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DIRECTORATE OF MINORITIES

MINORITIES WELFARE DEPARTMENT

MATHEMATICS

S.S.L.C Super Notes: - 2020 – 21

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1. **Surface area and volume: All Formulae**

Cuboid :

Lateral surface area = LSA = 2h (l + b)

Total surface area = TSA = 2 (lb + bh+ lh)

Volume = lbh.

Area of four walls of a room = 2h (l + b)

Diagonals of cuboid = $\sqrt{l^2 + b^2 + h^2}$

Cube :

Lateral surface area = LSA = $4a^2$

Total surface area = TSA = $6a^2$

Volume = a^3 (a is edge of cube)

Diagonal of cube = $\sqrt{3}$ a.

Cylinder :

Right circular cylinder

LSA (or) CSA = 2πrh

TSA = 2πrh + $2\pi r^2$ (or)

TSA = 2πr (r + h)

Volume = $\pi r^2 h$.

Hollow cylinder.

Thickness of cylinder = R –r.

Area of cross section = π $(R^2 - r^2)$

External CSA = 2πRh

Internal CSA = 2πrh.

TSA = External CSA + Internal CSA +area of two ends.

= 2πRh + 2πrh + 2π $(R^2 - r^2)$

Volume = π $(R^2 - r^2)$ h.

Right circular cone :

CSA (or) LSA = πrl

TSA = πr (r + l)

Volume = $\frac{1}{3}$ π r2 h

Slant height = $\sqrt{h^2 + r^2}$.

Frustum of a cone :

Slant height = $\sqrt{h^2 + (R - r)^2}$..

LSA = π (R + r) l.

TSA = π [R^2 + r^2 + (R + r) l]

Volume = $\frac{1}{3}$ πh [R^2 + r^2 + Rr].

Sphere:

CSA = $4\pi r^2$

TSA = $4\pi r^2$

Volume = $\frac{4}{3}$ πr^3.

Hemisphere:

CSA = $2\pi r^2$

TSA = $3\pi r^2$

Volume = $\frac{2}{3}$ πr^3.

2. Arithmetic Progression: nth terms of A.P

$$a_n = a+(n-1)d$$

1. Find the 20^{th} term from the last term of the AP: 3, 8, 13, …, 253.
   Solution: We have, last term = 1 = 253
   And, common difference d = 2^{nd} term – 1^{st} term = 8 – 3 = 5
   Therefore, 20^{th} term from end = 1 -(20 – 1) × d = 253 – 19 × 5 = 253 – 95 = 158.
2. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.
   Solution:
   Natural numbers between 101 and 999 divisible by both 2 and 5 are 110, 120, … 990.
   so, $a_1 = 110$, $d = 10$, $a_n = 990$
   We know, $a_n = a_1 + (n - 1)d$
   $990 = 110 + (n - 1) 10$
   $(n - 1) = 990–11010$
   $\Rightarrow n = 88 + 1 = 89.$
3. Find how many integers between 200 and 500 are divisible by 8.
   Solution:
   AP formed is 208, 216, 224, …, 496
   Here, $a_n = 496$, $a = 208$, $d = 8$
   $a_n = a + (n - 1) d$
   $\Rightarrow 208 + (n - 1) x 8 = 496$
   $\Rightarrow 8 (n - 1) = 288$
   $\Rightarrow n - 1 = 36$
   $\Rightarrow n = 37.$
4. How many terms of the AP 18, 16, 14, …. be taken so that their sum is zero?
   Solution:
   Here, $a = 18$, $d = -2$, $s_n = 0$
   Therefore, $n2 [36 + (n - 1) (- 2)] = 0$
   $\Rightarrow n(36 - 2n + 2) = 0$
   $\Rightarrow n(38 - 2n) = 0$
   $\Rightarrow n = 19.$
5. Which term of the AP: 3, 8, 13, 18, … , is 78?
   Solution:
   Let a_n be the required term and we have given AP
   3, 8, 13, 18, …..
   Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 78$

Now, $a_n = a + (n - 1)d$

$\Rightarrow 78 = 3 + (n - 1)\ 5$

$\Rightarrow 78 - 3 = (n - 1) \times 5$

$\Rightarrow 75 = (n - 1) \times 5$

$\Rightarrow 755 = n - 1$

$\Rightarrow 15 = n - 1$

$\Rightarrow n = 15 + 1 = 16$

Hence, 16^{th} term of given AP is 78.

Practice:

6. Find the 9th term from the end (towards the first term) of the A.P. 5, 9,13,185.(ans: 153).
7. How many two-digit numbers are divisible by 3?. (ans: 30)
8. Find the middle term of the A.P. 6,13,20,...,216. (Ans;111)
9. Find the 25th term of an arithmetic progression 2, 6, 10, 14, ...(ans: 98)
10. Find the 10th term of arithmetic progression 2, 7, 12 .... using the formula.(ans: 47).

3. Arithmetic Progression: Sum of nth terms.

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \& \quad S_n = \frac{n}{2}[a + l]$$

1. Find the sum of the A.P: 1, 3, 5, …….. 199.
   Solution: a=1, d=2 and last term l=199
   $a_n = a + (n-1)d$
   $\Rightarrow 199 = 1 + (n-1) \times 2$
   $\Rightarrow 2n = 200$
   $n = 100$
   $\therefore sum = \frac{n}{2}[a+l]$
   $= \frac{100}{2}[1+199]$
   $= 10000$

2. Find the sum of the series 51+50+49+----------+21.
   Solution: a=51, d=-1 and last term l=21
   $a_n = a + (n-1)d$
   $\Rightarrow 21 = 51 + (n-1) \times -1$
   $21 = 51 + 1 - n$
   $\Rightarrow n = 52 - 21$
   $n = 31$
   $\therefore sum = \frac{n}{2}[a+l]$
   $= \frac{31}{2}[51+21] = \frac{31}{2}[72]$
   $= 1116$

3. How many terms of the AP 18, 16, 14, …. be taken so that their sum is zero?
   Solution:
   Here, a = 18, d = -2, $s_n = 0$
   Therefore, $\frac{n}{2}[36 + (n - 1)(-2)] = 0$
   $\Rightarrow n(36 - 2n + 2) = 0$
   $\Rightarrow n(38 - 2n) = 0$
   $\Rightarrow n = 19$

4. Find the sum of first 22 terms of an AP in which $\boldsymbol{d} = 7$ and 22^{nd} term is 149.

Solution: Given,
Common difference, $d = 7$
22^{nd} term, $a_{22} = 149$
To find: Sum of first 22 term, S_{22}
By the formula of nth term, we know;
$a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$
$149 = a + 21 \times 7$
$149 = a + 147$
$a = 2$ = First term
Sum of nth term is given by the formula;
$S_n = n/2\ (a + a_n)$
$= 22/2\ (2 + 149)$
$= 11 \times 151$
$= 1661$

5. Find the sum of first 20 natural numbers which are divisible by 4.

Solution: The A.P which are divisible by 4 is 4, 8, 12, ……
Here we have to find a_n. $a=4$, $d=4$

$a_n=a+(n-1d)$
$a_{20}=4+19x4$
$a_{20}=4+76$
$a_{20}=80.$

$\therefore sum=\frac{n}{2}[a+l]$
$=\frac{20}{2}[4+80]$
$=10x84$
$=840.$

Practice :

6. Find the sum of first 50 natural numbers which are divisible by 5.
7. Find the sum of : 1+5+9+------------ up to 25 terms.
8. Find the sum of first 30 terms of the A,P 2, 6, 10, ………
9. How many terms of the A.P. 27,24,21,… should be taken so that their sum is zero?
10. Find the sum of 2+5+8+................ to 20 terms using the formula.

4. Coordinate geometry: Problems on distance formula.

Distance formula = $\sqrt{(x2-x1)^2+(y2-y1)^2}$.

1. Find the distance between the two points (2, 5) & (7, 6).
   Solution: here x_1=2, x_2=7, y_1=5 & y_2=6. Put all the values in the given formula.
   d= $\sqrt{(x2-x1)^2+(y2-y1)^2}$.
   = $\sqrt{(7-2)^2+(6-5)^2}$.
   = $\sqrt{(5)^2+(1)^2}$.
   = $\sqrt{25+1}$.
   =$\sqrt{26}$ sq.units
2. Prove that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.
   Solution:
   Let A (7, 10), B(-2, 5), C(3, -4) be the vertices of a triangle.

$$AB = \sqrt{(-2-7)^2+(5-10)^2}$$
$$= \sqrt{81+25} = \sqrt{106}$$
$$BC = \sqrt{(3+2)^2+(-4-5)^2} = \sqrt{25+81} = \sqrt{106}$$
$$AC = \sqrt{(3-7)^2+(-4-10)^2}$$
$$= \sqrt{16+196} = \sqrt{212}$$
$$AB = BC = \sqrt{106}$$

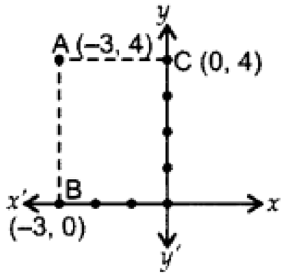
$\therefore$ ABC is an isosceles Δ. ..(i)

$$AB^2 + BC^2 = (\sqrt{106})^2 + (\sqrt{106})^2$$
$$= 106 + 106 = 212 = AC^2$$

   … [By converse of Pythagoras theorem
   ΔABC is an isosceles right angled triangle. …(ii) From (i) & (ii), Points A, B, C are the vertices of an isosceles right triangle.
3. Find that value(s) of x for which the distance between the points P(x, 4) and Q(9, 10) is 10 units. (2011D)
   Solution:
   PQ = 10 …Given
   $PQ^2 = 10^2 = 100$ … [Squaring both sides
   $(9-x)^2 + (10-4)^2 = 100$…(using distance formula
   $(9-x)^2 + 36 = 100$
   $(9-x)^2 = 100 - 36 = 64$
   $(9-x) = \pm 8$ …[Taking square-root on both sides

$9 - x = 8$ or $9 - x = -8$
$9 - 8 = x$ or $9 + 8 = x$
$x = 1$ or $x = 17$

4. Find the distance of the point (-3, 4) from the x-axis.



Solution:
B(-3, 0), A (-3, 4)
Here $x_1=-3$, $x_2=-3$, $y_1=0$ & $y_2=4$. Put all the values in the given formula.
d= $\sqrt{(x2-x1)^2+(y2-y1)^2}$.

$$\mathbf{AB} = \sqrt{(-3+3)^2+(4-0)^2}$$
$$\mathbf{AB} = \sqrt{(4)^2} = \mathbf{4}$$

5. Find distance between the points (0, 5) and (-5, 0).
Solution:
Here $x_1 = 0$, $y_1 = 5$, $x_2 = -5$ and $y_2 = 0$)
$$\therefore \quad d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-5-0)^2+(0-5)^2}$$
$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ units}$$

<u>Practice:</u>

6. Find the distance between the two points(-4, 0) & (0, 3).
7. Find the distance between the points(-3, 4) from its origin.
8. The point A(3, y) is equidistant from the points P(6, 5) and Q(0, -3). Find the value of y.
9. Find the distance between the points A(3, 6) and B(5, 7) using distance formula.
10. Find the distance between the co-ordinate of the points A(2, 3) and B(10, -3).

5. Quadratic equations: Formula method.

Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

1. Solve by using quadratic formula: x^2 -3x+1=0.
   Solution: a=1, b=-3, c=1
   Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   $$x=\frac{-(-3)\pm\sqrt{(-3)^2-4X1X1}}{2X1}$$
   $$x=\frac{3\pm\sqrt{9-4}}{2X1}$$
   $$x=\frac{3\pm\sqrt{5}}{2}$$
   $$x=\frac{3+\sqrt{5}}{2} \quad \text{or} \quad x=\frac{3-\sqrt{5}}{2}$$
2. Solve the quadratic equation by using the formula: x^2-6x-4=0
   Solution: a=1, b=-6, c=-4
   Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   $$x=\frac{-(-6)\pm\sqrt{(-6)^2-4X1X-4}}{2X1}$$
   $$x=\frac{6\pm\sqrt{36+16}}{2}$$
   $$x=\frac{6\pm\sqrt{52}}{2}=$$
   $$x=\frac{6+\sqrt{52}}{2} \quad \text{or} \quad x=\frac{6-\sqrt{52}}{2}$$
3. By using the quadratic formula, find the solutions: $6x^2$-7x-5=0.
   Solution: a=6, b=-7, c=-5.
   Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   $$x=\frac{-(-7)\pm\sqrt{(-7)^2-4X6X-5}}{2X6}$$
   $$x=\frac{7\pm\sqrt{49+120}}{12}$$
   $$x=\frac{7\pm\sqrt{169}}{12}=\frac{7\pm13}{12}$$
   $$x=\frac{7+13}{12} \quad \text{or} \quad x=\frac{7-13}{12}$$
   $$x=\frac{20}{12} \quad \text{or} \quad x=\frac{-6}{12}$$
   $$x=\frac{5}{3} \quad \text{or} \quad x= - \frac{-1}{2}$$

4. Solve the quadratic equation by formula: $2x^2+11x+5=0$.
Solution: a=2, b=11, c=5.

Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-11\pm\sqrt{(11)^2-4X2X5}}{2X5}$$

$$x=\frac{-11\pm\sqrt{121-40}}{10}$$

$$x=\frac{-11\pm\sqrt{81}}{10}=\frac{-11\pm9}{10}$$

$$x=\frac{-11+9}{10} \quad \text{or} \quad x=\frac{-11-9}{10}$$

$$x=\frac{-2}{10} \quad \text{or} \quad x=\frac{-20}{10}$$

$$x=-\frac{1}{5} \quad \text{or} \quad x=-2$$

5. Solve the quadratic equation using formula: $x^2-8x+15=0$.
Solution: a=2, b=11, c=5.

Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-11\pm\sqrt{(11)^2-4X2X5}}{2X5}$$

$$x=\frac{-11\pm\sqrt{121-40}}{10}$$

$$x=\frac{-11\pm\sqrt{81}}{10}=\frac{-11\pm9}{10}$$

$$x=\frac{-11+9}{10} \quad \text{or} \quad x=\frac{-11-9}{10}$$

$$x=\frac{-2}{10} \quad \text{or} \quad x=\frac{-20}{10}$$

$$x=-\frac{1}{5} \quad \text{or} \quad x=-2$$

Practice:

Solve the quadratic equation by using formula method

6. $2x^2+x-5=0$.
7. $x^2+2x+1=0$.
8. $5x^2+31x+6=0$.
9. $x^2-x-30=0$.
10. $4x^2-11x-3=0$.
11. $x^2+2x-5=0$.

6. Pair of linear equations in two variables: solve x & y.

1. Solve the equations by elimination method: $x+y= -2$ & $2x-y= 8$.
   Solution: let the given equations be $x+y= -2$ & $2x-y= 8$.
   $x+y= -2$ ---------(1)
   $2x-y= 8$ ----------(2)
   By eliminating add the above two equations.
   We get $x+y= -2$
   $2x-y= 8$
   $3x=6$
   $\boxed{x=2}$
   put above x value in any one equation we get y value
   equation (1) becomes $2+y=-2$
   $y=-2-2$
   $\boxed{y=-4}$

2. Solve: $x-y= 1$& $2x-3y= 5$.
   Solution: The given two equations are $x-y= 1$& $2x-3y= 5$.
   $x-y= 1$ -----------(1)
   $2x-3y= 5$ --------(2)
   For eliminating, multiple 2 to the equation (1) we get
   $2x-2y=2$
   $2x-3y=5$ subtract this two
   $\boxed{y=3}$
   put y value in equation (1) we get $x-(-3)=1$
   $\boxed{x=-2}$

3. Solve: $x-2y=2$ & $2x-y=-8$.
   Solution: The given two equations are $x-2y=2$ & $2x-y=-8$.
   $X-2y=2$ -----------(1)
   $2x-y=-8$--------(2)
   For eliminating, multiple 2 to the equation (1) we get
   $2x-4y=4$
   $2x-y=-8$ subtract this two
   put y value $\boxed{y=-3}$ in equation (1) we get $x-2(-3)=1$
   $\boxed{x=-5}$

4. Solve: $3x+2y= -5$ & $x-6y= -15$.

Solution: The given two equations are $x-2y=2$ & $2x-y=-8$.

$3x+2y= -5$ -----------(1)

$x-6y= -15$--------(2)

For eliminating, multiple 3 to the equation (2) we get

$3x+2y= -5$

$3x-18y= -45$ subtract this two

put y value $\boxed{y=20}$ in equation (2) we get $x-6(20)=-45$

$\boxed{x=75}$

5. Solve: $x-2y= 8$ & $2x-3y= 14$.

Solution: The given two equations are $x-2y= 8$ & $2x-3y= 14$.

$x-2y= 8$ -----------(1)

$2x-3y= 14$--------(2)

For eliminating, multiple 2 to the equation (1) we get

$2x-4y= 16$

$2x-3y= 14$ subtract this two

put y value $\boxed{y=-2}$ in equation (1) we get $x-2(-2)=8$

$\boxed{x=4}$

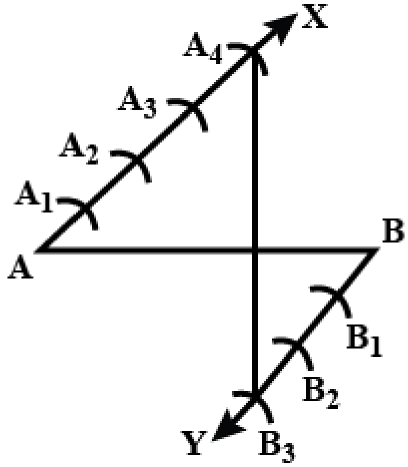
<u>Practice:</u>

Solve the following equations

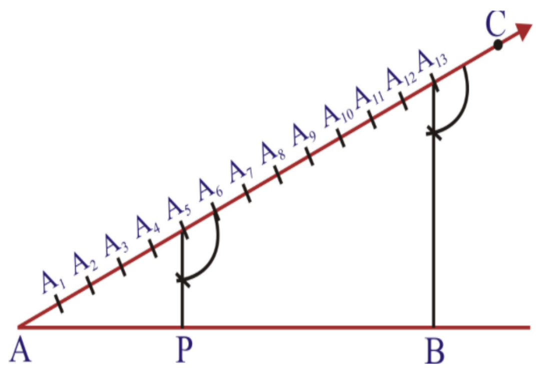
1) $x+2y= 10$ & $2x-4y= -4$.
2) $3x+y= -2$& $x+2y= 1$.
3) $x-y= 1$& $2x-3y= 5$.
4) $3x+4y=10$ & $x-8y= -6$.
5) $x+2y=9$ & $2x-y=3$.
6) $2x+y=9$ & $3x-2y= -4$.
7) $8x+2y=-2$ & $4x-6y=-22$.
8) $x-2y=8$ & $3x-6y=9$.
9) $x-5y=-14$ & $6x+y=9$.
10) $x-2y=2$ & $2x+y=-8$.
11) $x-2y=-9$ & $3x+y=1$.
12) $x+y= -7$ & $2x-3y= 1$.
13) $x-2y= -7$ & $3x+2y= 3$.
14) $4x-2y=16$ & $3x+y= 2$.
15) $x+4y= 2$ & $3x-6y = 18$.
16) $x-y= 5$ & $2x+y=- 11$.
17) $6x+y=1$ & $2x-y= 7$.
18) $x+y= 4$ & $2x-3y= 18$.
19) $x+y= -2$ & $2x+4y= -14$.
20) $2x+3y= -5$ & $4x+8y= -8$.
21) $x+2y=7$ & $3x-4y= -9$.

7. Constructions : Dividing the line segment

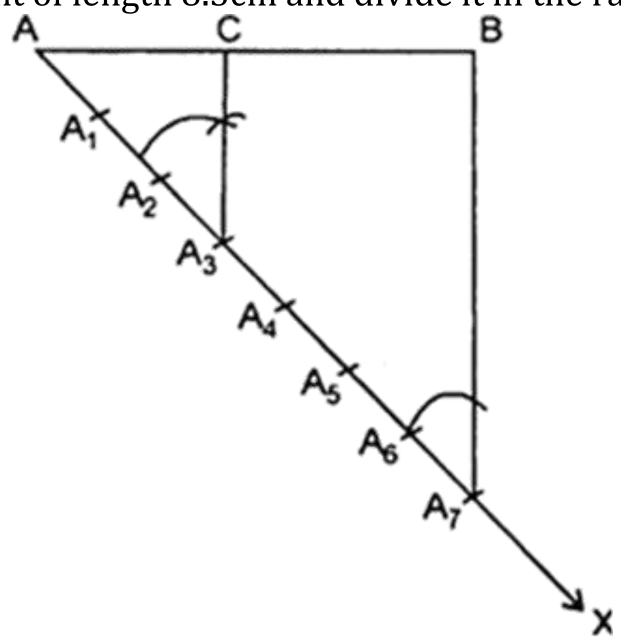
1. Draw a line segment of length 9cm and divide it in the ratio 2:3.



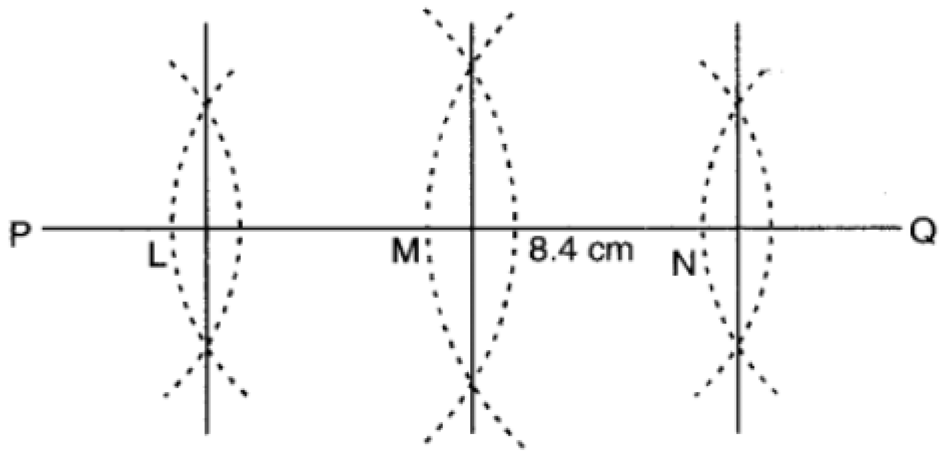
2. Draw a line segment of length 7.6cm and divide it in the ratio 5:8.



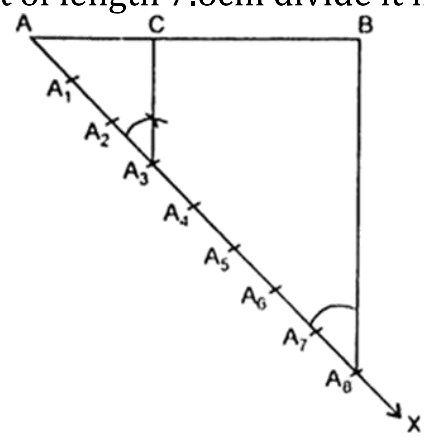
3. Draw a line segment of length 8.3cm and divide it in the ratio 2:5.



4. Draw a line segment PQ = 8.4 cm. Divide PQ into four equal parts using ruler and compass.



5. Draw a line segment of length 7.6cm divide it in the ratio 3:5.

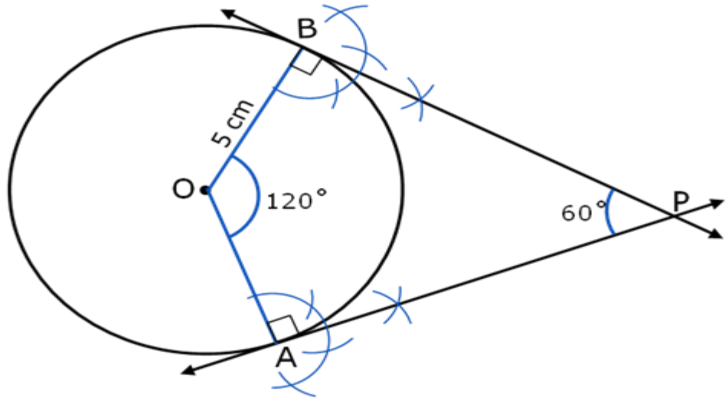


<u>Practice:</u>

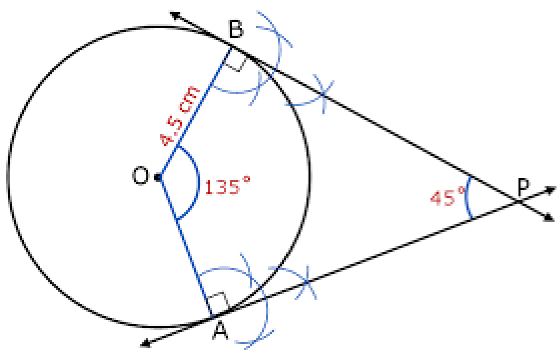
6. Draw a line of length 7cm, divide it in the ratio 2:4.
7. Draw a line segment then divide internally in the ratio of 3:7.
8. Draw a line segment AB=10cm & divide it in the ratio 5:8.
9. Draw a line of length 7.3cm and then divide it in the ratio 4:6.
10. Draw a line segment of AB=8cm and divide it in the ratio 3:2 by geometrical construction.
11. Construct a tangent to a circle of radius 4cm at any point P on its circumference.

8. Constructions : Tangent construction

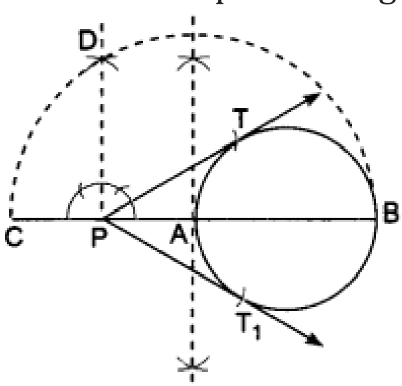
1. Construct tangents to a circle of radius 5cm such that the angle between the tangents is 60^0.



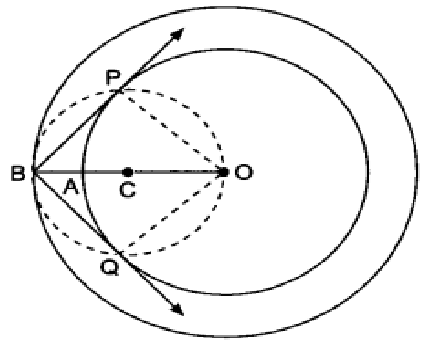
2. Construct a circle of radius 4.5cm, such that the angle between the two radii is 135^0.



3. Draw a circle with the help of a bangle. Take a point outside the circle. Construct the pair of tangents from this point to the circle.



4. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length.



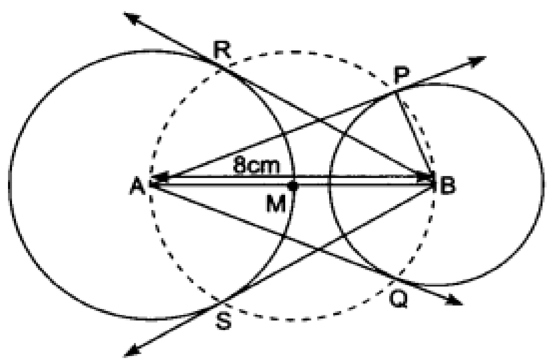
Justification:
In ΔBPO, we have
$\angle BPO = 90°$, $OB = 6$ cm and $OP = 4$ cm
$\therefore OB^2 = BP^2 + OP^2$ [Using Pythagoras theorem]

$$\Rightarrow BP = \sqrt{OB^2 - OP^2}$$

$$\Rightarrow BP = \sqrt{36-16} = \sqrt{20} \text{ cm} = 4.47 \text{ cm}$$

Similarly, BQ = 4.47 cm

5. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.



Justification:
On joining BP, we have $\angle BPA = 90°$, as $\angle BPA$ is the angle in the semicircle.
$\therefore AP \perp PB$
Since BP is the radius of given circle, so AP has to be a tangent to the circle. Similarly, AQ, BR and BS are the tangents.

6. Construct a pair of tangents to a circle of radius 6.2cm from an external point 3.8 cm away from the circle.
7. Construct a pair of tangents to a circle of radius 4cm from an external point 4 cm away from the circle.
8. Construct a tangent to a circle of radius 3.5cm from a point on the concentric circle of radius 7cm and measure its length.
9. Construct a pair of tangents to a circle of radius 5.5cm at the end point of radii. The angle between the two radii is 90^0.

9. Statistics : Mean, Median & Mode.

Mean for grouped data, x=$\frac{\sum fx}{n}$ (direct method)

Median for grouped data, median= LRL+$\left\{\frac{\frac{n}{2}-fc}{fm}\right\}$ x h

Mode for grouped data, Mode=LRL+$\left\{\frac{f1-f0}{2f1-f0-f2}\right\}$x h.

1. Find the mean, median and mode for the gollowing data.

C.I	10-20	20-30	30-40	40-50	50-60
f	5	2	3	6	4

To find the mean,

C.I	f	x	fx
10-20	5	15	75
20-30	2	25	50
30-40	3	35	105
40-50	6	45	270
50-60	4	55	220
	n=20		$\sum fx = 720$

x=$\frac{\sum fx}{n}$

x=$\frac{720}{20}$

Mean=36

To find the median, first we should find $\frac{n}{2}$, = $\frac{20}{2}$= 10

C.I	f	f_c
10-20	5	5
20-30	2	7
30-40	3	10
40-50	6	16
50-60	4	20
	n=20	

Median= LRL+$\left\{\frac{\frac{n}{2}-fc}{fm}\right\}$ x h   LRL=30, f_m=3, f_c=7 & h=1

=30+$\left\{\frac{10-7}{3}\right\}$ x 10 = 30+1x10

Median = 30+10= 40

To find the mode, note that f_1, f_0 & f_2.

C.I	f
10-20	5
20-30	2
30-40	3 f_0
40-50	6 f_1
50-60	4 f_2

Mode=LRL+$\left\{\frac{f1-f0}{2f1-f0-f2}\right\}$x h,   LRL=40, f_1=6, f_0=3 & f_2=4.

$=40+\left\{\frac{6-3}{12-3-4}\right\}X10 \Rightarrow 40+(\frac{3}{5})X10$

=40+6.

$\boxed{\text{Mode=46}}$

2. Find the mean, median and mode for the gollowing data.

C.I	2-6	7-11	12-16	17-21	22-26
f	7	13	8	7	5

To find the mean,

C.I	f	x	fx
2-6	7	4	28
7-11	13	9	117
12-16	8	14	112
17-21	7	19	133
22-26	5	24	120
	n=40		$\sum fx$ =510

$x=\frac{\sum fx}{n}$

$x=\frac{510}{40}$

$\boxed{\text{Mean=12.75}}$

To find the median, first we should find $\frac{n}{2}$, = $\frac{40}{2}$= 20

C.I	f	fc
2-6	7	7
7-11	13	20
12-16	8	28
17-21	7	35
22-26	5	40
	n=40	

$$\text{median} = LRL + \left\{\frac{\frac{n}{2} - fc}{fm}\right\} \times h \qquad LRL=7,\ f_m=13,\ f_c=7\ \&\ h=5$$

$$= 7 + \left\{\frac{20-13}{7}\right\} \times 5 \;=\; 7+5$$

Median = 12

To find the mode, note that f_1, f_0 & f_2.

C.I	f
2-6	7 f_0
7-11	13 f_1
12-16	8 f_2
17-21	7
22-26	5

$$\text{Mode} = LRL + \left\{\frac{f1 - f0}{2f1 - f0 - f2}\right\} \times h, \qquad LRL=7,\ f_1=13,\ f_0=7\ \&\ f_2=8.$$

$$= 7 + \left\{\frac{13-7}{26-7-8}\right\} X5 \;\Rightarrow 7 + \left(\frac{6}{11}\right) X10$$

$=7+5.4.$

$=12.4.$

3. Find the mean, median and mode for the gollowing data.

C.I	1-5	6-10	11-15	16-20	21-25
f	6	7	4	8	5

To find the mean,

C.I	f	x	fx
1-5	6	4	24
6-10	7	9	63
11-15	4	14	56
16-20	8	19	152
21-25	5	24	120
	n=30		$\sum fx$ =415

$$x = \frac{\sum fx}{n}$$

$$x = \frac{415}{30}$$

Mean=13.83

To find the median, first we should find $\frac{n}{2}$, $= \frac{30}{2} = 15$

C.I	f	fc
1-5	6	6

6-10	7	13
11-15	4	17
16-20	8	25
21-25	5	30
	n=30	

median= $LRL+\left\{\frac{\frac{n}{2}-fc}{fm}\right\} \times h$ LRL=11, f_m=4, f_c=13 & h=5

$=11+\left\{\frac{15-13}{4}\right\} \times 5 = 11+2.5$

Median = 13.5

To find the mode, note that f_1, f_0 & f_2.

C.I	f
1-5	6
6-10	7
11-15	4 f_0
16-20	8 f_1
21-25	5 f_2

Mode=$LRL+\left\{\frac{f1-f0}{2f1-f0-f2}\right\}$x h, LRL=16, f_1=8, f_0=4 & f_2=5.

$=16+\left\{\frac{8-4}{16-4-5}\right\}X5 \Rightarrow 16+(\frac{4}{7})X10$

=16+5.71.

=21.71.

Practice:

Find the mean, Median and Mode for the following data.

C.I	0-20	20-40	40-60	60-80	80-100
f	3	4	2	7	4

C.I	3-13	13-23	23-33	33-43	43-53	53-63
f	12	9	8	13	5	3

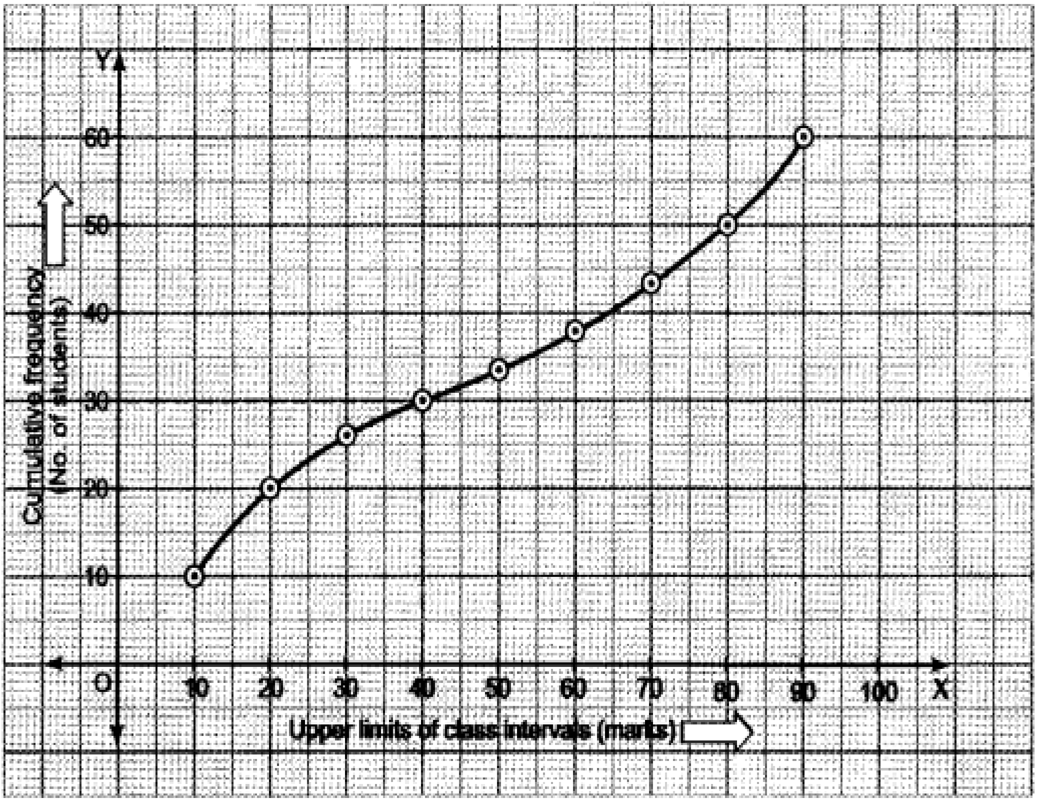
C.I	2-6	7-11	12-16	17-21	22-26
f	5	7	4	8	6

C.I	1-5	6-10	11-15	16-20	21-25
f	1	2	4	1	2

10. Statistics : Ogive graph.

1. Convert the following as less than type then draw its ogive.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No.of students	10	10	6	4	4	4	6	6	10



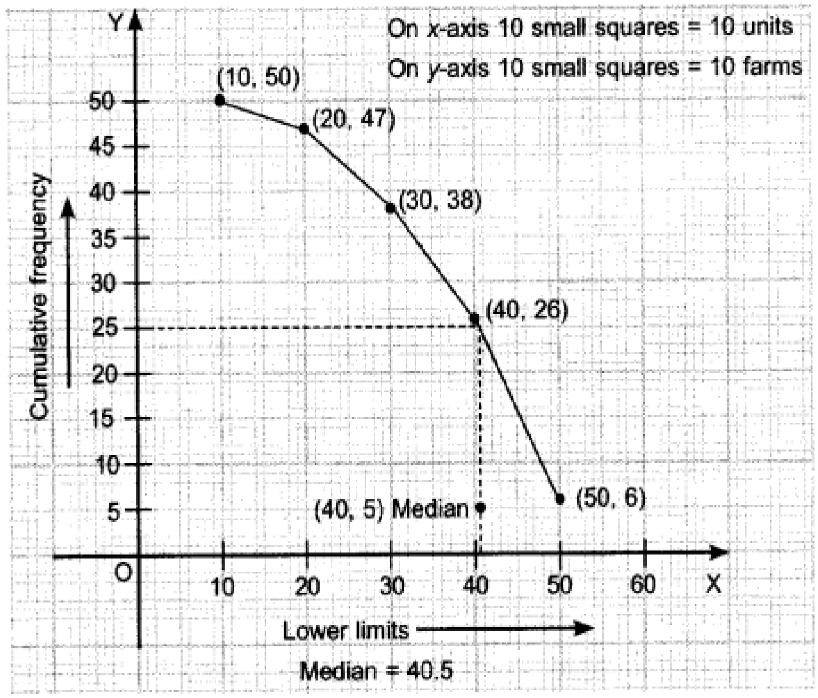
2.

The following table gives production yield of rice per hectare in some farms of a village:

Production yield (in kg/hectare)	10–20	20–30	30–40	40–50	50–60
No. of farms	3	9	12	20	6

Draw a 'more than type' ogive. Also, find median from the curve.

Production yield (in kg/ hectare) Class interval	Frequency	Production yield more than or equal to	c.f
10 – 20	3	10	50
20 – 30	9	20	47
30 – 40	12	30	38
40 – 50	20	40	26
50 – 60	6	50	6

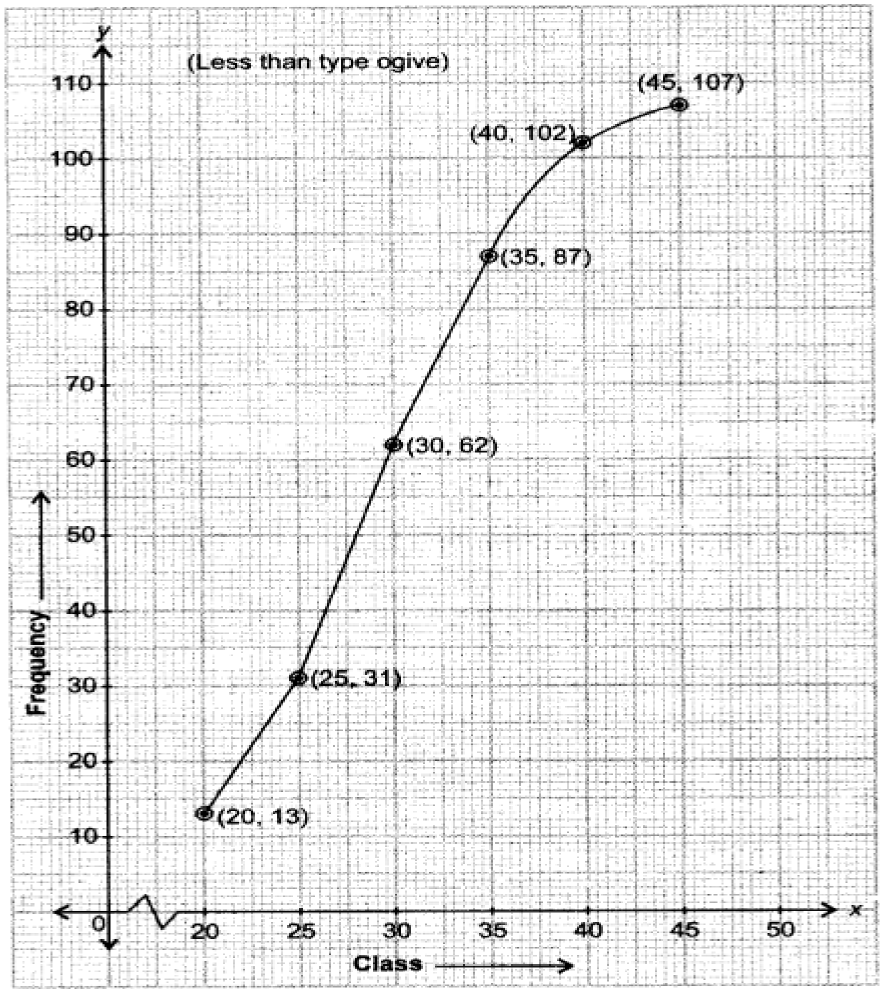


3. Draw a 'less than type' ogive for the following frequency distribution.

Class	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	13	18	31	25	15	5

Solution:

Class	Frequency
Less than 20	13
Less than 25	13 + 18 = 31
Less than 30	31 + 31 = 62
Less than 35	62 + 25 = 87
Less than 40	87 + 15 = 102
Less than 45	102 + 5 = 107



4.

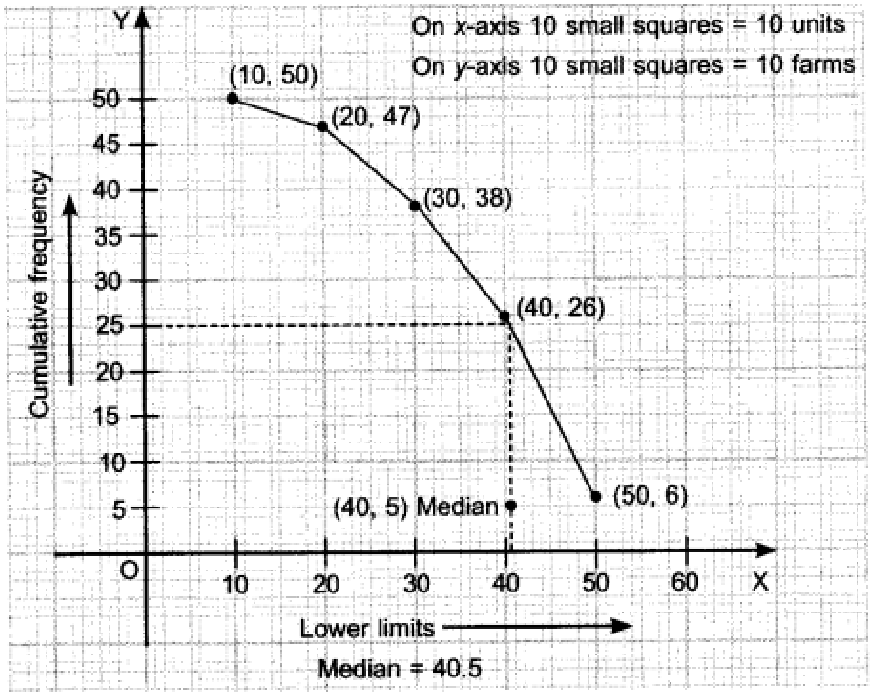
The following table gives production yield of rice per hectare in some farms of a village:

Production yield (in kg/hectare)	**10–20**	**20–30**	**30–40**	**40–50**	**50–60**
No. of farms	**3**	**9**	**12**	**20**	**6**

Draw a 'more than type' ogive. Also, find median from the curve.

Solution:

Production yield (in kg/ hectare) Class interval	**Frequency**	**Production yield more than or equal to**	***c.f***
10 – 20	3	10	50
20 – 30	9	20	47
30 – 40	12	30	38
40 – 50	20	40	26
50 – 60	6	50	6



Practice:

5.

No. of mangoes	50–52	53–55	56–58	59–61	62–64
No. of boxes	15	110	135	115	25

6.

Marks obtained	Less than 20	Less than 30	Less than 40	Less than 50
No. of students cumulative frequency	8	13	19	24

7.

Weight (in kg)	50–55	55–60	60–65	65–70	70–75	75–80
No. of candidates	13	18	45	16	6	2

8.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	16	14	24	26	x

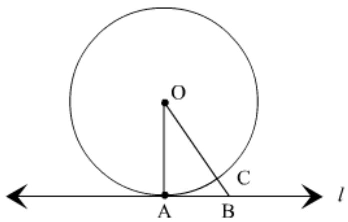
9.

Length (in mm)	109–117	118–126	127–135	136–144	145–153	154–162	163–171
No. of leaves	4	6	14	13	6	4	3

11. Circle: Theorems.

1. **Prove that "the tangent at any point of a circle is perpendicular to the radius through the point of contact".**

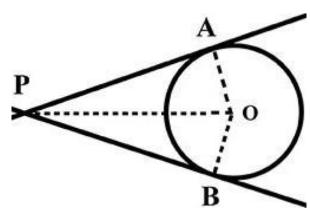
   Solution:

   Given: a circle C(0, r) and a tangent l at point A.
   To prove: OA ⊥ l
   Construction: Take a point B, other than A, on the tangent l. join OB. Suppose OB meets the circle in C.
   Proof: We know that, among all line segment joining the point O to a point on l, the perpendicular is shortest to l.
   OA=OC (Radius of the same circle)
   Now, OB=OC+BC.
   ∴ OB>OC
   ⇒OB>OA
   ⇒OA<OB
   B is an arbitrary point on the tangent l. Thus, OA is shorter than any other line segment joining O to any point on l.
   Here OA ⊥ l.

2. **Prove that "the lengths of the tangent drawn from an external point to the circle are equal".**

   Solution:
   Given: A circle with center O. PA & PB are two tangents drawn from an external point P.

   To prove: PA=PB
   Construction: Join OA, OB & OP.
   Proof: It is known that a tangent is at any point of a circle is perpendicular to the radius through the point of contact.
   OA⊥PA & OB⊥PB
   In ΔOPA & OPB, ∟OPA=∟OPB
   OA=OB (radii)
   OP=OP (common)
   Hence ΔOPA is congruent to ΔOPB. Therefore AP=PB.

12. Pair of linear equations in two variables: Graphical solution.

1. **Solve by graphically: x-y=4 & x+y=10.**

Solution: x-y=4-----------(i) & x+y=10---------(ii)

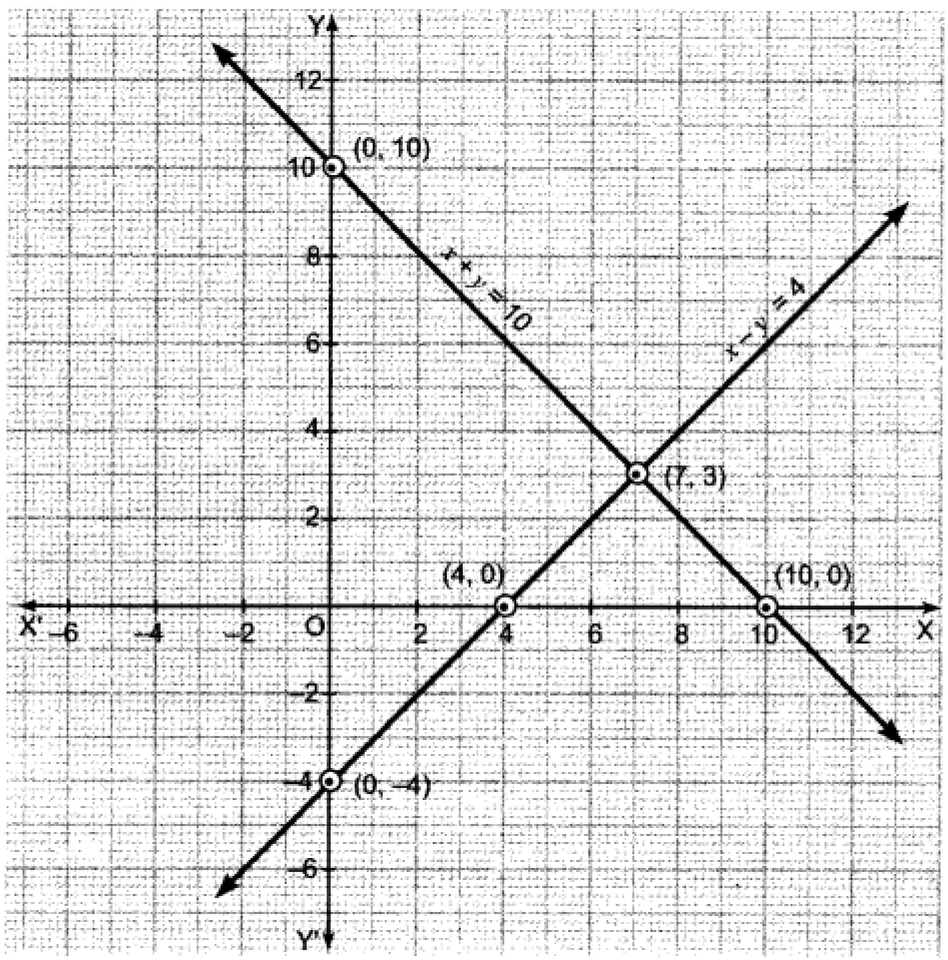
From equation (i), we have the following table:

x	0	4	7
y	– 4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have



Here, the two lines intersect at point (7,3) i.e., x = 7, y = 3.

2. **Show graphically the given system of equations $2x + 4y = 10$ and $3x + 6y = 12$ has no solution.**

Solution: $2x+4y=10$-----------(i) & $3x+6y=12$---------(ii)

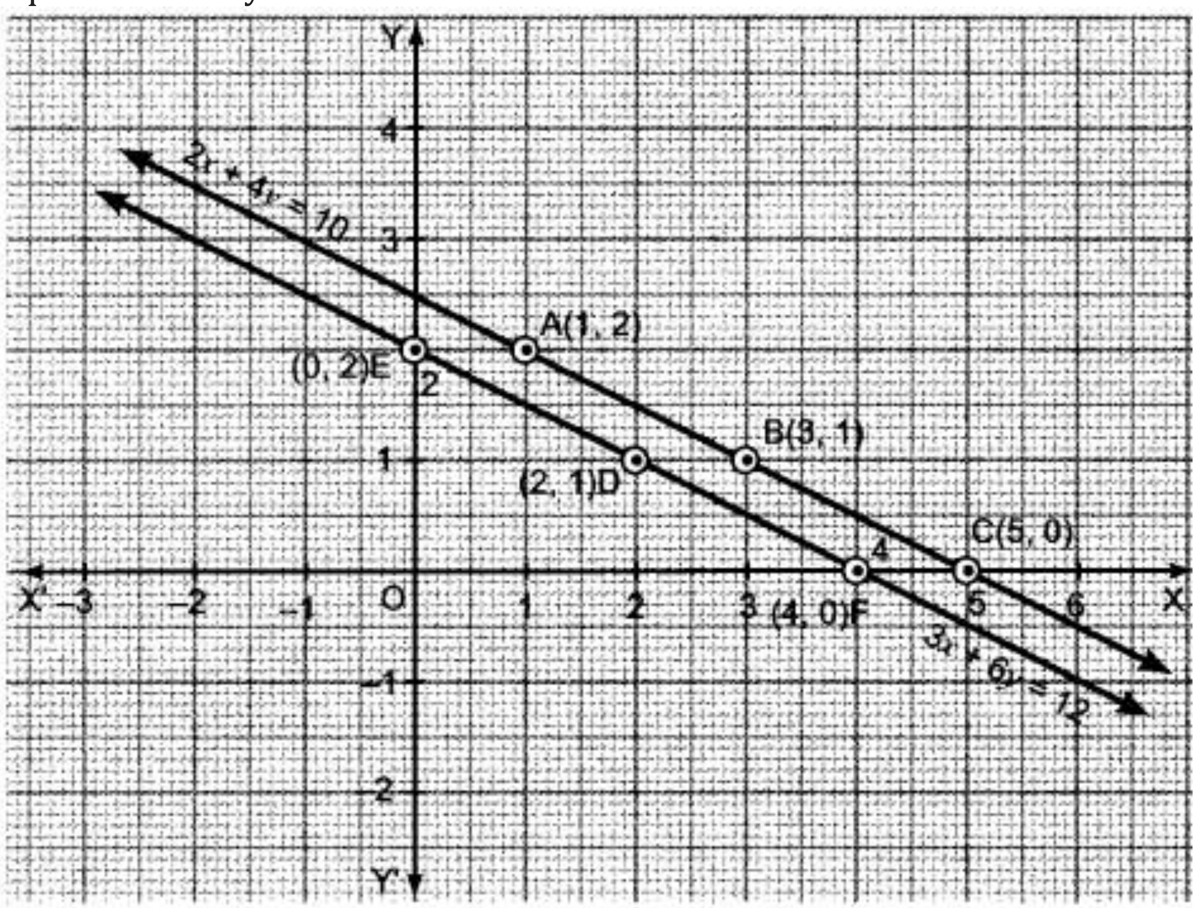
From equation (i), we have the following table:

x	1	3	5
y	2	1	0

From equation (ii), we have the following table:

x	2	0	4
y	1	2	0

Plot the points D (2, 1), E (0, 2) and F (4,0) on the same graph paper. Join D, E and F and extend it on both sides to obtain the graph of the equation $3x + 6y = 12$.



We find that the lines represented by equations $2x + 4y = 10$ and $3x + y = 12$ are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

3. Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$.

Solution:

We have, $2x + y = 6$-----------(i)

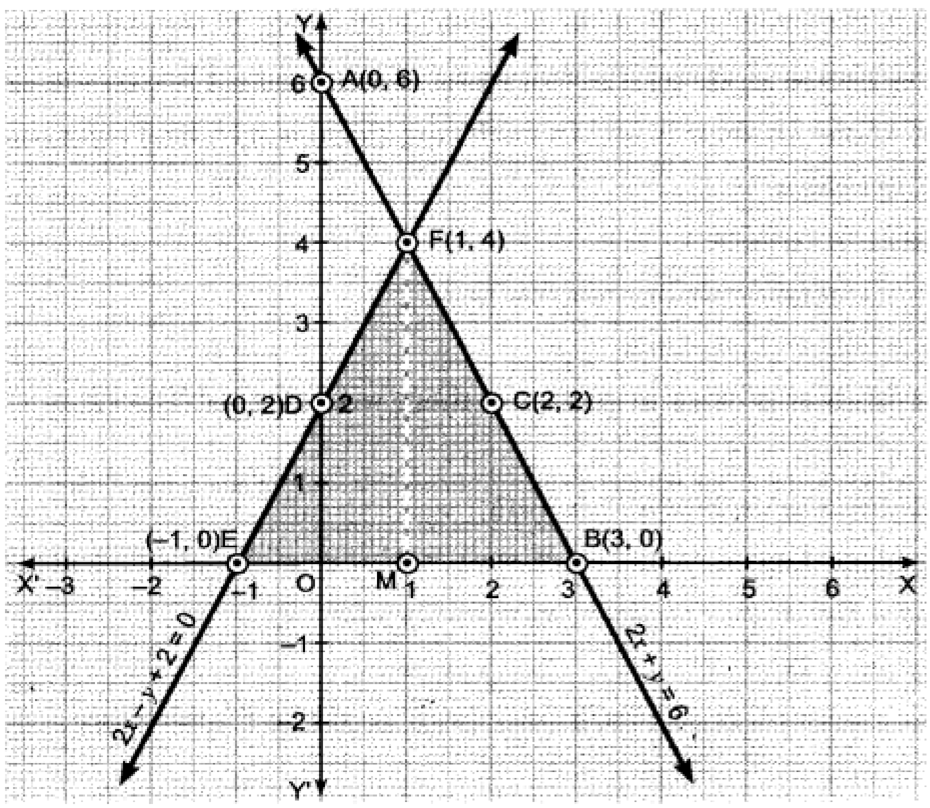
$2x-y=-2$ -------------(ii)

From equation (i), we have the following table:

x	0	3	2
y	6	0	2

From equation (ii), we have the following table:

x	0	− 1	1
y	2	0	4



4. Draw the graph of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$.

Solution: we have $x-y=-1$ ----------(i)

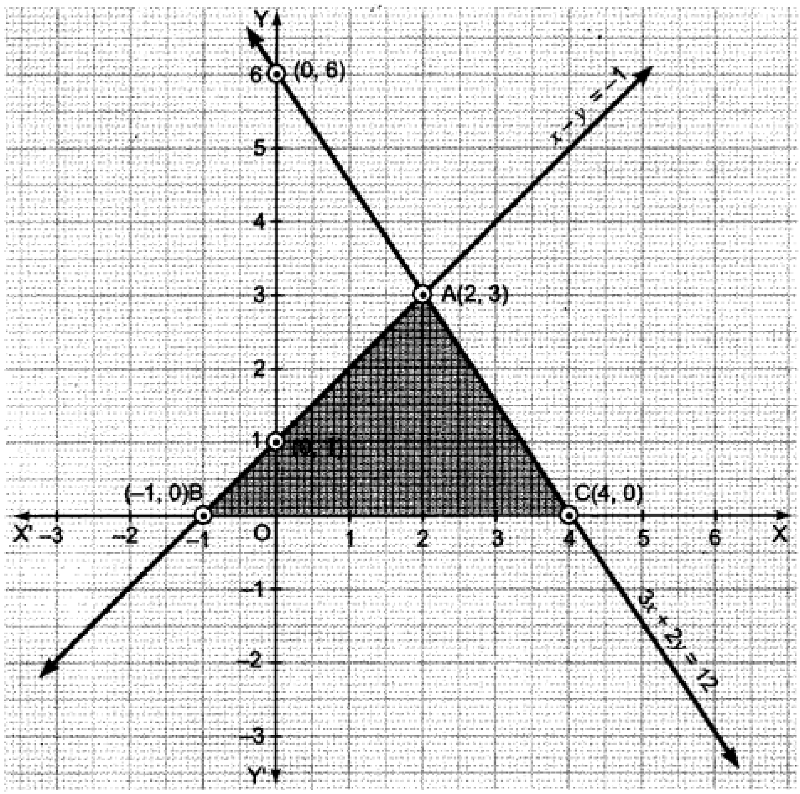
$3x+2y=12$-------(ii)

From equation (i), we have the following table:

x	− 1	0	2
y	0	1	3

From equation (ii), we have the following table:

x	0	4	2
y	6	0	3



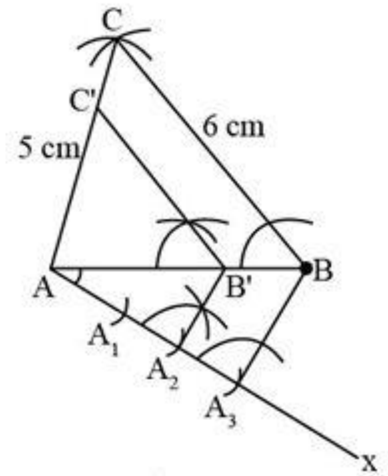
Practice: solve the following equations graphically

5. x+2y=9 & 2x-y=3.
6. x-2y=2 & 2x+y=-8.
7. x-2y=-9 & 3x+y=1.
8. x+2y=4 & 6x+y=13.
9. X+2y=1 & 2x+3y= -1.
10. X-2y= 8 & 2x-3y= 14.
11. x-y= 5 & 2x+y=- 11.
12. x+y= -7 & 2x-3y= 1.
13. x+4y= 2 & 3x-6y = 18.

13. <u>Constructions</u> : Constructions of similar triangles.

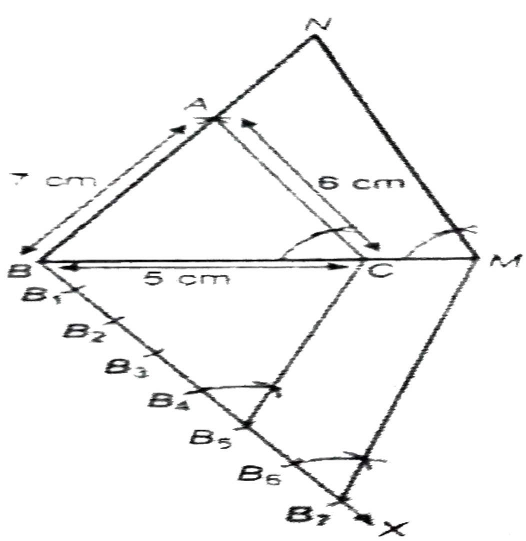
This construction is depends on two type of fractions, one is proper and another is improper fraction. Let's see both in different examples.

1. Construct a triangle with sides 4cm, 5cm & 6cm and then another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.



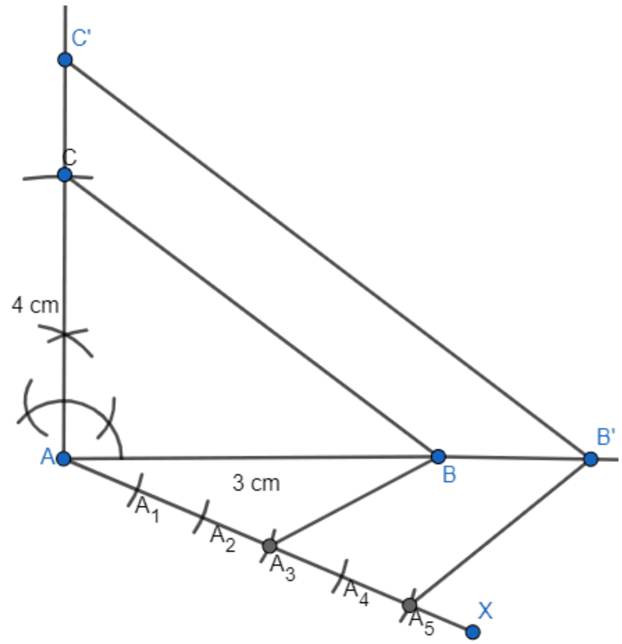
2. Construct a triangle with sides 5cm, 6cm & 7cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Solution:



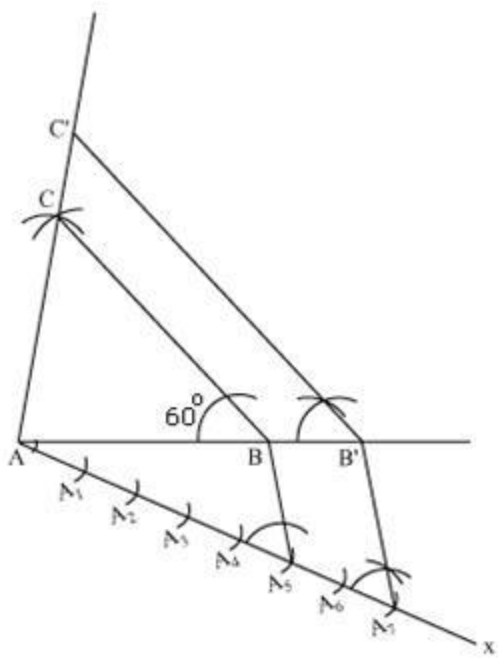
3. **Construct a right angled triangle with sides 3cm & 4cm and then another triangle whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle.**

Solution:



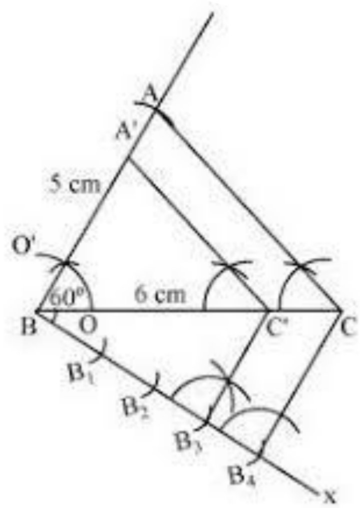
4. **Construct a triangle ABC with base AB=5cm, ∟ABC=60^0 & BC=7cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.**

Solution:



5. **Construct a triangle ABC with AB=5cm, ∟ABC=60^0 & BC=6cm and then another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle.**

Solution:

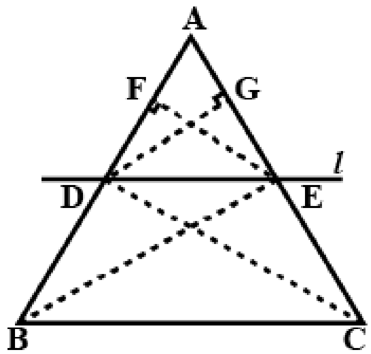


Practice:

6. Draw a triangle ABC with side BC=6cm, ∟B=60^0, ∟A=10^0 Then construct a triangle a triangle whose sides are $\frac{1}{3}$ times the corresponding sides of ΔABC.
7. Draw a triangle PQR with side QR=5cm, ∟Q=45^0, ∟P=105^0.Then construct a triangle a triangle whose sides are $\frac{5}{2}$ times the corresponding sides of ΔPQR.
8. Construct an isosceles triangle whose base is 5cm and altitude 3cm and then another triangle whose sides are $\frac{2}{5}$ times the corresponding sides of the isosceles triangle.
9. Construct a triangle with sides 3.5cm, 4cm & 5cm and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.
10. Construct a triangle with sides 3cm, 4cm & 6cm and then another triangle whose sides are $\frac{7}{4}$ of the corresponding sides of the first triangle.
11. Construct a right angled triangle with sides 5cm & 6cm and then another triangle whose sides are $2\frac{1}{2}$ of the corresponding sides of the first triangle.

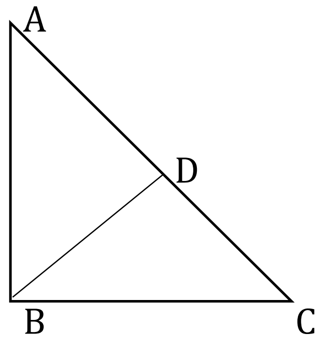
14. <u>TRIANGLES</u>: Theorems.

1. **<u>Basic proportionality theorem(B.P.T) or Thales Theorem</u>:**-**
   "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio".



Let **ABC** be the triangle.
The line **l** parallel to **BC** intersect **AB** at **D** and **AC** at **E**.
To prove: $\frac{DB}{AD}=\frac{CB}{AE}$
Join **BE**,**CD**
Draw **EF⊥AB**, **DG⊥CA**
Since **EF⊥AB**,
EF is the height of triangles **ADE** and **DBE**
Area of **△ADE**=$\frac{1}{2}$**×** base **×** height=$\frac{1}{2}$**xAD×EF**
Area of **△DBE**=$\frac{1}{2}$×**DB×EF**
$\frac{area of \Delta DBE}{area of \Delta ADE}=\frac{1/2\times DB\times EF}{1/2 x AD\times EF}\times=\frac{DB}{AD}$ ....(1)
Similarly,
$\frac{area of \Delta DBE}{area of \Delta ADE}=\frac{1/2\times CB\times EF}{1/2 x AE\times EF}\times=\frac{CB}{AE}$ ....(2)
But **ΔDBE** and **ΔDCE** are the same base **DE** and between the same parallel straight line **BC** and **DE.**
Area of **ΔDBE**= area of **ΔDCE** ....(3)
From (1), (2) and (3), we have
$\frac{DB}{AD}=\frac{CB}{AE}$
Hence proved.

2. **<u>Pythagoras theorem:</u>**
   In a right angles triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Data: In ΔABC, $\angle ABC = 90°$

To Prove: $AB^2 + BC^2 = CA^2$

Construction: Draw $BD \perp AC$.

Proof: **Statement** **Reason**

Statement	Reason
Compare ΔABC and ΔADB,	
$\angle ABC = \angle ADB = 90°$	(Q Data and construction)
$\angle BAD$ is common.	
$\therefore \Delta ABC \sim \Delta ADB$	(Q Equiangular triangles)
$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$	(Q A A similarity criteria)
$\therefore AB^2 = AC.AD$ ...... (1)	
Compare ΔABC and ΔBDC,	
$\angle ABC = \angle BDC = 90°$	(Q Data and construction)
$\angle ACB$ is common	
$\therefore \Delta ABC \sim \Delta BDC$	(Q Equiangular Triangles)
$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC} \Rightarrow =$	(Q AA similarity criteria)
$BC^2 = AC.DC$ .....(2)	

By adding (1) and (2) we get

$AB^2 + BC^2 = (AC.\,AD) + (AC.\,DC)$

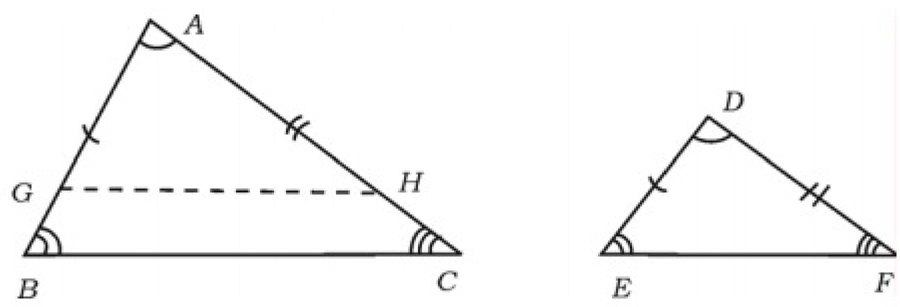
$AB^2 + BC^2 = AC\,(AD + DC)$

$AB^2 + BC^2 = AC.\,AC = AC^2$ [Q $AD + DC = AC$]

$\therefore AB^2 + BC^2 = AC^2$

3. AA similarity Criterion theorem:

"If two triangles are equiangular then their corresponding sides are in proportion"



Data: In ΔABC and ΔDEF

(i) ∠BAC = ∠EDF

(ii) ∠ABC = ∠DEF

To prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Construction: Mark points 'G' and 'H' on AB and AC such that

(i) AG = DE and (ii) AH = DF Join G and H

Proof:	Statement	Reason
Compare	ΔAGH and ΔDEF,	
	AG = DE	[Construction]
	∠GAH = ∠EDF	[Data]
	AH = DF	[Construction]
	∴ ΔAGH ≅ ΔDEF	[SAS]
	∴ ∠AGH = ∠DEF	[CPCT]
	But ∠ABC =∠DEF	[Data]
	⇒ ∠AGH = ∠ABC	[Axiom - 1]

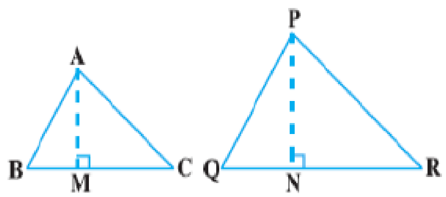
∴ GH || BC [If corresponding angles are equal then lines are ||.]

∴ In ΔABC $\frac{AB}{AG} = \frac{BC}{GH} = \frac{CA}{HA}$ [third corollary to Thales theorem]

Hence $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

4. Area Of Similar Triangle:

Prove that "The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides".



We need to prove that

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Now, $ar(ABC) = \frac{1}{2} \times BC \times AM$

and $ar(PQR) = \frac{1}{2} \times QR \times PN$

So, $\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ ... (1)

Now, in ΔABM and ΔPQN,

$\angle B = \angle Q$ (As $\Delta ABC \sim \Delta PQR$)

and $\angle M = \angle N$ (Each is $90°$)

So, $\Delta ABM \sim \Delta PQN$ (AA similarity criterion)

Therefore, $\dfrac{AM}{PN} = \dfrac{AB}{PQ}$ ... (2)

Also, $\Delta ABC \sim \Delta PQR$ (Given)

So, $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{CA}{RP}$ ... (3)

Therefore, $\dfrac{ar(ABC)}{ar(PQR)} = \dfrac{AB}{PQ} \times \dfrac{AM}{PN}$

[From (1) & (3)]

$= \dfrac{AB}{PQ} \times \dfrac{AB}{PQ}$ [From (2)]

$= \left(\dfrac{AB}{PQ}\right)^2$

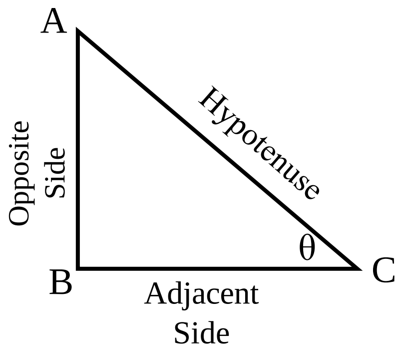
Now using (3) we get:

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

INTRODUCTION TO TRIGONOMETRY

TRIGONOMETRIC FUNCTIONS

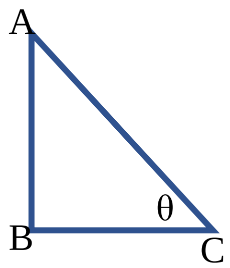
Si No	Function	Description	Ratios
1	Sin θ	$^{Opp}/_{Hyp}$	$^{AB}/_{AC}$
2	Cos θ	$^{Adj}/_{Hyp}$	
3	Tan θ	$^{Opp}/_{Adj}$	
4	Cot θ	$^{Adj}/_{Opp}$	
5	Sec θ	$^{Hyp}/_{Adj}$	
6	Cosec θ	$^{Hyp}/_{Opp}$	



INVERSE TRIGONOMETRIC FUNCTIONS

$\text{Sin}\ \theta = {}^{1}/_{\text{Cosec}\theta}$	$\text{Cos}\ \theta = {}^{1}/_{\text{Sec}\theta}$	$\text{Tan}\ \theta = {}^{1}/_{\text{Cot}\theta}$
$\text{Cosec}\ \theta = {}^{1}/_{\text{Sin}\theta}$	$\text{Sec}\ \theta = {}^{1}/_{\text{Cos}\theta}$	$\text{Cot}\ \theta = {}^{1}/_{\text{Tan}\theta}$

1. If 7Cosθ = 4 , Find the Value of other Trigonometric Functions

	**Acc to P T** $AC^2 = AB^2 + BC^2$ $AB^2 = AC^2 - BC^2$ $= 7^2 - 4^2$ $= 49 - 16 = 23$ $AB = \sqrt{23}$	$\text{Cos}\theta = {}^{Adj}/_{Hyp} = {}^{4}/_{7}$	$\text{Sin}\theta = {}^{Opp}/_{Hyp} = {}^{\sqrt{23}}/_{7}$	$\text{Tan}\ \theta = {}^{Opp}/_{Adj} = {}^{\sqrt{23}}/_{4}$
		$\text{Cot}\theta = {}^{Adj}/_{Opp} = {}^{4}/_{\sqrt{23}}$	$\text{Sec}\theta = {}^{Hyp}/_{Adj} = {}^{7}/_{4}$	$\text{Cosec}\theta = {}^{Hyp}/_{Opp} = {}^{7}/_{\sqrt{23}}$

For Practice : 1. If 5CosA=4 Write all other Trigonometric ratios.

2. If 3CosecA=7, Write all other Trigonometric ratios.

Values of Trigonometric Functions for Different angles

θ	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
Cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
Cosec	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TRIGNOMETRIC FUNCTIONS OF COMPLEMENTARY ANGLES

$\sin(90-\theta) = \cos\theta$	$\operatorname{cosec}(90-\theta) = \sec\theta$	$\tan(90-\theta) = \cot\theta$
$\cos(90-\theta) = \sin\theta$	$\sec(90-\theta) = \operatorname{cosec}\theta$	$\cot(90-\theta) = \tan\theta$

Solve the following

1. Find the value of $$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\times(\frac{1}{2})^2 + 4(\frac{2}{\sqrt{3}})^2 - 1}{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \frac{5\times(\frac{1}{4}) + 4(\frac{4}{3}) - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{1}$$

$$= \frac{5}{4} + \frac{16}{3} - 1 = \frac{79}{12} - 1 = \frac{67}{12}$$

For Practice : a. $6\sin^2 30^\circ + 5\cos^2 60^\circ = ?$

b. $\sin 60^\circ + \sec 45^\circ + \cos 60^\circ = ?$ c. $\frac{\tan 60^\circ + \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = ?$

2. Evaluate : $\frac{\tan 65^\circ}{\cot 25^\circ} = ?$

$$= \frac{\tan(90-25^\circ)}{\cot 25^\circ} = \frac{\cot 25^\circ}{\cot 25^\circ} = 1$$

3. P T : $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

$= \tan 48^\circ \times \tan 23^\circ \times \frac{1}{\cot 42^\circ} \times \frac{1}{\cot 67^\circ}$

$= \tan 48^\circ \times \tan 23^\circ \times \frac{1}{\tan 48^\circ} \times \frac{1}{\tan 23^\circ} = 1$

4. $$\frac{\sin 72^\circ}{\cos 12^\circ} = \frac{\sin 72^\circ}{\cos(90-72)^\circ} = \frac{\sin 72^\circ}{\sin 72^\circ} = 1$$

5. If $\sec 4A = \operatorname{cosec}(A - 20^0)$, where $4A$ is an acute angle, find the value of A.

$\sec 4A = \operatorname{cosec}(A-20)$

$\operatorname{cosec}(90 - 4A) = \operatorname{cosec}(A - 20)$

$90 - 4A = A - 20$

$90 + 20 = A + 4A$

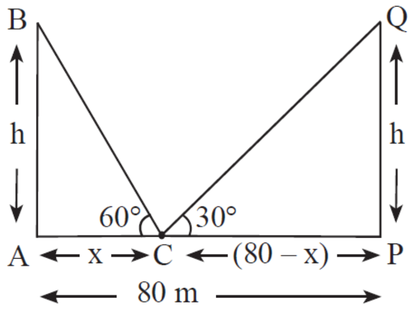
$110 = 5A$

$A = \frac{110}{5} = 22^\circ$

For Practice : a. P. T. $\tan 48^\circ . \tan 42^\circ . \tan 42^\circ . \tan 48^\circ = 1$ b . $\frac{\sin 36^\circ}{\cos 54^\circ} = ?$

APPLICATION OF TRIGONOMETRY

1. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.



in Right angle ΔABC $\lfloor C = 60°$.

$$\tan 60° = \frac{AB}{AC}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = \sqrt{3}x \rightarrow (1)$$

Again in right angle ΔPQC, $\lfloor C = 30°$.

$$\tan 30° = \frac{PQ}{PC} = \frac{h}{(80-x)}$$

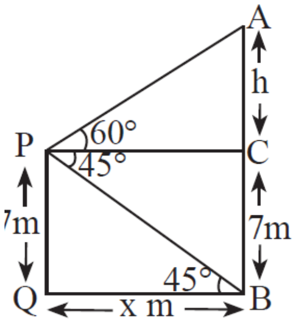
$$\frac{1}{\sqrt{3}} = \frac{h}{(80-x)}$$

$$h = \frac{(80-x)}{\sqrt{3}} \rightarrow (2)$$

From (1) and (2)

$$\sqrt{3}x = \frac{80-x}{\sqrt{3}}$$

2. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.



Let AB be the height of tower

AB = (h + 7) m and PQ be the height of building

In Right angle ΔPQB $\lfloor B = 45°$

$$\tan 45° = \frac{PQ}{BQ}$$

$$1 = \frac{PQ}{BQ} \quad [PQ = 7m]$$

BQ = 7 m

Again In Right angle ΔAPC $\lfloor P = 60°$

$$\tan 60° = \frac{AC}{PC}$$

$$\sqrt{3} = \frac{h}{PC}$$

$$h = PC\sqrt{3}\ (PC = BQ = 7m)$$

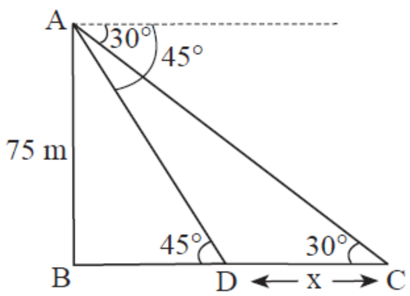
$$h = 7\sqrt{3}\ m$$

So, height of tower = AB = 7 + h

$$= 7 + 7\sqrt{3}$$

$$= 7(\sqrt{3}+1)\ m$$

3. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.



Let the height of light house be AB = 75 m and distance between two ships be DC = x

In Right angle ΔABD $\lfloor D = 45°$

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{75}{BD}$$

BD = 75 m

In Right angle ΔABC $\lfloor C = 30°$

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{BD + DC}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{75 + x}$$

$$75 + x = 75\sqrt{3}$$

$$x = 75\sqrt{3} - 75$$

$$x = 75(\sqrt{3} - 1)\ m$$

4. From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of a tower is 30° ,the angle of elevation of the top of water tank on top of the tower is 45° . Find (i) height of the tower (ii) depth of the tank

(i) Height of the tower .

$$\tan 30° = \frac{AC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{AC}{40}$$

$$AC = \frac{40}{\sqrt{3}} m$$

Depth of the tank is CD.

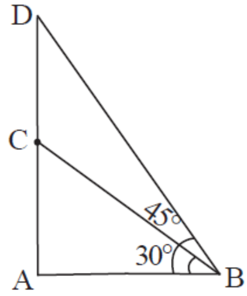
$$\tan 45° = \frac{AD}{AB}$$

$$1 = \frac{AD}{40}$$

$$AD = 40 \text{ m}$$

$$\therefore CD = AD - AC = 40 - \frac{40}{\sqrt{3}}$$

$$CD = 40\left(1 - \frac{1}{\sqrt{3}}\right) m$$



5. A tree is broken over by the wind forms a right angled triangle with the ground. IF the broken parts makes an angle of 60° ,with the ground and the top of the tree is now 20 m from its base, how tall was the tree.

In Δ^{le} ABC $\angle B = 90°$.

$$\cos 60° = \frac{AC}{BC} = \frac{20}{BC}$$

$$\frac{1}{2} = \frac{20}{BC}$$

$$BC = 40 \text{ m}$$

$$\tan 60° = \frac{AB}{AC}$$

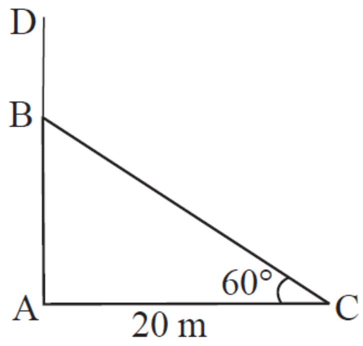
$$\sqrt{3} = \frac{AB}{20}$$

$$AB = 20\sqrt{3} \text{ m.}$$

$$\text{Height of tree} = AD = AB + BC$$

$$= 40 + 20\sqrt{3}$$

$$= 20(2 + \sqrt{3})$$



6. There is a small island in the middle of a 100 m wide river and a tall tree stands on the island . P and Q are points directly opposite to each other on two banks and in the line with the tree . If the angle of elevation of the top of the tree from P and Q are respectively 30^o and 45^o find the height of the tree

Let OA be the tree of height h metre.

In triangle POA and QOA, we have

$\tan 30^\circ = \frac{OA}{OP}$ and $\tan 45^\circ = \frac{OA}{OQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OP}$ and $1 = \frac{h}{OQ}$

$\Rightarrow OP = \sqrt{3}h$ and $OQ = h$

$\Rightarrow OP + OQ = \sqrt{3}h + h$

$\Rightarrow PQ = (\sqrt{3} + 1)h$

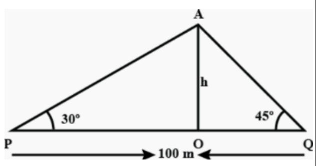
$\Rightarrow 100 = (\sqrt{3} + 1)h$ [$\because PQ = 100 m$]

$\Rightarrow h = \frac{100}{\sqrt{3} + 1} m$

$\Rightarrow h = \frac{100(\sqrt{3} + 1)}{2} m$

$\Rightarrow h = 50(1.732 - 1) m = 36.6 m$

Hence, the height of the tree is 36.6 m.



PRACTICE PAPER

1. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.
2. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.
3. From a point 20m away from the foot of a tower, the angle of elevation of top of the tower is 30°, Find the height of the tower.
4. An electric pole is 10m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the wire makes an angle of 45° with the horizontal through the foot of the pool, find the length of the wire.

SURFACE AREA & VOLUME

	C S A	T S A	VOLUME
Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Cone	πrl	$\pi r(r+l)$	$^1/_3\ \pi r^2 h$
Sphere	$4\ \pi r^2$		$^4/_3\ \pi r^3$
Hemisphere	$2\ \pi r^2$	$3\ \pi r^2$	$^2/_3\ \pi r^3$
Frustrum of Cone	$\pi(r_1 + r_2)l$	$\pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$	$^1/_3\ \pi(r_1^2 + r_2^2 + r_1 r_2)$
Cube	Surface Area = $6a^2$		a^3
Cuboid	Surface Area = $2(lb + bh + hl)$		$l \times b \times h$

1) What is the volume of a Cylinder having the area of its circular base 154Sq cm and height 10cm.

Ans : $\pi r^2 = 154$ Sq cm h = 10cm

$V = \pi r^2 h$

$= 154 \times 10 = \mathbf{1540cm^3}$

2) What is the volume of a Cylinder having the area of its circular base 22Sq cm and height 10cm.

Ans : $\pi r^2 = 22$ Sq cm h = 10cm

$V = \pi r^2 h = 22 \times 10$

$= \mathbf{220cm^3}$

3. The height and areas of circular bases of a cylinder and a cone are equal. If Volume of cylinder if 360cm^3, What would be the volume of Cone

Ans : Vol of Cone = $^1/_3 \times$ Vol of Cylinder

$= {}^1/_3 \times 360$

$= \mathbf{120cm^3}$

4. What is the formula to findout the Total surface area of a frustrum of cone?

Ans : $A = \pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$

5. Find the volume of a Cone whose height is 4cm and the diameter of its base is 21cm

Ans : h = 4cm r = $^d/_2 = {}^{21}/_2$

$V = {}^1/_3\ \pi r^2 h$

$= {}^1/_3 \times {}^{22}/_7 \times ({}^{21}/_2)^2 \times 4 = {}^1/_3 \times {}^{22}/_7 \times {}^{22}/_7 \times {}^{21}/_2 \times 4 = 22 \times 21 = \mathbf{462cm^3}$

6. Find the Curved surface area, Total surface area and Volume of a cylinder of height 7cm and radius of base 5cm

Ans : $h = 7cm$ $r = 5cm$

$CSA = 2\pi rh = 2 \times {}^{22}/_{7} \times 5 \times 7 = 2 \times 22 \times 5 = \mathbf{220cm^2}$

$TSA = 2\pi r(r + h) = 2 \times {}^{22}/_{7} \times 5\ (5 + 7) = 2 \times {}^{22}/_{7} \times 5 \times 12 = 2 \times 3.14 \times 60 = \mathbf{376.8cm^2}$

$Vol = \pi r^2 h = {}^{22}/_{7} \times 5 \times 5 \times 7 = 22 \times 25 = \mathbf{550cm^3}$

7. Find the CSA and TSA of a Cone whose Slant height is 14cm and base radius 5cm.

Ans : $l = 14cm$ $r = 5cm$

$CSA = \pi rl = {}^{22}/_{7} \times 5 \times 14 = 22 \times 5 \times 2 = \mathbf{220cm^2}$

$TSA = \pi r(r+l) = {}^{22}/_{7} \times 5(5 + 14) = {}^{22}/_{7} \times 5 \times 19 = 3.14 \times 95 = \mathbf{298.3cm^2}$

8. Find the surface area and Volume of a Sphere of diameter 28cm

Ans : $d = 28cm$ $r = 14cm$

Surface area $= 4\pi r^2 = 4 \times {}^{22}/_{7} \times 14 \times 14 = 4 \times 22 \times 2 \times 14 = \mathbf{2464cm^2}$

Volume $= {}^{4}/_{3}\ \pi r^3 = {}^{4}/_{3} \times {}^{22}/_{7} \times 14 \times 14 \times 14 = 1.33 \times 22 \times 2 \times 196 = \mathbf{11469.92cm^3}$

9. Find the TSA of a Frustrum of Cone of Slant height 10cm whose radii are 14cm and 7cm.

Ans : $l = 10cm$ $r_1 = 14cm$ $r_2 = 7cm$

$CSA = \pi(r_1 + r_2)l = {}^{22}/_{7}\ (14 + 7)10 = {}^{22}/_{7} \times 21 \times 10 = 22 \times 3 \times 10 = \mathbf{660cm^2}$

$TSA = A = \pi(r_1 + r_2)l + \pi(r_1^2 + r_2^2)$

$= {}^{22}/_{7}\ (14 + 7)10 + {}^{22}/_{7}\ (14^2 + 7^2)$

$= 660 + {}^{22}/_{7}\ (196 + 49)$

$= 660 + {}^{22}/_{7}(245)$

$= 660 + 22\ (35)$

$= 660 + 77 = \mathbf{1430cm^2}$

STATISTICS

1. Find the Mean, Median & Mode of the following data.

C I	1-10	11-20	21-30	31-40	41-50	51-60	61-70
f	2	5	7	5	6	3	2

C I	f	X	fX
0-10	2	5	10
10-20	5	15	75
20-30	7	25	175
30-40	5	35	175
40-50	6	45	270
50-60	3	55	165
60-70	2	65	130
N = 30		ΣfX = 1000	

Mean = $^{\Sigma fX}/_{N}$ = $^{1000}/_{30}$ = 33.3

C I	f	f_c
0-10	2	2
10-20	5	7
20-30	7	14
30-40	5	19
40-50	6	25
50-60	3	28
60-70	2	30
N = 30		

$^{N}/_{2} = {}^{30}/_{2} = 15$

$$\text{Median} = l + \frac{^{N}/_{2} - f_c}{f_m} \times h$$

$$= 30 + \frac{15 - 14}{5} \times 10$$

$$= 30 + {}^{1}/_{5} \times 10$$

$$= 30 + 2$$

Median = 32

C I	f
0-10	2
10-20	5 $\rightarrow f_0$
20-30	7 $\rightarrow f_1$
30-40	5 $\rightarrow f_2$
40-50	6
50-60	3
60-70	2

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_2 - f_0} \times h$$

$$= 20 + \frac{7 - 5}{2 \,.\, 7 - 5 - 5} \times 10$$

$$= 20 + {}^{2}/_{4} \times 10 = 20 + {}^{20}/_{4} = 20 + 5 = 25$$

2. Find the Mean, Median & Mode of the following data.

C I	25-35	35-45	45-55	55-65	65-75
f	7	8	16	10	9

C I	f	X	fX
25-35	7	30	210
35-45	8	40	320
45-55	16	50	800
55-65	10	60	600
65-75	9	70	630
N = 50		ΣfX = 2560	

Mean = $^{\Sigma fX}/_{N}$ = $^{2560}/_{50}$ = 51.2

C I	f	f_c
25-35	7	7
35-45	8	15
45-55	16	31
55-65	10	41
65-75	9	50
N = 50		

$^{N}/_{2} = {}^{50}/_{2} = 25$

$$\text{Median} = l + \frac{^{N}/_{2} - f_c}{f_m} \times h$$

$$= 45 + \frac{25 - 15}{16} \times 10$$

$$= 45 + {}^{10}/_{16} \times 10$$

$$= 45 + {}^{100}/_{16}$$

Median = 45 + 6.25 = 51.25

C I	f
25-35	7
35-45	8 → f_0
45-55	16 → f_1
55-65	10 → f_2
65-75	9

$$\textbf{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_2 - f_0} \times h = 45 + \frac{16 - 8}{2 \cdot 16 - 10 - 8} \times 10 = 45 + {}^{8}/_{14} \times 10$$

$$= 45 + {}^{80}/_{14} = 45 + 5.7 = \mathbf{50.7}$$

For Practice : Find the Mean, Median & Mode of the following data.

C I	1-5	6-10	11-15	16-20	21-25
f	3	5	4	2	6

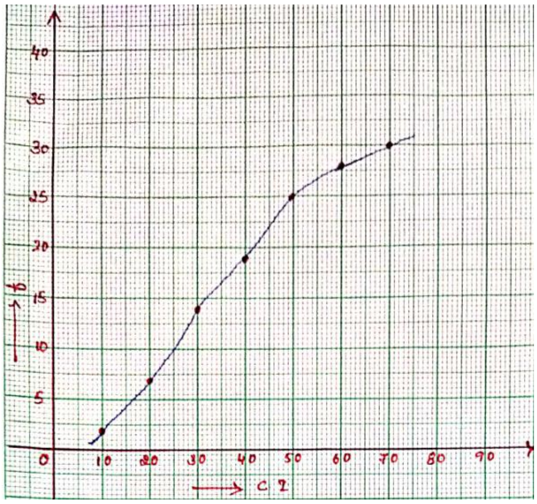
C I	16-20	21-25	26-30	31-35	36-40
f	5	6	8	4	7

C I	100-120	120-140	140-160	160-180	180-200
f	8	9	7	5	6

Drawing an O-give Curve

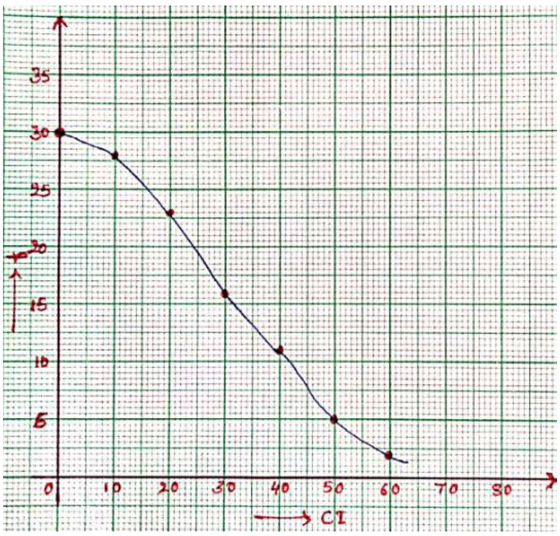
3. Draw a less than type O-give for the following data.

C I	f	Less than fc
0-10	2	2
10-20	5	7
20-30	7	14
30-40	5	19
40-50	6	25
50-60	3	28
60-70	2	30



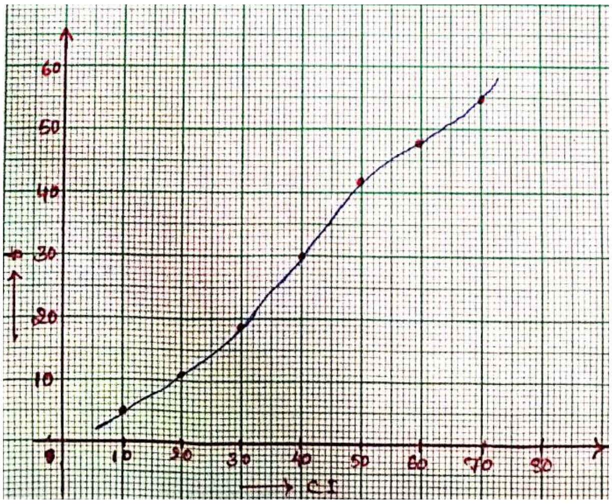
4. Draw a More than type O-give for the following data

C I	f	More than fc
0-10	2	30
10-20	5	28
20-30	7	23
30-40	5	16
40-50	6	11
50-60	3	5
60-70	2	2



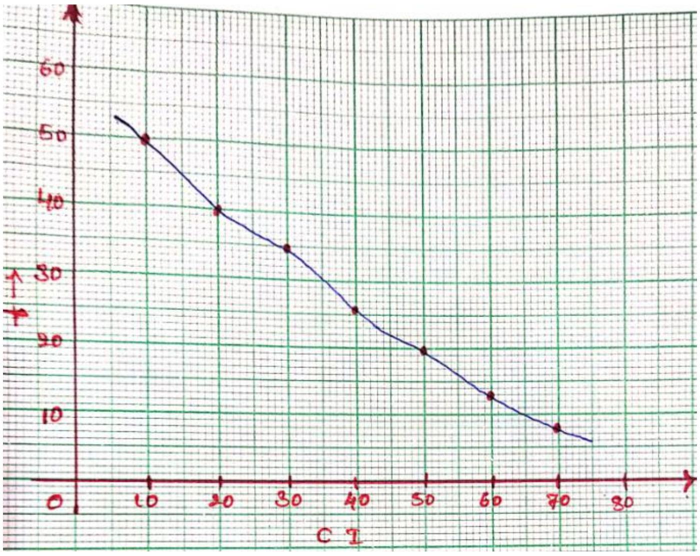
5. Draw Less than type O-give for the following data.

C I	fc
< 10	5
< 20	11
< 30	19
< 40	30
< 50	42
< 60	48
< 70	55



6. Draw a More than type O-give for the following data.

C I	fc
> 10	50
> 20	40
> 30	35
> 40	26
> 50	20
> 60	13
> 70	8



7. If the mean of 4, x, 6, 9 is 6, find the value of x.

$$\text{Mean} = \frac{\text{Sum of all terms}}{\text{Number of terms}} = \frac{4 + x + 6 + 9}{4}$$

$$6 = \frac{19 + x}{4} \longrightarrow 24 = 19 + x \longrightarrow x = 24-19 \longrightarrow x = 6$$

8. Write the Modal Class in the following data.

C I	f
0-10	2
10-20	5
20-30	7
30-40	5
40-50	6
50-60	3
60-70	2

as 7 is the highest frequency in the data, 20-30 is the modal class

9. Write the Median in 2, 5, 9, 7, 4, 11, 1

1, 2, 4, 5, 7, 9, 11
here middle term is 5.
Hence 5 is Median

Arithmetic Progression

1. General form of A.P is

   $a, a+d, a+2d, a+3d \ldots \ldots \ldots ..$

2. n^{th} term of an A.P is

   $a_n = a + (n-1)d$

3. n^{th} term from last of an A.P is

   $l - (n-1)d$

4. Common difference of an A.P is

   $d = a_2 - a_1 \quad (or) \quad d = a_3 - a_2$

5. The sum of first 'n' positive integer

   $S_n = \frac{n(n+1)}{2}$

6. The sum of 'n' odd natural numbers

   $S_n = n^2$

7. The sum of 'n' even natural numbers

   $S_n = n(n+1)$

8. Sum of first 'n' terms of an A.P is

   $S_n = \frac{n}{2}[2a + (n-1)d]$

9. Sum of AP, if first and last terms are given

   $S_n = \frac{n}{2}[a + a_n]$ **(or)** $S_n = \frac{n}{2}[a + l]$

10. In any progression

   $S_n - S_{n-1} = a_n$

1. Find 10^{th} term of the sequence $a_n = 2n - 5$

Soln: $a_n = 2n - 5$

$a_{10} = 2(10) - 5$

$a_{10} = 20 - 5$

$\therefore a_{10} = 15$

Drill work

1. If the n^{th} term of an AP $a_n = 3n - 2$. Find the 9^{th} term.
2. If the n^{th} term of an AP $a_n = 24 - 3n$. Find the 2^{th} term.
3. If the n^{th} term of an AP $a_n = 5n + 3$. Find the 3^{th} term.

2. Find the 10^{th} term of AP 2, 7, 12, ... using formula.

Soln: $a = 2,\ d = 5,\ n = 10$

w.k.t $a_n = a + (n-1)d$

$a_{10} = 2 + (10-1)5$

$a_{10} = 2 + 9 \times 5$

$a_{10} = 2 + 45$

$\therefore \mathbf{a_{10} = 47}$

Alternate Method

$a_{10} = a + 9d$

$a_{10} = 2 + 9 \text{ X } 5$

$a_{10} = 2 + 45$

$\therefore \mathbf{a_{10} = 47}$

Drill work

1. In an AP 21, 18, 15, ... find 35^{th} term.
2. In an AP 3, 8, 13, ... find 12^{th} term.
3. In an AP 10, 7, 4, ... find 18^{th} term.
4.

3. Find the sum of 2+5+8+……... to 20 terms

Soln: $a = 2, d = 3, n = 20$ and $S_n = ?$

w.k.t $S_n = \frac{n}{2}[2a + (n-1)d$

$S_{20} = \frac{20}{2}[2 \text{ x } 2 + (20-1) \text{ x } 3]$

$S_{20} = 10[4 + 19 \text{ x } 3]$

$S_{20} = 10[4 + 57]$

$S_{20} = 10 \text{ x } 61$

$\therefore \mathbf{S_{20} = 610}$

Drill work

1. Find the sum of first 20 terms of an AP 3, 7, 11, 15, …….
2. Find the sum of first 25 terms of an AP 5, 10, 15, 20, …….
3. Find the sum of first 18 terms of an AP 2, 7, 12, …….
4. Find the sum of: 1+5+9+ ……. up to 25 terms.
5. Find the sum terms of an AP 2, 7, 12, …….