SRSI PU COLLEGE, BELLUR

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Presenting

II-PU PASSING PACKAGE EASY CAPSULES.

(As per Reduced syllabus 2020-21)

SOLVED MODEL QUESTION PAPER

FOR THE SUBJECT:

"MATHEMATICS"

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SRSI PU COLLEGE, BELLUR

MATHEMATICS (35)

II PUC Model Question Paper (1) with answers – 2021 Max Marks: 100

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i. The question paper has five parts namely A, B, C, D and E. Answer all the parts.

ii. ii. Use the graph sheet for the question on linear programming in PART – E.

PART – A

I. **Answer ALL the questions:** $\mathbf{10 \times 1 = 10}$

1. Define an empty relation.
   Sol: A relation R in a set A is called as empty relation, if no element of A is related to any element of A. $R = \phi \subset R \times R$.

2. Write the domain of the function $\boldsymbol{y} = \mathbf{sec}{-}\mathbf{1}\ \boldsymbol{x}$.

Sol: $R - (-1, 1)$

3. If a matrix has 5 elements, what are the possible orders it can have?
   Sol: 5 × 1, 1 × 5

4. Find the values of $\boldsymbol{x}$ for which $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

Sol:

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$
$$\Rightarrow x^2 - 36 = 36 - 36$$
$$\Rightarrow x^2 - 36 = 0$$
$$\Rightarrow x^2 = 36$$
$$\Rightarrow x = \pm 6$$

5. If $\boldsymbol{y} = \mathbf{tan}\ \sqrt{\boldsymbol{x}}$, find $\boldsymbol{dy}/\ \boldsymbol{dx}$.

Sol: $dy/\ dx$ = sec2 $\sqrt{x}$. $\frac{1}{2\sqrt{x}}$

6. Find $\int(\mathbf{2}x^2 + e^x\ )\boldsymbol{dx}$.

Sol: $\int(\ \mathbf{2}x^2 + e^x\ )dx = 2\,\frac{x^3}{3} + e^x + c$

7. Define a negative vector

Sol: A vector whose magnitude is same as the given vector but has the opposite direction is called the negative vector

8. If a line makes $\mathbf{90}°$, $\mathbf{135}°$ and $\mathbf{45}°$ with the x, y and z – axis respectively, find its direction cosines.

Sol: Let direction cosines be l, m and n. l = cos 90° = 0, m = cos 135° = − 1 √2 , n = cos 45° = 1 √2 ∴ Direction cosines are, 0, $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

9. Define Optimal solution in a linear programming problem.

Sol: Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

10. If $(\boldsymbol{A}) = \frac{7}{13}, (\boldsymbol{B}) = \frac{9}{13}$ and $\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \frac{4}{13}$, evaluate $\boldsymbol{P}(\boldsymbol{A}|\boldsymbol{B})$

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Sol: $(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{4}{9}$

PART – B

II. Answer any TEN questions: **$10 \times 2 = 20$**

11. Let $*$ be a binary operation on Q defined by $\boldsymbol{a} * \boldsymbol{b} = \frac{ab}{2}$, $\forall \boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{Q}$. Show that $*$ is associative.

Sol: Condition for associativity : (a $*$ b) $*$ c = a $*$ (b $*$ c) For all $a, b, c \in Q$,

LHS = (a $*$ b) $*$ c RHS = a $*$ (b $*$ c)

$= (\frac{ab}{2}) * c = a * (\frac{bc}{2})$

$= (\frac{ab}{2})\frac{c}{2} = \frac{a}{2}(\frac{bc}{2})$

LHS = $\frac{abc}{4}$ RHS = $\frac{abc}{4}$ $\therefore$ From LHS and RHS, (a $*$ b) $*$ c = a $*$ (b $*$ c)

12. Find the principal value of $\cot^{-1}(\frac{-1}{\sqrt{3}})$.

Sol: Let $\cot^{-1}(\frac{-1}{\sqrt{3}}) = y$

$\cot y = \frac{-1}{\sqrt{3}} = -\cot(\frac{\pi}{3}) = \cot(\pi - \frac{\pi}{3})$ $\cot y = \cot(\frac{2\pi}{3})$ $y = \frac{2\pi}{3} \in (0, \pi)$

13. Find the area of the triangle whose vertices are $(-\mathbf{2}, -\mathbf{3})$, $(\mathbf{3}, \mathbf{2})$ **and** $(-\mathbf{1}, -\mathbf{8})$ using determinants

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2+8)+3(3+1)+1(-24+2)]$$

$$= \frac{1}{2}[-2(10)+3(4)+1(-22)]$$

$$= \frac{1}{2}[-20+12-22]$$

$$= -\frac{30}{2} = -15$$

$\therefore$ Area of triangle = 15 sq. units

14. Find $\frac{dy}{dx}$, if $\boldsymbol{y} = \mathbf{cos}(\boldsymbol{logx} + e^x)$, $\boldsymbol{x} > \mathbf{0}$

Sol: $\boldsymbol{y} = \mathbf{cos}(\boldsymbol{logx} + e^x)$

$\frac{dy}{dx}$ =sin(log x + e^x)($\frac{1}{x} + e^x$)

15. Find $\frac{dy}{dx}$, if $\sin^2 x + \cos^2 x = 1$

Sol : $\sin^2 x + \cos^2 x = 1$

2sinxcosx + 2cosy(-siny) $\frac{dy}{dx}$ =0

2cosysiny $\frac{dy}{dx}$ =-2sinxcosx

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$\frac{dy}{dx} = \frac{2sinxcosx}{2sinycosy}$

$\frac{dy}{dx} = \frac{sin2x}{sin2y}$

16. If $\boldsymbol{y} = x^3 + \mathbf{tan}\ \boldsymbol{x}$, then find $\frac{dy^2}{dx^2}$.

Sol : $\boldsymbol{y} = x^3 + \mathbf{tan}\ \boldsymbol{x}$

$\frac{dy}{dx} = 3x^2 + sec^2x$

$\frac{dy^2}{dx^2}$ = 6x + 2secx(secxtanx)

$\frac{dy^2}{dx^2}$= 6x + 2sec^2xtanx

17. Find the slope of the tangent to the curve $\boldsymbol{y} = x^3 - \boldsymbol{x}$ at $\boldsymbol{x} = \mathbf{2}$.

Sol : $\boldsymbol{y} = x^3 - \boldsymbol{x}$

$\frac{dy}{dx}$= $3x^2$ -1

$\frac{dy}{dx}\Big|_{x=2}$ = 3(4) -1

$\frac{dy}{dx}_{x=2}$ = 11

18. Find $\int_0^\pi (sin^2\ \frac{x}{2}\ - cos^2\ \frac{x}{2})$

I =$\int_0^\pi (sin^2\ \frac{x}{2}\ - cos^2\ \frac{x}{2})$ dx

= $\int_0^\pi (-cosx)dx$

=-sinx$]_0^\pi$

=-[sinπ – sin0]

= -[0 -0]

=0

19. find $\int x\ sec^2x\ dx$

Sol : I = $\int x\ sec^2x\ dx$

$$I = x\int \sec^2 x\,dx - \int\left\{\left\{\frac{d}{dx}x\right\}\int \sec^2 x\,dx\right\}dx$$

$$= x\tan x - \int 1\cdot \tan x dx$$

$$= x\tan x + \log|\cos x| + C$$

20. Find the order and degree of the differential equation, y'''+ $2y''$ + y' =0.

Sol: Order: 3; Degree:

21. Find the projection of the vector $\vec{a} = \hat{\imath} + 3\ \hat{\jmath} + 7\ \hat{k}$ on the vector $\vec{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$.

Sol : projection on a and b

$\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{1.7+3(-1)+ 7.8}{\sqrt{49+1+64}}$

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. $= \frac{7-3+56}{\sqrt{114}}$ =

$= \frac{60}{\sqrt{114}}$

22. Find the area of parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$

Sol : Area of parallelogram = $|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$= \hat{\imath}(-1+21) - \hat{\jmath}(1-6) + k(-7+2)$

$\vec{a} \times \vec{b} = 20\hat{\imath} + 5\hat{\jmath} - 5k$

$= |\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2}$

$= \sqrt{450} = \sqrt{225 \times 2}$

$= 15\sqrt{2}$ units

23. Find the equation of the plane with intercept 2, 3 and 4 on x, y and z axes respectively.

Sol : let the equation of the plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Given that a =2, b=3, c=4 $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

6x + 4y+ 3z =12

24. Assume that each child born in a family is equally likely to be a boy or girl. If a family has two children, what is the conditional probability that both are girls, given that the youngest is a girl.

Sol: Let b and g represent the boy and girl child respectively. $S = \{(b,b), (b,g), (g,b), (g,g)\}$Let, A = Event that both children are girls. ∴ $A = \{(g,g)\}$ Let B = Event that the youngest is a girl. ∴ To find: $(A|B)$ =? $A \cap B = \{(g,g)\}$

P(A) $= \frac{1}{4}$, P(B) $= \frac{1}{2}$, P(A ∩ B) $= \frac{1}{4}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{1/4}{1/2} = \frac{1}{2}$

PART-C

III. Answer any ten questions $10 \times 3 =$

25. Show that the relation R defined in the set A of all triangle asR = $\{(T_1, T_2): T_1 \textit{ is similar to } T_2\}$ is equivalence relation.

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Sol. Reflexive: $\forall$ T1 $\in$ A

T1~T1$\Rightarrow$ $(T_1, T_1) \in R$ ∴ R is reflexive relation

Symmetric: For any $T_1, T_2 \in A$

$\Rightarrow T_1 \sim T_2 \Rightarrow T_2 \sim T_1$

$\Rightarrow (T_2, T_1) \in R$ ∴ R is symmetric relation.

Transitive: For any $T_1, T_2, T_3 \in A$

If $(T_1, T_2) \in R$ & $(T_2, T_3) \in R$

$\Rightarrow T_1 \sim T_2$ & $T_2 \sim T_3$

$\Rightarrow T_1 \sim T_3$

$\therefore (T_1, T_3) \in R$ ∴ R is transitive relation

∴ R is equivalence relation.

26. For the matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

$(\mathbf{A} + \mathbf{A}')$ is a symmetric matrix

$(\mathbf{A} - \mathbf{A}')$ is a skew matrix

Sol : A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ A' = $\begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

TO SHOW THAT : (A + A') IS SYMMETIC MATRIX

Let P = A+A'

$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

P = $\begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

P' = $\begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

P=P'

TO SHOW THAT : Q = (A-A')

Let Q = (A –A')

$= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

Q= $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Q' = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q=-Q'

27. If x=2at^2 and y = at^4 , find $\frac{dy}{dx}$

Sol : x=2at^2

$\frac{dx}{dt}$ = 4at

y = at^4

$\frac{dy}{dt}$ = 4at^3

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{4at^3}{4at}$

$\frac{dy}{dx} = t^2$

28. find $\frac{dy}{dx}$, if $x^y = y^x$

sol : $x^y = y^x$

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Apply log both sideslog x^y = log y^x

y log x = x log y

$y \frac{1}{x} + \log x \frac{dy}{dx} = x \frac{1}{y} + \log y .1$

$\log x \frac{dy}{dx} = \frac{x}{y} + \log y - \frac{y}{x}$

$\frac{dy}{dx} = \frac{\frac{x}{y} + \log y - \frac{y}{x}}{\log x}$

29. **Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is**

strictly increasing (b) strictly decreasing.

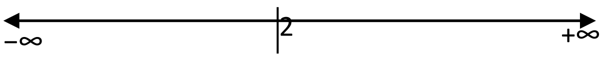
Sol. $f(x) = x^2 - 4x + 6$ $f'(x) = 2x - 4$

Let $f'(x) = 0$

=> $2x - 4 = 0$

$2x = 4$

$x = 2$



x = 2 divides the real line into two disjoint intervals, $(-\infty, 2)$ & $(2, \infty)$.If $x \in (-\infty, 2)$, $f'(x) = 2x - 4 < 0$

If $x \in (2, \infty)$, $f'(x) = 2x - 4 > 0$

∴ f is strictly decreasing in $(-\infty, 2)$ and strictly increasing in $(2, \infty)$

30. evaluate : $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$

Sol : $\mathbf{I} = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^3 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^3 x}{\sin^2 x \cos^2 x} dx$

$= \int \tan x . \sec x \, dx + \int \cot x . \operatorname{cosec} x \, dx$

$I = \sec x - \operatorname{cosec} x + c$

31.find = $\int \frac{(x-3)}{(x-1)^3} e^x \, dx$

$I = \int \frac{(x-1-2)}{(x-1)^3} e^x \, dx$

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$= \int [\frac{(x-1)}{(x-1)^3} + \frac{-2}{(x-1)^3}] e^x dx$

$= \int [\frac{1}{(x-1)^2} + \frac{-2}{(x-1)^3}] e^x dx$

f(x) = $\frac{1}{(x-1)^2} = (x-1)^{-2}$

f ’(x) = -2$(x-1)^{-3}$

f ’(x) = $\frac{-2}{(x-1)^3}$

We have, $\int [f(x) + f'(x)]e^x dx = e^x|(x) + c|$

I= $\frac{1}{(x-1)^2} e^x$+C

32.Evaluate : $\int \frac{1}{(x+1)(x+2)} dx$

Sol : consider $\frac{1}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

Multiply by (x+1)(x+2)

1= A(x+2) + B(x+2)

If x = − 2 ⇒ 1 = B(−1) If x = − 1 ⇒ 1 = A (1) B = − 1 A = 1

$\int \frac{1}{(x+1)(x+2)} dx = \int \frac{1}{(x+1)} dx + \int \frac{-1}{(x+2)} dx$

= log|x+1| + log|x+2| +C

= log$\left|\frac{x+1}{x+2}\right|$+C

33. **Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y – axis in the firstquadrant.**

Sol. $x^2 = 4y$

x = $\pm\sqrt{4y}$

x= +2 $\sqrt{y}$ [I quad]

Area = $\int_2^4 \sqrt{4y}\, dy$

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= $2\int_2^4 y^{1/2}\, dy$ $=2\frac{y^{3/2}}{3/2}\Big]_2^4$

$= \frac{4}{3}[4^{\frac{3}{2}} - 2^{\frac{3}{2}}]$

$= \frac{4}{3}[8-2\sqrt{2}]$

Area = $\frac{8}{3}$ $(4-\sqrt{2})$sq units

34. Solve : $\frac{dy}{dx} = e^{x+y}$

Sol : $\frac{dy}{dx} = e^x . e^y$

$\frac{dy}{e^y} = e^x$ dx

e^{-y} dy = e^x dx

Integrate on both sides

$\int e^{-y}dy = \int e^x\, dx$

$-e^{-y} = e^x + c$ $e^x + e^{-y} + c = 0$

$e^x + \frac{1}{e^y}$ +c =0

35. Let $\vec{a}$, $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ be three vectors such that $|\vec{\mathbf{a}}| = 3$, $|\vec{\mathbf{b}}| = 4$, $|\vec{\mathbf{c}}| = \mathbf{5}$ and each one of them beingperpendicular to the sum of the other two, find $|\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$.

Given: $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$

$\vec{a} \perp (\vec{b} + \vec{c})$

$\Rightarrow \vec{a}.(\vec{b} + \vec{c}) = 0$

$\Rightarrow \vec{a} . \vec{b} + \vec{a} . \vec{c} = 0$

$\vec{b} \perp (\vec{c} + \vec{a})$

$\Rightarrow \vec{b} . (\vec{c} + \vec{a}) = 0$

$\Rightarrow \vec{b} . \vec{c} + \vec{b} . \vec{a} = 0$

$\vec{c} \perp (\vec{a} + \vec{b})$

$\Rightarrow \vec{c} . (\vec{a} + \vec{b}) = 0$ $\Rightarrow \vec{c} . \vec{a} + \vec{c} . \vec{b} = 0$ To find : $|\vec{a} + \vec{b} + \vec{c}| = ?$

Consider $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\overrightarrow{\vec{a}.\vec{b}} + 2\overrightarrow{\vec{b}.\vec{c}} + 2\overrightarrow{\vec{c}.\vec{a}}$

$= 3^2 + 4^2 + 5^2 + \vec{a} . \vec{b} + \vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{b} . \vec{c} + \vec{c} . \vec{a} + \vec{c} . \vec{a}$

$= 9 + 16 + 25 + (\vec{a} . \vec{b} + \vec{c} . \vec{a}) + (\vec{a} . \vec{b} + \vec{b} . \vec{c}) + (\vec{c} . \vec{a} + \vec{b} . \vec{c})$

= 50 + 0 + 0 + 0 [given]

$|\vec{a} + \vec{b} + \vec{c}|^2 = 50$ $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$

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36. Show that the points $(-2\mathbf{i}+3\mathbf{j}+5\hat{\mathbf{k}})$, $\mathbf{B}(\mathbf{i}+2\mathbf{j}+3\hat{\mathbf{k}})$ and $\mathbf{C}(7\mathbf{i}-\hat{\mathbf{k}})$ are collinear

Sol. $\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + 2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ $\qquad$ $\overrightarrow{BC} = 7\hat{\imath} - \hat{k} - \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$\overrightarrow{AB} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ $\qquad$ $\overrightarrow{BC} = 6\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$

$\overrightarrow{BC} = 2\overrightarrow{AB}$

A,B,C are collinear

37. Find the equation of the plane through the intersection of the planes $\mathbf{3x - y + 2z - 4 = 0}$, $\mathbf{x + y + z - 2 = 0}$ and the point $(\mathbf{2}, \mathbf{2}, \mathbf{1})$.

Sol. Equation of any plane through intersection of planes 3 x − y + 2 z − 4 = 0 is

$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0 \rightarrow (1)$ where $\lambda \in R$

(1)passes through(2, 2, 1), $(6 - 2 + 2 - 4) + \lambda(2 + 2 + 1 - 1) = 0$ $2 + \lambda(3) = 0$

$\lambda = \frac{-2}{3}$

Substitute value of λ in (1), $(3x - y + 2z - 4)\frac{-2}{3}(x + y + z - 2) = 0$

➧ $9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$

$7x - 5y + 4z - 8 = 0$

38. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Sol. Let E : Man reports that six occurs in throwing a die.

S1:Six occurs

S2:Six doesnot occurs

$P(S1) = \frac{1}{6}$ $\qquad$ $P(S2) = \frac{5}{6}$

P(E/S1) = Probability that man reports that six occurs when six has actually occurred on die

= Probability that the man speaks the truth

$P(E/S1) = \frac{3}{4}$

P(E/S2) = Probability that the man does not speak truth = $1 - \frac{3}{4}$

$P(E/S2) = \frac{1}{4}$

$$P(S1/E) = \frac{P(S1)P(\frac{E}{S1})}{P(S1)P\left(\frac{E}{S1}\right)+P(S2)P(\frac{E}{S2})}$$

$$= \frac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}$$

$$\frac{1}{8}\times\frac{24}{8}$$

$P(S1/E) = \frac{3}{8}$

∴ Probability that the report of the man that six has occurred is actually a six is $\frac{3}{8}$

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PART – D

IV. Answer any SIX questions: **$6 \times 5 = 30$**

39. Check the injectivity and surjectivity of the function f : R → R defined by $\mathbf{f(x) = 3 - 4x}$. Is it a bijective function.

Sol. Injectivity [One-One]: ∀ x1, x2 ∈ R If f (x1) = f (x2)

3 − 4 x1 = 3 − 4 x2

−4 x1 = −4 x2

⇒ x1 = x2 ∴ f is injective function

Surjectivity [Onto] : For y ∈ R [Co − domain], ∃ x ∈ R such that f(x) = y

Consider, f(x) = y

3 − 4x = y

−4x = y − 3

4x = 3 − y

x = $\frac{3-y}{4}$ ∈ R [Domain]

Verification: f (x) = 3- 4 x

f ($\frac{3-y}{4}$) = 3 − 4 $\frac{3-y}{4}$

= 3 − 3 + y

f (3 − y 4) = y

∴ f is surjective function.

∴ f is bijective function.

40. if A = $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-3} \\ \mathbf{5} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$, B = $\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix}$ and C = $\begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \end{bmatrix}$ then compute A+B and B-C. Also verify that A+(B-C) = (A+B)-C

A +B = $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-3} \\ \mathbf{5} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ + $\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix}$ = $\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & \mathbf{4} \end{bmatrix}$

B-C = $\begin{bmatrix} \mathbf{3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{4} & \mathbf{2} & \mathbf{5} \\ \mathbf{2} & \mathbf{0} & \mathbf{3} \end{bmatrix}$ - $\begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \end{bmatrix}$ = $\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & \mathbf{0} \end{bmatrix}$

LHS = A+(B-C)

= $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-3} \\ \mathbf{5} & \mathbf{0} & \mathbf{2} \\ \mathbf{1} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$ + $\begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & \mathbf{0} \end{bmatrix}$

LHS= $\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-3} \\ \mathbf{9} & \mathbf{-1} & \mathbf{5} \\ \mathbf{2} & \mathbf{1} & \mathbf{1} \end{bmatrix}$

RHS = (A+B)-C

$\begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & \mathbf{4} \end{bmatrix}$ - $\begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{2} \\ \mathbf{0} & \mathbf{3} & \mathbf{2} \\ \mathbf{1} & \mathbf{-2} & \mathbf{3} \end{bmatrix}$

RHS = $\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{-3} \\ \mathbf{9} & \mathbf{-1} & \mathbf{5} \\ \mathbf{2} & \mathbf{1} & \mathbf{1} \end{bmatrix}$

LHS = RHS

A+(B-C) = (A+B)-C

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41. Solve the system of equations by matrix method: $2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$.

Sol:

The given system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 2(5) - 3(-5) + 3(5) = 10 + 15 + 15 = 40 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 5, A_{12} = 5, A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = 11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$\therefore A^{-1} = \frac{1}{|A|}(adjA) = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 2,$ and $z = -1$.

42. If $\boldsymbol{y} = (\tan^{-1} x)^2$, show that $(1 + x^2)^2\, y_2 + 2x(1 + x^2)y_1 = \mathbf{2}$.

Sol: $\boldsymbol{y} = (\tan^{-1} x)^2$

$y_1 = 2\tan^{-1} x . \frac{1}{1+x^2}$

$(1 + x^2)y_1 = 2\tan^{-1} x$

$(1 + x^2)\, y_2 + y1\,(0 + 2x) = 2\,(\frac{1}{1+x^2})$

$(1 + x^2)\, y_2 + 2xy_1 = 2\,(\frac{1}{1+x^2})$

$(1 + x^2)^2\, y_2 + 2x(1 + x^2)y_1 = \mathbf{2}$.

Hence proved.

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43. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute. When x = **10** $\boldsymbol{cm}$ and y = **6** $\boldsymbol{cm}$, find the rates of change of (a) the perimeter (b) the area of the rectangle.

Sol: Given: $\frac{dx}{dt}$ = −3cm/min $\quad$ $\frac{dy}{dt}$ = 2cm/min

(a) $\frac{dP}{dt}$=? $\quad$ (b) $\frac{dA}{dt}$=? at x = 10cm and y = 6cm

(a) Perimeter of rectangle, $\boldsymbol{P = 2x + 2y}$

$P = 2x + 2y$

$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$

= 2(−3) + 2(2)

= −6 + 4

$\frac{dP}{dt}$= −2 cm/min

(b) Area of rectangle, $\boldsymbol{A = xy}$

$A = xy$

$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$

= 10(2) + 6(−3)

= 20 − 18

$\frac{dA}{dt}$ = 2 cm2 /min

∴ The perimeter of rectangle is decreasing at the rate of 2 cm/min (Negative sign shows the decrease) The area of rectangle is increasing at the rate of 2 cm^2/min.

44. Find the integral of $\frac{1}{\sqrt{a^2-x^2}}$ with respect to x and hence evaluate $\frac{1}{\sqrt{9-25x^2}}$

Sol : I = $\frac{1}{\sqrt{a^2-x^2}}$

Let x= asin θ, then dx= acos $\theta\ d\theta$

Therefore $\int \frac{1}{\sqrt{a^2-x^2}}dx = \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2-a^2\sin^2\theta}}$

$= \int d\theta = \theta + C = \sin^{-1}\frac{x}{a}$ +C

$\int \frac{1}{\sqrt{9-25x^2}} = \int \frac{1}{\sqrt{\frac{9}{25}-x^2}}$

$= \frac{1}{5}\int \frac{1}{\sqrt{\frac{9}{25}-x^2}} = \frac{1}{5}\sin^{-1}\frac{x}{\frac{3}{5}}$ +c

$= \frac{1}{5}\sin^{-1}\frac{5x}{3}$ +c

45. Using the method of integration, find the area enclosed by the circle $x^2 + y^2 = a^2$

Sol: $x^2 + y^2 = a^2$

$y^2 = a^2 - x^2$

$y = \pm\sqrt{a^2 - x^2}$

$y = +\sqrt{a^2 - x^2}$[I quadrant]

Area = 4(Area of OABO)

$= 4\int_0^a |y|dx$

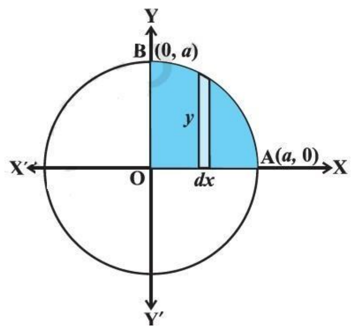
$= 4\int_0^a \sqrt{a^2 - x^2}dx$

$= 4[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}]_0^a$

$= 4[(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}) - (0+0)]$

$= 4[\frac{a^2}{2}(\frac{\pi}{2})]$

Area = πa^2 sq units



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46. Find the general solution of the differential equation x$\frac{dy}{dx}$ + $\mathbf{2y}$ = x^2 ($\mathbf{x \neq 0}$)

Sol. x$\frac{dy}{dx}$ + $\mathbf{2y}$ = x^2 (Divide by x)

$\frac{dy}{dx}$ + $\frac{2y}{x}$ = x

= x $\frac{dy}{dx}$+ Py = Q

I.F = $e^{\int Pdx}$

= $e^{\int \frac{1}{x} dx}$ = e^{logx^2}

I.F x^2

General solution,

y (I. F) = ∫ Q (I. F) dx + c

y(x^2) = ∫ x(x^2) dx + c

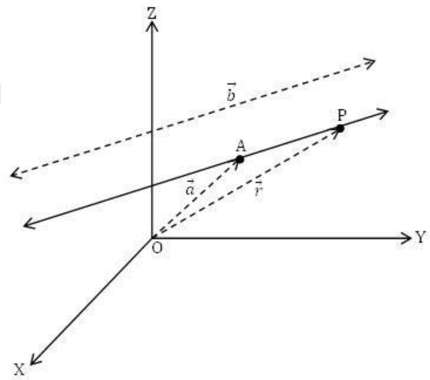
x^2 y = ∫ x^3dx + c

x^2 y= $\frac{x^4}{4}$+ c

$\frac{x^4}{4}$ - x^2 y +c=0

47. Derive the equation of the line in space passing through a given point and parallel to a given vector both in vector and Cartesian form

Sol: Given:



$A -$ given point and $\overrightarrow{OA} = a$

$l -$ line which passes through A

$P -$ arbitrary point on l

$\vec{b} -$ parallel vector to$\overrightarrow{AP}$

Vector form:

$r =$ Position vector of an arbitrary point on the line.

$a =$ Position vector of given point with respect to origin.

$\vec{b} =$ Vector parallel to the line which passes through given point.

$\lambda =$ Some real number.

Proof : $\overrightarrow{AP} = \vec{b}$

$\overrightarrow{AP} = \lambda \vec{b}$

$\overrightarrow{OP} - \overrightarrow{OA} = \lambda \vec{b}$

$\vec{r} - \vec{a} = \lambda \vec{b}$

$\vec{r} = \vec{a} + \lambda \vec{b}$

Cartesian form :

Given point A(x_1, y_1, z_1). Directional ratios of line l are a, b, c. P(x,y,z) be arbitray point.

$\vec{\boldsymbol{a}} = x_1 \hat{\boldsymbol{i}} + y_1 \hat{\boldsymbol{j}} + z_1 \hat{\boldsymbol{k}}$ $\vec{\boldsymbol{r}} = x \hat{\boldsymbol{i}} + y \hat{\boldsymbol{j}} + z \hat{\boldsymbol{k}}$

$\vec{\boldsymbol{b}} = a \hat{\boldsymbol{i}} + b \hat{\boldsymbol{j}} + c \hat{\boldsymbol{k}}$

Wkt, $\vec{r} = \vec{a} + \lambda \vec{b}$

$x \hat{\boldsymbol{i}} + y \hat{\boldsymbol{j}} + z \hat{\boldsymbol{k}} = (x_1 \hat{\boldsymbol{i}} + y_1 \hat{\boldsymbol{j}} + z_1 \widehat{\boldsymbol{k})} + \lambda (a \hat{\boldsymbol{i}} + b \hat{\boldsymbol{j}} + c \hat{\boldsymbol{k}})$

$x \hat{\boldsymbol{i}} + y \hat{\boldsymbol{j}} + z \hat{\boldsymbol{k}} = (x_1 + \lambda a) \hat{\imath} + (y_1 + \lambda b) \hat{\jmath} + (z_1 + \lambda c) \hat{k}$

$x = (x_1 + \lambda a)$ $y = y_1 + \lambda b$ $z = z_1 + \lambda c$

$x - x_1 = \lambda a$ $y - y_1 = \lambda b$ $z - z_1 = \lambda c$

$\frac{x - x_1}{a} = \lambda$ $\frac{y - y_1}{b} = \lambda$ $\frac{z - z_1}{c} = \lambda$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

48. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ & $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i)the problem is solved ii) exactly one of them solves the p

$P(A) = \frac{1}{2}$

P(B) = Probability of solving problem by B

$P(B) = \frac{1}{3}$

Given: A & B solve the problem independently.

$$\therefore P(AB) = P(A)P(B) = \frac{1}{2} \times \frac{1}{3}$$

$$P(AB) = \frac{1}{6}$$

$P(A') = 1 - P(A)$ $P(B') = 1 - P(B)$

$= 1 - \frac{1}{2}$ $= 1 - \frac{1}{3}$

$P(A') = \frac{1}{2}$ $P(B') = \frac{2}{3}$

Probability of that problem is solved,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$= P(A) + P(B) - P(AB)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$

$= \frac{3+2}{6} - \frac{1}{6}$

$= \frac{5}{6} - \frac{1}{6}$

$= \frac{4}{6}$ $\quad P(A \cup B) = \frac{2}{3}$

i) Probability that exactly one of them solves the problem, P (A). P (B′) + P (B). P (A′)

$= \frac{1}{2} \times \frac{2}{3} + \frac{1}{2} \times \frac{1}{3}$

$= \frac{2}{6} + \frac{1}{6}$

$= \frac{3}{6}$

$= \frac{1}{2}$

PART – E

V. Answer any ONE question: 1 × 10 = 10

49. Prove that $\int^{a} \mathbf{f(x)dx} = \int^{a} \mathbf{f(a-x)}$dx and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$ dx

Sol :

<u>To prove that</u>: $\int_0^a f(x)dx = \int_0^a f(a-x)$

Proof: Let $t = a - x$

$dt = -dx$

$-dt = dx$ When x = 0 ⇒ t = a

x =a ⇒ t=0

$\therefore \int_0^a f(a-x)dx = \int_a^0 f(t) - dt$

$= -\int_0^a f(t)dt$ $\left[\because \int f(x)\,dx = -\int f(x)\,dx\right]$

$\therefore \int_0^a f(a-x)dx = \int_0^a f(x)$

$\left[\because \int f(x)\,dx = \int f(t)\,dt\right]$

hence proved

To evaluate : $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ⟶ (1)

$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$

$\int_0^a f(x)\,dx = \int_0^a f(a-x)$

$\int_0^a \frac{\sqrt{a-ax}}{\sqrt{a-x}+\sqrt{a-(a-x)}}$

$\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}$ (2)

adding equation (1) and (2)

$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$

$= \int dx$

$= x]_0^a$

$2I = a-0$

$I = \frac{a}{2}$

b) find the value of k if $f(x) = \begin{cases} kx+1, & x \le \pi \\ \cos x, & x > \pi \end{cases}$ is continuous at x = π

Sol: f is continuous at x = π

LHL = RHL = Value

$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$

$\lim_{x\to\pi^-} kx+1 = \lim_{x\to\pi^-} \cos x$ kx +1

k(π) +1 = cos π

k π = 0 -1

$k = \frac{-1}{\pi}$

50. a) Maximize z = 4x + y subject to the constraint x + y ≤50 , 3x + y ≤90 , $x \geq 0$, $y \geq 0$ by graphical method

Sol: Convert inequalities to equalities.

$* \; x + y = 50 \rightarrow (1)$

x	0	50
y	50	0

Put $(0,0)$ in $x + y \leq 10$

$\Rightarrow 0 \leq 10$ (True)

Shaded region towards origin.

$* \; 3x + y = 90 \rightarrow (2)$

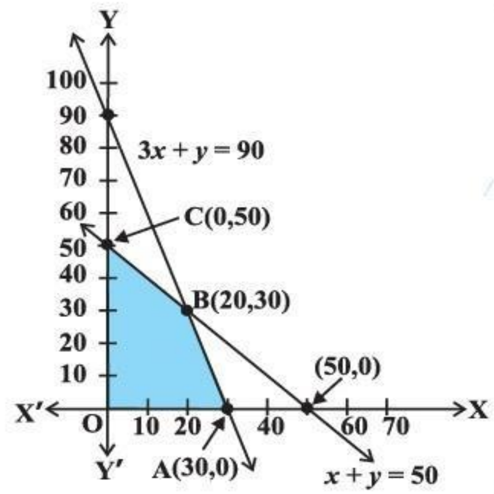
x	0	30
y	90	0

Put $(0,0)$ in $3x + y \leq 90$

$\Rightarrow 0 \leq 90$ (True)

Shaded region towards origin.

$* \; x, y \geq 0 \Rightarrow$ Feasible region lies in first quadrant.



Corner Point	Corresponding value of Z	
(0, 0)	0	
(30, 0)	**120** ←	Maximum
(20, 30)	110	
(0, 50)	50	

The maximum value of $z = 120$ at $(30, 0)$.

Sol:

b) if A $= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, satisfies the equation A^2 – 5A + 7I =0 , then find the inverse of A using this equation where I is the identity matrix of order 2.

Sol:

=6+1

|A| =7 $\neq$ 0

$\therefore$ A is non-singular $\therefore$ A^{-1} exists

$$A^2 - 5A + 7I = 0$$

$$A.A - 5A = -7I$$

Post multiply by A^{-1}

$$A.(AA^{-1}) - 5AA^{-1} = -7IA^{-1}$$

$$AI - 5I = -7A^{-1}$$

$$\frac{1}{-7}(A - 5I) = A^{-1}$$

$$A^{-1} = \frac{-1}{7}\left(\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}$$