CHITTI CREATIONS

5 SET MODEL QUESTION PAPERS WITH KEY ANSWERS INCLUDING BOARD PAPERS

10TH STANDARD MATHEMATICS(E,M)

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KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD
2020-21 MODEL PAPER – 2 with Key answers

Subject: Mathematics Subject code: 81E

Time : 3 hours Max.marks: 80

Answer the following questions. **8 x 1 = 8**

1. The Pair of lines $ax_1+b\,y_1+c_1=0$ and $ax_2+by_2+c_2=0$ are intersecting lines then the ratio of their coefficients is :
   a. $\frac{a1}{a2} \neq \frac{b1}{b2}$
2. 2, x, 14 are in Arithmetic progression, then the value of x is :
   d. 8
3. The standard form of quadratic equation is :
   b. $ax^2+bx+c=0$
4. Sin (90-θ) is equal to :
   a. Cos θ.
5. The value of tan 45^0 is :
   c. 1
6. In the given graph. The co-ordinate of point A is :
   d. (2, 0)
7. The Emperical relationship between the three measures of central tendency is :
   c. 3 Median = Mode + 2 Mean
8. In the given figure ST||QR then PS/SQ is equal to :
   a. $\frac{PT}{TR}$

Answer the following questions. **8 x 1 = 8**

9. Answer is 4
10. $\left(\frac{x3+x2}{2}, \frac{y3+y2}{2}\right)$
11. 90^0.
12. Total suface area of a right cylinder = 2∏rh(r+h) sq units.
13. Volume of a solid sphere= $\frac{4}{3}$∏r^3.
14. = πrl , where l is the **slant height** of the **cone**. Example 1: Find the lateral **surface area** of a right **cone** if the **radius** is 4 cm and the **slant height** is 5 cm. ?
15. In an arithmetic progression if an = 3n-2, then the second term is 3x2-2=4.
16. If, 15 cot A=8, then find the value of tan A=$\frac{15}{8}$.

Answer the following questions. **8 x 2 = 16**

17. x + y = 8 2x - y = 7

Consider the given equation.

x+y=8 …….. (1)

x-y=7 ……… (2)

On subtracting both equation (1) and (2), we get

3x=15

x=5

Now, put the value of y in equation (1), we get

3+y=8

y=5

Hence, the value of x is 3 and y is 5

18. Find the 10th term of arithmetic progression 2, 7, 12 .... using the formula.

The given A.P is 2, 7, 12 ...... we know an=a+(n-1)d.

Common Difference, d = 5.

First term, a = 2

Hence, 10th term is a+9d = 2+45 = 47.

19. Find the sum of 2+5+8+................ to 20 terms using the formula.

Given:- $2+5+8+\ldots\ldots.$

As we know that sum of n terms in an A.P. is given as-

$S_n = \frac{n}{2}[2a+(n-1)d] \ldots..(1)$

whereas, a and b are the first term and common difference of A.P.

From the given series-

$a = 2$

$d = 5 - 2 = 3$

$n = 20$

Therefore, from equation (1), we have

Hence the sum of 20 terms of the given series is 610.

20. Find the discriminant of the equation $3x^2 - 5x+2=0$ and hence write the nature of its roots.

Comparing with the form $ax^2 + bx + c=0$

we get:

a = 3; b = -5; c = 2

Discriminant, D = b^2 - 4ac

=> $(-5)^2 - 4(3)(2)$

=> 25 - 24

=> 1 > 0

D > 0

The discriminant is greater than 1, that means it is real and have distinct.

21. Solve $x^2 - 2x+3=0$ by using the quadratic formula.

answer:

a=1, b=-2, c=3

quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-2)\pm\sqrt{2^2-4x(1)x3}}{2x1}$

$x=\frac{2\pm\sqrt{4-12}}{2}$

$x=1+\sqrt{-8}$ or $x=1-\sqrt{-8}$

OR

Solve by Factorisation $x^2 + 5x+6=0$.

By factorization, $x^2 + 5x+6=0$

$X^2+3x+2x+6=0$

(x+3) (x+2)

X=-3 & x=-2

22. Find the distance between the points A(3, 6) and B(5, 7) using distance formula.

Answer: by distance formula $d=\sqrt{(x2-x1)^2+(y2-y1)^2}$

$$d=\sqrt{(5-3)^2+(7-6)^2}$$

$$d=\sqrt{(2)^2+(1)^2}$$

So the distance is $\sqrt{5}$ units

OR

Find the co-ordinates of the point P, which divides the line joining A(0, 0) and B(5, 10) in the ratio of 2:3.

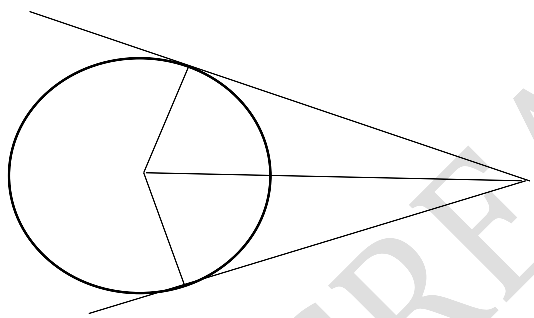
By section formula, we have $(x, y) = (\frac{mx2+nx1}{m+n}, \frac{my2+ny1)}{m+n})$ here m=2, n=3,

$$(x, y) = (\frac{2x5+3x0)}{2+3}, \frac{2x10+3x0}{2+3})$$

$$(x, y)= \frac{10}{5}, \frac{20}{5}$$

So coordinates are (2, 4)

23. Construct a tangent to a circle of radius 4cm at any point P on its circumference.



24. In the given figure, find the value of sin∝ + cosθ ?

Answer : by figure sin $\boldsymbol{\alpha}=\frac{3}{5}$ & cos$\boldsymbol{\theta}=\frac{3}{5}$

Therefore sin $\boldsymbol{\alpha}+$ cos$\boldsymbol{\theta}=\frac{3}{5}+\frac{3}{5}=\frac{6}{5}$

Answer the following questions. 9 x 3 = 27

25. A train travels 480 km at a uniform speed. If the speed had been 10km/h more, it would have taken 4 hours less for the same journey, find the speed of the train?

Let the speed of the train be x

The distance covered is 450km

So the time taken is $\boldsymbol{\frac{480}{x}}$ --------(1)

The speed is now 40kmph less, so the speed is x−40

The same distance so 450km

Time taken is $\boldsymbol{\frac{480}{x+10}}$

According to question,

The second case takes 4 more hours to reach a destination

So $\boldsymbol{\frac{480}{x}} = \boldsymbol{\frac{480}{x+40}} + 4$

Solve, bring this to standard form of quadratic equation

$4x^2-160x-4500=0$

$x^2+10x-1200=0$

$x^2+40x-30x-1200=0$

$x(x+40)-30(x+40)=0$

(x+40)(x-30)=0
x=40,−30
Speed cannot be negative so the speed of the train is 30kmph
OR

Find two consecutive odd positive integers, sum of whose squares is 290.

Let x an odd positive integer
Then, according to question
$x^2+(x+2)^2=290$
$2x^2+4x-286=0$
$x^2+2x-143=0$
$x^2+13x-11x-143=0$
$(x+13)(x-11)=0$
x=11 as x is positive
Hence required integers are 11 & 13.

26. Prove that {Cosec (90-θ) - Sin (90-θ)} {(Cosecθ - Sin θ) (tanθ + cotθ)} = 1

(sec θ-cos θ)(cosec θ-sin θ)(tan θ+cot θ)
(1/cos θ-cos θ)(1/sin θ-sin θ)(tan θ+1/tan θ)

$$\left(\frac{1-cos2\theta}{cos\theta}\right)\left(\frac{1-sin2\theta}{sin\theta}\right)\left(\frac{1-cos2\theta}{cos\theta}\right)\left(\frac{tan2\theta+1}{tan\theta}\right)$$

=1

OR

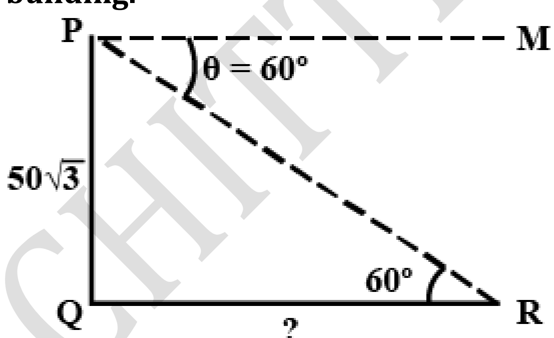
Prove that $\dfrac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \dfrac{(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)} = \dfrac{2}{2\,Sin2\theta}$

LHS, $\dfrac{(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)} + \dfrac{(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)}$

Take LCM, $\left(\dfrac{(\sin\theta - \cos\theta)2 + (\sin\theta + \cos\theta)2}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}\right.$

$$\frac{(\sin 2\theta \mp \cos 2\,\theta) + (\sin 2\theta \mp \cos 2\,\theta)}{(\sin 2\theta - \cos 2\,\theta)} = \frac{2}{2\,Sin2\theta}$$

27. From the top of a building $50\sqrt{3}$ m high the angle of depression of a car on the ground is observed to be 60^0 . Find the distance of the car from the Foot of a building.



Let QR be the distance of the car from the building.
Now, In ΔPQR
tan60°=QR/PQ= QR/$50\sqrt{3}$
$\sqrt{3}$=QR/$50\sqrt{3}$
QR=50m
The car is at 50m distant from the building.
Hence, the answer is 50m.

28. Find the area of triangle ABC, whose co-ordinates are A(4, -6), B(3, -2) and C(5, 2) then find the length of the median AD?

$(x_1, y_1)=(4, -6)$, $(x_2, y_2)=(3, -2)$, $(x_3, y_3)=(5, 2)$,

We know that area of triangle $=\frac{1}{2}(x_1(y_2-y_3)+x_2((y_3-y_1)+x_3(y_1-y_2))$.

$=\frac{1}{2}(4(-4)+3(8)+5(-4))$

$=\frac{1}{2}(-12)$

$=-6$

Therefore area of triangle ABC is 6 sq units.

$P(x, y)=(\frac{x1+x2}{2}, \frac{y1+y2}{2})= (\frac{3+5}{2}, \frac{-2+2}{2})= (4, 0)$

$d=\sqrt{(x2-x1)^2+(y2-y1)^2}$

$=\sqrt{(4-4)^2+(0+6)^2}=\sqrt{(6)^2}$ =36 units

29. Find the mean of the following data, by direct method.

C.I	**f**
1-5	**4**
5-9	**3**
9-13	**5**
13-17	**7**
17-21	**1**
	n=20

Answer : we know the formula that $x=\frac{\sum fxx}{n}=\frac{212}{20}$ =10.6

C.I	f	x	fx
1-5	4	3	12
5-9	3	7	21
9-13	5	11	55
13-17	7	15	105
17-21	1	19	19
	n=20		$\sum fxx=212$

So mean value is 10.6

OR

Find the mode of the following data.

C.I	f
0-10	**6**
10-20	**9**
20-30	**15**
30-40	**9**
40-50	**1**
	N=40

Answer: here n=40,
Then place the chart

C.I	f
0-10	**6**
10-20	**9**
20-30	**15**
30-40	**9**
40-50	**1**
	N=40

$f=15$, $f_0=9$, $f_1=15$, $f_2=9$, $h=10$

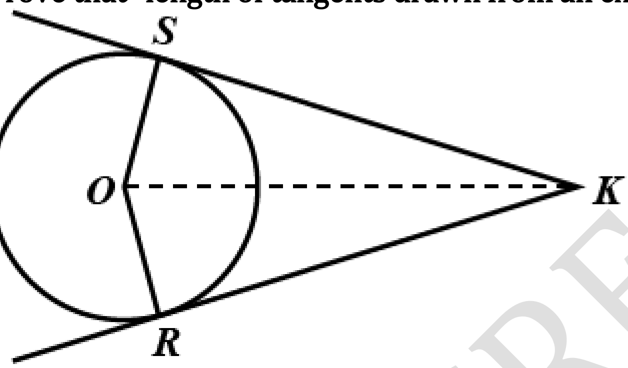
Mode = $l+\left(\frac{f1-f0}{2f1-f0-f2}\right)xh$

$=20+(6/12)x10$

$=20+5$

$=25$

30. Prove that "length of tangents drawn from an external point to a circle are equal.



To prove that: SK=RK

Proof:

Normal and tangent at a point on the circle are perpendicular to each other.

$\angle OSK=\angle ORK=90o$

Using Pythagoras Theorem,

$OK^2=OS^2+SK^2$............(i)

$OK^2=OR^2+RK^2$............(ii)

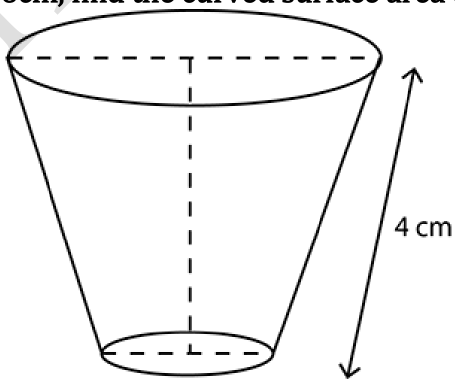
Subtracting (ii) from (i),

$OK^2-OK^2=OS^2+SK^2-OR^2-RK^2$

$\Longrightarrow SK2=RK2 \because OS=OR$

SK=RK

31. The slant height of a frustrum of a cone is 4cm and perimeters of its circular bases are 18cm and 6cm, find the curved surface area of the frustrum of a cone.



Given:
l=4 cm
circumference of the circular end = 18 cm.
$\Rightarrow 2\pi r1=18$
$\Rightarrow \pi \times r1=18/2=9$ ....(1)
Circumference of other circular end =6 cm
$\Rightarrow 2\pi r^2=6$
$\Rightarrow \pi r2=6/2=3$...(2)
Adding (1) and (2)
Curved surface area
$=\pi(r1+r2)l$
$=(9+3)\times 4$
$=48$ cm^2

OR

The circumference of the base of a cylinder is 132cm and its height is 25cm. Find the volume of the cylinder?

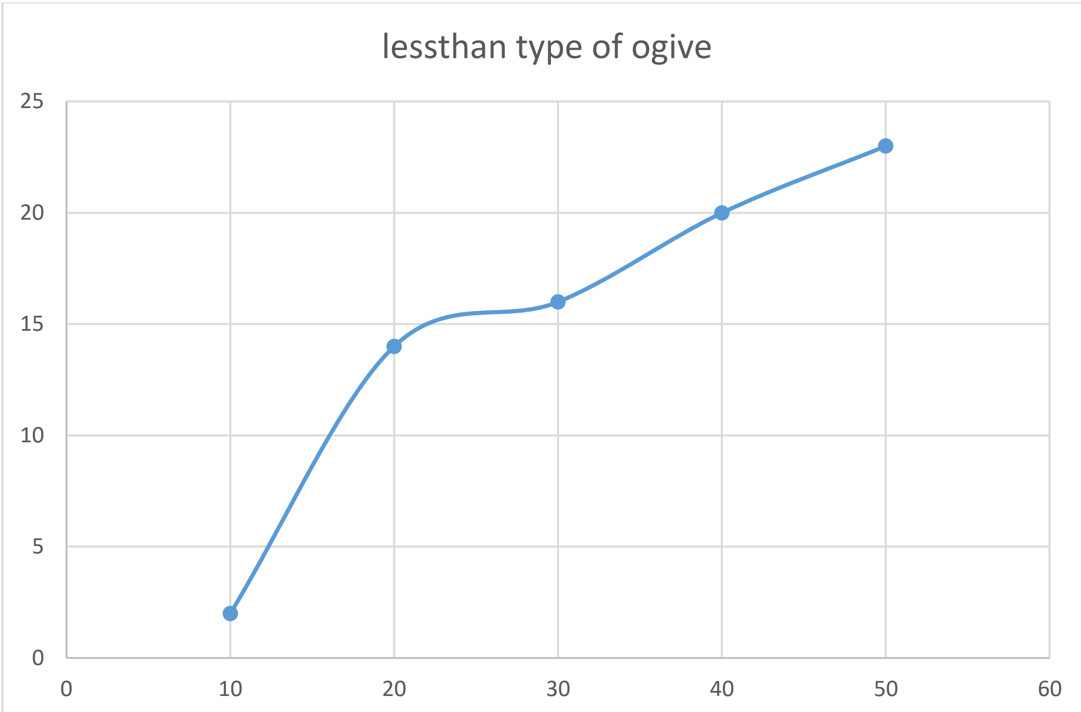
Let r be the radius of the cylinder , circumference =132cm.
$2\prod r=132$
r=21cm
then, volume of cylinder = $\prod r^2 h$.
=3.142x21x21x25
=34650 cm^3.

32. Draw a "less than type ogive" for the data given in the following table.

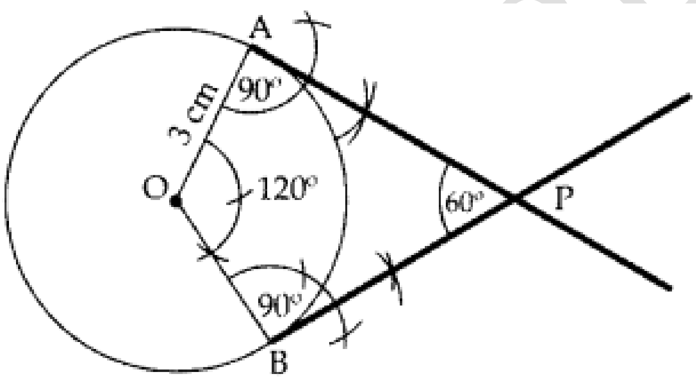
C.I	f
0-10	2
10-20	12
20-30	2
30-40	4
40-50	3

Answer:

C.I	f	fc	points
0-10	2	2	<10, 2
10-20	12	14	<20,14
20-30	2	16	<30, 16
30-40	4	20	<40, 20
40-50	3	23	<50, 23



33. Construct tangents to a circle of radius 3cm such that the angle between the tangents is 60^0 .



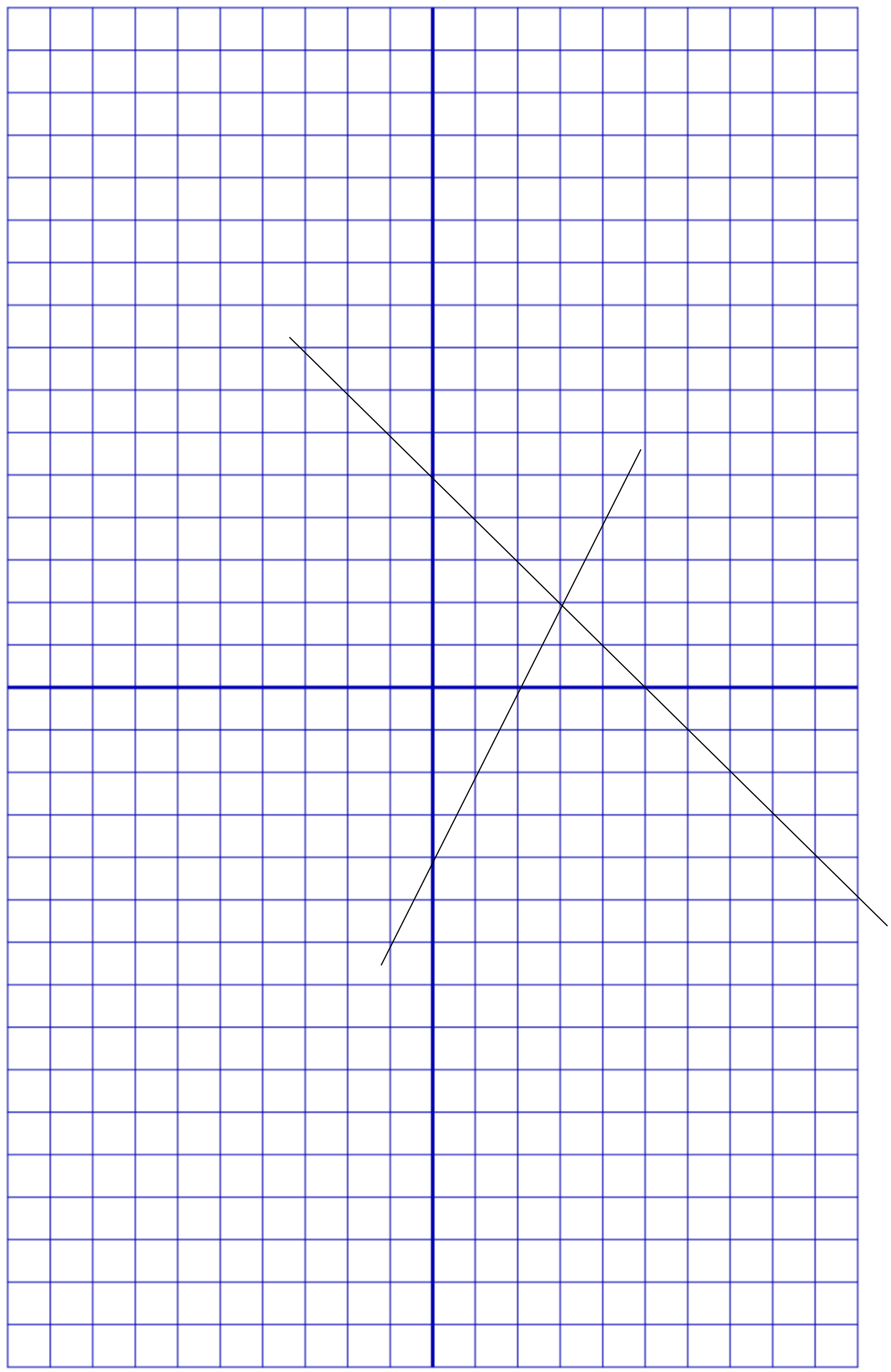
34. Find the Solution to the given pair of linear equations by graphical method.
x + y = 5, & 2x - y = 4.

From equation (i), we have the following table:

x	**0**	**2**	**4**
y	**5**	**3**	**1**

From equation (ii), we have the following table:

x	**0**	**2**	**4**
y	**-4**	**0**	**4**



So, two lines intersects at (2, 3) hence x=2 & y=3.

35. The third term of an arithmetic progression is 8 and its ninth term exceeds three times the third term by 2 find the sum of the first 19 terms.

Answer :

We know that the nth term of an A.P with first term a and common difference d is Tn=a+(n−1)d.

Here, it is given that the third term of an A.P is 8, therefore,

⇒T3=a+(3−1)d

⇒8=a+2d

⇒a+2d=8......(1)

It is also given that the ninth term of an A.P exceeds three times the third term by 2, therefore,

⇒T9=3T3+2=(3×8)+2=24+2=26

But

⇒T9=a+(9−1)d=a+8d, thus,

⇒a+8d=26.........(2)

Now, subtract equation 1 from equation 2 as follows:

⇒(a−a)+(8d−2d)=26−8

⇒6d=18

⇒d=618=3

Substitute d=3 in equation 1:

a+(2×3)=8⇒a+6=8⇒a=8−6=2

We also know that the sum of n terms of an A.P with first term a and common difference d is:

⇒Sn=2n[2a+(n−1)d]

⇒Substitute n=19, a=2 and d=3 in Sn=2n[2a+(n−1)d] as follows:

⇒S19=219[(2×2)+(19−1)3]=219[4+(18×3)]=219(4+54)=219×58=19×29=551

Hence, the sum of the first 19 terms of an A.P is S19=551.

OR

In an arithmetic progressive the sum of the three terms is 24, and their product is 480, write three terms of the arithmetic progression?

Solution: let the three terms be a-d, a, a+d

Its sum =24 product =480

a-d+ a,+a+d=24 a-dx a xa+d=480

a=8 $64-d^2=60$

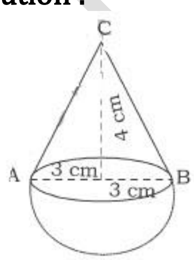
d=2

therefore the three terms are a-d a, a+d

6, 8, 10.

36. A toy is in the form of a cone mounted on a hemisphere with the some radius is as shown in the figure. If the diameter of the conical portion is 6cm and its height is 4cm, then find the surface area of the toy.

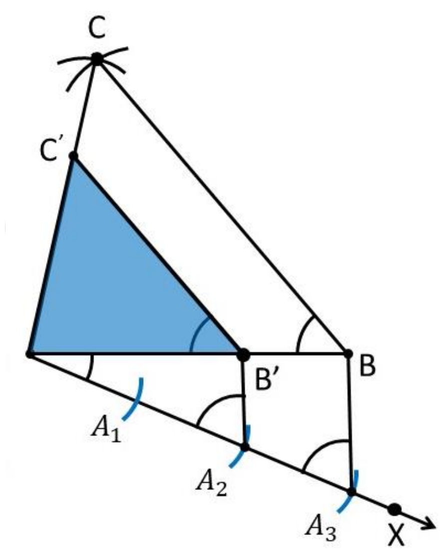
Solution :



Given :
For cone -
1. Height of the cone = 4cm
2. diameter of the cone = 6cm
3.radius of the cone = $\frac{6}{2}$ = 3cm
Slant height of the cone l = $\sqrt{r^2+h^2}$
$\Rightarrow$ l = $\sqrt{3^2+4^2}$
$\Rightarrow$ l = $\sqrt{9+16}$
$\Rightarrow$ l = 5
Lateral surface area of the cone = πrl
$\Rightarrow 3.14 \times 3 \times 5$
$\Rightarrow 3.14 \times 15$
$\Rightarrow$ 47,10 cm^2
For Hemisphere -
1. Diameter of the hemisphere = 6cm
2. Radius of hemisphere = $\frac{6}{2}$ = 3cm
Lateral surface area of hemisphere = $2\pi r^2$
$\Rightarrow 2 \times 3.14 \times 3^2$
$\Rightarrow 2 \times 3.14 \times 9$
$\Rightarrow 18 \times 3.14$
$\Rightarrow$ 56.52 cm^2
The surface are of toy = lateral surface area of cone + lateral surface area of hemisphere
$\Rightarrow$ 47.10 + 56.52
$\Rightarrow$ 103.62cm^3
$\therefore$ The total surface area of toy = 103.62cm^3

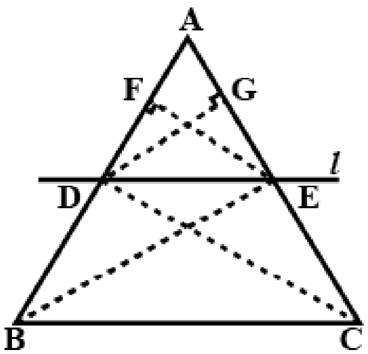
37. Construct a triangle ABC of its sides BC=4cm, AB=6cm and AC=4.5cm then construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle ABC.

Solution :



38. State and Prove "Basic proportionally theorem"

Basic Proportionality Theorem states that, if a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides of the triangle in proportion.



Let **ABC** be the triangle.

The line **l** parallel to **BC** intersect **AB** at **D** and **AC** at **E**.

To prove: $\frac{DB}{AD}=\frac{CB}{AE}$

Join **BE,CD**

Draw **EF⊥AB**, **DG⊥CA**

Since **EF⊥AB**,

EF is the height of triangles **ADE** and **DBE**

Area of △**ADE=1/2×** base **×** height=**1/2xAD×EF**

Area of △**DBE=1/2×DB×EF**

$\frac{areaof\Delta DBE}{areaof\Delta ADE}=\frac{1/2\times DB\times EF}{1/2xAD\times EF}\times=\frac{DB}{AD}$ ....(1)

Similarly,

$\frac{areaof\Delta DBE}{areaof\Delta ADE}=\frac{1/2\times CB\times EF}{1/2xAE\times EF}\times=\frac{CB}{AE}$ ....(2)

But **ΔDBE** and **ΔDCE** are the same base **DE** and between the same parallel straight line **BC** and **DE**.

Area of **ΔDBE**= area of **ΔDCE** ....(3)

From (1), (2) and (3), we have

$\frac{DB}{AD}=\frac{CB}{AE}$

Hence proved.

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2020-21 MODEL PAPER – 1 with Key answer

Subject: Mathematics Subject code: 81E

Time : 3 hours Max.marks: 80

Answer the following questions. **8 x 1 = 8**

1. **If the pair of Linear equations x + 2y = 3 and 2x + 4y = k are coincide then the value of 'k' is :**
   Answer: 6
2. **The nth term of an arithmetic progression is an = 4n + 5 then the 3rd term is :**
   Answer: 17
3. **If the roots of the quadratic equation x2 + 6x + k = 0 are equal, then the value of 'k' is :**
   Answer: 9
4. **The value of sin $60^0 \times$ cos 30^0 is :**
   Answer: $\frac{3}{4}$
5. **The distance of the co-ordinate p(4, 3) from the x- axis is :**
   Answer: 3 units
6. **A straight line intersecting a circle at two points is called :**
   Answer: a secant
7. **The volume of a cylinder is 300 m3 then the volume of a cone having the same radius and height as that of the cylinder is :**
   Answer: 100 cm^3.
8. **The surface area of a sphere of radius 7cm is :**
   Answer: 616 cm^2.
9. **How many solutions have the pair of linear equations 2x+3y-9=0 and 4x + 6y - 18 = 0?**
   Answer: $\frac{a1}{a2}=\frac{2}{4}$, $\frac{b1}{b2}=\frac{3}{6}$, $\frac{c1}{c2}=-\frac{9}{18}$,
   Hence all are equal. So lines are parallel (many solutions)
10. **Write the standard form of a quadratic equation.**
    Answer: $ax^2+bx+c=0$
11. **Find the value of tanθ - cot (90^0-θ).**
    Answer: tanθ-cot(90^0-θ).
    tanθ- tanθ =0
12. **In the figure ∠B=90^0 , ∠A = ∠C and BC=10cm, then find the value of tan 45^0 .**
    Answer: tan $45^0=\frac{AB}{BC}$
    tan$45^0=\frac{BC}{BC}$ (Because AB=BC)
    tan45^0=1
13. **Write the co-ordinates of the midpoint of the line segment joining the points A(x_1 , y_1) and B (x_2 , y_2).**
    Answer: p(x, y)=$\left[\frac{x1+x2}{2} , \frac{y1+y2}{2}\right]$
14. **Find the median of the scores 5, 8, 14, 16, 19 and 20 ?**

Answer: 5, 8, 14, 16, 19, 20. Median $= \frac{14+16}{2}$

$=\frac{30}{2}$

Median $= 15$

15. State 'Thale's theorem ?

Thales theorem states that "if a line is parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides of the triangle in proportion".

16. Write the formula to find the curved surface area of the frustum of a cone as shown in the figure?

Answer: C.S.A of frustum $= \prod(r_1+r_2)l$.

17. Find the 25th term of an arithmetic progression 2, 6, 10, 14, ...

Solution: here a=2, d=4, n=25

We should have to find a_n, we know an=a+(n-1)d

$a_{25}=2+(25-1)4$

$a_{25}=2+24x4$

$a_{25}=2+96$

$a_{25}=98$

Hence 25th term is this A.P is 98

18. Find the sum of first 20 terms of the arithmetic progression 3, 8, 13, ... using the formula.

Solution: here a=3, d=5, n=20

We should have to find sn, we know $S_n=\frac{n}{2}(2a+(n-1)d)$

$s_{20}=\frac{20}{2}(2x3+(20-1)5)$

$s_{20}=10(6+19x5)$

$s_{20}=10x101$

$s_{20}=1010$

Hence sum of first 20th term is this A.P is 1010

OR

Find the sum of the first 30 positive integers divisible by 6

Solution: integers which is divisible by 6 is 6, 12, 18, 24, ……..

Here a=6, d=6, n=30, we need to find S_{30}.

We know the formula, $S_n=\frac{n}{2}(2a+(n-1)d)$

$S_n=\frac{30}{2}(2X6+(30-1)X6)$

$S_n=15X(12+29)X6)$

$=2790$

Hence the sum of first 30 terms which is divisible by 6 is 2790.

19. Solve: 3x + y = 15 & 2x - y = 5.

Solution: Consider the given equation.

3x+y=15 …….. (1)

2x-y=5 ……… (2)

On subtracting both equation (1) and (2), we get

5x=20

x=4

Now, put the value of x in equation (1), we get

3(4)+y=15

12+ y=15 y=3

Hence, the value of x is 4 and y is 3

20. Solve by using quadratic formula: x^2 -3x+1=0.

Solution: a=1, b=-3, c=1

Quadratic formula is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-3)\pm\sqrt{3^2-4x(1)x1}}{2x1}$

$x=\frac{3\pm\sqrt{9-4}}{2}$

$x=\frac{3\pm\sqrt{5}}{2}$

21. Find the discriminant of the quadratic equation $2x^2$ -6x+3=0 and hence write the nature of roots.

Solution: $2x^2$ -6x+3=0

a=2, b=-6, c=3

$\Delta=b^2-4ac$.

Δ=36-4x2x3

Δ=36-24=12

$\Delta>0$, hence roots are different.

OR

Prove that the quadratic equation x2 +ax-4=0 has distinct, real roots.

Solution: x^2+ax-4=0

a=1, b=a, c=-4

$\Delta=b^2-4ac$.

$\Delta=a^2$-4x1x(-4)

$=a^2+16$

$\Delta>0$

So here roots are real & exists.

22. Find the distance between the co-ordinate of the points A(2, 3) and B(10, -3).

Solution: $(x_1, y_1)=(2, 3)$ & $(x_2, y_2)=(10, -3)$

We know the distance formula $d=\sqrt{(x2-x1)^2+(y2-y1)^2}$.

$d=\sqrt{(10-2)^2+(-3-3)^2}$

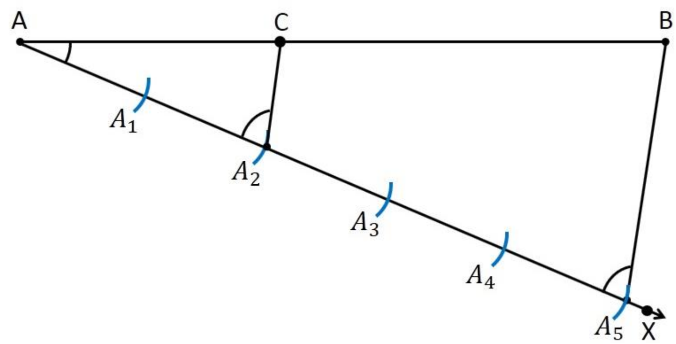
$d=\sqrt{(8)^2+(-6)^2}$

$d=\sqrt{64+36}$

d= 10 units.

23. Draw a line segment of AB=8cm and divide it in the ratio 3:2 by geomtrical construction.

Solution:



24. In the figure given below find the value of sinθ and cos∝ ?.

Answer: by figure sin $\boldsymbol{\theta}=\frac{5}{13}$ & cos $\boldsymbol{\alpha}=\frac{5}{13}$.

25. The sum of two natural numbers is 9 and the sum of their reciprocals is 9 20 . Find the numbers.

Solution :

The sum of two numbers is 9 and the sum of their reciprocal is 9/20.

Let the numbers be x and y respectively.

Sum of numbers is 9.

⇒ x + y = 9.... (i)

⇒ y = 9 - x .... (ii)

Sum of reciprocals is 1/2.

⇒ $\frac{1}{x}+\frac{1}{y}=\frac{9}{20}$.. (iii), put y=9-x in the equation (iii)

Now, on solving (iii), we get $x^2-9x+20=0$.

$x^2-5x-4x+20=0$

(x-5)(x-4)

X=5 & x=4

Putting the value of (i) and (ii) here, we get -

Hence, the required numbers are 5 and 4.

OR

The perimeter and area of a rectangular playground are 80m and 384m^2 respectively. Find the length and breadth of the playground.

Solution: let length of the rectangular playground be = l

Breadth of the rectangular playground be = b

2(l+b)=80 lxb=384

l+b=40 lxb=384

l=40-b

this gives (40-b)xb=384

$40b-b^2=384$

$b^2-40b+384=0$

by factorization

$b^2-24b-16b+384=0$

(b-24) (b-16)

b=24 & b=16

put the value of b in above equations we get l+16=40

l=24

so the length of the rectangular garden is 24m and breadth is 16m.

26. Prove that $\frac{\sin\theta}{1-\cos\theta}+\frac{\cos\theta}{1-\tan\theta}$=sinθ+cosθ

Solution: LHS $\frac{\sin\theta}{1-\cos\theta}+\frac{\cos\theta}{1-\tan\theta}$

$$\frac{\sin\theta}{1-1/Sin\theta}+\frac{\cos\theta}{1-\sin\theta/\cos\theta}=\frac{\sin2\theta}{\sin\theta-1}+\frac{\cos2\theta}{\cos\theta-\sin\theta}=\frac{\sin2\theta}{\sin\theta-\cos\theta}-\frac{\cos2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{\sin2\theta-\cos2\theta}{\sin\theta-\cos\theta}$$

$$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}=(\sin\theta+\cos\theta)$$ hence the proof

OR

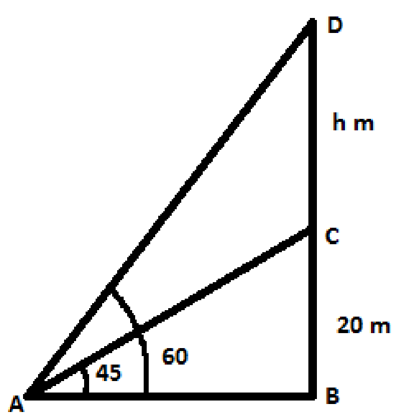
Prove that: $\frac{COS\theta-2COS3\theta}{2Sin3\theta-\sin\theta}$=cosθ

Solution: $\frac{COS\theta-2COS3\theta}{2Sin3\theta-\sin\theta}=\frac{COS\theta(1-2COS2\theta\_}{\sin\theta(2Sin2\theta-1)}=$ cot**θ**$\frac{1-2+2\sin2\theta}{2\sin2\theta-1}$

= cot($\frac{2\sin2\theta-1}{2\sin2\theta}$) =cot **θ**. Hence the proof.

27. From a point on the ground, the angles of elevation of the top and bottom of a transmission tower fixed at the top of a 20m high building are 60^0 and 45^0 .respectively. Find the height of the transmission tower.

Solution:



Let DC be the tower and BC be the building, then

∠CAB=45^o,∠DAB=60^o,BC=20 m

Let height of the tower, DC=h m.

In right △ABC,

tan45^o=$\frac{AB}{BC}$

1=$\frac{AB}{20}$

AB=20 m

In right △ABD,

tan60o=AB/BD

$\sqrt{3}=\frac{h+20}{20}$ = .>

h=20($\sqrt{3}$−1) m

28. Find the value of 'k'. If the co-ordinates of the points A(2, -2), B(-4, 2) and C(-7, k) are collinear.

Solution: $(x_1, y_1)=(2, -2)$, $(x_2, y_2)=(-4, 2)$, $(x_3, y_3)=(-7, k)$.

We know that area of triangle = $\frac{1}{2}(x_1(y_2-y_3)+x_2((y_3-y_1)+x_3(y_1-y_2))$.

$0 = \frac{1}{2}(2(2-k)+-4(k+2)+-7(-2-2))$

0 =-6k+4-8+28

6k=24

k=4

29. Calculate the 'mean' for the frequency distribution table given below, by direct method.

c.i	f
5-15	4
15-25	3
25-35	6
35-45	5
45-55	2

Solution: we know the formula that $x=\frac{\sum fxx}{n}=\frac{580}{20}=29$

C.I	f	x	fx
5-15	4	10	40
15-25	3	20	60
25-35	6	30	180
35-45	5	40	200
45-55	2	50	100
	n=20		$\sum fxx$=580

So mean value is 29

OR

Find the 'mode' of the frequency distribution table given below.

C.I	f
0-10	7
10-20	9
20-30	15
30-40	11
40-50	8
	N=50

Answer: here n=50,

Then place the chart

C.I	f
0-10	7
10-20	9
20-30	15
30-40	11
40-50	8
	N=50

$l=20$, $f_0=9$, $f_1=15$, $f_2=11$, $h=10$

Mode = $l+(\frac{f1-f0}{2f1-f0-f2})xh$

=20+(6/10)x15

=20+6

=26

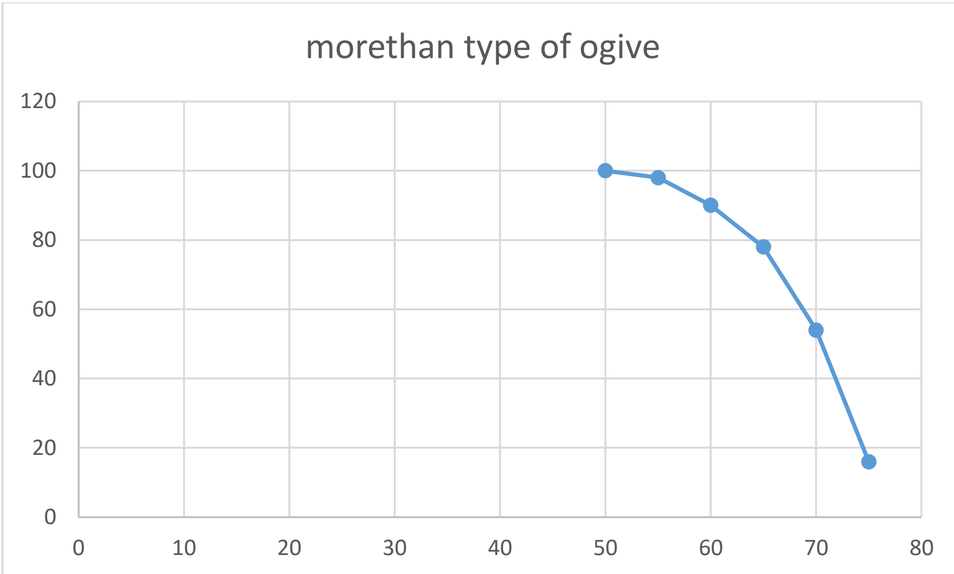
30. The following table gives the production yield per hectare of wheat of 100 farms of a village. Draw a 'more than type ogive' for the given data.

Production yield in kg/hectare	**Cumulative Frequency**
More than or equal to 50	**100**
More than or equal to 55	**98**
More than or equal to 60	**90**
More than or equal to 65	**78**
More than or equal to 70	**54**
More than or equal to 75	**16**

Solution:

Answer:

C.I	f	points
>50	100	50, 100
>55	98	55, 98
>60	90	60, 90
>65	78	65, 78
>70	54	70, 54
>75	16	75, 16

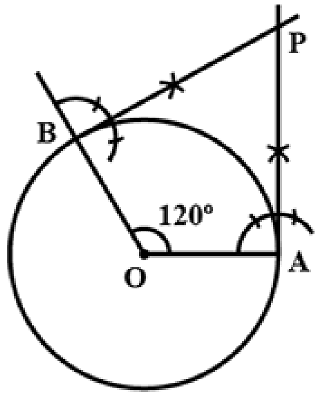


31. . Prove that "the tangent at any point of a circle is perpendicular to the radius through the point of contact".

Given: a circle with tangent XY at point of contact P.
To Prove: OP⊥XY
Proof: Let Q be a point on XY connect OQ
Suppose it touches the circle at R
Hence,
OQ>OR
OQ>OP   OP=OR (radius)
Same will be the case with all other points on the circle
Hence,
We get OP is the smallest line that connects XY.

32. Draw a pair of tangents to a circle of radius 4cm which are inclined to each other at an angle of 60^0 and write the measure of its length.

Solution: Draw a circle of radius 4cm, then construct 120^0 between the two radii ($180^0–60^0$).



33. A right circular metalic cone of height 20cm and base radius 5cm is melted and recast into a solid sphere. Find the radius of the sphere.

Solution: volume of the cone= volume of the sphre

$\Rightarrow \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi r_2^3 \Rightarrow 5\times5\times20 = 4\times r_2^3$

$\Rightarrow r_2 = \sqrt[3]{35 \times 5 \times 5} = 5\text{cm}$

∴ Radius of sphere is 5cm.

OR

A solid sphere of radius 3cm is melted and reformed by stretching it into a cylindrical shaped wire of length 9m. Find the radius of the wire.

Solution: sphere, r=3cm

Cylinder , l=9m=900cm.

Volume of cylinder = volume of sphere

$\pi r^2 h = \frac{4}{3}\pi r^3$

$r^2 x 900 = \frac{4}{3} x3x3x3$

$r^2 x900=36$

$r^2 = \frac{4}{100} = 0.2$ cm

radius of the wire is 0.2cm

34. Find the Solution to the given pair of linear equations by graphical method.
2x + y = 10, & x+ y = 6.

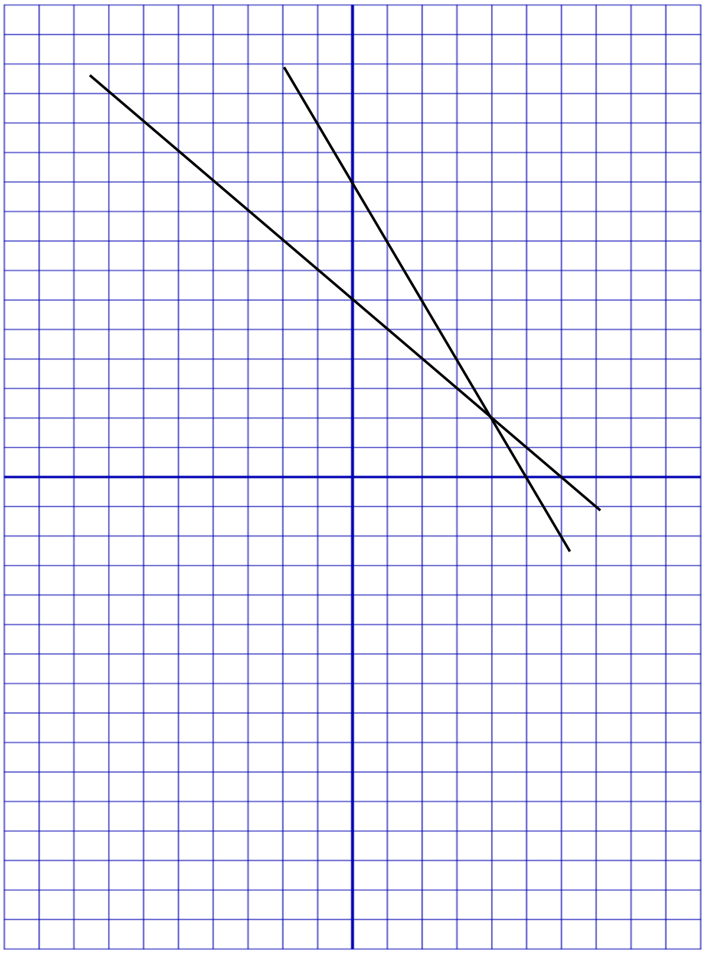
Solution:

From equation (i), we have the following table:

x	**0**	**2**	**4**
y	**10**	**6**	**2**

From equation (ii), we have the following table:

x	**0**	**2**	**4**
y	**6**	**4**	**2**



So the two lines are intersects at (4, 2). Hence x=4 & y=2

35. An arithmetic progression consists of 37 terms. The sum of the first 3 terms of it is 12 and the sum of its last 3 terms is 318, then find the first and last terms of the progression.

Solution: let first three terms be a, a+d, a+2d, and last three terms be a+34d, a+35d, a+36d.

According to problem,

a+a+d+a+2d=12	a+34d+a+35d+a+36d=318
3a+3d=12	3a+105d=318
a+d=4 ---→(1)	a+35d=106------------→(2)

subtract above two equations, we get 34d=102

d=3

put d value in any one equation we get a, a+3=4

a=1

therefore first term is 1 and last term is 1+36d=1+36x3=1+108=109

OR

The sum of the first 7 terms of an arithmetic progression is 140 and the sum of the next 7 terms of the same progression is 385 then find the arithmetic progression.

Solution: $S_7=140$ & $S_{14}=140+385$

$S_7=140$ $S_{14}=525$

we know $S_n=\frac{n}{2}(2a+(n-1)d)$ $S_n=\frac{n}{2}(2a+(n-1)d)$

$140=\frac{7}{2}(2a+(7-1)d)$ $525=\frac{14}{2}(2a+(14-1)d)$

$40=2a+6d$ -----------(1) $75=2a+13d$ --------(2)

Solve above two equations, $7d=35$

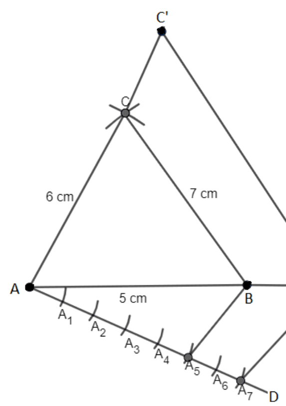
$d=5$

put d value in anyone equation we get $a=5$

so arithmetic progression is 5, 10, 15, ……..

36. Construct a triangle with sides 4cm, 5cm, and 6cm and then another triangle whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle.

Solution:



37. A toy is made in the shape of a cylinder with one hemisphere stuck to one end and a cone to the other end, as shown in the figure, the length of the cylindrical part of the toy is 20cm and its diameter is 10cm. If the slant height of the cone is 13cm. Find the surface area of the toy.

Solution: cylinder, r=5cm, h=20cm

Hemisphere, r=5cm

Cone, r=5c, & l=13cm

We should have to find surface area of the toy

= CSA of cylinder+ CSA of hemisphere+ CSA of Cone

$=2\pi rh+2\pi r^2+\pi rl.$

$=\pi(2rh+2r^2+rl)$

$=\pi r(2x20+2x5+x13)$

$=\pi r(40+10+13)$

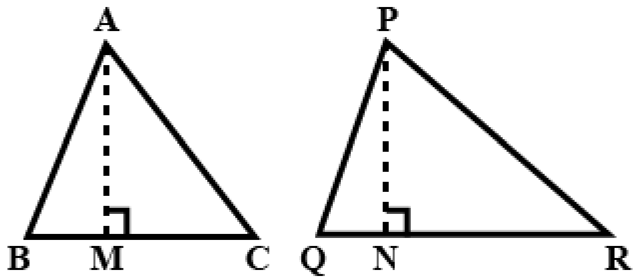
$=\frac{22}{7}(63)x5$

$=22x45$

$= 990\ cm^2$.

38.. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:



Let the two triangles be:

ΔABC and ΔPQR

Area of $\Delta ABC = \frac{1}{2} \times BC \times AM$……………..(1)

Area of $\Delta PQR = \frac{1}{2} \times QR \times PN$……………………..(2)

Dividing (1) by (2)

$\frac{ar(ABC)}{ar(PQR)} = \frac{QR \times PN}{BC \times AM}$ ------------(3)

In ΔABM and ΔPQN

$\angle B = \angle Q$ (Angles of similar triangles)

$\angle M = \angle N$ (Both 90)

Therefore, $\Delta ABM \sim \Delta PQN$

So, $\frac{AM}{AB} = \frac{PN}{PQ}$ ……………………..(4)

From 2 and 4

$\frac{ar(ABC)}{ar(PQR)} = \frac{QR \times BC}{PN \times AM}$

$\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} X \frac{AB}{PQ}$ ------------(5)

But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AB}{PQ}$

Hence $\frac{ar(ABC)}{ar(PQR)} = (\frac{AB}{PQ})^2 = (\frac{BC}{QR})^2 = (\frac{AB}{PQ})^2$.

As per reduced syllabus-2021
MODEL QUESTION PAPER-3

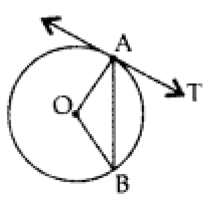
Subject: Mathematics | Subject code: 81E
Time : 3 hours | Max.marks: 80

Choose the correct answer given below ------- 1x8=8

1. If a=10 and d=10, then first four terms will be:
   a. 10, 30, 50, 60 b. 10, 20, 30, 40 c.10, 15, 20, 25 d. 10, 18, 20, 30
2. The cubic equation has degree
   a. 1 b.2 c. 0 d. 3
3. Graphically, the pair of equations 7x – y = 5; 21x – 3y = 10 represents two lines which are…..
   a. Intersecting at one point b. parallel c. intersecting at two points d. coincident
4. If COT A=$\frac{1}{4}$, Then TAN A is …..
   a. $\frac{1}{2}$ **b.** $\frac{1}{4}$ **c.** $\frac{4}{1}$ **d.** $\frac{3}{4}$
5. The distance of the point (4, 3) from the origin?
   a. 3 units b. 4 units c. 7 units d. 5 units
6. In an A.P, 1, 5, 9,….. which of the following is common difference ?
   a. 4 b. 2 b. 3 d. 1
7. In ΔABC DE || AB. If CD = 3 cm, EC = 4 cm, BE = 6 cm, then DA is equal to …
   a. 7.5cm b. 3cm c. 4.5cm d. 6cm
8. A cylindrical pencil sharpened at one edge is the combination of……
   a. Cylinder & hemisphere b. Cylinder & cone c. Cone & hemisphere d. two cylinder.

Answer the following questions 1 x8 =8

9. What will be the nature of the roots of the quadratic equation $5x^2 – 4x + 5 = 0$.
10. Find the 10^{th} term in the A.P 4, 8, 12…………..
11. Find the value of $\sec^2 42° – \text{cosec}^2 48°$.
12. If (1 + cos A) (1 – cos A) =3/4, find the value of sec A.
13. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then calculate the height of the wall.
14. If ΔABC ~ ΔPQR, perimeter of ΔABC = 32 cm, perimeter of ΔPQR = 48 cm and PR = 6 cm, then find the length of AC.
15. In the given figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then calculate ∠BAT.



16. Write the formula to total surface area of the cylinder.

Answer the questions 2x8=16

17. Find whether -150 is a term of the A.P. 17, 12, 7, 2, …?.

OR

Which term of the progression 4, 9, 14, 19, … is 109?

18. Solve the following quadratic equation by using formula:

$x^2 + 2x - 8 = 0$

19. Solve the equations : x+3y=6, 2x-3y=12
20. Evaluate: tan 15° . tan 25° , tan 60° . tan 65° . tan 75° – tan 30°.
21. ΔABC ~ ΔDEF. If AB = 4 cm, BC = 3.5 cm, CA = 2.5 cm and DF = 7.5 cm, find the perimeter of ΔDEF.
22. Express cot 75° + cosec 75° in terms of trigonometric ratios of angles between 0° and 30°.
23. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60°.
24. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then calculate the rise of water level (in cm).

Solve the following problems 3x9=27

25. A man earns ₹600 per month more than his wife. One-tenth of the man's salary and l/6th of the wife's salary amount to ₹1,500, which is saved every month. Find their incomes.

OR

The age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

26. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is 16656 cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹10 per cm^2. [Use π = 22/7]
27. If the sum of two natural numbers is 8 and their product is 15, find the numbers.
28. Find the mean of the following data.

Class	Frequency
less than 20	15
less than 40	37
less than 60	74
less than 80	99
less than 100	120

OR

Weekly income of 600 families is given below: Find the median

Income in (₹)	No. of families
0–1000	250
1000–2000	190
2000–3000	100
3000–4000	40
4000–5000	15
5000–6000	5

29. Prove that "the tangent at any point of a circle is perpendicular to the radius through the point of contact".
30. Find that value(s) of x for which the distance between the points P(x, 4) and Q(9, 10) is 10 units.

31. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹5 per 100 sq. cm. (Use π = 3.14).

32. If an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are 3/4 times the corresponding sides of the isosceles triangle.

33. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of top of the tower from the foot of the hill is 30°.If the tower is 50 m high, what is the height of the hill?

OR

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Solve 4x4=16

34. Solve pair of linear equations graphically : x+3y=6 & 2x-3y=12.

35. If the sum of first 7 terms of an A.P is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

OR

If S_n, denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$.

36. The following table gives the daily income of 50 workers of a factory. Draw both types ("less than type" and "greater than type") ogives.

Daily income (in ₹)	*No. of workers*
100–120	12
120-140	14
140–160	8
160–180	6
180–200	10

37. The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 115. Find the fraction.

Solve 5x1=5

38. State and Prove Pythagoras theorem.

KEY ANSWER-3

Subject: Mathematics Subject code: 81E

Time :3 hours Max.marks:80

1. **Answer: b**

   Explanation: a = 10, d = 10

   $a_1 = a = 10$

   $a_2 = a_1+d = 10+10 = 20$

   $a_3 = a_2+d = 20+10 = 30$

   $a_4 = a_3+d = 30+10 = 40$
2. **Option (b) 3**
3. **Option (b) parallel**
4. **Option (c)**
5. **Option (d) 5 units**
6. **Answer: 4.**
7. **Option (c) 4.5cm**
8. **Option (b) cylinder and cone.**
9. To find the nature, let us calculate $b^2 - 4ac$

   $b^2 - 4ac = 4^2 - 4 \times 5 \times 5$

   $= 16 - 100 = -84 < 0$, therefore it is not real.

10. Here a = 4, d = 4, we need to find a_{10}.

$a_{10}=a+9d$

$a_{10}=4+9x4$

$=4+36$

$=40$

11.

$\sec^2 42° - \mathrm{cosec}^2 48° = \sec^2 42° - \mathrm{cosec}^2 (90° - 42°)$

$= \sec^2 42° - \sec^2 42°$ [Using $\sec\theta = \mathrm{cosec}(90° - \theta)$]

12.

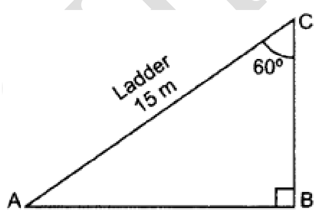
$(1 + \cos A)(1 - \cos A) = \frac{3}{4}$

$\therefore \quad 1 - \cos^2 A = \frac{3}{4}$

$1 - \frac{3}{4} = \cos^2 A$

$\frac{1}{4} = \cos^2 A \Rightarrow \sec^2 A = 4 \Rightarrow \sec A = \pm 2$

13.



$\angle BAC = 180° - 90° - 60o = 30°$

sin 30° = BCAC

12=BC15

2BC = 15

BC = 152m

14. Solution:

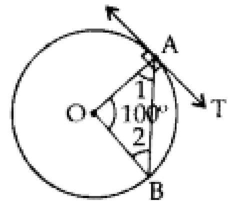
$\Delta ABC \sim \Delta PQR$ ...[Given

$\therefore \quad \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \dfrac{AC}{PR}$

$\Rightarrow \quad \dfrac{32}{48} = \dfrac{AC}{6} \qquad \Rightarrow \quad$ **AC = 4 cm**

15.

Solution:



$\angle 1 = \angle 2$

$\angle 1 + \angle 2 + 100° = 180°$

$\angle 1 + \angle 1 = 80°$

$\Rightarrow 2\angle 1 = 80°$

$\Rightarrow \angle 1 = 40°$

$\angle 1 + \angle BAT = 90°$

$\angle BAT = 90° - 40° = 50°$

16. The Surface Area of Cylinder = Curved Surface + Area of Circular bases

S.A. (in terms of π) = 2πr (h + r) sq.unit

17. Given: 1^{st} term, a = 17

Common difference, d = 12 – 17 = -5

n^{th} term, a_n = – 150 (Let)

$\therefore$ a + (n – 1) d = – 150

17 + (n – 1)(-5) = – 150

(n – 1) (-5) = – 150 – 17 = – 167

(n – 1) = −167−5

n = 1675 + 1 = 167+55=1725

n = 1725 ...[Being not a natural number

$\therefore$ -150 is not a term of given A.P.

OR

Given: A.P.: 20, 774,374,714

Here a = 20, d = 77−804=−34

For first negative term, a_n < 0

$\Rightarrow$ a + (n – 1)d < 0 $\Rightarrow$ 20 + (n – 1)(-34) < 0

$\Rightarrow$ −34(n – 1) < -20 $\Rightarrow$ 3(1 – 1) > 80

$\Rightarrow$ 3n – 3 > 80 $\Rightarrow$ 3n > 83

n > 834 $\Rightarrow$ n > 27.5

$\therefore$ Its negative term is 28th term.

18. x2+2x−8=0

The equation is of the form ax2+bx+c=0 where:

a=1,b=2,c=−8

The solutions are found using the formula

x=−b±√Δ2·a

x=(−2)±√36)2·1=(−2±6)2
x=−2+62=42=2 , x=2
x=−2−62=−82=−4 , x=−4

19. Consider the given equation.
x+3y=6 .... (1)
2x−3y=12 .... (2)
On adding both equation (1) and (2), we get
3x=18
x=6
Now, put the value of x in equation (1), we get
6+3y=6
3y=6−6
3y=0
y=0
Hence, the value of x is 6 and y is 0

20.
Solution:
tan 15°. tan 25°, tan 60°. tan 65°. tan 75° – tan 30°
= tan(90° – 75°) tan(90° – 65°). 3−√ . tan 65°. tan 75° – 13√

$$= \cot 75^\circ . \cot 65^\circ . \sqrt{3} . \frac{1}{\cot 65^\circ} . \frac{1}{\cot 75^\circ} - \frac{1}{\sqrt{3}}$$

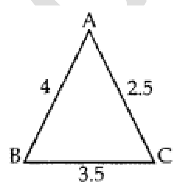
$$= \sqrt{3} - \frac{1}{\sqrt{3}} \quad \ldots \left[\begin{array}{l} \because \tan(90^\circ - A) = \cot A \\ \tan B = \dfrac{1}{\cot B} \end{array} \right.$$

$$= \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

21. Solution:
ΔABC – ΔDEF ...[Given

$$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AC}{DF}$$

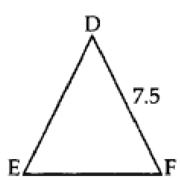
$$\frac{AB + BC + CA}{\text{Perimeter of } \Delta DEF} = \frac{AC}{DF}$$



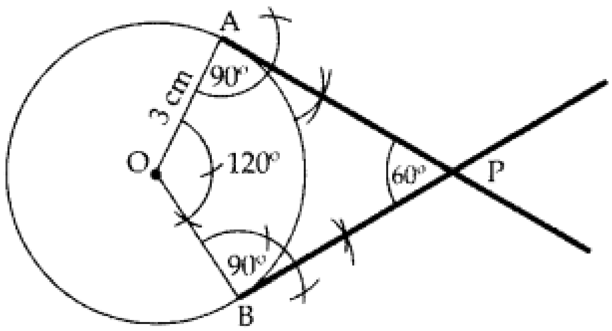
$$\frac{4 + 3.5 + 2.5}{\text{Perimeter of } \Delta DEF} = \frac{2.5}{7.5}$$

$$\frac{10}{\text{Perimeter of } \Delta DEF} = \frac{1}{3}$$

$$\therefore \textbf{Peri.}(\Delta \textbf{DEF}) = \textbf{30 cm}$$



22. Solution:
cot 75° + cosec 75°
= cot(90° – 15°) + cosec(90° – 15°)
= tan 15° + sec 15° ...[cot(90°-A) = tan A
cosec(90° – A) = sec A



23. ∴ PA & PB are the required tangents.

24. Solution:

Volume of Cylinder = Volume of Sphere

$$\pi R^2 h = \frac{4}{3}\pi r^3$$

$$(18)^2 h = \frac{4}{3} \times (9)^3 \ldots [\because R = \frac{36}{2} = 18 \text{ cm}; r = \frac{18}{2} = 9 \text{ cm}$$

$$\therefore \quad h = \frac{4}{3} \times \frac{9\times 9\times 9}{18\times 18} = 3 \text{ cm}$$

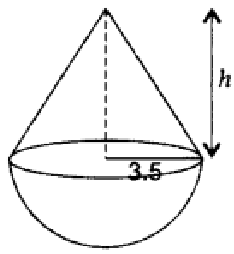
25. Solution:

Let wife's monthly income = ₹x
Then man's monthly income = ₹(x + 600)
According to the question,
$\frac{1}{10}(x + 600) + \frac{1}{6}(x)$ = ₹1,500
3(x+600)+5x/30 = ₹1,500
3x + 1,800 + 5x = ₹45,000
8x = ₹45,000 – ₹1,800
x = ₹343,200/8 = ₹5,400
Wife's income = ₹x = ₹5,400
Man's income = ₹(x + 600) = ₹6,000

OR

Let the sum of the ages of two children will be x and age of father will be y.
A.T.Q. ,
y = 2x ----------(i)
and , After 20 years,
x+40=y+20
==> x-y= -20
put y = 2x from (i),
x-2x = -20
==> x = 20
Now, put x= 20 in (i),
y = 2 x 20 = 40
Hence age of father will be 40 years.

26. Solution:



Let the height of cone = h
Radius of cone = Radius of hemisphere = r = 3.5 cm
Volume of solid wooden toy = Volume of hemisphere + Volume of cone

$\Rightarrow \quad 166\frac{5}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad \frac{1001}{6} = \frac{1}{3}\pi r^2 (2r + h)$

$\Rightarrow \quad \frac{1001}{6} = \frac{1}{3}\times\frac{22}{7} \times (3.5)^2(2 \times 3.5 + h)$

$\Rightarrow \quad \frac{1001}{6} = \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\ (7 + h)$

$\Rightarrow \quad \frac{1001}{6} = \frac{77}{6}(7 + h) \Rightarrow \quad \frac{1001}{77} = 7 + h$

$\Rightarrow \quad 13 = 7 + h \quad \Rightarrow \quad h = 6$

$\therefore$ **Height of toy** $= h + r = 6 + 3.5 =$ **9.5 cm**

Area of hemispherical part of toy $= 2\pi r^2$

$= \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\text{cm}^2 = 77\ \text{cm}^2$

$\therefore$ **Cost of painting** = ₹(77 × 10) = **₹770**

27. Solution:

Let the numbers be x and (8 – x).
According to the Question,
$x(8 - x) = 15$
$\Rightarrow 8x - x^2 = 15$
$\Rightarrow 0 = x^2 - 8x + 15$
$\Rightarrow x^2 - 5x - 3x + 15 = 0$
$\Rightarrow x(x - 5) - 3(x - 5) = 0$
$\Rightarrow (x - 3)(x - 5) = 0$
$x - 3 = 0$ or $x - 5 = 0$
$x = 3$ or $x = 5$
When x = 3, numbers are 3 and 5.
When x = 5, numbers are 5 and 3.

28.

Class	c.f.	C.I.	Freq. f_i	x_i	$d'_i = \frac{x_i - 50}{20}$	$f_i d'_i$
Less than 20	15	0-20	15	10	−2	−30 }−52
Less than 40	37	20-40	22	30	−1	−22
Less than 60	74	40-60	37	50	0	0
Less than 80	99	60-80	25	70	1	25 }67
Less than 100	120	80-100	21	90	2	42
			$\Sigma f_i = 120$			$\Sigma f_i d_i = 15$

Let $a = 50$

$$\therefore \quad \textbf{Mean} = a + \left(\frac{\Sigma f_i d'_i}{\Sigma f_i} \times h\right) = 50 + \left(\frac{15}{120} \times 20\right)$$

$$= 50 + 2.5 = \mathbf{52.5}$$

OR

Income (in ₹)	f_i	c.f.
0–1000	250	250
1000–2000	$\boxed{190}$	$\boxed{440}$
2000–3000	100	540
3000–4000	40	580
4000–5000	15	595
5000–6000	5	600
	$n = 600$	

$$\frac{n}{2} = \frac{600}{2} = 300$$

$\therefore$ Median class is 1000 – 2000

$$\textbf{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 1000 + \left(\frac{300 - 250}{190} \times 1000\right)$$

$$= 1000 + \frac{50,000}{190} = 1000 + 263.16$$

$$= \mathbf{1263.16\ (approx.)}$$

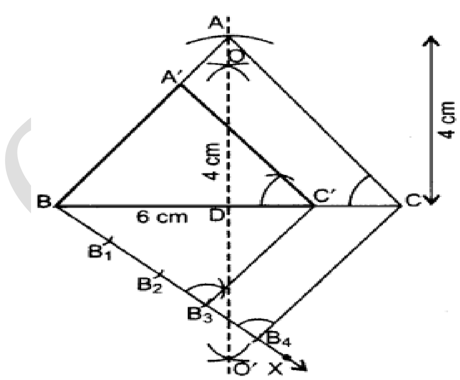
29. Given: a circle with tangent XY at point of contact P.

To Prove: OP⊥XY
Proof: Let Q be a point on XY connect OQ
Suppose it touches the circle at R
Hence,
OQ>OR
OQ>OP   OP=OR (radius)
Same will be the case with all other points on the circle
Hence,
We get OP is the smallest line that connects XY.

30.
Solution:
PQ = 10 …Given
$PQ^2 = 10^2 = 100$ … [Squaring both sides
$(9 – x)^2 + (10 – 4)^2 = 100$…(using distance formula
$(9 – x)^2 + 36 = 100$
$(9 – x)^2 = 100 – 36 = 64$
$(9 – x) = \pm 8$ …[Taking square-root on both sides
$9 – x = 8$ or $9 – x = -8$
$9 – 8 = x$ or $9+ 8 = x$
$x = 1$ or $x = 17$

31. Solution:
Let the side of cuboidal block (a) = 10 cm
Let the radius of hemisphere be r.
Side of cube = Diameter of hemisphere Largest possible diameter of hemisphere = 10 cm
∴ Radius, r = 102 = 5 cm
Total surface area = Total surface area of cube + Curved surface area of hemisphere- Area of base

$$= (6a^2 + 2\pi r^2 - \pi r^2) = 6a^2 + \pi r^2$$
$$= 6(10)^2 + 3.14 \times (5)^2 = 600 + 78.5$$
$$\Rightarrow \quad 678.5 \text{ cm}^2$$
$$\therefore \quad \text{Cost of painting} = \frac{678.5 \times 5}{100} = \frac{3392.50}{100}$$
$$= ₹33.9250 \text{ or } ₹\mathbf{33.93}$$



32.

33. Let AB be the tower and CD be the hill. Then, ∠ACB=30o,∠CAD=60o and AB=50 m.
Let **CD=x m**
In right △**BAC**, we have,

cot**30o=ABAC**
3=50AC
AC=503 m
In right **△ACD**, we have,
tan**60o=ACCD**
3=503x
x=50×3=150 m
Therefore, the height of the hill is 150 m.

34.

$$x + 3y = 6$$
$$\Rightarrow x = 6 - 3y$$

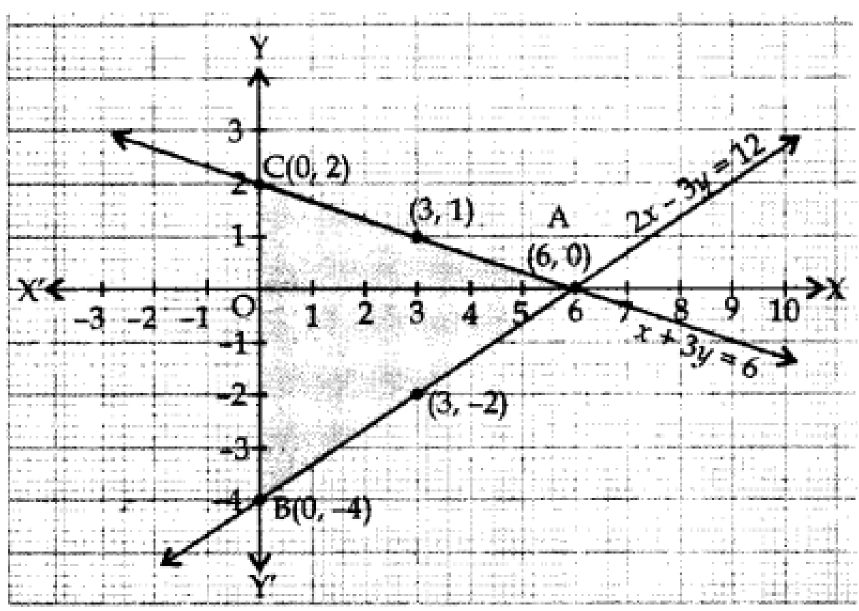
x	6	3	0
y	0	1	2

(6, 0), (3, 1), (0, 2)

$$2x - 3y = 12$$
$$\Rightarrow 2x = 12 + 3y$$
$$\Rightarrow x = \frac{12 + 3y}{2}$$

x	0	6	3
y	−4	0	−2

(0, −4), (6, 0), (3, 2)



35.

Solution:
Let 1st term = a, Common difference = d

Given: $S_7 = 49$, $S_{17} = 289$

As we know, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_7 = \frac{7}{2}(2a + 6d)$$

$$49 \times \frac{2}{7} = 2a + 6d \quad \Rightarrow \quad 2a + 6d = 14 \quad ...(i)$$

Now, $S_{17} = \frac{17}{2}(2a + 16d)$

$$289 \times \frac{2}{17} = 2a + 16d$$

$$2a + 16d = 34 \qquad ...(ii)$$

Solving (i) and (ii), we get

$$\begin{array}{rcr} 2a + 6d &=& 14 \\ -2a \pm 16d &=& \pm 34 \\ \hline -10d &=& -20 \\ \hline \end{array}$$

$\Rightarrow \quad d = 2$

Putting $d = 2$ in (i), we get $a = 1$

$$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2(1) + (n-1)2] = \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{2n^2}{2} = n^2 \text{ (Hence proved)}$$

OR

Solution:

Let a be the first term and d be the common difference of A.P.

$S_n = n2\ (2a + (n - 1)d)$

$\therefore S_{12} = 122\ (2a + (12 - 1)d)$

$S_{12} = 6\ [2a + 11d] = 124 + 66d$ ...(i)

$\therefore S_8 = 8n2\ (2a + (8 - 1)d)$

$S_8 = 4[2a + 7d] = 8a + 28d$ ... (ii)

$\therefore S_4 = 42\ (2a + (4 - 1)d)$

$S_4 = 2[2a + 3d) = 4a + 6d$ ...(iii)

Now, $S_{12} = 3(S_8 - S_4)$

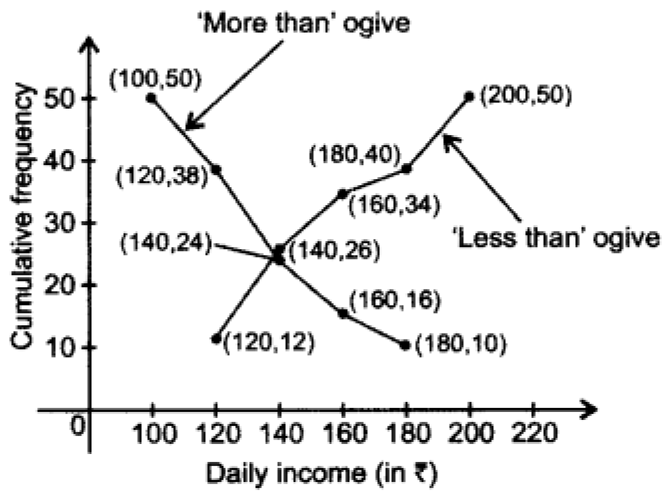
$12a + 660 = 3(8a + 28d - 4a - 6d)$ ... [From (i), (ii) & (iii)

$12a + 660 = 3(4a + 22d)$

$12a + 660 = 12a + 66d$ ...Hence proved

36.

For 'Less than' ogive Daily income (in ₹)	*No. of Workers (c.f.)*	*For 'more than' ogive Daily income (in ₹)*	*No. of Workers (c.f.)*
Less than 120	12	More than 100	50
Less than 140	26	More than 120	38
Less than 160	34	More than 140	24
Less than 180	40	More than 160	16
Less than 200	50	More than 180	10



37. Solution:

Let the denominator be x and the numerator be x – 3.

∴ Fraction =x−3x

New denominator = x + 1

According to the Question,

$$\Rightarrow \frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$$

$$\Rightarrow \frac{x-3}{x+1} = \frac{15x-45-x}{15x}$$

$$\Rightarrow \frac{x-3}{x+1} = \frac{14x-45}{15x}$$

$\Rightarrow 15x^2 - 45x = 14x^2 - 45x + 14x - 45$

$\Rightarrow 15x^2 - 14x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 14x + 45 = 0$

$\Rightarrow x^2 - 5x - 9x + 45 = 0$

$\Rightarrow x(x - 5) - 9(x - 5) = 0$

$\Rightarrow (x - 5)(x - 9) = 0$

$\Rightarrow x - 5 = 0$ or $x - 9 = 0$

$\Rightarrow x = 5$ or $x = 9$

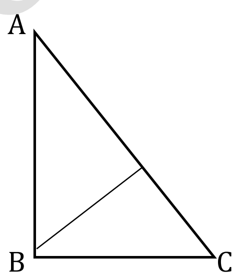
When x = 5, fraction = 5−35=25

When x = 9, fraction = 9−39=69=23

∴ Fraction = 25 or 23

38.

In a right angles triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. 1



Data : In ΔABC, $\angle ABC = 90°$

To Prove : $AB^2 + BC^2 = CA^2$ 1

Construction : Draw $BD \perp AC$.

Proof : **Statement** **Reason**

Compare ΔABC and ΔADB,

$\angle ABC = \angle ADB = 90°$ (Q Data and construction)

$\angle BAD$ is common.

$\therefore \Delta ABC \sim \Delta ADB$ (Q Equiangular triangles)

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$ (Q A A similarity criteria)

$\therefore AB^2 = AC.AD$ ...... (1) 1

Compare ΔABC and ΔBDC,

$\angle ABC = \angle BDC = 90°$ (Q Data and construction)

$\angle ACB$ is common

$\therefore \Delta ABC \sim \Delta BDC$ (Q Equiangular Triangles)

$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC} \Rightarrow =$ (Q AA similarity criteria)

$BC^2 = AC.DC$ .....(2) 1

By adding (1) and (2) we get

$AB^2 + BC^2 = (AC.\ AD) + (AC.\ DC)$

$AB^2 + BC^2 = AC\ (AD + DC)$

$AB^2 + BC^2 = AC.\ AC = AC^2$ [Q $AD + DC = AC$]

$\therefore AB^2 + BC^2 = AC^2$

As per reduced syllabus-2021
MODEL QUESTION PAPER-4

Subject: Mathematics — Subject code: 81E

Time : 3 hours — Max.marks: 80

Choose the correct answer given below ------- 1x8=8

1. The equation $(x - 2)^2 + 1 = 2x - 3$ is a
   a. linear b. quadratic c. cubic d. bi-quadratic
2. A pair of linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ is said to be inconsistent, if
   (a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
   (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (d) $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$
3. The n^{th} term of an A.P. is given by $a_n = 3 + 4n$. The common difference is
   b. 7 b. 3 c. 4 d. 1
4. The distance of the point P (4, 3) from the origin is.....
   b. 2 b.3 c.1 d. 5
5. If x tan 45° sin 30° = cos 30° tan 30°, then x is equal to
   b. $\sqrt{3}$ b. $\frac{1}{\sqrt{2}}$ c. $\frac{1}{2}$ d. 1
6. D and E are respectively the midpoints on the sides AB and AC of a triangle ABC and BC = 6 cm. If DE || BC, then the length of DE (in cm) is
   a. 2.5 b. 3 b. 5 d. 6
7. The length of tangent from an external point on a circle is……
   a. **always greater than the radius of the circle.**
   b. **always less than the radius of the circle.**
   c. **may or may not be greater than the radius of circle**
   d. **None of these.**
8. If A cone is cut through a plane parallel to its base and then the cone that is for medon one side of that plane is removed. The new part that is left over on the other side of the plane is called…
   b. Frustum of a cone b. Cone c. cylinder d. sphere

Answer the following questions 1 x8 =8

9. Find the nature of roots of quadratic equation $5x^2 - 4x + 5 = 0$.
10. A right circular cylinder of radius r cm and height h cm (h>2r) just encloses, what will be a sphere of diameter?.
11. Write the general form of arithmetic progression.
12. How many solution(s) does the pair of equations $x + 2y - 5=0$ & $-3x - 6y + 15=0$ have?
13. Write the distance formula.
14. If tan A= 1/3, What will be the cot A?.
15. State Pythagoras theorem.
16. How many tangents can be drawn to a circle from a point on the same circle?

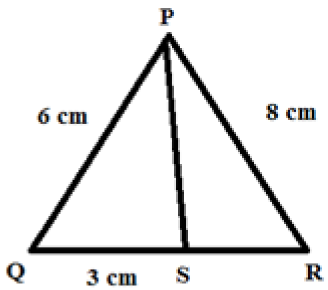
Answer the questions 2x8=16

17. Which term of the progression 4, 9, 14, 19, … is 109?

    OR

    Which term of the progression 20, 192, 183, 17 … is the first negative term?
18. Find Find the value of x: x+y=5 & 2x+3y=11.

19. If $\tan 4\theta = \cot(\theta - 10°)$, where 4θ and $(\theta - 10°)$ are acute angles then find value of θ.
20. If the distance between the points (4, p) and (1, 0) is 5, then find the value of p.
21. In triangle PQR, if PQ = 6 cm, PR = 8 cm, QS = 3 cm, and PS is the bisector of angle QPR, what is the length of SR?



22. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.
23. Find the number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm.
24. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [Take $\pi = 22/7$]

Solve the following problems 3x9=27

25. Solve for x:
    $36x^2 - 12ax + (a^2 - b^2) = 0$

OR

Find the value of p for which the roots of the equation $px(x - 2) + 6 = 0$, are equal.

26. Find the ratio in which the point $P(3/4, 5/12)$ divides the line segment joining the points $A(1/2, 3/2)$ and B(2, -5).

OR

Find the ratio in which y-axis divides the line segment joining the points A(5, -6), and B(-1, -4). Also find the coordinates of the point of division.

27. A shopkeeper buys some books for 80. If he had bought 4 more books for the same amount, each book would have cost ₹1 less. Find the number of books he bought.
28. A two digit number is seven times the sum of its digits. The number formed by reversing the digits is 18 less than the given number. Find the given number.
29. Prove that "the tangent at any point of a circle is perpendicular to the radius through the point of contact".
30. Find the value of:

$$\left[\frac{\tan 20°}{\operatorname{cosec} 70°}\right]^2 + \left[\frac{\cot 20°}{\sec 70°}\right]^2 + 2\tan 75° \tan 45° \tan 15°$$

31. Which term of the AP: 3, 8, 13, 18, … , is 78?
32. If the mean of the following distribution is 50, find the value of p:.

Class	Frequency
0–20	17
20–40	p
40–60	32
60–80	24
80–100	19

OR

Weekly income of 600 families is given below:

Income in (₹)	No. of families
0–1000	250
1000–2000	190
2000–3000	100
3000–4000	40
4000–5000	15
5000–6000	5

Find the median.

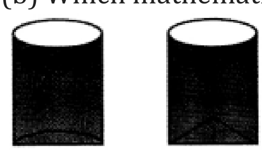
33. The lengths of leaves of a plant are measured correct to the nearest mm and the data obtained is represented as the following frequency distribution:

Length (in mm)	No. of leaves
110–115	2
115–120	6
120–125	10
125–130	13
130–135	6
135–140	3
140-145	2

Draw a 'more than type' ogive for the above data..

Solve 4x4=16

34. Draw the graphs of the equations x +2 y = 7 and 2x + 3y = 11.
35. A milkman was serving his customers using two types of mugs A and B of inner diameter 5 cm to Mug'A Mug'B' serve the customers. The height of the mugs is 10 cm.
He decided to serve the customers in 'B' type of mug.
(a) Find the volume of the mugs of both types.
(b) Which mathematical concept is used in the above problem?



OR

A solid cone of base radius 10 cm is cut into two parts through the mid-point of its height, by a plane parallel to its base. Find the ratio in the volumes of the two parts of the cone.

36. Prove that "in a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides".
37. Draw a triangle ABC with side BC = 7 cm, ∠B = 45° and ΔA = 105°. Then construct a triangle whose sides are 3/5times the corresponding sides of ΔABC.

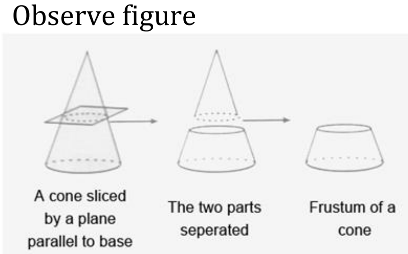
Solve 5x1=5

38. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take 3–√ = 1.732).

KEY ANSWER-4

Subject: Mathematics Subject code: 81E
Time :3 hours Max.marks:80

1. **Answer: B**
   Explanation: We have $(x - 2)^2 + 1 = 2x - 3$
   $\Rightarrow x^2 + 4 - 2 \times x \times 2 + 1 = 2x - 3$
   $\Rightarrow x^2 - 4x + 5 - 2x + 3 = 0$
   $\therefore x^2 - 6x + 8 = 0$, which is a quadratic equation.
2. Option (B).
3. Answer: c
   Explanation: We have $a_n = 3 + 4n$
   $\therefore a_{n+1} = 3 + 4(n + 1) = 7 + 4n$
   $\therefore d = a_{n+1} - a_n$
   $= (7 + 4n) - (3 + 4n)$
   $= 7 - 3$
   $= 4$
4. Option (d), we have to find distance between the points (4,3) & (0, 0).
   Using distance formula we get 5.
5. 1
6. $\Delta ABC \sim \Delta PQR$ ...[Given
   $\therefore \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \dfrac{AC}{PR}$
   $\Rightarrow \dfrac{32}{48} = \dfrac{AC}{6}$ $\Rightarrow$ **AC = 4 cm**
7. $\angle 1 = \angle 2$
   $\angle 1 + \angle 2 + 70° = 180°$
   $\angle 1 + \angle 1 = 180° - 70°$
   $2\angle 1 = 110° \Rightarrow \angle 1 = 55°$
   $\angle 1 + \angle TPQ = 90°$
   $55° + \angle TPQ = 90°$
   $\Rightarrow \angle TPQ = 90° - 55° = 35°$
8. Observe figure



9. To find the nature, let us calculate $b^2 - 4ac$
   $b^2 - 4ac = 4^2 - 4 \times 5 \times 5$
   $= 16 - 100$
   $= -84 < 0$
10. **Explanation:** The sphere is enclosed inside the cylinder, therefore the diameter of sphere is equal to the diameter of cylinder which is 2*r* cm.
11. $a_n=a+(n-1)d$.

12. there are many solutions.
13. The distance formula is d=$\sqrt{(x2-x1)2+(y2-y1)2}$.
14. If tan A=1/3, Then cot will be reverse of the tan. So answer is 3.
15. Pythagoras theorem state that "in a right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides".
16. One and only tangent can we drawn.
17. Given AP

    4, 9, 14, 19,..... 109
    Here, first term a = 4
    Common Difference d = (a2 - a1) = 9 - 4 = 5
    Last term an = 109
    We that the nth term of an AP is
    an = a + (n - 1) * d
    => 109 = 4 + (n - 1) * 5
    => 109 = 4 + 5n - 5
    => 109 = - 1 + 5n
    => 109 + 1 = 5n
    => 110 = 5n
    => 110 / 5 = n
    => 22 = n
    => n = 22
    Hence the nth term of an AP is 22.

OR

Given: a = 201
d = 192 – 201 → d = - 9
We have to find the first negative term i.e., The first term which is less than 0.
We know that,
nth term of an AP = a + (n - 1)d
→ 201 + (n - 1)(- 9) ≤ 0
→ 201 - 9n + 9 ≤ 0
→ 210 - 9n ≤ 0
→ 9n ≤ 210
→ n ≤ 23.3
→ n = 23 (Approx.)
Hence, the 23rd term is the first negative term in the given AP.

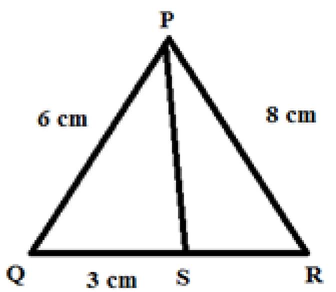
18. Consider the given equation.
x+y=5 …….. (1)
2x+3y=11……… (2)
On subtracting both equation (1) and (2), we get
x=4
Now, put the value of x in equation (1), we get
4+y=5
y=1
Hence, the value of x is 4 and y is 1

19. Given, tan 4θ=cot(θ−10°)
    This can be written as
    cot(90°− 4θ)=cot(θ−10°) ——(i)
    (∵ Tan θ = Cot(90°− θ))
    Hence, from (i) we have
    ⇒90°− 4θ= θ−10°

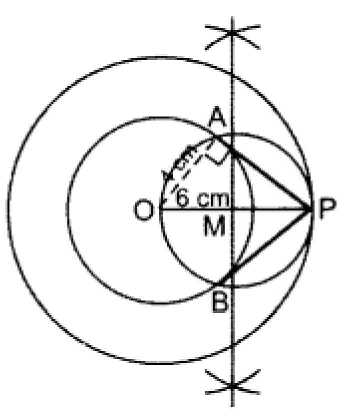
$\Rightarrow 5\theta = 100°$
$\Rightarrow \theta = 20°$

20. According to question:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$$
$$\Rightarrow \sqrt{(4-1)^2 + (p-0)^2} = 5$$
$$\Rightarrow \sqrt{9 + p^2} = 5$$
$$\Rightarrow 9 + p^2 = 25$$
$$\Rightarrow 9 + p^2 = 25$$
$$\Rightarrow p^2 = 16$$
$$\Rightarrow p = \pm 4$$



21.
Since, PS is the angle bisector of angle QPR
So, by angle bisector theorem,
QS/SR = PQ/PR
⇒ 3/SR = 6/8
⇒ SR = (3 X 8)/6 cm = 4 cm



22.

23. Solution:
Number of solid spheres

$$= \frac{\text{Volume of cylinder}}{\text{Volume of one solid sphere}}$$

$$= \left(\frac{\pi(2)^2(45)}{\frac{4}{3}\pi(3)^3}\right) \quad \ldots \left[\begin{array}{l}\because \textbf{Volume of Cylinder} = \pi r^2 h \\ \textbf{Volume of Sphere} = \frac{4}{3}\pi r^3\end{array}\right.$$

$$= \frac{2 \times 2 \times 45}{\frac{4}{3} \times 3 \times 3 \times 3} = 5$$

24. Solution:

Total surface area of hemisphere = 462 cm^2

$$3\pi r^2 = 462$$

$$3 \times \frac{22}{7} \times r^2 = 462$$

$$r^2 = \frac{462 \times 7}{3 \times 22} = 49$$

$r = +7$cm ...[**Radius cannot be negative**

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{2156}{3} = \mathbf{718.\bar{6}\ cm^3}$$

25. Solve for x:

$36x^2 - 12ax + (a^2 - b^2) = 0$ (2011OD)

Solution:

We have, $36x^2 - 12ax + (a^2 - b^2) = 0$

$\Rightarrow (36x^2 - 12ax + a^2) - b^2 = 0$

$\Rightarrow [(6x)^2 - 2(6x)(a) + (a)^2] - b^2 = 0$

$\Rightarrow (6x - a)^2 - (b)^2 = 0$ ...[$\because x^2 - 2xy + y^2 = (x - y)^2$

$\Rightarrow (6x - a + b)(6x - a - b) = 0$ „[$\because x^2 - y^2 = (x + y)(x - y)$

$\Rightarrow 6x - a + b = 0$ or $6x - a - b = 0$

$\Rightarrow 6x = a - b$ or $6x = a + b$

$\Rightarrow x = a-b/6$ or $a+b/6$.

OR

Solution:

We have , $px(x - 2) + 6 = 0$

$px^2 - 2px + 6 = 0, p \neq 0$

Two equal roots ...[Given

$b^2 - 4ac = 0$ ....[$a = p, b = -2p, c = 6$

$\therefore (-2p)^2 - 4(p)(6) = 0$

$4p^2 - 24p = 0 \Rightarrow 4p(p - 6) = 0$

$4p = 0$ or $p - 6 = 0$

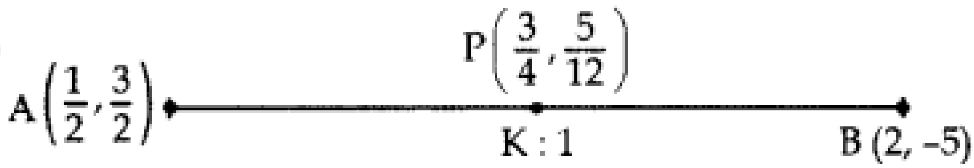
$p = 0$ (rejected) or $p = 6$

Since p cannot be equal to 0.

...[Standard form of a quad. eq. $ax^2 + bx + c = 0, a \neq 0$

$\therefore P = 6$

26. Solution:



Let P divide AB in the ratio of K : 1.

Applying section formula,

$$x = \frac{mx_2 + nx_1}{m+n} \Rightarrow \frac{3}{4} = \frac{K(2) + 1\left(\frac{1}{2}\right)}{K+1}$$

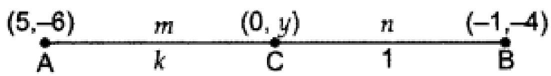
$$\frac{2K + \frac{1}{2}}{K+1} = \frac{3}{4} \Rightarrow 8K + 2 = 3K + 3$$

$$\Rightarrow 5K = 1 \Rightarrow K = \frac{1}{5}$$

∴ Required ratio = 1 : 5

OR

Solution:



Let AC: CB = m : n = k : 1.

$$\text{Coordinates of C} = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right) \quad \ldots(i)$$

Point C lies on y-axis ∴ $\frac{-m+5}{m+1} = 0$

$\Rightarrow -k + 5 = 0$ or $\boldsymbol{k = 5}$

∴ **Required ratio** = $k : 1$ = **5 : 1**

From (i), required point C,

$$\Rightarrow \left(\frac{-5+5}{5+1}, \frac{-20-6}{5+1}\right) = \left(0, \frac{-26}{6}\right) = \left(\mathbf{0}, \frac{\mathbf{-13}}{\mathbf{3}}\right)$$

27. Solution:

Let the number of books he bought = x

Increased number of books he had bought = x +4

Total amount = ₹80

According to the problem,

$$\frac{80}{x} - \frac{80}{x+4} = 1$$

$$\Rightarrow \frac{80(x+4-x)}{x(x+4)} = 1$$

⇒ x(x + 4) = 320

⇒ x^2 + 4x – 320 = 0

⇒ x^2 + 20x – 16x – 320 = 0

⇒ x(x + 20) – 16(x + 20) = 0

⇒ (x + 20) (x – 16) = 0

⇒ x + 20 = 0 or x – 16 = 0

⇒ x = -20 … (neglected) or x = 16

∴ Number of books he bought = 16.

28. Solution:

Let unit's place digit be x and ten's place digit bey.

Then original number = x + 10y

and reversed number = 10x + y

According to the Question,

x + 10y = 7(x + y)

x + 10y = 7x + 7y

⇒ 10y – 7y = 7x – x

$\Rightarrow 3y = 6x \Rightarrow y = 2x$ ...(i)
$(x + 10y) - (10x + y) = 18$
$x + 10y - 10x - y = 18$
$\Rightarrow 9y - 9x = 180$
$\Rightarrow y - x = 2$ ...[Dividing by 9
$\Rightarrow 2x - x = 2$ ...[From (i)
$\therefore x = 2$
Putting the value of 'x' in (i), we get $y = 2(2) = 4$
$\therefore$ Required number $= x + 10y$
$= 2 + 10(4) = 42$.

29. Given: a circle with tangent XY at point of contact P.
To Prove: OP⊥XY
Proof: Let Q be a point on XY connect OQ
Suppose it touches the circle at R
Hence,
OQ>OR
OQ>OP OP=OR (radius)
Same will be the case with all other points on the circle
Hence,
We get OP is the smallest line that connects XY.

30. Solution:

$$\left(\frac{\tan 20^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot 20^\circ}{\sec 70^\circ}\right)^2 + 2\tan 75^\circ \tan 45^\circ \tan 15^\circ$$

$$= \left(\frac{\tan(90^\circ - 70^\circ)}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\cot(90^\circ - 70^\circ)}{\sec 70^\circ}\right)^2 + 2\tan(90^\circ - 15^\circ) \cdot 1 \cdot \tan 15^\circ$$

$$= \left(\frac{\cot 70^\circ}{\operatorname{cosec} 70^\circ}\right)^2 + \left(\frac{\tan 70^\circ}{\sec 70^\circ}\right)^2 + 2\cot 15^\circ \cdot \frac{1}{\cot 15^\circ}$$

$$\ldots\left[\begin{array}{l} \because \tan(90^\circ - A) = \cot A \\ \cot(90^\circ - A) = \tan A \\ \tan A = \dfrac{A}{\cot A} \end{array}\right.$$

$$= \left(\frac{\frac{\cos 70^\circ}{\sin 70^\circ}}{\frac{1}{\sin 70^\circ}}\right)^2 + \left(\frac{\frac{\sin 70^\circ}{\cos 70^\circ}}{\frac{1}{\cos 70^\circ}}\right)^2 + 2$$

$$= \cos^2 70^\circ + \sin^2 70^\circ + 2$$

$$= 1 + 2 = 3 \qquad \ldots[\because \cos^2 A + \sin^2 A = 1$$

31. Solution:
Let a_n be the required term and we have given AP
3, 8, 13, 18, ...
Here, $a = 3$, $d = 8 - 3 = 5$ and $a_n = 78$
Now, $a_n = a + (n - 1)d$
$\Rightarrow 78 = 3 + (n - 1)\ 5$
$\Rightarrow 78 - 3 = (n - 1) \times 5$
$\Rightarrow 75 = (n - 1) \times 5$
$\Rightarrow 755 = n - 1$
$\Rightarrow 15 = n - 1$

$\Rightarrow$ n = 15 + 1 = 16

Hence, 16th term of given AP is 78..

32. Solution:

Class	Frequency (f_i)	X_i	f_iX_i
0–20	17	10	170
20–40	p	30	$30p$
40–60	32	50	1600
60–80	24	70	1680
80–100	19	90	1710
	$\Sigma f_i = 92+p$		$\Sigma f_iX_i = 5160 + 30p$

$\therefore$ Mean $= \dfrac{\Sigma f_iX_i}{\Sigma f_i}$

$$50 = \frac{5160 + 30p}{92 + p}$$

$\Rightarrow$ $4600 + 50p = 5160 + 30p$

$\Rightarrow$ $50p - 30p = 5160 - 4600$

$\Rightarrow$ $20p = 560$

$\Rightarrow$ $p = \dfrac{560}{20} = 28$ $\quad\therefore\quad$ $\mathbf{p = 28}$

OR

Solution:

Income (in ₹)	f_i	c.f.
0–1000	250	250
1000–2000	$\boxed{190}$	$\boxed{440}$
2000–3000	100	540
3000–4000	40	580
4000–5000	15	595
5000–6000	5	600
	$n = 600$	

$$\frac{n}{2} = \frac{600}{2} = 300$$

$\therefore$ Median class is 1000 – 2000

$$\textbf{Median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

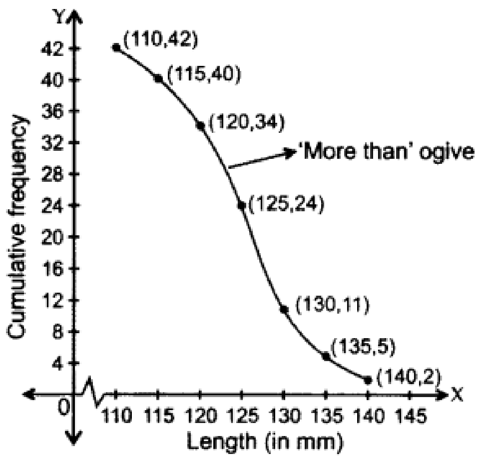
$$= 1000 + \left(\frac{300 - 250}{190} \times 1000\right)$$

$$= 1000 + \frac{50,000}{190} = 1000 + 263.16$$

$$= \mathbf{1263.16\ (approx.)}$$

33. Solution:

Length (in mm)	*No. of leaves (f)*	*(c.f.)*
More than 110	2	42
More than 115	6	40
More than 120	10	34
More than 125	13	24
More than 130	6	11
More than 135	3	5
More than 140	2	2

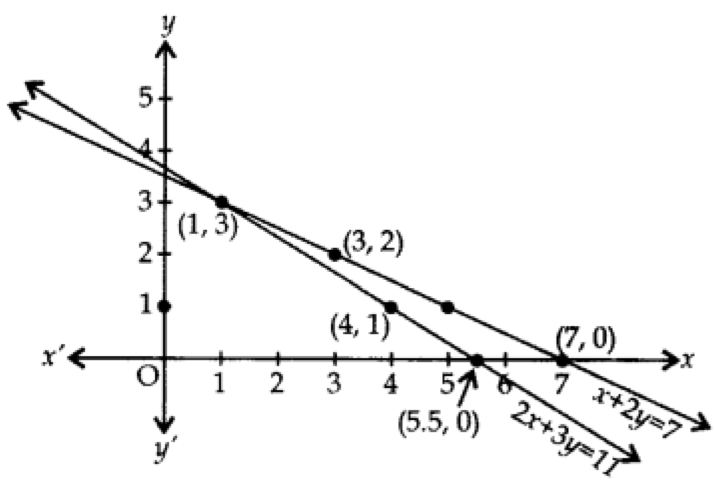


34.

$2x + 3y = 11,$	$1x + 2y = 7$
$2x = 11 - 3y$	$x = 7 - 2y$
$x = \dfrac{11 - 3y}{2}$	

x	5.5	4	1
y	0	1	3

x	7	3	1
y	0	2	3



So the two lines are intersects at (1, 3).
Hence the solutions are 1 & 3.

35. Solution:

(a) Let the radius of cylinder, hemi-sphere and cone be r cm
Let the height of cylinder and cone h_1 and h_2 respectively.

$$\text{Volume of mug of type 'A'} = \pi r^2 h_1 - \frac{2}{3}\pi r^3$$

$$= 3.14 \times 2.5 \times 2.5 \times 10 - \frac{2}{3} \times 3.14 \times (2.5)^3$$

$$= 196.25 - 32.71 = \mathbf{163.54\ cm^3}$$

$$\text{Volume of mug of type 'B'} = \pi r^2 h_1 - \frac{1}{3}\pi r^2 h_2$$

$$= 196.25 - \frac{1}{3} \times 3.14 \times 2.5 \times 2.5 \times 1.5$$

$$= 196.25 - 9.81 = \mathbf{186.44\ cm^3}$$

(*b*) Volume of solid figures (Mensuration).

OR

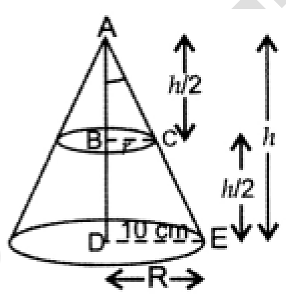
Solution:
Let BC = r cm and
DE = R = 10 cm
B and C are the mid-points of AD and AE respectively. ...[Given

$$BC\ (r) = \frac{1}{2}DE$$

...[Mid-point Theorem

$$BC = \frac{1}{2}(10) = 5\text{ cm}$$

$$= \frac{\text{Vol. of cone}}{\text{Vol. of frustum of a cone}} = \frac{\frac{1}{3}\pi r^2(AB)}{\frac{1}{3}\pi(BD)[R^2 + r^2 + Rr]}$$



$$= \frac{(5)^2(h/2)}{(h/2)[10^2 + 5^2 + 10 \times 5]}$$

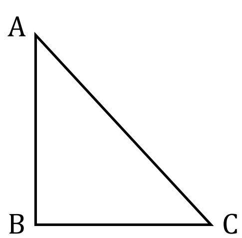
$$= \frac{25}{100 + 25 + 50} = \frac{25}{175} = \frac{1}{7}$$

$\therefore$ **Required ratio = 1 : 7**

$\Rightarrow$ **Volume of small cone**

= $\frac{1}{7}$ (Volume of frustum of a cone)

36. In a right angles triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. 1



Data : In ΔABC, ∠ABC = 90°

To Prove : $AB^2 + BC^2 = CA^2$ 1

Construction : Draw BD ⊥ AC.

Proof : **Statement** **Reason**

Compare ΔABC and ΔADB,

∠ABC = ∠ADB = 90° (Q Data and construction)

∠BAD is common.

∴ ΔABC ~ ΔADB (Q Equiangular triangles)

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AB}$ (Q A A similarity criteria)

∴ $AB^2 = AC.AD$ ...... (1) 1

Compare ΔABC and ΔBDC,

∠ABC =∠BDC =90° (Q Data and construction)

∠ACB is common

∴ ΔABC ~ ΔBDC (Q Equiangular Triangles)

$\Rightarrow \frac{BC}{DC} = \frac{AC}{BC} \Rightarrow =$ (Q AA similarity criteria)

$BC^2 = AC.DC$ .....(2) 1

By adding (1) and (2) we get

$AB^2 + BC^2 = (AC.\ AD) + (AC.\ DC)$

$AB^2 + BC^2 = AC\ (AD + DC)$

$AB^2 + BC^2 = AC.\ AC = AC^2$ [Q AD + DC = AC]

∴ $AB^2 + BC^2 = AC^2$

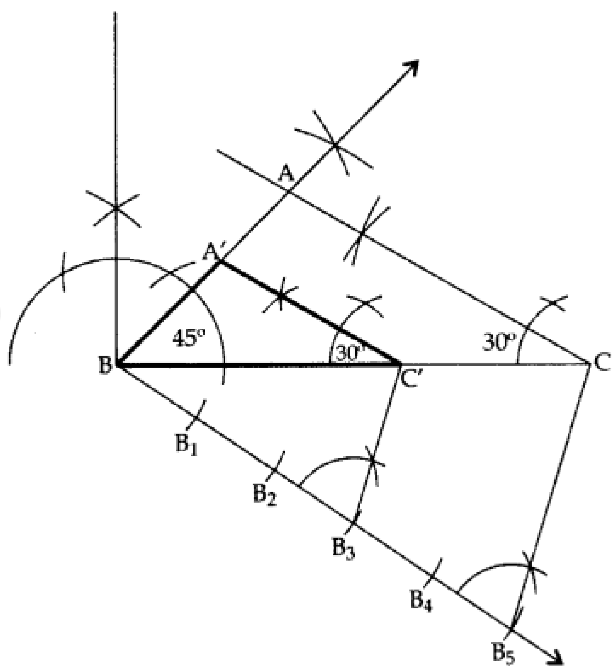
37. Solution:

In ΔABC, ∠A + ∠B + ∠C = 180° … [angle sum property of a Δ

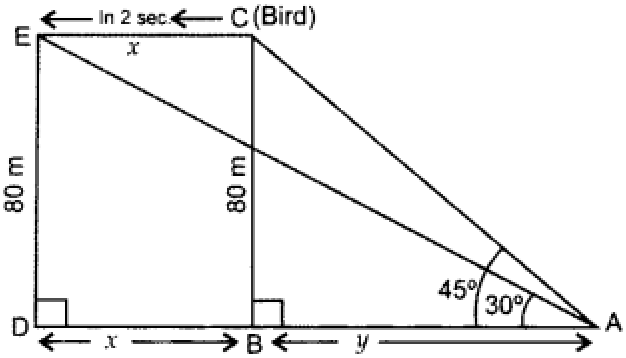
105° + 45° + C = 180°

∠C = 180° – 105° – 45o = 30°

BC = 7 cm



38. Solution:



Let BC be the tree

In rt. ΔABC, tan 45° = BCAB.

$$\Rightarrow 1 = \frac{80}{y} \qquad \Rightarrow \qquad y = 80 \text{ m} \qquad \ldots(i)$$

In rt. ΔADE, $\tan 30^\circ = \frac{DE}{AD}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$\Rightarrow \quad x + y = 80\sqrt{3}$$

$$\Rightarrow \quad x + 80 = 80\sqrt{3} \qquad \ldots\textbf{[From (}i\textbf{)}$$

$$\Rightarrow \quad x = 80\sqrt{3} - 80$$

$$\Rightarrow \quad x = 80\left(\sqrt{3} - 1\right)$$

$$\Rightarrow \quad x = 80(1.732 - 1) \qquad \ldots[\because \sqrt{3} = 1.732$$

$$\Rightarrow \quad x = 80(0.732)$$

$$\therefore \quad \textbf{CE, } x = \textbf{58.56 m}$$

$$\text{Hence, speed of bird} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{\text{CE}}{\text{Time}} = \frac{58.56 \text{ m}}{2 \text{ sec.}}$$

$$= \textbf{29.28 m/sec.}$$

As per reduced syllabus-2021
MODEL QUESTION PAPER-5

Subject: Mathematics Subject code: 81E
Time : 3 hours Max.marks: 80

Choose the correct answer given below ------- 1x8=8

1. Which of the following is a quadratic equation?
   a. $x^2 + 2x + 1 = (4 - x)^2 + 3$ **b. $-2x^2 = (5 - x)[2x - 25]$**
   c. $(k + 1)x^2 + 32x = 7$, where $k = -1$ **d. $x^3 - x^2 = (x - 1)^3$**
2. Graphically, the pair of equations $7x - y = 5$; $21x - 3y = 10$ represents two lines which are
   (a) intersecting at one point (b) parallel (c) intersecting at two points (d) coincident
3. If x and y are complementary angles, then
   (a) sin x = sin y (b) tan x = tan y (c) cos x = cos y (d) sec x = cosec y
4. If the height of a tower and the distance of the point of observation from its foot,both, are increased by 10%, then the angle of elevation of its top…..
   c. increased b. decreased c. remains same d. have no relation
5. The distance between the point P(1, 4) and Q(4, 0) is
   c. $\sqrt{3}$ b.4 c. 5 d. 6
6. Cumulative frequency curve is also called
   a. histogram b. ogive b. bar graph d. median
7. If angle between two radii of a circle is 130º, then the angle between the tangents at the ends of the radii is: ……
   e. 90º b. 60º c. 50º d. 120º
8. If A right circular cylinder of radius r cm and height h cm (h>2r) just encloses a sphere of diameter…
   c. r cm b. 2r cm c. h cm d. 2h cm

Answer the following questions 1 x8 =8

9. Find the nature of roots of quadratic equation $x^2 + x + 3 = 0$.
10. Write the formula to find the volume of cube.
11. If $a_n = a - 4$, then what will be the common difference?.
12. How many solutions does the pair of equations y = 0 and y = -5 have?
13. Write the section formula.
14. If tan A= 1/3, What will be the cot A?.
15. State Basic proportionality theorem.
16. How many tangents can be drawn to a circle from a point on the same circle?

Answer the questions 2x8=16

17. If $a_n = 5 - 11n$, find the common difference.
    OR
    For what value of p are $2p + 1$, 13, $5p - 3$, three consecutive terms of AP?
18. Solve: $x-2y=1$ & $x+2y=9$.
19. If $\sin(x - 20)° = \cos(3x - 10)°$, then find the value of x.
20. Find a relation between x and y such that the point P(x, y) is equidistant from the points A (2, 5) and B (-3, 7).
21. If the mode of a distribution is 8 and its mean is also 8, then find median.
22. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

23. Find the number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm.
24. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. [Take π = 22/7]

Solve the following problems 3x9=27

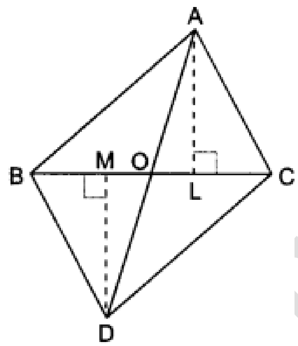
25. Using quadratic formula solve the following quadratic equation:
    $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

    OR

    If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.
26. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

    OR

    Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
27. If tan (A +B) = √3 and tan (A – B) = 13√; 0° < A + B ≤ 90°; A > B, find A and B.
28. Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 30° and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 15^{o}. (Use 15^{0}=0.27).
29. Prove that “the length of the tangent drawn from an external point to the circle are equal”.
30. In the given figure, ΔABC and ADBC are on the same base BC. If AD intersects BC at 0. Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$



31. If (1, 2), (4, y), (x, 6) and (3,5) are the vertices of a parallelogram taken in order, find x and y.?
32. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

    OR

    ABC is an equilateral triangle of side 2a. Find each of its altitudes.
33. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are 53 times the corresponding sides of the given triangle.

Solve 4x4=16

34. Draw the graphs of the equations x – y = 4 and x + y = 10.
35. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP.

    OR

    If s, denotes the sum of the first n terms of an AP, prove that $s_{30} = 3(s_{20} - s_{10})$.

36. A solid iron pole consists of a cylinder of height 220 cm and base diameter r = 8 cm 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm of iron has approximately 8 g mass. 60 cm (Use π = 3.14)
37. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	100–120	120–140	140–160	160–180	180–200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Solve 5x1=5

38. Prove that "Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides".

KEY ANSWER-5

Subject: Mathematics Subject code: 81E
Time :3 hours Max.marks:80

1. **Answer: d**

Answer: (d) $x^3 - x^2 = (x - 1)^3$

2. **Answer : b**
3. **Answer : d**
4. **Answer: (C)**

   Explanation: Since
   $\tan \theta = h/x$
   Where h is height and x is distance from tower,
   If both are increased by 10%, then the angle will remain unchanged.
5. **Answer: c**

   Reason: The required distance = √ (4−1)2+(0−4)2=√9+16 –> √25−−>5.
6. **Answer: (b) ogive.**
7. **Explanation:** If the angle between two radii of a circle is 130º, then the angle between tangents is 180º – 130º **=** 50º. (By the properties of circles and tangents)
8. **Option b**

   Because the sphere is enclosed inside the cylinder, therefore the diameter of sphere is equal to the diameter of cylinder which is 2*r* cm.
9. To find the nature, let us calculate $b^2 - 4ac$

$b^2 - 4ac = 1^2 - 4 \times 1 \times 3$
$= 1 - 12 = -11 < 0$, therefore it is not real.

10. Volume of cube= side x sidexside= axaxa= a^3.
11. Since a_n=a-4, then a_1=1-4=-3

    a_2=2-4=-2 therefore d=a_2-a_1=-2-(-3)= 1
12. Solution:

    y = 0 and y = -5 are Parallel lines, hence no solution.
13. Section formula is (x, y) =$\frac{mx2+nx1}{m+n}$. $\frac{my2+my1}{m+n}$
14. If tan A=1/3, then cot A=3/1.
15. It states that "If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.".
16. Only one.
17. Solution:

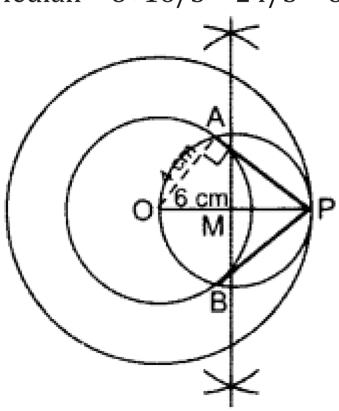
    We have $a_n = 5 - 11n$
    Let d be the common difference
    $d = a_{n+1} - a_n$
    $= 5 - 11(n + 1) - (5 - 11n)$
    $= 5 - 11n - 11 - 5 + 11n = -11$.

    OR

    since 20 + 1, 13, 5p – 3 are in AP.
    ∴ second term – First term = Third term – second term
    $\Rightarrow 13 - (2p + 1) = 5p - 3 - 13$
    $\Rightarrow 13 - 2p - 1 = 5p - 16$
    $\Rightarrow 12 - 2p = 5p - 16$
    $\Rightarrow -7p = -28$
    $\Rightarrow p = 4$
18. Consider the given equation.

x-2y=1 .... (1)
x+2y=9 .... (2)
On adding both equation (1) and (2), we get
2x=10
x=5
Now, put the value of x in equation (1), we get
5-2y=1
2y=4
y=2
Hence, the value of x is 5 and y is 2

19. Solution:
    $\sin (x - 20)° = \cos (3x - 10)°$
    $\Rightarrow \cos [90° - (x - 20)°] = \cos (3x - 10)°$
    By comparing the coefficient
    $90° - x° + 20° = 3x° - 10° = 110° + 10° = 3x° + x°$
    $120° = 4x°$
    $\Rightarrow 120∘4 = 30°$
20. Solution:
    Let P (x, y) be equidistant from the points A (2, 5) and B (-3, 7).
    ∴ AP = BP ...[Given
    $AP^2 = BP^2$ ...[Squaring both sides
    $(x - 2)^2 + (y - 5)^2 = (x + 3)^2 + (y - 7)^2$
    $\Rightarrow x^2 - 4x + 4 + y^2 - 10y + 25$
    $\Rightarrow x^2 + 6x + 9 + y^2 - 14y + 49$
    $\Rightarrow -4x - 10y - 6x + 14y = 9 + 49 - 4 - 25$
    $\Rightarrow -10x + 4y = 29$
    ∴ $10x + 29 = 4y$ is the required relation.
21. Solution:
    Mode = 8; Mean = 8; Median = ?
    Relation among mean, median and mode is
    3 median = mode + 2 mean
    3 × median = 8 + 2 × 8
    Median = 8+16/3 = 24/3 = 8.



22.
23. Solid sphere volume $\mathbf{v=34\pi r^3}$
$\mathbf{rv=34\times722\times33cm^3}$
$\mathbf{rv=4\times\pi\times9=36\pi cm3}$
Volume of cylinder $\mathbf{=\pi R^2h=\pi\times(24)2\times45cm}$
or $\mathbf{v=180\pi cm^3}$

n= no of spheres =**vv=36πcm3180πcm3**
⇒n=5

24. Total surface area of the hemisphere =462cm2
    Total surface area of the hemisphere =2πr2
    ⇒462=3πr2
    ⇒r=7 cm
    Volume of hemisphere =32πr3
    V=32×722×7^3
    V=718.67cm^3
25. Solution:
    We have, $p^2x^2 + (p^2 – q^2) x – q^2 = 0$
    Comparing this equation with $ax^2 + bx + c = 0$, we have
    $a = p^2$, $b = p^2 – q^2$ and $c = – q^2$
    ∴ $D = b^2 – 4ac$
    ⇒ $(p^2 – q)^2 – 4 \times p^2 \times (-q^2)$
    ⇒ $(p^2 – q^2)^2 + 4p^2q^2$
    ⇒ $(p^2 + q^3)^2 > 0$
    So, the given equation has real roots given by

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

and $$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

OR

Solution:
Since the equation $(a – b)x^2 + (b – c) x + (c – a) = 0$ has equal roots, therefore discriminant
$D = (b – c)^2 – 4(a – b) (c – a) = 0$
⇒ $b^2 + c^2 – 2bc – 4(ac – a^2 – bc + ab)$
⇒ $b^2 + c^2 – 2bc – 4ac + 4a^2 + 4bc – 4ab = 0$
⇒ $4a^2 + b^2 + c^2 – 4ab + 2bc – 4ac = 0$
⇒ $(2a)^2 + (- b)^2 + (-c)^2 + 2(2a) (-b) + 2(-b) (-c) + 2(-c) 2a = 0$
⇒ $(2a – b – c)^2 = 0$
⇒ $2a – b – c = 0$
⇒ $2a = b + c$. Hence Proved.

26. Solution:
    Let the first term be ‘a’ and common difference be ‘d’.
    Given, $a = 5$, $T_n = 45$, $s_n = 400$.
    Tn = a + (m – 1)d
    ⇒ 45 = 5 + (m – 1)d
    ⇒ (n – 1) d = 40 ....(i)
    s_n = n2 (a + T_n)
    ⇒ 400 = n2 (5 + 45)
    ⇒ n = 2 × 8 = 16 substituting the value of n in (i)
    ⇒ (16 – 1)d = 40
    ⇒ d = 4015 = 83

OR

Solution:
Let the first term be a and common difference be d.
Now, we have

$$a_{11} = 38 \Rightarrow a + (11 - 1)d = 38$$
$$\Rightarrow a + 10d = 38 \quad ...(i)$$
and $$a_{16} = 73 \Rightarrow a + (16 - 1)d = 73$$
$$\Rightarrow a + 15d = 73 \quad ...(ii)$$

Now subtracting (ii) from (i), we have

$$\begin{array}{r} \text{Now,}\quad a + 10d = 38 \\ \underline{_{-}a \underset{-}{+} 15d = \underset{-}{73}} \\ -5d = -35 \end{array} \quad \text{or} \quad 5d = 35$$

$$\therefore \quad d = \frac{35}{5} = 7$$

Putting the value of d in equation (i), we have
$a + 10 \times 7 = 38$
$\Rightarrow a + 70 = 38$
$\Rightarrow a = 38 - 70$
$\Rightarrow a = -32$
We have, $a = -32$ and $d = 7$
Therefore, $a_{31} = a + (31 - 1)d$
$\Rightarrow a_{31} = a + 30d$
$\Rightarrow (-32) + 30 \times 7$
$\Rightarrow -32 + 210$
$= a_{31} = 178$

27. Solution:
We have, $\tan(A + B) = \sqrt{3}$
$\Rightarrow \tan(A + B) = \tan 60°$
$\therefore A + B = 60°$ ...(i)
Again, $\tan(A - B) = 13\sqrt{}$
$\therefore A - B = 30°$ ... (ii)
Adding (i) and (ii), we have
$2A = 90°$
$\Rightarrow A = 45°$
Putting the value of A in (i), we have
$45° + B = 60°$
$\therefore B = 60° - 45o = 15°$
Hence, $A = 45°$ and $B = 15°$

28. Solution:
Let AB be the mountain of height h kilo metres. Let C be a point at a distance of x km, from the base of the mountain such that the angle of elevation of the top at C is 30°. Let D be a point at a distance of 10 km from C such that angle of elevation at D is of 15°.
In MBC (Fig. 11.22), we have

In ΔABC, $\tan 60° = \frac{AB}{BC}$ or $\sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad x\sqrt{3} = h \quad \ldots(i)$

In ΔABD, $\tan 30° = \frac{AB}{BD}$

i.e., $\quad \frac{1}{\sqrt{3}} = \frac{h}{x+40} \quad \ldots(ii)$

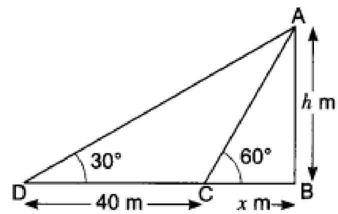
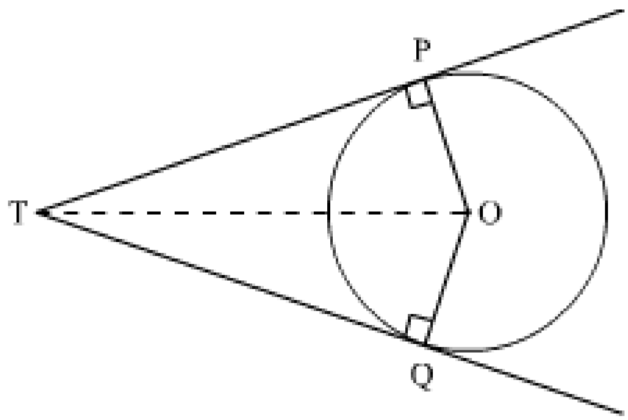


Fig. 11.23

Substituting x = √3h in equation (i), we get
⇒ 0.27 (√3h + 10) = h
= 0.27 × √3h + 0.27 × 10 = h
⇒ 2.7 = h – 0.27 × √3h
⇒ 27 = h (1 – 0.27 × √3)
⇒ 27 = h (1 – 0.46)
⇒ h = 2.70.54 = 5
Hence, the height of the mountain is 5 km.



29.
Given:
PT and TQ are two tangents drawn from an external point T to the circle C(O,r).
To prove: PT=TQ
Construction: Join OT.
Proof:
We know that, a tangent to circle is perpendicular to the radius through the point of contact.
Therefore, ∠OPT=∠OQT=90o
In ΔOPT and ΔOQT,
OT=OT
Radius of the circle =OP=OQ
∠OPT=∠OQT=900
Therefore, ΔOPT≅ΔOQT (RHS congruence criterion)
Therefore, PT=TQ
So, the length of the tangents drawn from an external point to a circle are equal.

30. Solution:

Given: ΔABC and ΔDBC are on the same base BC and AD intersects BC at O.

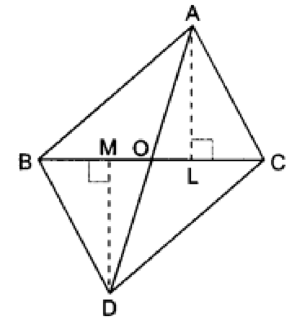


Fig. 7.24

To Prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw $AL \perp BC$ and $DM \perp BC$

Proof: In ΔALO and ΔDMO, we have

$\angle ALO = \angle DMO = 90°$ and

$\angle AOL = \angle DOM$ (Vertically opposite angles)

$\therefore \quad \Delta ALO \sim \Delta DMO$ (By AA-Similarity)

$\Rightarrow \quad \frac{AL}{DM} = \frac{AO}{DO}$ ....(*i*)

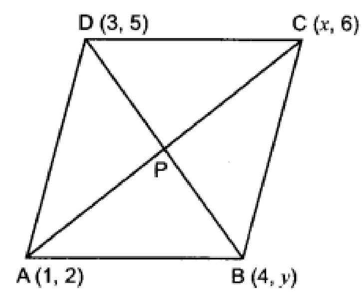
$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}BC \times AL}{\frac{1}{2}BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$ (Using (*i*))

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

31. Solution:

Let A(1, 2), B(4, y), C(x, 6) and D(3, 5) be the vertices of a parallelogram ABCD.

Since, the diagonals of a parallelogram bisect each other.



$\therefore \quad \left(\frac{x+1}{2}, \frac{6+2}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{x+1}{2} = \frac{7}{2}$

$\Rightarrow \quad x + 1 = 7$ or $x = 6$

$\Rightarrow \quad 4 = \frac{5+y}{2}$ or $5 + y = 8$ or $y = 8 - 5 = 3$

Hence, x = 6 and y = 3.

32. Solution:

Let AB be a vertical pole of length 6m and BC be its shadow and DE be tower and EF be its shadow. Join AC and DF.

Now, in ΔABC and ΔDEF, we have

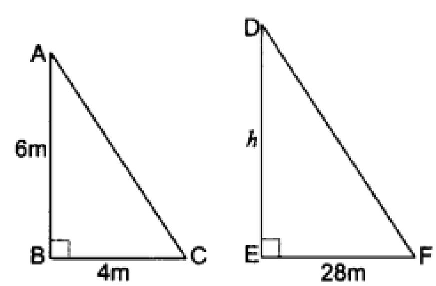


Fig. 7.12

$\angle B = \angle E = 90°$

$\angle C = \angle F$ (Angle of elevation of the Sun)

$\therefore \quad \Delta ABC \sim \Delta DEF$ (By AA criterion of similarity)

Thus, $\frac{AB}{DE} = \frac{BC}{EF}$

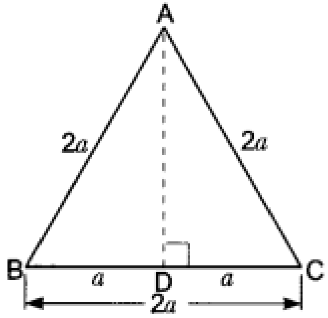
$\Rightarrow \quad \frac{6}{h} = \frac{4}{28}$ (Let $DE = h$)

$\Rightarrow \quad \frac{6}{h} = \frac{1}{7} \quad \Rightarrow h = 42$

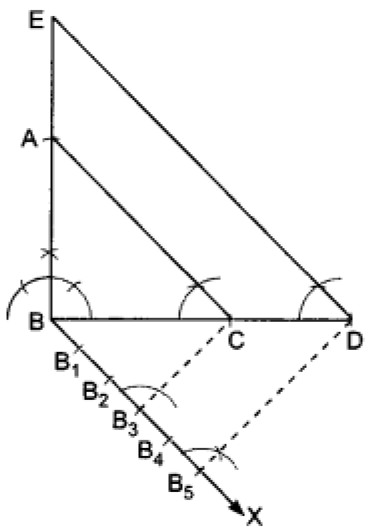
h = 42 Hence, height of tower, DE = 42m

OR

Solution:



Let ABC be an equilateral triangle of side 2a units.
We draw AD ⊥ BC. Then D is the mid-point of BC.
⇒ BC2 = 2a2 = a
Now, ABD is a right triangle right-angled at D.
⇒ $AB^2 = AD^2 + BD^2$ [By Pythagoras Theorem]
⇒ $(2a)^2 = AD^2 + a^2$
⇒ $AD^2 = 4a^2 - a^2 = 3a^2$
⇒ AD = $\sqrt{3}a$
Hence, each altitude = $\sqrt{3}a$ unit.



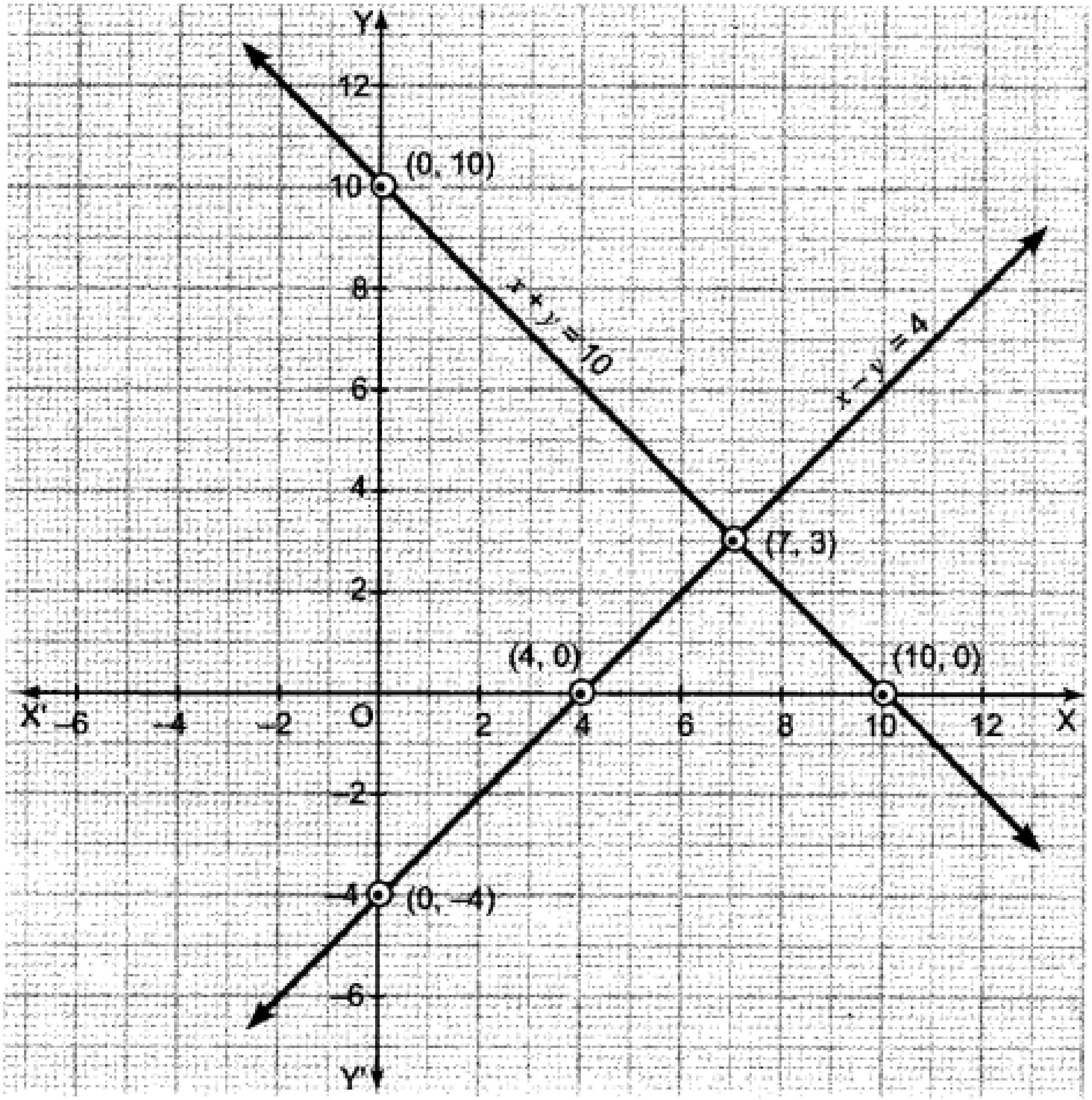
33.
34. From equation (i), we have the following table:

x	0	4	7
y	– 4	0	3

From equation (ii), we have the following table:

x	0	10	7
y	10	0	3

Plotting this, we have



Here, the two lines intersect at point (7,3) i.e., x = 7, y = 3.

35. Solution:

Let 'a' be the first term and 'd be the common difference.

nth term of AP is $a_n = a + (n-1)d$

and sum of AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of first 10 terms $= 210 = \frac{10}{2}[2a + 9d]$

$\Rightarrow \quad 42 = 2a + 9d \quad \Rightarrow \quad 2a + 9d = 42 \quad \ldots(i)$

15th term from the last $= (50 - 15 + 1)^{\text{th}} = 36^{\text{th}}$ term

$\Rightarrow \quad a_{36} = a + 35d$

Sum of last 15 terms $= 2565 = \frac{15}{2}[2a_{36} + (15-1)d]$

$\Rightarrow \quad 2565 = \frac{15}{2}[2(a + 35d) + 14d]$

$\Rightarrow \quad 2565 = 15[a + 35d + 7d]$

$\Rightarrow \quad a + 42d = 171 \quad \ldots(ii)$

$(i) - 2 \times (ii)$, we get

$9d - 84d = 42 - 342 \quad \Rightarrow \quad 75d = 300$

$\Rightarrow \quad d = \frac{300}{75} = 4$

Putting the value of d in (ii)

$42 \times 4 + a = 171 \quad \Rightarrow \quad a = 171 - 168$

$\Rightarrow \quad a = 3$

$\Rightarrow \quad a_{50} = a + 49d = 3 + 49 \times 4 = 199$

So, the AP formed is 3, 7, 11, 15, ... and 199.

OR

Solution:

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2a + 29d] \quad \Rightarrow \quad S_{30} = 30a + 435d \quad \ldots(i)$

$\Rightarrow \quad S_{20} = \frac{20}{2}[2a + 19d] \quad \Rightarrow \quad S_{20} = 20a + 190d$

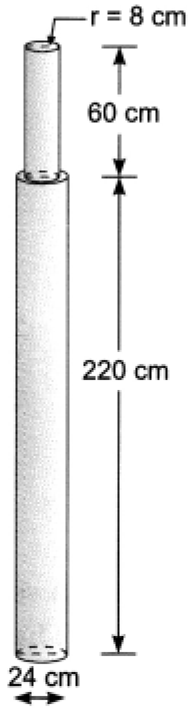
$S_{10} = \frac{10}{2}[2a + 9d] \quad \Rightarrow \quad S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30}$ [From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

36. Solution:



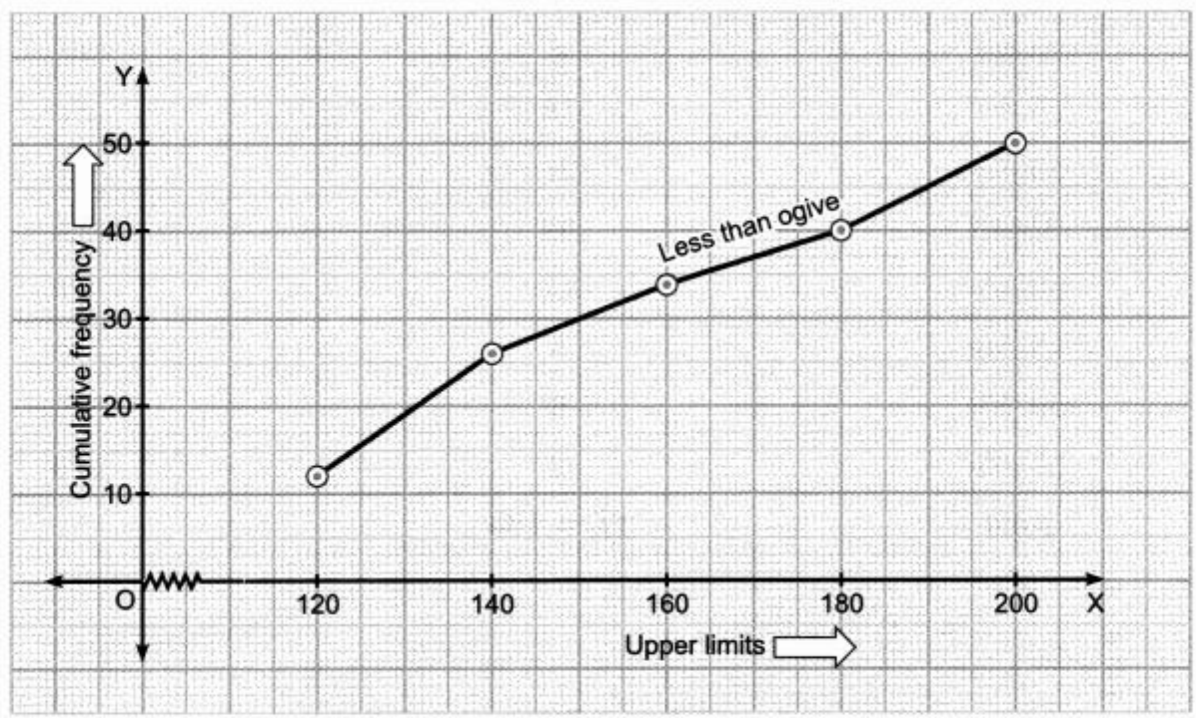
Let r_1 and h_1 be the radius and height of longer cylinder, respectively, and r_2, h_2 be the respective radius and height of smaller cylinder mounted on the longer cylinder.
Then we have,
$r_1 = 12$ cm, $h_1 = 220$ cm
$r_2 = 8$ cm, $h_2 = 60$ cm
Now, Volume of solid iron pole
= Volume of the longer cylinder + Volume of smaller cylinder
$= \pi r_1^2 h^1 + \pi r_2^2 h^2$
$= 3.14 R (12)^2 \times 220 + 3.14 R (8)^2 \times 60$
$= 3.14 \times 144 \times 220 + 3.14 \times 64 \times 60$
$= 99475.2 + 12057.6 = 111532.8$ cm^3
Hence, the mass of the pole =(111532.8 × 8) grams
= 111532.8×81000kg = 892.2624 kg.

37. Solution:

Converting given distribution to a less than type cumulative frequency distribution, we have,

Daily income (in ₹)	Cumulative frequency
Less than 120	12
Less than 140	12 + 14 = 26
Less than 160	26 + 8 = 34
Less than 180	34 + 6 = 40
Less than 200	40 + 10 = 50

Now, let us plot the points corresponding to the ordered pairs (120, 12), (140, 26), (160, 34), (180, 40), (200, 50) on a graph paper and join them by a freehand smooth curve.



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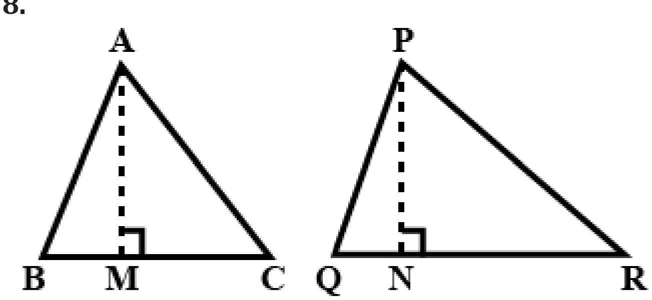

Let the two triangles be:

ΔABC and ΔPQR

Area of **ΔABC**=$\frac{1}{2}$×**BC×AM**................**(1)**

Area of **ΔPQR**=$\frac{1}{2}$×**QR×PN**........................**(2)**

Dividing (1) by (2)

$$\frac{ar(PQR)}{\text{ar(ABC)}}=\frac{\frac{1}{2}\times BC\times AM}{\frac{1}{2}\times QR\times PN}$$

$$\frac{ar(PQR)}{\text{ar(ABC)}}=\frac{BCXAM}{QRXPN}$$ ....**(1)**

In **ΔABM** and **ΔPQN**

$\angle$**B**=$\angle$**Q** (Angles of similar triangles)

$\angle$**M**=$\angle$**N (Both 90°)**

Therefore, **ΔABM~ΔPQN**

So, $\frac{AM}{AB}=\frac{PN}{PQ}$ ....**(2)**

From 1 and 2

$\frac{ar(PQR)}{ar(ABC)} = \frac{QR}{BC} \times \frac{PN}{AM}$

$\Rightarrow \frac{ar(PQR)}{ar(ABC)} = \frac{QR}{BC} \times \frac{PQ}{AB}$ ....**(3)**

$\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$ ....**($\Delta ABC \sim \Delta PQR$)**

Putting in (3) $\frac{ar(PQR)}{ar(ABC)} = \frac{PQ}{AB} \text{ X } \frac{PQ}{AB} = (\frac{PQ}{AB})^2$

$\Rightarrow \frac{ar(PQR)}{ar(ABC)} = (\frac{PQ}{AB})^2$.