec} A} \\ &= 1 + \frac{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)}{(1 + \operatorname{cosec} A)} \\ &= 1 + \operatorname{cosec} A - 1 \\ &= \operatorname{cosec} A \\ &= \text{RHS} \end{aligned}$$

$$38) \frac{\tan \theta}{1+\sec \theta} - \frac{\tan \theta}{1-\sec \theta} = 2 \operatorname{cosec} \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

38

$$\text{LHS} = \frac{\tan \theta}{1+\sec \theta} - \frac{\tan \theta}{1-\sec \theta}$$

$$= \tan \theta \left[\frac{1}{1+\sec \theta} - \frac{1}{1-\sec \theta} \right]$$

$$= \tan \theta \left[\frac{1-\sec \theta - 1 - \sec \theta}{(1+\sec \theta)(1-\sec \theta)} \right]$$

$$= \tan \theta \left[\frac{-2\sec \theta}{(1-\sec^2 \theta)} \right]$$

$$= \tan \theta \left[\frac{-2\sec \theta}{-\tan^2 \theta} \right]$$

$$= \left[\frac{\frac{2}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \right]$$

$$= 2 \frac{1}{\sin \theta}$$

$$= 2 \operatorname{cosec} \theta$$

$$= \text{RHS}$$

39) $\sec A = x + \frac{1}{4x}$ ಆದರೆ $\sec A + \tan A = 2x$ ಎಂದು ಸಾಧಿಸಿ.

$$\sec A = x + \frac{1}{4x}$$

ವರ್ಗಗೊಳಿಸಿದಾಗ

$$\sec^2 A = \left(x + \frac{1}{4x}\right)^2$$

$$\sec^2 A = x^2 + \frac{1}{16x^2} + 2(x)\left(\frac{1}{4x}\right)$$

$$1 + \tan^2 A = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} - \frac{2x}{4x}$$

$$\tan^2 A = \left(x - \frac{1}{4x}\right)^2$$

$$\tan A = x - \frac{1}{4x}$$

$$\sec A + \tan A = x + \frac{1}{4x} + x - \frac{1}{4x}$$

$$\sec A + \tan A = 2x$$

40) $\sec A = x + \frac{1}{4x}$ ಆದರೆ $\sec A - \tan A = \frac{1}{2x}$ ಎಂದು ಸಾಧಿಸಿ.

40

$$\sec A = x + \frac{1}{4x}$$

ವರ್ಗಗೊಳಿಸಿದಾಗ

$$\sec^2 A = \left(x + \frac{1}{4x}\right)^2$$

$$\sec^2 A = x^2 + \frac{1}{16x^2} + 2(x)\left(\frac{1}{4x}\right)$$

$$1 + \tan^2 A = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\tan^2 A = x^2 + \frac{1}{16x^2} - \frac{2x}{4x}$$

$$\tan^2 A = \left(x - \frac{1}{4x}\right)^2$$

$$\tan A = x - \frac{1}{4x}$$

$$\sec A - \tan A = x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$\sec A - \tan A = \frac{2}{4x}$$

$$\sec A - \tan A = \frac{1}{2x}$$

41) $\cos^4 A - \sin^4 A = 1 - \sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \cos^4 A - \sin^4 A$$

$$= (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A + \sin^2 A) (\cos^2 A - \sin^2 A)$$

$$= (\cos^2 A + \sin^2 A) (\cos^2 A - \sin^2 A)$$

$$= 1 (1 - \sin^2 A - \sin^2 A)$$

$$= 1 - \sin^2 A$$

$$= \text{RHS}$$

$$42) \frac{\cos \theta}{1-\tan \theta} + \frac{\sin \theta}{1-\cot \theta} = \sin \theta + \cos \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\cos \theta}{1-\tan \theta} + \frac{\sin \theta}{1-\cot \theta}$$

$$= \frac{\cos \theta}{1-\frac{\sin \theta}{\cos \theta}} + \frac{\sin \theta}{1-\frac{\cos \theta}{\sin \theta}}$$

$$= \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} + \frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}}$$

$$= \frac{\cos^2 \theta}{\cos \theta - \sin \theta} + \frac{\sin^2 \theta}{\sin \theta - \cos \theta}$$

$$= \frac{\cos^2 \theta}{\cos \theta - \sin \theta} - \frac{\sin^2 \theta}{\cos \theta - \sin \theta}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta}$$

$$= \frac{(\cos \theta + \sin \theta)(\cos \theta - \sin \theta)}{\cos \theta - \sin \theta}$$

$$= (\cos \theta + \sin \theta)$$

$$= \text{RHS}$$

43) $\frac{\tan \theta - \sin \theta}{\sin^3 \theta} = \frac{\sec \theta}{1 + \cos \theta}$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} - \sin \theta}{\sin^3 \theta}$$

$$= \frac{\frac{\sin \theta(1 - \cos \theta)}{\cos \theta}}{\sin^3 \theta}$$

$$= \frac{1}{\sin^2 \theta} \left[\frac{1 - \cos \theta}{\cos \theta} \right]$$

$$= \frac{1}{1 - \cos^2 \theta} \left[\frac{1 - \cos \theta}{\cos \theta} \right]$$

$$= \left[\frac{1 - \cos \theta}{(1 + \cos \theta)(1 - \cos \theta) \cos \theta} \right]$$

$$= \left[\frac{1}{(1 + \cos \theta) \cos \theta} \right]$$

$$= \frac{\sec \theta}{1 + \cos \theta}$$

$$= \text{RHS}$$

44) $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A \sec^2 A$ ಎಂದು ಸಾಧಿಸಿ.

$$\begin{aligned}\text{LHS} &= \sec^6 A - \tan^6 A \\ &= (\sec^2 A)^3 - \tan^6 A \\ &= (1 + \tan^2 A)^3 - \tan^6 A \\ &= 1 + \tan^6 A + 3 \tan^2 A (1 + \tan^2 A) - \tan^6 A \\ &= 1 + 3 \tan^2 A (1 + \tan^2 A) \\ &= 1 + 3 \tan^2 A \sec^2 A \\ &= \text{RHS}\end{aligned}$$

$$45) \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \sqrt{\frac{(1 - \cos \theta) \times (1 - \cos \theta)}{(1 + \cos \theta) \times (1 - \cos \theta)}}$$

$$= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}}$$

$$= \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}}$$

$$= \frac{1 - \cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$= \operatorname{cosec} \theta - \cot \theta$$

$$= \text{RHS}$$

46) $(1 - \operatorname{cosec} A)(1 - \sec A)(1 + \operatorname{cosec} A)(1 + \cos A) = \cos A$ ಎಂದು ಸಾಧಿಸಿ.

$$\text{LHS} = (1 - \operatorname{cosec} A)(1 - \sec A)(1 + \operatorname{cosec} A)(1 + \cos A) = \cos A$$

$$= (1 - \operatorname{cosec}^2 A) \left(1 - \frac{1}{\cos A}\right) (1 + \cos A)$$

$$= -\cot^2 A \left(\frac{\cos A - 1}{\cos A}\right) (1 + \cos A)$$

$$= (-\cot^2 A) \left(\frac{\cos^2 A - 1}{\cos A}\right)$$

$$= \frac{(-\cot^2 A) \times (-\sin^2 A)}{\cos A}$$

$$= \frac{\frac{\cos^2 A}{\sin^2 A} \times (\sin^2 A)}{\cos A}$$

$$= \cos A$$

$$= \text{RHS}$$

47) $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = \tan^2 \theta + \cot^2 \theta + 7$ ಎಂದು ಸಾಧಿಸಿ.

47

$$\begin{aligned} LHS &= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta) + (\cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta) \\ &= \sin^2 \theta + \cos^2 \theta + \operatorname{cosec}^2 \theta + \sec^2 \theta + 2 \sin \theta \frac{1}{\sin \theta} + 2 \cos \theta \frac{1}{\cos \theta} \\ &= 1+1+ \cot^2 \theta+ \tan^2 \theta + 1 + 2 + 2 \\ &= \tan^2 \theta + \cot^2 \theta + 7 \\ &= \text{RHS} \end{aligned}$$

$$48) \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = (1 + \sin A) \sec A \text{ ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= (\tan A + \sec A)$$

$$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= (1 + \sin A) \sec A$$

$$= \text{RHS}$$

$$49) \frac{\cos \theta}{1+\sin \theta} + \frac{1+\sin \theta}{\cos \theta} = 2\sec \theta \quad \text{ಎಂದು ಸಾಧಿಸಿ.}$$

49

$$\begin{aligned} LHS &= \frac{\cos \theta}{1+\sin \theta} + \frac{1+\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta + (1+\sin \theta)^2}{\cos \theta(1+\sin \theta)} \\ &= \frac{\cos^2 \theta + 1 + \sin^2 \theta + 2\sin \theta}{\cos \theta(1+\sin \theta)} \\ &= \frac{1+1+2\sin \theta}{\cos \theta(1+\sin \theta)} \\ &= \frac{2+2\sin \theta}{\cos \theta(1+\sin \theta)} \\ &= \frac{2(1+\sin \theta)}{\cos \theta(1+\sin \theta)} \\ &= 2\sec \theta = \text{RHS} \end{aligned}$$

$$50) \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} = \operatorname{cosec} \theta + \cot \theta \quad \text{ಎಂದು ಸಾಧಿಸಿ.}$$

$$\text{LHS} = \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1}$$

$$= \frac{\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\sin \theta} + \frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\sin \theta} - \frac{1}{\sin \theta}}$$

$$= \frac{\cot \theta - 1 + \operatorname{cosec} \theta}{\cot \theta + 1 - \operatorname{cosec} \theta}$$

$$= \frac{\cot \theta - (\operatorname{cosec}^2 \theta - \cot^2 \theta) + \operatorname{cosec} \theta}{\cot \theta + 1 - \operatorname{cosec} \theta}$$

$$= \frac{\cot \theta + \operatorname{cosec} \theta - [(\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta)]}{\cot \theta + 1 - \operatorname{cosec} \theta}$$

$$= \frac{(\cot \theta + \operatorname{cosec} \theta)[1 - (\operatorname{cosec} \theta - \cot \theta)]}{\cot \theta + 1 - \operatorname{cosec} \theta}$$

$$= \frac{(\cot \theta + \operatorname{cosec} \theta)(1 - \operatorname{cosec} \theta + \cot \theta)}{\cot \theta + 1 - \operatorname{cosec} \theta}$$

$$= (\cot \theta + \operatorname{cosec} \theta)$$

$$= \text{RHS}$$

ಎಸ್ ಹರ್ಷ & ಹೆಚ್.ಜಿ. ದೀಪಶ್ರೀ

ಧನ್ಯವಾದಗಳು

ಸರ್ಕಾರಿ ಪ್ರೌಢಶಾಲೆ, ಹೆಡತಲೆ, ನಂಜನಗೂಡು, ಮೈಸೂರು