

TARGET 80

ತ್ರಿಕೋಣಮಿತಿ ಅನ್ವಯ ನಾಮಸ್ವರ್ಗಳು

(2 ಅಥವಾ 3 ಅಂತರಾಳಗಾಗಿ)

1. $\sec^4\theta - \sec^2\theta = \tan^2\theta + \tan^4\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \sec^4\theta - \sec^2\theta \\ &= \sec^2\theta(\sec^2\theta - 1) \\ &= (1 + \tan^2\theta)\tan^2\theta \\ &= \tan^2\theta + \tan^4\theta \\ \therefore LHS &= RHS \end{aligned}$$

2. $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta} \\ &= \frac{\sin^2\theta + 1 + \cos^2\theta + 2 \times 1 \times \cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} \\ &= \frac{2}{\sin\theta} \\ &= 2\operatorname{cosec}\theta \\ \therefore LHS &= RHS \end{aligned}$$

3. $\frac{\cos(90-\theta)}{1+\sin(90-\theta)} + \frac{1+\sin(90-\theta)}{\cos(90-\theta)} = 2\operatorname{cosec}\theta$
ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\cos(90-\theta)}{1+\sin(90-\theta)} + \frac{1+\sin(90-\theta)}{\cos(90-\theta)} \\ &= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \\ &= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta} \end{aligned}$$

$$\begin{aligned} &= \frac{\sin^2\theta + 1 + \cos^2\theta + 2 \times 1 \times \cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{2 + 2\cos\theta}{(1+\cos\theta)\sin\theta} \\ &= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta} \\ &= \frac{2}{\sin\theta} \\ &= 2\operatorname{cosec}\theta \\ \therefore LHS &= RHS \end{aligned}$$

4. $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta} \\ &= \frac{\cos^2\theta + 1 + \sin^2\theta + 2 \times 1 \times \sin\theta}{(1+\sin\theta)\cos\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta} \\ &= \frac{1 + 1 + 2\sin\theta}{(1+\sin\theta)\cos\theta} \\ &= \frac{2 + 2\sin\theta}{(1+\sin\theta)\cos\theta} \\ &= \frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta} \\ &= \frac{2}{\cos\theta} \\ &= 2\sec\theta \\ \therefore LHS &= RHS \end{aligned}$$

5. $\frac{\sin(90-\theta)}{1+\cos(90-\theta)} + \frac{1+\cos(90-\theta)}{\sin(90-\theta)} = 2\sec\theta$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\sin(90-\theta)}{1+\cos(90-\theta)} + \frac{1+\cos(90-\theta)}{\sin(90-\theta)} \\ &= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} \\ &= \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cos^2\theta + 1 + \sin^2\theta + 2 \times 1 \times \sin\theta}{(1 + \sin\theta)\cos\theta} \\
 &= \frac{\sin^2\theta + \cos^2\theta + 1 + 2\sin\theta}{(1 + \sin\theta)\cos\theta} \\
 &= \frac{1 + 1 + 2\sin\theta}{(1 + \sin\theta)\cos\theta} \\
 &= \frac{2 + 2\sin\theta}{(1 + \sin\theta)\cos\theta} \\
 &= \frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta} \\
 &= \frac{2}{\cos\theta} \\
 &= 2\sec\theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

6. $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \cos\theta + \sin\theta$ එය වාස්තුවෙන් (MQP.1 – 2021, 3 marks)

පරිජාර :

$$\begin{aligned}
 LHS &= \frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} \\
 &= \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} \\
 &= \frac{\cos\theta}{\cos\theta-\sin\theta} + \frac{\sin\theta}{\sin\theta-\cos\theta} \\
 &= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta} \\
 &= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta} \\
 &= \frac{\cos\theta-\sin\theta}{\cos^2\theta-\sin^2\theta} \\
 &= \frac{\cos\theta-\sin\theta}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)} \\
 &= \frac{1}{\cos\theta-\sin\theta} \\
 &= \cos\theta + \sin\theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

7. $\frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta} = 2\cot\theta$ එය වාස්තුවෙන්.

(Preparatory – 2020, 3 marks)

පරිජාර :

$$\begin{aligned}
 LHS &= \frac{1+\cos\theta}{\sin\theta} - \frac{\sin\theta}{1+\cos\theta} \\
 &= \frac{(1+\cos\theta)^2 - \sin^2\theta}{\sin\theta(1+\cos\theta)} \\
 &= \frac{1^2 + \cos^2\theta + 2 \times 1 \times \cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 + \cos^2\theta + 2\cos\theta - \sin^2\theta}{\sin\theta(1+\cos\theta)} \\
 &= \frac{1 - \sin^2\theta + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)} \\
 &= \frac{\cos^2\theta + \cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)} \\
 &= \frac{2\cos^2\theta + 2\cos\theta}{\sin\theta(1+\cos\theta)} \\
 &= \frac{2\cos\theta(\cos\theta+1)}{\sin\theta(1+\cos\theta)} \\
 &= 2\cot\theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

8. $\frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} = 2\tan\theta$ එය වාස්තුවෙන්.
පරිජාර :

$$\begin{aligned}
 LHS &= \frac{1+\sin\theta}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} \\
 &= \frac{(1+\sin\theta)^2 - \cos^2\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{1^2 + \sin^2\theta + 2 \times 1 \times \sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{1 + \sin^2\theta + 2\sin\theta - \cos^2\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{1 - \cos^2\theta + \sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{1 - \cos^2\theta + \sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{\sin^2\theta + \sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{2\sin^2\theta + 2\sin\theta}{\cos\theta(1+\sin\theta)} \\
 &= \frac{2\sin\theta(\sin\theta+1)}{\cos\theta(1+\sin\theta)} \\
 &= 2\tan\theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

9. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \cdot \cosec\theta$ එය වාස්තුවෙන්.

පරිජාර :

$$\begin{aligned}
 LHS &= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} \\
 &= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}} \\
&= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta} \\
&= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)} \\
&= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)} \\
&= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta \cdot \sin\theta (\sin\theta - \cos\theta)} \\
&= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta \cdot \cos\theta)}{\cos\theta \cdot \sin\theta (\sin\theta - \cos\theta)} \\
&= \frac{1 + \sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta} \\
&= \frac{1}{\cos\theta \cdot \sin\theta} + \frac{\sin\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta} \\
&= \frac{1}{\cos\theta \cdot \sin\theta} + 1 \\
&= \sec\theta \cdot \cosec\theta + 1 \\
&= 1 + \sec\theta \cdot \cosec\theta
\end{aligned}$$

 $\therefore LHS = RHS$

10. $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ එයේ සඳහා :

$$\begin{aligned}
LHS &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} \\
&= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{1^2 - (\sin A)^2}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} \\
&= \sqrt{\left(\frac{1 + \sin A}{\cos A}\right)^2} \\
&= \frac{1 + \sin A}{\cos A}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
&= \sec A + \tan A
\end{aligned}$$

 $\therefore LHS = RHS$

11. $\sqrt{\frac{1+\sin A}{1-\sin A}} + \sqrt{\frac{1-\sin A}{1+\sin A}} = 2\sec A$ එයේ සඳහා :

$$\begin{aligned}
LHS &= \sqrt{\frac{1 + \sin A}{1 - \sin A}} + \sqrt{\frac{1 - \sin A}{1 + \sin A}} \\
&= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A}} + \sqrt{\frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{1^2 - (\sin A)^2}} + \sqrt{\frac{(1 - \sin A)^2}{1^2 - (\sin A)^2}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{1 - \sin^2 A}} + \sqrt{\frac{(1 - \sin A)^2}{1^2 - \sin^2 A}} \\
&= \sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}} + \sqrt{\frac{(1 - \sin A)^2}{\cos^2 A}} \\
&= \sqrt{\left(\frac{1 + \sin A}{\cos A}\right)^2} + \sqrt{\left(\frac{1 - \sin A}{\cos A}\right)^2} \\
&= \frac{1 + \sin A}{\cos A} + \frac{1 - \sin A}{\cos A} \\
&= \frac{1 + \sin A + 1 - \sin A}{\cos A} \\
&= \frac{2}{\cos A} \\
&= 2\sec A
\end{aligned}$$

 $\therefore LHS = RHS$

12. $\sqrt{\frac{1+\cos A}{1-\cos A}} = \cosec A + \cot A$ එයේ සඳහා :

$$\begin{aligned}
LHS &= \sqrt{\frac{1 + \cos A}{1 - \cos A}} \\
&= \sqrt{\frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A}} \\
&= \sqrt{\frac{(1 + \cos A)^2}{1^2 - (\cos A)^2}}
\end{aligned}$$

$$\begin{aligned}
&= \sqrt{\frac{(1 + \cos A)^2}{1 - \cos^2 A}} \\
&= \sqrt{\frac{(1 + \cos A)^2}{\sin^2 A}} \\
&= \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2} \\
&= \frac{1 + \cos A}{\sin A} \\
&= \frac{1}{\sin A} + \frac{\cos A}{\sin A} \\
&= \cosec A + \cot A
\end{aligned}$$

$$\therefore LHS = RHS$$

13. $\sqrt{\frac{1+\cos A}{1-\cos A}} + \sqrt{\frac{1-\cos A}{1+\cos A}} = 2\cosec A$ බංධුව
සාධිස්.

පරිභාර :

$$\begin{aligned}
LHS &= \sqrt{\frac{1 + \cos A}{1 - \cos A}} + \sqrt{\frac{1 - \cos A}{1 + \cos A}} \\
&= \sqrt{\frac{1 + \cos A \times 1 + \cos A}{1 - \cos A \times 1 + \cos A}} + \sqrt{\frac{1 - \cos A \times 1 - \cos A}{1 + \cos A \times 1 - \cos A}} \\
&= \sqrt{\frac{(1 + \cos A)^2}{1^2 - (\cos A)^2}} + \sqrt{\frac{(1 - \cos A)^2}{1^2 - (\cos A)^2}} \\
&= \sqrt{\frac{(1 + \cos A)^2}{1 - \cos^2 A}} + \sqrt{\frac{(1 - \cos A)^2}{1 - \cos^2 A}} \\
&= \sqrt{\frac{(1 + \cos A)^2}{\sin^2 A}} + \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}} \\
&= \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2} + \sqrt{\left(\frac{1-\cos A}{\sin A}\right)^2} \\
&= \frac{1 + \cos A}{\sin A} + \frac{1 - \cos A}{\sin A} \\
&= \frac{1 + \cos A + 1 - \cos A}{\sin A} \\
&= \frac{2}{\sin A} \\
&= 2\cosec A
\end{aligned}$$

$$\therefore LHS = RHS$$

14. $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2\cosec A$ බංධුව
සාධිස්.

පරිභාර :

$$\begin{aligned}
LHS &= \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} \\
&= \sqrt{\frac{\sec A - 1}{\sec A + 1} \times \frac{\sec A - 1}{\sec A - 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1} \times \frac{\sec A + 1}{\sec A + 1}} \\
&= \sqrt{\frac{(\sec A - 1)^2}{(\sec A)^2 - (1)^2}} + \sqrt{\frac{(\sec A + 1)^2}{(\sec A)^2 - (1)^2}} \\
&= \sqrt{\frac{(\sec A - 1)^2}{\sec^2 A - 1}} + \sqrt{\frac{(\sec A + 1)^2}{\sec^2 A - 1}} \\
&= \sqrt{\frac{(\sec A - 1)^2}{\tan^2 A}} + \sqrt{\frac{(\sec A + 1)^2}{\tan^2 A}} \\
&= \sqrt{\left(\frac{\sec A - 1}{\tan A}\right)^2} + \sqrt{\left(\frac{\sec A + 1}{\tan A}\right)^2} \\
&= \frac{\sec A - 1}{\tan A} + \frac{\sec A + 1}{\tan A} \\
&= \frac{\sec A - 1 + \sec A + 1}{\tan A} \\
&= \frac{2\sec A}{\tan A} \\
&= \frac{2}{\frac{\sin A}{\cos A}} \\
&= \frac{2}{\sin A} \\
&= 2\cosec A
\end{aligned}$$

$$\therefore LHS = RHS$$

15. $\frac{1+\cos A}{1-\cos A} = (\cosec A + \cot A)^2$ බංධුව සාධිස්.

(March – 2019, 2marks)

පරිභාර :

$$\begin{aligned}
LHS &= \frac{1 + \cos A}{1 - \cos A} \\
&= \frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} \\
&= \frac{(1 + \cos A)^2}{1^2 - (\cos A)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(1 + \cos A)^2}{1 - \cos^2 A} \\
&= \frac{(1 + \cos A)^2}{\sin^2 A} \\
&= \left(\frac{1 + \cos A}{\sin A} \right)^2 \\
&= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} \right)^2 \\
&= (\csc A + \cot A)^2 \\
\therefore LHS &= RHS
\end{aligned}$$

$$\begin{aligned}
&= \left(\frac{1 + \sin A}{\cos A} \right)^2 - \left(\frac{1 - \sin A}{\cos A} \right)^2 \\
&= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)^2 - \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A} \right)^2 \\
&= (\sec A + \tan A)^2 - (\sec A - \tan A)^2 \\
&= \sec^2 A + \tan^2 A + 2\sec A \cdot \tan A - (\sec^2 A \\
&\quad + \tan^2 A - 2\sec A \cdot \tan A) \\
&= \sec^2 A + \tan^2 A + 2\sec A \cdot \tan A - \sec^2 A - \\
&\quad \tan^2 A + 2\sec A \cdot \tan A \\
&= 4\sec A \cdot \tan A \\
\therefore LHS &= RHS
\end{aligned}$$

16. $\frac{1+\sin A}{1-\sin A} = (\sec A + \tan A)^2$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
LHS &= \frac{1 + \sin A}{1 - \sin A} \\
&= \frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A} \\
&= \frac{(1 + \sin A)^2}{1^2 - (\sin A)^2} \\
&= \frac{(1 + \sin A)^2}{1 - \sin^2 A} \\
&= \frac{(1 + \sin A)^2}{\cos^2 A} \\
&= \left(\frac{1 + \sin A}{\cos A} \right)^2 \\
&= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)^2 \\
&= (\sec A + \tan A)^2 \\
\therefore LHS &= RHS
\end{aligned}$$

17. $\frac{1+\sin A}{1-\sin A} - \frac{1-\sin A}{1+\sin A} = 4\sec A \cdot \tan A$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
LHS &= \frac{1 + \sin A}{1 - \sin A} - \frac{1 - \sin A}{1 + \sin A} \\
&= \frac{1 + \sin A}{1 - \sin A} \times \frac{1 + \sin A}{1 + \sin A} - \frac{1 - \sin A}{1 + \sin A} \times \frac{1 - \sin A}{1 - \sin A} \\
&= \frac{(1 + \sin A)^2}{1^2 - (\sin A)^2} - \frac{(1 - \sin A)^2}{1^2 - (\sin A)^2} \\
&= \frac{(1 + \sin A)^2}{1 - \sin^2 A} - \frac{(1 - \sin A)^2}{1 - \sin^2 A} \\
&= \frac{(1 + \sin A)^2}{\cos^2 A} - \frac{(1 - \sin A)^2}{\cos^2 A}
\end{aligned}$$

18. $\frac{1+\cos A}{1-\cos A} - \frac{1-\cos A}{1+\cos A} = 4\csc A \cdot \cot A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned}
LHS &= \frac{1 + \cos A}{1 - \cos A} - \frac{1 - \cos A}{1 + \cos A} \\
&= \frac{1 + \cos A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} - \frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A} \\
&= \frac{(1 + \cos A)^2}{1^2 - (\cos A)^2} - \frac{(1 - \cos A)^2}{1^2 - (\cos A)^2} \\
&= \frac{(1 + \cos A)^2}{1 - \cos^2 A} - \frac{(1 - \cos A)^2}{1 - \cos^2 A} \\
&= \frac{(1 + \cos A)^2}{\sin^2 A} - \frac{(1 - \cos A)^2}{\sin^2 A} \\
&= \left(\frac{1 + \cos A}{\sin A} \right)^2 - \left(\frac{1 - \cos A}{\sin A} \right)^2 \\
&= \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} \right)^2 - \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2 \\
&= (\csc A + \cot A)^2 - (\csc A - \cot A)^2 \\
&= \csc^2 A + \cot^2 A + 2\csc A \cdot \cot A - (\csc^2 A \\
&\quad + \cot^2 A - 2\csc A \cdot \cot A) \\
&= \csc^2 A + \cot^2 A + 2\csc A \cdot \tan A \\
&\quad - \csc^2 A - \cot^2 A + 2\csc A \cdot \cot A \\
&= 4\csc A \cdot \cot A \\
\therefore LHS &= RHS
\end{aligned}$$

19. $\frac{1-\tan^2 A}{1+\tan^2 A} = 1 - 2\sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
LHS &= \frac{1 - \tan^2 A}{1 + \tan^2 A} \\
&= \frac{1 - \tan^2 A}{\sec^2 A}
\end{aligned}$$

$$= \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{1}{\cos^2 A}}$$

$$= \frac{\cos^2 A - \sin^2 A}{\frac{1}{\cos^2 A}}$$

$$= \cos^2 A - \sin^2 A$$

$$= 1 - \sin^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A$$

$\therefore LHS = RHS$

20. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ එය සාධිෂ.

පරිජාර :

$$\begin{aligned} LHS &= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 \\ &= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A \\ &\quad + \cos^2 A + \sec^2 A + 2\cos A \cdot \sec A \\ &= \sin^2 A + \cos^2 A + \sec^2 A + \operatorname{cosec}^2 A \\ &\quad + 2\sin A \cdot \operatorname{cosec} A \\ &\quad + 2\cos A \cdot \sec A \\ &= 1 + 1 + \tan^2 A + 1 + \cot^2 A + 2\sin A \cdot \frac{1}{\sin A} \\ &\quad + 2\cos A \cdot \frac{1}{\cos A} \\ &= 3 + \tan^2 A + \cot^2 A + 2 + 2 \\ &= 7 + \tan^2 A + \cot^2 A \\ \therefore LHS &= RHS \end{aligned}$$

21. $\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$
එය සාධිෂ.

පරිජාර :

$$\begin{aligned} LHS &= \frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} \\ &= \frac{1}{\operatorname{cosec} A - \cot A} - \operatorname{cosec} A \\ &= \frac{1 - \operatorname{cosec} A(\operatorname{cosec} A - \cot A)}{\operatorname{cosec} A - \cot A} \\ &= \frac{1 - \operatorname{cosec}^2 A + \operatorname{cosec} A \cdot \cot A}{\operatorname{cosec} A - \cot A} \\ &= \frac{-\cot^2 A + \operatorname{cosec} A \cdot \cot A}{\operatorname{cosec} A - \cot A} \\ &= \frac{\cot A(-\cot A + \operatorname{cosec} A)}{\operatorname{cosec} A - \cot A} \\ &= \cot A \end{aligned}$$

$$\begin{aligned} RHS &= \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A} \\ &= \operatorname{cosec} A - \frac{1}{\operatorname{cosec} A + \cot A} \\ &= \frac{\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 1}{\operatorname{cosec} A + \cot A} \\ &= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A \cdot \cot A - 1}{\operatorname{cosec} A + \cot A} \\ &= \frac{\operatorname{cosec}^2 A - 1 + \operatorname{cosec} A \cdot \cot A}{\operatorname{cosec} A + \cot A} \\ &= \frac{\cot^2 A + \operatorname{cosec} A \cdot \cot A}{\operatorname{cosec} A + \cot A} \\ &= \frac{\operatorname{cosec} A + \cot A}{\cot A(\cot A + \operatorname{cosec} A)} \\ &= \frac{\operatorname{cosec} A + \cot A}{\operatorname{cosec} A + \cot A} \\ &= \cot A \\ \therefore LHS &= RHS \end{aligned}$$

22. $\frac{\tan A - \sin A}{\tan A + \sin A} = \frac{\sec A - 1}{\sec A + 1}$ එය සාධිෂ.

(September – 2020, 2 marks)

පරිජාර :

$$\begin{aligned} LHS &= \frac{\tan A - \sin A}{\tan A + \sin A} \\ &= \frac{\frac{\sin A}{\cos A} - \sin A}{\frac{\sin A}{\cos A} + \sin A} \\ &= \frac{\sin A \left(\frac{1}{\cos A} - 1 \right)}{\sin A \left(\frac{1}{\cos A} + 1 \right)} \\ &= \frac{\sec A - 1}{\sec A + 1} \\ \therefore LHS &= RHS \end{aligned}$$

23. $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$ එය සාධිෂ

පරිජාර :

$$LHS = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

අංත මුතු අද්‍යග්‍රීරේදනු $\sin A$ දින බාගිනිදාග

$$\begin{aligned} &= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} \\ &= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A} \\ &= \frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} \\ &= \cot A - \operatorname{cosec} A + 1 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1} \\
 &= \frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{\cot A - \operatorname{cosec} A + 1} \\
 &= \frac{\cot A + \operatorname{cosec} A[1 - (\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1} \\
 &= \frac{\cot A + \operatorname{cosec} A[1 - \operatorname{cosec} A + \cot A]}{\cot A - \operatorname{cosec} A + 1} \\
 &= \frac{\cot A + \operatorname{cosec} A[\cot A - \operatorname{cosec} A + 1]}{-\cot A - \operatorname{cosec} A + 1} \\
 &= \operatorname{cosec} A + \cot A \\
 \therefore LHS &= RHS
 \end{aligned}$$

24. $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \\
 \text{ಅಂಶ ಮತ್ತು ಭೇದಗಳಿರದನ್ನು } \cos A &\text{ ದಿಂದ ಭಾಗಿಸಿದಾಗ} \\
 &= \frac{\frac{\sin A}{\cos A} - \frac{\cos A}{\cos A} + \frac{1}{\cos A}}{\frac{\sin A}{\cos A} + \frac{\cos A}{\cos A} - \frac{1}{\cos A}} \\
 &= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A} \\
 &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A - (\sec A + \tan A)(\sec A - \tan A)}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A(1 - (\sec A - \tan A))}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A(1 - \sec A + \tan A)}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A(\tan A - \sec A + 1)}{\tan A - \sec A + 1}
 \end{aligned}$$

$$LHS = \tan A + \sec A$$

$$\begin{aligned}
 RHS &= \frac{1}{\sec A - \tan A} \\
 &= \frac{1}{\sec A - \tan A} \times \frac{\sec A + \tan A}{\sec A + \tan A} \\
 &= \frac{\sec A + \tan A}{\sec^2 A - \tan^2 A} \\
 &= \frac{\sec A + \tan A}{1}
 \end{aligned}$$

$$RHS = \tan A + \sec A$$

$$\therefore LHS = RHS$$

25. $\frac{\cos A - \sin A + 1}{\cos A + \sin A + 1} = \frac{1}{\tan A + \sec A}$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{\cos A - \sin A + 1}{\cos A + \sin A + 1} \\
 \text{ಅಂಶ ಮತ್ತು ಭೇದಗಳಿರದನ್ನು } \cos A &\text{ ದಿಂದ ಭಾಗಿಸಿದಾಗ} \\
 &= \frac{\frac{\cos A}{\cos A} - \frac{\sin A}{\cos A} + \frac{1}{\cos A}}{\frac{\cos A}{\cos A} + \frac{\sin A}{\cos A} + \frac{1}{\cos A}} \\
 &= \frac{1 - \tan A + \sec A}{1 + \tan A + \sec A} \\
 &= \frac{\sec A - \tan A + 1}{\sec A + \tan A + 1} \\
 &= \frac{\sec A - \tan A + (\sec^2 A - \tan^2 A)}{\sec A + \tan A + 1} \\
 &= \frac{\sec A - \tan A + (\sec A + \tan A)(\sec A - \tan A)}{\sec A + \tan A + 1} \\
 &= \frac{\sec A - \tan A(1 + \sec A + \tan A)}{\sec A + \tan A + 1} \\
 &= \sec A - \tan A \\
 RHS &= \frac{1}{\tan A + \sec A} \\
 &= \frac{1}{\tan A + \sec A} \times \frac{\tan A - \sec A}{\tan A - \sec A} \\
 &= \frac{\tan A - \sec A}{\tan^2 A - \sec^2 A} \\
 &= \frac{-(\sec A - \tan A)}{-(\sec^2 A - \tan^2 A)} \\
 &= \frac{\sec A - \tan A}{1} \\
 &= \sec A - \tan A \\
 \therefore LHS &= RHS
 \end{aligned}$$

26. $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{\cot A - \cos A}{\cot A + \cos A} \\
 &= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \\
 &= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \\
 &= \frac{\cos A \left(\frac{1}{\sin A} - 1 \right)}{\cos A \left(\frac{1}{\sin A} + 1 \right)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\sin A} - 1 \\
 &= \frac{1}{\sin A} + 1 \\
 &= \frac{\cosec A - 1}{\cosec A + 1} \\
 \therefore LHS &= RHS
 \end{aligned}$$

27. $\frac{\cot A + \cosec A - 1}{\cot A - \cosec A + 1} = \frac{1 + \cos A}{\sin A}$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{\cot A + \cosec A - 1}{\cot A - \cosec A + 1} \\
 &= \frac{\cot A + \cosec A - (\cosec^2 A - \cot^2 A)}{\cot A - \cosec A + 1} \\
 &= \frac{\cot A + \cosec A - (\cosec A + \cot A)(\cosec A - \cot A)}{\cot A - \cosec A + 1} \\
 &= \frac{\cot A + \cosec A[1 - (\cosec A - \cot A)]}{\cot A - \cosec A + 1} \\
 &= \frac{\cot A + \cosec A[1 - \cosec A + \cot A]}{\cot A - \cosec A + 1} \\
 &= \frac{\cot A + \cosec A[\cot A - \cosec A + 1]}{\cot A - \cosec A + 1} \\
 &= \cosec A + \cot A \\
 &= \frac{1 + \cos A}{\sin A} \\
 \therefore LHS &= RHS
 \end{aligned}$$

28. $(\sec A + \tan A)^2 = \frac{\cosec A + 1}{\cosec A - 1}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= (\sec A + \tan A)^2 \\
 &= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right)^2 \\
 &= \left(\frac{1 + \sin A}{\cos A} \right)^2 \\
 &= \frac{(1 + \sin A)^2}{\cos^2 A} \\
 &= \frac{(1 + \sin A)^2}{1 - \sin^2 A} \\
 &= \frac{(1 + \sin A)(1 + \sin A)}{(1 + \sin A)(1 - \sin A)} \\
 &= \frac{1 + \sin A}{1 - \sin A}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1 + \frac{1}{\cosec A}}{1 - \frac{1}{\cosec A}} \\
 &= \frac{\cosec A + 1}{\cosec A - 1} \\
 &= \frac{\cosec A}{\cosec A - 1} \\
 &= \frac{\cosec A + 1}{\cosec A - 1} \\
 \therefore LHS &= RHS
 \end{aligned}$$

29. $\tan^2 \theta + \cot^2 \theta + 2 = \sec^2 \theta \cdot \cosec^2 \theta$

ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \tan^2 \theta + \cot^2 \theta + 2 \\
 &= \sec^2 \theta - 1 + \cosec^2 \theta - 1 + 2 \\
 &= \sec^2 \theta + \cosec^2 \theta \\
 &= \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \cdot \sin^2 \theta} \\
 &= \frac{1}{\cos^2 \theta \cdot \sin^2 \theta} \\
 &= \sec^2 \theta \cdot \cosec^2 \theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

30. $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} \\
 &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \\
 &= \tan A \frac{1 - 2(1 - \cos^2 A)}{2\cos^2 A - 1} \\
 &= \tan A \frac{1 - 2 + 2\cos^2 A}{2\cos^2 A - 1} \\
 &= \tan A \frac{2\cos^2 A - 1}{2\cos^2 A - 1} \\
 &= \tan A \\
 \therefore LHS &= RHS
 \end{aligned}$$

31. $(\cosec A - \sin A)(\sec A - \cos A) =$

$\frac{1}{\tan A + \cot A}$ ಎಂದು ಸಾಧಿಸಿ.
ಪರಿಹಾರ :

$$LHS = (\cosec A - \sin A)(\sec A - \cos A)$$

$$\begin{aligned} &= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \\ &= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \\ &= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \\ &= \cos A \cdot \sin A \end{aligned}$$

$$\begin{aligned} RHS &= \frac{1}{\tan A + \cot A} \\ &= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} \\ &= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A}} \\ &= \frac{\cos A \cdot \sin A}{\sin^2 A + \cos^2 A} \\ &= \frac{\cos A \cdot \sin A}{1} \\ &= \cos A \cdot \sin A \\ \therefore LHS &= RHS \end{aligned}$$

32. $(\sin A + \sec A)^2 + (\cos A + \cosec A)^2 =$

$(1 + \sec A \cdot \cosec A)^2$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= (\sin A + \sec A)^2 + (\cos A + \cosec A)^2 \\ &= \left(\sin A + \frac{1}{\cos A} \right)^2 + \left(\cos A + \frac{1}{\sin A} \right)^2 \\ &= \left(\frac{\sin A \cdot \cos A + 1}{\cos A} \right)^2 + \left(\frac{\cos A \cdot \sin A + 1}{\sin A} \right)^2 \\ &= \frac{(\sin A \cdot \cos A + 1)^2}{\cos A^2} + \frac{(\sin A \cdot \cos A + 1)^2}{\sin^2 A} \\ &= \frac{(\sin A \cdot \cos A + 1)^2 \sin^2 A + \cos^2 A (\sin A \cdot \cos A + 1)^2}{\cos A^2 \cdot \sin^2 A} \\ &= \frac{(\sin A \cdot \cos A + 1)^2 (\sin^2 A + \cos^2 A)}{\cos A^2 \cdot \sin^2 A} \\ &= \frac{(\sin A \cdot \cos A + 1)^2 \cdot 1}{\cos A^2 \cdot \sin^2 A} \\ &= \left(\frac{\sin A \cdot \cos A + 1}{\cos A \cdot \sin A} \right)^2 \end{aligned}$$

$$\begin{aligned} &= \left(\frac{\sin A \cdot \cos A}{\cos A \cdot \sin A} + \frac{1}{\cos A \cdot \sin A} \right)^2 \\ &= \left(1 + \frac{1}{\cos A \cdot \sin A} \right)^2 \\ &= (1 + \sec A \cdot \cosec A)^2 \\ \therefore LHS &= RHS \end{aligned}$$

33. $\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \tan^2 A - \sin^2 A \\ &= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A \\ &= \frac{\sin^2 A - \sin^2 A \cdot \cos^2 A}{\cos^2 A} \\ &= \frac{\sin^2 A (1 - \cos^2 A)}{\cos^2 A} \\ &= \frac{\sin^2 A}{\cos^2 A} \cdot \sin^2 A \\ &= \tan^2 A \cdot \sin^2 A \\ \therefore LHS &= RHS \end{aligned}$$

34. $\cos^4 A - \sin^4 A = 1 - 2\sin^2 A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \cos^4 A - \sin^4 A \\ &= (\cos^2 A)^2 - (\sin^2 A)^2 \\ &= (\cos^2 A + \sin^2 A) (\cos^2 A - \sin^2 A) \\ &= 1 (\cos^2 A - \sin^2 A) \\ &= 1 - \sin^2 A - \sin^2 A \\ &= 1 - 2\sin^2 A \\ \therefore LHS &= RHS \end{aligned}$$

35. $\sec^6 A - \tan^6 A = 1 + 3\tan^2 A \cdot \sec^2 A$

ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \sec^6 A - \tan^6 A \\ &= (\sec^2 A)^3 - \tan^6 A \\ &= (1 + \tan^2 A)^3 - \tan^6 A \\ &= 1^3 + (\tan^2 A)^3 + 3 \times 1 \\ &\quad \times \tan^2 A (1 + \tan^2 A) - \tan^6 A \\ &= 1 + \tan^6 A + 3\tan^2 A \cdot \sec^2 A - \tan^6 A \\ &= 1 + 3\tan^2 A \cdot \sec^2 A \\ \therefore LHS &= RHS \end{aligned}$$

36. $\sqrt{\sec^2 A + \cosec^2 A} = \tan A + \cot A$

ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \sqrt{\sec^2 A + \cosec^2 A} \\ &= \sqrt{1 + \tan^2 A + 1 + \cot^2 A} \\ &= \sqrt{\tan^2 A + \cot^2 A + 2} \\ &= \sqrt{(\tan A + \cot A)^2} \\ &= \tan A + \cot A \\ \therefore LHS &= RHS \end{aligned}$$

37. $\frac{\cos A}{\sec A - \tan A} = 1 + \sin A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\cos A}{\sec A - \tan A} \\ &= \frac{\cos A}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}} \\ &= \frac{\cos A}{\frac{1 - \sin A}{\cos A}} \\ &= \cos A \times \frac{\cos A}{1 - \sin A} \\ &= \frac{\cos^2 A}{1 - \sin A} \\ &= \frac{1 - \sin^2 A}{1 - \sin A} \\ &= \frac{(1 + \sin A)(1 - \sin A)}{1 - \sin A} \\ &= 1 + \sin A \\ \therefore LHS &= RHS \end{aligned}$$

38. $\frac{\sin^2 A}{1 - \cos A} = \frac{1 + \sec A}{\sec A}$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\sin^2 A}{1 - \cos A} \\ &= \frac{1 - \cos^2 A}{1 - \cos A} \\ &= \frac{(1 + \cos A)(1 - \cos A)}{1 - \cos A} \\ &= 1 + \cos A \\ &= 1 + \frac{1}{\sec A} \\ &= \frac{\sec A + 1}{\sec A} \\ \therefore LHS &= RHS \end{aligned}$$

39. $\frac{\cosec^2 A + 1}{\cosec^2 A - 1} = \sec^2 A + \tan^2 A$ ಎಂದು

ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\cosec^2 A + 1}{\cosec^2 A - 1} \\ &= \frac{\frac{1}{\sin^2 A} + 1}{\frac{1}{\sin^2 A} - 1} \\ &= \frac{\frac{1 + \sin^2 A}{\sin^2 A}}{\frac{1 - \sin^2 A}{\sin^2 A}} \\ &= \frac{1 + \sin^2 A}{1 - \sin^2 A} \\ &= \frac{1 + \sin^2 A}{\cos^2 A} \\ &= \frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} \\ &= \sec^2 A + \tan^2 A \\ \therefore LHS &= RHS \end{aligned}$$

40. $\frac{\sin A}{1 - \cos A} = \cosec A + \cot A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= \frac{\sin A}{1 - \cos A} \\ &= \frac{\sin A}{1 - \cos A} \times \frac{1 + \cos A}{1 + \cos A} \\ &= \frac{\sin A(1 + \cos A)}{1^2 - \cos^2 A} \\ &= \frac{\sin A(1 + \cos A)}{\sin^2 A} \\ &= \frac{1 + \cos A}{\sin A} \\ &= \frac{1}{\sin A} + \frac{\cos A}{\sin A} \\ &= \cosec A + \cot A \\ \therefore LHS &= RHS \end{aligned}$$

41. $(\tan A \times \sin A) + \cos A = \sec A$ ಎಂದು

ಸಾಧಿಸಿ. (june – 2019, 2 marks)

ಪರಿಹಾರ :

$$\begin{aligned} LHS &= (\tan A \times \sin A) + \cos A \\ &= \left(\frac{\sin A}{\cos A} \times \sin A \right) + \cos A \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sin^2 A}{\cos A} + \cos A \\
 &= \frac{\sin^2 A + \cos^2 A}{\cos A} \\
 &= \frac{1}{\cos A} \\
 &= \sec A \\
 \therefore LHS &= RHS
 \end{aligned}$$

42. $x = p \tan \theta + q \sec \theta$ മുള്ളായി $x = p \sec \theta + q \tan \theta$

അതിൽ $x^2 - y^2 = q^2 - p^2$ എന്ദു സാധിച്ചി.

(june - 2020, 3 marks)

പരിഹാര :

$$\begin{aligned}
 LHS &= x^2 - y^2 \\
 &= (p \tan \theta + q \sec \theta)^2 - (p \sec \theta + q \tan \theta)^2 \\
 &= (p \tan \theta)^2 + (q \sec \theta)^2 - 2p \tan \theta \cdot p \sec \theta \\
 &\quad - [(p \sec \theta)^2 + (q \tan \theta)^2 \\
 &\quad - 2p \sec \theta \cdot p \tan \theta] \\
 &= p^2 \tan^2 \theta + q^2 \sec^2 \theta - 2pq \tan \theta \cdot \sec \theta - [p^2 \sec^2 \theta \\
 &\quad + q^2 \tan^2 \theta - 2pq \sec \theta \cdot \tan \theta] \\
 &= p^2 \tan^2 \theta + q^2 \sec^2 \theta - 2pq \tan \theta \cdot \sec \theta \\
 &\quad - p^2 \sec^2 \theta - q^2 \tan^2 \theta + 2pq \sec \theta \cdot \tan \theta \\
 &= p^2 \tan^2 \theta + q^2 \sec^2 \theta - p^2 \sec^2 \theta - q^2 \tan^2 \theta \\
 &= p^2 \tan^2 \theta - p^2 \sec^2 \theta + q^2 \sec^2 \theta - q^2 \tan^2 \theta \\
 &= p^2 (\tan^2 \theta - \sec^2 \theta) + q^2 (\sec^2 \theta - \tan^2 \theta) \\
 &= p^2 (-1) + q^2 (1) \\
 &= q^2 - p^2 \\
 \therefore LHS &= RHS
 \end{aligned}$$

43. $\frac{\cot^2(90^\circ - \theta)}{\tan^2 \theta - 1} + \frac{\cosec^2 \theta}{\sec^2 \theta - \cosec^2 \theta} = \frac{1}{\sin^2 \theta - \cos^2 \theta}$
എന്ദു സാധിച്ചി. (june - 2020, 3 marks)

പരിഹാര :

$$\begin{aligned}
 LHS &= \frac{\cot^2(90^\circ - \theta)}{\tan^2 \theta - 1} + \frac{\cosec^2 \theta}{\sec^2 \theta - \cosec^2 \theta} \\
 &= \frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\cosec^2 \theta}{\sec^2 \theta - \cosec^2 \theta} \\
 &= \frac{\tan^2 \theta}{\tan^2 \theta - 1} + \frac{\cosec^2 \theta}{\sec^2 \theta - \cosec^2 \theta} \\
 &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} - 1} + \frac{\frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta} - \frac{1}{\sin^2 \theta}} \\
 &= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{\sin^2 \theta - \cos^2 \theta}{\cos^2 \theta}} + \frac{\frac{1}{\sin^2 \theta}}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta \cdot \sin^2 \theta}} \\
 &= \frac{\sin^2 \theta}{\cos^2 \theta} \times \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{1}{\sin^2 \theta} \times \frac{\cos^2 \theta \cdot \sin^2 \theta}{\sin^2 \theta - \cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sin^2 \theta}{\sin^2 \theta - \cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta} \\
 &= \frac{1}{\sin^2 \theta - \cos^2 \theta} \\
 \therefore LHS &= RHS
 \end{aligned}$$

44. $\frac{\cos \theta - 2 \cos^3 \theta}{2 \sin^3 \theta - \sin \theta} = \cot \theta$ എന്ദു സാധിച്ചി.

(MQP. 1 - 2021, 3 marks)

പരിഹാര :

$$\begin{aligned}
 LHS &= \frac{\cos \theta - 2 \cos^3 \theta}{2 \sin^3 \theta - \sin \theta} \\
 &= \frac{\cos \theta (1 - 2 \cos^2 \theta)}{\sin \theta (2 \sin^2 \theta - 1)} \\
 &= \cot \theta \frac{1 - 2 \cos^2 \theta}{2(1 - \cos^2 \theta) - 1} \\
 &= \cot \theta \frac{1 - 2 \cos^2 \theta}{2 - 2 \cos^2 \theta - 1} \\
 &= \cot \theta \frac{\frac{1 - 2 \cos^2 \theta}{1 - 2 \cos^2 \theta}}{\frac{1 - 2 \cos^2 \theta}{1 - 2 \cos^2 \theta}} \\
 &= \cot \theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

45. $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} = \sin \theta + \cos \theta$ എന്ദു സാധിച്ചി.

(MQP. 1 - 2021, Preparatory - 2020, 3 marks)

പരിഹാര :

$$\begin{aligned}
 LHS &= \frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} \\
 &= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} \\
 &= \frac{\sin \theta}{\frac{\sin \theta - \cos \theta}{\sin \theta}} + \frac{\cos \theta}{\frac{\cos \theta - \sin \theta}{\cos \theta}} \\
 &= \sin \theta \times \frac{\sin \theta}{\sin \theta - \cos \theta} + \cos \theta \times \frac{\cos \theta}{\cos \theta - \sin \theta} \\
 &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta} \\
 &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} - \frac{\cos^2 \theta}{\sin \theta - \cos \theta} \\
 &= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \\
 &= \frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta} \\
 &= \sin \theta + \cos \theta \\
 \therefore LHS &= RHS
 \end{aligned}$$

46. $\{\text{Cosec}(90 - \theta) - \sin(90 - \theta)\}\{\text{Cosec}\theta - \sin\theta\tan\theta + \cot\theta\} = 1$ ಎಂದು ಸಾಧಿಸಿ.

(MQP. 2 – 2021, 3 marks)

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \{\text{Cosec}(90 - \theta) - \sin(90 - \theta)\}\{\text{Cosec}\theta - \sin\theta(\tan\theta + \cot\theta)\} \\
 &= \{\sec\theta - \cos\theta\}\{\text{Cosec}\theta - \sin\theta\}(\tan\theta + \cot\theta)\} \\
 &= \left\{ \frac{1}{\cos\theta} - \cos\theta \right\} \left\{ \left(\frac{1}{\sin\theta} - \sin\theta \right) \left(\tan\theta + \frac{1}{\tan\theta} \right) \right\} \\
 &= \left\{ \frac{1 - \cos^2\theta}{\cos\theta} \right\} \left\{ \left(\frac{1 - \sin^2\theta}{\sin\theta} \right) \left(\frac{1 + \tan^2\theta}{\tan\theta} \right) \right\} \\
 &= \left\{ \frac{\sin^2\theta}{\cos\theta} \right\} \left\{ \left(\frac{\cos^2\theta}{\sin\theta} \right) \left(\frac{\sec^2\theta}{\tan\theta} \right) \right\} \\
 &= \left\{ \frac{\sin^2\theta}{\cos\theta} \right\} \left\{ \left(\frac{\cos^2\theta}{\sin\theta} \right) \left(\frac{1}{\frac{\sin\theta}{\cos\theta}} \right) \right\} \\
 &= \frac{\sin^2\theta}{\cos\theta} \times \frac{\cos^2\theta}{\sin\theta} \times \frac{1}{\frac{\sin\theta}{\cos\theta}} \times \frac{\cos\theta}{\sin\theta} \\
 &= \frac{\sin^2\theta}{\sin^2\theta} \\
 &= 1 \\
 \therefore LHS &= RHS
 \end{aligned}$$

47. $\frac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \frac{(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)} = \frac{2}{(2\sin^2\theta - 1)}$ ಎಂದು ಸಾಧಿಸಿ. (MQP. 2 – 2021, 3 marks)

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \frac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \frac{(\sin\theta + \cos\theta)}{(\sin\theta - \cos\theta)} \\
 &= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)} \\
 &= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta} \\
 &= \frac{\sin^2\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} \\
 &= \frac{1 + 1}{\sin^2\theta - (1 - \sin^2\theta)} \\
 &= \frac{2}{\sin^2\theta - 1 + \sin^2\theta} \\
 &= \frac{2}{2\sin^2\theta - 1} \\
 \therefore LHS &= RHS
 \end{aligned}$$

48. $\text{Cosec}A(1 - \cos A)(\text{Cosec}A + \cot A) = 1$ ಎಂದು ಸಾಧಿಸಿ.

(Gadag dist level exam – 2021, 3 marks)

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \text{Cosec}A(1 - \cos A)(\text{Cosec}A + \cot A) \\
 &= \frac{1}{\sin A} (1 - \cos A) \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} \right)
 \end{aligned}$$

$$= \frac{1}{\sin A} (1 - \cos A) \left(\frac{1 + \cos A}{\sin A} \right)$$

$$\begin{aligned}
 &= \frac{1 - \cos^2 A}{\sin^2 A} \\
 &= \frac{\sin^2 A}{\sin^2 A} \\
 &= 1 \\
 \therefore LHS &= RHS
 \end{aligned}$$

49. $\sec A(1 - \sin A)(\sec A + \tan A) = 1$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= \sec A(1 - \sin A)(\sec A + \tan A) \\
 &= \frac{1}{\cos A} (1 - \sin A) \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} \right) \\
 &= \frac{1}{\cos A} (1 - \sin A) \left(\frac{1 + \sin A}{\cos A} \right) \\
 &= \frac{1 - \sin^2 A}{\cos^2 A} \\
 &= \frac{\cos^2 A}{\cos^2 A} \\
 &= 1
 \end{aligned}$$

$$\therefore LHS = RHS$$

50. $(\cot A \times \cos A) + \sin A = \text{cosec} A$ ಎಂದು ಸಾಧಿಸಿ.

ಪರಿಹಾರ :

$$\begin{aligned}
 LHS &= (\cot A \times \cos A) + \sin A \\
 &= \left(\frac{\cos A}{\sin A} \times \cos A \right) + \sin A \\
 &= \frac{\cos^2 A}{\sin A} + \sin A \\
 &= \frac{\cos^2 A + \sin^2 A}{\sin A} \\
 &= \frac{1}{\sin A} \\
 &= \text{cosec} A
 \end{aligned}$$

$$\therefore LHS = RHS$$

ಶ್ರೀ ನಾಗರಾಜ ಬ ಹಳ್ಳಕೆಲ

ಪ.ಪ್ರೈ.ಶಾಲೆ ಜಂತಲ ಶೀರ್ಯಾರ

ತಾ॥ ಮುಂದರಗಿ

ವೋ : ೯೦೧೩೫೫೫೦

mail:hallikeri.nagaraj567@gmail.com

