# Chapter - 1 : Arithmetic Progressions

## Quick Review

- **Sequence :**
  - **(i)** A sequence is an ordered arrangement of numbers according to a definite rule.
  - **(ii)** A sequence containing a finite number of terms is called a finite sequence.
  - **(iii)** A sequence containing infinite number of terms is called an infinite sequence.
  - **(iv)** The sum of terms of a sequence is called the series of the corresponding sequence.
  - **(v)** The series of finite number of terms 'n' is denoted by $S_n$.

    *i.e.,* $S_n = T_1 + T_2 + \text{..........................} + T_n$
- **Arithmetic Progression (A.P.) :**
  - **(i)** An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the proceeding term, except the first term.
  - **(ii)** The difference between the two successive terms of an A.P. is called the common difference.
  - **(iii)** Each of the number in the list of arithmetic progression is called a term of an A.P.
  - **(iv)** The arithmetic progression containing finite number of terms is called a finite arithmetic progression.
  - **(v)** Let a list of numbers $a_1, a_2, a_3$ ............... from an A.P., then the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3$ .......... give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of $k$.
  - **(vi)** The general form of an A.P. is $a, a + d, a + 2d, a + 3d, \text{..............}, a + (n - 1)d$.
  - **(vii)** If the A.P. "$a, a + d, a + 2d \text{............}, l$" is reversed to "$l, l - d, l - 2d \text{.................}, a$," then the common difference of reversed A.P. changes in sign only of common difference of original A.P.
  - **(viii)** The $n^{th}$ term of an A.P. is the difference of the sum to first n terms and the sum to first (n – 1) terms of it.

    i.e., $T_n = S_n - S_{n-1}$
  - **(ix)** The general terms of A.P. is given by :

    $$T_n = a + (n-1)d,$$

    where a is the first term, d is the common difference and n is the total number of terms.

    Again common difference, $d = \dfrac{T_p - T_q}{p - q}$
  - **(x)** Sum of n terms of an A.P. is given by : $S_n = \dfrac{n}{2}[2a + (n-1)d]$,

    where $a$ is the first term, $d$ is the common difference and $n$ is the total number of terms.
  - **(xi)** Sum of $n$ terms of an A.P. is also given by : $S_n = \dfrac{n}{2}[a + l]$,

    where $a$ is the first term and $l$ is the last term.
  - **(xii)** The general term of an A.P. $l, l - d, l - 2d$............ is given by :

    $$a = l + (n-1)(-d),$$

    where $l$ is the last term, $d$ is the common difference and $n$ is the number of terms.
  - **(xiii)** The arithmetic progression containing infinite number of terms is called an infinite arithmetic progression.
  - **(xiv)** Let there be an A.P. with first term '$a$' and common difference '$d$'. If there are $m$ terms in the A.P., then

    $$\begin{aligned} n^{th} \text{ term from the end} &= (m - n + 1)^{th} \text{ term from the beginning} \\ &= a + (m - n)d \\ \text{Also } n^{th} \text{ term from the end} &= \text{last term} + (n-1)(-d) \\ &= l - (n-1)d, \text{ where } l \text{ is last term.} \end{aligned}$$

**(xv) Various terms of an A.P. can be chosen in the following manner :**

| Numbers of terms | Term | Common difference |
|---|---|---|
| **3** | $a - d, a, a + d$ | $d$ |
| **4** | $a - 3d, a - d, a + d, a + 3d$ | $2d$ |
| **5** | $a - 2d, a - d, a, a + d, a + 2d$ | $d$ |
| **6** | $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ | $2d$ |

➢ **Arithmetic Series :** A series whose terms are in A.P. is called an arithmetic series.

❑❑

# Chapter - 2 : Triangles

## Quick Review

➢ Two figures are said to be congruent if they have the same shape and the same size.

➢ Those figures which have the same shape but not necessarily the same size are called similar figures.
Hence, we can say that all congruent figures are similar but the similar figures are not always congruent.

➢ **Similarity of Polygons :** Two polygons having same number of sides are similar, if :

**(i)** their corresponding sides are proportional i.e., in the same ratio.

**(ii)** their corresponding angles are equal.

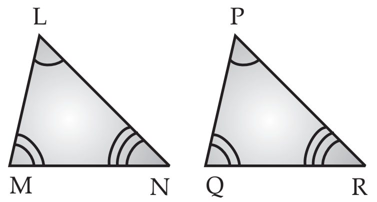
➢ **Similarity of Triangles :** Two triangles are similar, if :

**(i)** their corresponding sides are proportional.

**(ii)** their corresponding angles are equal.

If $\Delta ABC$ and $\Delta DEF$ are similar, then this similarity can be written as : $\Delta ABC \sim \Delta DEF$.

➢ **Criteria for Similarity of Triangles :**




In $\Delta LMN$ and $\Delta PQR$, if

**(i)** $\angle L = \angle P, \angle M = \angle Q, \angle N = \angle R$ **(ii)** $\frac{LM}{PQ} = \frac{MN}{QR} = \frac{LN}{PR}$

then $\Delta LMN \sim \Delta PQR$,

**(i)** **AAA-Criterion :** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio. Hence the triangles are similar.

**Remark :** If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angle will also be equal. Therefore, AAA similarity criterion can also be stated as follows :

**AA-Criterion :** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

**(ii)** **SSS-Criterion :** If in two triangles the sides of one triangle are proportional to the sides of another triangle, then their corresponding angles are equal and hence the two triangles are similar.

**(iii)** **SAS-Criterion :** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

➢ **Some Theorems Based on Similarity of Triangles :**

**(i)** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. It is known as 'Basic Proportionality Theorem' or 'Thales Theorem'.

**(ii)** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the Converse of Basic Proportionality Theorem.

**(iii)** If two triangles are similar, then the ratio of area of these triangles is equal to the ratio of squares of their corresponding sides.

➢ Two figures are similar if and only if they have same shape, but not necessarily the same size.

➢ **Areas of similar triangles are proportional to :**

**(i)** Squares of their corresponding sides.

**(ii)** Squares of their corresponding altitudes.

**(iii)** Squares of their corresponding medians.

**(iv)** Squares of their corresponding circum-radii.

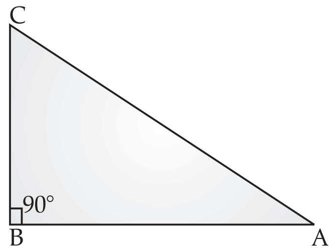
**(v)** Squares of their corresponding angular bisectors.

**(vi)** Squares of their corresponding in-radii.

The converse of the above six statements also hold good.

➢ **Pythagoras theorem :** In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

➢ **According to Pythagoras theorem :** $AC^2 = AB^2 + BC^2$




➢ **Converse of Pythagoras theorem :** "If the square of the largest side of a triangle is equal to the sum of the squares on the other two sides, then those two sides contain a right angle.

➢ **Pythagorian Triplets :** If the three numbers, which are the measures of three sides of a right angled triangle are natural numbers, then they are called Pythagorian triplets. e.g.,

| | |
|---|---|
| 3, 4, 5 | 6, 8, 10 |
| 5, 12, 13 | 15, 36, 39 |
| 7, 24, 25 | 14, 48, 50 |
| 11, 60, 61 | 44, 240, 244 |

➢ Pythagorean triplets can be found using the following general form.

For natural numbers : $2n, (n^2 - 1), (n^2 + 1)$, where n may be even or odd.

For odd natural numbers : n, $\frac{1}{2}n^2 - 1, \frac{1}{2}n^2 + 1$ , where n is odd.

From the above general forms any number of pythagorian triplets can be generated by giving value to 'n'.

➢ "It is impossible to write any power beyond the second as the sum of two similar powers."

There are no a, b, c $\in$ N for which $a^n + b^n = c^n$, where $n \in$ N and n > 2.

From the above general forms any number of pythagorian triplets can be generated by giving value to '$n$'.

❑❑

# Chapter - 3 : Pair of Linear Equations in Two Variable

## Quick Review

➢ **Linear Equation in two variables :** An equation of the form $ax + by + c = 0$, where $a$, $b$ and $c$ are real numbers and $a$ and $b$ are both not zero, is called a linear equation in two variables $x$ and $y$.

General form of a pair of linear equations in two variables is :

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0,$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that

$a_1, b_1 \neq 0$ and $a_2, b_2 \neq 0$.

*e.g.,*
$$3x - y + 7 = 0,$$
$$7x + y = 3$$

are linear equations in two variables $x$ and $y$.

➢ **There are two methods of solving simultaneous linear equations in two variables :**

**1.** Graphical method,

**2.** Algebraic method.

**1.** **Graphical Method :**

**(i)** Express one variable say $y$ in terms of the other variable $x$, $y = ax + b$, for the given equation.

**(ii)** Take three values of independent variable $x$ and find the corresponding values of dependent variable $y$, take integral values only.

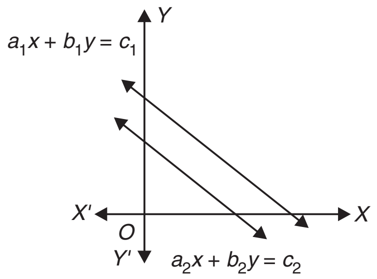
**(iii)** Plot these values on the graph paper in order to represent these equations.

**(iv)** If the lines intersect at a distinct point, then point of intersection is the unique solution of the two equations. In this case, the pair of linear equations is consistent.

**(v)** If the lines representing the linear equations coincides, then system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.

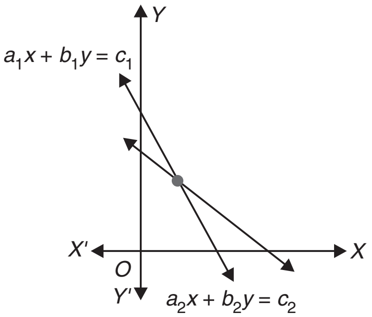
**(vi)** If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.

**Parallel Lines :**



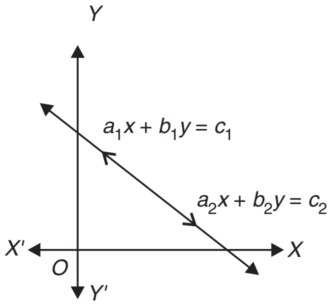

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

**Intersecting Lines :**




$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2},$$

**Coincident Lines :**




$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

➢ If $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0,$

**is a pair of linear equations in two variables *x* and *y* such that :**

**(i)** If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.

**(ii)** If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.

**(iii)** If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.

**Possibilities of Solutions and Inconsistency :**

| Pair of lines | $\frac{a_1}{a_2}$ | $\frac{b_1}{b_2}$ | $\frac{c_1}{c_2}$ | Compare the ratios | Graphical representation | Algebraic interpretation | Conditions for solvability |
|---|---|---|---|---|---|---|---|
| $x - 2y = 0$<br>$3x - 4y - 20 = 0$ | $\frac{1}{3}$ | $\frac{-2}{-4}$ | $\frac{0}{-20}$ | $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | Intersecting lines | Exactly one solution<br>Unique solution | System is consistent |
| $2x + 3y - 9 = 0$<br>$4x + 6y - 18 = 0$ | $\frac{2}{4}$ | $\frac{3}{6}$ | $\frac{-9}{-18}$ | $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ | Coincident lines | Infinitely many solutions | System is consistent |
| $x + 2y - 4 = 0$<br>$2x + 4y - 12 = 0$ | $\frac{1}{2}$ | $\frac{2}{4}$ | $\frac{-4}{-12}$ | $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | Parallel lines | No solution | System is inconsistent |

**2. Algebraic Method :** We can solve the linear equations algebraically by substitution method, elimination method and cross-multiplication method.

**1. Substitution Method :**

**(i)** Find the value of one variable say *y* in terms of the other variable *i.e., x* from either of the equations.

**(ii)** Substitute this value of *y* in other equation and reduce it to an equation in one variable.

**(iii)** Solve the equation so obtained and find the value of *x*.

**(iv)** Put this value of *x* in one of the equations to get the value of variable *y*.

**2. Elimination Method :**

**(i)** Multiply given equations with suitable constants, make either the *x*-coefficient or the *y*-coefficient of the two equations equal.

**(ii)** Subtract or add one equation from the other to get an equation in one variable.

**(iii)** Solve the equation so obtained to get the value of the variable.

**(iv)** Put this value in any one of the equation to get the value of the second variable.

**Note :**

**(a)** If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

**(b)** If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.,* it is inconsistent.

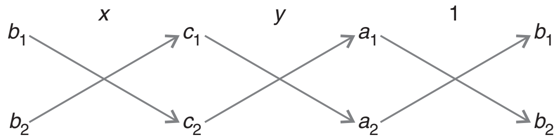
**3. Cross-multiplication Method :** If two simultaneous linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given, then a unique solution is given by :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

or

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

**Note :** To obtain the above result, following diagram may be helpful :




The arrows between the two numbers indicate that they are to be multiplied. The product with upward arrows are to be subtracted from the product with downward arrows.

**Remember :**

**1.** If the equations

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

are in the form

$$a_1x + b_1y = -c_1 \text{ and } a_2x + b_2y = -c_2$$

Then, we have by cross-multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

➢ **Equations reducible to a Pair of Linear Equations in two variables :** Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

**Steps to be followed for solving word problems**

| S. No. | Problem type | Steps to be followed |
|---|---|---|
| 1. | Age Problems | If the problem involves finding out the ages of two persons, take the present age of one person as $x$ and of the other as $y$. Then '$a$' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after '$b$' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them. |
| 2. | Problems based on Numbers and Digits | Let the digit in unit's place be $x$ and that in ten's place be $y$. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes $y$ and in ten's place becomes $x$. The two digit number becomes $10x + y$. Formulate the equations and then solve them. |
| 3. | Problems based on Fractions | Let the numerator of the fraction be $x$ and denominator be $y$, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for $x$ and $y$ to get the value of the fraction. |
| 4. | Problems based on Distance, Speed and Time | We know that Speed $= \frac{\text{Distance}}{\text{Time}}$, Distance = Speed × Time and Time $= \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be $x$ km/h and speed of stream be $y$ km/h. Then, the speed of boat downstream $= x + y$ km/h and speed of boat upstream $= x - y$ km/h. |

| | | |
|---|---|---|
| **5.** | Problems based on commercial Mathematics | For solving specific questions based on commercial mathematics, the fare of 1 full ticket may be taken as ₹ $x$ and the reservation charges may be taken as ₹ $y$, so that one full fare = $x + y$ and one half fare = $\frac{x}{2} + y$.<br>To solve the questions of profit and loss, take the cost price of 1st article as ₹ $x$ and that of 2nd article as ₹ $y$.<br>To solve the questions based on simple interest, take the amount invested as ₹ $x$ at some rate of interest and ₹ $y$ at some other rate of interest. |
| **6.** | Problems based on Geometry and Mensuration | Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle.<br>In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary. |

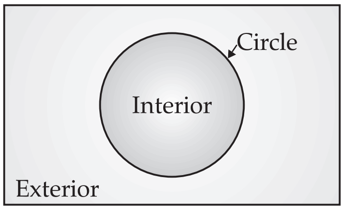
❑❑

# Chapter - 4 : Circles

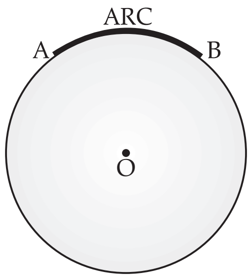
## Quick Review

**Properties of Chord :**

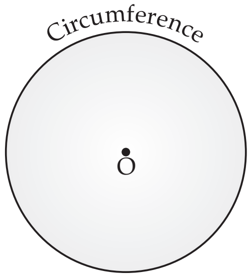
- A circle is a collection (set) of all those points in a plane, each one of which is at a constant distance from a fixed point in the plane.
- The fixed point is called the centre and the constant distance is called the radius of the circle.
- All the points lying inside a circle are called its interior points and all those points which lie outside the circle are called its exterior points.
- The collection (set) of all interior points is called the interior of circle while the collection of all exterior points of a circle is called the exterior of the circle.



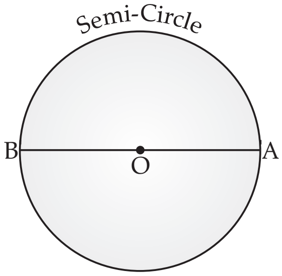

- A line segment joining two points on a circle is called the chord of the circle.
- A chord passing through the centre of the circle is called a diameter of the circle which is also the longest chord of the circle.
- Diameter of a circle = 2 × Radius.
- A (continuous) part of a circle is called an arc of the circle. The arc of a circle is denoted by the symbol '⌒'.



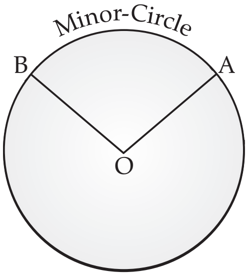

- **Circumference :** The whole arc of a circle is called the circumference of the circle.




➢ **Semi-circle :** One-half of the whole arc of a circle is semi-circle of the circle.




➢ **Minor and Major Arcs :** An arc less than one-half of the whole arc of a circle is called a minor arc of the circle and an arc greater than one-half of the whole arc of a circle is called a major arc of the circle.




**Major arc**

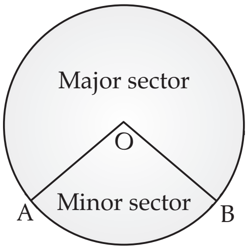
➢ **Central Angle :** Any angle whose vertex is centre of the circle is called a central angle.

➢ **Degree measure of an Arc :** The degree measure of a minor arc is the measure of the central angle subtended by the arc.

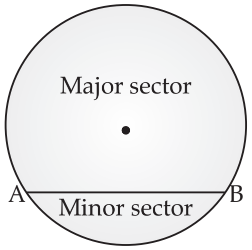
➢ **Congruent Circles :** Two circles are said to be congruent if and only if either of them can be superimposed on the other so as to cover it exactly.

➢ **Congruent Arcs :** Two arcs of a circle (or of congruent circles) are congruent if either of them can be superimposed on the other so as to cover it exactly.

➢ **Sector of a Circle :** The part of the plane region enclosed by an arc of a circle and its two bounding radii is called a sector of a circle.

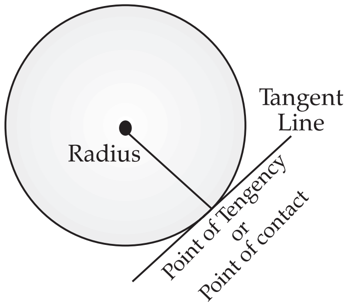



➢ **Segment of a Circle :** A chord of a circle divides it into two parts. Each part is called a segment. The part containing the minor arc is called the minor segment and the part containing the major arc is called the major segment.

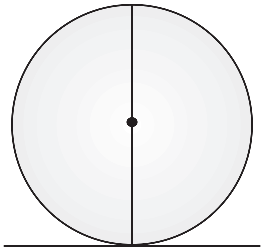



**Properties of Tagents**

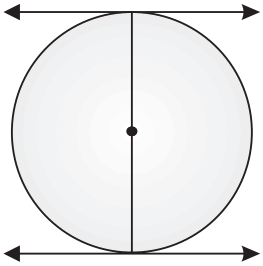
- **Secant :** A straight line which intersects a circle at two distinct points is called a secant.
- **Tangent :** A tangent to a circle is a line that intersects the circle at one point only.
- **Point of contact :** The point where a tangent touches the circle is called the point of contact.
- The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point outside the circle.
- At any point on the circle there can be one and only one tangent.




- The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- In a circle, the perpendicular to the radius at its non-centre end is the tangent to the circle.
- The perpendicular to the tangent at the point of contact passes through the centre of the circle.

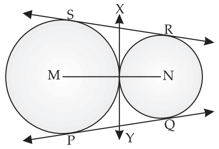
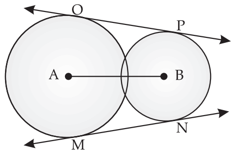
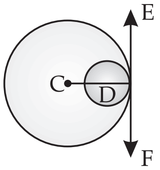
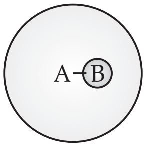
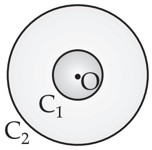


- Tangents drawn at the ends of a diameter are parallel to each other.



- **The tangents drawn from an external point to a circle :**
  **(i)** are equal
  **(ii)** subtend equal angles at the centre.
  **(iii)** are equally inclined to the line joining the centre and the external point.

➢ Let us summarise the data regarding common tangents. Study the given table.

| Sl. No. | Circle | Figures | Relation between *d*, *R* and *r* | Number of DCT's | Number of TCT's | Number of CT's |
|---|---|---|---|---|---|---|
| 1. | Circles touching externally |  | $d = R + r$ | 2<br>$PQ, RS$ | 1<br>$XY$ | 3 |
| 2. | Intersecting circles |  | $d < R + r$ and $d > R - r$ | 2<br>$MN, PQ$ | - | 2 |
| 3. | Circles touching internally |  | $d = R - r$ | 1<br>$EF$ | - | 1 |
| 4. | Non-concentric circles |  | $d < R - r$ | - | - | - |
| 5. | Concentric circles |  | $d = 0$ | - | - | - |

❑❑

# Chapter - 5 : Areas Related to Circles

## Quick Review

➢ A circle is a collection of all points in a plane which a re at a constant distance from a fixed point in the same plane.

➢ A line segment joining the centre of the circle to a point on the circumference of a circle is called its radius.

➢ A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle.

➢ A part of a circumference of circle is called an arc.

➢ A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.

➢ The region bounded by an arc of a circle and two radii at its end points is called a sector.

➢ A chord divides the interior of a circle into two parts, each called a segment.

➢ An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.

➢ An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.

➢ Circles having the same centre but different radii are called concentric circles.

- Two circles (or arcs) are said to be congruent if on placing one over the other such that they cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.
- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

  **1.** Circumference (perimeter) of a circle = $\pi d$ or $2\pi r$, where $d$ is diameter and $r$ is the radius of the circle.

  **2.** Area of a circle = $\pi r^2$.

  **3.** Area of a semi-circle = $\frac{1}{2}\pi r^2$.

  **4.** Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)r$

  **5.** Area of a ring or an annulus = $\pi(R + r)(R - r)$. Where R is the outer radius.

- Length of arc $AB = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$. where q is the angle subtend at centre by the arc.
- Area of a sector = $\frac{\pi r^2\theta}{360°}$ or Area of sector = $\frac{1}{2}(l \times r)$. where $l$ is the length of arc.
- Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2\sin\theta$.
- Area of major segment = Area of the circle – Area of minor segment = $\pi r^2$ – Area of minor segment.
- If a chord subtend a right angle at the centre, then

  Area of the corresponding segment = $\left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$
- If a chord subtend an angle of 60° at the centre, then

  Area of the corresponding segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$.
- If a chord subtend an angle of 120° at the centre, then
- Area of the corresponding segment = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$.
- Distance moved by a wheel in 1 revolution = Circumference of the wheel.
- Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.
- Perimeter of a sector = $\frac{\pi r\theta}{180°} + 2r$.

# Chapter - 6 : Construction

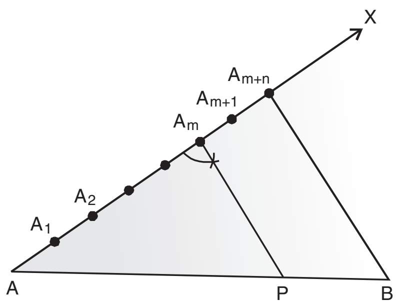
## Quick Review

**Properties of Chord :**

- The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.

  To divide a line segment internally

  **Tally is given in a ratio *m : n*, where both *m* and *n* are positive integers, we follow the following steps :**



**Step 1.** Draw a line segment *AB* of given length by using a ruler.

**Step 2.** Draw any ray *AX* making an acute angle with *AB*.

**Step 3.** Along *AX* mark off $(m + n)$ points $A_1, = A_2, \ldots\ldots, A_m, A_{m+1}, \ldots\ldots, A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1} A_{m+n}$.

**Step 4.** Join $BA_{m+n}$.

**Step 5.** Through the point $A_m$ draw a line parallel to $A_{m+n} B$ by making an angle equal to $\angle AA_{m+n} B$ at $A_m$. *i.e.*, $\angle AA_mP$.

This line meets *AB* at point *P*.

The point *P* is the required point which divides *AB* internally in the ratio $m : n$.

➢ **Construction of triangles similar to a given triangle :**

**(a) Steps of constructions, when $m < n$ :**

**Step I.** Construct the given triangle *ABC* by using the given data.

**Step II.** Take any one of the three sides of the given triangle as base. Let *AB* be the base of the given triangle.

**Step III.** At one end, say *A*, of base *AB*. Construct an acute $\angle BAX$ below the base *AB*.

**Step IV.** Along *AX* mark off *n* points $A_1, A_2, A_3, \ldots, A_n$ such that $AA_1 = A_1A_2 = \ldots\ldots = A_{n-1} A_n$.

**Step V.** Join $A_nB$.

**Step VI.** Draw $A_mB'$ parallel to $A_nB$ which meets *AB* at $B'$.

**Step VII.** From $B'$ draw $B'C' \parallel CB$ meeting *AC* at $C'$.

Triangle $AB'C'$ is the required triangle each of whose sides is $\left(\dfrac{m}{n}\right)^{th}$ of the corresponding side of $\Delta ABC$.

**(b) Steps of construction, when $m > n$ :**

**Step I.** Construct the given triangle by using the given data.

**Step II.** Take any one of the three sides of the given triangle and consider it as the base. Let *AB* be the base of the given triangle.

**Step III.** At one end, say *A*, of base *AB*. Construct an acute angle $\angle BAX$ below the base *AB* i.e., on the opposite side of the vertex *C*.

**Step IV.** Along *AX* mark off *m* (larger of m and *n*) points $A_1, A_2, A_3, \ldots\ldots A_m$ such that $AA_1 = A_1A_2 = \ldots\ldots = A_{m-1}A_m$.

**Step V.** Join $A_nB$ to *B* and draw a line through $A_m$ parallel to $A_nB$, intersecting the extended line segment *AB* at $B'$.

**Step VI.** Draw a line through $B'$ parallel to *BC* intersecting the extended line segment *AC* at $C'$.

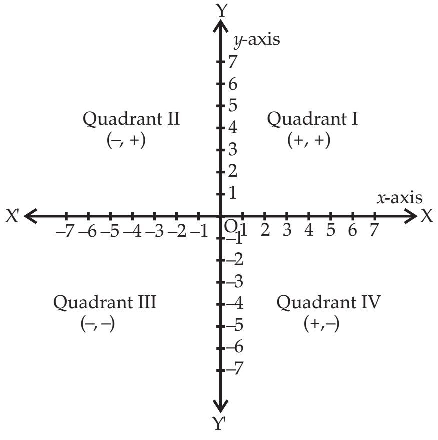
$\Delta AB'C'$ so obtained is the required triangle.

❑❑

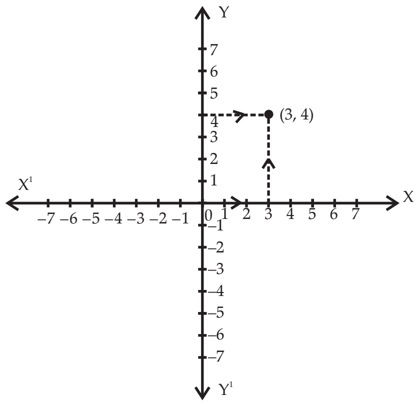
# Chapter - 7 : Co-ordinate Geometry

## Quick Review

➢ Two perpendicular number lines intersecting at point zero are called co-ordinate axes. The horizontal number line is the x-axis (denoted by X′OX) and the vertical one is the y-axis (denoted by Y′OY).



- The point of intersection of x-axis and y-axis is called origin and denoted by 'O'.
- Cartesian plane is plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate plane or xy-plane.
- The x-co-ordinate (or abscissa) of a point is its perpendicular distance from y-axis.
- The y-co-ordinate (or ordinate) of a point is its perpendicular distance from x-axis.
- The point where the x-axis and the y-axis intersect is co-ordinate points (0, 0).
- If the abscissa of a point is x and the ordinate of the point is y, then (x, y) are called the co-ordinates of the point.
- The axis divide the cartesian plane into four parts called the quadrants (one-fourth part), numbered I, II, III and IV anticlockwise from OX.
- The co-ordinates of a point on the x-axis are of the form (x, 0) and that of the point on y-axis are (0, y).
- Sign of co-ordinates depicts the quadrant in which it lies. The co-ordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.
- Three points A, B and C are collinear if the distances AB, BC, CA are such that the sum of two distances is equal to the third.
- Three points A, B and C are the vertices of an equilateral triangle if the distances AB = BC = CA.
- Three points A, B and C are collinear if the area of DABC is zero.
- The points A, B and C are the vertices of an isosceles triangle if the distances AB = BC or BC = CA or CA = AB.
- Three points A, B and C are the vertices of a right triangle, if $AB^2 + BC^2 = CA^2$.
- **For the given four points A, B, C and D :**
  **(i)** AB = BC = CD = DA; AC = BD $\Rightarrow$ ABCD is a square.
  **(ii)** AB = BC = CD = DA; AC $\neq$ BD $\Rightarrow$ ABCD is a rhombus.
  **(iii)** AB = CD, BC = DA; AC = BD $\Rightarrow$ ABCD is a rectangle.
  **(iv)** AB = CD, BC = DA; AC $\neq$ BD $\Rightarrow$ ABCD is a parallelogram.
- Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.
- Diagonals of rhombus and square bisect each other at right angle.
- Four given points are collinear, if the area of quadrilateral is zero.
- If $x \neq y$, then (x, y) $\neq$ (y, x) and if (x, y) = (y, x), then x = y.
- The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .
- The distance of a point P(x, y) from origin is $\sqrt{x^2 + y^2}$ .
- **To plot a point P(3, 4) in the cartesian plane. Draw :**
  **(i)** a distance of 3 units along X-axis.
  **(ii)** a distance of 4 units along Y-axis.




- Co-ordinates of a point which divides the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ in the ratio m : n internally are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

- Co-ordinates of mid-point of the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- The ratio of the vertical distance to the horizontal distance is called slope.

$$\text{Slope} = \frac{\text{Vertical distance}}{\text{Horizontal distance}}$$

- The slope of a line is the tangent of the angle of its inclination. It is generally denoted by m.

or, $$m = \tan\theta$$

- **(i)** The x-axis subtends an angle 0° with itself. Hence, slope of $x$-axis is $m = \tan 0° = 0$.

  **(ii)** The y-axis subtends an angle 90° with the $x$-axis. Hence, slope of $y$-axis is $m = \tan 90°$ = not defined.

  **(iii)**

| Value of angle of inclination | Value of slope |
|---|---|
| $\theta = 0°$ | 0 |
| $0° < \theta < 90°$ | Positive number |
| $\theta = 90°$ | Not defined |
| $90° < \theta < 180°$ | Negative number |

- If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two fixed points on a cartesian plane, then slope of a straight line passing through A and B is given by,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Parallel lines have equal slopes i.e. $m_1 = m_2$
- Slope of x-axis = 0
- Slope of y-axis is not defined.
- If two lines are mutually perpendicular, then the product of their slopes is –1. i.e. $m_1 m_2 = -1$
- Three points A, B and C are collinear if slope of AB = slope of AC.
- The equation of a line with slope m and whose y-intecept is 'c' is given by

$$y = mx + c.$$

❑❑

# Chapter - 8 : Real Numbers

## Quick Review

- **Algorithm :** An algorithm means a series of well defined steps which gives a procedure for solving a type of problem.
- **Lemma :** A lemma is a proven statement used for proving another statement.
- **Euclid's Division Lemma :** For given positive integers a and b, there exist unique integers q and r, satisfying

$$a = bq + r, \quad 0 \le r < b.$$

  **The steps to find the HCF of two positive integers by Euclid's division algorithm are given below :**

  **(i)** Let two integers are a and b such that a > b.

  **(ii)** Take greater number a as dividend and the number b as divisor.

  **(iii)** Now, find whole numbers '$q$' and '$r$' as quotient and remainder respectively.

$$a = bq + r, \ 0 \le r < b.$$

  **(iv)** If r = 0, b is the HCF of a and b. If $r \neq 0$, then take r as divisor and b as dividend.

  **(v)** Repeat the step (iii), till the remainder is zero, the divisor thus obtained at this stage is the required HCF.

- **I.** We state Euclid division/Algorithm for positive integers only it can be extended for all integers except zero, $b \neq 0$.

  **II.** When 'a' and 'b' are two positive integers such that

$$a = bq + r, 0 \le r < b, \text{ then HCF } (a, b) = \text{HCF } (b, r).$$

- **Fundamental Theorem of Arithmetic :** Every composite number can be expresed as a product of primes and this decomposition is unique, apart from the order in which the prime factors occur. This method is also called prime factorization or canonical factorization method.

- **Prime Factorization Method to find HCF and LCM :**
  **(i)** First find all the prime factors of given numbers.
  **(ii)** The product of least power of each common factor among all the prime factors is the required HCF and the product of greatest power of each prime factor in the number is the required LCM.
- **If p and q are two positive integers, then :**

$$\text{HCF}(p, q) \times \text{LCM}(p, q) = p \times q.$$

  Let p be a prime number. If p divides $a^2$, then p divides a, where a is a positive integer.
- **Rational Number :** The number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is known as rational number.
- **Irrational Number :** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, p are irrational numbers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n and m are non negative integers. Then x has a decimal expansion which terminates after k places of decimals, where k is the largest of m and n.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n and m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.
- The sum or difference of a rational and irrational number is irrational.
- The product and quotient of a non–zero rational and irrational number is irrational.
- The product of three numbers is not equal to the product of their HCF and LCM, i.e.,

$$\text{HCF}(p, q, r) \times \text{LCM}(p, q, r) \neq p \times q \times r,$$

  where p, q, r are positive integers.
  However, the following results hold good for three numbers p, q and r :

$$\text{LCM}(p, q, r) = \frac{p \times q \times r \times \text{HCF}(p,q,r)}{\text{HCF}(p,q) \times \text{HCF}(q,r) \times \text{HCF}(p,r)}$$

$$\text{HCF}(p, q, r) = \frac{p \times q \times r \times \text{LCM}(p,q,r)}{\text{LCM}(p,q) \times \text{LCM}(q,r) \times \text{LCM}(p,r)}$$

- **Prime number :** A positive integer 'p' is considered a prime number, if
  **(i)** $p > 1$ and
  **(ii)** p does not have factors other than 1 and p.
- **Composite Number :** A number greater than 1 and not a prime number is a Composite number.
- **Co-primes :** Two numbers 'a' and 'b' are said to be co-prime if the only common divisor of a and b is 1.
- If a prime number p divides $a^2$, then p divides a, where a is a positive integer.
- If a prime number divides the product of two integers, then it divides at least one of them.

❑❑

# Chapter - 9 : Polynomials

## Quick Review

- An algebraic expression in which the variables involved have only non-negative integral exponents are called Polynomials.
- A polynomial p (x) in one variable x is an algebraic expression in x of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_2 x^2 + a_1 x + a_0,$$

  where $a_0, a_1, a_2, \ldots\ldots\ldots$an are constants and $a_n \neq 0$.
  $a_0, a_1, a_2, \ldots\ldots$ an are respectively the coefficients of $x_0, x_1, x_2, \ldots\ldots\ldots x^n$ and $n$ is called the degree of the polynomial.
- This form of polynomial is known as "Standard Form of Polynomial."
- Highest power of variable in a polynomial is the degree of polynomial.
- A polynomial of one term is called a monomial.
- A polynomial of two terms is called a binomial.
- A polynomial of three terms is called a trinomial.
- A polynomial of degree zero is called a constant polynomial.

- A polynomial of degree two is called a quadratic polynomial.
- A polynomial of degree three is called a cubic polynomial.
- A polynomial having all coefficients zero is called a zero polynomial. Also, the constant polynomial '0' is called the zero polynomial and the degree of the zero polynomial is not defined.
- **Value of a Polynomial :** At $x = a$, the value of polynomial $p(x)$ is $p(a)$.
- **Zero of a Polynomial :** Zero of a polynomial p(x) is as a number a such that p(a) = 0.
  **(i)** '0' may be a zero of a polynomial.
  **(ii)** Every linear polynomial in one variable has a unique zero.
  **(iii)** Maximum number of zeroes of a polynomial is equal to its degree.
  **(iv)** A non-zero constant polynomial has no zero.
  **(v)** Every real number is a zero of the zero polynomial.
  If $p(x)$ is a polynomial, then $p(x) = 0$ is a polynomial equation.
- **Remainder Theorem :** If $p(x)$ be any polynomial of degree greater than or equal to 1 and a be any real number, then If $p(x)$ is divided by the linear polynomial $x - a$, the remainder is $p(a)$
  Dividend = (Divisor × Quotient) + Remainder
  **(i)** It is important to observe that the remainder theorem hold good only when the divisor is a binomial and in the linear form $(x - a)$
  **(ii)** If $p(x)$ is divided by $(x + a)$, then the remainder is $p(-a)$.
  **(iii)** If $p(x)$ is divided by $(ax + b)$, then the remainder is .
- **Factor Theorem :** If $p(x)$ is a polynomial of degree $x \geq 1$ and a is any real number, then
  **(i)** $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$
  **(ii)** $p(a) = 0$, if $(x - a)$ is factor of $p(x)$.
  **Note :**
  **(i)** $(x + a)$ is a factor of polynomial $p(x)$, if $p(-a) = 0$.
  **(ii)** $(ax - b)$ is a factor of polynomial $p(x)$, if = 0.
  **(iii)** $(x - a)(x - b)$ is a factor of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$.
- **Algebraic Identities :**
  **(i)** $(x + y)^2 = x^2 + 2xy + y^2$
  **(ii)** $(x - y)^2 = x^2 - 2xy + y^2$
  **(iii)** $x^2 - y^2 = (x + y)(x - y)$
  **(iv)** $(x + a)(x + b) = x^2 + (a + b)x + ab$
  **(v)** $(x + y + z)2 = x2 + y2 + z2 + 2xy + 2yz + 2zx$
  **(vi)** $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
  **(vii)** $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
  **(viii)** $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
  **(ix)** If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
  **(x)** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
  **(xi)** $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- **Division Algorithm for Polynomials :** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that
  $$p(x) = g(x) \times q(x) + r(x),$$
  where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.
  **Note :** (i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.

❑❑

# Chapter - 10 : Quadratic Equations

## Quick Review

- A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where $a, b, c$ are real numbers and $a \neq 0$.
- The value of x that satisfies an equation is called the solution or root of the equation.
- Any quadratic equation can be converted to the form $(x + a)^2 - b^2 = 0$ by adding and subtracting some term. This method of finding the roots of quadratic equation is called the method of completing the square.
- For the equation $ax^2 + bx + c = 0$, $a \neq 0$, expression $b^2 - 4ac$ is known as discriminant.

- A real number $\alpha$ is said to be a solution/root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + ba + c = 0$.
- **A quadratic equation can be solved by following algebraic methods :**
  **(i)** Splitting the middle term **(ii)** Completing squares
  **(iii)** Quadratic formula
- **Method of splitting the middle term of the equation $ax^2 + bx + c = 0$, where $a \neq 0$ :**
  **(i)** Form the product *"ac"*
  **(ii)** Find a pair of numbers $b_1$ and $b_2$ whose product is *"ac"* and whose sum is *"b"* (if you can't find such numbers, it can't be factored).
  **(iii)** Split the middle term using $b_1$ and $b_2$, that express the term $bx$ as $b_1x + b_2x$. Now factor by grouping pairs of terms.
- Roots of the quadratic equation can be found by equating each linear factor to zero, product of two numbers is zero, if either or both of them are zero.
- Method of completing the square of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

  **(i)** Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

  **(ii)** Multiplying and dividing the coefficient of x by 2

$$x^2 + 2\frac{b}{2a}x + \frac{c}{a} = 0$$

  **(iii)** Adding and subtracting $\frac{b^2}{4a^2}$

$$x^2 + 2.\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \qquad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

  If $\sqrt{b^2 - 4ac} > 0$, then by taking square root of both sides, we have

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

  $\therefore x =$ is known as quadratic formula.

  This method is also called Sridharacharya's method.
- **Nature of the roots of a quadratic equation :**
- The nature of roots depends on the values of $b^2 - 4ac$, which is called discriminant of the quadratic equation and symbolically denoted by delta (D).
  **(i)** If $\Delta = b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  **(ii)** If $\Delta = b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  **(iii)** If $\Delta = b^2 - 4ac < 0$, the quadratic equation has no real roots (imaginary roots).
- Roots of $ax^2 + bx + c = 0$, $a \neq 0$ are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$, where $\sqrt{b^2 - 4ac} > 0$
- Roots of $ax^2 + bx + c = 0$, $a \neq 0$ are $\frac{-b}{2a}$ and $\frac{-b}{2a}$, where $\sqrt{b^2 - 4ac} = 0$.
- If $m$ and $n$ are the roots of the quadratic equation $ax^2 + bx + c = 0$.

$$\Rightarrow \qquad \text{Sum of the roots} = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2} \qquad \Rightarrow m + n = -\frac{b}{a}$$

$$\text{and product of roots} = \frac{\text{Constant term}}{\text{Coeff. of } x^2}. \qquad \Rightarrow mn = \frac{c}{a}$$

- If roots $m$ and $n$ are given, then the standard form of the equation is $x^2$ – (sum of the roots) $x$ + product of the roots $= 0$

  i.e., $x^2 - (m + n)x + mn = 0$

❑❑

# Chapter - 11 : Introduction to Trigonometry

## Quick Review

- An equation is called an identity if it is true for all values of the variables(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity it is true for all values of the angle.

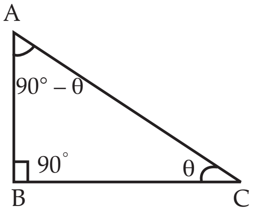
  We can see from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

  Also, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of the trigonometric ratio of an angle does not depend on the size of the triangle. It only depends on the angle.
- **Trigonometric Ratios of Some Specific Angles :**

| θ | **sin θ** | **cos θ** | **tan θ** | **cosec θ** | **sec θ** | **cos θ** |
|---|---|---|---|---|---|---|
| 0° | 0 | 1 | 0 | $\infty$ | 1 | $\infty$ |
| 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ | 2 | $\frac{2}{\sqrt{3}}$ | $\sqrt{3}$ |
| 45° | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |
| 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{2}{\sqrt{3}}$ | 2 | $\frac{1}{\sqrt{3}}$ |
| 90° | 1 | 0 | $\infty$ | 1 | $\infty$ | 0 |

- **Complementary Angles :**
- Two angles are said to be complementary, if their sum is 90°. Thus (in fig.), ∠A and ∠C are complementary angles.




- **Trigonometric Ratios of Complementary Angles :**

  We have, BC = Base, AB = Perpendicular,

  AC = Hypotenuse, with respect to angle ∠C = q

$$\sin\theta = \frac{AB}{AC},\ \cos\theta = \frac{BC}{AC},\ \tan\theta = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{AC}{AB},\ \sec\theta = \frac{AC}{BC},\ \cot\theta = \frac{BC}{AB}$$

  Agian, BC = Perpendicular, AB = Base

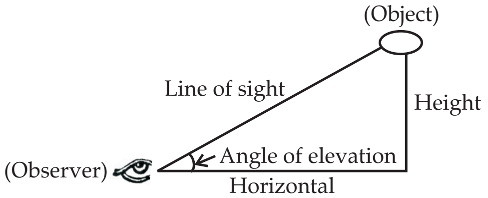
  AC = Hypotenuse, with respect to ∠A = (90° – θ)

$$\sin(90° - \theta) = \frac{BC}{AC} = \cos\theta \quad \text{cosec}(90° - \theta) = \frac{AC}{BC} = \sec\theta$$

$$\cos(90° - \theta) = \frac{AB}{AC} = \sin\theta \quad \sec(90° - \theta) = \frac{AC}{AB} = \text{cosec}\,\theta$$

$$\tan(90° - \theta) = \frac{BC}{AB} = \cot\theta \quad \cot(90° - \theta) = \frac{AB}{BC} = \tan\theta$$

- **Trigonometric Identities :**
  - **(i)** $\sin^2\theta + \cos^2\theta = 1$; $\sin^2\theta = 1 - \cos^2\theta$ ; $\cos^2\theta = 1 - \sin^2\theta$, for $0° \le \theta \le 90°$
  - **(ii)** $1 + \tan^2\theta = \sec^2\theta$; $\sec^2\theta - 1 = \tan^2\theta$; $\sec^2\theta - \tan^2\theta = 1$, for $0° \le \theta < 90°$
  - **(iii)** $1 + \cot^2\theta = \text{cosec}^2\theta$; $\text{cosec}^2\theta - 1 = \cot^2\theta$; $\text{cosec}^2\theta - \cot^2\theta = 1$, for $0° \le \theta < 90°$
- The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.
- The **angle of elevation** of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

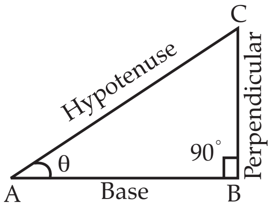



❑❑

# Chapter - 12 : Some Application of trigonometry

## Quick Review

- In fig., a right angled triangle $\angle ABC$ is given. $\angle ABC = 90°$ and $\angle BAC = \theta$, is an acute angle. Here side AB which is adjacent to $\angle A$ is base, side BC opposite to $\angle A$ is perpendicular and the side AC is hypotenuse which is opposite to right angle B.




The trigonometric ratios of $\angle A$ in right angled triangle ABC is defined as

sine of $\angle A$ = $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AC}$

cosine of $\angle A$ = $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

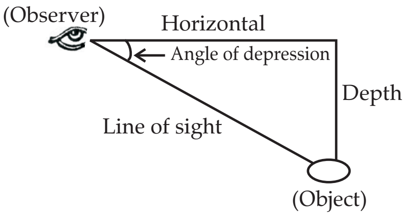
tangent of $\angle A$ = $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB}$

cosecant of $\angle A$ = $\text{cosec}\,\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{AC}{BC} = \dfrac{1}{\sin\theta}$

secant of $\angle A$ = $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB} = \dfrac{1}{\cos\theta}$

cotangent of $\angle A$ = $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{AB}{BC} = \dfrac{1}{\tan\theta}$

The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.




- **Remember : (i)** Angle of elevation and angle of depression are always measured with the horizontal.
  **(ii)** The angle of elevation of an object as seen by the observer is same as the angle of depression of the observe as seen from the object.
  **(iii)** If the height of the observer is not given, then the observer is taken as a point.
- The height or length of an object or the distance between two distinct objects can be determined by the help of trigonometric ratios.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.
- The two heights above and below the ground level in case of reflection from the water surface are equal.

❑❑

# Chapter - 13 : Statistics

## Quick Review

- Statistics deals with the collection, presentation and analysis of numerical data.
- **Three measures of central tendency are :**
  **(i)** Mean, **(ii)** Median and
  **(iii)** Mode
- **Mean :** The mean of n quantities $x_1, x_2, x_3, \ldots\ldots, x_n$

$$= \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \text{S}\,\frac{x_i}{n}$$

where, the Greek letter Σ(sigma) means 'Summation of'.
- **Median :** It is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or descending order of their magnitudes.
- It divides the arranged series in two equal parts i.e., 50 % of the observations lie below the median and the remaining are above the median.
- **Mode :** The item which occurs most frequently i.e., maximum number of times is called mode.
- **Mean :**
  **(a) For Raw Data :**
  If n observations $x_1, x_2, \ldots, x_n$ are given, then their arithmetic mean is given by :

$$x = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  **(b) For Ungrouped Data :**
- If there are n distinct observations $x_1, x_2, \ldots, x_n$ of variable x with frequencies $f_1, f_2, \ldots, f_n$ respectively, then the arithmetic mean is given by :

$$x = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

**For Grouped Data :**

**(i)** To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point.

**(ii) Direct Method :**

$$\text{Mean } (x) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where the $x_i$ (class mark) is the mid-point of the $i^{th}$ class interval and $f_i$ is the corresponding frequency.

**(iii) Assumed Mean Method or Short-cut Method :**

$$\text{Mean } (x) = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$$

where a is the assumed mean and $d_i = x_i - a$ are the deviations of $x_i$ from a for each i.

**(iv) Step-Deviation Method :**

$$\text{Mean } (x) = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right),$$

where, a is the assumed mean, h is the class-size and $u_i = \frac{x_i - a}{h}$.

- **Median :**

**(a) For Ungrouped Data :**

If n is odd, $\text{Median} = \left(\frac{n+1}{2}\right)^{th}$ term

If n is even, $\text{Median} = \dfrac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n+1}{2}\right)^{th} \text{term}}{2}$

**(b) For Grouped Data :**

- Let $n = f_1 + f_2 + f_3 + ... + f_n$. First of all find $\frac{n}{2}$ and then the class in which $\frac{n}{2}$ lies. This class is known as the median class. Median of the given distribution lies in this class.
- **Median of the grouped data can be calculated using the formula :**

$$\text{Median (Me)} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h,$$

where, l = lower limit of median class, f = frequency of median class, n = number of observations, c.f. = cumulative frequency of the class preceding the median class, h = class-size or width of the class-interval.

- **Mode of Grouped Data :**

**(i)** Mode is the observation which occurred maximum times. In ungrouped data, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the modal class. The mode of the data is a value inside the modal class.

**(ii)** Mode of the grouped data can be calculated by using the formula :

$$\text{Mode (M)} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$

where, l = lower limit of the modal class, h = width or size of the class-interval, $f_1$ = frequency of the modal class, $f_0$ = frequency of the class preceding the modal class, $f_2$ = frequency of the class succeeding the modal class.

**Note :**

**(a)** If the series has only one mode, then it is known as Unimodal.

**(b)** If the series has two modes, then it is known as Bimodal.

**(c)** If the series has three modes, then it is known as Trimodal.

**(d)** Mode may or may not be defined for a given series.

- **Empirical Relation between Mean, Median and Mode :**

**(i)** Mode = 3 Median – 2 Mean

**(ii)** Median = $\frac{1}{3}$ Mode + $\frac{2}{3}$ Mean

**(iii)** Mean = $\frac{3}{2}$ Median – $\frac{1}{2}$ Mode

**Note :** For calculating the mode and median for grouped data, it should be ensured that the class-intervals are continuous before applying the formula. Same condition also apply for construction of an ogive. Further, in case of ogives, the scale may not be the same on both the axes.

- **Cumulative Frequency Distribution :**

**(i)** Cumulative frequency of a particular value of the variable (or class) is the sum (total) of all the frequencies up to than value (or class).

**(ii)** There are two types of cumulative frequency distributions :

**(a)** Cumulative frequency distribution of less than type.

**(b)** Cumulative frequency distribution of more than type.

➢ **For example :**

| Class interval (marks) | Frequency (No. of Students) | Cumulative frequency (c.f.) | Less than type | | More than type | |
|---|---|---|---|---|---|---|
| | | | Marks Out of 50 | c.f. less than type | Marks Out of 50 | c.f. less than type |
| 0 – 10 | 2 | 2 | Less than 10 | 2 = 2 | 0 or More than 0 | 60 = 60 |
| 10 – 20 | 10 | 12 | Less than 20 | 2 + 10 = 12 | More than 10 | 60 – 2 = 58 |
| 20 – 30 | 25 | 37 | Less than 30 | 12 + 25 = 37 | More than 20 | 58 – 10 = 48 |
| 30 – 40 | 20 | 57 | Less than 40 | 37 + 20 = 57 | More than 30 | 48 – 25 = 23 |
| 40 – 50 | 3 | 60 | Less than 50 | 57 + 3 = 60 | More than 40 | 23 – 20 = 3 |

**Cumulative frequency curve or an Ogive curve :** The graphical representation of a cumulative frequency distribution is called the cumulative frequency curve or ogive.

There are two methods to construct ogives :

1. **Less than ogive :**

   In this method, an ogive is cumulated upward. Scale the cumulative frequencies along the y-axis and exact upper limits along the x-axis. The scale along the y-axis should be such as may accommodate the total frequency.

   **Step I.** Form the cumulative frequency table.

   **Step II.** Mark the actual upper class limits along the x-axis.

   **Step III.** Mark the cumulative frequency of the respective classes along the y-axis.

   **Step IV.** Plot the points (upper limits, corresponding cumulative frequency.)

   By joining these points on the graph by a free hand curve, we get an ogive of 'less than' type.

2. **More than ogive :**

   In this method, an ogive is cumulated downward. Scale the cumulative frequencies along the y-axis and the exact lower limits along the x-axis.

   **Step I.** Scale the cumulative frequencies along the y-axis and the actual lower limits along the x–axis.

   **Step II.** Plot the ordered pairs (lower limit, corresponding cumulative frequency). To complete an ogive, we also plot the ordered pair (upper limit of the highest class, 0).

   **Step III.** Join these plotted points by a smooth curve. The curve so obtained is the required 'more than' type ogive.

3. **Median :**

   Ogive can be used to estimate the median of the data. There are two methods to get the median :

   **(i)** Mark a point corresponding to $\frac{N}{2}$, where $N$ is the total frequency on cumulative frequency axis (*i.e.*, $y$-axis).

   Draw a line parallel to $x$-axis to cut the ogive at a point. From this point draw a line perpendicular to the $x$-axis to get another point. The abscissa of this point gives median.

   **(ii)** Draw both the ogives (less than and more than ogive) on the same graph paper which cut each other at a point. From this point draw a line perpendicular to the $x$-axis, to get another point. The point at which it cuts $x$-axis, gives the median.

❑❑

# Chapter - 14 : Probability

## Quick Review

➢ **Coin, Die and Playing Cards**

**(i) Coin :** It has two sides, viz. head and tail. If we have been more than one coin, coins are regarded as distinct if not otherwise stated.

**(ii) Die :** A die has six faces marked 1, 2, 3, 4, 5 and 6. If we have more than one die, all dice are regarded as different, if not otherwise stated.

**(iii) Playing Cards :** Its pack has 52 cards. There are four suits, viz., spade, heart, diamond and club each having 13 cards. There are two colours red (heart and diamond) and black (spade and club) each having 26 cards. In 13 cards of each unit, there are 3 face cards viz., king, queen and jack, so there are in all 12 face cards in a pack of playing cards. Also, there are 16 honours cards, 4 of each suit viz., ace, king, queen and jack.

- **Sample Space :**

  The set of all possible outcomes of a random experiment is called a sample space of that experiment. It is usually denoted by $S$. The elements of $S$ are called sample points.

  If $E_1, E_2, E_3, ........, E_n$ are the possible outcomes (or elementary events) of a random experiment, then $S = \{E_1, E_2, ........, E_n\}$ is the sample sspace associated to it.

  **Random Experiment :** If an experiment conducted repeatedly under the identical conditions does not give necessarily the same result, then the experiment is called random experiment. The result of the experiment is called outcome.

  **Event :** A subset of the sample space associated with a random experiment is called an event.

- **Different Types of Events :**

  **(i) Simple Event :** If an event has only one sample point of the sample space, then it is called a simple (or elementary) event.

  **(ii) Compound Event :** When an event is composed of a number of simple events, then it is called a compound event.

  **(iii) Null Event :** An event having no sample point is called null event. It is denoted by $\phi$. It is also known as impossible event.

  **(iv) Sure Event :** The event which is certain to occur is said to be the sure event.

  **(v) Equally Likely Events :** Events are called equally likely, when we do expect the happening of one event in preference to the other.

  **(vi) Mutually Exclusive Events :** A set of events is said to be mutually exclusive, if the happening of one excludes the happening of the other *i.e.*, $A \cap B = \phi$.

  **(vii) Exhaustive Events :** A set of events is said to be exhaustive, if the performance of the experiment always results in the occurrence of at least one of them.

‹ If $A$ is an event of an experiment whose sample space is $S$, then its probability $P(a)$ is given by

$$P(A) = \frac{(n(A)}{n(S)} = \frac{\text{Number of favourable cases}}{\text{Total number of exhaustive cases}}$$

- For an event $E$, $P(\overline{E}) = 1 - P(E)$, where the event $\overline{E}$ representing 'not $E$' is the complement of the event $E$.
- For $A$ and $B$ two possible outcomes of an event,

  **(i)** If $P(a) > P(b)$, then event $A$ is more likely to occur than event $B$.

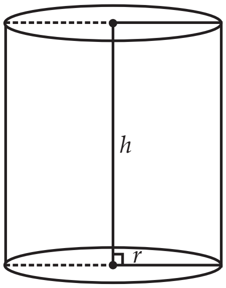
  **(ii)** If $P(a) = P(b)$, then events $A$ and $B$ are equally likely to occur.
- Probability is a quantitative measure of certainity.
- Any activity associated to certain outcome is called a random experiment, *e.g.*, (i) tossing a coin. (ii) throwing a dice, (iii) selecting a card.
- Outcome associated with an experiment is called an event, *e.g.*, (i) getting a head on tossing a coin. (ii) getting a face card when a card is drawn from a pack of 52 cards.
- The event whose probability is one is called as sure/certain event.
- The event with only one possible outcome is called an elementary event.
- In a given experiment, if two or more events are equally likely to occur or if they have equal probabilities, then they are called equally likely events.
- Probability of an event lies between 0 and 1.
- Probability can never be negative.
- A pack of playing cards consist of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits
- are spades, hearts, diamonds and clubs.
- The sum of the probabilities of all elementary events of an experiment is 1.
- Two events $A$ and $B$ are said to be complements of each other if the sum of their probabilities is 1.

❑❑

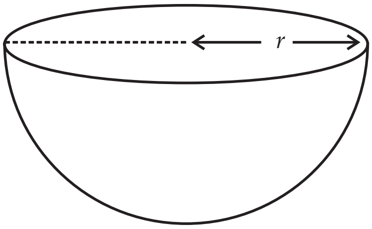
# Chapter - 15 : Surface Area and Volumes

## Quick Review

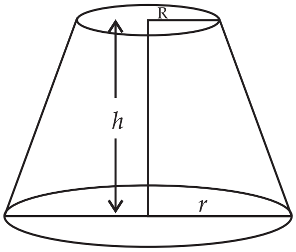
- A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.




- A hemisphere is half of a sphere.




- If a right circular cone is cut off by a plane parallel to its base, then the portion of the cone between the plane and the base of the cone is called a frustum of the cone.




- Cylinders are sort of prisms with circular bases.
- If a rectangle revolves about one of its sides and completes a full rotation, the solid formed is called a right circular cylinder.
- **In the cylinder :**
  **(i)** It has two congruent and parallel circular bases.
  **(ii)** It has a curved surface joining the edges of the two bases.
  **(iii)** The line segment joining the centres of the two bases is perpendicular to base. It is know as the height of the cylinder and also called the axis of the cylinder.
- A cone is a solid object that tapers smoothly from a flat circular base to a point is called vertex.
- If a right angled triangle is revolved about one of the side containing the right triangle, the solid formed is called a right circular cone.
- If a plane cuts a right circular cone parallel to its base and the upper smaller cone is removed, then the remaining part of the cone containing the base is called frustum of the cone.
- If a circular disc is rotated about one of its diameters, the solid thus generated is called a sphere.
- When a solid is melted and converted to another solid, volume of both the solids remains the same, assuming there is no wastage in the conversion. The surface area of the two solids may or may not be the same.
- The solids having the same curved surface do not necessarily have the same volume.
- **Right circular cylinder :**

Area of base or top face $= \pi r^2$

Area of curved surface or lateral surface area $=$ perimeter of the base $\times$ height $= 2\pi rh$

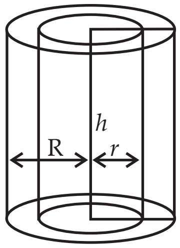
Total surface area (including both ends) $= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

Volume $=$ Area of the base $\times$ height $= \pi r^2 h$

Here, r is the radius of base and h is the height.

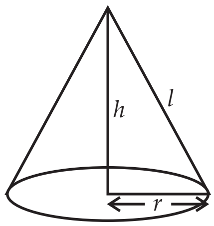
- **Right circular hollow cylinder :**

Total surface area $= (\text{External surface}) + (\text{Internal surface}) + \text{Area of base and top}$
$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$
$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$
$= 2\pi(R + r)(h + R - r)$
Curved surface area $= 2\pi Rh + 2\pi rh = 2\pi h(R + r)$
Volume of the material used $= (\text{External volume}) - (\text{Internal volume})$
$= \pi R^2h - \pi r^2h = \pi h(R^2 - r^2)$




- **Right circular cone :**

Slant height (l) $= \sqrt{h^2 + r^2}$
Area of curved surface $= \pi rl$
$= \pi r\sqrt{h^2 + r^2}$
Total surface area $= \text{Area of curved surface} + \text{Area of base}$
$= \pi rl + \pi r^2$
$= \pi r(l + r)$
Volume $= \frac{1}{3}\pi r^2 h$

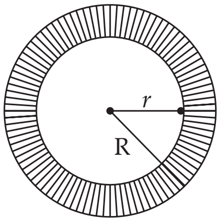



- **Sphere :**

Surface area $= 4\pi r^2$
Volume $= \frac{4}{3}\pi r^3$

- **Spherical shell :**

Surface area (outer) $= 4\pi R^2$
Volume of material $= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$

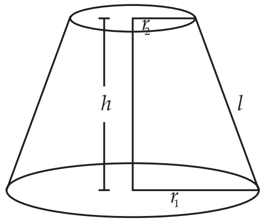



- **Hemisphere :**

Area of curved surface $= 2\pi r^2$
Total surface area $= \text{Area of curved surface} + \text{Area of base}$
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$
Volume $= \frac{2}{3}\pi r^3$

- **Frustum of a cone :**

Total surface area $= \pi l[r_1^2 + r_2^2 + l(r_1 + r_2)]$
Volume of the material $= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$




Curved surface area $= \pi l[l(r_1 + r_2)]$

❑❑