DR ACADEMY
DO RIGHT FOR GENUINE EDUCATION

KCET EXAMINATION – 2020
SUBJECT : MATHEMATICS

DATE :- 30-07-2020   **TIME : 02.30 PM TO 03.50 PM**

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1. If $2^x + 2^y = 2^{x+y}$, then $\dfrac{dy}{dx}$ is

   a) 2^{y-x} b) -2^{y-x} c) 2^{x-y} d) $\dfrac{2^y - 1}{2^x - 1}$

Ans. b

Sol. $\dfrac{dy}{dx} = \dfrac{2^x\left(1 - 2^y\right)}{2^y\left(2^x - 1\right)}$

$= \dfrac{2^x - \left(2^x - 2^y\right)}{2^x + 2^y - 2^y}$

$= -2^{y-x}$

2. If $f(x) = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$, then $f'\left(\sqrt{3}\right)$ is

   a) $-\dfrac{1}{2}$ b) $\dfrac{1}{2}$ c) $\dfrac{1}{\sqrt{3}}$ d) $-\dfrac{1}{\sqrt{3}}$

Ans. b

Sol. $f(x) = 2\tan^{-1} x$

$f'(x) = \dfrac{2}{1+x^2}$

$f'\left(\sqrt{3}\right) = \dfrac{2}{4} = \dfrac{1}{2}$

3. The right hand and left hand limit of the function

$$f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

are respectively
   a) 1 and 1   b) 1 and -1
   c) -1 and -1   d) -1 and 1

Ans. b

Sol. $\underset{x\to 0^+}{Lt} f(x) = \underset{h\to 0}{Lt} \dfrac{e^{1/h} - 1}{e^{1/h} + 1}$

$= \underset{h\to 0}{Lt} \dfrac{1 - e^{-1/h}}{1 + e^{-1/h}} = 1$

$\underset{h\to 0^-}{Lt} f(x) = \underset{h\to 0}{Lt} \dfrac{e^{-1/h} - 1}{e^{-1/h} + 1}$

$= \dfrac{0-1}{0+1} = -1$

4. If $y = 2x^{n+1} + \dfrac{3}{x^n}$, then $x^2\dfrac{d^2y}{dx^2}$ is

   a) $6n(n+1)y$ b) $n(n+1)y$
   c) $x\dfrac{dy}{dx} + y$ d) y

Ans. b

Sol. $\dfrac{dy}{dx} = 2(n+1)x^n - \dfrac{3n}{x^{n+1}}$

$\dfrac{d^2y}{dx^2} = 2n(n+1)x^{n-1} + \dfrac{3n(n+1)}{x^{n+2}}$

$\dfrac{x^2 d^2y}{dx^2} = n(n+1)\left[2x^{n+1} + \dfrac{3}{x^n}\right]$

$= n(n+1)y$

5. If the curves $2x=y^2$ and $2xy=K$ intersect perpendicularly, then the value of K^2 is
   a) 4   b) $2\sqrt{2}$ c) 2   d) 8

Ans. d

Sol. $2x = y^2$, $2xy = k$

$\dfrac{dy}{dx} = \dfrac{1}{y}$, $\dfrac{dy}{dx} = \dfrac{-y}{x}$

$m_1 m_2 = -1$

$\dfrac{1}{y} \times \dfrac{y}{n} = -1$

$X = 1$

$\therefore y^2 = 2$

$4x^2y^2 = k^2$

$k^2 = 4(1)(2)$

$k^2 = 8$

6. if $(xe)^y = e^y$, then $\dfrac{dy}{dx}$ is

   a) $\dfrac{\log x}{(1 + \log x)^2}$ b) $\dfrac{1}{(1 + \log x)^2}$

   c) $\dfrac{\log x}{(1 + \log x)}$ d) $\dfrac{e^x}{x(y - 1)}$

Ans. a

Sol. $(xe)^y = e^x$

Take log on both sides
$y(1 + \log x) = x$

$$y = \frac{x}{1 + \log x}$$

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

7. If the side of a cube is increased by 5%, then the surface area of a cube is increased by
a) 10%   b) 60%   c) 6%   d) 20%

Ans. a

Sol. $\frac{\delta x}{x} \times 100 = 5\%$

$$\frac{\delta S}{S} \times 100 = \frac{12x}{6x^2} \times \delta x \times 100$$

$$= 2 \times \frac{\delta x}{x} \times 100$$

$$= 2 \times 5 = 10\%$$

8. The value of $\int \frac{1 + x^4}{1 + x^6} dx$ is

a) $\tan^{-1} x + \tan^{-1} x^3 + C$

b) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^3 + C$

c) $\tan^{-1} x - \frac{1}{3} \tan^{-1} x^3 + C$

d) $\tan^{-1} x + \frac{1}{3} \tan^{-1} x^2 + C$

Ans. b

Sol. $\int \frac{(x^4 - x^2 + 1) + x^2}{(x^2 + 1)(x^4 - x^2 + 1)} dx$

$$= \int \frac{1}{x^2 + 1} dx + \frac{1}{3} \int \frac{3x^2}{x^6 + 1} dx \quad \text{Let } x^3 = t, \quad 3x^2 dx = dt$$

$$= \text{Tan}^{-1} x + \frac{1}{3} \tan^{-1}(x^3) + c$$

9. The maximum value of $\frac{\log_e x}{x}$, if x>0 is

a) e   b) 1   c) $\frac{1}{e}$ d) $-\frac{1}{e}$

Ans. c

Sol. $f^1(x) = \frac{1 - \log x}{x^2}$

$$f^1(x) = 0$$

x=e

$$f^{11}(e) < 0$$

$\therefore$ on maximum value $= \frac{1}{e}$

10. The value of $\int e^{\sin x} \sin 2x \, dx$ is

a) $2e^{\sin x}(\sin x - 1) + C$

b) $2e^{\sin x}(\sin x + 1) + C$

c) $2e^{\sin x}(\cos x + 1) + C$

d) $2e^{\sin x}(\cos x - 1) + C$

Ans. a

Sol.   Let sinx=t

cosx dx=dt

$$2\int e^t \, t \, dt = 2e^t [t - 1] + c$$

$$= 2e^{\sin x}[\sin x - 1] + c$$

11. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1} x \, dx$ is

a) π b) $\frac{\pi}{2}$ c) 1   d) $\frac{\pi^2}{2}$

Ans. b

Sol. $I = \int_{-1/2}^{1/2} \cos^{-1}(x) dx \ldots\ldots (1)$

$$I = \int_{-1/2}^{1/2} \cos^{-1}(-x) dx$$

$$I = \int_{-1/2}^{1/2} \pi - \cos^{-1} x \, dx \ldots\ldots (2)$$

(1) + (2)

$$2I = \int_{-1/2}^{1/2} \pi \, dx$$

$$2I = \pi(1)$$

$$I = \pi / 2$$

12. If $\int \frac{3x + 1}{(x-1)(x-2)(x-3)} dx$

$= A \log|x - 1| + B \log|x - 2| + C \log|x - 3| + C$, then the values of A, B and C are respectively.
a) 5, -7, -5   b) 2, -7, -5
c) 5, -7, 5   d) 2, -7, 5

Ans. d

Sol. $\frac{3x + 1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$3x + 1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Taking $x = 1, 2, 3$

$A = 2, B = -7, C = 5$

13. The value of $\int_0^1 \dfrac{\log(1+x)}{1+x^2}dx$ is

   a) $\dfrac{\pi}{2}\log 2$ b) $\dfrac{\pi}{4}\log 2$

   c) $\dfrac{1}{2}$ d) $\dfrac{\pi}{8}\log 2$

Ans. d

Sol. Let $x=\tan\theta$

   $dx = \sec^2\theta\,d\theta$

   $I = \int_0^{\frac{\pi}{4}} \dfrac{\log(1+\tan\theta)}{\sec^2\theta} \times \sec^2\theta\,d\theta$

   $I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta)\,d\theta .....(1)$

   $I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\dfrac{\pi}{4}-\theta\right)\right)d\theta$

   $= \int_0^{\frac{\pi}{4}} \log\left(\dfrac{2}{1+\tan\theta}\right)d\theta$

   $I = \log 2\int_0^{\frac{\pi}{4}} 1\,d\theta - I$

   $2I = \log 2.\dfrac{\pi}{4}$

   $I = \dfrac{\pi}{8}\log 2$

14. The area of the region bounded by the curve $y^2=8x$ and the line y=2x is

   a) $\dfrac{16}{3}$ sq.units b) $\dfrac{4}{3}$ sq.units

   c) $\dfrac{3}{4}$ sq.units d) $\dfrac{8}{3}$ sq.units

Ans. b

Sol. $A = \int_0^2 \left(2\sqrt{2}\sqrt{x} - 2x\right)dx$

   $= \left(\dfrac{4\sqrt{2}}{3}x^{3/2} - x^2\right)_0^2$

   $= \left(\dfrac{4\,2^{1/2}}{3}2^{3/2} - 4\right)$

   $= \dfrac{4}{3}$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \dfrac{\cos x}{1+e^x}dx$ is

   a) 2 b) 0 c) 1 d) -2

Ans. c

Sol. $I = \int_{-\pi/2}^{\pi/2} \dfrac{\cos x}{1+e^x}dx ....(1)$

   $I = \int_{-\pi/2}^{\pi/2} \dfrac{\cos x}{1+e^{-x}}dx$

   $I = \int_{-\pi/2}^{\pi/2} \dfrac{e^x(\cos x)}{1+e^x}dx .....(2)$

   (1) + (2)

   $2I = \int_{-\pi/2}^{\pi/2} \cos x\,dx$

   $= (\sin x)_{-\pi/2}^{\pi/2}$

   $2I = 2$

   $I=1$

16. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is

   a) 1 b) 2 c) 3 d) 4

Ans. a

Sol. $C_1 y = k_1 e^x . e^{cy}$

   $y = \dfrac{k_1 e^{cy}}{c_1}e^x$

   $y=ke^x$

   order = 1

17. The general solution of the differential equation $x^2dy-2xydx=x^4\cos x\,dx$ is

   a) $y=x^2\sin x+cx^2$ b) $y=x^2\sin x+c$

   c) $y=\sin x+cx^2$ d) $y=\cos x+cx^2$

Ans. a

Sol. $\dfrac{dy}{dx} - \dfrac{2}{x}y = x^2\cos x$

   $I.F = e^{\int \frac{-2}{x}dx} = \dfrac{1}{x^2}$

   G.S $y.\dfrac{1}{x^2} = \int \dfrac{1}{x^2}\left(x^2\cos x\right)dx$
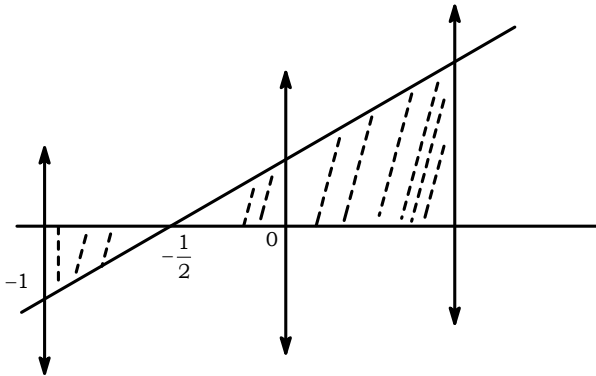
   $\dfrac{y}{x^2} = \sin x + c$

   $y = x^2\sin x + cx^2$

18. The area of the region bounded by the line y=2x+1, x-axis and the ordinates x=-1 and x=1 is

a) $\dfrac{9}{4}$ b) 2   c) $\dfrac{5}{2}$ d) 5

Ans. c

Sol. Area $= \dfrac{1}{2} \times \dfrac{1}{2} \times 1 + \dfrac{1}{2} \times \dfrac{3}{2} \times 3$
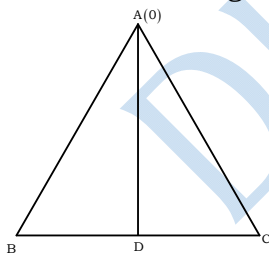

$$= \dfrac{1}{4} + \dfrac{9}{4}$$

$$= \dfrac{5}{2}$$

19. The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a ΔABC. The length of the median through A is

a) $\dfrac{\sqrt{14}}{2}$ b) 14   c) 7   d) $\sqrt{14}$

Ans. d

Sol.   Consider A as origin

$\overrightarrow{AD} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2} = \hat{i} + 2\hat{j} + 3\hat{k}$

Length of $AD = \sqrt{1 + 4 + 9} = \sqrt{14}$

20. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\dfrac{\theta}{2}$ is

a) $|\vec{a} + \vec{b}|$ b) $\dfrac{|\vec{a} + \vec{b}|}{2}$ c) $\dfrac{|\vec{a} - \vec{b}|}{2}$ d) $|\vec{a} - \vec{b}|$

Ans. c

Sol. $|\vec{a} - \vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}.\vec{b}$

$$= 1 + 1 - 2\cos\theta$$

$$= 2(1 - \cos\theta)$$

$$|\vec{a} - \vec{b}|^2 = 2.2\sin^2(\theta/2)$$

$$|\vec{a} - \vec{b}| = 2\sin(\theta/2)$$

21. The curve passing through the point (1, 2) given that the slope of the tangent at any point (x, y) is $\dfrac{3x}{y}$ represents

a) Circle   b) Parabola
c) Ellipse   d) Hyperbola

Ans. d

Sol. $\dfrac{dy}{dx} = \dfrac{2x}{y}$

$ydy = 2xdx$

$\dfrac{y^2}{2} = x^2 + c$ passes through (1, 2)

C=1

$\therefore \dfrac{y^2}{2} = x^2 + 1$

$\dfrac{x^2}{1} - \dfrac{y^2}{2} = -1$

Represents hyperbola

22. If $|\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2 = 144$ and $|\vec{a}| = 6$ then $|\vec{b}|$ is equal to

a) 6   b) 3   c) 2   d) 4

Ans. c

Sol. $|\vec{a}|^2 |\vec{b}|^2 = 144$

$|\vec{b}|^2 = 4$

$|\vec{b}| = 2$

23. The point (1, -3, 4) lies in the octant
a) Second b) Third   c) Fourth d) Eighth

Ans. c

Sol. Fourth octant

24. If the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}, 2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + 2\hat{k}$ are coplanar, then the value of λ is
a) 6   b) -5   c) -6   d) 5

Ans. a

Sol. $\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$2(2-1) + 3(4+\lambda) + 4(-2-\lambda) = 0$

$\lambda = 6$

25. The distance of the point (1, 2, -4) from the line $\dfrac{x-3}{2} = \dfrac{y-3}{3} = \dfrac{z+5}{6}$ is

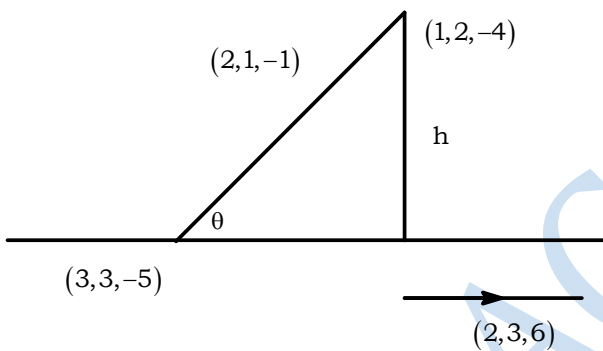a) $\dfrac{293}{7}$ b) $\dfrac{\sqrt{293}}{7}$ c) $\dfrac{293}{49}$ d) $\dfrac{\sqrt{293}}{49}$

Ans. b

Sol. $\cos\theta = \dfrac{4+3-6}{\sqrt{6}.\times 7} = \dfrac{1}{7\sqrt{6}}$

$\sin\theta = \dfrac{h}{\sqrt{6}}$

$h = \sqrt{6}\sqrt{1-\cos^2\theta}$

$= \dfrac{\sqrt{293}}{7}$

26. The sine of the angle between the straight line $\dfrac{x-2}{3} = \dfrac{3-y}{-4} = \dfrac{z-4}{5}$ and the plane $2x - 2y + z = 5$ is

a) $\dfrac{3}{\sqrt{50}}$ b) $\dfrac{3}{50}$ c) $\dfrac{4}{5\sqrt{2}}$ d) $\dfrac{\sqrt{2}}{10}$

Ans. G

Sol. $\vec{b}(3,-4,5), \vec{n} = (2,-2,1)$

$\sin\theta = \dfrac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|} = \dfrac{1}{\sqrt{50}}$

27. If a line makes an angle of $\dfrac{\pi}{3}$ with each of x and y-axis, then the acute angle made by z-axis is

a) $\dfrac{\pi}{4}$ b) $\dfrac{\pi}{6}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{\pi}{2}$

Ans. a

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\dfrac{1}{4} + \dfrac{1}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - 1/2$

$= 1/2$
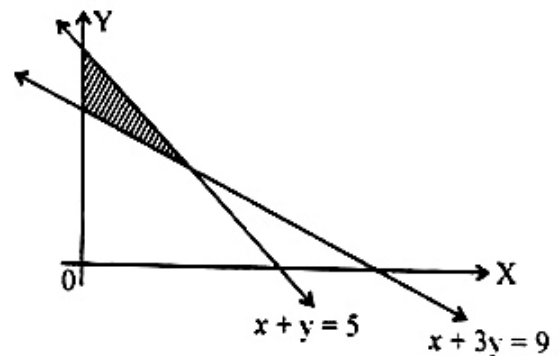
$\cos\gamma = \dfrac{1}{\sqrt{2}}$

$\gamma = \pi/4$

28. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $z = px + qy$, where p, q>0. Condition on p and q so that the minimum of z occurs at (3, 0) and (1, 1) is

a) $p = 2q$ b) $p = \dfrac{q}{2}$ c) $p = 3q$ d) $p = q$

Ans. b

Sol. z=px+qy

Z occurs minimum at (3, 0), (1, 1)

3p=p+q

2p=q

29. The feasible region of an LPP is shown in the figure. If $Z = 11x + 7y$, then the maximum value of Z occurs at

a) $(0,5)$ b) $(3,3)$ c) $(5,0)$ d) $(3,2)$

Ans. d

Sol. Corner points (0,3), (0,5), (3,2)

∴ maximum value at (3,2)

30. A die is thrown 10 times, the probability that an odd number will come up atleast one time is

a) $\dfrac{1}{1024}$ b) $\dfrac{1023}{1024}$ c) $\dfrac{11}{1024}$ d) $\dfrac{1013}{1024}$

Ans. b

Sol. $n = 10, p = \dfrac{3}{6} = \dfrac{1}{2}, q = \dfrac{1}{2}$

$p(x \geq 1) = 1 - p(x = 0)$

$$= 1 - \left[10C_0 \left(\frac{1}{2} \right)^{10} \right]$$

$$= 1 - \frac{1}{2^{10}}$$

$$= 1 - \frac{1}{1024}$$

$$= \frac{1023}{1024}$$

31. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$, then $P(A'/B)$ is

   a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{1}{12}$

Ans. a

Sol. $p(A'/B) = \frac{p(A' \cap B)}{p(B)}$

$$= \frac{p(B) - p(A \cap B)}{p(B)}$$

$$= \left(\frac{1}{2} - \frac{1}{6} \right) 2$$

$$= 2 \left[\frac{3-1}{6} \right] = \frac{2}{3}$$

32. Events E_1 and E_2 from a partition of the sample space S. A is any event such that $P(E_1) = P(E_2) = \frac{1}{2}$, $P(E_2 / A) = \frac{1}{2}$ and

   $P(A / E_2) = \frac{2}{3}$, then $P(E_1 / A)$ is

   a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) 1   d) $\frac{1}{4}$

Ans. a

Sol. $\frac{P(E_2 \cap A)}{P(A)} = \frac{1}{2}$, $\frac{P(A \cap E_2)}{P(E_2)} = \frac{2}{3}$

$$P(A) = \frac{2}{3}, \qquad P(A \cap E_2) = \frac{1}{3}$$

$$A \cap (E_1 \cup E_2) = A$$

$$P(A \cap E_1) + P(A \cap E_2) = P(A)$$

$$P(A \cap E_1) = \frac{1}{3}$$

$$p(E_1 / A) = \frac{P(E_1 \cap A)}{P(A)} = \frac{1/3}{2/3} = 1/2$$

33. The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is

   a) $\frac{1}{12}$ b) $\frac{1}{4}$ c) $\frac{1}{24}$ d) $\frac{1}{8}$

Ans. b

Sol. $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{3}$

   Probability of the problem solved by any two
$$= P(A)P(B)P(\bar{C}) + P(A)P(\bar{B})P(C) + P(\bar{A})P(B)P(C)$$

$$= \frac{1}{4}$$

34. If $n(A) = 2$ and total number of possible relations from Set A to set B is 1024, then $n(B)$ is

   a) 512   b) 20   c) 10   d) 5

Ans. d

Sol. $2 \times p \qquad 10$

   $2 = 1024 = 2$

   $2p = 10$

   $n(B) = p = 5$

35. The value of $\sin^2 51° + \sin^2 39°$ is

   a) 1   b) 0   c) $\sin 12°$ d) $\cos 12°$

Ans. a

Sol. $\sin^2 51 - \cos^2 39 + 1$

   $\cos(51 + 39)\cos(5) - 39) + 1$

   $\cos 90 \times \cos 1241 = 0 + = 1$

36. If $\tan A + \cot A = 2$, then the value of $\tan^4 A + \cot^4 A =$

   a) 2   b) 1   c) 4   d) 5

Ans. a

Sol. $\text{TanA} + \frac{1}{\text{TanA}} = 2$

   $(\text{TanA} - 1)^2 = 0$

   $\text{TanA} = 1, A = 45^0$

   $\text{Tan}^4 45^0 + \cos^4 . 45 = 2$

37. If $A = \{1,2,3,4,5,6\}$, then the number of subsets of A which contain atleast two elements is
a) 64   b) 63   c) 57   d) 58

Ans. c

Sol. $P(A) = 2^6 = 64$

$P(A) - 7 = 64 - 7 = 57$

38. If $z = x + iy$, then the equation $|z+1| = |z-1|$ represents
a) a circle   b) a parabola
c) x-axis   d) y-axis

Ans. d

Sol. $(x+1)^2 + y^2 = (x-1)^2 + y^2$

$4x = 0$

$x = 0, \text{ y} - \text{axis}$

39. The value of $^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is
a) 0   b) 1   c) $^{17}C_{10}$ d) $^{17}C_3$

Ans. a

Sol. $^nC_r + {}^nC_{r-1} = {}^{(n+1)}C_r$

$^{16}C_9 + {}^{16}C_{10} - \left({}^{16}C_6 + {}^{16}C_7\right)$

$^{17}C_{10} - {}^{17}C_7$

$^{17}C_{17-10} - {}^{17}C_7 = 0 \left(\because {}^nC_r = {}^nC_{n-r}\right)$

40. The number of terms in the expansion of $(x + y + z)^{10}$ is
a) 66   b) 142   c) 11   d) 110

Ans. a

Sol. $^{(n+r-1)}C_{r-1}$

$n = 10, r = 3$

$^{(10+3-1)}C_{31} = {}^{12}C_2 = \dfrac{12 \times 11}{2} = 66$

41. If $P(n): 2^n < n!$

Then the smallest positive integer for which P(n) is true if
a) 2   b) 3   c) 4   d) 5

Ans. c

Sol. $2^n < 4! \Rightarrow 16 < 24$

42. The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if
a) $ll' + mm' = 0$ b) $lm' = ml'$
c) $lm + l'm' = 0$ d) $lm' + ml' = 0$

Ans. a

Sol. $m_1 = -l/m \qquad m_2 = -l'/m'$

$m_1 \times m_2 = -1$

$\left(\dfrac{-l}{m}\right)\left(\dfrac{-l'}{m'}\right) = -1$

$ll' + mm' = 0$

43. If the parabola $x^2 = 4ay$ passes through the point (2, 1), then the length of the latus rectum is
a) 1   b) 4   c) 2   d) 8

Ans. b

Sol. $x^2 = 4ay \Rightarrow 2^2 = 4a \times 1 \Rightarrow a = 1$

$4a = 4 \times 1 = 4$

44. If the sum of n terms of an A.P is given by $S_n = n^2 + n$, then the common difference of the A.P is
a) 4   b) 1   c) 2   d) 6

Ans. c

Sol. $d = S_2 - 2S_1$

45. The negation of the statement "For all real numbers x and y, x + y = y + x" is
a) For all real numbers x and y, $x + y \neq y + x$
b) For some real numbers x and y, $x + y = y + x$
c) For some real number x and y, $x + y \neq y + x$
d) For some real numbers x and y, $x - y = y - x$

Ans. c

Sol. By concept

46. The standard deviation of the data 6, 7, 8, 9, 10 is
a) $\sqrt{2}$ b) $\sqrt{10}$ c) 2   d) 10

Ans. a

Sol. $6, 7, 8, 9, 10, \qquad \bar{x} = 8, n = 5$

$\text{S.D} = \sqrt{\dfrac{1}{n}\left(x_i^2\right) - x^{-2}} = \sqrt{66 - 64} = \sqrt{2}$

47. $\displaystyle\lim_{x \to 0}\left(\dfrac{\tan x}{\sqrt{2x+4} - 2}\right)$ is equal to
a) 2   b) 3   c) 4   d) 6

Ans. a

Sol. $\displaystyle\lim_{x \to 0}\dfrac{\tan x\left(\sqrt{2x+4} + 2\right)}{2x + 4 - 4}$

$$\lim_{x \to 0} \frac{\tan x}{x} \times \lim_{x \to 0} \frac{\left(\sqrt{2x+4}+2\right)}{2}$$

$$1 \times \frac{2+2}{2} = 2$$

48. If a relation R on the set {1, 2, 3} be defined by R={(1, 1)}, then R is
   a) Reflexive and symmetric
   b) Reflexive and transitive
   c) Symmetric and transitive
   d) Only symmetric

Ans. c

Sol. By concept

49. Let $f:[2,\infty) \to R$ be the function defined $f(x) = x^2 - 4x + 5$, then the range of f is
   a) $(-\infty, \infty)$ b) $[1, \infty)$ c) $(1, \infty)$ d) $[5, \infty)$

Ans. b

Sol. $y = f(x) = x^2 - 4x + 5$

$$= (x-2)^2 + 1$$

$$1 + (x-2)^2 \geq 0 + 1$$

$$y \geq 1 \Rightarrow y \in [1, \infty)$$

50. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $P(A) = 2P(B) = 3P(C)$, then $P(B)$ is equal to

   a) $\frac{1}{11}$ b) $\frac{2}{11}$ c) $\frac{3}{11}$ d) $\frac{4}{11}$

Ans. c

Sol. $\frac{P(A)}{6} = \frac{2P(B)}{6} = \frac{3P(C)}{6}$

$$\frac{P(A)}{6} = \frac{P(B)}{3} = \frac{P(C)}{2} = K$$

$$P(A) = 6K, \ P(B) = 3K, \ P(C) = 2(K)$$

$$P(A) + P(B) + P(C) = 1 \Rightarrow 11K = 1, \ K = \frac{1}{11}$$

$$P(B) = \frac{3}{11}$$

51. The domain of the function defined by $f(x) = \cos^{-1}\sqrt{x-1}$ is
   a) [1, 2]   b) [0, 2]   c) [-1, 1]   d) [0, 1]

Ans. a

Sol. $0 \leq \sqrt{x-1} \leq 1$

$$0 \leq (x-1) \leq 1$$

$$1 \leq x \leq 1+1 \Rightarrow [1,2]$$

52. The value of $\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right)$ is
   a) 0   b) 1   c) -0
   d) Does not exist

Ans. a

Sol. $\cos\left(\frac{\pi}{2}\right) = 0$ $\left(\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

53. If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then A^4 is equal to
   a) A   b) 2A   c) I   d) 4A

Ans. c

Sol. $A.A = A^2 = I$

$$A^4 = A^2.A^2 = I - I = I$$

$$A^4 = I$$

54. If $A = \{a, b, c\}$, then the number of binary operations on A is
   a) 3   b) 3^6 c) 3^3 d) 3^9

Ans. d

Sol. By Conception. n^{n^2}

$$= 3^{3^2} = 3^9$$

55. If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the matrix a is

   a) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

   b) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

   c) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

   d) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

Ans. b

Sol. $BA = I \Rightarrow B = A^{-1}$

56. If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$ then
   a) $f(1) = 0$ b) $f(2) = 0$
   c) $f(0) = 0$ d) $f(-1) = 0$

Ans. c

Sol. By substitution

57. If A and B are square matrices of same order and B is a skew symmetric matrix, then A'BA is
    a) Symmetric matrix
    b) Null matrix
    c) Diagonal matrix
    d) Skew symmetric matrix

Ans. d

Sol. $\left(A'BA\right)^{+} = (BA)^{+}(A')'$ $B^{+} = -3$

$= A'B^{T}A$

$= A'(-B)A$

$= -A'BA$

58. If A is a square matrix of order 3 and |A|=5, then |A adj.A| is
    a) 5   b) 125   c) 25   d) 625

Ans. b

Sol. $(Adj)|= A|^{n} = 5^{3} = 125$

59. If $f(x)\begin{cases} \dfrac{1-\cos Kx}{x\sin x}, & \text{If } x \neq 0 \\ \dfrac{1}{2}, & \text{If } x = 0 \end{cases}$ is continuous at x=0, then the value of K is

    a) $\pm\dfrac{1}{2}$ b) 0

    c) ± 2 d) ± 1

Ans. d

Sol. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x\sin x}$, $f(0) = \dfrac{1}{2}$

by L,Hoptial rule

$\dfrac{k\sin x}{x\cos x + \sin x} = \lim\limits_{x\to 0}\dfrac{k^{2}\cos x}{-x\sin x + \cos x + \cos x}$

$= \dfrac{k^{2}\times 1}{1+1} = \dfrac{1}{2}$

$k = \pm 1$

60. If $a_{1}a_{2}a_{3}......a_{9}$ are in A.P. then the value of $\begin{vmatrix} a_{1} & a_{2} & a_{3} \\ a_{4} & a_{5} & a_{6} \\ a_{7} & a_{8} & a_{9} \end{vmatrix}$ is

    a) $\dfrac{9}{2}(a_{1} + a_{9})$ b) $a_{1} + a_{9}$

    c) $\log_{e}(\log_{e} e)$ d) 1

Ans. c

Sol. By verification

$8,1,2.............9,\begin{vmatrix} a_{1} & a_{2} & a_{3} \\ a_{4} & a_{5} & a_{6} \\ a_{7} & a_{8} & a_{9} \end{vmatrix} 20$

$\log(\log e) = \log 1 = 0$