INDEX

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit 1: Arithmetic Progression

I. Multiple choice questions.

1) The n^{th} term of an A.P whose first term 'a' and common difference 'd' is

A) $a+(n+1)d$ B) $a+(n-1)d$ C) $a-(n+1)d$ D) $a-(n-1)d$

2) Which of the following list of numbers is an A.P?

A) 1,3,6,8---- B) 1,4,9---- C) 2,4,8,16---- D) 1,3,5,7---- (common difference is 2)

3) The common difference of the A.P 2, 0, −2, −4,----

A) 0 B) 2 C) −2 D) −4 (0-2=-2-0=-4+2=-2)

4) The sum of first 'n' natural numbers of an A.P is

A) $\frac{n(n+1)}{2}$ B) $\frac{n(n-1)}{2}$ C) n^2 D) $n(n+1)$

5) The value of 'x' if 7, x, 23 are in A.P is

A) 30 B) 18 C) 15 D) 9 (X=7+15/2=30/2=15)

6) If the nth term of an A.P is $a_n= 8-3n$, then its common difference is

A) −5 B) −3 C) 3 D) 5

7) The 13^{th} term of an A.P whose first term and common difference respectively are $\frac{3}{2}$ and $\frac{2}{3}$ is

A) $\frac{6}{5}$ B) $\frac{11}{2}$ C) $\frac{17}{2}$ D) $\frac{19}{2}$

8) The result obtained on making half the sum of 7^{th} and 9^{th} term of an A.P is

A) 6^{th} term B) 8^{th} term C) 10^{th} term D) common difference

9) In an A.P the first term is 'm' and common difference is 2m then its 5^{th} term is

A) 5m B) 8m C) 9m D) 10m (a_5=a+4d=m+4x2m=m+8m=9m)

10) In an A.P first term is 'a' and common difference is 'd' the correct relation in the following is

A) $a_6 = a_4 + 4d$ B) $a_8 = a_5 + 3d$ C) $a_{10} = a_3 + 4d$ D) $a_5 = a_3 + d$

II. One mark questions.

1)In an A.P if $S_{10} = 35$ and $S_9 = 28$ find a_{10}.

Ans:- $a_{10}=S_n-S_{n-1}=S_{10}-S_{10-1}=S_{10}-S_9=35-28=7$

2) Find the sum of first 25 odd natural numbers.

Ans:-The number series 1, 3, 5, 7, 9,. . . . 49.

The first term a = 1

The common difference d = 2

Total number of terms n = 25

$S_n = n/2 \times (a + a_n)$

$= 25/2 \times (1 + 49)$

$= (25 \times 50)/ 2$

$= 1250/2= 625$

3) Find the common difference of the A.P 1, −1, −3, −5 ----- Ans:- $d=a_2-a_1=-1-1=-2$

4) Write the formula used to find the sum of first 'n' terms of the A.P whose first term 'a' and common difference 'd'. Ans:- $S_n=n/2\{a+(n-1)d\}$

5) Find the 20^{th} term of the A.P 12, 7, 2-------

$\longrightarrow a = 12$
$\longrightarrow d = 7 - 12 = -5$
$\Longrightarrow T_n = 12 + (n-1)(-5)$
$\Longrightarrow T_n = 12 - 5n + 5$
$\Longrightarrow T_n = 17 - 5n$
$\Longrightarrow T_{20} = 17 - 5(20)$
$\Longrightarrow T_{20} = 17 - 100$
$\Longrightarrow \boxed{T_{20} = -83}$

• Hence , The 20th term of A.P. is -83.

6) In an A.P first term is 'K' and common difference is 'm'. Find its $(n-3)^{rd}$ terms.

Ans:-k+(n-4)m

7) The interior angles of a triangle are in A.P in which the first term and common differences are equal. Find the measure of bigger angle if the smaller one is 30^o.

Ans:- sum of all n interior angles of triangle = 180°

the angles are in A. P. with the smallest angle = 30°

Common difference = $d=a=30^0$ the angles are a, a + d and a+2d

The bigger angle=a+2d=30+2x30=30+60=90^0

8) Find the sum of first 10 terms of an A.P in which the half of the sum of first and last term is 80.

Ans:- (a+l)/2 =80

a+l =160

S_{10} = n/2(a+l)=10/2(a+l) =5x160 =800

III. Two mark questions.

1) Which term of the A.P 3,8,13,18,------- is 78 .

a = 3, d = 8 – 3 = 5, an = 78, n =?

an = a + (n – 1) d

78 = 3+(n—1)(5) = 3 + 5n – 5 = 5n – 2

5n = 78 + 2 ∴ n = 80/5 ∴ n = 16

2) How many two-digit numbers which are divided by 3.

We know, first two digit number divisible by 3 is 12 and last two digit number divisible by 3 is 99. Thus, we get 12,15,18,...,99 which is an AP
Here, a=12,d=3
Let there be n terms. Then,

an=99
a+(n−1)d=99
12+(n−1)3=99
n=29+1=30
Therefore, two digit numbers divisible by 3 are 30.

3) Find the 20^{th} term from the last term of the A.P 3,8,13 ----253.

a_n=253, d=−5
∴a_{20}=253+19d=253−19(5)=253−95=158.

4) Find the sum of first 20 terms of the A.P 1,4,7 ---

Sn=n/2[2a+(n−1)d]
S_{24}=24/2[2(5)+(24−1)3]=12[10+23×3]=948

5) Find the sum of first 15 terms of the A.P having the n^{th} term is 3+4n.

n^{th} term = 3+4 n
1st term = 3+4(1)=7, 2nd term = 3+4(2)=11, 3rd term 3+4(3)=15
7, 11, 15 ....

a=7, d=4 n=15
Sn=n/2[2a+(n−1)d]
$S_{15=}$15/2[2(7)+14(4)]=15/2x2[7+28]=525

6) Find the sum of first 15 positive integers are divisible by 6 (using formula)

First forty positive integers which are divisible by 6 are
6,12,18,24,.... to 40 terms
Here, a=6,d=12−6=6, and n=40.
S_{40}=n/2[2a+(n−1)d]
=40/2[2×6+(40−1)×6]=20[12+39+6]=20[12+234]=20×246=4920.

7) The angles of a triangle are in an A.P. The smallest angle is 30^o. Find all the angles of a triangle.

It is given that the smallest angle is 30^0.

Then other angle are 30^0+d and 30^0+2d.

Since, in a triangle, the sum of three interior angles is 180^0, therefore,
$30^0+30^0+d +30^0+2d=180^0, \Rightarrow 90^0+3d=180^0 \Rightarrow 3d=180^0-150^0 \Rightarrow 3d=30^0 \Rightarrow d=10^0$
Now, the other angles are:
$a=30^0$ $30^0+d=30^0+30^0=60^0$ and $30^0+2d=30^0+2(30^{0)} = 30^0+60^0= 90^0$

8) The sum of 20 terms of an A.P is 820. If the first term is 3. Find the common difference.

S_n= n/2[2a+(n-1)d]

S_n=820,n=20,a=3

820=20/2[2×3+(20-1)d]

820=10[6+19d] =6+19d =82-6=19d

76=19d

d=4

9) If 2x, x + 10, 3x + 2 are in an A.P. Find the value of x.

2x, x+10, 3x+2 are in A.P

=> (x+10) - 2x = (3x+2) - (x+10) [the common difference!]

=> -x + 10 = 2x - 8

=> 3x = 18

=> x = 6

IV. Three mark questions.

1) Find the A.P whose third term is 16 and 7th term exceeds the 5th term by 12.

Now , $a_3 = 16$

a + 2d = 16 ------ equation 1

Also , $a_7 = a + 6d$ $a_5 = a + 4d$

▸ Now , it is given that 7th term exceeds the 5th term by 12 .

➡ $a_7 - a_5 = 12$

➡ a + 6d - (a + 4d) = 12

➡ a + 6d - a - 4d = 12

➡2d = 12 = 12/2d = 6

☯ Substitute the values in equation 1 now ,

➡ a + 2 × 6 = 16

➡ a + 12 = 16 = 16 - 12 = 4

A.P. = a, + a + d, a + 2d, + a : + 3d, + ...

A.P. = 4, 10, 16 , 22, ...

2) If 10 times the 10th term of an A.P is equal to 15 times the 15th term. Show that 25th term of the A.P is zero.

$10a_{10} = 15a_{15}$

10[a + (10 – 1)d] = 15[a + (15 – 1)d]

[an = a + (n - 1)d]

10[a + 9d] = 15[a + 14d]

2[a + 9d] = 3[a + 14d]

[Dividing by 5 on both sides]

2a + 18d = 3a + 42d

2a - 3a = 42d - 18d

- a = 42d – 18d

-a = 24d

a = - 24d ………….(1)

25th term :

an = a + (n - 1)d

a_{25} = a + (25 - 1)d = a + 24d = -24d + 24d [From eq 1]

a25 = 0 (zero)

3) How many terms of the A.P 1,4,7 ----- should be taken so that their sum is 51.

a=1, d=3 Sn=51

Sn=n/2 {2a+(n-1) d}

=>51=n/2 {2.1+(n-1) 3}=n/2 {3n-1}

=>$3n^2$ -n=102

=> $3n^2$-n-102=0

$3n^2$-18n+17n-302=0

3n(n-6)+17(n-6)=0

(n-6)=0 or(3n-17)=0

n=6 or n=-17/3

but n =-17/3 is not possible, so n=6

4) The first term of an A.P is 5, the last term is 45 and the sum is 400. Find the number of terms and common difference.

a = 5, l = 45 and Sn = 400

Sn = n/2 [a + l]

400 = n/2 [5 + 45] = n/2 [5 + 45]

400 = n/2 × 50

400 = 25n

n = 400/25

n = 16, a_{16} = 45

a + (16 - 1)d = 45

a + 15d = 45

5 + 15d = 45

15d = 45 - 5

15d = 40

d = 40/15

d = 8/3

5) The sum of three terms of an A.P is 12 and their product is 48. Find the terms

Let the three consecutive terms be a-d , a , and a+d

Sum of 3 consecutive terms = 12

➙ (a - d) + a + (a + d) = 12

➙ a + a + a - d + d = 12

➙ 3a = 12 = 12/3 = 4

Product of 3 consecutive terms = 48

➙ (a - d) × a × (a + d) = 48

we know that (a + b)(a - b) = $a^2 - b^2$, then ➙ ($a^2 - d^2$) × a = 48

➙ ($4^2 - d^2$) × 4 = 48 ➙ 16 - d^2 = 48/4 ➙ 16 - d^2 = 12 ➙ -d^2 = 12 - 16 = -4

➙ d = $\pm\sqrt{4}$ ➙ d = $\pm$2

three consecutive terms when d = +2 are (a - d) = 4 -2 = 2 a = 4 (a + d) = 4 + 2 = 6

three consecutive terms , when d = -2 (a - d) = 4 -(-2) = 6 a = 4 (a + d) = 4 - 2 = 2

6) In an A.P if $a_n = 5 – 2n$. Find the sum of first 30 terms.

a_1=5–2(1)=5-2=3 a_2=5-2(2)=5-4=1 a_3=5-2(3)=5-6= -1

a=3, d=1-3= -2

S_{30}=n/2[2a+(n-1)d]

=30/2[2(3)+29(-2)]=15[6-58]=15x-52= -780

V. 4 mark questions.

1) The third term of an A.P is 8 and the 9th term of the A.P exceeds three times the third term by 2. Find the sum of its first 19 terms.

a_3=8

$a_1 + 2d = 8$ -----(1)

$a_9 = 3 \times a_3 + 2$

$a_9 = 3 \times 8 + 2$

$a_9 = 24 + 2$

$a_9 = 26$

$a_1 + 8d = 26$ ------(2)

➡ **Solving equation 1 and equation 2 by elimination method,**

$a_1 + 8d = 26$

$a_1 + 2d = 8$

6d = 18

d = 18/6= 3

$a_1 + 2 \times 3 = 8$

$a_1 + 6 = 8$

$a_1 = 8 - 6 =2$

$$S_n = \frac{n}{2}(2a_1 + (n-1) \times d)$$

S_{19} = 19/2 (2 × 2 + (19 - 1) × 3)

S_{19} = 9.5 × (4 + 54) = 551

2) Find three numbers of the A.P whose sum is 24 and sum of their squares is 200.

a' be middle term and 'd' be common difference

(a-d)+a+(a+d)=24

3a=24⇒a=8

and $(a-d)^2+a+(a-d)^2=200$

i.e,$3a^2+2d^2=200⇒3×64+2d=200$

$⇒2d^2=8$ ⇒d=2

i.e, the numbers are 6,8,10

3) Divide 32 into four parts which are in A.P such that the product of extremes to the product of means is 7:15. Find the four parts.

Let 32 be divided into four parts such as,

(a-3d),(a-d),(a+d),(a+d) and (a+3d).

(a-3d)+(a-d)+(a+d) +(a+3d)=32

4a = 32

$a = \frac{32}{4} = 8$

Now According to the question,

$\frac{product\ of\ extremes}{product\ of\ means} = \frac{7}{15}$

So here the extremes are (a-3d)×(a+3d).

Means are (a-d) ×(a+d).

Therefore,

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

By putting the value of a =8,we get

$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$

$128d^2 = 512$

$d^2 = \frac{512}{128} = 4$

$d = 2$

The four parts are 2,6,10 and 14 respectively.

4) In an A.P whose first term is 2. The sum of first five terms is one fourth of the sum of the next five terms. Show that a_{20} = – 112.

a = 2
$[a_1 + a_2 + a_3 + a_4 + a_5] = 1/4\ [a_6 + a_7 + a_8 + a_9 + a_{10}]$
{(a) + (a + d) + (a + 2d) + (a + 3d) + (a + 4d)} = 1/4 {(a+5d) + (a+6d) + (a+7d) + (a+8d) + (a+9d)}
5a + 10d = 1/4 (5a + 35d)
20a + 40d = 5a + 35d
15a = -5d
15 (2) = -5d.
d = 30 / -5= -6
a_{20}=a + 19d = 2 + 19 (-6) = 2 - 114 = -112

5) In an A.P, the sum of first term, third term and fifth term is 39 and the sum of second, fourth and sixth terms is 51. Find the 10^{th} term of the A.P.

$a_1 + a_3 + a_5$= 39-------------- (1)

$a_2 + a_4 + a_6$ = 51------------ (2)

In equation (1)

a + a + 2d + a + 4d = 39

3a + 6d = 39

a + 2d = 13 (divide the equation by 3) ------(3)

In equation (2)

a + a + d + a + 3d + a + 5d = 51

3a + 9d = 51

a + 3d = 17 (divide the equation by 3)-------(4)

subtracting (3) from (4)we getd=4

In the equation (3)

a + 2d = 13

a = 9

a_{10}= a + 9d = 9+ 36= 45

VI. 5mark questions.

1) The sum of first 10 terms of the A.P is 175 and sum of next 10 terms is 475. Find the first term and common difference.

S_{10} = 175
S_{10} + S'_{10} = S_{20}
S'_{10} = 475
S_{20} = 475 + 175 = 650
S_{10} = 175
n = 10
Sn = n/2 [2a + (n-1)d]
175 = 10/2 [2a + 9d]
175 = 5 [2a + 9d]
175/5 = 2a + 9d
2a + 9d = 35 ---> (1)
S_{20} = 650
n = 20
Sn = n/2 [2a + (n-1)d]
650 = 20/2 [2a + 19d]
650 = 10 [2a + 19d]
650/10 = 2a + 19d
2a + 19d = 65 -----> (2)
(2) - (1)
2a + 19d = 65
- 2a + 9d = 35

10d = 30
d = 30/10 = 3
2a + 9d = 35
2a + 27 = 35
2a = 8

a = 8/2 = 4
a = 4, d = 3

2) The sum of three terms of an A.P is 21 and the product of the first and third term exceeds the second term by 6. Find the sum of 20 terms of the A.P.

Sum of three terms of an A.P. is 21

Let the three terms in AP are (a - d), a, (a + d)

(a – d) + a + (a + d) = 21

3a = 21

a = 21/3

a = 7 ………..(1)

(a – d) (a + d) = a + 6

$a^2 - d^2 = a + 6$

$7^2 - d^2 = 7 + 6$

[From eq.1, a = 7]

$49 - d^2 = 13$

$d^2 = 36$

$d = \sqrt{36}$

$d = \pm 6$

If d = 6 , then

First term (a – d) = 7 – 6 = 1 Third term (a + d) = 7 + 6 = 13 Second term a = 7

If d = - 6 , then

First term, (a – d) = 7 – (-6) = 7 + 7 = 13 Third term (a + d) = 7 + (-6) = 7 - 6 = 1 Second term a = 7

3) In an A.P the sum of first 9 terms is 14 more than 5 times the 8^{th} term, 8^{th} and 2^{nd} terms are in the ratio 11:2. Find the sum of first 20 terms of the A.P.

$t_8 : t_2 = 11 : 2 \quad --- [\text{Given}]$

$\Longrightarrow \frac{t_8}{t_2} = \frac{11}{2}$

$\Longrightarrow t_8 = \frac{11}{2} \times t_2$

$\Longrightarrow t_8 = \frac{11\,t_2}{2}$

$\Longrightarrow t_8 = \frac{11 \times [a + (2-1)d]}{2}$

$\Longrightarrow t_8 = \frac{11 \times (a + d)}{2}$

$\Longrightarrow t_8 = \frac{11a + 11d}{2} \quad --- (1)$

Now, we know that,

Now, from the given condition,

$S_9 = 5 \times t_8 + 14$

$\implies \frac{9}{2}(2a + 8d) = \frac{55a + 55d}{2} + 14$

$\implies \frac{18a + 72d}{2} = \frac{55a + 55d + 28}{2}$

$\implies 18a + 72d = 55a + 55d + 28$

$\implies 18a + 72d - 55a - 55d = 28$

$\implies 18a - 55a + 72d - 55d = 28$

$\implies -37a + 17d = 28 \quad --- (2)$

Now, we know that,

$t_n = a + (n - 1) \times d \quad --- [\text{Formula}]$

Now,

$t_8 : t_2 = 11 : 2 \quad --- [\text{Given}]$

$\implies \frac{t_8}{t_2} = \frac{11}{2}$

$\implies \frac{a + (8 - 1)d}{a + (2 - 1)d} = \frac{11}{2}$

$\implies \frac{a + 7d}{a + d} = \frac{11}{2}$

$\implies 2(a + 7d) = 11(a + d)$

$\implies 2a + 14d = 11a + 11d$

$\implies 14d - 11d = 11a - 2a$

$\implies 3d = 9a$

$\implies d = \frac{9a}{3}$

$\implies \boxed{d = 3a} \quad --- (3)$

Now, by substituting d = 3a in equation (2), we get,

$-37a + 17d = 28 \quad --- (2)$

$\implies -37a + 17 \times 3a = 28$

$\implies -37a + 51a = 28$

$\implies 14a = 28$

$\implies a = \frac{28}{14}$

$\implies \boxed{a = 2}$

Now, we know that,

$\implies S_{20} = \frac{20}{2}[2a + (20 - 1)d]$

$\Longrightarrow \; S_{20} = 10(2a + 19d)$

$\Longrightarrow \; S_{20} = 10(2a + 19 \times 3a) \quad - - - \text{[From (3)]}$

$\Longrightarrow \; S_{20} = 10 \times (2a + 57a)$

$\Longrightarrow \; S_{20} = 10 \times 59a$

$\Longrightarrow \; S_{20} = 10 \times 59 \times 2$

$\Longrightarrow \; S_{20} = 590 \times 2$

$\Longrightarrow \; \boxed{S_{20} = 1180}$

∴ The sum of the first 20 terms of the AP is 1180.

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit 2: Triangles

I. Multiple choice questions.

1) Sides of two similar triangles are in the ratio 4:9 then areas of these triangles are in the ratio.

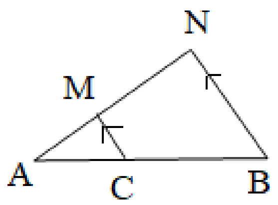
A) 2 : 3 B) 4 : 9 C) 81 : 16 D) 16 : 81

2) At a certain time of the day, a man 6 feet tall casts his shadow 8 feet long, then the length of the shadow cast by a building 45 feet high at the same time which is next to the man is

A) 60 feet B) 45 feet C) 48 feet D) 90 feet

3) In the adjoining figure Δ ABN ~ Δ AMC. The ratio of sides AM and AN is 2:5 then CM : BN is

A) 5 : 2 B) 2 : 5 C) 1 : 2 D) 2 : 3



4) The measures representing the sides of a right angled triangle are

A) 2,3,5 B) 6,8,10 C) 8,4,6 D) 6,8,9

II. One mark questions.

1) Write the statement of Basic proportionality theorem.

Ans:- If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

2) Write the statement of converse of Basic proportionality theorem.

Ans:- If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

3) Write the statement of Pythagoras theorem.

Ans:- In a right angled triangle the square on the hypotenuse is equal to the squares of the other two sides.

4) In a right angles triangle ABC, ∠B = 90°, AC = 17 cm and AB = 8 cm find the length of BC.

Ans:- In ◺ ABC ∠B=90°

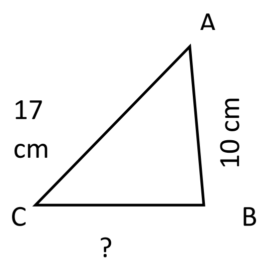
$AB^2+BC^2=AC^2$

$8^2+BC^2=17^2$

$16+BC^2=289$

$BC^2=289-64$

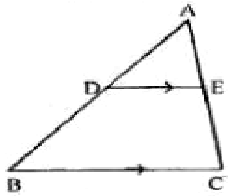
$=225$



BC= $\sqrt{225}$ =15 cm

III. 2 mark questions.

1) In the adjoining figure DE ∥ BC, BD = 7 cm, AD = 5c.m and AC = 18 cm, find AE and CE.



Ans:- In △ADE and △ ABC , AC=18 cm, AD=5 cm, BD= 7 cm

∠A =∠A (∵common angle)

∠D =∠ B (∵corresponding angles)

∴ ΔADE∼ΔABC(∵ A.A. Criteria)

$\frac{AE}{AC} = \frac{AD}{AB}$

$\frac{AE}{18} = \frac{5}{12}$ (∵ AB=AD+DB)

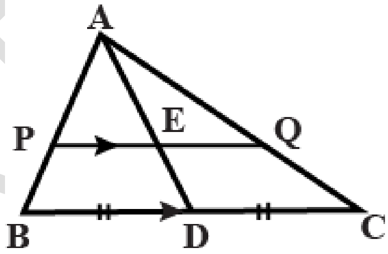
AE = $\frac{5}{12}$ x18 = 7.5 cm

CE= AC – AE

= 18 – 7.5

= 10.5 cm

2) In ΔABC, PQ ∥BC and BD = DC prove that PE = EQ.



Ans:- PQ||BC ,BD=DC

In Δ APE and Δ ABD

∠A =∠A (∵common angle)

∠P =∠B (∵ PE||BD, corresponding angles)

∴ ΔAPE ∼ΔABD (∵ A.A. Criteria)

$\frac{AE}{AD} = \frac{PE}{BD}$--------1

Similarly

ΔAEQ ∼ΔADC

$\frac{AE}{AD} = \frac{EQ}{DC}$ --------2

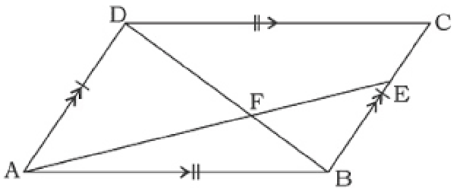
From equations 1 and 2

$\frac{PF}{BD} = \frac{EQ}{DC}$ (∵ Axiom 1)

PE= $\frac{EQ}{DC}$ x BD

∴ PE=EQ (∵ $BD = DC$)

3) The diagonal BD of parallelogram ABCD intersects AE at F. E is any point on BC. Prove that DF.EF = FB.FA.



Ans:- In Δ ADF and Δ EFB

∠A =∠E

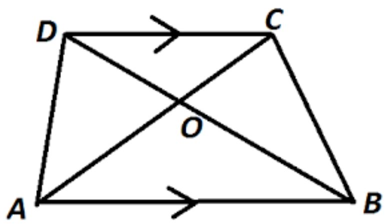
∠D =∠B (∵ AD||BC, Alternate angles)

∴ ΔADF ∼ΔEFB (∵ A.A. Criteria)

$\frac{DF}{FB} = \frac{FA}{EF}$

⟹ DF·EF = FA·FB

4) In the trapezium ABCD, AB ∥DC, AB = 2CD and area (Δ AOB) = 84 cm^2. Find the area of ΔCOD.



Ans:- In the trapezium ABCD, AB ∥DC, AB = 2CD , area (Δ AOB) = 84 cm^2

area of ΔCOD=?

In Δ AOB and Δ COD

∠A =∠C

$\angle B = \angle D$ (∵ AB||DC, Alternate angles)

Δ AOB $\sim \Delta$ COD(∵ A.A. Criteria)

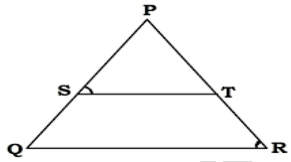
$\frac{A(COD)}{A(AOB)} = (\frac{CD}{AB})^2 = (\frac{CD}{2CD})^2 = \frac{CD2}{4CD2} = \frac{1}{4}$

$\frac{A(COD)}{84} = \frac{1}{4}$

A(COD) $= \frac{1}{4}$ x 84

= 21 cm^2

5) In the adjoining figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$.



Prove that PQR is an isosceles triangle.

Ans:- Data:- In diagram $\frac{Ps}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$

→ $\frac{PS}{SQ} = \frac{PT}{TR}$ (∵ $data$)

ST||QR (∵ $converse\ of\ Thales\ theorem$)

$\angle PST = \angle PQR$ (∵ $corresponding\ angles$) ----------1

$\angle PST = \angle PRQ$(∵ $data$)--------------------2

From 1 and 2

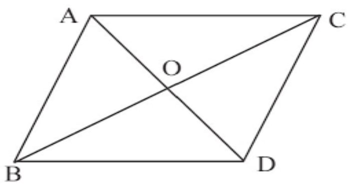
$\angle PQR = \angle PRQ$ (∵ Axiom 1)

∴ ΔPQR is an Isosceles triangle. (∵ $two\ sides\ and\ two\ angles\ are\ equal$)

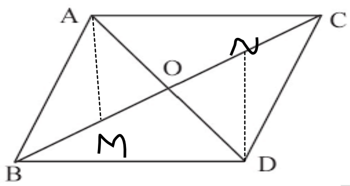
IV. 3 mark questions.

1) In the adjoining figure, ABC and DBC are two triangles on the same base BC.

If AD intersect BC at O. Show that $\frac{area(\Delta ABC)}{area(\Delta DBC)} = \frac{AO}{DO}$



Ans:-



Data:- ΔABCandΔBDC standing on the same base BC

Construction:- AM⊥BC,

DN⊥BC

In ΔAOM andΔ DON

$\angle M = \angle N = 90^0$ (∵ Construction)

$\angle AOM = \angle DON$ (∵ Vertically opposite angles)

∴ ΔAOM $\sim \Delta$DON (∵ A.A. Criteria)

$\frac{AM}{DN} = \frac{AO}{D0}$ ------------ 1

$\frac{A(ABC)}{A(BDC)} = \frac{½\ X\ BC\ X\ AM}{½\ X\ BC\ X\ DN}$ (∵ $Area\ of\ triangle = ½\ XbXh$)

$= \frac{AM}{DN}$

$\frac{A(ABC)}{A(BDC)} = \frac{AO}{D0}$(∵ from equation 1)

2) BL and CM are medians of a ΔABC right angled at A. Prove that 4 ($BL^2 + CM^2$) = $5BC^2$

Ans:- In ΔABC, $\angle A = 90^0$

AL=CL, AM=MB

→ $In \triangle$ALB, $\angle A = 90^0$

$BL^2=AL^2+AB^2$ (∵ pythagoras theorem)

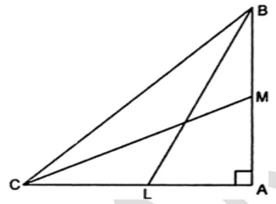
$BL^2 =(\frac{AC}{2})^2+AB^2$

$BL^2 =\frac{AC2}{4}+AB^2$

$4\ BL^2 =AC^2 + 4AB^2$ ------------------- 1

In ⊿ACM, ∠A=90^0

$CM^2=AM^2+AC^2$ (∵ pythagoras theorem)



$CM^2 =(\frac{AB}{2})^2 +AC^2$(∵ AB is midpoint of M)

$CM^2 =\frac{AB2}{4}+AC^2$

$4\ CM^2 =AB^2 + 4AC^2$ ------------------- 2

In ΔABC, ∠A=90^0

$AB^2+AC^2=BC^2$ (∵ pythagoras theorem) -------3

Adding equations 1 and 2

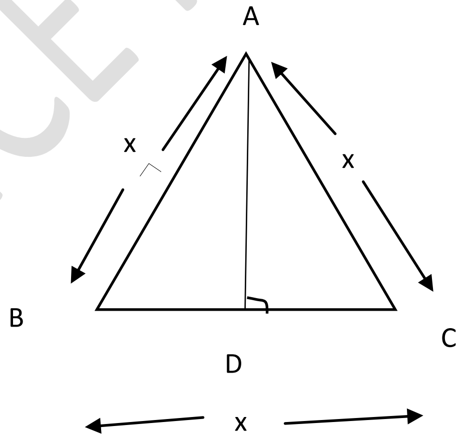
$4\ BL^2 + 4\ CM^2 = AC^2 + 4AB^2 + AB^2 + 4AC^2$

$4(BL^2 + CM^2) = 5\ AB^2 + 5\ AC^2 = 5(AB^2 + AC^2)$

$4(BL^2 + CM^2) = 5BC^2$ (∵ equation 3)

3) In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans:- AB=BC=AC=x units



AD⊥BC

BD=DC =$\frac{x}{2}$ (∵ perpendicular bisects the base)

∴ In ⊿ ADC

$AC^2=AD^2+DC^2$ (∵ pythagoras theorem)

$X^2 = AD^2 + (\frac{x}{2})^2 = AD^2 + \frac{x2}{4} = \frac{4AD2+x2}{4}$

$4X^2= 4\ AD^2 +x^2$

$4X^2- x^2 =4\ AD^2$

$3X^2 =4\ AD^2$

$3(side)^2 =4(height)^2$

4) If the areas of two similar triangles are equal, then they are congruent prove.

Ans:- Data:- ΔABC ~ΔPQR

A(ABC) = A(PQR)

$\frac{A(ABC)}{A(PQR)} = (\frac{AB}{PQ})^2= (\frac{BC}{QR})^2 =(\frac{AC}{PR})^2$

$1= (\frac{AB}{PQ})^2= (\frac{BC}{QR})^2 =(\frac{AC}{PR})^2$

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}=\frac{AC}{PR} =1$

∴ AB=PQ, BC=QR, AC=PR

∴ ΔABC ≅ ΔPQR (∵ S.S.S. congruency)

V. 4 mark questions. (Prove these theorems)

1) "In two triangles; corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar" Prove this.

2) The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides." Prove this.

VI. 5 mark questions. (State and prove these theorems)

1) State and prove "Basic proportionality theorem" (Thales theorem).

2) State and prove "Pythagoras theorem".

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit-3:- Pair of linear equations in two variables

I. Multiple choice questions.

1. If two equations have exactly one solution and are in the form $\frac{a1}{a2} \neq \frac{b1}{b2}$ then they are

a. Coincident lines b. Intersecting lines c. Transversal lines d. Parallel lines

2. If two equations have no solutions and are in the form $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$ then they are

a. Coincident lines b. Intersecting lines c. Parallel lines d. All of the above

3. In the general form of pair of linear equations $a_1x+b_1y+c1=0$ and $a_2x+b_2y+c_2=0$ where a_1, a_2, b_1, b2 and c_1, c_2 are

a. Whole numbers b. Real numbers c. Integers d. Co-primes

4. x+2y-4=0 and 2x+4y-12=0 then the lines are

a. Coincide b. Parallel c. Intersect d. None of the above

5. If the lines 3x+2ky-2=0and 2x+5y+1=0 are parallel then the value of k is

a. $\frac{4}{15}$ b. $\frac{15}{4}$ c. $\frac{4}{5}$ d. $\frac{5}{4}$

6. The solution of the equations x-y=2 and x+y=4 are

a. 3,1 b. 4,3 c. 5,1 d. -1, -3

II. One mark questions.

1. The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 2 more balls of the same kind for Rs 1300. Represent this situation algebraically.

Ans:- 3x+6y=3900

x+2y=1300

2. Check whether the pair of equations $x + 3y = 6$ and $2x – 3y = 12$ is consistent.

The given pair can be written as

x+3y−6=0 and 2x−3y−12

Here$a_1=1,b_1=3,c_1=-6$...........eq1 and $a_2=2,b_2=-3,c_2=-12$.........eq2

$\therefore a_2/a_1 = 2/1, b_2/b^1 = -6/3 = -2/1$ Hence $a_2/a_1 \neq b_2/b_1$

Thus the given pair of equation is consistent.

III. Two mark questions.

1) On comparing the ratios $\frac{a1}{a2}, \frac{b1}{b2}$ and $\frac{c1}{c2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident.

i) $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$

Ans. $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$
On comparing these equation with
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
We get
$a_1 = 5$, $b_1 = -4$, and $c_1 = 8$
$a_2 = 7$, $b_2 = 6$ and $c_2 = -9$
$a_1/a_2 = 5/7$,
$b_1/b_2 = -4/6$ and
$c_1/c_2 = 8/-9$
Hence, $a_1/a_2 \neq b_1/b_2$
Therefore, both the lines intersect at one point

ii)) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$

$9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$
Comparing these equations with
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
We get
$a_1 = 9$, $b_1 = 3$, and $c_1 = 12$
$a_2 = 18$, $b_2 = 6$ and $c_2 = 24$
$a_1/a_2 = 9/18 = 1/2$
$b_1/b_2 = 3/6 = 1/2$ and
$c_1/c_2 = 12/24 = 1/2$
Hence, $a_1/a_2 = b_1/b_2 = c_1/c_2$
Therefore, both the lines are coincident

IV. Find the value of x and y by graphical method for the following equations .

1. $x-y+1=0$ and $3x+2y-12=0$

$x - y + 1 = 0$

$y = x + 1$

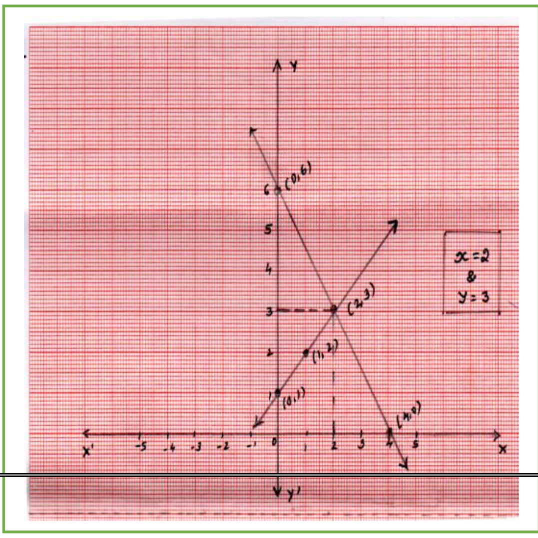
x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$

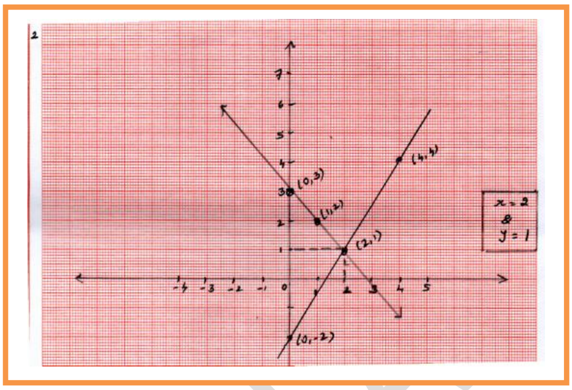
$2y = 12 - 3x$

$Y = \frac{12-3x}{2}$

x	0	2	4
y	6	3	0



2. x+y=3 and 3x-2y=4



x+ y = 3

y = 3 – x

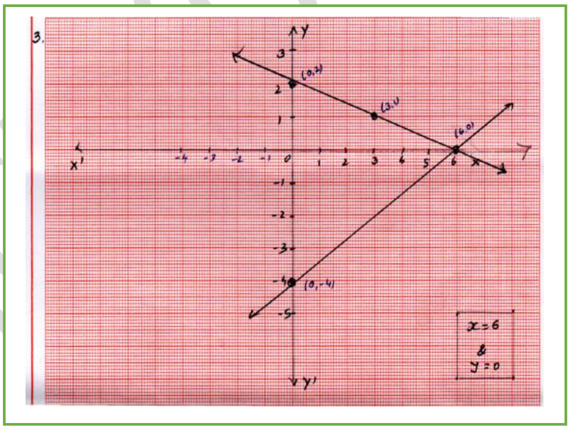
3x-2y = 4

2y = 3x – 4

x	0	1	2
y	3	2	1

x	0	2	4
y	-2	1	4

3. x+3y=6 and 2x-3y=12



X + 3y = 6

3y = 6 – x

$Y=\frac{6-x}{3}$

2x – 3y = 12

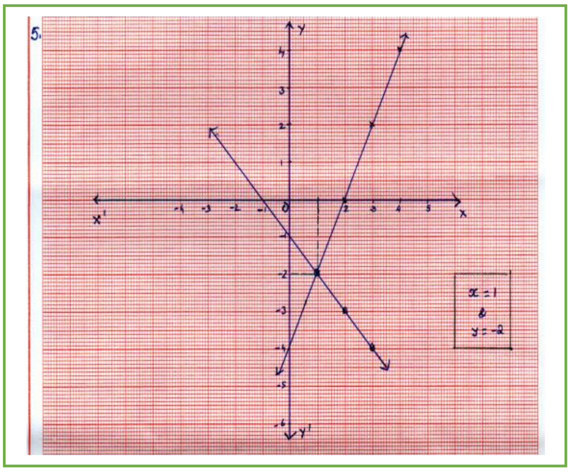
3y = 2x – 12

$Y=\frac{2x-12}{3}$

x	0	3	6
y	2	1	0

x	0	3	6
y	-4	-2	0

4. 2x-y-4=0 and x+y+1=0



2x – y – 4 = 0

Y = 2x – 4

x + y + 1 = 0

y = -x – 1

X	2	3	1
y	0	2	-2

x	1	2	0
y	-2	-3	-1

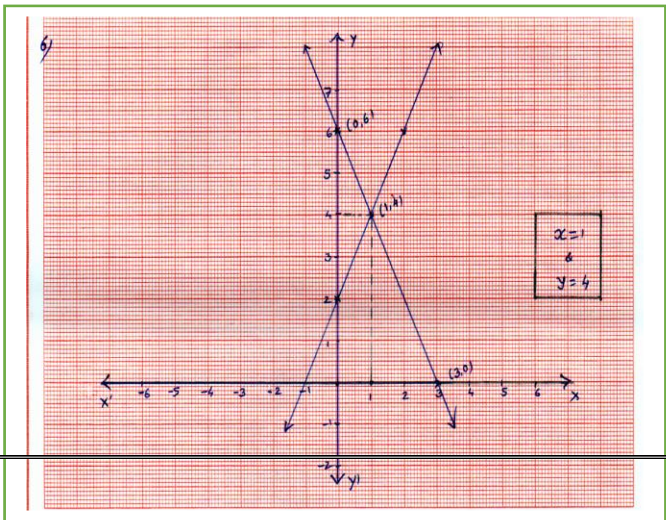
5. 2x+y=6 and 2x-y+2=0

2x + y = 6

Y = 6 – 2x

2x – y + 2 = 0

y = 2x + 2



x	0	1	3
y	6	4	0

x	0	1	2
y	2	4	6

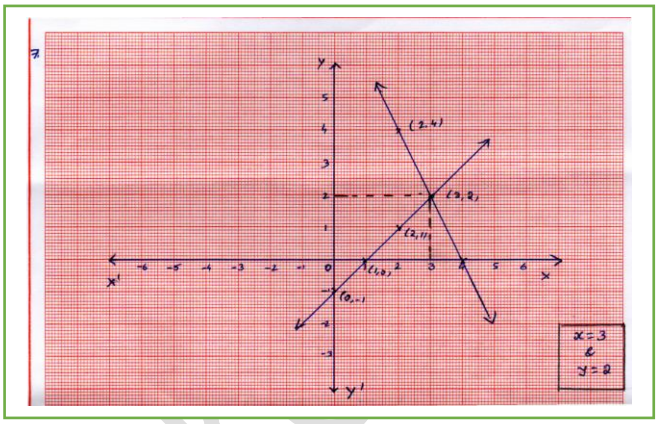
6. x-y=1 and 2x+y=8

x – y = 1   2x + y = 8

y = x -1   y = 8 – 2x

x	0	1	2
y	-1	0	1

x	3	4	2
y	2	0	4



7. 2x-y-2=0 and 4x-3y-24=0

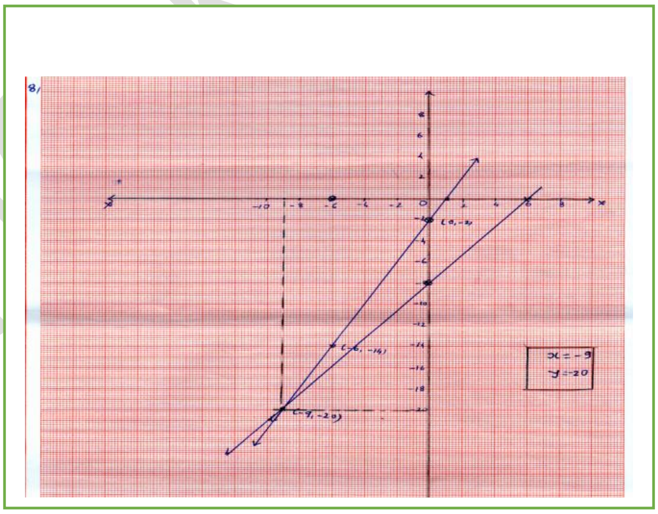
2x – y – 2 = 0   4x – 3y – 24 = 0

Y = 2x – 2   3y = 4x – 24

$y = \frac{4x-24}{3}$

x	0	1
y	-2	0

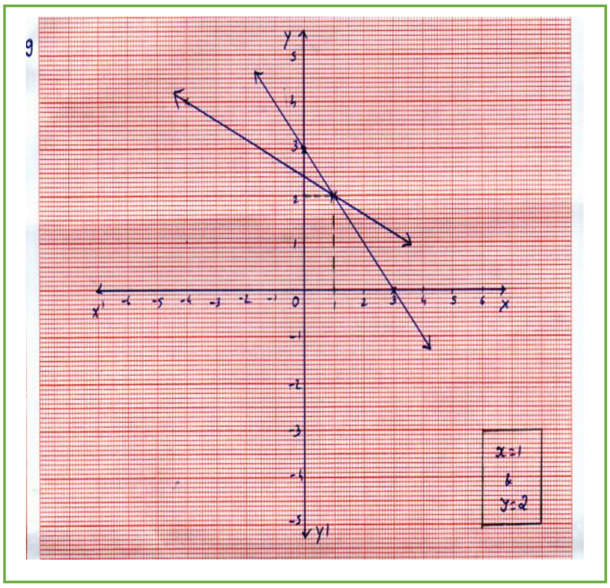
x	0	6
y	-8	0



8. x+y=3 and 2x+5y=12

x + y = 3   2x+5y=12

y = 3 – x $y = \frac{12-2x}{5}$



x	0	1	3
y	3	2	0

x	1	-4	6
y	2	4	0

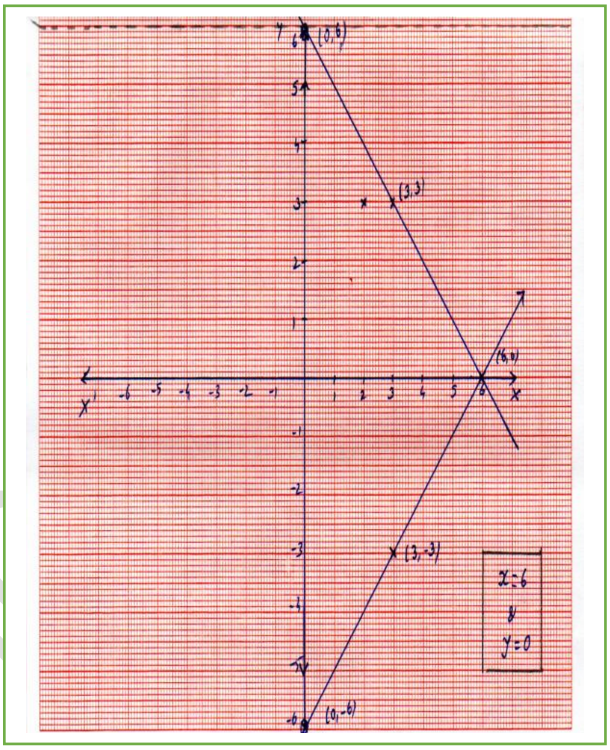
9. x+y=6 and x-y=6

x + y = 6 x – y = 6

y = 6 – x y = x – 6

x	0	6	3
y	6	0	3

x	0	6	3
y	-6	0	-3



V. For the pair of linear equations find their solution by elimination method.

1. 3x-5y=-1 and x-y=-1

3x-y=-1

3x-3y=-3

3x-5y=-1

3x-3y=-3

-. + +

-2y=2

y=-1

substituting y=-1 in eq 1

3x-5(-1)=-1

3x+5=-1

3x=-6

x=-2

2. x+2y=-1 and 2x-3y=12

Multiply eq-(1) by 2*

We get,

→2x+4y+2=0-(3)

Now,

Add Eq-(3)&(2)*

we get, y=2-(4)

•Using eq-(4) in eq-(1) or eq(2)•

I prefer using in eq-(1)

→x+2(2)=-1
→x=-1-4
→x=-5

---

x=-5'and y=2

3. 2x+3y=9 and 3x+4y=5

2 x + 3 y = 9

3 x + 4 y = 5

Eliminate y

multiply (1)by -4

Multiply (2) by 3

-8 x -12 y = -36

9 x +12 y = 15

Add the two equations

1 x = -21

x = -21

put value of x in (1)

2 x + 3 y = 9

-42 + 3 y = 9

3 y = 9 + 42

3 y = 51

y = 17

4. x-y+1=0 and 3x+2y-12=0

Solve these equations:-)

$\mathbf{x - y + 1 = 0}.$

$=> \mathbf{x - y = -1}.............(1).$

$\mathbf{3x + 2y - 12 = 0}.$

$=> \mathbf{3x + 2y = 12}..............(\mathbf{2}).$

Solving by elimination method

Multiply equation (1) by 3.

$=> 3x - 3y = -3..........(3).$

Now, Substract in equation (2) and (3).
3x + 2y = 12.
3x - 3y = -3.
(-)...(+)......(+).

=> 5y = 15.
$=> y = 3.$
↪➡ Put the value of ‘ y’ in equation (1).
=> x - 3 = -1.
=> x = -1 + 3.
$=> x = 2.$

5. x-y=1 and 2x+y=8
x - y = 1 .....(1)

2x + y = 8 .....(2)

Adding (1) and (2),

x - y + 2x + y = 1 + 8

→ 3x = 9

→ x = 9 ÷ 3 = 3 .....(3)

From (3) in (1),

3 - y = 1

→ y = 3 - 1 = 2

6. x+y =6 and x-y=6
x+y=6 ----------(1)

x-y=6 -----------(2)

add both equation

x + y=6

x -y=6

2x=12

x= 12/2

x=6

putting the value of x in question 1st

x+y=6

6+y=6

y=6-6

$y=0$

Ans- $X=6$, $y=0$

7. $x-y=1$ and $2x+y=8$
$x - y = 1$ .....(1)

$2x + y = 8$ .....(2)

Adding (1) and (2),

$x - y + 2x + y = 1 + 8$

$\rightarrow 3x = 9$

$\rightarrow x = 9 \div 3 = 3$ .....(3)

From (3) in (1),

$3 - y = 1$

$\rightarrow y = 3 - 1 = 2$

VI. Solve the following by constructing linear equation.

1. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Let the two numbers be $10x+y$ and $10y+x$ where x and y are positive integers.

Adding the two numbers we get

$11x+11y=66$.

$x+y=6$.....1

Also, difference of the two digits it 2 so,

$x-y=2$.....2

Adding eq. 1 and 2,

$2x=8$

$x=4$

Substituting x,

$4-y=2$

$y=2$

So the number can be $10x+y$ or $10y+x$.

So the required numbers are 24 and 42.

So there are two such numbers.

2. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find herspeed of rowing in still water and the speed of the current.

Ans. Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.
Speed of Ritu while rowing Upstream = $(x – y)$ km/h
Downstream = $(x + y)$ km/h
$2(x + y) = 20$
$\Rightarrow x + y = 10$ ... (i)

2(x – y) = 4
⇒ x – y = 2 ... (ii)
Adding equation (i) and (ii), we get x=6
Putting this equation in (i), we get
y = 4
Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

3. 2 women and 5 men can together finish an embroidery work in 4 days, while 3women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone and also time taken by 1man alone.

Ans. Let the number of days taken by a woman and a man be x and y respectively.
Therefore, work done by a woman in 1 day = 1/x
According to the question,
4(2/x + 5/y) = 1
2/x + 5/y = 1/4
3(3/x + 6/y) = 1
3/x + 6/y = 1/3
Putting 1/x = p and 1/y = q in these equations, we get
2p + 5q = 1/4
By cross multiplication, we get
p/–20 – (–18) = q/–9 – (–18) = 1/144 – 180
p/–2 = q/–1 = 1/–36
p/–2 = – 1/36 and q/–1 = 1/–36
p = 1/18 and q = 1/36
p = 1/x = 1/18 and q = 1/y = 1/36
x = 18 and y = 36
Hence, number of days taken by a woman = 18 and number of days taken by a man = 36

4. The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, he buys another bat and 2 more balls of the same kind for Rs 1300. Find the cost of each ball and bat separately.

let the cost of bat be x

and the cost of ball be y.

acc. to the Question:

3x+6y= 3900----------------(1)

x +3y = 1300-----------------(2). {×2]

=> 2x+ 6y = 2600

now by elimination method we get ,

3x+6y= 3900

- 2x + 6y = 2600

____

x =1300

from (2) we get,

1300 +3y= 1300

3y= 1300-1300

y= 0

therefore , the value of x& y is 1300 and 0 respectively.

5. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Ans. Let the first angle be= x

And the second angle be = y
As both angles are supplementary so that sum will 180
$x + y = 180$
$x = 180 – y$ ... (i)
Given; difference is 18 degree
Therefore
$x – y = 18$
Putting the value of x we get
$180 – y – y = 18$
$– 2y = – 162$
$y = – 162/–2$
$y = 81$
Putting the value back in equation (i), we get
$x = 180 – 81 = 99$ Hence, the angles are 99° and 81°.

6. Five years ago, hari was thrice as old as ramu. Ten years later hari will be twice as old as ramu. How old ar ehari and ramu.

Let the present age of hari be = x
Let the present age of ramu be = y
According to the given information,
$(x – 5) = 3(y – 5)$
$x – 3y = – 10$ ... (i)
$(x + 10y) = 2(y + 10)$
$x – 2y = 10$ ... (ii)
Subtracting equation (i) from equation (ii), we get
$y = 20$ ... (iii)
Putting this value in equation (i), we get
$x – 60 = – 10$
$x = 50$
Hence, age of hari = 50 years and age of ramu = 20 years.

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

UNIT -4:- CIRCLES

I. Multiple choice questions.

1. Maximum number of tangents drawn to a circle from an external point is
A. 2 B. 3 C. 4 D. 5
2. A straight which intersects a circle at two distinct points is
A. Tangent B. Chord C. Secant D. Diameter
3. The angle between a tangent to a circle and the radius through the point of contact is
A. 60° B. 90° C. 120° D. 180°
4. Number of tangents can be drawn at any point on a circle is
A. 1 B. 2 C. 3 D. Many
5. The lengths of tangents drawn from an external point to the circle are
A. Equal B. Not equal C. sometimes are equal D. none
6. Tangents drawn at extremities of the diameter of a circle are
A. Perpendicular B. parallel C. Equal D. Not equal
7. A line through point of contact and passing through the centre of circle is known as
A. Tangent B. Secant C. Chord D. Segment

8. If the angle between the two tangents to a circle is 40^0 , then the angle between the radii is

A. 90^0 B. 100^0 C. 140^0 D. 180^0

9. Distance between two parallel tangents of a circle of radius 3.5cm is

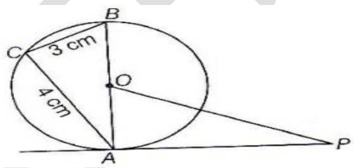
A. 3.5cm B. 7cm C. 10cm D. 14cm.

II. VSAQ (one mark questions).

1. How many tangents can a circle have? **Ans:-**Infinetely many
2. What is called the intersecting point of a circle and a tangent? **Ans:-Point of Contact**
3. How many parallel tangents at most a circle can have? **Ans:-2**
4. What is angle between a tangent of a circle and its radius? **Ans:-90°**
5. What is the name of line intersecting a circle in two points? **Ans:-Secant**
6. What is the name of two circles having a common centre? **Ans:-Concentric Circles**
7. How many lines passes through a point on the circle ? **Ans:-One**
8. How many tangents can be drawn at the ends of diameter of a circle? **Ans:-Two**

III. 2 marks questions.

1. *PA* is a tangent to the circle with center *O*. If *BC* = 3 cm, *AC* = 4 cm, and $\Delta ACB \sim \Delta PAO$ then find *OA* and $\frac{OP}{AP}$



IN ΔABC, ∠ACB = 90° (Angle in semi circle)

$AB^2 = AC^2 + BC^2$ (Pythagoras theorem)

$AB^2 = 4^2 + 3^2$

AB = $\sqrt{16+9} = \sqrt{25}$ = 5cm

∴OA = $\frac{AB}{2} = \frac{5}{2}$ = 2.5cm (r=$\frac{d}{2}$)

ΔACB ~ΔPAO (Given)

$$\frac{OP}{AP} = \frac{AB}{AC}$$

$$\frac{OP}{AP} = \frac{5}{4}$$

2. The length of common chord of two intersecting circles is 30 cm. If the diameters of these two circles are 50 cm and 34 cm, then calculate the distance between their centers.

Data: ∠SQR= 38° PQ and PR are tangents

In Quadrilateral PQOR,∠Q=∠R =90° (Radius ⊥ Tangent at touching point)

∠O =90° (Data)

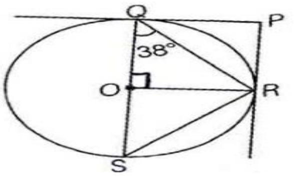
∠O + ∠R + ∠Q + ∠P = 360°

90° + 90° + 90° + ∠QPR =360°

∠QPR = 360° - 270° = 90°

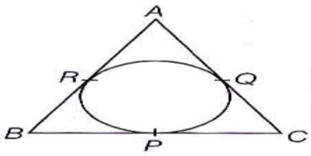
In ΔPQR, PQ = PR (tangents drawn from an external point)

∴∠PQR = ∠PRQ = x (opposite angles of equal sides)
∴ x + x ∠QPR = 180° (Sum of ∠'s of ∆)
2x + 90° = 180°
x = $\frac{90°}{2}$ = <u>45°</u>
<u>∴∠PQR = 45°&∠PRQ = 45°</u>
3 . PQ and PRare tangents at *Q* and *R*, respectively. If ∠*SQR* = 38°, then find ∠QPR, ∠PRQ, ∠QSR and ∠PQR

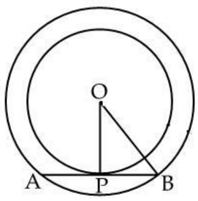


In ∆QSR, ∠QRS = 90° (Angle in semi circle)
∠SQR + ∠QRS+ ∠QSR = 180°
38° + 90° + ∠QSR = 180°
∠QSR = 180° - 128° = <u>52</u>°

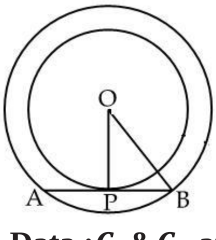
4. In the adjoining figure, an isosceles ∆*ABC* with *AB* = *AC*, circumscribes a circle. Prove that point of contact *P* bisects the base *BC*.



AB = AC ------ (1) (Data)
AR = AQ ------ (2) (tangents drawn from an external point)
(1) – (2) →AB – AR = AC – AQ
BR = QC ------ (3)
BR = BP & QC = PC (Axiom 1)
5.Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.



Data :C_1&C_2 are concentric circles with centre 'O',AB is the Chord of C_2 touches C_1.
<u>To Prove</u> : - AP = PB
<u>Proof</u> :- OP ⊥ AB (Radius ⊥ Tangent at touching point)
∴<u>AP = PB</u> (line drawn from center of a circle ⊥ ly bisects Chord)
6 . In two concentric circles, a chord of the larger circle touches the smaller circle. If the length of this chord is 8 cm and the diameter of the smaller circle is 6 cm, then find the diameter of the larger circle.



Data :C_1&C_2 are concentric circles
OP ⊥ AB-----------------1 (Radius ⊥ Tangent at touching point)
AP = PB = 4cm (⊥from center of a circle to its Chord bisects it).
∴ In ∆OPB , ∠P = 90° (from 1)

$OB^2 = OP^2 + PB^2$ **(Pythagoras theorem)**

$OB^2 = 3^2 + 4^2 = 9 + 16 = 25$

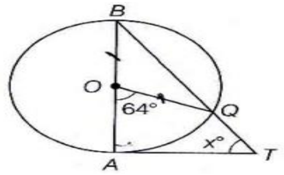
OB = $\sqrt{25}$ = 5cm

R = 5cm , ∴D = 2R = 2(5) = 10 cm

∴Diameter of the bigger circle is 10cm.

IV. 3 Mark questions.

1. In the given figure, *AB* is a diameter of the circle with center *O* and *AT* is a tangent. Calculate the numerical value of *x*.



∠AOQ = 64° (Given)

∠AOQ + ∠BOQ = 180°

∠BOQ = 180° - 64° = 116° -------- (1)

In ΔBOQ , OB = OQ (radii of same circle)

∴∠OBQ = ∠OQB --------- (2)(Sum of ∠'s of triangle)

∠OBQ +∠OBQ + 116° = 180°

2∠OBQ = 180° - 116° = 64°

∠OBQ = $\frac{64°}{2}$ = 32 --------(3)

∴ *In* Δ BAT, ∠A = 90° (Radius ⊥ Tangent)

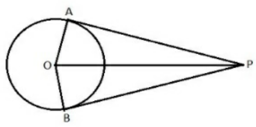
∠B + ∠A + ∠T =180°

32° + 90° + x° = 180°

x° = 180° -122°

x° = 58°

2.Two tangents *PA* and *PB* are drawn to the circle with center *O*, such that ∠∠*APB* =120°. Prove that *OP* = *2AP*.



ΔAOP ≅ ΔBOP (R.H.S Postulate)

∠APO = ∠BPO = 60°

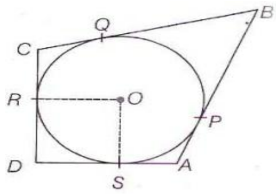
In ΔOAP,∠A = 90°

Cos60°= $\frac{AP}{OP}$ = $\frac{1}{2}$

$\frac{1}{2}$OP = AP

∴OP = 2AP

3. In the given figure, ∠*ADC* = 90°, *BC* = 38 cm, *CD* = 28 cm and *BP* = 25 cm, then the radius of the circle.



Data :-∠*ADC* = 90°, *BC* = 38 cm, *CD* = 28 cm and *BP* = 25 cm

→BQ = 25cm (tangent from an external point)

CQ = BC – BQ = 38 – 25 13cm

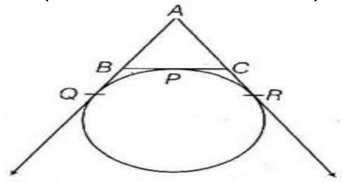
∴CQ = CR = 13cm (Tangents from an external point)

DR = CD – CR = 28 – 13

DR = 15cm----------------1

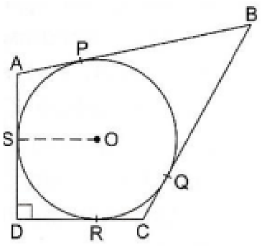
In Quadrilateral ORDS,∠R = ∠S = 90° (radius ⊥ tangent at a point of contact)
∠D = 90° (Data)
∠O = 90° (sum of ainterior angles in Quadrilateral is 360°)
OR = OS (radii of same circle)
∴ ORDS is a square.
∴ OS = OR = DR
OS = OR = 15cm (from (1))
∴Radius of given circle is <u>15cm.</u>

4. A circle touches the side *BC* of a Δ*ABC* at *P* and *AB* and *AC* when produced at *Q* and *R* respectively as shown in the figure. Show that *AQ* = ½ (Perimeter of Δ*ABC*)



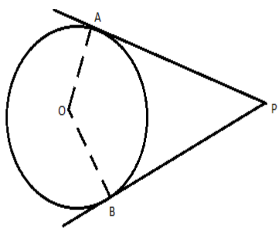
AQ = AR ---------(1) (tangents drawn from an external point)
Similarly BQ = BP, PC =CR ---------(2)
Perimeter of ΔABC = AB + BC + AC = AB + BQ + CR+ AC (from(2))
= AQ + AR = AQ + AQ (from(1))
= 2AQ
<u>$\frac{1}{2}$ (perimeter of ΔABC) = AQ</u>

5.A circle touches all the four sides of a quadrilateral ABCD. Prove that AB+CD = BC+DA



AP = AS, BP = BQ, SD = DR, CQ = CR-------------------- 1
(tangents drawn from an external point are equal)
L.H.S, AB + CD = AP + PB + DR + RC
= AS + BQ + SD + CQ (from (1))
= AS + SD + BQ + CQ
<u>AB + CD = AD + BC = R.H.S</u>

6. **Prove that the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the center.**



Data :- AP and BP are tangents to a circle with center 'O' .
To prove that :- ∠AOB + ∠APB = 180°
Proof :- In Quadrilateral OAPB ∠A = ∠B = 90°
∠A + ∠B + ∠O + ∠P = 360°(sum of angles in a Quadrilateral)

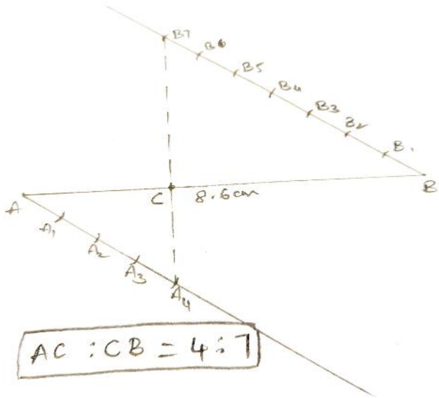
$90° + 90° + \angle O + \angle P = 360°$
$\angle O + \angle P = 360° - 180° = 180$
$\underline{\angle AOB + \angle APB = 360°}$

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

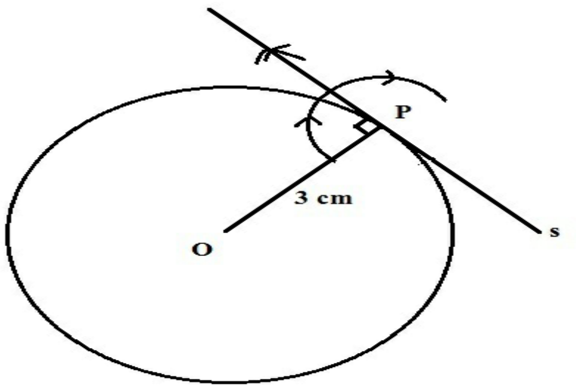
UNIT -5:- Constructions

I. Two marks questions.

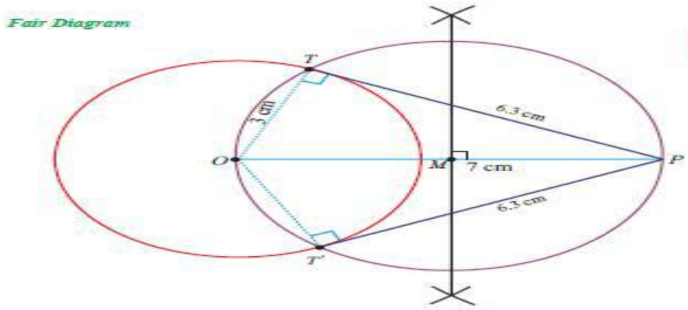
1. Draw a line segment of length 8.6 cm and divide it in the ratio 4:7 and measure the parts.

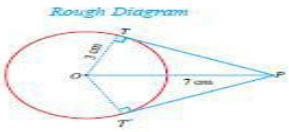


2. Draw a circle of radius 3 cm and construct a tangent at a point 'P' on the circle.

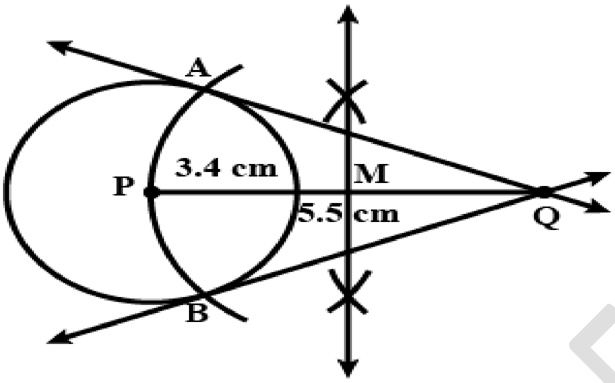


3. Draw a circle of radius 3 cm, construct a tangent which is 7 cm away from the centre of the circle. Measure the length of the tangent and verify.

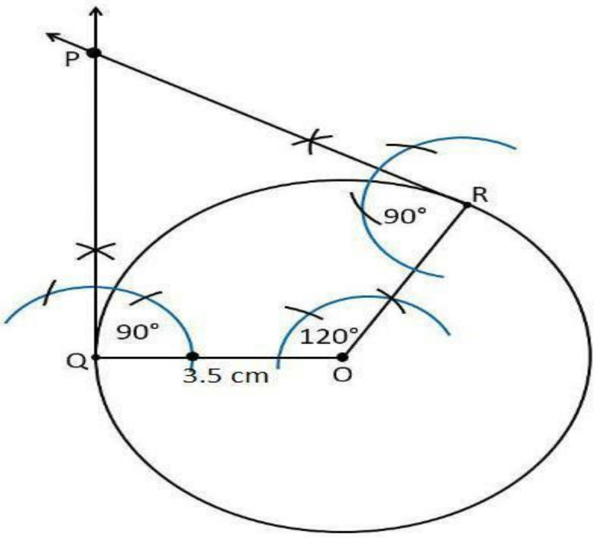




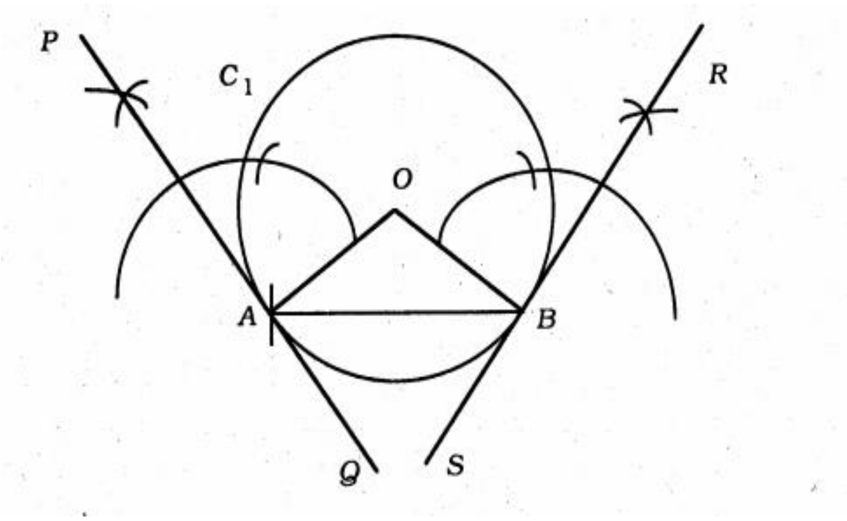
4. Draw a circle of radius 3.5 cm, construct two tangents from an external point which is 5.5 cm away from the circle.



5. Draw a circle of radius 4 cm and draw two radii which inclines an angle of 60^0. Construct tangents at the ends of radii to the circle.

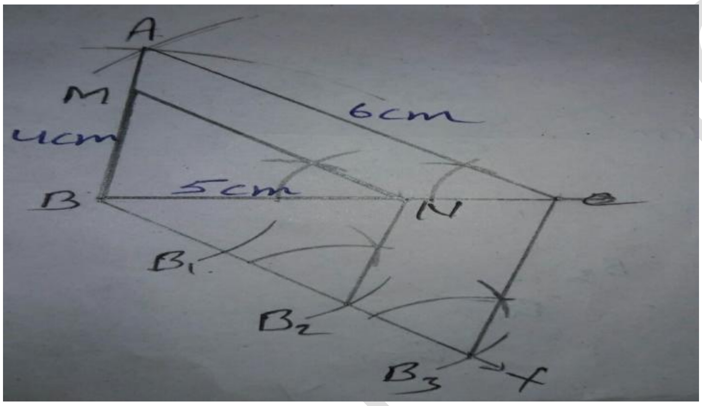


6. Draw a circle of radius 3 cm and a chord 5 cm, construct a tangent at one end of the chord.

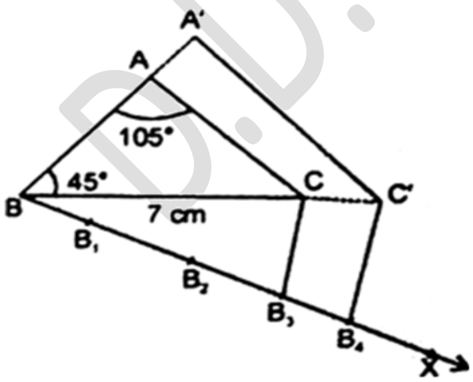


II. Three marks questions.

1. Construct a triangle of sides 4 cm, 6 cm and 7 cm and then construct a triangle similar to it whose sides are ⅔ of the corresponding sides of the first triangle.

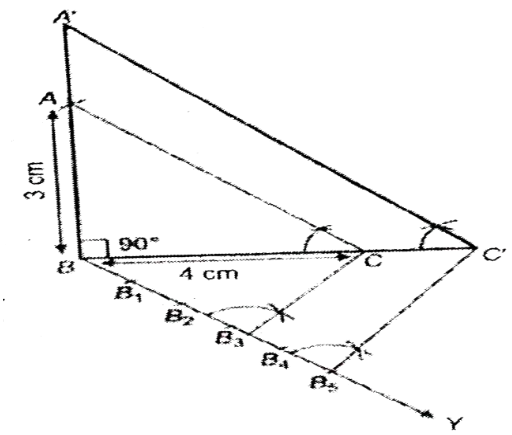


3.Draw a triangle ABC given BC=7cm, ∟B=45^0, ∟A=105^0 then construct a triangle whose sides are $^4/_3$times the corresponding sides of the triangle ABC.



4. Draw a right angled triangle in which the sides other than the hypotenuse are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $^3/_5$times the corresponding sides

of the given triangle.



OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

UNIT -6:- Co-ordinate Geometry

I. Choose the correct answer for the fallowing questions.

1. The Coordinates of the origin is

   A) (1,1) B) (0,0) C) (0,1) D) (1,0)

2. Area of the triangle formed by three collinear points is

   A) 0 sq.units B)1 sq.units C)2 sq.units D)4 sq.units

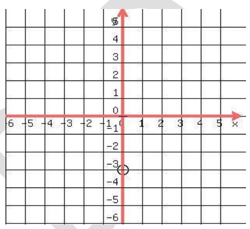
3. The perpendicular distance of point P (3,-5) from x axis is

   A) 4 units B) 1 Unit C) 3 units D) 5 units

4. If the distance between origin and the point p (x, y) is

   A) x + y B) x + y C) $\sqrt{x^2 - y^2}$ D) $\sqrt{x^2 + y^2}$

5. The coordinates of the point P in the given graph is



   A) (1,3) B) (-3,-1) C) (0,-3) D) (-3,0)

6. The coordinates of a point P on the x-axis is

   A) (x,0) B) (0,y) C) (0,0) D) (0,-y)

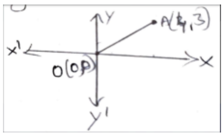
II. one - mark questions.

7. Write the coordinates of the midpoint of a line segment formed by joining the points P(x_1,y_1) and Q(x_2,y_2) Ans:-M(x,y) = ($\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$)

8. Find the perpendicular distance of the point K(5,12) from y-axis Ans:-5 units

9. Find the distance of the origin from the point P(m,n). Ans:-d = $\sqrt{m^2+n^2}$units

10. Calculate the length of the line segment OA in the given graph.



(x_1, y_1)=(0,0) (x_2, y_2) =(4,3)

length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

OA =$\sqrt{(4-0)^2+(3-0)^2}$ = $\sqrt{4^2+3^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5units

11. Write the formula to find the area of the triangle formed by the points A(x,y),B(x,y) & C(x,y).

A = $\frac{1}{2}[x_1(y_2-y_1)+x_2(y_{3-}y_1)+x_3(y_1-y_2)]$sq.units

12. Find the distance between the points (0,3) and (4,0)

(x_1, y_1) = (0,3) (x_2, y_2) = (4,0)

length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(4-0)^2+(0-3)^2}$ = $\sqrt{4^2+3^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5units

13. Find the coordinates of the midpoint of the line segment formed by joining the points (2,3) and (4,5)

(x_1, y_1) = (2,3) (x_2, y_2) = (4,5)

Mid point = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$= $\left(\frac{2+4}{2},\frac{3+5}{2}\right)$= $\left(\frac{6}{2},\frac{8}{2}\right)=(3,4)$

III. 2/3-mark questions.

14. Find the distance between (-5, 7) and (-1, 3).

(x_1, y_1) = (-5,7) (x_2, y_2) = (-1,3)

Distance = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

= $\sqrt{[-5-(-1)]^2+(7-3)^2}$ = $\sqrt{(-5+1)^2+(7-3)^2}$ = $\sqrt{(-4)^2+4^2}$

= $\sqrt{16+16}$ = 4$\sqrt{2}$units.

15. Check whether points (1, 1), (2, 2) an (3,3) are collinear.

x_1, y_1 () = (-5,7) (x_2, y_2) = (-1,3) (x_3, y_3) = (-5,7)

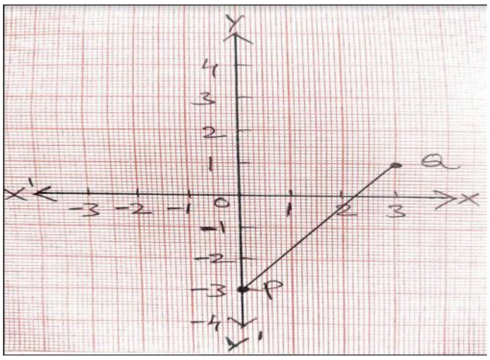
A = $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$= \frac{1}{2}[1(2-3)+2(3-1)+3(1-2)]$ $= \frac{1}{2}[1(-1)+2(2)+3(-1)]$

$= \frac{1}{2}(-1+4-3) = \frac{1}{2}(0) = 0$ **sq. units**

(This means that the area of the triangle formed by these points is zero. But no triangle has area of zero units practically which means that these points are collinear.)

16. Find the distance between P & Q in the given graph .



Coordinates of point P = (0,-3) = (x_1, y_1) Coordinates of point Q = (3,1) = (x_2, y_2)

Distance from P to Q is PQ = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$= \sqrt{(3-0)^2+(1-(-3))^2}$ $= \sqrt{3^2+4^2}$ $= \sqrt{9+16}$ $=\sqrt{25}$ **= 5 units.**

17. If the distance between the points (R,-4) and (-2,0) is 5 units, then find the value of R.

Data : (x_1, y_1) = (R,-4), (x_2, y_2) = (-2,0) Distance d = 5 units

d = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$5 = $\sqrt{(-2-R)^2+(0-(-4)^2}$

$= \sqrt{(-2-R)^2+4^2}$ **squaring both sides, we get**

$5^2 = (-2-R)^2$ **+16**

$(-2-R)^2$ **= 25-16 =9-2-R =** $\pm\sqrt{9}$ **=**± 3 $\therefore -R = 3+2$

$R = -5$ **Or -2-R =-3 -R = -3+2 <u>R =1</u>**

18. Find the radius of the circle whose centre is (3,4) and a point on its circumference is (-3,-4)

Let A(3,4) =(x_1, y_1) = and B = $(-3,-4) = (x_2, y_2)$

The line joining the centre and a point on circumference is nothing but the radius of the circle

∴radius = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3-3)^2+(-4-4)^2}$ $= \sqrt{(-6)^2+(-8)^2}$ $= \sqrt{36+64}$ $= \sqrt{100}$

∴ radius of the circle = <u>10 units .</u>

16. Find the coordinates of the point which divides the line joining the points (0,-3) and (5,2) internally in the ratio 2:3.

(x_1, y_1) = (0,-3) (x_2, y_2) = (5,2) (x_3, y_3) = (2,3)

Let the point be P (x,y)

$\therefore$ P(x,y) = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

= $\left(\frac{2\times5+3\times0}{2+3}, \frac{2\times2+3\times-3}{2+3}\right)$ = $\left(\frac{10+0}{5}, \frac{4-9}{5}\right)$ = $\left(\frac{10}{5}, \frac{-5}{5}\right)$

$\therefore$The point is P(x,y) = <u>(2,-1)</u>

17. Find the value of m if the points (m,2),(-3,4) and (7,-1) are collinear.

As these points are collinear, the area of the triangle formed so is zero (x_1, y_1) = (m,2)

(x_2, y_2) = (-3,4) (x_3, y_3) = (7,-1)

A = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

0 = $\frac{1}{2}[m(4-(-1)) + (-3)(-1-2) + 7(2-4)]$

0 = $\frac{1}{2}[m(5) + (-3)(-3) + 7(-2)]$

0 = $\frac{1}{2}[5m + 9 - 14]$

0 = $\frac{1}{2}[5m - 5]$

0 = $\frac{5}{2}(m-1)$ $\therefore 5(m-1) = 0$ m-1=0 $\therefore$ m=1

19. Find the area of the triangle whose vertices are (-3,-5), (-4,6) and (1,-1).

(x_1, y_1) = (-3,-5) (x_2, y_2) = (-4,6) (x_3, y_3) = (1,-1)

A = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

= $\frac{1}{2}[-3(6-(-1)) + (-4)(-1-(-5)) + 1(-5-6)]$ = $\frac{1}{2}[-3(7) + (-4)(4) + 1(-11)]$

= $\frac{1}{2}[-21 - 16 - 11]$ = $\frac{1}{2} \times -48$ = -24 =24 sq.units

V. Three marks questions.

20. Find the type of the triangle formed by the points (3, 1), (7,4) & (11,1)and justify your answer.

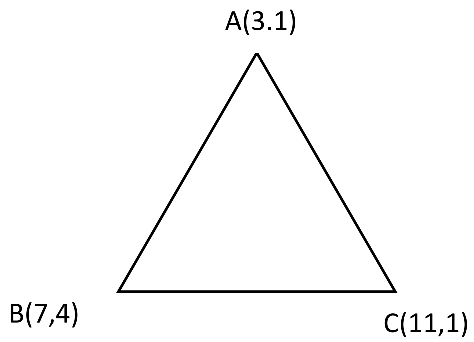
<u>Length of side AB</u>

(x_1, y_1) = (3,1) (x_2, y_2) = (7,4)

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(7-3)^2 + (4-1)^2}$ = $\sqrt{(4)^2 + (3)^2}$

= $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units.



Length of side BC

(x_1, y_1) = (7,4) (x_2, y_2) = (11,1)

BC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(11-7)^2 + (1-4)^2}$ = $\sqrt{(4)^2 + (-3)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units.

Length of side AC

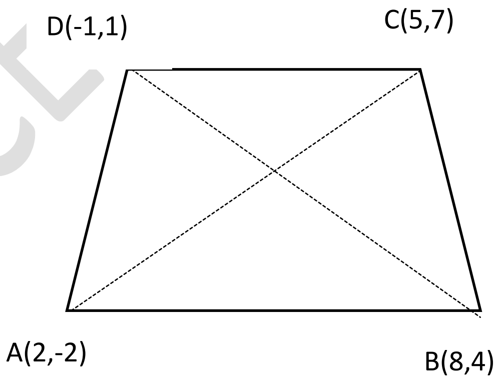
(x_1, y_1) = (3,1) (x_2, y_2) = (11,1)

AC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(11-3)^2 + (1-1)^2}$ = $\sqrt{(8)^2 + (0)^2}$= $\sqrt{64}$ = 8 = 8 units.

∴The triangle formed is an *Isosceles triangle* as two sides are of equal length.

21. Name the type of the Quadrilateral with reason, which is formed by joining the points A(2,-2), B(8,4), C(5,7) and D(-1,1) .



Length of side AB

(x_1, y_1) = (2,-2) (x_2, y_2) = (8,4)

AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(8-2)^2 + (4-(-2))^2}$ = $\sqrt{(6)^2 + (6)^2}$

= $\sqrt{36+36}$ = $\sqrt{36 \times 2}$= 6$\sqrt{2}$ units.

Length of side BC

(x_1, y_1) = (8,4) (x_2, y_2) = (5,7)

BC = $\sqrt{(5-8)^2 + (7-4)^2}$ = $\sqrt{(-3)^2 + (3)^2}$ = $\sqrt{9+9}$ = $\sqrt{9 \times 2}$= 3$\sqrt{2}$ units.

Length of side CD

(x_1, y_1) = (5,7) (x_2, y_2) = (-1,1)

CD = $\sqrt{(-1-5)^2 + (1-7)^2}$ = $\sqrt{36+36}$ = $\sqrt{36 \times 2}$ = 6$\sqrt{2}$ units.

Length of side AD

(x_1, y_1) = (2,-2) (x_2, y_2) = (-1,1)

AD = $\sqrt{(2-(-1))^2 + (-2-1)^2}$= $\sqrt{(3)^2 + (-3)^2}$ = $\sqrt{9+9}$ = $\sqrt{9 \times 2}$ = 3$\sqrt{2}$ units.

Length of diagonal AC

(x_1, y_1) = (2,2) (x_2, y_2) = (5,7)

AD = $\sqrt{(2-5)^2 + (-2-7)^2}$

= $\sqrt{(-3)^2+(-9)^2}$= $\sqrt{9+81}$ = $\sqrt{90}$ = $\sqrt{9\times10}$ = 3$\sqrt{10}$ units.

<u>Length of diagonal BD</u>

(x_1, y_1) = (-8,4) **(x_2, y_2) = (-1,1)**

BD = $\sqrt{(-1-8)^2+(1-4)^2}$

= $\sqrt{(-9)^2+(-3)^2}$ = $\sqrt{81+9}$ = $\sqrt{90}$ = $\sqrt{9\times10}$ = 3$\sqrt{10}$ units.

Here opposite sides are equal and diagonals are also equal ∴given quadrilateral is a *Rectangle.*

22. In what ratio is the line segment joining the points (2,-3) and (5,6) is divided by y-axis

A point on y-axis is in the form (0,y)

∴(x_1, y_1) = (2,-3) **(x_2, y_2) = (5,6)** **P(x,y) = (0,y)** **$m_1 : m_2$ = ?**

∴ P(x,y) = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

(0,y) = $\left(\frac{m_1\times5+m_2\times2}{m_1+m_2}, \frac{m_1\times6+m_2\times-3}{m_1+m_2}\right)$

(0,y) =$\left(\frac{5m_{1+2m_2}}{m_1+m_2}, \frac{6m_1-3m_2}{m_1+m_2}\right)$

$\frac{5m_{1+2m_2}}{m_1+m_2}$=0

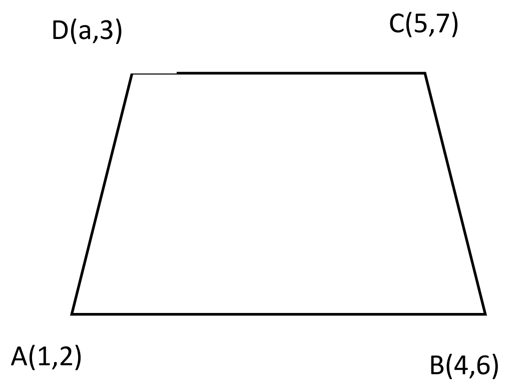
$5m_1$+$2m_2$=0 **$5m_1$=-$2m_2$**

$\frac{m_1}{m_2}$ = -$\frac{2}{5}$ **∴ m_1:m_2= -2 :5**

23. If (1,2),(4,6),(5,7) and (a,3) are the vertices of a parallelogram taken in order, find 'a`.

Opposite sides are equal in a parallelogram.

∴AB = CD → (1)



<u>Length of side AB</u>

(x_1, y_1) = (1,2) **(x_2, y_2) = (4,6)**

AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

= $\sqrt{(4-1)^2+(6-2)^2}$ = $\sqrt{(3)^2+(4)^2}$

= $\sqrt{9+16}$ = $\sqrt{25}$ =5 units.

<u>Length of side CD</u>

(x_1, y_1) = (5,7) **(x_2, y_2) = (a,3)**

CD = $\sqrt{(5-a)^2+(7-3)^2}$ = $\sqrt{(5-a)^2+16}$

But AB = CD ,from (1)

5 = $\sqrt{(5-a)^2+16}$ squaring both sides

5^2 = $\sqrt{(5-a)^2+16}^2$ 25 = $(5-a)^2$ + 16

$(5-a)^2$ =25- 16 = 9

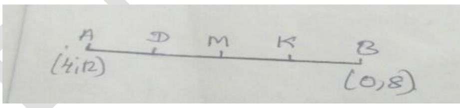
5 – a = 3 or 5a =-3

∴-a = 3-5 -a = -3-5

a = 2 a = 8

24. Find the coordinate points which divides the line joining the points (4,12) and (0,8) into four equal parts.

Let A(4,12), B(0,8) M is the mid point of AB .



∴ M = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (x_1,y_1) = (4,12) (x_2,y_2) = (0,8)

= $\left(\frac{4+10}{2},\frac{12+8}{2}\right)$ = $\left(\frac{4}{2},\frac{20}{2}\right)$ = (2,10)

'D` is the mid point of AM

Coordinates of D = $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (x_1,y_1) = (4,12) (x_2,y_2) = (2,10)= $\left(\frac{4+2}{2},\frac{12+10}{2}\right)$= $\left(\frac{6}{2},\frac{22}{2}\right)$ = (3,11)

'K ` is the mid point of MB

K= $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ x_1,y_1) = (2,10) (x_2,y_2) = (0,8)

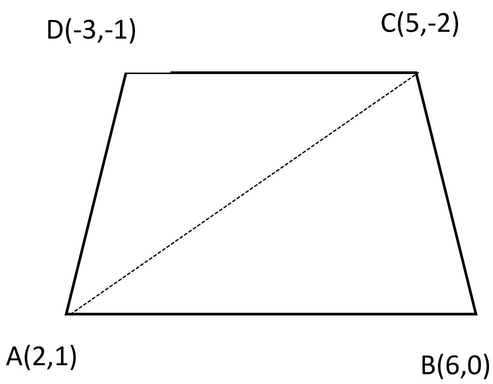
= $\left(\frac{2+0}{2},\frac{10+8}{2}\right)$ = $\left(\frac{2}{2},\frac{18}{2}\right)$ = (1,9)

∴The required coordinates are D (3,11) , M(2,10) and K (1,9)

25. Find the area of the quadrilateral formed by the points (2,1), (6,0), (5,-2) and (-3,-1) taken in order.

From the figure it is observed that the area of the Quadrilateral ABCD is the sum of the Areas of ΔABC and ΔADC

Area of **ΔABC**



(x_1,y_1) = (2,1)(x_2,y_2) = (6,0) (x_3,y_3) = (5,-2)

A = $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

= $\frac{1}{2}[2(0-(-2))+6(-2-1)+5(1-0)]$

$= \frac{1}{2}[2(2) + 6(-3) + 5(1)]$ $= \frac{1}{2}[4 - 18 + 5]$

$= \frac{1}{2} \times -9$ = -4.5 =4.5 sq.units

Area of **ΔADC**

(x_1, y_1) = (2,1) **(x_2, y_2) = (-3,-1)** **(x_3, y_3) = (5,-2)**

A $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[2(-1 - (-2)) + (-3)(-2 - 1) + 5(1 - (-1))]$

$= \frac{1}{2}[2(1) + (-1)(-3) + 5(2)]$ $= \frac{1}{2}[2 + 9 + 10] = \frac{1}{2} \times 21$ = 10.5

∴Area of ΔADC = 10.5 sq.units

∴ Area of Quadrilateral ABCD = $Area\ of\ \Delta$ABC + Area of ΔADC

= 4.5 + 10.5 = <u>15 sq.units</u>

26. Find a point on x-axis which is equidistant from the points (2,-5) and (-2,9)

A point on x-axis is in the form (x,0)

Here the length of AM and BM are same

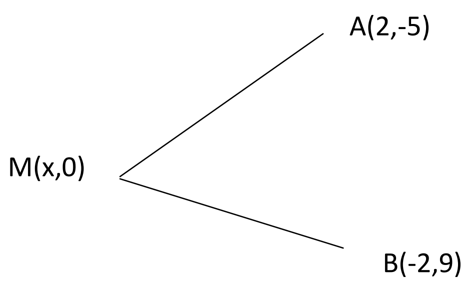
∴AM = BM

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-(-2))^2 + (0-9)^2}$

$\sqrt{(x-2)^2 + 5^2} = \sqrt{(x+2)^2 + (-9)^2}$ **Squaring both sides,we get**

$(x-2)^2 + 25 = (x+2)^2 + 81$



x^2 + 4 -4x +25 = x^2 + 4 +4x +81

29 – 4x = 4x + 85

-4x – 4x = 85 – 29

-8x = 56 **x = $\frac{56}{-8}$** **x = -7**

Required point is <u>(-7,0)</u>

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit -7 :-Quadratic equation

I Choose the correct answer for the fallowing questions.

1. **If the roots of $ax^2 + bx + c = 0$ are equal then,**

A) $\frac{b}{2a} = \frac{2c}{b}$ B) $b^2 + 4ac = 0$ C) $\frac{b}{2a} = \frac{b}{2c}$ D) a = b

2. If one root of $px^2 + qx + r = 0$ is reciprocal of the other root then,

A) p = q B) q = r C) p = r D) p = q = r

3. The sum of the roots of $3x^2 + 6x + 3 = 0$ is

A) 2 B) -3 C) 1 D) -2

4. If one root of $2x^2 + kx + 4 = 0$ is -2, then the value of k is

A) 12 B) -6 C) 6 D) -12

5. The nature of the roots of $2x^2 – 4x - 3 = 0$ is

A) Real & distinct B) real & equal C) no real roots D) imaginary roots

6. The roots of quadratic equation $3x^2 – 6x = 0$ are

A) (0,2) B) (3,6) C) (0,-2) D) (0,6)

7. The sum of the squares of two consecutive natural numbers is 20. Representing this statement in the form of quadratic equation is,

A) $X^2 + (x + 1)^2=20$ B) $x^2 – (x – 1)^2 = 20$ C) $(x + 1)^2 – x^2 =20$ D) $x^2+(x+1)^2+20=0$

II. One mark questions.

8.Write the standard form of quadratic equation. Ans:-ax2 + bx + c =0

9.Write the formula to find the roots of $ax^2 + bx + c =0$. Ans:-$x = \frac{-b \pm \sqrt{(b^2 - 4ac}}{2a}$

10. The quadratic equation whose roots are x = 2 & x = -3. Ans:-$x^2+x-6=0$

11.If the roots of $6x^2 – 24x + c = 0$ are equal, then find the value of c. Ans:- 24

12.Determine whether -3 is a root of the equation $3x^2 + 5x – 8 = 0$. Ans:- No

III. Two mark questions.

13. Check whether, y(y+7) + 9 = (y+7)(y-7) form a quadratic equation.

- ⋆ $y(y+7)+9=(y+7)(y-7)$
- ⋆ $y^2+7y+9=y^2-49$
- ⋆ $y^2-y^2+7y+9+49=0$
- ⋆ $7y+58=0$

$\therefore y(y+7)+9=(y+7)(y-7)$ is not a quadratic equation.

14.If the roots of $9x^2 - 3kx + 4 = 0$ are equal, then find the value of k

$9x^2 - 3kx + k = 0$

a = 9 b = - 3 k c = k .

$b^2 = 4ac$

$(-3k)^2 = 4(9)(k)$

$\Rightarrow 9k^2 = 36k$

$\Rightarrow k^2 = 36k / 9$

$\Rightarrow k^2 = 4k$

$\Rightarrow k = 4$

The value of k is 4 .

15.Find the roots of $3x^2 + 11x + 8 = 0$ by factorization method.

$3x^2+11x+8=0$

$3x^2+8x+3x+8=0$

$3x(x+8)+3(x+8)=0$

$(3x+3)(x+8)=0$

16.Check the nature of the roots of $2x^2 + 5x + 5 = 0$.

$\Rightarrow 2x^2 + 5x + 5.$

$\Rightarrow\Delta$ = discriminant Or $b^2 - 4ac$.

$\Rightarrow = (5)^2 - 4(2)(5) = 25 - 40 = -15.$

$\Rightarrow\Delta < 0$ Roots are imaginary.

17.A chess board contains 64 equal squares & the area of each square is 6.25 cm^2. A border around the board is 2cm wide. Find the length of each side of the chess board.

we know that a chess board contains 64 equal squares.
given the area of one square = $6.25cm^2$
total area of 64 equal squares = $64 \times 6.25 = 400cm^2$
therefore side × side = $400cm^2$
==> $side^2 = 400cm^2$
==> side = √400
==> side = 20cm
the side of the board is 20cm.
ATQ, the width of the border around the board is 2cm.
hence, the length of the side of the chess board Along with the border is = 20 + 2 + 2 (as border is all around so the length will increase from both sides)
= 24cm

18. Find the quadratic equations whose roots are $2\sqrt{3}$ and - 2

Let the quadratic equation be

$ax^2+bx+c=0, a\neq0$ and

$it's\ zeroes\ be\ \alpha$
$and\ \beta.$
$Here, \alpha = 2+\sqrt{3}, and\ \beta = 2-\sqrt{3}$
$Sum\ of\ the\ roots$
$= \alpha + \beta$
$= 2+\sqrt{3}+2-\sqrt{3}$
$= 4$

$Product\ of\ the\ roots$
$= \alpha\beta$
$= (2+\sqrt{3})(2-\sqrt{3})$
$= 2^2 - (\sqrt{3})^2$
$= 4 - 3$
$= 1$

Therefore,

The quadratic equation is $ax^2+bx+c=0$ is

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\implies x^2 - 4x + 1 = 0$

19.The area of a triangular plot is 156cm^2. The base of the plot is 2cm more than twice its height. Form a quadratic equation.

Let the breadth of the plot be x metres
It is given that the length of the plot is one more than twice its breadth
Length=(2x+1) metres
Now. Area of the plot =528m^2
Length× Breadth=528m^2
⇒(2x+1)×x=528
⇒2x2+x−528=0
This is the required quadratic equation

20.Solve by using formula:

a) $3x^2 - 7x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$$ Substitute the values into the formula.

$$= \frac{7 \pm \sqrt{49+72}}{6}$$

$$= \frac{7 \pm \sqrt{49+72}}{6}$$ Simplify the radicand.

$$= \frac{7 \pm \sqrt{121}}{6}$$

$$= \frac{7 \pm 11}{6}$$

$$x = \frac{7+11}{6} \qquad x = \frac{7-11}{6}$$

$$= \frac{18}{6} \qquad = \frac{-4}{6}$$

$$= 3 \qquad = \frac{-2}{3}$$

IV. Three mark questions.

21. If one root of the equation x^2 + px + 12= 0 is 4, while the equation x^2 + px + q has equal roots. Find the value of q.

$Given\ 4\ is\ one\ root\ of\ quadratic$
$equation\ x^2 + px + 12 = 0$

Substitute x = 4 in the equation, we get

$\implies 4^2 + p \times 4 + 12 = 0$

Divide each term by 4, we get

$\implies 4 + p + 3 = 0$
$\implies p + 7 = 0$
$\implies p = -7 \ --(1)$

Now,

$Comparex^2 + px + q = 0\ with\ ax^2 + bx + c = 0,$
$we\ get$
$a = 1,\ b = p,\ c = q$
$Discreminant(D) = 0$

Given roots are equal

$b^2 - 4ac = 0$
$\implies p^2 - 4 \times 1 \times q = 0$
$\implies (-7)^2 - 4q = 0\ [from\ (1)]$
$\implies 49 = 4q$
$\implies q = \frac{49}{4}$

22.The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle.

- **Perimeter = 60cm**
- **hypotenuse = 25cm**

Let ABC be the Given right angled triangle

Let , Base = Xcm

hypotenuse = 25cm

Perimeter = 60cm

AB+BC+AC = 60

→ AB +x+25=60

→ AB =35-x

By Pythagoras theorem

h^2= p^2+b^2

→ 25^2= $(35-x)^2$+x^2

→ $2x^2-70x+600=0$

→ $x^2-35x+300=0$

→ $x^2-20x-15x+300=0$

→ $x(x-20)-15(x-20)=0$

→ $(x-20)(x-15)=0$

$x=20$ or $x=15$

If $x= 20$ Then,

$AB = 35-x = 35-20 = 15$

$BC = x = 20$

Area = $1/2 X b X h$

$=1/2 \times 20 \times 15= 1/2 \times 300$

$= 150cm^2$

23.Sum of the areas of two squares is 468 m^2. I f the difference of their perimeter Is 24 m, find the sides of two squares.

Let the sides of the two squares are x m and y m.
so, their perimeter will be 4x and 4yrespectively and their areas will be x^2 and y^2respectively.
[as you know perimeter of square = 4 × side length and area of square = (side length)²]
It is given that $4x - 4y = 24$ [Difference of perimeter]
or $x - y = 6$
$x = y + 6$.........(1)
Also, $x^2 + y^2 = 468$ [sum of squares is 468]
=> $(6+y)^2 + y^2 = 468$ [put eq (1)]
=> $36 + y^2+12y + y^2 = 468$
=> $2y^2 + 12y - 432 = 0$
=> $y^2 + 6y - 216 = 0$
=> $y^2 +18y - 12y - 216 = 0$
=> $y(y + 18)(y - 12) = 0$
=> $y = - 18$ or 12.
However, side of a square cannot be negative.
Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

25.The sum of the squares of two consecutive natural numbers is 365. Find the numbers.

Let the first number = x

Then the second number = x+1

According to the question ,

$(x)^2+(x+1)^2= 365$

$x^2+x^2+1+2x = 365$

$2x^2+2x+1 = 365$

$2x^2+2x = 364$

$2x^2+2x-364 = 0$
$2x^2-26x+28x-364 = 0$
$2x(x-13)+28(x-13)$
$(x-13)(2x+28) = 0$
$x-13 \Rightarrow x = 13$
$2x+28 \Rightarrow x = -28/2$
But the value of x can't be negative so $x = 13$ $x+1 = 13+1 = 14$
The required numbers are = 13 and 14

24.The perimeter of rectangular field is 28 cm and its area is 48 cm^2. Find its length and breadth.

P=28cm, A=42 cm^2
Perimeter= $2(l+b)$
$28= 2(l+b)$
$l+b=14$
$l=14-b$ --------------1

Area= $l \times b$
$48= (14-b)b$ (from 1)
$48=14b-b^2$
$b^2-14b+48=0$
$b^2-8b-6b+48=0$
$b(b-8)-6(b-8)=0$

b=6 0r b=8
substitute b=6 in eq. 1
$l= 14-6=8$
l=8 and b=6

25.The sum of the ages of a father and his son is 45 years. Five years ago the product of their age was 124. Determine their present ages.

Let father's age be *x* years. Then
➽ Present age of son = $(45 – x)$
[5 years ago their ages were]
Father's age = $(x – 5)$ years
Son's age = $(45 – x – 5) = (40 – x)$ years
Product of ages is 124.
$(x – 5)(40 – x) = 124$
$x(40 – x) – 5(40 – x) = 124$
$40x – x^2 – 200 + 5x = 124$
$45x – x^2 = 124 + 200$
$45x – x^2 = 324$
$x^2 – 45x + 324=0$
Now, break this by middle term splitting method.
$x^2 – 45x + 324=0$
$x^2 – 36x – 9x + 324=0$
$x(x – 36) – 9(x – 36)=0$
$(x – 9)(x – 36)=0$ ⇨ $(x – 9) = 0$ or, $(x – 36) = 0$
$x = 9$ or $x = 36$
Father's age cannot be 9 years so we will take x = 36 years.
So,➪ Father's age = x = 36 years➪ Son's age = 45 – x = 45 – 36 = 9 years.

26.A two-digit number is four times the sum of their digits. It is also equal to 2 times the product of their digits. Find the number.

let the two digits be x & y

10x + y = the number

A two digit number is 4 times the sum of its digits

10x + y = 4(x+y)

10x + y = 4x + 4y

10x - 4x = 4y - y

6x = 3y

divide both sides by 3

2x = y

" and twice the product of its digit."

10x + y = 2xy

Replace y with 2x

10x + 2x = 2 (x)2x

$12x = 4x^2$

Divide both sides by 4x

3 = x

Theny = 2(3)= 6

The number = 36

You can confirm this in both statement:

36 = 4(3+6)and

36 = 2(3*6)

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit-8 : Introduction to Trignometry

I. Choose the most appropriate answer for the following questions.

1. $(1+\cos\theta)(1-\cos\theta) =$

a) $\sin^2\theta$ b) $\tan^2\theta$ c) 1   d) 0

2. sin A . cosA . tanA + cosA. sinA. cotA =

a) $\sin^2A - \cos^2A$ b) $\tan^2A + \cot^2A$ c) $\sin^2A + \cos^2A$ d) $\sin^2A + \tan^2A$

3. If $1 - \cos^2\theta = \frac{3}{4}$ then the value ofsinθ

a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) 1   d) 0

4. 2 cos θ =1 andθ is an acute angle then the value of 'θ'

a) 0^0 b) 30^0 c) 45^0 d) 60^0

5. If $\sin\theta = \frac{3}{5}$ then the value of $\operatorname{cosec}\theta$

a) $\frac{4}{5}$ b) $\frac{5}{3}$ c) $\frac{4}{3}$ d) $\frac{5}{4}$

6. If sinθ = cosθ then the value of θ

a) 0^0 b) 30^0 c) 45^0 d) 90^0

7. maximum value of sinθ is

a) $\frac{2}{\sqrt{3}}$ b) $\frac{\sqrt{3}}{2}$ c) 1 d) $\sqrt{2}$

8. The value of $\cos 48^0 - \sin 42^0$ is

a) 0 b) $\frac{1}{4}$ c) 1 d) $\frac{1}{2}$

9. If 13 sinθ = 5 then the value of tanθ

a) $\frac{5}{12}$ b) $\frac{12}{5}$ c) $\frac{12}{13}$ d) $\frac{5}{13}$

10. The value of $\frac{\tan 65}{\cot 25}$

a) $\sqrt{2}$ b) 0 c) 1 d) $\frac{1}{\sqrt{2}}$

II. very short answer questions (1 mark)

1. Find the value of $\sin 90^0 + \tan 45^0$ Ans:- $\sin 90^0 + \tan 45^0 = 1 + 1 = 2$

2. Find the value of sin θ x cosec θ . Ans:- $\sin\theta \times \frac{1}{\sin\theta} = 1$

3 .If $\sqrt{3}\cot A = 1$ then find the value of acute angle A.

Ans:- $\cot A = \frac{1}{\sqrt{3}}$

$= \cot 60^0 = \frac{1}{\sqrt{3}}$

$\therefore A = 60^0$

4. Find the value of $\operatorname{cosec} 31^0 - \sec 59^0$.

Ans:- $= \operatorname{cosec}(90^0 - 59^0) - \sec 59^0$

$= \sec 59^0 - \sec 59^0 = 0$

5. Find the value of $\frac{1 - \tan 45}{1 + \tan 45}$ Ans:- $= \frac{1-1}{1+1} = \frac{0}{2}$

6. Find the value of sin θ x cosec θ . Ans:- $\sin\theta \times \operatorname{cosec}\theta = \sin\theta \times \frac{1}{\sin\theta} = 1$

7. If sin θ $= \frac{2}{\sqrt{3}}$ and cos θ $= \frac{3}{\sqrt{3}}$ then find the value oftan θ . Ans:- $tan\theta = \frac{sin\theta}{cos\theta} = \frac{\frac{2}{\sqrt{3}}}{\frac{3}{\sqrt{3}}} = \frac{2}{3}$

8. If cos A=sinB then show that A+B=90^0.

Ans:- cos A=sinB

Sin(90^0-A)=sinB

90^0-A=B

90^0 = A+B

9. Evaluate tan 23^0 . tan67^0

Ans:- tan(90^0-67^0) tan 67^0

= cot67^0. tan67^0 = $\frac{1}{\tan 670}$ x tan67^0 = 1

10. Show that(1+$\tan^2$ θ).$\cos^2$ θ=1 Ans:- $\sec^2\theta$ x$\cos^2$ θ = $\frac{1}{\cos^2\theta}$ x $\cos^2$ θ =1

III. 2 marks questions.

1. If tan2A = cot(A-18^0) and2A is an acute angle then find A.

cot(90^0 – 2A)= cot(A-18^0)

90^0 – 2A= A-18^0

90^0+18^0 = A +2A

3A= 108^0

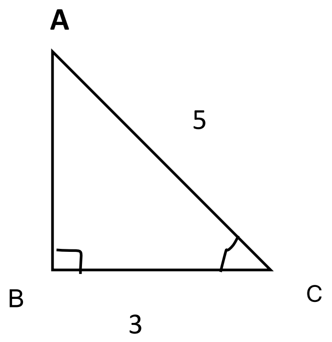
A=36^0

2. Show that (tan A x sinA) + cosA = secA

Ans:- $= \frac{sinA}{cosA}$ x sinA + cos A

$= \frac{sin^2A}{cosA}$ + cos A = $\frac{sin^2A+cos^2A}{cosA}$ $= \frac{1}{cosA}$ = secA ($\sin^2$A + $\cos^2$A=1)

3.If cosθ =0.6 then prove that 5sin θ - 3 tanθ =0 (Hint: $0.6 = \frac{6}{10}$)



Ans:- cos θ = $\frac{6}{10} = \frac{3}{5}$ $AC^2 = AB^2+BC^2$

$5^2 = AB^2+3^2$

AB^2 = 25-9=16

AB =4

L.H.S = 5sin θ - 3 tanθ

$5x\frac{4}{5} - 3x\frac{4}{3}$ = 4-4=0 =R.H.S

4. **Evaluate $\sin 18^0 - \cos 72^0 + \cos 18^0 - \sin 72^0$.**

Ans:- $\sin(90^0-72^0) - \cos 72^0 + \cos(90^0-72^0) - \sin 72^0$

$= \cos 72^0 - \cos 72^0 + \sin 72^0 - \sin 72^0 = 0$

5. **If $A=60^0$, $B=30^0$ then show that $\cos(A+B) = \cos A.\cos B - \sin A.\sin B$**

Ans:- $\cos(A+B) = \cos A.\cos B - \sin A.\sin B$

$\text{Cos}(60^0 + 30^0) = \cos 60^0 . \text{c0s}30^0 - \sin 60^0 . \sin 30^0$

$\text{Cos } 90^0 = \frac{1}{2} \text{x} \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \text{x} \frac{1}{2}$

$0 = 0$

6. **If $A=60^0$, $B=30^0$ then show that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A . \tan B}$**

Ans:- $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A . \tan B}$

$\tan(60^0 - 30^0) = \frac{\tan 60^0 - \tan 30^0}{1+\tan 60^0 . \tan 30^0}$

$\tan 30^0 = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+\sqrt{3} \ X \frac{1}{\sqrt{3}}}$

$\frac{1}{\sqrt{3}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

IV. Prove that the following questions. (3 marks)

1. $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Ans:- $\sqrt{\frac{1+\sin A}{1-\sin A}} \text{x} \sqrt{\frac{1-\sin A}{1-\sin A}} = \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1-\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

2. **If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, Here $0^0 < (A+B) \le 90^0$; then find the value of A and B.**

Ans:- $\tan(A+B) = \sqrt{3}$

$\tan 60^0 = \sqrt{3}$

$A+B=60^0$ ----- 1

$\tan(A-B) = \frac{1}{\sqrt{3}}$

$\tan 30^0 = \frac{1}{\sqrt{3}}$

$A-B=30^0$ -----2

$A+B=60^0$

$A-B=30^0$

$2A = 90$

$A = 45^0$

eq. 1 $\rightarrow 45^0 + B = 60^0$

$B = 15^0$

3. $\frac{\sin(90-\theta)}{1+\sin\theta} + \frac{\cos\theta}{1-\cos(90-\theta)} = 2\sec\theta$

Ans:- L.H.S = $\frac{sin(90-\theta)}{1+\sin\theta} + \frac{cos\theta}{1-cos(90-\theta)}$

$= \frac{cos\theta}{1+\sin\theta} + \frac{cos\theta}{1-sin\theta} = \frac{cos\theta\,(1-\sin\theta)+cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \frac{cos\theta - \cos\theta.\sin\theta + cos\theta + \cos\theta.\sin\theta)}{1^2 - sin^2\theta}$

$= \frac{2cos\theta}{cos^2\theta} = \frac{2}{\cos\theta}$=2secθ

4. $\tan^2A - \sin^2A = \tan^2A.\sin^2A$

Ans:- $\tan^2A - \sin^2A = \frac{sin^2A}{\cos^2A} - \sin^2 A = \sin^2 A\,(\frac{1}{\cos^2 A} - 1) = \sin^2 A\,(\frac{1-\cos^2 A}{\cos^2 A}) = \tan^2A\,.\,\sin^2A$

5. $\frac{1-cos\theta}{1+\cos\theta}$ = $(\text{cosec}\,\theta - \cot\theta)^2$

Ans:- $\frac{1-cos\theta}{1+\cos\theta} \times \frac{1-cos\theta}{1-\cos\theta} = \frac{(1-cos\theta)^2}{1^2-\cos^2\theta} = \frac{1^2+cos^2\theta-2\cos\theta}{sin^2\theta} = \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - \frac{2cos\theta}{\sin^2\theta}$

$= \text{cosec}^2\,\theta + \cot^2\theta - \frac{2cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} = \text{cosec}^2\,\theta + \cot^2\theta - 2\cot\theta\,.\,\text{cosec}\theta = (\text{cosec}\,\theta - \cot\theta)^2$

6. $\frac{sin\theta}{1+\cos\theta} + \frac{1+cos\theta}{sin\theta}$ = 2 cosec θ

Ans:- $\frac{sin\theta}{1+\cos\theta} + \frac{1+cos\theta}{sin\theta} = \frac{sin^2\theta+(1+cos\theta)^2}{(1+\cos\theta)\sin\theta} = \frac{sin^2\theta+1^2+cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta} = \frac{1+1+2cos\theta}{(1+cos\theta)sin\theta}$

$= \frac{2+2cos\theta}{(1+cos\theta)sin\theta} = \frac{2(1+cos\theta)}{(1+cos\theta)sin\theta} = \frac{2}{sin\theta}$ = 2 cosec θ

7. x= a sec θ + b tan θ andy= a tan θ + b sec θthen prove that $x^2 - y^2 = a^2 - b^2$

Ans:- $x^2 - y^2 = a^2 - b^2 = (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + 2a\sec\theta.\,b\tan\theta - a^2\tan^2\theta - b^2\sec^2\theta - 2a\tan\theta.\,b\sec\theta$

$= a^2\sec^2\theta + b^2\tan^2\theta - a^2\tan^2\theta - b^2\sec^2\theta$

$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\tan^2\theta - \sec^2\theta)$ == $(\sec^2\theta - \tan^2\theta)(a^2 - b^2) = 1(a^2 - b^2) = (a^2 - b^2)$

8. $(\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2A + \cot^2A$

Ans:- L.H.S = $(\sin A + \text{cosec}A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A.\,\text{cosec}A + \cos^2 A + \sec^2 A + 2\cos A.\sec A$

$= \sin^2 A + \cos^2 A + \text{cosec}^2 A + \sec^2 A + 2(\sin A.\,\text{cosec}A + \cos A.\sec A)$

$= 1 + \text{cosec}^2 A + \sec^2 A + 2(1+1)$

$= 1 + \text{cosec}^2 A + \sec^2 A + 2(2) = 5 + 1 + \cot^2 A + 1 + \tan^2 A = 7 + \cot^2 A + \tan^2 A$

V. Prove that the following questions. (4 marks)

1. $\frac{sin(90-A)}{1-tanA} + \frac{cos(90-A)}{1-cotA}$ = cosA +sinA

Ans:- $\frac{cosA}{1-tanA} + \frac{sinA}{1-cotA} = \frac{cosA}{1-\frac{sinA}{cosA}} + \frac{sinA}{1-\frac{cosA}{sinA}} = \frac{cosA}{\frac{cosA-sinA}{cosA}} + \frac{sinA}{\frac{sinA-cosA}{sinA}} = \frac{cos^2 A}{cosA-sinA} + \frac{sin^2 A}{sinA-cosA}$

$= \frac{cos^2 A - sin^2 A}{cosA - sinA} = \frac{(cosA+sinA)(cosA-sinA)}{(cosA-sinA)}$ = cosA+sinA

2. $\frac{cosecA+1}{cosecA-1}$ = (secA+tanA)^2

Ans:- $\frac{cosecA+1}{cosecA-1}$ x $\frac{cosecA+1}{cosecA+1} = \frac{(cosecA+1)^2}{cosec^2A-1^2} = \frac{cosec^2A+1^2+2\,cosecA}{cot^2A} = \frac{cosec^2 A}{cot^2A} + \frac{1}{cot^2A} + \frac{2\,cosecA}{cot^2A}$

$= \frac{1}{\sin^2 A}$ x $tan^2A + tan^2A$ + 2x $\frac{1}{sinA}$ xtan^2A

$= \frac{1}{\sin^2A}\ X\ \frac{sin^2 A}{cos^2A} + tan^2A + \frac{1}{sinA}$ x $\frac{sin^2 A}{cos^2A}$ = sec^2 A +tan^2A +2 tanA.secA = $(secA + tanA)^2$

3. If secA=x+$\frac{1}{4x}$ thensecA-tanA =$\frac{1}{2x}$

Ans:- secA-tanA =$\frac{1}{2x}$

X+$\frac{1}{4x}$ – tanA =$\frac{1}{2x}$

tanA= X+$\frac{1}{4x}$ - $\frac{1}{2x} = \frac{4x^2+1-2}{4x} = \frac{4x^2-1}{4x}$

∴ secA – tanA=(x+ $\frac{1}{4x}$) – ($\frac{4x^2-1}{4x}$) =$\frac{4x^2+1}{4x}$ - $\frac{4x^2-1}{4x} = \frac{4x^2+1-4x^2+1}{4x} = \frac{2}{4x} = \frac{1}{2x}$

4. $\frac{tan\theta}{1+cot\theta} + \frac{cot\theta}{1-tan\theta}$ =1+tanθ+cotθ

Ans:- $\frac{tan\theta}{1+cot\theta} + \frac{cot\theta}{1-tan\theta} = \frac{tan\theta(1-tan\theta)+cot\theta(1+cot\theta)}{(1+cot\theta)(1-tan\theta)} = \frac{tan\theta - \tan^2\theta + cot\theta + \cot^2\theta)}{(1+cot\theta)(1-tan\theta)} = \frac{tan\theta + cot\theta + \cot^2\theta - \tan^2\theta}{1-tan\theta + \cot\theta - cot\theta\ tan\theta}$

$= \frac{tan\theta + cot\theta + \cot^2\theta - \tan^2\theta}{1-tan\theta + cot\theta - cot\theta tan\theta} = \frac{(tan\theta+cot\theta)+(cot\theta-tan\theta)((cot\theta+tan\theta)}{1-tan\theta+cot\theta-1} = \frac{(tan\theta+cot\theta)(1+cot\theta-tan\theta)}{cot\theta - tan\theta}$= cot θ+tanθ+1

5.$\sqrt{sec^2\theta + cosec^2\theta}$ = tan θ + cot θ

Ans:- L.H.S= $\sqrt{sec^2\theta + cosec^2\theta} = \sqrt{\frac{1}{cos^2\theta} + \frac{1}{sin^2\theta}} = \sqrt{\frac{sin^2\theta+cos^2\theta}{cos^2\theta.sin^2\theta}} = \sqrt{\frac{1}{cos^2\theta.sin^2\theta}} = \frac{1}{cos\theta\ .sin\theta}$

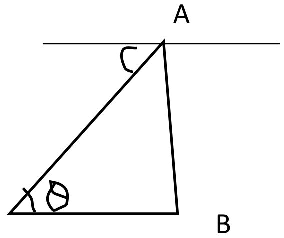
R.H.S = tan θ + cot θ = $\frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta} = \frac{sin^2\theta+cos^2\theta}{cos\theta.sin\theta} = \frac{1}{cos\theta.sin\theta}$

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit-9:- Some applications of Trignometry

I. 2 Marks questions.

1. Find the angle of depression, when a person standing on the ground is observed from the tip of the tower $50\sqrt{3}$m high, who is standing $50\sqrt{3}$m away from the foot of the tower.



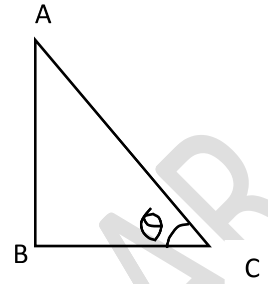
Ans:- θ= angle of dispersion =?

tan θ=$\frac{AB}{BC} = \frac{50\sqrt{3}}{50\sqrt{3}}$ =1

$\tan\theta = \tan 45^0 \therefore \theta = 45^0$

2. A tower stands vertically on the ground from a point on the ground which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60^0. Find the height of the tower.

Ans:- θ= angle of elevation=60^0 AB= height of the tower=?



$\tan\theta = \frac{AB}{BC}$

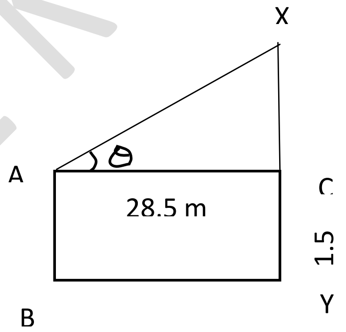
$\tan 60^0 = \frac{AB}{15}$

$\sqrt{3} = \frac{AB}{15}$

$\therefore AB = 50\sqrt{3}$m

3. An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45^0. What is the height of the chimney?

Ans:- AB=1.5 m= CY BY= 28.5 m



θ= angle of elevation=45^0

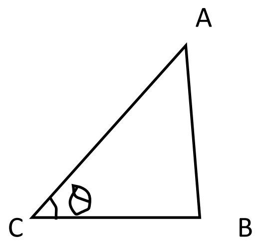
In triangle ACY, $\tan\theta = \frac{XC}{AC}$

$\tan 45^0 = \frac{XC}{28.5}$

$1 = \frac{XC}{28.5}$

XC= 28.5 m Height of chimney XY = XC + CY= 28.5+1.5=30 m

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30^0. Find the height of the tower.



Ans:- θ= angle of elevation=30^0

AB= height of the tower=?

$\tan\theta = \frac{AB}{BC}$

$\tan 30^0 = \frac{AB}{30}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30}$

$30 = \sqrt{3}AB$

$AB = \frac{30}{\sqrt{3}} = \frac{3x10}{\sqrt{3}} = \frac{\sqrt{3}x\sqrt{3}x10}{\sqrt{3}} = 10\sqrt{3}$ m

5. A tower stands vertically on the ground from a point on the ground which is 50 m away from the foot of the tower, the angle of elevation of the top of the tower 60^0. Find the height of the tower.

θ= angle of elevation=60^0

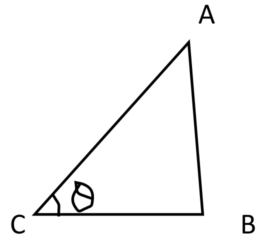
AB= height of the tower=?

tan θ=$\frac{AB}{BC}$

tan60^0=$\frac{AB}{50}$

$\sqrt{3}$= $\frac{AB}{50}$

AB =50$\sqrt{3}$ m



6. Two wind mills of height 50 m and 40 m are on either side of the field. A person observes the top of the wind mills from a point on the ground in between the towers. The angle of elevation was found to be 45^0 in both the cases, find the distance between the wind mills.

Ans:- Distance between the wind mills AB= AP+BP=?

In triangle APY, tan θ=$\frac{AX}{AP}$

tan45^0 =$\frac{50}{AP}$

1= $\frac{50}{AP}$

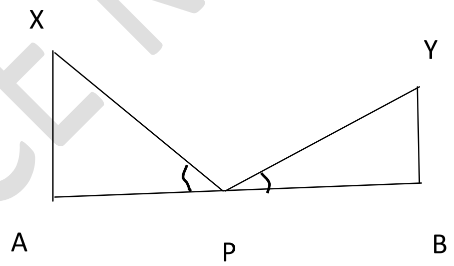
AP=50 m

In triangle BPY, tan θ=$\frac{BY}{BP}$

tan45^0 =$\frac{40}{BP}$

1= $\frac{40}{BP}$

BP=40 m   Distance between the wind mills AB= AP+BP = 50+40=90 m



7. A ladder 15 m long just reaches the top of vertical wall. If the ladder makes an angle of 60^0 with the wall, find the height of the wall.

Ans:- AC= 15 m   AB=?

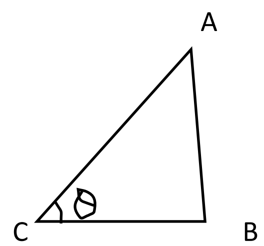
sin θ=$\frac{AB}{AC}$

sin60^0 =$\frac{AB}{15}$

$\frac{\sqrt{3}}{2}$=$\frac{AB}{15}$

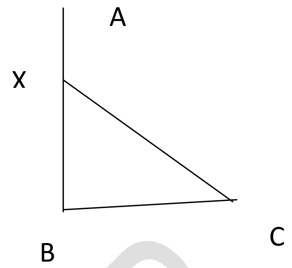
15$\sqrt{3}$ = 2AB

AB=$\frac{15\sqrt{3}}{2}$ m



II. 3 Marks questions.

1. A tree breaks due to storm and the broken part bends that the top of the tree touches the ground making an angle of 30^0 with it the distance between the foot of the tree to the point where the top touches the ground is 8m. How tall was the tree.



Ans:- XC= Brocken part of tree=? **AX=XC**

$\tan\theta=\frac{BX}{BC}$ $\frac{1}{\sqrt{3}}=\frac{BX}{8}$

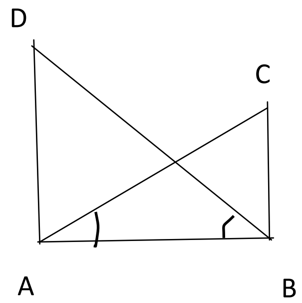
$\tan 30^0=\frac{BX}{8}$ $BX=\frac{8}{\sqrt{3}}$

$\sin\theta=\frac{BX}{XC}$ $\frac{1}{2}=\frac{\frac{8}{\sqrt{3}}}{XC}$

$\sin 30^0=\frac{\frac{8}{\sqrt{3}}}{XC}$ $XC=\frac{16}{\sqrt{3}}$

Height of the tree= $\frac{8}{\sqrt{3}}+\frac{16}{\sqrt{3}}=\frac{8+16}{\sqrt{3}}=\frac{24}{\sqrt{3}}=\frac{8x3}{\sqrt{3}}=\frac{8x\sqrt{3}x\sqrt{3}}{\sqrt{3}}=8\sqrt{3}\ m$

2. The angle of elevation of the top of the building from the foot of tower is 30^0 and the angle of elevation of the top of the tower from the foot of the building is 60^0. If the tower is 50m high. Find the height of the building.



Ans:- AD= Height of tower = 50 m **In triangle ABC,** $\tan\theta=\frac{BC}{AB}$

BC= height of the building =? $\tan 30^0=\frac{BC}{\frac{50}{\sqrt{3}}}$

In triangle ABD, $\tan\theta=\frac{AD}{AB}$ $\frac{1}{\sqrt{3}}=\frac{BC}{\frac{50}{\sqrt{3}}}$

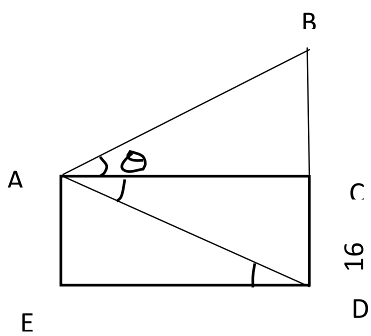
$\tan 60^0=\frac{50}{AB}$ $\sqrt{3}\ BC=\frac{50}{\sqrt{3}}$

$\sqrt{3}=\frac{50}{AB}$ $BC=\frac{50}{\sqrt{3}X\sqrt{3}}$

$AB=\frac{50}{\sqrt{3}}$ $BC=\frac{50}{3}$ **m**

3.From the top of a building 16m high. The angular elevation of the top of a hill is 60^0 and the angular depression of the foot of the hill is 30^0. Find the height of the hill.

Ans:- In triangle ADE, , $\tan\theta=\frac{AE}{DE}$ **In triangle ABC,** $\tan\theta=\frac{BC}{AC}$



$\tan 30^0=\frac{16}{DE}$ $\tan 60^0=\frac{BC}{16\sqrt{3}}$

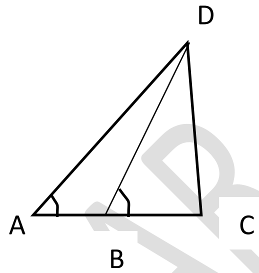
$\frac{1}{\sqrt{3}}=\frac{16}{DE}$ $\sqrt{3}=\frac{BC}{16\sqrt{3}}$

$DE=16\sqrt{3}$ **m** $BC=16x\sqrt{3}x\sqrt{3}$

BC= 16x3=48 m

Height of tower BD=BC+DC=48+16=64 m

4. The angle of elevation of the top of a tower from a point A on the ground is 30^0 moving a distance of 20m towards foot of the tower to a point B, the angle of elevation increases to 60^0. Find the height of the tower and the distance of tower from the point A. $(\sqrt{3} = 1.732)$



Ans:- AB=20m, BC=x m

In triangle BDC,

$\tan\theta = \frac{DC}{BC}$

$\tan 60^0 = \frac{DC}{x}$

$\sqrt{3} = \frac{DC}{x}$

$DC = \sqrt{3}\,x$

In triangle ADC

$\tan\theta = \frac{DC}{AC}$

$\tan 30^0 = \frac{\sqrt{3}x}{20+x}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{20+x}$

$20+x = \sqrt{9}\,x$

$20+x = 3x$

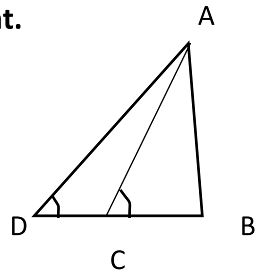
$2x = 20$

$X = 10$ m

Height of the tower $DC = \sqrt{3}\,x = 10\sqrt{3}$ m

Distance of tower from point A = AC=AB+BC=20+10=30 m

5. A man in a boat rowing away from a light house 150m high takes 2 minutes to change the angle of elevation of the top of the light house from 60^0 to 45^0. Find the speed of the boat.



Ans:- AB –Light house

In triangle ABC

$\tan\theta = \frac{AB}{BC}$

$\tan 60^0 = \frac{150}{BC}$

$\sqrt{3} = \frac{150}{BC}$

$BC = 150\sqrt{3}$ m

In triangle ABD

$\tan\theta = \frac{AB}{BD}$

$\tan 45^0 = \frac{150}{BC+CD}$

$1 = \frac{150}{150\sqrt{3}+CD}$

$50\sqrt{3} + CD = 150$

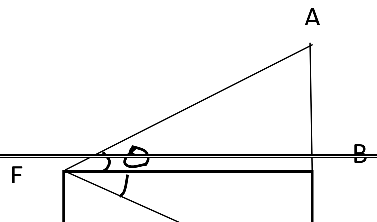
$CD = 150 - 50\sqrt{3} = 50(3-\sqrt{3}) = 50(\sqrt{3}X\sqrt{3} - \sqrt{3}) = 50\sqrt{3}(\sqrt{3} -$

1

$\text{Speed} = \frac{\text{distance travelled}}{time\ taken} = \frac{50\sqrt{3}(\sqrt{3}-1)}{2} = 25\sqrt{3}(\sqrt{3}-1)$ m/s

6. From the top of 7 m high building, the angle of elevation of the top of a tower is 60^0 and angle of depression to its foot is 45^0. Find the height of the tower.

Ans:- DE=BC=7m, AC=?



In triangle EDC **In triangle ABE**

tan θ=$\frac{ED}{DC}$ tan θ=$\frac{AB}{BE}$

tan45^0 =$\frac{7}{DC}$ tan60^0 =$\frac{AB}{7}$

1 =$\frac{7}{DC}$ $\sqrt{3}$=$\frac{AB}{7}$

DC= 7m AB =7$\sqrt{3}$ m

Height of tower =AB+BC=7$\sqrt{3}$ + 7 =7($\sqrt{3}$ +1) m (BC=DE)

III. 4 Marks questions.

1. A boy observes the tip of a tower fixed on the top of a building of height 14m from a point on the ground, then the angle of elevation is 45^0. While walking towards the building again he observes the tip and base of the tower from another point, now if angles of elevation are 60^0 and 30^0 respectively. Find the height of the tower and the distance he walked.

Ans:- In triangle BCX,tan θ=$\frac{BX}{BC}$ In triangle ABC, tan θ=$\frac{AB}{BC}$

tan30^0 =$\frac{14}{BC}$

$\frac{1}{\sqrt{3}}$ = $\frac{14}{BC}$

BC= 14$\sqrt{3}$ m

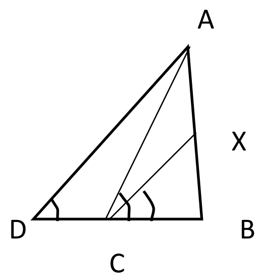
tan60^0 =$\frac{AX+BX}{14\sqrt{3}}$

$\sqrt{3}$ =$\frac{AX+14}{14\sqrt{3}}$

14$\sqrt{3}$x$\sqrt{3}$ = AX+14

14x3= AX+14

AX= 42-14=28 m



In triangle ABD, tan θ=$\frac{AB}{BD}$

tan45^0 =$\frac{42}{14\sqrt{3}+CD}$

1=$\frac{42}{14\sqrt{3}+CD}$

14$\sqrt{3}$ +CD= 42

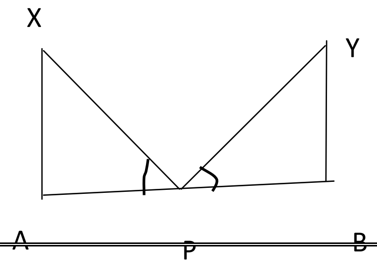
CD= 42 -14$\sqrt{3}$ =14x3-14$\sqrt{3}$ =14x$\sqrt{3}$x$\sqrt{3}$ -14$\sqrt{3}$ =14$\sqrt{3}$($\sqrt{3}$ - 1) m

2. Two poles of equal heights standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road, the angle of elevation of the top of the poles are 60^0 and 30^0 respectively. Find the height of the poles and the distances of the point from the poles.

Ans:- In triangle AXP, tan θ=$\frac{AX}{AP}$:- In triangle BYP, tan θ=$\frac{BY}{BP}$

tan60^0=$\frac{AX}{AP}$ tan30^0=$\frac{BY}{BP}$

$\sqrt{3}$ =$\frac{AX}{AP}$ $\frac{1}{\sqrt{3}}$ =$\frac{BY}{BP}$



$\sqrt{3}$ AP=AX -------1

AX=BY

$\sqrt{3}$ AP= $\frac{BP}{\sqrt{3}}$

BP= $\sqrt{3}$ x$\sqrt{3}$AP=3 AP

AB=AP+BP

BY= $\frac{BP}{\sqrt{3}}$-------2

80= AP+3AP

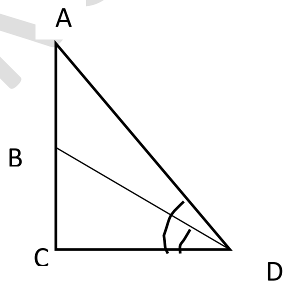
80=4AP

AP=$\frac{80}{4}$ =20 ft.

BP= 80-20=60 ft

3. A statue, 1.6m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point the angle of elevation of the top of the pedestal is 45^0. Find the height of the pedestal.

Ans:- BC=pedestal AB=statue=1.6 m



In triangle DBC, tan θ=$\frac{BC}{DC}$

tan45^0=$\frac{BC}{DC}$

1=$\frac{BC}{DC}$

DC=BC

In triangle ADC, tan θ=$\frac{AC}{BC}$

tan60^0=$\frac{AC}{BC}$

$\sqrt{3}$ = $\frac{AC}{BC}$

AC= $\sqrt{3}$ BC

1.6 +BC=$\sqrt{3}$ BC

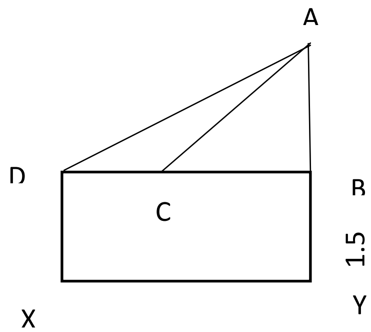
1.6=$\sqrt{3}$ BC – BC

1.6= BC($\sqrt{3}$ -1)

Height of pedestal BC=$\frac{(\sqrt{3}\ -1)}{1.6}$ m

4. A 1.5 m tall boy is standing at some distance from a 30m tall building. The angle of elevation from his eyes to the top of the building increases from 30^0 to60^0 as he walks towards the building. Find the distance he walked towards the building.

Ans:- XD =BY=1.5m AY= 30 m AB=30-1.5=28.5 m



In triangle ABC, tan θ=$\frac{AB}{BC}$

tan60^0=$\frac{28.5}{BC}$

$\sqrt{3}$ = $\frac{28.5}{BC}$

BC= $\frac{28.5}{\sqrt{3}}$ = $\frac{3x9.5}{\sqrt{3}}$ = 9.5$\sqrt{3}$ m

In triangle ABD, tan θ=$\frac{AB}{BD}$

tan30^0=$\frac{28.5}{BD}$

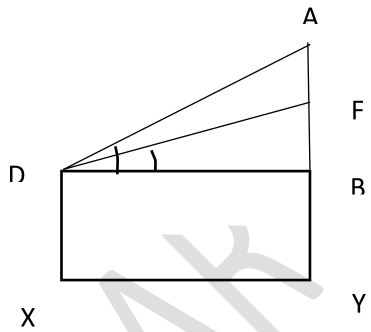
$\frac{1}{\sqrt{3}}$ = $\frac{28.5}{9.5\sqrt{3}+DC}$

$9.5\sqrt{3} + DC$ = 28.5$\sqrt{3}$

DC= 28.5$\sqrt{3}$ - $9.5\sqrt{3}$

=$\sqrt{3}$ (28.5 -9.5)=$19\sqrt{3}$ m

5. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2.m from the ground. The angle of elevation of the balloon from the eyes of the girl at that instant is 60^0. After some time the angle of elevation reduces to 30^0. Find the distance travelled by the balloon during the interval.



Ans:- In triangle ABE, , tan θ=$\frac{AB}{BE}$

$\tan 60^0 = \frac{88.2}{BE}$

$\sqrt{3} = \frac{88.2}{BE}$

BE=$\frac{88.2}{\sqrt{3}}$

$= \frac{29.4x3}{\sqrt{3}} = 29.4\sqrt{3}$

In triangle BEF, tan θ=$\frac{BF}{BE}$

$\tan 30^0 = \frac{BF}{29.4\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{BF}{29.4\sqrt{3}}$

BF= $\frac{29.4\sqrt{3}}{\sqrt{3}} = 29.4\sqrt{3}$m

distance travelled by the balloon= 88.2 -29.4=58.8 m

OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit -10:-Statistics

I. Multiple choice questions :-

1. The mean value of scores 3,4,8,6,9,12 is

a. 7　　**b. 8**　　**c. 9**　　**d. 42**

2. If the mean of 10,15,19,20 and m+1 is 20, then 'm' is

a. 30　　**b. 35**　　**c. 65**　　**d.100**

3. The median of scores 81,95,106,38,95,104 and 28 is

a. 106　　**b.81**　　**c. 104**　　**d. 95**

4. The midpoint of interval (10-20) is

a.15　　**b. 14**　　**c. 12**　　**d. 10**

5. The practical relation between mean, median and mode is written as

a. median= mode – mean　　**b. mode=mean + 2 median**

c. 3 median=mode+2 mean　　**d. 2 mean= 3 mode – median**

II. One mark questions:-

1. Write the formula to find the mean of classified data in direct method.

Ans:- Mean =$\frac{\Sigma fx}{N}$ OR Mean =$\frac{\Sigma fixi}{fi}$

2. Write the formula to find the mode of a classified data.

Ans:- Mode = l + [$\frac{f1-f0}{2f1-f0-f2}$]xh

3. Find the mode of the data 8, 12, 9, 3, 5, 12.

Ans:- 3,5,8,9,<u>12,12</u>

Mode = 12

4. Find the class interval from this distribution which contains mode.

C.I.	0-5	5-10	10-15	15-20	20-25
f	3	8	4	9	2

Ans:- 15-20

5. The mean and median of a distribution are 10 and 11 respectively. Find the mode.

Ans:- 3 median=mode+2 mean

3x11=mode+2x20

33= mode +20

Mode= 33-20 = 13

III. Two Mark questions:-

1. Find the mean for the data by direct method

C.I.	1-5	5-9	9-13	13-17	17-21
f	2	3	5	3	2

Ans:-

C.I	fi	xi	fixi
1-5	2	3	6
5-9	3	7	21
9-13	5	11	55
13-17	3	15	45
17-21	2	19	38
	Σfi=15		$\Sigma fixi = 165$

Mean $=\frac{\Sigma fixi}{fi}$

$=\frac{165}{15}$

=11

2. Find the mean for the following frequency distribution table.

x	10	20	30	40	50
f	1	2	3	2	2

Ans:

xi	fi	fixi
10	1	10
20	2	40
30	3	90
40	2	80
50	2	100
	Σfi=10	$\Sigma fixi = 320$

Mean $=\frac{\Sigma fixi}{fi}$

$=\frac{320}{10} = 32$

IV. Three marks questions:-

1. Find the mode for the frequency distribution table given below.

C.I.	4-10	10-16	16-22	22-28	28-34	34-40
f	4	5	3	6	2	1

l=22

h=6 Mode = l + [$\frac{f1-f0}{2f1-f0-f2}$]xh

f_1=6 = 22+ [$\frac{6-3}{2(6)-3-2}$]x6

f_0=3 = 22 +[$\frac{3}{12-5}$] x6

f_2=2 = 22 + [$\frac{3}{7}$] x6 = 22 +[[$\frac{18}{7}$] = 24.57

2. calculate the median for the given data.

C.I.	1-4	4-7	7-10	10-13	13-16	16-19
f	6	30	40	16	4	4

Ans:-

C.I.	f	c.f.
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100
	N=100	

l=7 Median = l + [$\frac{\frac{n}{2}-cf}{f}$] xh

n=100 = 7 + [$\frac{50-36}{40}$] x3

n/2=50

h= 3 = 7 + $\frac{14}{40}$ x3

cf=36, f=40 = 7 + $\frac{21}{20}$ = 7+1.05=8.05

3. The following table gives the age of 300 people in a village. Find the arithmetic mean of their ages.

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
No. of people	20	50	80	120	20	10

Ans:-

C.I	fi	xi	fixi
10-20	20	15	300
20-30	50	25	1250
30-4-	80	35	2800
40-50	120	45	5400
50-60	20	55	1100
60-70	10	65	650
	Σfi=300		$\Sigma fixi = 11500$

Mean =$\frac{\Sigma fixi}{fi}$

=$\frac{11500}{300}$

= 38.33 years ≈ $38\ years$

4. Draw a 'more than' type of ogive for the data given below.

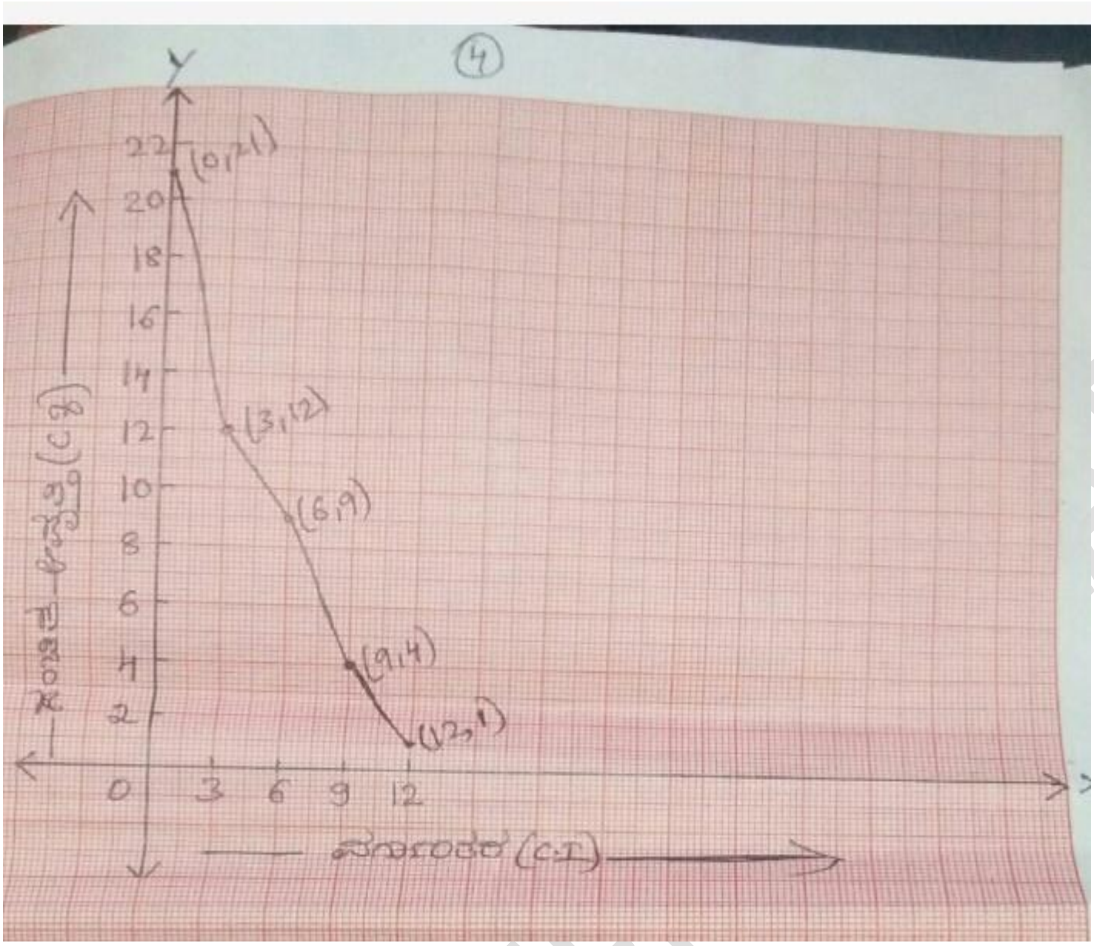
C.I.	0-3	3-6	6-9	9-12	12-15
f	9	3	5	3	1

Ans:-

C.I.	f	c.f.	Points
0-3	9	21	(0,21)
3-6	3	12	(3,12)
6-9	5	9	(6,9)
9-12	3	4	(9,4)
12-15	1	1	(12,1)

Scale:- X-axis :- 1cm=3 units

Y-axis:- 1cm= 2 units



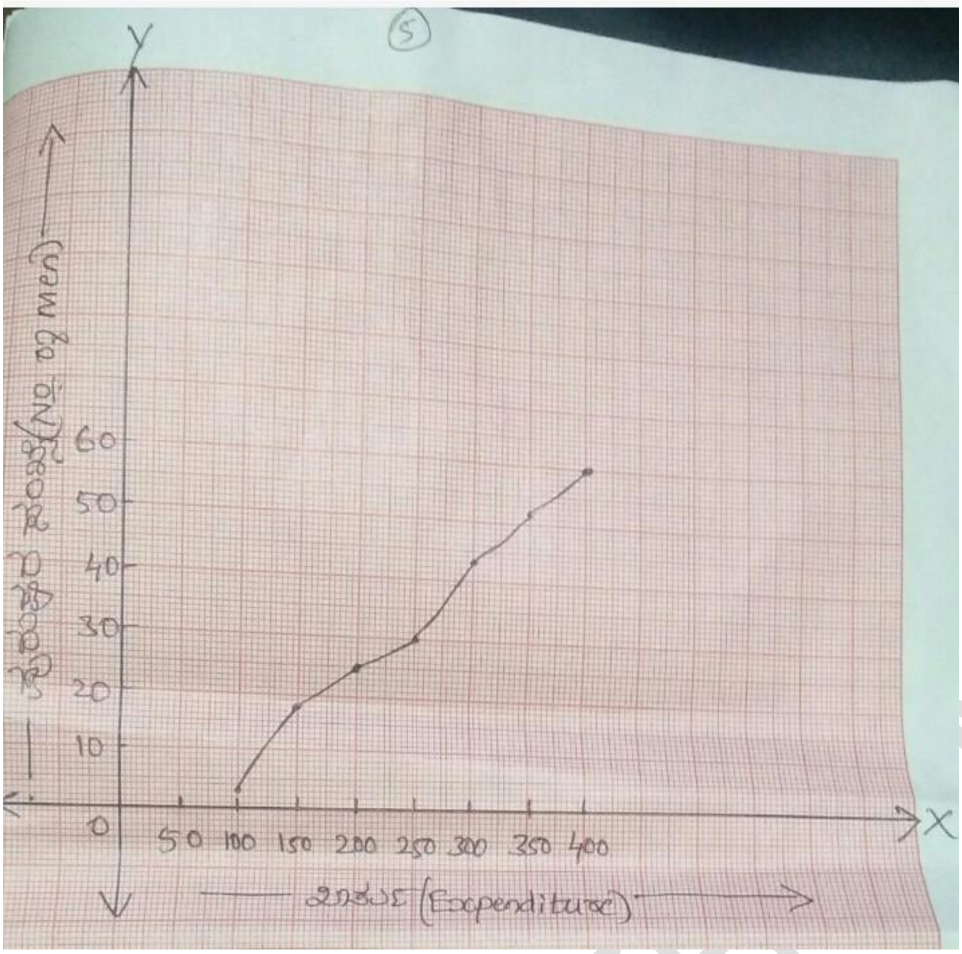
5. Following table shows that 60 mens weekly expenditure. Draw a 'less than' type of ogive for the data given below.

Expenditure	No. of men
Less than 100	3
Less than 150	18
Less than 200	25
Less than 250	30
Less than 300	43
Less than 350	52
Less than 400	60

Scale:- X-axis :- 1cm= 50

Y-axis:- 1cm= 10

Expenditure	No. of men	Points
Less than 100	3	(100,3)
Less than 150	18	(150,18)
Less than 200	25	(200,25)
Less than 250	30	(250,30)
Less than 300	43	(300,43)
Less than 350	52	(350,52)
Less than 400	60	(400,60)



OFFICE OF D.D.P.I , KOLAR DISTRICT, KOLAR

Unit -11:-SURFACE AREA AND VOLUME

I. Multiple choice questions one mark each.

1) The height of the cylinder is 20 cm and radius is 7cm the volume of a cylinder is.

A) 3080 cm^3 B) 3080 cm^2 C) 3080 cm D) 3080 cm^4

2) A rectangular tank is 25 m long and 9.5m deep. If 600 m^3 of water to be drawn off the tank the level of water goes down by 1.5 m then the width of the tank is.

A) 18m B) 17m C) 16m D) 19m

3) The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their radii.

A) 1:27 B) 3:27 C) 9:81 D) 3:9

4) A cylindrical pencil sharpened at one end it is a combination of.

A) Two cylinders   B) Hemisphere and cylinder   <u>C) Cone and cylinder</u>   D) Frustum of a cone and cylinder

5) The total surface area of a hemispherical solid having 7 cm radius is.

<u>A) 462 cm^2</u>   B) 294 cm^2 C) 588 cm^2 D) 154 cm^2

6) The surface area of a sphere is 616cm^2 its radius is.

<u>A) 7 cm</u>   B) 14 cm   C) 21 cm   D) 28 cm

7) A cylinder and cone are of same base radius and of same height. The ratio of their volumes is.

A) 2:1   <u>B) 3:1</u>   C) 2:3   D) 3:2

8) If two solid hemispheres of same radius are joined together along their bases. Then surface area of this new solid is.

A) $3\pi r^2$ <u>B) 4</u>πr^2 C) $5\pi r^2$ D) $6\pi r^2$

II. ONE MARK QUESTIONS.

1) How many balls each of radius 3 cm can be made by melting a bigger ball whose diameter is 48 cm?

Radius of small ball - 3cm , vol - $\frac{4}{3}$ x π x 3 = 4π
radius of bigger ball 24 cm , vol = $\frac{4}{3}$ x π x 24 =32π

no. of small balls can be created = $\frac{vol of big ball}{vol of small ball}$=$\frac{32\pi}{4\pi}$= 8 balls

2) A spherical ball of lead has been melted and made in to identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?

Let the radius of big ball be 'r' cm
∴ Radius of smaller ball = r/2 cm.
➻ Volume of sphere = 4/3 πr^3
➻ Volume of big spherical ball = 4/3 πr^3
➻ Volume of smaller spherical ball = 4/3 $\pi(r/2)^3$
= Volume of big ball/Volume of smaller ball
= (4/3 πr^3)/[4/3 π (r^3/8)]
= (4/3 πr^3)/[πr^3/6]
= (4/3)/(1/6)   [Cancelling πr^3]
= (4/3) × 6
= 4 × 2
= 8
∴ <u>Number of balls can be made = 8</u>

3) **Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of the given hemisphere?**

Let 'r' be the radius of the hemisphere.
Volume of hemisphere = Surface area of hemisphere
$(2/3)\pi r^3 = 3\pi r^2$
$(2/3)\pi r^3 / r^2 = 3\pi$
$(2/3)r = 3$
Radius of hemisphere, r = 3 × 3/2
Radius of hemisphere, r = 9/2
Radius of hemisphere,r = 9/2 units
Diameter = 2 × Radius
Diameter of hemisphere = 2 × (9/2)
Diameter of hemisphere = 9 units.

III. TWO MARK QUESTIONS.

1)How many litters of water flows out of a pipe of crass section area 5cm² in one minute if the speed of the water in the pipe is 30 cm/s. (1 litter=1000 cm³)

Area of cross section of pipe = 5 cm²
Speed of water flowing out of the pipe = 30 cm/sec
Volume of the water flows in 1 minute = Area of the cross section × 30 × 60
= 5 × 30 × 60 = 9000 cm³
Since, 1000 cm³ = 1 litre **Then, 9000 cm³ = 9 litres**
Hence, 9 litres of water flows out from a pipe.

2) The surface area of sphere is 2464 cm² find its volume.

Surface area of sphere= $4 \times \pi \times r^2$

Surface area=2464cm square

$4 \times \frac{22}{7} \times r^2 = 2464$

$r^2 = \frac{2464 \times 7}{3 \times 7}$

$r^2 = 14^2$

$r = 14$

Volume= $4/3 \times \pi \times r^3$

4/3×22/7×14×14×14

=11498.66 cm³

3) Eight metallic spheres each of radius 2 mm are melted and recast in to a single sphere. Calculate the radius of the new sphere.

Radius= 2mm

volume if one metallic sphere = 4/3 πr^3

= 4/3 × 3.14 × 2^3

= 4/3 × 3.14 × 8 = 33.49 mm^3

Volume of eight metallic sphere having radius 2mm = 33.49 × 8 = 267.94 mm^3

Since eight spheres are melted and casted into single sphere.

That means single sphere has same volume as that of volume of eight spheres

•°• Volume of new sphere = 267.94mm^3

4/3 πr^3 = 267.94

4/3 × 3.14 × r^3 = 267.94

r^3 = 267.94/4.18

r^3 = 64.100 $\approx$64

r^3 = 64

$r^3=4^3$

r = 4mm

Therefore, radius of new sphere is 4mm

4) A right circular con of radius 3cm has a curved surface area 47.1 cm^2. Find the volume of cone ($\pi = 3.14$)

Radius of cone (r) = 3 cm,

Curved surface area of a cone= 47.1 cm^2

Curved surface area of a cone= πrl

47.1 = πrl

47.1 = 3.14 × 3 × l

l = 47.1 / (3.14 × 3)

l = 15.7 / 3.14= 1570/314= 5 cm

slant height (l) = 5 cm

Height of a cone (h) = $\sqrt{(\mathbf{l}^2) - (\mathbf{r}^2)}$

h = $\sqrt{(5^2) - (3^2)}$

h = $\sqrt{(25 - 9}= 16$ = 4 cm

h = 4 cm

Volume of cone= ⅓($\pi r^2 h$)

= ⅓(3.14 × 3 × 3 × 4)

= 3.14 × 12= 37.68 cm^3

Hence, the volume of a cone is 37.68 cm^3.

5) The volume of a sphere is 38808 cm^3 find its diameter and surface area.

volume of the sphere = 38808cm^3

formula to find the volume of a sphere is 4/3πr^3

therefore 4/3πr^3 = 38808cm^3

➡ 4/3 × 22/7 × r^3 = 38808cm^3

➡ 88/21 × r^3 = 38808cm^3

➡ r^3 = 38808/1 × 21/88

➡ $r^3 = 441 \times 21$
➡ $r^3 = 9261$
➡ $r = \sqrt[3]{21 \times 21 \times 21} = 21cm$

Diameter d= 2r= 2x21=42cm
now, it's curved surface area = $4\pi r^2$
= 4 × 22/7 × 21 × 21= 88/7 × 441= 88 × 63= 5544cm^2
hence, the curved surface area of the sphere is 5544cm^2

6) A cylinder ,cone and a hemisphere have the same base and the same height. Find the ratio between their volumes.

Volume of hemisphere = $2/3 \pi r^3$ = V1

Volume of cylinder = $\pi r^2 H$ = V2

Volume of Cone = $⅓ \pi r^2 H$ = V3

We know that radius of hemisphere is its height which means that radius of all given solids are equal to their height since their radius are also given equal.

V1/V2/V3=2/3*πH^3 (since r=h) / πH^3 / 1/3*πH^3

=> 2/3 /1 / 1/3

=>2/3:1:1/3

=> 1:1/3:2/3

Multiplying by 3

=> 3:1:2 is the ratio of volumes of cylinder : Cone : hemisphere

IV. THREE MARK QUESTIONS.

1) The ratio between the radius of the base and the height of the cylinder is 2: 3. If its volume is 1617 cm3, the total surface area of the cylinder is?

- Ratio between the radius of the base and the height of the cylinder is 2: 3.
- volume is 1617 cm^3.

- Total surface area of the cylinder = ?

Let the radius of the base and the height of the cylinder be 2x and 3x respectively.
Now,
Volume of Cylinder = $\pi r^2 h$
=) 1617 = $\pi \times (2x)^2 \times (3x)$
=) 1617 × 7/22 = $4x^2 \times 3x$
=) 514.5 = $12x^3$
=) x^3 = 514.5/12
=) x^3 = 42.875

=) $x = \sqrt[3]{42.875}$)

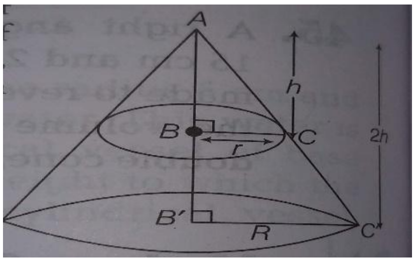
=) x = 3.5

- **<u>Radius</u> = 2x = 2×3.5 = <u>7 cm.</u>**
- **<u>Height</u> = 3x = 3 × 3.5 = <u>10.5 cm.</u>**

<u>Total Surface Area of Cylinder = 2πr (h + r)</u>

=) T.S.A. of Cylinder = 2 × (22/7) × 7 (10.5 + 7)= 44 (17.5)= 770 cm^3.

2) A solid cone of a base radius 10 cm is cut into two parts through the midpoint of its height by a plane parallel to its base. Find the ratio of the volume of the two parts of the cone.



Let r & R be the radius of the lower part of the frustum.

Height of a cone , AB’ = 10 cm

Height of a Smaller cone, AB = 5 cm

[Cut through the midpoint of its height]

From the figure,

AB = h = 5

AB’ = 2h = 10

BC = r

B'C = R

In ΔABC & ΔAB’C’ ,

∠ABC = ∠AB’C’ (each 90°)

∠ACB = ∠AC’B’ (corresponding angles)

ΔABC ~ ΔAB’C’ [By AA Similarity]

BC/B'C’ = AB/AB’

[Corresponding sides of a similar triangles are proportional]

r/R = 5 /10

r/R = ½

R = 2r

Volume of the upper part (Smaller cone) = ⅓ $\pi r^2 h$

Volume of solid cone = ⅓ π $R^2$2h

= ⅓ π $(2r)^2$ 2h = ⅓ π × $4r^2$ × 2h = 8/3$\pi r^2 h$

Volume of lower part (frustum) = volume of solid cone - volume of Smaller cone = 8/3$\pi r^2 h$ - ⅓ $\pi r^2 h$ = 7/3 $\pi r^2 h$

Volume of lower part (frustum) = 7/3 $\pi r^2 h$

Volume of the upper part (Smaller cone)/ Volume of lower part (frustum) =

$\frac{1}{3}\pi r^2h$ / $\frac{7}{3}\pi r^2h$= 1/7

Hence, the ratio of volume of two parts of the cone is 1 : 7 .

3) A solid metallic sphere of diameter 8 cm is melted and drawn into a cylindrical wire of uniform width if the length of the wire is 12 m find its width.

Diameter of sphere = 8m
Radius of sphere = Diameter/2 = 8/2 =4 m
Volume of sphere = $\frac{4}{3}\pi r^3$
= $\frac{4}{3}\pi(4)^3$
= $\frac{4}{3}\pi \times 64$
Length of wire is 12 m
Since wire is in the shape of cylinder
Volume of wire = $\pi r^2 h$
= $\pi r^2 \times 12$
Since Sphere is melted to make wire so, volume will remain same .

$\frac{4}{3}\pi \times 64 = \pi r^2 \times 12$

$\frac{4}{3} \times 64 = r^2 \times 12$

$85.33 = r^2 \times 12$

$\frac{85.33}{12} = r^2$

$7.11 = r^2$

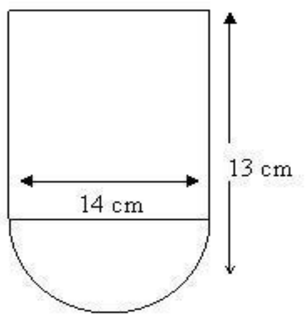
$\sqrt{7.11} = r$

$2.66 = r$

Radius of wire = 2.66m
Width = Diameter = Radius x2 = 2.66x2 =5.32 m
Hence the width of the wire is 5.32 m

V. FOUR MARK QUESTIONS.

1) The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of cylinder is 1628 cm^2 find the volume of the cylinder.



TSA = 2πr(h+r)

1628 = 2πr × 37 (from q)

814 = 22/7 × 37 × r

814×7/22 = 37r

37 × 7 = 37r

r = 7 cm

So h = 37-7 = 30 cm

Volume of cylinder= $\pi r^2 h$

V = 22 / 7 × 7 × 7 × 30= 22 × 7 × 30= 4620 cm^3

2) A vessel is in the form of a hemispherical bowl is surmounted by a hallow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the inner total surface area of the vessel.

=> Diameter of hemisphere = Diameter of cylinder = 14 cm.

=> Radius of hemisphere = Radius of cylinder = 7 cm.

=> Height of cylinder = Total height - radius of hemisphere

=> 13 - 7 = 6 cm.

=> Curved surface area of cylinder = 2πrh

=> Curved surface area of hemisphere = $2\pi r^2$

Curved surface area of cylinder = 2πrh

$$\implies 2 \times \frac{22}{7} \times 7 \times 6$$

$$\implies 264 \text{ cm}^2$$

Now, Curved surface area of hemisphere = $2\pi r^2$

$$\implies 2 \times \frac{22}{7} \times 7 \times 7$$

$$\implies 308 \text{ cm}^2$$

∴ Inner surface area of vessel = Curved surface area of cylinder + curved surface area of hemisphere

=> 264 + 308

=> 572 cm^2

<u>So, the inner surface area of vessel = 572 cm^2</u>

3) 504 cones each of diameter 3.5 cm and height 3 cm are melted and recast into a metallic sphere. Find the diameter of the sphere and also find the surface area.

Number of cones = 504

Diameter of cone = 3.5cm **Radius of cone = 1.75cm** **Height = 3cm**

Volume of each cone

$=\frac{1}{3} \times \pi \times r^2 \times h$

=1/3 x 22/7 x 1.75x1.75 x3

= 22x0.25x1.75= 9.625 cm3

Volume of all cones = 504 x9.625= 4851 cm3

After melting,

Volume of sphere = Volume of cones

Volume of sphere = 4851

$\frac{4}{3} \times \pi \times r^3 = 4851$

r^3= 1157.625

r= 10.5 cm

Diameter of sphere = 2*Radius

= 2x10. 5= 21 cm

VI. FIVE MARK QUESTIONS.

1) A vessel in the form of an inverted cone is filled with water to the brim. Its height is 32 cm and diameter of the base is 25.2 cm. six equal solid cones are dropped in it, so that they are fully submerged. As a result one fourth of water in the original cone over flows. What is the volume of each of the solid cones submerged?

Height of Cone = 32 cm Diameter of Cone = 25.2 cm Radius of Cone = 12.6 cm

Now if we will find the Volume of Cone in Liters then we will be able to find the Volume of each cone.

Volume of Cone = $\pi r^2 h/3$

V = π (12.6)(12.6)(32)/3 cm^3

V = 1693.44π cm^3

Converting cm^3 to Liters.

V = 1.69344π L

Now, As given one-fourth over flow.

Volume of small cones =1/4Volume of water×1/6

V = 1/24 × 1.69344

V = 0.07056 L

Again, Converting L to cm^3 V = 70.56 cm^3

Volume of each cone = V = 70.56 cm^3

2) From a solid cylinder of height 36 cm and radius 14 cm, a conical cavity of radius 7 cm and height 24 cm drilled out. Find the volume and the total surface area of the remaining solid.

Vol. 20944 cm^3 , S.A. 4796 cm^2

Vol. of Cylinder = 22/7x $Radius^2$ x Height , Total SURFACE AREA OF CYLINDER = 2x(22/7)xRADIUS(radius + height)

Vol. of cone = (1/3) (22/7) ($Radius^2$) x Height

Lateral Surface area of cone = (22/7)x radius x slant height

Vol. of Cylinder= (22/7) x 14 x 14 x36

Vol of Cone = (1/3) (22/7) x7x7x24

Vol. of cylinder- Vol. of cone = (22/7) (14x14x36 - (7x7x24/3)

= 22/7(7056- 392)

= 22/7X6664

=20944 cm^3

Tsa of remaining solid = 44/7X14(14+36) + 22/7x7x25 - (22/7 x 7 x7)

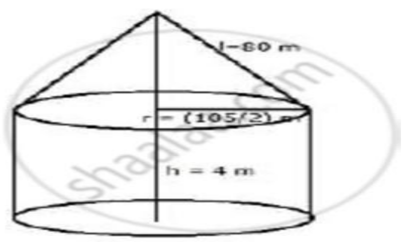
=4400- 154 + 550

= 4796 cm^2

l = $\sqrt{h2 + r2}$

l = $\sqrt{625}$= 25 cm

3) A circus tent is cylindrical to a height of 4m and conical above it. If its diameter is 105 m and its slant height is 80 m. calculate the total surface area of canvas required. Also find the total cast of canvas used at rate of Rs 15 per meter of the width is 1.5 m.



For cylinder, Diameter is 105 m

then radius is 105/2m

Now height=4 m

For cone, l=80 m and r= 105/2

Total surface area of the tent is the sum of lateral suface of cone and cylinder

=2πrh+πrl

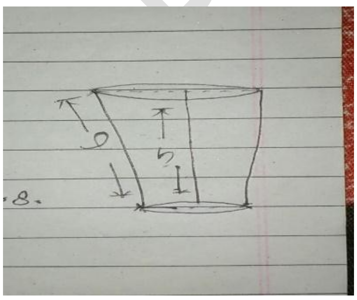
=1320+13200=14520m²

Width of canvas used = 1.5 m

Length of canvas = 14520/1.5 = 9680m

Therefore, total cost of canvas at the rate of rupees 15 per metre = 9680 × 15 = ₹ 145200

4) Bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radius of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used the making the bucket. (use π=3.14)



Bucket is in the form of the frustum of a cone.

Capacity of Frustum of a cone = volume of the frustum of a cone = 12308.8 cm^3

Bigger radius (r_1) = 20 cm   Smaller radius (r_2) = 12 cm

Volume of Frustum of cone =$(\frac{1}{3}\pi)h(r_1^2 + r_2^2 + r_1r_2)$

12308.8 = $(\pi/3)h(20\times20 + 12\times12 + 20\times12)$

12308.8$\times$ 3 = π h(400 + 144+240)

12308.8$\times$3 = 22/7 h(784)

h = 12308.8$\times$3 $\times$7 / (22 $\times$ 784)

h = 12308.8$\times$3 / (22 $\times$ 112)

h = 6,154.4 $\times$ 3 / 11$\times$ 112

h= 18,463.2/1232 = 14.99

h=15 (approximately)

Height of Frustum of cone (h) = 15 cm

Slant height (l) of a frustum cone =$\sqrt{\mathbf{h}^2 + (\mathbf{r1} - \mathbf{r2})^2}$

l = $\sqrt{15^2 + (20 - 12)^2}$=$\sqrt{225 + (8)^2}$= $\sqrt{225 + (64)}$ =$\sqrt{289}$ =17 cm

Surface area of a frustum cone = πl (r1 + r2)

= π $\times$ 17 (20+12)

= 22/7 $\times$ 17(32)

= 11968/7 = 1,709.7cm^2

Cost of making 1 cm^2 = ₹ 10

Cost of making 1709.7 cm^2 = ₹ (1709.7 $\times$ 10) = ₹ 17097

Cost of making 1709.7 cm^2 = ₹ 17097

Hence, the height of the bucket is 15 cm and the cost of making the bucket is ₹ 17097.