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MATHS

English Medium

Part-1

All Solutions

Based on new syllubus

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Contents

Part –1

1 Arithmetic progression

1.2 Arithmetic Progressions:

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

i)	**1, 2, 3, 4 ....**	each term is 1 more than the term preceding it.
ii)	**100, 70, 40, 10 ....**	each term is 30 less than the term preceding it.
iii)	**-3, -2, -1, 0 ....**	each term is obtained by adding 1 to the term preceding it.
iv)	**3, 3, 3, 3 ....**	all the terms in the list are 3 , i.e., each term is obtained by adding(or subtracting) 0 to the term preceding it.
v)	**-1, -1.5,-2.0,-2.5 ....**	each term is obtained by adding – 0.5 to (i.e., subtracting 0.5 from) the term preceding it.

This fixed number is called the common difference of the AP. Remember that it can be positive, negative or zero.

Let us denote the first term of an AP by a_1, second term by a_2, . . ., nth term by a_n and the common difference by d. Then the AP becomes $a_1, a_2, a_3, . . ., a_n$.

So, $a_2 - a_1 = a_3 - a_2 = . . . = a_n - a_{n-1} = d$

$a, a + d, a + 2d, a + 3d, . . .$

Represents an arithmetic progression where a is the first term and d the common difference. This is called the general form of an AP.

Finite AP.:

In an AP there are only a finite number of terms. Such an AP is called a finite AP. Each of these Arithmetic Progressions (APs) has a last term.

a) The heights (in cm) of some students of a school standing in a queue in the morning assembly are 147 , 148, 149, . . ., 157.

b) The balance money (in Rs) after paying 5 % of the total loan of Rs 1000 every month is 950, 900, 850, 800, . . ., 50.

c) The total savings (in Rs) after every month for 10 months when Rs50 are saved each month are 50, 100, 150, 200, 250, 300, 350, 400, 450, 500.

Infinite AP.:

In an AP there are infinite number of terms. Such an AP is called a infinite AP. Each of these Arithmetic Progressions (APs) do not have last term.

a) 3, 7, 11, ...

b) 1, 4, 7, 10, . . .

c) −10, −15, −20, ...

Note: You will If we know the first term a' and the common difference d' then we can write an AP.

Example 1: $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$........... **write the first term a and the common difference d.**

Here, $a_1 = \frac{3}{2}$ $d = a_2 - a_1 = \frac{1}{2} - \frac{3}{2} = -1$ $a_3 - a_2 = -\frac{1}{2} - \frac{1}{2} = -1$

Example 2 : Which of the following list of numbers form an AP? If they form an AP, write the next two terms

i) 4, 10, 16, 22 ....

ii) 1, −1, −3, −5 ....

iii) −2, 2, −2, 2 ....

iv) 1, 1, 1, 2, 2, 2, 3, 3, 3 ....

Solution :

i) 4, 10, 16, 22 ....

$a_2 - a_1 = 10 - 4 = 6$
$a_3 - a_2 = 16 - 10 = 6$
$a_3 - a_2 = 22 - 16 = 6$
i.e., $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference d = 6. The next two terms are: 22 + 6 = 28 and 28 + 6 = 34.

ii) 1, −1, −3, −5 ....

$a_2 - a_1 = -1 - 1 = -2$
$a_3 - a_2 = -3 - (-1) = -2$
$a_3 - a_2 = -5 - (-3) = -2$
i.e., $a_{k+1} - a_k$ is the same every time.
So, the given list of numbers forms an AP with the common difference d = -2. The next two terms are: – 5 + (– 2) = – 7 and – 7 + (– 2) = – 9

iii) -2, 2, -2, 2 ....

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$
$a_3 - a_2 = -2 - 2 = -4$
$a_3 - a_2 = 2 - (-2) = 2 + 2 = 4$
Here, $a_{k+1} \neq a_k$ So, the given list of numbers does not form an AP.

iv) 1, 1, 1, 2, 2, 2, 3, 3, 3 ....

$a_2 - a_1 = 1 - 1 = 0$
$a_3 - a_2 = 1 - 1 = 0$
$a_3 - a_2 = 2 - 1 = 1$
$a_2 - a_1 = a_3 - a_2 \neq a_3 - a_2$ So, the given list of numbers does not form an AP.

EXERCISE 1.1

1. **In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?**

i). The taxi fare after each km when the fare is Rs15 for the first km and Rs 8 for each additional km

The first term $a_1 = 15$, $a_2 = 15 + 8 = 23$,$a_3 = 23 + 8 = 31$ ....

Here, each term is obtained by adding a common difference = 8, except first term.

ii). The amount of air present in a cylinder when a vacuum removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

Let the initial volume of the air present in the cyclinder be V.

The remaining air in the cylinder after using vacuum pump first time V - $\frac{1}{4} = \frac{3V}{4}$,

Remaining air in the cylinder after using vacuum pump second time

$\frac{3V}{4} - \frac{3V}{4} \times \frac{1}{4} = \frac{3V}{4} - \frac{3V}{16} = \frac{9V}{16}$ and so on.

Here, the terms are V, $\frac{3V}{4}, \frac{9V}{16}$ .. ..

$a_2 - a_1 = \frac{3V}{4} - V = -\frac{V}{4}$

$a_3 - a_2 = \frac{9V}{16} - \frac{3V}{4} = \frac{9V}{16} - \frac{12V}{16} = -\frac{3V}{16}$

$\therefore a_2 - a_1 \neq a_3 - a_2$

Hence, it does not form an AP

iii). The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

The cost of digging for the first meter = Rs 150
Cost of digging for the second meter = 150+50 = Rs 200
Cost of digging for the third meter = 200+50 = Rs 250
Cost of digging for the fourth meter = 250+50 = Rs 300
Thus the list of numbers is 150, 200, 250, 300........

Here, we can find the common difference = 50
So it forms an AP.

iv). The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8 % per annum.

We know that amount A = $P\left(1 + \frac{r}{100}\right)^n$

Here, P = 10,000; r = 8%, n = 1,2,3…

Amount in first year = $10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} = 100 \times 108$ = Rs 10800

Amount in second year = $10000\left(1 + \frac{8}{100}\right)^2 = 10000 \times \frac{108}{100} \times \frac{108}{100} = 108 \times 108$ = Rs 11664

Thus the list of numbers is 10000, 10800, 11664 .. ..

$a_2 - a_1 = 10800 - 10000 = 800$

$a_3 - a_2 = 11664 - 10800 = 864$

There for $a_2 - a_1 \neq a_3 - a_2$

Hence it does not form an AP.

2. **Write first four terms of the AP, when the first term a and the common difference d are given as follows:**

i) a = 10, d = 10

a_1 = 10,
$a_2 = a_1 + d$ = 10 + 10 = 20
$a_3 = a_2 + d$ = 20 + 10 = 30
$a_4 = a_3 + d$ = 30 + 10 = 40
Thus the first four terms of an AP are 10, 20, 30, 40

ii) a = -2, d = 0

a_1 = -2,
$a_2 = a_1 + d$ = -2 + 0 = -2
$a_3 = a_2 + d$ = -2 + 0 = -2
$a_4 = a_3 + d$ = -2 + 0 = -2
Thus the first four terms of an AP are -2, -2, -2, -2,

iii) a = 4, d = -3

a_1 = 4,
$a_2 = a_1 + d$ = 4 - 3 = 1
$a_3 = a_2 + d$ = 1 - 3 = -2
$a_4 = a_3 + d$ = -2 - 3 = -5
Thus the first four terms of an AP are 4, 1, -2, -5

iv) a = -1, d = $\frac{1}{2}$

a_1 = -1,
$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$
$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$
$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$
Thus the first four terms of an AP are -1, $-\frac{1}{2}$, 0, $\frac{1}{2}$

v) a = -1.25, d = -0.25

a_1 = -1.25
$a_2 = a_1 + d$ = -1.25 - 0.25 = -1.50
$a_3 = a_2 + d$ = -1.50 - 0.25 = - 1.75
$a_4 = a_3 + d$ = -1.75 + 0.25 = -2.00
Thus the first four terms of an AP are –1.25, -1.50, -1.75, -2.00

3. For the following APs, write the first term and the common difference:

i) 3, 1, - 1, - 3.......

The first term a = 3,
Common difference d = a_2 - a_1 = 1 – 3 = -2

ii) -5, -1, 3, 7.......

The first term a = 5
Common difference d = a_2 - a_1 = -1 – (-5) = -1 + 5 = 4

iii) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots\ldots$

The first term a = $\frac{1}{2}$
Common difference d = a_2 - a_1 = $\frac{1}{2} - \frac{1}{2} = 0$

iv) 0.6, 1.7, 2.8, 3.9, ...

The first term a = 0.6
Common difference d = a_2 - a_1 = 1.7 – 0.6 = 1.1

4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms

i) 2, 4, 8, 16 ....

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ ....

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = \frac{1}{2}$

The next 3 terms of this AP are, $\frac{7}{2} + \frac{1}{2} = 4;\ 4 + \frac{1}{2} = \frac{9}{2};\ \frac{9}{2} + \frac{1}{2} = 5$

iii) -1.2, -3.2, -5.2, -7.2 .....

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$

$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = -2$

The next 3 terms of this AP are, $-7.2 - 2 = -9.2$; $-9.2 - 2 = -11.2$; $-11.2 - 2 = -13.2$

iv) -10, -6, -2, 2 ......

$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = 4$

The next 3 terms of this AP are $2 + 4 = 6$; $6 + 4 = 10$; $10 + 4 = 14$

i) **3, 3+ $\sqrt{2}$, 3 + 2$\sqrt{2}$, 3 + 3$\sqrt{2}$, ...**

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 + \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = \sqrt{2}$

The next 3 terms of this AP are $3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$; $3 + 5\sqrt{2}$; $3 + 6\sqrt{2}$

ii) **0.2, 0.22, 0.222, 0.2222 ....**

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

iii) **0, -4, -8, -12 ....**

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = -4$

The next 3 terms of this AP are $-12 - 4 = -16$; -20; -24

iv) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\ldots$

$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = 0$

The next 3 terms of this AP are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$

v) **1, 3, 9, 27 ....**

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

vi) **a, 2a, 3a, 4a ....**

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = a$

The next 3 terms of this AP are $5a, 6a, 7a$

vii) **a, a^2, a^3, a^4.......**

$a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

viii) **$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$........**

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = \sqrt{2}$

The next 3 terms of this AP are $\sqrt{50}, \sqrt{72}, \sqrt{98}$

ix) **$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$..........**

$a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

x) **$1^1, 3^2, 5^2, 7^2, \ldots\ldots$**

$a_2 - a_1 = 3^2 - 1^1 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

Here, $a_2 - a_1 \neq a_3 - a_2$

There fore the given list of numbers does not form an AP.

xv) $1^1, 5^2, 7^2, 73, \ldots\ldots\ldots$

$a_2 - a_1 = 5^2 - 1^1 = 25 - 1 = 24$

$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 7^2 = 73 - 49 = 24$

Here, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

Therefore the given list of numbers forms an AP with common difference $d = 24$

The next 3 terms of this AP are 73 + 24 = 97, 97 + 24 = 121, 121 + 24 = 145

1.3 n^{th} Term of an AP

The first term of an AP is a' Common difference is d' then the n^{th} term is

$$a_n = a + (n-1)d$$

n^{th} term from the last n [l–last term , d – Common difference

$$l - (n-1)d$$

Example 3 : Find the 10th term of the AP : 2, 7, 12, . . .

Solution : a = 2, d = 7 - 2 = 5 and n = 10

$a_n = a + (n-1)d$

$a_{10} = 2 + (10-1)5$

$a_{10} = 2 + (9)5$

$a_{10} = 2 + 45$

$a_{10} = 47$

Example 4 : Which term of the AP : 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

Solution: a = 21, d = 18 - 21 = -3 and a_n = -81. Now we have to find n

$a_n = a + (n-1)d$

$-81 = 21 + (n-1)(-3)$

$-81 = 21 - 3n + 3$

$-81 = 24 - 3n$

$3n = 24 + 81 = 105$

$n = 35$

which term is Zero?

$0 = 21 + (n-1)(-3)$

$0 = 21 - 3n + 3$
$3n = 24$
$n = 8$
8th term is Zero

Example 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solutin: $a + (n-1)d = a_n$
$a + (3-1)d = 5$
$a + 2d = 5$ ------- (1)
$a + (7-1)d = 9$
$a + 6d = 9$ --------(2)

a	+	**2d**	=	**5**
a	+	**6d**	=	**9**
		–4d	=	**- 4**

$\Rightarrow d = 1$
$\Rightarrow a + 2(1) = 5 \Rightarrow a + 2 = 5 \Rightarrow a = 5 - 2 = 3$
∴ **AP: 3, 4, 5, 6, – – –**

Alternate Method:
$d = \frac{a_p - a_q}{p-q}$
$a_p = a_7$; $a_q = a_3$
$d = \frac{a_7 - a_3}{7-3} = \frac{9-5}{7-3} = \frac{4}{4} = 1$
$a = a_p + (p-1)d$ a
$a = a_7 + (7-1)1$
$a = 9 + (7-1)1$
$a = 9 + 6 = 3$

Example 6 : Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .

Solution: $a = 5$, $d = 11 - 5 = 6$

$a + (n-1)d = a_n$
$5 + (n-1)6 = 301$
$5 + 6n - 6 = 301$
$6n - 1 = 301$
$6n = 301 + 1$
$6n = 302$
$n = \frac{302}{6} = \frac{151}{3}$
Here n is not an integer
There fore 301 is not a term of the list of numbers 5, 11, 17, 13 ....

Example 7 : How many two-digit numbers are divisible by 3?

Solution: 12, 15, 18 99

$a = 12$, $d = 3$, $a_n = 99$
$a + (n-1)d = a_n$
$12 + (n-1)3 = 99$
$12 + 3n - 3 = 99$
$3n + 9 = 99$
$3n = 99 - 9$
$3n = 90 \Rightarrow n = 30$
There for 30 two digit numbers are divisible by 3.

Example 8 : Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Solution: $a = 10$, $d = 7 - 10 = -3$, $l = -62$

$l = a + (n-1)d$
n^{th} term from the last $= l - (n-1)d$
$= -62 - (11-1)(-3)$
$= -62 + 33 - 3$
$= -62 + 30$
$= -32$

Example 9 : A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

Solution: The formula to calculate the simple intrest $\mathbf{I} = \frac{PRT}{100}$

So, the interest at the end of the 1st year = $\frac{1000x8x1}{100}$ = **Rs 80**

the interest at the end of the 2nd year = $\frac{1000x8x2}{100}$ = **Rs 160**

the interest at the end of the 3rd year = $\frac{1000x8x3}{100}$ = **Rs 240**

There fore the terms are 80, 160, 240, – – –

Here a_2 - a_1 = a_3 - a_2 = d = 80

It is an AP as the difference between the consecutive terms in the list is 80,
The interest at the end of 30 years $\mathbf{a_n}$; **a = 80, d = 80, n = 30**

$a_n = a + (n-1)d$
$a_{30} = 80 + (30-1)80$
$a_n = 80 + 29x80$
$a_n = 80 + 2320$
$a_n = \text{Rs } 2400$

Example 10 : In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Solution: The number of rose plants in the 1st, 2nd, 3rd, . . ., rows are :23, 21, 19,– – –

Here a_2 - a_1 = a_3 - a_2 = -2
There fore it is an AP. a = 23, d = -2, a_n = 5, n = ?

$a + (n-1)d = a_n$
$23 + (n-1)(-2) = 5$
$23 - 2n + 2 = 5$
$-2n + 25 = 5$
$-2n = 5 - 25$
$-2n = -20$
$n = 10$

So, there are 10 rows in the flower bed. = 10.

EXERCISE 1.2

1. **Fill in the blanks in the following table, given that a is the first term, d the common difference and a the nth term of the AP:**

	a	*d*	*n*	a_n
(i)	7	3	8	**28**
(ii)	− 18	**2**	10	0
(iii)	**46**	− 3	18	− 5
(iv)	− 18.9	2.5	**10**	3.6
(v)	3.5	0	105	**3.5**

i) $\mathbf{a_n = a + (n-1)d}$
$a_8 = 7 + (8-1)3$
$a_8 = 7 + 7x3$
$a_8 = 7 + 21$

$a_8 = 28$

i) $a_n = a + (n-1)d$

$0 = -18 + (10-1)d$

$0 = -18 + 9d$

$9d = 18$

$d = 2$

ii) $\mathbf{a_n = a + (n-1)d}$

$-5 = a + (18-1)(-3)$

$-5 = a - 17x3$

$-5 = a - 51$

$a = 46$

iii) $\mathbf{a_n = a + (n-1)d}$

$3.6 = -18.9 + (n-1)(2.5)$

$3.6 = -18.9 + 2.5n - 2.5$

$3.6 = -21.4 + 2.5n$

$2.5n = 3.6 + 21.4$

$n = \frac{25}{2.5} = \frac{250}{25} = 10$

iv) $\mathbf{a_n = a + (n-1)d}$

$a_n = 3.5 + (105-1)(0)$

$a_n = 3.5 + 104x0$

$a_n = 3.5$

2. Choose the correct choice in the following and justify :

(i) 30th term of the AP: 10, 7, 4, . . . , is

$a_n = a + (n-1)d$

$d = a_2 - a_1 = 7 - 10 = -3$

$a_{30} = 10 + (30-1)(-3)$

$a_{30} = 10 + (29)(-3)$

$a_{30} = 10 - 87$

$a_{30} = -77$

(A) 97 (B) 77 **(C) -77** (D) -87

(ii) 11th term of an AP: -3, $-\frac{1}{2}$, 2, ... is

$a_n = a + (n-1)d$

$d = a_2 - a_1 = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$

$a_{11} = -3 + (11-1)\left[\frac{5}{2}\right]$

$a_{11} = -3 + (10)\left[\frac{5}{2}\right]$

$a_{11} = -3 + 25$

$a_{11} = 22$

(A) 28 **(B) 22** (C) -38 (D) $-48\frac{1}{2}$

3. In the following APs, find the missing terms in the boxes :

i) 2, $\boxed{14}$, 26

ii) $\boxed{18}$ 13, $\boxed{8}$ 3,

iii) 5, $\boxed{6\frac{1}{2},}$ $\boxed{8}$, $9\frac{1}{2}$,

iv) -4, $\boxed{-2}$, $\boxed{0}$, $\boxed{2}$, $\boxed{4}$,, 6

v) $\boxed{53,}$, 38, $\boxed{23,}$, $\boxed{8,}$, $\boxed{-7,}$, -22

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Solution: $a_n = a + (n - 1)d$

$d = a_2 - a_1 = 8 - 3 = 5$; $a = 3$; $a_n = 78$; $n = ?$

$78 = 3 + (n - 1)5$

$78 = 3 + 5n - 5$

$78 = 5n - 2$

$5n = 78 + 2$

$5n = 80$

$n = 16$

5. Find the number of terms in each of the following APs :

Solution: i) **7, 13, 19 .... 205**

$a_n = a + (n - 1)d$

$d = a_2 - a_1 = 13 - 7 = 6$; $a = 7$; $a_n = 205$; $n = ?$

$205 = 7 + (n - 1)6$

$205 = 7 + 6n - 6$

$205 = 6n + 1$

$6n = 205 - 1$

$6n = 204$

$n = \frac{204}{6}$

$n = 34$

(ii) 18, $15\frac{1}{2}$, 13 .... -47

$a_n = a + (n - 1)d$

$d = a_2 - a_1 = 15\frac{1}{2} - 18 = -\frac{5}{2}$; $a = 18$; $a_n = -47$; $n = ?$

$-47 = 18 + (n - 1)\left[-\frac{5}{2}\right]$

$-47 = 18 - \frac{5}{2}n + \frac{5}{2}$

$-47 = \frac{36 - 5n + 5}{2}$

$-47 = \frac{41 - 5n}{2}$

$-94 = 41 - 5n$

$-5n = -94 - 41$

$-5n = -135$

$n = 27$

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Solution: $a_n = a + (n - 1)d$

$d = a_2 - a_1 = -3$; $a = 11$; $a_n = 150$; $n = ?$

$-150 = 11 + (n - 1)(-3)$

$-150 = 11 - 3n + 3$

$-150 = 14 - 3n$

$-3n = -150 - 14$

$-3n = -164$

$n = \frac{164}{3}$

n is not an integer. So, −150 is not a term of the AP: 11, 8, 5, 2, ..

7. **Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.**

Solution: $a_n = a + (n-1)d$

$a_{11} = 38, a_{16} = 73, a_{31} = ?$

$a + (11-1)d = 38$

$a + 10d = 38$ ----------------(1)

$a + (16-1)d = 73$

$a + 15d = 73$ ----------------(2)

from (1) and (2)

$\mathbf{a + 10d = 38}$
$\mathbf{a + 15d = 73}$
$-5d = -35$

$d = \frac{-45}{-3} = 7$

$(1) \Rightarrow a + 10x7 = 38$

$\Rightarrow a + 70 = 38$

$\Rightarrow a = 38 - 70$

$\Rightarrow a = -32$

$a_{31} = -32 + (31-1)7$

$a_{31} = -32 + (30)7$

$a_{31} = -32 + 210$

$a_{31} = 178$

Alternate Method:

$d = \frac{a_p - a_q}{p-q}$

$a_p = a_{16}$; $a_q = a_{11}$

$d = \frac{a_{16} - a_{11}}{16-11} = \frac{73-38}{5} = \frac{35}{5} = 7$

$a_n = a_p + (n-p)d$ a

$a_{31} = a_{16} + (31-16)7$

$a_{31} = 73 + (15)7$

$a_{31} = 73 + 105 = 178$

8. **An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.50**

Solution: $a + (n-1)d = a_n$

$n = 50, a_3 = 12, a_n = 106$ $a_{29} = ?$

$a + (50-1)d = 106$

$a + 49d = 106$ -------- (1)

$a + 2d = 12$ -------- -(2)

$\mathbf{a + 49d = 106}$
$\mathbf{a + 2d = 12}$
$\mathbf{47d = 94}$

$\Rightarrow d = 2$

Substitute d = 2 in eqn (2)

$a + 2(2) = 12$

$a + 4 = 12$

$a = 12 - 4$

$a = 8$

$a_{29} = 8 + (29-1)2$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56$

$a_{29} = 64$

Alternate Method:

$d = \frac{a_p - a_q}{p-q}$

$a_p = a_{50}$; $a_q = a_3$

$d = \frac{a_{50} - a_3}{50-3} = \frac{106-12}{47} = \frac{94}{47} = 2$

$a_n = a_p + (n-p)d$ a

$a_{29} = a_3 + (29-3)2$

$a_{31} = 12 + (26)2$

$a_{31} = 12 + 52 = 64$

9. **If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?**

Solution: $a_3 = 4$, $a_9 = -8$
$a_n = a + (n-1)d$
$a_3 = a + (3-1)d$
$4 = a + 2d$ ------------(i)
$a_9 = a + (9-1)d$
$-8 = a + 8d$ ---------- (ii)
Substract (i) from (ii),we get
$-12 = 6d \Rightarrow d = -2$
From equation (i),
$4 = a + 2(-2)$
$4 = a - 4$
$a = 8$
If $a_n = 0$,
$a_n = a + (n-1)d$
$0 = 8 + (n-1)(-2) = 8 - 2n + 2$
$2n = 10$
$n = 5$
So, the 5^{th} term is 0

Alternate method:
$d = \frac{a_p - a_q}{p-q}$
$a_p = a_9$; $a_q = a_3$
$d = \frac{a_9 - a_3}{9-3} = \frac{-8-4}{6} = \frac{-12}{6} = -2$
$a = a_p + (p-1)d$ a
$a = a_3 + (3-1)(-2)$
$a_{31} = 4 + (2)(-2)$
$a_{31} = 4 - 4 = 0$

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Solution: $a_n = a + (n-1)d$
$a_{17} = a + (17-1)d$
$a_{17} = a + 16d$
Similarlly, $a_{10} = a + 9d$
But , $a_{17} - a_{10} = 7$
$(a + 16d) - (a + 9d) = 7$
$7d = 7$
$d = 1$

11. Which term of the AP: 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Solution: AP: 3, 15, 27, 39, …
$a = 3$,
$d = a_2 - a_1 = 15 - 3 = 12$
$a_{54} = a + (54-1)d$
$a_{54} = 3 + (53)(12)$
$a_{54} = 3 + 636 = 639$
$132 + 639 = 771$
Now we find which term is 771
$a_n = 771$.
$a_n = a + (n-1)d$
$771 = 3 + (n-1)12$
$768 = (n-1)12$
$(n-1) = 64$
$n = 65$
There fore 65^{th} term is 132 more than 54^{th} term.

Or

n'th term is 132 more than 54^{th} term.
$n = 54 + \frac{132}{12}$
$= 54 + 11 = 65^{th}$ term.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Solution: Let the first terms of an AP's be a' and b'. Common difference – d

For the first AP,

$a_{100} = a + (100 - 1)\,d$
$a_{100} = a + 99d$
$a_{1000} = a + (1000 - 1)\,d$
$a_{1000} = a + 999d$

For 2nd AP,

$a_{100} = b + (100 - 1)\,d$
$a_{100} = b + 99d$
$a_{1000} = b + (1000 - 1)\,d$
$a_{1000} = b + 999d$

The difference of 100^{th} terms is 100

There for $(a + 99d) - (b + 99d) = 100$

$a - b = 100$ ------------------- (i)

The difference of 1000^{th} terms is ?

$(a + 999d) - (b + 999d) = a - b$

From equation (i) ,

$a_1 - a_2 = 100$

So, the difference of 1000^{th} terms is 100.

13. How many three-digit numbers are divisible by 7?

Solution: The first 3 digit number which is divisible by 7 is a = 105 and d = 7

The last 3 digit number which is divisible by 7 is $a_n = 994$

There for AP: 105, 112, 119, …994

n = ?

$a_n = a + (n - 1)\,d$
$994 = 105 + (n - 1)\,7$
$889 = (n - 1)\,7$
$(n - 1) = 127$
$n = 128$

There for 128 three digit numbers are divisible by 7.

Or

The 3-digit numbers which are divisible by 7 are 105, 112, 119, ... 994 .

These numbers are in AP:

$a = 105$ and $d = 7$, $a_n = 994$

$\Rightarrow a + (n - 1)\,d = 994$
$\Rightarrow 105 + (n - 1) \times 7 = 994$
$\Rightarrow 7(n - 1) = 889$
$\Rightarrow n - 1 = 127$
$\Rightarrow n = 128$

14. How many multiples of 4 lie between 10 and 250?

Solution: Multiples of 4 lie between 10 and 250 are 12, 16, 20, 24, …248

$a = 12$, $d = 4$, $a_n = 248$
$a_n = a + (n - 1)\,d$

$248 = 12 + (n - 1) \times 4$
$248 = 12 + 4n - 4$
$248 = 8 + 4n$
$4n = 248 - 8$
$4n = 240$
$n = 60$
Hence, ther are 60 multiples of 4 lie between 10 and 250.

15. For what value of n, are the nth terms of two APs: 63, 65, 67 , . . . and 3, 10, 17, . . . equal?

Solution: $a = 63$, $d = a_2 - a_2 = 65 - 63 = 2$
$a_n = a + (n - 1) d$
$a_n = 63 + (n - 1) 2 = 63 + 2n - 2$
$a_n = 61 + 2n$ ----------- (i)
3, 10, 17, …
$a = 3$, $d = a_2 - a_2 = 10 - 3 = 7$
$a_n = 3 + (n - 1) 7$
$a_n = 3 + 7n - 7$
$a_n = 7n - 4$ ------------ (ii)
According to question , n^{th} term of both AP's are equal.
$\Rightarrow 61 + 2n = 7n - 4$
$\Rightarrow 61 + 4 = 5n \Rightarrow 5n = 65$
$\Rightarrow n = 13$
Hence, the 13^{th} the two given AP's are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

$a_3 = 16$
$a + (3 - 1) d = 16$
$a + 2d = 16$ ---------- (i)
$a_7 - a_5 = 12$
$[a + (7 - 1) d] - [a + (5 - 1) d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
$d = 6$
From equation (i) ,
$a + 2 (6) = 16$
$a + 12 = 16$
$a = 4$
Then the required AP is 4, 10, 16, 22, …

17. Find the 20th term from the last term of the AP: 3, 8, 13, . . ., 253.

Given AP: 3, 8, 13, …, 253
n^{th} term from the last $= l - (n - 1)d$
$l = 253$, $a = 3$, $d = 5$
n^{th} term from the last $= 253 - (20 - 1)5$
$= 253 - (19)5$
$= 253 - 95$
$= 253 - 95 = 158$

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

$a_n = a + (n - 1)\,d$
$a_4 = a + (4 - 1)\,d$
$a_4 = a + 3d$
Similarlly,
$a_8 = a + 7d$
$a_6 = a + 5d$
$a_{10} = a + 9d$
But , $a_4 + a_8 = 24$
$a + 3d + a + 7d = 24$
$2a + 10d = 24$
$a + 5d = 12$ ------------(i)
$a_6 + a_{10} = 44$
$a + 5d + a + 910d = 44$
$2a + 14d = 44$
$a + 7d = 22$ -----------(ii)
By substracting (ii) from **(i)**,
$2d = 22 - 12$
$2d = 10$
$d = 5$
Substituting $d = 5$ in equation (i) ,
$a + 5d = 12$
$a + 5\,(5) = 12$
$a + 25 = 12$
$a = -13$
$a_2 = a + d = - 13 + 5 = -8$
$a_3 = a_2 + d = - 8 + 5 = -3$
Hence the first three terms are -13, -8, and -3.

19. Subbia Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

The annual salary received by Subba Rao in the years 1995 onwards are
5000, 5200, 5400,----7000
Hence, these numbers forms an AP.
$a = 5000$, $d = 200$, $a_n = 7000$.
$a_n = a + (n - 1)\,d$
$7000 = 5000 + (n - 1)\,200$
$200(n - 1) = 2000$
$(n - 1) = 10$
$n = 11$
Thus the 11th years of his service or in 2005, Subba Rao received an annual salary of Rs 7000.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly savings by Rs 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

$a = 5$, $d = 1.75$, $a_n = 20.75$, $n = ?$
$a_n = a + (n - 1)\,d$
$20.75 = 5 + (n - 1) \times 1.75$

$15.75 = (n - 1) \times 1.75$

$15.75 = 1.75n - 1.75$

$1.75n = 15.75 + 1.75$

$1.75n = 17.50$

$n = \frac{17.50}{1.75} = \frac{1750}{175} = 10$

1.4 Sum of First n Terms of an AP

- First term - a Common difference - d

$$S = \frac{n}{2}[2a + (n - 1)d]$$

- When the first and the last terms of an AP are given and the common difference is not given

$$S = \frac{n}{2}[a + l]$$

Example 11 : Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution: Here a = 8, d = 3 - 8 = -5, n = 22.

$S = \frac{n}{2}[2a + (n - 1)d]$

$S = \frac{22}{2}[2x8 + (22 - 1)(-5)]$

$S = 11[16 + 21(-5)]$

$S = 11[16 - 105]$

$S = 11x-89 = -979$

Example 12 : If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.

Solution: Here, $S_{14} = 1050$, n = 14, a = 10

$S = \frac{n}{2}[2a + (n - 1)d]$

$1050 = \frac{14}{2}[2x10 + (14 - 1)d]$

$1050 = 7[20 + 13d]$

$1050 = 140 + 91d$

$91d = 1050 - 140$

$91d = 910$

$d = \frac{910}{91} = 10$

$a_n = a + (n - 1) d$

$a_{20} = 10 + (20 - 1)10$

$a_{20} = 10 + 19x10$

$a_{20} = 10 + 190$

$a_{20} = 200$

Example 13 : How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Solution: a = 24, d = 21-24 = -3, Sn = 78 , We have to find 'n'

$S = \frac{n}{2}[2a + (n - 1)d]$

$78 = \frac{n}{2}[2x24 + (n - 1)(-3)]$

$78 = \frac{n}{2}[48 - 3n + 3]$

$156 = n[48 - 3n + 3]$

$156 = 51n - 3n^2$

$52 = 17n - n^2$

$n^2 – 17n + 52 = 0$

$n^2 – 13n – 4n + 52 = 0$

$n(n – 13) – 4(n – 13) = 0$

$(n – 13) (n – 4) = 0$

$n = 13$ ಅಥವಾ $n = 4$

Example 14 : Find the sum of :

(i) the first 1000 positive integers (ii) the first n positive integers

Solution: (i) **Let S = 1 + 2 + 3 +...... + 1000**

$S = \frac{n}{2}[2a + (n - 1)d]$

$S = 500[2 + 999]$

$S = 500[1001]$

$S = 500500$

(i) **Let S = 1 + 2 + 3 +...... + n**

$S = \frac{n}{2}[2a + (n - 1)d]$

$S = \frac{n}{2}[2x1 + (n - 1)1]$

$S = \frac{n}{2}[2 + n - 1]$

$S = \frac{n}{2}[n + 1]$

Example 15 : Find the sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$.

Solution: $a_n = 3 + 2n$

$a_1 = 3 + 2x1 = 3 + 2 = 5$

$a_2 = 3 + 2x2 = 3 + 4 = 7$

$a_3 = 3 + 2x3 = 3 + 6 = 9$

There for AP is : 5, 7, 9, – – –

$a = 5, d = 2, n = 24$

$S = \frac{n}{2}[2a + (n - 1)d]$

$S = \frac{24}{2}[2x5 + (24 - 1)2]$

$S = 12[10 + 23x2]$

$S = 12[10 + 46]$

$S = 12x56$

$S = 672$

Example 16 : A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

(i) the production in the 1st year (ii) the production in the 10th year

(iii) the total production in first 7 years

Solution:i) Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1st, 2nd, 3rd, . . ., years will form an AP.
Let us denote the number of TV sets manufactured in the n^{th} year by a_n

$a_3 = 600$, $a_7 = 700$,
$a + 2d = 600$
$a + 6d = 700$
By solving the equation we get,
$d = 25$ and $a = 550$
(i) Therefore, production of TV sets in the first year is $= 550$
(ii) Production of TV sets in the 10th year is : $a_{10} = a + 9d$
$a_{10} = 550 + 9x25$
$= 550 + 225 = 775$
(iii) The total production of TV sets in first 7 years is
$S = \frac{n}{2}[2a + (n - 1)d]$
$S = \frac{7}{2}[2x550 + (7 - 1)25]$
$S = \frac{7}{2}[1100 + 6x25]$
$S = \frac{7}{2}[1100 + 150]$
$S = \frac{7}{2}[1250]$
$S = 7x625 = 4375$

Exercise 1.3

1. **Find the sum of the following APs:**

i) 2, 7, 12 ...... to 10 terms

$a = 2$, $d = a_2 - a_1 = 7 - 2 = 5$, $n = 10$
$S_n = \frac{n}{2}[2a + (n - 1) d]$
$S_{10} = \frac{10}{2}[2(2) + (10 - 1) \times 5]$
$S_{10} = 5[4 + (9) \times (5)]$
$S_{10} = 5 \times 49 = 245$

ii) -37, -33, -29 ...... to 12 terms

$a = -37$
$d = a_2 - a_1 = (-33) - (-37 = - 33 + 37 = 4$
$n = 12$
$S_n = \frac{n}{2}[2a + (n - 1) d]$
$S_{12} = \frac{12}{2}[2(-37) + (12 - 1) \times 4]$
$S_{12} = 6[-74 + 11 \times 4]$
$S_{12} = 6[-74 + 44]$
$S_{12} = 6(-30) = -180$

iii) 0.6, 1.7, 2.5 ..... to 100 terms

$a = 0.6$
$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$
$n = 100$
$S_n = \frac{n}{2}[2a + (n - 1) d]$
$S_{100} = \frac{100}{2}[1.2 + (99) \times 1.1]$

$S_{100} = 50[1.2 + 108.9]$
$S_{100} = 50[110.1]$
$S_{100} = 5505$

iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ --------- to 11 terms

$a = \frac{1}{15}$
$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$
$n = 11$
$S_n = \frac{n}{2}[2a + (n - 1) d]$
$S_{11} = \frac{11}{2}[2x\frac{1}{15} + (11 - 1) \times \frac{1}{60}]$
$S_{11} = \frac{11}{2}[\frac{2}{15} + \frac{10}{60}]$
$S_{11} = \frac{11}{2}[\frac{8+10}{60}]$
$S_{11} = \frac{11}{2}[\frac{18}{60}]$
$S_{11} = \frac{11}{2}[\frac{3}{10}] = \frac{33}{20}$

2. Find the sums given below :

i) $7 + 10\frac{1}{2} + 14 +$ -------- $+ 84$

$a = 7, l = 84$
$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$
$l = a + (n - 1)d$
$84 = 7 + (n - 1) \times \frac{7}{2}$
$77 = (n - 1) \times \frac{7}{2}$
$154 = 7n - 7$
$7n = 161$
$n = 23$
$S_n = \frac{n}{2}(a + l)$
$S_{23} = \frac{23}{2}(7 + 84)$
$= \frac{23}{2} x 91 = \frac{2093}{2}$
$= 1046\frac{1}{2}$

ii) 34 + 32 + 30 + ... + 10

$a = 34, d = a_2 - a_1 = 32 - 34 = -2, l = 10$
$l = a + (n - 1) d$
$10 = 34 + (n - 1)(-2)$
$-24 = (n - 1)(-2)$
$12 = n - 1$
$n = 13$
$S_n = \frac{n}{2}(a + l)$
$S_{13} = \frac{13}{2}(34 + 10)$
$S_{13} = \frac{13}{2} x44$
$S_{13} = 13 \times 22$
$S_{13} = 286$

iii) -5 + (-8) + (-11)+ .... + (-230)

$a = -5, l = -230, d = a_2 - a_1 = (-8) - (-5) = -8 + 5 = -3$
$l = a + (n - 1)d$

$-230 = -5 + (n-1)(-3)$
$-225 = (n-1)(-3)$
$(n-1) = 75$
$n = 76$
$S_n = \frac{n}{2}(a + l)$
$S_{76} = \frac{76}{2}[(-5) + (-230)]$
$S_{76} = 38(-235)$
$S_{76} = -8930$

3. **In an AP:**

i) Given $a = 5, d = 3, a_n = 50$ find n and Sn

$a = 5, d = 3, a_n = 50$
$a_n = a + (n-1)d,$
$\Rightarrow 50 = 5 + (n-1) \times 3$
$\Rightarrow 3(n-1) = 45$
$\Rightarrow n - 1 = 15$
$\Rightarrow n = 16$
$S_n = \frac{n}{2}(a + a_n)$
$S_{16} = \frac{16}{2}(5 + 50) = 440$
$S_{16} = 8(55) = 440$

ii) Given $a = 7, a_{13} = 35$ find d and S_{13}

$a = 7, a_{13} = 35$
$a_n = a + (n-1)d,$
$\Rightarrow 35 = 7 + (13-1)d$
$\Rightarrow 12d = 28$
$\Rightarrow d = 28/12 = 2.33$
$S_n = \frac{n}{2}(a + a_n)$
$S_{13} = \frac{13}{2}(7 + 35)$
$S_{13} = \frac{13}{2}(42) = 13 \times 21$
$S_{13} = 273$

iii) Given $a_{12} = 37, d = 3$ find a and S_{12}

$a_{12} = 37, d = 3$
$a_n = a + (n-1)d,$
$\Rightarrow a_{12} = a + (12-1)3$
$\Rightarrow 37 = a + 33$
$\Rightarrow a = 4$
$S_n = \frac{n}{2}(a + a_n)$
$S_{12} = \frac{12}{2}(4 + 37)$
$S_{12} = 6(41)$
$S_{12} = 246$

iv) Given $a_3 = 15, S_{10} = 125$ find d and a_{10}

$a_3 = 15, S_{10} = 125$
$a_n = a + (n-1)d,$
$a_3 = a + (3-1)d$
$15 = a + 2d$ ------------ (i)
$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$
$125 = 5(2a + 9d)$
$25 = 2a + 9d$ ----------- (ii)
Substract equation (i) from (2)
$30 = 2a + 4d$ ----------- (iii)
Substract (ii) ಜಿಡಿಂಚ (iii)
$-5 = 5d$
$d = -1$
From equation **(i)**,
$15 = a + 2(-1)$
$15 = a - 2$
$a = 17$
$a_{10} = a + (10 - 1)d$
$a_{10} = 17 + (9)(-1)$
$a_{10} = 17 - 9 = 8$

v) Given $d = 5, S_9 = 75$ find a and a_9

$d = 5, S_9 = 75$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$75 = \frac{9}{2}[2a + (9 - 1)5]$
$75 = \frac{9}{2}(2a + 40)$
$75 = 9(a + 20)$
$75 = 9a + 180$
$9a = 75 - 180$
$a = \frac{-35}{3}$
$a_n = a + (n - 1)d$
$a_9 = a + (9 - 1)(5)$
$= \frac{-35}{3} + 8(5)$
$= \frac{-35}{3} + 40$
$= \frac{-35+120}{3} = \frac{85}{3}$

vi) Given $a = 2, d = 8, Sn = 90$ find n and a_n

$a = 2, d = 8, S_n = 90$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$90 = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow 180 = n(4 + 8n - 8)$
$\Rightarrow 180 = n(8n - 4)$
$\Rightarrow 180 = 8n^2 - 4n$
$\Rightarrow 8n^2 - 4n - 180 = 0$
$\Rightarrow 2n^2 - n - 45 = 0$
$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$
$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$
$\Rightarrow (2n - 9)(2n + 9) = 0$
$n = 5$ (Positive number)
There for $a_5 = 8 + 5 \times 4 = 34$

vii) Given $a = 8, an = 62, Sn = 210$ find n and d

a = 8, a_n = 62, S_n = 210

$S_n = \frac{n}{2}(a + a_n)$

$210 = \frac{n}{2}(8 + 62)$

⇒ 35n = 210

⇒ $n = \frac{210}{35} = 6$

$a_n = a + (n-1)d$

62 = 8 + 5d

⇒ 5d = 62 - 8 = 54

⇒ $d = \frac{54}{5} = 10.8$

viii) Given an = 4, d = 2, Sn = -14 find n and a

$a_n = 4$, d = 2, $S_n = -14$

$a_n = a + (n-1)d$

4 = a + (n − 1)2

4 = a + 2n − 2

a + 2n = 6

a = 6 − 2n ---------- **(i)**

$S_n = \frac{n}{2}(a + a_n)$

$-14 = \frac{n}{2}(a + 4)$

−28 = n (a + 4)

−28 = n (6 − 2n + 4) {From equation **(i)**}

−28 = n (− 2n + 10)

$-28 = -2n^2 + 10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

n (n − 7) + 2(n − 7) = 0

(n − 7) (n + 2) = 0

Either n − 7 = 0 or n + 2 = 0

n = 7 or ~~n = −2~~

From equation **(i)** ,

a = 6 − 2n

a = 6 − 2(7)

a = 6 − 14

a = −8

ix) Given a = 3, n = 8, S = 192 find d

a = 3, n = 8, S = 192

$S_n = \frac{n}{2}[2a + (n-1)d]$

$192 = \frac{8}{2}[2 \times 3 + (8-1)d]$

192 = 4 [6 + 7d]

48 = 6 + 7d

42 = 7d

d = 6

x) Given l = 28, S = 144 and there are 9 terms. Find the value of a

l = 28, S = 144, n = 9

$S_n = \frac{n}{2}(a + l)$

$144 = \frac{9}{2}(a + 28)$
$(16) \times (2) = a + 28$
$32 = a + 28$
$a = 4$

4. **How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?**

$a = 9$
$d = a_2 - a_1 = 17 - 9 = 8$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$636 = \frac{n}{2}[2 \times a + (8 - 1) \times 8]$
$636 = \frac{n}{2}[18 + (n - 1) \times 8]$
$636 = n[9 + 4n - 4]$
$636 = n(4n + 5)$
$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
$n(4n + 53) - 12(4n + 53) = 0$
$(4n + 53)(n - 12) = 0$
$4n + 53 = 0$ or $n - 12 = 0$
$n = \cancel{(-53/4)}$ or $n = 12$
$\Rightarrow n = 12$

5. **The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

$a = 5$, $l = 45$, $S_n = 400$
$S_n = \frac{n}{2}(a + l)$
$400 = \frac{n}{2}(5 + 45)$
$400 = \frac{n}{2}(50)$
$25n = 400$
$n = 16$
$l = a + (n - 1)d$
$45 = 5 + (16 - 1)d$
$40 = 15d$
$d = \frac{40}{15} = \frac{8}{3}$

6. **The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?**

$a = 17$, $l = 350$, $d = 9$
$l = a + (n - 1)d$
$350 = 17 + (n - 1)9$
$333 = (n - 1)9$
$(n - 1) = 37$
$S_n = \frac{n}{2}(a + l)$
$S_{38} = \frac{38}{2}(17 + 350)$
$S_{38} = 19 \times 367$
$S_{38} = 6973$

7. **Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.**

$d = 7$, $a_{22} = 149$, $S_{22} = ?$
$a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$

$149 = a + 21 \times 7$
$149 = a + 147$
$a = 2$
$S_n = \frac{n}{2}(a + a_n)$
$S_{22} = \frac{22}{2}(2 + 149) = 11 \times 151$
$S_{22} = 1661$

8. **Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.**

$a_2 = 14$, $a_3 = 18$, $d = a_3 - a_2 = 18 - 14 = 4$
$a_2 = a + d$
$14 = a + 4$
$a = 10$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$
$= \frac{51}{2}[20 + (50) \times 4]$
$= \frac{51}{2}[20 + 200]$
$= \frac{51}{2}[220] = 51 \times 110 = 5610$

9. **If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.**

$S_7 = 49$, $S_{17} = 289$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_7 = \frac{7}{2}[2a + (n - 1)d]$
$S_7 = \frac{7}{2}[2a + (7 - 1)d]$
$49 = \frac{7}{2}[2a + 6d]$
$7 = (a + 3d)$
$a + 3d = 7$ ----------- **(i)**
Similarlly,
$S_{17} = \frac{17}{2}[2a + (17 - 1)d]$
$289 = \frac{17}{2}(2a + 16d)$
$17 = (a + 8d)$
$a + 8d = 17$ --------- **(ii)**
Substract equation (ii) from (i)
$5d = 10 \Rightarrow d = 2$
From equation **(i)**
$a + 3(2) = 7$
$a + 6 = 7 \Rightarrow a = 1$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$= \frac{n}{2}[2(1) + (n - 1) \times 2] = \frac{n}{2}(2 + 2n - 2)$
$= \frac{n}{2}(2n) = n^2$

10. Show that a_1, a_2, a_3 ..... a_n ,......form an AP where a_n is defined as below
(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$
Also, find the sum of the first 15 terms in each case.

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$
$a_2 = 3 + 4(2) = 3 + 8 = 11$
$a_3 = 3 + 4(3) = 3 + 12 = 15$
$a_4 = 3 + 4(4) = 3 + 16 = 19$
$\Rightarrow a_2 - a_1 = 11 - 7 = 4$
$a_3 - a_2 = 15 - 11 = 4$
$a_4 - a_3 = 19 - 15 = 4$
So, the given sequence forms an AP with first term =7 and common difference = 4
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{15} = \frac{15}{2}[2(7) + (15 - 1) \times 4]$
$= \frac{15}{2}[(14) + 56]$
$= \frac{15}{2}(70) = 15 \times 35 = 525$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$
$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$
$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$
$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$
$\Rightarrow a_2 - a_1 = -1 - 4 = -5$
$a_3 - a_2 = -6 - (-1) = -5$
$a_4 - a_3 = -11 - (-6) = -5$
So, the given sequence forms an AP with first term = 4 and common difference = -5
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{15} = \frac{15}{2}[2(4) + (15 - 1)(-5)]$
$= \frac{15}{2}[8 + 14(-5)]$
$= \frac{15}{2}(8 - 70) = \frac{15}{2}(-62) = 15(-31) = -465$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n^{th} terms.

$S_n = 4n - n^2$
First term $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$
Sum of first two terms
$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$
$a_2 = S_2 - S_1 = 4 - 3 = 1$
$d = a_2 - a = 1 - 3 = -2$
n^{th} term $a_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$
$= 3 - 2n + 2$
$= 5 - 2n$
So, the third term $a_3 = 5 - 2(3) = 5 - 6 = -1$
10^{th} term $a_{10} = 5 - 2(10) = 5 - 20 = -15$

12. **Find the sum of the first 40 positive integers divisible by 6.**

6, 12, 18, 24 …

This is an AP with common difference = 6 and the first term = 6

a = 6, d = 6, S_{40} = ?

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{40} = \frac{40}{2}[2(6) + (40 - 1)\,6]$

$= 20[12 + (39)(6)]$

$= 20(12 + 234) = 20 \times 246 = \mathbf{4920}$

13. **Find the sum of the first 15 multiples of 8.**

The numbers multiples of 8 are

8, 16, 24, 32…

These numbers form an AP with common difference 8 and the first term 8

a = 8, d = 8, S_{15} = ?

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{15} = \frac{15}{2}[2(8) + (15 - 1)8]$

$= \frac{15}{2}[6 + (14)(8)]$

$= \frac{15}{2}[16 + 112] = \frac{15}{2}(128) = 15 \times 64 = 960$

14. **Find the sum of the odd numbers between 0 and 50.**

The odd numbers between 0 and 50

1, 3, 5, 7, 9 … 49

This is an AP with common difference 1 and the first term 2

a = 1, d = 2, $l = 49$

$l = a + (n - 1)\,d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$

$S_n = \frac{n}{2}(a + l)$

$S_{25} = \frac{25}{2}(1 + 49)$

$= \frac{25}{2}(50) = (25)(25) = 625$

15. **A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?**

This is an AP with common difference 50 and the first term 200

a = 200, d = 50

The penalty payable for the delay of 30 days = S_{30}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{30} = \frac{30}{2}[2(200) + (30 - 1)\,50]$

$= 15\,[400 + 1450] = 15\,(1850)$

= Rs 27750

16. **A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.**

Let the first prize = a

The amount of 2nd prize = a − 20
The amount of 3rd prize = a − 40
This is an AP with common difference -20 and the first term a
$d = -20,\ S_7 = 700$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\frac{7}{2}[2a + (7 - 1)d] = 700$
$\frac{7}{2}[2a + 6d] = 700$
$7\,[a + 3d] = 700$
$a + 3d = 100$
$a + 3(-20) = 100$
$a - 60 = 100$
$a = 160$
So, the values of prizes Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

17. **In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?**

1, 2, 3, 4, 5………………..12
This is an AP with common difference 1 and the first term 1
$a = 1,\ d = 2 - 1 = 1$
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{12} = \frac{12}{2}[2(1) + (12 - 1)(1)]$
$= 6\,(2 + 11) = 6\,(13) = 78$
Hence, the trees planted by the students of each section = 78
There for the trees planted by the students of 3 sections = 78 x 3 = 234

18. **A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 1.4. What is the total length of such a spiral made up of thirteen consecutive semi circles (Take $\pi = \frac{22}{7}$)**

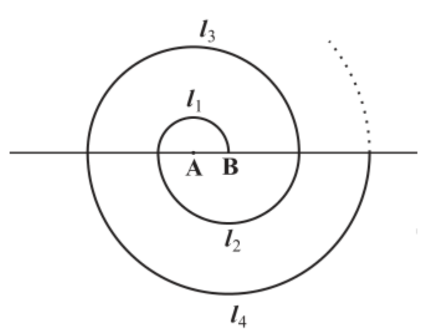


fig 1.4

[Hint: length of successive semi circles is $l_1, l_2, l_3, l_4,$ …..with centers as A, B,A,B ….. $l_1, l_2, l_3, l_4,$ …..]

The length of the semi circles $= \pi r$

$l_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$l_2 = \pi(1) = \pi$ cm

$l_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

$l_1, l_2, l_3 \ldots$ are the lengths of semicircles

$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots$

$d = l_2 - l_1 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$a = \frac{\pi}{2}$cm

$S_n = \frac{n}{2}[2a + (n-1)d]$

There for the total length of such a spiral made up of thirteen consecutive semi circles

$S_{13} = \frac{13}{2}[2x\frac{\pi}{2} + (13-1)\frac{\pi}{2}] = \frac{13}{2}[\pi + 6\pi]$

$= \frac{13}{2}(7\pi) = \frac{13}{2} \times 7 \times \frac{22}{2} = 143$ cm

19. **200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 1.5). In how many rows are the 200 logs placed and how many logs are in the top row?**

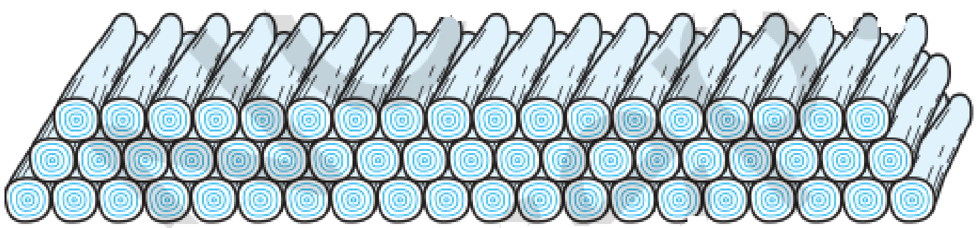


fig 1.5

The logs are in an AP

20, 19, 18…

$a = 20, d = a_2 - a_1 = 19 - 20 = -1$

$S_n = 200$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$200 = \frac{n}{2}[40 - n + 1]$

$400 = n(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

$(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n-1)d$

$a_{16} = 20 + (16-1)(-1) \Rightarrow a_{16} = 20 - 15 = 5$

Similarly,

$a_{25} = 20 + (25-1)(-1) = 20 - 24$

$= -4$ (negetive number is not possible)

Hence the number of rows is 16 and the number of logs in the top row is 5

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 1.6).

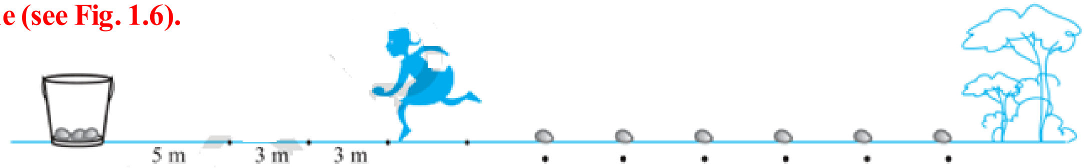


fig 1.6

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

The distances from the bucket to potatos 5, 8, 11, 14…

They have to run twice, then the distacnes run by the competiror 10, 16, 22, 28, 34,……….

$a = 10$, $d = 16 - 10 = 6$, $S_{10} = ?$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2(10) + (10 - 1)(6)]$

$= 5[20 + 54] = 5(74) = 370$

Hence, the distance the competitor has to run is 370km

Summery:

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference
- The general form of an AP : a, a + d, a + 2d, a + 3d......
- In an AP if there are only a finite number of terms. Such an AP is called a finite AP. Such AP has a last term.
- The AP has infinite number of terms is called infinite Arithmetic Progression. Such APs do not have a last term.
- The first term – a and the common difference is d then the nth term of an AP
  $a_n = a + (n - 1)d$
- The nth term from the last [last term – l , common difference – d]
  $l - (n - 1)d$
- a is the first term, d is the common difference then sum to nth term
  $S = \frac{n}{2}[2a + (n - 1)d]$
- If common difference is unknown then the sum to nth term
  $S = \frac{n}{2}[a + l]$ { l – the last term }

2 Triangles

2.2 Similar Figures

Two polygons of the same number of sides are similar, if

All the corresponding angles are equal and

All the corresponding sides are in the same ratio (or proportion).

EXERCISE 2.1

1. Fill in the blanks using the correct word given in brackets

i) All circles are ______________ (congruent, similar)
ii) All squares are _____________(similar, congruent)
iii) All_____________triangles are similar. (isosceles, equilateral)
iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are____________and (b) their corresponding sides are_____________(equal, proportional)

2. Give two different examples of pair of

(i) similar figures. (ii) non-similar figures.

3. State whether the following quadrilaterals are similar or not:

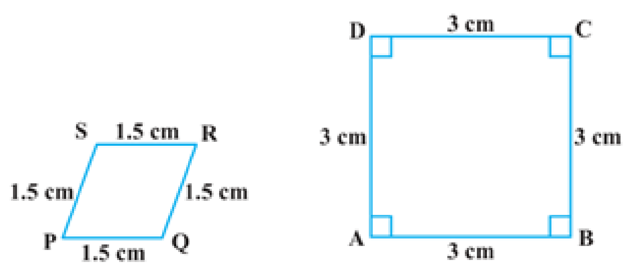


Fig 2.8

Solutions:

1. Fill in the blanks using the correct word given in brackets

v) All circles are **similar** (congruent, similar)
vi) All squares are **similar** (similar, congruent)
vii) All similar triangles are **equilateral**. (isosceles, equilateral)
viii) Two polygons of the same number of sides are similar, if (a) their corresponding angles are **equal** and (b) their corresponding sides are **proportional** (equal, proportional)

1. **Give two different examples of pair of**

   (i) similar figures:

   Pair of circles

   Pair of squares

   (ii) non-similar figures.

   A triangle and a square

   A rectangle and a Quadrilateral

4. **State whether the following quadrilaterals are similar or not:**

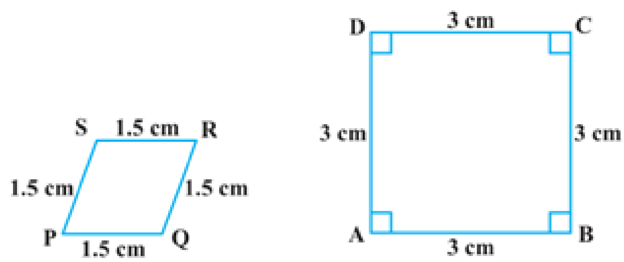


Fig 2.8

The corresponding angles are not equal. Hence, they are not similar

2.3 Similarity of Triangles

Basic proportionality theorem[Thales theorem]

Theorem 2.1

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio

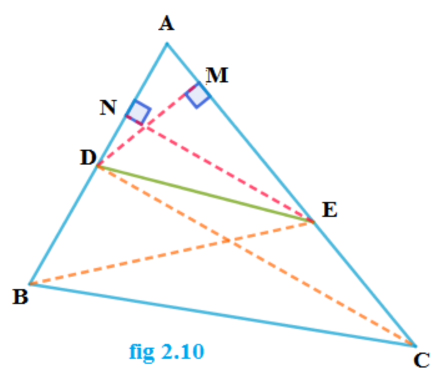


fig 2.10

Data: In ΔABC, the line drawn parallel to BC intersects AB and AC at D and E .

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$,

Construction: Join BE and CD. Draw $DM \perp AC$ and $EN \perp AB$.

Proof :

$$\frac{Area(\Delta ADE)}{Area(\Delta BDE)} = \frac{\frac{1}{2}\times AD \times EN}{\frac{1}{2}\times DB \times EN} = \frac{AD}{DB} \quad [\because \text{Area of triangle} = \frac{1}{2} \text{ x Base x Height}]$$

$$\frac{Area(\Delta ADE)}{Area\ (\Delta CED)} = \frac{\frac{1}{2}\times AE \times DM}{\frac{1}{2}\times EC \times DM} = \frac{AE}{EC}$$

ΔBDE and ΔDEC stand on the same base DE and in between $BC \parallel DE$

$\therefore$ Area (ΔBDE) = Area (ΔDEC) --- (3)

$\therefore$ From (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Theorem 2.2

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Example 1 :If a line intersects sides AB and AC of a ABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$ (See fig 2.13)

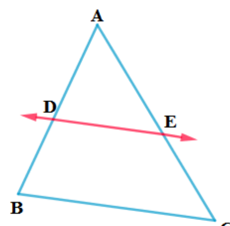


Fig 2.13

Solution: $DE \parallel BC$ (Data)

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{Theorem } 2.1)$$

$$\text{or } \frac{DB}{AD} = \frac{EC}{AE}$$

$$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$$

$$\frac{AB}{AD} = \frac{AC}{AE}$$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad (\because \text{ Taken reciprocals})$$

Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non parallel sides AD and BC respectively such that EF is parallel to (See fig 2.14) Show that $\frac{AE}{ED} = \frac{BF}{FC}$

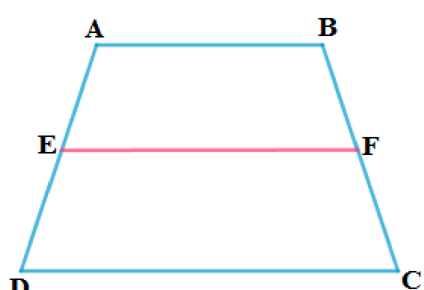


Fig 2.14

solution: Join AC to intersect EF at G. (See fig 2.15)

$AB \parallel DC$ ಮತ್ತು $EF \parallel AB$ (Given)

So, $EF \parallel DC$ (Lines parallel to the same line are parallel to each other)

Now in ΔADC,

$EG \parallel DC$ ($\because EF \parallel DC$)

$\therefore \frac{AE}{ED} = \frac{AG}{GC}$ (Theorem 2.1) ----(1)

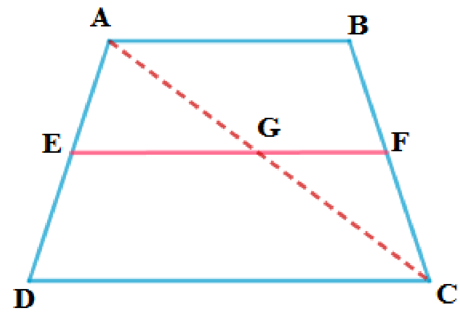
Similarly, from ΔCAB,

$\frac{CG}{AG} = \frac{CF}{BF}$

i.e., $\frac{AG}{GC} = \frac{BF}{FC}$ ----(2)

$\therefore$ From (1), (2) axiom (1),

$\frac{AE}{ED} = \frac{BF}{FC}$



Example 3 : In Fig. 2.16, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$ prove that ΔPQR is an isosceles triangle.

Solution:

Given that, $\frac{PS}{SQ} = \frac{PT}{TR}$

$\therefore ST \parallel QR$ ($\because$ Theorem 2.2)

$\Rightarrow \angle PST = \angle PQR$ ($\because$ Corresponding angles) ----(1)

But, $\angle PST = \angle PRQ$ ($\because$ Given) -------(2)

So, $\angle PRQ = \angle PQR$ [from (1), (2) and axiom (1)]

Therefore, $PQ = PR$ ($\because$ Sides opposite the equal angles)

i.e., PQR is an isosceles triangle.

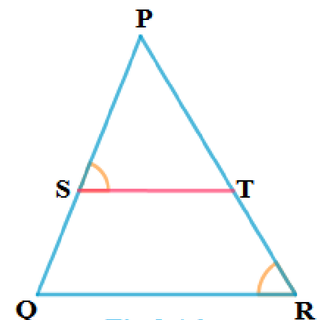


Fig 2.16

Exercise 2.2

1. **In Fig. 2.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).**

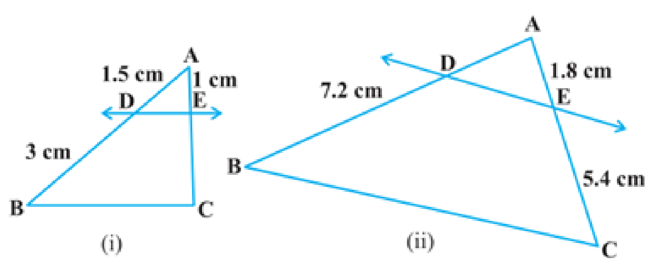


Fig. 2.17

(i) In triangle ΔABC, $DE \parallel BC$ (Given)

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Thales theorem]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$ $\Rightarrow EC = \frac{3x1}{1.5} = \frac{30}{15} = 2$ cm.

(ii) In ΔABC, $DE \parallel BC$ (Given)

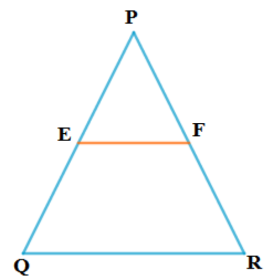
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Thales theorem]

$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

$\Rightarrow$ AD = $\frac{1.8x7.2}{5.4} = \frac{18x7.2}{54}$ = 2.4 cm.

2. **E and F are points on the sides PQ and PR respectively of a PQR. For each of the following cases, state whether EF || QR :**
(i) PE = 3.9cm EQ = 3cm PF = 3.6cm FR = 2.4cm
(ii) PE = 4cm QE = 4.5cm PF = 8cm RF = 9cm
(iii) PQ = 1.28cm PR = 2.56cm PE = 0.18cm PF = 0.36cm

Solution:



(i) PE = 3.9 cm, EQ = 3 cm ,PF = 3.6 cm, FR = 2,4 cm (Given)

$\therefore \frac{PE}{EQ} = \frac{3.9}{3} = \frac{39}{3}$ = 1.3 [Thales theorem]

And $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{36}{24} = 1.5$

Therefore , $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8cm, RF = 9cm

$\therefore \frac{PE}{QE} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$ [Thales theorem]

And, $\frac{PF}{RF} = \frac{8}{9}$

Therefore , $\frac{PE}{QE} = \frac{PF}{RF}$

Hence, EF∥QR

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm
Here, EQ = PQ - PE = 1.28 - 0.18 = 1.10 cm

And, FR = PR - PF = 2.56 - 0.36 = 2.20 cm

$\therefore \frac{PE}{QE} = \frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$ ... (i)

and, $\frac{PE}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ ... (ii)

$\therefore \frac{PE}{QE} = \frac{PE}{FR}$

There fore, EF∥QR

3. **In Fig. 2.18, if LM || CB and LN || CD, prove that**
$\frac{AM}{AB} = \frac{AN}{AD}$

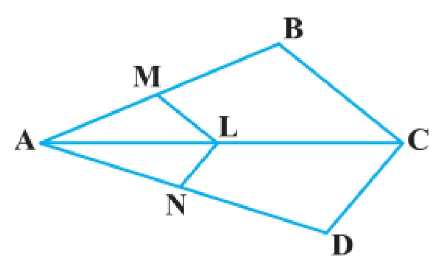


Fig 2.18

In the given fig, LM || CB,

$\frac{AM}{AB} = \frac{AL}{AC}$ ... (i) [corollary of BPT]

Similarlly, LN || CD,

$\therefore \frac{AN}{AD} = \frac{AL}{AC}$ ... (ii) [corollary of BPT]

From (i) and (ii) ,

$\frac{AM}{MB} = \frac{AN}{AD}$

4. **In Fig. 2.19, DE || AC and DF || AE. Prove that**
$\frac{BF}{FE} = \frac{BE}{EC}$

In ΔABC, $DE \parallel AC$ (Given)

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ --------(1) [Thales theorem]

In ΔABC, $DF \parallel AE$ (Given)

$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ -----------(2) [Thales theorem]

From equation **(i)** and **(ii)**

$\frac{BE}{EC} = \frac{BF}{FE}$

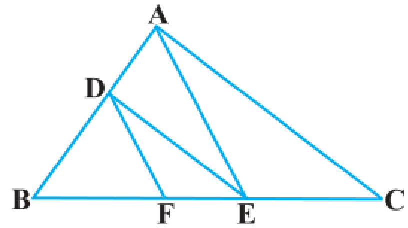


Fig 2.19

5. **In Fig. 2.20, DE || OQ and DF || OR. Show that EF || QR**

In ΔPQO, $DE \parallel OQ$ (Given)

$\therefore \frac{PD}{DO} = \frac{PE}{EQ}$ -------------- (1) [Thales theorem]

In ΔPOR, $DF \parallel OR$ (Given)

$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ ---------------(2) [Thales theorem]

$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$ [From equation **(1)** and **(2)**]

$\therefore$ In ΔPQR, $EF \parallel QR$. [Converse of BPT]

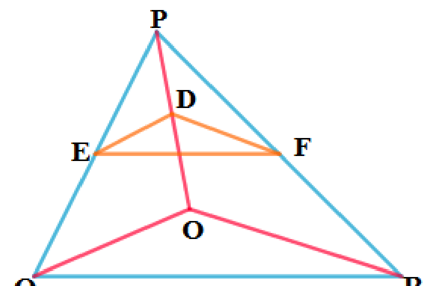


Fig 2.20

6. **In Fig. 2.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.**

In ΔOPQ, $AB \parallel PQ$ (Given)

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ -------------------(1) [Thales theorem]

In ΔOPR, $AC \parallel PR$ (Given)

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ -------------------(2) [Thales theorem]

$\frac{OB}{BQ} = \frac{OC}{CR}$ [From equation **(1)** and **(2)**]

$\therefore$ ΔOQR ನಲ್ಲಿ, $BC \parallel QR$. [Converse of BPT]

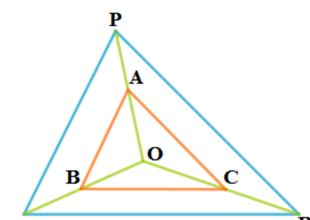


Fig 2.21

7. **Using Theorem 2.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).**

Data: : In ΔABC, D is the mid-point of AB $\Rightarrow AD=DB$.

The parallel line DE to BC drawn from D intersects AC at E

To prove: E is the mid-point of AC.

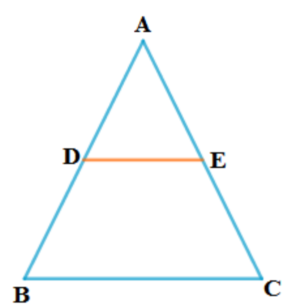
Proof: D is the mid-point of AB.

$\therefore AD = DB \quad \Rightarrow \frac{AD}{BD} = 1$ ----------------------(1)

In ΔABC, $DE \parallel BC$,

$\therefore \frac{AD}{BD} = \frac{AE}{EC}$ [Thales theorem] $\Rightarrow 1 = \frac{AE}{EC}$ [From equation (1)]

$\therefore AE = EC \Rightarrow$ E is the mid-point of AC



8. **Using Theorem 2.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).**

Data: In ΔABC, D and E are the mid-points of AB and AC

⇒AD=BD and AE=EC.

To prove: DE || BC

Proof: D is the mid-point of AB (Given)

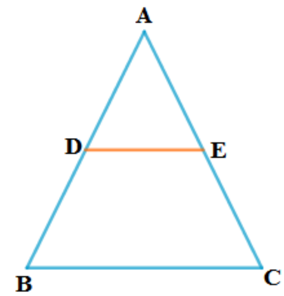
∴ AD = DB ⇒ $\frac{AD}{BD} = 1$ ------------------------ (1)

E is the mid-point of AC (Given)

∴ AE=EC ⇒$\frac{AE}{EC} = 1$ -------------------------(2)

$\frac{AD}{BD} = \frac{AE}{EC}$ [From equation **(1)** and **(2)]**

∴ DE || BC [By BPT]



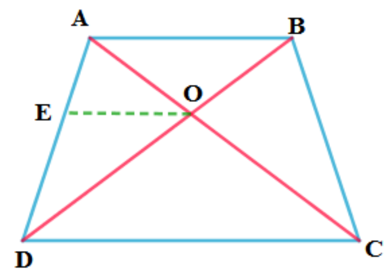
9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{CO} = \frac{CO}{DO}$

Data: In trapezium ABCD, AB∥BC, and AC and BD intersects each other at O.

To Prove: $\frac{AO}{CO} = \frac{CO}{DO}$

Construction: Draw EO from O such that EO || DC || AB

Proof: In ΔADC, OE || DC (Construction)



$\frac{AE}{ED} = \frac{AO}{CO}$ -------------------------(1) [By BPT]

In ΔABD, OE || AB (Construction)

$\frac{ED}{AE} = \frac{DO}{BO}$ [By BPT]

⇒$\frac{AE}{ED} = \frac{BO}{DO}$ ----------------------(2) [Taken reciprocals]

$\frac{AO}{CO} = \frac{BO}{DO}$ [From equation (1) and (2)]

⇒$\frac{AO}{BO} = \frac{CO}{DO}$

10. **The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium**

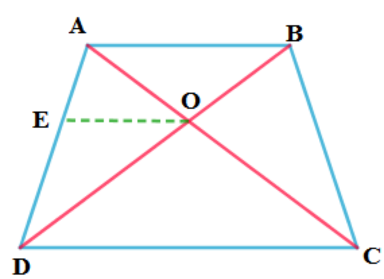
Given: In ABCD, AC and BD intersects at O

Such that $\frac{AO}{BO} = \frac{CO}{DO}$

To prove: ABCD is a trapezium

Construction: Draw EO through O such that EO || AB which Intersects AD at E

Proof: In ΔDAB, EO || AB



∴ $\frac{ED}{AE} = \frac{DO}{BO}$ [BPT] ⇒ $\frac{AE}{ED} = \frac{BO}{DO}$ ------------------------ (1) [taken reciprocals]

Similarlly, $\frac{AO}{BO} = \frac{CO}{DO}$ (Given)

⇒ $\frac{AO}{CO} = \frac{BO}{DO}$ ------------------(2)

∴ $\frac{AE}{ED} = \frac{AO}{CO}$ [From equation **(1)** and **(2)]**

EO || DC ಮತ್ತು EO || AB [converse of BPT]

⇒ AB || DC.

∴ABCD is a trapezium

2.4 Criteria for Similarity of Triangles

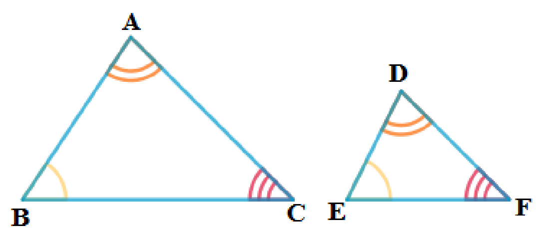


Fig 2.22

It must be noted that as done in the case of congruency of two triangles, the similarity of two triangles should also be expressed symbolically, using correct correspondence of their vertices. For example, for the triangles ABC and DEF of Fig. 2.22, we cannot write ABC ~ EDF or ABC ~ FED. However, we can write BAC ~ EDF

Theorem 2.3

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

AAA

This criterion is referred to as the AAA (Angle–Angle–Angle) cr it er ion of similarity of two triangles.

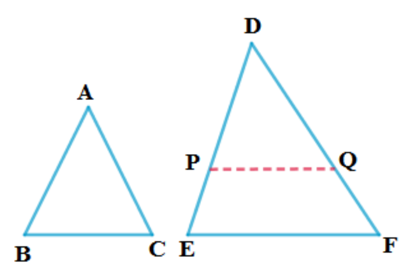


Fig 2.24

Data: In ΔABC and ΔDEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ (<1) and $\Delta ABC \sim \Delta DEF$

Construction: Cut DP = AB from DE and DQ = AC from DF and join PQ

Proof: In ΔABC and ΔDPQ,

AB = DP [Construction]

AC = DQ [Construction]

$\angle A = \angle D$ [data]

$\therefore \Delta ABC \cong \Delta DPQ$ [SAS Congruency rule]

$\Rightarrow$ **BC = PQ ---------(1)** and

$\Rightarrow \angle B = \angle P$ [CPCT] But $\angle B = \angle E$ [Given]

$\therefore \angle P = \angle E$

$\therefore$ **PQ‖EF** [Since corresponding angles are equal]

$\therefore \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$ [by Corolary of BPT]

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ [By construction and (1)]

$\therefore \Delta ABC \sim \Delta DEF$

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

This may be referred to as the AA similarity criterion for two triangles.

Theorem 2.4 SSS

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.

This criterion is referred to as the SSS (Side–Side–Side) similarity criterion for two triangles.

Data: In ΔABC and ΔDEF,

$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ (<1) ---------(1)

To Prove: $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

And $\Delta ABC \cong \Delta DEF$

Construction: Cut DP = AB from DE and DQ = AC from DF.Join PQ

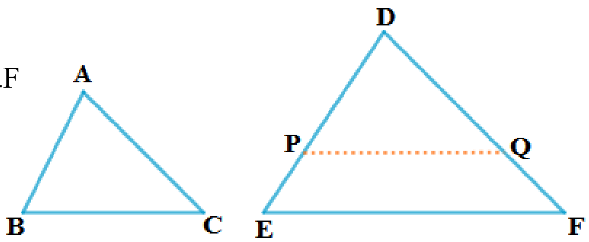


Fig 2.26

Proof : $\frac{AB}{DE} = \frac{AC}{DF}$ [Given]

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [$\because$ DP = AB, DQ = AC]

$\therefore$ PQ∥EF [corollary of Converse of BPT in Δ DEF]

$\Rightarrow$ **$\angle P = \angle E$ ಮತ್ತು $\angle Q = \angle F$**

$\therefore$ $\Delta DPQ \sim \Delta DEF$ [AA Similarity criteria]

$\therefore \frac{DP}{DE} = \frac{PQ}{EF}$ [Corresponding sides of similar triangles]

$\Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ ------(1) [AB = DP Construction]

But, $\frac{AB}{DE} = \frac{BC}{EF}$ -------(2) [Given]

$\Rightarrow \frac{PQ}{EF} = \frac{BC}{EF}$ [$\because$ (1) ಮತ್ತು (2) ರಿಂದ]

$\Rightarrow$BC = PQ

In ΔABC and ΔDPQ ,

BC = PQ [Proved]

AB = DP [Construction]

AC = DQ [Construction]

$\therefore \Delta ABC \cong \Delta DPQ$ [SSS Congruency rule]

Hence, $\angle A = \angle D$, $\angle B = \angle P$ and $\angle C = \angle Q$

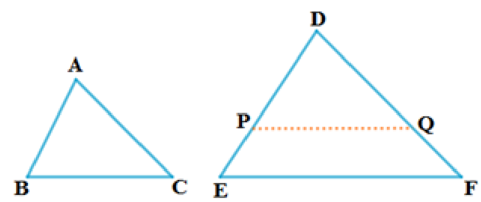
$\Rightarrow \angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ and $\Delta ABC \cong \Delta DEF$

Theorem 2.5

Theorem 2.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar

Given: In ΔABC and ΔDEF, $\angle A = \angle D$ and

$\frac{AB}{DE} = \frac{AC}{DF}$ (< 1) ------------ (1)



To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Cut DP = AB from DE and DQ = AC from DF.Join PQ

Proof: In ΔABC and ΔDPQ,

AB = PQ [By Construction]

AC = DF [By Construction]

$\angle A = \angle D$ [Given]

$\Delta ABC \cong \Delta DPQ$ [By SAS Congruency rule]----------(2)

From eqn (1) we get,

$\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ [AB = DP and AC = DQ]

⇒ PQ∥EF [By converse of corollary of BPT]

$\Rightarrow \angle P = \angle E$, $\angle Q = \angle F$ [Corresponding angles]

∴ $\Delta DPQ \sim \Delta DEF$ [by AA similarity ctriteria] --------(3)

⇒ $\Delta ABC \cong \Delta DEF$ [From equation (2) and (3)]

Example 4 : In Fig. 2.29, if PQ ∥ RS, prove that $\Delta POQ \sim \Delta SOR$

Solution: PQ∥RS [given]

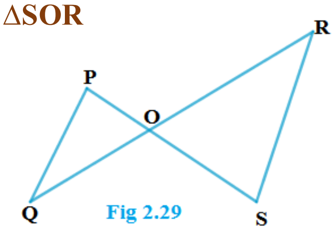


Fig 2.29

∴ $\angle P = \angle S$ [Alternate angles]

$\angle Q = \angle R$ [Alternate angles]

And $\angle POQ = \angle SOR$ [vertically opposite angles]

∴$\Delta POQ \sim \Delta SOR$ [AAA similarity criteria]

Example 5 : Observe Fig. 2.30 and then find P.

Solution: In ΔABC and ΔPQR,

$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}, \frac{BC}{CQ} = \frac{6}{12} = \frac{1}{2}$

And $\frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$

$\Rightarrow \frac{AB}{RQ} = \frac{BC}{CQ} = \frac{CA}{PR}$

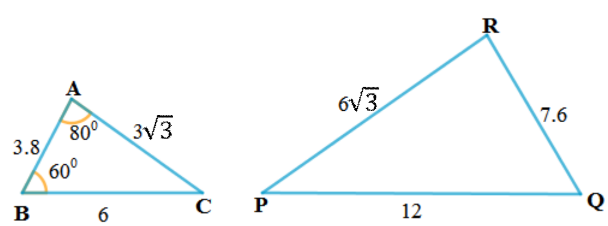


Fig 2.30

∴ $\Delta ABC \sim \Delta RQP$ [SSS similarity criteria]

$\angle C = \angle P$ [Corresponding angles of similar triangles]

But $\angle C = 180 - \angle A - \angle B$ [The sum of interior angles of a triangle is 180^0]

$= 180^0 - 80^0 - 60^0 = 40^0 \Rightarrow \angle P = 40^0$

Example 6 : In Fig. 2.31, OA . OB = OC . OD. Show that ∠A = ∠C ಮತ್ತು ∠B = ∠D.

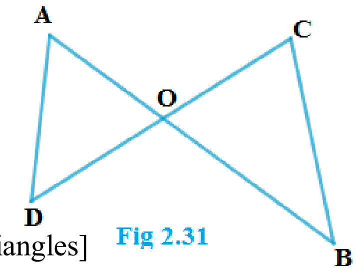


Fig 2.31

Solution: OA.OB = OC.OD [Given]

$\Rightarrow \frac{OA}{OC} = \frac{OD}{OB}$ ------ (1)

∠AOD = ∠COB [Vertically opposite angles] -----(2)

From equation (1) and (2),
ΔAOD ~ ΔCOB [SAS similarity criteria]
∴ ∠A = ∠C and ∠D = ∠B [Corresponding angles of similar triangles]

Example 7 : A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution:

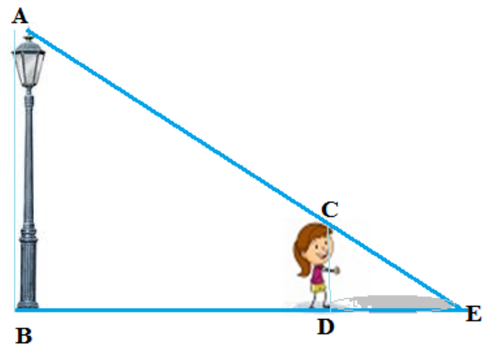


Fig 2.32

AB is a Lamp post. CD i the hs hight of the girl
DE is the length of the shadow of the girl.

Let DE = *x*' m
Now, BD = 1.2m × 4 = 4.8m
In ΔABE and ΔCDE,

∠B = ∠D = 90^0
And ∠E = ∠E [Common angle]
∴ ΔABE ~ ΔCDE [AA similarity criteria]

$\therefore \frac{BE}{DE} = \frac{AB}{CD}$

$\Rightarrow \frac{4.8+x}{x} = \frac{3.6}{0.9}$ ($\because 90cm = \frac{90}{100} m = 0.9m$)

⇒ 4.8+x = 4x
⇒ 3x = 4.8
⇒x = 1.6
Hence, the length of her shadow after 4 seconds is 1.6m

Example 8 : In Fig. 2.33, CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ ΔPQR, prove that :
(i) ΔAMC ~ ΔPNR
(ii) $\frac{CM}{RN} = \frac{AB}{PQ}$
iii) ΔCMB ~ ΔRNQ

Solution:

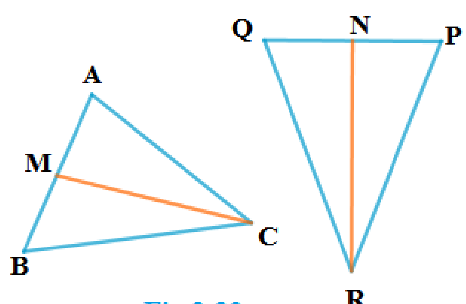


Fig 2.33

i) ΔABC ~ ΔPQR

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ----------(1)

and ∠A = ∠P , ∠B = ∠Q , ∠C = ∠R ------ (2)

But, AB = 2AM and PQ = 2PN [CM and RN are the medians]

$\Rightarrow \frac{2AM}{2PN} = \frac{CA}{RP}$

$\Rightarrow \frac{AM}{PN} = \frac{CA}{RP}$ -------------(3)

But, ∠A = ∠P [From (2)] ---------(4)

From (3) and (4),

ΔAMC ~ ΔPNR [SAS similarity criteria] --(5)

ii) From (5) $\frac{CM}{RN} = \frac{CA}{RP}$ --------(6)

But, $\frac{CA}{RP} = \frac{AB}{PQ}$ [From (1)] --------(7)

$\therefore \frac{\mathbf{CM}}{\mathbf{RN}} = \frac{\mathbf{AB}}{\mathbf{PQ}}$ --------(8)

(iii) $\frac{AB}{PQ} = \frac{BC}{QR}$ ------ (9)

$\Rightarrow \frac{CM}{RN} = \frac{BC}{QR}$ [From (8)]

$\Rightarrow \frac{CM}{RN} = \frac{AB}{PQ} = \frac{2\mathbf{B}M}{2QN}$ [CM and RN are the medians]

$\Rightarrow \frac{CM}{RN} = \frac{\mathbf{B}M}{QN}$ -------- (10)

$\Rightarrow \frac{CM}{RN} = \frac{\mathbf{B}C}{QR} = \frac{\mathbf{B}M}{QN}$ [From (9) and (10)]

∴ ΔCMB ~ ΔRNQ [SSS similarity criteria]

[Note: you can solve (ii) and (iii) using same method as solved for (i)]

Exercise 2.3

1) State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :.

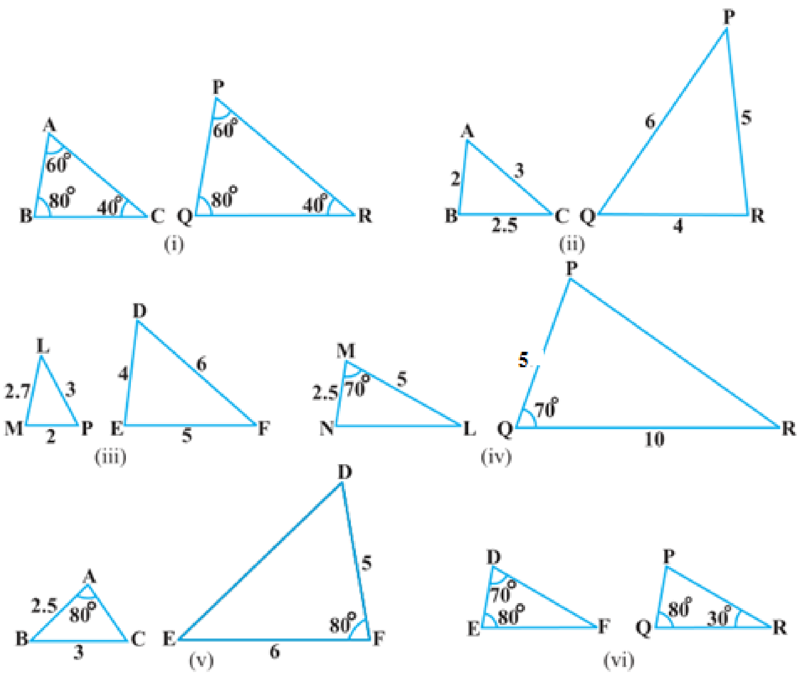


Fig. 2.34

(i) In ΔABC and ΔPQR,

∠A = ∠P = 60° [Given] ; ∠B = ∠Q = 80° [Given]; ∠C = ∠R = 40° [Given]
∴ ΔABC ~ ΔPQR [AAA similarity criteria]
(ii) **In ΔABC and ΔPQR ,**

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

∴ ΔABC ~ ΔQRP [SSS similarity criteria]
(iii)In ΔLMP and ΔDEF,

LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$

$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$

Here, $\frac{MP}{DE} = \frac{PL}{DF} \neq \frac{LM}{EF}$

∴ ΔLMP and ΔDEF are not similar
(iv) In ΔMNL and ΔQPR,

$\frac{MN}{QP} = \frac{ML}{QR} = \frac{1}{2}$

∠M = ∠Q = 70°
∴ ΔMNL ~ ΔQPR [SAS similarity criteria]
(v) In ΔABC and ΔDEF,

AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80°

$\Rightarrow \frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

and, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

⇒ ∠B ≠ ∠F
⇒ ΔABC and ΔDEF are not similar
(vi) In ΔDEF,

∠D + ∠E + ∠F = 180° [Sum of the interior angles of a triangle]
⇒ 70° + 80° + ∠F = 180°
⇒ ∠F = 180° - 70° - 80°
⇒ ∠F = 30°
In ΔPQR,

∠P + ∠Q + ∠R = 180 [Sum of the interior angles of a triangle]
⇒ ∠P + 80° + 30° = 180°
⇒ ∠P = 180° - 80° -30°
⇒ ∠P = 70°
In ΔDEF and ΔPQR ,

∠D = ∠P = 70°
∠F = ∠Q = 80°

$\angle F = \angle R = 30°$

$\Rightarrow \Delta DEF \sim \Delta PQR$ [AAA similarity criteria]

2. In Fig. 2.35, $\Delta OBA \sim \Delta ODC$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$

Solution:

DOB is a straight line

$\therefore \angle DOC + \angle COB = 180°$

$\Rightarrow \angle DOC = 180° - 125° = 55°$

In ΔDOC,

$\angle DCO + \angle CDO + \angle DOC = 180°$

$\Rightarrow \angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO = 180^0 - 125° = 55^0$

$\Delta ODC \sim \Delta OBA$. [Given]

$\therefore \angle OAB = \angle OCD$ [CPCT]

$\Rightarrow \angle OAB = 55°$

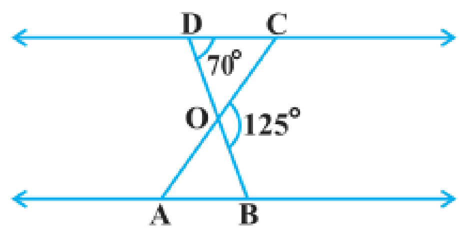


Fig. 2.35

3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles. Show that $\frac{OA}{OC} = \frac{OB}{OD}$

In ΔBOA and ΔDOC,

$\angle ABO = \angle CDO$ [AB || CD, Alternate angles]

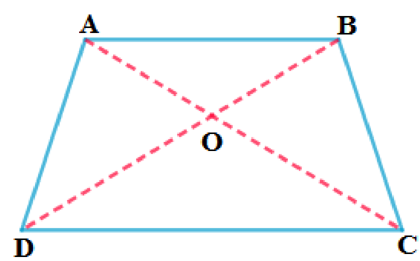
$\angle BAO = \angle DCO$ [AB || CD, Alternate angles]

$\angle BOA = \angle DOC$ [Vertically opposite angles]

$\therefore \Delta BOA \sim \Delta DOC$ [AAA Similarity criteria]

$\therefore \frac{OC}{OA} = \frac{OD}{OB}$ [By CPCT]

$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$ [Taken on reciprocals]



4. **In Fig. 2.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ Show that $\Delta PQS \sim \Delta TQR$**

In ΔPQR,

$\angle 1 = \angle 2$ [Given]

$\therefore PQ = PR$ --------------(1)

But, $\frac{QR}{QS} = \frac{QT}{PR}$ [Given]

$\frac{QR}{QS} = \frac{QT}{PQ}$ [From (1)] ------------(2)

In ΔPQS and ΔTQR,

$\frac{QR}{QS} = \frac{QT}{PQ}$ [From eqn (2)]

$\angle Q = \angle Q$ [Common angle]

$\therefore \Delta PQS \sim \Delta TQR$ [SAS similarity criteria]

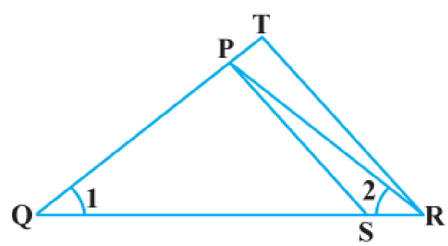


Fig. 2.36

5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$

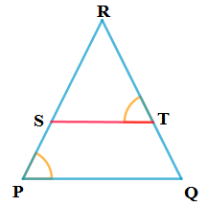
Solution:

In ΔRPQ and ΔRST,

∠RTS = ∠QPS [Given]

∠R = ∠R [Common angle]

∴ ΔRPQ ~ ΔRTS [AA similarity criteria]



6. In Fig. 2.37, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC

Solution:

ΔABE ≅ ΔACD [Given]

∴ AB = AC -----------(1) [By CPCT]

and AD = AE -----------(2) [By CPCT]

ΔADE ಮತ್ತು ΔABCಗಳಲ್ಲಿ,

Dividing (2) by (1)

$$\frac{AD}{AB} = \frac{AE}{AC}$$

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [SAS Similarity criteria]

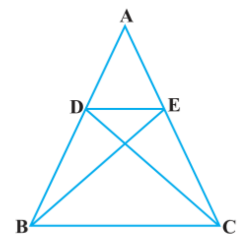


Fig. 2.37

7. In Fig. 2.38, altitudes AD and CE of ABC intersect each other at the point P. Show that

i) ΔAEP ~ ΔCDP

ii) ΔABD ~ ΔCBE

iii) ΔAEP ~ ΔADB

iv) ΔPDC ~ ΔBEC

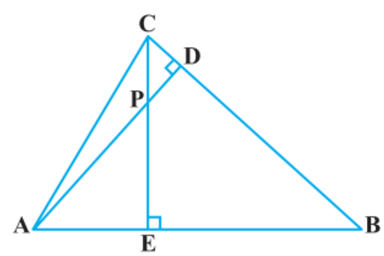


Fig. 2.38

(i) In ΔAEP and ΔCDP,

∠AEP = ∠CDP = 90°

∠APE = ∠CPD [Vertically opposite angles]

∴ ΔAEP ~ ΔCDP[AA similarity criteria]

(ii) In ΔABD and ΔCBE,

∠ADB = ∠CEB = 90°

∠ABD = ∠CBE [Common angle]

∴ ΔABD ~ ΔCBE [AA similarity criteria]

(iii) In ΔAEP and ΔADB,

∠AEP = ∠ADB = 90^0

∠PAE = ∠DAB [Common angle]

∴ ΔAEP ~ ΔADB [AA similarity criteria]

(iv) In ΔPDC and ΔBEC,

∠PDC = ∠BEC = 90°

∠PCD = ∠BCE [Common angle]

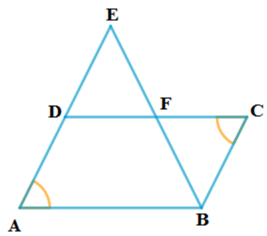
∴ ΔPDC ~ ΔBEC [AA similarity criteria]

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB

Solution:

In ΔABE and ΔCFB,

∠A = ∠C [Opposite angles of a parallelogram]



$\angle AEB = \angle CBF$ [AE || BC,Alternate angles]

$\therefore \Delta ABE \sim \Delta CFB$ [AA similarity criteria]

9. In Fig. 2.39, ΔABC and ΔAMP are two right triangles, right angled at B and M respectively. Prove that:

i) $\Delta ABC \sim \Delta AMP$

ii) $\frac{CA}{PA} = \frac{BC}{MP}$

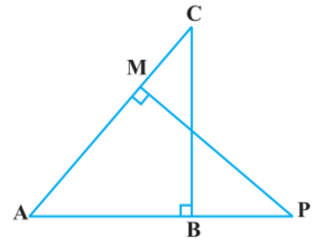


Fig. 2.39

(i) **In ΔABC and ΔAMP ,**

$\angle A = \angle A$ [Common angle]

$\angle ABC = \angle AMP = 90°$

$\therefore$ $\Delta ABC \sim \Delta AMP$ [AA similarity criteria]

(ii) $\Delta ABC \sim \Delta AMP$ [Proved in(i)]

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ [the corresponding sides of similar triangles are proportional]

10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta EFG$, show that:

i) $\frac{CD}{GH} = \frac{AC}{FG}$

ii) $\Delta DCB \sim \Delta HGE$

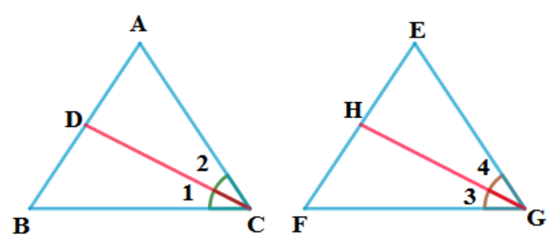
iii) $\Delta DCA \sim \Delta HGF$

(i) **$\Delta ABC \sim \Delta FEG$ [Given]**

$\therefore \angle A = \angle F$, $\angle ACB = \angle FGE$ [Corresponding angles of similar triangles]

CD is the bisector of $\angle ACB$, GH is the bisector of $\angle FGE$

$\therefore$ $\angle ACD = \angle FGH$



In ΔACD and ΔFGH,

$\angle A = \angle F$

$\angle ACD = \angle FGH$

$\therefore \Delta ACD \sim \Delta FGH$ [AA similarity criteria]

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta ABC \sim \Delta FEG$ [Given]

$\therefore \angle B = \angle E$, and $\angle ACB = \angle FGE$ [Corresponding angles of similar triangles]

CD is the bisector of $\angle ACB$, GH is the bisector of $\angle FGE$

$\therefore \angle DCB = \angle HGE$

Now, In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$

$\angle B = \angle E$

$\therefore$ $\Delta DCB \sim \Delta HGE$ [AA similarity criteria]

(iii) $\Delta ABC \sim \Delta FEG$ [Given]

$\therefore \angle A = \angle F$, $\angle ACB = \angle FGE$ [Corresponding angles of similar triangles]

CD is the bisector of $\angle ACB$, GH is the bisector of $\angle FGE$

$\therefore \angle ACD = \angle FGH$

In ΔDCA and ΔHGF,

$\angle ACD = \angle FGH$

$\angle A = \angle F$

$\therefore \Delta DCA \sim \Delta HGF$ [AA similarity criteria]

11. In Fig. 2.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD⊥BC and EF⊥AC , prove that $\Delta ABD \sim \Delta ECF$

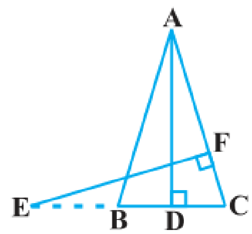


Fig. 2.40

ABC is an isosceles triangle

AB = AC

$\Rightarrow \angle B = \angle C$ [Opposite sides of equal angles]

$\Rightarrow \angle ABD = \angle ECF$ ------ (1)

In ΔABD and ΔECF,

$\angle ADB = \angle EFC = 90^0$ [AD⊥BC, EF⊥AC]

$\Rightarrow \angle ABD = \angle ECF$ [From (1)]

$\therefore \Delta ABD \sim \Delta ECF$ [AA similarity criteria]

12. Sides AB and BC and median AD of a triangle ABC are respectively propor- tional to sides PQ and QR and median PM of PQR (see Fig.2.41). Show that $\Delta ABC \sim \Delta PQR$

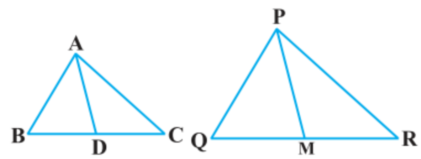


Fig. 2.41

Given: In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

To prove: $\Delta ABC \sim \Delta PQR$

Proof: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$ [D and M are the mid-points of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\Rightarrow \Delta ABD \sim \Delta PQM$ [SSS similarity criteria]

$\therefore \angle ABD = \angle PQM$ [Corresponding angles of similar triangles]

$\Rightarrow \angle ABC = \angle PQR$ -----------(1)

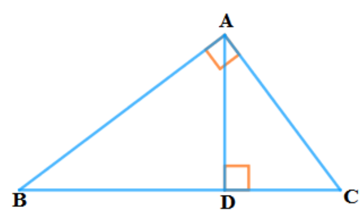
In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{BC}{QR}$ [Given]

$\angle ABC = \angle PQR$ [From (1)]

$\therefore \Delta ABC \sim \Delta PQR$ [SAS similarity criteria]

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.



In ΔADC and ΔBAC,

$\angle ADC = \angle BAC$ [Given]

$\angle ACD = \angle BCA$ [Common angle]

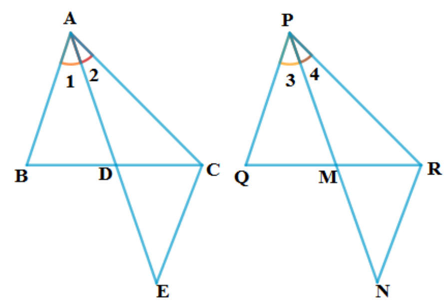
$\therefore \Delta ADC \sim \Delta BAC$ [AA similarity criteria]

$\therefore \frac{CA}{CB} = \frac{CD}{CA}$ [The corresponding sides of the similar triangles are proportional]

$\Rightarrow CA^2 = CB.CD$.

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR

Given: In ΔABC and ΔPQR,
AD and PM are the medians drawn to
BC and QR respectively.
and $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

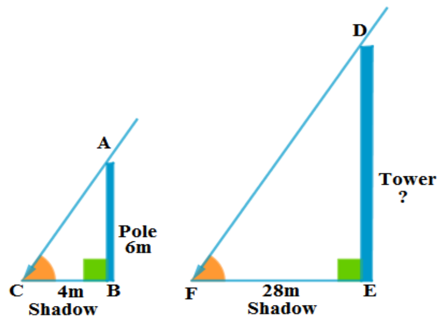


To prove: ΔABC ~ ΔPQR
Construction: Produce AD to E such that
AD = DE, join CE
and produce PM to N such that PM = MN, join RN

Proof: In ΔABD and ΔCDE,
AD = DE [Construction]
BD = DC [AD is Median]
∠ADB = ∠CDE [Vertically opposite angles]
∴ ΔABD ≅ ΔCDE [SAS Congruency rule]
⇒ AB = CE [By CPCT] -------------(i)
Similarlly, In ΔPQM and ΔMNR,
⇒ PQ = RN [By CPCT] -------------(ii)
But, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$ [Given]
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$ [From (i) and (ii)]
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$
$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [∵ 2AD = AE ಮತ್ತು 2PM = PN]
∴ ΔACE ~ ΔPRN [SSS similarity criteria]
∴ ∠2 = ∠4
Similarlly, ∠1 = ∠3
∴ ∠1 + ∠2 = ∠3 + ∠4
⇒ ∠A = ∠P --------------(iii)
In ΔABC and ΔPQR,
$\frac{AB}{PQ} = \frac{AC}{PR}$ [Given]
∠A = ∠P [From (iii)]
∴ ΔABC ~ ΔPQR [SAS similarity criteria]

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Length of the vertical Pole = AB = 6m
Length of the shadow casts by the Pole = BC = 4 m
Length of the shadow casts by the Tower = EF = 28 m
Let the height of the tower = DE = *h* ‘m
In ΔABC and ΔDEF,



$\angle C = \angle F$ [The angles make by sun at same time]

$\angle B = \angle E = 90°$

$\therefore \Delta ABC \sim \Delta DEF$ [AA similarity criteria]

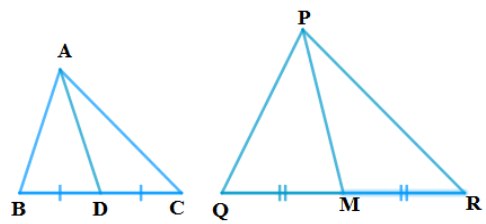
$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ [corresponding sides of the similar triangles]

$\therefore \frac{6}{h} = \frac{4}{28}$

$\Rightarrow h = 6\times\frac{6x28}{4} = 6 \times 7 \Rightarrow h = 42$ m

$\therefore$ Height of the tower = 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$



$\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ -------------- (1)

and $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ **--------- (2)**

AD and PM are the Medians

$\therefore BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ ----------(3)

From equations **(i)** and **(iii)**, we get

$\therefore \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QM}$ ----------(4)

In ΔABD and ΔPQM,

$\angle B = \angle Q$ [From (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [From (iv)]

$\therefore \Delta ABD \sim \Delta PQM$ [SAS similarity criteria]

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$

2.5 Areas of Similar Triangles

Theorem 2.6

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

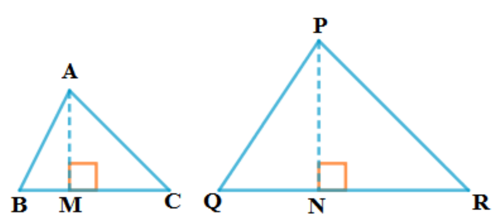


Fig 2.42

Given: $\Delta ABC \sim \Delta PQR$

To Prove: $\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$

Construction: Draw $AM \perp BC$ and $PN \perp QR$

Proof: $\frac{Area(ABC)}{Area(PQR)} = \frac{\frac{1}{2} x\, BC\, x\, AM}{\frac{1}{2} x\, QR\, x\, PN} = \frac{BC\, x\, AM}{QR\, x\, PN}$ - - -(1) [Area of triangle=$\frac{1}{2}$xbasexheight]

In ΔABM and ΔPQN,

$\angle B = \angle Q$ [Corresponding angles of the similar triangle]

$\angle M = \angle N = 90^0$ [Construction]

$\therefore \Delta ABM \sim \Delta PQN$ [AA similarity criteria]

$\Rightarrow \frac{AM}{PN} = \frac{AB}{PQ}$ ------- (2)

But, $\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}$ ---- (3)

$\Rightarrow \frac{AM}{PN} = \frac{BC}{QR}$ [From (2) and (3)]

$\therefore \frac{Area(ABC)}{Area(PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$ --------- [From (1) and (3)]

$\Rightarrow \frac{Area(ABC)}{Area(PQR)} = \left(\frac{BC}{QR}\right)^2$

$\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$ [From (3)]

Example 9 : In Fig. 2.43, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$

Solution: XY∥AC [Given]

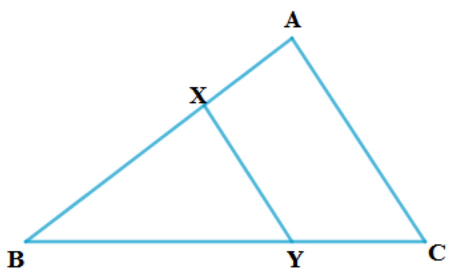


Fig 2.43

$\angle BXY = \angle A$ [Corresponding angles]

$\angle BYX = \angle C$ [Corresponding angles]

$\therefore \Delta ABC \sim \Delta XBY$ [AA similarity criteria]

$\frac{Area(ABC)}{Area(XBY)} = \left(\frac{AB}{XB}\right)^2$ [Theorem 2.6] -------- (1)

But, Area(ABC) = 2 Area (XBY) [Given]

$\frac{Area(ABC)}{Area(XBY)} = \frac{2}{1}$ --------- (2)

$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$ [from (1) and (2)]

$\frac{AB}{XB} = \frac{\sqrt{2}}{\sqrt{1}}$

Or $\frac{XB}{AB} = \frac{1}{\sqrt{2}}$ [Taken reciprocals]

$\Rightarrow 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$

$\frac{AB-XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} \Rightarrow \frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

Exercise 2.4

1. **Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.**

$\Delta ABC \sim \Delta DEF$ [Given]

Area ΔABC = 64 cm^2 and area ΔDEF = 121 cm^2; EF = 15.4 cm

$\frac{Area(ABC)}{Area(DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ [$\because \Delta ABC \sim \Delta DEF$] ------------(i)

$\frac{64}{121} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{8^2}{11^2} = \frac{BC^2}{EF^2} \Rightarrow \frac{8}{11} = \frac{BC}{EF} \Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

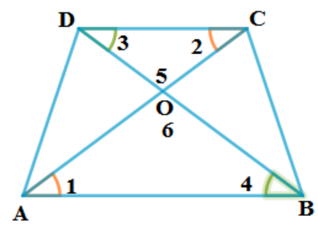
$\Rightarrow BC = \frac{8}{11} x 15.4 \quad \Rightarrow BC = 8 \times 1.4$

$\Rightarrow BC = 11.2$ cm

2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles ΔAOB and ΔCOD

Solution: In trapezium ABCD, AB || DC,

Diagonals AC and BD intersect each other at O



In ΔAOB and ΔCOD,

$\angle 1 = \angle 2$ [Alternate angles]

$\angle 3 = \angle 4$ [Alternate angles]

$\angle 5 = \angle 6$ [Vertically opposite angles]

∴ ΔAOB ~ ΔCOD [AAAsimilarity criteria]

$\Rightarrow \frac{\text{Area}(AOB)}{\text{Area}(COD)} = \frac{AB^2}{CD^2} = \frac{(2CD)^2}{CD^2}$ [∴ AB = 2CD]

$\Rightarrow \frac{\text{Area}(AOB)}{\text{Area}(COD)} = \frac{4CD^2}{CD^2} = \frac{4}{1}$

∴ The ratio of the areas of triangles ΔAOB and ΔCOD is = 4:1

3. In Fig. 2.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ವಿ(ABC)}{ವಿ(DBC)} = \frac{AO}{DO}$

Construction: Draw AP⊥ BC and DM ⊥BC

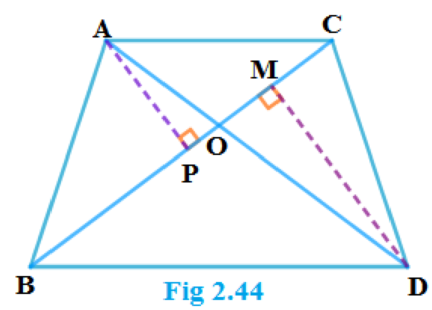


Fig 2.44

Proof: $\frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{\frac{1}{2}BCxAP}{\frac{1}{2}BCxDM} = \frac{AP}{DM}$ --------- (1)

In ΔAPO and ΔDMO,

$\angle APO = \angle DMO = 90^0$

$\angle AOP = \angle DOM$ [Vertically opposite angles]

∴ ΔAPO ~ ΔDMO [AA similarity criteria]

$\Rightarrow \frac{AP}{DM} = \frac{AO}{DO}$ -------------- (2)

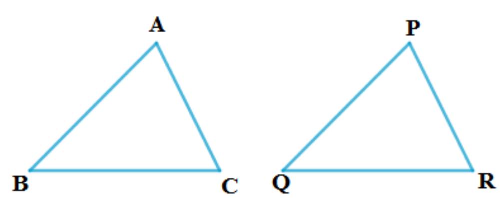
$\Rightarrow \frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{AO}{DO}$ [From (1) and (2)]

4. If the areas of two similar triangles are equal, prove that they are congruent.

Given : ΔABC ~ ΔPQR and Area ΔABC = AreaΔPQR

To prove: ΔABC ≅ ΔPQR

Proof: ΔABC ~ ΔPQR



$\Rightarrow \frac{\text{Area}(ABC)}{\text{Area}(PQR)} = \frac{BC^2}{QR^2}$

$\Rightarrow 1 = \frac{BC^2}{QR^2}$ [Area(ABC)= Area(PQR)]

$\Rightarrow BC^2 = QR^2 \Rightarrow BC = QR$

Similarlly, AB = PQ and AC = PR

∴ ΔABC ≅ ΔPQR [SSS congruency rule]

5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

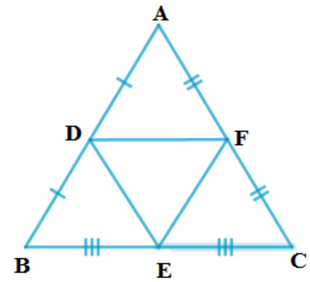
Solution:

In ΔABC, D,E and F are the mid-points of AB, BC and AC respectively

∴ $DF = \frac{1}{2}BC$, $DE = \frac{1}{2}AC$, ಮತ್ತು $EF = \frac{1}{2}AB$[Mid-point theorem]

In ΔDEF and ΔCAB,

$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$



$\therefore \Delta DEF \sim \Delta CAB$

$\therefore \frac{\text{Area}(DEF)}{\text{Area}(CAB)} = \frac{DE^2}{AC^2}$

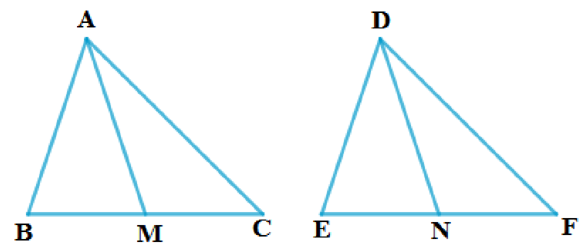
$\Rightarrow \frac{\text{Area}(DEF)}{\text{Area}(CAB)} = \frac{\left(\frac{1}{2}AC\right)^2}{AC^2}$

$\Rightarrow \frac{\text{Area}(DEF)}{\text{Area}(ABC)} = \frac{1}{4}$ [AreaΔABC = AreaΔCAB]

$\Rightarrow$ Area(DEF) : Area(ABC) = 1: 4

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Solution: $\Delta ABC \sim \Delta DEF$ [Given]



$\therefore \frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{AB^2}{DE^2}$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{BM}{EN}$ ---------(1)

In ΔABM and ΔDEN,

$\angle B = \angle E$ [$\Delta ABC \sim \Delta DEF$]

$\frac{AB}{DE} = \frac{BM}{EN}$ [from (1)]

$\therefore \Delta ABM \sim \Delta DEN$ [SAS similarity criteria]

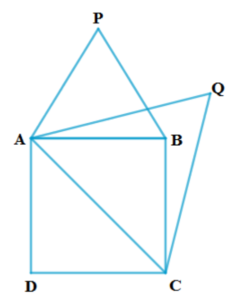
$\Rightarrow \frac{AB}{DE} = \frac{AM}{DN}$

$\therefore \frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{AB^2}{DE^2} = \frac{AM^2}{DN^2}$

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals

Solution: ΔAPB and ΔAQC are equilateral triangles

$\therefore \Delta APB \sim \Delta AQC$ [AAA Similarity criteria]



$\therefore \frac{\text{Area}(AQC)}{\text{Area}(APB)} = \frac{AC^2}{AB^2}$

$\Rightarrow \frac{\text{Area}(AQC)}{\text{Area}(APB)} = \frac{(\sqrt{2}AB)^2}{AB^2}$ [Diagonal of a square = $\sqrt{2}$side]

$\Rightarrow \frac{\text{Area}(AQC)}{\text{Area}(APB)} = \frac{2}{1}$

$\Rightarrow \text{Area}(APB) = \frac{1}{2} \times \text{Area}(AQC)$

Tick the correct answer and justify :

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

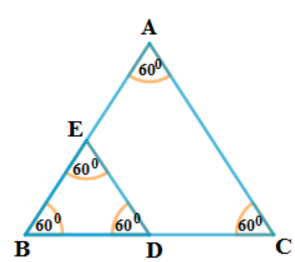
A) 2 : 1 B) 1 : 2 C) 4 : 1 D) 1 : 4

ΔABC and ΔBDE are equilateral triangle. D is the mid-point of BC

$\therefore BD = DC = \frac{1}{2}BC$

Let the sides of ΔABC = $2a$

$\Rightarrow$ the sides of ΔBDE = a



$\Delta ABC \sim \Delta BDE$

$\therefore \frac{\text{Area}(ABC)}{\text{Area}(BDE)} = \frac{BC^2}{BD^2}$

$\Rightarrow \frac{\text{Area}(ABC)}{\text{Area}(BDE)} = \frac{(2a)^2}{a^2} = \frac{4a^2}{a^2} = \frac{4}{1}$

$\therefore$ **Ans: (C) 4:1**

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio
A) 2 : 3 B) 4 : 9 C) 81 : 16 D) 16 : 81

$\Delta ABC \sim \Delta DEF$ and $\frac{BC}{EF} = \frac{4}{9}$

$\therefore \frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{BC^2}{EF^2}$

$\Rightarrow \frac{\text{Area}(ABC)}{\text{Area}(DEF)} = \frac{4^2}{9^2} = \frac{16}{81}$

∴ **Ans : (D) 16:81**

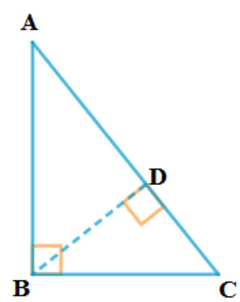
2.6 Pythagoras Theorem

Theorem 2.7
If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Theorem 2.8
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Theorem 2.9
In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Theorem 2.8: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: In $\triangle ABC$, $\angle B = 90^0$

To Prove: $AC^2 = AB^2 + BC^2$

Consruction: Draw $BD \perp AC$

Proof: In ΔADB and ΔABC

$\angle B = \angle D = 90^0$

$\angle A = \angle A$ [Common angle]

$\Delta ADB \sim \Delta ABC$ [AA similarity criteria]

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$

$\Rightarrow AD.AC = AB^2$ --------- (1)

In ΔBDC and ΔABC

$\angle B = \angle D = 90^0$

$\angle C = \angle C$ [Common angle]

$\Delta BDC \sim \Delta ABC$ [AA similarity criteria]

$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow$ CD.AC = BC^2 ---------- (2)

AD.AC + CD.AC = $AB^2 + BC^2$ [By adding (1) and (2)]

$\Rightarrow$ AC (AD+CD) = $AB^2 + BC^2$

$\Rightarrow$ AC × AC = $AB^2 + BC^2$

$\Rightarrow$ AC2 = $AB^2 + BC^2$

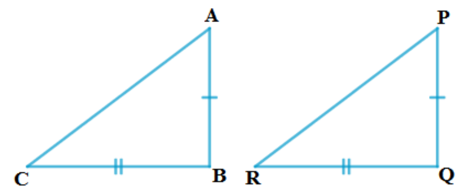
Theorem2.9: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: In ΔABC, $AC^2 = AB^2 + BC^2$

To prove: $\angle B = 90^0$

Construction: Draw ΔPQR such that

$\angle Q = 90^0$ and PQ = AB, QR = BC



Proof:

In ΔPQR,

$PR^2 = PQ^2 + QR^2$ [by Pythogoras theorem]

$PR^2 = AB^2 + BC^2$ [Construction] ------------ (1)

But, $AC^2 = AB^2 + BC^2$ [Given] ------------(2)

∴ AC = PR [from (1) and (2)] ------------(3)

AB = PQ [Construction]

BC = QR [Construction]

AC = PR [from (3)]

∴ $\Delta ABC \cong \Delta PQR$ [SSS congruency rule]

∴ $\angle B = \angle Q$ [By CPCT]

But, $\angle Q = 90^0$ [Construction]

∴ $\mathbf{\angle B = 90^0}$

Example10: In Fig. 2.48, $\angle ACB = 90°$ and $CD \perp AB$ prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$

Solution: $\Delta ACD \sim \Delta ABC$ [Theorem 2.7]

∴ $\frac{AC}{AB} = \frac{AD}{AC} \Rightarrow AC^2 = AD.AB$ ----------------- (1)

$\Delta BCD \sim \Delta BAC$ [Theorem 2.7]

∴ $\frac{BC}{BA} = \frac{BD}{BC} \Rightarrow BC^2 = BA.BD$ ----------------(2)

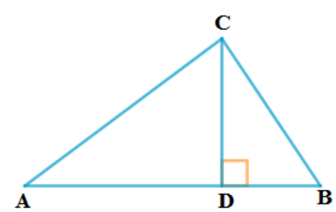


Fig 2.48

dividing (2) by (1)

$$\frac{BC^2}{AC^2} = \frac{BA}{AB} \times \frac{BD}{AD} = \frac{BD}{AD}$$

Example 11 : A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 m above the ground. Find the length of the ladder.

Solution:

Let AB is a ladder, CA is a wall and A is a window

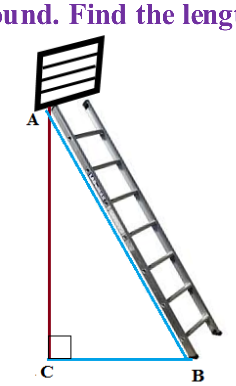
∴BC = 2.5m ಮತ್ತು CA = 6m

$AB^2 = BC^2 + CA^2$ [by Pythagoras theorem]

$AB^2 = (2.5)^2 + 6^2$

$AB^2 = 6.25 + 36$

$AB^2 = 42.25$



AB = 6.5

The length of the ladder 6.5m

Example 12: In Fig. 2.50, if AD⊥BC, prove that $AB^2 + CD^2 = BD^2 + AC^2$

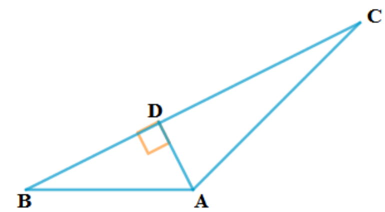


Fig 2.50

Solution: In ΔADC, ∠ ADC = 90^0

∴ $AC^2 = AD^2 + CD^2$ -------(1) [by Pythagoras theorem]

In ΔADB, ∠ ADB = 90^0

∴ $AB^2 = AD^2 + BD^2$ -------(2) [by Pythagoras theorem]

Substract (2) from (1)

$AB^2 - AC^2 = BD^2 - CD^2$

$AB^2 + CD^2 = BD^2 + AC^2$

Example 13 : BL and CM are medians of a triangle ABC right angled at A. Prove that $4\,(BL^2 + CM^2) = 5\,BC^2$

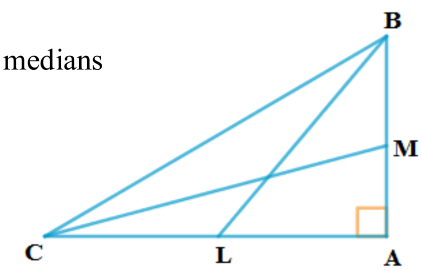


Fig 2.51

Solution: In ΔABC, ∠A = 90^0 , BL and CM are the medians

In ΔABC,

$BC^2 = AB^2 + AC^2$ [Pythagoras Theorem] -----(1)

In ΔABL,

$BL^2 = AL^2 + AB^2$ [Pythagoras Theorem]

$\Rightarrow BL^2 = \left(\frac{AC}{2}\right)^2 + AB^2$ [L is the mid-point of AC]

$\Rightarrow 4BL^2 = AC^2 + 4AB^2$ --------------------------(2)

In ΔCMA,

$CM^2 = AC^2 + AM^2$

$\Rightarrow CM^2 = AC^2 + \left(\frac{AB}{2}\right)^2$ [M is the mid-point of AB]

$\Rightarrow CM^2 = AC^2 + \frac{AB^2}{4}$

$\Rightarrow 4CM^2 = 4AC^2 + AB^2$ -----------------------(3)

Adding (2) and (3)

$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$

$\Rightarrow 4(BL^2 + CM^2) = 5\,BC^2$ [from(1)]

Example 14 : O is any point inside a rectangle ABCD (see Fig. 2.52). Prove that $OB^2 + OD^2 = OA^2 + OC^2$

Solution: Draw PQ through the point O such that PQ∥AD∥ BC

∴ PQ⊥AB and PQ⊥DC [∠B = 90^0 and ∠C = 90^0]

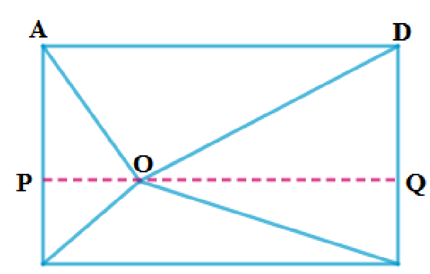


Fig 2.52

In ΔOPB, ∠OPB = 90^0

$OB^2 = BP^2 + OP^2$ -------------- (1)

In ΔOQD,

$OD^2 = OQ^2 + DQ^2$ ---------------(2)

In ΔOQC,

$OC^2 = OQ^2 + CQ^2$ ---------------(3)

In ΔOAP,

$OA^2 = AP^2 + OP^2$ --------------(4)

Adding (1) and (2),

$OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$

$OB^2 + OD^2 = CQ^2 + OP^2 + OQ^2 + AP^2$ [BP=CQ and DQ=AP]

$OB^2 + OD^2 = CQ^2 + OQ^2 + OP^2 + AP^2$

$\mathbf{OB^2 + OD^2 = OC^2 + OA^2}$ [from (3) and (4)]

Exercise 2.5

1. **Sides of triangles are given below. Determine which of them are right triangles.In case of a right triangle, write the length of its hypotenuse.**
   i) 7cm, 24cm, 25cm
   ii) 3cm, 8cm, 6cm
   iii) 50cm, 80cm, 100cm
   iv) 130cm, 12cm, 5cm

   (i) 7 cm, 24 cm and 25 cm.

   Squaring the numbers 49, 576, and 625.

   $49 + 576 = 625$

   $(7)^2 + (24)^2 = (25)^2$

   ∴ Converse of Pythagoras theorem, It is a right triangle.

   Length of the hypotenuse = 25cm

   (ii) 3 cm, 8 cm and 6 cm.

   Squaring the numbers, 9, 64, and 36.

   $9 + 36 \neq 64$

   $\Rightarrow 3^2 + 6^2 \neq 8^2$

   It is not a right triangle

   (iii) 50 cm, 80 cm and 100 cm.

   Squaring the numbers 2500, 6400 and 10000.

   $2500 + 6400 \neq 10000$

   $\Rightarrow 50^2 + 80^2 \neq 100^2$

   It is not a right triangle

   (iv) 13 cm, 12 cm and 5 cm.

   Squaring the numbers 169, 144, and 25.

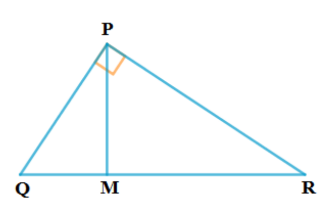
   $144 + 25 = 169$

   $\Rightarrow 12^2 + 5^2 = 13^2$

   ∴ Converse of Pythagoras theorem, It is a right triangle.

   Length of the hypotenuse = 13cm

2. **PQR is a triangle right angled at P and M is a point on QR such that PM⊥QR. Show that $PM^2 = QM . MR$**

   Solution: In ΔPQM, $\angle P = 90^0$ and PM⊥QR

   ∴ ΔPQM ~ ΔRPM [Theorem 2.7]

   $\Rightarrow \frac{PM}{MR} = \frac{QM}{PM}$

   $\Rightarrow PM^2 = QM. MR$

3. **In Fig. 2.53, ABD is a triangle right angled at A and AC⊥BD. Show that**

i) $AB^2 = BC.BD$
ii) $AC^2 = BC.DC$
iii) $AD^2 = BD.CD$ ಎಂದು ಸಾಧಿಸಿ

(i) In ΔADB, $\angle A = 90^0$, $AC \perp BD$
$\Rightarrow \Delta ABD \sim \Delta\ CBA$ [Theorem 2.7]
$\Rightarrow \frac{AB}{BC} = \frac{BD}{AB} \Rightarrow AB^2 = BC.BD$
(ii) In ΔADB, $\angle A = 90^0$, $AC \perp BD$
$\Rightarrow \Delta ABC \sim \Delta\ DAC$ [Theorem 2.7]
$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC} \Rightarrow AC^2 = BC.DC$
(iii) **In ΔADB,** $\angle A = 90^0$, $AC \perp BD$
$\Rightarrow \Delta ABD \sim \Delta\ CAD$ [Theorem 2.7]
$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD} \Rightarrow AD^2 = BD \times CD$

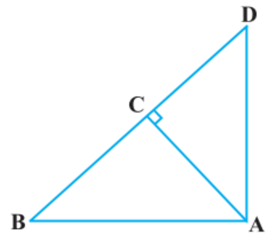
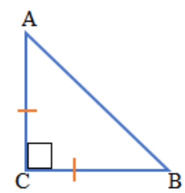


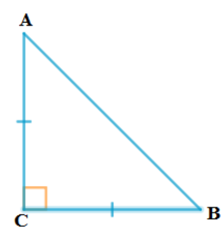
Fig. 2 .53

4. **ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$**



Solution: In $\Delta ABC, \angle C = 90^0$
$AC = BC$ [Given]
$AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]
$AB^2 = AC^2 + AC^2$ [$AC = BC$]
$AB^2 = 2AC^2$

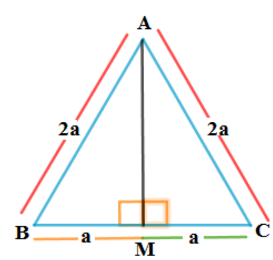
5. **ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.**



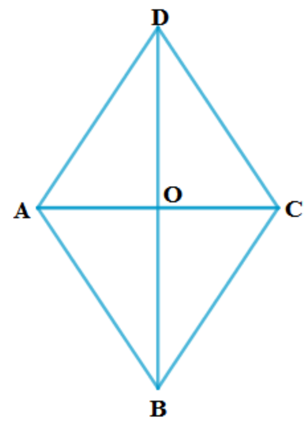
Solution: In ΔABC,
$AC = BC$ and $AB^2 = 2AC^2$ [Given]
$AB^2 = AC^2 + AC^2$
$AB^2 = AC^2 + BC^2$ [$AC = BC$]
$\therefore$**ΔABC is a right triangle**

6. **ABC is an equilateral triangle of side 2a. Find each of its altitudes.**

Solution: ABC is an equilateral triangle
$AB = BC = CA = 2a$.
Draw height $AM \perp BC$
In ΔABM,
$\Rightarrow BM = MC = a$ [ΔABC is an equilateral, $AM \perp BC$ Bisects BC]
$\angle AMB = 90^0$ [$AM \perp BC$ is the height]
$AB^2 = AD^2 + BD^2$
$\Rightarrow AD^2 = (2a)^2 - a^2$
$\Rightarrow AD^2 = 4a^2 - a^2$
$\Rightarrow AD = \sqrt{3}a$



7. **Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.**



Solution:ABCD is a rhombus

∴ AC and BD bisects each other perpendicularly at O

∴ AO = CO = $\frac{AC}{2}$ and BO = DO = $\frac{BD}{2}$

In ΔAOB, ∠AOB = 90°

$AB^2 = AO^2 + BO^2$ [by Pythagoras theorem]

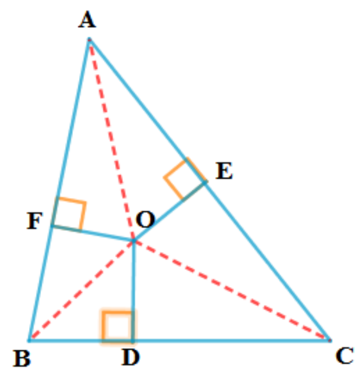
$\Rightarrow AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

$\Rightarrow AB^2 = \frac{AO^2}{4} + \frac{BO^2}{4}$

$\Rightarrow 4AB^2 = AC^2 + BD^2$

$AB^2 + AD^2 + DC^2 + BC^2 = AC^2 + BD^2$ [AB=BC=CD=DA]

8. **In Fig. 2.54, O is a point in the interior of a triangle ABC, OD⊥BC, OE⊥AC and OF⊥AB . Show that**
i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$
ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$ ಎಂದು ಸಾಧಿಸಿ.



Solution: Join OA, OB and OC

(i) In ΔAOF, ∠OFA = 90°

∴ $OA^2 = OF^2 + AF^2$ [By Pythagoras theorem]

In ΔBOD, ∠ODB = 90°

∴ $OB^2 = OD^2 + BD^2$ [By Pythagoras theorem]

In ΔCOE, ∠OEC = 90°

∴ $OC^2 = OE^2 + EC^2$ [By Pythagoras theorem]

Adding all three,

$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

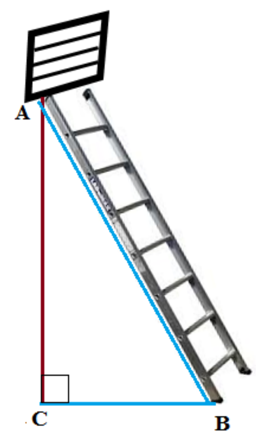
$\mathbf{OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2}$.

(ii) $AF^2 + BD^2 + EC^2 = \underline{OA^2 - OF^2} + \underline{OB^2 - OD^2} + \underline{OC^2 - OE^2}$

$AF^2 + BD^2 + EC^2 = \underline{OA^2 - OE^2} + \underline{OC^2 - OD^2} + \underline{OB^2 - OF^2}$

∴ $\mathbf{AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2}$.

9. **A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.?**



Solution: Height of the wall CA = 8m, Height of the ladder AB = 1(

$AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$10^2 = 8^2 + BC^2$

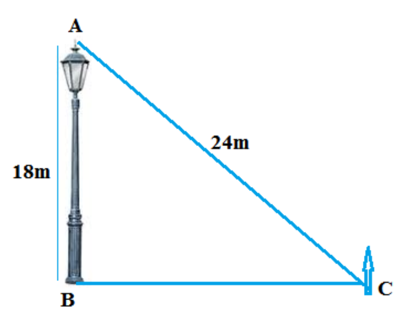
$BC^2 = 100 - 64$

$BC^2 = 36$

BC = 6m

The foot of the ladder is 6m away from the base of the wall

10. **A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?**



Solution: Length of the Pole AB= 18m,

Length of the wire AC = 24m the wire.

$AC^2 = AB^2 + BC^2$ [By Pythagoras theorem]

$24^2 = 18^2 + BC^2$

$BC^2 = 576 - 324$

$BC^2 = 252$

$BC = 6\sqrt{7}$m

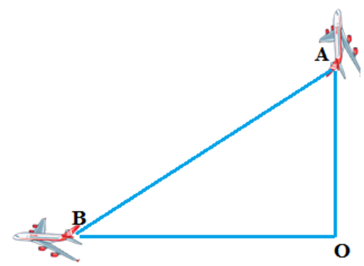
11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Solution: The speed of the aeroplane which flies due north = 1000 km/hr

∴ The distance travelled in $1\frac{1}{2}$ hours = 1500 km

The speed of the aeroplane which flies due west = 1200km/hr

∴ The distance travelled in $1\frac{1}{2}$ hours = 1800 km



In ΔAOB, $\angle AOB = 90°$

$AB^2 = AO^2 + OB^2$ [by Pythagoas theorem]

$\Rightarrow AB^2 = (1500)^2 + (1800)^2$

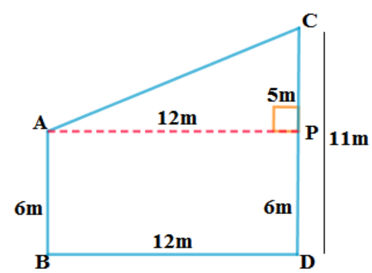
$\Rightarrow AB = \sqrt{2250000 + 3240000}$

$\Rightarrow AB = \sqrt{5490000}$

$\Rightarrow AB = 300\sqrt{6}$ km

12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Solution: Pole CD= 11m and Pole AB = 6m



∴ CP = 11 - 6 = 5 m

Distance between two poles BD = 12m = AP

ΔAPC, $\angle CPC = 90°$

$AP^2 + PC^2 = AC^2$

$AC^2 = (12m)^2 + (5m)^2$

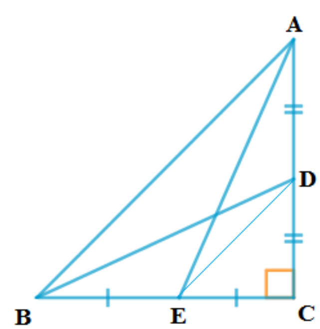
$AC^2 = (144+25)m^2 = 169\ m^2$

AC = 13m

∴ The distance between the top = 13 m.

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$

Solution: In ΔACE, $\angle ACE = 90^0$



∴ $AC^2 + CE^2 = AE^2$ .... **(i)**

In ΔBCD, $\angle BCD = 90^0$

∴ $BC^2 + CD^2 = BD^2$ .... **(ii)**

Adding (1) and (2),

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ ... **(iii)**

In ΔCDE, $\angle DCE = 90^0$

$DE^2 = CD^2 + CE^2$ ------------ (iv)

In ΔABC, $\angle ACB = 90^0$

$AB^2 = AC^2 + CB^2$ ----------- (v)

Substitute (iv) and (v) in (iii)

$\mathbf{DE^2 + AB^2 = AE^2 + BD^2}$.

14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD (see Fig. 2.55). Prove that $2\,AB^2 = 2\,AC^2 + BC^2$

Solution: In ΔABC, AD ⊥ BC and DB = 3CD

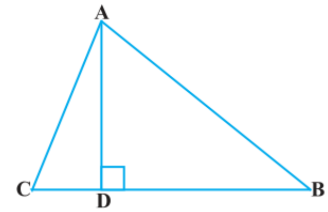


Fig. 2.55

In right angle triangles ADB and ADC,

$AB^2 = AD^2 + BD^2$ ... (i) [By Pythagoras theorem]

$AC^2 = AD^2 + DC^2$ ... (ii) [By Pythagoras theorem]

Substract (i) from (ii),

$AB^2 - AC^2 = DB^2 - DC^2$

$AB^2 - AC^2 = 9CD^2 - CD^2$ [∴ BD = 3CD]

$AB^2 - AC^2 = 8 \times \left(\frac{BC}{4}\right)^2$ [∵BC = DB + CD = 3CD + CD = 4CD]

$\therefore AB^2 - AC^2 = \frac{BC^2}{2}$

$\Rightarrow 2(AB^2 - AC^2) = BC^2$

$\Rightarrow 2AB^2 - 2AC^2 = BC^2$

$\therefore 2AB^2 = 2AC^2 + BC^2$.

15. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$ Prove that $9AD^2 = 7AB^2$

Solution: In an equilateral triangle ABC,

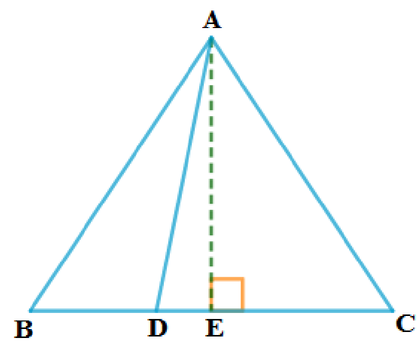
Let AB = BC = AC = a $BD = \frac{BC}{3} = \frac{a}{3}$, Draw AE⊥BC

$\Rightarrow BE = EC = \frac{BC}{2} = \frac{a}{2}$

$\Rightarrow AE^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow AE = \frac{\sqrt{3}a}{2}$

$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$



In ΔADE, ∠AED = 90^0

$AD^2 = AE^2 + DE^2$ [by Pythagoras Theorem]

$AD^2 = \left(\frac{\sqrt{3}a}{2}\right)^2 + \left(\frac{a}{6}\right)^2$

$AD^2 = \frac{3a^2}{4} + \frac{a^2}{36} = \frac{27a^2 + a^2}{36} = \frac{28a^2}{36} = \frac{7a^2}{9}$

$\Rightarrow AD^2 = \frac{7}{9}AB^2$

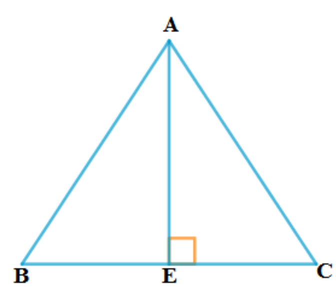
$\Rightarrow \mathbf{9\,AD^2 = 7\,AB^2}$

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Solution: In an equilateral triangle ABC,

Let AB = BC = AC = a . AE⊥BC .

$\Rightarrow BE = EC = \frac{BC}{2} = \frac{a}{2}$



In ΔABE, ∠AEB = 90^0

$AB^2 = AE^2 + BE^2$ [by Pythagoras Theorem]

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$a^2 = AE^2 + \frac{a^2}{4}$

$\Rightarrow AE^2 = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

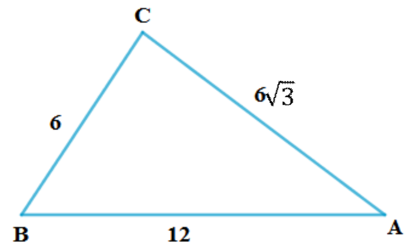
$4AE^2 = 3a^2$

$\Rightarrow 4 \times (\text{height})^2 = 3 \times (\text{Side})^2$

(ತ) **Tick the correct answer and justify :**

In $\triangle ABC$, $AB = 6\sqrt{3}$cm, AC = 12 cm and BC = 6 cm$\triangle ABC$ ಯಲ್ಲಿ $AB = 6\sqrt{3}$cm, AC = 12cm the angle B is,

A) 120^0 B) 60^0 C) 90^0 D) 45^0



$AB = 6\sqrt{3}$cm, AC = 12 cm, and BC = 6 cm

$AB^2 = 108$, $AC^2 = 144$ and $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

$108 + 36 = 144$

$\therefore \angle B = 90°$

The ans is **C).90^0**

2.7 Summary

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true
3. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equaland (ii) their corresponding sides are in the same ratio (i.e., proportion).
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
6. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
7. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
9. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).
10. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
11. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
12. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides (Pythagoras Theorem).
13. If in a triangle, square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Pair of Linear Equations in two Variables

3

Linear equation with one variable: The algebraic equation of the type $\mathbf{ax + b = 0}$ ($\mathbf{a \neq 0}$ and $\mathbf{b}$ are real numbers, $\mathbf{x -}$ variable is called linear equation of one variable. These type of equations having only one solution.

Example : $\mathbf{2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = \frac{-5}{2}}$

3.2 Pair of Linear Equations in Two Variables

2x + 3y = 5 ;

x - 2y - 3 = 0 and

x - 0y = 2, ⇒ x = 2

An equation which can be put in the form ax + by + c = 0, where a, b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables x and y. A solution of such an equation is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.

In fact, this is true for any linear equation, that is, each solution (x, y) of a linear equation in two variables, ax + by + c = 0, corresponds to a point on the line representing the equation, and vice versa.

2x + 3y=5; x - 2y - 3 = 0

These two linear equations are in the same two variables x and y. Equations like these are called a **pair of linear equations in two variables.**

The general form for a pair of linear equations in two variables x and y is,

$a_1x + b_1x + c_1 = 0$ and $a_2x + b_2x + c_2 = 0$

Here, a_1, b_1, c_1, a_2, b_2, c_2 are real numbers

Some examples of pair of linear equations in two variables are:

(i) 2x + 3y - 7 = 0 ; 9x - 2y + 8 = 0

(ii) 5x = y ; -7x + 2y + 3 = 0

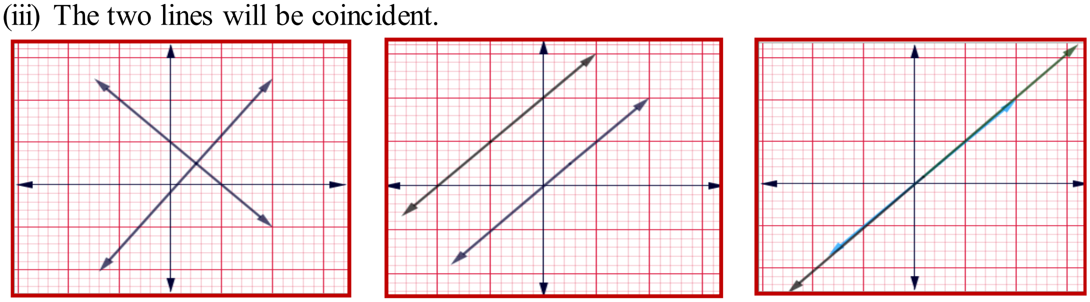
(iii) x + y = 7 ; 17 = y

Two lines in a plane, only one of the following three possibilities can happen:

(i) The two lines will intersect at one point.

(ii) The two lines will not intersect, i.e., they are parallel.

(iii) The two lines will be coincident.



Example 1: Akhila goes to a fair with Rs 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically).

Solution: The pair of equations formed is :

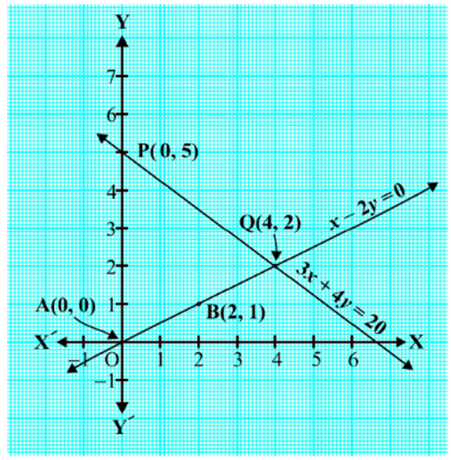
$y = \frac{1}{2}x \Rightarrow 2y = x$

$\Rightarrow x - 2y = 0$ (1) and $3x + 4y = 20$ (2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

x	0	2
$y = \frac{x}{2}$	2	1

x	0	4	8
$y = \frac{20-3x}{4}$	5	2	-1



Example 2 : Romila went to a stationery shop and purchased 2 pencils and 3 erasers for Rs 9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs 18. Represent this situation algebraically and graphically.

Solution : Let us denote the cost of 1 pencil by x' and one eraser by y'. Then the algebraic representation is given by the following equations:

$2x + 3y = 9$ -------------(1)

$4x + 6y = 18$ -----------(2)

$(1) \Rightarrow 3y = 9 - 2x$

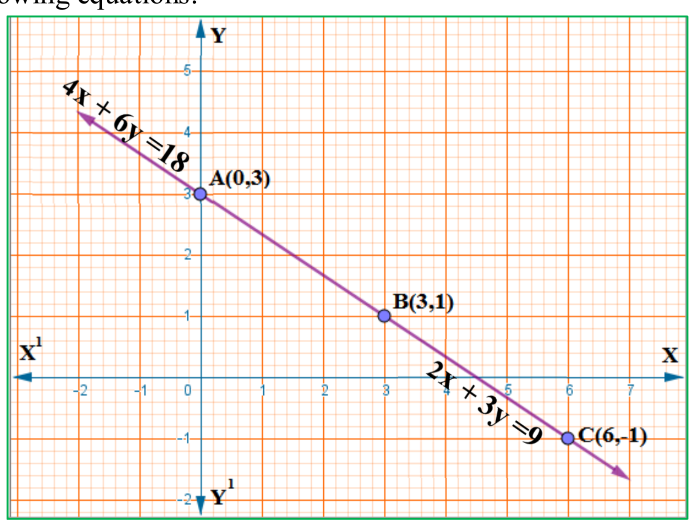
$y = \frac{9-2x}{3}$

x	0	3	6
$y = \frac{9-2x}{3}$	3	1	-1

$(2) \Rightarrow 6y = 18 - 4x$

$y = \frac{18-4x}{6}$

x	0	3	6
$y = \frac{18-4x}{6}$	3	1	-1



Example 3 : Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ Represent this situation geometrically.

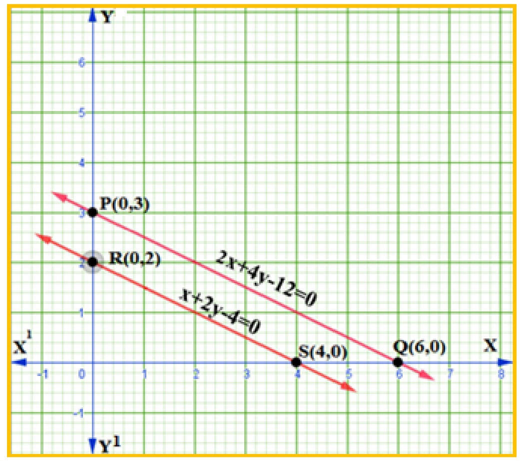
Solution : Two solutions of each of the equations : $x + 2y = 4$ and $2x + 4y = 12$

$x + 2y = 4 \Rightarrow 2y = 4 - x \Rightarrow y = \frac{4-x}{2}$

x	0	4
$y = \frac{4-x}{2}$	2	0

$2x + 4y = 12 \Rightarrow y = \frac{12-2x}{4}$

x	0	6
$y = \frac{12-2x}{4}$	3	0



Exercise 3.1

1. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then.Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Let the present age of Aftab $= x$ years and the present age of his daughter $= y$ years.

Age of Aftab before7years $= (x - 7)$ and the age of his daughter before 7 years $= (y - 7)$

Then the algebraic representation is,

$x - 7 = 7(y - 7) \Rightarrow x - 7 = 7y - 49 \Rightarrow x - 7y = -42$

Age of Aftab after 3 years= $(x + 3)$ years and age of his daughter after 3 years= $y + 3$

Then the algebraic representation is,

$x + 3 = 3(y + 3) \Rightarrow x + 3 = 3y + 9 \Rightarrow x - 3y = 9 - 3 \Rightarrow x - 3y = 6$

Solutions of each of the equations :

$x - 7y = -42 \Rightarrow 7y = x + 42 \Rightarrow y = \frac{x+42}{7}$

x	−7	0	7
$y = \frac{x+42}{7}$	5	6	7

$x = 0 \Rightarrow y = \frac{0+42}{7} = \frac{42}{7} = 6$

$x = 7 \Rightarrow y = \frac{7+42}{7} = \frac{49}{7} = 7$

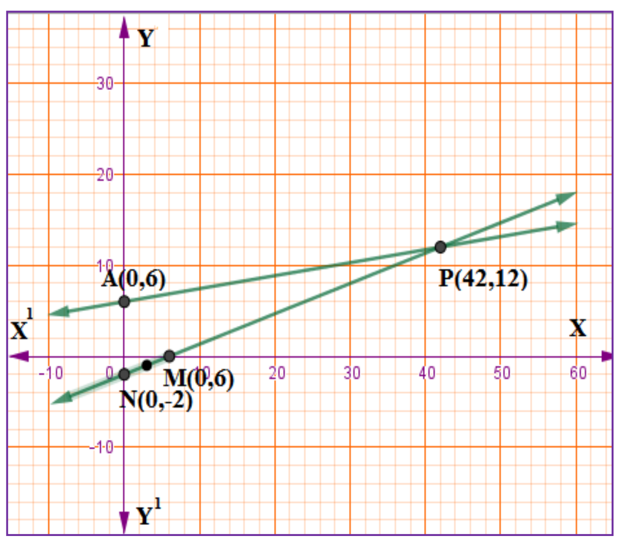
$x - 3y = 6 \Rightarrow y = \frac{x-6}{3}$

x	6	3	0
$y = \frac{x-6}{3}$	0	−1	−2

$x = 6 \Rightarrow y = \frac{6-6}{3} = \frac{0}{3} = 0$

$x = 3 \Rightarrow y = \frac{3-6}{3} = \frac{-3}{3} = -1$

$x = 0 \Rightarrow y = \frac{0-6}{3} = \frac{-6}{3} = -2$



The two lines are intersecting each other. There fore there is a unique solution. The coordinates of intersecting point are $(42, \ 12)$

2. The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs1300. Represent this situation algebraically and geometrically.

Let no.of bats $=x$, Let no.of balls $=y$. Then the algebraic representation is,

$3x+6y=3900$ and $x+3y=1300$

$3x+6y=3900 \Rightarrow 6y=3900-3x \Rightarrow y=\frac{3900-3x}{6}$

x	300	100	–100
$\mathbf{y=\frac{3900-3x}{6}}$	500	600	700

$x=300 \Rightarrow y=\frac{3900-3(300)}{6}=\frac{3000}{6}=500$

$x=100 \Rightarrow y=\frac{3900-3(100)}{6}=\frac{3600}{6}=600$

$x=-100 \Rightarrow y=\frac{3900-3(-100)}{6}=\frac{4200}{6}=700$

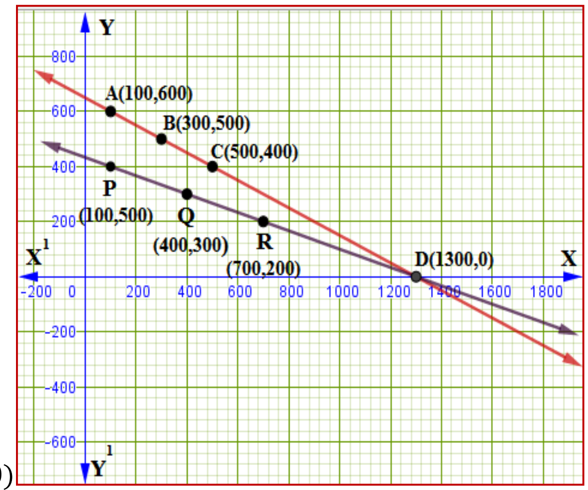
$x+3y=1300 \Rightarrow y=\frac{1300-x}{3}$

$\mathrm{x}=400 \Rightarrow \mathrm{y}=\frac{1300-400}{3}=\frac{900}{3}=300$

$\mathrm{x}=700 \Rightarrow \mathrm{y}=\frac{1300-700}{3}=\frac{600}{3}=200$

$\mathrm{x}=1000 \Rightarrow \mathrm{y}=\frac{1300-1000}{3}=\frac{300}{3}=100$

xx	400	700	1000
$\mathbf{y=\frac{1300-x}{3}}$	300	200	100



The two lines are intersecting each other. There fore there is a unique solution. The coordinates of intersecting point are $(1300, 0)$

3. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Let the cost of apples/ kg = Rs x, the cost of grapes/kg = Rs y

Then the algebraic representation is,

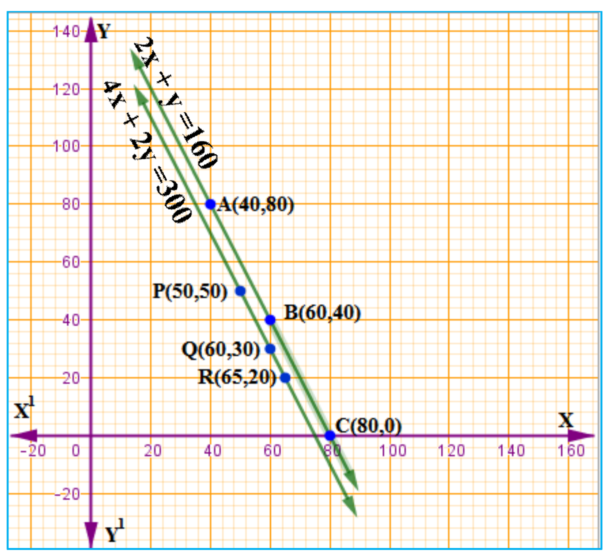
$2x+y=160;\qquad 4x+2y=300$

$2x+y=160 \Rightarrow y=160-2x$

x	50	60	70
$\mathbf{y=160-x}$	60	40	20

$4x+2y=300 \Rightarrow y=\frac{300-4x}{2}$

$\boldsymbol{x}$	70	80	75
$\boldsymbol{y=\frac{300-4x}{2}}$	10	–10	0



Both lines are parallel to each other. There fore there are no solutions for thes equations.

3.3. Graphical Method of Solution of a Pair of Linear Equations

Consistent pair : A pair of linear equations in two variables, which has a solution, is called a consistent pair of linear equations.

Dependent pair : A pair of linear equations which are equivalent has infinitely many distinct common solutions. Such a pair is called a dependent pair of linear equations in two variables.

Inconsistent pair : A pair of linear equations which has no solution, is called an inconsistent pair of linear equations.

$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$ ಎಂಬ ಎರಡು ಸಮೀಕರಣಗಳಿಗೆ,

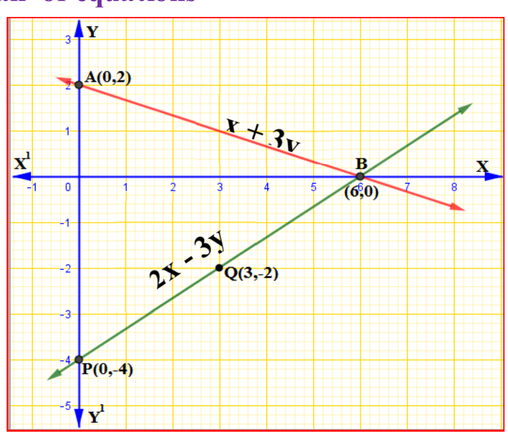
Comparing the ratios	Representing on graph	Algebraic solution	Consistency
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting each other	Unique solution	Consistant
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	coincident lines.	Infinite sulutions	dependent
$\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solutions	Inconsistent

Example 4 : Check graphically whether the pair of equations

1) $x + 3y = 6$ (1) and

$2x - 3y = 12$ (2)

is consistent. If so, solve them graphically.



$x + 3y = 6 \Rightarrow 3y = 6 - x \Rightarrow y = \frac{6-x}{3}$

$$\boldsymbol{x = 0 \Rightarrow y = \frac{6-0}{3} = \frac{6}{3} = 2}$$
$$\boldsymbol{x = 6 \Rightarrow y = \frac{6-6}{3} = \frac{0}{3} = 0}$$

x	0	6
$y = \frac{6-x}{3}$	2	0

$2x - 3y = 12 \Rightarrow 3y = 2x - 12 \quad \Rightarrow y = \frac{2x-12}{3}$

$$\boldsymbol{x = 0 \Rightarrow y = \frac{2(0)-12}{3} = \frac{-12}{3} = -4}$$
$$\boldsymbol{x = 3 \Rightarrow y = \frac{2(3)-12}{3} = \frac{-6}{3} = -2}$$

x	0	3
$y = \frac{2x-12}{3}$	−4	−2

Both lines are intersecting at the point (6,0). Therefore the solution of the equation is $x = 6$ and $y = 0 \Rightarrow$ The equations are consistant pair.

Example 5 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:

$\mathbf{5x - 8y + 1 = 0}$ **(1)**

$\mathbf{3x - \frac{24}{5}y + \frac{3}{5} = 0}$ **(2)**

Multiplying equation (2) by $\frac{5}{3}$

$3\left(\frac{5}{3}\right)x - \frac{24}{5}\left(\frac{5}{3}\right)y + \frac{3}{5}\left(\frac{5}{3}\right) = 0$

$5x - 8x + 1 = 0$

But, this is the same as Equation (1). Hence the lines represented by Equations (1)and (2) are coincident. Therefore, Equations (1) and (2) have infinitely many solutions.

Example 6 : Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Let us denote the number of pants by x and the number of skirts by y.

Then the equations are:

$y = 2x - 2$ (1)

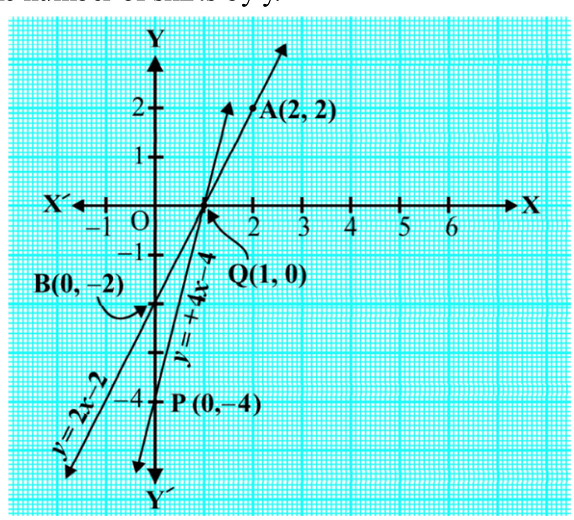
$y = 4x - 4$ (2)

$$y = 2x - 2$$
$$x = 2 \Rightarrow y = 2(2) - 2 = 4 - 2 = 2$$
$$x = 1 \Rightarrow y = 2(1) - 2 = 2 - 2 = 0$$
$$x = 0 \Rightarrow y = 2(0) - 2 = 0 - 2 = -2$$

x	2	1	0
$y = 2x - 2$	2	0	−2

$$y = 4x - 4$$
$$x = 0 \Rightarrow y = 4(0) - 4 = 0 - 4 = -4$$
$$x = 1 \Rightarrow y = 4(1) - 4 = 4 - 4 = 0$$

x	0	1
$y = 4x - 4$	−4	0



The two lines intersect at the point (1, 0).

So, x = 1, y = 0 is the required solution of the pair of linear equations, i.e., the number of pants she purchased is 1 and she did not buy any skirt.

Exercise 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.
(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.
(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen

(i) Let the number of girls be x number of boys be y

$x+y=10$ (1) ; $x-y=4$ (2)

(1) $x+y=10 \Rightarrow y=10-x$

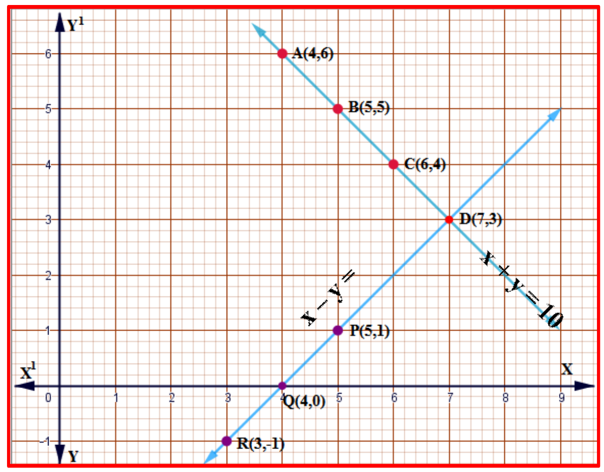
$x=5 \Rightarrow y=10-5=5$
$x=4 \Rightarrow y=10-4=6$
$x=6 \Rightarrow y=10-6=4$

x	5	4	6
$y=10-x$	5	6	4

(2)$\Rightarrow x-y=4 \Rightarrow y=x-4$

$x=5 \Rightarrow y=5-4=1$
$x=4 \Rightarrow y=4-4=0$
$x=3 \Rightarrow y=3-4=-1$

x	5	4	3
$y=x-4$	1	0	–1



Two lines are intersecting at the point $(7, 3)$

$\therefore$ The solution is: $x=7$, $y=3$

$\Rightarrow$Number of Girs = 7, Number of boys = 3

(ii) Let the cost of a pencil be Rs x, and the cost of a pen is Rs y Then the equation are:

$5x+7y=50$ and $7x+5y=46$

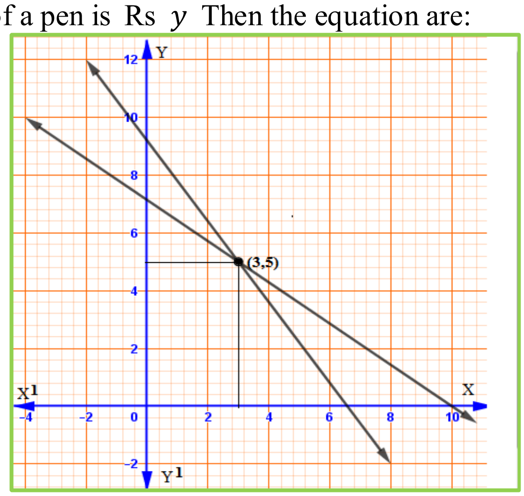
$5x+7y=50 \Rightarrow y=\frac{50-5x}{7}$

$x=3 \Rightarrow y=\frac{50-5(3)}{7}=\frac{50-15}{7}=\frac{35}{7}=5$

$x=10 \Rightarrow y=\frac{50-5(10)}{7}=\frac{50-50}{7}=0$

$x=-4 \Rightarrow y=\frac{50-5(-4)}{7}=\frac{70}{7}=10$

x	3	10	–4
$y=\frac{50-5x}{7}$	5	0	10



$7x+5y=46$

$\Rightarrow 5y=46-7x \Rightarrow y=\frac{46-7x}{5}$

$x=8 \Rightarrow y=\frac{46-7(8)}{5}=\frac{46-56}{5}=\frac{-10}{5}=-2$

$x=3 \Rightarrow y=\frac{46-7(3)}{5}=\frac{46-21}{5}=\frac{25}{5}=5$

$x=-2 \Rightarrow y=\frac{46-7(-2)}{5}=\frac{46+14}{5}=\frac{60}{5}=12$

x	8	3	–2
$y=\frac{300-4x}{2}$	–2	5	12

Two lines are intersecting at the point $(3, 5)$. $\therefore$ The solution is: $x=3$, $y=5$

Therefore the cost of pencil = Rs 3; The cost of pen = Rs 5

1) On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, $\frac{c_1}{c_2}$ find out whether the lines representing the following pair of linear equations intersect at a point, parallel or coincident.

(i) $5x - 4y + 8 = 0$ (ii) $9x + 3y + 12 = 0$ (iii) $6x - 3y + 10 = 0$
$7x + 6y - 9 = 0$ $18x + 6y + 24 = 0$ $2x - y + 9 = 0$

(i) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get
$a_1 = 5,\ b_1 = -4,\ c_1 = 8$ and $a_2 = 7,\ b_2 = 6,\ c_2 = -9$

$\frac{a_1}{a_2} = \frac{5}{7}, \quad \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \quad \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ The pair of lines intersect at a point and have unique solution.

(ii) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get
$a_1 = 9,\ b_1 = 3,\ c_1 = 12$ and $a_2 = 18,\ b_2 = 6,\ c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2} \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The pair of lines are coincident.and have infinite solution.

(iii) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

Comparing these with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we get
$a_1 = 6,\ b_1 = -3,\ c_1 = 10$ and $a_2 = 2,\ b_2 = -1,\ c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = 3, \quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3, \quad \frac{c_1}{c_2} = \frac{10}{9} \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ The pair of lines are parallel and have no solution.

2) On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing the following pair of linear equations are consistent or inconsistent?

(i) $3x + 2y = 5;\ 2x - 3y = 7$
(ii) $2x - 3y = 8;\ 4x - 6y = 9$
(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$
(iv) $5x - 3y = 11;\ -10x + 6y = -22$
(v) $\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

(i) $3x + 2y = 5;\ 2x - 3y = 7$

$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{2}{-3} \quad \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

⇒This pair of linear equations are consistent.

(ii) $2x - 3y = 8;\ 4x - 6y = 9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{8}{9} \quad \therefore \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

⇒This pair of equations are inconsistent

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7;\ 9x - 10y = 14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{5}{3} \times \frac{1}{-10} = -\frac{1}{6} \quad \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

⇒This pair of linear equations are consistent.

(iv) $5x - 3y = 11; \quad -10x + 6y = -22$

$\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = -\frac{1}{2} \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

⇒This pair of linear equations are consistent and have infinite solutions.

(v) $\frac{4}{3}x + 2y = 8; \quad 2x + 3y = 12$

$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3} \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

⇒This pair of linear equations are consistent and have infinite solutions.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, \quad 2x + 2y = 10$

(ii) $x - y = 8, \quad 3x - 3y = 16$

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, \quad 4x - 3y - 5 = 0$

(i) $x + y = 5; \quad 2x + 2y = 10$

$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2} \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ These are coincident lines and have infinite solutions. Hence the pair is consistent.

$x + y = 5 \Rightarrow y = 5 - x$

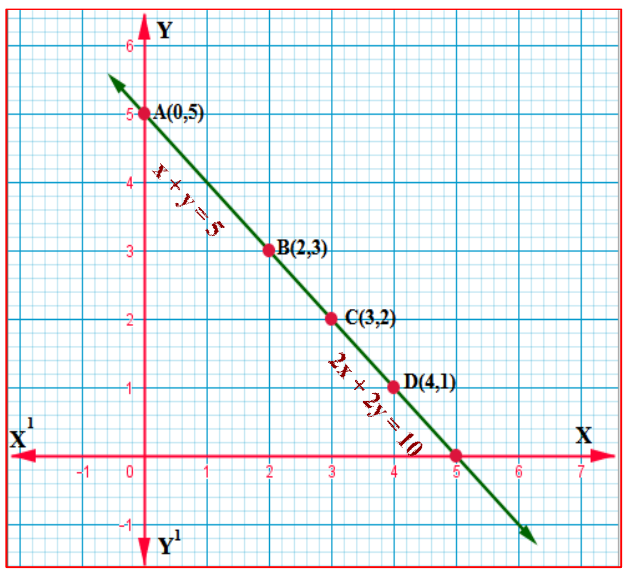
$x = 2 \Rightarrow y = 5 - 2 = 3$
$x = 3 \Rightarrow y = 5 - 3 = 2$
$x = 4 \Rightarrow y = 5 - 4 = 1$

x	2	3	4
$y = 5 - x$	3	2	1

$2x + 2y = 10 \Rightarrow y = \frac{10-2x}{2}$

$x = 2 \Rightarrow y = \frac{10-2(2)}{2} = \frac{10-4}{2} = 3$
$x = 3 \Rightarrow y = \frac{10-2(3)}{2} = \frac{10-6}{2} = 2$
$x = 4 \Rightarrow y = \frac{10-2(4)}{2} = \frac{10-8}{2} = 1$

x	2	3	4
$y = \frac{10-2x}{2}$	3	2	1



(ii) $x - y = 8; \quad 3x - 3y = 16$

$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2} \quad \therefore \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴ These are parallel lines and have no solutions.Hence the pair is inconsistent.

(iii) $2x + y - 6 = 0; \quad 4x - 2y - 4 = 0$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2} \quad \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴these lines are consistent and intersect each other. These lines have unique solution (2.2). $2x + y - 6 = 0 \Rightarrow y = 6 - 2x$

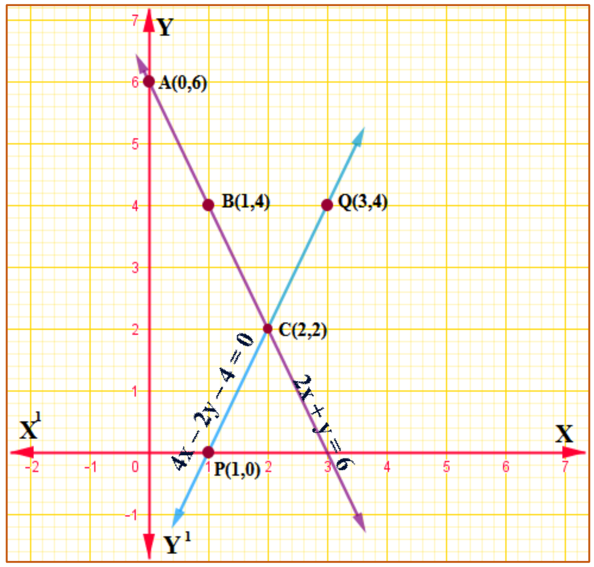
$x = 0 \Longrightarrow y = 6 - 2(0) = 6 - 0 = 6$
$x = 1 \Longrightarrow y = 6 - 2(1) = 6 - 2 = 4$
$x = 2 \Longrightarrow y = 6 - 2(2) = 6 - 4 = 2$

x	0	1	2
$y = 6 - 2x$	6	4	2

$4x - 2y - 4 = 0 \Longrightarrow y = \frac{4x-4}{2}$

$x = 1 \Longrightarrow y = \frac{4(1)-4}{2} = \frac{4-4}{2} = \frac{0}{2} = 0$
$x = 2 \Longrightarrow y = \frac{4(2)-4}{2} = \frac{8-4}{2} = \frac{4}{2} = 2$
$x = 3 \Longrightarrow y = \frac{4(3)-4}{2} = \frac{12-4}{2} = \frac{8}{2} = 4$

x	1	2	3
$y = \frac{4x-4}{2}$	0	2	4



(iv) $2x - 2y - 2 = 0$; $4x - 3y - 5 = 0$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-2}{-3} = \frac{2}{3}$ $\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ these lines are consistent and intersect each other. These lines have unique solution.

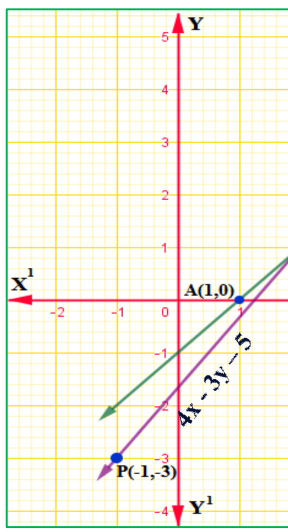
$2x - 2y - 2 = 0 \Longrightarrow y = \frac{2x-2}{2}$

$x = 1 \Longrightarrow y = \frac{2(1)-2}{2} = \frac{2-2}{2} = \frac{0}{2} = 0$
$x = 2 \Longrightarrow y = \frac{2(2)-2}{2} = \frac{4-2}{2} = \frac{2}{2} = 1$
$x = 3 \Longrightarrow y = \frac{2(3)-2}{2} = \frac{6-2}{2} = \frac{4}{2} = 2$

x	1	2	3
$y = \frac{2x-2}{2}$	0	1	2

$4x - 3y - 5 = 0 \Longrightarrow y = \frac{4x-5}{3}$

$x = 2 \Longrightarrow y = \frac{4(2)-5}{3} = \frac{8-5}{3} = \frac{3}{3} = 1$
$x = 5 \Longrightarrow y = \frac{4(5)-5}{3} = \frac{20-5}{3} = \frac{15}{3} = 5$
$x = -1 \Longrightarrow y = \frac{4(-1)-5}{3} = \frac{-4-5}{3} = -3$



x	2	5	–1
$y == \frac{4x-5}{3}$	1	5	–3

5. **Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

Let the width of the Garden $= x$, Length $= y$.

Length: $y = x + 4$

x	0	8	16
$y = x + 4$	4	12	20

$x = 0 \Longrightarrow y = 0 + 4 = 4$
$x = 8 \Longrightarrow y = 8 + 4 = 12$
$x = 16 \Longrightarrow y = 16 + 4 = 20$

Half the perimeter: $\frac{2x+2y}{2} = 36 \Longrightarrow x + y = 36$

$x + y = 36 \Longrightarrow y = 36 - x$

x	0	16	36
$y = 36 - x$	36	20	0

$x = 0 \Longrightarrow y = 36 - 0 = 36$
$x = 16 \Longrightarrow y = 36 - 16 = 20$
$x = 36 \Longrightarrow y = 36 - 36 = 0$

∴ these lines are consistent and intersect each other. These lines have unique solution (16,20)

⇒Width = 16m Length = 20m

6. **Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:**
(i) Intersecting lines (ii) Parallel lines (iii) Coinsident lines

(i) Given equation is $2x + 3y - 8 = 0$

If the lines are intersecting then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefor the second equation is $2x + 4y - 6 = 0$

$\frac{a_1}{a_2} = \frac{2}{2} = 1,\quad \frac{b_1}{b_2} = \frac{3}{4} \quad \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) If the lines are parallel then $\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefor the the second line is $4x + 6y - 8 = 0$

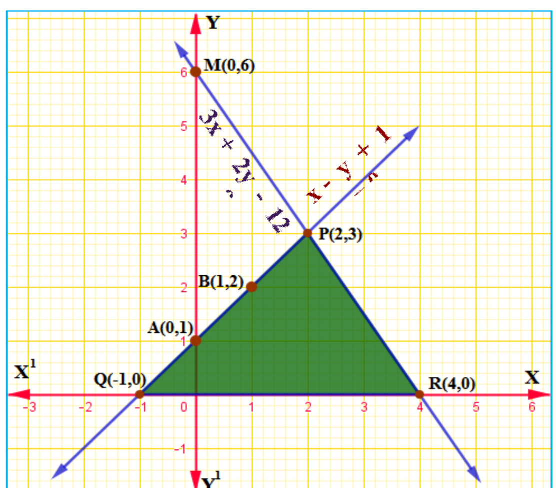
$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},\quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2},\quad \frac{c_1}{c_2} = \frac{-6}{-8} = \frac{3}{4} \quad \Rightarrow \quad \frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(iii) If the lines are coincident then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therfor the second line is $6x + 9y - 24 = 0$

$\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3},\quad \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3},\quad \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3} \Rightarrow \frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

6. **Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.**



$x - y + 1 = 0 \Longrightarrow y = x + 1$

x	0	1	2
$y = x + 1$	1	2	3

$x = 0 \Longrightarrow y = 0 + 1 = 1$
$x = 1 \Longrightarrow y = 1 + 1 = 2$
$x = 2 \Longrightarrow y = 2 + 1 = 3$

$3x + 2y - 12 = 0 \Longrightarrow y = \frac{12-3x}{2}$

x	0	2	4
$y = \frac{12-3x}{2}$	6	3	0

The coordinates of the vertices of the triangle are $(2,3)$, $(-1,0)$, $(4,0)$

3.4 Algebraic Methods of Solving a Pair of Linear Equations

In the previous section, we discussed how to solve a pair of linear equations graphically. The graphical method is not convenient in cases when the point representing the solution of the linear equations has non-integral coordinates

3.4.1 Substitution Method :

Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.

Step 2 : Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.

Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

We have substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why the method is known as the substitution method.

Example 7: Solve the following pair of equations by substitution method:.

$\mathbf{7x - 15y = 2}$ **(1)**

$\mathbf{x + 2y = 3}$ **(2)**

Equation (2) $\Longrightarrow x + 2y = 3$

$\Longrightarrow x = 3 - 2y$ (3)

Substitute the value of x in equation (1) we get,

$7(3 - 2y) - 15y = 2 \Rightarrow 21 - 14y - 15y = 2$

$-29y = 2 - 21 \Rightarrow y = \frac{-19}{-29} = \frac{19}{29}$

Substitute $y = \frac{19}{29}$ in equation (3),

$x = 3 - 2\left(\frac{19}{29}\right) = 3 - \frac{38}{29} = \frac{87-38}{29} = \frac{49}{29}$

∴The solution is, $x = \frac{49}{29}$, $y = \frac{19}{29}$

Example 8: Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then.Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Let the present age of Aftab be x Years, The present age of his daughter be y' years

The age of Aftab before 7 Years $= x - 7$ Years

The age of his daughter before 7 years $= y - 7$ yeares

$x - 7 = 7(y - 7) \Longrightarrow x - 7y + 42 = 0$ (1)

After 3 years, his age = $x + 3$ years and his daughter's age = y +3 years

$x + 3 = 3(y + 3) \Longrightarrow x - 3y = 6$ (2)

Equation (2) $\Longrightarrow x = 3y + 6$ (3)

Substituting the value of x in (1) we get,

$3y + 6 - 7y + 42 = 0$

$4y = 48 \Rightarrow y = 12$

Substituting the value of y in equation we get,

$x = 3(12) + 6 = 36 + 6 = 42$

Therfor the age of Aftab and his daughter is 42 and 12 respectively.

Example 9 : Let us consider Example 2 in Section 3.3, i.e., the cost of 2 pencils and3 erasers is Rs 9 and the cost of 4 pencils and 6 erasers is Rs 18. Find the cost of each pencil and each eraser.

Let the cost of pencil be Rs x and the cost of rubber be Rs y , the the equations are

$2x + 3y = 9$ (1)

$4x + 6y = 18$ (2)

Equation (1) $\Longrightarrow 2x = 9 - 3y \Longrightarrow x = \frac{9-3y}{2}$ (3)

Substituting x in equation (2) we get,

$4\left(\frac{9-3y}{2}\right) + 6y = 18$
$18 - 6y + 6y = 18$
$18 = 18$
This statement is true for all values of y. However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x. This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

Example:10 Let us consider the Example 3 of Section 3.2. Will the rails cross each other?

$x + 2y - 4 = 0$ (1)
$2x + 4y - 12 = 0$ (2)
Equation (1) $\Longrightarrow x = 4 - 2y$ (3)
Substituting x in equation (2) we get,
$2(4 - 2y) + 4y - 12 = 0$
$8 - 4y + 4y - 12 = 0$
$8 - 12 = 0$
$-4 = 0$
which is a false statement. Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

Exercise 3.3

1) **Solve the following pair of linear equations by substitution method.**

(i) $\boldsymbol{x + y = 14}$ **(1)**
$\boldsymbol{x - y = 4}$ **(2)**
Equation (1) $\Longrightarrow x = 14 - y$ (3)
Substituting x in equation (2) we get,
$14 - y - y = 4 \Rightarrow 14 - 2y = 4$
$-2y = 4 - 14 \Rightarrow -2y = -10 \Rightarrow y = \frac{-10}{-2} = 5$
Substituting $y = 5$ in equation (3)
$x = 14 - y = 14 - 5 \Rightarrow x = 9$
$\therefore x = 9,\ y = 5$

(ii) $\boldsymbol{s - t = 3}$ **(1)**
$\boldsymbol{\frac{s}{3} + \frac{t}{2} = 6}$ **(2)**
Equation (1) $\Longrightarrow \mathrm{s} = 3 + \mathrm{t}$ (3)
Substituting s in equation (2) we get,
$\frac{3+t}{3} + \frac{t}{2} = 6 \Rightarrow \frac{6+2t+3t}{6} = 6$
$\Rightarrow 6 + 5t = 36 \Rightarrow 5t = 36 - 6 \Rightarrow t = \frac{30}{5}$
Substituting $t = 6$ in equation (3)
$s = 3 + t \Rightarrow s = 3 + 6 \Rightarrow s = 9$
$\therefore s = 9,\ t = 6$

(iii) $\boldsymbol{3x - y = 3}$ **(1)**
$\boldsymbol{9x - 3y = 9}$ **(2)**
Equation (1) $\Longrightarrow y = 3x - 3$ Substituting y in equation (2) we get,
$9x - 3(3x - 3) = 9 \Rightarrow 9x - 9x + 9 = 9$
$9 = 9$

This statement is true for all values of y. However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x. This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

(iv) $0.2x + 0.3y = 1.3$
$0.4x + 0.5y = 2.3$

$0.2x + 0.3y = 1.3 \qquad (1) \times 10$
$0.4x + 0.5y = 2.3 \qquad (2) \times 10$
$2x + 3y = 13 \qquad (3)$
$4x + 5y = 23 \qquad (4)$

Equation (3) $\Longrightarrow 2x = 13 - 3y \Longrightarrow x = \frac{13-3y}{2} \qquad (5)$

Substituting x in equation (4) we get,

$4\left(\frac{13-3y}{2}\right) + 5y = 23$
$26 - 6y + 5y = 23$
$26 - 23 = y \Rightarrow = 3$, Substituting $y = 3$ in equation (5)

$x = \frac{13-3(3)}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$

$\therefore x = 2, \quad y = 3$

(v) $\sqrt{2}x + \sqrt{3}y = 0 \qquad (1)$
$\sqrt{3}x - \sqrt{8}y = 0 \qquad (2)$

Equation (1) $\Longrightarrow \sqrt{2}x = -\sqrt{3}y \Longrightarrow x = -\frac{\sqrt{3}\ y}{\sqrt{2}} \qquad (3)$

Substituting x in equation (2) we get,

$\sqrt{3}\left(-\frac{\sqrt{3}\ y}{\sqrt{2}}\right) - \sqrt{8}y = 0 \Rightarrow -\frac{3y}{\sqrt{2}} - \sqrt{4 \times 2}\ y = 0$

$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}\ y = 0 \Rightarrow y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

$y = 0$, Substituting $y = 0$ in equation (3)

$x = -\frac{\sqrt{3}\ (0)}{\sqrt{2}} = 0$

$\therefore x = 0, \quad y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{2} = -2 \qquad (1)$
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \qquad (2)$

Equation (1) $\times 2 \Longrightarrow 3x - 5y = -4 \qquad (3)$

Equation (2) $\times 6 \Longrightarrow 2x + 3y = 13 \qquad (4)$

Equation (3) $\Longrightarrow 3x = 5y - 4 \Longrightarrow x = \frac{5y-4}{3} \qquad (5)$

Substituting x in equation (4) we get,

$2\left(\frac{5y-4}{3}\right) + 3y = 13 \Rightarrow \frac{10y-8+9y}{3} = 13$

$19y - 8 = 39 \Rightarrow 19y = 39 + 8 \Rightarrow 19y = 47 \Rightarrow y = \frac{47}{19}$

Substituting $y = \frac{47}{19}$ in equation (5)

$x = \frac{5\left(\frac{47}{19}\right)-4}{3} = \frac{235-76}{19} \times \frac{1}{3} \Rightarrow x = \frac{159}{19} \times \frac{1}{3} \Rightarrow x = \frac{53}{19}$

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of ‘m’ for which y = mx + 3.

$2x + 3y = 11$ (1)
$2x - 4y = -24$ (2)
Equation (2) $\Longrightarrow 2x = 4y - 24 \Longrightarrow x = 2y - 12$ (3)
Substituting x in equation (1) we get,
$2(2y - 12) + 3y = 11$
$4y - 24 + 3y = 11 \Rightarrow 7y = 11 + 24 \Rightarrow 7y = 35 \Rightarrow y = 5$
substituting $y = 5$ in equation (3)
$x = 2x5 - 12 = 10 - 12 = -2$
$\therefore x = -2, \quad y = 5$
$y = mx + 3$
$5 = m(-2) + 3$
$5 - 3 = -2m \Rightarrow -2m = 2 \Rightarrow m = \frac{2}{-2} = -1$

3. Form the pair of linear equations for the following problems and find their solution by substitution method

(i) **The difference between two numbers is 26 and one number is three times the other. Find them.**

Let the first number be x, the second number be y, then $y > x$. By the given condition the equations are
$y - x = 26$ (1)
$y = 3x$ (2)
Substituting the value of y in equation (1) we get,
$3x - x = 26 \Rightarrow 2x = 26$
$x = 13$, Substitute $x = 13$ in equation (2)
$y = 3(13) = 39$
$\therefore x = 13, \quad y = 39$

(ii)The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Let the larger angle be , the smaller angle be y. If the angles are supplementary then sum of two angles is 180^0 , By the given conditions the equations are,
$x + y = 180^0$ (1)
$x = y + 18^0$ (2)
Substituting the value of x in equation (1) we get,
$y + 18^0 + y = 180^0 \Rightarrow 2y = 162^0$
$y = 81^0$ Substitute $y = 81^0$ in equation (2)
$x = 81^0 + 18^0 = 99^0$
$\therefore x = 99^0 , \quad y = 81^0$

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Let the cost of a bat = Rs x , the cost of a ball = Rs y. the equations are
$7x + 6y = 3800$ (1)
$3x + 5y = 1750$ (2)
Equation (1) $\Longrightarrow 7x = 3800 - 6y \Longrightarrow x = \frac{3800-6y}{7}$ (3)
Substituting the value of x in equation (2)we get
$3\left(\frac{3800-6y}{7}\right) + 5y = 1750$
$\frac{11400-18y+35y}{7} = 1750 \Rightarrow 11400 + 17y = 12250$

$\Rightarrow\ 17y = 12250 - 11400\ \Rightarrow\ 17y = 850\ \Rightarrow y = \frac{850}{17} = 50$

Substitute $y\ =\ 50$ in equation (3)

$x = \frac{3800-6(50)}{7} = \frac{3800-300}{7} = \frac{3500}{7} = 500$

$\therefore$ The cost of a bat = Rs 500 , The cost of a ball = Rs 50

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25km?

Let the fixed charges be Rs x, the charges/km be Rs y. Then the equations are,

$x + 10y = 105$ (1)

$x + 15y = 155$ (2)

Equation (1) $\Longrightarrow x = 105 - 10y$ (3)

Substituting the value of x in equation (2)we get

$105 - 10y + 15y = 155$

$105 + 5y = 155\ \Rightarrow\ 5y = 155 - 105\ \Rightarrow\ y = \frac{50}{5} = 10$

Substitute $y = 10$ in equation (3)

$x = 105 - 10(10) = 105 - 100 = 5$

$\therefore$ the fixed charges is Rs 5 and charges per km is Rs 10

The total charges to travel 25km is $x + 25y = 5 + 25(10)\ = 5 + 250\ =$ Rs 255

(v) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$ Find the fraction.

Let the fraction be $\frac{x}{y}$. By the given condition,

$\frac{x+2}{y+2} = \frac{9}{11} \Longrightarrow 11x + 22 = 9y + 18$

$\Longrightarrow 11x - 9y = 18 - 22 \Longrightarrow 11x - 9y = -4$ (1)

$\frac{x+3}{y+3} = \frac{5}{6} \Longrightarrow 6x + 18 = 5y + 15$

$\Longrightarrow 6x - 5y = 15 - 18 \quad \Longrightarrow\ 6x - 5y = -3$ (2)

Equation (1) $\Longrightarrow\ 11x = -4 + 9y \Longrightarrow x = \frac{-4+9y}{11}$ (3)

Substituting the value of x in equation (2)we get

$6\left(\frac{-4+9y}{11}\right) - 5y = -3\ \Rightarrow\ \frac{-24+54y-55y}{11} = -3$

$\Rightarrow\ -24 - y = -33\ \Rightarrow\ -y = -33 + 24\ \Rightarrow\ -y = -9$

Subtitute $y = 9$ in equation (3)

$x = \frac{-4+9(9)}{11} = \frac{-4+81}{11} = \frac{77}{11} = 7$

The fraction $\frac{x}{y} = \frac{7}{9}$

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Let age of Jocob $= x$, and age of his son $= y$ By the given condition the equations are

After 5 years $x + 5 = 3(y + 5)$

$\Longrightarrow x + 5 = 3y + 15 \Longrightarrow x - 3y = 10$ (1)

Before 5 years $x - 5 = 7(y - 5)$

$\Longrightarrow x - 5 = 7y - 35 \Longrightarrow x - 7y = -30$ (2)

Equation (1) $\Longrightarrow\ x = 10 + 3y$ (3)

Substituting the value of x in equation (2)we get

$10 + 3y - 7y = -30 \Rightarrow 10 - 4y = -30$

$\Rightarrow -4y = -30 - 10 \Rightarrow -4y = -40 \Rightarrow y = \frac{-40}{-4} = 10$

Substitute $y = 10$ in equation (1),

$x = 10 + 3(10) = 10 + 30 = 40$

$\therefore$ Age of Jocob $= 40$ and age of his son $= 10$

3.4.2 Elimination Method

Example 11 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ` 2000 per month, find their monthly incomes

Solution: Let the incomes of two persons be Rs 9x and Rs 7x respectively.The expenditure be Rs 4y and Rs 3y, then we get the equations,

$9x - 4y = 2000$ (1)

$7x - 3y = 2000$ (2)

Multiply Equation (1) by 3 and Equation (2) by 4 to make the coefficients of y equal.

$9x - 4y = 2000$ (1) x 3

$7x - 3y = 2000$ (2) x 4

$27x - 12y = 6000$	(3)
$28x - 12y = 8000$	(4)
$-x = -2000$	

$\Rightarrow x = 2000$

Substitute x = 2000 in equation (1), we get

$9(2000) - 4y = 2000 \Rightarrow 18000 - 2000 = 4y \Rightarrow 4y = 16000 \Rightarrow y = 4000$

$\therefore$ The monthly incomes of two persons are = Rs 18000 and Rs 14000

The method used in solving the example above is called the elimination method, because we eliminate one variable first, to get a linear equation in one variable. In the example above, we eliminated y. We could also have eliminated x. Try doing it that way.

You could also have used the substitution, or graphical method, to solve this problem. Try doing so, and see which method is more convenient.

Example 12 : Use elimination method to find all possible solutions of the following pair of linear equations

$2x + 3y = 8$ (1)

$4x + 6y = 7$ (2)

Multiply Equation (1) by 2 to make the coefficients of x equal.

$2x + 3y = 8$ (1) x 2

$4x + 6y = 16$ (3)

Substracting (2) from (1) , we get

$4x + 6y = 16$	(3)
$4x + 6y = 7$	(2)
$0 = 9$	

0 = 9, which is a false statement. Therefore, the pair of equations has no solution

Example 13 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Let the two digits number = $10x + y$

Number after reversing the digits $= 10y + x$

$\therefore 10x + y + 10y + x = 66 \Rightarrow 11x + 11y = 66 \Rightarrow x + y = 6$ (1)

We are also given that the digits differ by 2, therefore,

$x - y = 2$ (2)

substract (2) from (1)

$x + y = 6$	(1)
$x - y = 2$	(2)
$2y = 4$	

$\Rightarrow y = 2$

Substitute $y = 2$ in equation (1)

$x + 2 = 6 \Rightarrow x = 4$

Therfor the number is $10x + y = 10 \times 4 + 2 = 42$

$\Rightarrow$Two numbers are 42 and 24

Exercise 3.4

1. **Solve the following pair of linear equations by the elimination method and the substitution method :**

(i) $x + y = 5$ ಮತ್ತು $2x - 3y = 4$

Eliminating method:

$x + y = 5$ (1)

$2x - 3y = 4$ (2)

Multiply Equation (1) by 2 to make the coefficients of x equal.

$2x + 2y = 10$ (3)

Substracting (2) from (1),

$2x + 2y = 10$	(3)
$2x - 3y = 4$	(2)
$5y = 6$	

$\Rightarrow y = \frac{6}{5}$

Substitute $y = \frac{6}{5}$ in equation (1),

$x + \frac{6}{5} = 5 \Rightarrow 5x + 6 = 25 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

Substituting Method:

$x + y = 5$ (1)

$2x - 3y = 4$ (2)

(1) $\Rightarrow y = 5 - x$

Substitute $y = 5 - x$ in (2)

$\Rightarrow 2x - 3(5 - x) = 4$ (3)

$\Rightarrow 2x - 15 + 3x = 4 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

Substitute $x = \frac{19}{5}$ in (1)

$\frac{19}{5} + y = 5 \Rightarrow 19 + 5y = 25 \Rightarrow 5y = 25 - 19 \Rightarrow y = \frac{6}{5}$

$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

Eliminating Method:

$3x + 4y = 10$ (1)

$2x - 2y = 2$ (2)

Multiply Equation (2) by 2 to make the coefficients of y equal.

$2x - 2y = 2$ (2) x 2

$4x - 4y = 4$ (3)

Adding equation (1) and (2)

$3x + 4y = 10$	(1)
$4x - 4y = 4$	(3)
$7x = 14$	

$\Rightarrow x = 2$

Substitute x = 2 in (1)

$3(2) + 4y = 10 \Rightarrow 6 + 4y = 10 \Rightarrow 4y = 10 - 6 \Rightarrow 4y = 4 \Rightarrow y = 1$

$\therefore x = 2, y = 1$

Substituting Method:

$3x + 4y = 10$ (1)

$2x - 2y = 2$ (2)

(2) $\Rightarrow -2y = -2x + 2 \Rightarrow y = x - 1$

Substitute $y = x - 1$ in equation (1)

$3x + 4(x - 1) = 10 \Rightarrow 3x + 4x - 4 = 10 \Rightarrow 7x = 10 + 4 \Rightarrow 7x = 14 \Rightarrow x = 2$

Substitute x = 2 in (1)

$2(2) - 2y = 2 \Rightarrow 4 - 2y = 2 \Rightarrow -2y = 2 - 4 \Rightarrow -2y = -2 \Rightarrow y = 1$

$\therefore x = 2, y = 1$

(iii) $3x - 5y - 4 = 0$ ಮತ್ತು $9x = 2y + 7$

Eliminating method:

$3x - 5y - 4 = 0 \quad \Rightarrow 3x - 5y = 4$ (1)

$9x = 2y + 7 \quad \Rightarrow 9x - 2y = 7$ (2)

Multiply Equation (1) by 3 to make the coefficients of x equal.

$9x - 15y = 12$ (3)

Substracting (2) from (3)

$9x - 15y = 12$	(3)
$9x - 2y = 7$	(2)
$-13y = 5$	

$-13y = 5 \Rightarrow y = -\frac{5}{13}$

Substitute $y = -\frac{5}{13}$ in (1)

$3x - 5\left(-\frac{5}{13}\right) = 4 \Rightarrow 3x + \frac{25}{13} = 4 \Rightarrow 39x + 25 = 52 \Rightarrow 39x = 27 \Rightarrow x = \frac{27}{39} = \frac{9}{13}$

$\therefore x = \frac{9}{13}$ and $y = -\frac{5}{13}$

Substituting Method:

$3x - 5y - 4 = 0 \quad \Rightarrow 3x - 5y = 4$ (1)

$9x = 2y + 7 \quad \Rightarrow 9x - 2y = 7$ (2)

(1) ⇒ $-5y = 4 - 3x \Rightarrow 5y = 3x - 4 \Rightarrow y = \frac{3x-4}{5}$ (3)

Substitute $y = \frac{3x-4}{5}$ in (2)

$9x - 2\left(\frac{3x-4}{5}\right) = 7 \Rightarrow 9x - \left(\frac{6x-8}{5}\right) = 7 \Rightarrow 45x - 6x + 8 = 35 \Rightarrow 39x = 27 \Rightarrow x = \frac{27}{39} = \frac{9}{13}$

Substitute $x = \frac{9}{13}$ in (1),

$3\left(\frac{9}{13}\right) - 5y = 4 \Rightarrow 27 - 65y = 52 \Rightarrow -65y = 52 - 27 \Rightarrow y = -\frac{25}{65} \Rightarrow y = -\frac{5}{13}$

$\therefore x = \frac{9}{13}$ and $y = -\frac{5}{13}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Eliminating Method:

$\frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow 3x + 4y = -6$ (1)

$x - \frac{y}{3} = 3 \Rightarrow 3x - y = 9$ (2)

Substracting (1) from (2)

$3x + 4y = -6$	(1)
$3x - y = 9$	(2)
$+5y = -15$	

$\Rightarrow y = -3$

Substitute $y = -3$ in (1)

$3x + 4(-3) = -6 \Rightarrow 3x - 12 = -6 \Rightarrow 3x = 6 \Rightarrow x = 2$

$\therefore x = 2$ and $y = -3$

Substituting Method:

$3x + 4y = -6$ (1)

$3x - y = 9$ (2)

(2) ⇒ $-y = 9 - 3x \Rightarrow y = 3x - 9$ (3)

Substitute $y = 3x - 9$ in (1)

$3x + 4(3x - 9) = -6 \Rightarrow 3x + 12x - 36 = -6 \Rightarrow 15x = 30 \Rightarrow x = 2$

Substitute $x = 2$ in (3)

$y = 3(2) - 9 \Rightarrow y = 6 - 9 \Rightarrow y = -3$

$\therefore x = 2$ and $y = -3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Let the fraction = $\frac{x}{y}$

According to question, $\frac{x+1}{y-1} = 1 \Rightarrow x + 1 = y - 1 \Rightarrow x - y = -2$ (1)

and $\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1$ (2)

Substract (1) from (2)

$x - y = -2$	(1)
$2x - y = 1$	(2)
$-x = -3$	

$\Rightarrow x = 3$

Substitute $x = 3$ in (1)

$3 - y = -2 \Rightarrow -y = -2 - 3 \Rightarrow y = 5$

The fraction $= \frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Let the age of Nuri = x and that of Sonu = y .According to question

$(x - 5) = 3(y - 5)$

$x - 3y = -10$ (1) and

$(x + 10y) = 2(y + 10)$

$x - 2y = 10$ (2)

Substract (1) from (2)

$x - 3y = -10$	(1)
$x - 2y = 10$	(2)
$-y = -20$	

$\Rightarrow y = 20$

Substitute $y = 20$ in (1)

$x - 60 = -10 \Rightarrow x = 50$

∴ The age of Nuri = 50 years and the age of Sonu = 20years

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Let the two digits number = xy , According to question,

$x + y = 9$ (1)

$2(10y + x) = 9(10x + y)$

$20y + 2x = 90x + 9y$

$88x - 11y = 0$

$\Rightarrow 8x - y = 0$ (2)

By adding (1) and (2)

$x + y = 9$	(1)
$8x - y = 0$	(2)
$9x = 9$	

$\Rightarrow x = 1$

Substitute $x = 1$ in (1)

$1 + y = 9 \Rightarrow y = 8$

Therefor the number is $xy = 18$

(iv) Meena went to a bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received

Let the number of Rs 50 notes = x and the number of Rs 100 notes = y. According to qn

$x + y = 25$ (1) and

$50x + 100y = 2000$

$\Rightarrow x + 2y = 40$ (2)

Substract (1) from (2)

x + 2y = 40	(2)
x + y = 25	(1)
y = 15	

Substitute y = 15 in (1)

x + 15 = 25 ⇒ x = 25 – 15 ⇒ x = 10

Therefor the number of Rs 50 notes = 10 and the number of Rs 100 notes = 15

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Let the fixed charges for first 3 days = Rs x

The additional charges for remaing days = Rs y According to question

$x + 4y = 27$ (1)

$x + 2y = 21$ (2)

Substract (1) from (2)

x + 2y = 21	(2)
x + 4y = 27	(1)
- 2y = -6	

⇒y = 3

Substitute y = 3 in (1)

x + 4x3 = 27 ⇒ x + 12 = 27

⇒ x = 27 – 12 ⇒ x = 15

The fixed charges = Rs 15 and the additional charges = Rs 3

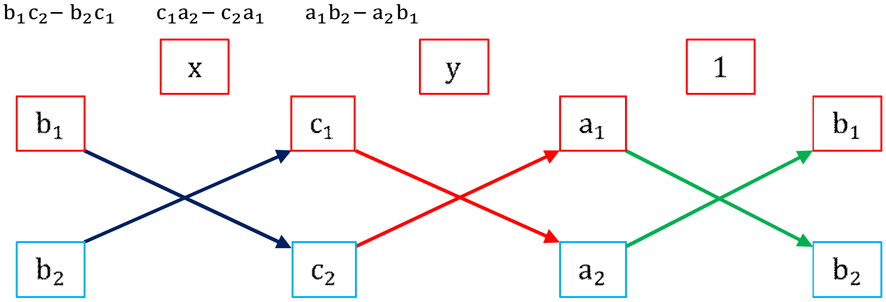
3.4.3 Cross - Multiplication Method

Equations are:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \quad [a_1b_2 - a_2b_1 \neq 0]$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

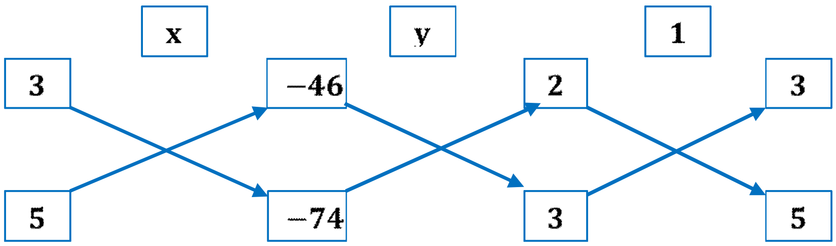


Example14: From a bus stand in Bangalore , if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is Rs 46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is Rs 74. Find the fares from the bus stand to Malleswaram, and to Yeshwanthpur.

Solution: Let Rs x be the fare from the bus stand in Bangalore to Malleswaram, and Rs y to Yeshwanthpur. From the given information, we have

$2x + 3y = 46, \Rightarrow 2x + 3y - 46 = 0$ (1)

$3x + 5y = 74 \Rightarrow 3x + 5y - 74 = 0$ (2)



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{3(-74) - 5(-46)} = \frac{y}{(-46)3 - (-74)2} = \frac{1}{2(5) - 3(3)}$$

$$\Rightarrow \frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9}$$

$$\Rightarrow \frac{x}{8} = \frac{y}{10} = 1 \Rightarrow \frac{x}{8} = 1 \Rightarrow x = 8$$

$$\Rightarrow \frac{y}{10} = 1 \Rightarrow y = 10$$

Hence, the fare from the bus stand in Bangalore to Malleswaram is Rs 8 and the fare to Yeshwanthpur is Rs 10.

Example 15 : For which values of p does the pair of equations given below has unique solution?

$4x + py + 8 = 0$

$2x + 2y + 2 = 0$

Here, $a_1 = 4, b_1 = p, c_1 = 8$ and $a_2 = 2, b_2 = 2, c_2 = 2$

Now for the given pair to have a unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{4}{2} \neq \frac{p}{2} \Rightarrow p \neq 4$

Therefore, for all values of p, except 4, the given pair of equations will have a unique solution

Example 16 : For what values of k will the following pair of linear equations have infinitely many solutions?

$kx + 3y - (k - 3) = 0$

$12x + ky - k = 0$

For a pair of linear equations to have infinitely many solutions:

Here, $a_1 = k$, $b_1 = 3$, $c_1 = -(k-3)$ and $a_2 = 12$, $b_2 = k$, $c_2 = -k$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{k}{12} = \frac{3}{k} = \frac{-(k-3)}{-k} \Rightarrow \frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$$

$$\frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{k} = \frac{-(k-3)}{-k}$$

$\Rightarrow 3k = k^2 - 3k \Rightarrow 6k = k^2 \Rightarrow (6k - k^2) = 0 \Rightarrow k(6 - k) = 0 \Rightarrow k = 0$ ಅಥವಾ $6 - k = 0 \Rightarrow k = 6$

Therefore, the value of k, that satisfies both the conditions, is k = 6. For this value, the pair of linear equations has infinitely many solutions.

Exercise 3.5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -3$ and $a_2 = 3$, $b_2 = -9$, $c_2 = -2$

$\frac{a_1}{a_2} = \frac{1}{3}$; $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$; $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore the given pair of linear equations are parallel and not intersecting each other. Hence the pair has no solution.

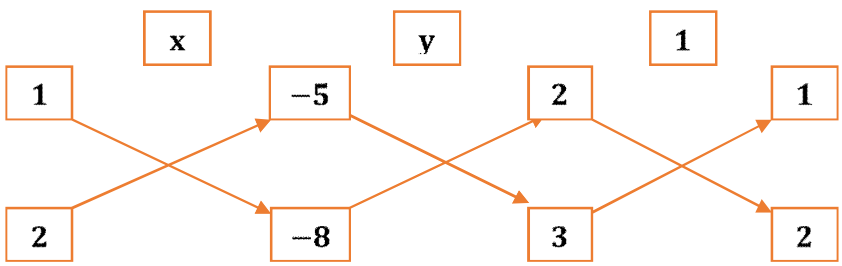
(ii) $2x + y = 5 \Rightarrow 2x + y - 5 = 0$

$3x + 2y = 8 \Rightarrow 3x + 2y - 8 = 0$

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -8$

$\frac{a_1}{a_2} = \frac{2}{3}$; $\frac{b_1}{b_2} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$ $\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore the pair of linear equations has unique solution



$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{1(-8) - 2(-5)} = \frac{y}{(-5)3 - (-8)2} = \frac{1}{2(2) - 3(1)}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3} \Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$$

$\Rightarrow \frac{x}{2} = \frac{1}{1} \Rightarrow x = 2$

$\frac{y}{1} = \frac{1}{1} \Rightarrow y = 1$

Therefore $x = 2$ and $y = 1$

iii) $3x - 5y = 20 \Rightarrow 3x - 5y - 20 = 0$

$6x - 10y = 40 \Rightarrow 6x - 10y - 40 = 0$

Here, $a_1 = 3$, $b_1 = -5$, $c_1 = -20$ and $a_2 = 6$, $b_2 = -10$, $c_2 = -40$

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$; $\frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore the given pair of linear equations are coincident and the pair has infinite solutions.

(iv) $x - 3y - 7 = 0$

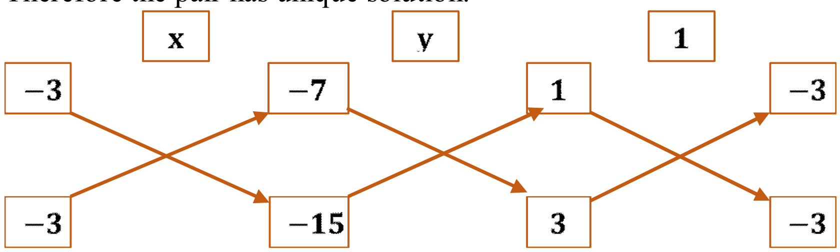
$3x - 3y - 15 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -7$ and $a_2 = 3$, $b_2 = -3$, $c_2 = -15$

$\frac{a_1}{a_2} = \frac{1}{3}$; $\frac{b_1}{b_2} = \frac{-3}{-3} = 1$; $\frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore the pair has unique solution.



$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{(-3)(-15)-(-3)(-7)} = \frac{y}{(-7)3-(-15)1} = \frac{1}{1(-3)-3(-3)}$

$\Rightarrow \frac{x}{45-21} = \frac{y}{-21+15} = \frac{1}{-3+9}$

$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$ $\Rightarrow \frac{x}{24} = \frac{1}{6} \Rightarrow 6x = 24 \Rightarrow x = 4$

$\frac{y}{-6} = \frac{1}{6} \Rightarrow 6y = -6 \Rightarrow y = -1$

There fore $x = 4$ and $y = -1$

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$

$(a - b)x + (a + b)y = 3a + b - 2 \Rightarrow x + (a + b)y - (3a + b - 2) = 0$

For a pair of linear equations to have infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$ and $a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(3a + b - 2)$

$\frac{a_1}{a_2} = \frac{2}{(a-b)}$; $\frac{b_1}{b_2} = \frac{3}{(a+b)}$; $\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{(3a+b-2)}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{2}{(a-b)} = \frac{3}{(a+b)} \Rightarrow 2(a + b) = 3(a - b)$

$\Rightarrow 2a + 2b = 3a - 3b$

$\Rightarrow a = 5b$ (1)

$\frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{(a+b)} = \frac{7}{(3a+b-2)}$

$\Rightarrow 3(3a + b - 2) = 7(a+b)$ $\Rightarrow 9a + 3b - 6 = 7a + 7b$

$\Rightarrow 2a - 4b = 6 \Rightarrow a - 2b = 3$ (2)

From (1) and (2)

(2)$\Rightarrow 5b - 2b = 3$ $\Rightarrow 3b = 3$ $\Rightarrow b = 1$ $[\because a = 5b]$

$a = 5b \Rightarrow a = 5x1 \Rightarrow a = 5$

$\Rightarrow$ If $a = 5$ and $b = 1$ the pair of linear equations has infinite solutions.

ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1 \Rightarrow 3x + y - 1 = 0$

$(2k - 1)x + (k - 1)y = 2k + 1 \Rightarrow (2k - 1)x + (k - 1)y - (2k + 1) = 0$

For a pair of linear equations to have no solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

ಇಲ್ಲಿ $a_1 = 3$, $b_1 = 1$, $c_1 = -1$ ಮತ್ತು $a_2 = (2k - 1)$, $b_2 = (k - 1)$, $c_2 = -(2k + 1)$

$\frac{a_1}{a_2} = \frac{3}{(2k-1)}$; $\frac{b_1}{b_2} = \frac{1}{(k-1)}$; $\frac{c_1}{c_2} = \frac{-1}{-(2k+1)} = \frac{1}{(2k+1)}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{3}{(2k-1)} = \frac{1}{(k-1)}$

$\Rightarrow 3(k-1) = (2k-1) \Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 2$

Therefore, if $k = 2$ then the pair of linear equations have no solution.

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods :

$8x + 5y = 9$ (1)

$3x + 2y = 4$ (2)

Substitution Method:

$8x + 5y = 9 \Rightarrow 5y = 9 - 8x \Rightarrow y = \frac{9-8x}{5}$

Equation (2) $\Rightarrow 3x + 2y = 4 \Rightarrow 3x + 2\left(\frac{9-8x}{5}\right) = 4$

$\Rightarrow 3x + \frac{18-16x}{5} = 4$ Multiplying by 5

$15x + 18 - 16x = 20$

$\Rightarrow -x = 20 - 18 \Rightarrow -x = 2 \Rightarrow x = -2$

Substitute $x = -2$ in (1)

$3(-2) + 2y = 4 \Rightarrow -6 + 2y = 4$

$\Rightarrow 2y = 4 + 6 \Rightarrow 2y = 10 \Rightarrow y = 5$

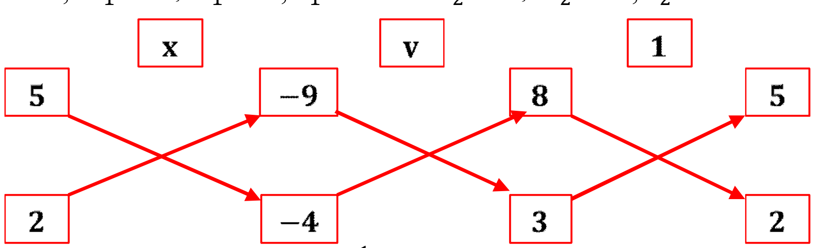
Therefore the solution is: $x = -2$ ಮತ್ತು $y = 5$

Cross multiplication Method:

$8x + 5y = 9 \Rightarrow 8x + 5y - 9 = 0$ (1)

$3x + 2y = 4 \Rightarrow 3x + 2y - 4 = 0$ (2)

Here, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -4$



$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{x}{(5)(-4)-(2)(-9)} = \frac{y}{(-9)3-(-4)8} = \frac{1}{8(2)-3(5)}$

$\Rightarrow \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$

$\Rightarrow \frac{x}{-2} = \frac{y}{5} = 1 \Rightarrow \frac{x}{-2} = 1 \Rightarrow x = -2$

$\frac{y}{5} = 1 \Rightarrow y = 5$

Therefore the solution: $x = -2$ ಮತ್ತು $y = 5$

4. Form the pair of linear equations in the following problems and find their solutions (if they exists) by any algebraic method.

i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

Let the fixed charge = x and daily food charges = y. According to question

$x + 20y = 1000$ (1)

$x + 26y = 1180$ (2)

Substracting equation (1) from (2)

$x + 26y = 1180$	(2)
$x + 20y = 1000$	(1)
$6y = 180$	

$\Rightarrow y = 30$

Substitute $y = 30$ in equation (1)

$x + 20 \times 30 = 1000 \Rightarrow x = 1000 - 600 \Rightarrow x = 400$

Therefore the fixed charges = Rs 400 and daily food charges = Rs 30

(ii) A fraction becomes $\frac{1}{3}$ when 1 is substracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Let the given fraction = $\frac{x}{y}$ According to questions,

$\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x - y = 3$ (1)

$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x - y = 8$ (2)

By substracting equation(1) from (2)

$4x - y = 8$	(2)
$3x - y = 3$	(1)
$x = 5$	

Substituting $x = 5$ in equation (1)

$15 - y = 3 \Rightarrow y = 12$

Therefore the fraction = $\frac{5}{12}$

iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Let the number of right answer = x and the number of wrong answer = y

According to question,

$3x - y = 40$ (1) and

$4x - 2y = 50 \Rightarrow 2x - y = 25$ (2)

By substracting equation(1) from (2)

$2x - y = 25$	(2)
$3x - y = 40$	(1)
$-x = -15$	

$\Rightarrow x = 15$

Substituting $x = 15$ in equation (1)

$3(15) - y = 40 \Rightarrow -y = 40 - 45 \Rightarrow -y = -5 \Rightarrow y = 5$

Therefore the right answers = 15 ; Wrong answers = 5; Total questions = 20

iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Let the speed of car A = x km/h and speed of car B = y km/h

The speed of both car travel in same direction = (x - y) km/h

The speed of both car travel in opposite direction = $(u + v)$ km/h

According to question,

$5(x - y) = 100 \Rightarrow x - y = 20$ (1)

$1(x + y) = 100 \Rightarrow x + y = 100$ (2)

By adding equation (1) and (2)

$x - y = 20$	(2)
$x + y = 100$	(1)
$2x = 120$	

$\Rightarrow x = 60$

Substituting x = 60 in equation (1)

$60 - y = 20 \Rightarrow -y = -40 \Rightarrow y = 40$

Therefore the speed of car A and B = 60 km/h and 40 km/h

v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Let the area $= xy$ According to question,

$(x - 5)(y + 3) = xy - 9$

$\Rightarrow xy + 3x - 5y - 15 = xy - 9 \Rightarrow 3x - 5y = 6$ (1)

$(x + 3)(y + 2) = xy + 67$

$\Rightarrow xy + 2x + 3y + 6 = xy + 67 \Rightarrow 2x + 3y = 61$ (2)

$3x - 5y = 6 \Rightarrow 3x = 6 + 5y \Rightarrow x = \frac{6 + 5y}{3}$

Substituting the value of x in equation (1), we get

$2\left(\frac{6 + 5y}{3}\right) + 3y = 61 \Rightarrow \frac{12 + 10y}{3} + 3y = 61$ Multiplying by 3,

$\Rightarrow 12 + 10y + 9y = 183 \Rightarrow 19y = 183 - 12 \Rightarrow 19y = 171 \Rightarrow y = 9$

Substituting y = 9 in equation (1),

$3x - 5(9) = 6 \Rightarrow 3x - 45 = 6 \Rightarrow 3x = 51 \Rightarrow x = 17$

Therefore length of the rectangle = 17 units and bredth = 15 units

3.5 Equations Reducible to a Pair of Linear Equations in Two Variables

Example 17: Solve the pair of equations

$\frac{2}{x} + \frac{3}{y} = 13; \quad \frac{5}{x} - \frac{4}{y} = -2$

Solution: $\frac{2}{x} + \frac{3}{y} = 13 \Rightarrow 2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13$ (1)

$\frac{5}{x} - \frac{4}{y} = -2 \Rightarrow 5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2$ (2)

Let $\frac{1}{x} =$ p and $\frac{1}{y} =$ q

(1) $\Rightarrow$ $2p + 3q = 13$ (3)

(2) $\Rightarrow$ $5p - 4q = -2$ (4)

From (3) $2p + 3q = 13 \Rightarrow 3q = 13 - 2p \Rightarrow q = \frac{13-2p}{3}$

Substitute $q = \frac{13-2p}{3}$ in equation (4) we get,

$5p - 4\left(\frac{13-2p}{3}\right) = -2 \Rightarrow 5p - \left(\frac{52-8p}{3}\right) = -2$ Multiplying by 3,

$\Rightarrow 15p - 52 + 8p = -6 \Rightarrow 23p = 46 \Rightarrow p = 2$

Substitute $p = 2$ in (1) we get,

$2(2) + 3q = 13 \Rightarrow 4 + 3q = 13 \Rightarrow 3q = 9 \Rightarrow q = 3$

$\Rightarrow$ substituting the value of p and q,

$\frac{1}{x} = p \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$

$\frac{1}{y} = q \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

Example 18 : Solve the following pair of equations by reducing them to a pair of linear equations

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} + \frac{3}{y-2} = 1$$

Solution: $\frac{5}{x-1} + \frac{1}{y-2} = 2 \Rightarrow 5\left(\frac{1}{x-1}\right) + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1 \Rightarrow 6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1$

Let $\frac{1}{x-1} = p$; $\frac{1}{y-2} = q$

$5p + q = 2$ (1)

$6p - 3q = 1$ (2)

From (1) $\Rightarrow q = 2 - 5p$ Substituting in (2) we get

$6p - 3(2 - 5p) = 1$

$\Rightarrow 6p - 6 + 15p = 1 \Rightarrow 21p = 7 \Rightarrow p = \frac{1}{3}$

Substitute $p = \frac{1}{3}$ in (1),

$5\left(\frac{1}{3}\right) + q = 2 \Rightarrow q = 2 - \frac{5}{3} \Rightarrow q = \frac{1}{3}$

$\frac{1}{x-1} = p \Rightarrow \frac{1}{x-1} = \frac{1}{3} \Rightarrow 3 = x - 1 \Rightarrow x = 4$

$\frac{1}{y-2} = q \Rightarrow \frac{1}{y-2} = \frac{1}{3} \Rightarrow 3 = y - 2 \Rightarrow y = 5$

Example 19:A boat goes 30 km upstream and 44 km downstream in10 hours. In 13 hours, it can go40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

Solution: Let speed of the boat in still water = x km/h and the speed of the stream = y km/h

Then the speed of the boat down stream = (x + y) km/h ಮತ್ತು

Speed of the boat in upstram = (x - y) km/h

time = $\frac{\text{Distance}}{\text{Speed}}$

In the first case, when the boat goes 30 km upstream, let the time taken, in hour be T_1

Then $T_1 = \frac{30}{x-y}$

Let T_2 be the time, in hours, taken by the boat to go 44 km downstream

Then $T_2 = \frac{44}{x+y}$

The total time taken $(T_1 + T_2) \Rightarrow \frac{30}{x-y} + \frac{44}{x+y} = 10$ (1)

In the second case, in 13 hours it can go 40 km upstream and 55 km downstream we get the equation,

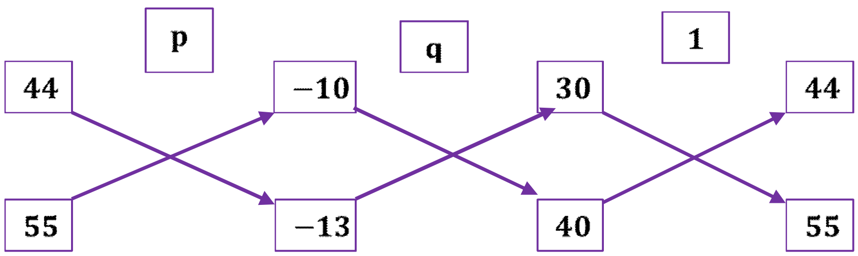
$\frac{40}{x-y} + \frac{55}{x+y} = 13$ (2)

Let $\frac{1}{x-y} = p$; $\frac{1}{x+y} = q$,

(1) $\Rightarrow$ $30p + 44q = 10 \Rightarrow 30P + 44q - 10 = 0$ (3)

(2) $\Rightarrow$ $40p + 55q = 13 \Rightarrow 40p + 55q - 13 = 0$ (4)

Here, $a_1 = 30$, $b_1 = 44$, $c_1 = -10$ and $a_2 = 40$, $b_2 = 55$, $c_2 = -13$



$\frac{p}{b_1c_2 - b_2c_1} = \frac{q}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{p}{(44)(-13)-(55)(-10)} = \frac{q}{(-10)40-(-13)30} = \frac{1}{30(55)-40(44)}$

$\Rightarrow \frac{p}{-572+550} = \frac{q}{-400+390} = \frac{1}{1650-1760}$

$\Rightarrow \frac{p}{-22} = \frac{q}{-10} = \frac{1}{-110}$

$\Rightarrow \frac{p}{-22} = \frac{1}{-110}$

$p = \frac{-22}{-110} \Rightarrow p = \frac{1}{5}$

$\frac{q}{-10} = \frac{1}{-110}$

$\Rightarrow q = \frac{-11}{-110} \Rightarrow q = \frac{1}{11}$

$\Rightarrow \frac{1}{x-y} = p \Rightarrow \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5$

$\frac{1}{x+y} = q \Rightarrow \frac{1}{x+y} = \frac{1}{11} \Rightarrow x + y = 11$

Adding the equations we get,

$2x = 16 \Rightarrow x = 8$

$8 - y = 5 \Rightarrow y = 3$

Therefore the speed of the boat = km/h and speed of the stream = 3km/h

Exercise 3.6

1. Solve the following pairs of equations by reducing them to a pair of linear equations

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$; $\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

Let $\frac{1}{x} = p$ and $\frac{1}{y} = q$

$\frac{1}{2x} + \frac{1}{3y} = 2 \Rightarrow \frac{p}{2} + \frac{q}{3} = 2 \Rightarrow 3p + 2q - 12 = 0$ (1)

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6} \Rightarrow \frac{p}{3} + \frac{q}{2} = \frac{13}{6} \Rightarrow 2p + 3q - 13 = 0$ (2)

Here, $a_1 = 3,\ b_1 = 2,\ c_1 = -12$ and $a_2 = 2,\ b_2 = 3,\ c_2 = -13$

$\frac{p}{b_1c_2 - b_2c_1} = \frac{q}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{p}{(2)(-13)-(3)(-12)} = \frac{q}{(-12)2-(-13)3} = \frac{1}{3(3)-2(2)}$

$\Rightarrow \frac{p}{-26+36} = \frac{q}{-24+39} = \frac{1}{9-4}$

$\Rightarrow \frac{p}{10} = \frac{q}{15} = \frac{1}{5}$

$\Rightarrow \frac{p}{10} = \frac{1}{5} \Rightarrow 5p = 10 \Rightarrow p = 2$

$\frac{q}{15} = \frac{1}{5} \Rightarrow 5q = 15 \Rightarrow q = 3$

$\frac{1}{x} = p \Rightarrow \frac{1}{x} = 2 \Rightarrow x = \frac{1}{2}$ ಮತ್ತು $\frac{1}{y} = q \Rightarrow \frac{1}{y} = 3 \Rightarrow y = \frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2;\quad \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \Rightarrow 2p + 3q = 2 \Rightarrow 2p + 3q - 2 = 0$ (1)

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \Rightarrow 4p - 9q = -1 \Rightarrow 4p - 9q + 1 = 0$ (2)

Here, $a_1 = 2,\ b_1 = 3,\ c_1 = -2$ and $a_2 = 4,\ b_2 = -9,\ c_2 = 1$

$\frac{p}{b_1c_2 - b_2c_1} = \frac{q}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\Rightarrow \frac{p}{(3)(1)-(-9)(-2)} = \frac{q}{(-2)4-(1)2} = \frac{1}{2(-9)-4(3)}$

$\Rightarrow \frac{p}{3-18} = \frac{q}{-8-2} = \frac{1}{-18-12}$

$\Rightarrow \frac{p}{-15} = \frac{q}{-10} = \frac{1}{-30}$

$\Rightarrow \frac{p}{-15} = \frac{1}{-30} \Rightarrow -30p = -15 \Rightarrow p = \frac{1}{2}$

$\frac{q}{-10} = \frac{1}{-30} \Rightarrow -30q = -10 \Rightarrow q = \frac{1}{3}$

$\frac{1}{\sqrt{x}} = p \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$ ಮತ್ತು

$\frac{1}{\sqrt{y}} = q \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow \sqrt{y} = 3 \Rightarrow y = 9$

(iii) $\frac{4}{x} + 3y = 14;\quad \frac{3}{x} - 4y = 23$

Let $\frac{1}{x} = p$

$4p + 3y = 14$ (1) x 3

$3p - 4y = 23$ (2) x 4

$12p + 9y = 42$ (3)

$12p - 16y = 92$ (4)

Substracting equation(3) from equation(4) we get,

$-25y = 50 \Rightarrow y = -2$

Substitute $y = 2$ in equation (1)

$4p + 3(-2) = 14$

$\Rightarrow 4p = 20 \Rightarrow p = 5$

$\frac{1}{x} = p \Rightarrow \frac{1}{x} = 5 \Rightarrow x = \frac{1}{5}$

Therefore $x = \frac{1}{5}$ and $y = -2$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2; \quad \frac{6}{x-1} - \frac{3}{y-2} = 1$

Let $\frac{1}{x-1} = p\ ; \frac{1}{y-2} = q$

$5p + q = 2 \quad (1)$

$6p - 3q = 1 \quad (2)$

$(1)x3 = 15p + 3q = 6 \quad (3)$

Adding (2) and (3) we get,

$6p - 3q = 1$

$15p + 3q = 6$

$21p = 7 \Rightarrow p = \frac{1}{3}$

Substituting $p = \frac{1}{3}$ in (1),

$\frac{5}{3} + q = 2 \Rightarrow q = 2 - \frac{5}{3} = \frac{1}{3}$

$\frac{1}{x-1} = p \Rightarrow \frac{1}{x-1} = \frac{1}{3} \Rightarrow 3 = x - 1 \Rightarrow x = 4$

$\frac{1}{y-2} = q \Rightarrow \frac{1}{y-2} = \frac{1}{3} \Rightarrow 3 = y-2 \Rightarrow y = 5$

(v) $\frac{7x-2y}{xy} = 5; \quad \frac{8x+7y}{xy} = 15$

$\frac{7x}{xy} - \frac{2y}{xy} = 5; \quad \frac{8x}{xy} + \frac{7y}{xy} = 15$

$\Rightarrow \frac{7}{y} - \frac{2}{x} = 5; \quad \frac{8}{y} + \frac{7}{x} = 15$

Let $\frac{1}{y} = p\ ; \frac{1}{x} = q$,

$7p - 2q = 5 \quad (1)$

$8p + 7q = 15 \quad (2)$

$(1) \Rightarrow 7p = 5 + 2q \Rightarrow p = \frac{5 + 2q}{7}$

Substituting in (2) we get,

$8\left(\frac{5+2q}{7}\right) + 7q = 15$

$\frac{40 + 16q}{7} + 7q = 15$ Multiplying by 7,

$40 + 16q + 49q = 105$

$65q = 65 \Rightarrow q = 1$

Substituting $q = 1$ in equation (1) ,

$7p - 2(1) = 5$

$\Rightarrow 7p = 7 \Rightarrow p = 1$

$\frac{1}{y} = p \Rightarrow \frac{1}{y} = 1 \Rightarrow y = 1$

$\frac{1}{x} = q \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

(vi) 6x + 3y = 6xy; 2x + 4y = 5xy

$6x + 3y = 6xy; \quad 2x + 4y = 5xy$

Divide both equations by **xy** we get

$\frac{6x}{xy} + \frac{3y}{xy} = \frac{6xy}{xy}; \quad \frac{2x}{xy} + \frac{4y}{xy} = \frac{5xy}{xy}$

$\Rightarrow \frac{6}{y} + \frac{3}{x} = 6; \ \frac{2}{y} + \frac{4}{x} = 5$

Let $\frac{1}{y} = p$; $\frac{1}{x} = q$

$6p + 3q = 6$ (1)

$2p + 4q = 5$ (2)

Multply equation (2) by 3,

$6p + 12q = 15$ (3)

Substract (1) from (3),

$6p + 12q = 15$
$6p + 3q = 6$
$9q = 9$

$\Rightarrow q = 1$

Substitute $q= 1$ in equation (2) we get,

$2p + 4(1) = 5 \Rightarrow 2p = 1$

$\Rightarrow p = \frac{1}{2}$

$\frac{1}{y} = p \Rightarrow \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$

$\frac{1}{x} = q \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4; \quad \frac{15}{x+y} - \frac{5}{x-y} = -2$

Let $\frac{1}{x+y} = p$; $\frac{1}{x-y} = q$

$10p + 2q = 4$ (1)

$15p - 5q = -2$ (2)

(1) $\Rightarrow 5p + q = 2$ (3)

(3) $\Rightarrow q = 2 - 5p$ (4)

Substitute (4) in (2) we get,

$15p - 5(2-5p) = -2$

$\Rightarrow 15p - 10 + 25p = -2$

$\Rightarrow 40p = 8$

$\Rightarrow p = \frac{8}{40} \Rightarrow \frac{1}{5}$

Substitute $p = \frac{1}{5}$ in (3) we get,

$5\left(\frac{1}{5}\right) + q = 2 \Rightarrow 1 + q = 2$

$\Rightarrow$ **q = 1**

$\frac{1}{x+y} = p \Rightarrow \frac{1}{x+y} = \frac{1}{5}$

$\Rightarrow$ **x + y = 5** **(5)**

$\frac{1}{x-y} = q \Rightarrow \frac{1}{x-y} = 1$

$\Rightarrow$ **x – y = 1** **(6)**

By adding (5) and (6)

$x + y = 5$
$x - y = 1$
$2x = 6$

$\Rightarrow x = 3$

Substituting $x = 3$ in equation (5), we get

$3 + y = 5 \Rightarrow y = 5 - 3 \Rightarrow y = 2$

Therefore the solutions are $x = 3,\ y = 2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$; $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

Let $\frac{1}{3x+y} = p$; $\frac{1}{3x-y} = q$

$P + q = \frac{3}{4} \Rightarrow \mathbf{4p + 4q = 3}$ (1)

$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8} \Rightarrow \mathbf{4p - 4q = -1}$ (2)

Substract (2) from (1)

$4p + 4q = 3$
$4p - 4q = -1$
$8q = 4$

$\Rightarrow q = \frac{4}{8} \Rightarrow q = \frac{1}{2}$

Substitute $q = \frac{1}{2}$ in (1) we get,

$4p + 4\left(\frac{1}{2}\right) = 3 \Rightarrow 4p + 2 = 3 \Rightarrow 4p = 1 \Rightarrow p = \frac{1}{4}$

$\frac{1}{3x+y} = p \Rightarrow \frac{1}{3x+y} = \frac{1}{4} \Rightarrow \mathbf{3x + y = 4}$ **(3)**

$\frac{1}{3x-y} = q \Rightarrow \frac{1}{3x-y} = \frac{1}{2} \Rightarrow \mathbf{3x - y = 2}$ **(4)**

By adding (3) and (4), we get

$3x + y = 4$
$3x - y = 2$
$\mathbf{6x = 6}$

$\Rightarrow x = 1$

Substituting $x = 1$ in equation (3),

$3(1) + y = 4 \Rightarrow y = 4 - 3 \Rightarrow y = 1$

Therefore the solutions are $x = 1,\ y = 1$

2. Formulate the following problems as a pair of equations, and hence find their solutions.

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

Let the speed of Ritu in still water = x km/h the speed of the stream = y km/h

The speed of Ritu in upstream = $(x - y)$ km/h

The speed of Ritu in downstream = $(x + y)$ km/h, According to question

$2(x + y) = 20$

$\Rightarrow x + y = 10$ (1)

$2(x - y) = 4$

$\Rightarrow x - y = 2$ (2)

By adding equation (1) and (2) ,

$x + y = 10$
$x - y = 2$
$2x = 12$

$\Rightarrow x = 6$

Substituting $x = 6$ in (1)

$6 + y = 10 \Rightarrow y = 10 - 6 \Rightarrow y = 4$

Therefore the speed of Ritu in still water = 6 km/h and the speed of the water = 4 km/h.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

Let the time taken by women to finish embroidery work = x days and

the time taken by men = y days

Therefore the work finished by one women in a day = $\frac{1}{x}$ and

the work finished by a man in day = $\frac{1}{y}$. According to question,

$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$; $\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

Let $\frac{1}{x}$ = p ; $\frac{1}{y}$ = q

$2p + 5q = \frac{1}{4} \Rightarrow 8p + 20q = 1$ (1)

$3p + 6q = \frac{1}{3}$

$\Rightarrow 9p + 18q = 1$ (2)

(1) $\Rightarrow 8p = 1 - 20q \Rightarrow p = \frac{1-20q}{8}$

Substituting $p = \frac{1-20q}{8}$ in (2)

$9\left(\frac{1-20q}{8}\right) + 18q = 1 \Rightarrow \frac{9-180q}{8} + 18q = 1$

$\Rightarrow 9 - 180q + 144q = 8$ Multiplying by 8

$\Rightarrow -36q = -1 \Rightarrow q = \frac{1}{36}$

Substituting $q = \frac{1}{36}$ in (1),

$8p + 20\left(\frac{1}{36}\right) = 1 \Rightarrow 8p + \frac{20}{36} = 1 \Rightarrow 8p + \frac{5}{9} = 1$

$\Rightarrow 72p + 5 = 9 \Rightarrow 72p = 4 \Rightarrow p = \frac{4}{72} \Rightarrow p = \frac{1}{18}$

$\frac{1}{x} = p \Rightarrow \frac{1}{x} = \frac{1}{18}$ $\Rightarrow$ **x = 18**

$\frac{1}{y} = q \Rightarrow \frac{1}{y} = \frac{1}{36}$ $\Rightarrow$ **y = 36**

Therefore the time taken by a women to finish the work = 18 days and a man = 36 days

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Let the speed of the train = x km/h and the speed of the bus y km/h

According to question,

$\frac{60}{x} + \frac{240}{y} = 4$ (1)

$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ (2)

Let $\frac{1}{x}$ = p and $\frac{1}{y}$ = q

$60p + 240q = 4 \Rightarrow 15p + 60q = 1$ (3)

$100p + 200q = \frac{25}{6} \Rightarrow 600p + 1200q = 25$

$\Rightarrow 24p + 48q = 1$ (4)

(3) $\Rightarrow 15p = 1 - 60q \Rightarrow P = \frac{1-60q}{15}$

Substitute $P = \frac{1-60q}{15}$ in (4) ,

$24\left(\frac{1-60q}{15}\right) + 48q = 1$

$\Rightarrow \frac{24-1440q}{15} + 48q = 1$ Multiplying by 15,

$24 - 1440q + 720q = 15 \Rightarrow -720q = -9 \Rightarrow \mathbf{q = \frac{1}{80}}$

Substitute $q = \frac{1}{80}$ in (3),

$15p + 60\left(\frac{1}{80}\right) = 1$

$15p + \frac{3}{4} = 1 \Rightarrow 15p = 1 - \frac{3}{4} \Rightarrow 15p = \frac{1}{4} \Rightarrow \mathbf{p = \frac{1}{60}}$

$\frac{1}{x} = p \Rightarrow \frac{1}{x} = \frac{1}{60} \Rightarrow \mathbf{x = 60}$

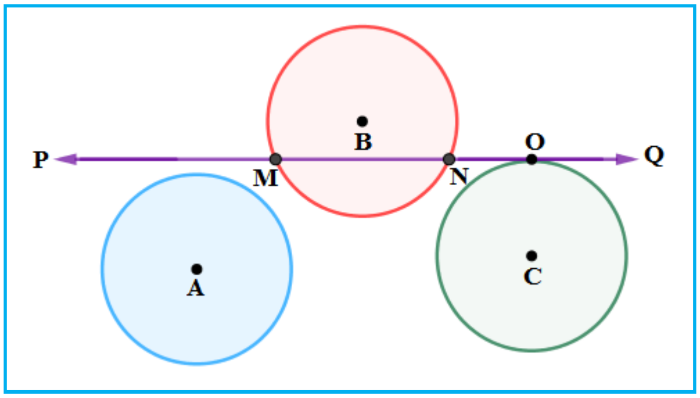
$\frac{1}{y} = q \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

Therefore the speed of the train = 60 km/h and the speed of the bus = 80 km/h.

Summery:

1. Two linear equations in the same two variables are called a pair of linear equations in two variables. The most general form of a pair of linear equations is
   $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
2. A pair of linear equations in two variables can be represented, and solved, by the:
   (i) graphical method (ii) algebraic method
3. Graphical Method :
   The graph of a pair of linear equations in two variables is represented by two lines.
   (i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is consistent.
   (ii) If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is dependent (consistent).
   (iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is inconsistent
4. Algebraic Methods : We have discussed the following methods for finding the solution(s) of a pair of linear equations :
   (i) Substitution Method
   (ii) Elimination Method
   (iii) Cross-multiplication Method
5. If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the the following situations can arise :
   $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ In this case, the pair of linear equations is consistent
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ In this case, the pair of linear equations is inconsistent
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ In this case, the pair of linear equations is dependent and consistent
6. There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations

4 Circles



Non-intersecting Line: The line PQ and the circle have no common point. In this case, PQ is called a non-intersecting line.PQ is non-intersecting line for the circle of center A

Secant: There are two common points M and N that the line PQ and the circle have. In this case, we call the line PQ a secant of the circle of center B

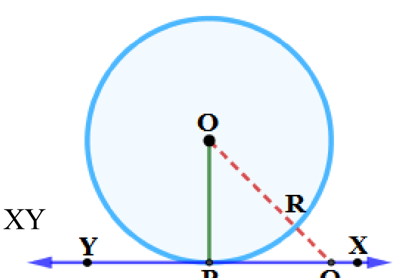
Tangent: There is only one point O which is common to the line PQ and the circle. In this case, the line is called a tangent to the circle of center C

4.2 Tangent to a Circle

Tangent to a circle is a line that intersects the circle at only one point. There is only one tangent to a circle at a point. The common point of the tangent and the circle is called the point of contact.

Theorem 4.1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given: A circle with centre O and tangent XY at a point P.

To Prove: $OP \perp XY$

Consruction: Take any point Q, other than P on the tangent XY and join OQ

Proof: Hence, Q is a point on the tangent XY, other than the point of contact P. So Q lies outside the circle..

[∵There is only one point of contact to a tangent]

Let OQ intersect the circle at R
∴ OP = OR [∵ Radius of the same circle]
Now, OQ = OR + RQ
⇒OQ > OR
⇒OQ> OP [∵OP = OR]
Therefore, OP is the shortest distance to the tangent from the center O
∴ OP⊥XY [∵ Perpendicular distance is always the shortest distance]

Remarks :

1. By theorem above, we can also conclude that at any point on a circle there can be one and only one tangent.

2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

Exercise 4.1

1. **How many tangents can a circle have?**

Answer: Infinite

2. **Fill in the blanks :**

i) A tangent to a circle intersects it in __________ point (s).
Answer: One
(ii) A line intersecting a circle in two points is called a __________ .
Answer: Secant
iii) A circle can have __________ parallel tangents at the most.
Answer: Two [Note: we can draw only two(pair) parallel tangents each other. But we can draw infinite parallel pair of tangents]
iv) The common point of a tangent to a circle and the circle is called ________.
Answer: Point of cantact

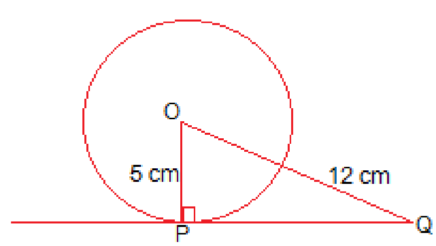
3. **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :**

a) 12 cm b) 13 cm c) 8.5 cm d) $\sqrt{119}$ cm

Answer:

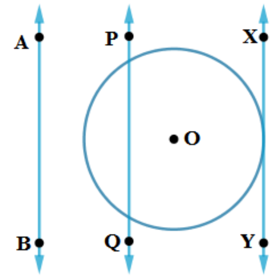
The line drawn from the point of contact to the center of the circle is perpendicular to the tangent.

⇒OP ⊥ PQ
In ΔOPQ,
$OQ^2 = OP^2 + PQ^2$[Pythagoras Theorem]
⇒ $(12)^2 = 5^2 + PQ^2$
⇒$PQ^2 = 144 - 25$
⇒$PQ^2 = 119$
⇒PQ = $\sqrt{119}$ cm
(d) $\sqrt{119}$ cm



4. **Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

AB – A line
PQ – A secant
XY – A tangent



4.3 Number of Tangents from a Point on a Circle

Case 1 : There is no tangent to a circle passing through a point lying inside the circle.

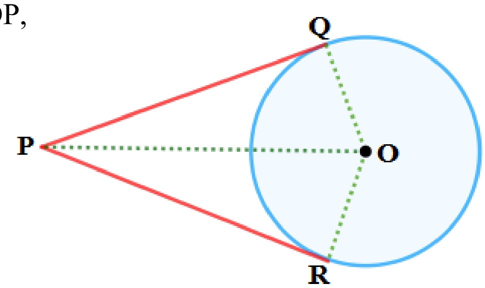
Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.

Case 3 : There are exactly two tangents to a circle through a point lying outside the circle.

Theorem 4.2

The lengths of tangents drawn from an external point to a circle are equal.

ದತ್ತ: PQ and PR are the two tangents drawn from an external point P to a circle of center O. JoinOP, OQ, OR



T Prove: PQ = PR
Proof: In right angle triangle OQP and ORP,
OQ = OR [Radius of the same circle]
OP = OP [Common side]
∴, $\Delta OQP \cong \Delta ORP$ [RHS]
∴, PQ = PR [CPCT]

Example 1 : Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

We are given two concentric circles C_1 and C_2 with centre O and a chord AB of the larger circle C_1 which touches the smaller circle C_2 at the point P (see Fig. 4.8).
We need to prove that AP = BP.
Let us join OP. Then, AB is a tangent to C_2 at Pand OP is its radius. Therefore, by Theorem 4.1,
Therefore $OP \perp AB$ [From theorem 4.1]
Now AB is a chord of the circle C_1 and $OP \perp AB$

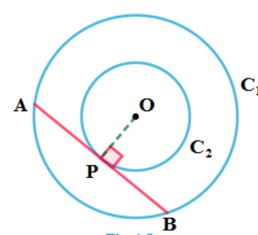


Fig 4.8

Therefore, OP is the bisector of the chord AB, as the perpendicular from the centre bisects the chord,
⇒ AP = BP

Example 2 : Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that PTQ = 2 OPQ.

Solution: TP and TQ are the two tangents drawn from an external point T to the circle with centre O

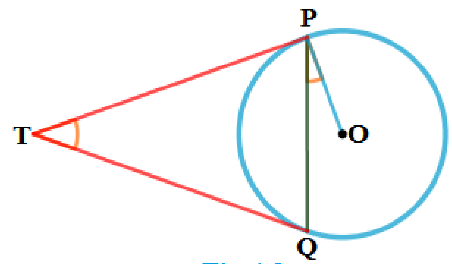


Fig 4.9

To Prove: $\angle PTQ = 2\angle OPQ$

Let $\angle PTQ = \theta$ (1)

TP = TQ [∵Theorem 4.2]

Therefore TPQ is an isosceles triangle.

$\angle TPQ = \angle TQP = \frac{1}{2}[180 - \theta]$

$\Rightarrow \angle TPQ = \angle TQP = 90^0 - \frac{1}{2}\theta$ (2)

$\angle OPT = 90^0$ (3)

$\angle OPQ = \angle OPT - \angle TPQ$

$\Rightarrow \angle OPQ = 90^0 - \left(90^0 - \frac{1}{2}\theta\right)$ [∵ from(2)and(3)]

$\Rightarrow \angle OPQ = \frac{1}{2}\theta$

$\Rightarrow \angle PTQ = 2\angle OPQ$ [∵From (1)]

Example 3 : PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T (see Fig. 4.10). Find the length TP.

Solution: Join OT Let it intersect PQ at the point R Then ΔTPQ is isosceles and TO is the angle bisector of $\angle PTQ$.

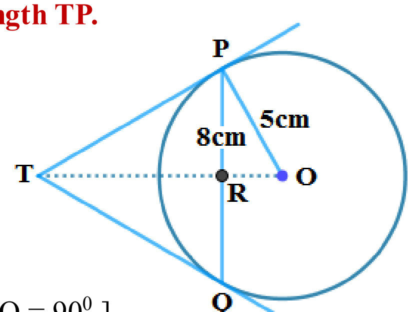


Fig 4.10

$\therefore OT \perp PQ$ therefore OT bisects PQ

$\Rightarrow$ PR = RQ = 4 cm.

$\therefore RO = \sqrt{5^2 - 4^2} \Rightarrow RO = \sqrt{25 - 16} \Rightarrow RO = \sqrt{9}$

$\Rightarrow$ RO = 3cm

$\angle OPR + \angle TPR = 90^0$ (1) [∵In ΔPRO, $\angle PRO = 90^0$]

$\angle PTR + \angle TPR = 90^0$ (2) [∵In ΔPTR, $\angle PRT = 90^0$]

From (1) and (2),

$\angle OPR = \angle PTR$ (3)

$\therefore \Delta PRO$ and ΔPTR right triangles are similar [AA similarity criteria]

$\Rightarrow \frac{PT}{OP} = \frac{PR}{OR} \Rightarrow \frac{PT}{5} = \frac{4}{3} \Rightarrow PT = \frac{4x5}{3} = \frac{20}{3}$

EXERCISE 4.2

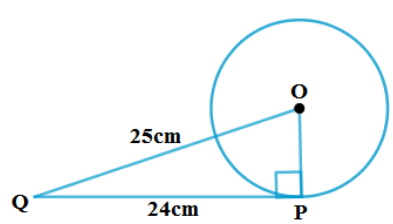
In Q.1 to 3, choose the correct option and give justification.

1. **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is**

OP ⊥ PQ and ΔOPQ is a right angle triangle.

OQ = 25 cm and PQ = 24 cm

In ΔOPQ, By Pythagoras theorem,



$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576 \Rightarrow OP^2 = 49 \Rightarrow OP = 7$ cm

ಉತ್ತರ: (A) 7 cm.

A) 7 cm B) 12 cm C) 15 cm D) 24.5 cm

2. In Fig. 4.11, if TP and TQ are the two tangents to a circle with centre O so that POQ = 110°, then PTQ is equal to

TP and TQ are the tangents to a circle at P and Q OP and OQ are radius of the circle at point of contacts P and Q

∴ OP ⊥ TP and OQ ⊥ TQ

∠OPT = ∠OQT = 90°

In Quadrilateral POQT,

∠PTQ + ∠OPT + ∠POQ + ∠OQT = 360°

⇒ ∠PTQ + 90° + 110° + 90° = 360°

⇒ ∠PTQ = 70°

⇒Ans (B) 70°.

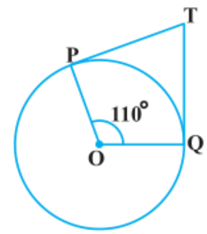


Fig 4.11

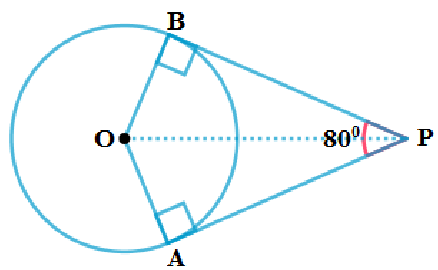
A) 60 **B) 70** C) 80 D) 90

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then POA is equal to

OA and OB are the radius drawn at the point of contact of the tangents BP and BQ

∴ OA ⊥ PA and OB ⊥ PB

∠OBP = ∠OAP = 90°



In Quadrilateral AOBP,

∠AOB + ∠OBP + ∠OAP + ∠APB = 360°

⇒ ∠AOB + 90° + 90° + 80° = 360°

⇒ ∠AOB = 100°

Now, In ΔOPB and ΔOPA,

AP = BP [Tangents drawn from an external point]

OA = OB [Radius of the same circle]

OP = OP [Common]

∴ ΔOPB ≅ ΔOPA [SSS congruence rule]

⇒∠POB = ∠POA

∠AOB = ∠POB + ∠POA

⇒ 2 ∠POA = ∠AOB

⇒ ∠POA = 50°

⇒ (A) 50°

A) 50^0 B) 60^0 C) 70^0 D) 80^0

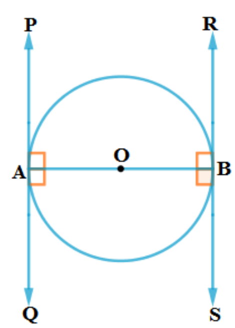
4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

AB is a diameter. PQ and RS are the tangents drawn to the circle at point A and B

OA and OB are the radius drawn at point of contact.

∴ OA ⊥ PQ and OB ⊥ RS

∠OAP = ∠OAQ = ∠OBR = ∠OBS = 90°



In the fig,

∠OBR = ∠OAQ [Alternate angles]

∠OBS = ∠OAP [Alternate angles]

⇒ PQ∥ RS

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

AB is the tangent drawn to the circle with radius O

To Prove:The perpendicular at P passes through the center O.

If possible,let the perpendicular passing through some other

Point say Q

Join QP and OP

OP is the radius at point of contact AB is the tangent

$\therefore OP \perp AB \Rightarrow \angle OPA = 90^0$

But, $\angle RPA = 90°$ ($PQ \perp AB$)

$\Rightarrow$ Which is possible only when points P and Q coinside.

$\therefore$ the perpendicular at the point of contact to the tangent to a circle passes through the centre.

6. **The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle**

AB is the tangent to the circle at point B. OB is the radius at point of contact

$\therefore OB \perp AB$

OA = 5cm and AB = 4 cm [Given]

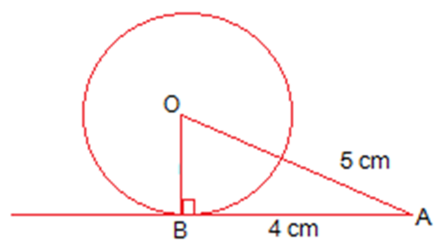
In ΔABO,

$OA^2 = AB^2 + BO^2$ [Pythagoras theorem]

$\Rightarrow 5^2 = 4^2 + BO^2 \Rightarrow BO^2 = 25 - 16$

$\Rightarrow BO^2 = 9 \Rightarrow BO = 3$

$\therefore$ Radius = 3 cm.



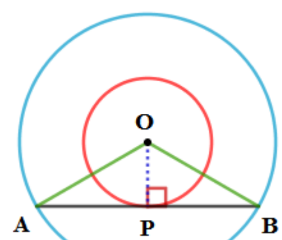
7. **Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

Two concentric circles of radius 5cm and 3cm drawn with common center O

AB is the chord of circle with radius 5cm such that it touches the circle of radius 3cm at P

$\therefore$ AB is the tangent to the smaller circle at P

$\Rightarrow OP \perp AB$



$\therefore$ AP = PB [The perpendicular drawn from the center to the chord bisects the chord]

$OA^2 = AP^2 + OP^2$ [By Pythagoras theorem]

$\Rightarrow 5^2 = AP^2 + 3^2 \Rightarrow AP^2 = 25 - 9 \Rightarrow AP = 4$,

$AB = 2AP = 2 \times 4 = 8$ cm

$\therefore$ The length of the chord = 8 cm.

8. **A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 4.12). Prove that AB + CD = AD + BC**

From the figure,

DR = DS [Tangents from the external point D] (1)

AP = AS [Tangents from the external point A] (2)

BP = BQ [Tangents from the external point B] (3)

CR = CQ [Tangents from the external point C] (4)

(1) + (2) + (3) + (4)

DR + AP + BP + CR = DS + AS + BQ + CQ

$\Rightarrow$ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

$\Rightarrow$AB + CD = AD + BC

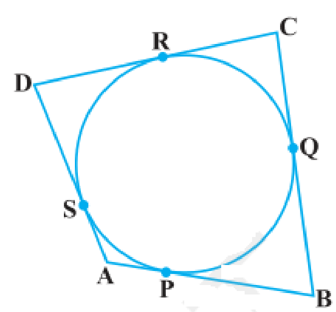


Fig 4.12

9. **In Fig. 4.13, XY and X^1Y^1 are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X^1Y^1 at B. Prove that $\angle AOB = 90°$.**

Let the tangent AB touches the circle at C. Join OC.

In ΔOPA and ΔOCA,

OP = OC [radius of the same circle]

AP = AC [The tangents from an external point A]

AO = AO [Common]

$\therefore \Delta OPA \cong \Delta OCA$ [SSS congruence rule]

$\Rightarrow \angle POA = \angle COA$ (1)

Similarlly,

$\Delta OQB \cong \Delta OCB$

$\angle QOB = \angle COB$ (2)

POQ is the diameter $\therefore \angle POQ = 180^0$

$\Rightarrow \angle POA + \angle COA + \angle COB + \angle QOB = 180°$

From (1) and (2) we get,

$2\angle COA + 2\angle COB = 180°$

$\Rightarrow \angle COA + \angle COB = 90° \quad \Rightarrow \angle AOB = 90°$

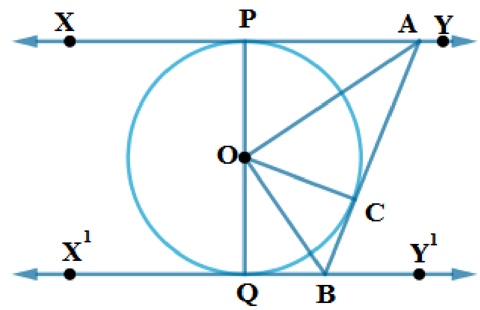


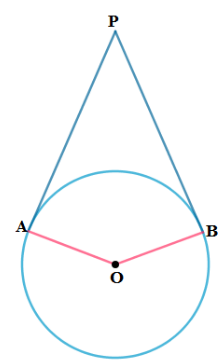
Fig 4.13

10. **Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre**

Let PA and PB are the tangents drawn from an external point to the circle with center O

Join OA and OB

To Prove: $\angle APB + \angle BOA = 180°$



Proof: $OA \perp PA$ [Radius at point of contact to the circle]

$\therefore \angle OAP = 90°$

Similarlly,

$OB \perp PB \quad \therefore \angle OBP = 90°$

In quadrilateral OAPB,

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$[Sum of interior agles]

$\Rightarrow 90° + \angle APB + 90° + \angle BOA = 360°$

$\Rightarrow \angle APB + \angle BOA = 180°$

11. **Prove that the parallelogram circumscribing a circle is a rhombus.**

Given: ABCD is a parallelogram circumscribing a circle.

To prove: AB = BC = CD = DA

Proof: ABCD ಒಂದು ಸಮಾಂತರ ಚತುರ್ಭುಜ.

$\therefore$ AB = CD (1)

$\therefore$ BC = AD (2)

We know that the tangent drawn from an external point to the circles are equal

Therefore, DR = DS, AP = AS, BP = BQ, and CR = CQ

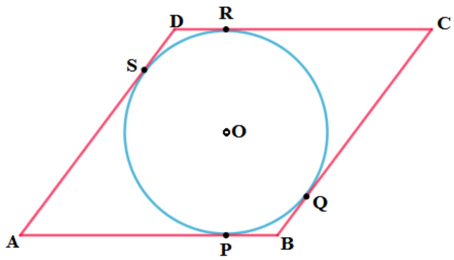
Adding all these, we get

DR + CR + BP + AP = DS + CQ + BQ + AS

⇒ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

⇒ AB + CD = AD + BC (3)

Substituting (1) and (2) in (3),



2AB = 2BC ⇒ AB = BC (4)

From equation (1), (2) and (4),

AB = BC = CD = DA

∴ ABCD is a Rhombus

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 4.14). Find the sides AB and AC.

In ΔABC,

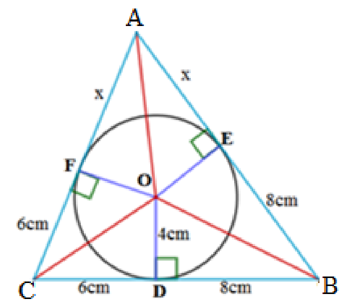


Fig 4.14

CF = CD = 6cm [Tangents from an external point]

BE = BD = 8cm [Tangents from an external point]

AE = AF = x [Tangents from an external point]

⇒a = AB = AE + EB = $x + 8$

b = BC = BD + DC = 8 + 6 = 14

c= CA = CF + FA = 6 + x

$S = \frac{AB+BC+CA}{2} = \frac{x+8+14+6+x}{2} = \frac{2x+28}{2}$ ⇒ **S = 14 + x**

Area of ΔABC = $\sqrt{s\,(s-a)(s-b)(s-c)}$

$= \sqrt{14+x\,[14+x-(x+8)](14+x-14)[14+x-(6+x)]}$

$= \sqrt{(14+x)\,[14+x-x-8](14+x-14)[14+x-6-x]}$

$= \sqrt{(14+x)\,(6)(x)(8)}$

$= \sqrt{(14+x)\,48x}$ cm^2 (1)

Similarlly, Area of ΔABC= Area of ΔOCB+ Area of ΔOBA+ Area of ΔOAC

$= \frac{1}{2}BC.OD + \frac{1}{2}AB.OE + \frac{1}{2}AC.OF$

$= \frac{1}{2}(14x4) + \frac{1}{2}(8+x)4 + \frac{1}{2}(6+x)4 = 28 + 16 + 2x + 12 + 2x$

$= (56 + 4x)$cm^2 (2)

From (1) and (2),

$\sqrt{(14+x)\,48x} = 56 + 4x$

$48x\,(14 + x) = (56 + 4x)^2$ [Squaring on both sides]

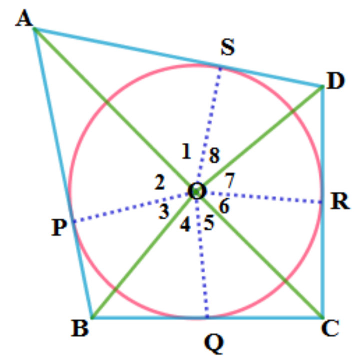
$\Rightarrow 48x = \frac{[4(14+x)]^2}{14+x}$

$\Rightarrow 48x = 16\,(14 + x) \Rightarrow 48x = 224 + 16x \Rightarrow 32x = 224 \Rightarrow x = 7$ cm

Therefore, AB = x + 8 = 7 + 8 = 15 cm

CA = 6 + x = 6 + 7 = 13 cm

13. **Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**



Given: ABCD is a quadrilateral circumscribing a circle with center O. Let the circle touches the quadrilateral at points P,Q,R and S

To Prove: $\angle AOB + \angle COD = 180^0$ and $\angle AOD + \angle BOC = 180^0$

Construction: Join OP, OQ, OR and OS.

Proof: The tangents drawn from an external point to the circle substend equal angle at the center.

$\Rightarrow \angle 1 = \angle 2;\ \angle 3 = \angle 4;\ \angle 5 = \angle 6;\ \angle 7 = \angle 8$

But,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) + (\angle 5 + \angle 6) + (\angle 7 + \angle 8) = 360^o$

$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360^o$

$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^o$

$\Rightarrow \angle AOB + \angle COD = 180^0$

Similarlly, $\angle AOD + \angle BOC = 180^0$

$\therefore$ **Opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**

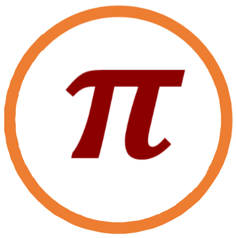
Summary

1. **The tangent to a circle is perpendicular to the radius through the point of contact.**
2. **The length of the tangents from an external point to the circle are equal.**

5 Area Related to circles

5.2 Perimeter and Area of a Circle — A Review

The distance covered by travelling once around a circle is its perimeter, usually called its circumference. You also know from your earlier classes, that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi'). In other words,



$\dfrac{\text{circumference}}{\text{diameter}}$ → Circumference $= 2\pi r$

Area of the triangle $= \pi r^2$

The cost of fencing a circular field at the rate of Rs 24 per metre is Rs 5280. The field is to be ploughed at the rate of Rs 0.50 per m^2. Find the cost of ploughing the field. (Take $\pi = \frac{22}{7}$)

Solution: Length of the fence (in metres) $= \frac{\text{Total Cost}}{\text{Rate}} = \frac{5280}{24} = 220$ ಮೀ.

So, circumference of the field $= 220$ m. Therefore, if r metres is the radius of the field,

then, $2\pi r = 220$ Or $2 \times \frac{22}{7} \times r = 220 \Rightarrow r = \frac{220 \times 7}{2 \times 22} = 35$m

Therefore, area of the field $= \pi r^2 = \frac{22}{7} \times 35^2 = (22 \times 5 \times 35)m^2$

Total cost of ploughing the field at the rate of 0.50/sqm $= (22 \times 5 \times 35) \times 0.5 =$ ರೂ 1925

Exercise 5.1

[Unless stated otherwise, use $\pi = \frac{22}{7}$]

1. **The radii of two circles are 19 cm and 9 cm respectively.Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.**

Let the required radius = R. Therefore the circumference $C = 2\pi R$

The circumference of the circle of radius 19 cm $= 2\pi \times 19 = 38\pi$ cm

The circumference of the circle of radius $= 2\pi \times 9 = 18\pi$ cm

The sum of the circumference of two circles $= 38\pi + 18\pi = 56\pi$ cm

$\Rightarrow 2\pi R = 56\pi$ cm [According to question] $\Rightarrow 2R = 56$ cm $\Rightarrow R = 28$ cm

2. **The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.**

Let the required radius = R. Therefore the area $= \pi R^2$

The area of the circle of radius 8 cm $= \pi \times 8^2 = 64\pi$ cm^2

The area of the circle of radius 6 cm $= \pi \times 6^2 = 36\pi$ cm^2

The sum of the areas of two circles $= 64\pi$ cm$^2 + 36\pi$ cm$^2 = 100\pi$ cm^2

According to question,

$\pi R^2 = 100\pi$ cm$^2 \Rightarrow R^2 = 100$cm$^2 \Rightarrow R^2 = 100$ cm$^2 \Rightarrow R = 10$ cm

3. **Fig. 5.3 depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide.Find the area of each of the five scoring regions.**

The diameter of the Golden colour Circle = 21 cm				
1st circle	2nd circle	3rd circle	4th circle	5th circle
$r_1 = 10.5$ cm	$r_2 = 21$ cm	$r_3 = 31.5$	$r_4 = 42$	$r_5 = 52.5$
$A_1 = \pi r_1^2$	$A_2 = \pi r_2^2$	$A_3 = \pi r_3^2$	$A_4 = \pi r_4^2$	$A_5 = \pi r_5^2$
$\pi (10.5)^2$	$\pi(21)^2$	$\pi(31.5)^2$	$\pi(42)^2$	$\pi(52.5)^2$
346.5 cm^2	1386 cm^2	3118.5 cm^2	5544 cm^2	8662.5 cm^2

Area of Golden colour $= \pi r_1^2 = \pi (10.5)^2 = 346.5$ cm^2

Area of Red colour = [Area of 2nd – Area of 1st]

= 1386 - 346.5 cm^2 = 1039.5 cm^2

Area of blue colour = [Area of 3rd – Area of 2nd]

= 3118.5 - 1386 cm^2 = 1732.5 cm^2

Area of black colour = [Area of 4th – Area of 3rd]

= 5544 - 3118.5 cm^2 = 2425.5 cm^2

Area of white colour = [Area of 5th – Area of 4th]

= 8662.5 cm^2 - 5544 cm^2 = 3118.5cm^2

4. **The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?**

The diameter of the wheels of a car = 80 cm

Circumference of the wheel $C = 2\pi r = 2r \times \pi = 80\pi$ cm

The Car traveled a distance in 10 minutes $= (66 \times 1000 \times 100 \times 10)/60 = 110000$ cm

Therefore no of revolution $= \dfrac{\text{ಚಲಿಸಿದ ದೂರ}}{C} = \dfrac{110000}{80\pi} = \dfrac{110000 \times 7}{80 \times 22} = 4375$

5. **Tick the correct answer in the following and justify your choice : If the perimeter and the area of a circle are numerically equal, then the radius of the circle is**
A) 2 Units B) π Units C) 4 Units D) 7 Units

Radius of the circle = r

∴ Circumference(Perimeter) $= 2\pi r$ ∴ Area $= \pi r^2$

According to question,

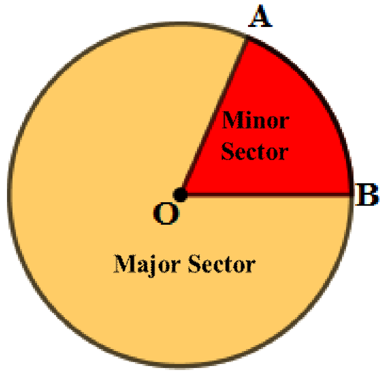
Perimeter = Area

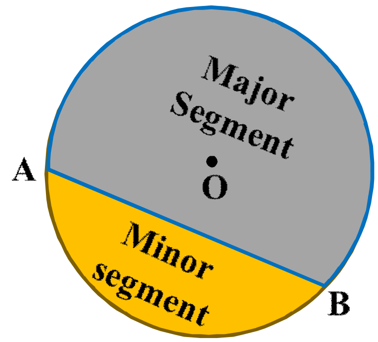
$2\pi r = \pi r^2 \Rightarrow 2 = r$

∴ **A) 2 Units**

5.3 Areas of Sector and Segment of a Circle

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle and the portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.





some relations (or formulae) to calculate their areas.

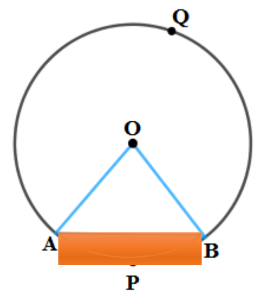
Let OAPB be a sector of a circle with centre O and radius r (see Fig. 5.6). Let the degree measure of ∠AOB be θ,

If the angle at the center is 360^0, then the area of the sector = πr^2

So, when the degree measure of the angle at the Centre is 1, area of the sector = $\frac{\pi r^2}{360}$

Therefore, when the degree measure of the angle at the centre is θ,

Area of the sector = $\frac{\pi r^2}{360}$ x θ ⇒ $\frac{\theta}{360}$ x πr^2



Area of the sector of angle θ = $\frac{\theta}{360}$ x πr^2

Length of the arc of a sector of angle θ = $\frac{\theta}{360}$ x $2\pi r$

Areaof SegmentAPB → **Area of OAPB Sector – Area of ΔOAB**

Area of OAQB Major sector → **πr^2 - Area of Minor sectorOAPB**

Area of AQB Major segment → **πr^2 – Area of APB Minor segment**

Example 2 : Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use = 3.14)

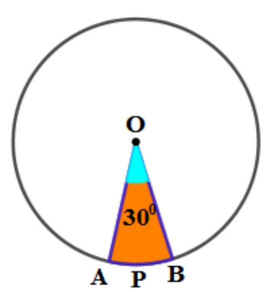
Solution: Given sector is OAPB.

Area of the sector OAPB = $\frac{\theta}{360}$ x πr^2

⇒ $\frac{30}{360}$ x 3.14 x 4 x 4 = $\frac{12.56}{3}$ ≈ 4.19 cm^2

Area of the corresponding major sector

= πr^2 - Area of sector OAPB = (3.14 x 16 - 4.19) cm^2 ≈ 46.1 cm^2



Alternate Method:

Area of the corresponding major sector = $\frac{360-\theta}{360}$ x πr^2

= $\frac{360-30}{360}$ x 3.14 x 4 x 4= 46.05 ≈ 46.1 cm^2

Example 3 : Find the area of the segment AYB shown in Fig. 5.9, if radius of the circle is 21 cm and $\angle AOB = 120^0$ (Use $\pi = \frac{22}{7}$)

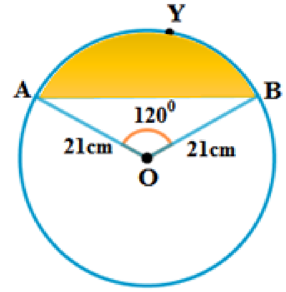


Fig 5.9

Solution: Area of the segment

= Area of sector OAYB – Area of ΔOAB -------- (1)

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

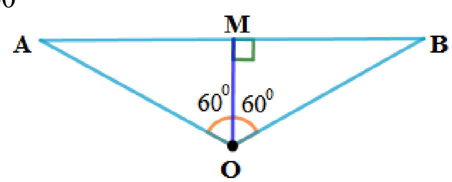
$= \frac{120}{360} \times \frac{22}{7} \times 21 \times 21 = 462 cm^2$

To find the area of ΔOAB, draw $OM \perp AB$ as shown in fig.5.10

Note that OA = OB

Therefore, by RHS congruence $\Delta AMO \cong \Delta BMO$

So, M is the mid-point of AB and $\angle AOM = \angle BOM = 60^0$



In ΔOAM, $\frac{OM}{OA} = Cos 60^0 \Rightarrow \frac{OM}{21} = \frac{1}{2} \Rightarrow OM = \frac{21}{2}$ cm

In ΔOAM, $\frac{AM}{OA} = Sin 60^0 \Rightarrow \frac{AM}{21} = \frac{\sqrt{3}}{2} \Rightarrow AM = \frac{21\sqrt{3}}{2}$ cm

$\Rightarrow AB = 2AM \Rightarrow 21\sqrt{3}$cm

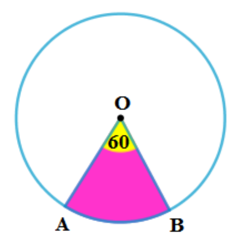
$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} = \frac{441\sqrt{3}}{4} cm^2$ (3)

Are of the segment $= 462 - \frac{441\sqrt{3}}{4} = \frac{462 \times 4 - 441\sqrt{3}}{4} = \frac{21}{4}(88 - 21\sqrt{3}) cm^2$

Exercise 5.2

[Unless stated, otherwise use $\pi = \frac{22}{7}$]

1. **Find the area of a sector of a circle with radius 6 cm if angle** of the sector is 60°.

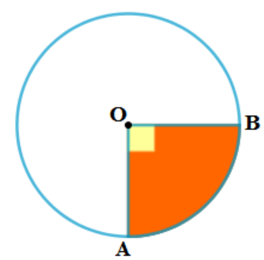


Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of the sector of angle $60^0 = \frac{60}{360°} \times \pi r^2 cm^2$

$= \frac{1}{6} \times 6 \times 6 \times \frac{22}{7} = \frac{132}{7} cm^2$

2. **Find the area of a quadrant of a circle whose circumference is 22 cm**



Quadrant of a circle = Angle of sector 90^0

Circumference $C = 2\pi r = 22$ cm

Radius $r = \frac{22}{2\pi}$ cm $= \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$ cm

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

Area of the sector of angle $90° = \frac{90}{360°} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} cm^2$

3. **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

The minute hand is the radius of the circle. $\Rightarrow$ Radius(r) = 14 cm

The angle of rotation formed by minute hand in 1 hour = 360°

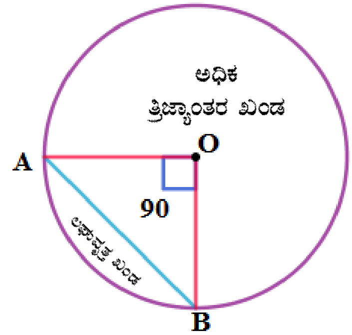
$\therefore$ The angle of rotation in 5 minutes $= \frac{360°}{60} \times 5 = 30°$

Area of the sector of angle $\theta = \frac{\theta}{360°} \times \pi r^2$

$\therefore$ Area of the sector of angle $30^0 = \frac{30}{360°} \times \frac{22}{7} \times 14 \times 14 = \frac{1}{12} \times \frac{22}{7} \times 14 \times 14$

$= \frac{1}{3} \times 22 \times 7 = \frac{154}{3} cm^2$

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use = 3.14)



Radius of the circle = 10 cm

The angle of the Major sector = 360° - 90° = 270°

Area of the major sector $= \frac{270}{360°} \times \pi\, r^2$ cm^2

$= \frac{3}{4} \times 3.14 \times 10 \times 10 = 75 \times 3.14$ cm^2 = 235.5 cm^2

In right angle ΔAOB , OA = 10 cm, OB = 10 cm

Area of $\Delta AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50$ cm^2 --- (1)

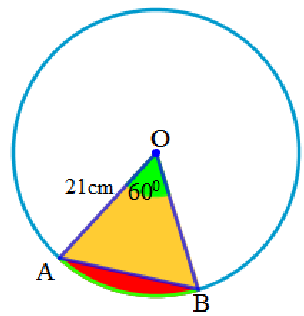
The angle of the Minor sector = 90°

Area of the minor sector $= \frac{90}{360°} \times \pi\, r^2$ cm^2

$= \frac{1}{4} \times 3.14 \times 10 \times 10 = 25 \times 3.14$ cm^2 $= 25 \times 3.14$ cm^2 = 78.5 cm^2 -------- (2)

Area of minor segment = (2) – (1) = 78.5 cm^2 - 50 cm^2 = 28.5 cm^2

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc (ii) area of the sector formed by the arc
(iii) area of the segment formed by the corresponding chord



Radius of the circle = 21 cm

(i) The length of the Arc AB $= \frac{\theta}{360°} \times 2\pi r$

Arc AB $= \frac{60}{360°} \times 2 \times \frac{22}{7} \times 21 = \frac{1}{6} \times 2 \times 22 \times 3 = \mathbf{22cm}$

(ii) The angle formed by arc AB = 60°

Area of the sector of angle $60^0 = \frac{60}{360°} \times \pi\, r^2$ cm^2

$= \frac{60}{360°} \times \frac{22}{7} \times 21 \times 21$ cm^2 $= \frac{1}{6} \times 22 \times 3 \times 21$ cm^2 $= \frac{1}{2} \times 22 \times 21$ cm^2

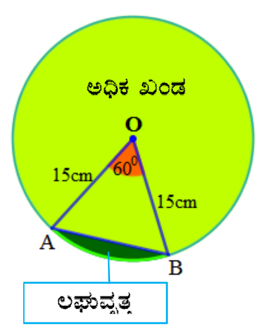
= 11x21 cm^2 = **231 cm^2**

(iii) The area of the equilateral $\Delta AOB = \frac{\sqrt{3}}{4} \times (OA)^2 = \frac{\sqrt{3}}{4} \times (21)^2 = \frac{441\sqrt{3}}{4}$ cm^2

Hence the required area = Area of the sector formed by the Arc – area of ΔAOB

$= \left(\mathbf{231} - \frac{\mathbf{441\sqrt{3}}}{\mathbf{4}}\right)\mathbf{cm^2}$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $= \sqrt{3} = 1.73$)



Radius of the circle = 15 cm

In triangle ΔAOB, $\angle AOB$ and $\angle A = \angle B = 60^0$ [$\because$ OA = OB = 15cm]

$\therefore \Delta AOB$ is an equilateral triangle.

The area of $\Delta AOB = \frac{\sqrt{3}}{4} \times (OA)^2 = \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4}$ cm^2

$= \frac{225 \times 1.73}{4}$ cm^2 = **97.3 cm^2**

The angle formed by the arc AB = 60°

$\therefore$ The area of the sector formed by the arc AB $= \frac{60}{360°} \times \pi\, r^2$ cm^2

$= \frac{60}{360°} \times (3.14) \times 15 \times 15$ cm^2 $= \frac{1}{2} \times 3.14 \times 5 \times 15$ cm^2 = 1.57 x 75 cm^2 = **117.75 cm^2**

Area of the minor segment = Area of the sector formed by the arc AB – Area of ΔAOB

= 117.75 - 97.3 = **20.4 cm^2**

Area of the major segment = Area of the circle – Area of minor segment

$= \pi r^2 - 20.4\ cm^2 = 3.14 \times 15\times15 - 20.4 = 3.14 \times 225 - 20.4$

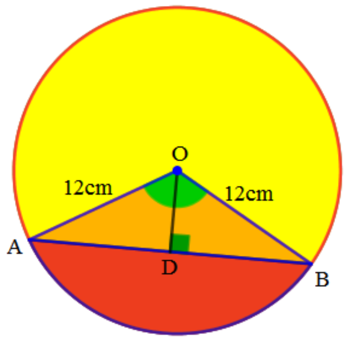
$= 706.5 - 20.4$ = $\mathbf{686.1 cm^2}$

7. **A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$).**

Radius of the circle(r) = 12 cm

Draw $AB \perp OD \Rightarrow OD$ bisects AB

$\Rightarrow \angle A = 180° - (90° + 60°) = 30°$



$\cos 30° = \frac{AD}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{12} \Rightarrow AD = 6\sqrt{3}$ cm

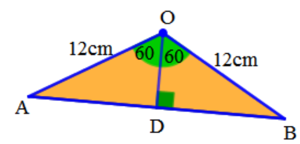
$\Rightarrow AB = 2 \times AD = 12\sqrt{3}$ cm

$\sin 30° = \frac{OD}{OA} \Rightarrow \frac{1}{2} = \frac{OD}{12} \Rightarrow OD = 6$ cm

The area of $\Delta AOB = \frac{1}{2} \times AB \times OD$

$= \frac{1}{2} \times 12\sqrt{3} \times 6\ cm^2 = 36\sqrt{3}\ cm = 36 \times 1.73$ = $\mathbf{62.28\ cm^2}$

The angle of minor sector = 120°



∴ Area of the minor sector = $\frac{120°}{360°} \times \pi r^2\ cm^2$

$= \frac{120°}{360°} \times 3.14 \times 12 \times 12\ cm^2 = \frac{1}{3} \times 3.14 \times 12 \times 12\ cm^2$

$= 3.14 \times 4 \times 12\ cm^2 = 3.14 \times 48\ cm^2$ = $\mathbf{150.72\ cm^2}$

∴ Area of the minor segment = Area of the minor sector – Area of ΔAOB

$= 150.72\ cm^2 - 62.28\ cm^2$ = $\mathbf{88.44\ cm^2}$

8. **A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 5.11). Find**

(i) The area of that part of the field in which the horse can graze

(ii) The increase in the grazing area if the rope were10 m long instead of 5 m. (Use = 3.14)

Given, the side of the square = 15 m

The length of the rope [Radius of the arc(r)] = 5 m

The radius of the field in which the horse can graze = 5 m.

Fig. 12.11

(i) Area of the field graze by the horse

[Horse is tied at the corner of the square.So, it graze only quadrant of the circle of radius 5m]

$= \frac{\pi r^2}{4} = \frac{3.14 \times 5^2}{4} = \frac{78.5}{4}$ = $\mathbf{19.625\ m^2}$

(ii) The length of the rope is 10m then, the area graze by the horse $= \frac{\pi r^2}{4} = \frac{3.14 \times 10^2}{4} = \frac{314}{4}$ = $\mathbf{78.5\ m^2}$

Therefore increase in grazing area

$= 78.5\ m^2 - 19.625\ m^2$ = $\mathbf{58.875\ m^2}$

9. **A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 5.12. Find :**

(i) the total length of the silver wire required

(ii) the area of each sector of the brooch.

Number of diameters = 5; Legnth of the diameter = 35 mm

∴ Radius (r) = 35/2 mm

(i) The total lenth of wire required

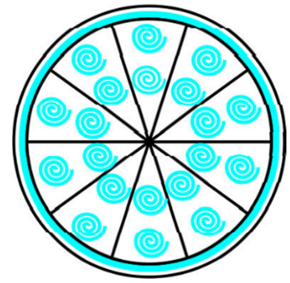


Fig 5.12

= Perimeter of the brooch + length of 5 diameter

$= 2\pi r + (5\times35)$ mm

$= (2 \times \frac{22}{7} \times \frac{35}{2}) + 175$ mm $= 110 + 175$ mm **= 285 mm**

(ii) Number of sectors = 10

Therefore area of each sector $= \frac{\pi r^2}{10}$

$= \frac{\frac{22}{7} x \left(\frac{35}{2}\right)^2}{10} = \frac{\frac{22}{7} x \frac{35}{2} x \frac{35}{2}}{10} = \frac{\frac{3850}{4}}{10} = \frac{385}{4}$ mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig. 5.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

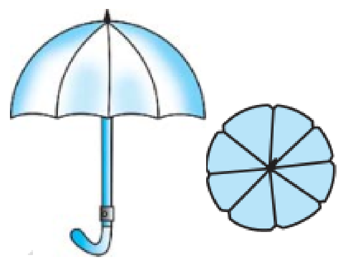


Fig 5.13

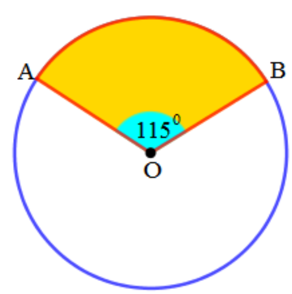
Total ribs in the umbrella = 8

The radius of the umbrella when it to be flat = 45 cm

The area between the two consecutive ribs = $\frac{\text{Total Area}}{\text{number of ribs}}$

$= \frac{\pi r^2}{8} = \frac{\frac{22}{7} x\ 45^2}{8} = \frac{44550}{56} = \frac{22275}{28}$ cm^2 **= 795.5 cm^2**

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.



The angle of the sector formed by the wiper = 115°

Radius of the sector = length of the wiper = 25 cm

Area of the sector formed by the wiper $= \frac{115°}{360°} \times \pi r^2$ cm^2

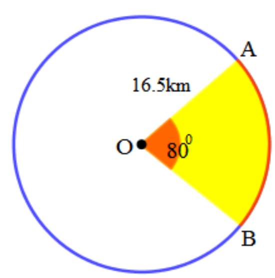
$= \frac{115°}{360°} \times \frac{22}{7}$ x 25 x 25 cm^2

$= \frac{23}{72} \times \frac{22}{7}$ x 625 cm^2 $= \frac{23}{36} \times \frac{11}{7}$ x 625 cm^2 $= \frac{158125}{252}$ cm^2

The total area coveed by blades of two wipers

$= 2 \times \frac{158125}{252}$ cm^2 $= \frac{158125}{126}$ **= 1254.96 cm^2**

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use =3.14)



Let the lighthouse be at O

Radius of the sector = length of the beam r = 16.5 km

Angle of the sector formed by the beam = 80°

The area of the sector which light spreads = Area of the sector $= \frac{80°}{360°} \times \pi r^2$ km^2

$= \frac{2}{9} \times 3.14$ x 16.5x16.5km^2 $= \frac{2}{9} \times 3.14$ x 272.25km^2 **= 189.97 km^2**

13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs 0.35 per cm^2 0.35 per cm^2 (Use $\sqrt{3} = 1.7$).

The number of equal designs = 6; The radius of the cover = 28 cm
Making Cost of the design = ರೂ 0.35 / cm^2

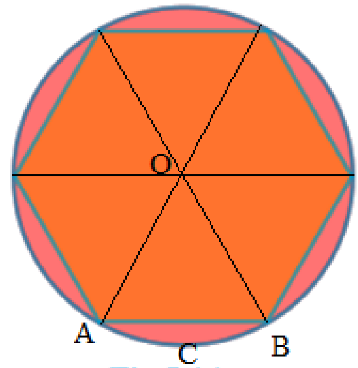


Fig 5.14

The angle of each sector $= \frac{360°}{6} = 60°$

ΔAOB ಯಲ್ಲಿ OA = OB [Radius of the same circle]

$\therefore \angle A = \angle B = 60^0$

$\therefore$Area of the equilateral $\Delta AOB = \frac{\sqrt{3}}{4} x\ (OA)^2$

$= \frac{\sqrt{3}}{4} x\ (28)^2 = 1.7x\ 7\ x\ 28$ $= \mathbf{333.2\ cm^2}$

Area of the sector OACB $= \frac{60°}{360°} \times \pi\ r^2\ cm^2$

$= \frac{1}{6} \times \frac{22}{7} x\ 28^2\ cm^2 = \frac{1}{6} \times 22x\ 4x28\ cm^2$

$= \frac{1}{3} \times 22x\ 2x28\ cm^2 = 410.67 cm^2$

Area of the design = Area of the sector OACB - Area of the ΔAOB

$= 410.67\ cm^2 - 333.2\ cm^2 = 77.47\ cm^2$

$\therefore$ The total area of 6 designs = $6 \times 77.47\ cm^2 = 464.82\ cm^2$

$\therefore$ Total cost of making designs = $464.76\ cm^2 \times$ ರೂ $0.35\ /cm^2$ = **Rs 162.68**

14. Tick the correct answer in the following :

Area of a sector of angle p (in degrees) of a circle with radius R is

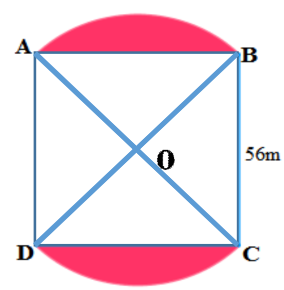
A) $\frac{P}{180} x\ 2\pi r$ B) $\frac{P}{180} x\ 2\pi r^2$ C) $\frac{P}{360} x\ 2\pi R$ D) $\frac{P}{720} x\ 2\pi R^2$

The area of the sector of angle p $= \frac{p°}{360°} \times \pi\ R^2\ cm^2 = \frac{p}{360°} \times \pi\ R^2\ x\ \frac{2}{2} = \frac{P}{720} x\ 2\pi R^2$

Answer **(D)** $\mathbf{\frac{P}{720} x\ 2\pi R^2}$

5.4 Areas of Combinations of Plane Figures

In Fig. 5.15, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



Solution: Area of the square lawn = $56\ x\ 56\ m^2$ (1)

Let The radius = OA = OB = x m

Therefore, $x^2 + x^2 = 56^2$ [By Pythagoras theorem $OA^2 + OB^2 = AB^2$]

$\Rightarrow 2x^2 = 56\ x\ 56$

$\Rightarrow x^2 = 56\ x\ 28$ (2)

Now, Area of the sector OAB

$= \frac{90}{360} x\ \pi x^2 = \frac{1}{4} x \frac{22}{7} x\ 28\ x\ 56$ [From eqn (2)] (3)

Total Area = [Area of sector OAB + Area of sector ODC + Area of ΔOAD + Area of ΔOBC]

$= \frac{90}{360} x \frac{22}{7} x28x56 + \frac{90}{360} x \frac{22}{7} x28x56 + \frac{1}{4} x56x56 + \frac{1}{4} x56x56$

$= 22x56 + 22x56 + 14x56 + 14x56$

$= 56(22+ 22 + 14+14) = 56(22+ 22 + 14+14) = 56x72 = 4032m^2$

Example 5: Find the area of the shaded region in Fig. 5.16, where ABCD is a square of side 14 cm

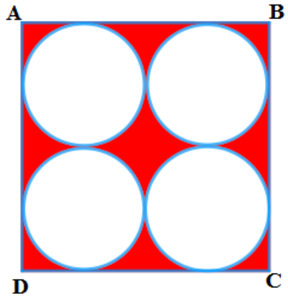


Fig 5.16

Solution : Area of square ABCD = 14 x 14 cm^2 = 196 cm

Diaameter of each circle = $\frac{14}{2}$ = 7cm

So, radius of the circle = $\frac{7}{2}$ cm

So, area of each circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$ cm^2

Therefore area of four circles = $4 \times \frac{77}{2} = 154cm^2$

Therefore area of shaded region = (196 – 154)= $42cm^2$

ಉದಾಹರಣೆ 6: ABCD ಯು 10 cm ಬಾಹುವುಳ್ಳ ಚೌಕವಾಗಿದೆ ಮತ್ತು ಪ್ರತಿ ಚೌಕದ ಬಾಹುವು ವ್ಯಾಸವಾಗಿರುವಂತೆ ಅರ್ಧವೃತ್ತವನ್ನು ಎಳೆದಿದೆ. ಚಿತ್ರ 5.17 ರಲ್ಲಿ ಛಾಯೆಗೊಳಿಸಿದ ವಿನ್ಯಾಸದ ವಿಸ್ತೀರ್ಣವನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ. (π = 3.14 ಎಂದು ಬಳಸಿ)

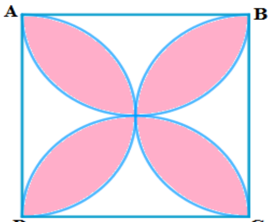


Fig 5.17

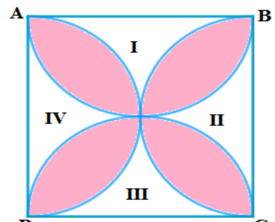


Fig 5.18

Area I + Area II = Area ABCD – Area of the two semicircles circle of radius 5 cm

⇒Area of ABCD – Area of the circle of radius5 cm = $a^2 - \pi r^2$

⇒$10 \times 10 - 3.14 \times 5^2 = 100 - 3.14 \times 25 = 100 - 78.5 = 21.5cm^2$

Area III + Area IV = $21.5cm^2$

Therefore, Area of shaded region = Area ABCD – Area [I + II + III + IV]

= $100 - 2 \times (21.5) = 100 - 43 = 57cm^2$

Exercise 5.3

[Unless stated otherwise, use $\pi = \frac{22}{7}$]

1. Find the area of the shaded region in Fig. 5.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.

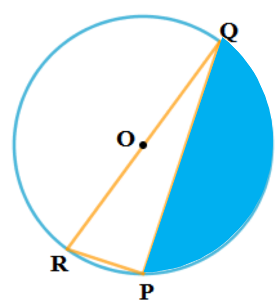


Fig 5.19

PQ = 24 cm and PR = 7 cm

∠P = 90° [Angle of semi circle]

∴ Hypotenuse QR = Diameter of the circle

$QR^2 = PR^2 + PQ^2$ [Pythagoras theorem in Δ PRQ]

⇒ $QR^2 = 7^2 + 24^2$ ⇒ $QR^2 = 49 + 576$

⇒ $QR^2 = 625$ ⇒ QR = 25 cm

∴ Radius of the circle = $\frac{25}{2}$ cm

Area of semi circle $= \frac{\pi R^2}{2} = \frac{\frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}}{2} = \frac{13750}{56}$ cm^2 = $\mathbf{\frac{6875}{28}}$ **cm^2 = 245.54 cm^2**

Area of ΔPQR = $\frac{1}{2} \times PR \times PQ$

$= \frac{1}{2} \times 7 \times 24$ cm^2 = **84 cm^2**

∴ **Area of shaded region** = 245.54 cm^2 - 84 cm^2 = **161.54 cm^2**

[Or $\frac{6875}{28} - 84 = \frac{6875-2352}{28} = \frac{4523}{28}$ cm^2]

2. **Find the area of the shaded region in Fig. 5.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and AOC = 40°.**

Radius of the inner circle = 7 cm

Radius of the outer circle = 14 cm

The angle of the sector = 40°

Area of the sector OAC = $\frac{40°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{9} \times \frac{22}{7} \times 14^2$ cm^2 $= \frac{1}{9} \times 22 \times 2 \times 14$ cm^2 $= \mathbf{\frac{616}{9}}$ **cm^2**

Area OBD = Area of the sector = $\frac{40°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{9} \times \frac{22}{7} \times 7^2$ cm^2 $= \frac{1}{9} \times 22 \times 7$ cm^2 $= \mathbf{\frac{154}{9}}$ **cm^2**

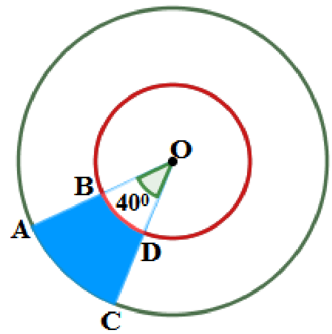


Fig 5.20

∴ **Area of shaded region**

= Area of the sector OAC - Area of sector OBD

$= \left(\frac{616}{9} - \frac{154}{9}\right)$cm^2 $= \frac{462}{9}$ cm^2 $= \mathbf{\frac{154}{3}}$ **cm^2**

3. **Find the area of the shaded region in Fig. 5.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.**

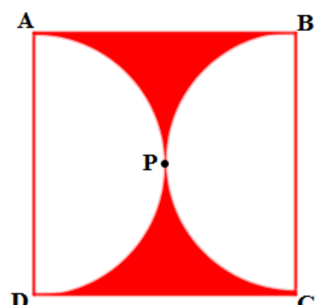


Fig 5.21

Side of the square = 14 cm

Diameter of the semi circle = 14 cm

∴ Radius of the semi circle = 7 cm

Area of the square = 14 × 14 = 196 cm^2

Area of the semi circle $= \frac{\pi R^2}{2} = \frac{\frac{22}{7} \times 7 \times 7}{2} = \frac{154}{2} = 77$ cm^2

Area of two semicircle = 2 × 77 cm^2 = 154 cm^2

∴ **Area of shaded region** = 196 cm^2 - 154 cm^2 = **42 cm^2**

4. **Find the area of the shaded region in Fig. 5.22, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.**

OAB is a equilateral triangle, Therefore each angle = 60°.

Radius of the circle = 6 cm.

Side of the triangle = 12 cm.

Area of the equilateral triangle $= \frac{\sqrt{3}}{4}(OA)^2$

$= \frac{\sqrt{3}}{4}(12)^2 = \sqrt{3} \times 3 \times 12 = \mathbf{36\sqrt{3}}$ **cm^2**

Area of the circle $= \pi R^2 = \frac{22}{7} \times 6^2 = \frac{22 \times 36}{7}$ cm^2 $= \mathbf{\frac{792}{7}}$ **cm^2**

The area of the sector of angle 60° $= \frac{60°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{6} \times \frac{22}{7} \times 6^2$ cm^2 $= \frac{22 \times 6}{7}$ cm^2 $= \mathbf{\frac{132}{7}}$ **cm^2**

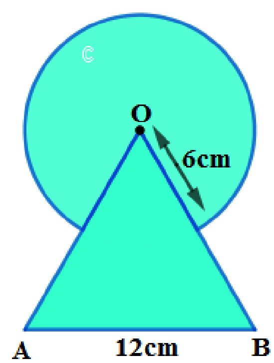


Fig 5.22

∴ **Area of the shaded region**

= Area of equilateral triangle + Area of the circle – Area of the sector

$= \left(\mathbf{36\sqrt{3}} + \mathbf{\frac{792}{7}} - \mathbf{\frac{132}{7}}\right)$ **cm^2** $= \left(\mathbf{36\sqrt{3} + \frac{660}{7}}\right)$ cm^2

5. **From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 5.23. Find the area of the remaining portion of the square.**

The side of the square = 4 cm; Radius of the circle = 1 cm

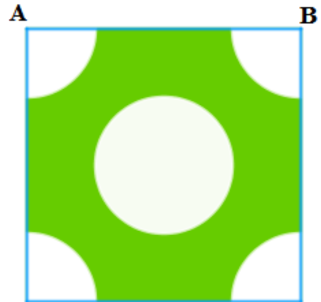


Fig 5.23

Area of the square = $(\text{Side})^2 = 4^2 = 16\ \text{cm}^2$

Area of each quadrant $= \frac{\pi R^2}{4}\ \text{cm}^2 = \frac{\frac{22}{7} \times 1^2}{4} = \frac{11}{14}\ \text{cm}^2$

∴ Area of four quadrant $= 4 \times \frac{11}{14}\ \text{cm}^2 = \mathbf{\frac{22}{7}\ cm^2}$

Area of the circle $= \pi R^2\ \text{cm}^2 = \frac{22}{7} \times 1^2 = \frac{22}{7}\ \text{cm}^2$

Area of the square $=$ Side$^2 = 4^2 = 16\text{cm}^2$

$\left(\frac{22}{7} + \frac{22}{7}\right)\text{cm}^2 = \mathbf{\frac{44}{7}\ cm^2}$

∴ **Area of shaded region** = Area of square – [Area of four quadrants+ area of circle]

$= 16 - \left(\frac{22}{7} + \frac{22}{7}\right)\text{cm}^2 = \left(\frac{112-44}{7}\right)\text{cm}^2 = \mathbf{\frac{68}{7}\ cm^2}$

6. **In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in Fig. 5.24. Find the area of the design.**

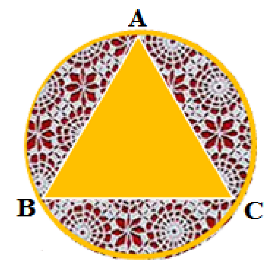
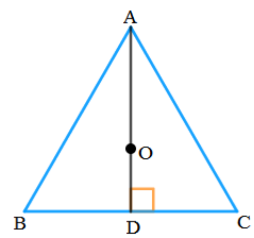


Fig 5.24



Radius of the circle = 32 cm

AD is the median drawn through the center O ⇒ $BD = \frac{AB}{2}$

∴ Radius of the circle $AO = \frac{2}{3}AD$[AD is the median]

$\Rightarrow \frac{2}{3}AD = 32\ \text{cm} \Rightarrow \mathbf{AD = 48\ cm}$

In triangle ΔADB,

$AB^2 = AD^2 + BD^2$ [By Pythagoras theorem]

$\Rightarrow AB^2 = 48^2 + \left(\frac{AB}{2}\right)^2 \Rightarrow AB^2 = 2304 + \frac{AB^2}{4}$

$\Rightarrow \frac{3AB^2}{4} = 2304 \Rightarrow AB^2 = 3072 \Rightarrow \mathbf{AB = 32\sqrt{3}\ cm}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}(AB)^2 = \frac{\sqrt{3}}{4}(32\sqrt{3})^2 = \mathbf{768\sqrt{3}\ cm^2}$

Area of the circle $= \pi R^2 = \frac{22}{7} \times 32 \times 32 = \mathbf{\frac{22528}{7} cm^2}$

∴Area of the design = Area of the circle - Area of ΔABC

$= \left(\mathbf{\frac{22528}{7} - 768\sqrt{3}}\right)\ \mathbf{cm^2}$

7. **In Fig. 5.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.**

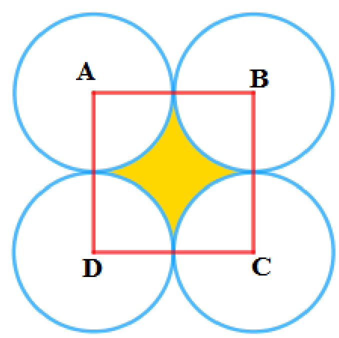


Fig 5.25

Side of the Square = 14 cm

∴ Radius of each circle $= \frac{14}{2} = 7$ cm

Area of square ABCD $= 14^2 = 196\ \text{cm}^2$

Area of the quadrant $= \frac{\pi R^2}{4}\ \text{cm}^2 = \frac{\frac{22}{7} \times 7^2}{4} = \frac{154}{4}\ \text{cm}^2 = \frac{77}{2}\ \text{cm}^2$

∴ Area of four quadrant $= 4 \times \frac{77}{2}\ \text{cm}^2 = 154\ \text{cm}^2$

∴ **Area of shaded region**

= Area of the square ABCD - 4 Area of four quadrant

$= 196\ \text{cm}^2 - 154\ \text{cm}^2 = \mathbf{42\ cm^2}$

8. Fig. 5.26 depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :

(i) The distance around the track along its inner edge

(ii) The area of the track.

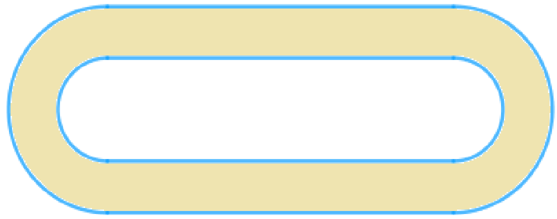
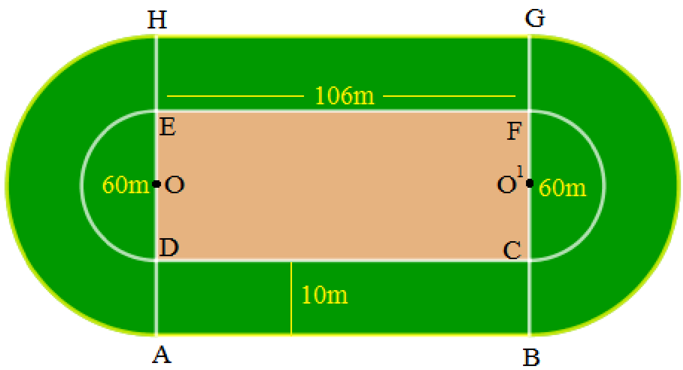


Fig 5.26



Width of the track = 10 m

Distance between parallel lines DE = CF = 60 m, Length of each parallel line = 106 m

Radius of inner circle r = OD = O'C = $\frac{60}{2}$ = 30 m

Radius of outer circle R = OA = O'B = 30 + 10 m = 40 m

AB = CD = EF = GH = 106 m

(i) The distance around the track along its inner edge

= CD + EF + 2 × (Circumference of inner semi-circle)

= 106 + 106 + (2 × πr) m = 212 + (2 × $\frac{22}{7}$ × 30) m = 212 + $\frac{1320}{7}$ m = $\mathbf{\frac{2804}{7}}$ **m**

(ii) Area of the running track

= Area ABCD + Area EFGH + 2 xArea of outer semi-circle - 2 x Inner semi-circle

= (AB × CD) + (EF × GH) + 2 × $\left(\frac{\pi R^2}{2}\right)$ - 2 × $\left(\frac{\pi r^2}{2}\right)$ m^2

= (106 × 10) + (106 × 10) + 2 × $\frac{\pi}{2}(R^2 - r^2)$ m^2 = 1060 + 1060 + $\frac{22}{7}$ × 700 m^2

= [1060 + 1060 +(22 × 100)] m^2 = [2120 +2200] m^2 = **4320 m^2**

9. In Fig. 5.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

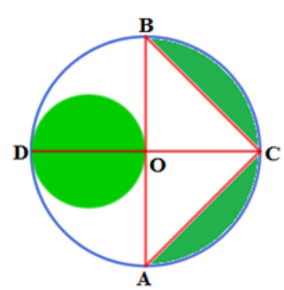


Fig 5.27

Radius of the greater circle R = 7 cm

Radius of the smaller circle r = $\frac{7}{2}$ cm

Height of ΔBCA = OC = 7 cm; Base of ΔBCA = AB = 14 cm

Area of ΔBCA = $\frac{1}{2}$× AB × OC = $\frac{1}{2}$× 7 × 14 = **49 cm^2**

Area of greater circle = πR^2 = $\frac{22}{7}$ × 7^2 = **154 cm^2**

Area of greater semi-circle = $\frac{154}{2}$ cm^2 = **77 cm^2**

Area of smaller circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$**cm^2**

Area of shaded aregion

= Area of greater semi-circle - Area of ΔBCA + Area of smaller circle

$= \left(77 - 49 + \frac{77}{2}\right)$ cm^2 = $\left(\frac{154-98+77}{2}\right)$ cm^2 = $\left(\frac{133}{2}\right)$ cm^2 **= 66.5 cm^2**

10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 5.28). Find the area of the shaded region. (Use = 3.14 and π = 3.14 and $\sqrt{3}$ = 1.73205)

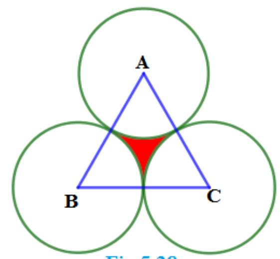


Fig 5.28

ABC is an equilateral triangle $\therefore \angle A = \angle B = \angle C = 60°$

Area of ΔABC = 17320.5 cm^2 $\Rightarrow \frac{\sqrt{3}}{4} \times (AB)^2 = 17320.5$

$\Rightarrow AB^2 = 17320.5 \times \frac{4}{1.73205} \Rightarrow AB^2 = 4 \times 10^4 \Rightarrow AB = 200$ cm

Radius of the circle $= \frac{200}{2}$ cm = 100 cm

Area of three equal sector of angle 60^0 =3x $\frac{60°}{360°} \times \pi r^2$ cm^2

=3x $\frac{1}{6}$ ×3.14 x 100^2 cm^2 = $\frac{1}{2}$ ×3.14 x 100^2 cm^2 = $\frac{31400}{2}$ cm^2 = 15700 cm^2

Area of Shaded region =Area of ΔABC - Area of three equal sector

= (17320.5 – 15700) cm^2 **= 1620.5 cm^2**

11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 5.29). Find the area of the remaining portion of the handkerchief.

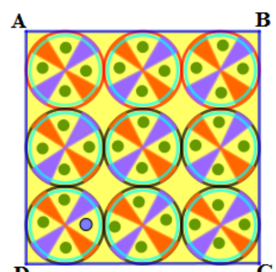


Fig 5.29

Number of circles = 9; Radius of each circle = 7 cm

There are three circles touch each other

$\therefore$ Side of the square = 3 × diameter of the circle = 3 × 14 = 42 cm

Area of the square = 42 × 42 cm^2 = 1764 cm^2

Area of 9 equal circle = $9\pi r^2$ = 9x $\frac{22}{7}$ × 7 × 7 = 1386 cm^2

The area of remaing part of the handkerchief

= Area of the square - Area of 9 equal circle = 1764 - 1386 = **37**

12. In Fig. 5.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

i) quadrant OACB ii) shaded region.

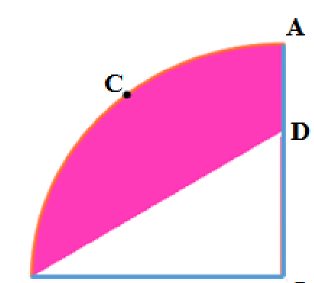


Fig 5.30

Radius of the quadrant of the circle = 3.5 cm = $\frac{7}{2}$ cm

(i) Area of OACB quadrant = $\frac{\pi R^2}{4}$ cm^2 = $\frac{\frac{22}{7} x \frac{7}{2} x \frac{7}{2}}{4}$ cm^2= $\frac{77}{8}$ **cm^2**

(ii) Area of ΔBOD $= \frac{1}{2} \times \frac{7}{2} \times 2$ cm^2= $\frac{7}{2}$ **cm^2**

Area of shaded region

= Area of OACB – Area of Δ BOD

$= \left(\frac{77}{8} - \frac{7}{2}\right)$ cm^2= $\left(\frac{77}{8} - \frac{28}{8}\right)$ cm^2 = $\left(\frac{49}{8}\right)$ cm^2 **= 6.125 cm^2**

13. In Fig. 5.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use = 3.14)

Side of the square = OA = AB = 20 cm

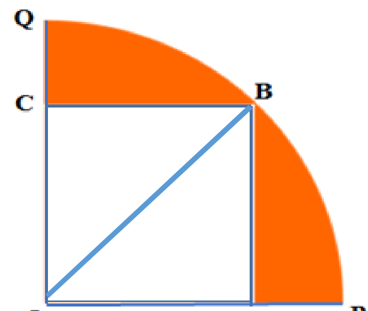


Fig 5.31

Radius of the quadrant of the circle = OB

OAB is a right angle triangle

∴ in right angle ΔOAB,

$OB^2 = AB^2 + OA^2$ [By Pythagoras theorem]

$\Rightarrow OB^2 = 20^2 + 20^2 \Rightarrow OB^2 = 400 + 400$

$\Rightarrow OB^2 = 800 \Rightarrow OB = 20\sqrt{2}$ cm

The area of the quadrant of circle $= \frac{\pi R^2}{4}$ cm^2

$= \frac{3.14 \times (20\sqrt{2})^2}{4}$ cm^2 $= \frac{3.14 \times 400 \times 2}{4}$ cm^2 $= 3.14 \times 200$ cm^2 $= 628$ cm^2

ಚೌಕದ ವಿಸ್ತೀರ್ಣ $= 20 \times 20 = 400$ cm^2

Area of shaded region

= Area of quadrant of circle - Area of the square = 628 - 400 cm^2 = **228 cm^2**

14. **AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 5.32). If AOB = 30°, find the area of the shaded region**

Radius of the greater circle R = 21 cm and radius of smaller circle r =

The angle formed by two concentric are = 30°

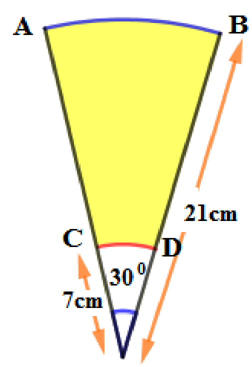


Fig 5.32

Area of outer sector $= \frac{30°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{12} \times \frac{22}{7} \times 21 \times 21$ cm^2 $= \frac{1}{4} \times 22 \times 3 \times 7$ cm^2

$= \frac{1}{2} \times 11 \times 3 \times 7$ cm^2 $= \mathbf{\frac{231}{2}}$ **cm^2**

Area of inner sector $= \frac{30°}{360°} \times \pi r^2$ cm^2

$= \frac{1}{12} \times \frac{22}{7} \times 7 \times 7$ cm^2 $= \frac{1}{6} \times 11 \times 7$ cm^2 $= \mathbf{\frac{77}{6}}$ **cm^2**

Area of shaded region

= Area of outer sector – Area of inner sector

$= \left(\frac{231}{2} - \frac{77}{6}\right)$cm^2 $= \left(\frac{693}{6} - \frac{77}{6}\right)$cm^2 $= \left(\frac{616}{6}\right)$cm^2 $= \mathbf{\frac{308}{3}}$ **cm^2**

15. **In Fig. 5.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.**

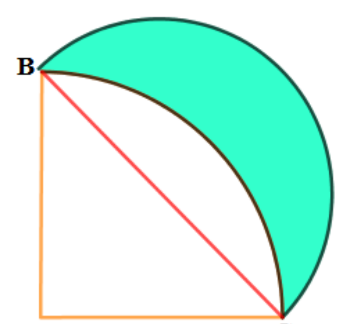


Fig 5.33

The radius of quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is the diameter of semi circle

Now, ABC is a right angle triangle

$\therefore BC^2 = AB^2 + AC^2$ [Pythgoras theorem]

$\Rightarrow BC^2 = 14^2 + 14^2 \Rightarrow BC = 14\sqrt{2}$ cm

Radius semi circle $= \frac{14\sqrt{2}}{2}$cm $= 7\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times 14 \times 14$ cm^2 $= 7 \times 14 \times 14 =$ **98 cm^2**

The area of the quadrant of circle $= \frac{\pi R^2}{4}$ cm^2 $= \frac{\frac{22}{7} \times 14 \times 14}{4}$ cm^2 = **154 cm^2**

Area of semi circle $= \frac{\pi R^2}{2} = \frac{\frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}}{2}$ = **154 cm^2**

Area of shaded region

= Area of semi circle + Area of ΔABC- Area of quadrant of circle

$= 154 + 98 - 154 \text{ cm}^2$ $= \mathbf{98\ cm^2}$

16. Calculate the area of the designed region in Fig. 5.34 common between the two quadrants of circles of radius 8 cm each.

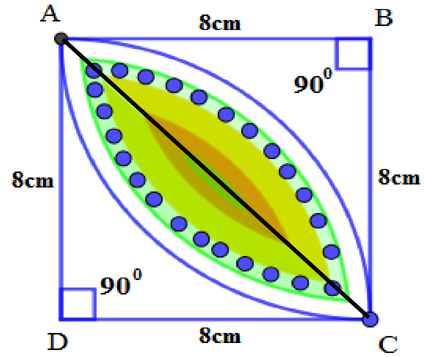


Fig 5.34

AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC $= \frac{1}{2} \times 8 \times 8 = \mathbf{32\ cm^2}$

Area of Quadrant AECB = Area of quadrant AFCD

$= \frac{\pi R^2}{4} \text{ cm}^2 = \frac{\frac{22}{7} \times 8 \times 8}{4} = \frac{352}{7} \mathbf{\ cm^2}$

Area of shaded region

= (Area of quadrant AECB – Area ofΔABC)

+ (Area of quadrant AFCD – Area of ΔADC)

$= \left(\frac{352}{7} - 32\right) + \left(\frac{352}{7} - 32\right) \text{ cm}^2$

$= 2 \times \left(\frac{352}{7} - 32\right) \text{ cm}^2 = 2 \times \left(\frac{352-224}{7}\right) \text{ cm}^2$

$= 2 \times \left(\frac{128}{7}\right) \text{ cm}^2 = \frac{256}{7} \mathbf{\ cm^2}$

Summary:

1. Circumference of the circle = $2\pi r^2$
2. Area of the circle = πr^2
3. The radius of the circle r the angle measure with θ
   Then the Length of the Arc of the sector = $\frac{\theta}{360} \times 2\pi r$
4. The radius of the circle r the angle measure with θ
   Then the area of the sector = $\frac{\theta}{360} \times \pi r^2$
5. Area of segment of a circle = Area of the corresponding sector - Area of the corresponding triangle.

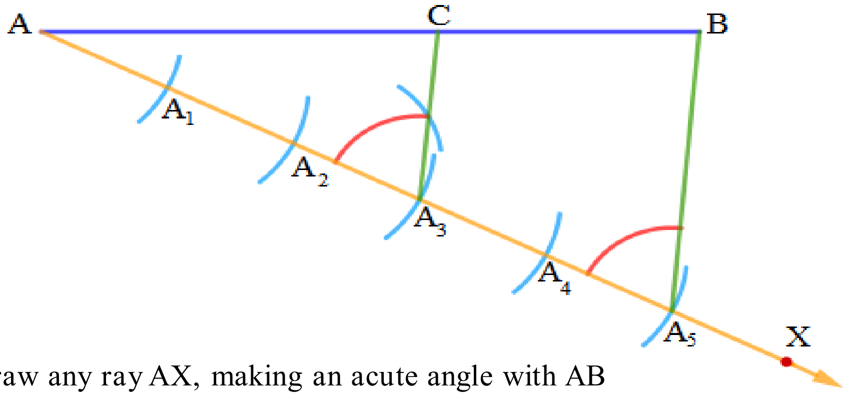
6 Constructions

6.2 Division of a Line Segment

Construction 6.1: To divide a line segment in a given ratio.

Divede a line segment AB in the ratio m:n

Example: Devide the line segment AB in the ratio 3:2



Step-1: Draw any ray AX, making an acute angle with AB

(Can draw above or below the given line)

Step-2: Locate 5 (= m + n) points A_1, A_2, A_3, A_4 and A_5 on AX so that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

Step-3: Join BA_5

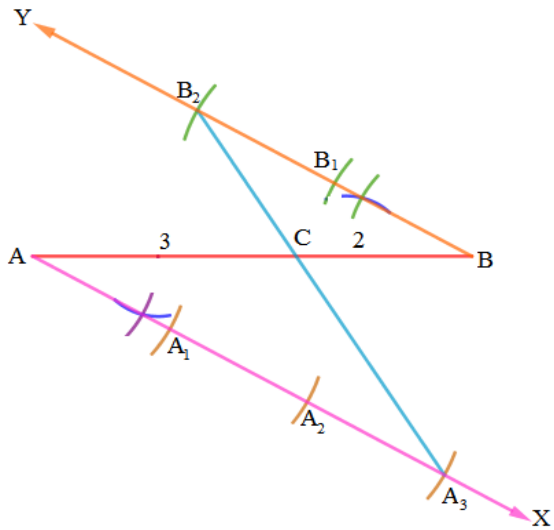
Step-4: Through the point A_3 (m = 3), draw a line parallel to A_5B (by making an angle equal to AA_5B) at A_3 intersecting AB at the point C (see Fig.). Then, AC : CB = 3 : 2

Justification:

$A_3C \parallel A_5B \Rightarrow \frac{AA_3}{A_3A_5} = \frac{AC}{CB}$ [Basic proportionality theorem]$\Rightarrow \frac{AA_3}{A_3A_5} = \frac{3}{5-3} = \frac{3}{2} \Rightarrow 3:2$

Now AC : CB = 3 : 2

Alternate Method:



Step 1: Draw any ray AX making an acute angle with AB .

Step 2: Draw a ray BY parallel to AX by making ∠ABY equal to ∠BAX

Step 3: Locate the points A_1, A_2, A_3 (m = 3) on AX and B_1, B_2 (n = 2) on BY such that $AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2$

Step 4: Join A_3B_2. Let it intersect AB at a point C

Justification:

In Δ AA_3C and Δ BB_2C ∠ AC A_3 = BC B_2 (Vertically opposite angles)

∠ CAA_3 = CBB_2 (Alternate angles)

Δ AA_3C ~ Δ BB_2C (AA similarity criteria)

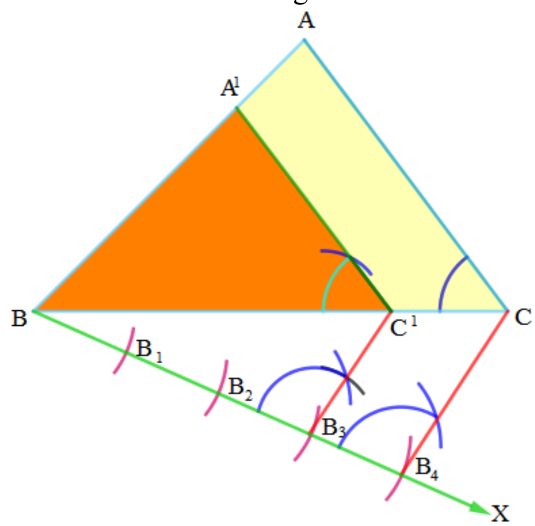
$\Rightarrow \frac{AA_3}{BB_2} = \frac{AC}{BC}$ [BPT] $\Rightarrow \frac{AA_3}{BB_2} = \frac{3}{2} \Rightarrow \frac{AC}{BC} = \frac{3}{2} \Rightarrow AC : BC = 3:2$

Construction 6.2:

To construct a triangle similar to a given triangle as per given scale factor.

Example1:Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding side of the triangle ABC [i,e. of scale factor $\frac{3}{4}$]

Solution: Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.



Step-1: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A

Step-2: Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step-3: Join B_4C and draw a line through B_3 the 3rd point, (3 being smaller of 3 and 4in $\frac{3}{4}$)parallel toB_4C to intersect BC at C^1

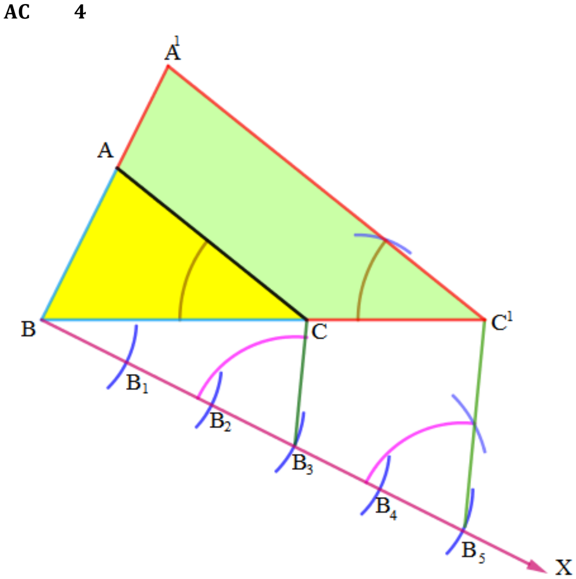
Step-3: Draw a line through C^1 parallel to the line CA to intersect BA at A^1

Justification:

$\frac{BC^1}{C^1C} = \frac{3}{1}$ $\therefore \frac{BC}{BC^1} = \frac{3+1}{3} = \frac{4}{3}$ $\Rightarrow \frac{BC^1}{BC} = \frac{3}{4}$

$C^1A^1 \parallel CA$ $\therefore \Delta A^1BC^1 \sim \Delta ABC$ $\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{4}$

Example 2 : Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding side of the triangle ABC [i,e. of scale factor $\frac{5}{3}$]



Step1: Construct any ΔABC. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step 2: Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) B_1, B_2, B_3, B_4 and B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$

Step 3: Join B_3(the 3rd point, 3 being smaller of 3 and 5 in $\frac{5}{3}$) to C and draw a through B_5 parallel to B_3C intersect BC at C^1

Step 4: Draw a line through C^1 parallel to the line CA to intersect BA at A^1 [Note: Extended BA]

Justification:

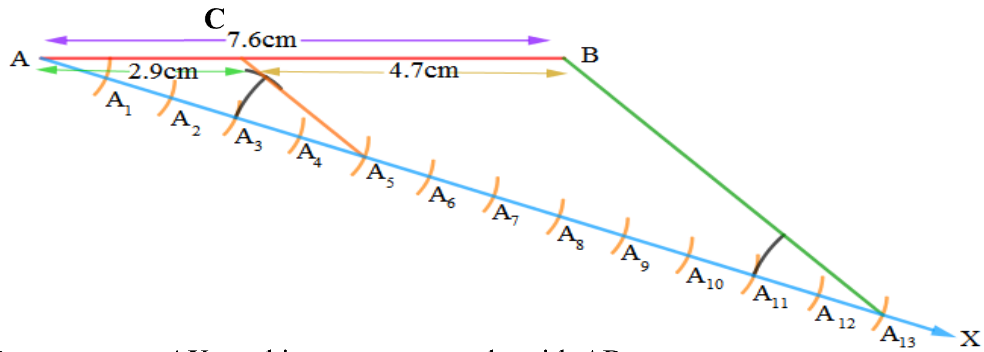
$\Delta ABC \sim \Delta A'BC \Rightarrow \frac{AB}{A^1B} = \frac{AC}{A^1C^1} = \frac{BC}{BC^1}$

But, $\frac{BC}{BC^1} = \frac{BB_3}{BB_5} = \frac{3}{5}$ $\therefore \frac{BC^1}{BC} = \frac{5}{3}$ $\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{5}{3}$

Exercise 6.1

In each of the following, give the justification of the construction also:

1. **Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.**



Step-1: Draw any ray AX, making an acute angle with AB
(Can draw above or below the given line)

Step-2: Locate 13(5+8) points $A_1, A_2,$ ---A_{12}, A_{13}, on AX so that $AA_1 = A_1A_2 = A_2A_3$---$A_{12}A_{13}$

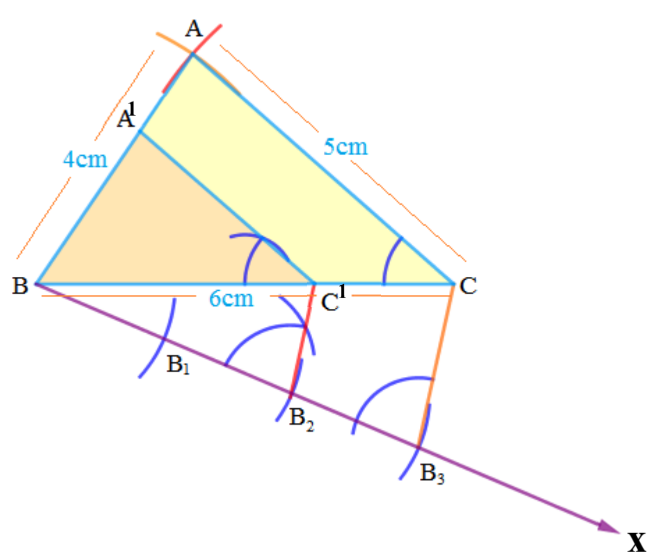
Step-3: Join BA_{13}

Step-4: Through the point A_5 (m = 5), draw a line parallel to A_{13} B (by making an angle equal to AA_{13} B) at A_5 intersecting AB at the point C (see Fig.). Then, AC : CB = 5 : 8

Justification:

$A_5C \parallel A_{13}B \Rightarrow \frac{AA_3}{A_3A_5} = \frac{AC}{CB}$ [BPT] $\Rightarrow \frac{AA_3}{A_3A_5} = \frac{5}{13-5} = \frac{5}{8} \Rightarrow 5:8$

2. **Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.**



Step-1: Construct any ΔABC.Draw any ray BX making an acute angle with BC on the side opposite to the vertex A

Step-2: Locate 3 (the greater of 3 and 4 in $\frac{2}{3}$) points B_1, B_2, B_3 on BX so that $BB_1 = B_1B_2 = B_2B_3$

Step-3: Join B_3C and draw line through to B_3C to intersect BC at C^1

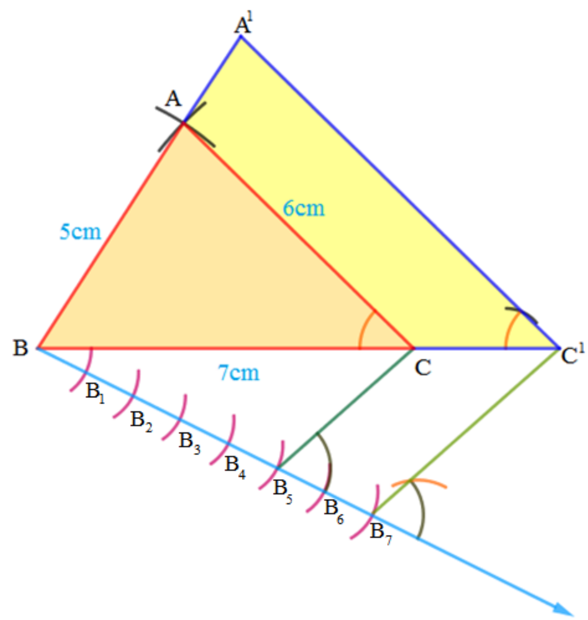
Step – 4: Draw a line through C^1 parallel to the line CA to intersect BA at A^1

Justification:

$\frac{BC^1}{C^1C} = \frac{2}{1}$ $\therefore \frac{BC}{BC^1} = \frac{2+1}{2} = \frac{3}{2} \Rightarrow \frac{BC^1}{BC} = \frac{2}{3}$

$\Rightarrow C^1A^1 \parallel CA$ $\therefore \Delta A^1BC^1 \sim \Delta ABC$

3. **Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose ssides are $\frac{7}{5}$ of the corresponding sides of first.**



Step-1: Construct a triangle BC= 7cm, AB=5cm and AC =6cm

Step-2: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step-3: Locate 7 points (the greater of 7 and 5 in $\frac{7}{5}$) B_1, B_2, B_3, B_4 ---- B_7 on BX such that $B_1B_2 = B_2B_3 = B_3B_4 = ---- = B_6B_7$

Step–4: Join B_5C(the 5th point, 5 being smaller of 5 and 7 in $\frac{7}{5}$) to C and draw a line through B_7 parallel to B_5C intersect BC at C^1 [Note:Extended BC]

Step-5: Draw a line through C^1 parallel to the line CA to intersect BA at A^1[Note: Extended BA]

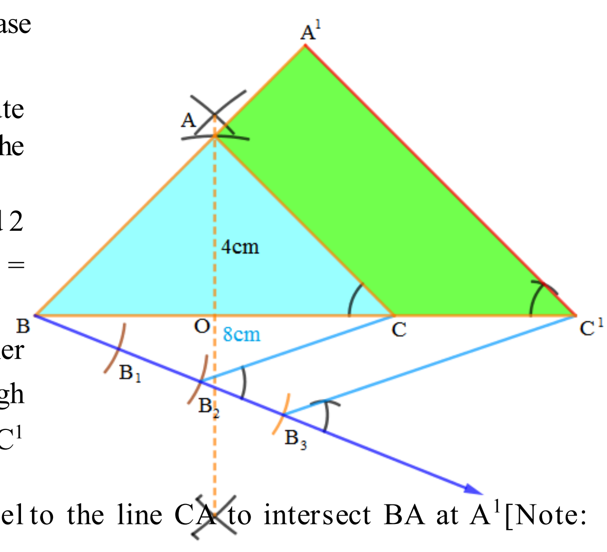
Justification:

$\Delta ABC \sim \Delta A'BC' \Rightarrow \frac{AB}{A^1B} = \frac{AC}{A^1C^1} = \frac{BC}{BC^1}$

But, $\frac{BC}{BC^1} = \frac{BB_3}{BB_5} = \frac{5}{7}$

$\therefore \frac{BC^1}{BC} = \frac{7}{5} \Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{7}{5}$

4. **Construct an isosceles triangle whose base is 4cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.**



Step-1: Construct an Isoceles triangle with base 8cm and altitude 4cm.,

Step-2: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step-3: Locate 3 points (the greater of 3 and 2 in $\frac{3}{2}$) B_1, B_2, B_3 on BX such that $BB_1 = B_1B_2 = B_2B_3$

Step-4: Join B_2(the 5th point, 5 being smaller of 5 and 7 in $\frac{7}{5}$) to C and draw a line through B_3 parallel to B_2C intersect BC at C^1 [Note:Extended BC]

Step-5: Draw a line through C^1 parallel to the line CA to intersect BA at A^1[Note: Extended BA]

Justification:

Δ **ABC** $\sim$ Δ **A′BC** $\Rightarrow \frac{AB}{A^1B} = \frac{AC}{A^1C^1} = \frac{BC}{BC^1}$

But, $\frac{BC}{BC^1} = \frac{BB_3}{BB_5} = \frac{2}{3} \therefore \frac{BC^1}{BC} = \frac{3}{2} \Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{2}$

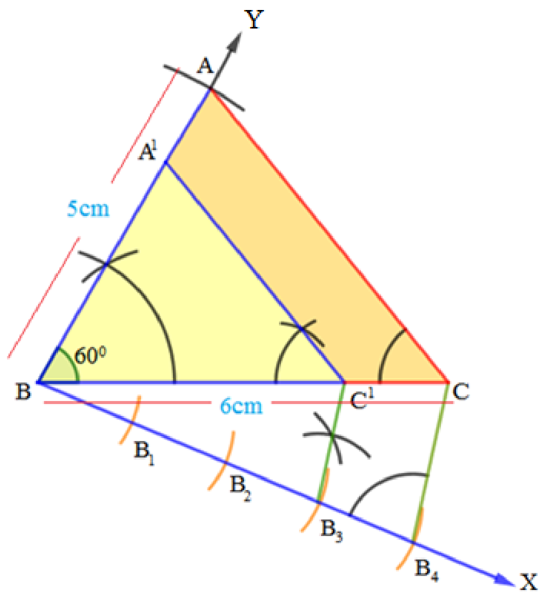
5. Draw a triangle ABC with side BC = 6cm, AB = 5cm, and $\angle ABC = 60^0$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Step-1: Construct a ΔABC with side BC = 6cm, AB = 5cm and $\angle ABC = 60^0$

Step-2: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step-3: Locate 4 points (the greater of 4 and 3 in $\frac{3}{4}$) B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

Step-4: Join B_4(the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) to C and draw a line through B_3 parallel to B_4C intersect BC at C^1



Justification:

$\frac{BC^1}{C^1C} = \frac{3}{1} \quad \therefore \frac{BC}{BC^1} = \frac{3+1}{3} = \frac{4}{3} \Rightarrow \frac{BC^1}{BC} = \frac{3}{4}$

$C^1A^1 \parallel CA \quad \therefore \Delta A^1BC^1 \sim \Delta ABC$

$\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{4}$

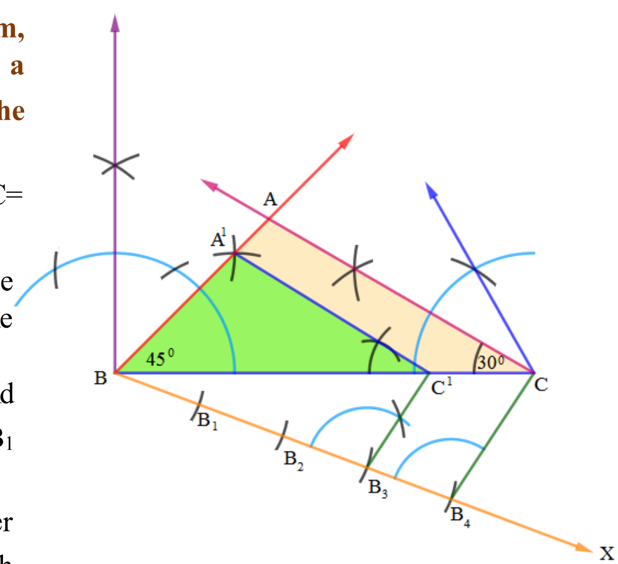
6. Draw a triangle ABC with side BC = 7cm, $\angle B = 45^0$, $\angle A = 105^0$ then construct a triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

Step-1: Construct a ΔABC with sideb BC= 7cm, $\angle B = 45^0$, $\angle A = 105^0$

Step -2: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step-3: Locate 4 points (the greater of 4 and 3 in $\frac{3}{4}$) B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

Step-4: Join B_4(the 4th point, 4 being greater of 3 and 4 in $\frac{3}{4}$) to C and draw a line through B_3 parallel to B_4C intersect BC at C^1



Step-5: Draw a line through C^1 parallel to the line CA to intersect BA at A^1

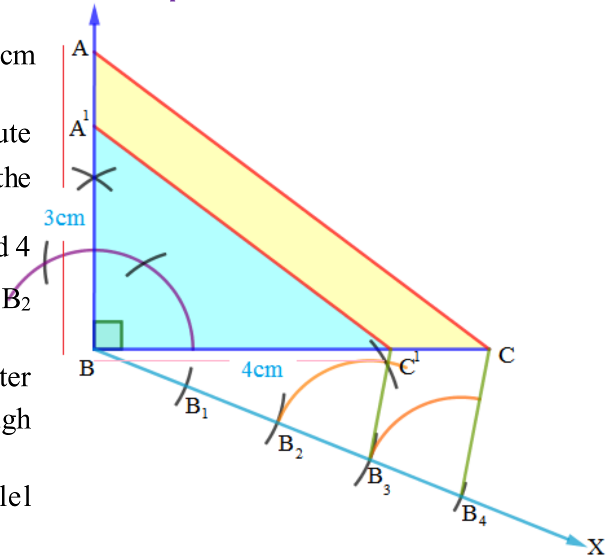
Justification:

$\frac{BC^1}{C^1C} = \frac{3}{1} \therefore \frac{BC}{BC^1} = \frac{3+1}{3} = \frac{4}{3} \Rightarrow \frac{BC^1}{BC} = \frac{3}{4}$

$C^1A^1 \parallel CA \quad \therefore \Delta A^1BC^1 \sim \Delta ABC$

$\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{4}$

7. Draw a right triangle in which the sides(other than hypotenuse) are lengths 4cm and 3cm. Then construct another triangle whose sides are $\frac{3}{4}$ times corresponding sides of the given triangle.



Step-1: Construct right with sides 4cm and 3cm (exept hypotenuse)

Step-2: Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

Step-3: Locate 4 points (the greater of 3 and 4 in $\frac{3}{4}$) B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

Step-4: Join B_4(the 4th point, 4 being greater of 4 and 3 in $\frac{3}{4}$) to C and draw a line through B_3 parallel to B_4C intersect BC at C^1

Step-4: Draw a line through C^1 parallel to the line CA to intersect BA at A^1

Justification:

$\frac{BC^1}{C^1C} = \frac{3}{1} \therefore \frac{BC}{BC^1} = \frac{3+1}{3} = \frac{4}{3} \Rightarrow \frac{BC^1}{BC} = \frac{3}{4}$

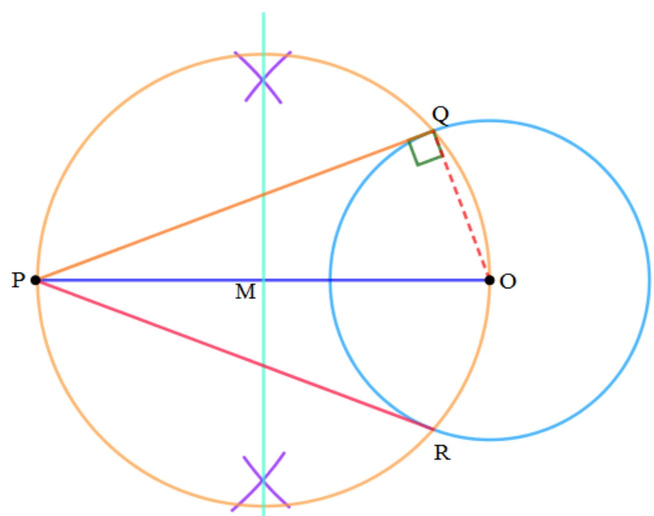
$C^1A^1 \parallel CA \quad \therefore \Delta A^1BC^1 \sim \Delta ABC$

$\Rightarrow \frac{A^1B}{AB} = \frac{BC^1}{BC} = \frac{A^1C^1}{AC} = \frac{3}{4}$

6.3 Construction of Tangents to a Circle

To construct the tangents to a circle from a point outside it

We are given a circle with centre O and a point P outside it. We have to construct the two tangents from P to the circle.



Step 1: Join PO and bisect it. Let M be the mid- point of PO

Step 2: Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R.

Step 3: Join PQ and PR

Then PQ and PR are the required two tangents

Justification:

Join OQ, $\angle PQO$ is an angle in semi circle

$\therefore \angle PQO = 90^0 \Rightarrow PQ \perp OQ$, OQ is the radius of given circle. Therefore PQ is the tangent to the circle.

Similarly PR also the tangent to the circle.

Exercise 6.2

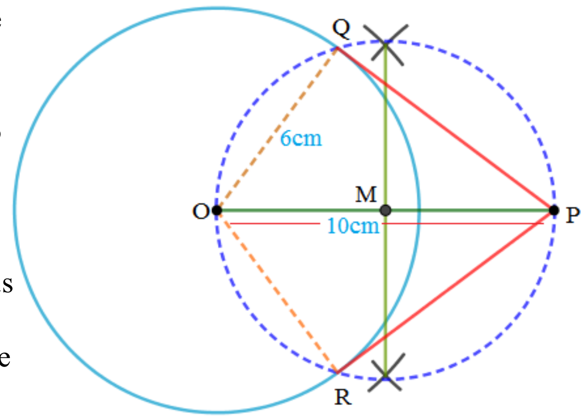
In each of the following, give also the justification of the construction:

1. **Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.**

Step-1: Draw PO = 10cm. Join PO and bisect it. Let M be the mid- point of PO

Step-2: Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q and R

Step-3: Join PQ and PR measure the length Then PQ and PR are the required two tangents



Justfication:

Join O, Q $\angle PQO$ is an angle in semi circle

$\therefore \angle PQO = 90^0 \Rightarrow PQ \perp OQ$, OQ is the radius of given circle.Therefore PQ is the tangent to the circle. Similarly PR also the tangent to the circle.

Δ PQO is a right angle triangle

$OP^2 = OQ^2 + PQ^2$ [Pythagoras theorem]

$\Rightarrow PQ^2 = 10^2 - 6^2$

$\Rightarrow PQ^2 = 64 \Rightarrow PQ = 8$ cm

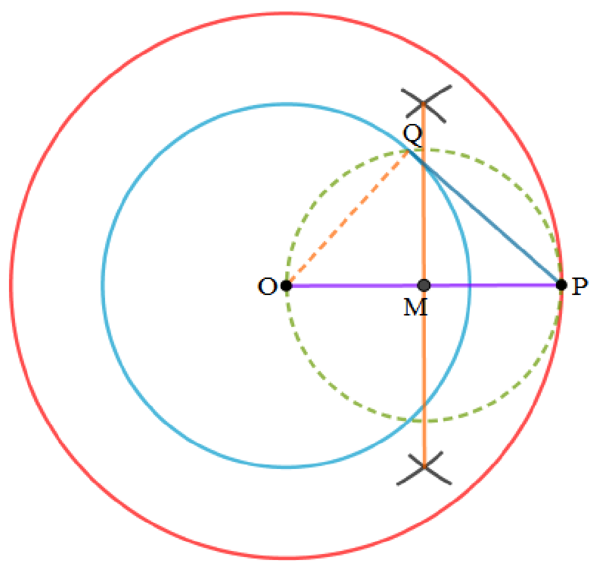
2. **Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.**

Step-1: Draw two concentric circles at O with radius 4cm and 6cm. Locate a point P on greater circle and join PO. Bisect it. Let M be the mid- point of PO

Step-2: Taking M as centre and MO as radius, draw a circle. Let it intersect the given circle at the points Q

Step-3: Join PQ and mesure the length. It is $\approx$4.5cm

Then PQ a$\approx$4.5cm is the required tangent



Justification:

Join O, Q $\angle PQO$ is an angle in semi circle

$\therefore \angle PQO = 90^0 \Rightarrow PQ \perp OQ$, OQ is the radius of given circle.Therefore PQ is the tangent to the circle.

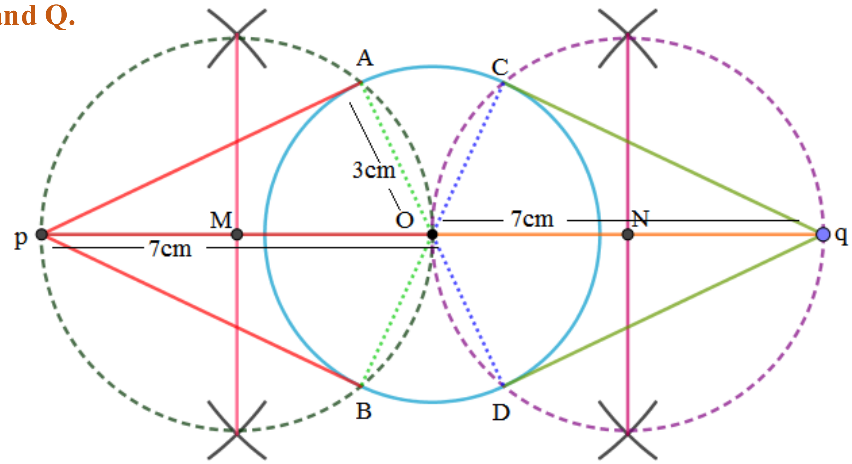
Δ PQO is a right angle triangle

$OP^2 = OQ^2 + PQ^2$ [Pythagoras theorem]

$\Rightarrow PQ^2 = 6^2 - 4^2$

$\Rightarrow PQ^2 = 20 \Rightarrow PQ = 4.47$ cm

3. **Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.**

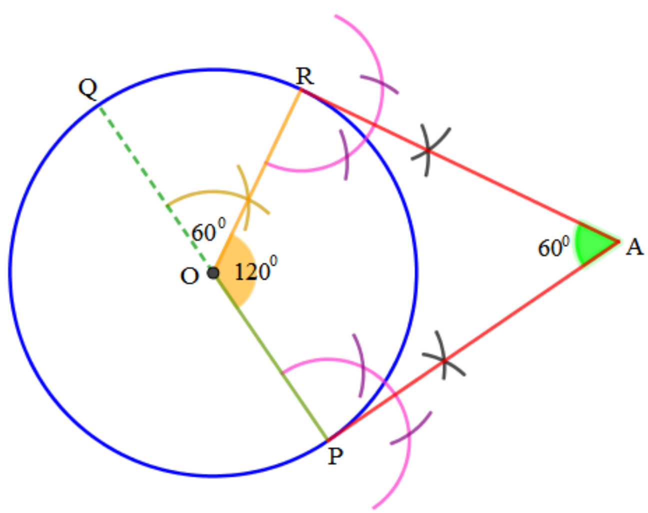


Step-1: Construct a circle of radius 3cm at O. Draw a diameter and extend the diameter 7cm on both sides. Name the end point as p and q. Draw perpendicular bisector of Op and Oq and they intersect at M and N.
Step-2: From centers M and N draw circles of radius MO and NO. These circles intersect the given circle at A,B and C,D.
Step-3: Join p to A and B and q to C and D.
Now, pA,pB, qC and qD are the required tangents.
Justification:
Join p, A ∠pAO is angle on semi circle.
$\therefore \angle pAO = 90^0 \Rightarrow pA \perp OA$
OA is the radius of the given circle. So, pA has to be a tangent.
Similarly pB, qC, qD are the tangents.

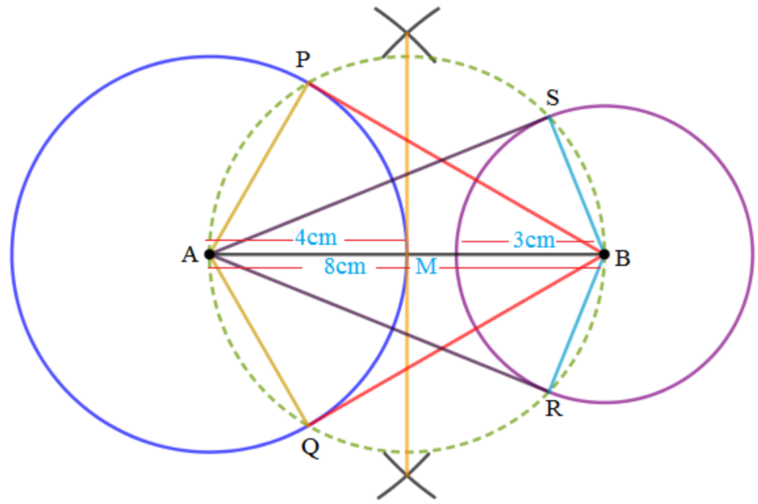
4. **Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.**

Step-1: Construct angle 120^0 [180^0- 60^0] at the center.For this draw a diameter and construct $\angle ROQ = 60^0$. Then we get $\angle ROP = 120^0$ at the center.
Step-2: Draw two perpendiculars at P and R and the intersect at A.
Now, AP and AR are the required tangents

Justification:
$AR \perp OR$ [Construction] $\Rightarrow \angle ARO = 90^0$
And OR is the radius at point of contact.
$\therefore$ AR is the tangent. Similarly AP is also the tangent to the circle.



5. **Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.**



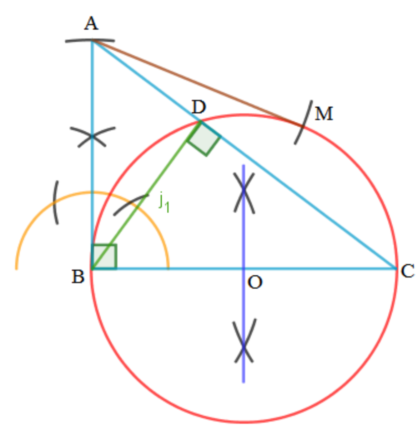
Step-1: Draw AB = 8cm. Draw circles of radii 4cm and 3cm at A and B.
Step-2: Draw perpendicular bisector to AB, it intersect AB at M
Step-3: Draw a circle with center M and passes through A and B. It intersects a circle of radius 4cm at P,Q and the circles of radius 3cm at S,R.
Step-4: Join BP, BQ,AS and AR. These are required tangents.

Justification:

Join AP and BS
$\angle APB$ and $\angle ASB$ are the angles on semi-circles
$\therefore \angle APB = 90^0$ and $\angle ASB = 90^0$

Since, AP and BS are the radius, BP and AS have to be the tangents to the circle.
Similarlly BQ and AR are also the tangents.

6. **Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and B = 90°. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.**



Step-1:Draw the line segments BC= 8cm.and AB =6cm perpendicular to each other. Join AC. Thus, ΔABC is a right angle triangle.
Step-2: Draw perpendicular to BC which meets BC at O
Step-3: With O as center and OB as radius draw a circle which intersects AC at D then $\angle BDC = 90^0$. Thus BD is perpendicular to AC.
Step-4: With a as center and AB as radius draw an arc cutting the circle at M
Step-5: Join AM. Thus AB and AM are required tangents.

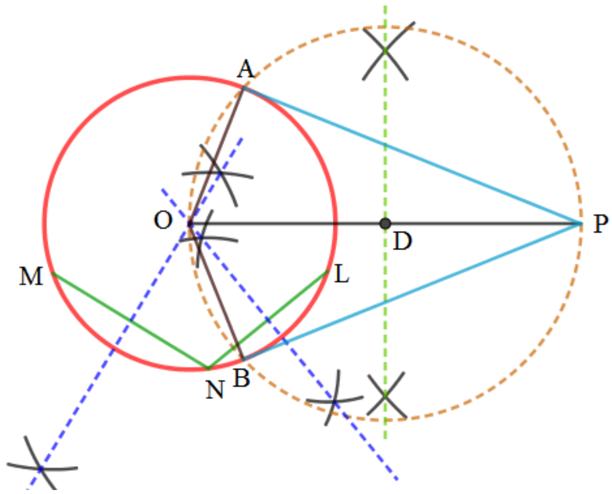
Justification:

In ΔABC, $\angle ABC = 90^0$

$\Rightarrow \Delta ABC$ is right angle triangle.
$\Rightarrow AB \perp BO$ and BO is the radius. So , AB has to be a tangent to the circle.
AB = AM [Construction]
$\therefore$ AM is also a tangent.

7. **Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.**

Step-1: Draw a circle by using a bangle.
Step-2: Draw two chords MN and NL
Step-3: Draw perpendicular bisector of MN and NL they intersect at point O which is the center of the circle.
Step-4: Take a point P outside the circle join OP and bisect it.
Step-5: Let D be the midpoint of OP. Taking D as center and OD as radius, draw a circle which intersects the given circle at A and B.
Step-6: Join AP and BP. Thus AP and BP are the required tangents from P



Justification:
Join OA,OB. $\angle OAP$ is the angle lying in the semi-circle.
$\therefore \angle OAP = 90^0 \Rightarrow AP \perp OA$,
Since, OA is the radius of the circle with center O. So AP has to be a tangent to the circle.
Similarlly BP is also a tangent.

Summary

In this chapter, you have learnt how to do the following constructions:

1. **To divide a line segment in a given ratio**
2. **To construct a triangle similar to a given triangle as per a given scale factor which may be less than 1 or greater than 1.**
3. **To construct the pair of tangents from an external point to a circle.**

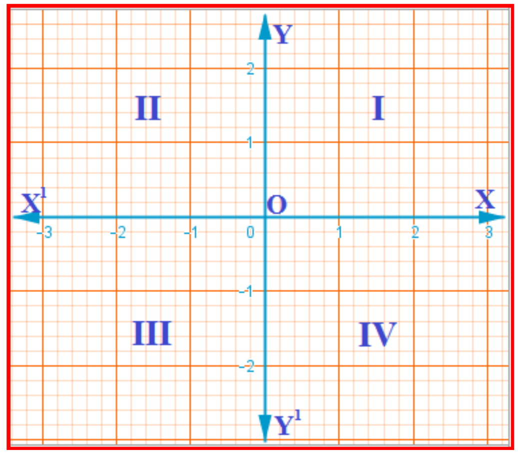
7 Coordinate Geometry

Coordinate axes:

A set of a pair of perpendicular axes $X^{|}OX$ and $YOY^{|}$

Horizontal line: $X^{1}OX$, X - Axis

Vertical line: YOY^{1}, Y - Axis



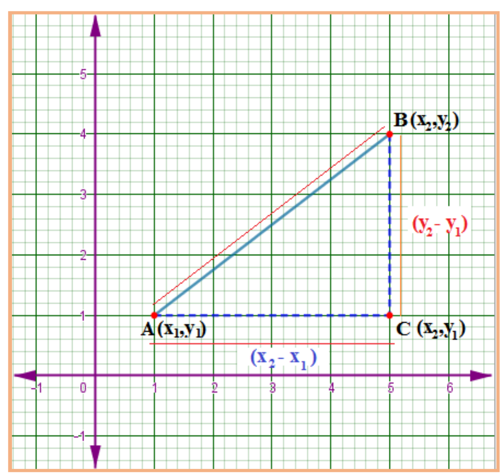
The intersection point of X and Y axes is called the Orgin 'O'

The distance of a point from the y-axis is called its x-coordinate, or abscissa. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The coordinates of a point on the x-axis are of the form (x, 0), and of a point on the y-axis are of the form (0, y).

The Coordinate axes divides the plane in to four parts. They are called quadrants.

The coordinaes of the orgin is (0, 0)

7.2 Distance Formula



The distance between two points on X-axis or on the straight line paralle to X-axis is

Distance $= x_2 - x_1$

The distance between two points on Y-axis or on the straight line paralle to Y-axis is

Distance $= y_2 - y_1$

$AB^2 = AC^2 + BC^2$

The distance between two points which are neither on X or Y axis nor on the line paralle to X orY axis

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

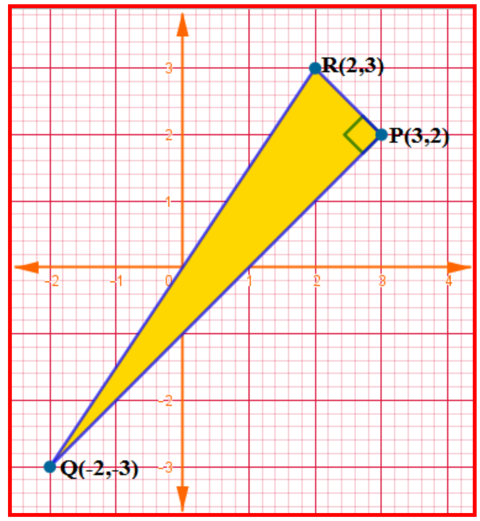
The distance between the point P(x,y) and the orgin

$$d = \sqrt{x^2 + y^2}$$

Example 1:Do the points (3, 2), (–2, –3) and (2, 3) form a triangle? If so, name the type of triangle formed.

$P(3,2),\quad Q(-2,-3),\quad R(2,3)$

Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$PQ = \sqrt{(3-(-2))^2+(2-(-3))^2}$

$= \sqrt{(3+2)^2+(2+3)^2}$

$= \sqrt{(5)^2+(5)^2} = \sqrt{25+25} = \sqrt{50} = 7.07$

$QR = \sqrt{(-2-2)^2+(-3-3)^2}$

$= \sqrt{(-4)^2+(-6)^2} = \sqrt{16+36} = \sqrt{52} = 7.21$

$PR = \sqrt{(3-2)^2+(2-3)^2} = \sqrt{(1)^2+(-1)^2}$

$= \sqrt{1+1} = \sqrt{2} = 1.41$

Since the sum of any two of these distances is greater than the third distance, therefore the point P,Q and R form a triangle.

Also, $PQ^2 + PR^2 = QR^2$ by the converse of Pythagoras theorem $\angle P = 90^O$ we have

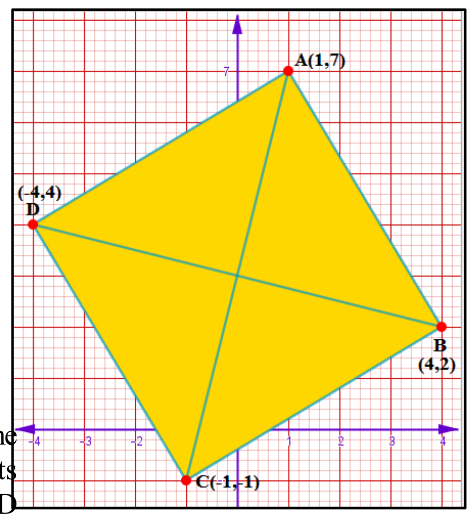
Therefore, PQR is a right triangle.

Example2: Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

Solution: A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)

$AB = \sqrt{(4-1)^2+(2-7)^2} = \sqrt{(3)^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(-1-4)^2+(-1-2)^2} = \sqrt{(-5)^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$



$CD = \sqrt{(-4-(-1))^2+(4-(-1))^2}$

$= \sqrt{(-4+1)^2+(4+1)^2}$

$= \sqrt{(-3)^2+(5)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(1-(-4))^2+(7-4)^2}$

$= \sqrt{(1+4)^2+(3)^2} = \sqrt{(5)^2+(3)^2} = \sqrt{25+9} = \sqrt{34}$

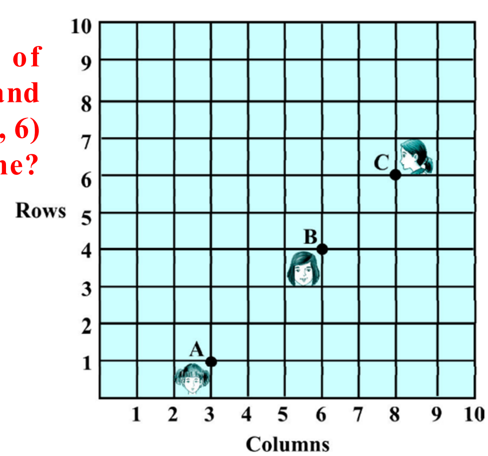
$AC = \sqrt{(-1-1)^2+(-1-7)^2}$

$= \sqrt{(-2)^2+(-8)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(-4-4)^2+(4-2)^2}$

$= \sqrt{(-8)^2+(2)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Thereore, ABCD is a square



Example 3 : Fig. 7.6 shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at A (3, 1), B(6, 4) and C(8, 6) respectively. Do you think they are seated in a line? Give reasons for your answer.

$AB = \sqrt{(6-3)^2+(4-1)^2} = \sqrt{(3)^2+(3)^2}$

$= \sqrt{9+9} = \sqrt{18} = \sqrt{9\times 2} = \mathbf{3\sqrt{2}}$

$BC = \sqrt{(8-6)^2+(6-4)^2} = \sqrt{(2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = \mathbf{2\sqrt{2}}$

$AC = \sqrt{(8-3)^2+(6-1)^2} = \sqrt{(5)^2+(5)^2}$

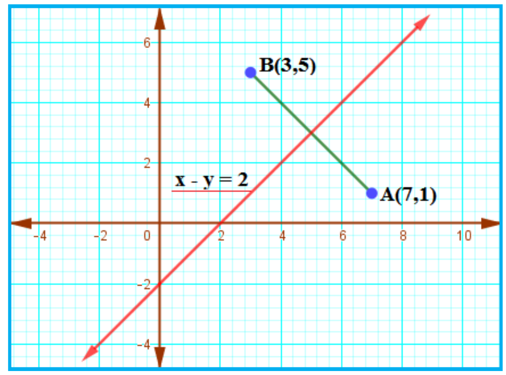
$= \sqrt{25+25} = \sqrt{50} = \sqrt{25\times 2} = \mathbf{5\sqrt{2}}$

$AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

Since,, **AB + BC = AC** we can say that the points A, B and C are collinear.

Therefore, they are seated in a line

Example 4 : Find a relation between x and y such that the point (x , y) is equidistant from the points (7, 1) and (3, 5).



Let the point P (x, y) is equi distance from the points A (7, 1) and B (3, 5)

$PA = PB \Rightarrow PA^2 = PB^2$

$PA = \sqrt{(x-7)^2+(y-1)^2}$

$PB = \sqrt{(x-3)^2+(y-5)^2}$

$AP^2 = BP^2 \Rightarrow \left(\sqrt{(x-7)^2+(y-1)^2}\right)^2$

$= \left(\sqrt{(x-3)^2+(y-5)^2}\right)^2$

$(x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

$x^2 + 7^2 - 2(x)(7) + y^2 + 1^2 - 2(y)(1) = x^2 + 3^2 - 2(x)(3) + y^2 + 5^2 - 2(y)(5)$

$x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

$x^2 - x^2 - 14x + 6x + y^2 - y^2 - 2y + 10y = 34 - 50$

$-8x + 8y = -16 \quad \div -8 \quad \Rightarrow x - y = 2$

Which is the require relation.

Remark : Note that the graph of the equation $x - y = 2$ is a line. From your earlier studies, you know that a point which is equidistant from A and B lies on the perpendicular bisector of AB. Therefore, the graph of $x - y = 2$ is the perpendicular bisector of AB

Example 5 : Find a point on the y-axis which is equidistant from the points A(6, 5) and B(– 4, 3).

We know that a point on the $y-$axis is of the form $(0, y)$. $P\ (0, y)$ So, let the point P(0, y) be equidistant from A and B. Then PA = PB

$(6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$

$36 + 5^2 + y^2 - 2(5)(y) = 16 + 3^2 + y^2 - 2(3)(y)$

$36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y$

$y^2 - y^2 - 10y + 6y = 25 - 61 \Rightarrow -4y = -36$

$y = \frac{-36}{-4} = 9$ **Therefore the required point is** $(\mathbf{0}, \mathbf{9})$

$PA = \sqrt{(6-0)^2+(5-9)^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52}$

$PB = \sqrt{(-4-0)^2+(3-9)^2} = \sqrt{(-4)^2+(-6)^2} = \sqrt{16+36} = \sqrt{52}$

Exercise 7.1

1. **Find the distance between the following pairs of points :**

i) (2, 3), (4, 1) ii) (-5, 7), (-1, 3) iii) (a, b), (-a, -b)

i) $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (4, 1)$

x_1	y_1	x_2	y_2
2	3	4	1

Formula $\mathbf{d = \sqrt{(x_2 - x_1)^2+(y_2 - y_1)^2}}$

$d = \sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2}$

$d = \sqrt{4+4} = \sqrt{2 \times 4} = 2\sqrt{2}$ Units

ii) $(x_1, y_1) = (-5, 7)$, $(x_2, y_2) = (-1, 3)$

x_1	y_1	x_2	y_2
-5	7	-1	3

$d = \sqrt{(-1-[-5)^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2}$

$d = \sqrt{16+16} = \sqrt{2 \times 16} = 4\sqrt{2}$ Units

iii) $(x_1, y_1) = (a, b)$, $(x_2, y_2) = (-a, -b)$

$d=\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2}$

$d=\sqrt{4a^2+4b^2}=\sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$ Units

x_1	y_1	x_2	y_2
a	b	-a	-b

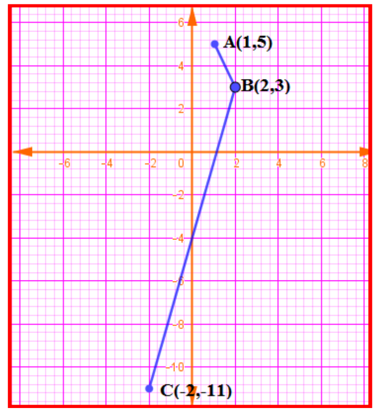
2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

$(x, y) = (36, 15)$

$d=\sqrt{x^2+y^2} = \sqrt{36^2+15^2}=\sqrt{1296+225}=\sqrt{1521} = \mathbf{39}$ **Units**

We can find the distance between the two towns A and B.Suppose town A is at the Orgin, then the town has to be at (36,15). The distance between these two towns is 39km **(1, 5),**

3. Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.



A (1, 5), B (2, 3) ಮತ್ತು C (-2, -11)

$AB=\sqrt{(2-1)^2+(3-5)^2}=\sqrt{(1)^2+(-2)^2}$

$=\sqrt{1+4}=\sqrt{5}$

$BC=\sqrt{(-2-2)^2+(-11-3)^2}$

$=\sqrt{(-4)^2+(-14)^2}$

$=\sqrt{16+196}=\sqrt{212}$

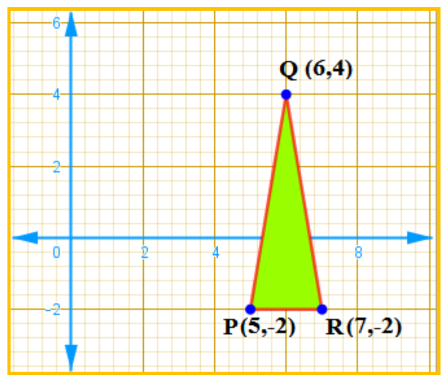
$AC=\sqrt{(-2-1)^2+(-11-5)^2}$

$=\sqrt{(-3)^2+(-16)^2}=\sqrt{9+256}=\sqrt{265}$

$AB+BC\neq AC$

∴ **These are non-collinear points**

4. Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.



Formula $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$PQ=\sqrt{(6-5)^2+(4-(-2))^2}$

$=\sqrt{(1)^2+(6)^2}=\sqrt{1+36}=\sqrt{37}$ (i)

$QR=\sqrt{(7-6)^2+(-2-4)^2}$

$=\sqrt{(1)^2+(-6)^2}=\sqrt{1+36}=\sqrt{37}$ (ii)

$PR=\sqrt{(7-5)^2+(-2-[-2])^2}$

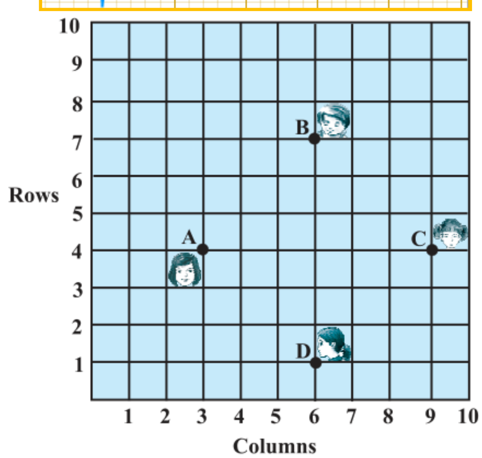
$=\sqrt{(2)^2+(0)^2}=\sqrt{4}=2$ (iii)

(i), (ii), (iii) ⇒ PQ = QR,

Since,Two sides of the tringle are equal.

Hence, ΔPQR is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



The coordinates of the points A,B,C and D are,

A(3,4) , B(6,7) , C(9,4), D(6,1)

$AB=\sqrt{(6-3)^2+(7-4)^2} = \sqrt{(3)^2+(3)^2}$

$=\sqrt{9+8}=\sqrt{18}=3\sqrt{2}$ **(i)**

$BC=\sqrt{(9-6)^2+(4-7)^2} = \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(ii)**

$\text{CD} = \sqrt{(6-9)^2+(1-4)^2} = \sqrt{(-3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(iii)**

$\text{DA} = \sqrt{(6-3)^2+(1-4)^2} = \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(iv)**

$\text{AB} = \text{BC} = \text{CD} = \text{DA}$

Diagonal $\text{AC} = \sqrt{(9-3)^2+(4-4)^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36} = 6$ **(v)**

Diagonal $\text{BD} = \sqrt{(6-6])^2+(7-1)^2} = \sqrt{(0)^2+(6)^2} = \sqrt{36} = 6$ **(vi)**

$\text{AC} = \text{BD}$

Thus, $\text{AB} = \text{BC} = \text{CD} = \text{DA}$, diagonals: $\text{AC} = \text{DB}$

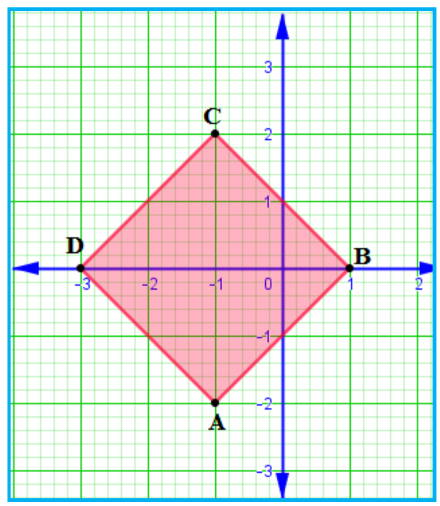
Since all the four sides and diagonals are equal.

Hence, ABCD is a square. So, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

i) (-1, -2), (1, 0), (-1, 2), (-3, 0) ii) (-3, 5), (3, 1), (0, 3), (-1, -4) iii) (4, 5), (7, 6), (4, 3), (1, 2)

i) $\mathbf{A(-1,-2)}$, $\mathbf{B(1,0)}$, $\mathbf{C(-1,2)}$, $\mathbf{D(-3,0)}$



$\text{AB} = \sqrt{\left(1-(-1)\right)^2+\left(0-(-2)\right)^2}$

$= \sqrt{(1+1)^2+(0+2)^2} = \sqrt{(2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$\text{BC} = \sqrt{(-1-1)^2+(2-0)^2} = \sqrt{(-2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$\text{CD} = \sqrt{\left(-3-(-1)\right)^2+(0-2)^2} = \sqrt{(-3+1)^2+(-2)^2}$

$= \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$\text{DA} = \sqrt{\left(-3-(-1)\right)^2+\left(0-(-2)\right)^2}$

$= \sqrt{(-3+1)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$\text{AB} = \text{BC} = \text{CD} = \text{DA}$

$\text{AC} = \sqrt{\left(-1-(-1)\right)^2+\left(2-(-2)\right)^2} = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{(0)^2+(4)^2} = \sqrt{16} = 4$

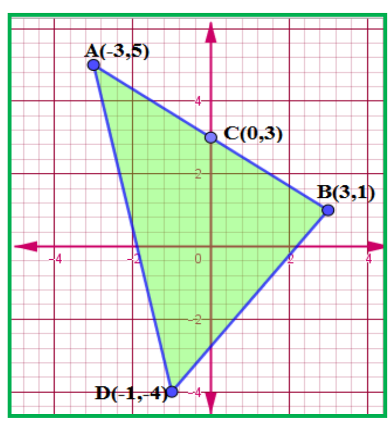
$\text{BD} = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16} = 4$

Thus, $\text{AC} = \text{BD}$

Since, the four sides AB, BC, CD and DA are equal and the diagonals $\text{AC} = \text{DB}$ are equal.

So the quadrilateral ABCD is a square.

ii) A(-3, 5), B(3, 1), C(0, 3), D(-1, -4)



$\text{AB} = \sqrt{\left(3-(-3)\right)^2+\left(1-(-3)\right)^2}$

$= \sqrt{(3+3)^2+(1+3)^2} = \sqrt{(6)^2+(4)^2}$

$= \sqrt{36+16} = \sqrt{52}$

$\text{BC} = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{(-3)^2+(2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\text{CD} = \sqrt{(-1-0)^2+(-4-3)^2}$

$= \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49}$

$= \sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$

$\text{DA} = \sqrt{\left(-3-(-1)\right)^2+(-4-5)^2}$

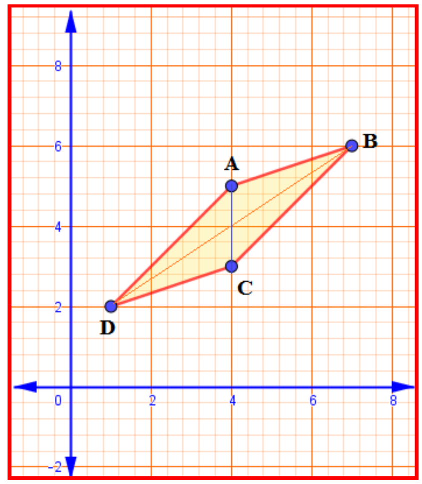
$= \sqrt{(-3+1)^2+(-9)^2} = \sqrt{(-2)^2+(-9)^2}$

$= \sqrt{4+81} = \sqrt{85}$

$\text{AB} \neq \text{BC} \neq \text{CD} \neq \text{DA}$

Since, the four sides AB, BC, CD and DA are not equal.Hence these poist does not form a quadrilateral.

iii) A(4, 5), B(7, 6), C(4, 3), D(1, 2)



Formula: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\text{AB} = \sqrt{(7-4)^2+(6-5)^2}$

$= \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

$\text{BC} = \sqrt{(4-7)^2+(3-6)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$

$\text{CD} = \sqrt{(1-4)^2+(2-3)^2}$

$= \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

$\text{DA} = \sqrt{(1-4)^2+(2-5)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$

$\text{AB} = \text{CD}, \quad \text{BC} = \text{DA}$

$\text{AC} = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

$\text{BD} = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = \sqrt{4\times 13} = 2\sqrt{13}$

$\text{AC} \neq \text{DB}$

Thus opposite sides are equal. $\text{AB} = \text{CD}$, & $\text{BC} = \text{DA}$

But diagonals are not equal. $\text{AC} \neq \text{DB}$ ∴ The given points are forming a parallelogram.

7. **Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).**

We know that a point on the X – axis is of the form (x, 0).

Let the point P (x, 0) is equi distance from the points A(2, -5) and B(-2, 9)

AP = BP

$(x-2)^2 + (0-(-5))^2 = (x-(-2))^2 + (0-9)^2$

$(x-2)^2 + 5^2 = (x+2)^2 + (-9)^2$

$x^2 + 2^2 - 2(x)(2) + 25 = x^2 + 2^2 + 2(x)(2) + 81$

$-4x + 25 = 4x + 81$

$-4x - 4x = 81 - 25 \Rightarrow -8x = 56 \Rightarrow x = \frac{56}{-8} = -7$

Thus, the required point is $(-7, 0)$

8. **Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.**

$(x_1, y_1) = (2, -3), \quad (x_2, y_2) = (10, y), \quad d = 10$

x_1	y_1	x_2	y_2
2	-3	10	y

Formula: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$10 = \sqrt{(10-2)^2+(y-(-3))^2} = \sqrt{(8)^2+(y+3)^2}$

$10^2 = 64 + (y+3)^2 \Rightarrow 100 - 64 = (y+3)^2$

$\Rightarrow (y+3)^2 = 36 \Rightarrow y+3 = \pm\sqrt{36} \Rightarrow y+3 = \pm 6$

$\Rightarrow y = 6 - 3 = 3 \quad \text{or} \quad x = -6 - 3 = -9$

9. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.

The point Q (0, 1) is equi distance from the points P (5, -3) and R (x, 6)

$PQ = QR \Rightarrow PQ^2 = PR^2$

$PQ = \sqrt{(5-0)^2+(-3-1)^2} = \sqrt{(5)^2+(-4)^2} = \sqrt{25+16} = \sqrt{41}$

$QR = \sqrt{(x-0)^2+(6-1)^2} = \sqrt{(x)^2+(5)^2} = \sqrt{x^2+25}$

$PQ^2 = PR^2 \Rightarrow \left(\sqrt{x^2+25}\right)^2 = \left(\sqrt{41}\right)^2$

$x^2+25 = 41 \Rightarrow x^2 = 41-25 \Rightarrow x^2 = 16 \Rightarrow x = \pm\sqrt{16} \Rightarrow x = \pm 4$

The coordinate of the point R is $(4,6)$ or $(-4,6)$

If the coordinates of R is $(4,6)$ then,

$QR = \sqrt{(4-0)^2+(6-1)^2} = \sqrt{(4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(4-5)^2+(6-(-3))^2} = \sqrt{(-1)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$

If the coordinates of R is $(-4,6)$ then,

$QR = \sqrt{(-4-0)^2+(6-1)^2} = \sqrt{(-4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(-4-5)^2+(6-(-3))^2} = \sqrt{(-9)^2+(6+3)^2} = \sqrt{81+81} = \sqrt{81\times 2} = 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).

The point P (x, y) is equidistance from the points A (3, 6) and B (-3, 4).

$PA = PB \Rightarrow PA^2 = PB^2$

$PA = \sqrt{(x-3)^2+(y-6)^2}$

$PB = \sqrt{(x-(-3))^2+(y-4)^2}$

$AP^2 = BP^2 \Rightarrow \left(\sqrt{(x-3)^2+(y-6)^2}\right)^2 = \left(\sqrt{(x-(-3))^2+(y-4)^2}\right)^2$

$(x-3)^2+(y-6)^2 = (x+3)^2+(y-4)^2$

$x^2+3^2-2(x)(3)+y^2+6^2-2(y)(6) = x^2+3^2+2(x)(3)+y^2+4^2-2(y)(4)$

$x^2+9-6x+y^2+36-12y = x^2+9+6x+y^2+16-8y$

$x^2-x^2-6x-6x+y^2-y^2-12y+8y = 25-45$

$-12x-4y = -20 \quad \div -4$

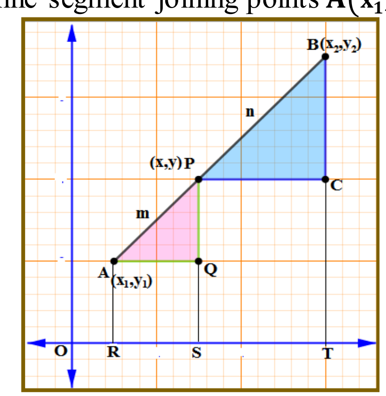
$3x+y-5=0$ This is the required relation

$3x+y-5=0$ is representing a straight line

Thus the point equidistance from the point A and B on the perpendicular bisector of AB

7.3 Section Formula

The coordinates of the point P(x, y) which divides the line segment joining points $\mathbf{A}(x_1, y_1)$ and $\mathbf{B}(x_2, y_2)$, internally, in the ratio $m_1 : m_2$ are



$$\mathbf{P(x,y)} = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

The mid-point of a line segment divides the line segment in the ratio 1: 1. Then the coordinates of the midpoint of the line segment,

$$\mathbf{P(x,y)} = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Example 6 : Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

x_1	y_1	x_2	y_2
4	-3	8	5

$(x_1, y_1) = (4, -3), \ (x_2, y_2) = (8,5), m_1 : m_2 = 3:1$

$x = \frac{m_1 x_2 + m_2 \ x_1}{m_1+m_2} = \frac{3(8)+1(4)}{3+1} = \frac{24+4}{4} = \frac{28}{4} = 7$

$y = \frac{m_1 y_2 + m_2 \ y_1}{m_1+m_2} = \frac{3(5)+1(-3)}{3+1} = \frac{15-3}{4} = \frac{12}{4} = 3$

Therefore the required point is $(7,3)$

Example 7 : In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

$P(x, y) = (-4, 6), \ A(x_1, \ y_1) = (-6, 10), B(x_2, \ y_2) = (3, -8), m_1 = ?, m_2 = ?$

$(x, y) = \left(\frac{m_1 x_2 + m_2 \ x_1}{m_1+m_2}, \ \frac{m_1 y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$(-4,6) = \left(\frac{m_1(3) + m_2 \ (-6)}{m_1+m_2}, \ \frac{m_1(-8) + m_2 \ (10)}{m_1+m_2}\right)$

$-4 = \frac{3m_1 - 6 \ m_2}{m_1+m_2}$ Or $\quad 6 = \frac{-8m_1 + 10 \ m_2}{m_1+m_2}$

$-4m_1 - 4m_2 = 3m_1 - 6 \ m_2$

$-4m_1 - 3m_1 = -6 \ m_2 + 4m_2$

$-7m_1 = -2m_2$

$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7} \Rightarrow m_1 : m_2 = 2:7$

We should verify that the ratio satisfies the y-coordinate also.

$\frac{-8m_1+10 \ m_2}{m_1+m_2} = \frac{-8(2)+10 \ (7)}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7

Example: Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).

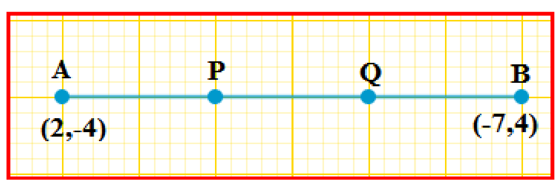
Let P and Q be the trisection points of AB. ⇒ AP = PQ = QB

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula,

$A(x_1, y_1) = (2, -2), B(x_2, y_2) = (-7, 4)$

$m_1 = 1, \quad m_2 = 2$

$P(x, y) = \left(\frac{m_1 x_2 + m_2 \ x_1}{m_1+m_2}, \ \frac{m_1 y_2 + m_2 \ y_1}{m_1+m_2}\right)$



$= \left(\frac{1(-7)+2(2)}{1+2}, \ \frac{1(4)+2(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$

$= \left(\frac{-3}{3}, \ \frac{0}{3}\right) = (-1, 0)$

Now, Q also divides AB internally in the ratio 2 : 1. So, the coordinates of Q are

$A(x_1, y_1) = (2, -2), B(x_2, y_2) = (-7, 4)$

$m_1 = 2, \quad m_2 = 1$

$Q(x, y) = \left(\frac{m_1 x_2 + m_2 \ x_1}{m_1+m_2}, \ \frac{m_1 y_2 + m_2 \ y_1}{m_1+m_2}\right)$

$= \left(\frac{2(-7)+1(2)}{2+1}, \ \frac{2(4)+1(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \ \frac{6}{3}\right) = (-4, \ \ 2)$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are (–1, 0) and (– 4, 2).

Example 9 : Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.

We know that a point on the Y − axis is of the form $(0, y)$. Let the ratio be k : 1

$A(x_1, y_1) = (5, -6)$, $B(x_2, y_2) = (-1, -4)$, $m_1 = k$, $m_2 = 1$

$$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$$

$$(0, y) = \left(\frac{k(-1) + 1(5)}{k+1}\ ,\ \frac{k(-4) + 1\ (-6)}{k+1}\right)$$

$$\Rightarrow 0 = \frac{-k+5}{k+1} \ \Rightarrow\ -k + 5 = 0\ \Rightarrow\ k = 5\ \Rightarrow \text{ The ratio is } 5:1$$

$$y = \frac{5(-4) + 1\ (-6)}{5+1} = \frac{-20-6}{5+1} = \frac{-26}{6} = \frac{-13}{3}$$

∴ The coordinates of the point of intersection $\left(0, \frac{-13}{3}\right)$

Example 10 : If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Solution: We know that diagonals of a parallelogram bisect each other.

So, the coordinates of the mid-point of AC = coordinates of the mid-point of BD

The coordinates of the Midpoint $= \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$

$$\left(\frac{9+6}{2}\ ,\ \frac{4+1}{2}\right) = \left(\frac{p+8}{2}\ ,\ \frac{3+2}{2}\right) \Rightarrow \frac{15}{2} = \frac{p+8}{2}$$

$$30 = 2p + 16\ \Rightarrow\ 2p = 30 - 16\ \Rightarrow\ p = \frac{14}{2}\ \Rightarrow\ p = 7$$

Exercise 7.2

1. **Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.**

Let the Coordinates of the Points be(x,y)

$m_1 : m_2 = 2:3$ $(x_1,\ y_1) = (-1,\ 7)$, $(x_2,\ y_2) = (4, -3)$,

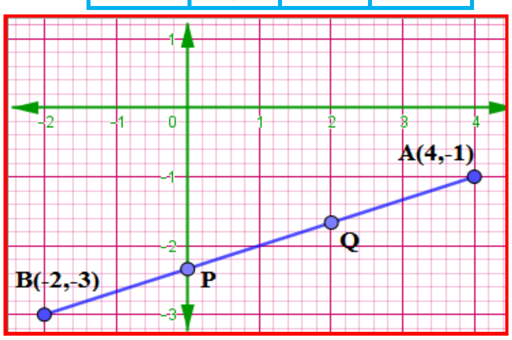
x_1	y_1	x_2	y_2
-1	7	4	-3

$$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$$

$$= \left(\frac{2(4)+3(-1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right) = \left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = \left(\frac{5}{5}, \frac{15}{5}\right) \Rightarrow (x, y) = (1, 3)$$

2. **Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).**

x_1	y_1	x_2	y_2
-1	7	4	-3



Let P and Q are the trisection points of AB

⇒ AP = PQ = QB

∴ The point P divides AB internally in the ratio 1 : 2

$A(x_1, y_1) = (4, -1)$, $B(x_2, y_2) = (-2, -3)$,

$m_1 = 1$, $m_2 = 2$

∴ The coordinates of P is,

$$P(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$$

$$= \left(\frac{1(-2)+2(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2}\right)$$

$$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right) = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$$

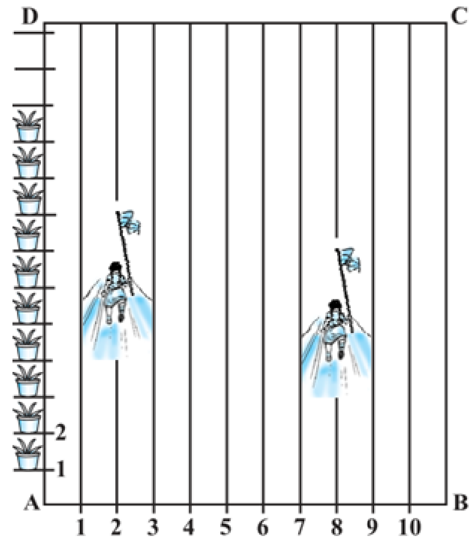
The point Q divides AB internally in the ratio 2 : 1

$A(x_1, y_1) = (4, -1)$, $B(x_2, y_2) = (-2, -3)$; $m_1 = 2$, $m_2 = 1$

$Q(x,y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$ [Using section formula]

$= \left(\frac{2(-2)+1(4)}{2+1},\ \frac{2(-3)+1(-1)}{2+1}\right) = \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right) = \left(\frac{0}{3},\ \frac{-7}{3}\right) = \left(0,\ \frac{-7}{3}\right)$

3. **To conduct Sports day activities in your rectangular shaped school groun ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in fig 7.12. Niharika runs $\frac{1}{4}$th the distance AD on the 2^{nd} line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eight line and posts a red flag. What is the distance between both the flags? If Rashmi has post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?**



Solution: The distance of green flag posted by Niharika on the 2^{nd} line

$= \frac{1}{4} \times AD = \frac{1}{4} \times 100 = 25$ m

The distance of red flag posted by Preet on the 8^{th} line

$= \frac{1}{5} \times AD = \frac{1}{5} \times 100 = 20$ m

Coordinates of Green flag $= (2,25) = (x_1, y_1)$

Coordinates of red flag $= (8,20) = (x_2, y_2)$

x_1	y_1	x_2	y_2
2	25	8	20

The distance between flags $d =$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(8-2)^2 + (20-25)^2} = \sqrt{(6)^2 + (-5)^2} = \sqrt{36+25} = \sqrt{61}$ m

The coordinates of blue flag, if Rashmi post in between these two flags be

$(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right) = \left(\frac{8+2}{2}\ ,\ \frac{20+25}{2}\right) = \left(\frac{10}{2}\ ,\ \frac{45}{2}\right) = (5\ ,22.5)$

4. **Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).**

$P(x,y) = (-1,6)$, $A(x_1,\ y_1) = (-3,10)$, $B(x_2,\ y_2) = (6,-8)$, $m_1 = ?$, $m_2 = ?$

x_1	y_1	x_2	y_2
-3	10	6	-8

$(x,y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$(-1,6) = \left(\frac{m_1 6) + m_2\ (-3)}{m_1+m_2}\ ,\ \frac{m_1(-8) + m_2\ (10)}{m_1+m_2}\right)$

$-1 = \frac{6m_1 - 3\ m_2}{m_1+m_2}$ Or $6 = \frac{-8m_1 + 10\ m_2}{m_1+m_2}$

$-m_1 - m_2 = 6m_1 - 3\ m_2$

$-m_1 - 6m_1 = -3\ m_2 + m_2$

$-7m_1 = -2m_2$

$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7}$

$m_1 : m_2 = 2 : 7$ We should verify that the ratio satisfies the y-coordinate also

$\frac{-8m_1 + 10\ m_2}{m_1+m_2} = \frac{-8(2)+10\ (7)}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$

Therefore, the point (– 1, 6) divides the line segment joining the points A(– 3, 10) and

B(6, – 8) in the ratio 2 : 7

5. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

We know that a point on the X − axis is of the form $(x, 0)$ Let the ratio be $k : 1$

$A(x_1, y_1) = (1, -5)$, $B(x_2, y_2) = (-4, 5)$ $m_1 = k$, $m_2 = 1$

$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}, \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$(x, 0) = \left(\frac{k(-4) + 1(1)}{k+1}, \frac{k(5) + 1\ (-5)}{k+1}\right)$

$0 = \frac{5k-5}{k+1}$

$5k - 5 = 0 \Rightarrow 5k = 5$

$k = 1$, the ratio is $1 : 1$

$x = \frac{1(-4) + 1(1)}{1+1} = \frac{-4+1}{2} = \frac{-3}{2}$

∴ The coordinates of the point of division = $\left(\frac{-3}{2}, 0\right)$

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Solution: Let A(1,2), B(4,y), C(x,6) and D(3,5) are the vertices of the parallelogram.

Since ABCD is a parallelogram

Therefore diagonals AC and BD bisects each other.

So, the coordinates of both AC and BD are same.

∴ Mid point of AC = Mid point of BD = $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$\left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\left(\frac{x+1}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

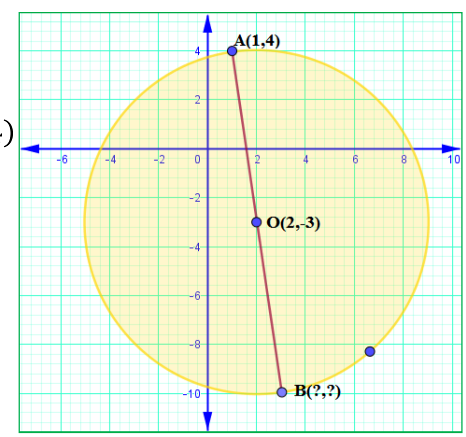
$\frac{x+1}{2} = \frac{7}{2}$, $\frac{5+y}{2} = \frac{8}{2}$

$x + 1 = 7$, $5 + y = 8$

$x = 7 - 1$, $y = 8 - 5$

$x = 6$, $y = 3$

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).



The center of the Circle is the mid-point of the diameter

∴ $(x, y) = (2, -3)$, $A(x_1, y_1) = ?$, $B(x_2, y_2) = (1, 4)$

$(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$(2, -3) = \left(\frac{1 + x_1}{2}, \frac{4 + y_1}{2}\right)$

$\frac{1 + x_1}{2} = 2$, $\frac{4 + y_1}{2} = -3$

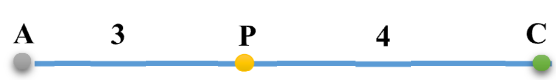
$1 + x_1 = 4$, $4 + y_1 = -6$

$x_1 = 4 - 1$, $y_1 = -6 - 4$

$x_1 = 3$, $y_1 = -10$

∴ The coordinates of a point A is $(3, -10)$

8. If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP $= \frac{3}{7}$ AB and P lies on the line segment AB



Given AP $= \frac{3}{7}$ AB

P divides AB in the ratio 3:4

x_1	y_1	x_2	y_2
-2	-2	2	-4

$\Rightarrow$ AP: PB = 3: 4

$Q(x,y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$= \left(\frac{3(2)+4(-2)}{3+4},\ \frac{3(-4)+4(-2)}{3+4}\right) = \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

9. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts

The point X divides AB in the ratio 1: 3

The coordinates of X is,

x_1	y_1	x_2	y_2
-2	2	2	8

$(x,y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$= \left(\frac{1(2)+3(-2)}{1+3},\ \frac{1(8)+3(2)}{1+3}\right) = \left(\frac{2-6}{4}, \frac{8+6}{4}\right) = \left(\frac{-4}{4}, \frac{14}{4}\right) = \left(-1, \frac{7}{2}\right)$

The point Y is the mid-point of AB

The coordinates of Y

$(x,y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right) = \left(\frac{2-2}{2}, \frac{8+2}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,\ 5)$

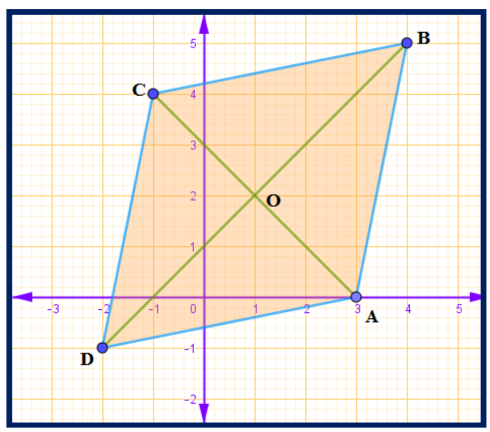
The point Z divides AB in the ratio 3: 1

The coordinates of Z is,

$(x,y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$= \left(\frac{3(2)+1(-2)}{3+1},\ \frac{3(8)+1(2)}{3+1}\right) = \left(\frac{6-2}{4}, \frac{24+2}{4}\right) = \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order[Hint: Area of rhombus = $\frac{1}{2}$(product of its diagonals)]



$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{(-4)^2+(4)^2}$

$= \sqrt{16+16} = \sqrt{16\times 2} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{36+36} = \sqrt{36\times 2} = 6\sqrt{2}$

The area of the rhombus $= \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$

$= \frac{24(\sqrt{2})^2}{2} = 12(2) = 24$ square units.

7.4 Area of a Triangle

Area of triangle $= \frac{1}{2}\times$ base $\times$ height

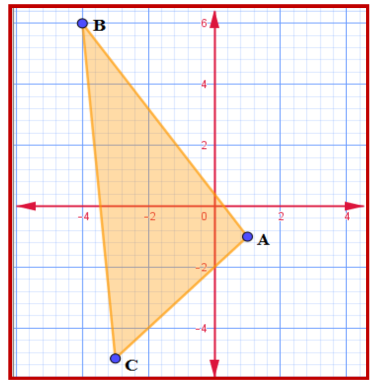
By Heron's Formula Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$, Here, $s = \frac{a+b+c}{2}$

a, b and c are the sides of the triangle.

We could find the lengths of the three sides of the triangle using distance formula. But this could be tedious, particularly if the lengths of the sides are irrational number. Then we can use the following formula to find the area of the triangle.

$$\textbf{Area of the triangle } = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

Example 11:Find the area of a triangle whose vertices are (1, –1),(– 4, 6) and (–3, –5).



A (1, -1), B(-4, 6) ಮತ್ತು C (-3, -5)

Area $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}[1(6-(-5))+(-4)(-5-(-1))+(-3)(-1-6)]$

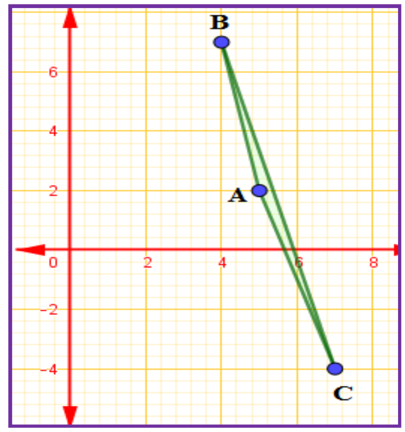
$=\frac{1}{2}[1(6+5)+(-4)(-5+1)+(-3)(-7)]$

$=\frac{1}{2}[11+16+21]$

$=\frac{1}{2}(48)=24$

Area of the triangle is $=24$ Square units

Example 12 : Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C (7, – 4).



A (5, 2), B (4, 7) and C (7, -4)

Area $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}[5(7-(-4))+4(-4-2)+7(2-7)]$

$=\frac{1}{2}[5(7+4)+4(-6)+7(-5)]$

$=\frac{1}{2}[55-24-35]$

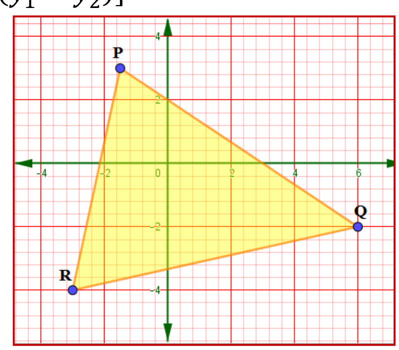
$=\frac{1}{2}(55-59)$

$=\frac{1}{2}(-4)=-2$

Since area is a measure, which cannot be negative, we will take the numerical value of – 2, i.e., 2. Therefore, the area of the triangle = 2 square units.

Example 13 : Find the area of the triangle formed by the points P(–1.5, 3), Q(6, –2) and R(–3, 4).

Area of the triangle$=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$



$=\frac{1}{2}[(-1.5)(-2-4)+6(4-3)+(-3)(3-(-2))]$

$=\frac{1}{2}[(-1.5)(-6)+6(1)+(-3)(3+2)]$

$=\frac{1}{2}[9+6-15]$ $=\frac{1}{2}(15-15)$

$=\frac{1}{2}(0)=0$

If the area of a triangle is 0 square units, then its vertices will be collinear.

Example 14 : Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.

Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$

$\frac{1}{2}[2(k-(-3))+4(-3-3)+6(3-k)]=0$

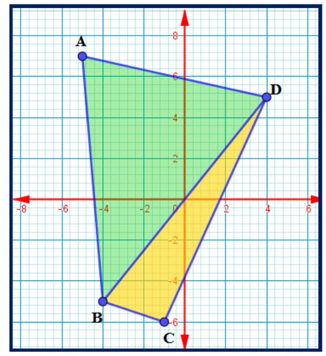
$\frac{1}{2}[2(k+3)+4(-6)+6(3-k)]=0$

$\frac{1}{2}[2k+6-24+18-6k]=0$

$\frac{1}{2}(-4k)=0$ $\Rightarrow$ k = 0

Example 15 : If A(–5, 7), B(– 4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

By joining B to D, we will get two triangles ABD and BCD



$\therefore$ Area $\Delta ABD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-5)(-5-5) + (-4)(5-7) + 4(7-(-5))]$

$= \frac{1}{2}[(-5)(-10) + (-4)(-2) + 4(7+5)] = \frac{1}{2}[50 + 8 + 48]$

$= \frac{1}{2}(106) = 53$ Sq.units

$\therefore$ Area $\Delta BCD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-4)(-6-5) + (-1)(5-(-5)) + 4(-5-(-6))]$

$= \frac{1}{2}[(-4)(-11) + (-1)(5+5) + 4(-5+6)]$

$= \frac{1}{2}[44 - 10 + 4] = \frac{1}{2}(38) = 19$ Sq.units

Therefore the area of quadrilateral ABCD = 53 + 19 = 72 Sq.units

Exercise 7.3

1. **Find the area of the triangle whose vertices are :**

i) (2, 3), (-1, 0), (2, -4) **ii) (-5, -1), (3, -5) (5, 2)**

i) (2, 3), (-1, 0), (2, -4)

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(0-(-4)) + (-1)(-4-3) + 2(3-0)]$

$= \frac{1}{2}[2(4) + (-1)(-7) + 2(3)] = \frac{1}{2}[8+7+6] = \frac{1}{2}(21) = \frac{21}{2}$ Sq.units

ii) (-5, -1), (3, -5) (5, 2)

Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-5)(-5-2) + 3(2-(-1)) + 5(-1-(-5))]$

$= \frac{1}{2}[(-5)(-7) + 3(2+1) + 5(-1+5)] = \frac{1}{2}[35+9+20] = \frac{1}{2}(64)$

$= 32$ Sq.units.

2. **In each of the following find the value of 'k', for which the points are collinear.**

i) (7, -2), (5, 1), (3, k) **ii) (8, 1), (k, -4) (2, -5)**

i) (7, -2), (5, 1), (3, k)

Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2}[7(1-k) + 5(k-(-2)) + 3(-2-1)] = 0$

$\frac{1}{2}[7(1-k) + 5(k+2) + 3(-3)] = 0$

$\frac{1}{2}[7 - 7k + 5k + 10 - 9] = 0$

$\frac{1}{2}(-2k+8) = 0$

$-2k = -8 \Rightarrow k = \frac{-8}{-2} = 4$

ii) (8, 1), (k, -4) (2, -5)

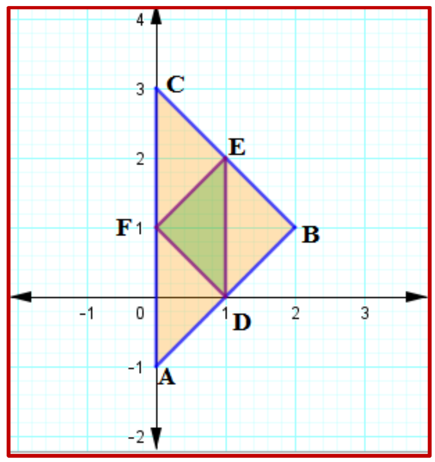
Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2}[8(-4-(-5))+k(-5-1)+2(1-(-4))]=0$
$\frac{1}{2}[8(-4+5)+k(-6)+2(1+4)]=0$
$\frac{1}{2}[8(1)+k(-6)+2(5)]=0$
$\frac{1}{2}[8-6k+10]=0$
$\frac{1}{2}(-6k+18)=0$
$-6k=-18 \Rightarrow k=\frac{-18}{-6}=3$

3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Let A(0, -1), B(2, 1) and C(0, 3) be the vertices of the triangle ABC
D, E and F are the mid-point of AB,BC and AC



The coordinates of D
$(x, y)=\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)=\left(\frac{2+0}{2}, \frac{1-1}{2}\right)$
$=\left(\frac{2}{2}, \frac{0}{2}\right)=(1, 0)$
The coordinates of E
$(x, y)=\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)=\left(\frac{0+2}{2}, \frac{3+1}{2}\right)$
$=\left(\frac{2}{2}, \frac{4}{2}\right)=(1, 2)$
The coordinates of F
$(x, y)=\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)=\left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$
$=\left(0, \frac{2}{2}\right)=(0, 1)$
The area of ΔDEF with vertices $D(1, 0)$, $E(1, 2)$ and $F(0, 1)$
$=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$
$=\frac{1}{2}[1(2-1)+1(1-0)+0(0-2)]=\frac{1}{2}[1(1)+1+0]$
$=\frac{1}{2}[1+1]=\frac{1}{2}(2)=1$ Sq.units
The area of given triangle $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$
$=\frac{1}{2}[0(1-3)+2(3-(-1))+0(-1-1)]=\frac{1}{2}[0+2(3+1))+0]$
$=\frac{1}{2}[0+8+0]=\frac{1}{2}(8)=4$ Sq.units
The ratio of the ΔABC and $\Delta DEF=4:1$

4. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).

A(-4, -2), B(-3, -5), C(3, -2) and D(2, 3)
By joining B to D, we will get two triangles ABD and BCD
∴ Area ABD

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-4)(-5 - 3) + (-3)(3 - (-2)) + 2(-2 - (-5))]$

$= \frac{1}{2}[(-4)(-8) + (-3)(3 + 2) + 2(-2 + 5)]$

$= \frac{1}{2}[32 - 15 + 6] \ = \frac{1}{2}(23) \ = \frac{23}{2}$ Sq.units

$\therefore$ AreaBCD

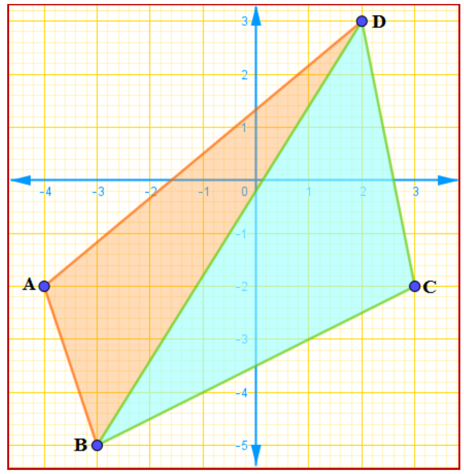
$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-3)(-2 - 3) + 3(3 - (-5)) + 2(-5 - (-2))]$

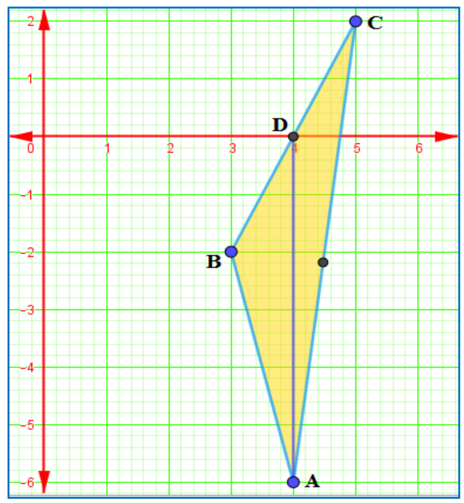
$= \frac{1}{2}[(-3)(-5) + 3(3 + 5) + 2(-5 + 2)]$

$= \frac{1}{2}[15 + 24 - 6] \ = \frac{1}{2}(33) \ = \frac{33}{2}$ Sq.units

$\therefore$ Area of ABCD $= \frac{23}{2} + \frac{33}{2} = \frac{56}{2} = 28$ Sq.units



5. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for ABC whose vertices area (4 – 6), B(3, –2) and C(5, 2).



Coordinates of D, the midpoint of BC

$(\mathbf{x}, \mathbf{y}) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{5+3}{2}, \frac{2-2}{2}\right)$

$= \left(\frac{8}{2}, \frac{0}{2}\right) = (\mathbf{4}, \ \mathbf{0})$

AreaΔABD

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4(-2 - 0) + 3(0 - (-6)) + 4(-6 - (-2))]$

$= \frac{1}{2}[4(-2) + 3(6) + 4(-6 + 2)]$

$= \frac{1}{2}[-8 + 18 - 16] \ = \frac{1}{2}(18 - 24)$

$= \frac{1}{2}(-6) = -3$ Sq.units

Since area is a measure, which cannot be negative, we will take the numerical value of – 3, i.e., 3. Therefore, the area of the triangle = 3square units.

Area ΔADC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4(0 - 2) + 4(2 - (-6)) + 5(-6 - 0)]$

$= \frac{1}{2}[4(-2) + 4(2 + 6) + 5(-6)] \quad = \frac{1}{2}[-8 + 32 - 30] \quad = \frac{1}{2}(-6) = -3$ ಚದರಮಾನಗಳು

Since area is a measure, which cannot be negative, we will take the numerical value of – 3, i.e., 3. Therefore, the area of the triangle = 3square units.

Hence, the mid-point of a triangle divides it into two triangles of equal areas.

7.5 Summary

1. The distance between two given points $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. The distance from the orgin to the given points $d = \sqrt{x^2 + y^2}$

3. Section formula :P is the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2,\ y_2)$
   If the point P divides the line in the ratio $m{:}\,n$ then the coordinates of P
   $P(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1 + m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1 + m_2}\right)$
4. If P is the midpoint of AB, it divides in the ratio 1:1
   P £À ¤zÉÃð±ÁAPÀUÀ¼ÀÄ $P(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$
5. Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

8 Real Numbers

Euclid's division algorithm, as the name suggests, has to do with divisibility of integers. Stated simply, it says any positive integer a can be divided by another positive integer b in such a way that it leaves a remainder r that is smaller than b.

8.2 Euclid's Division Lemma

Theorem 8.1

(Euclid's Division Lemma) : Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, 0 r < b.

A lemma is a proven statement used for proving another statement

Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576.

4052	**12576**	**3**
	12156	
	420	

$12576 = 4052 \times 3 + 420$

420	4052	9
	3780	
	272	

$4052 = 420 \times 9 + 272$

272	420	1
	272	
	148	

$420 = 272 \times 1 + 148$

148	272	1
	148	
	124	

$272 = 148 \times 1 + 124$

124	148	1
	124	
	24	

$148 = 124 \times 1 + 24$

24	124	5
	120	
	4	

$124 = 24 \times 5 + 4$

4	24	6
	24	
	0	

$24 = 4 \times 6 + 0$ ∴ **The HCF of 4052 and 12576 is 4**

Example 2 : Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Solution :Let 'a' be a possitive even number,

(i) $a = 2q + r$ here, $0 \leq r < 2$ $\Rightarrow r$ must be 0 or 1

But 'a' is a positive even number. So $r = 0$

$\therefore a = 2q + 0 \Rightarrow a = 2q$

(ii)Let 'a'be a positive odd integer , $r \neq o$ $\Rightarrow r = 1$ $\therefore a = 2q + 1$

Example 3 : Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Let a' and b are be a positive integer and $a > b$

By Division algorithm,

$a = bq + r$; $0 \leq r < b$

If $b = 4$,

$a = (4x2) + r, 0 \leq r < 4 \therefore$ $r = 0, 1, 2, 3$

i) If $r = 0$,

$a = 4q \Rightarrow a = 2(2q)$ this is divisible by 2. Therefore this is an even number.

ii) If $r = 1$,

$a = 4q + 1 \Rightarrow a = 2(2q) + 1$ this is not divisible by 2. Therefore this is an odd number.

iii)If $r = 2$,

$a = 4q + 2 \Rightarrow a = 2(2q + 1)$ This is divisible by 2. Therefore this is an even number.
iv) If $r = 3$,
$a = 4q + 3 \Rightarrow a = 2(2q+1) + 1$ this is not divisible by 2. Therefore this is an odd number.
$\therefore$ Any positive integer is of the form $4q + 1$ Or $4q + 3$ where q is some integer.

Example 4 : A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?

$420 = 130 \times 3 + 30$

$130 = 30 \times 4 + 10$

$30 = 10 \times 3 + 0$

So, the HCF of 420 and 130 is 10.

Therefore the sweetseller can make stacks of 10 for both kinds of burfi

Exercise 8.1

1. **Use Euclid's division algorithm to find the HCF of :**
(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

(i) 135 ಮತ್ತು 225

135	225	1
	135	
	90	

90	135	1
	90	
	45	

45	90	2
	90	
	0	

$225 = 135 \times 1 + 90$ $\quad 135 = 90 \times 1 + 45$ $\quad 90 = 45 \times 2 + 0$

$\therefore$ HCF = 45

(ii) 196 ಮತ್ತು 38220

196	38220	195
	38220	
	0	

$38220 = 196 \times 195 + 0$

$\therefore$ HCF = 196

(iii) 867 ಮತ್ತು 255

255	867	3
	765	
	102	

102	255	2
	204	
	51	

51	102	2
	102	
	0	

$867 = 255 \times 3 + 102$ $\quad 255 = 102 \times 2 + 51$ $\quad 102 = 51 \times 2 + 0$

$\therefore$ HCF = 51

2. **Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.**

Let a be a positive integer and let $b = 6$

Therefore by division algorithm

$a = bq + r$ [$0 \le r < b$]

$\Rightarrow a = 6q + r$ [$r = 0,1,2,3,4,5$]

(i) if $r = 0$, $a = 6q \Rightarrow$ divisible by 2. $\therefore$ 6q is even

(ii) if $r = 1$, $a = 6q + 1 \Rightarrow$ not divisible by 2 $\therefore 6q + 1$ is odd

(iii) if $r = 2$, $a = 6q + 2 \Rightarrow$ divisible by 2. $\therefore 6q + 2$ is even

(iv) if $r = 3$, $a = 6q + 3 \Rightarrow$ not divisible by 2 . $\therefore$ $6q + 3$ is odd

(v) if $r = 4$, $a = 6q + 4 \Rightarrow$ divisible by 2. $\therefore$ $6q + 4$ is even

(iv) if $r = 5$, $a = 6q + 5 \Rightarrow$ not divisible by 2 . $\therefore$ $6q + 5$ is odd

any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

3. **An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**

(iii) 867 ಮತ್ತು 255

32	616	19
	608	
	8	

8	32	4
	32	
	0	

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

$\therefore$ HCF = 8

They can march maximum 8 columns.

4. **Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.**

[Hint : Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Any positive integer divisible by 3, we get the remainder 0,1 or 2

$\Rightarrow$ a is of the form $3q$, $3q + 1$ or $3q + 2$

i) if $a = 3q$,

$a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$ **($m = 3q^2$)**

ii) if $a = 3q + 1$,

$a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2) + 1 = 3m+ 1$ **($m = 3q^2 + 2$)**

iii)if $a = 3q + 2$,

$a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 \Rightarrow a^2 = 9q^2 + 12q + 3 + 1$

$\Rightarrow 3(3q^2 + 4q +1) + 1 = 3m + 1$ **($m = 3q^2 + 4q +1$)**

From (i) (ii) and (iii)

We say, square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

5. **Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8**

Let a be a positive inteer and let $b = 3$

By Euclid's division algorithm,

$a = bq + r$ [$0 \leq r < b$] $\Rightarrow a = 3q + r$ [$r = 0,1,2$]

(i)if $r = 0$, $a = 3q$

$\Rightarrow a^3 = (3q)^3 \Rightarrow a^3 = 9q^3 \Rightarrow 9m$ [$\because m = q^3$]

(ii)if $r = 1$, $a = 3q + 1$

$a^3 = (3q + 1)^3 \Rightarrow a^3 = 27q^3 + 3x(3q)^2x1 + 3x3qx1 + 1 \Rightarrow a^3 = 27q^3 + 27q^2 + 9q + 1$

$\Rightarrow a^3 = 9(3q^3 + 9q + q) + 1 \Rightarrow a^3 = 9m + 1$ [$\because m = 3q^3 + 9q + q$]

(iii)if $r = 2$, $a = 3q + 2$

$a^3 = (3q + 2)^3 \Rightarrow a^3 = 27q^3 + 54q^2 + 18q + 8$

$\Rightarrow a^3 = 9(3q^3+ 6q^2 + 2q) + 8 \Rightarrow a^3 = 9m + 8$ [$\because m = 3q^3+ 6q^2 + 2q$]

$\therefore$ We say, the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8

8.3 The Fundamental Theorem of Arithmetic

Theorem 8.2 (Fundamental Theorem of Arithmetic) : Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime the prime factors occur

The Fundamental Theorem of Arithmetic says that every composite number can be factorised as a product of primes. Actually it says more. It says that given any composite number it can be factorised as a product of prime numbers in a'unique' way, except for the order in which the primes occur. That is, given any composite number there is one and only one way to write it as a product of primes, as long as we are not particular about the order in which the primes occur. So, for example, we regard $2 \times 3 \times 5 \times 7$ as the same as $3 \times 5 \times 7 \times 2$, or any other possible order in which these primes are written.

Example 5 : Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution: If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 4^n would contain the prime 5. This is not possible because $4^n = (2)^{2n}$; so the only prime in the factorisation of 4^n is 2.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 4^n.

So, there is no natural number n for which 4^n ends with the digit zero.

Example 6 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution: $6 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

HCF (6,20) = 2 and LCM (6, 20) = 2 x 2 x 3 x 5 = 60

Any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

Example 7 : Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Solution: We can write the prime factors of 96 and 404 are as follows

$96 = 2^5 \times 3$; $404 = 2^2 \times 101$

$HCF(96,404) = 2^2 = 4$

$\therefore$ LCM (96, 404) $= \frac{96 \times 404}{4} = 9696$

Example 8 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

$6 = 2 \times 3$; $72 = 2^3 \times 3^2$; $120 = 2^3 \times 3 \times 5$

$\therefore$ HCF (6, 72, 120) $= 2^1 \times 3^1 = 2 \times 3 = 6$

$\therefore$ LCM (6, 72, 120) $= 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

Exercise 8.2

1. **Express each number as a product of its prime factors:**
   (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429
   (i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$
   (ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$
   (iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$
   (iv) $5005 = 5 \times 7 \times 11 \times 13$
   (v) $7429 = 17 \times 19 \times 23$
2. **Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.**
   (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54.
   (i) $26 = 2 \times 13$
   $91 = 7 \times 13$
   HCF = 13; LCM $= 2 \times 7 \times 13 = 182$
   Product of two numbers $= 26 \times 91 = 2366$
   LCM x HCF = $13 \times 182 = 2366$
   ∴ LCM x HCF = Product of two numbers

   (ii) $510 = 2 \times 3 \times 5 \times 17$
   $92 = 2 \times 2 \times 23$
   HCF = 2; LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
   Product of two numbers $= 510 \times 92 = 46920$
   LCM x HCF = $2 \times 23460 = 46920$
   ∴ LCM x HCF = Product of two numbers

   (iii) $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
   $54 = 2 \times 3 \times 3 \times 3$
   HCF $= 2 \times 3 = 6$; LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$
   Product of two numbers $= 336 \times 54 = 18144$
   LCM x HCF = $6 \times 3024 = 18144$
   ∴ LCM x HCF = Product of two numbers
3. **Find the LCM and HCF of the following integers by applying the prime factorisation method (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25**
   (i) $12 = 2 \times 2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$
   HCF = 3; LCM $= 2 \times 2 \times 3 \times 5 \times 7 = 420$
   (ii) $17 = 1 \times 17$; $23 = 1 \times 23$; $29 = 1 \times 29$
   HCF = 1; LCM $= 1 \times 17 \times 19 \times 23 = 11339$
   (iii) $8 = 1 \times 2 \times 2 \times 2$; $9 = 1 \times 3 \times 3$; $25 = 1 \times 5 \times 5$
   HCF = 1; LCM $= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$
4. **Given that HCF (306, 657) = 9, find LCM (306, 657).**
   LCM x HCF = Product of two numbers
   ∴ LCM (306, 657) = $\frac{306 \times 657}{9}$ = 22338
5. **Check whether 6^n can end with the digit 0 for any natural number n.**
   Here, n is a natural number.

If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5. That is, the prime factorisation of 4^n would contain the prime 5. This is not possible because the prime factors of 6 are 2 and 3.
Therefore 5 is not a factor of 6. $\Rightarrow 6^n = (2 \text{ x } 3)^n$

So, there is no natural number n for which 6^n ends with the digit zero.

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers

6. $7 \times 11 \times 13 + 13 = 13\,(7\text{x}11 + 1) = 13(77 + 1) = 13\,(78) = 13\text{x}2\text{x}3\text{x}13$
The product of two or more than two prime numbers is a composite number.
Therefore $7 \text{ x } 11 \text{ x } 13 + 13$ is a composite number.
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5(1008 + 1) = 5(1009)$
The product of two or more than two prime numbers is a composite number.
Therefore $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number

7. **There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**
To find the time they meet again in the same point, we have to find the LCM of time
$18 = 2 \times 3 \times 3$; $12 = 2 \times 2 \times 3$
LCM $= 2 \times 2 \times 3 \times 3 = 36$
Therefore after 36 minutes they meet again at the starting point.

8.4 Revisiting Irrational Numbers

A number which can not be expressed in the form of $\frac{p}{q}$ is called irrational number.Here, $p, q \in Z, q \neq 0$

Theorem 8.3: Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.

Theorm 8.4: $\sqrt{2}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{2}$ is rational.
$\Rightarrow \sqrt{2} = \frac{p}{q}$ [$p,q \in Z$, $q \neq 0$ and $(p,q)=1$]
So, there is no other common factors for p and q other than 1
Now, $\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2}q = p$ Squaring on both sides we get,
$(\sqrt{2}q)^2 = p^2 \quad \Rightarrow 2q^2 = p^2 \qquad (1)$
$\Rightarrow 2$ divides $p^2 \Rightarrow 2$, divides p . [By theorem]
$\therefore$ Let $p = 2m$,
(1) $\Rightarrow 2q^2 = (2m)^2 \Rightarrow q^2 = 2m^2$
$\Rightarrow 2$,divides $q^2 \Rightarrow 2$, divides q [By theorem]
$\therefore$ 2 is the common factor for both p and q
This contradicts that there is no common factor of p and q .
Therefore our assumption is wrong. So, $\sqrt{2}$ is a an irrational number.

Example 9 : Prove that $\sqrt{3}$ is irrational.

Proof: Let us assume, to the contrary, that $\sqrt{3}$ is rational.

$\Rightarrow \sqrt{3} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{3} = \frac{p}{q} \Rightarrow \sqrt{3}q = p$ Squaring on both sides we get,

$(\sqrt{3}q)^2 = p^2 \Rightarrow 3q^2 = p^2$ (1)

$\Rightarrow$ 3 divides $p^2 \Rightarrow$ 3 divides p [By theorem]

∴ Let p = 3m,

(1) $\Rightarrow 3q^2 = (3m)^2 \Rightarrow q^2 = 3m^2$

$\Rightarrow$3 divides $q^2 \Rightarrow$ 3 divides q [By theorem]

∴ 3 is the common factor for both p and q , This is not possible.

Therefore our assumption is wrong. So, $\sqrt{3}$ is a an irrational number.

- **The sum or difference of a rational and an irrational number is irrational and**
- **The product and quotient of a non-zero rational and irrational number is irrational.**

Example 10 : Show that $5 - \sqrt{3}$ is irrational

Proof: Assume that $5 - \sqrt{3}$ is a rational number.

$\Rightarrow 5 - \sqrt{3} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \quad \Rightarrow \frac{5q-p}{q} = \sqrt{3}$

Here, $\frac{5q-p}{q}$ is a rational number but $\sqrt{3}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $5 - \sqrt{3}$ is an irrational number.

Example 11 : Show that $3\sqrt{2}$ is irrational.

Proof: Assume that $3\sqrt{2}$ is a rational number.

$\Rightarrow 3\sqrt{2} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{2} = \frac{p}{3q}$

Here, $\frac{p}{3q}$ s a rational number but $\sqrt{2}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $3\sqrt{2}$ is an irrational number ಆದ್ದರಿಂದ $3\sqrt{2}$ ಒಂದು

Exercise 8.3

1. **Prove that $\sqrt{5}$ is irrational.**

Proof: Let us assume, to the contrary, that $\sqrt{5}$ is rational.

$\Rightarrow \sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{5} = \frac{p}{q} \Rightarrow \sqrt{5}q = p$, squaring on both sides we get,

$(\sqrt{5}q)^2 = p^2 \Rightarrow 5q^2 = p^2$ (1)

$\Rightarrow$ 5 divides $p^2 \Rightarrow$ 5 divide s p [By theorem]

∴ Let p = 5m,

(1) $\Rightarrow 5q^2 = (3m)^2 \Rightarrow q^2 = 5m^2$

$\Rightarrow$5 divides $q^2 \Rightarrow$ 5 divides q [By theorem]

∴ 5 is the common factor for both p and q ; this is not possible

Therefore our assumption is wrong. So, $\sqrt{5}$ is a an irrational number.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Proof: Assume that $3 + 2\sqrt{5}$ is a rational number.

$\Rightarrow 3 + 2\sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow 2\sqrt{5} = \frac{p}{q} - 3 \Rightarrow \sqrt{5} = \frac{p-3q}{2q}$

Here, $\frac{p-3q}{2q}$ is a rational number but $\sqrt{5}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $3 + 2\sqrt{5}$ is an irrational number.

1. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

(i) $\frac{1}{\sqrt{2}}$

Proof: Assume that $\frac{1}{\sqrt{2}}$ is a rational number.

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{p}{q} \Rightarrow \sqrt{2} = \frac{2p}{q}$

Here, $\frac{2p}{q}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

Therefore our assumption is wrong. $\therefore \frac{1}{\sqrt{2}}$ is an irrational number.

(ii) $7\sqrt{5}$

Proof: Assume that $7\sqrt{5}$ is a rational number.

$7\sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{5} = \frac{p}{7q}$

Here, $\frac{p}{7q}$ is a rational number, but $\sqrt{5}$ is an irrational. This is impossible.

Therefore our assumption is wrong. $\therefore 7\sqrt{5}$ is an irrational number.

(iii) $6 + \sqrt{2}$

Proof: Assume that $6 + \sqrt{2}$ is a rational number

$\Rightarrow 6 + \sqrt{2} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 and (p,q)=1]

$\Rightarrow \sqrt{2} = \frac{p}{q} - 6 \quad \Rightarrow \sqrt{2} = \frac{p-6q}{2}$

Here, $\frac{p-6q}{2}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

Therefore our assumption is wrong. $\therefore 6 + \sqrt{2}$ is an irrational number.

8.5 Revisiting Rational Numbers and Their Decimal Expansion:

Theorem 8.5: Let x be a rational number whose decimal expansion terminates.Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime, and the prime factorisation of q is of the form $2^n.5^m$, where n, m are non-negative integers.

Theorem 8.6 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form 2n5m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Theorem 8.7 : Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Exercise 8.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^3}$ (vii) $\frac{23}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

(i) $\frac{13}{3125}$ - Factorising the denominator $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 2^0 \times 5^5$

Here, The factors of 3125 is of the form $2^n.5^m$. So, **this** has a terminating decimal expansion.

(ii) $\frac{17}{8}$ - Factorising the denominator $8 = 2 \times 2 \times 2 = 2^3 x 5^0$

Here, The factors of 8 is of the form $2^n.5^m$. So, **this** has a terminating decimal expansion.

(iii) $\frac{64}{455}$ - Factorising the denominator $455 = 5 \times 7 \times 13$

Here, The factors of 455 is 5x7x13 is not in the form 2^n x 5^m

So, **this** has non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600}$ - Factorising the denominator $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^6 \times 5^2$

Here, The factors of 1600 is of the form $2^n.5^m$.

So, **this** has a terminating decimal expansion.

(v) $\frac{29}{343}$ - Factorising the denominator $343 = 7 \times 7 \times 7 = 7^3$

Here, The factors of 343 is not in the form $2^n \times 5^m$

So, **this** has non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$ - denominator is of the form 2^n x 5^m

So, **this** has a terminating decimal expansion.

(vii) $\frac{23}{2^2 5^7 7^5}$ denominator is not in the form 2^n x 5^m

So, this has non-terminating repeating decimal expansion.

(viii) $\frac{6}{15}$ $\Rightarrow \frac{6}{15} = \frac{2}{5}$ dinominator 2^0 x 5^1 is of the form 2^n x 5^m

So, **this** has a terminating decimal expansion.

(ix) $\frac{35}{50}$ $\Rightarrow \frac{35}{50} = \frac{7}{10} = \frac{7}{2x5}$ dinominator 2^1 x 5^1 is of the form 2^n x 5^m

So, **this** has a terminating decimal expansion.

x) $\frac{77}{210}$ $\Rightarrow \frac{77}{210} = \frac{11}{30} = \frac{11}{2x3x5}$ dinominator not in the form 2^n x 5^m

So, **this** has non-terminating repeating decimal expansion.

2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

(i) $\frac{13}{5^5} \Rightarrow \frac{13}{5^5} \text{ x } \frac{2^5}{2^5} = \frac{15x32}{105} = \frac{416}{100000} = 0.00416$

(ii) $\frac{17}{8} \Rightarrow \frac{17}{8} = \frac{17}{2^3} \text{ x } \frac{5^3}{5^3} = \frac{17x125}{1000} = 2.125$

(iii) $\frac{15}{1600} = \frac{15}{2^6 5^2} = \frac{15 x 5^4}{2^6 x 5^6} = \frac{15x625}{1000000} = 0.009375$

(iv) $\frac{23}{2^3 5^3} \Rightarrow \frac{23}{2^3 5^3} = \frac{23x5}{2^3 5^3}\ \frac{115}{1000} = 0.115$

(v) $\frac{6}{15} \Rightarrow \frac{6}{15} = \frac{2}{5} x \frac{2}{2} = \frac{4}{10} = 0.4$

(vi) $\frac{35}{50} \Rightarrow \frac{35}{50} = \frac{7}{10} = 0.7$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$ what can you say about the prime factors of q?

(i) 43.123456789 (ii) 0.120120012000120000... (iii) $43.\overline{123456789}$

(i) 43.123456789 - has a terminating decimal expansion.

Therefore this in a rational number and is of the form $\frac{p}{q}$

and q is of the form $2^n x 5^m \Rightarrow \frac{43123456789}{1000000000} = \frac{43123456789}{2^9 5^9}$

(ii) 0.120120012000120000...

this has non-terminating and non- repeating decimal expansion.

Therefore this is an irrational number

(iii) $43.\overline{123456789}$ - this has non-terminating repeating decimal expansion.

Therefore this in a rational number and is of the form $\frac{p}{q}$

Let $x = 43.\overline{123456789}$ (1)

$\Rightarrow 1000000000x = 43123456789\overline{123456789}.....$ (2)

(1) – (2) = 999999999x = 43123456746

$x = \frac{43123456746}{999999999}$ this is of the form $\frac{p}{q}$. i.e., the factors of 999999999 is not in the form 2^n x 5^m

Summery:

- **Euclid's Division Lemma:** Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r, \quad 0 \le r < b$
- **Euclid's division algorithm:** This is based on Euclid's division lemma. According to this, the HCF of any two positive integers a and b, with a > b, is obtained as follows:

  Step 1 : Apply the division lemma to find q and r where $a = bq + r,\ 0 \le r < b$

  Step 2 : If r = 0, the HCF is b. If $r \neq 0$ apply Euclid's lemma to b and r.

  Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b). Also, HCF(a, b) = HCF(b, r).
- **The Fundamental Theorem of Arithmetic :**
- Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
- If p is a prime and p divides a^2, then p divides a, where a is a positive integer.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).