Government of Karnataka

Karnataka Residential Educational Institutions Society - Bengaluru

10TH MATHEMATICS STUDY MATERIAL 2020-2021

INSTRUCTIONS TO THE STUDENTS/TEACHERS

Dear teachers/students,

1. This study material is the substitute of Text book.
2. At first go through the text book chapter wise to develop a basic concept of the various topics of the chapter.
3. Study material topic wise relating it with the text book to develop lateral thinking.
4. Give more attention on the theorems and graphs, constructions given in the study material.
5. First practice to solve the previous years question papers.
6. Practice the CBSE/state board model/sample question papers to clear your concept more and more to the topic concerned.
9. Make a group of 4 to 5 student to prepare and to solve problems.
10. In case of any difficulty in understanding the topic take the help of concern subject teacher.
11. This study material has been made in such a way that students will be fully guided to prepare for the exam in the most effective manner, securing higher grades problems.
12. To make the students understand the chapter completely, each chapter has been divided in to individual topics and each key points has easy to understand.
13. Majority of questions is answered in this study material and did not solve the exercises questions which will be given by your teacher.
14. Using keys points assume/imagine to construct the different types of questions which will be given in board examinations.

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ARITHMETIC PROGRESSION

Introduction:

You must have observed that in nature, many things follows a certain pattern such as the petals of a sunflower the holes of a honeycomb The grain in a maize cob, the spirals in a pineapple and in a pine etc.,,...and also, if we observe in our regular lives, we come across, Arithmetic progression quite often, for example Roll numbers of a student's in a class, days in a week or months in a year. This pattern of series and sequence has been generalised in maths as progression.

Example 1 :

Shalika puts ?100 into her daughter money box when she was 1 year old and increased the amount by ? 50 every year. The amount of money (in ?)in the box on the the 1^{st} ,2^{nd} ,3^{rd},4^{th},....... Birthdays were

100,150,200,250......respectively.

In the example above we observe some pattern and we found that the succeeding terms are obtained by adding a fixed numbers.

In this chapter we shall discuss one of these pattern in which succeeding term are obtained by a fixed number to the preceding terms.

We shall also see how to find there n^{th} term and the sum of n consecutive and use this knowledge in solving some daily life problems.

1.2 Arithmetic progression:

Consider the following list of numbers

* 1, 2,3, 5....... * 10,20,30,40........

* 20, 40, 60....... * -1, 0, 1, 2, 3....

Each of the numbers in the list is called a **term**

<u>Definition</u> :

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

This fixed number is called the **common difference (c.d)** of the arithmetic progression.

It can be **positive, negative or zero**.

Common difference in the A.P

In this progression for a given series the terms used are the first term a_1 second terms a_2. . . . .n^{th} terms are a_n. The common difference between the two terms and the n^{th} terms is

$$d = a_2 - a_1 = a_3 - a_2 = \text{-----} = a_n - a_{n-1}$$

where d is common difference, it can be **+ve, -ve and zero**

For example:

1) The heights (in cm) of some students of a school standing in a queue in the morning assembly are

147,149,151,153........159

In this first term **a** is 147 and common difference **d**=2

*So in above example we observe that a, a+d, a+2d, a+3d... ......represents an arithmetic progression where **a** is the first term and **d** is the common difference this is called **"the general form of an A.P"**

*If the sequence contains finite terms called **finite A.P (**contains the terms a_n) otherwise called **infinite arithmetic progression**

* Now ,to know about an A.P the minimum information that you know is first term and the common difference . If you know these two terms **a and d** we can calculate any terms in an A.P.

For instance if the first term is 6 and C.D(d) is 3 then A.P is 6,9,12,15,........

If a=6 and d=-3 then the A.P is 6,3,0,-3 and so on.

Sometimes if the sequence is given ,we can easily obtain the numbers of terms by seeing first term and common difference.

For example: The list of numbers in an A.P is 6,9,12,15

we have $a_2 - a_1 = 9 - 6 = 3$

$a_3 - a_2 = 12 - 9 = 3$

$a_4 - a_3 = 15 - 12 = 3$

Here the difference of any two consecutive terms in each is **3**.Hence first term is **6** and **d=3**

In general for an A.P $a_1, a_2, a_3 \ldots\ldots\ a_n$

we have $d = a_{k+1} - a_k$

where a_{k+1} and a_k are the $(k+1)^{th}$ and the k^{th} terms respectively.

n^{th} term of an A.P:

We know that general form of an A.P

$a_1, a_2, a_3, a_4, \ldots\ldots$ **is** $a, a+d, a+2d, a+3d, \ldots\ldots\ldots a+(n-1)d$

Suppose, if we have finite terms in an A.P i.e $a_1, a_2, a_3, a_4, \ldots\ldots\ldots\ldots\ a_n$ then it's n^{th} term of an A.P. is given by

a,a+d,a+2d,a+3d,……………..a+(n-1)d

So, the n^{th} term a_n of the A.P with first term a and common difference d is given by

$$\boxed{a_n = a+(n-1)d}$$

Note: If there are n terms in an A.P. then a_n represents the last term which is sometimes also denoted by l, i.e **l= a+(n-1)d**

Examples: 1) Find the 10^{th} term of the A.P.2,7,12…..

Sol: Here a=2, d=7-2=5 and n=10

We have $a_n = a+(n-1)d$

$a_{10} = 2+(10-1)5 = 2+45 = 47.$

$\therefore$ **10^{th}** term of the given A.P.is 47.

2) Which term of the A.P: 21, 18, 15……. is -81?

Solution: Here a=21, d=18-21=-3 and a_n=-81

And we have to find n

$a_n = a+(n-1)d$

$-81=21+(n-1)(-3)$

$-81= 24-3n$

$-105=-3n$

n=35

$\therefore$ The 35^{th} term of the given A.P is **-81**.

3**) Determine the A.P whose 3^{rd} term is 5 and 7^{th} term is 9**.

Solution: We have

$a_3 = a+(3-1)d = a+2d=5$ ----------(1)

$a_7 = a+(7-1)d = a+6d=9$ ----------(2)

Solving (1) and (2) we get **a=3, d=1**

Hence the required A.P.is **3,4,5,6,7,8,9**

1) In which of the following situations, does the list of numbers involved make an arithmetic progression and why?

(i) The taxi fare after each km when the is Rs. 15 for the first km Rs. 8 for each additional km.

Solution : It can be observed that

Taxi fare for 1^{st} km = 15

2^{nd} km = 15+8=23

3^{rd} km = 23+8=31

Clearly 15, 23, 31, 39, ………. Forms an A.P because every term is 8 more than preceding term.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ th of the air Remaining in the cylinder at a time.

Solution: Let the initial volume of air in a cylinder be V lit. In each stroke , the vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

In other words, after every stroke, only 1 - $\frac{1}{4}$ th = $\frac{3}{4}$ th part of air will remain.

There fore, Volumes will be $V, (\frac{3V}{4}), (\frac{3V}{4})^2, (\frac{3V}{4})^3$............

Clearly, it can be observed the adjacent terms of this series do not have the same difference between them. **Therefore, this is not an A.P.**

2) Write first four terms of the A.P. when the first term a & the common difference d are given as follow.

i) a= 10, d = 10

Solution: $a_1 = a = d$, $a_2 = a_1 + d = 10+10=20$

$a_3 = a_2 + d - 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

∴ First four terms of this A.P will be 10, 20, 30 ,40

ii) a = -1, d = ½

$a_1 = a = -1$, $a_2 = a_1 + d = -1 + ½ =- ½$

$a_3 = a_2 + d = -{}^1/_2 + ½ = 0$,

$a_4 = a_3 + d$

= 0 + ½

= ½

∴ First four terms of this A.P will be -1, - ½, 0 & ½

Now to know about an A.P the minimum information that you know is first term & the common difference of you know these two terms a & d we can calculate any terms in an A.P.

For instance if the first term is 6 & c.d (d) is 3 then the A.P is 6, 9, 12, 15 ....

If a = 6 & d = -3 then the A.P is 6, 3, 0, -3 and so on.

Sometimes if the sequence is given we can clearly obtain the number of terms by seeing first & common difference.

For example , The list of numbers in an A.P is 6, 9, 12, 15

we have $a_1 - a_1 = 9 - 6 = -3$

$a_3 - a_2 = 12 - 9 = -3$

$a_4 - a_3 = 15 - 12 = -3$

Hence the difference of any two consecutions terms in each case is 3.

Hence first term is 6 & c-d=3

In general for an A.P $a_1, a_2, a_3, \ldots\ldots\ldots a_0$

We have $d = a_{k+1} - a_k$

Where a_{k+1} & a_k are the (k+1) and k^{th} terms respectively.

Sum of first n terms of an A.P.

For any progression, the sum of n terms can be easily calculated. For an A.P. the sum of the first n terms can be calculated if the first term and the total terms is known.

Let us consider a simple example. We consider the problem given to Gauss, to solve when he was just 10 years old. He was asked to find the sum of the positive integers from 1 to 100.He immediately replied that the sum is 5050.Can you guess how did he do? He wrote:

S=1+2+3+......................+99+100

And then, reversed the numbers to write

S= 100+99+.................+3+2+1

Adding these two, we get

2S= (100+1) + (99+2) +..............................+(3+98)+(2+99)+(1+100)

2S= 101+101+.....................+101+101 (100 times)

S= 100x101/2

S= 5050 i.e. **The sum = 5050**

We will now use the same techniques to find the terms of the first n terms of an A.P.

a, a+d, a+2d, ...

The n^{th} terms of this A.P is **a + (n-1)d.** Let Sn denote the sum of the first n terms of the A.P. we have,

$S_n = a + (a+d) + (a+2d) + \ldots\ldots\ldots\ldots + [a+(n-1)d]$ ....(1)

Rewriting the terms in reverse order, we have

$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots\ldots\ldots\ldots +(a+d)+ a$..........(2)

On adding (1) & (2), term-wise we get

$2S_n = \{[2a+(n-1)d] + [2a+(n-1)d] + \ldots\ldots\ldots\ldots + [2a+(n-1)d] + [2a+(n-1)d]\}$ ....n times.

or $2S_n = n\,[2a+(n-1)d]$

$S_n = {}^{n}/_{2}\,[2a+(n-1)d]$

So, **the sum of the first n terms of an AP in given by**

$$\boxed{S_n = \frac{n}{2}\,[2a + (n-1)d]}$$

We can also write this $s = {}^{n}/_{2}\,[a+a+(n-1)d]$

$$\boxed{S_n = \frac{n}{2}[a + a_n] \quad \text{or} \quad S_n = \frac{n}{2}[a + l]}$$

This form of the result is useful when the first & the last terms of an AP are given & the common difference is not given.

2) For the following APs, write the first term & common difference

(i) 3, 1, -1, -3, ...

Here first term $a = 3$

Common difference $d = a_2 - a_1$

$= 1 - 3$

$d = \mathbf{-2}$

(ii) ${}^{1}/_{3}$, ${}^{5}/_{3}$, ${}^{9}/_{3}$, ${}^{13}/_{3}$, ...

Here first term $a = {}^{1}/_{3}$

Common difference, $d = a_2 - a_1$

$= {}^{5}/_{3} - {}^{1}/_{3}$

$\mathbf{d = {}^{4}/_{3}}$

3) Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	A	d	n	a_n
(i)	7	3	8	-
(ii)	-18	-	10	0
(iii)	-	-3	18	-5

(iv)	-18.9	2.5	-	3.6
(v)	3.5	0	105	-

Ans : (i) $a=7, d=3, n=8, a_n=?$

We know that

For an A.P. $a_n = a+(n-1)d$

$= 7+(8-1)3$

$= 7+7X3=7+21$

$= 28$

Hence $\mathbf{a_n = 28}$

(ii) Given that

$a = -18, n=10, a_n=0, d=?$

we know that

$a_n = a+(n-1)d$

$0 = 18+(10-1)d$

$18= 9d \Rightarrow d= {}^{18}/_{9} = 2$

Hence, **d =2**

(iii) Given that

$d=-3, n=18, a_n=-5$

we know that

$a_n = a+(n-1)d$

$-5 = a+(18-1)(-3)$

$-5 = a+(17)(-3)$

$-5= a-51$

$a = 51-5=46$

Hence **a=46**

(iv) Given that

$a = -18.9, d = 2.5, n = ?, a_n = 3.6$

we know that

$a_n = a+(n-1)d$

$3.6 = -18.9+(n-1)\, 2.5$

$3.6 + 18.9 = (n-1)\, 2.5$

$22.5 = (n-1)\, 2.5$

$n-1 = \frac{22.5}{2.5} = 9$

$n = 9+1 = 10,$

Hence **n = 10**

(v) $a = 3.5, d = 0, n=10.5, a_n=-?$

we know that

$a_n = a+(n-1)d$

$a_n = 3.5+(10.5-1)\, 0$

Hence an = 3.5

4) The angles of a triangle are in AP. The greatest angle is twice the least. Find all angles of a triangle.

Solution: Let the angles of a triangle be A, B & C such that $A < B < C$ & A, B & C are in A.P

Using property of triangle, sum of all internal angles is 180^0

$A + B + C = 180^0 \rightarrow$ (i)

$2B = A + C$ .... (ii) (Since A, B & C are in A.P)

$C = 2A$ .... (iii) (Given)

From (i) & (ii), we get

$2B + B = 180^0$, $B = 60^0$

From (ii), we have

$120^0 = A + C$

$120^0 = A + 2A$ .... From (iii)

$A = 40^0$, $C = 80^0$

Hence, the angles are 40^0, 60^0, & 80^0

5) In the following APs, find the missing terms in the Hence.

(i) 2, ----, 36

Solution: Given that

$a = 2$, $a_3 = 36$, $a_2 = ?$, $n=3$

we know that

$a_n = a+(n-1)d$

$a_3 = a+(3-1)d$

$36 = 2+2d$

$36-2 = 2d$

$34 = 2d$

$d = \frac{34}{2} = 17$

or

$d = a_2 - a_1 = a_3 - a_2$

$a_1 + d - 2 = 36 - (a_1 + d)$

$2 + d - 2 = 36 - 2 - d$

$d + d = 34$

$2d = 34$

$d = \frac{34}{2}$

$\mathbf{d = 17}$

Hence, $a_2 = a + d$

$= 2 + 17$

$\mathbf{a_2 = 19}$

Try these ii) 5, -----, 11 iii) 3, -----, -----, 9 iv) 7, -----, -------, ------, 19

6) Find the 31st term of an AP whose 11th term is 38 & the 16th term is 73

Solution : Given that

$a_{11} = 38$, $a_{16} = 73$, $a_{31} = ?$

We know that

$a_n = a+(n-1)d$

$a_{11} = a+(11-1)d$

38 = a+10d -------------- (1)

$a_{16} = a+(16-1)d$

73 = a+15d

73 = a+10d+5d ----------- (2)

From (1) & (2), we have subtracting (1) from (2) we rotain

73 = 38+5d		73 = a + 15d
5d = 73-38	**or**	38 = a + 10d
5d = 35		35 = 5d
d = 35/5= 7		d =35/5 = 7
d = 7		d = 7

Substitute the value of d in equation (1)

38 = a+10X7

38 = a+70

a = 38-70

a = -32

$a_{31} = -32 + (31-1)7$

= -32 + 30X7

= -32+210

$a_{31} = 178$

Hence 31^{st} term is **<u>178</u>**

6) **The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.**

Solution: we know that

$a_n = a+(n-1)d$

$a_{17} = a+(17-1)d$

$a_{17} = a+16d$

$a_{17} = a+16d$

Similarly $a_{10} = a+9d$

It is given that

$a_{17} = a_{10}+7$

a_{17}-a_{10}=7

a+16d – a -9d = 7

7d = 7

d = 1

d = 1

∴ The common difference is 1

7) Two APs have the some common difference. The difference between their 100th term is 100. What is difference between their 1000th term?

Solution: Let the first term of these APs be a_1 & a_2 respectively & the common difference of these APs be & d.

For first A.P

$a_{100} = a_1+(100-1)d = a_1+99d$

$a_{1000} = a_1+(1000-1)d = a_1+999d$

For second A. P

$a_{100} = a_2+(100-1)d = a_2+99d$

$a_{1000} = a_2+(1000-1)d = a_2+999d$

Given that difference between 100th term of these APs = 100

Therefore $(a_1+99d) – (a_2+99d) = 100$

$a_1 – a_2 = 100$ (1)

Difference between 1000th terms of these A.Ps

$(a_1+999d) – (a_2+999d) = a_1 – a_2$

From equation----(1)

This difference a_1-a_2=100

∴ The difference between 1000th terms of these A.P will be 100

9) How many three digit numbers are divisible by 7?

Soliution: First three digit number that will be divisible by 7 = 105

Next number = 102 + 7 = 112

Therefore 105, 112, 119, …….. are terms of an A.P having first term is 105 & common difference as 7

The maximum possible three digit number is 999. The remainder will be 5, clearly 999-5 = 994 is the maximum possible three digit number that will be divisible by 7

The series is as follows ; 105, 112, 119, ……………994

Let 994 be the nth term of this A.P

$a = 105$, $d = 7$, $a_n = 994$, $n = ?$

$a_n = a + (n-1)d$, $994 = 105 + (n-1)7 = ?$

$994 - 105 = (n-1)\ 7$

$889 = (n-1)\ 7$

$n-1 = 127$ **$n = 128$**

10) In an AP i) given $a=5, d=3, a_n=50$, find n and s_n.

Solution: Given that $a=5, d=3, a_n=50$

As $a_n = a + (n-1)d$

$50=5+(n-1)3$

$45=(n-1)3$

$15=n-1 \Rightarrow n=16$

$s_n = \frac{n}{2}[a + a_n]$

$s_{16} = \frac{16}{2}[5 + 50]$

$=8x55$

S_n =440.

ii) Given $a=7$, $a_{13}=35$, find d and s_{13}

Solution: Given that $a=7$, $a_{13}=35$

As $a_n = a + (n-1)d$

$a_{13}=a+(13-1)d$

$35=7+12d$

$28 = 12d \Rightarrow d = \frac{7}{3}$

$s_n = \frac{n}{2}[a + a_n]$

$s_{13} = \frac{n}{2}[a + a_{13}]$

$=\frac{13}{2}[7 + 35]$

$S_{13} = \frac{13x42}{2} = 13x21 =$ **273.**

11) How many terms of the AP: 9, 17, 25……….must be taken to give a sum of 636?

Solution: Let there be n terms of this AP.

For this AP, a=9, d= $a_2 - a_1 = 17 - 9 = 8$

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

$$636 = \frac{n}{2}[2x9 + (n-1)8]$$

$$636 = \frac{n}{2}[18 + (n-1)8]$$

$636 = n\ [9+4n-4] = n[5+4n]$

$$4n^2 + 5n - 636 = 0$$

$$4n^2 + 53n - 48n - 636 = 0$$

$$n(4n+53) - 12(4n + 53) = 0$$

$$(4n + 53)(n - 12) = 0$$

Either $4n + 53 = 0$ $or\ n - 12 = 0$

n= $\frac{-53}{4}$ or n=12 n cannot be $\frac{-53}{4}$. as the number of terms can neither be negative nor fractional, therefore **n=12.**

12) The first term and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution: Given that a=17, l = 350, d=9

Let there be n terms in the A.P

$$l = a + (n - 1)d$$

$$350 = 17 + (n - 1)9$$

$$333 = (n - 1)9$$

$$(n - 1) = 37 \qquad n = 38$$

$$s_n = \frac{n}{2}(a + l)$$

$$s_n = \frac{38}{2}(17 + 350) = 19(367) = \mathbf{6973}$$

13) If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution: Given that, $s_7 = 49, s_{17} = 289$

$$s_n = \frac{n}{2}[2a + (n - 1)d]$$

$$s_7 = \frac{7}{2}[2a + (7 - 1)d]$$

$$49 = \frac{7}{2}[2a + 6d] = 7(a + 3d)$$

$$a + 3d = 7 \ldots\ldots(i)$$

Similarly,

$$s_{17} = \frac{17}{2}[2a + (17 - 1)d]$$

$$289 = \frac{17}{2}[2a + 16d] = 17(a + 8d)$$

$$a + 8d = 17 \ldots\ldots(ii)$$

Subtracting equation (i) from equation (ii),

$5d = 10 \quad \Rightarrow \quad d = 2$

From equation (i), $a+3(2) = 7$, $a+6 = 7$

$a = 1$

14) If the sum of the first n terms of an AP is $4n-n^2$, Find S_1 ? Find the sum of first two terms ? What is the second term ? Similarly , find the 3^{rd}, 10^{th} and the n^{th} term.

Solution: Given that,

$S_n = 4n-n^2$

First term, $a=S_1 =4(1)-(1)^2=4-1=3$

Sum of the first two terms $=S_2 = 4(2)-(2)^2=8-4=4$

Second term, $a_2=S_2-S_1=4-3=1$

$d=a_2-a_1=1-3=-2$

$$a_n = a + (n - 1)d$$

$= 3+(n-1)(-2)$

$=3-2n+2$

$a_n = 5-2n$

$a_3=5-2(3)=5-6=-1$

$a_{10}=5-2(10) = 5-20 = -15$

Hence the sum of first two terms is 4. The second term is 1. And 3^{rd}, 10^{th}, n^{th} terms are -1, -15, and 5-2n respectively.

15) A sum of ?700 is to be used to give seven cash prize to students of a school for their overall academic performance. If each prize is ?20 less than its preceding prize, Find the value of each of the prizes.

Solution: Let the cost of first prize be ‘p’ cost of second prize = p-20 and

the cost of third prize = p-40 it can be observed that cost of these prizes are in AP having common difference as -20 and the first term as p

$S_7 = 700$, d = -20, n = 7, a = p

$$s_n = \frac{n}{2}[2a + (n-1)d]$$

$$700 = \frac{7}{2}[2p + (7-1)(-20)]$$

$700 = \frac{7}{2}[2p-120]$ ➔ 1400=7(2p-120) ➔ 1400/7= 2(p-60)

200=2(p-60)

100=p-60

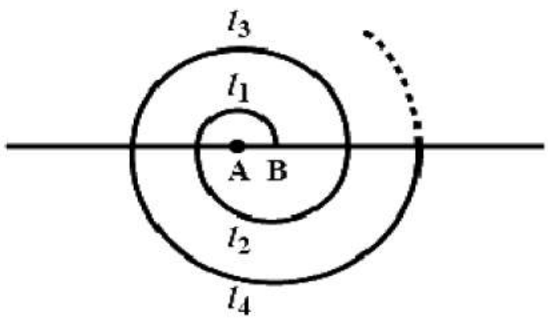
➔ p=160

∴ The value of each of the prizes was Rs.160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, Rs 40.

16) A spiral is made up of successive semicircles, with centers alternately at A and B, starting with centers at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles?

n=13, r_1=0.5cm, r_2=1cm

r_3=1.5cm…..



Semi-perimeter of circle =πr

The length of the circle $l_1=\pi(0.5) = \frac{\pi}{2}$ cm.

The length of the circle $l_2=\pi(1) = \pi$

The length of the circle $l_3=\pi(1.5) = \frac{3\pi}{2}$

l_1, l_2, l_3 the length of semicircle are in A.

$\frac{\pi}{2}$,π, $\frac{3\pi}{2}$, 2 π......

Hence a= $\frac{\pi}{2}$, d= $\frac{\pi}{2}$

$S_{13}=13/2[2\ (\frac{\pi}{2})+(13-1)\ (\frac{\pi}{2})\ ]$

$\Rightarrow 13/2[\pi+6\pi]$

$\Rightarrow\ \frac{13}{2}\times 7\pi$

$\Rightarrow\ \frac{91}{2}\ \pi$

$\Rightarrow\ \frac{91\ X\ 22}{2\ X\ 7}$

$\Rightarrow$ **143 cm .**

<u>Practice problems</u>:

1. In an A.P. of 5 terms , the sum of middle three terms is 24 and the product of first and last terms is 48. Write the A.P.
2. The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the A.P.
3. The 12^{th} term of an A.P. is -13 and the sum of first 4 terms is 24 then find the sum of first 10 terms.
4. If 7 times the seventh term of an A.P. is equal to 11 times the 11^{th} term then P.T. 18^{th} term is 0.
5. In an A.P. If $a_n=3n-2$, then find a_{20} and S_{20}.
6. In A.P. the sum of first, third and fifth terms is 24 and that of second, fourth, and sixth terms is 33.Find the A.P.
7. The sum of three consecutive terms of an A.P. is 36 and the sum of their squares is 440. Find the terms.
8. The first term of an A.P. is 2 and the sum of first 5 terms is $1/4^{th}$ of the sum of next 5 terms. Show that $a_{20}=-112$.
9. The sum of first 11 terms of an A.P. is 44 and the sum of next 11 terms is 55. Find the A.P.
10. Find sum of an A.P. containing 35 terms and whose middle term is 100.
11. The third term of an A.P. containing 50 terms is 12 and the last term is 106. Find 25^{th} term.
12. If the first term of an A.P. is 5, last term is 45 and the sum of all terms is 400.Find the number of terms and common difference.
13. Find the sum of first 30 positive integers which are divisible by 4.
14. If 2x + 1, $x^2 + x +1$, $3x^2 - 3x +3$ are consecutive terms of an AP. Find the value of x.

15. The ratio of first 'n' terms of two A.P.'s is 7n+1:4n+27.Find the ratio of their 11th terms.
16. If the sum of 'P' terms of an A.P. is $3p^2+4p$.Find the n^{th} term.
17. The 8^{th} term of an A.P. is half of its 2^{nd} term and 11^{th} term exceeds one third pf its 4^{th} term by 1.Find the 20^{th} term.
18. Divide 96 into 4 parts which are in A.P and the ratio b/n the product of their means to the product of their extremes is 15:7.Find the A.P.
19. If the angles of a quadrilateral are in AP and the ratio of smallest and biggest angle is 1 : 3. Find the angles of the quadrilateral.
20. Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and 16^{th} term is 73.

Important instructions to solve the applied problems on A.P.

- **First we have to understand the question carefully.**
- **Convert the statements given in the question into equations.**
- **Find 'a' in terms of 'd' or 'd' in terms of 'a' from one equation and substitute it into another equation then we get values of 'a' and 'd'. (or we can also use elimination method of solving pair of linear equations to find 'a' and 'd').**
- **If we get 'a' and 'd' , then we can find any term and sum of terms by using formulae.**

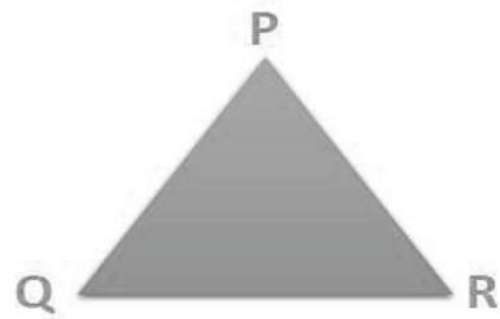
****

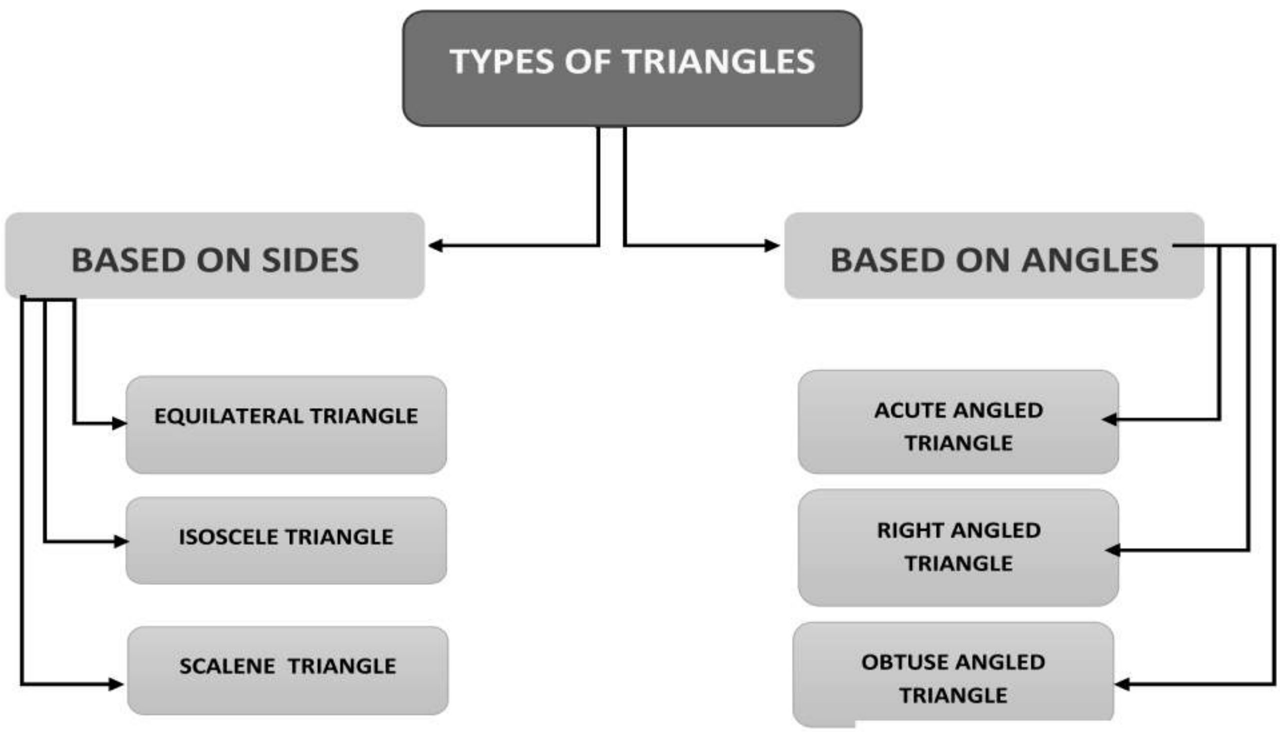
TRIANGLES

Definition:- A closed figure bounded by three line segments is called Triangle.

Triangle has:

1) Three vertices → P,Q, R
2) Three sides → PQ , QR, RP
3) Three angles $\angle P$, $\angle Q$, and $\angle R$

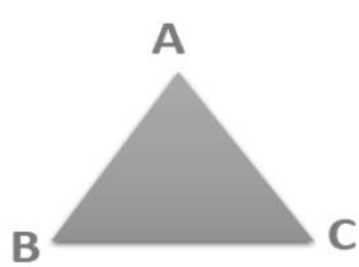




TYPE OF TRIANGLE BASED ON SIDES

EQUILATERAL TRIANGLES:

A triangle having equal sides and equal angles are called equilateral triangles.



Sides AB = BC = CA

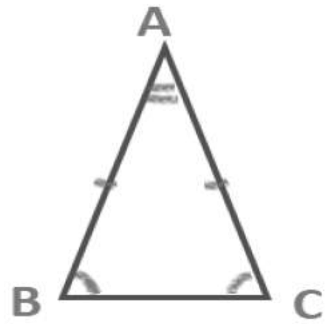
Angles: $\angle A = \angle B = \angle C$

Isosceles triangle;

A triangle having any two equal sides is called Isosceles triangle .

AB = AC ≠ BC

$\angle B = \angle C$

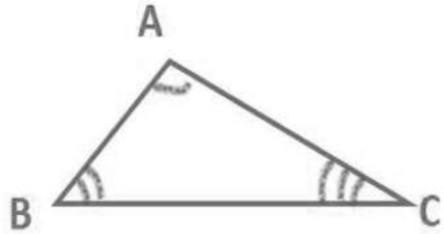


Scalene triangle:

A triangle with different sides is called Scalene triangle.

AB≠ BC ≠ CA

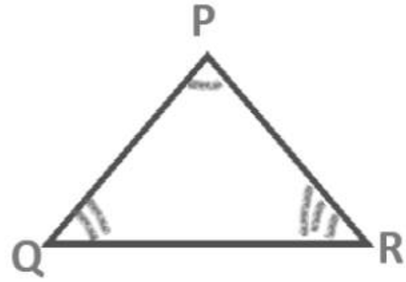
$\angle A \neq \angle B \neq \angle C$



TYPE OF TRIANGLE BASED ON ANGLES

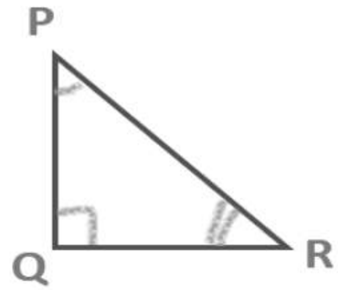
Acute angled triangle:

If a triangle having all 3 angles are less than 90^0 (i.e 0^0 –90^0)] , then it is called Acute angled triangle.



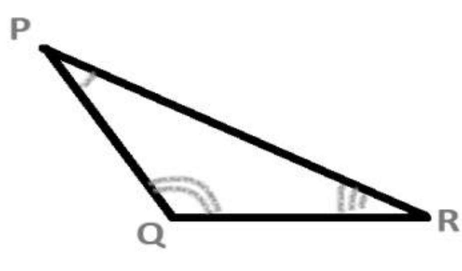
Right angled triangle:

If a triangle having any of the angle is right angle (equal to 90^0) , then it is called Right angled triangle.



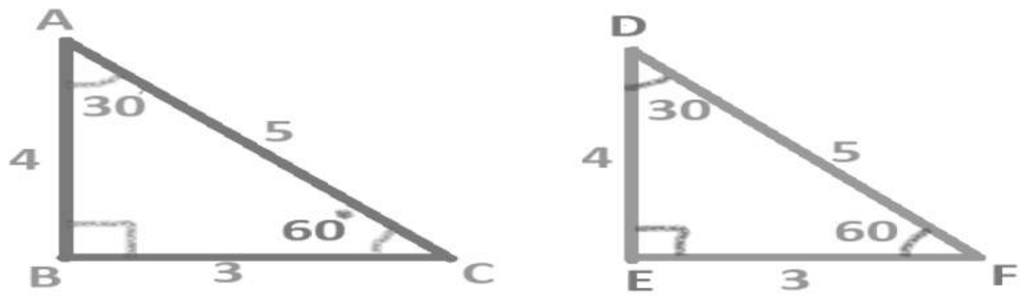
Obtuse angled triangle;

If a triangle having any of the angle is Obtuse angle (more than 90^0 , less than 180^0) is called Obtuse angled triangle.



CONGRUENT FIGURES:

Two figures are said to be congruent if they must have **same shape** and **same size.**



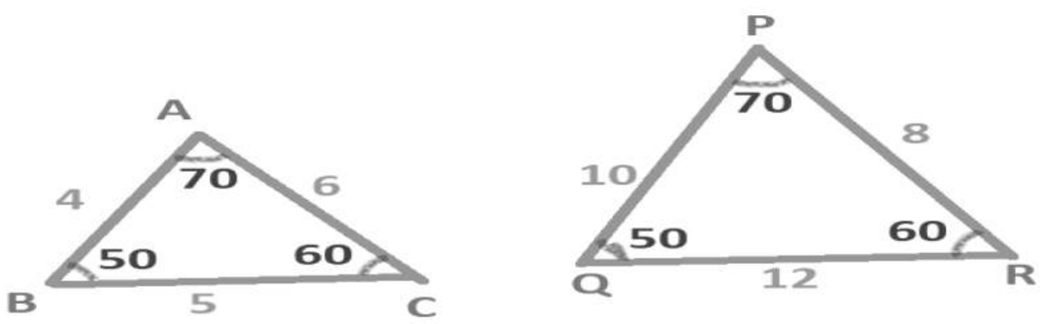
These two triangles having,

SHAPE	**SIZE**
$\angle A = \angle D = 30^0$	AB = DE = 4cm
$\angle B = \angle C = 90^0$	BC = EF = 3cm
$\angle C = \angle F = 60^0$	AC = DF = 5cm

∴ Two figures having same shape and same size.

Hence they are congruent figures.

$\Delta ABC \cong \Delta DEF$

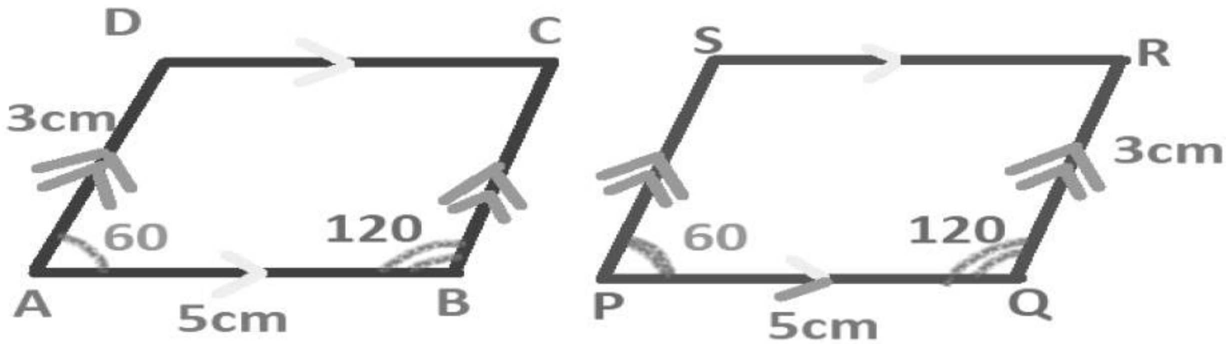


These two triangles having,

SHAPE	**SIZE**
$\angle A = \angle P = 70^0$	AB ≠ PQ
$\angle B = \angle Q = 50^0$	BC ≠ QR
$\angle C = \angle R = 60^0$	AC ≠ RP

∴ Two figures having same shape but not size

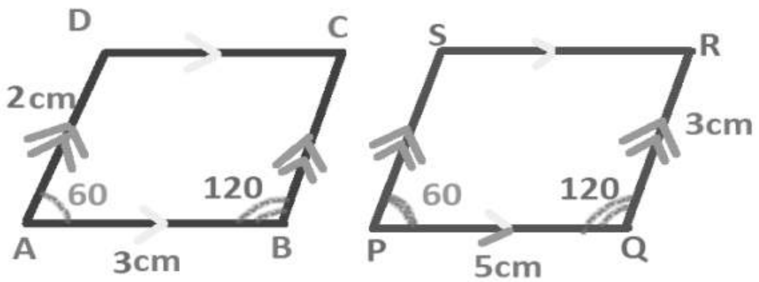
Hence they are not congruent figures. $\Delta ABC \ncong \Delta DEF$



Shape	**size**
$\angle A = \angle P = 60^0$	AB = PQ = 5cm
$\angle B = \angle Q = 120^0$	BC = QR = 3cm
$\angle C = \angle R = 60^0$	CD = PS = 5cm
$\angle D = \angle S = 120^0$	DA = PS= 3cm

Above figures having same shape and same size

Therefore, they are congruent.



$\angle A = \angle P = 60^0$	AB ≠ PQ
$\angle B = \angle Q = 120^0$	BC ≠ QR
$\angle C = \angle R = 60^0$	CD ≠ PS
$\angle D = \angle S = 120^0$	DA ≠ PS

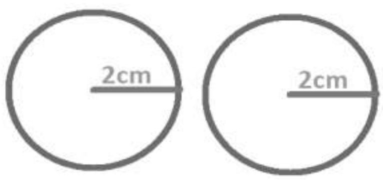
Above figures having same shape but not same size.

Therefore , they are not congruent.

SIMILAR FIGURES ;

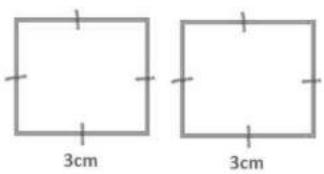
Two figures having same shape and difference in their size (not necessarily same size).

Congruent figures | **Similar figures**

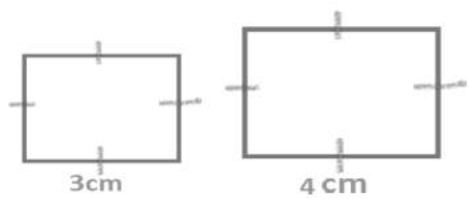


Two circles are congruent

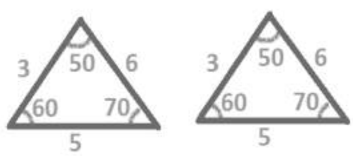
Two circles are similar



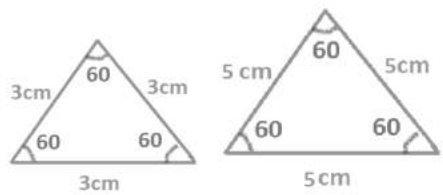
Two squares are congruent



Two squares are similar



Two triangles are congruent



Two triangles are similar

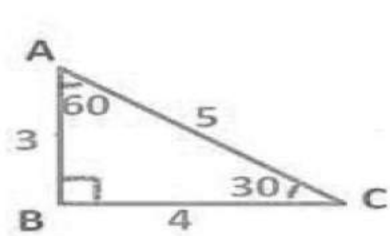
Note : All congruent figures are similar , But all similar figures may or may not be congruent.

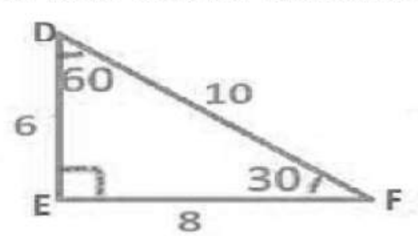
Similarity of Triangles:

Two triangles are similar, if

1) Their corresponding angles are equal.
2) Their corresponding sides are in the same ratio (or proposition)

Ex



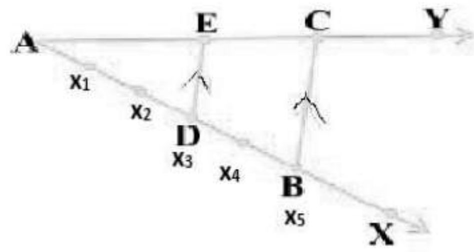


$\frac{AB}{DE} = \frac{3}{6} = \frac{1}{2}$ $\frac{BC}{EF} = \frac{4}{8} = \frac{1}{2}$ $\frac{AC}{DF} = \frac{5}{10} = \frac{1}{2}$

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=\frac{1}{2}$$

$$\Delta ABC \sim \Delta DEF$$

Activity :



Draw any angle XAY and on its one arm AX , mark points x1, x2, x3, x4, x5 such that

$Ax_1 = x_1x_2 = x_2x_3 = x_3\,x_4 = x_4\,x_5$

Also , through x_5 draw a line intersect AY at C.

Also, through x_3 draw a line parallel to BC ,

i.e., DE II BC

Now , the measurement of $\frac{AD}{DB}=\frac{3}{2}$,

Also measure AE and EC .

When we observe the measurements $\frac{AE}{EC}=\frac{3}{2}$

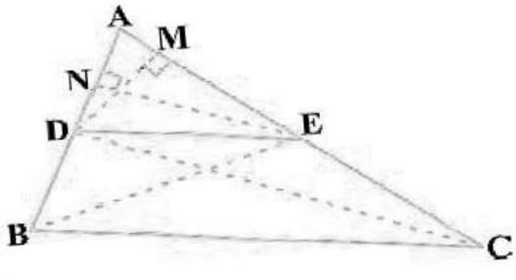
From Above activity we can see that

In Δ ABC , DE II BC Then $\frac{AD}{DB}=\frac{AE}{EC}$

(This result given by a famous Greek mathematician Thales. It is also called as Basic proportionality theorem.)

Theorem: Basic Proportionality theorem / Thales theorem

Statement :- If a line drawn parallel to one side of a triangle then it divides other two sides proportionately (Same ratio).



Data : In triangle ABC , DE II BC

To prove : $\frac{AD}{DB}=\frac{AE}{EC}$

Construction : Join BE and CD and then draw

DM $\perp$ AE , EN $\perp$ AD.

Proof :

$$\frac{Area\,of\,\Delta ADE}{Area\,of\,\Delta BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN}$$ (area of triangle =1/2xbase x height)

$$\therefore \frac{ar\,of\,\Delta ADE}{ar\,of\,\Delta BDE} = \frac{AD}{BD} \longrightarrow 1$$

Also , $$\frac{Area\,of\,\Delta ADE}{Area\,of\,\Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

$$\therefore \frac{ar\,of\,\Delta ADE}{ar\,of\,\Delta CDE} = \frac{AE}{EC}$$

W.k.t, **ar** $\Delta CDE = ar\,\Delta BDE$

(Triangles standing on the same base b/w same parallels are equal in their areas.)

$$\frac{Ar\,of\,ADE}{Ar\,of\,BDE} = \frac{AE}{EC} \longrightarrow 2$$

From Equation 1 & 2

$$\frac{AD}{BD} = \frac{AE}{EC}$$

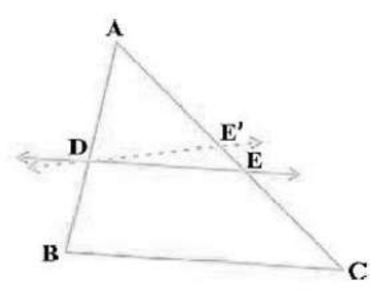
Conclusion: - If a line is drawn parallel to one side of triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Converse of Basic Proportionality Theorem

Statement : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If In Δ ABC , $\frac{AD}{DB} = \frac{AE}{EC}$

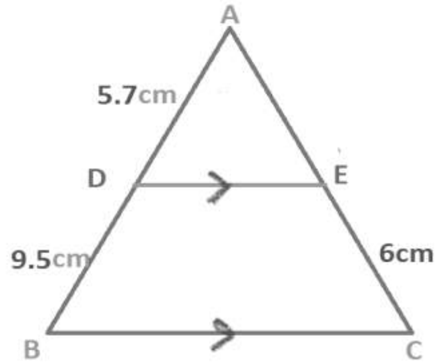
Then DE II BC



Workde examples ;

1) **In Δ ABC , DE is parallel to BC , AD = 5.7cm , BD = 9.5cm EC = 6cm. Find AE.**

Sol:- In ABC , DE II BC



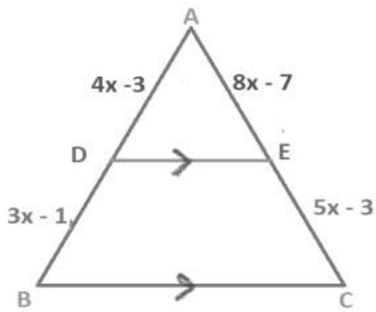
By B.P.T. $\frac{AD}{DB} = \frac{AE}{EC}$

$$AE = \frac{AD \times EC}{DB} = \frac{5.7 \times 6}{9.5} = \frac{18}{5} = 3.6 \text{ cm.}$$

AE = 3.6cm.

2) **In Δ ABC , D and E are points on AB and AC respectively . Such that DE BC , if AD = 4x- 3 , BD = 3x – 1 , AE = 8x – 7 and CE = 5x – 3. Find the value of 'x' .**

Sol:- In Δ ABC , DE II BC



$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

By cross multiplication

$(4x – 3) (5x – 3) = (8x – 7) (3x – 1)$

$20x^2 – 12x – 15x + 9 = 24x^2 – 8x – 21x + 7$

$24x^2 – 8x – 21x + 7 – 20x^2 +12x + 15x – 9 = 0$

$4x^2 – 2x – 2 = 0$ Divide this equation by 2 , we get ,

$2x^2 -x – 1 = 0$ By factorisation $2x^2 – 2x + 1x – 1 = 0$

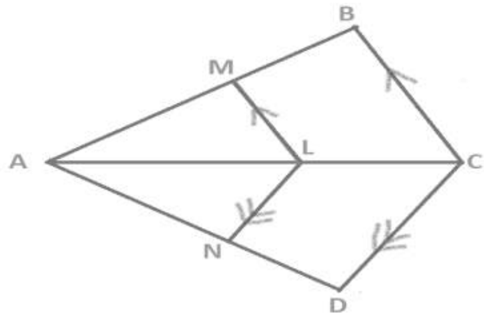
$2x(x – 1) + 1(x – 1) = 0$

$(x – 1) (2x + 1) = 0$

$X – 1 = 0$ or $2x + 1 = 0$

$x = 1$ or $x = \frac{-1}{2}$

3) In Fig LM II CB and LN II CD .



Prove that $\frac{AM}{AB} = \frac{AN}{AD}$

Sol:- In Δ ABC , LM II CB

$\frac{AL}{AC} = \frac{AM}{AB}$ - - - - - > (1) (Corollary of B.P.T.)

In Δ ACD , LN II CD

$\frac{AL}{AC} = \frac{AN}{AD}$ - - - - - > (2) (Corollary of B.P.T.)

From eq (1) and (2)

$\frac{AM}{AB} = \frac{AN}{AD}$ Hence proved .

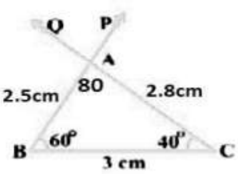
Criteria of similarity of triangles

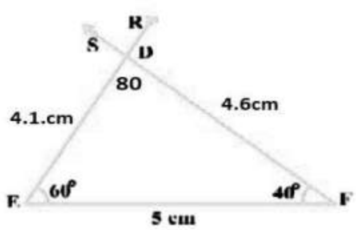
In triangles are similar if ,

1) Their corresponding angles are equal

2) Their corresponding sides are in the same ratio. (or proportion)

Activity : Draw two line segments BC and EF of two different lengths say 3cm and 5cm respectively at the points B and C respectively construct angles 60^0 and 40^0 . Also at the points E and F construct angles 60^0 and 40^0 respectively.





In Δ ABC and Δ DEF

$$\left.\begin{array}{l} \angle A = \angle D = 80^0 \\ \angle B = \angle E = 60^0 \\ \angle C = \angle F = 40^0 \end{array}\right\} \text{Corresponding angles are equal}$$

$$\frac{AB}{DE} = \frac{2.5}{4.1} X \frac{10}{10} = \frac{25}{41} = 0.60$$

$$\left.\begin{array}{l}\frac{AC}{DF} = \frac{2.8}{4.6} X \frac{10}{10} = \frac{28}{46} = 0.60 \\ \frac{AB}{DE} = \frac{3}{5} = 0.60\end{array}\right\}$$ Corresponding sides are in same ratio

In two triangles corresponding angles are equal then their corresponding sides are in the same ratio.

Theorem ; AAA (Angle – Angle – Angle) criterion of similarity of triangles.

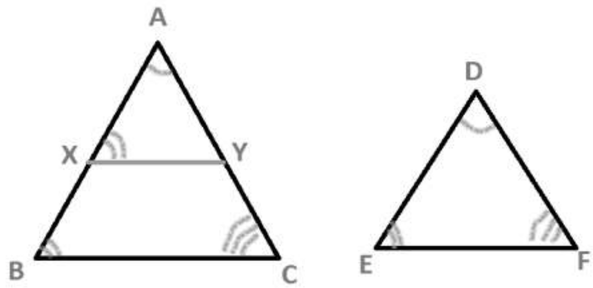
Statement : If in two triangles , corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

OR

Two triangles are equiangular then their corresponding sides are in same ratio (proportional)

Data :- In Δ ABC and Δ DEF

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$



To prove :- $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Construction :- Mark 'X' on AB and 'Y' on AC such that AX = DE , AY = DF and join XY

Proof :- In Δ AXY and Δ DEF

$$\left.\begin{array}{l}AX = DE \\ AY = DF\end{array}\right\}$$ Construction

$\angle A = \angle D$ Data

Δ AXY $\cong$ Δ DEF (S.A.S congruence rule)

$$\left.\begin{array}{l}\therefore \angle AXY = \angle DEF \\ XY = EF\end{array}\right\}$$ C.P.C.T.

WKT , $\angle AXY = \angle DEF = \angle ABC$

$\therefore \angle AXY = \angle ABC$

$\therefore$ XY ॥ BC - - - - - - (1) $\because$ $\angle AXY$ and $\angle ABC$ are corresponding angles

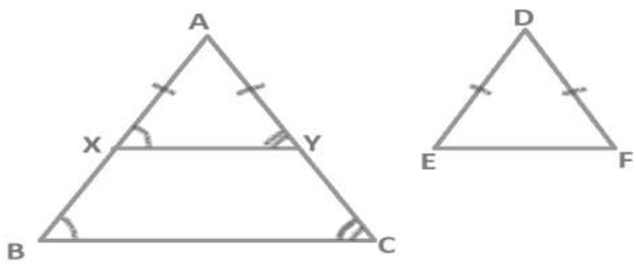
In triangle ABC , XY ॥ BC

$\frac{AX}{AB} = \frac{AY}{AC} = \frac{XY}{EF}$ Corollary of B.P.T.

$\frac{DE}{AB} = \frac{DF}{AC} = \frac{EF}{BC}$ From construction and eq (1)

Theorem :- SSS (Side – Side – Side) similarity criterion for triangles.

Statement :- If in two triangles sides of one triangle are proportional (in the same ratio) to the sides of the other triangle, then their corresponding angles are equal and hence two triangles are similar.



In Δ ABC and Δ DEF

If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$

Then $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

Theorem :- SAS (side – angle – side) Similarity criterion for triangles.

Statement : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional , then the two triangles are similar.

In Δ ABC and Δ DEF If $\angle A = \angle D$

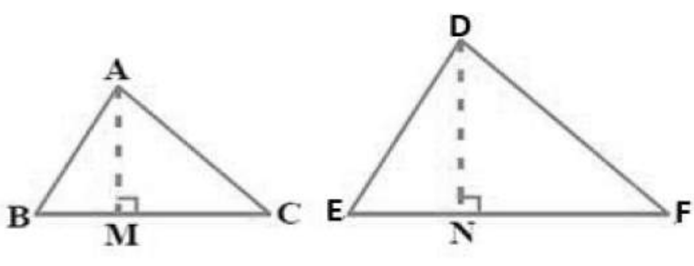
$\frac{AB}{DE} = \frac{AC}{DF}$

Areas of similar triangles

Statement :- **The ratio of the areas of two similar triangles is equal to the square of the ratios of their corresponding sides .** or

Prove that the areas of similar triangles are proportional to the squares of the corresponding sides.

Data : - Δ ABC ~ Δ DEF



$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

To prove : - $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{BC}{EF}\right)^2$

Construction : - Draw AM $\perp$ BC and DN $\perp$ EF

Proof :- $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times EF \times DN}$ (area of triangle =1/2xbase x height)

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{BC \times AM}{EF \times DN} \quad ----->(1)$$

Also , in Δ ABM and Δ DEN

$\angle M = \angle N = 90^0$

$\angle B = \angle E$ By data

Δ ABM ~ Δ DEN

$\frac{AB}{DE} = \frac{AM}{DN}$ (AA Similarity criterion)

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ From data

$\therefore \frac{BC}{EF} = \frac{AM}{DN}$ ------> (2)

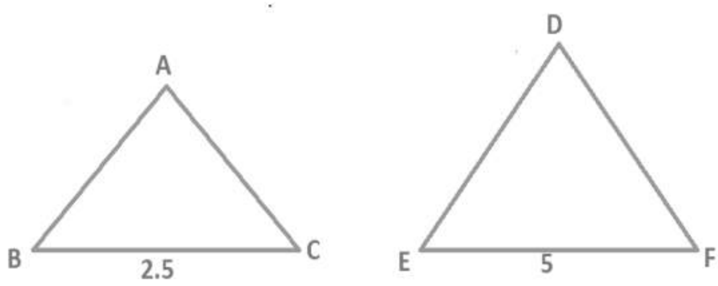
From eq 1 and eq 2

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{BC \times BC}{EF \times EF} = \frac{BC^2}{EF^2}$$

= Bydata. $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF}$ $\left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2 = \left(\frac{AB}{DE}\right)^2$

Ex 1):- Δ ABC ~ Δ DEF, $\frac{BC}{EF} = \frac{2.5}{5}$. if the area of ABC = 120 sq.cm. Find the area of Δ DEF.

Solution:- Δ ABC ~ Δ DEF



$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \left(\frac{BC}{EF}\right)^2$

$= \frac{(2.5)^2}{5^2} = \frac{6.25}{25}$

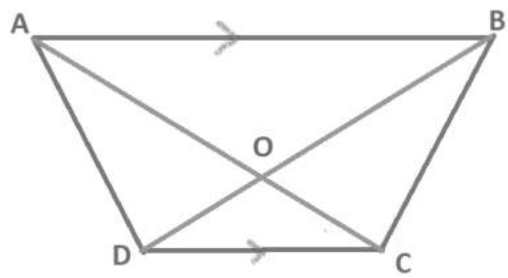
$\therefore \frac{ar\ ABC}{ar\ DEF} = \frac{6.25}{25} = \frac{1}{4}$

ar Δ DEF = 4 X ar $\Delta ABC = 4\ X\ 120 = 480\ sq.cm$

ar Δ DEF = 480 sq.cm.

Ex 2):- A trapezium ABCD has its sides AB $\parallel$ CD and its diagonals intersect at 'o' . If side AB is twice CD . Find the ratio of the Δ AOB to the Δ COD.

Sol:- By data AB = 2CD



In Δ AOB and Δ COD

$\angle A = \angle C$, $\angle B = \angle D$ Alternate angles

$\angle AOB = \angle COD$ V.O.A.

Δ AOB and Δ COD are equiangular triangles .

Δ AOB ~ Δ COD (AA Similarity criteria)

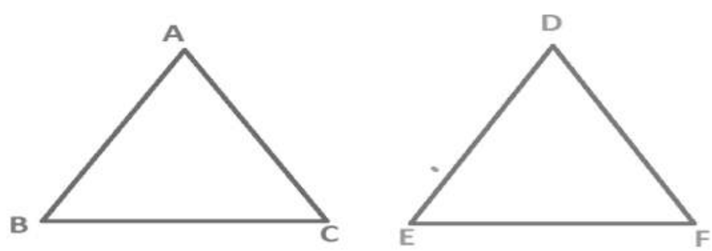
$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \left(\frac{AB}{CD}\right)^2$ By theorem

$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \left(\frac{2CD}{CD}\right)^2$

$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{4}{1}$ $\therefore$ ar Δ AOB : ar Δ COD = 4 : 1

Ex 3):-If the areas of two similar triangles are equal prove that they are congruent.

Solu:- Δ ABC ~ Δ DEF



Area of Δ ABC = Area of Δ DEF

$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = 1$ ------------ > (1)

WKT, Δ ABC ~ Δ DEF

$\frac{ar\ \Delta ABC}{ar\ \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

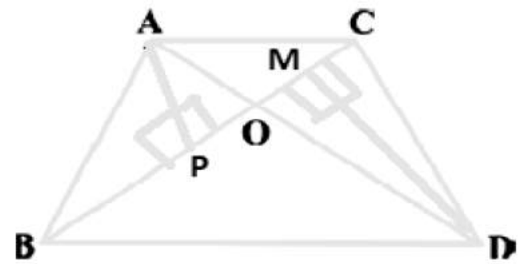
$\frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ = 1 from equation 1

$\frac{AB^2}{DE^2}$ = 1 => $AB^2 = DE^2$ => AB = DE

$Similarly\ , BC = EF\ \ and\ AC = DF$

$\triangle$ ABC $\cong$ $\triangle$ DEF (SSS Congruence rule)

Ex 4):- In Fig. ABC and DBC are two triangles on the same base BC . If AD intersects BC at O . Show that $\frac{ar\,\triangle ABC}{ar\,\triangle DBC} = \frac{AO}{DO}$



Sol:- Draw AP $\perp$ BC and DM $\perp$ BC

$$\frac{Area\ of\ \triangle ABC}{Area\ of\ \triangle DBC} = \frac{\frac{1}{2} \times BC \times AP}{\frac{1}{2} \times BC \times DM}$$

$$\frac{Area\ of\ \triangle ABC}{Area\ of\ \triangle DBC} = \frac{AP}{DM} \text{ -------- > (1)}$$

In $\triangle$ AOP and $\triangle$ DOM

$\angle P = \angle M = 90^0$

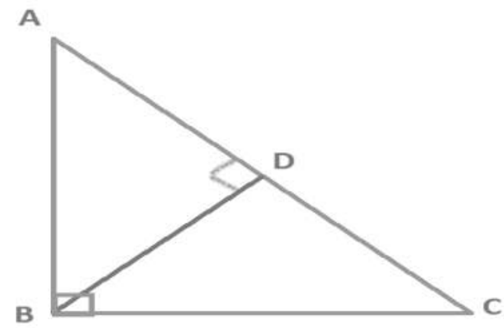
$\angle O = \angle O$ V.O.A

$\triangle$ AOP ~ $\triangle$ DOM

$\frac{AP}{DM} = \frac{AO}{DO}$ -------- > (2) AA Similarity criterion

From equation (1) and (2) $\frac{Area\ of\ \triangle ABC}{Area\ of\ \triangle DBC} = \frac{AO}{DO}$

Theorem :- If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.



Data ; - In $\triangle$ ABC , $\angle ABC = 90^0$

To prove : - $\triangle$ ABC ~ $\triangle$ ABD ~ $\triangle$ BDC

Construction : - Draw BD $\perp$ AC

Proof :- In $\triangle$ ABC and $\triangle$ ABD

$\angle B = \angle D = 90^0$

$\angle A = \angle A$ Common angle

$\triangle$ ABC ~ $\triangle$ ABD -------------- > (1) (A.A. Similarity criteria)

Also in $\triangle$ ABC and $\triangle$ BDC

$\angle B = \angle D = 90^0$

$\angle C = \angle C$ Common angle

Δ ABC ~ Δ BDC ---------- > (2) (A.A. Similarity criteria)

From equation (1) and (2)

Δ ABC ~ Δ ABD

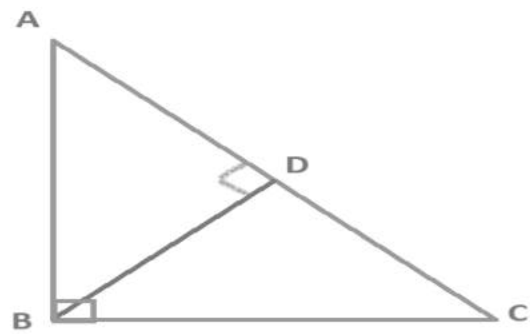
Δ ABC ~ Δ BDC

Δ ABC ~ Δ ABD ~ Δ BDC

From above we can conclude **" If a perpendicular is drawn from the vertex of the right angle of a right angle of a right triangle to the hypotenuse then triangle on both sides of the perpendicular are similar to the whole triangle and to each other".**

Theorem : Pythagoras theorem

Statement : In a right triangle , the square on the hypotenuse is equal to the sum of the squares on other two sides.



Data : In Δ ABC , $\angle ABC = 90^0$

To prove : $AC^2 = AB^2 + BC^2$

Construction :- Draw BD $\perp$ AC

Proof :- In Δ ABC and Δ ABD

$\angle B = \angle D = 90^0$

$\angle A = \angle A$ Common angle

Δ ABC ~ Δ ABD (A.A. Similarity criterion)

So , $\frac{AC}{AB} = \frac{AB}{AD}$ (sides are proportional)

$AB^2 = AD . AC$ ------------ > (1)

Also In Δ ABC and Δ BDC

$\angle B = \angle D = 90^0$

$\angle C = \angle C$ Common angle

Δ ABC ~ Δ BDC (A.A.Similarity criterion)

So , $\frac{AC}{BC} = \frac{BC}{CD}$ (Sides are proportional)

$BC^2 = AC \,.\, CD$ -------------- (2)

Adding eq (1) and (2)

$AB^2 + BC^2 = AD.AC + AC.CD$

$= AC (AD + CD)$

$= AC \,.\, AC$

$AB^2 + BC^2 = AC^2$

Converse of Pythagoras theorem:

Statement : In a triangle , if square of one side is equal to the sum of the squares of the other two sides , then the angle opposite to the first side is a right angle.

****

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Equation: A statement that the values of two mathematical expressions are equal (indicated by the sign =)

Or

Equality between two quantities is called equation

Linear equation: An equation involving a variable having degree one.

Ex: $ax+b=0$ Where a&b are constants and $a\neq 0$ $a,b \in R$

Linear equation in two variables.

An equation of the form $ax+by+c=0$ where a b & c are real numbers and a&b are both non zero real numbers is called a linear equation in two variables x&y.

The solution of a linear equation is not affected when.

i) The same number is added (or subtracted from) to both the sides of the equation.

ii) Multiply & divide both sides of the equation by the same non-zero number.

General form of a pair of linear equation in two variables:

$$a_1x+b_1y+c_1=0$$

$$a_2x+b_2y+c_2=0$$

where $a_1, a_2. b_1, b_2, c_1$ & c_2 are real numbers such that $a1\neq 0$, $b1\neq 0$, $a_2\neq 0$, $b_2\neq 0$ may c_1 & c_2 be zero .

Ex: $2x-y+7=0$

$4x+y=6$ are Linear equations in two variables x&y

There are two method of solving simultaneous linear equation in two variables:

1. Graphical method 2. Algebraic method.

1. <u>Graphical method:</u>

a) Express one variable say y in terms of the other variable x, or vice versa

i.e $y= ax+c$ for the given equation

Ex: 1) $3x+y-2=0$ ii) $2x+3y-7 = 0$

Y = 2-3x $\qquad\qquad$ y = $\frac{7-2x}{3}$

b) As we say like above, x is independent variable and y is dependent variable so take three values of independent variable x and find the corresponding values of dependent variable y (take integral values only)

For Ex: (1) if x = 0 ; Y = 2-3x $\Rightarrow$ y = 2-3 (0) = 2

x = 1 ; Y = 2-3x $\Rightarrow$ y = 2-3(1) = -1

x = 2 ; Y = 2-3x $\Rightarrow$ y =2-3(2) = -4

c) Plot these values on the graph paper in order to represent these equations.

d) If the lines representing the pair of linear equations intersect at a point, then the point of intersection is the unique solution of the two equations (in this case pair of linear equations is consistent).

e) If the lines representing the pair of linear equations coincide, then system of equation has infinitely many solutions (in this case, the pair of linear equations is consistent & dependent).

f) If the lines representing the pair of linear equations are parallel, then the system of equation has no solution and is called inconsistent.

<u>We may verify above statement by using some examples:</u>

Ex: (3) $\quad$ 4x – y = 4

3x + 2y =14

Consider $\quad$ 4x –y = 4 $\qquad\qquad$ 3x+2y = 14

Y= 4x-4 $\qquad\qquad$ y= $\frac{14-3x}{2}$

If x = 0; $\quad$ y=4(0)-4=-4 $\qquad$ if x=0; $\quad$ y=14-3(0)/2

X = 1; $\quad$ y=4(1)-4=0 $\qquad$ = 7

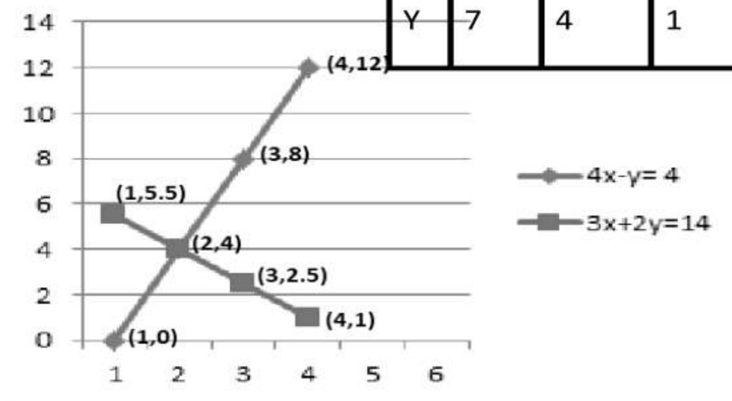
X= 2; $\quad$ y=4(2)-4=4 $\qquad$ x=2; $\quad$ y=14-3(2)/2

$\longrightarrow$ 4x-y =4 $\qquad$ = 4

X=4; y= 14-3(4)/2=1

X	0	1	2
Y	-4	0	4

X	0	2	4
Y	7	4	1



As obtained lines intersect each other at (2, 4) Hence x = 2 and y =2

Form the above examples

$a_1 = 4$; $b_1 = -1$; $c_1 = -4$

$a_2 = 3$; $b_2 = 2$; $c_2 = -14$

$\Rightarrow$ $a_1/a_2 = 4/3$ & $b_1/b_2 = -1/2$ $c_1/c_2 = -4/-14 = 2/7$

$\Rightarrow$ $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

Therefore Pair of linear equations in two variables is consistent & unique solutions.

Ex: (4) : x+y = 5

3x+3y = 5

Solution:- consider x+y = 5

Y=5-x

If x=0; y=5-0=5

X=1; y=5-1=4

X=2; y=5-2=3

consider : 3x+3y = 15

3y = 15-3x

$y = \frac{15-3x}{3}$

if x=0; y = 15-3(0)/3=5

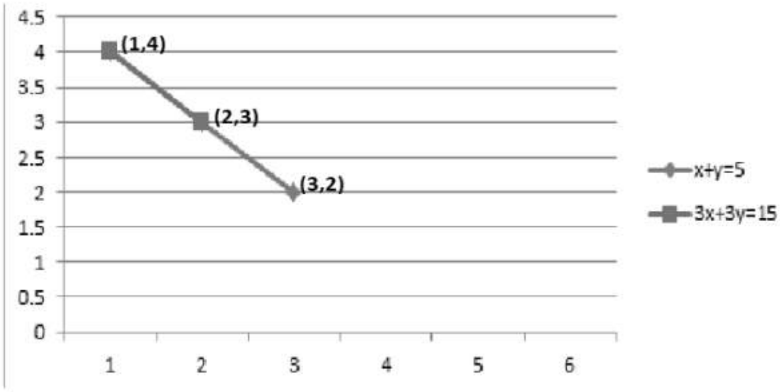
x=1; y =15-3(1)/3=4

X=2; y = 15-3(2)/3=3

X	0	1	2
Y	5	4	3

X	0	1	2
Y	5	4	3

x+y=5 3x+3y=15



Since the lines are coincide, so the pair of linear equation is consistent with infinitely many solutions.

$A_1 = 1$; $b_1 = 1$; $c_1 = -5$

$A_2 = 3$; $b_2 = 3$; $c_2 = -15$

$A_1/a_2 = 1/3$; $b_1/b_2 = 1/3$; $c_1/c_2 = -5/-15 = 1/3$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore from the above we may concluded that such pair of linear equations in two variables is **consistent** with infinitely many solutions.

Ex: (5) $2x + y - 5 = 0$

$4x + 2y - 6 = 0$

Solution:- consider $2x + y - 5 = 0$ $4x + 2y - 6 = 0$

$Y = 5 - 2x$ $2y = 6 - 4x$ $y = \frac{6-4x}{2}$

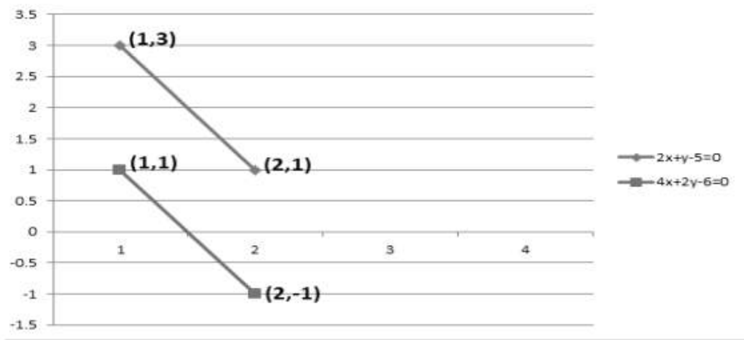
Ifx=0 ; y=5-2(0)=5 If x=0; y=6-4(0)/2=3

X=1 y=5-2(1)= 3 X=1; y=6-4(1)/2=1

X=2 y=5-2(2)= 1 X=2; y=6-4(2)/2= -1

X	0	1	2
Y	5	3	1

X	0	1	2
Y	3	1	-1



The lines are not intersected anywhere, i.e lines are parallel, so the pair of linear equation in two variable is inconsistent and no solution.

$a_1=2;$ $b_1=1;$ $c_1=-5$

$a_2=4;$ $b_2=2;$ $c_2=-6$

$a_1/a_2=2/4=1/2$ $b_1/b_2=1/2$ $c_1/c_2=-5/-6=5/6$

$\Rightarrow$ $$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore such pair of linear equations in two variables is **inconsistent** and no solutions.

If $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ are any pair of linear equations in two variables x&y such that

i) if $a_1/a_2\neq b_1/b_2$, then the pair of linear equations is consistent with a unique solution. (graph of pair of linear equation in two variables is intersecting)

ii) if $a_1/a_2 = b_1/b_2 = c_1/c_2$, then the pair of linear equations is consistent with infinitely many solutions

iii)if $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, then the pair of linear equations possibilities of solutions is inconsistent with no solutions inconsistency.

Pair of linear equation	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Comparation	Graphical representation	Algebraic interpretation	Conditions for solvability
4x-y=4 3x+2y=14	$\frac{4}{3}$	$\frac{-1}{2}$	$\frac{2}{7}$	$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$	Intersecting	Unique solutions or one solution	System is consistent

X+y = 5 3x+3y = 15	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	Coincide each other	Infinitely many solutions	System is consistent
2x+y-5=0 4x+2y-6=0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

Algebraic method:

We can solve the linear equations algebraically by substitution method, Elimination method & cross multiplication method.

Elimination method:

a) Multiply given equation with suitable constants, to make either the x-co-efficient or the y- co-efficient of the two equations equal.

b) Subtract or and one equation from the other to get an equation in one variable .

c) Solve the equation so obtained to get the value of the variable.

d) Put this value in any of the equation to get the value of the second variable.

Note: (i) if in Step (2) , we obtain a true equation involving no variable, then the original pair of equation has infinitely many solutions.

(ii) if in step(2) we obtain a false equation involving no variable, then the original pair of equations has no solutions i.e it is inconsistent.

Ex 8: solve for x & y x-y=1 & 2x+y=8 by elimination method.

Solution: given x-y=1 --------(1)

2x+y=8 ----------(2)

Add equation (1) & (2)

We get $2x+y=8$

$\underline{X-y=1}$

$3x=9$

$\boxed{X=3//}$

Substitute x=3 in equation (1)

$x-y = 1$

$3-y = 1$

$Y=3-1 \Rightarrow Y=2.$

Ex 9: Solve ; 2x+3y=11

X+2y=7 by elimination method

Solution: Given: $2x+3y=11$ ----------(1)

$X+2y=7$ -----------(2)

Multiply equation (1) by 1 & equation (2) by 2 to make the co-efficient of x equal.

We get $2x+3y=11$ -------(1)

$2x+4y=14$ -------(4)

Subtract equation (3) from equation (4)

$\cancel{2x}+4y=14$

$\underline{\cancel{2x}+3y=11}$

$\boxed{Y=3}$

Substitute y=3 in equation (2), we get

$X+2(3)=7$

$X=7-6$

X=1 y =3

Additional problems / practice question:

1) solve: $-x+2y-1=0$

2x+3y-12=0 by elimination method

2) solve: 4x+3y=12

2x+5y= -1 by elimination method.

On comparing the ratios a_1/a_2, b_1/b_2, & c_1/c_2 find out whether the lines representing the following pairs of linear equations intersect at a point or parallel or coincident.

(i) x+y = 14

X-y=4

Here $a_1=1$; $b_1=1$; $c_1=14$

$a_2=1$; $b_2=-1$; $c_2=4$

$a_1/a_2=1/1$; $b_1/b_2=1/-1=-1$

⇨ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

⇨ Therefore Lines are intersect

ii) 2x-4y=1

6x-12y=3

Here $a_1=2$, $b_1=-4$, $c_1=1$

$a_2=6$, $b_2=-12$, $c_2=3$

$a_1/a_2=2/6=1/3$: $b_1/b_2=-4/-12=1/3$: $c_1/c_2=1/3$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Therefore lines are coincide

iii) x+2y-4 = 0

2x=4y-12=0

$a_1=1$; $b_1=2$; $c_1=-4$

$a_2=2$: $b_2=4$; $c_2=-12$

$a_1/a_2=1/2$; $b_1/b_2=2/4=1/2$; $c_1/c_2=-4/12=1/3$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Therefore lines are parallel.

Additional problems:

1) 5x-3y=11

-10x+6y=-22

2) 3x+2y=5

2x-3y=7

3) 3/2x + 5/3y=7

9x-10y=14

On comparing the ratios of a_1/a_2, b_1/b_2, & c_1/c_2 find out whether the following pair of linear equations are consistent or inconsistent.

i) 0.2x+0.3y = 1.3

0.4x+0.5y =2.3

Here $a_1=0.2$; $b_1=0.3$; $c_1=1.3$

$A_2=0.4$; $b_2=0.5$; $c_2=2-3$

$A_1/a_2=0.2/0.4=1/2$; $b_1/b_2=0.3/0.5=3/5$; $c_1/c_2=1.3/2.3=13/23$

⇨ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Therefore pair of linear equation are consistent

ii) 4x-2y=10

8x-4y=20

Here $a_1=4$ $b_1=-2$, $c_1=-10$

$A_2=8$ $b_2=-4$, $c_2=-20$

$A_1/a_2=4/8=1/2$; $b_1/b_2=-2/-4=1/2$; $c_1/c_2=-10/-20=1/2$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Therefore pair of linear equation are consistent

iii) x-3y-3=0

3x-9y-2=0

Here $a_1=1$ $b_1=-3$ $c_1=13$

$a_2=3$ $b_2=-9$ $c_2=-2$

$a_1/a_2=1/3$; $b_1/b_2=-3/-9=1/3$ $c_1/c_2=-3/-2$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Therefore pair of linear equations are inconsistent .

Steps to be followed for solving word problems:

S.No	Problem type	Steps to be followed
1.	Age Problems	If the problem involves find in out the ages of two persons, take the present age of one person as x and of the other a y. then 'a' years ago, age of 1^{st} person was 'x-a' years and that of 2^{nd} person was 'y-a' and after 'b' years, age of 1^{st} person will be 'x+b' years and that of 2^{nd} person will be 'y+b' years. Formulate the equations and then solve them.
2	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. the two-digit number is given by 10y+x. on interchanging the positions of the digits,the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes 10x+y. Formulate the equations and then solve them.
3	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is x/y Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4	Problems based on Distance, Speed and Time	We know that speed =Distance/Time ⇨ Distance =speed *time and Time= Distance/Speed To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. then the speed of boat downstream =x+y km/h and speed of boat upstream=x-y km/h
5	Problems based on commercial Mathematics	For solving specific questions based on ommercial mathematics, the fare of 1ull ticket may be taken as Rs.x and the reservation charges may be taken as Rs. Y, So that one ull fare=x+y and one half fare =x/2+y To solve the questions of profit and loss, take the cost price of 1^{st} article as Rs x and that of 2^{nd} article as Rs y. To solve the questions based on simple interest, take the amount invested as Rs. X at some rate of interest and Rs. Y at some other rate of interest.
6	Problems based on Geometry and Mensuration	Make use of angle sum property of a triangle (<A+<B +<C $=180^0$) in case of triangle. In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral opposite angles are supplementary.

Ex 1): The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadths is increased by 2 units. Find the perimeter of the rectangle.

Solution: Let length of given rectangle be x and breadth be y

Therefore area of rectangle = xy

$(x-5)(y+3)=xy-9$

$3x-5y=6$ ---------(1)

Or

According to the second condition.

$(x+3)(y+2) = xy+67$

$2x+3y=61$ ---------(2)

Or

Multiplying equation (1) by 3 and euation (2) by 5 and then adding.

$9x-15y=18$

$10x+15y=305$

⇨ $X=\frac{323}{19}=17$

Substituting this value of x in equation (1),

$3(17)-5y=6$

$5y=51-6$

$Y=9$

Hence, perimeter = $2(x+y)=2(17+9)=52$ units.

Ex: solve for x and y :

$2(3x-y)=5xy$ -----------------(1)

$2(x+3y)=5xy$ ----------------(2)

Divide equation (1) and (2) by xy,

$\frac{6}{y}-\frac{2}{x}=5$ -----------------(3)

And $\frac{2}{y}+\frac{6}{x}=5$ ----------------(4)

Let $\frac{1}{y}=a$ and $\frac{1}{x}=b$,

Then equations (3) and (4) become

6a-2b=5 -----------(5)

2a+6b=5 -----------(6)

Multiplying equation (5) by 3 and then adding with equation (6)

20a=20

a=1

Substituting this value of a in equation (5),

b=$\frac{1}{2}$

Now $\frac{1}{y}$ =a=1

Or y=1

And $\frac{1}{x}$ =b=$\frac{1}{2}$

Or, x=2

Hence, x=2, y=1

Ex 2): a fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Solution: let the fraction be $\frac{x}{y}$, then according to the question,

X+$\frac{2}{y}$ +2=$\frac{9}{11}$ or 11x+22=9y+18

Or 11x-9y+4=0 -------(1)

And x+$\frac{3}{y}$ +3=$\frac{5}{6}$

Or 6x-5y+3=0 ---------(2)

On comparing with ax+by+c=0

We get a_1=11, b_1 = -9, $c_{1=}$4

A_2=6, b_2 = -5, & $c_{3=}$3

Now, $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$$\frac{x}{(-9)(3)-(-5)(4)} = \frac{y}{(4)(6)-(11)(3)} = \frac{1}{(11)(-5)-(6)(-9)}$$

$$\frac{x}{-27+20}=\frac{y}{24-33}=\frac{1}{-55+549}$$

$$\frac{x}{-7}=\frac{y}{-9}=\frac{1}{-1}$$

⇨ hence, x=7, y=9 therefore fraction = 7/9

Additional problems:

1. 4 chairs and 3 tables cost Rs. 2100 and 5 chairs and 2 tables cost RS 1750. Find the cost of one chair and one table separately.
2. Solve for x and y:

   2x-y+3=0

   3x-5y+1=0
3. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.
4. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes ½, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes 3/2, find the fraction

****

CIRCLES

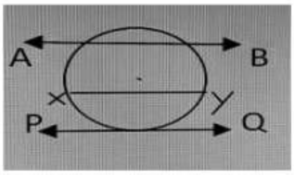
- **Chord:**A line segment in which its end points lie on the circle.
- **Secant:** A line which intersects the circle at two distinct points is called secant.
- **Tangent**: The line which intersects the circle at only one point is called tangent.

In the adjoining figure.

AB-> secant

XY-> chord

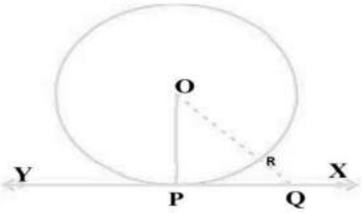
PQ->tangent



Theorem—1

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Data: XY is a tangent to a circle with center 'O' at P and OP is the radius.
To prove: $OP \perp XY$
Construction: - Take a point Q on XY other than P and join OQ intersects the circle at R.
Proof: - OQ > OR
But OP = OR (∵ radii of the same circle)
So OQ > radius OP



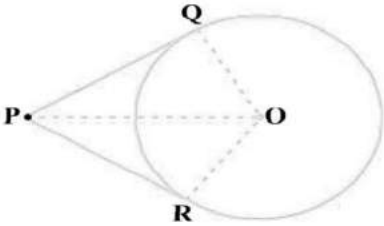
Since this happens for every point on the line XY except the point P,
OP is the shortest of all the distance between the point O and tangent XY.
So $OP \perp XY$
Hence the proof

Theorem—2

The lengths of tangents to a circle drawn from an external point are equal.

Data: - PQ and PR are the two tangents drawn from an external point P to circle with center 'o'

To prove: PQ = PR



Construction: - Draw OP, OQ, and OR

Proof: - In ΔOQP and ΔORP

OQ = OR (∵radii of same circle)

OP = OP (∵ common side)

$\angle Q = \angle R = 90^0$ (∵ tangent perpendicular to radius through point of contact)

$\angle OQP = \angle ORP$ (∵RHS theorem)

PQ = PR (CPCT)

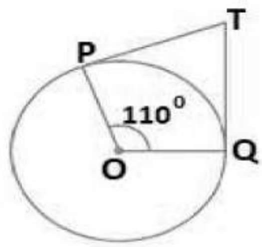
Hence the proof.

Points to remember

- **There is only one tangent at a point of a circle**
- **Only two tangents can be drawn to a circle from an external point.**
- **No tangents can be drawn from a point inside the circle.**
- **The tangent at any point of a circle is perpendicular to the radius through the point of contact.**
- **Tangents drawn from an external point to circle are equal.**

Solved problems

1. **In the given fig find $\angle PTQ$?**



Solution: In quadrilateral PTQO

$\angle OPT = \angle OQT = 90^0$ (∵ radius ⊥ to tangent)

SO $\angle POQ + \angle PTQ = 180^0$

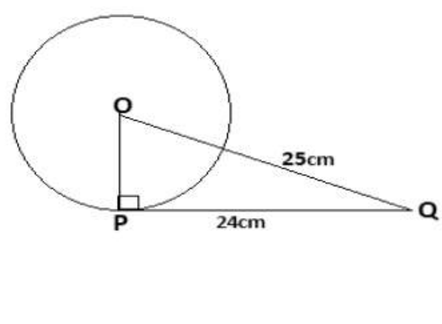
$110^0 + \angle PTQ = 180^0$

$\angle PTQ = 180^0 - 110^0$

$\angle PTQ = 70^0$

2. **From a point Q , the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. find the radius of the circle.**

Solution: In ΔOPQ , $\angle OPQ = 90^0$



$OQ^2 = OP^2 + PQ^2$

$25^2 = OP^2 + 24^2$

$625 - 576 = OP^2$

$OP = \sqrt{49} = 7cm$

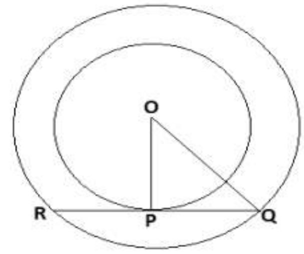
OP=7cm.

3. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle .

Solution: In the fig OQ = 5cm (radius of larger circle) OP= 3cm (radius of smaller circle)

$OP \perp PQ$ (radius is $\perp$ to tangent through the point of contact)

So $\angle OPQ = 90^0$



by Pythagoras theorem $OQ^2 = OP^2 + PQ^2$

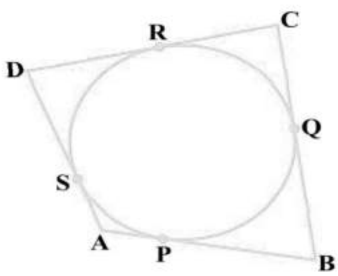
$5^2 = 3^2 + PQ^2$

$PQ^2 = 25-9$

PQ= 4 cm

RQ= PQ+RP= 4+4=8cm

4. A quadrilateral ABCD is drawn to circumscribe a circle . Prove that AB+CD=AD+BC



In the Fig

AP = AS [tangent from the same point to a circle are equal]

BP = BQ

CQ = CR

DR = DS

LHS = AB+CD

= (AP+BP)+(DR+CR)

= (AS+BQ)+(DS+CQ)

= (AS+DS)+(BQ+CQ)

= AD+BC=**RHS**

Practice test

- The straight line which touches the circle at only one point is ____________

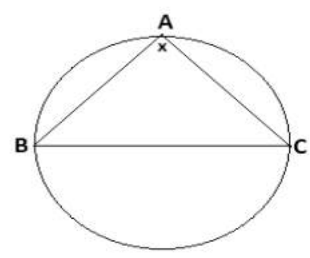
  a. radius b. tangent c. secant d. line segment

- Maximum number of tangents drawn to a circle from an external point are ______

  a. 1 b . 3 c. 2 d. infinite

- In the fig BC is the diameter then the value of x is

  a. 90^0 b. 150^0 c. 180^0 d. 160^0



- In the adjoining fig ∠AOP=60^0 the ∠APO=

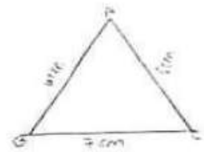
  a. 120^0 b. 90^0 c. 60^0 d. 30^0

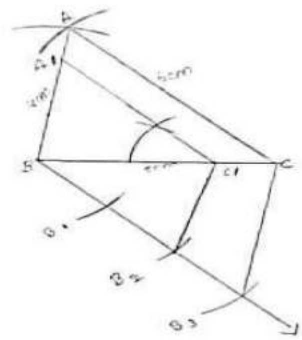
- The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm. find the radius of the circle.

- Prove that the tangents drawn from an external point to a circle are equal.

CONSTRUCTIONS

1. Construct a triangle of sides 4cm, 5cm and 6cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

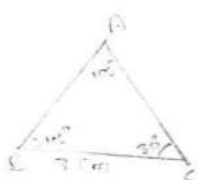


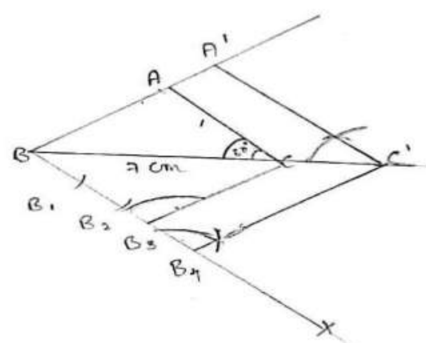


Steps to construction

- Construct ΔABC with given measurements.
- Draw any ray BX making an acute angle with BC on the side opposite to vertex A
- Locate 3 points (greater of 2 and 3 in $\frac{2}{3}$) B1, B2 ,& B3 on BX so that $BB_1= B_1B_2= B_2B_3$
- Join B3 (from denominator in the 2/3) to C and draw a line through B2 parallel to B_3C to intersect BC at C^1
- Draw a line through C^1 parallel to the line CA to intersect BA at A^1
- Then ΔA^1BC^1 is the required triangle

2. Draw a triangle ABC with base BC = 7cm $\angle B = 45°$ $\angle A = 105°$ then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ABC



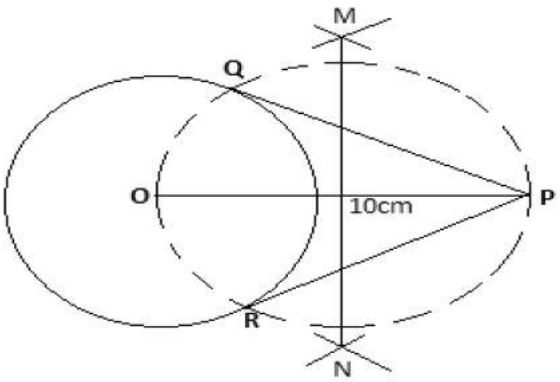


Steps to construction

- Construct ? ABC with BC = 7cm ?B = 45° and ?c = 30°
- Draw any ray BX making an acute angle with BC on the side opposite to vertex A.
- Locate 4 points (greater of $\frac{4}{3}$) B_1, B_2, B_3 and B_4 on BX so that BB_1=B_1B_2=B_2B_3= B_3 B_4
- Join B_3 (from denominator of 4/3) to C and draw a line through B_4parallel to B_3C to intersect extended BC at C'.
- Draw a line through C' parallel to the line CA to intersect extended BA at A'
- Then A'BC' is the required triangle.

Construction of tangents to a circle

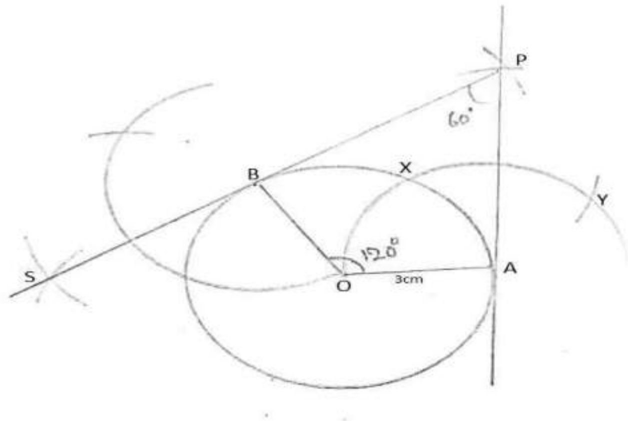
1. Draw a circle of radius 4cm, from a point 10cm away from its centre construct the pair of tangents to the circle and measure their lengths



Steps of constructions

- Draw a circle of radius 4cm with centre O
- Draw a circle OP from centre measuring 10cm
- Construct a perpendicular bisector to OP
- With C as centre and CP as radius draw a circle
- This circle intersects the previous circle at A and B
- Join PA and PB which are the required tangents
- Measure PA & PB

2. Drawapairoftangentstoacircleofradius5cmwhichareinclinedtoeachother atan angle of60°.



Steps of constructions:

- Draw a circle of radius 4cm with centre O
- Construct 120° in the centre (given 60°= 180°-60°=120°
- With O as centre OA as radius draw a semicircle With same radius cut that semicircle with two arcs.
- Draw perpendicular bisectors at X & Y
- Join PA
- Follow the Same steps from OB
- Join SB to P
- PA and PB are the required tangents

Practice test

a) Draw a circle of radius 3cm from a point 7cm away from its centre.

b) Draw a pair of tangents to a circle of radius 4cm which are inclined to each other at an angle of 50°

c) Construct a triangle with radius 5cm, 6cm and 7cm and then another triangle whose sides are 7/5 of the corresponding sides of the first triangle.

d) Draw a triangle ABC with sides BC=6cm, AB=5cm $\angle ABC$ = 60° thus construct a triangle whose sides are 3/4 of the corresponding sides of triangle ABC.

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COORDINATE GEOMETRY

* The distance of a point from the y-axis is called its **x-coordinate**, or **abscissa**.

* The distance of a point from the x-axis is called its **y-coordinate**, or **ordinate**.

*The coordinates of a point on the x-axis are of the form $(x, 0)$,

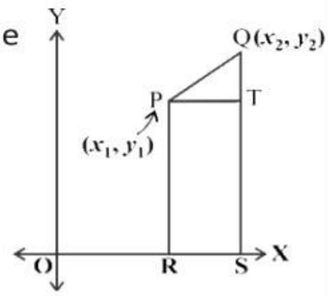
*The coordinates of a point on the y-axis are of the form $(0, y)$.

* coordinates of origin are $(0,0)$,

Distance formula :

Distance formula is used to find distance between any two points on the Cartesian plane.

let$P(x_1, y_1)$ and $Q(x_2, y_2)$. are any two points on the Cartesian plane



Draw PR and QS perpendicular to the x-axis.
Draw PT perpendicular to QS
Then, $OR = x_1$, $OS = x_2$. So, $RS = x_2 - x_1 = PT$.

Also, $SQ = y_2$, $ST = PR = y_1$. So, $QT = y_2 - y_1$.

Now, applying the Pythagoras theorem in Ä PTQ, we get

$$PQ^2 = PT^2 + QT^2$$

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Therefore, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

*The distance of a point P(x,y)from the origin O(0,0) is given by $OP = \sqrt{x^2 + y^2}$

Example problems:

1) Find the distance between the pair of points (2,3)(4,1)

Solution: (2,3) ,(4,1)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4-2)^2 + (1-3)^2}$$

$d = 2\sqrt{2}$ units

Try this: 1) (-5,7) , (-1,3) 2) (a,b) , (-a, -b)

2) Do the points (3,2) , (-2,-3) and (2,3) form a triangle ?

If so, name the type of triangle formed.

Solution : distance between the points (3,2)(-2,-3)

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-2-3)^2 + (-3-2)^2}$

= $\sqrt{50}$

d=7.07(approx.)

distance between the points (-2-3)and (2,3)

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(-2-(-2))^2 + (-3-(-3))^2}$

= $\sqrt{52}$

d=7.21 (approx.)

distance between the points (3,2)and (2,3)

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

= $\sqrt{(2-3)^2 + (3-2)^2}$

= $\sqrt{2}$

d=1.41 (approx.)

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle.

Also $(\sqrt{52})^2 = (\sqrt{50})^2 + (\sqrt{2})^2$ ($\because$ converse of Pythagoras theorem)

52=50+2

52=52

$\because$ PQR is a right angled triangle

Try this: 1) check whether (5,-2), (6,4) and (7,-2) are the vertices of isosceles triangle.

3) Find the values of y for which the distance between the points P(2,-3) and Q(10,y) is 10 units

Solution:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(10-2)^2 + (y-(-3))^2}$$

$$10 = \sqrt{(8)^2 + (y+3)^2} \quad \text{S.B.S}$$

$$10^2 = 64+y^2+6y+9$$

$$100-73= y^2+6y$$

$$y^2+6y-27=0$$

$$y^2+9y-3y-27=0$$

$$y(y+9)-3(y+9)=0$$

$$(y+9)(y-3)=0$$

$$Y+9=0 \quad \text{or} \quad y-3=0$$

$$Y=-9 \quad \text{or} \quad y=3$$

4) Determine if the points (1,5),(2,3) and (-2,-11) are collinear.

Solution: distance between (1,5) and (2,3)

$$d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(1)^2 + (-2)^2}$$

$$= \sqrt{5} units$$

distance between (2,3) and (-2,-11)

$$d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-4)^2 + (-14)^2}$$

$$= \sqrt{212} units$$

distance between (1,5) and (-2,-11)

$$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-3)^2 + (-16)^2}$$

$$= \sqrt{265} units$$

since $\mathbf{d_3 \neq d_1+d_2}$ the points are non collinear

5) Name the type of quadrilateral formed by the points (4,5)(7,6)(4,3)(1,2)

Solution : A(4,5) B(7,6) C(4,3) D(1,2)

AB $=\sqrt{(7-4)^2+(6-5)^2} = \sqrt{9+1} = \sqrt{10}$

BC $=\sqrt{(4-7)^2+(3-6)^2} = \sqrt{9+9} = \sqrt{18}$

CD = $\sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$

AD $=\sqrt{(4-1)^2+(5-2)^2} = \sqrt{9+9} = \sqrt{18}$

Diagonal BD $=\sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = \sqrt{52}$

Diagonal AC $=\sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+4} = 2$

Opposite sides of this quadrilateral are of the same length. However ,the diagonals are of different lengths. Therefore the given points are the vertices of a parallelogram.

Try this: 1) Name the type of quadrilateral formed by the points (-1,-2) , (1,0) , (-1,2) ,(-3,0)

6) Find the point on the x-axis which is equidistant from (2,-5) and (-2,9)

Solution : To find the point on x-axis, ,therefore its y-coordinate will be 0. Let the point on x-axis be (x,0)

Consider A=(x,0), B=(2,-5), C=(-2,9)

Given that AC=AB

$$\sqrt{(-2-x)^2+(9-0)^2} = \sqrt{(2-x)^2+(-5+0)^2}$$

S.B.S

$(-2)^2+(-x)^2+2(-2)(-x)+81 = 2^2+x^2-2(2)(x)+25$

$4+x^2+4x+81 = 4+x^2-4x+25$

$4x+4x=25-81$

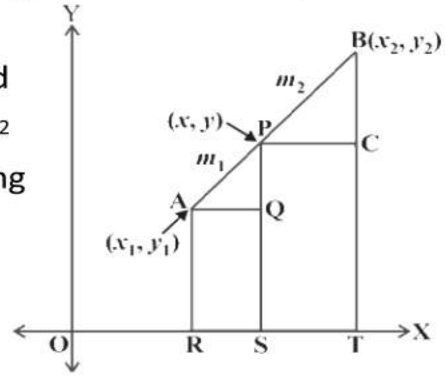
$8x = -56$

$X = -7$

$\therefore$ The point is (-7,0)

Section formula

Section formula is used to find the ratio in which a line segment is divided by a point internally or externally



Let AB is a straight line with coordinates A(x_1 , y_1) and B(x_2,y_2). Let the point P(x,y) divides AB in the Ratio m_1:m_2

Draw perpendiculars from P,A and B to the x-axis meeting at S,R and T . Draw AQ and PC Parallel to the x-axis.

Comparing Δ APQ and Δ BPC:

By A A criterion Δ APQ ~Δ BPC

Therefore $\frac{PA}{PB} = \frac{AQ}{PC} = \frac{PQ}{BC}$ ---------- (1)

Now, AQ = RS = OS–OR = $x-x_1$

PC = ST = OT– OS = x_2-x

PQ=PS–QS = PS–AR = $y-y_1$

BC=BT–CT = BT–PS = y_2-y

Substituting these values in (1), we get

$$\frac{m_1}{m_2} = \frac{x-x_1}{x_2-x} = \frac{y-y_1}{y_2-y}$$

Taking $\frac{m_1}{m_2} = \frac{x-x_1}{x_2-x}$ we get X = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

Similarly, taking $\frac{m_1}{m_2} = \frac{y-y_1}{y_2-y}$ we get y = $\frac{m_1y_2+m_2y_1}{m_1+m_2}$

So, the coordinates of the point P (x , y) which divides the line segment joining the points A(x_1 , y_1) and B (x_2 , y_2), internally in the ratio m_1:$m_2$are

$$P(x,y)= \left[\frac{m_1x_2+m_2x_1}{m_1+m_2} , \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

This is known as **section formula.**

Mid –point formula:

If the point P(x, y) divides the line segment A(x_1,y_1) and B(x_2,y_2) in the radio 1:1 then the section formula becomes mid-point formula

P(x,y) = $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

Example problems:

1) **Find the coordinates of the point which divides the line joining the points (-1,7) and (4,-3) in the ratio 2:3**

Solution : P(x,y) = $\left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$

P(x,y) = $\left[\frac{2(4)+3(-1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right]$

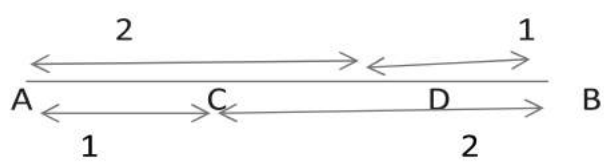
P(x,y) = (1, -3)

2) Find the coordinates of the points of trisection the line segment joining (4,-1) (-2,-3)

Solution : trisection means dividing a line segment into three equal parts

Or

Dividing a line segment in the ratio 1:2 and 2:1 internally



C(x,y)= $\left[\frac{m_1x_2+m_2x_1}{m_{1+}m_2}, \frac{m_1y_2+m_2y_1}{m_{1+}m_2}\right]$

C(x,y) = $\left[\frac{2(-2)+1(4)}{2+1}, \frac{2(-3)+1(-1)}{2+1}\right]$

C(x,y) = $\left[\frac{-4+4}{3}, \frac{-6-1}{3}\right]$

C(x,y) = $\left[0, \frac{-7}{3}\right]$

$$D(x,y) = \left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

$$D(x,y) = \left[\frac{1(-2)+2(4)}{2+1}, \frac{1(-3)+2(-1)}{2+1}\right]$$

$$D(x,y) = \left[\frac{-2+8}{3}, \frac{-3-2}{3}\right]$$

$$D(x,y) = \left[2, \frac{-5}{3}\right]$$

Therefore the coordinates of points of trisections are $\left[0, \frac{-7}{3}\right]$ and $\left[2, \frac{-5}{3}\right]$

3) find the ration in which the line segment joining the points (-3,10) and (6,-8) is divided by (-1,6)

Solution : $P(x,y)=(-1,6)$ $A(x_1,y_1)=(-3,10)$, $B(x_2,y_2)=(6,-8)$, $m_1:m_2=?$

$$P(x,y) = \left[\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right]$$

$$D(x,y) = \left[\frac{m_1(6)+m_2(-3)}{m_1+m_2}, \frac{m_1(-8)+m_2(10)}{m_1+m_2}\right]$$

$$D(x,y) = \left[\frac{6m_1-3m_2}{m_1+m_2}, \frac{-8m_1+10m_2}{m_1+m_2}\right]$$

$$-1 = \frac{6m_1-3m_2}{m_1+m_2} \quad \text{and} \quad 6 = \frac{-8m_1+10m_2}{m_1+m_2}$$

$-m_1-m_2 = 6m_1-3m_2$ $\qquad$ $6m_1+6m_2 = -8m_1+10m_2$

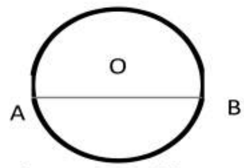
$-m_1-6m_1 = -3m_2+m_2$ $\qquad$ $6m_1+8m_1 = 10m_2-6m_2$

$-7m_1 = -2m_2$ $\qquad$ $14m_1 = 4m_2$

$$\therefore \frac{m_1}{m_2} = \frac{2}{7}$$

4) Find the coordinates of a point A, where AB is the diameter of a circle whose Centre is (2,-3) and B is (1,4)

solution :given O(2,-3) B(1,4)



since AB is diameter , the Centre of the circle become mid-point of line Segment AB

$\therefore O(x,y) = [\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}]$

$(2, -3) = [\frac{x_1+1}{2}, \frac{y_1+4}{2}]$

$2 = \frac{x_1+1}{2}$ and $-3 = \frac{y_1+4}{2}$

$4=x,+1$ $\qquad$ $-6=y,+4$

$4 -1 = x_1$ $\qquad$ $-6-4 = y_1$

$3=x_1$ $\qquad$ $-10=y_1$

$\therefore A\ (x, y)= (3,-10)$

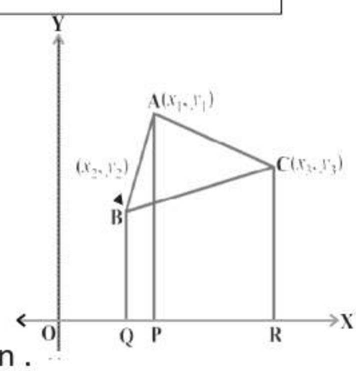
AREA OF A TRIANGLE

Let ABC be any triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Draw AP, BQ and CR perpendiculars from A, B and C, respectively, to the x-axis. Clearly ABQP, APRC and BQRC are all trapezia.

Now, it is clear that

Area of Δ ABC = area of trapezium ABQP + area of trapezium APRC – area of trapezium BQRC.

Area of trapezium = ½(sum of parallel sides) (distance between them)



Area of Δ ABC $=\frac{1}{2}[(BQ + AP)QP + \frac{1}{2}(AP+CR)PR - \frac{1}{2}(BQ+CR)QR]$

$= \frac{1}{2}[(y_2+y_1)(x_1-x_2)+\frac{1}{2}(y_1+y_2)(x_3-x_1)-\frac{1}{2}(y_2+y_3)(x_3-x_2)]$

$= \frac{1}{2}(x_1(y_2-y_3)+x_2(y_3-y_3)+x_3(y_1-y_2)$

Thus the area of Δ ABC is the numerical value of the expression .

$A = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Example problems:

1)Find the area of the triangle whose vertices are (2,3) (-1,0) (2,-4)

Solution : (2,3) (-1,0) (2,-4)

Area of $\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Area of $\Delta = \frac{1}{2}[2(0-(-4))+(-1)(-4-3)+2(3-0)]$

Area of $\Delta = \frac{1}{2}[2(4)+(-1)(-7)+2(3)]$

Area of $\Delta = \frac{1}{2}(8+7+6)$

Area of $\Delta = \frac{1}{2}(21)$

Area of $\Delta = \frac{21}{2}$ sq units.

2) Find the value of 'k' for which the points are collinear

(7-2) (5,1) , (3,k)

Solution : for collinear points Area of Δ is equal '0' (zero)

Area of $\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$0 = \frac{1}{2}[7(1-k)+5(-1(-2))+3(-2-1)]$

$0 = \frac{1}{2}[7-7k+5k+10-9]$

$0 = \frac{1}{2}(-2k+8)$

$0 = -2k+8$

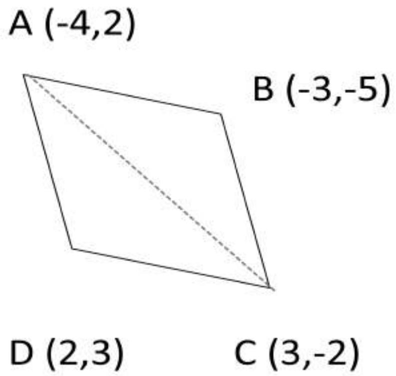
$2k=8$

$k=4.$

3) find the area of quadrilateral whose vertices, taken in order are

(-4,-2) (-3,-5) (3,-2)and (2,3)

Solution: let the vertices of quadrilateral be A(-4,-2) , B(-3,-5), C(3,-2) and (2,3)



Join AC and divide quadrilateral into two triangle ABC and ADC

Area of Δ $=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

Area of Δ ABC $=\frac{1}{2}\left[(-4)(5-(-2))+3(-2+2)+3((-2)-(-5))\right]$

$=\frac{1}{2}-4\left[(-5+2)+(-3)(0)+3(-2+5)\right]$

$=\frac{1}{2}\left[-4(-3)+(0)+3(3)\right]$

$=\frac{1}{2}(12+0+9)$

$=\frac{1}{2}(21)=\frac{21}{2}$ sq units.

Area of Δ ADC $=\frac{1}{2}\left[(-4)((-2)-(-3))+3((2)-(-2))+2(-2)-(-2))\right]$

$=\frac{1}{2}\left[-4(-5)+3(5)+2(0)\right]$

$=\frac{1}{2}(20+15+0)$

$=\frac{1}{2}(35)=\frac{35}{2}$ sq units

Area of quadrilateral ABCD = area of Δ ABC + area of Δ ADC

$=\frac{21}{2}+\frac{35}{2}$

$=\frac{21+35}{2}=\frac{56}{2}=28$ sq units.

Remarks: We can also solve this by joining diagonal BD.

Points to remember:

1) Distance between any two points A(x_1, y_1) and B(x_2, y_2) is

AB= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or AB= $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Distance between a point p(x,y) and the origin O(0,0) is

OP= $\sqrt{(x)^2 + (y)^2}$

2) Section formula: : P(x,y) = $\left[\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right]$

3) Mid point formula: P(x,y)= $\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

4)Area of trapezium = ½ (sum of parallel sides) (perpendicular distance between them)

5)Area of triangle with vertices A(x_1, y_1) , B(x_2, y_2) and C(x_3 , y_3) is

Area of Δ ABC = $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

Practice test

1.Find the distance between the points (-5,7) and (-1,3)

2. Find the coordinates of the point which divides the line segment joining the points A(4, - 3) and B(8,5) in the ratio 3:1 internally

3. The vertices of a triangle ABC are A(-3,2), B(-1,-4) and C(5,2) if M and N are the midpoints of AB and AC respectively show that 2MN =BC

4 The points A(1,1), B(3,2) and C(5,3) cannot be the vertices of the triangle ABC justify.

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QUADRATIC EQUATIONS

- Linear equation :- An equation involving a variable whose highest degree is one is called a linear equation.

  Ex. $2x+4=0$, $k-5=0$

  A linear equation has only one root.

- Quadratic equation:- An equation involving a variable whose highest degree is 2 is called a quadratic equation.

  Ex. $a^2-25=0$, $2x^2-18=0$, $x^2-x-6=0$, $a^2-3a+2=0$

  A quadratic equation has only two roots

- Pure quadratic equation:- Quadratic equation involving a variable only in second degree is pure quadratic equation.

  Ex. $x^2-9=0$, $4y^2-9=0$

  The standard form of pure quadratic equation is $\mathbf{ax^2+c=0,}$ where a and c are real numbers and $a\neq0$.

Adfected quadratic equation: Quadratic equation involving a variable in second degree as well as in first degree is called a adfected quadratic equation.

Ex : $x^2+3x-10=0$, $2x^2-3x+1=0$, $4x-3x^2+2=0$

The standard form of adfected quadratic equation is $\mathbf{ax^2+bx+c=0}$ where a, b and c are real numbers and $a\neq0$.

Quadratic equation: A quadratic equation in the variable x is an equation of the form $\mathbf{ax^2+bx+c=0,}$ where a, b, c are real numbers and $a\neq0$

Ex : $2x^2+x-300=0$, $x^2-3x-10=0$, $2x-x+1/8=0$

Standard form of a quadratic equation:

$$ax^2+bx+c=0$$

Where a, b and c are real numbers and $a\neq0$.

Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$

LHS $(x-2)^2+1$

$= x^2-4x+4+1$

(ii) $x(2x+3)=x^2+1$

LHS $x(2x+3)$

$= 2x^2+3x$

$= x^2-4x+5$

$(x-2)^2+1 \quad = 2x-3$

$x^2-4x+5 \quad = 2x-3$

$x^2-4x-2x+5+3 = 0$

$\mathbf{x^2-6x+8=0}$

This is in the form of $ax^2+bx+c=0$

So It is a quadratic equation

$x(2x+3) \quad = x^2+1$

$2x^2+3x \quad = x^2+1$

$2x^2-x^2+3x-1 \quad = 0$

$\mathbf{x^2+3x+1 \quad = 0}$

This is in the form of $ax^2+bx+c=0$

So It is a quadratic equation.

(iii) $(x+1)^2=2(x-3)$

$x^2+1+2x=2x-6$

$x^2+2x+1-2x+6=0$

$\mathbf{x^2+7=0}$

The degree of the variable is 2.

This is a Quadratic Equation.

(iv) $x^2-2x=(-2)(3-x)$

$x^2-2x=-6+2x$

$x^2-2x-2x+6=0$

$\mathbf{x^2-4x+6=0}$

It is of the form $ax^2+bx+c=0$

It is a Quadratic Equation.

(v) $(2x-1)(x-3)=(x+5)(x-1)$

$2x(x-3)-1(x-3)=x(x-1)+5(x-1)$

$2x^2-6x-x+3= x^2-x+5x-5$

$2x^2-7x+3-x^2-4x+5=0$

$\mathbf{x^2-11x+8=0}$

It is of the form $ax^2+bx+c=0$

It is a Quadratic Equation.

(vi) $x^2+3x+1=(x-2)^2$

$x^2+3x+1=x^2+4-4x$

$x^2- x^2+3x+4x+1-4=0$

$\mathbf{7x-3=0}$

It is not of the form $ax^2+bx+c=0$. It is not a Quadratic Equation.

(vii) $x^2=8$

$x^2-8=0$

The degree of the variable is 2.

This is a Quadratic Equation.

(viii) $x(x+6)=0$

$x^2+6x=0$

The degree of the variable is 2.

This is a Quadratic Equation.

For Practice:

Check whether the following are quadratic equations:

i. $x(x+1)+8=(x+2)(x-2)$
ii. $(x+2)^3=x^3-4$
iii. $(x-2)(x+1)=(x-1)(x^2+3)$
iv. $(x+2)^2=2x(x^2-1)$
v. $x^3-4x^2-x+1=(x-2)^3$
vi. $(x+9)(x-9)=0$

Represent the following situations in the form of Quadratic Equation:

i. Sum of a numbers and its reciprocal is $5\frac{1}{5}$

Solution : Let the number be =y

Reciprocal of the number = $\frac{1}{y}$

(number)+(its reciprocal)=$5\frac{1}{5}$

$Y+\frac{1}{y}=\frac{26}{5}$

$Y^2+\frac{1}{y}=\frac{26}{5}$

$5(Y^2+1)=26y$

$5Y^2+5=26y$

$5Y^2-26+5=0$

It is in the form of $ax^2+bx+c=0$.

ii. The lengh of a rectangular form (m) is one more then twice its breadth and its area is $528\ m^2$

Solution :-

let the breadth of the rectangular park be= x meter.

Its length is one more than twice its breadth =(2x+1) meter

Area of rectangle = length x breadth

$$528=(2x+1)(x)$$

$$528=2x^2+x$$

$$\mathbf{2x^2+x-528=0}$$

It is in the form of $ax^2+bx+c=0$.

iii. Rohans mother is 26 yers older then him. The product of there ages (in years) 3 years form now will be 360.

Solution:-

let Rohans present age be = x years

Since Rohans mother is 26 years older than him.

Her age will be (x+26)

After 3 years, Rohans age will be (x+3) and his mother age will be x+26+3=(x+29) years.

Product of their ages after 3 years =360

$(x+3)\ (x+29)=360$

$x(x+29)+3(x+29)=360$

$x^2+29x+3x-87-360=0$

$x^2+32-273=0$

This is in the form of $ax^2+bx+c=0$

iv. A train travels a distance of 480 km at a uniform speed. If the speed had been 8km / h, less then it would have taken 3 hours more two cover the some distance.

Solution:

let the speed of the train be =x km/h

Distance covered =480 km

Wkt , speed = $\frac{d}{t_1}$

$$t_1 = \frac{d}{s} = \frac{480}{x} \textit{hours} \qquad \text{---------(1)}$$

If the speed is less by 8km/h, then thee speed =(x-8) km/h and distance covered =480 km

$$s = \frac{d}{t_2}$$

$t_2 = \frac{d}{s}$

$t_2 \quad \frac{480}{x-8} hours$ -----------(2)

The difference in time =3 hours

Subtracting equation (1) from (2)

$\frac{480}{x-8} - \frac{480}{x} = 3$ hours

$\frac{480x - 480(x-8)}{x(x-8)} = 3$

$\frac{480x - 480x + 3840}{x^2 - 8x} = 3$

$3(x^2-8x) = 3840$

$x^2-8x = \frac{3840}{3}$

$x^2-8x = 1280$

$x^2-8x-1280 = 0$

It is in the form of $ax^2+bx+c=0$

For practice

Represent the following situations in the form of Quadratic Equation.

(i) The product of two consecutive positive integer is 306.

(ii) The base of a triangle is 4cm longer than its altitude. If the area of the triangle is 48 sq cm.

(iii) The length of rectangular field is 3 times its breadth If the area of the field is147 sq m.

(iv) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124.

Solution of a quadratic equation :

A real number α is called a root of the quadratic equation $ax^2+bx+c=0$, where $a \neq 0$ if $a\alpha^2+b\alpha+c=0$

$x = \alpha$ is a solution of the quadratic equation.

Note:-

1. The zeroes of the quadratic polynomial ax^2+bx+c and the roots of the quadratic equation $ax^2+bx+c=0$ are the same.
2. A quadratic polynomial can have at most two zeroes.
3. A quadratic equation can have at most two roots.

Finding the roots of a quadratic equation by factorization method

Steps for the finding roots by using factor method.

Step1:

Write the given equation in standard form i.e. $ax^2+bx+c=0$ then find the values of a,b and c.

Step:2:

Find the product of *a* and *c* and write it as a sum of its two factor such that sum is equal to *b*. i.e. *ac=pxq* and *p+q=b* where *p* and *q* are factors of *ac*.

Step3:

Put the value of *b* obtained from step 2 in given equation an write it LHS as product of two linear factors.

Step4:

Now, equate each factor to zero to get desired roots of given quadratic equation.

Find the roots of the equator $x^2-3x+2=0$ by factorization method

Solution:- $x^2-3x+2=0$

- Resolve the expression $x^2-2x-1x+2=0$
- Factorize $x(x-2)-1(x-2)=0$
- Take common factor $(x-2)(x-1)=0$
- Equate each factor to zero $x-2=$ $x-1=0$
- The roots are $x-2=0$ or $x-1=0$

$x=2$ $x=1$

Find the roots of the following quadratic equations by factorization.

i) $2x^2-5x+3=0$

solution: $2x^2-5x+3=0$

$2x^2-2x-3x+3=0$

$2x(x-1)-3(x-1)=0$

$(2x-3)(x-1)=0$

$2x-3=0$ or $x-1=0$

$2x=3$ or $x=1$

$x=\frac{3}{2}$ or $x=1$

ii) $x^2-3x-10=0$

solution: $x^2-3x-10=0$

$x^2-5x+2x-10=0$

$x(x-5)+2(x-5)=0$

$(x+2)(x-5)=0$

$x+2=0$ or $x-5=0$

$x=2$ or $x=5$

i) $\sqrt{2}x^2+7x+5\sqrt{2}=0$

Solution: $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\sqrt{2}x\left(x+\sqrt{2}\right)+5\left(x+\sqrt{2}\right)=0$

$(\sqrt{2}x+5)(x+\sqrt{2})=0$

$\sqrt{2}x+5=0 \quad x+\sqrt{2}=0$

$x=-\sqrt{2} \qquad \sqrt{2}x=-5$

$x=-\frac{5}{\sqrt{2}}$

Roots are $-\sqrt{2}$ and $-\frac{5}{\sqrt{2}}$

(iv) $100x^2-20x+1=0$

Solution: $100x^2-10x-10x+1=0$

$10x(10x-1)-1(10x-1)=0$

$(10x-1)(10x-x)=0$

$10x-1=0$ or $10x-1=0$

$X=\frac{1}{10}$ or $x=\frac{1}{10}$

The roots are $\frac{1}{10}$ and $\frac{1}{10}$

(v) $4k(3k-1)=5$

Solution : $4k(3k-1)=5$

$12k^2-4k-5=0$

$12k^2-10k+6k-5=0$

$2k(6k-5)+1(6k-5)=0$

$(2k+1)(6k-5)=0$

$2k+1=0$ or $6k-5=0$

$K=-\frac{1}{2}$ or $k=\frac{5}{6}$

iv) $a^2 -5a+6=0$

solution: $a^2 -5a+6=0$

$a^2-3a-2a+6=0$

$a(a-3)-2(a-3)=0$

$(a-2)(a-3)=0$

$a-2=0$ or $a-3=0$

$a=2$ or $a=3$

For practice:

(i) $x^2 +15x+50=0$

(ii) $6x^2 -x-2=0$

(iii) $3x^2 -2\sqrt{6}x+2=0$

(iv) $2x^2 +x-6=0$

(v) $2x^2 -x+1/8=0$

(vi) $6-p^2 =p$

(vii) $6a^2 +a=5$

(viii) $0.2t^2 -0.04t=0.03$

Solving quadratic equations by formula method.

Consider the general form of quadratic equation

$ax^2+bx+c=0$

Dividing by ‘a’ on both sides

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Add $\left(\frac{b}{2a}\right)^2$ on both sides

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The roots are $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

Note:

Roots of the equation $ax^2+bx+c=0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{if } b^2-4ac \geq 0$$

Solve the following equations by using quadratic formula.

(i) $x^2+7x+12=0$

Solution : $x^2+7x+12=0$

This is in the form of $ax^2+bx+c=0$

a=1, b=7 and c=12

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 - 48}}{2}$$

$$x=\frac{-7\pm\sqrt{1}}{2}$$

$$x=\frac{-7+1}{2} \quad \text{or} \quad x=\frac{-7-1}{2}$$

$$x=\frac{-6}{2} \quad \text{or} \quad x=\frac{-8}{2}$$

$\therefore$ x=-3 or x=-4

(ii) x^2-3x-10=0

Solution : x^2-3x-10=0

This is in the form of $ax^2+bx+c=0$

a=1, b=-3 and c=-10

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-10)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9+40}}{2}$$

$$x=\frac{3\pm\sqrt{49}}{2}$$

$$x=\frac{3+7}{2} \quad \text{or} \quad x=\frac{3-7}{2}$$

$$x=\frac{10}{2} \quad \text{or} \quad x=-\frac{4}{2}$$

$\therefore$ x=5 or x=-2

(iii) $3x^2$-5x+2=0

Solution : $3x^2$-5x+2=0

This is in the form of $ax^2+bx+c=0$

a=3, b=-5 and c=2

$$X=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(2)}}{2(3)}$$

$$x=\frac{5\pm\sqrt{25+24}}{6}$$

$$x = \frac{5 \pm \sqrt{49}}{6}$$

$$x = \frac{5 \pm 7}{6}$$

$$x = \frac{5+7}{6} \quad \text{or} \quad x = \frac{5-7}{6}$$

$$x=\frac{12}{6} \quad \text{or} \quad x=\frac{-2}{6}$$

$$\therefore \; x=2 \quad \text{or} \quad x=\frac{-1}{3}$$

(iv) $2x^2+x-4=0$

Solution : $2x^2+x-4=0$

This is in the form of $ax^2+bx+c=0$

$a=2$, $b=1$ and $c=-4$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(1)^2-4(2)(-4)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1+32}}{4}$$

$$x= \frac{-1 \pm \sqrt{33}}{4}$$

$\therefore$ The roots are $\frac{-1+\sqrt{33}}{4}$, $x = \frac{-1-\sqrt{33}}{4}$

(iv) $4x^2+4\sqrt{3}x+3=0$

Solution :

$4x^2+4\sqrt{3}x+3=0$

This is in the form of $ax^2+bx+c=0$

$a=4$, $b=4\sqrt{3}$ and $c=3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(4\sqrt{3}) \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{48-48}}{8}$$

$$x = \frac{-4\sqrt{3}}{8}$$

$$x = -\frac{\sqrt{3}}{2}$$

∴ The root are $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(vi) $x - \frac{1}{x} = 3$, $x \neq 0$

Solution: $x - \frac{1}{x} = 3$, $x \neq 0$

multiplying through by 'x'

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

$a=1$, $b=-3$, $c=-1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9+4}}{2}$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$x = \frac{3+\sqrt{13}}{2}$ or $X = \frac{3-\sqrt{13}}{2}$.

(vii) $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$, $x \neq -4, 7$

Solution: $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

(x-7-x-4) 30 =11(x+4) (x-7)

(-11)30 =11 (x^2+4x-7x-28)

- 330 = 11 (x^2-3x -28)

x^2-3x -28= -30

x^2-3x -28+30=0

x^2-3x +2=0

This is in the form $ax^2+bx+c=0$

a=1, b=-3 and c=2

$$x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x=\frac{(-3) \pm \sqrt{(-3)^2-4(1)(2)}}{2(1)}$$

$$x=\frac{3 \pm \sqrt{9-8}}{2}$$

$$x=\frac{3 \pm \sqrt{1}}{2}$$

$$x=\frac{3 \pm 1}{2}$$

$x=\frac{3 \pm 1}{2}$ or $x=\frac{3-1}{2}$

$x=\frac{4}{2}$ or $x=\frac{2}{2}$

x=2 or x=1

For practice:

Solve by using formula:

i) $2x^2+x+4=0$ ii) $x^2+4x+5=0$ iii) $2x^2-2\sqrt{2}x+1=0$ iv) $2p^2-p=15$

v) $8r^2=r+0$ vi) $k^2-6k=1$ vii) $\frac{1}{x-2}+\frac{2}{x-1}=\frac{2}{x}$ viii) $m^2-2m=2$

ix) $2y^2+6y=3$ x) P=5-p2 xi) $\frac{1}{x}-\frac{1}{x-3}=3$, $x\neq 0,2$

To solve the problems word based on quardratic equations:

1. If the square of number added to 3 times the number the sum is 28 find the number

Solution :- let the number be =x

Square of the number =x^2

3 times the number $=3x$

Square of the number + 3 times the number =28

$x^2+3x=28$

So $x=-7$ or $x=4$

The required numbers are -7 or 4.

2) The sum of the reciprocal of Rehmans ages (in years) 3years ago and 5years from now is $\frac{1}{3}$ find his present age.

Solution:

Let Rehmans present age be = x years

3years ago, Rehmans age was=(x-3)years

5years from now, Rehmans age will be(x+5) years

The sum of their reciprocals $=\frac{1}{3}$

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3}$$

$$\Rightarrow 3(2x+2)=1(x^2+2x-15)$$

$$\Rightarrow x^2-4x-21=0$$

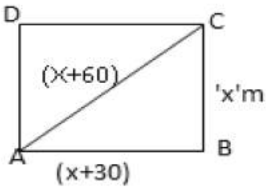
So $x=7$ or $x=-3$

$\therefore$ The present age of Rehaman = 7 years.

3) The diagonal of a rectangular field 60 meters more than the shorter side. if the longer side is 30 meters more than the shorter side find the sides of the field.

Solution:

Let the shorter side be=x meters



Then longer side =30 m more than shorter side

Longer side =(x+30)m

Diagonal =60 m more than shorter side

Diagonal=(x+60)

According Pythagoras theorem

$AC^2=AB^2+BC^2$

$(x+60)^2=(x+30)^2+x^2$

$\Rightarrow x^2+120x+3600=x^2+60x+900+x^2$

$\Rightarrow x^2-60x-2700=0$

So x=90 or x=-30(which can't be length)

shorter side=x m=90 km

longer side x+30=90+30=120m

Diagonals =x+60=90+60=150m

4) A train travels 360 km at uniform speed if the speed had been 5km/h more , it would have taken 1 hour less for the same journey find the speed of the train.

Solution:

Let the speed of the train be x km/h

Distance covered =360 km

WKT, time $t_1 = \frac{d}{s} = \frac{360}{x}$ hours

if the speed is increased by 5 km/h

then its speed =(x+5)km/h

distance covered =360 km

time $t_2 = \frac{d}{s} = \frac{360}{x+5}$ hours

$t_1 - t_2 = 1$

$$\therefore \frac{360}{x} - \frac{360}{x+5} = 1$$

$$\Rightarrow \frac{360(x+5) - 360x}{x(x+5)} = 1$$

$\Rightarrow 360x+1800-360x=x^2+5x$

$\Rightarrow x^2+5x-1800=0$

So x =-45 or x=40

$\therefore$ The speed of the train is 40 km/hr .

5) An express train takes 1 hour less than a passenger train to travels 132 km between Mysore and Bangalore (without taking into consideration the time the time they stop at intermediate station). If the average speed of the express train is 11 km/hr more than that of the passenger train ,find the average speed of the two trains.

Solution:-

Let the average speed of passenger train be x km/hr

Distance travelled =132 km

Time taken $t_1 = \frac{d}{s} = \frac{132}{x}$ hours

The average speed of the express train is 11 km/hr more than passenger train be =(x+11) km/hr

Time taken $t_{2}= \frac{d}{s} = \frac{360}{x+11}$ hours

∴ t_2-t_1=1 hour

$\frac{132}{x} - \frac{132}{x+11} = 1$

$\frac{132(x+11)-132x}{x(x+11)} = 1$

$132x+1452-132x=x^2+11x$

$x^2+11x-1452=0$

So x= - 44 x=33

∴ The average speed of the passenger train x=33km/hr.

The average speed of express train=x+11=33+11=44km/hr

6) The age of Kavya and Karthik are 11 years and 14 years. In how many years time will the products of their ages be 304.

Solution:-

Let the number of years = x

Present age of Kavya=11 years.

after x years ,Kavya's age = x+11

Similarly, after 14 years , Karthik age will be (x+14)

Product of their ages after x years =304

(x+11)(x+14)=304

x(x+14)+11(x+14)=304

x^2+14 x+11x+154-304=0

x^2+25x -150=0

So x=-30, or x=5

∴ In 5 years, the product of their ages will be 304.

7) A Dealer sells an article for Rs.24 and gains as much percent as the cost price of the article Find the cost price of the article.

Solution:-

Let the cost price of the article =Rs x

Then gain=x%

Gain= $xX\frac{x}{100}$

Gain = $\frac{x^2}{100}$

Selling price = cost price +Gain

$$= x + \frac{x^2}{100}$$

But selling price Rs. 24

$\therefore x + \frac{x^2}{100}$ =24

$100x + x^2$ =2400

$\therefore x^2+100x-2400=0$

So X=-120 or X=20

∴ The cost price of the article = Rs.20

For practise:-

1. The age of the man is twice the square of the age of her son. Eight years hence, the age of the man will be 4 years more than three times the age of his son. Find their present age.

2. The base of a triangle is 4cm longer than its altitude .If the area of the triangle is 48 sq cm. Find its base and altitude.

3.A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to reverse downstream to the to the same spot .Find the speed of the stream.

4. The difference of squares of two numbers is 180.The square of the smaller number is 8 times the larger number. Find the two numbers.

5. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

6. Sum of the areas of two squares is 468 m^2. If the difference of their perimeter is 24 cm, find the sides of the two squares.

<u>Nature of roots of a quadratic equation</u>

Case i:- Consider the equation

$x^2 - 2x + 1 = 0$

This is in the form $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4-4}}{2}$$

$$x = \frac{2 \pm \sqrt{0}}{2}$$

$$x = \frac{2}{2} = 1$$

x=1 or x=1

,',Roots are equal.

Case ii) Consider the equation

$$x^2 - 2x - 3 = 0$$

This is in the form $ax^2 + bx + c = 0$

a=1, b=-2, c=-3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+12}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2}$$

$$x = \frac{2 \pm \sqrt{16}}{2} = x = \frac{2 \pm 4}{2}$$

$x=\frac{2+4}{2}$ or $x=\frac{2-4}{2}$

$x=\frac{6}{2}$ or $x=\frac{-2}{2}$

x=3 x=-1

roots are distinct.

Case iii.) Consider the equation

$$x^2 - 2x + 3 = 0$$

This is in the form $ax^2 + bx + c = 0$

a=1, b=-2, c=3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 12}}{2}$$

$$x = \frac{2 \pm \sqrt{-4 \times 2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{-2}}{2}$$

$$x = \frac{2(1 \pm \sqrt{-2})}{2}$$

$x = 1 + \sqrt{-2}$, $x = 1 - \sqrt{-2}$

,', Roots are real or imaginary

From the above cases:-

➔ Nature of the roots of quadratic equation depend upon the value b^2-4ac.

➔ In the equation $ax^2 + bx + c = 0$ the expression b^2-4ac is called the discriminant.

Discriminant ($b^2 - 4ac$)	Nature of the roots
$b^2 - 4ac$ =0	Roots are real and equal
$b^2 - 4ac$ >0	Roots are real and Distinct
$b^2 - 4ac$ <0	No real roots

Examples:-

1.)Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$ and hence find the nature of its roots?

Solution:-

$2x^2 - 4x + 3 = 0$

This is in the form $ax^2 + bx + c = 0$

$a = 2, b = -4, c = 3$

$= b^2 - 4ac$

$= (-4)^2 - 4(2)(3)$

=16-24=-8

∴, $b^2 - 4ac$ <0

∴, The given equation has no real roots

2.Find the nature of the roots of the following quadratic equations .If the real roots exist find them.

i.) $2x^2 - 3x + 5 = 0$

This is in the form $ax^2 + bx + c = 0$

$a = 2, b = -3, c = 5$

$= b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

=9-40=-31

∴, $b^2 - 4ac$ <0

∴, The given equation has no real roots

ii.) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

This is in the form $ax^2 + bx + c = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

$=b^2-4ac$

$=\left(-4\sqrt{3}\right)^2-4(3)(4)$

=16(3)-48

=48-48=0

,', b^2-4ac =0

,', The given equation has real roots

We know that,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4\sqrt{3})\pm\sqrt{\left(-4\sqrt{3}\right)^2-4(3)(4)}}{2(3)}$$

$$x=\frac{4\sqrt{3}\pm\sqrt{48-48}}{6}$$

$$x=\frac{4\sqrt{3}}{6}$$

$$x=\frac{2\sqrt{3}}{3}$$

$$x=\frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}}$$

$$x=\frac{2}{\sqrt{3}},$$

The two equal roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$

iii.) $2x^2-5x-1=0$

This is in the form $ax^2+bx+c=0$

$a=2, b=-5, c=-1$

$=b^2-4ac$ =$(-5)^2$-(2)(-1)

=25+8=33

,', b^2-4ac >0 The roots are real and distinct.

iv) $a^2 + 4a + 4 = 0$

This is in the form $ax^2 + bx + c = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

$= b^2 - 4ac$

$= (4)^2 - 4(1)(4)$

=16-16=0

$\therefore$, $b^2 - 4ac$ =0

$\therefore$, The given roots are equal

$a^2 + 4a + 4 = 0$

$a^2 + 2a + 2a + 4 = 0$

$a(a + 2) + 2(a + 2) = 0$

$(a + 2)(a + 2) = 0$

$a = -2$ or $a = -2$

The two equal roots are $a = -2$ or $a = -2$

3. Find the value of k so that the equation $2x^2 + kx + 3 = 0$ So that they have two equal roots.

Solution:- $2x^2 + kx + 3 = 0$

This is in the form $ax^2 + bx + c = 0$

$a = 2, b = k, c = 3$

$= b^2 - 4ac$

$= (k)^2 - 4(2)(3)$

$=k^2-24$

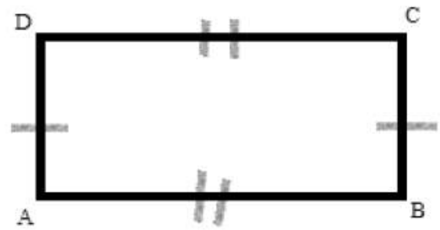
$=k=\pm\sqrt{24}$

$=k=\pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

$k=\pm 2\sqrt{6}$

4. Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 m^2?If so find its length and breadth.

Solution:- Yes, it is possible design a rectangular mango grove whose length is twice its breadth.



➔ Let the length of the rectangle be l m and breadth be b m

➔ Length is twice it's breadth l=2b

800=2b*b

$800=2b^2$

$b^2=\frac{800}{2}=400$

$b=\sqrt{400}=20$

b=20 m

,', breadth of the Rectangle =20m

Length of the rectangle 2b=2(20)=40 m

To practice:

1.Find the nature of the following quadratic equation i.) $x^2-2x+3=0$ ii.) $2x^2+5x-1=0$

iii.) $2x^2-6x+3=0$ iv.) $y^2-7y+2=0$

2. Find the value of k for each of the following quadratic equations So that they have two equal roots

3. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so find its length and breadth.

4. For what values of m the roots of two equations $x^2+mx+4=0$ are 1.) Equal 2.)Distinct.

Practice Test

1.Check whether the following equations are quadratic or not.

i. $x^2-1/x^2=5$ ii. (x-1) (x+2)=(x-3) (x+1)

2.Find the roots of the following equations by using factorization.

i. $x^2-10x+21=0$ ii. $5x+1/x=6, x\neq0$

3. Solve the following quadratic equations by using completing square method.

i. $5x^2-6x+2=0$ ii. $2x^2+x-4=0$

4. Solve the following quadratic equations by using formula method.

i. $12x^2+5x-3=0$ (vii) $\frac{x-4}{x-5}-\frac{x-6}{x-7}=\frac{10}{3}, x \neq 5,7$

5. Find the discriminant of the quadratic equation $x^2-4x+1=0$.

6. The hypotenuse of right angled triangle is 6m more than twice the shortest side. If the third side is 2m less than the hypotenuse, then find all sides of the triangle.

7. If the roots of the equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, then prove that $2a=b+c$.

8. One year ago, a man was 8 times as old as his son. Now, his age is equal to the square of his son's +age. Find their present age.

POINTS TO REMEMBER:

1. Linear equation involving a variable whose highest degree is one.
2. A linear equation has one root.
3. Quadratic equation involving a variable whose highest degree is two

- The standard form of quadratic equation is $ax^2+bx+c=0$. Where $a \neq 0$ and a,b,and c are real numbers.
- A quadratic equation only two roots.
- A real number α is called a root of the quadratic equation $ax^2+bx+c=0$, where $a \neq 0$ if $a\alpha^2+b\alpha+c=0$

  So $x = \alpha$ is a solution of the quadratic equation.
- The roots of the quadratic equation $ax^2+bx+c=0$, $a \neq 0$ can obtained by using formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ if $b^2-4ac \geq 0$

- The nature of roots of the quadratic equation $ax^2+bx+c=0$

  In the equation $ax^2+bx+c=0$ the expression b^2-4ac is called the discriminant.

****

INTRODUCTION TO TRIGONOMETRY

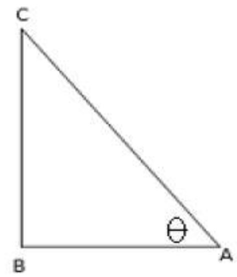
The word **TRIGONOMETRY** is derived from Greek words 'tri'(meaning **three**), 'gon'(meaning **sides**) and 'metron'(meaning **measure**).

Trigonometry can be defined as "A branch of mathematics which deals with the study of relationships between sides and angles of a triangle".

TRIGONOMETRIC RATIOS:

In trigonometry we have 6 standard ratios which gives us the relationship

between sides and angles.



Let us consider a right angled triangle ABC in which $\angle ABC = 90^0$

Let $\angle CAB = \theta$ (read as **theta**) which is acute angle.

Now observe the positions of sides of triangle with respect to θ, BC is called

Opposite side and AB is called adjacent side. We already know that AC is hypotenuse (Opposite side to right angle) .

In triangles chapter we learnt that for some constant angles, the ratio of a pair

Of sides is always constant. In any triangle we have mainly 6- ratios of pair of sides and those ratios are named as follows.

i) The ratio of opposite side to hypotenuse is called **Sine** of respective angle.

i.e. **Sine of angle** θ is $\sin\theta = \frac{Opposite\ side}{Hypotenuse} = \frac{BC}{AC}$

ii) The ratio of adjacent side to hypotenuse is called **Cosine** of respective angle.

i.e. **Cosine of angle** θ is $\cos\theta = \frac{Adjacent\ side}{Hypotenuse} = \frac{AB}{AC}$

iii) The ratio of opposite side to adjacent side is called **Tangent** of respective angle.

i.e. **Tangent of angle** θ is $\tan\theta = \frac{Opposite\ side}{Adjacent\ side} = \frac{BC}{AB}$

iv) The ratio of hypotenuse to opposite side is called Cosecant of respective angle.

i.e. **Cosecant of angle** θ is $\operatorname{cosec}\theta = \frac{Hypotenuse}{Opposite\ side} = \frac{AC}{BC}$

v) The ratio of hypotenuse to adjacent side is called Secant of respective angle.

i.e. **Secant of angle** θ is $\sec\theta = \dfrac{Hypotenuse}{\text{Adjacent side}} = \dfrac{AC}{AB}$

vi) The ratio of adjacent side to opposite side is called Cotangent of respective angle.

i.e. **Cotangent of angle** θ is $\cot\theta = \dfrac{Adjacent\ side}{Opposite\ side} = \dfrac{AB}{BC}$

Reciprocal relations of trigonometric ratios:

By observing the definitions of trigonometric ratios we can conclude that,

i) $\sin\theta = \dfrac{1}{\cos ec\theta}$ or $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

ii) $\cos\theta = \dfrac{1}{\sec\theta}$ or $\sec\theta = \dfrac{1}{\cos\theta}$

iii) $\tan\theta = \dfrac{1}{\cot\theta}$ or $\cot\theta = \dfrac{1}{\tan\theta}$

Ratios of trigonometric ratios:

Consider $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\dfrac{oppositeside}{hypotenuse}}{\dfrac{adjacentside}{hypotenuse}} = \dfrac{oppositeside}{adjacentside} = \tan\theta$

$$\boxed{\therefore \mathbf{tan}\,\theta = \dfrac{\sin\theta}{\cos\theta}}$$

Also $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\dfrac{adjacentside}{hypotenuse}}{\dfrac{oppositeside}{hypotenuse}} = \dfrac{adjacentside}{oppositeside} = \cot\theta$

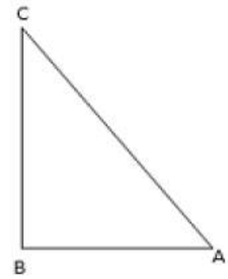
$$\boxed{\therefore \mathbf{cot}\,\theta = \dfrac{\cos\theta}{\sin\theta}}$$

Note: 1) The values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same.

2) If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.

- **Let us discuss some examples based on trigonometric ratios.**

Example 1) In a ΔABC, right angled at B, write the trigonometric ratios with respect to $\angle A$ and $\angle C$.



Solution: In ΔABC $\angle B$=90^0, AC is hypotenuse

With respect to $\angle A$

AB is adjacent side

And BC is opposite side.

Sin A=$\frac{BC}{AC}$

Cos A=$\frac{AB}{AC}$

Tan A=$\frac{BC}{AB}$

Cosec A=$\frac{AC}{BC}$

Sec A=$\frac{AC}{AB}$

Cot A=$\frac{AB}{BC}$

With respect to $\angle C$

BC is adjacent side and

AB is opposite side.

Sin C =$\frac{AB}{AC}$

Cos C =$\frac{BC}{AC}$

Tan C =$\frac{AB}{BC}$

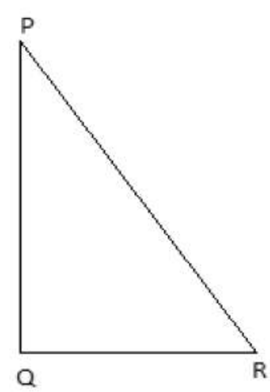
Cosec C =$\frac{AC}{AB}$

Sec C =$\frac{AC}{BC}$

Cot C =$\frac{BC}{AB}$

2) In ΔPQR right angled at Q, PQ=12 cm, PR=13cm write i) sin R, tan R

(ii) cos P , sec P



Solution: In ΔPQR, $\angle Q$ =90^0

By Pythagoras theorem $PR^2=PQ^2+QR^2$ $\therefore$ $QR^2=PR^2-PQ^2$

$QR^2= 13^2-12^2$

$QR^2=169-144$

$QR^2=25$

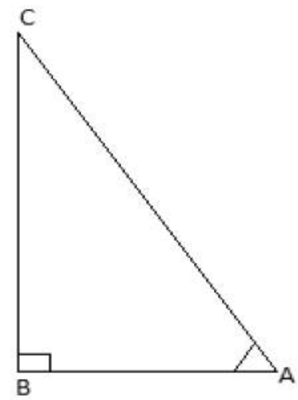
$QR=5$ cm

i) Sin R= $\frac{PQ}{PR}=\frac{12}{13}$ and tan R= $\frac{PQ}{QR}=\frac{12}{5}$

ii) Cos P= $\frac{PQ}{PR}=\frac{12}{5}$ and sec P= $\frac{PR}{PQ}=\frac{5}{12}$

3) Given SecA = $\frac{13}{12}$, calculate all other trigonometric ratios.

Solution: Let us consider a right angled triangle in which $\angle ABC = 90^0$



Given that Sec A= $\frac{13}{12}=\frac{AC}{AB}\left(SecA=\frac{Hypotenuse}{Adjacentside}\right)$

$\therefore$ AC = 13k, AB= 12k

By Pythagoras theorem, we have

$AC^2=AB^2+BC^2 \Rightarrow BC^2=AC^2-AB^2$

$BC^2=(13k)^2-(12k)^2=169k^2-144k^2=25k^2$

$\therefore$ BC=5k.

$\therefore$ Sin A = $\frac{BC}{AC}=\frac{5k}{13k}=\frac{5}{13}$ Cosec A = $\frac{AC}{BC}=\frac{13k}{5k}=\frac{13}{5}$

Cos A = $\frac{AB}{AC}=\frac{12k}{13k}=\frac{12}{13}$ Sec A = $\frac{AC}{AB}=\frac{13k}{12k}=\frac{13}{12}$

Tan A = $\frac{BC}{AB}=\frac{5k}{12k}=\frac{5}{12}$ Cot A = $\frac{AB}{BC}=\frac{12k}{5k}=\frac{12}{5}$

Practice problems:

1. In ΔPQR, right angled at Q, PQ= 12cm, and PR = 13cm find

   i) SinR, CosR and ii) SinP , CosP

2. If sinA = $\frac{3}{4}$, calculate Cos A and tan A.

3. If Cotθ = $\frac{7}{8}$, evaluate $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

4. If 3CotA =4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = Cos^2 A - \sin^2 A$

5. If CosecΘ= $\frac{5}{3}$ and 'Θ' is an acute angle, Show that $\frac{4sin\theta - 7cos\theta}{3sin\theta + 2cos\theta}$ = 40.

6. Express all the trigonometric ratios in terms of sinΘ.

7. Express all the trigonometric ratios in terms of cosΘ.

8. Express all the trigonometric ratios in terms of tanΘ.

NOTE: The value of SinΘ or CosΘ never exceeds 1, whereas the value of SecΘ or CosecΘ is always greater than or equal to 1.

Trigonometric ratios of some specific angles:

We already know that "**The values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same"** so let us learn the values of trigonometric ratios for some specific angles.

	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND
Cosec	ND	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	ND
Cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Example 1) Evaluate $Sin60^0 Cos30^0 + Sin30^0 Cos60^0$.

Solution: $Sin60^0 Cos30^0 + Sin30^0 Cos60^0$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1.$$

Example 2) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 30^0$.

Solution: $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 30^0$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{1}{4} = \frac{5}{2}.$$

Example 3) Evaluate $\frac{1-\tan^2 45}{1+\tan^2 45}$.

Solution: $\frac{1-\tan^2 45}{1+\tan^2 45} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0.$

Example 4) If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 1/\sqrt{3}$, $0^0 < A+B \leq 90^0$; $A>B$, Find A and B.

Solution: Given that,

$\tan(A+B) = \sqrt{3}$ -------- (1)

$\tan(A-B) = 1/\sqrt{3}$ -------- (2)

From (1) and (2)

$A+B = 60^0$ ------------------- (3)

$A-B = 30^0$ ------------------- (4)

From (3) and (4)

$A+B = 60^0$	Put $A=45^0$ in equation (3),
$\underline{A-B = 30^0}$	$45^0 + B = 60^0$
$2A = 90^0$	$B = 60^0 - 45^0$
$A = 45^0$	$B = 15^0$

Practice problems:

1) Evaluate

i) $\frac{2\sin 30 - 2\cos 30}{3\cot^2 45}$

ii) $\dfrac{\sin 30+\tan 45-\operatorname{cosec}60}{\sec 30+\cos 60+\cot 45}$

2)If $\sqrt{3}\tan\theta$ = 1 and θ is acute, find the value of sin3θ + cos2θ.

3)If A=$60^0$and B=30^0, then prove that Cos (A+B) = CosA.CosB – SinA.SinB.

4)If B=15^0, then prove that 4Sin2B.Cos4B.Sin6B=1.

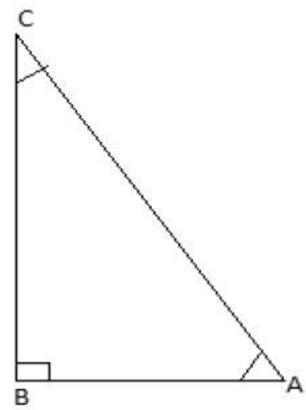
5)For A=30^0,45^0 and 60^0 **verify** i) Sin2A =2SinA.CosA ii) Cos2A = Cos^2A-Sin^2A.

<u>Trigonometric Ratios of Complementary angles:</u>

Complementary angles: The two angles are said to be complementary if their sum equals to 90°.

n a Δ ABC right angled at B, ∠A+∠C=90°

If ∠A is a referencing angle, then



Sin A= $\dfrac{BC}{AC}$ cos A= $\dfrac{AB}{AC}$ tan A= $\dfrac{BC}{AB}$

cosec A= $\dfrac{AC}{BC}$ Sec A= $\dfrac{AC}{AB}$ cot A= $\dfrac{AB}{BC}$

Similarly, if ∠C is a referencing angle, then

Sin C= $\dfrac{AB}{AC}$ cos C= $\dfrac{BC}{AC}$ tan C= $\dfrac{AB}{BC}$

cosec C= $\dfrac{AC}{AB}$ Sec C= $\dfrac{AC}{BC}$ Cot C= $\dfrac{BC}{AB}$

By observing the above ratios we have,

Sin C = Cos A and Cos C = Sin A

Tan C = Cot A and Cot C = tan A

Cosec C = Sec A and Sec C = Cosec A

But we know ∠A+∠C=90° $\Rightarrow \angle C = 90^0 - \angle A$

$\therefore$ **Sin (90-A) = Cos A or Cos (90-A) = Sin A**

Tan (90-A) = Cot A or Cot (90-A) = tan A

Cosec (90-A) = Sec A or Sec (90-A) = Cosec A

➢ **Let us solve some examples based on complementary angles,**

Example 1) Evaluate $\frac{\sin 23}{\cos 67}$

Solution: $\frac{\sin 23}{\cos 67}$ = $\frac{\sin 23}{\cos(90-23)}$ = $\frac{\sin 23}{\sin 23}$ = **1**

Example 2) Evaluate cosec 32- Sec 58

Solution: Cosec 32 –Sec 58 = Cosec 32 – Sec (90-32)

= Cosec 32 –Cosec 32 = **0**

Example 3) $\frac{\sin 48}{\cos 42} - \frac{\tan 36}{\cot 54}$

Solution: $\frac{\sin 48}{\cos 42} - \frac{\tan 36}{\cot 54}$ = $\frac{\sin 48}{\sin 48} - \frac{\tan 36}{\tan 36}$ = 1-1 =**0**

Example 4) If Sec4A=Cosec(A-20) where 4A is an acute angle, find the value of A.

Solution: Sec 4A = Cosec (A-20)

Cosec (90-4A) = Cosec (A-20) $\because$ SecA = Cosec (90-A)

$\Rightarrow$ 90-4A = A-20

$\Rightarrow$ 5A = 110

$\Rightarrow$ A = 22^0

Example 5) If A, B and C are interior angles of triangle ABC, then show that

sin $\left(\frac{B+C}{2}\right)$ = Cos $\left(\frac{A}{2}\right)$

Solution: Given that A, B and C are interior angles of triangle.

$\therefore$ A+B+C = $180^0 \Rightarrow$ B+C = 180 –A

Divide by 2 on both sides,

$$\left(\frac{B+C}{2}\right)=\left(\frac{180-A}{2}\right)$$

$$\left(\frac{B+C}{2}\right)=\left(90-\frac{A}{2}\right)$$

Apply sine on both sides, we get

$$Sin\left(\frac{B+C}{2}\right) = Sin\left(90-\frac{A}{2}\right)$$

$$Sin\left(\frac{B+C}{2}\right) = Cos\left(\frac{A}{2}\right).$$ $\because$ CosA = Sin (90-A)

Practice problems:

1) Show that $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$

2) Show that $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0 = 0$

3) If $\sin 3A = \cos(A-26^0)$, where 3A is an acute angle, find the value of A.

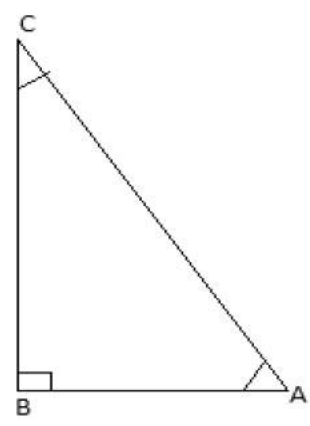
4) Show that Sin40Cos50+Cos40Sin50 = 1

5) If tan2A= Cot (A-30), find the value of A.

Trigonometric Identities:

Trigonometric Identities are the equations containing trigonometric ratios and which are true for all the values of angles involved.

Let us derive some basic trigonometric identities and use it further to prove other useful trigonometric identities.



In ΔABC, $\angle ABC = 90^0$

$AB^2+BC^2=AC^2$ --------(1) (by Pythagoras theorem)

Dividing each term of (1) by AC^2, we get

$AB^2+BC^2=AC^2$

$$\frac{AB^2}{AC^2}+\frac{BC^2}{AC^2}=\frac{AC^2}{AC^2}$$

$$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

$(\cos A)^2 + (\sin A)^2 = 1$

$$\boxed{\cos^2 A + \sin^2 A = 1}$$

This is true for all A such that $0^0 \le A \le 90^0$

Let's us now divide equation (1) by AB^2 we get,

$$\frac{AB^2}{AB^2}+\frac{BC^2}{AB^2}=\frac{AC^2}{AB^2}$$

$$(\frac{AB}{AB})^2 + (\frac{BC}{AB})^2 = (\frac{AC}{AB})^2$$

$$\boxed{1 + \mathrm{Tan}^2 A = \mathrm{Sec}^2 A}$$

This is true for all A such that $0^0 \le A \le 90^0$

Let us see what we get on dividing eqn (1) by BC^2

$$\frac{AB^2}{BC^2}+\frac{BC^2}{BC^2}=\frac{AC^2}{BC^2}$$

$$(\frac{AB}{BC})^2 + (\frac{BC}{BC})^2 = (\frac{AC}{BC})^2$$

$$\boxed{\mathrm{Cot}^2 A + 1 = \mathrm{Cosec}^2 A}$$

This is true for all A such that $0^0 \le A \le 90^0$

Three identities are:

1. **Cos A ^{2}+ Sin A ^{2}=1**
2. **1+tan^2 A= Sec^2 A**

Using these identities we can express each trigonometric ratio in terms of other trigonometric ratios, we can also determine the value of other trigonometric ratios.

- **Let us prove some more trigonometric identities:**

1) Prove that $\left(\frac{1+\cos\theta}{1-\cos\theta}\right)=(\cos ec\theta+\cot\theta)^2$

Proof: L.H.S. = $\frac{1+\cos\theta}{1-\cos\theta}$

Multiply and divide by (1+Cos θ) (conjugate of denominator)

$$=\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$

$$=\frac{(1+\cos\theta)^2}{1-\cos^2\theta} = \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta}$$

$$=\frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}+\frac{2\cos\theta}{\sin^2\theta} = \mathrm{Cosec}^2\Theta + \cot^2\Theta + 2\cot\Theta.\ \mathrm{cosec}\Theta$$

= $(\text{Cosec}\theta+\text{Cot}\theta)^2$

2) Show that (tan A X Sin A) +Cos A = Sec A.

Proof: L.H.S. = $(\tan A \times \sin A) + CosA$

$$= \frac{SinA}{CosA} \times \sin A + \text{CosA} \quad = \frac{Sin^2 A}{CosA} + CosA$$

$$= \frac{Sin^2 A + Cos^2 A}{CosA} = \frac{1}{CosA} = \textbf{SecA}$$

3) Show that $\frac{\tan\theta}{1-\cot\theta} + \frac{Cot\theta}{1-\tan\theta} = 1 + Sec\theta.Co\sec\theta$

Proof: L.H.S. = $\frac{\tan\theta}{1-\cot\theta} + \frac{Cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{Cos\theta}{Sin\theta}} + \frac{\frac{Cos\theta}{Sin\theta}}{1-\frac{Sin\theta}{Cos\theta}}$$

$$= \frac{\frac{Sin\theta}{Cos\theta}}{\frac{Sin\theta - Cos\theta}{Sin\theta}} + \frac{\frac{Cos\theta}{Sin\theta}}{\frac{Cos\theta - Sin\theta}{Cos\theta}}$$

$$= \frac{Sin^2\theta}{Cos\theta(Sin\theta - Cos\theta)} + \frac{Cos^2\theta}{Sin\theta(Cos\theta - Sin\theta)}$$

$$= \frac{Sin^2\theta}{Cos\theta(Sin\theta - Cos\theta)} - \frac{Cos^2\theta}{Sin\theta(Sin\theta - Cos\theta)}$$

$$= \frac{Sin^3\theta - Cos^3\theta}{Sin\theta.Cos\theta(\sin\theta - Cos\theta)}$$

$$= \frac{(Sin\theta - Cos\theta)(Sin^2\theta + Sin\theta.Cos\theta + Cos^2\theta)}{Sin\theta.Cos\theta(Sin\theta - Cos\theta)}$$

$$= \frac{1 + Sin\theta.Cos\theta}{Sin\theta.Cos\theta} \quad = \frac{1}{Sin\theta.Cos\theta} + \frac{Sin\theta.Cos\theta}{Sin\theta.Cos\theta}$$

= Secθ. Cosecθ +1.

- **Prove the following identities.**

1) $\sqrt{\dfrac{SecA+\tan A}{SecA-\tan A}}=\dfrac{1+SinA}{CosA}$

2) $\sec^4A - \sec^2A = \tan^4A + \tan^2A$

3) $\cos^4A - \sin^4A = 2\cos^2A - 1$

4) (1-CosecA) (1-SecA)(1+CosecA)(1+CosA) = CosA

5) tanA+ CotA = SecA.CosecA

6) $\dfrac{Sec\theta-Cosec\theta}{Sec\theta+Cosec\theta}=\dfrac{\tan\theta-1}{\tan\theta+1}$

7) $(1+\tan\Theta)^2+(1+\cot\Theta)^2 = (\sec\Theta+\operatorname{cosec}\Theta)^2$.

8) $(\sin\Theta+\operatorname{cosec}\Theta)^2+(\cos\Theta+\sec\Theta)^2 = 7+\tan^2\Theta+\cot^2\Theta$.

Instructions to Prove trigonometric identities:

Dear students we have many types of trigonometric identities and there are many

ways to prove them. Here we are giving you some tips which make the proof easier.

- If there is a common factor in either L.H.S. or R.H.S. then factorise it and you may get a basic trigonometric identity.

- If there are binomials in denominator then multiply and divide the expression by CONJUGATE of denominator.(as we done in ex1).

- Try to get same denominators if there are fractions. (as we done in Ex3).

- Split up a fraction into two separate fractions. (as we done in Ex3)

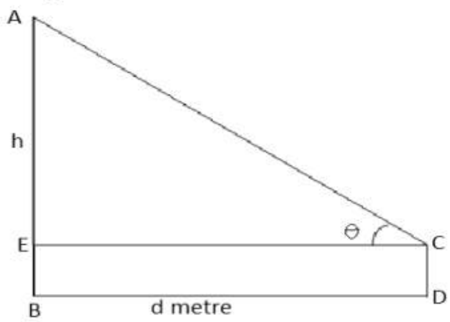
- Try to write every trigonometric function in terms of Sine and Cosine.

****

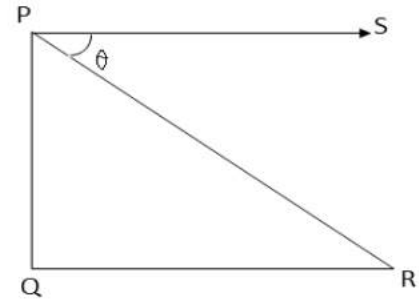
SOME APPLICATIONS OF TRIGONOMETRY

Dear students we learnt about basic concepts of trigonometry in our previous chapter. Now we shall see the applications of trigonometry in this chapter and specially the use of trigonometric in finding heights and distances.

Before learning applications we have to learn some basic terms related to heights and distances.



Fig(i)



fig(ii)

In figure (i) AB is a tower of height 'h' meter and 'CD' is a person standing at a distance of 'd' meter from AB. Now the person is observing the top of the tower, so the line CA drawn from the eye of observer to the object is called **line of sight** and the line CE drawn from eye of observer and parallel to ground is called **horizontal line.** The $\angle ACE$ formed by the line of sight with horizontal line is called **angle of elevation.**

In figure (ii) PQ is a building stand on the ground and a person standing at P observing a point on the ground at R. here the line PR is called the line of sight , PS is called horizontal line and $\angle SPR$ the angle formed by line of sight with horizontal line is called angle of depression.

Line of Sight : The line drawn from the eye of an observer to the object viewed is called line of sight.

Horizontal line: The line drawn from the eye of observer and parallel to ground is called horizontal line.

Angle of elevation: The upward angle formed by the line of sight with horizontal line is called angle of elevation.

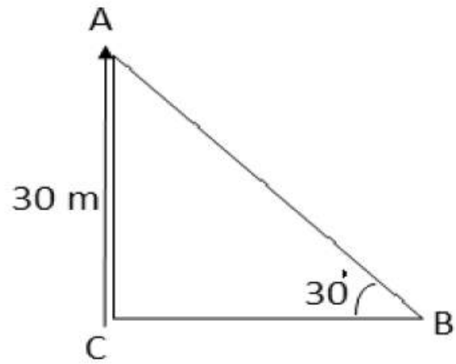
Angle of depression: The downward angle formed by the line of sight with horizontal line is called angle of depression.

NOTE: 1) The angle of elevation and the angle of depression of line of sight must measure with horizontal line.

➢ **Let us discuss some examples.**

Example 1) The angle of elevation of the top of a tower from a point on the ground which is 30m away from the foot of the tower is 30^0. Find the height of the tower.

Solution: Let AB= 'h' m be the height of the tower and 'C' be the point at which observer exists. In $\triangle$ABC, BC= 30, $\angle C = 30^0$ and AB= h m (to be find out) are related by trigonometric ratio tanΘ.



$$\tan C = \frac{AB}{BC} \Rightarrow \text{Tan } 30 = \frac{h}{30}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30} \Rightarrow h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

= 10$\sqrt{3}$ m.

The height of the tower is h= 10$\sqrt{3}m$.

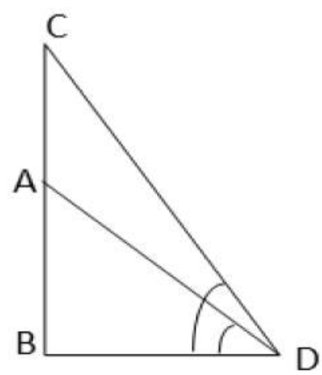
Example2) From a point on the ground, the angles of elevation of the bottom and the top of transmission tower fixed at the top of a 20 m high building are 450 and 60^0 respectively. Find the height of the tower.

Solution: Let AB=20m be the height of the building.

And AC = 'h' m height of the transmission

tower fixed at the top of AB.

'D' be the point at which observer exists.



In $\triangle$ABD, $\tan 45 = \frac{AB}{BD} \Rightarrow 1 = \frac{AB}{BD} \Rightarrow AB = BD$.

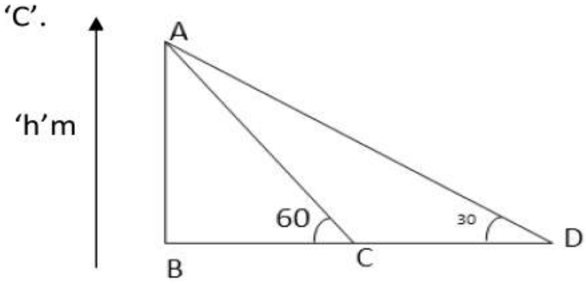
In $\triangle$DBC, $\tan 60 = \frac{BC}{BD} \Rightarrow \sqrt{3} = \frac{h+20}{20} \Rightarrow h = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)m$

The height of the tower is h= 20($\sqrt{3}-1)m$.

Example3) A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^0. From another point 20m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30^0. Find the height of the tower and width of the canal.

Solution: Let AB= 'h' m be the height of the tower and BC = 'x' m be the width of the canal.

'D' be the another point 20m away from 'C'.



In $\triangle$ABC, tanC = $\frac{AB}{BC}$ $\Rightarrow$Tan60 = $\frac{h}{x}$

$\Rightarrow\sqrt{3} = \frac{h}{x} \Rightarrow$ h = x$\sqrt{3}$----(1)

In $\triangle$ABD, tanD = $\frac{AB}{BD}$

$\Rightarrow$Tan30 = $\frac{h}{x+20}$

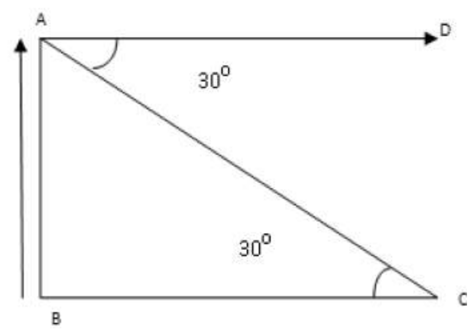
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20} \Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{x+20} \Rightarrow x+20 = 3x \Rightarrow x = 10m$$

Substitute x= 10m in (1), we get h= 10$\sqrt{3}$ m.

The height of the tower is h= 10 $\sqrt{3}m$ and the width of the canal is x= 10m.

Example 4) From a ship's mast head 100m high, the angle of depression of a boat is observed to be 30^0. Find the distance of the boat from the ship.

Solution: Let AB = 100m be the height of the mast.
D



And 'C' be the point at which boat exists.

$\angle DAC = 30^0 = \angle BCA$ ($\because$ Alternate angles)

tanC = $\frac{AB}{BC}$ 100m

$\Rightarrow$ Tan 30 = $\frac{100}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BC}$

BC = 100$\sqrt{3}$ m.

Distance of the boat from the ship is 100$\sqrt{3}$ m.

Example 5) From the top of a cliff 40m high the angles of depression of two objects on the ground due west of cliff are 45^0 and 30^0. Find the distance of the objects .

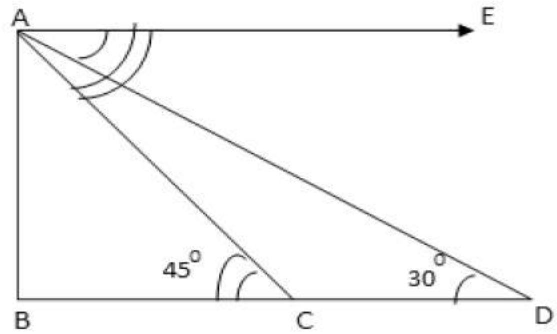
Solution: Let AB= 40m be the height of cliff.

'C' and 'D' be the two different objects on the ground.

$\angle EAD = \angle ADB = 30^0$ $\because$ Alternate angles

$\angle EAC = \angle ACB = 45^0$ $\because$ Alternate angles

In $\triangle$ABC,



tanC = $\frac{AB}{BC}$

$\Rightarrow$ Tan 45 = $\frac{AB}{BC} \Rightarrow 1 = \frac{40}{BC}$

$\therefore$
BC= 40m.

In $\triangle$ADB,

tanD = $\frac{AB}{BD} \Rightarrow$ Tan 30 = $\frac{40}{40+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{40+x}$

40+x = 40$\sqrt{3} \Rightarrow x = 40(\sqrt{3}-1)m$.

The distance between the objects is 40($\sqrt{3}-1)m$

Example 6) A 1.5 m tall boy is standing at some distance from a 30m tall building. The angle of elevation from his eyes to the top of the building increases from 30^0 to 60^0 as he walks towards the building. Find the distance he walked towards the building.

Solution: Let AB= 1.5m be the toll boy.

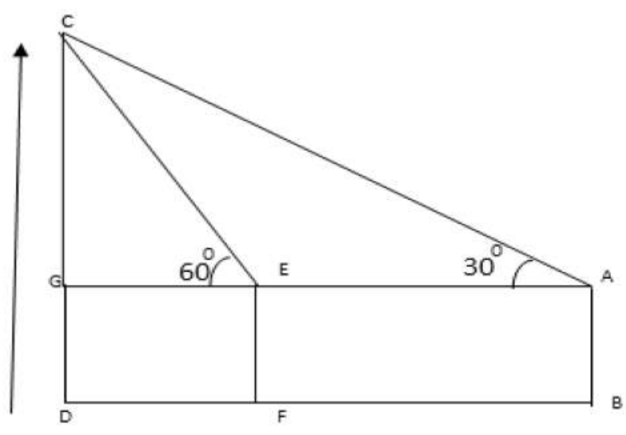
CD= 30m be the height of the building.

BF=AE = 'x' m be the distance travelled.

AB=EF=GD=1.5m

CG= CD-DG $\Rightarrow CG = 30 - 1.5 = 28.5m$

In $\triangle$CEG,



tanE = $\frac{CG}{EG}$ 30m

$\Rightarrow$ Tan 60 = $\frac{28.5}{EG} \Rightarrow \sqrt{3} = \frac{28.5}{EG}$

$\therefore$ EG = $\frac{28.5}{\sqrt{3}}m$

In $\triangle$CAG,

tanA = $\frac{CG}{AG}$

$\Rightarrow$ Tan 30 = $\frac{28.5}{AG} \Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{AG}$

AG = 28.5 $\sqrt{3}$ m.

$\therefore$ AE+EG = 28.5 $\sqrt{3}$

$\Rightarrow AE = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$ ($\because$ EG = $\frac{28.5}{\sqrt{3}}m$)

$\therefore$ AE = $\frac{85.5 - 28.5}{\sqrt{3}} = \frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 19\sqrt{3}m$

$\therefore$ **The distance walked towards the building is 19 $\sqrt{3}$ m.**

Example 7) From the top of a 7m high building, the angle of elevation of the top of a cable tower is 60^0 and the angle of depression of its foot is 45^0. Determine the height of the tower.

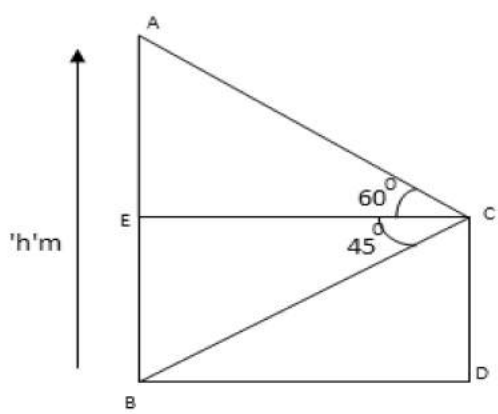
Solution: Let AB= 'h' m be the height of the tower.

CD= 7m is a building.

CD=BE=7m

In $\triangle$DBC,

tanB = $\frac{CD}{BD}$



$\Rightarrow$ Tan 45 = $\frac{7}{BD} \Rightarrow 1 = \frac{7}{BD}$

BD = 7m= CE.

In $\triangle$AEC,

tanC = $\frac{AE}{EC} \Rightarrow$ Tan 60 = $\frac{h-7}{7}$

$\Rightarrow \sqrt{3} = \frac{h-7}{7}$ h =7(1+ $\sqrt{3}$) m.

The height of the tower is $7(1+\sqrt{3})$ m.

Example 8) The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6m.

Solution: Let AB = 'h' m be the height of the tower

BC= 4m and BD= 9m.

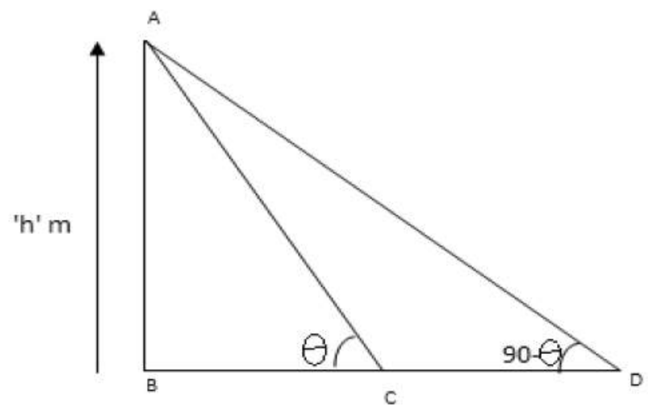
In $\triangle ABC$, $\tan\Theta=\frac{h}{4}$----(1)

In $\triangle ABD$, $\tan(90-\Theta)=\frac{h}{9}$ ----(2)

Multiply (1) and (2)

$\tan\Theta.\tan(90-\Theta) = \frac{h}{4}\cdot\frac{h}{9}$

$$\Rightarrow 1 = \frac{h^2}{36} \Rightarrow h^2 = 36 \Rightarrow h = 6m$$



Hence proved.

PRACTICE PROBLEMS:

1) A tower casts a shadow of 300ft long, when th sun's altitude (elevation) is 30^0. Find the height of the tower.

2) A flagstaff stands upon the top of building. At a distance of 75m, the angles of elevation of tops of the flagstaff and building are 45^0 and 30^0. Find the height of the flagstaff.

3) The angle of elevation of the top of a tower is 30^0. On walking 300m nearer the elevation is found to be 60^0. Find the height of the tower.

4) From the top of a cliff 180 m high the angles of depression of the top and bottom of a tower are observed to be 30^0 and 60^0. Find the height of the tower.

5) From a point 100m above a lake, the angle of elevation of an object is 300 and the angle of depression of its image in the water is 450. Find the height of the object above the lake.

Instructions to solve application problems on trigonometry:

- Understand the question clearly .
- Draw the suitable geometrical figure.
- Note down the known things(given lengths and angles of elevation/ depression) and the things to be find out.
- Chose the triangle which contain the unknown thing to be find out.
- Apply the suitable trigonometric ratio of given angle which relates unknown thing with known things(Sine, Cosine and tangent).

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STATISTICS

The branch of mathematics which deals with the study of collection, analysis, interpretation and representation of data is called Statistics.

Data: The collection of information in numerical form is called data.

There are mainly two types of data,

i) Ungrouped data: The random collection of data is called Ungrouped data.

ii) Grouped data: The data categorized into groups based on some measure is called grouped data.

Central tendency: Central tendency is a descriptive summary of a dataset through a single value that reflects the centre of the data distribution.

We have three measures of central tendency,

i) **Mean:** The ratio of sum of values of all observations to the total number of observations is called Mean.

Mean = $\dfrac{Sum of all observations}{Total number of observations}$

ii) **Median:** The middle most observation of an orderly arranged data distribution is called Median.

If there are 'n' scores, then $(\frac{n}{2})th$ score will be median if 'n' is odd and average of $(\frac{n}{2})th$ and $(\frac{n+1}{2})th$ score will be median if 'n' is even.

iii) **Mode:** The most repeated observation in a dataset is called Mode.

A data set may have unimode, bimode or multimodes.

Mean for grouped data:

i) **By direct method**:

If $x_1, x_2, x_3, \ldots\ldots x_n$ are observations with respective frequencies $f_1, f_2, f_3, \ldots\ldots f_n$, then mean will be calculated as ,

Mean = $\dfrac{f1x1 + f2x2 + f3x3 + \cdots \ldots\ldots + fnxn}{f1 + f2 + f3 + \cdots \ldots\ldots + fn}$

$$\boxed{\therefore \text{X} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}}$$

- **Let us take one grouped data distribution and calculate mean by all these three methods.**

Example 1) Calculate the mean of the following data distribution:

C-I	0-10	10-20	20-30	30-40	40-50
f	7	4	3	6	5

Solution: i) By direct method:

C-I	f	Class mark(x)	fx
0-10	7	5	35
10-20	4	15	60
20-30	3	25	75
30-40	6	35	210
40-50	5	45	225

$\sum f = 25$ $\sum fx = 605$

$$\therefore Mean = \frac{\sum fx}{\sum f} = \frac{605}{25}$$

$\therefore$ **Mean = 24.2**

Mode for grouped data:

The formula to find mode of the grouped data is,

Mode= $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Where l- lower limit of modal class

h- Size of class interval

f_1- frequency of the modal class

f_0 – frequency of the class preceding the modal class

f_2- frequency of the class succeeding the modal class.

Example 1) Calculate the mode of the following data distribution.

C-I	0-5	5-10	10-15	15-20	20-25
F	5	3	6	2	4

Solution: Here the maximum class frequency is 6 and the class corresponding to this Frequency is 10-15

$\therefore$ Lower limit of modal class l=10,

Size of class interval h= 5

Frequency of the modal class $f_1 = 6$

Frequency of the class preceding the modal class $f_0 = 3$

Frequency of the class succeeding the modal class $f_2 = 2$

$$\therefore \quad \text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$\text{Mode} = 10 + \left[\frac{6-3}{2(6)-3-2}\right] \times 5$$

$$\text{Mode} = 10 + \left[\frac{3}{7}\right] \times 5$$

$\therefore$ **Mode = 12.14**

<u>Median for grouped data:</u>

The formula to find median for grouped data is,

$$\textbf{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

Where, l-lower limt of median class

n- number of observations

cf- cumulative frequency of class preceding the median class

f- frequency of median class

h- size of class interval

Example 1) Calculate the median of the following data distribution.

C-I	5-15	15-25	25-35	35-45	45-55	55-65
f	6	11	21	23	14	5

Solution:

C-I	f	Cf
5-15	6	6
15-25	11	17
25-35	21	38
35-45	23	61
45-55	14	75
55-65	5	80

N=80

Consider $(\frac{n}{2})th$ score = $(\frac{80}{2})th$ score=40^{th} score which comes in the class interval (35-45)

which have the cumulative frequency 61 greater than and nearest to $\frac{n}{2}$ i.e. 40.

Lower limt of median class l =35

Cumulative frequency of class preceding the median class cf= 38

Frequency of median class f = 23

Size of class interval h = 10

Median = $l+\left[\frac{\frac{n}{2}-cf}{f}\right]\times h = 35+\left[\frac{\frac{80}{2}-38}{23}\right]\times 10 = 35+\left[\frac{2}{23}\right]\times 10 = 35+0.86$ = **35.86**

Example 2) If the median of the data distribution given below is 28.5, find the values of x and y.

Class Interval	Frequency
0-10	5
10-20	X
20-30	20
30-40	15
40-50	Y
50-60	5
Toatl	60

Solution:

C-I	f	Cf
0-10	5	5
10-20	X	5+x
20-30	20	25+x
30-40	15	40+x
40-50	Y	40+x+y
50-60	5	45+x+y

Median is 28.5 means the class (20-30) is the median class.

$\Rightarrow$ l=20, f= 20, cf=5+x, n=60 =(45+x+y) and h=10.

Median = $l + \left[\dfrac{\frac{n}{2} - cf}{f} \right] \times h$

$$\Rightarrow 28.5 = 20 + \left[\dfrac{\frac{60}{2} - (5 + x))}{20} \right] \times 10$$

$$\Rightarrow 28.5 - 20 = \left[\dfrac{30 - 5 - x}{2} \right]$$

$$\Rightarrow 8.5 = \left[\frac{25-x}{2}\right]$$

$$\Rightarrow 17 = 25 - x$$

$\therefore$ **x=8**

Substitute x= 8 in 45+x+y = 60

We get **y=7**.

Note: There is a empirical relationship between the three measures of central tendency:

3Median = Mode + 2mean

Practice problems: Calculate mean, median and mode for the following data.

1)

C-I	0-5	5-10	10-15	15-20	20-25
F	5	6	3	4	2

2)

C-I	1-3	3-5	5-7	7-9	9-11
F	4	7	6	3	5

3)

C-I	1-10	11-20	21-30	31-40	41-50
F	8	6	5	2	4

4)

C-I	0-100	100-200	200-300	300-400	400-500
F	12	14	8	7	9

Graphical representation of cumulative frequency distribution:

As we all know, pictures speak better than words. **A graphical representation helps us in understanding given data at a glance.** In our previous classes we have represented the data through pictographs, bar graphs, histograms and frequency polygons. Let us now represent a cumulative frequency distribution graphically which is known as **ogive**.

Ogive: The term **ogive** is pronounced as **'ojeev'** and is derived from the word **Ogee**. **An ogee is a shape consisting of a concave arc flowing into a convex arc, so forming an S-shaped curve with vertical ends.** In architecture, the ogee shape is one of the characteristics of the 14^{th} and 15^{th} century Gothic styles.

The ogive is a graph of a cumulative distribution, which explains data values on the horizontal axis(x-axis) and cumulative frequency on the vertical axis(y-axis).

We have 2 types of ogive graph: 1) Less than type and 2) More than type.

1) **Less than type ogive:** The graph in which upper limit of class interval taken along x-axis and cumulative frequency taken along y-axis is called less than type ogive.

2) **More than type ogive:** The graph in which lower limit of class interval taken along x-axis and cumulative frequency taken along y-axis is called more than type ogive.

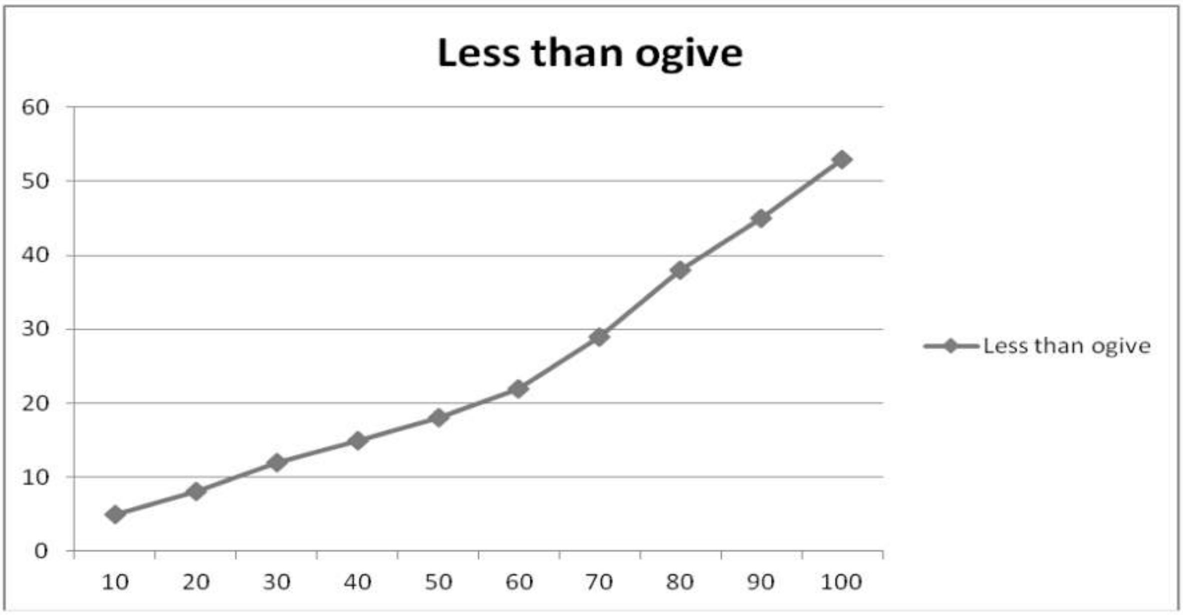
Let us draw both less than type and more than type graphs for a data distribution of marks obtained ,out of 100 by 53 studentsin a certain examination.

C-I	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
F	5	3	4	3	3	4	7	9	7	8

i) **Less than type:** In less than type we take only upper limits along x-axis and cumulative frequency along y-axis.

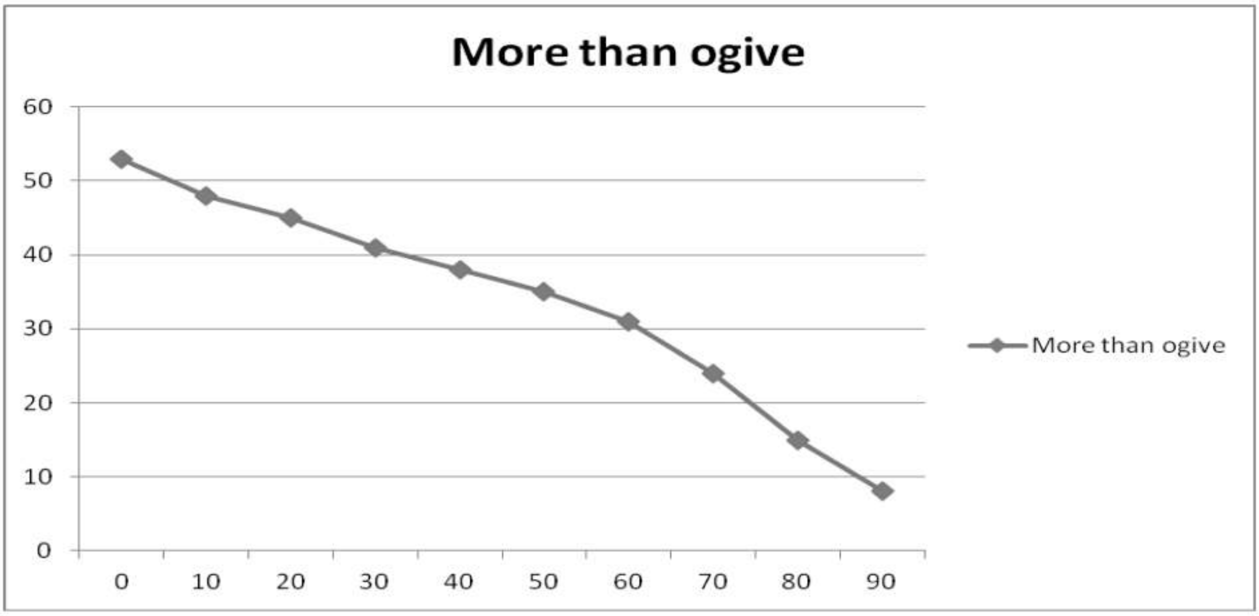
Marks	Cf
Less than 10	5
Less than 20	5+3=8
Less than 30	8+4=12
Less than 40	12+3=15
Less than 50	15+3=18

Less than 60	18+4=22
Less than 70	22+7=29
Less than 80	29+9=38
Less than 0	38+7=45
Less than 100	45+8=53



More than type: In more than type we take only lower limits along x-axis and cumulative frequency along y-axis.

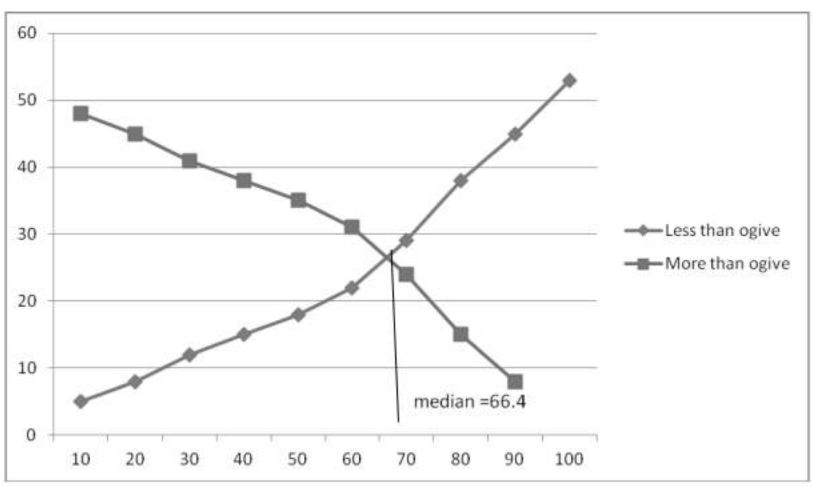
Marks	cf
More than 0	53
More than 10	53-5=48
More than 20	48-3=45
More than 30	45-4=41
More than 40	41-3=38
More than 50	38-3=35
More than 60	35-4=31
More than 70	31-7=24
More than 80	24-9=15
More than 90	15-7=8



Note: 1) We can obtain the Median by ogive graph by locating $(\frac{n}{2})th$ score on Y-axis and from this point, draw a line parallel to the X-axis cutting the curve at a point and from that point draw a perpendicular to the X-axis. The point of intersection of this perpendicular with X-axis is the Median of the data.

2)We can also obtain the Median by another method i.e. draw both ogives (less than type and more than type) on the same axis then two ogives will intersect at a point. From this point, if we draw the perpendicular on the X-axis, the point at which it cuts the X- axis gives us Median.

We can draw both less than type and more than type in a single graph as follow and here from point of intersection of two ogives draw a perpendicular to x-axis.



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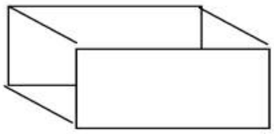
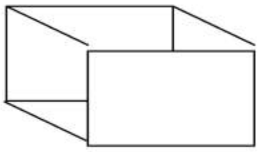
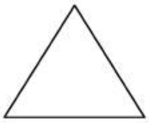
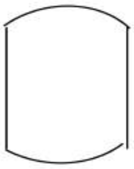
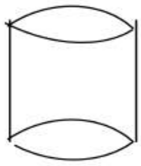
SURFACE AREAS AND VOLUMES

Plane figures **solid figures**

1.  

2.  

3.  

4.  

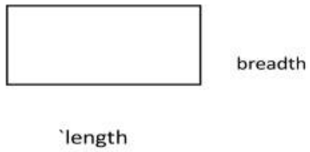
Plane figures having surface area solid figures having surface area and volume

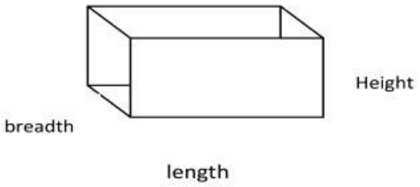
- Surface area: - The amount of space occupied by two dimensions.

  Area can be measured in square units.

- Volume: - The amount of space occupied by three dimensions.

  Volume can be measured in cubic units.

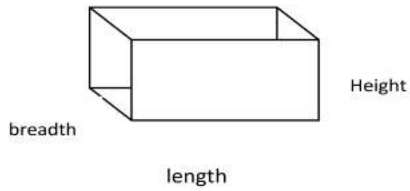


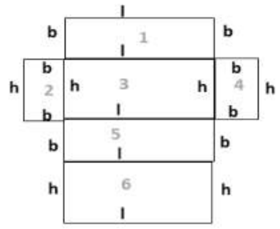


Area= Length * breadth Volume= Length * breadth * height

Volume of any object = Area of its base X height

- Cuboid: A cuboid having three dimensions namely length, breadth, and height.





Surface area of cuboid= area of rectangle 1 + rectangle 2 + rectangle 3 + rectangle 4+ rectangle 5 + rectangle 6

$= (l \times b)+(b \times h) +(l \times h)+(b \times h)+(l \times b)+(l \times h)$

$=lb+bh+lh+bh+lb+lh$

$= 2lb+2bh+2hl$

Surface area of cuboid= 2(lb+bh+hl)

For lateral surface area, we only find the area of the four faces leaving the bottom and top faces.

Lateral surface area=area of rectangle 2 + rectangle 3 + rectangle 4 + rectangle 6

$= (bxh) +(lxh)+(bxh)+(lxh)$

$= 2bh + 2lh$

Lateral surface area of cuboid = 2h(l+b)

Also volume of cuboid = area of its base x height

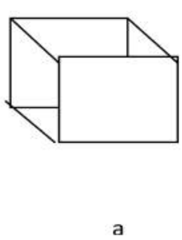
Volume of cuboid= lxbxh

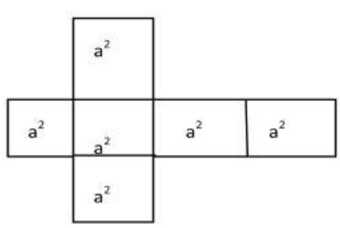
Note:- 1. Total surface area of cuboid= 2(lb+bh+hl)

2. Lateral surface area of cuboid= 2h(l+b)

3. Volume of cuboid = lxbx h

- Cube=A cuboid whose length, breadth and height are all equal.





Length = a

Breadth = a

Height = a

Total surface area of cube=area cube1+ cube 2+ cube 3+ cube 4+ cube 5+ cube 6

$$= a^2 + a^2 + a^2 + a^2 + a^2 + a^2$$

Total surface area of cube= $6a^2$ (a-> where length of the side)

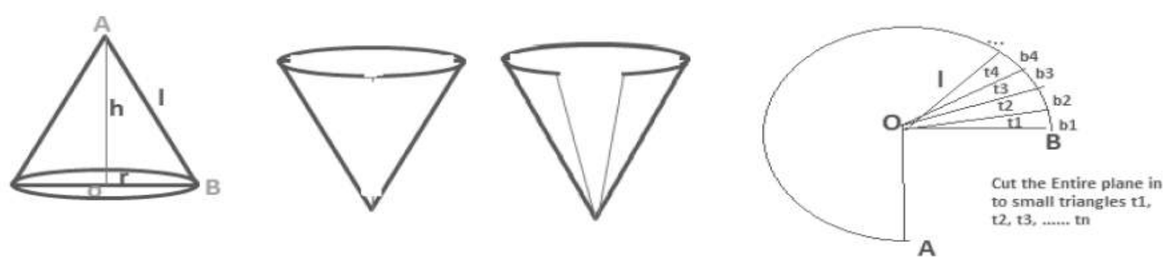
Lateral surface area of cube= $4a^2$ (a-> where length of the side)

Volume of cube= lxbxh

= axaxa

Volume of cube= a^3

- Cone = if a right angled triangle revolved one of its side containing right angle, the new solid generated is called cone



In triangle AOB, $\angle AOB = 90^0$ Area of entire plane= Area of $T_1 + T_2 + T_3 + \ldots\ldots T_n$

$AB^2 = AO^2 + OB^2$

$= \frac{1}{2}x(b_1xl) + \frac{1}{2}x(b_2xl) + \frac{1}{2}x(b_3xl) + \ldots\ldots \frac{1}{2}x\,(b_nxl)$

$l^2 = h^2 + r^2$

$= \frac{1}{2} \times l \times (b_1 + b_2 + b_3 \ldots\ldots b_n)$

So $l = \sqrt{h^2 + r^2}$

$= \frac{1}{2} \times l$ x (length of entire curved boundary of plane)

Curved surface area of cone= $\frac{1}{2} \times l$ x (perimeter of base of the cone)

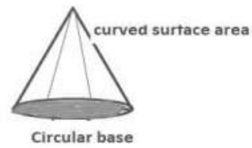
$= \frac{1}{2} \times l \times (2\pi r)$

C.S.A of cone = πrl

Total surface area of cone = C.S.A of cone + area of circular base

$= \pi rl + \pi r^2$

$= \pi r(l+r)$



T S A of cone = $\pi r(l+r)$

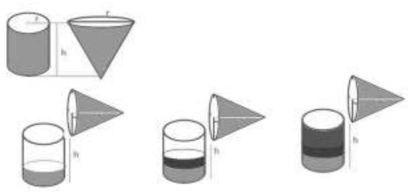
where r is its base radius and l its slant height and h is the height of the cone

$l^2 = r^2 + h^2$

$l = \sqrt{h^2 + r^2}$

Volume of a right circular cone:

Activity :- Must take right circular cylinder and a right circular cone of the same base radius and the same height



From above activity we can conclude that

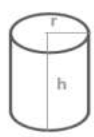
Volume of cylinder = 3x volume of cone

Volume of cone $= \dfrac{\text{Volume of cylinder}}{3}$

Volume of cone = $\frac{\pi r^2 h}{3}$ where r = radius of cone

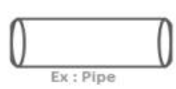
h = height of the cone

- Cylinder: - A rectangle is revolved one of its side which remains fixed, solid generated is called cylinder



Two types of cylindee

1.hollow cylinder



2. Solid cylinder

The hollow cylinder formed by the lateral surface only

Ex: pipe

The solid cylinder is the region bounded by two circular plane surface and also the lateral surface

Ex: A Golden roller, Pen

Length of rectangle l = circumference of base of cylinder

Breadth of rectangle b = Height of cylinder h

C. S. A OF cylinder = area of rectangle sheet

= lxb

=Perimeter of the base of cylinder x h

= 2 πr^2xh

Total surface area of cylinder = area of base B_1+ CSA of cylinder + area of base B_2

$= \pi r^2 + 2\pi rh + \pi r^2$

$= 2\pi r^2 + 2\pi rh$

$= 2\pi r(r + h)$

Total surface area of cylinder = $2\pi r(r + h)$ where r ->radius of cylinder

h -> height of cylinder

Volume of a right circular cylinder :

Volume of cylinder :- area of its base x height

= B x h

= $\pi r^2 h$ area of circular base B= πr^2

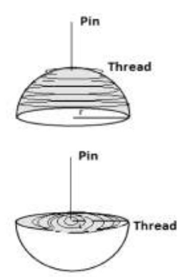
Volume of cylinder = $\pi r^2 h$ cubic units

- Sphere:- A aphere is a solid described by the rotation of a semicircle about a fixed diameter.

Ex :- Shot put, Ball

Sphere

Surface area of a sphere



Activity:-1. Consider a sphere of radius ‘r’

2. cut the solid sphere into two equal halves

3. fix a pin at the top most point of hemisphere

4. starting from the centre point of the curved surface of the hemisphere wind a uniform thread so as to cover the whole curved surface of the hemisphere

5. Measure the length of the thread

6. similarly, fix a pin at the centre of the plane circular surface

7. starting from the centre, wind the thread of same thickness to cover the whole circular surface

8. unwind and measure the lengths of the threads

9. compare the lengths

By above activity we can conclude that the length of the thread required to cover the curved surface is twice the length required to cover the circular plane surface.

C.S.A of hemisphere = 2x area of plane circular sphere

$= 2 \times \pi r^2$

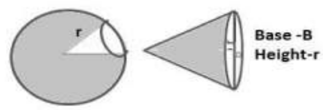
C.S.A of hemisphere = $2\pi r^2$

Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Surface area of whole sphere = $2\pi r^2 \times 2\pi r^2 = 4\pi r^2$

Surface area of whole sphere = $4\pi r^2$ where r-> radius of sphere

- **Volume of a sphere:**



A solid sphere is cut into small of pieces of cone, whose height is equal to the radius of the sphere and each having circular base

Volume of sphere= vol of cone $c_1 + c_2 + c_3 + \ldots\ldots\ldots + c_n$

Volume of sphere= vol of cone c_1 + vol of cone c_2 + vol of cone c_3+...........+ vol of cone c_n

$$= \left(\frac{1}{3} \times B_1 \times r\right) + \left(\frac{1}{3} \times B_2 \times r\right) + \left(\frac{1}{3} \times B_3 \times r\right) \ldots\ldots\ldots\ldots\ldots + \left(\frac{1}{3} \times B_n \times r\right)$$

$$= \frac{1}{3}\, r\, [B_1 + B_2 + B_3 \ldots\ldots\ldots B_n]$$

$$= \frac{1}{3}\, r \times 4\pi r^2$$

Volume of sphere $= \frac{4}{3}\pi r^3$ cubic units

Volume of hemisphere = $\frac{1}{2}$ x vol of sphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3 \text{ cubic units}$$

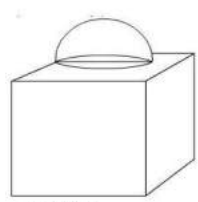
<u>**Surface area of a combination of solids**</u>

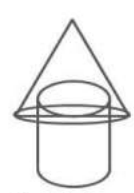
In our day to day life we come across a number of solids made up of combinations of two or more of the basic solids given below.


(i) water tanker


(ii)Testtube


(iii) icecream playing top(latter)


(iv) Dome


(v) Rocket


(vi)capsule/tablet


(vii)warehouse/godown

Now we want to find out surface areas and volumes of combination of solids

Water tank is combination of cylinder and 2 hemispheres

1. Surface area of water tanker = C.S.A of hemisphere + C.S.A of cylinder+ C,S.A of hemisphere

$$=2\pi r^2 + 2\pi rh + 2\pi r^2$$

$$= 4\pi r^2 + 2\pi rh$$

Volume of water tanker = vol of hemisphere + vol of cylinder + vol of hemisphere

$$= \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$$

Test-tube is combination of cylinder and one hemisphere

2. Surface area of Test tube = C.S.A of hemisphere + C.S.A of cylinder

$$= 2\pi r^2 + 2\pi rh$$

Volume of Test tube = Vol of cylinder + vol of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

3. Playing top (latter) :- combination of cone with hemisphere

Surface area of playing top = C.S.A of cone + C.S.A of cylinder

$$= \pi rl + 2\pi r^2$$

Volume of playing top = Vol of cone + vol of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

4. Dome :- Combination of cube with hemisphere

Surface area of Dome = T.S.A of cube + C.S.A of cylinder – base area of hemisphere

$$= 6a^2 + 2\pi r^2 - \pi r^2$$

Volume of Dome = Vol of Dome + vol of hemisphere

$$= a^3 + \frac{2}{3}\pi r^3$$

1. **Rocket :- combination of cylinder with cone**

Surface area of Rocket = C.S.A of Cylinder + Base area of Cylinder +C.S.A of cone+(base area Cone – base area Cylinder)

$$= 2\pi r_2 h + \pi r_2^{\ 2} + \pi r_1 l + \pi r_1^{\ 2} - \pi r_2^{\ 2}$$

Volume of Rocket = Vol of cylinder + vol of Cone.

$$= \pi r_2 h + \frac{1}{3}\pi r_1^2 h$$

6. Capsule : combination of cylinder with two hemispheres

Surface area of capsule = C.S.A of Cylinder + C.S.A of hemisphere+ C.S.A of hemisphere

$$= 4\pi r^2 + 2\pi rh$$

Volume of capsule = vol of hemisphere + vol of cylinder + vol of hemisphere

$$= \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$$

7. Ware house :- Combination of cuboid with hemi cylinder

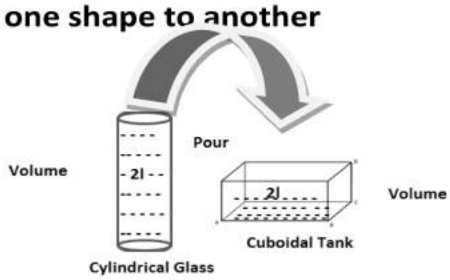
Surface area of Ware house = C.S.A of cuboid + ½ C.S.A of Cylinder + Base area of cuboid

$$= 2h(l+b) + \frac{1}{2}\pi r^2 h + (l \times b)$$

Volume of ware house = vol of cuboid + ½ vol of cylinder

$$= (l \times b \times h) + \frac{1}{2}\pi r^2 h$$

Conversion of solid form one shape to another

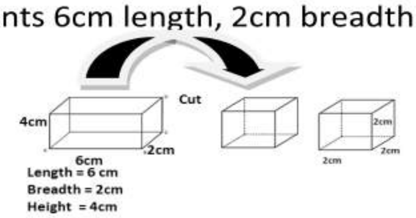


When we pour 2l of water from cylindrical glass to cuboidal tank the volume (2l of water) does not change it remains equal in their state

So Volume of cylinder = Volume of cuboid

Ex : How many cubes of 2cm length has been cutoff from the cuboid having measurements 6cm length, 2cm breadth and 4cm height

Sol



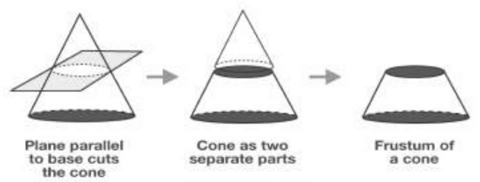
so Volume of cubes = Volume of cuboid

$n(a^3) = l \times b \times h$

$n = \frac{6 \times 2 \times 4}{2 \times 2 \times 2} = 6$

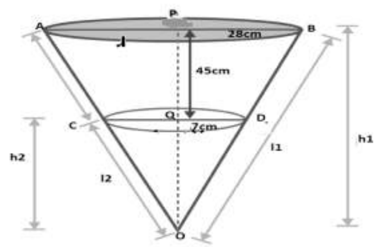
So 6 cubes can be cutout from the cuboid

- **Frustum of a cone** : Given a right circular cone, which is sliced (cutoff) through by a plane parallel to its base, when the smaller conical position is removed the resulting solid is called Frustum of a right circular cone.



Frustum of cone

In our day to day life we seen many solids knowingly water glass washing tub, joker's cap, ice cap...etc



In above fig slant height OA=OB=l_1 The radii of the ends of a frustum of a cone 45cm height are 28cm and 7cm

Height of cone AOB is h_1 = OP

Height of cone COD is OQ= h_2

slant height of cone COD is OC = OD = l_2

radius of cone AOB is r_1 = 28cm

radius of COD is r_2 = 7cm

height of the frustum h = 45cm

Also $h_1 = 45 + h_2$ ------1

w.k.t OPB ~ OQD

OP/OQ =PB/QD

$h_1 / h_2 = 28/7 = 4$ --------------2

from eqn 1 and @

h_2=15cm and h_1= 60cm

The respective slant height l_1 and l_2 of cone AOB and COD are given by

$l_2 = \sqrt{r1^2 + h1^2} = \sqrt{7^2 + 15^2} = 16.55cm$ $\quad$ $l_1 = \sqrt{r2^2 + h2^2} = \sqrt{28^2 + 60^2} = 66.20cm$

Volume of frustum = volume of cone OAB – volume of cone OCD

$= (\frac{1}{3} X\ 22/7\ X\ 28^2 X\ 60 - \frac{1}{3} X\ 22/7\ X\ 7^2\ X 15)$

$= 48.510\ cm^3$

Now we know that h_2=15cm and h_1= 60cm

l_2 = 16.55cm l_1= 66.20cm

r_1 = 28cm r_2 = 7cm

curved surface area of frustum = C.S.A of cone AOB – C.S.A of Cone COD

$= \pi r_1 l_1 - \pi r_2$

= 22/7 X 28X 66.20 - 22/7 X 7 X16.55

= 5461.5 cm^2

Total surface area of frustum = C.S.A of frustum + Area of top + area of bottom

$= 5461.5 + \pi r^2{}_1 + \pi r^2{}_2$

= 5461.5+2464+154 = 8079.5cm^2

Note : Let h b ethe height , l the slant height and r_1 and r_2 the radii of the ends of the frustum of a cone , then we can directly find the volume, the C.S.A and T.S.A of frastum by using the formula given below.

i) Volume of frustum of the cone = $1/3\pi h(r^2{}_1 + r^2{}_2 + r_1 r_2)$

ii) C.S.A of frustum of cone $= \pi(r_1 + r_2)l$ where $l = \sqrt{(h^2 +(r_1 - r_2)^2)}$

iii) T.S.A of Frustum of cone $= \pi(r_1 + r_2)l + \pi r^2{}_1 + \pi r^2{}_2$

Examples

1. Length of the diagonal of a square is 4√2 then find length.

Solution : In a square ABCD AB=BC=CD=AD=x and diagonal AC =4√2

In triangle ∠ ABC = 90^0

AC = AB + BC

$(4\sqrt{2})^2 = x^2 + x^2$

32 = $2x^2$

x^2 =16

x=4

Length of the square is x=4

2. A cone of height 24cm and radius of the base 6cm find its volume

Solution : Height of the cone h = 24cm

Radius of the base r = 6cm

Volume of the cone = 1/3 X $\pi r^2 h$

= 1/3 * 22/7 * 6 * 6* 24

= 6336/7

= 905.14 cm^2

3. A cylinder of height 8 cm and perimeter of the base is 44cm , find its total surface area.

Solution : height of cylinder h =8cm

Perimeter of the cylinder = 44cm

$2 \pi r = 44$

r = 44 *7/22*2

r=7cm

Total surface area of cylinder = $2\pi r(r+h)$

= 2* 22/7 * 7(7+8)

= 660cm^2

4. The surface area of the sphere is 616 $cm^{2,}$ find its radius.

Solution: Surface area of the sphere = 616 cm^2

Surface area of the sphere = $4\pi r^2$

616 = 4 * 22/7 * r^2

r^2 = 616*7/88 = 49

r=7cm

radius of the sphere = 7cm.

5. Two cubes each of volume 64cm^3 are joined end to end. Find the surface area of resulting cuboid

Solution: A cube has all its side equal

So volume of cube= 64 cm^3

$a^3 = 4^3$

a = 4cm

length of the side of the cube is 4cm

two cubes are joined length of cuboid l = 8cm

breadth of cuboid b = 4cm

height of cuboid h = 4cm

surface area of cuboid = 2(lb+bh+hl)

= 2((8*4) + (4*4) + (4*8))

=2(80)

=160cm^2

6. A toy is in the form of cone of radius 3.5cm mounted on a hemisphere od same radius the total height of toy is 15.5cm find the total surface area of the toy.

Solution : A cone mounted on hemisphere , then it seems radius of the sphere

OA=OB=3.5cm

Height of the cone OC = BC-OB

= 15.5 – 3.5

OC = 12cm

So T.S.A of the toy = C.S.A of cone + C.S.A of hemisphere

$= \pi rl + 2\pi r^2$

= 22/7 * 35 *12.5 + 2 * 22/7 * 3.5 * 3.5

= (11 *12.5) + (22*3.5)

= 137.5 + 77

= 214.5cm^2

7. A cup is in the form of a hemisphere surrounded by a cylinder the height of the cylindrical portion is 8cm and the total height of the cup is 11.5cm find the total surface area of the cup

Solution : Hemisphere surrounded by a cylinder

Height of the cylinder h = 8cm

Radius of hemisphere r= 11.5 – 8 = 3.4cm

T.S.A of the cup = C.S.A of cylinder + C.S.A of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h+r)$

= 2 * 22/ 7 * 3.5 * (8+ 3.5)

= 22(11.5)

= 253 cm^2

8. A cone and a hemisphere are joined on either sides of s cylinder . these solids have radius 7cm each, if the total height of the solid is 61cm and the height of the cylinder 30cm, find the cost of painting the outer surface of the solid at the rate of 10 per 100 cm^2

Solution : radius of the solid r = 7cm

Height of the cone h_1 = 61-37 =24cm

Height of the cylinder h_2 = 30cm

Slant height of cone $l = \sqrt{r^2 + h_1^2} = \sqrt{7^2 + 24^2}$ =25cm

So surface area of the solid = C.S.A of cone +C.S.A of cylinder + C.S.A of hemisphere

$= \pi rl + 2\pi rh_2 + 2\pi r^2$

$= \pi r[l + 2h_2 + 2r]$

= 22/7 * 7[25 + 60 + 14]

= 22(99)

Surface area of solid = 2178 cm^2

So cost of painting the outer surface of the solid at the rate rs 10 per 100 cm^2

So total cost of painting = 2178/10= 217.8

9. The diameter of metallic sphere is 4.2cm . it is melted and recast in to a right circular cone of height 8.4cm find the radius of the base of the cone

Solution : radius of sphere r1=2.1cm

Radius of cone r2 =

Height of the cone h= 8.4cm

Vol of cone = vol of sphere

$1/3\ \pi r_2^2 h = 4/3\ \pi r_1^3$

$8.4 * r_2^2 = 4 * 2.1 * 2.1 * 2.1$

$r_2^2 = 4 * 2.1 * 2.1 * 2.1/8.4$

$r_2^2 = (2.1)^2$

Radius of the base of cone $r_2 = 2.1$cm

10. A conical plask is full of water the plask has base radius 3cm and height 15cm. the water is poured into cylindrical glass tube of uniform inner radius 1.5cm placed vertically and closed at lower end. Find the height of water in the glass tube.

Solution : Radius of conical flask r_1 -3cm

Radius of cylindrical glass r_2 = 1.5cm

Height of the conical flask h_1 = 15cm

Height of cylindrical flask h_2 =

Volume of cylindrical glass = volume of conical flask

$\pi r_2^2 h_2 = 1/3\ \pi r_1^2 h_1$

$(1.5)^2 * h_2 = 1/3 * 3 * 3 * 15$

$h_2 = 3 * 1.5/ 1.5 * 1.5$

$h_2 = 20$cm

11. Volume of a frustum is 48510cm^3 Its height is 45cm and radius of one circular base 7cm. find the radius of the other circular base of the frustum.

Solution: vol of frustum v = 48150cm^3

Height of the frustum =

Radius of one circular base r_1 = 7cm

Radius of other circular base r_2 =

vol of frustum v = 48150cm^3

$1/3\pi h(r^2_1 + r^2_2 + r_1 r_2) = 48150$

$1/3 * 3.142 * 45 (7^2 + r_2^2 + 7r_2) = 48150$ $r_2^2 + 35r_2 - 28r_2$ $-980=0$

$49 + r_2^2 + 7r_2 = 48150 * 7 /15 * 22$ $r_2(r_2 + 35) - 28(r_2 + 35) = 0$

$r_2^2 + 7r_2 - 980 = 0$ $(r_2 + 35)(r_2 - 28) = 0$

Radius of other circular base r_2= 28cm $r_2 - 28 = 0$

12 . the slant height of the frustum of a cone is 4cm and perimeter is of its circular bases are 18cm and 6cm respectively. Find the C. S. A and T. S. A of the frustum.

Soln: perimeter of the upper circular base= 18cm

$2\pi r_1 = 18$

$r_1 = 18/2\pi$ $l=\sqrt{h^2 + (6/\pi)^2}$

$r_1 = 9/\pi$ s.b.s $16 = h^2 + 36/\pi^2$

Perimeter of lower circular base = 6cm $h^2 = 16 - 36*7/22$

$2\pi r_2$ = 6cm $= 352 - 252/22$

$r_2 = 3/\pi$ $h^2 = 100/22$

Slant height of frustum l = 4cm

CSA of frustum = $\pi (r_1 + r_2)l$

$= \pi (9/\pi + 3/\pi)\, l$

$= \pi (12/\pi)\, 4$

CSA of frustum = 48 cm^2

TSA of frustum = CSA of frustum + area of top + area of bottom

$= 48 + \pi r_1^2 + \pi r_2^2$

$= 48 + \pi ((9/\pi)^2 + (3/\pi)^2)$

$= 48 + \pi (81/\pi^2 + 9/\pi^2)$

$= 48 + \pi (90/\pi^2)$

$= 48 + (90*7/22)$

$= 1056 + 630/22$

$= 1686/22$

$= 76.63$ cm^2

13. An open metal bucket is in the shape of frustum of a cone, mounted on a hollow cylindrical base made of same metallic sheet. The diameter of two circular ends of the

bucket are 45cm and 25cm. the total vertical height of the bucket is 40cm and that of the cylindrical base is 6cm. Find the area of metallic sheet used to make bucket where we do not take into account of handle of the bucket. Also find volume of water that bucket can hold?

Solution : radius of upper base of the bucket r_1 = 45/2cm

radius of lower base of the bucket r_2 = 25/2cm

height of the frustum of a cone h_1 = 34cm

height of the cylindrical base h_2 = 6cm

slant height of the frustum $l=\sqrt{h_1^2 + (r_1-r_2)^2}$

$=\sqrt{34^2 + 10^2}$

$= \sqrt{1156 + 100}$

= 35.44cm

Area of sheet used to make bucket=CSA of Frustum+area of base+CSA of cylindrical base of bucket

$= \pi (r_1+r_2)l + \pi r_2^2 + 2\pi r_2 h_2$

= 22/7[35(35.44) +625/4 z=150]

= 22/7 [1239 +156.25 + 150]

= 22/7(1545.25)

Area of sheet used to make bucket = 4856.50cm^2

Volume of bucket = $1/3\pi h_1(r_1^2 + r_2^2 + r_1 r_2)$

= 1/3 *22/7 * 34 (2025/4 + 625/4 + 1125/4)

= 22 * 34/21 * 3775/4

= 11*17*3775/21 = 33615.48cm^3 1litre=10000cm^3

Volume of bucket = 33.62l

Remembering formulas

1. Surface area of cuboid = 2 (lb+bh+hl)

   C.S.A of cuboid = 2h(l+b)

   Volume of cuboid = lbh

2. Surface area of cube = $6a^2$ where a is length of the side

   C. S. A of cube = $4a^2$

   Volume of cube = a^3

3. Curved surface area of the cylinder = $2 \pi rh$

   Total surface area of the cylinder = $2 \pi r (r+h)$

   Volume of cylinder = $\pi r^2 h$

4. Curved surface area of cone = πrl where $l = \sqrt{r^2+h^2}$

   Total surface area of cone = $\pi r(r+l)$

   Volume of cone = $1/3\pi r^2 h$

5. C. S. A of hemispehre = $2\pi r^2$

   T.S.A of hemisphere = $3\pi r^2$

   T.S.A of sphere = $4\pi r^2$

   Volume of sphere = $4/3\pi r^3$

   Volume of hemisphere = $2/3\pi r^3$

6. Curved surface area of frustum of cone = $\pi (r_1+r_2) l$

   Total surface area of frustum of cone = $\pi (r_1+r_2) l + hr_1^2+hr_2^2$

   Volume of frustum of cone = $1/3 \pi h(r_1^2+r_2^2+r_1r_2)$

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