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CLASS 10th

MATHEMATICS

Basic and Standard

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7. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1 2012]

We have $90 = 9 \times 10 = 9 \times 2 \times 5$

$= 2 \times 3^2 \times 5$

and $144 = 16 \times 9$

$= 2^4 \times 3^2$

$\text{HCF} = 2 \times 3^2 = 18$

$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$

CHAPTER 1

REAL NUMBERS

SUMMARY

1. **Algorithm :** An algorithm means a series of well defined step which gives a procedure for solving a type of problem.
2. **Lemma :** A lemma is a proven statement used for proving another statement.
3. **Fundamental Theorem of Arithmetic :** Every composite number can be expressed (factorised) as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.
4. If p is prime number and p divides a^2, then p divides a, where a is a positive integer.
5. If x be any rational number whose decimal expansion terminates, then we can express x in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n \times 5^m$, where n and m are non-negative integers.
6. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which terminates.
7. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^n \times 5^m$, where n and m are non-negative integers, then x has a decimal expansion which is non-terminating repeating (recurring).
8. For any two positive integers p and q, HCF $(p, q) \times$ LCM $(p, q) = p \times q$.
9. For any three positive integers p, q and r,

$$\text{LCM } (p, q, r) = \frac{p \times q \times r \times \text{HCF}(p, q, r)}{\text{HCF}(p, q) \times \text{HCF}(q, r) \times \text{HCF}(p, r)}$$

$$\text{HCF } (p, q, r) = \frac{p \times q \times r \times \text{LCM}(p, q, r)}{\text{LCM}(p, q) \times \text{LCM}(q, r) \times \text{LCM}(p, r)}$$

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The sum of exponents of prime factors in the prime-factorisation of 196 is

(a) 3 (b) 4

(c) 5 (d) 2

Ans : [Board 2020 OD Standard]

Prime factors of 196,

$$196 = 4 \times 49$$
$$= 2^2 \times 7^2$$

The sum of exponents of prime factor is $2 + 2 = 4$.

Thus (b) is correct option.

2. The total number of factors of prime number is

(a) 1 (b) 0

(c) 2 (d) 3

Ans : [Board 2020 Delhi Standard]

There are only two factors (1 and number itself) of any prime number.

Thus (c) is correct option.

3. The HCF and the LCM of 12, 21, 15 respectively are

(a) 3, 140 (b) 12, 420

(c) 3, 420 (d) 420, 3

Ans : [Board 2020 Delhi Standard]

We have
$$12 = 2 \times 2 \times 3$$
$$21 = 3 \times 7$$
$$15 = 3 \times 5$$

$$\text{HCF}(12, 21, 15) = 3$$
$$\text{LCM } (12, 21, 15) = 2 \times 2 \times 3 \times 5 \times 7 = 420$$

Thus (c) is correct option.

4. The decimal representation of $\frac{11}{2^3 \times 5}$ will

(a) terminate after 1 decimal place

(b) terminate after 2 decimal place

(c) terminate after 3 decimal places

(d) not terminate

Ans : [Board 2020 SQP Standard]

We have $$\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5^1}$$

Denominator of $\frac{11}{2^3 \times 5}$ is of the form $2^m \times 5^n$, where m, n are non- negative integers. Hence, $\frac{11}{2^3 \times 5}$ has terminating decimal expansion.

Now $$\frac{11}{2^3 \times 5} = \frac{11}{2^3 \times 5} \times \frac{5^2}{5^2}$$

$$= \frac{11 \times 5^2}{2^3 \times 5^3} = \frac{11 \times 25}{10^3} = 0.275$$

So , it will terminate after 3 decimal places.

Thus (c) is correct option.

5. The LCM of smallest two digit composite number and smallest composite number is

(a) 12 (b) 4

(c) 20 (d) 44

Ans : [Board 2020 SQP Standard]

Smallest two digit composite number is 10 and smallest composite number is 4.

$$\text{LCM}\ (10, 4) = 20$$

Thus (c) is correct option.

6. HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, then the other number is

(a) 36 (b) 35

(c) 9 (d) 81

Ans : [Board 2020 OD Basic]

Let y be the second number.

Since, product of two numbers is equal to product of LCM and HCM,

$$54 \times y = \text{LCM} \times \text{HCF}$$

$$54 \times y = 162 \times 27$$

$$y = \frac{162 \times 27}{54} = 81$$

Thus (b) is correct option.

7. HCF of 144 and 198 is

(a) 9 (b) 18

(c) 6 (d) 12

Ans : [Board 2020 Delhi Basic]

Using prime factorization method,

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^4 \times 3^2$$

and $$198 = 2 \times 3 \times 3 \times 11$$

$$= 2 \times 3^2 \times 11$$

$$\text{HCF}(144, 198) = 2 \times 3^2 = 2 \times 9 = 18$$

Thus (b) is correct option.

8. 225 can be expressed as

(a) 5×3^2 (b) $5^2 \times 3$

(c) $5^2 \times 3^2$ (d) $5^3 \times 3$

Ans : [Board 2020 Delhi Basic]

By prime factorization of 225, we have

$$225 = 3 \times 3 \times 5 \times 5$$

$$= 3^2 \times 5^2 \text{ or } 5^2 \times 3^2$$

Thus (c) is correct option.

9. The decimal expansion of $\frac{23}{2^5 \times 5^2}$ will terminate after how many places of decimal?

(a) 2 (b) 4

(c) 5 (d) 1

Ans : [Board 2020 OD Basic]

$$\frac{23}{2^5 \times 5^2} = \frac{23 \times 5^3}{2^5 \times 5^2 \times 5^3}$$

$$= \frac{23 \times 125}{2^5 \times 5^5} = \frac{2875}{(10)^5}$$

$$= \frac{2875}{100000} = 0.02875$$

Hence, $\frac{23}{2^5 \times 5^2}$ will terminate after 5 five decimal places.

Thus (c) is correct option.

10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

Ans : [Board 2020 Delhi Standard]

Rational number,

$$\frac{14587}{1250} = \frac{14587}{2^1 \times 5^4} = \frac{14587}{2^1 \times 5^4} \times \frac{2^3}{2^3}$$

$$= \frac{14587 \times 8}{2^4 \times 5^4} = \frac{116696}{(10)^4}$$

$$= 11.6696$$

Hence, given rational number will terminate after four decimal places.

Thus (d) is correct option.

11. $2.\overline{35}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a natural number

Ans : [Board 2020 Delhi Basic]

$2.\overline{35}$ is a rational number because it is a non terminating repeating decimal.

Thus (b) is correct option.

12. $2\sqrt{3}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) a whole number

Ans : [Board 2020 OD Basic]

Let us assume that $2\sqrt{3}$ is a rational number.

Now $2\sqrt{3} = r$ where r is rational number

or $\sqrt{3} = \frac{r}{2}$

Now, we know that $\sqrt{3}$ is an irrational number, So, $\frac{r}{2}$ has to be irrational to make the equation true. This is a contradiction to our assumption. Thus, our assumption is wrong and $2\sqrt{3}$ is an irrational number.

Thus (c) is correct option.

13. The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

Ans :

Product of a non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ which is irrational.

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Thus (a) is correct option.

14. For some integer m, every even integer is of the form

(a) m (b) $m+1$

(c) $2m$ (d) $2m+1$

Ans :

We know that even integers are 2, 4, 6, ...

So, it can be written in the form of $2m$ where m is a integer.

$$m = ..., -1, 0, 1, 2, 3, ...$$

$$2m = ..., -2, 0, 2, 4, 6, ...$$

Thus (c) is correct option.

15. For some integer q, every odd integer is of the form

(a) q (b) $q+1$

(c) $2q$ (d) $2q+1$

Ans :

We know that odd integers are 1, 3, 5, ...

So, it can be written in the form of $2q+1$ where q is integer.

$$q = ..., -2, -1, 0, 1, 2, 3, ...$$

$$2q+1 = ..., -3, -1, 1, 3, 5, 7, ...$$

Thus (d) is correct option.

16. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2

(c) x^3y^3 (d) x^2y^2

Ans :

We have $a = x^3y^2 = x \times x \times x \times y \times y$

$$b = xy^3 = x \times y \times y \times y$$

$$\text{HCF}(a,\ b) = \text{HCF}(x^3y^3, xy^3)$$

$$= x \times y \times y = xy^2$$

HCF is the product of the smallest power of each common prime factor involved in the numbers.

Thus (b) is correct option.

17. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2

(c) a^3b^2 (d) a^3b^3

Ans :

We have $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

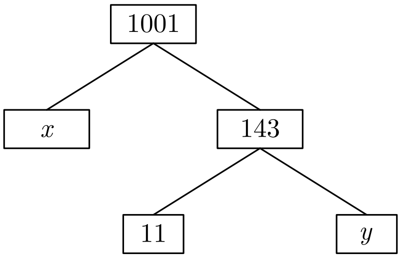
$$\text{LCM}(p,\ q) = \text{LCM}(ab^2, a^3b)$$

$$= a \times b \times b \times a \times a = a^3 b^2$$

LCM is the product of the greatest power of each prime factor involved in the numbers.

Thus (c) is correct option.

18. The values of x and y in the given figure are



(a) 7, 13 (b) 13, 7

(c) 9, 12 (d) 12, 9

Ans :

$$1001 = x \times 143 \Rightarrow x = 7$$

$$143 = y \times 11 \Rightarrow y = 13$$

Hence $x = 7,\ y = 13$

Thus (a) is correct option.

19. The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100

(c) 504 (d) 2520

Ans :

Factor of 1 to 10 numbers

$$1 = 1$$

$$2 = 1 \times 2$$

$$3 = 1 \times 3$$

$$4 = 1 \times 2 \times 2$$

$$5 = 1 \times 5$$

$$6 = 1 \times 2 \times 3$$

$$7 = 1 \times 7$$

$$8 = 1 \times 2 \times 2 \times 2$$

$$9 = 1 \times 3 \times 3$$

$$10 = 1 \times 2 \times 5$$

$$\text{LCM}(1 \text{ to } 10) = \text{LCM}(1,2,3,4,5,6,7,8,9,10)$$

$$= 1 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$

$$= 2520$$

Thus (d) is correct option.

20. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

(a) an even number (b) an odd number

(c) an odd prime number (d) a prime number

Ans :

$p_1^2 - p_2^2$ is an even number.

Let us take $p_1 = 5$

and $p_2 = 3$

Then, $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

Thus (a) is correct option.

21. The rational form of $0.2\overline{54}$ is in the form of $\frac{p}{q}$ then $(p+q)$ is

(a) 14 (b) 55

(c) 69 (d) 79

Ans :

Let, $x = 0.2\overline{54}$, then

$$x = 0.2545454 \text{ ....} \quad \text{...(1)}$$

Multiplying equation (1) by 100, we get

$$100x = 25.4545. \text{ ....} \quad \text{...(2)}$$

Subtracting equation (1) from equation. (2), we get

$$99x = 25.2 \Rightarrow x = \frac{252}{990} = \frac{14}{55}$$

Comparing with $\frac{p}{q}$, we get

$$p = 14$$

and $q = 55$

Hence, $p + q = 14 + 55 = 69$

Alternative :

$$0.2\overline{54} = \frac{254 - 2}{990} = \frac{252}{990} = \frac{14}{55}$$

Thus (c) is correct option.

22. The rational number of the form $\frac{p}{q}$, $q \neq 0$, p and q are positive integers, which represents $0.1\overline{34}$ i.e., (0.1343434 ....) is

(a) $\frac{134}{999}$ (b) $\frac{134}{990}$

(c) $\frac{133}{999}$ (d) $\frac{133}{990}$

Ans :

$$0.1\overline{34} = \frac{134-1}{990} = \frac{133}{990}$$

Thus (d) is correct option.

23. Which of the following will have a terminating decimal expansion?

(a) $\frac{77}{210}$ (b) $\frac{23}{30}$

(c) $\frac{125}{441}$ (d) $\frac{23}{8}$

Ans :

For terminating decimal expansion, denominator must the form of $2^m \times 5^n$ where n, m are non-negative integers.

Here, $\frac{23}{8} = \frac{23}{2^3}$

Here only 2 is factor of denominator so terminating.

Thus (d) is correct option.

24. If $x = 0.\overline{7}$, then $2x$ is

(a) $1.\overline{4}$ (b) $1.\overline{5}$

(c) $1.\overline{54}$ (d) $1.\overline{45}$

Ans :

We have $x = 0.\overline{7}$

$10x = 7.\overline{7}$

Subtracting, $9x = 7$

$x = \frac{7}{9}$

$2x = \frac{14}{9} = 1.555 \text{ ....}$

$= 1.\overline{5}$

25. Which of the following rational number have non-terminating repeating decimal expansion?

(a) $\frac{31}{3125}$ (b) $\frac{71}{512}$

(c) $\frac{23}{200}$ (d) None of these

Ans :

$3125 = 5^5 = 5^5 \times 2^0$

$512 = 2^9 = 2^9 \times 5^0$

$200 = 2^3 \times 5^2$

Thus 3125, 512 and 200 has factorization of the form $2^m \times 5^n$ (where m and n are whole numbers). So given fractions has terminating decimal expansion.

Thus (d) is correct option.

26. The number $3^{13} - 3^{10}$ is divisible by

(a) 2 and 3 (b) 3 and 10

(c) 2, 3 and 10 (d) 2, 3 and 13

Ans :

$$3^{13} - 3^{10} = 3^{10}(3^3 - 1) = 3^{10}(26)$$
$$= 2 \times 13 \times 3^{10}$$

Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.

Thus (d) is correct option.

27. 1. The L.C.M. of x and 18 is 36.

2. The H.C.F. of x and 18 is 2.

What is the number x?

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

LCM $\times$ HCF $=$ First number $\times$ second number

Hence, required number $= \frac{36 \times 2}{18} = 4$

Thus (d) is correct option.

28. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and $\text{LCM}(a, b, c) = 2^3 \times 3^2 \times 5$, then n is

(a) 1 (b) 2

(c) 3 (d) 4

Ans :

Value of n must be 2.

Thus (b) is correct option.

29. The least number which is a perfect square and is divisible by each of 16, 20 and 24 is

(a) 240 (b) 1600

(c) 2400 (d) 3600

Ans :

The LCM of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect square number. 1600 is not multiple of 240.

Thus (d) is correct option.

30. $n^2 - 1$ is divisible by 8, if n is

(a) an integer (b) a natural number

(c) an odd integer (d) an even integer

Ans :

Let, $a = n^2 - 1$

For $n^2 - 1$ to be divisible by 8 (even number), $n^2 - 1$ should be even. It means n^2 should be odd i.e. n should be odd.

If n is odd, $n = 2k + 1$ where k is an integer

$$a = (2k+1)^2 - 1$$
$$= 4k^2 + 4k + 1 - 1$$
$$= 4k^2 + 4k$$
$$a = 4k(k+1)$$

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At $k = -1$, $a = 4(-1)(-1+1) = 0$

which is divisible by 8.

At $k = 0$, $a = 4(0) + (0+1) = 0$

which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.

Thus (c) is correct option.

31. When 2^{256} is divided by 17 the remainder would be

(a) 1 (b) 16

(c) 14 (d) None of these

Ans : (a) 1

When 2^{256} is divided by 17 then,

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$$\frac{2^{256}}{2^4+1} = \frac{(2^4)^{64}}{(2^4+1)}$$

By remainder theorem when $f(x)$ is divided by $x + a$ the remainder is $f(-a)$.

Here, $f(x) = (2^4)^{64}$ and $x = 2^4$ and $a = 1$

Hence, remainder $f(-1) = (-1)^{64} = 1$

Thus (a) is correct option.

32. Assertion : $\frac{13}{3125}$ is a terminating decimal fraction.

Reason : If $q = 2^m 5^n$ where m, n are non-negative integers, then $\frac{p}{q}$ is a terminating decimal fraction.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $3125 = 5^5 = 5^5 \times 2^0$

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Since the factors of the denominator 3125 is of the form $2^0 \times 5^5$, $\frac{13}{3125}$ is a terminating decimal

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

Thus (a) is correct option.

33. Assertion : 34.12345 is a terminating decimal fraction.

Reason : Denominator of 34.12345, when expressed in the form $\frac{p}{q}$, $q \neq 0$, is of the form $2^m \times 5^n$, where m and n are non-negative integers.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$34.12345 = \frac{3412345}{100000} = \frac{682469}{20000} = \frac{682469}{2^5 \times 5^4}$$

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Its denominator is of the form $2^m \times 5^n$, where $m = 5$ and $n = 4$ which are non-negative integers.

Thus both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

34. Assertion : The HCF of two numbers is 5 and their product is 150, then their LCM is 30

Reason : For any two positive integers a and b, HCF (a, b) + LCM $(a, b) = a \times b$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans : (c) Assertion (A) is true but reason (R) is false.

We have,

$$\text{LCM}(a,b)\times\text{HCF}(a,b) = a\times b$$

$$\text{LCM}\times 5 = 150$$

$$\text{LCM} = \frac{150}{5} = 30$$

Thus (c) is correct option.

Fill in the Blank Questions

35. If every positive even integer is of the form $2q$, then every positive odd integer is of the form .... where q is some integer.

Ans :

$2q+1$

36. The exponent of 2 in the prime factorisation of 144, is ....

Ans :

4

37. $\sqrt{2}, \sqrt{3}, \sqrt{7}$, etc. are .... numbers.

Ans :

Irrational

38. Every point on the number line corresponds to a .... number.

Ans :

Real

39. The product of three numbers is .... to the product of their HCF and LCM.

Ans :

Not equal

40. If p is a prime number and it divides a^2 then it also divides ...., where a is a positive integer.

Ans :

a

41. Every real number is either a .... number or an .... number.

Ans :

Rational, irrational

42. Numbers having non-terminating, non-repeating decimal expansion are known as ....

Ans :

Irrational numbers

Very Short Answer Questions

43. What is the HCF of smallest primer number and the smallest composite number?

Ans : [Board 2018]

Smallest prime number is 2 and smallest composite number is 4. HCF of 2 and 4 is 2.

44. Write one rational and one irrational number lying between 0.25 and 0.32.

Ans : [Board 2020 SQP Standard]

Given numbers are 0.25 and 0.32.

Clearly $\quad 0.30 = \frac{30}{100} = \frac{3}{10}$

Thus 0.30 is a rational number lying between 0.25 and 0.32. Also 0.280280028000.....has non-terminating non-repeating decimal expansion. It is an irrational number lying between 0.25 and 0.32.

45. If HCF(336, 54) = 6, find LCM(336, 54).

Ans : [Board 2019 OD]

$$\text{HCF}\times\text{LCM} = \text{Product of number}$$

$$6\times\text{LCM} = 336\times 54$$

$$\text{LCM} = \frac{336\times 54}{6}$$

$$= 56\times 54 = 3024$$

Thus LCM of 336 and 54 is 3024.

46. Explain why 13233343563715 is a composite number?

Ans : [Board Term-1 2016]

The number 13233343563715 ends in 5. Hence it is a multiple of 5. Therefore it is a composite number.

47. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then calculate the least prime factor of $(a+b)$.

Ans : [Board Term-1 2014]

Here a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. The least prime factor of $(a+b)$ would be 2.

48. What is the HCF of the smallest composite number and the smallest prime number?

Ans : [Board Term-

The smallest prime number is 2 and the smallest composite number is $4 = 2^2$.

Hence, required HCF is $(2^2,\ 2) = 2$.

49. Calculate the HCF of $3^3 \times 5$ and $3^2 \times 5^2$.

Ans : [Board 2007]

We have
$$3^3 \times 5 = 3^2 \times 5 \times 3$$
$$3^2 \times 5^2 = 3^2 \times 5 \times 5$$
$$\text{HCF } (3^3 \times 5,\ 3^2 \times 5^2) = 3^2 \times 5$$
$$= 9 \times 5 = 45$$

50. If HCF $(a, b) = 12$ and $a \times b = 1{,}800$, then find LCM (a, b).

Ans :

We know that
$$\text{HCF}\,(a, b) \times \text{LCM}\,(a, b) = a \times b$$

Substituting the values we have
$$12 \times \text{LCM}\,(a, b) = 1800$$

or,
$$\text{LCM}\,(a,\ b) = \frac{1{,}800}{12} = 150$$

51. What is the condition for the decimal expansion of a rational number to terminate? Explain with the help of an example.

Ans : [Board Term-1 2016]

The decimal expansion of a rational number terminates, if the denominator of rational number can be expressed as $2^m 5^n$ where m and n are non negative integers and p and q both co-primes.

e.g.
$$\frac{3}{10} = \frac{3}{2^1 \times 5^1} = 0.3$$

52. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

Ans : [Board Term-1 2016]

Since
$$\frac{1}{7} \times \frac{7}{100} = \frac{1}{100} = 0.01.$$

Thus smallest rational number is $\frac{7}{100}$

53. What type of decimal expansion does a rational number has? How can you distinguish it from decimal expansion of irrational numbers?

Ans : [Board Term-1 2016]

A rational number has its decimal expansion either terminating or non-terminating, repeating An irrational numbers has its decimal expansion non-repeating and non-terminating.

54. Calculate $\frac{3}{8}$ in the decimal form.

Ans : [Board 2008]

We have
$$\frac{3}{8} = \frac{3}{2^3} = \frac{2 \times 5^3}{2^3 \times 5^3}$$
$$= \frac{375}{10^3} = \frac{375}{1{,}000}$$
$$= 0.375$$

55. The decimal representation of $\frac{6}{1250}$ will terminate after how many places of decimal?

Ans : [Board 2009]

We have
$$\frac{6}{1250} = \frac{6}{2 \times 5^4} = \frac{6 \times 2^3}{2 \times 2^3 \times 5^4}$$
$$= \frac{6 \times 2^3}{2^4 \times 5^4} = \frac{6 \times 2^3}{(10)^4}$$
$$= \frac{48}{10000} = 0.0048$$

Thus $\frac{6}{1250}$ will terminate after 4 decimal places.

56. Find the least number that is divisible by all numbers between 1 and 10 (both inclusive).

Ans : [Board 2010]

The required number is the LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
$$\text{LCM} = 2 \times 2 \times 3 \times 2 \times 3 \times 5 \times 7$$
$$= 2520$$

57. Write whether rational number $\frac{7}{75}$ will have terminating decimal expansion or a non-terminating decimal.

Ans : [Board Term-1 2017, SQP]

We have
$$\frac{7}{75} = \frac{7}{3 \times 5^2}$$

Since denominator of given rational number is not of form $2^m \times 5^n$, Hence, It is non-terminating decimal expansion.

TWO MARKS QUESTIONS

58. If HCF of 144 and 180 is expressed in the form $13m - 16$. Find the value of m.

Ans : [Board 2020 SQP Standard]

According to Euclid's algorithm any number a can be written in the form,

$$a = bq + r \text{ where } 0 \leq r < b$$

Applying Euclid's division lemma on 144 and 180 we have

$$180 = 144 \times 1 + 36$$

$$144 = 36 \times 4 + 0$$

Here, remainder is 0 and divisor is 36. Thus HCF of 144 and 180 is 36.

Now $\quad 36 = 13m - 16$

$$36 + 16 = 13m$$

$$52 = 13m \Rightarrow m = 4$$

59. Find HCF and LCM of 404 and 96 and verify that HCF $\times$ LCM $=$ Product of the two given numbers.

Ans : [Board 2018]

We have $\quad 404 = 2 \times 2 \times 101$

$$= 2^2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$= 2^5 \times 3$$

$$\text{HCF}(404, 96) = 2^2 = 4$$

$$\text{LCM}(404, 96) = 101 \times 2^5 \times 3 = 9696$$

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

Also, $\quad 404 \times 96 = 38784$

Hence, HCF $\times$ LCM $=$ Product of 404 and 96

60. Find HCF of the numbers given below:
$k, 2k, 3k, 4k$ and $5k$,, where k is a positive integer.

Ans : [Board Term-1 2015, Set-FHN8MGD]

Here we can see easily that k is common factor between all and this is highest factor Thus HCF of $k, 2k, 3k, 4k$ and $5k$, is k.

61. Find the HCF and LCM of 90 and 144 by the method of prime factorization.

Ans : [Board Term-1 2012]

We have $\quad 90 = 9 \times 10 = 9 \times 2 \times 5$

$$= 2 \times 3^2 \times 5$$

and $\quad 144 = 16 \times 9$

$$= 2^4 \times 3^2$$

$$\text{HCF} = 2 \times 3^2 = 18$$

$$\text{LCM} = 2^4 \times 3^2 \times 5 = 720$$

62. Given that HCF $(306, 1314) = 18$. Find LCM $(306, 1314)$

Ans : [Board Term-1 2013]

We have $\quad \text{HCF}(306, 1314) = 18$

$$\text{LCM}(306, 1314) = ?$$

Let $a = 306$ and $b = 1314$, then we have

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$$

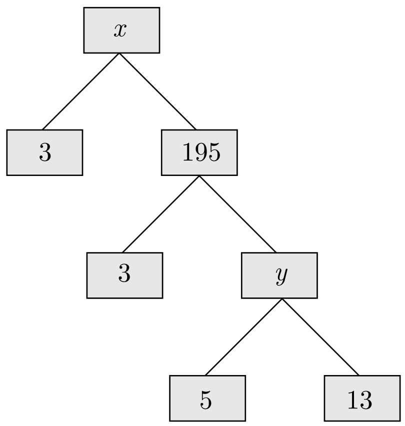
Substituting values we have

$$\text{LCM}(a, b) \times 18 = 306 \times 1314$$

$$\text{LCM}(a, b) = \frac{306 \times 1314}{18}$$

$$\text{LCM}(306, 1314) = 22{,}338$$

63. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015]

We have $\quad y = 5 \times 13 = 65$

and $\quad x = 3 \times 195 = 585$

64. Explain why $(7 \times 13 \times 11) + 11$ and $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$ are composite numbers.

Ans : [Board Term-1 2012, Set-64]

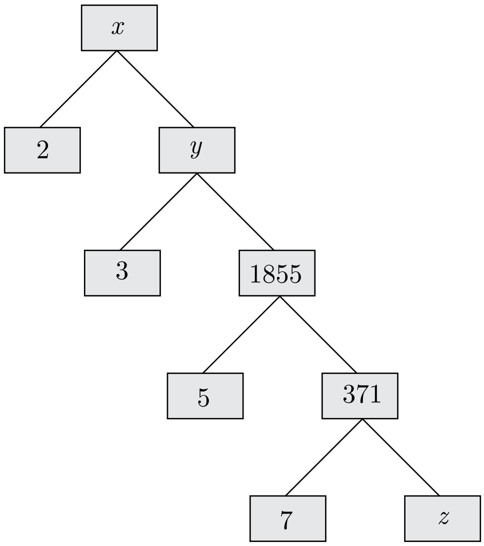
$$(7 \times 13 \times 11) + 11 = 11 \times (7 \times 13 + 1)$$
$$= 11 \times (91 + 1)$$
$$= 11 \times 92$$

and

$$(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 3$$
$$= 3(7 \times 6 \times 5 \times 4 \times 2 \times 1 + 1)$$
$$= 3 \times (1681) = 3 \times 41 \times 41$$

Since given numbers have more than two prime factors, both number are composite.

65. Complete the following factor tree and find the composite number x



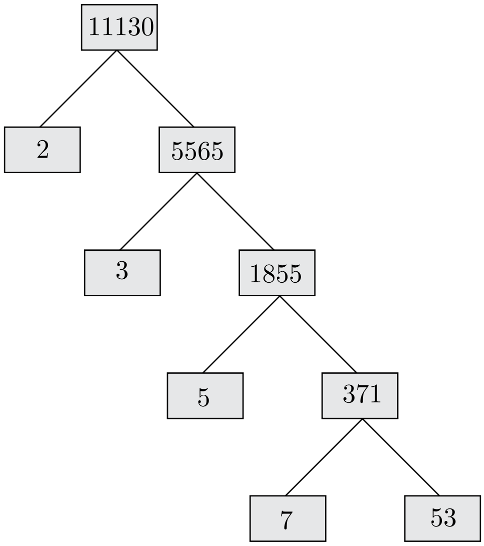
Ans : [Board Term-1 2015, Set DDE-M]

We have $z = \dfrac{371}{7} = 53$

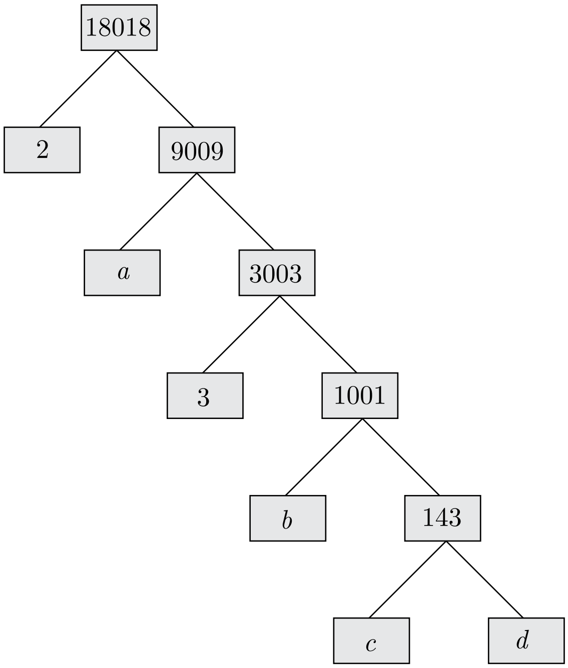
$y = 1855 \times 3 = 5565$

$x = 2 \times y = 2 \times 5565 = 11130$

Thus complete factor tree is as given below.



66. Find the missing numbers a, b, c and d in the given factor tree:



Ans : [Board Term-1 2012]

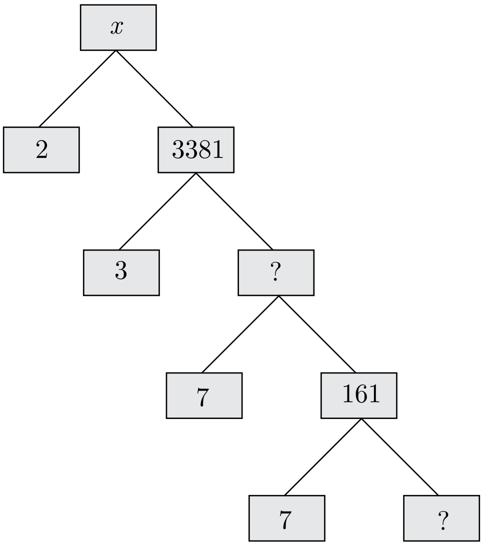
We have $a = \dfrac{9009}{3003} = 3$

$b = \dfrac{1001}{143} = 7$

Since $143 = 11 \times 13,$

Thus $c = 11$ and $d = 13$ or $c = 13$ and $d = 11$

67. Complete the following factor tree and find the composite number x.



Ans : [Board Term-1 2015, 2014]

We complete the given factor tree writing variable y and z as following.

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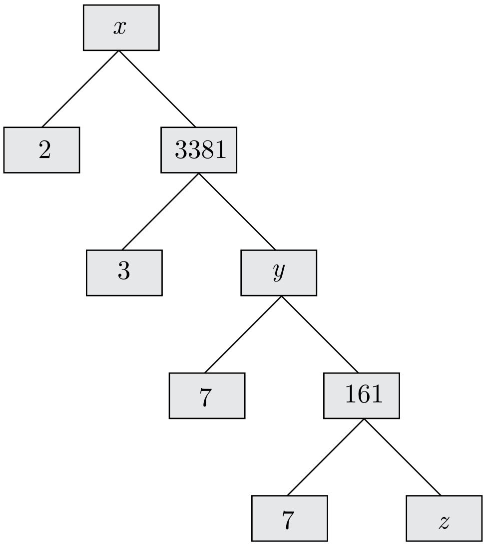

We have $$z = \frac{161}{7} = 23$$

$$y = 7 \times 161 = 1127$$

Composite number, $x = 2 \times 3381 = 6762$

68. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number.

Ans : [Board Term-1 2016-17 Set; 193RQTQ, 2015, DDE-E]

A prime number (or a prime) is a natural number greater than 1 that cannot be formed by multiplying two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1, involve 5 itself. However, 6 is composite because it is the product of two numbers (2×3) that are both smaller than 6. Every composite number can be written as the product of two or more (not necessarily distinct) primes.

$$\begin{aligned} 3 \times 12 \times 101 + 4 &= 4(3 \times 3 \times 101 + 1) \\ &= 4(909 + 1) \\ &= 4(910) \\ &= 2 \times 2 \times (10 \times 7 \times 13) \\ &= 2 \times 2 \times 2 \times 5 \times 7 \times 13 \\ &= \text{a composite number} \end{aligned}$$

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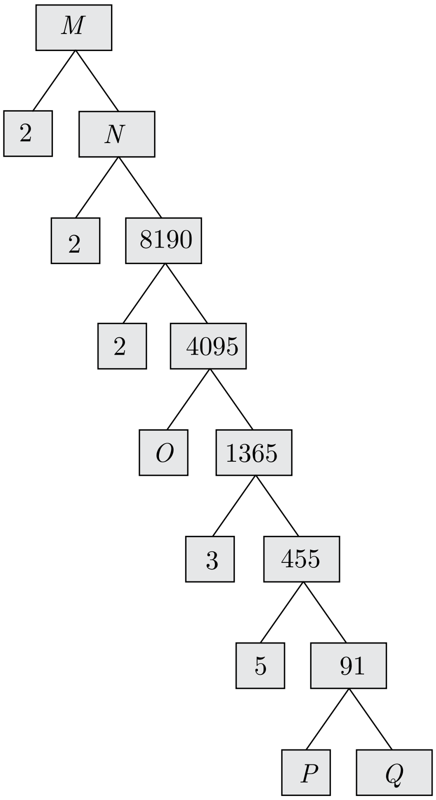
69. Complete the factor-tree and find the composite number M.



Ans : [Board Term-1 2013]

We have $$91 = P \times Q = 7 \times 13$$

So $P = 7, Q = 13$ or $P = 13, Q = 7$

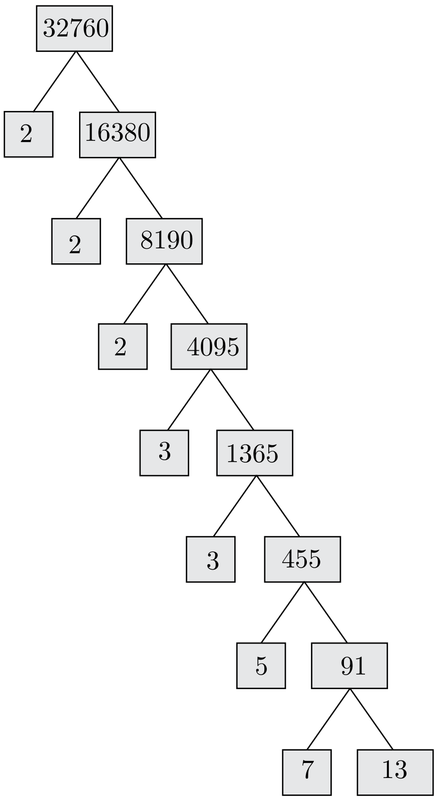
$$O = \frac{4095}{1365} = 3$$

$$N = 2 \times 8190 = 16380$$

Composite number,

$$M = 16380 \times 2 = 32760$$

Thus complete factor tree is shown below.



70. Find the smallest natural number by which 1200 should be multiplied so that the square root of the product is a rational number.

Ans : [Board Term-1 2016, 2015]

We have $1200 = 12 \times 100$

$$= 4 \times 3 \times 4 \times 25$$

$$= 4^2 \times 3 \times 5^2$$

Here if we multiply by 3, then its square root will be $4 \times 3 \times 5$ which is a rational number. Thus the required smallest natural number is 3.

71. Can two numbers have 15 as their HCF and their LCM? Give reasons.

Ans : [Board Te

LCM of two numbers should be exactly divisible by their HCF. Since, 15 does not divide 175, two numbers cannot have their HCF as 15 and LCM as 175.

72. Check whether 4^n can end with the digit 0 for any natural number n.

Ans : [Board Term-1 2015, Set-FHN8MGD; NCERT]

If the number 4^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 4^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $4^n = 2^{2n}$ is 2. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 4^n. So, there is no natural number n for which 4^n ends with the digit zero. Hence 4^n cannot end with the digit zero.

73. Show that 7^n cannot end with the digit zero, for any natural number n.

Ans : [Board Term-1 2012, Set-63]

If the number 7^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of 7^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $7^n = (1 \times 7)^n$ is 7. So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 7^n. So, there is no natural number n for which 7^n ends with the digit zero. Hence

7^n cannot end with the digit zero.

74. Check whether $(15)^n$ can end with digit 0 for any $n \in N$.

Ans : [Board Term-1 2012]

If the number $(15)^n$, for any n, were to end with the digit zero, then it would be divisible by 5 and 2.

That is, the prime factorization of $(15)^n$ would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(15)^n = (3 \times 5)^n$ are 3 and 5. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(15)^n$. Since there is no prime factor 2, $(15)^n$ cannot end with the digit zero.

75. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly.

Ans : [Board Term-1 2016]

Here we have to determine the HCF of all length which can measure all dimension.

$$\text{Length,} \quad l = 8 \text{ m } 50 \text{ cm} = 850 \text{ cm}$$
$$= 50 \times 17 = 2 \times 5^2 \times 17$$
$$\text{Breadth,} \quad b = 6 \text{ m } 25 \text{ cm} = 625 \text{ cm}$$
$$= 25 \times 25 = 5^2 \times 5^2$$
$$\text{Height,} \quad h = 4 \text{ m } 75 \text{ cm} = 475 \text{ cm}$$
$$= 25 \times 19 = 5^2 \times 19$$
$$\text{HCF}(l, b, h) = \text{HCF } (850, 625, 475)$$
$$= \text{HCF } (2 \times 5^2 \times 17,\ 5^2,\ 5^2 \times 19)$$
$$= 5^2 = 25 \text{ cm}$$

Thus 25 cm rod can measure the dimensions of the room exactly. This is longest rod that can measure exactly.

76. Show that $5\sqrt{6}$ is an irrational number.

Ans : [Board Term-1 2015]

Let $5\sqrt{6}$ be a rational number, which can be expressed as $\frac{a}{b}$, where $b \neq 0$; a and b are co-primes.

$$\text{Now} \quad 5\sqrt{6} = \frac{a}{b}$$
$$\sqrt{6} = \frac{a}{5b}$$
$$\text{or,} \quad \sqrt{6} = \text{rational}$$

But, $\sqrt{6}$ is an irrational number. Thus, our assumption is wrong. Hence, $5\sqrt{6}$ is an irrational number.

77. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence write its decimal expansion without actual division.

Ans : [Board Term-1 2012, NCERT Exemplar]

$$\text{We have} \quad 500 = 25 \times 20$$
$$= 5^2 \times 5 \times 4$$
$$= 5^3 \times 2^2$$

Here denominator is 500 which can be written as $2^2 \times 5^3$.

Now decimal expansion,

$$\frac{257}{500} = \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3}$$
$$= 0.514$$

78. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Ans : [K.V.S.]

We have $\sqrt{2} = \sqrt{\frac{200}{100}}$ and $\sqrt{3} = \sqrt{\frac{300}{100}}$

We need to find a rational number x such that

$$\frac{1}{10}\sqrt{200} < x < \frac{1}{10}\sqrt{300}$$

Choosing any perfect square such as 225 or 256 in between 200 and 300, we have

$$x = \sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{5}{3}$$

Similarly if we choose 256, then we have

$$x = \sqrt{\frac{256}{100}} = \frac{16}{10} = \frac{8}{5}$$

79. Write the rational number $\frac{7}{75}$ will have a terminating decimal expansion. or a non-terminating repeating decimal.

Ans : [Board 2018 SQP]

$$\text{We have} \quad \frac{7}{75} = \frac{7}{3 \times 5^2}$$

The denominator of rational number $\frac{7}{75}$ can not be written in form $2^m 5^n$ So it is non-terminating repeating decimal expansion.

80. Show that 571 is a prime number.

Ans :

$$\text{Let} \quad x = 571$$
$$\sqrt{x} = \sqrt{571}$$

Now 571 lies between the perfect squares of $(23)^2 = 529$ and $(24)^2 = 576$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Here 571 is not divisible by any of the above numbers, thus 571 is a prime number.

81. If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, where a and b are prime numbers than verify $\text{LCM}(p,q) \times \text{HCF}(q,q) = pq$

Ans : [Sample Paper 2017]

We have $p = a^2b^3 = a \times a \times b \times b \times b$

and $q = a^3b = a \times a \times a \times b$

Now $\text{LCM}(p,q) = a \times a \times a \times b \times b \times b$

$= a^3b^3$

and $\text{HCF}(p,q) = a \times a \times b$

$= a^2b$

$$\text{LCM}(p,q) \times \text{HCF}(p,q) = a^3b^3 \times a^2b$$
$$= a^5b^4$$
$$= a^2b^3 \times a^3b$$
$$= pq$$

THREE MARKS QUESTIONS

82. An army contingent of 612 members is to march behind an army band of 48 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans : [Board 2020 Delhi Basic]

Let the number of columns be x which is the largest number, which should divide both 612 and 48. It means x should be HCF of 612 and 48.

We can write 612 and 48 as follows

$$612 = 2 \times 2 \times 3 \times 3 \times 5 \times 17$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$\text{HCF}(612,\ 28) = 2 \times 2 \times 3 = 12$$

Thus HCF of 104 and 96 is 12 i.e. 12 columns are required.

Here we have solved using Euclid's algorithm but you can solve this problem by simple mehtod of HCF.

83. Given that $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

Ans : [Board 2020 SQP Standard]

Assume that $2\sqrt{5} - 3$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $2\sqrt{5} - 3 = \dfrac{p}{q}$

where $q \neq 0$ and p and q are co-prime integers.

Rewriting the above expression as,

$$2\sqrt{5} = \frac{p}{q} + 3$$
$$\sqrt{5} = \frac{p+3q}{2q}$$

Here $\frac{p+3q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{5}$ should be a rational number. But $\sqrt{5}$ is irrational. This contradicts the given fact that $\sqrt{5}$ is irrational. Hence $2\sqrt{5} - 3$ is an irrational number.

84. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 Delhi]

Assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$\frac{2+\sqrt{3}}{5} = \frac{p}{q}$$
$$2+\sqrt{3} = \frac{5p}{q}$$
$$\sqrt{3} = \frac{5p}{q} - 2$$
$$\sqrt{3} = \frac{5p-2q}{q}$$

Since, p and q are co-prime integers, then $\frac{5p-2q}{q}$ is a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong. Therefore $\frac{2+\sqrt{3}}{5}$ is an irrational number.

85. Given that $\sqrt{3}$ is irrational, prove that $(5+2\sqrt{3})$ is an irrational number.

Ans : [Board 2020 Delhi Basic]

Assume that $(5+2\sqrt{3})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p

and q are co-prime integers and $q \neq 0$.

Now $$5+2\sqrt{3} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

$$2\sqrt{3} = \frac{p}{q} - 5$$

$$\sqrt{3} = \frac{p-5q}{2q}$$

Here $\frac{p-5q}{2q}$ is rational because p and q are co-prime integers, thus $\sqrt{3}$ should be a rational number. But $\sqrt{3}$ is irrational. This contradicts the given fact that $\sqrt{3}$ is irrational. Hence $(5+2\sqrt{3})$ is an irrational number.

86. Prove that $2+5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Ans : [Board 2019 OD]

Assume that $2+5\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

$$2+5\sqrt{3} = \frac{p}{q}, \quad q \neq 0$$

$$5\sqrt{3} = \frac{p}{q} - 2$$

$$5\sqrt{3} = \frac{p-2q}{q}$$

$$\sqrt{3} = \frac{p-2q}{5q}$$

Here $\sqrt{3}$ is irrational and $\frac{p-2q}{5q}$ is rational because p and q are co-prime integers. But rational number cannot be equal to an irrational number. Hence $2+5\sqrt{3}$ is an irrational number.

87. Given that $\sqrt{2}$ is irrational, prove that $(5+3\sqrt{2})$ is an irrational number.

Ans : [Board 2018]

Assume that $(5+3\sqrt{2})$ is a rational number. Therefore, we can write it in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$.

Now $$5+3\sqrt{2} = \frac{p}{q}$$

where $q \neq 0$ and p and q are integers.

Rewriting the above expression as,

$$3\sqrt{2} = \frac{p}{q} - 5$$

$$\sqrt{2} = \frac{p-5q}{3q}$$

Here $\frac{p-5q}{3q}$ is rational because p and q are co-prime integers, thus $\sqrt{2}$ should be a rational number. But $\sqrt{2}$ is irrational. This contradicts the given fact that $\sqrt{2}$ is irrational. Hence $(5+3\sqrt{2})$ is an irrational number.

88. Write the smallest number which is divisible by both 306 and 657.

Ans : [Board 2019 OD]

The smallest number that is divisible by two numbers is obtained by finding the LCM of these numbers Here, the given numbers are 306 and 657.

$$306 = 6 \times 51 = 3 \times 2 \times 3 \times 17$$

$$657 = 9 \times 73 = 3 \times 3 \times 73$$

$$\text{LCM}(306,\ 657) = 2 \times 3 \times 3 \times 17 \times 73$$

$$= 22338$$

Hence, the smallest number which is divisible by 306 and 657 is 22338.

89. Show that numbers 8^n can never end with digit 0 of any natural number n.

Ans : [Board Term-1 2015, NCERT]

If the number 8^n, for any n, were to end with the digit zero, then it would be divisible by 5 and 2. That is, the prime factorization of 8^n would contain the prime 5 and 2. This is not possible because the only prime in the factorization of $(8)^n = (2^3)^n = 2^{3n}$ is 2. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of $(8)^n$. Since there is no prime factor 5, $(8)^n$ cannot end with the digit zero.

90. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Ans : [Board Term-1 2011]

The required answer will be HCF of 144 and 90.

$$144 = 2^4 \times 3^2$$

$$90 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144,\ 90) = 2 \times 3^2 = 18$$

Thus each stack would have 18 cartons.

91. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together?

Ans : [Board Term-1 2011, Set-44]

The required answer is the LCM of 9, 12, and 15 minutes.

Finding prime factor of given number we have,

$$9 = 3 \times 3 = 3^2$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$\text{LCM}(9, 12, 15) = 2^2 \times 3^2 \times 5$$

$$= 150 \text{ minutes}$$

The bells will toll next together after 180 minutes.

92. Find HCF and LCM of 16 and 36 by prime factorization and check your answer.

Ans :

Finding prime factor of given number we have,

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\text{HCF}(16, 36) = 2 \times 2 = 4$$

$$\text{LCM}(16, 36) = 2^4 \times 3^2$$

$$= 16 \times 9 = 144$$

Check :

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

or, $$4 \times 144 = 16 \times 36$$

$$576 = 576$$

Thus $$\text{LHS} = \text{RHS}$$

93. Find the HCF and LCM of 510 and 92 and verify that HCF $\times$ LCM = Product of two given numbers.

Ans : [Board Term-1 2011]

Finding prime factor of given number we have,

$$92 = 2^2 \times 23$$

$$510 = 30 \times 17 = 2 \times 3 \times 5 \times 17$$

$$\text{HCF}(510, 92) = 2$$

$$\text{LCM}(510, 92 = 2^2 \times 23 \times 3 \times 5 \times 14$$

$$= 23460$$

$$\text{HCF}(510, 92) \times \text{LCM}(510, 92)$$

$$= 2 \times 23460 = 46920$$

Product of two numbers $= 510 \times 92 = 46920$

Hence, HCF$\times$LCM = Product of two numbers

94. The HCF of 65 and 117 is expressible in the form $65m–117$. Find the value of m. Also find the LCM of 65 and 117 using prime factorization method.

Ans : [Board Term-1 2011, Set-40]

Finding prime factor of given number we have.

$$117 = 13 \times 2 \times 3$$

$$65 = 13 \times 5$$

$$\text{HCF}(117, 65) = 13$$

$$\text{LCM}(117, 65) = 13 \times 5 \times 3 \times 3 = 585$$

$$\text{HCF} = 65m - 117$$

$$13 = 65m–117$$

$$65m = 117 + 13 = 130$$

$$m = \frac{130}{65} = 2$$

95. Express $\left(\frac{15}{4} + \frac{5}{40}\right)$ as a decimal fraction without actual division.

Ans : [Board Term-1 2011, Set-74]

We have $$\frac{15}{4} + \frac{5}{40} = \frac{15}{4} \times \frac{25}{25} + \frac{5}{40} \times \frac{25}{25}$$

$$= \frac{375}{100} + \frac{125}{1000}$$

$$= 3.75 + 0.125 = 3.875$$

96. Express the number $0.3\overline{178}$ in the form of rational number $\frac{a}{b}$.

Ans : [Board Term-1 2011, Set-A1]

Let $$x = 0.3\overline{178}$$

$$x = 0.3178178178$$

$$10{,}000x = 3178.178178...$$

$$10x = 3.178178....$$

Subtracting, $$9990x = 3175$$

or, $$x = \frac{3175}{9990} = \frac{635}{1998}$$

97. Prove that $\sqrt{2}$ is an irrational number.

Ans : [Board Term-1 2011, NCERT]

Let $\sqrt{2}$ be a rational number.

Then $$\sqrt{2} = \frac{p}{q},$$

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where p and q are co-prime integers and $q \neq 0$ On squaring both the sides we have,

$$2 = \frac{p^2}{q^2}$$

or, $$p^2 = 2p^2$$

Since p^2 is divisible by 2, thus p is also divisible by 2.

Let $p = 2r$ for some positive integer r, then we have

$$p^2 = 4r^2$$

$$2q^2 = 4r^2$$

or, $$q^2 = 2r^2$$

Since q^2 is divisible by 2, thus q is also divisible by 2.

We have seen that p and q are divisible by 2, which contradicts the fact that p and q are co-primes. Hence, our assumption is false and $\sqrt{2}$ is irrational.

98. If p is prime number, then prove that $\sqrt{p}$ is an irrational.

Ans : [Board Term-1 2013]

Let p be a prime number and if possible, let $\sqrt{p}$ be rational

Thus $$\sqrt{p} = \frac{m}{n},$$

a161

where m and n are co-primes and $n \neq 0$.
Squaring on both sides, we get

$$p = \frac{m^2}{n^2}$$

or, $$pn^2 = m^2 \quad ...(1)$$

Here p divides pn^2. Thus p divides m^2 and in result p also divides m.

Let $m = pq$ for some integer q and putting $m = pq$ in eq. (1), we have

$$pn^2 = p^2q^2$$

or, $$n^2 = pq^2$$

Here p divides pq^2.Thus p divides n^2 and in result p also divides n.

[$\because$ p is prime and p divides $n^2 \Rightarrow p$ divides n]

Thus p is a common factor of m and n but this contradicts the fact that m and n are primes. The contradiction arises by assuming that $\sqrt{p}$ is rational.

Hence, $\sqrt{p}$ is irrational.

99. Prove that $3+\sqrt{5}$ is an irrational number.

Ans :

Assume that $3+\sqrt{5}$ is a rational number, then we have

$$3+\sqrt{5} = \frac{p}{q}, \quad q \neq 0$$

$$\sqrt{5} = \frac{p}{q} - 3$$

VIDEO Click Here

$$\sqrt{5} = \frac{p-3q}{q}$$

Here $\sqrt{5}$ is irrational and $\frac{p-3q}{q}$ is rational. But rational number cannot be equal to an irrational number. Hence $3+\sqrt{5}$ is an irrational number.

100. Prove that $\sqrt{5}$ is an irrational number and hence show that $2-\sqrt{5}$ is also an irrational number.

Ans : [Board Term-1 2011]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b}, \quad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{5}$$

Squaring both the sides, we have

a163

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

Let us assume that $2-\sqrt{5}$ be rational equal to a, then we have

$$2-\sqrt{5} = a$$

$$2 - a = \sqrt{5}$$

Since we have assume $2-a$ is rational, but $\sqrt{5}$ is not rational. Rational number cannot be equal to an irrational number. Thus $2-\sqrt{5}$ is irrational.

101. Show that exactly one of the number $n, n+2$ or $n+4$ is divisible by 3.

Ans : [Sample Paper 2017]

If n is divisible by 3, clearly $n+2$ and $n+4$ is not divisible by 3.

If n is not divisible by 3, then two case arise as given below.

Case 1: $n=3k+1$

$$n+2 = 3k+1+2 = 3k+3 = 3(k+1)$$

and $$n+4 = 3k+1+4 = 3k+5 = 3(k+1)+2$$

We can clearly see that in this case $n+2$ is divisible by 3 and $n+4$ is not divisible by 3. Thus in this case only $n+2$ is divisible by 3.

Case 1: $n=3k+2$

$$n+2 = 3k+2+2 = 3k+4 = 3(k+1)+1$$

and $$n+4 = 3k+2+4 = 3k+6 = 3(k+2)$$

We can clearly see that in this case $n+4$ is divisible by 3 and $n+2$ is not divisible by 3. Thus in this case only $n+4$ is divisible by 3.

Hence, exactly one of the numbers $n, n+2, n+4$ is divisible by 3.

FOUR MARKS QUESTIONS

102. Prove that $\sqrt{3}$ is an irrational number.

Ans : [Board 2020 OD Basic]

Assume that $\sqrt{3}$ is a rational number. Therefore, we can write it in the form of $\frac{a}{b}$ where a and b are co-prime integers and $q \neq 0$.

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

Now $$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 3b^2$ we have

$$3b^2 = 9c^2$$

$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

103. Prove that $\sqrt{5}$ is an irrational number.

Ans : [Board 2020 OD Standard]

Assume that $\sqrt{5}$ be a rational number then we have

$$\sqrt{5} = \frac{a}{b},$$

where a and b are co-primes and $b \neq 0$.

$$a = b\sqrt{5}$$

Squaring both the sides, we have

$$a^2 = 5b^2$$

Thus 5 is a factor of a^2 and in result 5 is also a factor of a.

Let $a = 5c$ where c is some integer, then we have

$$a^2 = 25c^2$$

Substituting $a^2 = 5b^2$ we have

$$5b^2 = 25c^2$$

$$b^2 = 5c^2$$

Thus 5 is a factor of b^2 and in result 5 is also a factor of b.

Thus 5 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{5}$ is rational number is wrong. Hence $\sqrt{5}$ is irrational.

104. Find HCF and LCM of 378, 180 and 420 by prime factorization method. Is HCF $\times$ LCM of these numbers equal to the product of the given three numbers?

Ans :

Finding prime factor of given number we have,

$$378 = 2 \times 3^3 \times 7$$

$$180 = 2^2 \times 3^2 \times 5$$

$$420 = 2^2 \times 3 \times 7 \times 5$$

$$\text{HCF}(378,\ 180,\ 420) = 2 \times 3 = 6$$

$$\text{LCM}(378, 180, 420) = 2^2 \times 3^3 \times 5 \times 7$$

$$= 2^2 \times 3^3 \times 5 \times 7 = 3780$$

$$\text{HCF}\times\text{LCM} = 6\times 3780 = 22680$$

Product of given numbers

$$= 378 \times 180 \times 420$$

$$= 28576800$$

Hence, HCF $\times$ LCM $\neq$ Product of three numbers.

105. State Fundamental theorem of Arithmetic. Find LCM of numbers 2520 and 10530 by prime factorization by 3.

Ans : [Board Term-1 2016]

a138

The fundamental theorem of arithmetic (FTA), also called the unique factorization theorem or the unique-prime-factorization theorem, states that every integer greater than 1 either is prime itself or is the product of a unique combination of prime numbers.

OR

Every composite number can be expressed as the product powers of primes and this factorization is unique.

Finding prime factor of given number we have,

$$2520 = 20 \times 126 = 20 \times 6 \times 21$$

$$= 2^3 \times 3^2 \times 5 \times 7$$

$$10530 = 30 \times 351 = 30 \times 9 \times 39$$

$$= 30 \times 9 \times 3 \times 13$$

$$= 2 \times 3^4 \times 5 \times 13$$

$$\text{LCM}(2520, 10530) = 2^3 \times 3^4 \times 5 \times 7 \times 13$$

$$= 294840$$

106. Can the number 6^n, n being a natural number, end with the digit 5 ? Give reasons.

Ans : [Board Term-1 2015]

a139

If the number 6^n for any n, were to end with the digit five, then it would be divisible by 5. That is, the prime factorization of 6^n would contain the prime 5. This is not possible because the only prime in the factorization of $6^n = (2 \times 3)^n$ are 2 and 3. The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorization of 6^n. Since there is no prime factor 5, 6^n cannot end with the digit five.

107. State Fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

Ans : [Board Term-1 2015]

a140

Fundamental theorem of Arithmetic : Every integer greater than one ither is prime itself or is the product of prime numbers and that this product is unique. Up to the order of the factors.

LCM of two numbers should be exactly divisible by their HCF. In other words LCM is always a multiple of HCF. Since, 24 does not divide 540 two numbers cannot have their HCF as 24 and LCM as 540.

$$\text{HCF} = 24$$

$$\text{LCM} = 540$$

$$\frac{LCM}{HCF} = \frac{540}{24} = 22.5 \text{ not an integer}$$

108. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Ans : [Board Term-1 2015, 2012]

We have $$n^3 - n = n(n^2 - 1)$$

$$= (n-1)n(n+1)$$

$$= (n-1)n(n+1)$$

Thus $n^3 - n$ is product of three consecutive positive integers.

Since, any positive integers a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

Let $a, a+1, a+2$ be any three consecutive integers.

Case I : $a = 3q$

If $a = 3q$ then,

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$

Product of two consecutive integers $(3q+1)$ and $(3q+2)$ is an even integer, say $2r$.

Thus $$a(a+1)(a+2) = 3q(2r)$$

$$= 6qr, \text{ which is divisible by 6.}$$

Case II : $a = 3q+1$

If $a = 3q+1$ then

$$a(a+1)(a+2) = (3q+1)(3q+2)(3q+3)$$
$$= (2r)(3)(q+1)$$
$$= 6r(q+1)$$

which is divisible by 6.

Case III : $a = 3q+2$

If $a = 3q+2$ then

$$a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= 3(3q+2)(q+1)(3q+4)$$

Here $(3q+2)$ and $= 3(3q+2)(q+1)(3q+4)$

$$= \text{multiple of 6 every } q$$
$$= 6r \text{ (say)}$$

which is divisible by 6. Hence, the product of three consecutive integers is divisible by 6 and $n^3 - n$ is also divisible by 3.

109. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

Ans : [Board Term-1 2012 Set-25]

We have $\quad n^2 - n = n(n-1)$

Thus $n^2 - n$ is product of two consecutive positive integers.

Any positive integer is of the form $2q$ or $2q+1$, for some integer q.

a144

Case 1 : $n = 2q$

If $n = 2q$ we have

$$n(n-1) = 2q(2q-1)$$
$$= 2m,$$

where $m = q(2q-1)$ which is divisible by 2.

Case 1 : $n = 2q+1$

If $n = 2q+1$, we have

$$n(n-1) = (2q+1)(2q+1-1)$$
$$= 2q(2q+1)$$
$$= 2m$$

where $m = q(2q+1)$ which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

110. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 + 2\sqrt{3}$ is also an irrational number.

Ans : [Board Term-1 2012]

Assume that $\sqrt{3}$ be a rational number then we have

$$\sqrt{3} = \frac{a}{b}, \qquad (a, b \text{ are co-primes and } b \neq 0)$$

$$a = b\sqrt{3}$$

Squaring both the sides, we have

$$a^2 = 3b^2$$

Thus 3 is a factor of a^2 and in result 3 is also a factor of a.

Let $a = 3c$ where c is some integer, then we have

$$a^2 = 9c^2$$

Substituting $a^2 = 9b^2$ we have

$$3b^2 = 9c^2$$
$$b^2 = 3c^2$$

Thus 3 is a factor of b^2 and in result 3 is also a factor of b.

Thus 3 is a common factor of a and b. But this contradicts the fact that a and b are co-primes. Thus, our assumption that $\sqrt{3}$ is rational number is wrong. Hence $\sqrt{3}$ is irrational.

Let us assume that $7 + 2\sqrt{3}$ be rational equal to a, then we have

$$7 + 2\sqrt{3} = \frac{p}{q} \qquad q \neq 0 \text{ and } p \text{ and } q \text{ are co-primes}$$

$$2\sqrt{3} = \frac{p}{q} - 7 = \frac{p - 7q}{q}$$

or $$\sqrt{3} = \frac{p - 7q}{2q}$$

Here $p - 7q$ and $2q$ both are integers, hence $\sqrt{3}$ should be a rational number. But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence our assumption is not correct and $7 + 2\sqrt{3}$ is irrational.

111. Show that there is no positive integer n, for which $\sqrt{n-1} + \sqrt{n-1}$ is rational.

Ans : [Board Term-1 2012]

Let us assume that there is a positive integer n for which $\sqrt{n-1} + \sqrt{n-1}$ is rational and equal to $\frac{p}{q}$, where p and q are positive integers and $(q \neq 0)$.

a165

$$\sqrt{n-1} + \sqrt{n-1} = \frac{p}{q} \qquad \text{...(1)}$$

or, $$\frac{q}{p} = \frac{1}{\sqrt{n-1}+\sqrt{n+1}}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(\sqrt{n-1}+\sqrt{n+1})(\sqrt{n-1}-\sqrt{n+1})}$$

$$= \frac{\sqrt{n-1}-\sqrt{n+1}}{(n-1)-(n+1)}$$

or $$\frac{q}{p} = \frac{\sqrt{n-1}-\sqrt{n+1}}{-2}$$

$$\sqrt{n+1}-\sqrt{n-1} = \frac{2q}{p} \quad ...(2)$$

Adding (1) and (2), we get

$$2\sqrt{n+1} = \frac{p}{q}+\frac{2q}{p} = \frac{p^2+2q^2}{pq} \quad ...(3)$$

Subtracting (2) from (1) we have

$$2\sqrt{n-1} = \frac{p^2-2q^2}{pq} \quad ...(4)$$

From (3) and (4), we observe that $\sqrt{n+1}$ and $\sqrt{n-1}$ both are rational because p and q both are rational. But it possible only when $(n+1)$ and $(n-1)$ both are perfect squares. But they differ by 2 and two perfect squares never differ by 2. So both $(n+1)$ and $(n-1)$ cannot be perfect squares, hence there is no positive integer n for which $\sqrt{n-1}+\sqrt{n+1}$ is rational.

CHAPTER 2

POLYNOMIALS

ONE MARK QUESTIONS

Multiple Choice Questions

1. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

VIDEO Click Here

Ans : [Board 2020 Delhi Basic]

We have $p(x) = kx^2 + 3x + k$

Since, 2 is a zero of the quadratic polynomial

$$p(2) = 0$$

$$k(2)^2 + 3(2) + k = 0$$

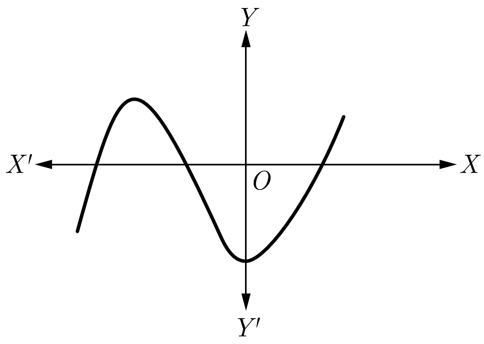
$$4k + 6 + k = 0$$

$$5k = -6 \Rightarrow k = -\frac{6}{5}$$

Thus (d) is correct option.

2. The graph of a polynomial is shown in Figure, then the number of its zeroes is

b173



(a) 3 (b) 1

(c) 2 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, the graph cuts the x-axis at 3 points, the number of zeroes of polynomial $p(x)$ is 3.

Thus (a) is correct option.

3. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4

(c) 2 (d) 3

VIDEO Click Here

Ans : [Board 2020 OD Basic]

A cubic polynomial has maximum 3 zeroes because its degree is 3.

Thus (d) is correct option.

4. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10

(c) -7 (d) -2

Ans : [Board 2020 Delhi Standard]

b175

We have $p(x) = x^2 + 3x + k$

If 2 is a zero of $p(x)$, then we have

$$p(2) = 0$$

$$(2)^2 + 3(2) + k = 0$$

$$4 + 6 + k = 0$$

$$10 + k = 0 \Rightarrow k = -10$$

Thus (b) is correct option.

5. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Ans : [Board 2020 Delhi Standard]

Let α and β be the zeroes of the quadratic polynomial, then we have

b176

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

Now $$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6$$

Thus (a) is correct option.

6. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then value of k is

(a) 3 (b) -3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

b177

Ans : [Board 2020 OD Basic]

Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes, $\alpha \cdot \frac{1}{\alpha} = \frac{\text{constant}}{\text{coefficient of } x^2}$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Thus (a) is correct option.

7. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Ans : [Board 2020 OD Standard]

We have $p(x) = x^2 - 3x - m(m+3)$

b178

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2 - 3(-m) - m(m+3)$$

$$= m^2 + 3m - m^2 - 3m = 0$$

Thus $x = -m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2 - 3(m+3) - m(m+3)$$

$$= (m+3)[m+3-3-m]$$

$$= (m+3)[0] = 0$$

Thus $x = m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.

Thus (b) is correct option.

8. The value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously, is

(a) 2 (b) -2

(c) -1 (d) 1

b179

Ans :

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x = 1$.

Thus (d) is correct option.

9. If α and β are zeroes and the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ is

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$

b180

(c) 4 (d) 15

Ans :

We have $f(x) = x^2 - x - 4$

$$\alpha + \beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1} - 4$$

Now $$\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

Thus (a) is correct option.

10. The value of the polynomial $x^8 - x^5 + x^2 - x + 1$ is

(a) positive for all the real numbers

(b) negative for all the real numbers

(c) 0

(d) depends on value of x

Ans :

We have $f(x) = x^8 - x^5 + x^2 - x + 1$

$f(x)$ is always positive for all $x > 1$

For $x = 1$ or 0, $f(x) = 1 > 0$

For $x < 0$ each term of $f(x)$ is positive, thus $f(x) > 0$. Hence, $f(x)$ is positive for all real x.

Thus (a) is correct option.

11. Lowest value of $x^2 + 4x + 2$ is

(a) 0 (b) -2

(c) 2 (d) 4

b182

Ans :

$$x^2 + 4x + 2 = (x^2 + 4x + 4) - 2$$

$$= (x+2)^2 - 2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2 - 2$ is -2 when $x + 2 = 0$.

Thus (b) is correct option.

12. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is

(a) 2 (b) -2

(c) 4 (d) -4

b183

Ans :

Sum of the zeroes, $6 = \frac{3k}{2}$

$$k = \frac{12}{3} = 4$$

Thus (c) is correct option.

13. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then the value of p is

(a) ± 9 (b) ± 12

(c) ± 15 (d) ± 18

b184

Ans :

We have $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition, we have

$$(\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$p^2 = 144 + 180 = 324 \Rightarrow p = \pm 18$$

14. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

b185

Ans :

If a is zero of quadratic polynomial $f(x)$, then

$$f(a) = 0$$

So, $f(-3) = (k-1)(-3)^2 + (-3)k + 1$

$$0 = (k-1)(9) - 3k + 1$$

$$0 = 9k - 9 - 3k + 1$$

$$0 = 6k - 8$$

$$k = \frac{8}{6} = \frac{4}{3}$$

Thus (a) is correct option.

15. A quadratic polynomial, whose zeroes are -3 and 4, is

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Ans :

b186

We have $\alpha = -3$ and $\beta = 4$.

Sum of zeros $\alpha + \beta = -3 + 4 = 1$

Product of zeros, $\alpha \cdot \beta = -3 \times 4 = -12$

So, the quadratic polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 1 \times x + (-12)$$

$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Thus (c) is correct option.

16. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(a) $a = -7, b = -1$

(b) $a = 5, b = -1$

(c) $a = 2, b = -6$

(d) $a = 0, b = -6$

b187

Ans :

If a is zero of the polynomial, then $f(a) = 0$.

Here, 2 and -3 are zeroes of the polynomial $x^2 + (a+1)x + b$

So, $f(2) = (2)^2 + (a+1)(-3) + b = 0$

$$4 + 2a + 2 + b = 0$$

$$6 + 2a + b = 0$$

$$2a + b = -6 \qquad ...(1)$$

Again, $f(-3) = (-3)^2 + (a+1)2 + b = 0$

$$9 - 3(a+1) + b = 0$$

$$9 - 3a - 3 + b = 0$$

$$6 - 3a + b = 0$$

$$-3a + b = -6$$

$$3a - b = 6 \qquad ...(2)$$

Adding equations (1) and (2), we get

$$5a = 0 \Rightarrow a = 0$$

Substituting value of a in equation (1), we get

$$b = -6$$

Hence, $a = 0$ and $b = -6$.

Thus (d) is correct option.

17. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal

Ans :

Let $$f(x) = x^2 + 99x + 127$$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = 99$ and $c = 127$.

Sum of zeroes $$\alpha + \beta = \frac{-b}{a} = -99$$

Product of zeroes $$\alpha\beta = \frac{c}{a} = 127$$

Now, product is positive and the sum is negative, so both of the numbers must be negative.

Alternative Method :

Let $$f(x) = x^2 + 99x + 127$$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = 99$ and $c = 127$.

Now by discriminant rule,

$$D = \sqrt{b^2 - 4ac}$$

$$= \sqrt{(99)^2 - 4 \times 1 \times 127}$$

$$= \sqrt{9801 - 508} = \sqrt{9293}$$

$$= 96.4$$

So, the zeroes of given polynomial,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-99 \pm \sqrt{96.4}}{2}$$

Now,as $$99 > 96.4$$

So, both zeroes are negative.

Thus (b) is correct option.

18. The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal

Ans :

Let $$f(x) = x^2 + kx + k,\ k \neq 0$$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = k$ and $c = k$.

Again, let if α, β be the zeroes of given polynomial then,

$$\alpha + \beta = -k$$

$$\alpha\beta = k$$

Case 1: If k is negative, then $\alpha\beta$ is negative. It means α and β are of opposite sign.

Case 2: If k is positive, then $\alpha + \beta$ must be negative and $\alpha\beta$ must be positive and α and β both negative.

Hence, α and β cannot both positive.

Thus (a) is correct option.

19. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have same sign

(d) c and b have the same sign

Ans :

Let $$f(x) = ax^2 + bx + c$$

Let α and β are zeroes of this polynomial

Then, $$\alpha + \beta = -\frac{b}{a}$$

and $$\alpha\beta = \frac{c}{a}$$

Since $\alpha = \beta$, then α and β must be of same sign i.e. either both are positive or both are negative. In both case

$$\alpha\beta > 0$$

$$\frac{c}{a} > 0$$

Both c and a are of same sign.

Thus (c) is correct option.

20. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

Ans :

Let $\qquad f(x) = x^2 + ax + b$

and let the zeroes of $f(x)$ are α and β,

As one of zeroes is negative of other,

sum of zeroes $\qquad \alpha + \beta = \alpha + (-\alpha) = 0 \quad ...(1)$

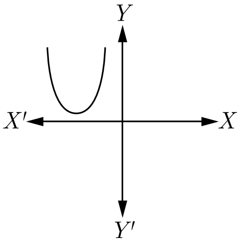
and $\qquad \alpha\beta = \alpha \cdot (-\alpha) = -\alpha^2 \; ...(2)$

Hence, the given quadratic polynomial has no linear term and the constant term is negative.

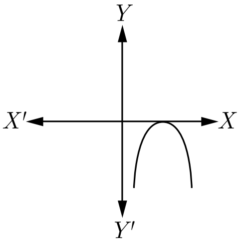
Thus (a) is correct option.

21. Which of the following is not the graph of a quadratic polynomial?

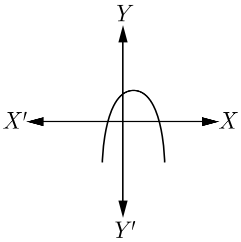
(a)



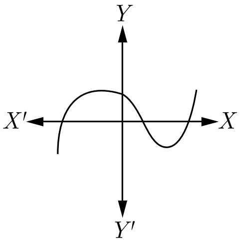
(b)



(c)



(d)



Ans :

As the graph of option (d) cuts x-axis at three points. So, it does not represent the graph of quadratic polynomial.

Thus (d) is correct option.

22. Assertion : $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$.

Reason : Irrational zeros (roots) always occurs in pairs.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As irrational roots/zeros always occurs in pairs therefore, when one zero is $(2-\sqrt{3})$ then other will be $2+\sqrt{3}$. So, both A and R are correct and R explains A.

Thus (a) is correct option.

23. Assertion : If one zero of poly-nominal $p(x) = (k^2+4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason : If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$ i.e. α is a zero of $p(x)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$$

$$1 = \frac{4k}{k^2+4}$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0 \Rightarrow k = 2$$

Assertion is true Since, Reason is not correct for Assertion.

Thus (b) is correct option.

24. Assertion : $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is a polynomial of degree 3.

Reason : The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

The highest power of x in the polynomial $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is 4. Degree is 4. So, A is incorrect but R is correct.

Thus (d) is correct option.

25. Assertion : $x^3 + x$ has only one real zero.

Reason : A polynomial of nth degree must have n real zeroes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

A polynomial of nth degree at most can have n real zeroes. Thus reason is not true.

b196

Again, $x^3 + x = x(x^2 + 1)$

which has only one real zero because $x^2 + 1 \neq 0$ for all $x \in R$.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

26. Assertion : If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.

Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As the polynomial is $x^2 - 2kx + 2$ and its zeros are equal but opposition sign, sum of zeroes must be zero.

$$\text{sum of zeros} = 0$$

$$\frac{-(-2k)}{1} = 0 \Rightarrow k = 0$$

b197

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

27. A .... polynomial is of degree one.

Ans :

b198

Linear

28. A cubic polynomial is of degree..........

Ans :

Three

b199

29. Degree of remainder is always .... than degree of divisor.

Ans :

Smaller/less

b200

30. Polynomials of degrees 1, 2 and 3 are called .... , ... and .... polynomials respectively.

Ans :

linear, quadratic, cubic

b201

31. .... is not equal to zero when the divisor is not a factor of dividend.

Ans :

Remainder

b202

32. The zeroes of a polynomial $p(x)$ are precisely the $x-$coordinates of the points, where the graph of $y = p(x)$ intersects the .... axis.

Ans :

x

b203

33. The algebraic expression in which the variable has non-negative integral exponents only is called ....

Ans :

Polynomial

b204

34. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most .... zeroes.

Ans :

3

b205

35. A .... is a polynomial of degree 0.

Ans :

Constant

b206

36. The highest power of a variable in a polynomial is called its ....

Ans :

Degree

b207

37. A polynomial of degree n has at the most .... zeroes.

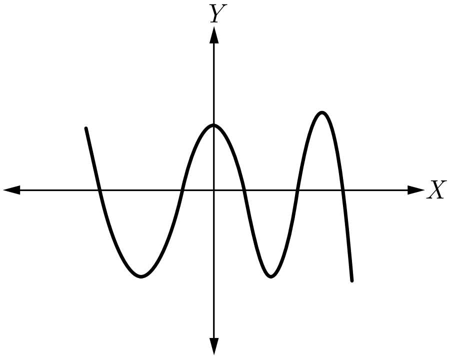
Ans :

n

b208

38. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows.

b209



The number of zeroes of $p(x)$ is ....

Ans : [Board 2020 SQP Standard]

The graph of the given polynomial $p(x)$ crosses the x-axis at 5 points. So, number of zeroes of $p(x)$ is 5.

39. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other then the value of k is ....

Ans : [Board 2020 SQP Standard]

We have $(k-1)x^2 - 10x + 3 = 0$

Let one root be α, then another root will be $\frac{1}{\alpha}$

Now $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{3}{(k-1)}$

$$1 = \frac{3}{(k-1)}$$

$$k - 1 = 3 \Rightarrow k = 4$$

b210

Very Short Answer Questions

40. If α and β are the roots of $ax^2 - bx + c = 0 \, (a \neq 0)$, then calculate $\alpha + \beta$.

Ans : [Board Term-1 2014]

We know that

$$\text{Sum of the roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

b101

Thus $\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$

41. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Ans : [Board Term-1 2010]

$$p(x) = 4x^2 - 12x + 9$$

$$= 4x^2 - 6x - 6x + 9$$

$$= 2x(2x-3) - 3(2x-3)$$

b103

$$= (2x-3)(2x-3)$$

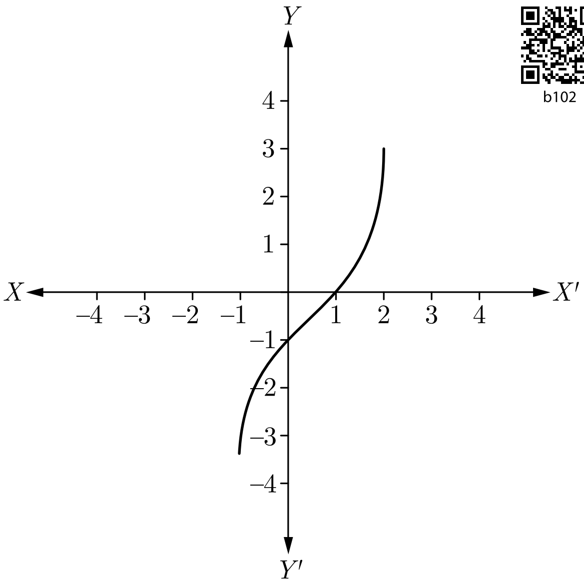
Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$x = \frac{3}{2}, \frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$.

42. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.

b102



Ans : [Board Term-1 2013]

The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

43. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans : [Board 2009]

We have $p(x) = 3x^2 - kx - 6$

b104

$$\text{Sum of the zeroes} = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $3 = -\frac{(-k)}{3} \Rightarrow k = 9$

44. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $f(x) = x^2 - 7x - 8$

b105

Let other zero be k, then we have

Sum of zeroes, $-1 + k = -\left(\frac{-7}{1}\right) = 7$

or $k = 8$

TWO MARKS QUESTIONS

45. If zeroes of the polynomial $x^2+4x+2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1 2016]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{2}{\alpha} = 2$$

or, $2a = 2$

Thus $a = 1$

46. Find all the zeroes of $f(x) = x^2 - 2x$.

Ans : [Board Term-1 2013]

We have $f(x) = x^2 - 2x$

$= x(x-2)$

Substituting $f(x) = 0$, and solving we get $x = 0, 2$

Hence, zeroes are 0 and 2.

47. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1 2013]

We have $p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$

Substituting $p(x) = 0$, we have

$$(\sqrt{3}x - 2)(x - 2\sqrt{3}) \;\; p(x) = 0$$

Solving we get $x = \frac{2}{\sqrt{3}}, 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

48. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1 2016]

Sum of zeroes, $\alpha + \beta = 6$

Product of zeroes $\alpha\beta = 9$

Now $p(x) = x^2 - (\alpha+\beta)x + \alpha\beta$

Thus $= x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $p(x) = x^2 - 6x + 9$

$= (x-3)(x-3)$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

49. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1 2012, Set-35]

Sum of zeroes, $\alpha + \beta = \frac{21}{8}$

Product of zeroes $\alpha\beta = \frac{5}{16}$

Now $p(x) \;\; x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - \frac{21}{8}x + \frac{5}{16}$

or $p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

50. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1 2012]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $p(x) \;\; x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - 3x - \frac{2}{5}$

$= \frac{1}{5}(5x^2 - 15x - 2)$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

51. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1 2012]

We have $\frac{m}{n} + \frac{n}{m} = \frac{m^2+n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn}$ (1)

Sum of zeroes $m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\frac{m}{n} + \frac{n}{m} = \frac{(m+n)^2 - 2mn}{mn}$$

$$= \frac{\left(-\frac{11}{3}\right)^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}}$$

$$= \frac{121 + 4 \times 3 \times 2}{-4 \times 3}$$

or $$\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$$

52. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

Ans : [Board Term-1 2012]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

b114

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$$

Hence $p^2 + q^2 = \frac{37}{4}$.

53. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Ans : [Board Term-1 2012]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

b115

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

54. Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

Ans : [Board Term-1 2012]

We have $p(x) = kx^2 - 4x + k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$

b116

$$k(-1)^2 - 4(-1) + k = 0$$

$$k + 4 + k = 0$$

$$2k + 4 = 0$$

$$2k = -4$$

Hence, $k = -2$

55. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Ans : [Board Term-1 2015]

We have $p(x) = x^2 - 4\sqrt{3}\,x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha + \beta = 4\sqrt{3}$

b117

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$.

56. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Ans : [Board Term-1 2013]

We have $p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a \Rightarrow b = -2a = -2$

57. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Ans : [Board Term-1 2015]

We have $f(x) = x^2 - 6x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$$(6)^2 - 2k = 40$$

$$36 - 2k = 40$$

b119

$$-2k = 4$$

Thus $k = -2$

58. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1 2012]

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

b120

Sum of zeroes $0 = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

59. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1 2012]

Let, the zero of $2x^2 + 3x + \lambda$ be $\dfrac{1}{2}$ and β.

b121

Product of zeroes $\dfrac{c}{a}$, $\dfrac{1}{2}\beta = \dfrac{\lambda}{2}$

or, $\beta = \lambda$

and sum of zeroes $-\dfrac{b}{a}$, $\dfrac{1}{2} + \beta = -\dfrac{3}{2}$

or $\beta = -\dfrac{3}{2} - \dfrac{1}{2} = -2$

Hence $\lambda = \beta = -2$

Thus other zero is -2.

60. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1 2013, Set FFC]

We have $f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

b122

Sum of zeroes, $\alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$$\alpha + \beta = 1 \qquad ...(1)$$

Given $\alpha - \beta = 9 \qquad ...(2)$

Solving (1) and (2) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $k = 20$

61. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1 2012, Set-39]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

b123

Sum of zeroes $\alpha + \beta = \dfrac{5}{2}$

Product of zeroes $\alpha\beta = -\dfrac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $2\alpha + 2\beta = \dfrac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $p = -5$

Product of zeroes, $2\alpha 2\beta = \dfrac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

62. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \dfrac{1}{2}\alpha\beta$.

Ans : [KVS Practice Test 2017]

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha + \beta = -[-(k-6)] = k-6$$

$$\alpha\beta = 2(2k-1)$$

b124

Now $\alpha + \beta = \frac{1}{2}\alpha\beta$

Thus $k+6 = \frac{2(2k-1)}{2}$

or, $k-6 = 2k-1$

$$k = -5$$

Hence the value of k is -5.

THREE MARKS QUESTIONS

63. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Ans : [Board 2020 Delhi Standard]

Let α and β be zeros of the given polynomial $ax^2 + bx + c$.

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Let $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ be the zeros of new polynomial then we have

b216

Sum of zeros, $s = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$$= \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

Product of zeros, $p = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$

Required polynomial,

$$g(x) = x^2 - sx + p$$

$$g(x) = x^2 + \frac{b}{c}x + \frac{a}{c}$$

$$cg(x) = cx^2 + bx + a$$

$$g'(x) = cx^2 + bx + a$$

64. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Ans : [Board Term-1 2013, LK-59]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$$= 50 - 50$$

b125

or $p(2) = 0$

(2) 3, $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$= 54 - 99 + 51 - 6$$

$$= 105 - 105$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

65. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha + \beta = -\frac{-5}{a} = \frac{5}{a}$

b126

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

66. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

b127

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

67. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2 2015]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

b128

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$

$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$

$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

68. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1 2011]

We have $p(y) = 6y^2 - 7y + 2$

Sum of zeroes $\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

Product of zeroes $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

b129

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$g(x) = y^2 - \frac{7}{2}y + 3$$

$$= \frac{1}{2}[2y^2 - 7y + 6]$$

69. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1 2011]

We have $p(x) = 4x^2 + 4x - 3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$

b130

$$= 1 + 2 - 3 = 0$$

and $p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right) + 4\left(-\frac{3}{2}\right) - 3$

$$= 9 - 6 - 3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2} \qquad \text{Verified}$$

70. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x+3$, $3x^2+7x+2$, $4x^3+3x^2+2$, $x^3+\sqrt{3x}+7$, $7x+\sqrt{7}$, $5x^3-7x+2$, $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$, ax^3+bx^2+cx+d, $x+\frac{1}{x}$.

b219

Answer the following question :

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

Ans : [Board 2020 OD Standard]

(i) $x^3+\sqrt{3x}+7, 2x^2+3-\dfrac{5}{x}$ and $x+\dfrac{1}{x}$ are not polynomials.

(ii) $3x^2+7x+2$ is only one quadratic polynomial.

71. Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1 2015]

We have $\quad p(x)x^2-2\sqrt{2}\,x = 0$

$$x(x-2\sqrt{2}) = 0$$

b131

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $\quad 2\sqrt{2} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $\quad 0 = \dfrac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

72. Find the zeroes of the quadratic polynomial $5x^2+8x-4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1 2013, Set LK-59]

We have $\quad p(x) = 5x^2+8x-4 = 0$

$$= 5x^2+10x-2x-4 = 0$$

$$= 5x(x+2)-2(x+2) = 0$$

$$= (x+2)(5x-2)$$

Substituting $p(x) = 0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2+\frac{2}{5} = \frac{-8}{5}$$

b132

$$\text{Product of zeroes} = (-2)\times\left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

Sum of zeroes $\quad -\dfrac{b}{a} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \dfrac{-8}{5}$

Product of zeroes $\quad \dfrac{c}{a} = \dfrac{\text{Constan term}}{\text{Coefficient of } x^2} = -\dfrac{4}{5}$

Hence Verified.

73. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta = 24$ and $\alpha-\beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1 2011, Set-44]

We have $\quad \alpha+\beta = 24 \qquad ...(1)$

$$\alpha-\beta = 8 \qquad ...(2)$$

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

b133

Subtracting (1) from (2) we have

$$2\beta = 16 \Rightarrow \beta = 8$$

Hence, the quadratic polynomial

$$p(x) = x^2-(\alpha+\beta)x+\alpha\beta$$

$$= x^2-(16+8)x+(16)(8)$$

$$= x^2-24x+128$$

74. If α, β and γ are zeroes of the polynomial $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.

Ans : [KVS practice Test 2017, Board 2010]

We have $\quad p(x) = 6x^3+3x^2-5x+1$

Since α, β and γ are zeroes polynomial $p(x)$, we have

$$\alpha+\beta+\gamma = -\frac{b}{c} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

b134

and $\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$

Now $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6} \times \frac{6}{-1} = 5$$

Hence $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

75. When $p(x) = x^2 + 7x + 9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1 2011]

We have $p(x) = x^2 + 7x + 9$

$q(x) = x + 2$

$r(x) = -1$

b153

Now $p(x) = g(x)q(x) + r(x)$

$$x^2 + 7x + 9 = g(x)(x+2) - 1$$

or, $g(x) = \frac{x^2 + 7x + 10}{x + 2}$

$$= \frac{(x+2)(x+5)}{(x+2)} = x + 5$$

Thus $g(x) = x + 5$

76. Find the value for k for which $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by $x + 7$.

Ans : [Board Term 2010]

We have $f(x) = x^4 + 10x^3 + 25x^2 + 15x + k$

If $x + 7$ is a factor then -7 is a zero of $f(x)$ and $x = -7$ satisfy $f(x) = 0$.

Thus substituting $x = -7$ in $f(x)$ and equating to zero we have,

$$(-7)^4 + 10(-7)^3 + 25(-7)^2 + 15(-7) + k = 0$$

$$2401 - 3430 + 1225 - 105 + k = 0$$

$$3626 - 3535 + k = 0$$

$$91 + k = 0$$

$$k = -91$$

b167

77. On dividing the polynomial $4x^4 - 5x^3 - 39x^2 -$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. Find the polynomial $g(x)$.

b170

Ans : [Board Term 2009]

Dividend = (Divisor × Quotient) + Remainder

$$4x^4 - 5x^3 - 39x^3 - 46x - 2$$
$$= g(x)(x^2 - 3x - 5) + (-5x + 8)$$

$$4x^2 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$$
$$= g(x)(x^2 - 3x - 5)$$

$$4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$

Hence, $g(x) = 4x^2 + 7x + 2$

78. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2 + px + 45$ is equal to 144, find the value of p.

Ans : [Board 2008]

We have $f(x) = x^2 + px + 45$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $\alpha + \beta = -p$

Product of zeroes $\alpha\beta = 45$

b171

Given, $(\alpha - \beta)^2 = 144$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

Substituting value of $\alpha + \beta$ and $\alpha\beta$ we get

$$(-p)^2 - 4 \times 45 = 144$$

$$p^2 - 180 = 144$$

$$p^2 = 144 + 180 = 324$$

Thus $p = \pm\sqrt{324} = \pm 18$

Hence, the value of p is ± 18.

FOUR MARKS QUESTIONS

79. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q.

Ans : [Board Term-1 2015]

We have $f(x) = x^4 + 7x^3 + 7x^2 + px + q$

Now $x^2 + 7x + 12 = 0$

b135

$$x^2+4x+3x+12 = 0$$
$$x(x+4)+3(x+4) = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4, -3$$

Since $f(x) = x^4+7x^3+7x^2+px+q$ is exactly divisible by $x^2+7x+12$, then $x=-4$ and $x=-3$ must be its zeroes and these must satisfy $f(x)=0$

So putting $x=-4$ and $x=-3$ in $f(x)$ and equating to zero we get

$$f(-4):\ (-4)^4+7(-4)^3+7(-4)^2+p(-4)+q = 0$$
$$256-448+112-4p+q = 0$$
$$-4p+q-80 = 0$$
$$4p-q = -80 \qquad ...(1)$$
$$f(-3):\ (-3)^4+7(-3)^3+7(-3)^2+p(-3)+q = 0$$
$$81-189+63-3p+q = 0$$
$$-3p+q-45 = 0$$
$$3p-q = -45 \qquad ...(2)$$

Subtracting equation (2) from (1) we have

$$p = -35$$

Substituting the value of p in equation (1) we have

$$4(-35)-q = -80$$
$$-140-q = -80$$
$$-q = 140-80$$

or
$$-q = 60$$
$$q = -60$$

Hence, $p=-35$ and $q=-60$.

80. If α and β are the zeroes of the polynomial $p(x) = 2x^2+5x+k$ satisfying the relation, $\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$, then find the value of k.

Ans : [Board Term-1 2012]

We have $p(x) = 2x^2+5x+k$

Sum of zeroes, $\alpha+\beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{k}{2}$

b136

According to the question,

$$\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$$
$$\alpha^2+\beta^2+2\alpha\beta-\alpha\beta = \frac{21}{4}$$
$$(\alpha+b)^2-\alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2-\frac{k}{2} = \frac{21}{4}$$
$$\frac{k}{2} = \frac{25}{4}-\frac{21}{4}$$
$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

81. If α and β are the zeroes of polynomial $p(x) = 3x^2+2x+1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1 2010, 2012]

We have $p(x) = 3x^2+2x+1$

Since α and β are the zeroes of polynomial $3x^2+2x+1$, we have

$$\alpha+\beta = -\frac{2}{3}$$

b137

and
$$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = \frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}$$
$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$
$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$
$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1$$

$$= x^2 - 2x + 3$$

82. If α and β are the zeroes of the polynomial x^2+4x+3, find the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

Ans : [Board Term-1 2013]

b138

We have $p(x) = x^2+4x+3$

Since α and β are the zeroes of the quadratic polynomial x^2+4x+3,

So, $\alpha+\beta = -4$

and $\alpha\beta = 3$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = 1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

83. If α and β are zeroes of the polynomial $p(x) = 6x^2-5x+k$ such that $\alpha-\beta = \frac{1}{6}$, Find the value of k.

Ans : [Board 2007]

b139

We have $p(x) = 6x^2-5x+k$

Since α and β are zeroes of

$$p(x) = 6x^2-5x+k,$$

Sum of zeroes, $\alpha+\beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha-\beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2}\times\frac{1}{3}$$

Hence, $k = 1$.

84. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2+a)x^2+61x+6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2+a)x^2+61x+6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta \frac{1}{\beta} = \frac{6a}{a^2 + a}$

b140

or, $1 = \frac{6}{a+1}$

$a + 1 = 6$

$a = 5$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \frac{-61 \pm \sqrt{(-61)^2 - 4 \times 30 \times 30}}{2 \times 30}$

$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

85. If α and β are the zeroes the polynomial $2x^2 - 4x + 5$, find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$ (iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^2 + \beta^2$

Ans : [Board 2007]

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

b169

and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= 2^2 - 2 \times \frac{5}{2}$$

$$= 4 - 5 = -1$$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$

$$= 2^2 - \frac{4 \times 5}{2}$$

$$4 - 10 = -6$$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$= 2^3 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$$

CHAPTER 3

PAIR OF LINEAR EQUATION IN TWO VARIABLES

ONE MARK QUESTIONS

Multiple Choice Questions

1. The value of k for which the system of linear equations $x+2y=3,\ 5x+ky+7=0$ is inconsistent is

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$

(c) 5 (d) 10

c183

Ans : [Board 2020 OD Standard]

We have $x+2y-3=0$

and $5x+ky+7=0$

If system is inconsistent, then

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5}=\frac{2}{k}\Rightarrow k=10$$

Thus (d) is correct option.

2. The value of k for which the system of equations $x+y-4=0$ and $2x+ky=3$, has no solution, is

(a) -2 (b) $\neq 2$

(c) 3 (d) 2

Ans : [Board 2020 Delhi Standard]

c184

We have $x+y-4=0$

and $2x+ky-3=0$

Here, $\frac{a_1}{a_1}=\frac{1}{2},\ \frac{b_1}{b_2}=\frac{1}{k}$ and $\frac{c_1}{c_2}=\frac{-4}{-3}=\frac{4}{3}$

Since system has no solution, we have

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{k}\neq\frac{4}{3}$$

$$k=2 \text{ and } k\neq\frac{3}{4}$$

Thus (d) is correct option.

3. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$$3x-y-5=0$$

$$6x-2y-p=0$$

(a) all real values except 10 (b) 10

(c) 5/2 (d) 1/2

Ans :

c185

We have, $3x-y-5=0$

and $6x-2y-p=0$

Here, $a_1=3,\ b_1=-1,\ c_1=-5$

and $a_2=6,\ b_2=-2,\ c_2=-p$

Since given lines are parallel,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{-1}{-2}\neq\frac{-5}{-p}$$

$$p\neq 5\times 2\Rightarrow p\neq 10$$

4. The 2 digit number which becomes $\frac{5}{6}$th of itself when its digits are reversed. The difference in the digits of the number being 1, then the two digits number is

(a) 45 (b) 54

(c) 36 (d) None of these

Ans :

If the two digits are x and y, then the number is $10x+y$.

c186

Now $\frac{5}{6}(10x+y)=10y+x$

Solving, we get $44x+55y$

$$\frac{x}{y}=\frac{5}{4}$$

Also $x-y=1$. Solving them, we get $x=5$ and $y=4$. Therefore, number is 54.

Thus (b) is correct option.

5. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is

(a) 36 (b) 63

(c) 48 (d) 84

Ans :

Let x be units digit and y be tens digit, then number will be $10y+x$

Then, $x = 2y$ ...(1)

If 36 be added to the number, the digits are reversed, i.e number will be $10x+y$.

$$10y+x+36 = 10x+y$$

$$9x-9y = 36$$

$$x-y = 4 \quad ...(2)$$

c187

Solving (1) and (2) we have $x=8$ and $y=4$.

Thus number is 48.

Thus (c) is correct option.

6. If $3x+4y:x+2y=9:4$, then $3x+5y:3x-y$ is equal to

(a) 4 : 1 (b) 1 : 4

(c) 7 : 1 (d) 1 : 7

Ans :

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$

Hence, $12x+16y = 9x+18y$

or $3x = 2y$

$$x = \frac{2}{3}y$$

c188

Substituting $x=\frac{2}{3}y$ in the required expression we have

$$\frac{3x\frac{2}{3}y+5y}{3x\frac{2}{3}y-y} = \frac{7y}{y} = \frac{7}{1} = 7:1$$

Thus (c) is correct option.

7. A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given frac[illegible]

(a) 2 (b) 3

(c) 5 (d) 15

c189

Ans :

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y+3 \quad ...(1)$$

and $$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y-6 \quad ...(2)$$

Solving (1) and (2), we have $x = 15$, $y=3$,

Thus (d) is correct option.

8. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of $x+y$ is

(a) 10 (b) 11

(c) 12 (d) 13

c190

Ans :

The numbers that can be formed are xy and yx. Hence, $(10x+y)+(10y+x)=11(x+y)$. If this is a perfect square than $x+y=11$.

9. The pair of equations $3^{x+y}=81$, $81^{x-y}=3$ has

(a) no solution

(b) unique solution

(c) infinitely many solutions

(d) $x=2\frac{1}{8}, y=1\frac{7}{8}$

Ans :

c191

Given, $3^{x+y} = 81$

$$3^{x+y} = 3^4$$

$$x+y = 4 \quad ...(1)$$

and $81^{x-y} = 3$

$$3^{4(x-y)} = 3^1$$

$$4(x-y) = 1$$

$$x-y = \frac{1}{4} \quad ...(2)$$

Adding equation (1) and (2), we get

$$2x = 4+\frac{1}{4} = \frac{17}{4}$$

$$x = \frac{17}{8} = 2\frac{1}{8}$$

From equation (1), we get

$$y = \frac{15}{8} = 1\frac{7}{8}$$

Thus (d) is correct option.

10. The pair of linear equations $2kx+5y=7$, $6x-5y=11$

has a unique solution, if

(a) $k \neq -3$ (b) $k \neq \frac{2}{3}$

(c) $k \neq 5$ (d) $k \neq \frac{2}{9}$

Ans :

Given the pair of linear equations are

c192

$$2kx + 5y - 7 = 0$$

and $$6x - 5y - 11 = 0$$

On comparing with

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

we get, $a_1 = 2k,\ b_1 = 5,\ c_1 = -7$

and $a_2 = 6,\ b_2 = -5,\ c_2 = -11$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2k}{6} \neq \frac{5}{-5}$$

$$\frac{k}{3} = \neq -1$$

$$k \neq -3$$

Thus (a) is correct option.

11. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Ans :

Given,equations are

c193

$$x + 2y + 5 = 0$$

and $$-3x - 6y + 1 = 0$$

Here, $a_1 = 1, b_1 = 2,\ c_1 = 5$

and $a_2 = -3,\ b_2 = -6,\ c_2 = 1$

Now $$\frac{a_1}{a_2} = -\frac{1}{3},\ \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},\ \frac{c_1}{c_2} = \frac{5}{1}$$

Now, we observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Thus (d) is correct option.

12. If a pair of linear equations is consistent, then the lines will be

(a) parallel

(b) always coincident

(c) intersecting or coincident

(d) always intersecting

c194

Ans :

Condition for a consistent pair of linear equations

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

[intersecting lines having unique solution]

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$ [coincident or dependent]

Thus (c) is correct option.

13. The pair of equations $y = 0$ and $y = -7$ has

(a) one solution

(b) two solutions

(c) infinitely many solutions

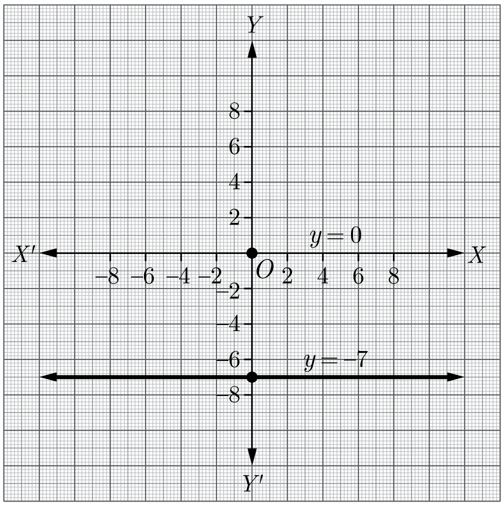
(d) no solution

c195

Ans :

The given pair of equations are

$$y = 0 \quad y = -7$$



The pair of both equations are parallel to x-axis and we know that parallel lines never intersects. So, there is no solution of these lines.

Thus (d) is correct option.

14. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel (b) intersecting at (b, a)

(c) coincident (d) intersecting at (a, b)

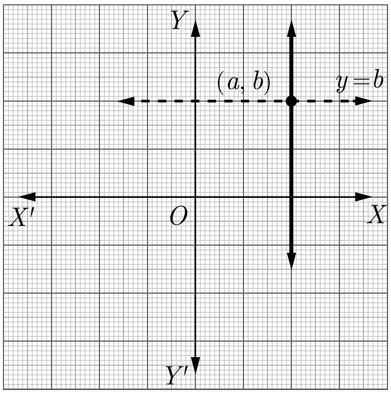
Ans :

The pair of equations

c196

$$x = a$$

and $$y = b$$



Graphically represents lines which are intersecting at (a, b).

Thus (d) is correct option.

15. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines ?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) -2

Ans :

Given, equations,

c197

$$3x - y + 8 = 0$$

and $$6x - ky + 16 = 0$$

Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(1)$$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From equation (1),

$$\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$$

$$\frac{1}{k} = \frac{1}{2} \quad \left[\text{since } \frac{3}{6} = \frac{8}{16} = \frac{1}{2}\right]$$

$$k = 2$$

Thus (c) is correct option.

16. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$

(c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Ans :

We have $3x + 2ky - 2 = 0$

c198

and $2x + 5y + 1 = 0$

Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = 1$

From equation (i), we have

$$\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

Considering, $$\frac{3}{2} = \frac{2k}{5} \quad \left[\frac{3}{2} \neq \frac{-2}{1} \text{ in any case}\right]$$

$$k = \frac{15}{4}$$

Thus (c) is correct option.

17. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have is

(a) 3 (b) -3

(c) -12 (d) no value

Ans :

The given lines are, $cx - y = 2$

c199

and $6x - 2y = 3$

Condition for infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad ...(i)$$

Here, $a_1 = c,\ b_1 = -1,\ c_1 = -2$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -3$

From equation (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $$\frac{c}{6} = \frac{1}{2}$$

and $$\frac{c}{6} = \frac{2}{3}$$

$$c = 3$$

and $$c = 4$$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Thus (d) is correct option.

18. One equation of a pair of dependent linear equations $-5x+7y=2$ The second equation can be

(a) $10x+14y+4=0$ (b) $-10x-14y+4=0$

(c) $-10x+14y+4=0$ (d) $10x-14y=-4$

Ans :

For dependent linear equation,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

c200

Checking for option (a):

$$\frac{-5}{10} \neq \frac{7}{14}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$ So, option (a) is rejected.

Checking for option (b):

$$\frac{-5}{-10} \neq \frac{7}{-14}$$

So, option (b) is also rejected.

Checking for option (c):

$$\frac{-5}{-10} = \frac{7}{14} \neq \frac{-2}{4}$$

So, option (b) is also rejected

Checking for option (d):

$$\frac{-5}{10} = \frac{7}{-14} = \frac{-2}{4}$$

Thus (d) is correct option.

19. If $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3

(c) 3 and 1 (d) -1 and -3

Ans :

Since, $x = a$ and $y = b$ is the solution of the equations $x-y = 2$ and $x+y = 4$, then these values will satisfy that equation

$$a-b = 2 \quad ...(1)$$

and $$a+b = 4 \quad ...(2)$$

Adding equations (1) and (2), we get

$$2a = 6$$

c201

$$a = 3$$

Substituting $a = 3$ in equation (2),we have

$$3+b = 4 \Rightarrow b = 1$$

Thus $a = 3$ and $b = 1$.

Thus (c) is correct option.

20. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20

(c) 15 and 35 (d) 25 and 25

Ans :

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

c202

Now, by given conditions,

$$x+y = 50 \quad ...(1)$$

Also, $$x \times 1 + y \times 2 = 75$$

$$x+2y = 75 \quad ...(2)$$

Subtracting equation (1) form equation (2), we get

$$(x+2y)-(x+y) = 75-50$$

$$y = 25$$

From equation (i), $x = 75-2x(25)$

Then, $$x = 25$$

Thus (d) is correct option.

21. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively.

(a) 4 and 24 (b) 5 and 30

(c) 6 and 36 (d) 3 and 24

Ans :

Let the present age of father $= x$ years

and the present age of son $= y$ years

Four years hence, it has relation by given condition

$$(x+4) = 4(y+4)$$

$$x-4y = 12 \quad ...(1)$$

As the father's age is six times his son's age, so we have

$$x = 6y \quad ...(2)$$

Putting the value of x from equation (2) in equation (1), we get

$$6x-4y = 12$$

$$2y = 12$$

$$y = 6$$

c203

From equation (1), $x = 6 \times 6$

Then, $x = 36$

Hence, present age of father is 36 year and age of son is 6 year.

Thus (c) is correct option.

22. **Assertion :** Pair of linear equations : $9x+3y+12=0$, $8x+6y+24=0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2=0$ have infinitely many solutions, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

From the given equations, we have

c204

$$\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2} \text{ i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. **Assertion :** $x+y-4=0$ and $2x+ky-3=0$ has no solution if $k=2$.

Reason : $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are consistent if $\frac{a_1}{a_2} \neq \frac{k_1}{k_2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

For assertion, given equation has no solution if

$$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

c205

$$k = 2\left[\frac{1}{2} \neq \frac{4}{3}\text{holds}\right]$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Fill in the Blank Questions

24. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is ....

Ans :

c206

consistent

25. An equation whose degree is one is known as a .... equation.

Ans :

c207

linear

26. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is

Ans :

c208

inconsistent

27. A pair of linear equations has .... solution(s) if it is represented by intersecting lines graphically.

Ans :

c209

unique

28. Every solution of a linear equation in two variables is a point on the .... representing it.

Ans :

c210

line

29. If a pair of linear equations has infinitely many solutions, then its graph is represented by a pair of .... lines.

Ans :

coincident

30. A pair of linear equations is .... if it has no solution.

Ans :

inconsistent

31. A pair of .... lines represent the pair of linear equations having no solution.

Ans :

parallel

32. If a pair of linear equations has solution, either a unique or infinitely many, then it is said to be ....

Ans :

consistent

33. If the equations $kx - 2y = 3$ and $3x + y = 5$ represent two intersecting lines at unique point, then the value of k is ....

Ans :

For unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Here, $a_1 = k$, $b_1 = -2$, $a_2 = 3$ and $b_2 = 1$

Now $$\frac{k}{3} \neq -\frac{2}{1}$$

or, $$k \neq -6$$

Very Short Answer Questions

34. Find whether the pair of linear equations $y = 0$ and $y = -5$ has no solution, unique solution or infinitely many solutions.

Ans :

The given variable y has different values. Therefore the pair of equations $y = 0$ and $y = -5$ has no solution.

35. If $am = bl$, then find whether the pair of linear equations $ax + by = c$ and $lx + my = n$ has no solution, unique solution or infinitely many solutions.

Ans :

Since, $am = bl$, we have

$$\frac{a}{l} = \frac{b}{m} \neq \frac{c}{n}$$

Thus, $ax + by = c$ and $lx + my = n$ has no solution.

36. If $ad \neq bc$, then find whether the pair of linear equations $ax + by = p$ and $cx + dy = q$ has no solution, unique solution or infinitely many solutions.

Ans :

Since $$ad \neq bc \text{ or } \frac{a}{c} \neq \frac{b}{d}$$

Hence, the pair of given linear equations has unique solution.

37. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

Ans :

The equation of one line is $4x + 3y = 14$. We know that if two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or $$\frac{4}{a_2} = \frac{3}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{a_2}{b_2} = \frac{4}{3} = \frac{12}{9}$$

Hence, one of the possible, second parallel line is $12x + 9y = 5$.

TWO MARKS QUESTIONS

38. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

Ans : [Board 2019 OD]

We have $$x + 2y - 5 = 0 \quad ...(1)$$

and $$3x + ky + 15 = 0 \quad ...(2)$$

Comparing equation (1) with $a_1x + b_1y + c_1 = 0$, and equation (2) with $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 1$, $a_2 = 3$, $b_1 = 2$, $b_2 = k$, $c_1 = -5$ and $c_2 = 15$

Since, given equations have unique solution, So,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e. $$\frac{1}{3} \neq \frac{2}{k}$$

$$k \neq 6$$

Hence, for all values of k except 6, the given pair of equations have unique solution.

39. If $2x+y=23$ and $4x-y=19$, find the value of $(5y-2x)$ and $(\frac{y}{x}-2)$.

Ans : [Board 2020 OD Standard]

We have $2x+y = 23$ ...(1)

$4x-y = 19$ ...(2)

Adding equation (1) and (2), we have

$$6x = 42 \Rightarrow x = 7$$

Substituting the value of x in equation (1), we get

$$14+y = 23$$

$$y = 23-14 = 9$$

c217

Hence, $5y-2x = 5\times 9 - 2\times 7$

$$= 45-14 = 31$$

and $\frac{y}{x}-2 = \frac{9}{7}-2 = \frac{9-14}{7} = \frac{-5}{7}$

40. Find whether the lines represented by $2x+y=3$ and $4x+2y=6$ are parallel, coincident or intersecting.

Ans : [Board Term-1 2016, MV98HN3]

Ans :

Here $a_1=2, b_1=1, c_1=-3$ and $a_1=4, b_2=2, c_2=-6$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

then the lines are parallel.

c105

Clearly $\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

41. Find whether the following pair of linear equation is consistent or inconsistent:

$3x+2y=8, \ \ 6x-4y=9$

Ans : [Board Term-1 2016]

We have $\frac{3}{6} \neq \frac{2}{-4}$

c106

i.e., $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the pair of linear equation is consistent.

42. Is the system of linear equations $2x+3y-9=0$ and $4x+6y-18=0$ consistent? Justify your answer.

Ans : [Board Term-1 2012]

For the equation $2x+3y-9=0$ we have

$$a_2 = 2, b_1 = 3 \text{ and } c_1 = -9$$

and for the equation, $4x+6y-18=0$ we have

$$a_2 = 4, b_2 = 6 \text{ and } c_2 = -18$$

Here $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$

c107

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{-9}{-18} = \frac{1}{2}$

Thus $\frac{c_1}{c_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, system is consistent and dependent.

43. Given the linear equation $3x+4y=9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines

(2) coincident lines.

Ans : [Board Term-1 2016, Set-O4YP6G7]

(1) For intersecting lines $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

c108

So, one of the possible equation $3x-5y=10$

(2) For coincident lines $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, one of the possible equation $6x+8y=18$

44. For what value of p does the pair of linear equations given below has unique solution ?

$4x+py+8=0$ and $2x+2y+2=0$.

Ans : [Board Term-1 2012]

We have $4x+py+8 = 0$

c109

$2x+2y+2 = 0$

The condition of unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, $\frac{4}{2} \neq \frac{p}{2}$ or $\frac{2}{1} \neq \frac{p}{2}$

Thus $p \neq 4$. The value of p is other than 4 it may be 1,2,3, -4.....etc.

45. For what value of k, the pair of linear equations $kx-4y=3$, $6x-12y=9$ has an infinite number of solutions ?

Ans : [Board Term-1 2012]

We have $kx - 4y - 3 = 0$

and $6x - 12y - 9 = 0$

where, $a_1 = k, b_1 = 4, c_1 = -3$

$a_2 = 6, b_2 = -12, c_2 - 9$

c110

Condition for infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9}$$

Hence, $k = 2$

46. For what value of k, $2x + 3y = 4$ and $(k+2)x + 6y = 3k + 2$ will have infinitely many solutions ?

Ans : [Board Term-1 2012]

We have $2x + 3y - 4 = 0$

and $(k+2)x + 6y - (3k+2) = 0$

c111

Here $a_1 = 2, b_1 = 3, c_1 = -4$

and $a_2 = k + 2, b_2 = 6, c_3 = -(3k+2)$

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or, $$\frac{2}{k+2} = \frac{3}{6} = \frac{4}{3k+2}$$

From $\frac{2}{k+2} = \frac{3}{6}$ we have

$3(k+2) = 2 \times 6 \Rightarrow (k+2) = 4 \Rightarrow k = 2$

From $\frac{3}{6} = \frac{4}{3k+2}$ we have

$3(3k+2) = 4 \times 6 \Rightarrow (3k+2) = 8 \Rightarrow k = 2$

Thus $k = 2$

47. For what value of k, the system of equations $kx + 3y = 1$, $12x + ky = 2$ has no solution.

Ans : [Board Term-1 2011, NCERT]

The given equations can be written as

$kx + 3y - 1 = 0$ and $12x + ky - 2 = 0$

Here, $a_1 = k, b_1 = 3, c_1 = -1$

and $a_2 = 12, b_2 = k, c_2 = -2$

The equation for no solution if

c112

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

or, $$\frac{k}{12} = \frac{3}{k} \neq \frac{-1}{-2}$$

From $\frac{k}{12} = \frac{3}{k}$ we have $k^2 = 36 \Rightarrow k \pm 6$

From $\frac{3}{k} \neq \frac{-1}{-2}$ we have $k \neq 6$

Thus $k = -6$

48. Solve the following pair of linear equations by cross multiplication method:

$$x + 2y = 2$$

$$x - 3y = 7$$

Ans : [Board Term-1 2016]

We have $x + 2y - 2 = 0$

$x - 3y - 7 = 0$

c133

Using the formula

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

we have $$\frac{x}{-14-6} = \frac{y}{-2+7} = \frac{1}{-3-2}$$

$$\frac{x}{-20} = \frac{y}{5} = \frac{-1}{5}$$

$$\frac{x}{-20} = \frac{-1}{5} \Rightarrow x = 4$$

$$\frac{y}{5} = \frac{-1}{5} \Rightarrow y = -1$$

49. Solve the following pair of linear equations by substitution method:

$$3x + 2y - 7 = 0$$
$$4x + y - 6 = 0$$

Ans : [Board Term-1 2015]

We have $3x + 2y - 7 = 0$ ...(1)

$4x + y - 6 = 0$ ...(2)

From equation (2), $y = 6 - 4x$ ...(3)

Putting this value of y in equation (1) we have

$$3x + 2(6 - 4x) - 7 = 0$$
$$3x + 12 - 8x - 7 = 0$$
$$5 - 5x = 0$$
$$5x = 5$$

c134

Thus $x = 1$

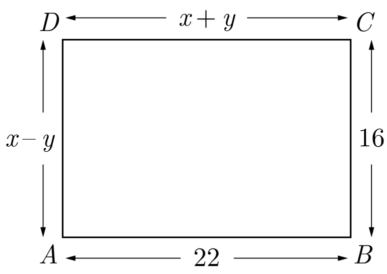
Substituting this value of x in (2), we obtain,

$$y = 6 - 4 \times 1 = 2$$

Hence, values of x and y are 1 and 2 respectively.

50. In the figure given below, $ABCD$ is a rectangle. Find the values of x and y.

Ans : [Board Term-1 2012, Set-30]



From given figure we have

$x + y = 22$ ...(1)

and $x - y = 16$ ...(2)

Adding (1) and (2), we have

c135

$$2x = 38$$
$$x = 19$$

Substituting the value of x in equation (1), we get

$$19 + y = 22$$
$$y = 22 - 19 = 3$$

Hence, $x = 19$ and $y = 3$.

51. Solve : $99x + 101y = 499$, $101x + 99y = 501$

Ans : [Board Term-1 2012, Set-55]

We have $99x + 101y = 499$ ...(1)

$101x + 99y = 501$ ...(2)

Adding equation (1) and (2), we have

$$200x + 200y = 1000$$

$x + y = 5$ ...(3)

Subtracting equation (2) from equation (1), we get

$$-2x + 2y = -2$$

$x - y = 1$ ...(4)

Adding equations (3) and (4), we have

$$2x = 6 \Rightarrow x = 3$$

Substituting the value of x in equation (3), we get

c136

$$3 + y = 5 \Rightarrow y = 2$$

52. Solve the following system of linear equations by substitution method:

$$2x - y = 2$$
$$x + 3y = 15$$

Ans : [Board Term-1 2012]

We have $2x - y = 2$ ...(1)

$x + 3y = 15$ ...(2)

From equation (1), we get $y = 2x - 2$ ...(3)

Substituting the value of y in equation (2),

$$x + 6x - 6 = 15$$

or, $7x = 21 \Rightarrow x = 3$

c137

Substituting this value of x in (3), we get

From equation (1), we have

$$y = 2 \times 3 - 2 = 4$$

$$x = 3 \text{ and } y = 4$$

53. Find the value(s) of k for which the pair of Linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions.

Ans : [Board Term-1 2017]

We have $\quad kx + y = k^2$

and $\quad x + ky = 1$

$$\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$$

For infinitely many solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{1} = \frac{1}{k} = \frac{k^2}{1} = k^2 = 1$$

$$k = \pm 1$$

THREE MARKS QUESTIONS

54. Solve the following system of equations.

$$\frac{21}{x} + \frac{47}{y} = 110, \frac{47}{x} + \frac{21}{y} = 162, x, y \neq 0$$

Ans :

We have $\quad \frac{21}{x} + \frac{47}{y} = 110$

$$\frac{47}{x} + \frac{21}{y} = 162$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$. then given equation become

$$21u + 47v = 110 \quad ...(1)$$

and $\quad 47u + 21v = 162 \quad ...(2)$

Adding equation (1) and (2) we get

$$68u + 68v = 272$$

$$u + v = 4 \quad ...(3)$$

Subtracting equation (1) from (2) we get

$$26u - 26v = 52$$

$$u - v = 2 \quad ...(4)$$

Adding equation (3) and (4), we get

$$2u = 6 \Rightarrow u = 3$$

Substituting $u = 3$ in equation (3), we get $v = 1$.

Thus $x = \frac{1}{u} = \frac{1}{3}$ and $y = \frac{1}{v} = \frac{1}{1} = 1$

55. A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator- Find the fraction.

Ans : [Board 2019 Delhi]

Let the fraction be $\frac{x}{y}$. According to the first condition,

$$\frac{x-2}{y} = \frac{1}{3}$$

$$3x - 6 = y$$

$$y = 3x - 6 \quad ...(1)$$

According to the second condition,

$$\frac{x}{y-1} = \frac{1}{2}$$

$$2x = y - 1$$

$$y = 2x + 1 \quad ...(2)$$

From equation (1) and (2), we have

$$3x - 6 = 2x + 1 \Rightarrow x = 7$$

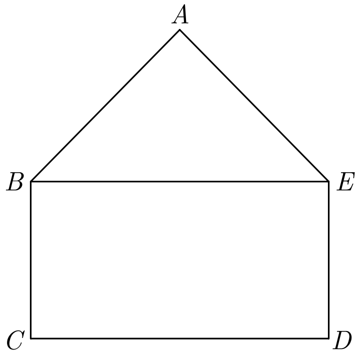
Substitute value of x in equation (1), we get

$$y = 3(7) - 6$$

$$= 21 - 6 = 15$$

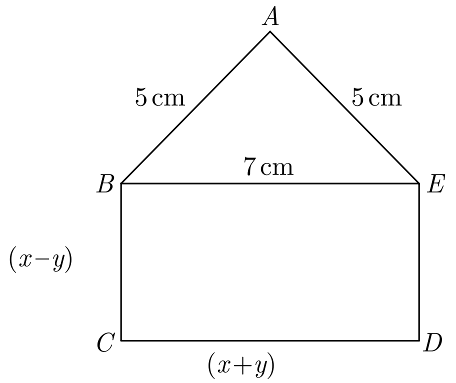
Hence, fraction is $\frac{7}{15}$.

56. In the figure, $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. $AB = 5$ cm, $AE = 5$ cm, $BE = 7$ cm, $BC = x - y$ and $CD = x + y$. If the perimeter of $ABCDE$ is 27 cm. Find the value of x and y, given $x, y \neq 0$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



We have $CD = BE$

$$x + y = 7 \quad ...(1)$$

Also, perimeter of $ABCDE$ is 27 cm, thus

$$AB + BC + CD + DE + AE = 27$$

$$5 + (x - y) + (x + y) + (x - y) + 5 = 27$$

$$3x - y = 17 \quad ...(2)$$

Adding equation (1) and (2) we have

$$4x = 24 \Rightarrow x = 6$$

Substituting $x = 6$ in equation (1) we obtain

$$y = 7 - x = 7 - 6 = 1$$

Thus $x = 6$ and $y = 1$.

57. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Ans : [Board Term-1 2013]

Let the length of the garden be x m and its width be y m.

Perimeter of rectangular garden

$$p = 2(x + y)$$

Since half perimeter is given as 36 m,

$$(x + y) = 36 \quad ...(1)$$

Also, $x = y + 4$

or $x - y = 4 \quad ...(2)$

For $x + y = 36$

$$y = 36 - x$$

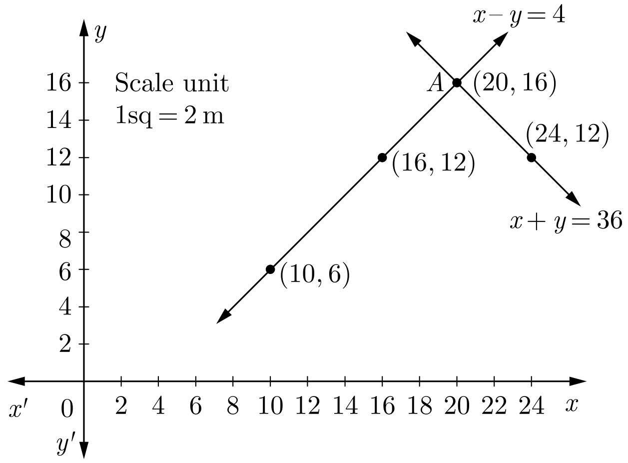
x	20	24
y	16	12

For $x - y = 4$

or, $y = x - 4$

x	10	16	20
y	6	12	16

Plotting the above points and drawing lines joining them, we get the following graph. we get two lines intersecting each other at (20, 16)



Hence, length is 20 m and width is 16 m.

58. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is :

(a) intersecting lines

(b) parallel lines

(c) coincident lines.

Ans : [Board Term-1 2014, Set-B]

Given, linear equation is $2x + 3y - 8 = 0$ ...(1)

(a) For intersecting lines, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

To get its parallel line one of the possible equation may be taken as

$$5x + 2y - 9 = 0 \quad (2)$$

(b) For parallel lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

One of the possible line parallel to equation (1) may be taken as

$$6x + 9y + 7 = 0$$

(c) For coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

To get its coincident line, one of the possible equation may be taken as

$$4x + 6y - 16 = 0$$

59. Solve the pair of equations graphically :
$4x - y = 4$ and $3x + 2y = 14$

Ans : [Board Term-1 2014]

c115

We have $4x - y = 4$

or, $y = 4x - 4$

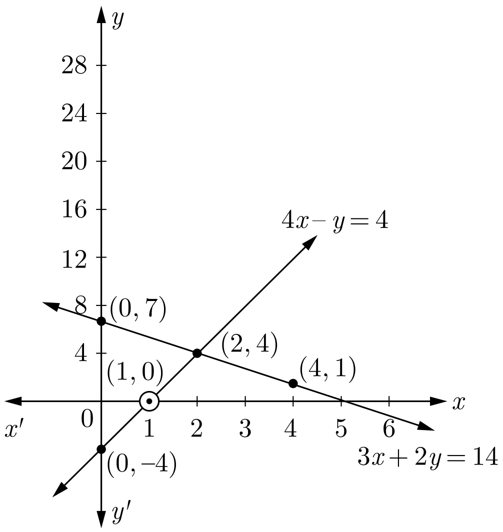
x	0	1	2
y	-4	0	4

and $3x + 2y = 14$

or, $y = \frac{14 - 3x}{2}$

x	0	2	4
y	7	4	1

Plotting the above points and drawing lines joining them, we get the following graph. We get two obtained lines intersect each other at (2, 4).



Hence, $x = 2$ and $y = 4$.

60. Determine the values of m and n so that the following system of linear equation have infinite number of solutions :

$$(2m - 1)x + 3y - 5 = 0$$
$$3x + (n - 1)y - 2 = 0$$

Ans : [Board Term-1 2013, VKH6FFC; 2011, Set-66

We have $(2m - 1)x + 3y - 5 = 0$ ...(1)

Here $a_1 = 2m - 1, b_1 = 3, c_1 = -5$

$3x + (n - 1)y - 2 = 0$ ...(2)

Here $a_2 = 3, b_2 = (n - 1), c_2 = -2$

For a pair of linear equations to have infinite number of solutions,

c116

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2m - 1}{3} = \frac{3}{n - 1} = \frac{5}{2}$$

or $2(2m - 1) = 15$ and $5(n - 1) = 6$

Hence, $m = \frac{17}{4}, n = \frac{11}{5}$

61. Find the values of α and β for which the following pair of linear equations has infinite number of solutions : $2x + 3y = 7;\ 2\alpha x + (\alpha + \beta)y = 28$.

Ans : [Board Term-1 2011]

We have $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$.

For a pair of linear equations to be consistent and having infinite number of solutions,

c117

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$\frac{2}{2\alpha} = \frac{7}{28}$$

$$2\alpha \times 7 = 28 \times 2 \Rightarrow \alpha = 4$$

$$\frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$7(\alpha + \beta) = 28 \times 3$$

$$\alpha + \beta = 12$$

$$\beta = 12 - \alpha = 12 - 4 = 8$$

Hence $\alpha = 4$, and $\beta = 8$

62. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$x - 5y = 6$ and $2x - 10y = 12$.

Ans : [Board Term-1 2011]

We have $x - 5y = 6$ or $x = 5y + 6$

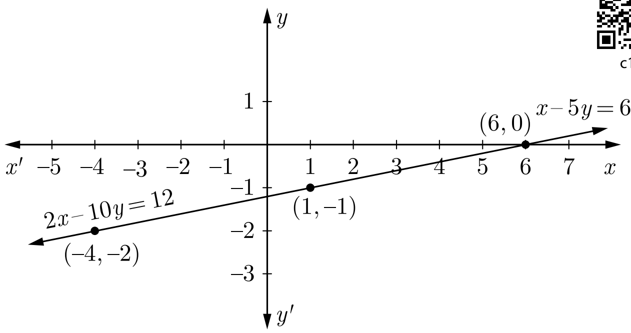
x	6	1	-4
y	0	-1	-2

and $2x - 10y = 12$ or $x = 5y + 6$

x	6	1	-4
y	0	-1	-2

Plotting the above points and drawing lines joining them, we get the following graph.

c118



Since the lines are coincident, so the system of linear equations is consistent with infinite many solutions.

63. For what value of p will the following system of equations have no solution ?

$(2p-1)x+(p-1)y = 2p+1;\ y+3x-1=0$

Ans : [Board Term-1 2011, Set-28]

We have $\quad (2p-1)x+(p-1)y-(2p+1) = 0$

Here $a_1 = 2p-1, b_1 = p-1$ and $c_1 = -(2p+1)$

Also $\quad 3x+y-1 = 0$

Here $a_2 = 3, b_2 = 1$ and $c_2 = -1$

c119

The condition for no solution is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{2p+1}{1}$$

From $\frac{2p-1}{3} = \frac{p-1}{1}$ we have

$$3p-3 = 2p-1$$

$$3p-2p = 3-1$$

$$p = 2$$

From $\frac{p-1}{1} \neq \frac{2p+1}{1}$ we have

$$p-1 \neq 2p+1 \text{ or } 2p-p \neq -1-1$$

$$p \neq -2$$

From $\frac{2p-1}{3} \neq \frac{2p+1}{1}$ we have

$$2p-1 \neq 6p+3$$

$$4p \neq -4$$

$$p \neq -1$$

Hence, system has no solution when $p = 2$

64. Find the value of k for which the following pair of equations has no solution :

$x+2y=3, (k-1)x+(k+1)y=(k+2)$.

Ans : [Board Term-1 2011, Set-52]

For $x+2y=3$ or $x+2y-3=0$,

c120

$$a_1 = 1, b_1 = 2, c_1 = -3$$

For $(k-1)x+(k+1)y = (k+2)$

or $(k-1)x+(k+1)y-(k-2)=0$

$$a_2 = (k-1), b_2 = (k+1), c_2 = -(k+2)$$

For no solution, $\quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{1}{k-1} = \frac{2}{k+1} \neq \frac{3}{k+2}$$

From $\frac{1}{k-1} = \frac{2}{k+1}$ we have

$$k+1 = 2k-2$$

$$3 = k$$

Thus $k = 3$.

65. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their respective ages.

Ans : [Board Term-1 2015]

Let age of father and son be x and y respectively.

$$x+y = 40 \quad ...(1)$$

$$x = 3y \quad ...(2)$$

Solving equations (1) and (2), we get

c139

$$x = 30 \text{ and } y = 10$$

Ages are 30 years and 10 years.

66. Solve using cross multiplication method:

$$5x+4y-4 = 0$$

$$x-12y-20 = 0$$

Ans : [Board Term-1 2015]

We have $\quad 5x+4y-4 = 0 \quad ...(1)$

$$x-12y-20 = 0 \quad ...(2)$$

By cross-multiplication method,

$$\frac{x}{b_2c_1-b_1c_2} = \frac{y}{c_1a_2-c_2a_1} = \frac{1}{b_1b_2-a_2b_1}$$

$$\frac{x}{-80-48} = \frac{y}{-4+100} = \frac{1}{-60-4}$$

$$\frac{x}{-128} = \frac{y}{96} = \frac{1}{64}$$

c140

$$\frac{x}{-128} = \frac{1}{-64} \Rightarrow x = 2$$

and $$\frac{y}{96} = \frac{1}{-64} \Rightarrow y = \frac{-3}{2}$$

Hence, $x = 2$ and $y = \frac{-3}{2}$

67. The Present age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Ans : [Board Term-1 2012, Set-39]

Let the sum of the ages of the 2 children be x and the age of the father be y years.

Now $$y = 2x$$

$$2x - y = 0 \quad ...(1)$$

and $$20 + y = x + 40$$

$$x - y = -20 \quad ...(2)$$

Subtracting (2) from (1), we get

$$x = 20$$

c166

From(1), $$y = 2x = 2 \times 20 = 40$$

Hence, the age of the father is 40 years.

68. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Ans : [Board Term-1 2016, 2015]

Let fixed charge be x and per day food cost be y

$$x + 20y = 3000 \quad ...(1)$$

$$x + 25y = 3500 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5y = 500 \Rightarrow y = 100$$

c141

Substituting this value of y in (1), we get

$$x + 20(100) = 3000$$

$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

69. Solve for x and y :

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$x - \frac{y}{3} = 3$$

Ans : [Board Term-1 2015, NCERT]

We have $$\frac{x}{2} + \frac{2y}{3} = -1$$

$$3x + 4y = -6 \quad ...(1)$$

and $$\frac{x}{1} - \frac{y}{3} = 3$$

$$3x - y = 9 \quad ...(2)$$

Subtracting equation (2) from equation (1), we have

$$5y = -15 \Rightarrow y = -1$$

Substituting $y = -3$ in eq (1), we get

c142

$$3x + 4(-3) = -6$$

$$3x - 12 = -6$$

$$3x = 12 - 6 \Rightarrow x = 2$$

Hence $x = 2$ and $y = -3$.

70. Solve the following pair of linear equations by the substitution and cross - multiplication method :

$$8x + 5y = 9$$

$$3x + 2y = 4$$

Ans : [Board Term-1 2015, SYFH4D]

We have $$8x + 5y = 9$$

or, $$8x + 5y - 9 = 0 \quad ...(1)$$

and $$3x + 2y = 4$$

or, $$3x + 2y - 4 = 0 \quad ...(2)$$

Comparing equation (1) and (2) with $ax + by + c = 0$,

$$a_1 = 8, b_1 = 5, c_1 = -9$$

and $\quad a_2 = 3, b_2 = 2, c_2 = -4$

By cross-multiplication method,

$$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{\{(5)(-4)-(2)(-9)\}} = \frac{y}{\{(-9)(3)-(-4)(8)\}}$$

$$= \frac{1}{\{8 \times 2 - 3 \times 5\}}$$

or, $\quad \frac{x}{-2} = \frac{1}{1}$ and $\frac{y}{5} = \frac{1}{1}$

$$x = -2 \text{ and } y = 5$$

c143

We use substitution method.
From equation (2), we have

$$3x = 4 - 2y$$

or, $\quad x = \frac{4-2y}{3} \quad$ ...(3)

Substituting this value of y in equation (3) in (1), we get

$$8\left(\frac{4-2y}{3}\right) + 5y = 9$$

$$32 - 16y + 15y = 27$$

$$-y = 27 - 32$$

Thus $\quad y = 5$

Substituting this value of y in equation (3)

$$x = \frac{4-2(5)}{3} = \frac{4-10}{3} = -2$$

Hence, $x = -2$ and $y = 5$.

71. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it ?

Ans : [Board Term-1 2013]

Let the man can finish the work in x days and the boy can finish work in y days.

Work done by one man in one day $= \frac{1}{x}$

And work done by one boy in one day $= \frac{1}{y}$

$$\frac{2}{x} + \frac{7}{y} = \frac{1}{4} \quad \text{...(1)}$$

and $\quad \frac{4}{x} + \frac{4}{y} = \frac{1}{3} \quad$ ...(2)

Let $\frac{1}{x}$ be a and $\frac{1}{y}$ be b, then we have

$$2a + 7b = \frac{1}{4} \quad \text{...(3)}$$

and $\quad 4a + 4b = \frac{1}{3} \quad$ ...(4)

Multiplying equation (3) by 2 and subtract equation (4) from it

$$10b = \frac{1}{6}$$

c144

$$b = \frac{1}{60} = \frac{1}{y}$$

Thus $\quad y = 60$ days.

Substituting $b = \frac{1}{60}$ in equation (3), we have

$$2a + \frac{7}{60} = \frac{1}{4}$$

$$2a = \frac{1}{4} - \frac{7}{60}$$

$$a = \frac{1}{15}$$

Now $\quad \frac{1}{15} = \frac{1}{x}$

Thus $x = 15$ days.

72. In an election contested between A and B, A obtained votes equal to twice the no. of persons on the electoral roll who did not cast their votes and this later number was equal to twice his majority over B. If there were 1,8000 persons on the electoral roll. How many votes for B.

Ans : [Board Term-1 2012, Set-56]

Let x and y be the no. of votes for A and B respectively.
The no. of persons who did not vote is $18000 - x - y$.

We have $\quad x = 2(18000 - x - y)$

$$3x + 2y = 36000 \quad \text{...(1)}$$

and $\quad (18000 - x - y) = 2(x - y)$

or $\quad 3x - y = 18000 \quad$ ...(2)

Subtracting equation (2) from equation (1),

$$3y = 18000$$

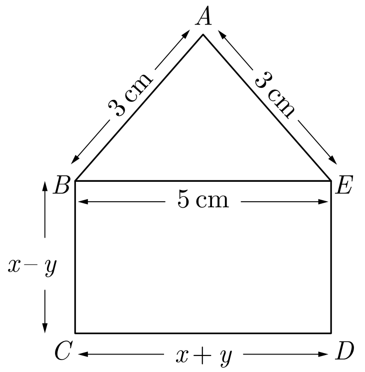
$$y = 6000$$

c179

Hence vote for B is 6000.

73. In the figure below $ABCDE$ is a pentagon with $BE || CD$ and $BC || DE$. BC is perpendicular to DC.

If the perimeter of $ABCDE$ is 21 cm, find the values of x and y.



Ans : [Board Term-1 2011]

Since $BC || DE$ and $BE || CD$ with $BC \perp DC$, $BCDE$ is a rectangle.

$$BE = CD,$$

$$x + y = 5 \qquad ...(1)$$

and $$DE = BE = x - y$$

Since perimeter of $ABCDE$ is 21,

c145

$$AB + BC + CD + DE + EA = 21$$

$$3 + x - y + x + y + x - y + 3 = 21$$

$$6 + 3x - y = 21$$

$$3x - y = 15$$

Adding equations (1) and (2), we get

$$4x = 20 \qquad ...(2)$$

$$x = 5$$

Substituting the value of x in (1), we get

$$y = 0$$

Thus $x = 5$ and $y = 0$.

74. Solve for x and y :

$$\frac{x+1}{2} + \frac{y-1}{3} = 9 \ ; \ \frac{x-1}{3} + \frac{y+1}{2} = 8.$$

Ans : [Board Term-1 2011, Set-52]

We have $$\frac{x+1}{2} + \frac{y-1}{3} = 9$$

c146

$$3(x+1) + 2(y-1) = 54$$

$$3x + 3 + 2y - 2 = 54$$

$$3x + 2y = 53 \qquad (1)$$

and $$\frac{x-1}{3} + \frac{y+1}{2} = 8$$

$$2(x-1) + 3(y+1) = 48$$

$$2x - 2 + 3y + 3 = 48$$

$$2x + 3y = 47 \qquad (2)$$

Multiplying equation (1) by 3 we have

$$9x + 6y = 159 \qquad (3)$$

Multiplying equation (2) by 2 we have

$$4x + 6y = 94 \qquad (4)$$

Subtracting equation (4) from (3) we have

$$5x = 65$$

or $$x = 13$$

Substitute the value of x in equation (2),

$$2(13) + 3y = 47$$

$$3y = 47 - 26 = 21$$

$$y = \frac{21}{3} = 7$$

Hence, $x = 13$ and $y = 7$

75. Solve for x and y :

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \text{ where } x \neq 1,\ y \neq 2.$$

Ans : [Board Term-1 2011]

We have $$\frac{6}{x-1} - \frac{3}{y-2} = 1 \qquad (1)$$

$$\frac{5}{x-1} - \frac{1}{y-2} = 2, \qquad (2)$$

Let $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. then given equations become

$$6p - 3q = 1 \qquad ...(3)$$

and $$5p - q = 2 \qquad ...(4)$$

Multiplying equation (4) by 3 and adding in equation (3), we have

$$21p = 7$$

$$p = \frac{7}{21} = \frac{1}{3}$$

Substituting this value of p in equation (3), we have

$$6\left(\frac{1}{3}\right) - 3q = 1$$

$$2 - 3q = 1 \Rightarrow q = \frac{1}{3}$$

c147

Now, $\frac{1}{x-1} = p = \frac{1}{3}$

or, $x - 1 = 3 \Rightarrow x = 4$

and $\frac{1}{y-2} = q = \frac{1}{3}$

or, $y - 2 = 3 \Rightarrow y = 5$

Hence $x = 4$ and, $y = 5$.

76. Seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number.

Ans : [Board Term-1 2017]

Let the ten's and unit digit by y and x respectively,
So the number is $10y + x$
The number when digits are reversed becomes $10x + y$

c174

Thus $7(10y + x) = 4(10x + y)$

$$70y + 7x = 40x + 4y$$

$$70y - 4y = 40x - 7x$$

$$2y = x \quad ...(1)$$

or $x - y = 3 \quad ...(2)$

From (1) and (2) we get

$$y = 3 \text{ and } x = 6$$

Hence the number is 36.

77. Solve the following pair of equations for x and y :

$$\frac{a^2}{x} - \frac{b^2}{y} = 0, \frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b, \qquad x \neq 0; y \neq 0.$$

Ans : [Board Term-1 2011]

We have $\frac{a^2}{x} - \frac{b^2}{y} = 0$

$$\frac{a^2 b}{x} + \frac{b^2 a}{y} = a + b = a + b$$

Substituting $p = \frac{1}{x}$ and $q = \frac{1}{y}$ in the given equations,

$$a^2 p - b^2 q = 0 \quad ...(1)$$

$$a^2 bp + b^2 aq = a + b \quad ...(2)$$

Multiplying equation (1), by a

$$a^3 p - b^2 aq = 0 \quad ...(3)$$

Adding equation (2) and equation (3),

$$(a^3 + a^2 b)p = a + b$$

c148

or, $p = \frac{(a+b)}{a^2(a+b)} = \frac{1}{a^2}$

Substituting the value of p in equation (1),

$$a^2\left(\frac{1}{a^2}\right) - b^2 q = 0 \Rightarrow q = \frac{1}{b^2}$$

Now, $p = \frac{1}{x} = \frac{1}{a^2} \Rightarrow x = a^2$

and $q = \frac{1}{y} = \frac{1}{b^2} \Rightarrow y = b^2$

Hence, $x = a^2$ and $y = b^2$

78. Solve for x and y :

$$ax + by = \frac{a+b}{2}$$

$$3x + 5y = 4$$

Ans : [Board Term-1 2011, Set-44]

We have $ax + by = \frac{a+b}{2}$

or $2ax + 2by = a + b \quad ...(1)$

and $3x + 5y = 4 \quad ...(2)$

Multiplying equation (1) by 5 we have

$$10ax + 10by = 5a + 5b \quad ...(3)$$

Multiplying equation (2) by $2b$, we have

$$6bx + 10by = 8b \quad ...(4)$$

Subtracting (4) from (3) we have

$$(10a - 6b)x = 5a - 3b$$

c149

or $x = \frac{5a - 3b}{10a - 6b} = \frac{1}{2}$

Substitute $x = \frac{1}{2}$ in equation (2), we get

$$3 \times \frac{1}{2} + 5y = 4$$

$$5y = 4 - \frac{3}{2} = \frac{5}{2}$$

$$y = \frac{5}{2 \times 5} = \frac{1}{2}$$

Hence $x = \frac{1}{2}$ and $y = \frac{1}{2}$.

79. Solve the following pair of equations for x and y :

$4x+\frac{6}{y}=15, 6x-\frac{8}{y}=14$

and also find the value of p such that $y=px-2$.

Ans : [Board Term-1 2011, Set-60]

We have

$$4x+\frac{6}{y} = 15 \quad (1)$$

$$6x-\frac{8}{y} = 14, \quad (2)$$

Let $\frac{1}{y}=z$, the given equations become

$$4x+6z = 15 \quad ...(3)$$

$$6x-8z = 14 \quad ...(4)$$

Multiply equation (3) by 4 we have

$$16x+24z = 60 \quad (5)$$

Multiply equation (4) by 3 we have

$$18x-24z = 24 \quad (6)$$

Adding equation (5) and (6) we have

$$34x = 102$$

$$x = \frac{102}{34}=3$$

c150

Substitute the value of x in equation (3),

$$4(3)+6z = 15$$

$$6z = 15-12=3$$

$$z = \frac{3}{6}=\frac{1}{2}$$

Now

$$z = \frac{1}{y}=\frac{1}{2} \Rightarrow y = 2$$

Hence $x = 3$ and $y=2$.

Again

$$y = px-2$$

$$2 = p(3)-2$$

$$3p = 4$$

Thus

$$p = \frac{4}{3}$$

80. A chemist has one solution which is 50 % acid and a second which is 25 % acid. How much of each should be mixed to make 10 litre of 40 % acid solution.

Ans : [Board Term-1 2015, JRTSY]

Let 50 % acids in the solution be x and 25 % of other solution be y.

Total volume in the mixture

$$x+y = 10 \quad ...(1)\ 1$$

and

$$\frac{50}{100}x+\frac{25}{100}y = \frac{40}{100}\times 10$$

$$2x+y = 16 \quad ...(2)\ 1$$

Subtracting equation (1) from (2) we have

$$x = 6$$

Substituting this value of x in equation (1) we get

c177

$$6+y = 10$$

$$y = 4$$

Hence, $x = 6$ and $y=4$.

81. Find whether the following pair of linear equations has a unique solutions. If yes, find the solution :

$$7x-4y = 49, 5x-6y=57.$$

Ans : [Board Term-1 2011]

We have

$$7x-4y = 49 \quad (1)$$

$$5x-6y = 57 \quad (2)$$

Comparing with the equation $a_1x+b_1y=c_1$,

$$a_1 = 7, b_1=-4, c_1=49$$

$$a_2 = 5, b_2=-6, c_2=57$$

c151

Since,

$$\frac{a_1}{a_2} = \frac{7}{5} \text{ and } \frac{b_1}{b_2}=\frac{4}{6}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, system has a unique solution.

Multiply equation (1) by 5 we get

$$35x-20y = 245 \quad (3)$$

Multiply equation (2) by 7 we get

$$35x-42y = 399 \quad (4)$$

Subtracting (4) by (3) we have

$$22y =-154$$

$$y =-7$$

Putting the value of y in equation (2),

$$5x-6(-7) = 57$$

$$5x = 57-42=15$$

$$x = 3$$

Hence $x = 3$ and $y=-7$

FOUR MARKS QUESTIONS

82. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y-x=8$, $5y-x=14$ and $y-2x=1$.

Ans : [Board 2020 Delhi Standard]

c223

We have $2y-x = 8$

L_1 : $x = 2y-8$

y	0	4	5
$x=2y-8$	-8	0	2

$5y-x = 14$

L_2 : $x = 5y-14$

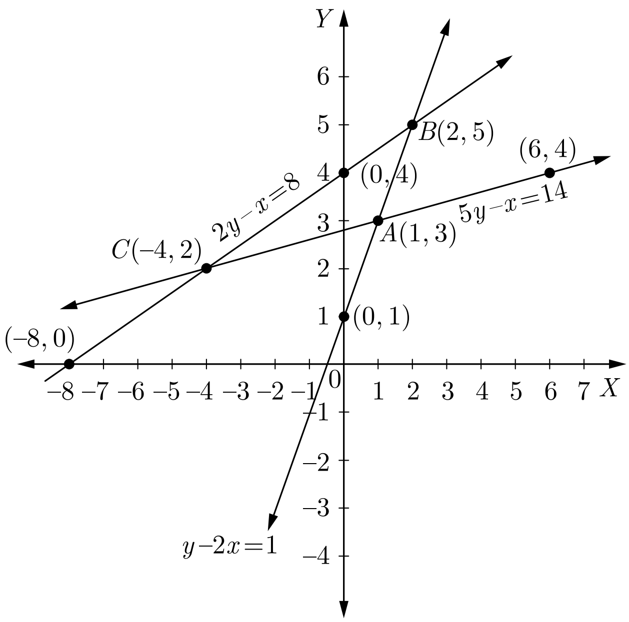
y	3	4	2
$x=5y-14$	1	6	-4

and $y-2x = 1$

L_3 : $y = 1+2x$

x	0	1	2
$y=1+2x$	1	3	5

Plotting the above points and drawing lines joining them, we get the graphical representation:



Hence, the coordinates of the vertices of a triangle ABC are $A(1,3)$, $B(2,5)$ and $C(-4,2)$.

83. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Ans : [Board 2020 Delhi Standard]

Let x be the speed of the boat in still water and y be the speed of the stream.

Relative Speed of boat in upstream will be $(x-y)$ and relative speed of boat in downstream will be $(x+y)$.

According to question, we have

c224

$$\frac{20}{x+y} = 2$$

$$x+y = 10 \quad ...(1)$$

and $$\frac{4}{x-y} = 2$$

$$x-y = 2 \quad ...(2)$$

Adding equation (1) and (2), we have

$$2x = 12 \Rightarrow x = 6 \text{ km/hr}$$

Substituting the value of x is equation (1) we have,

$$6+y = 10 \Rightarrow y = 10-6 = 4 \text{ km/hr}$$

Thus speed of a boat in still water is 6 km/hr and speed of the stream 4 km/hr.

84. It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for four hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. How long would it take for each pipe to fill the pool separately?

Ans : [Board 2020 OD Standard]

Let x be time taken to fill the pool by the larger diameter pipe and y be the time taken to fill the pool by the smaller diameter pipe.

According to question,

$$\frac{1}{x}+\frac{1}{y} = \frac{1}{12} \quad ...(1)$$

and $$\frac{4}{x}+\frac{9}{y} = \frac{1}{2} \quad ...(2)$$

Multiplying equation (1) by 9 and subtracting from equation (2), we get

$$\frac{5}{x} = \frac{9}{12}-\frac{1}{2} = \frac{1}{4}$$

c225

$$x = 20$$

Substituting the value of x in equation (1), we have

$$\frac{1}{20}+\frac{1}{y} = \frac{1}{12}$$

$$\frac{1}{y} = \frac{1}{12}-\frac{1}{20} = \frac{5-3}{60}$$

$$\frac{1}{y} = \frac{2}{60} = \frac{1}{30} \Rightarrow y = 30$$

Hence, time taken to fill the pool by the larger and smaller diameter pipe are 20 hrs and 30 hrs respectively.

85. For what value of k, which the following pair of linear equations have infinitely many solutions:
$2x+3y=7$ and $(k+1)x+(2k-1)y=4k+1$

Ans : [Board 2019 Delhi Standard]

c226

We have $\quad 2x+3y = 7$

and $\quad (k+1)x+(2k-1)y = 4k+1$

Here $\quad \frac{a_1}{a_2} = \frac{2}{k+1}, \frac{b_1}{b_2} = \frac{3}{(2k-1)}$

and $\quad \frac{c_1}{c_2} = \frac{-7}{-(4k+1)} = \frac{7}{(4k+1)}$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have

$$\frac{2}{k+1} = \frac{7}{4k+1}$$

$$2(4k+1) = 7(k+1)$$

$$8k+2 = 7k+7$$

$$k = 5$$

Hence, the value of k is 5, for which the given equation have infinitely many solutions.

86. Find c if the system of equations $cx+3y+(3-c)=0; 12x+cy-c=0$ has infinitely many solutions?

Ans : [Board 2019 Delhi]

c227

We have $\quad cx+3y+(3-c) = 0$

$$12x+cy-c = 0$$

Here, $\quad \frac{a_1}{a_2} = \frac{c}{12}, \frac{b_1}{b_2} = \frac{3}{c}, \frac{c_1}{c_2} = \frac{3-c}{-c}$

For infinite many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

For $\frac{a_1}{a_2} = \frac{c_1}{c_2}$ we have,

$$\frac{c}{12} = \frac{3-c}{-c}$$

$$-c^2 = 36-12c$$

$$-c^2+12c-36 = 0$$

$$c^2-12c+36 = 0$$

$$c^2-6c-6c+36 = 0$$

$$c(c-6)-6(c-6) = 0$$

$$(c-6)(c-6) = 0 \Rightarrow c = 6$$

and for $\frac{b_1}{b_2} = \frac{c_1}{c_2}$,

$$\frac{3}{c} = \frac{3-c}{-c}$$

$$-3c = 3c-c^2$$

$$c^2-6c = 0$$

$$c(c-6) = 0 \Rightarrow c = 6 \text{ or } c \neq 0$$

Hence, the value of c is 6, for which the given equations have infinitely many solutions.

87. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Ans : [Board 2019 Delhi]

c228

Let x be the age of father and y be the sum of the ages of his children.

After 5 years,

$$\text{Father's age} = (x+5) \text{ years}$$

Sum of ages of his children $= (y+10)$ years

According to the given condition,

$$x = 3y \qquad ...(1)$$

and $\quad x+5 = 2(y+10)$

or, $\quad x-2y = 15 \qquad ...(2)$

Solving equation (1) and (2), we have

$$3y - 2y = 15 \Rightarrow y = 15$$

Substituting value of y in equation (1), we get

$$x = 3 \times 15 = 45$$

Hence, father's present age is 45,

88. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Ans : [Board 2019 Delhi]

Let t be the time taken by the smaller diameter top. Time for larger tap diameter will be $t-2$.

Total time taken $= 1\frac{7}{8} = \frac{15}{8}h$.

Portion filled in one hour by smaller diameter tap will $\frac{1}{t}$ and by lager diameter tap will be $\frac{1}{t-2}$

According to the problem,

c229

$$\frac{1}{t} + \frac{1}{t-2} = \frac{8}{15}$$

$$\frac{t-2+t}{t(t-2)} = \frac{8}{15}$$

$$15(2t-2) = 8t(t-2)$$

$$30t - 30 = 8t^2 - 16t$$

$$8t^2 - 46t + 30 = 0$$

$$4t^2 - 23t + 15 = 0$$

$$4t^2 - 20t - 3t + 30 = 0$$

$$(4t-3)(t-5) = 0 \Rightarrow t = \frac{3}{4} \text{ or } t = 5$$

If $t = \frac{3}{4}$ then $t - 2 = \frac{3}{4} - 2 = \frac{-5}{4}$

Since, time cannot be negative, we neglect $t = \frac{3}{4}$

Therefore, $t = 5$

and $t - 2 = 5 - 2 = 3$

Hence, time taken by larger tap is 3 hours and time taken by smaller is 5 hours

89. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Ans : [Board 2019 Delhi]

Let x be the speed of boat in still water and y be the speed of stream.

Relative speed of boat in downstream will be $x+y$ and relative speed of boat in upstream will be $x-y$.

Time taken to go 30 km upstream,

$$t_1 = \frac{30}{x-y}$$

Time taken to go 44 km downstream,

c230

$$t_2 = \frac{40}{x+y}$$

According to the first condition we have

$$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad ...(1)$$

Similarly according to the second condition we have

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \quad ...(2)$$

Let $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = \upsilon$, then we have

$$30u + 44\upsilon = 10 \quad ...(3)$$

$$40u + 55\upsilon = 13 \quad ...(4)$$

Multiplying equation (3) by 4 and equation (4) by 3 and then subtracting we have

$$11v = 1 \Rightarrow \upsilon = \frac{1}{11}$$

Multiplying equation (3) by 5 and equation (4) by 4 and then subtracting we have

$$-10u = -2 \quad ...(4)$$

$$u = \frac{1}{5}$$

Now $u = \frac{1}{x-y} = \frac{1}{5}$

$$x - y = 5 \quad (5)$$

and $\upsilon = \frac{1}{x+y} = \frac{1}{11}$

$$x + y = 11 \quad (6)$$

Adding equation (5) and (6), we get

$$2x = 16 \Rightarrow x = 8$$

Substitute value of x in equation (5), we get

$$8 - y = 5 \Rightarrow y = 3$$

Hence speed of boat in still water is 8 km/hour and and speed of stream is 3 km/hour.

90. Sumit is 3 times as old as his son. Five years later he shall be two and a half times as old as his son. How old is Sumit at present?

Ans : [Board 2019 OD]

Let x be Sumit's present age and y be his son's present age.

According to given condition,

$$x = 3y$$

After five years, ...(1)

$$\text{Sumit's age} = x+5$$

and $\quad$ $$\text{His son's age} = y+5$$

Now, again according to given condition,

c231

$$x+5 = 2\tfrac{1}{2}(y+5)$$

$$x+5 = \frac{5}{2}(y+5)$$

$$2(x+5) = 5(y+5)$$

$$2x+10 = 5y+25$$

$$2x = 5y+15$$

$$2(3y) = 5y+15 \qquad \text{[from eq (1)]}$$

$$6y = 5y+15$$

$$y = 15$$

Again, from eq (1)

$$x = 3y = 3\times 15 = 45$$

Hence, Sumit's present age is 45 years.

91. For what value of k, will the following pair of equations have infinitely many solutions:

$2x+3y=7$ and $(k+2)x-3(1-k)y=5k+1$

Ans : [Board 2019 OD]

We have $\quad$ $$2x+3y = 7 \qquad ...(1)$$

and $\quad$ $$(k+2)x-3(1-k)y = 5k+1 \qquad ...(2)$$

Comparing equation (1) with $a_1x+b_1y=c_1$ and equation (2) by $a_2x+b_2y=c_2$ we have

$$a_1 = 2,\ b_1=3,\ c_1=7$$

and $\quad$ $$a_2 = (k+2),\ b_2=-3(1-k),\ c_1=5k+1$$

Here, $\quad$ $$\frac{a_1}{a_2} = \frac{2}{k+2},$$

$$\frac{b_1}{b_2} = \frac{3}{-3(1-k)},\ \frac{c_1}{c_2} = \frac{7}{5k+1}$$

For a pair of linear equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

c232

So, $\quad$ $$\frac{2}{k+2} = \frac{3}{-3(1-k)} = \frac{7}{5k+1}$$

$$\frac{2}{k+2} = \frac{3}{-3(1-k)}$$

$$2(1-k) = -(k+2)$$

$$2-2k = -k-2 \Rightarrow k = 4$$

Hence, for $k=4$, the pair of linear equations has infinitely many solutions.

92. The total cost of a certain length of a piece of cloth is ₹200. If the piece was 5 m longer and each metre of cloth costs ₹2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Ans : [Board 2019 OD]

Let x be the length of the cloth and y be the cost of cloth per meter.

Now $\quad$ $$x\times y = 200$$

$$y = \frac{200}{x} \qquad ...(1)$$

According to given conditions,

1. If the piece were 5 m longer
2. Each meter of cloth costed ₹ 2 less

c233

i.e., $\quad$ $$(x+5)(y-2) = 200$$

$$xy-2x+5y-10 = 200$$

$$xy-2x+5y = 210$$

$$x\left(\frac{200}{x}\right)-2x+5\left(\frac{200}{x}\right) = 210$$

$$200-2x+\frac{1000}{x} = 210$$

$$\frac{1000}{x}-2x = 10$$

$$1000-2x^2 = 10x$$

$$x^2+25x-20x-500 = 0$$

$$x(x+25)-20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = -25,\ 20$$

Neglecting $x=-25$ we get $x=20$.

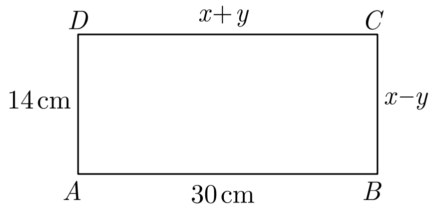
Now from equation (1), we have

$$y = \frac{200}{x} = \frac{200}{20} = 10$$

Hence, length of the piece of cloths is 20 m and rate per meter is ₹ 10.

93. In Figure, $ABCD$ is a rectangle. Find the values of

x and y.



Ans : [Board 2018]

Since $ABCD$ is a rectangle, we have

$$AB = CD \text{ and } BC = AD$$

Now $\quad x+y = 30 \quad ...(1)$

$$x-y = 14 \quad ...(2)$$

Adding equation (1) and (3) we obtain,

$$2x = 44 \Rightarrow x = \frac{44}{2} = 22$$

c234

Substituting value of x in equation (1) we have

$$22 + y = 30$$

$$y = 30 - 22 = 8$$

$$x = 22\,\text{cm and } y = 8\,\text{cm}$$

94. For Uttarakhand flood victims two sections A and B of class contributed Rs. 1,500. If the contribution of X-A was Rs. 100 less than that of X-B, find graphically the amounts contributed by both the sections.

Ans : [Board Term-1 2016]

Let amount contributed by two sections X-A and X-B be Rs. x and Rs. y.

c121

$$x+y = 1{,}500 \quad ...(1)$$

$$y-x = 100 \quad ...(2)$$

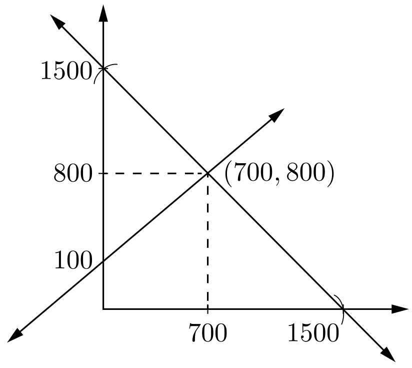
From (1) $y = 1500 - x$

x	0	700	1,500
y	1,500	800	0

From (2) $y = 100 + x$

x	0	700
y	100	800

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point (700, 800) Hence X-A contributes 700 Rs and X-B contributes 800 Rs.

95. Determine graphically whether the following pair of linear equations :

$$3x - y = 7$$

$$2x + 5y + 1 = 0 \text{ has :}$$

a. unique solution

b. infinitely many solutions or

c. no solution.

Ans : [Board Term-1 2015]

We have $\quad 3x - y = 7$

or $\quad 3x - y - 7 = 0 \quad (1)$

Here $a_1 = 3, b_1 = 1, c_1 = -7$

$$2x + 5y + 1 = 0 \quad (2)$$

Here $a_2 = 2, b_2 = 5, c_2 = 1$

Now $\quad \frac{a_1}{a_2} = \frac{3}{2},\ \frac{b_1}{b_2} = \frac{-1}{5}$

c122

Since $\frac{3}{2} \neq \frac{-1}{5}$, thus $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, given pair of linear equations has a unique solution.

Now line (1) $\quad y = 3x - 7$

x	0	2	3
y	-7	-1	2

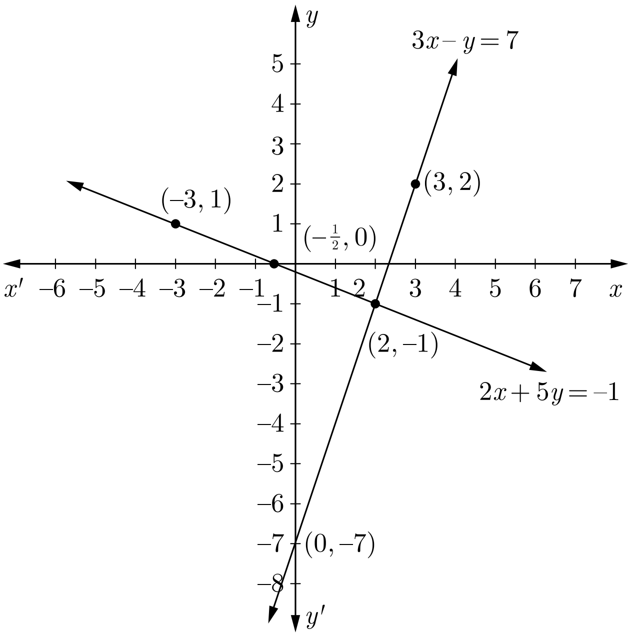
and line (2)

$$2x + 5y + 1 = 0$$

or, $$y = \frac{-1 - 2x}{5}$$

x	2	-3
y	-1	1

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(2, -1)$.
Hence $x = 2$ and $y = -1$

96. Draw the graphs of the pair of linear equations : $x + 2y = 5$ and $2x - 3y = -4$

Also find the points where the lines meet the $x-$axis.

Ans : [Board Term-1 2015]

We have $$x + 2y = 5$$

c123

or, $$y = \frac{5 - x}{2}$$

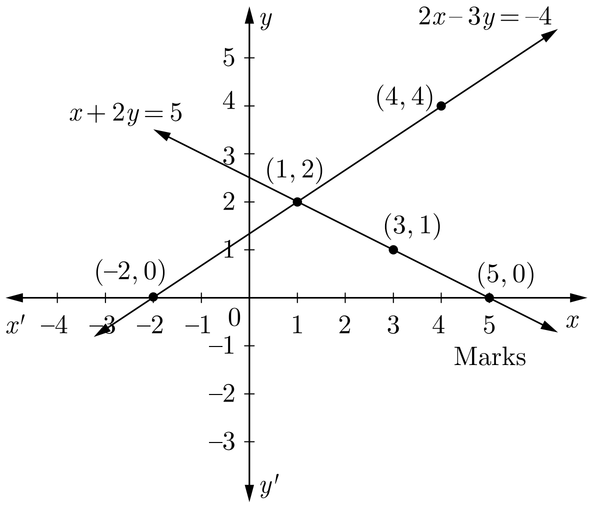
x	1	3	5
y	2	1	0

and $$2x - 3y = -4$$

or, $$y = \frac{2x + 4}{3}$$

x	1	4	-2
y	2	4	0

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two lines meet x-axis at $(5, 0)$ and $(-2, 0)$ respectively.

97. Solve graphically the pair of linear equations : $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$

Find the co-ordinates of the vertices of the triangular region formed by these lines and $x-$axis. Also, calculate the area of this triangle.

Ans : [Board Term-1 2015]

We have $$3x - 4y + 3 = 0$$

c124

or, $$y = \frac{3x + 3}{4}$$

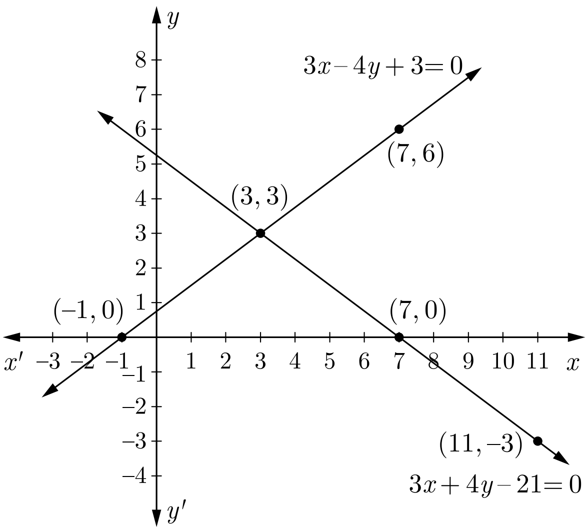
x	3	7	-1
y	3	6	0

and $$3x + 4y - 21 = 0$$

or, $$y = \frac{21 - 3x}{4}$$

x	3	7	11
y	3	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly, the two lines intersect at point $(3, 3)$.

(a) These lines intersect each other at point$(3, 3)$.
Hence $x = 3$ and $y = 3$

(b) The vertices of triangular region are $(3, 3)$, $(-1, 0)$ and $(7, 0)$.

(c) Area of $\Delta = \frac{1}{2} \times 8 \times 3 = 12$

Hence, Area of obtained Δ is 12 sq unit.

98. Aftab tells his daughter, '7 years ago, I was seven times as old as you were then. Also, 3 years from now, I shall be three times as old as you will be.' Represent this situation algebraically and graphically.

Ans : [Board Term-1 2015, NCERT]

Let the present age of Aftab be x years and the age of daughter be y years.

$$\text{7 years ago father's(Aftab) age} = (x - 7)\text{years}$$

$$\text{7 years ago daughter's age} = (y - 7) \text{ years}$$

According to the question,

$$(x - 7) = 7(y - 7)$$

or, $$(x - 7y) = -42 \quad (1)$$

$$\text{After 3 years father's(Aftab) age} = (x + 3) \text{ years}$$

$$\text{After 3 years daughter's age} = (y + 3) \text{ years}$$

According to the condition,

$$x + 3 = 3(y + 3)$$

or, $$x - 3y = 6 \quad (2)$$

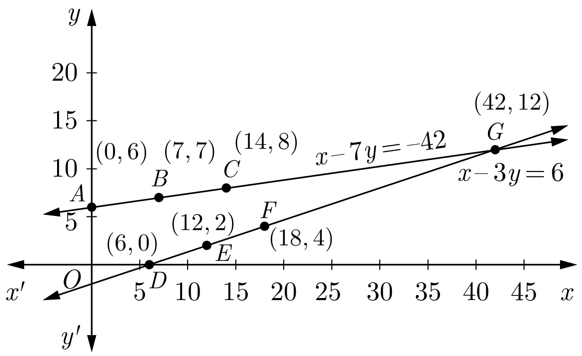
From equation(1) $\quad x - 7y = -42$

x	0	7	14
$y = \frac{x + 42}{7}$	6	7	8

From equation (2) $x - 3y = 6$

x	6	12	18
$y = \frac{x - 6}{3}$	0	2	4

Plotting the above points and drawing lines joining them, we get the following graph.



Two lines obtained intersect each other at $(42, 12)$

Hence, father's age $= 42$ years

and daughter's age $= 12$ years

99. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs. 160. After a month, the cost of 4kg of apples and 2kg of grapes is Rs. 300. Represent the situations algebraically and geometrically.

Ans : [Board Term-1 2013, Set DDE-E, NCERT]

Let the cost of 1 kg of apples be Rs.x and cost of 1 kg of grapes be Rs. y.

The given conditions can be represented given by the following equations :

$$2x + y = 160 \quad ...(1)$$

$$4x + 2y = 300 \quad ...(2)$$

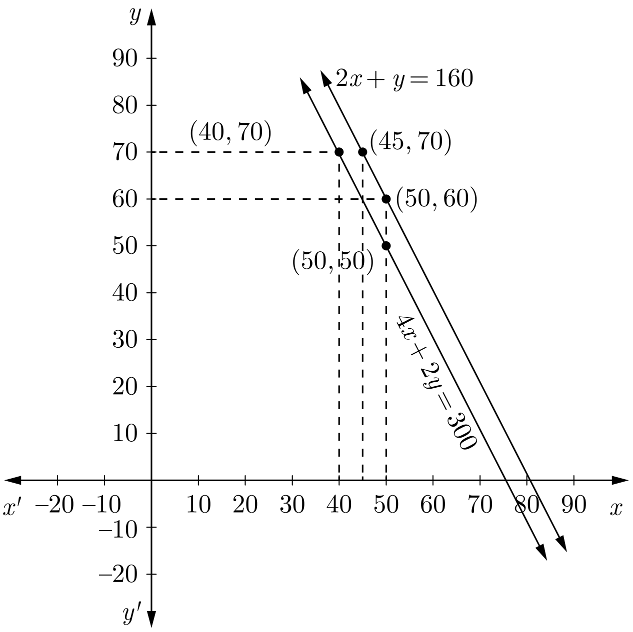
From equation (1) $\quad y = 160 - 2x$

x	50	45
y	60	70

From equation (2) $y = 150 - 2x$

x	50	40
y	50	70

Plotting these points on graph, we get two parallel line as shown below.



100. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Ans : [Board Term-1 2013 NCERT]

We have $x - y + 1 = 0$ ...(1)

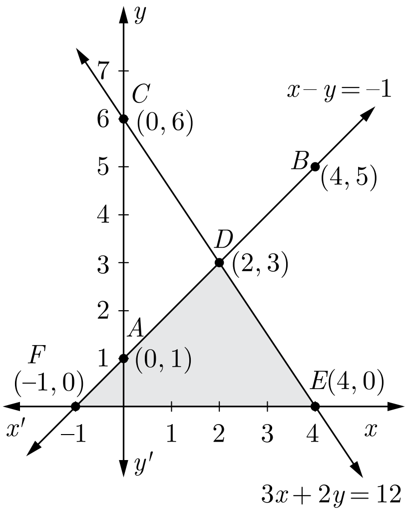
x	0	4	2
$y = x + 1$	1	5	3

and $3x + 2y - 12 = 0$ ...(2)

x	0	2	4
$y = \frac{12 - 3x}{2}$	6	3	0

Plotting the above points and drawing lines joining them, we get the following graph.

c127



Clearly, the two lines intersect at point $D(2,3)$. Hence, $x = 2$ and $y = 3$ is the solution of the given pair of equations. The line CD intersects the x-axis at the point $E(4,0)$ and the line AB intersects the x-axis at the points $F(-1,0)$. Hence, the co-ordinates of the vertices of the triangle are $D(2,3)$, $E(4,0)$ and $F(-1,0)$.

101. Solve the following pair of linear equations graphically:

$2x + 3y = 12$ and $x - y = 1$

Find the area of the region bounded by the two lines representing the above equations and y-axis.

Ans : [Board Term-1 2012, Set-58]

We have $2x + 3y = 12 \Rightarrow y = \frac{12 - 2x}{3}$

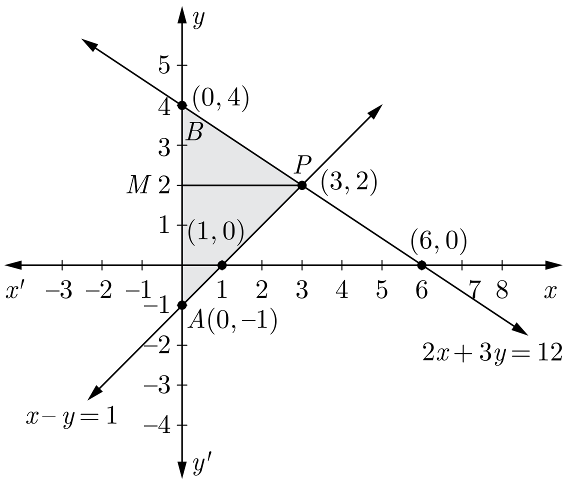
x	0	6	3
y	4	0	2

We have $x - y = 1 \Rightarrow y = x - 1$

x	0	1	3
y	1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.

c128



Clearly, the two lines intersect at point $p(3,2)$.

Hence, $x = 3$ and $y = 2$

Area of shaded triangle region,

$$\text{Area of } \Delta PAB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times PM$$

$$= \frac{1}{2} \times 5 \times 3$$

$$= 7.5 \text{ square unit.}$$

102. Solve the following pair of linear equations graphically:

$x + 3y = 12,\ 2x - 3y = 12$

Also shade the region bounded by the line $2x - 3y = 2$ and both the co-ordinate axes.

Ans : [Board Term-1 2013 FFC, 2012, Set-35, 48]

We have $\quad x + 3y = 6 \Rightarrow y = \dfrac{6 - x}{3}$ ...(1)

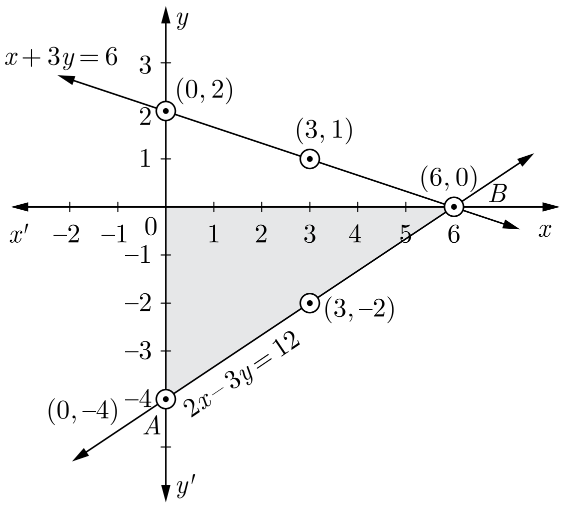
x	3	6	0
y	1	0	2

and $\quad 2x - 3y = 12 \Rightarrow y = \dfrac{2x - 12}{3}$

x	0	6	3
y	-4	0	-2

Plotting the above points and drawing lines joining them, we get the following graph.

c129



The two lines intersect each other at point $B(6,0)$.

Hence, $x = 6$ and $y = 0$

Again ΔOAB is the region bounded by the line $2x - 3y = 12$ and both the co-ordinate axes.

103. Solve the following pair of linear equations graphically:

$x - y = 1,\ 2x + y = 8$

Also find the co-ordinates of the points where the lines represented by the above equation intersect $y - axis$.

Ans : [Board Term-1 2012, Set-56]

We have $\quad x - y = 1 \Rightarrow y = x - 1$

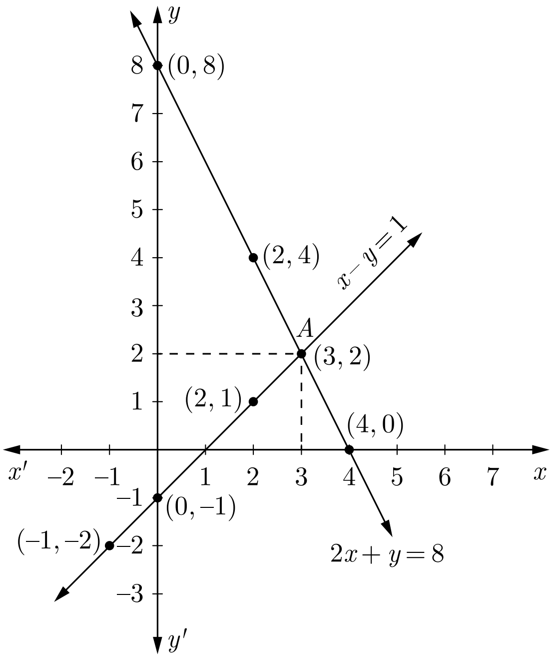
x	2	3	-1

y	1	2	-2

and $2x+y=8 \Rightarrow y=8-2x$

x	2	4	0
y	4	0	8

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $A(3,2)$. Thus solution of given equations is $x=3, y=2$.

Again, $x-y=1$ intersects $y-axis$ at $(0,-1)$

and $2x+y=8$ $y-axis$ at $(0,8)$.

104. Draw the graph of the following equations:

$$2x-y=1,\ x+2y=13$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

Ans : [Board Term-1 2012 Set-52]

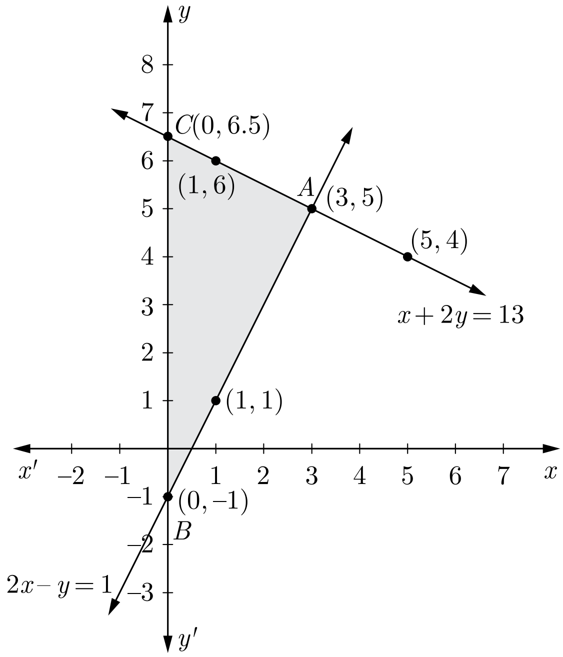
We have $2x-y=1 \Rightarrow y=2x-1$

x	0	1	3
y	-1	1	5

and $x+2y=13 \Rightarrow y=\dfrac{13-x}{2}$

x	1	3	5
y	6	5	4

Plotting the above points and drawing lines joining them, we get the following graph.



Clearly two obtained lines intersect at point $A(3,5)$.

Hence, $x=3$ and $y=5$

ABC is the triangular shaded region formed by the obtained lines with the y-axis.

105. Solve the following pair of equations graphically:

$$2x+3y=12,\ x-y-1=0.$$

Shade the region between the two lines represented by the above equations and the X-axis.

Ans : [Board Term-1 2012, Set-48]

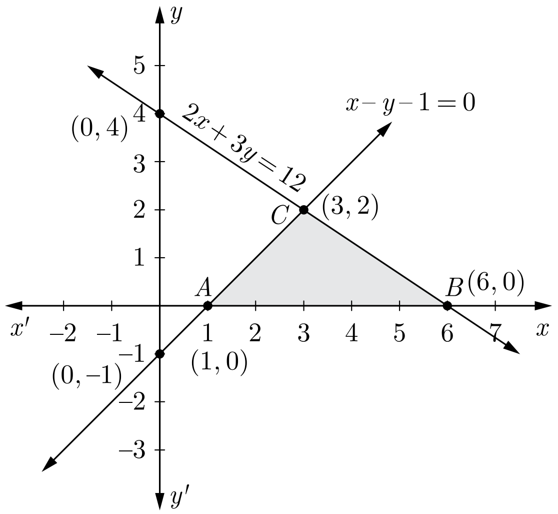
We have $2x+3y=12 \Rightarrow y=\dfrac{12-2x}{3}$

x	0	6	3
y	4	0	2

also $x-y=1 \Rightarrow y=x-1$

x	0	1	3
y	-1	0	2

Plotting the above points and drawing lines joining them, we get the following graph.



The two lines intersect each other at point $(3,2)$,
Hence, $x = 3$ and $y = 2$.
ΔABC is the region between the two lines represented by the given equations and the X-axis.

106. 4 chairs and 3 tables cost Rs 2100 and 5 chairs and 2 tables cost Rs 1750. Find the cost of none chair and one table separately.

Ans : [Board Term-1 2015]

Let cost of 1 chair be Rs x and cost of 1 table be Rs y According to the question,

$$4x + 3y = 2100 \quad ...(1)$$

$$5x + 2y = 1750 \quad ...(2)$$

Multiplying equation (1) by 2 and equation (2) by 3,

$$8x + 6y = 4200 \quad ...(3)$$

$$15x + 6y = 5250 \quad ...(iv)$$

Subtracting equation (3) from (4) we have

$$7x = 1050$$

$$x = 150$$

c152

Substituting the value of x in (1), $y = 500$

Thus cost of chair and table is Rs 150, Rs 500 respectively.

107. Solve the following pair of equations :

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Ans : [Board Term-1 2015]

We have $$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Substitute $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$$2X + 3Y = 2 \quad ...(1)$$

$$4X - 9Y = -1 \quad ...(2)$$

Multiplying equation (1) by 3, and adding in (2) we get

$$10X = 5 \Rightarrow X = \frac{5}{10} = \frac{1}{2}$$

c153

Thus $$\frac{1}{\sqrt{x}} = \frac{1}{2} \Rightarrow x = 4$$

Putting the value of X in equation (1), we get

$$2 \times \frac{1}{2} + 3y = 2$$

$$3Y = 2 - 1$$

$$Y = \frac{1}{3}$$

Now $$Y = \frac{1}{3} \Rightarrow \frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$$

Hence $x = 4$, $y = 9$.

108. Solve for x and y :

$$2x - y + 3 = 0$$

$$3x - 5y + 1 = 0$$

Ans : [Board Term-1 2015]

We have $$2x - y + 3 = 0 \quad ...(1)$$

$$3x - 5y + 1 = 0 \quad ...(2)$$

Multiplying equation (1) by 5,and subtracting (2) from it we have

$$7x = -14$$

$$x = \frac{-14}{7} = -2$$

c154

Substituting the value of x in equation (1) we get

$$2x - y + 3 = 0$$

$$2(-2) - y + 3 = 0$$

$$-4 - y + 3 = 0$$

$$-y - 1 = 0$$

$$y = -1$$

Hence, $x = -2$ and $y = -1$.

109. Solve $x+y=5$ and $2x-3y=4$ by elimination method and the substitution method.

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $x+y = 5$ ...(1)

and $2x-3y = 4$ ...(2)

Multiplying equation (1) by 3 and adding in (2) we have

$$3(x+y)+(2x-3y) = 3\times 5+4$$

or, $$3x+3y+2x-3y = 15+4$$

c155

$$5x = 19 \Rightarrow x = \frac{19}{5}$$

Substituting $x = \frac{19}{5}$ in equation (1),

$$\frac{19}{5}+y = 5$$

$$y = 5-\frac{19}{5} = \frac{25-19}{5} = \frac{6}{5}$$

Hence, $x = \frac{19}{5}$ and $y = \frac{6}{5}$

By Substituting Method :

We have, $x+y = 5$ ...(1)

and $2x-3y = 4$ ...(2)

From equation (1), $y = 5-x$ ...(3)

Substituting the value of y from equation (3) in equation (2),

$$2x-3(5-x) = 4$$

$$2x-15+3x = 4$$

$$5x = 19$$

$$x = \frac{19}{5}$$

Substituting this value of x in equation (3), we get

$$y = 5-\frac{19}{5} = \frac{6}{5}$$

Hence $x = \frac{19}{5}$ and $y = \frac{6}{5}$

110. Solve for x and y :

$$3x+4y = 10$$

$$2x-2y = 2$$

Ans : [Board Term-1 2015]

By Elimination Method :

We have, $3x+4y = 10$ ...(1)

and $2x-2y = 2$ ...(2)

Multiplying equation (2) by 2 and adding in (1),

$$(3x+4y)+2(2x-2y) = 10+2\times 2$$

or, $$3x+4y+4x-4y = 10+4$$

or, $$7x = 14 \Rightarrow x = 2$$

c156

Hence, $x = 2$ and $y = 1$.

By Substitution Method :

We have $3x+4y = 10$ ...(1)

and $2x-2y = 2$ ...(2)

From equation (2) $2y = 2x-2$

or, $y = x-1$ ...(3)

Substituting this value of y in equation (1),

$$3x+4(x-1) = 10$$

$$7x = 14 \Rightarrow x = 2$$

From equation (3), $y = 2-1 = 1$

Hence, $x = 2$ and $y = 1$

111. Solve $3x-5y-4=0$ and $9x = 2y+7$ by elimination method and the substitution method.

Ans : [Board Term-1 2012]

By Elimination Method :

We have, $3x-5y = 4$ ...(1)

and $9x = 2y+7$ ...(2)

Multiplying equation (1) by 3 and rewriting equation (2) we have

$$9x-15y = 12 \quad ...(3)$$

$$9x-2y = 7 \quad ...(4)$$

Subtracting equation (4) from equation (3),

$$-13y = 5$$

$$y = -\frac{5}{13}$$

c157

Substituting value of y in equation (1),

$$3x-5\left(\frac{-5}{13}\right) = 4$$

$$3x = 4-\frac{25}{13}$$

$$x = \frac{27}{13 \times 3} = \frac{9}{13}$$

Hence $x = \frac{9}{13}$ and $y = -\frac{5}{13}$

By Substituting Method :

We have $3x - 5y = 4$ ...(1)

and $9x = 2y + 7$ ...(2)

$$y = \frac{9x-7}{2} \quad ...(3)$$

Substituting this value of y (3) in equation (1),

$$3x - 5 \times \left(\frac{9x-7}{2}\right) = 4$$

$$6x - 45x + 35 = 8$$

$$-39x = -27$$

$$x = \frac{9}{13}$$

Substituting $x = \frac{9}{13}$ in equation (3),

$$y = \frac{9 \times \frac{9}{13} - 7}{2} = \frac{81 - 91}{2 \times 13}$$

$$= -\frac{10}{26} = -\frac{5}{13}$$

Hence, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

112. A train covered a certain distance at a uniform speed. If the train would have been 10 km/hr scheduled time. And, if the train were slower by 10 km/hr, it would have taken 3 hr more than the scheduled time. Find the distance covered by the train.

Ans : [Board Term-1 2012, NCERT]

Let the actual speed of the train be s and actual time taken t.

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= st \text{ km}$$

c158

According to the given condition, we have

$$st = (s+10)(t-2)$$

$$st = st - 2s + 10t - 20$$

$$2s - 10t + 20 = 0$$

$$s - 5t = -10 \quad (1)$$

and $st = (s-10)(t+3)$

$$st = st + 3s - 10t - 30$$

$$3s - 10t = 30 \quad ...(2)$$

Multiplying equation (1) by 3 and subtracting equation (2) from equation (1),

$$3 \times (s - 5t) - (3s - 10t) = -3 \times 10 - 30$$

$$-5t = -60 \Rightarrow t = 12$$

Substituting value of t equation (1),

$$s - 5 \times 12 = -10$$

$$s = -10 + 60 = 50$$

Hence, the distance covered by the train

$$= 50 \times 12 = 600 \text{ km.}$$

113. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Ans : [Board Term-1 2012]

Let the incomes of two persons be $11x$ and $7x$.

Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad ...(1)$$

and $7x - 5y = 400$ ...(2)

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad ...(3)$$

and $63x - 45y = 3600$ ...(4)

Subtracting, above equation we have

$$-8x = -1600$$

or, $x = \frac{-1,600}{-8} = 200$

c159

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

114. A and B are two points 150 km apart on a highway. Two cars start A and B at the same time. If they move in the same direction they meet in 15 hours. But if they move in the opposite direction, they meet in 1 hours. Find their speeds.

Ans : [Board Term-1 2012]

Let the speed of the car I from A be x and speed of the car II from B be y.

Same Direction :

Distance covered by car I

$= 150 + (\text{distance covered by car II})$

$$15x = 150 + 15y$$

$$15x - 15y = 150$$

$$x - y = 10 \quad ...(1)$$

c160

Opposite Direction :

Distance covered by car I + distance covered by car II

$$= 150 \text{ km}$$

$$x + y = 150 \quad ...(2)$$

Adding equation (1) and (2), we have $x = 80$.

Substituting $x = 80$ in equation (1), we have $y = 70$.

Speed of the car I from $A = 80$ km/hr and speed of the car II from $B = 70$ km/hr.

115. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction.

Ans : [Board Term-1 2012]

Let the fraction be $\frac{x}{y}$ then we have

c161

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$2x - 4 = y + 1$$

$$2x - y = 5 \quad ...(1)1$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$2x + 8 = 3y - 9]$$

$$2x - 3y = -17 \quad ...(2)\ 1$$

Subtracting equation (2) from equation (1),

$$2y = 22 \Rightarrow y = 11$$

Substituting this value of y in equation (1) we have,

$$2x - 11 = 5$$

$$x = 8$$

Hence, $$\text{Fraction} = \frac{8}{11}$$

116. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

Ans : [Board Term-1 2012]

Let the number of red balls be x and white balls be y. According to the question,

$$\frac{y}{2} = \frac{1}{3}x \text{ or } 2x - 3y = 0 \quad ...(1)$$

and $$3(x + y) - 7y = 6$$

or $$3x - 4y = 6 \quad ...(2)$$

Multiplying equation (1) by 3 and equation (2) by we have

$$6x - 9y = 0 \quad ...(3)$$

$$6x - 8y = 12 \quad ...(4)$$

Subtracting equation (3) from (4) we have

$$y = 12$$

c162

Substituting $y = 12$ in equation (1),

$$2x - 36 = 0$$

$$x = 18$$

Hence, number of red balls $= 18$

and number of white balls $= 12$

117. A two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 or by multiplying the difference of digits by 16 and adding 3. Find the number.

Ans : [Board Term-1 2012]

Let the digits of number be x and y, then number will $10x + y$.

According to the question, we have

$$8(x + y) - 5 = 10x + y$$

$$2x - 7y + 5 = 0 \quad ...(1)$$

also $$16(x - y) + 3 = 10x + y$$

$$6x - 17y + 3 = 0 \quad ...(2)$$

Comparing the equation with $ax + by + c = 0$ we get

$$a_1 = 2, b_1 = -1, c_1 = 5$$

$$a_2 = 6, b_2 = -17, c_2 = 3$$

Now $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$$

$$\frac{x}{(-7)(3) - (-17)(5)} = \frac{y}{(5)(6) - (2)(3)}$$

$$= \frac{1}{(2)(-17) - (6)(-7)}$$

c163

$$\frac{x}{-21+85} = \frac{y}{30-6} = \frac{1}{-34+42}$$

$$\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$$

$$\frac{x}{8} = \frac{y}{3} = 1$$

Hence, $x = 8, y = 3$

So required number$= 10 \times 8 + 3 = 83$.

118. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 square units if length is increased by 3 units and breadth is increased by 2 units. find the perimeter of the rectangle.

Ans : [Board Term-1 2012, Set-48]

Let length of given rectangle be x and breadth be y, then area of rectangle will be xy.

According to the first condition we have

$$(x-5)(y+3) = xy - 9$$

or, $3x - 5y = 6$ ...(1)

According to the second condition, we have

$$(x+3)(y+2) = xy - 67$$

or, $2x + 5y = 61$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 5 and then adding,

$$9x - 15y = 18$$

$$10x + 15y = 305$$

$$x = \frac{323}{19} = 17$$

c164

Substituting this value of x in equation (1),

$$3(17) - 5y = 6$$

$$5y = 51 - 6$$

$$y = 9$$

Hence, perimeter $= 2(x+y) = 2(17+9) = 52$ units.

119. Solve for x and y : $2(3x - y) = 5xy, 2(x + 3y) = 5xy$.

Ans : [Board Term-1 2012, Set-25]

We have $2(3x - y) = 5xy$ ...(1)

$2(x + 3y) = 5xy$ ...(2)

Divide equation (1) and (2) by xy,

$\frac{6}{y} - \frac{2}{x} = 5$ ...(3)

and $\frac{2}{y} + \frac{6}{x} = 5$ ...(4)

Let $\frac{1}{y} = a$ and $\frac{1}{x} = b$, then equations (3) and (4) become

$6a - 2b = 5$ ...(5)

$2a + 6b = 5$ ...(6)

Multiplying equation (5) by 3 and then adding with equation (6),

$$20a = 20$$

$$a = 1$$

c165

Substituting this value of a in equation (5),

$$b = \frac{1}{2}$$

Now $\frac{1}{y} = a = 1 \Rightarrow y = 1$

and $\frac{1}{x} = b = \frac{1}{2} \Rightarrow x = 2$

Hence, $x = 2, y = 1$

120. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Ans : [Board Term-1 2012, Set-68, NCERT]

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now $(x+3)(y-1) = xy$

$$xy + 3y - x - 3 = xy$$

$-x + 3y - 3 = 0$ ...(1)

and $(x-3)(y+2) = xy$

$$xy - 3y + 2x - 6 = xy$$

$2x - 3y - 6 = 0$ ...(2)

Multiply equation (1) 2 we have

$-2x + 6y - 6 = 0$ ...(3)

Adding equation (2) and (3) we have

$$3y-12 = 0$$

$$y = 4$$

c167

Substitute $y = 4$ in equation (1)

$$-x+12-3 = 0$$

$$x = 9$$

Total students $xy = 9\times 4 = 36$

Total students in the class is 36.

121. The ages of two friends ani and Biju differ by 3 years. Ani's father Dharam is twice as old as ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 year. Find the ages of Ani and Biju.

Ans : [Board Term-1 2012, Set-64]

Let the ages of Ani and Biju be x and y, respectively. According to the given condition,

$$x-y = \pm 3 \qquad ...(1)$$

Also, age of Ani's father Dharam $= 2x$ years

And age of Biju's sister $= \frac{y}{2}$ years

c168

According to the given condition,

$$2x-\frac{y}{2} = 30$$

$$4x-y = 60 \qquad ...(2)$$

Case I : When $x-y = 3$ ...(3)

Subtracting equation (3) from equation (2),

$$3x = 57$$

$$x = 19 \text{ years}$$

Putting $x=19$ in equation (3),

$$19-y = 3$$

$$y = 16 \text{ years}$$

Case II : When $x-y = -3$ ...(4)

Subtracting equation (iv) from equation (2),

$$3x = 60+3$$

$$3x = 63$$

$$x = 21 \text{ years}$$

Subtracting equation (4), we get

$$21-y = -3$$

$$y = 24 \text{ years}$$

Hence, Ani's age = 19 years or 21 years Biju age = 16 years or 24 years.

122. One says, "Give me a hundred, friend! I shall then become twice as rich as you." The other replies, "If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital.

Ans : [Board Term-1 2012, Set-54]

Let the amount of their respective capitals be x and y.

According to the given condition,

$$x+100 = 2(y-100)$$

$$x-2y = -300 \qquad ...(1)$$

and $$6(x-10) = y+10$$

$$6x-y = 70 \qquad ...(2)$$

Multiplying equation (2) by 2 we have

$$12x-2y = 140 \qquad ...(3)$$

Subtracting (1) from equation (3) we have

$$11x = 440$$

$$x = 40$$

c169

Substituting $x = 40$ in equation (1),

$$40-2y = -300$$

or, $$2y = 340$$

$$y = 170$$

Hence, the amount of their respective capitals are 40 and 170.

123. A fraction become $\frac{9}{11}$ if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes $\frac{5}{6}$. Find the fraction.

Ans : [Board Term-1 2012, Set-60]

Let the fraction be $\frac{x}{y}$, then according to the question,

$$\frac{x+2}{y+2} = \frac{9}{11}$$

c170

$$11x+22 = 9y+18$$

or, $$11x-9y+4 = 0 \qquad ...(1)$$

and $$\frac{x+3}{y+3} = \frac{5}{6}$$

or, $6x - 5y + 3 = 0$ ...(2)

Comparing with $ax + by + c = 0$

we get $a_1 = 11, b_1 = 9, c_1 = 4,$

$a_2 = 6, b_2 = -5$, and $c_2 = 3$

Now, $$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - b_2b_1}$$

$$\frac{x}{(-9)(3) - (-5)(4)} = \frac{y}{(4)(6) - (11)(3)}$$

$$= \frac{1}{(11)(-5) - (9)(-9)}$$

or, $$\frac{x}{-27 + 20} = \frac{y}{24 - 33} = \frac{1}{-55 + 54}$$

$$\frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$$

Hence, $x = 7, y = 9$

Thus fraction is $\frac{7}{9}$.

124. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

Ans : [Board Term-1 2012]

Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr.

Speed of boat up stream $= (x - y)$ km/hr.

Speed of boat down stream $= (x + y)$ km/hr.

$$\frac{30}{x - y} + \frac{28}{x + y} = 7$$

and $$\frac{21}{x - y} + \frac{21}{x + y} = 5$$

c171

Let $\frac{1}{x - y}$ be a and $\frac{1}{x + y}$ be b, then we have

$30a + 28b = 7$ ...(1)

$21a + 21b = 5$ ...(2)

Multiplying equation (1) by 3 and equation (2) by 4 we have

$90a + 84b = 21$ ...(3)

$84a + 84b = 20$ ...(4)

Subtracting (4) from (3) we have,

$6a = 1$

$a = \frac{1}{6}$

Putting this value of a in equation (1),

$$30 \times \frac{1}{6} + 28b = 7$$

$$28b = 7 - 30 \times \frac{1}{6} = 2$$

$$b = \frac{1}{14}$$

Thus $x + y = 14$ ...(5)

Now, $$a = \frac{1}{x - y} = \frac{1}{6}$$

or, $x - y = 6$ ...(6)

and $x + y = 14$

Solving equation (5) and (6), we get

$x = 10, y = 4$

Hence, speed of the boat in still water = 10km/hr

and speed of the stream = 4 km/hr.

125. A boat covers 32 km upstream and 36 km downstream in 7 hours. Also, it covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of the boat in still water and that of the stream.

Ans : [Board Term-1 2012, Set-48]

Let the speed of the boat be x km/hr and the speed of the stream be y km/hr.

According to the question,

c172

$$\frac{32}{x - y} + \frac{36}{x + y} = 7$$

and $$\frac{40}{x - y} + \frac{48}{x + y} = 9$$

Let $\frac{1}{x - y} = A, \frac{1}{x + y} = B$, then we have

$32A + 36B = 7$ ...(1)

and $40A + 48B = 9$ ...(2)

Multiplying equation (1) by 5 and (2) by 4, we have

$160A + 180B = 35$ ...(3)

and $160A + 192B = 36$ ...(4

Subtracting (4) from (3) we have

$-12B = -1$

$$B = \frac{1}{12}$$

Substituting the value of B in (2) we get

$$40A + 48\left(\frac{1}{12}\right) = 9$$

$$40A + 4 = 9$$
$$40A = 5$$
$$A = \frac{1}{8}$$

Thus $A = \frac{1}{8}$ and $B = \frac{1}{12}$

Hence $A = \frac{1}{8} = \frac{1}{x-y}$

$$x - y = 8 \quad ...(5)$$

and $B = \frac{1}{12} = \frac{1}{x+y}$

$$x + y = 12 \quad ...(6)$$

Adding equations (5) and (6) we have,

$$2x = 20$$
$$x = 10$$

Substituting this value of x in equation (1),

$$y = x - 8 = 10 - 8 = 2$$

Hence, the speed of the boat in still water = 10 km/hr and speed of the stream = 2 km/hr.

126. For what values of a and b does the following pair of linear equations have infinite number of solution ?

$2x + 3y = 7, a(x+y) - b(x-y) = 3a + b - 2$

Ans : [Board Term-1 2015]

We have $2x + 3y - 7 = 0$

Here $a_1 = 2, b_1 = 3, c_1 = -7$

and $a(x+y) - b(x-y) = 3a + b - 2$

$$ax + ay - bx + by = 3a + b - 2$$
$$(a-b)x + (a+b)y - (3a+b-2) = 0$$

Here $a_2 = a - b, b_2 = a + b, c_2 = -(3a+b-2)$

For infinite many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a-b} = \frac{3}{a+b} = \frac{-7}{(3a+b-2)}$$

From $\frac{2}{a-b} = \frac{7}{3a+b-2}$ we have

c173

$$2(3a+b-2) = 7(a-b)$$
$$6a + 2b - 4 = 7a - 7b$$
$$a - 9b = -4 \quad ...(1)$$

From $\frac{3}{a+b} = \frac{7}{3a+b-2}$ we have

$$3(3a+b-2) = 7(a+b)$$
$$9a + 3b - 6 = 7a + 7b$$
$$2a - 4b = 6$$
$$a - 2b = 3 \quad ...(2)$$

Subtracting equation (1) from (2),

$$-7b = -7$$
$$b = 1$$

Substituting the value of b in equation (1),

$$a = 5$$

Hence, $a = 5, b = 1$.

127. At a certain time in a deer, the number of heads and the number of legs of deer and human visitors were counted and it was found that there were 39 heads and 132 legs.

Find the number of deer and human visitors in the park.

Ans : [Board Term-1 2015]

Let the no. of deer be x and no. of human be y. According to the question,

$$x + y = 39 \quad ...(1)$$

and $4x + 2y = 132 \quad ...(2)$

Multiply equation (1) from by 2,

$$2x + 2y = 78 \quad ...(3)$$

Subtract equation (3) from (2),

$$2x = 54$$
$$x = 27$$

c175

Substituting this value of x in equation (1)

$$27 + y = 39$$
$$y = 12$$

So, No. of deer = 27 and No. of human = 12

128. Find the value of p and q for which the system of equations represent coincident lines $2x + 3y = 7$,

$(p+q+1)x+(p+2q+2)y=4(p+q)+1$

Ans : [Board Term-1 2012, Set-42]

We have $2x+3y = 7$

$$(p+q+1)x+(p+2q+2)y = 4(p+q)+1$$

Comparing given equation to $ab+by+c=0$ we have

$a_1=2, b_1=3, c_1=-7$

$a_2=p+q+1, b_2=p+2q+2, c_2=-4(p+q)-1$

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{p+q+1} = \frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$$

From $\frac{3}{p+2q+2} = \frac{7}{4(p+q)+1}$ we have

$$7p+14q+14 = 12p+12q+3$$

$$5p-2q-11 = 0 \quad ...(1)$$

From $\frac{2}{p+q+1} = \frac{7}{4(p+q)+1}$ we have

c176

$$8(p+q)+2 = 7p+7q+7$$

$$8p+8q+2 = 7p+7q+7$$

$$p+q-5 = 0 \quad ...(2)$$

Multiplying equation (2) by 5 we have

$$5p+5q-25 = 0 \quad ...(3)$$

Subtracting equation (1) from (3) we get

$$7q = 14$$

$$q = 2$$

Hence, $p = 3$ and $q=2$.

129. The length of the sides of a triangle are $2x+\frac{y}{2}$, $\frac{5x}{3}+y+\frac{1}{2}$ and $\frac{2}{3}x+2y+\frac{5}{2}$. If the triangle is equilateral , find its perimeter.

Ans : [Board Term-1 2012]

For an equilateral Δ,

$$2x+\frac{y}{2} = \frac{5x}{3}+y+\frac{1}{2} = \frac{1}{2}x+2y+\frac{5}{2}$$

Now $$\frac{4x+y}{2} = \frac{10x+6y+3}{6}$$

$$12x+3y = 10x+6y+3$$

$$2x-3y = 3 \quad ...(1)$$

Again, $$2x+\frac{y}{2} = \frac{2}{3}x+2y+\frac{5}{2}$$

$$\frac{4x+y}{2} = \frac{4x+12y+15}{6}$$

$$12x+3y = 4x+12y+15$$

$$8x-9y = 15 \quad ...(2)$$

Multiplying equation (1) by 3 we have

$$6x-9y = 9 \quad ...(1)$$

Subtracting it from (2) we get

$$2x = 6 \Rightarrow x = 3$$

Substituting this value of x into (1), we get

c178

$$2\times 3-3y = 3$$

or, $$3y = 3 \Rightarrow y = 1$$

Now substituting these value of x and y

$$2x+\frac{y}{2} = 2\times 3+\frac{1}{2} = 6.5$$

The perimeter of equilateral triangle = side $\times$ 3

$$= 6.5\times 3 = 19.5 \text{ cm}$$

Hence, the perimeter of Δ = 19.5 m

130. When 6 boys were admitted and 6 girls left, the percentage of boys increased from 60% to 75%. Find the original no. of boys and girls in the class.

Ans : [Board Term-1 2015]

Let the no. of boys be x and no. of girls be y.

No. of students $= x+y$

Now $$\frac{x}{x+y} = \frac{60}{100} \quad ...(1)$$

and $$\frac{x+6}{(x+6)+(y-6)} = \frac{75}{100} \quad ...(2)$$

From (1), we have

c180

$$100x = 60x+60y$$

$$40x-60y = 0$$

$$2x-3y = 0$$

$$2x = 3y \quad (3)$$

From (2) we have

$$100x+600 = 75x+75y$$

$$25x-75y = -600$$

$$x - 3y = -24 \quad ...(4)$$

Substituting the value of $3y$ from (3) in to (4) we have,

$$\begin{aligned} x - 2x &= -24 \Rightarrow x = 24 \\ 3y &= 24 \times 2 \\ y &= 16 \end{aligned}$$

Hence, no. of boys is 24 and no. of girls is 16.

131. A cyclist, after riding a certain distance, stopped for half an hour to repair his bicycle, after which he completes the whole journey of 30 km at half speed in 5 hours. If the breakdown had occurred 10 km farther off, he would have done the whole journey in 4 hours. Find where the breakdown occurred and his original speed.

Ans : [Board Term-1 2013, Set-32]

Let x be the distance of the place where breakdown occurred and y be the original speed,

c181

$$\frac{x}{y} + \frac{30 - x}{\frac{y}{2}} = 5$$

or

$$\begin{aligned} \frac{x}{y} + \frac{60 - 2x}{y} &= 5 \\ x + 60 - 2x &= 5y \\ x + 5y &= 60 \quad ...(1) \end{aligned}$$

and

$$\begin{aligned} \frac{x+10}{y} + \frac{30 - (x+10)}{\frac{y}{2}} &= 4 \\ \frac{x+10}{y} + \frac{60 - 2(x+10)}{y} &= 4 \\ x + 10 + 60 - 2x - 20 &= 4y \\ -x + 50 &= 4y \\ x + 4y &= 50 \quad (2) \end{aligned}$$

Subtract equation (2) from (1), $y = 10$ km/hr.

Now from (2),

$$\begin{aligned} x + 40 &= 50 \\ x &= 10 \text{ km} \end{aligned}$$

Break down occurred at 10 km and original speed was 10 km/hr.

132. The population of a village is 5000. If in a year, the number of males were to increase by 5% and that of a female by 3% annually, the population would grow to 5202 at the end of the year. Find the number of males and females in the village.

Ans : [Board Term-1 2012, Set-60]

Let the number of males be x and females be y

Now

$$x + y = 5,000 \quad ...(1)$$

and

$$\begin{aligned} x + \frac{5}{100}x + y + \frac{3y}{100} &= 5202 \\ \frac{5x + 3y}{100} + 5000 &= 5202 \end{aligned}$$

c182

$$\begin{aligned} 5x + 3y &= (5202 - 5000) \times 100 \\ 5x + 3y &= 20200 \quad (2) \end{aligned}$$

Multiply (1) by 3 we have

$$3x + 3y = 15,000 \quad ...(3)$$

Subtracting (2) from (3) we have

$$2x = 5200 \Rightarrow x = 2600$$

Substituting value of x in (1) we have

$$\begin{aligned} 2600 - y &= 5000 \\ y &= 2400 \end{aligned}$$

Thus no. of males is 2600 and no. of females is 2400.

CHAPTER 4

QUADRATIC EQUATIONS

ONE MARK QUESTIONS

Multiple Choice Questions

1. The sum and product of the zeroes of a quadratic polynomial are 3 and -10 respectively. The quadratic polynomial is

(a) $x^2-3x+10$ (b) $x^2+3x-10$

(c) $x^2-3x-10$ (d) $x^2+3x+10$

Ans : [Board 2020 Delhi Basic]

Sum of zeroes, $\alpha+\beta=3$

and product of zeroes, $\alpha\beta=-10$

d243

Quadratic polynomial,

$$p(x)=x^2-(\alpha+\beta)+\alpha\beta$$
$$=x^2-3x-10$$

Thus (c) is correct option.

2. If the sum of the zeroes of the quadratic polynomial $kx^2+2x+3k$ is equal to their product, then k equals

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Ans : [Board 2020 OD Basic]

We have $p(x)=kx^2+2x+3k$

Comparing it by ax^2+bx+c, we get $a=k$, $b=2$ and $c=3k$.

Sum of zeroes, $\alpha+\beta=-\frac{b}{a}=-\frac{2}{k}$

d244

Product of zeroes, $\alpha\beta=\frac{c}{a}=\frac{3k}{k}=3$

According to question, we have

$$\alpha+\beta=\alpha\beta$$
$$-\frac{2}{k}=3\Rightarrow k=-\frac{2}{3}$$

Thus (d) is correct option.

3. If α and β are the zeroes of the polynomial x^2+2x+1, then $\frac{1}{\alpha}+\frac{1}{\beta}$ is equal to

(a) -2 (b) 2

(c) 0

Ans : [Board 2020 Delhi Basic]

Since α and β are the zeros of polynomial x^2+2x+1,

Sum of zeroes, $\alpha+\beta=-\frac{2}{1}=-2$

d245

and product of zeroes, $\alpha\beta=\frac{1}{1}=1$

Now, $$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=-\frac{2}{1}=-2$$

Thus (a) is correct option.

4. If α and β are the zeroes of the polynomial $2x^2-13x+6$, then $\alpha+\beta$ is equal to

(a) -3 (b) 3

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

d246

Ans : [Board 2020 Delhi Basic]

We have $p(x)=2x^2-13x+6$

Comparing it with ax^2+bx+c we get $a=2$, $b=-13$ and $c=6$

Sum of zeroes $\alpha+\beta=-\frac{b}{a}=-\frac{(-13)}{2}=\frac{13}{2}$

Thus (c) is correct option.

5. The roots of the quadratic equation $x^2-0.04=0$ are

(a) ±0.2 (b) ±0.02

(c) 0.4 (d) 2

Ans : [Board 2020 OD Standard]

We have $x^2-0.04=0$

d247

$$x^2=0.04$$
$$x=\pm\sqrt{0.04}$$
$$x=\pm0.2.$$

Thus (a) is correct option.

6. If ½ is a root of the equation $x^2+kx-\frac{5}{4}=0$, then the value of k is

(a) 2 (b) -2

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

d248

Ans :

We have $x^2 + kx - \frac{5}{4} = 0$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

Thus
$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$
$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$
$$\frac{1 + 2k - 5}{4} = 0$$
$$2k - 4 = 0 \Rightarrow k = 2$$

Thus (a) is correct option.

7. Each root of $x^2 - bx + c = 0$ is decreased by 2. The resulting equation is $x^2 - 2x + 1 = 0$, then

(a) $b = 6, c = 9$ (b) $b = 3, c = 5$

(c) $b = 2, c = -1$ (d) $b = -4, c = 3$

Ans :

For $x^2 - bx + c = 0$ we have
$$\alpha + \beta = b$$
$$\alpha\beta = c$$

Now $\alpha - 2 + \beta - 2 = \alpha + \beta - 4 = b - 4$
$$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) + 4$$
$$= c - 2b + 4$$

d249

For $x^2 - 2x + 1 = 0$ we have
$$2 = b - 4 \Rightarrow b = 6$$

and
$$1 = c - 2b + 4$$
$$= c - 2 \times 6 + 4$$
$$= c - 8$$
$$c = 1 + 8 = 9$$

Thus (a) is correct option.

8. Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0 (b) 4

(c) 8 (d) 0, 8

Ans :

We have $2x^2 - kx + k = 0$

Comparing with $ax^2 + bx + c = 0$ we $a = 2$, $b = -k$ and $c = k$.

For equal roots, the discriminant must be zero.

Thus
$$b^2 - 4ac = 0$$
$$(-k)^2 - 4(2)k = 0$$
$$k^2 - 8k = 0$$
$$k(k - 8) = 0 \Rightarrow k = 0,\ 8$$

d250

Hence, the required values of k are 0 and 8.

Thus (d) is correct option.

9. If the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots, then

(a) $mp = nq$ (b) $mq = np$

(c) $mn = pq$ (d) $mq = \sqrt{np}$

Ans :

For equal roots, $b^2 = 4ac$
$$4(mp + nq)^2 = 4(m^2 + n^2)(p^2 + q^2)$$
$$m^2q^2 + n^2p^2 - 2mnpq = 0$$
$$(mq - np)^2 = 0$$
$$mq - np = 0$$
$$mq = np$$

d251

Thus (b) is correct option.

10. The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are

(a) $k \geq 2$ (b) $k \leq 2$

(c) $k \geq -2$ (d) $2 \leq k \leq -2$

Ans :

We have, $x^2 + kx + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = k$ and $c = 1$.

For linear factors, $b^2 - 4ac \geq 0$
$$k^2 - 4 \times 1 \times 1 \geq 0$$
$$(k^2 - 2^2) \geq 0$$
$$(k - 2)(k + 2) \geq 0$$

d252

$$k \geq 2 \text{ and } k \leq -2$$

Thus (d) is correct option.

11. If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

d253

Ans :

If one root is α, then the other $\frac{1}{\alpha}$.

Product of roots, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

Thus (d) is correct option.

12. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d255

Ans :

We have $2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots i.e., imaginary roots.

Thus (c) is correct option.

13. The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) 1, 8 (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

Ans :

We have $x^{2/3} + x^{1/3} - 2 = 0$

d256

Substituting $x^{1/3} = y$ we obtain,

$$y^2 + y - 2 = 0$$

$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, -8.

Thus (d) is correct option.

14. $(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

Ans :

We have $(x^2 + 1)^2 - x^2 = 0$

d257

$$x^4 + 1 + 2x^2 - x^2 = 0$$

$$x^4 + x^2 + 1 = 0$$

$$(x^2)^2 + x^2 + 1 = 0$$

Let $x^2 = y$ then we have

$$y^2 + y + 1 = 0$$

Comparing with $ay^2 + by + c = 0$ we get $a = 1$, $b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4 = -3$$

Since, $D < 0$, $y^2 + y + 1 = 0$ has no real roots.

i.e. $x^4 + x^2 + 1 = 0$ or $(x^2 + 1)^2 - x^2 = 0$ has no real roots.

Thus (c) is correct option.

15. The equation $2x^2 + 2(p+1)x + p = 0$, where p is real, always has roots that are

(a) Equal

(b) Equal in magnitude but opposite in sign

(c) Irrational

(d) Real

Ans :

We have $2x^2 + 2(p+1)x + p = 0$,

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = 2(p+1)$ and $c = p$.

Now $b^2 - 4ac = [2(p+1)]^2 - 4(2p)$

d258

$$= 4(p+1)^2 - 8p$$

$$= 4p^2 + 8p + 4 - 8p$$

$$= 4(p^2 + 1)$$

For any real value of p, $4(p^2 + 1)$ will always be positive as p^2 cannot be negative for real p. Hence, the discriminant $b^2 - 4ac$ will always be positive.

When the discriminant is greater than 0 or is positive, then the roots of a quadratic equation are real.

Thus (d) is correct option.

16. The condition for one root of the quadratic equation

$ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$ (d) $c^2 = 9a - b^2$

d259

Ans :

Sum of zeroes $\alpha + 2\alpha = -\frac{b}{a}$

$$3\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{3a}$$

Product of zeroes $\alpha \times 2\alpha = \frac{c}{a}$

$$2\alpha^2 = \frac{c}{a}$$

$$2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

$$2ab^2 - 9a^2c = 0$$

$$a(2b^2 - 9ac) = 0$$

Since, $a \neq 0$, $2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

Thus (b) is correct option.

17. If $x^2 + y^2 = 25$, $xy = 12$, then x is

(a) $(3, 4)$ (b) $(3, -3)$

(c) $(3, 4, -3, -4)$ (d) $(3, -3)$

Ans :

d260

We have $x^2 + y^2 = 25$

and $xy = 12$

$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$

$$x^4 + 144 - 25x^2 = 0$$

$$(x^2 - 16)(x^2 - 9) = 0$$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

Thus (c) is correct option.

18. The quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d261

Ans :

We have $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

Here $a = 2$, $b = -3\sqrt{2}$, $c = \frac{9}{4}$

Discriminant $D = b^2 - 4ac$

$$= (-3\sqrt{2})^2 - 4 \times 2 \times \frac{9}{4}$$

$$= 18 - 18 = 0$$

Thus, $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has real and equal roots.

Thus (b) is correct option.

19. The quadratic equation $x^2 + x - 5 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d262

Ans :

We have $x^2 + x - 5 = 0$

Here, $a = 1$, $b = 1$, $c = -5$

Now, $D = b^2 - 4ac$

$$= (1)^2 - 4 \times 1 \times (-5)$$

$$= 21 > 0$$

So $x^2 + x - 5 = 0$ has two distinct real roots.

Thus (a) is correct option.

20. The quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

d263

Ans :

We have $x^2 + 3x + 2\sqrt{2} = 0$

Here, $a = 1$, $b = 3$ and $c = 2\sqrt{2}$

Now, $D = b^2 - 4ac$

$$= (3)^2 - 4(1)(2\sqrt{2})$$

$$= 9 - 8\sqrt{2} < 0$$

Hence, roots of the equation are not real.
Thus (c) is correct option.

21. The quadratic equation $5x^2 - 3x + 1 = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d264

Ans :

We have $5x^2 - 3x + 1 = 0$

Here $a = 5,\ b = -3,\ c = 1$

Now, $D = b^2 - 4ac = (-3)^2 - 4(5)(1)$

$= 9 - 20 < 0$

Hence, roots of the equation are not real.
Thus (c) is correct option.

22. The quadratic equation $x^2 - 4x + 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d265

Ans :

We have $x^2 - 4x + 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = 3\sqrt{2}$

Now $D = b^2 - 4ac = (-4)^2 - 4(1)(3\sqrt{2})$

$= 16 - 12\sqrt{2}$

$= 16 - 12 \times (1.41)$

$= 16 - 16.92 = -0.92$

$b^2 - 4ac < 0$

Hence, the given equation has no real roots.
Thus (c) is correct option.

23. The quadratic equation $x^2 + 4x - 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d266

Ans :

We have $x^2 + 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = 4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac = (4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots,
Thus (a) is correct option.

24. The quadratic equation $x^2 - 4x - 3\sqrt{2} = 0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d267

Ans :

We have $x^2 - 4x - 3\sqrt{2} = 0$

Here $a = 1,\ b = -4$ and $c = -3\sqrt{2}$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(1)(-3\sqrt{2})$

$= 16 + 12\sqrt{2} > 0$

Hence, the given equation has two distinct real roots.
Thus (a) is correct option.

25. The quadratic equation $3x^2 + 4\sqrt{3}\,x + 4$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

d268

Ans :

We have $3x^2 + 4\sqrt{3}\,x + 4 = 0$

Here, $a = 3,\ b = 4\sqrt{3}$ and $c = 4$

Now $D = b^2 - 4ac = (4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

Hence, the equation has real and equal roots.
Thus (b) is correct option.

26. Which of the following equations has 2 as a root?
(a) $x^2 - 4x + 5 = 0$ (b) $x^2 + 3x - 12 = 0$
(c) $2x^2 - 7x + 6 = 0$ (d) $3x^2 - 6x - 2 = 0$

Ans :

(a) Substituting, $x = 2$ in $x^2 - 4x + 5$, we get

$$(2)^2-4(2)+5 = 4-8+5 = 1 \neq 0$$

So, $x = 2$ is not a root of

$$x^2-4x+5 = 0$$

d269

(b) Substituting, $x = 2$ in $x^2+3x-12$, we get

$$(2)^2+3(2)-12 = 4+6-12 = -2 \neq 0$$

So, $x = 2$ is not a root of $x^2+3x-12 = 0$.

(c) Substituting, $x = 2$ in $2x^2-7x+6$, we get

$$\begin{aligned} 2(2)^2-7(2)+6 &= 2(4)-14+6 \\ &= 8-14+6 \\ &= 14-14 = 0. \end{aligned}$$

So, $x = 2$ is a root of the equation $2x^2-7x+6 = 0$.

(d) Substituting, $x = 2$ in $3x^2-6x-2$, we get

$$3(2)^2-6(2)-2 = 12-12-2 = -2 \neq 0$$

So, $x = 2$ is not a root of

$$3x^2-6x-2 = 0.$$

Thus (c) is correct option.

27. Which of the following equations has the sum of its roots as 3 ?

(a) $2x^2-3x+6=0$

(b) $-x^2+3x-3=0$

(c) $\sqrt{2}\,x^2-\frac{3}{\sqrt{2}}x+1=0$

(d) $3x^2-3x+3=0$

Ans :

Sum of the roots,

$$\alpha+\beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

Option a : $\alpha+\beta = -\left(\frac{-3}{2}\right) = \frac{3}{2} \neq 3$

d270

Option b : $\alpha+\beta = -\left(\frac{3}{-1}\right) = 3$

Option c : $\alpha+\beta = -\left(\frac{\frac{3}{\sqrt{2}}}{\sqrt{2}}\right) = \frac{3}{2} \neq 3$

Option d $\alpha+\beta = -\left(\frac{-3}{3}\right) = 1 \neq 3$

Thus (b) is correct option.

28. Assertion : $4x^2-12x+9=0$ has repeated roots.

Reason : The quadratic equation $ax^2+bx+c=0$ have repeated roots if discriminant $D>0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Reason is false because if $D=0$, equation has repeated roots.

Assertion $4x^2-12x+9 = 0$

d271

$$\begin{aligned} D &= b^2-4ac \\ &= (-12)^2-4(4)(9) \\ &= 144-144 = 0 \end{aligned}$$

Roots are repeated.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

29. Assertion : The equation $x^2+3x+1=(x-2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2+bx+c=0$ where $a \neq 0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have, $x^2+3x+1 = (x-2)^2 = x^2-4x+4$

$$x^2+3x+1 = x^2-4x+4$$

$$7x-3 = 0$$

d272

It is not of the form $ax^2+bx+c=0$

(d) Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

30. Assertion : The values of x are $-\frac{a}{2}, a$ for a quadratic equation $2x^2+ax-a^2=0$.

Reason : For quadratic equation $ax^2+bx+c=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $2x^2 + ax - a^2 = 0$

$$x = \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a + 3a}{4} = \frac{2a}{4}, \frac{-4a}{4}$$

$$x = \frac{a}{2}, -a$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

31. Assertion : The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of k is $\pm\frac{8}{3}$.

Reason : The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $8x^2 + 3kx + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$$

For equal roots, $D = 0$

$$9k^2 - 64 = 0$$

$$9k^2 = 64$$

$$k^2 = \frac{64}{9} \Rightarrow k = \pm\frac{8}{3}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

32. Assertion : The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

Reason : If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $x^2 + 2x + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = -4 < 0$$

Roots are imaginary.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Fill in the Blank Questions

33. A real number α is said to be .... of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

Ans :

root

34. For any quadratic equation $ax^2 + bx + c = 0$, $b^2 - 4ac$, is called the .... of the equation.

Ans :

discriminant

35. If the discriminant of a quadratic equation is zero, then its roots are .... and ....

Ans :

real, equal

36. If the discriminant of a quadratic equation is greater than zero, then its roots are .... and ....

Ans :

real, distinct

37. A polynomial of degree 2 is called the .... polynomial.

Ans :

quadratic

38. A quadratic equation cannot have more than .... roots.

Ans :

two

39. Let $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$, be a quadratic equation, then this equation has no real roots if and only if ....

Ans :

$b^2 < 4ac$

40. If the product ac in the quadratic equation $ax^2 + bx + c$ is negative, then the equation cannot have .... roots.

Ans :

Non-real

41. The equation of the form $ax^2 + bx = 0$ will always have .... roots.

Ans :

real

42. A quadratic equation in the variable x is of the form

$$ax^2 + bx + c = 0,$$

where a, b, c are real numbers and a ....

Ans :

$\neq 0$

43. The roots of a quadratic equation is same as the .... of the corresponding quadratic polynomial.

Ans :

zero

44. Value of the roots of the quadratic equation, $x^2 - x - 6 = 0$ are ....

Ans : [Board 2020 OD Basic]

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x-3)(x+2) = 0 \Rightarrow x = 3 \text{ and } x = -2$$

45. If quadratic equation $3x^2 - 4x + k = 0$ has equal roots, then the value of k is ....

Ans : [Board 2020 Delhi Basic]

Given, quadratic equation is $3x^2 - 4x + k = 0$

Comparing with $ax^2 + bx + c = 0$, we get $a = 3$, $b = -4$ and $c = k$

For equal roots, $b^2 - 4ac = 0$

$$(-4)^2 - 4(3)(k) = 0$$

$$16 - 12k = 0$$

$$k = \frac{16}{12} = \frac{4}{3}$$

Very Short Answer Questions

46. Find the positive root of $\sqrt{3x^2 + 6} = 9$.

Ans : [Board Term-2, 2015]

We have $\sqrt{3x^2 + 6} = 9$

$$3x^2 + 6 = 81$$

$$3x^2 = 81 - 6 = 75$$

$$x^2 = \frac{75}{3} = 25$$

Thus $x = \pm 5$

Hence 5 is positive root.

47. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

Ans : [Board Term-2, Delhi 2015]

We have $3x^2 + 2kx - 3 = 0$

Substituting $x = -\frac{1}{2}$ in given equation we get

$$3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\frac{3}{4} - k - 3 = 0$$

$$k = \frac{3}{4} - 3$$

$$= \frac{3-12}{4} = \frac{-9}{4}$$

Hence $k = \frac{-9}{4}$

48. Find the roots of the quadratic equation $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

Ans : [Board Term-2, 2012, 2011]

We have $\sqrt{3}x^2 - 2x - \sqrt{3} = 0$

$$\sqrt{3}x^2 - 3x + x - \sqrt{3} = 0$$

$$\sqrt{3}x(x - \sqrt{3}) + 1(x - \sqrt{3}) = 0$$

$$(x-\sqrt{3})(\sqrt{3}\,x+1) = 0$$

Thus $$x = \sqrt{3}, \frac{-1}{\sqrt{3}}$$

49. Find the value of k, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

Ans : [Board Term-2, 2016]

We have $$kx^2 - 14x + 8 = 0$$

d104

Let one root be α and other root be 6α.

Sum of roots, $$\alpha + 6\alpha = \frac{14}{k}$$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots , $$\alpha(6\alpha) = \frac{8}{k} \text{ or } 6\alpha^2 = \frac{8}{k} \quad ...(2)$$

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$

$$6 \times \frac{4}{k^2} = \frac{8}{k}$$

$$\frac{3}{k^2} = \frac{1}{k}$$

$$3k = k^2$$

$$3k - k^2 = 0$$

$$k[3-k] = 0$$

$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

50. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

Ans : [Board Term-2 Foreign-2, 2017]

We have $$6x^2 - x - k = 0$$

Substituting $x = \frac{2}{3}$, we get

d105

$$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

$$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$$

$$\frac{8}{3} - \frac{2}{3} - k = 0$$

$$\frac{8-2}{3} - k = 0$$

$$2 - k = 0$$

Thus $k = 2$.

51. Find the value(s) of k if the quadratic equation $3x^2 - k\sqrt{3}\,x + 4 = 0$ has real roots.

Ans : [SQP 2017]

If discriminant $D = b^2 - 4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $$3x^2 - k\sqrt{3}\,x + 4 = 0$$

Comparing with $ax^2 + bx + c = 0 = 0$ we get

$$a = 3, b = -k\sqrt{3} \text{ and } c = 4$$

d106

For real roots $$b^2 - 4ac \geq 0$$

$$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$

$$3k^2 - 48 \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

TWO MARKS QUESTIONS

52. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

Ans : [Board 2019 Delhi]

We have $$x^2 + 4x + k = 0.$$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 4, c = k$.

Since, given the equation has real roots,

d308

$$D \geqslant 0$$

$$b^2 - 4ac \geqslant 0$$

$$4^2 - 4 \times 1 \times k \geqslant 0$$

$$4k \leqslant 16$$

$$k \leqslant 4$$

53. Find the value of k for which the roots of the equations $3x^2 - 10x + k = 0$ are reciprocal of each other.

Ans : [Board 2019 Delhi]

We have $$3x^2 - 10x + k = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$

we get $a = 3,\ b = -10, c = k$

Let one root be α so other root is $\frac{1}{\alpha}$.

Now product of roots $\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

d309

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Hence, value of k is 3.

54. Find the value of k such that the polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of its zeros equal to half of their product.

Ans : [Board 2019 Delhi]

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\alpha + \beta = -\frac{-(k+6)}{1} = k+6$

Product of roots, $\alpha\beta = \frac{2(2k+1)}{1} = 2(2k+1)$

According to given condition,

d310

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k+6 = \frac{1}{2}[2(2k+1)]$$

$$k+6 = 2k+1 \Rightarrow k = 5$$

Hence, the value of k is 5.

55. Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

Ans : [Board 2019 OD]

We have $2x^2 - 4x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2\ b = -4,\ c = 3$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(2) \times (3)$

$= -8 < 0$ or $(-\text{ve})$

d311

Hence, the given equation has no real roots.

56. Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Ans : [Board Term-2, 2012]

We have $6x^2 - x - 2 = 0$

$$6x^2 + 3x - 4x - 2 = 0 \qquad (3 \times 4 = 2 \times 6)$$

$$3x(2x+1) - 2(2x+1) = 0$$

$$(2x+1)(3x-2) = 0$$

$$3x - 2 = 0 \text{ or } 2x + 1 = 0$$

$$x = \frac{2}{3} \text{or } x = -\frac{1}{2}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

d107

57. Find the roots of the following quadratic equation :

$15x^2 - 10\sqrt{6}\,x + 10 = 0$

Ans : [Board Term-2, 2012]

We have $15x^2 - 10\sqrt{6}\,x + 10 = 0$

$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$

$$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$

$$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$$

$$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$$

d108

Thus $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

58. Solve the following quadratic equation for x :

$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

Ans : [Board Term-2, 2013, 2012]

We have $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

$$4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}\,x + 2) - \sqrt{3}(\sqrt{3}\,x + 2) = 0$$

$$(\sqrt{3}\,x + 2)(4x - \sqrt{3}) = 0$$

d109

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

59. Solve for x : $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Ans : [Board Term-2 Foreign 2015]

We have

$$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$$

$$x^2 - \sqrt{3}\,x - 1x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(x - 1) = 0$$

d110

Thus $x = \sqrt{3}, x = 1$

60. Find the roots of the following quadratic equation :

$(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

Ans : [Board Term-2 2012]

We have
$$(x+3)(x-1) = 3\left(x-\frac{1}{3}\right)$$
$$x^2+3x-x-3 = 3x-1$$
$$x^2-x-2 = 0$$
$$x^2-2x+x-2 = 0$$
$$x(x-2)+1(x-2) = 0$$
$$(x-2)(x+1) = 0$$

d111

Thus $x = 2, -1$

61. Find the roots of the following quadratic equation :
$$\frac{2}{5}x^2-x-\frac{3}{5} = 0$$

Ans : [Board Term-2, 2012]

We have
$$\frac{2}{5}x^2-x-\frac{3}{5} = 0$$
$$\frac{2x^2-5x-3}{5} = 0$$
$$2x^2-5x-3 = 0$$
$$2x^2-6x+x-3 = 0$$
$$2x(x-3)+1(x-3) = 0$$
$$(2x+1)(x-3) = 0$$

d112

Thus $x = -\frac{1}{2},\ 3$

62. Solve the following quadratic equation for x :
$$4x^2-4a^2x+(a^4-b^4) = 0$$

Ans : [Delhi Term-2, 2015]

We have
$$4x^2-4a^2x+(a^4-b^4) = 0$$

Comparing with $Ax^2+Bx+C = 0$ we have
$$A = 4, B = -4a^2, C = (a^4-b^4)$$
$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$= \frac{4a^2\pm\sqrt{(-4a^2)^2-4\times4(a^4-b^4)}}{2\times4}$$
$$= \frac{4a^2\pm\sqrt{16a^2-16a^4+16b^4}}{8}$$

d113

$$= \frac{4a^2\pm\sqrt{16b^4}}{8}$$

or,
$$x = \frac{4a^2\pm4b^2}{8} = \frac{a^2\pm b^2}{2}$$

Thus either $x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$

63. Solve the following quadratic equation for x :
$$9x^2-6b^2x-(a^4-b^4) = 0$$

Ans : [Delhi Term-2, 2015]

We have
$$9x^2-6b^2x-(a^4-b^4) = 0$$

Comparing with $Ax^2+Bx+C = 0$ we have
$$A = 9, B = -6b^2, C = -(a^4-b^4)$$
$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

d114

$$x = \frac{6b^2\pm\sqrt{(-6b^2)^2-4\times9\times\{(a^4-b^4)\}}}{2\times9}$$
$$= \frac{6b^2\pm\sqrt{36b^4+36a^4-36b^4}}{18}$$
$$= \frac{6b^2\pm\sqrt{36a^4}}{18} = \frac{6b^2\pm6a^2}{18}$$

Thus $x = \frac{a^2+b^2}{3}, \frac{b^2-a^2}{3}$

64. Solve the following equation for x :
$$4x^2+4bx-(a^2-b^2) = 0$$

Ans : [Board Term-2, OD 2012]

We have
$$4x^2+4bx+b^2-a^2 = 0$$
$$(2x+b)^2-a^2 = 0$$
$$(2x+b+a)(2x+b-a) = 0$$

d115

Thus $x = \frac{-(a+b)}{2}$ and $x = \frac{a-b}{2}$

65. Solve the following quadratic equation for x :
$$x^2-2ax-(4b^2-a^2) = 0$$

Ans : [Board Term-2, 2015]

We have
$$x^2-2ax-(4b^2-a^2) = 0$$
$$x^2-2ax+a^2-4b^2 = 0$$
$$(x-a)^2-(2b)^2 = 0$$

d116

$$(x-a+2b)(x-a-2b) = 0$$

Thus $x = a-2b, x = a+2b$

66. Solve the quadratic equation, $2x^2 + ax - a^2 = 0$ for x.

Ans : [Board Term-2 Delhi 2014]

We have $$2x^2 + ax - a^2 = 0$$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 2, B = a, C = -a^2$$

d117

Now $$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2}$$

$$= \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a \pm \sqrt{9a^2}}{4} = \frac{-a \pm 3a}{4}$$

$$x = \frac{-a+3a}{4}, \frac{-a-3a}{4}$$

Thus $x = \frac{a}{2}, -a$

67. Find the roots of the quadratic equation $4x^2 - 4px + (p^2 - q^2) = 0$

Ans : [Board Term-2, 2014]

We have $$4x^2 - 4px + (p^2 - q^2) = 0$$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 4, b = -4p, c = (p^2 - q^2)$$

The roots are given by the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4p \pm \sqrt{16p^2 - 4 \times 4 \times (p^2 - q^2)}}{2 \times 4}$$

$$= \frac{4p \pm \sqrt{16p^2 - 16p^2 + 16q^2}}{8}$$

$$= \frac{4p \pm 4q}{8}$$

d118

Thus roots are $\frac{p+q}{2}, \frac{p-q}{2}$.

68. Solve for x (in terms of a and b) :

$$\frac{a}{x-b} + \frac{b}{x-a} = 2, x \neq a, b$$

Ans : [Board Term-2 Foreign 2016]

We have $$\frac{a(x-a) + b(x-b)}{(x-b)(x-a)} = 2$$

d120

$$a(x-a) + b(x-b) = 2[x^2 - (a+b)x + ab]$$

$$ax - a^2 + bx - b^2 = 2x^2 - 2(a+b)x + 2ab$$

$$2x^2 - 3(a+b)x + (a+b)^2 = 0$$

$$2x^2 - 2(a+b)x - (a-b)x + (a+b)^2 = 0$$

$$[2x - (a+b)][x - (a+b)] = 0$$

Thus $$x = a+b, \frac{a+b}{2}$$

69. Solve for x : $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Ans : [Board Term-2 Foreign 2016]

We have

$$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3}$$
$$= 0$$

d121

$$\sqrt{3}x[x - \sqrt{6}] + \sqrt{2}[x - \sqrt{6}] = 0$$

$$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2}) = 0$$

Thus $$x \; \sqrt{6}, -\sqrt{\frac{2}{3}}$$

70. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

Ans : [Board Term-2 Delhi 2016]

We have $$ax^2 + 7x + b = 0 \qquad (1)$$

Substituting $x = \frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

d122

$$4a + 42 + 9b = 0$$

$$4a + 9b = -42 \qquad (2)$$

and substituting $x = -3$ in (1) we obtain

$$9a - 21 + b = 0$$

$$9a + b = 21 \qquad (3)$$

Solving (2) and (3), we get $a=3$ and $b=-6$

71. Solve for x : $\sqrt{6x+7}-(2x-7)=0$

Ans : [Board Term-2 OD 2016]

We have $\sqrt{6x+7}-(2x-7)=0$

or, $\sqrt{6x+7}=(2x-7)$

d123

Squaring both sides we get

$$6x+7=(2x-7)^2$$
$$6x+7=4x^2-28x+49$$
$$4x^2-34x+42=0$$
$$2x^2-17x+21=0$$
$$2x^2-14x-3x+21=0$$
$$2x(x-7)-3(x-7)=0$$
$$(x-7)(2x-3)=0$$

Thus $x=7$ and $x=\frac{2}{3}$.

72. Find the roots of $x^2-4x-8=0$ by the method of completing square.

Ans : [Board Term-2, 2015]

We have $x^2-4x-8=0$

$$x^2-4x+4-4-8=0$$
$$(x-2)^2-12=0$$
$$(x-2)^2=12$$
$$(x-2)^2=(2\sqrt{3})^2$$
$$x-2=\pm 2\sqrt{3}$$
$$x=2\pm 2\sqrt{3}$$

d124

Thus $x=2+2\sqrt{3},\ 2-2\sqrt{3}$

73. Solve for x : $\sqrt{2x+9}+x=13$

Ans : [Board Term-2 OD 2016]

We have $\sqrt{2x+9}+x=13$

$$\sqrt{2x+9}=13-x$$

d127

Squaring both side we have

$$2x+9=(13-x)^2$$
$$2x+9=169+x^2-26x$$
$$0=x^2+169-26x-9-2x$$
$$x^2-28x+160=0$$
$$x^2-20x-8x+160=0$$
$$x(x-20)-8(x-20)=0$$
$$(x-8)(x-20)=0$$

Thus $x=8$ and $x=20$.

74. Find the roots of the quadratic equation $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$

Ans : [Board Term-2 OD 2017]

We have $\sqrt{2}\,x^2+7x+5\sqrt{2}=0$

$$\sqrt{2}\,x^2+2x+5x+5\sqrt{2}=0$$
$$\sqrt{2}\,x(x+\sqrt{2})+5(x+\sqrt{2})=0$$
$$(x+\sqrt{2})(\sqrt{2}\,x+5)=0$$

d128

Thus $x=-\sqrt{2}$ and $=-\frac{5}{\sqrt{2}}=-\frac{5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=-\frac{5}{2}\sqrt{2}$

75. Find the value of k for which the roots of the quadratic equation $2x^2+kx+8=0$ will have the equal roots ?

Ans : [Board Term-2 OD Compt., 2017]

We have $2x^2+kx+8=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=2, b=k, \text{ and } c=8$$

d129

For equal roots, $D=0$

$$b^2-4ac=0$$
$$k^2-4\times 2\times 8=0$$
$$k^2=64$$
$$k=\pm\sqrt{64}$$

Thus $k=\pm 8$

76. Solve for x : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

Ans : [Board Term-II Foreign 2017 Set-2]

We have $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

$$\sqrt{3}\,x^2+3x+7x+7\sqrt{3}=0$$
$$\sqrt{3x}(x+\sqrt{3})+7(x+\sqrt{3})=0$$
$$(x+\sqrt{3})(\sqrt{3}\,x+7)=0$$

d130

Thus $x=-\sqrt{3}$ and $x=-\frac{7}{\sqrt{3}}$

77. Find k so that the quadratic equation $(k+1)x^2-2(k+1)x+1=0$ has equal roots.

Ans : [Board Term-2, 2016]

We have $(k+1)x^2-2(k+1)x+1=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(k+1), B=-2(k+1), C=1$

If roots are equal, then $D=0$, i.e.

$$B^2 = 4AC$$

$$4(k+1)^2 = 4(k+1)$$

$$k^2+2k+1 = k+1$$

$$k^2+k = 0$$

$$k(k+1) = 0$$

$$k = 0, -1$$

d194

$k=-1$ does not satisfy the equation, thus $k=0$

78. If 2 is a root of the equation $x^2+kx+12=0$ and the equation $x^2+kx+q=0$ has equal roots, find the value of q.

Ans : [Board Term 2 SQP 2016]

We have $x^2+kx+12=0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2+2k+12 = 0$$

, $$2k+16 = 0$$

$$k = -8$$

d195

Substituting $k=-8$ in $x^2+kx+q=0$ we have

$$x^2-8x+q = 0$$

For equal roots,

$$(-8)^2-4(1)q = 0$$

$$64-4q = 0$$

$$4q = 64 \Rightarrow q = 16$$

79. Find the values of k for which the quadratic equation $9x^2-3kx+k=0$ has equal roots.

Ans : [Board Term-2 Delhi, OD 2014]

We have $9x^2-3kx+k=0$

Comparing with $ax^2+bx+c=0$ we get

d196

$$a=9, b=-3k, c=k$$

Since roots of the equation are equal, $b^2-4ac=0$

$$(-3k)^2-(4\times 9\times k) = 0$$

$$9k^2-36k = 0$$

$$k^2-4k = 0$$

$$k(k-4) = 0 \Rightarrow k = 0 \text{ or } k=4$$

Hence, $k = 4$.

80. If the equation $kx^2-2kx+6=0$ has equal roots, then find the value of k.

Ans : [Board Term-2, 2012]

We have $kx^2-2kx+6=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=k, b=-2k, c=6$$

Since roots of the equation are equal, $b^2-4ac=0$

$$(-2k)^2-4(k)(6) = 0$$

$$4k^2-24k = 0$$

$$4k(k-6) = 0$$

$$k = 0, 6$$

d197

But $k\neq 0$, as coefficient of x^2 can't be zero.

Thus $k = 6$

81. Find the positive value of k for which $x^2-8x+k=0$, will have real roots.

Ans : [Board Term-2, 2014]

We have $x^2-8x+k=0$

Comparing with $Ax^2+Bx+C=0$ we get

d214

$$A=1, B=-8, C=k$$

Since the given equation has real roots, $B^2-4AC>0$

$$(-8)^2-4(1)(k) \geq 0$$

$$64-4k \geq 0$$

$$16-k \geq 0$$

$$16 \geq k$$

Thus $k\leq 16$

82. Find the values of p for which the quadratic equation $4x^2+px+3=0$ has equal roots.

Ans : [Board Term-2, 2014]

We have $4x^2+px+3=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=4, b=p, c=3$$

d198

Since roots of the equation are equal,

$$b^2 - 4ac = 0$$

$$p^2 - 4 \times 4 \times 3 = 0$$

$$p^2 - 48 = 0$$

$$p^2 = 48$$

$$p = \pm 4\sqrt{3}$$

83. Find the nature of the roots of the quadratic equation : $13\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Ans : [Board Term-2, 2012]

We have $\quad 13\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 13\sqrt{3}, b = 10, c = \sqrt{3}$$

d199

$$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})(\sqrt{3})$$

$$= 100 - 156$$

$$= -56$$

As $D < 0$, the equation has not real roots.

THREE MARKS QUESTIONS

84. Solve the following equation: $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

Ans : [Board 2020 SQP Standard]

We have $\quad \frac{1}{x} - \frac{1}{x-2} = 3 \qquad (x \neq 0, 2)$

d291

$$\frac{x-2-x}{x(x-2)} = 3$$

$$\frac{-2}{x(x-2)} = 3$$

$$3x(x-2) = -2$$

$$3x^2 - 6x + 2 = 0$$

Comparing it by $ax^2 + bx + c$, we get $a = 3,\ b = -6$ and $c = 2$.

Now, $\quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{6 \pm 2\sqrt{3}}{6}$$

$$= \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$$

85. Find the values of k for which the quadratic equation $x^2 + 2\sqrt{2k}\,x + 18 = 0$ has equal roots.

Ans : [Board 2020 SQP Standard]

We have $x^2 + 2\sqrt{2k}\,x + 18 = 0$

Comparing it by $ax^2 + bx + c$, we get $a = 1,\ b = 2\sqrt{2k}$ and $c = 18$.

Given that, equation $x^2 + 2\sqrt{2}\,kx + 18 = 0$ has equal roots.

$$b^2 - 4ac = 0$$

$$(2\sqrt{2}\,k)^2 - 4 \times 1 \times 18 = 0$$

d293

$$8k^2 - 72 = 0$$

$$8k^2 = 72$$

$$k^2 = \frac{72}{8} = 9$$

$$k = \pm 3$$

86. If α and β are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

Ans : [Board 2020 Delhi Basic]

We have $\quad p(x) = x^2 - 4x - 5$

Comparing it by $ax^2 + bx + c$, we get $a = 1,\ b = -4$ and $c = -5$

Since, given α and β are the zeroes of the polynomial,

Sum of zeroes, $\quad \alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{1} = 4$

and product of zeroes, $\quad \alpha\beta = \frac{c}{a} = \frac{-5}{1} = -5$

Now, $\quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

d294

$$= (4)^2 - 2(-5)$$

$$= 16 + 10 = 26$$

87. Find the quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. Hence find the zeroes.

Ans : [Board 2020 OD Basic]

Sum of zeroes $\alpha+\beta = -3$ ...(1)

and product of zeroes $\alpha\beta = 2$

Thus quadratic equation is

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$
$$x^2-(-3)x+2 = 0$$
$$x^2+3x+2 = 0$$

d296

Thus quadratic equation is $x^2+3x+2=0$.

Now above equation can be written as

$$x^2+2x+x+2 = 0$$
$$x(x+2)+(x+2) = 0$$
$$(x+2)(x+1) = 0$$

Hence, zeroes are -2 and -1.

88. If α and β are the zeroes of the polynomial $f(x)=5x^2-7x+1$ then find the value of $\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

Ans : [Board 2020 OD Basic]

Since, α and β are the zeroes of the quadratic polynomial $f(x)=5x^2-7x+1$,

Sum of zeros, $\alpha+\beta = -\left(\frac{-7}{5}\right) = \frac{7}{5}$ ...(1)

Product of zeros, $\alpha\beta = \frac{1}{5}$ ...(2)

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(\frac{7}{5}\right)^2-2\times\frac{1}{5}}{\frac{1}{5}}$$

$$= \frac{49-2\times5}{5} = \frac{39}{5}$$

d300

89. Find the zeroes of the quadratic polynomial $6x^2-3-7x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2020 Delhi Basic]

We have $p(x) = 6x^2-3-7x$

For zeroes of polynomial, $p(x)=0$,

$$6x^2-7x-3 = 0$$
$$6x^2-9x+2x-3 = 0$$
$$3x(2x-3)+1(2x-3) = 0$$
$$(2x-3)(3x+1) = 0$$

d301

Thus $2x-3 = 0$ and $3x+1 = 0$

Hence $x = \frac{3}{2}$ and $x = -\frac{1}{3}$

Therefore $\alpha=\frac{3}{2}$ and $\beta=-\frac{1}{3}$ are the zeroes of the given polynomial.

Verification :

Sum of zeroes, $\alpha+\beta = \frac{3}{2}+\left(-\frac{1}{3}\right)$

$$= \frac{3}{2}-\frac{1}{3} = \frac{7}{6}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = -\frac{1}{2}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

90. Find the zeroes of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2020 Delhi Basic]

Let, $p(x) = x^2+7x+10$

For zeroes of polynomial $p(x) = 0$,

$$x^2+7x+10 = 0$$

d302

$$x^2+5x+2x+10 = 0$$

$$x(x+5)+2(x+5) = 0$$

$$(x+5)(x+2) = 0$$

So, $x = -2$ and $x = -5$

Therefore, $\alpha = -2$ and $\beta = -5$ are the zeroes of the given polynomial.

Verification:

Sum of zeroes, $\alpha+\beta = -2+(-5)$

$$= -7 = \frac{-7}{1}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = (-2)(-5) = 10$

$$= \frac{10}{1}$$

$$= \frac{\text{contant term}}{\text{coefficient of } x^2}$$

91. Solve for x : $\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}$ $x \neq -4,\ -7$.

Ans : [Board 2020 OD Standard]

We have $\frac{1}{x+4}-\frac{1}{x+7} = \frac{11}{30}$

d305

$$\frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$

$$\frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$

$$\frac{3}{x^2+11x+28} = \frac{11}{30}$$

$$11x^2+121x+308 = 90$$

$$11x^2+121x+218 = 0$$

Comparing with $ax^2+bx+c = 0$, we get $a = 11$, $b = 121$ and $c = 218$ we obtain

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-121\pm\sqrt{14641-9592}}{22}$$

$$x = \frac{-121\pm\sqrt{5049}}{22}$$

$$= \frac{-121\pm 71.06}{22}$$

$$x = \frac{-49.94}{22}, \frac{-192.06}{22}$$

$$x = -2.27, -8.73.$$

92. Solve for x :

$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}; x \neq 1, -2, 2$

Ans : [Board Term-2 OD 2016]

We have $\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$

$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$

$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$

$$5x^2+19x-30 = 0$$

$$(5x-6)(x+5) = 0$$

$$x = -5, \frac{6}{5}$$

d131

93. Solve for x :

$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -\frac{3}{2}$

Ans : [Board Term-2, Delhi 2016]

d132

We have

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0$$

$$2x(2x+3)+(x-3)+(3x+9) = 0$$

$$4x^2+6x+x-3+3x+9 = 0$$

$$4x^2+10x+6 = 0$$

$$2x^2+5x+3 = 0$$

$$(x+1)(2x+3) = 0$$

Thus $x = -1, x = -\frac{3}{2}$

94. Solve for x : $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0,\ \frac{2}{3},\ 2.$

Ans : [Board Term-2, Foreign 2016]

We have $\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}$

d133

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$

$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$

$$(x-2)(4x-3) = 2x^2-3x$$
$$4x^2-11x+6 = 2x^2-3x$$
$$2x^2-8x+6 = 0$$
$$x^2-4x+3 = 0$$
$$(x-1)(x-3) = 0$$

Thus $x = 1,3$

95. Solve the following quadratic equation for x :

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$

Ans : [Board Term-2 OD 2016]

We have
$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 = 0$$
$$x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1 = 0$$
$$x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right) = 0$$
$$\left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) = 0$$

d134

Thus
$$x = \frac{-a}{a+b},\frac{-(a+b)}{a}$$

96. Solve for x :

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3};x\neq 1,2,3$$

Ans : [Board Term-2 OD 2016]

We have
$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$
$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$
$$3 = (x-1)(x-3)$$
$$x^2-4x+3 = 3$$
$$x^2-4x = 0$$
$$x(x-4) = 0$$

d135

Thus $x = 0$ or $x=4$

97. Solve for x : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

Ans : [Board Term-2, OD 2015, Foreign 2014]

We have
$$\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3} = 0$$
$$\sqrt{3}\,x^2-\sqrt{3}\sqrt{3}\sqrt{2}\,x+\sqrt{2}\,x-\sqrt{2}\sqrt{2}\sqrt{3} = 0$$
$$\sqrt{3}\,x(x-\sqrt{3}\sqrt{2})+\sqrt{2}\,(x-\sqrt{2}\sqrt{3}) = 0$$
$$\sqrt{3}\,x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}] = 0$$
$$(x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2}) = 0$$

d136

Thus $x = \sqrt{6} = -\sqrt{\frac{2}{3}}$

98. Solve for x : $2x^2+6\sqrt{3}\,x-60=0$

Ans : [Board Term-2, OD 2015]

We have
$$2x^2+6\sqrt{3}\,x-60 = 0$$
$$x^2+3\sqrt{3}\,x-30 = 0$$
$$x^2+5\sqrt{3}\,x-2\sqrt{3}\,x-30 = 0$$
$$x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3}) = 0$$
$$(x+5\sqrt{3})(x-2\sqrt{3}) = 0$$

d137

Thus $x = -5\sqrt{3},2\sqrt{3}$

99. Solve for x : $x^2+5x-(a^2+a-6) = 0$

Ans : [Board Term-2 Foreign Set I 2015]

We have
$$x^3+5x-(a^2+a-6) = 0$$
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

d138

Thus
$$x = \frac{-5\pm\sqrt{25+4(a^2+a-6)}}{2}$$
$$= \frac{-5\pm\sqrt{25+4a^2+4a-24}}{2}$$
$$= \frac{-5\pm\sqrt{4a^2+4a+1}}{2}$$
$$= \frac{-5\pm(2a+1)}{2}$$
$$= \frac{2a-4}{2},\frac{-2a-6}{2}$$

Thus $x= a-2,\ x=-(a+3)$

100. Solve for x : $x^2-(2b-1)x+(b^2-b-20) = 0$

Ans : [Board Term-2 Foreign 2015]

We have $x^2-(2b-1)x+(b^2-b-20)=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=1, B=-(2b-1), C=(b^2-b-20)$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{(2b-1)\pm\sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{4b^2-4b+1-4b^2+4b+80}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{81}}{2}=\frac{(2b-1)\pm 9}{2}$$

$$=\frac{2b+8}{2},\frac{2b-10}{2}$$

$$=b+4,\ b-5$$

d139

Thus $x=b+4$ and $x=b-5$

101. Solve for x : $\frac{16}{x}-1=\frac{15}{x+1}$; $x\neq 0,-1$

Ans : [Board Term-2, OD 2014]

We have $\frac{16}{x}-1=\frac{15}{x+1}$

$$\frac{16}{x}-\frac{15}{x+1}=1$$

$$16(x+1)-15x=x(x+1)$$

$$16x+16-15x=x^2+x$$

$$x+16=x^2+x$$

$$x^2-16=0$$

$$x^2=16$$

$$x=\pm 4$$

d140

Thus $x=-4$ and $x=+4$

102. Solve the quadratic equation $(x-1)^2-5(x-1)-6=0$

Ans : [Board Term-2, 2015]

We have $(x-1)^2-5(x-1)-6=0$

$$x^2-2x+1-5x+5-6=0$$

$$x^2-7x+6-6=0$$

$$x^2-7x=0$$

$$x(x-7)=0$$

d141

Thus $x=0,7$

103. Solve the equation for x : $\frac{4}{x}-3=\frac{5}{2x+3}$; $x\neq 0,\frac{-3}{2}$

Ans : [Board Term-2 Delhi 2014]

We have $\frac{4}{x}-3=\frac{5}{2x+3}$

$$\frac{4}{x}-\frac{5}{2x+3}=3$$

$$\frac{4(2x+3)-5x}{x(2x+3)}=3$$

$$8x+12-5x=3x(2x+3)$$

$$3x+12=6x^2+9x$$

$$6x^2+6x-12=0$$

$$x^2+x-2=0$$

$$x^2+2x-x-2=0$$

$$x(x+2)-(x+2)=0$$

$$(x+2)(x-1)=0$$

d142

Thus $x=1,-2$

104. Find the roots of the equation $2x^2+x-4=0$, by the method of completing the squares.

Ans : [Board Term-2, OD 2014]

We have $2x^2+x-4=0$

$$x^2+\frac{x}{2}-2=0$$

$$x^2+2x\left(\frac{1}{4}\right)-2=0$$

d143

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get

$$x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\frac{\sqrt{33}}{4}$$

Thus roots are $x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$

105. Solve for x : $9x^2 - 6ax + (a^2 - b^2) = 0$

Ans : [Board Term-2 2012]

We have $\quad 9x^2 - 6ax + a^2 - b^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 9, B = -6a, C = (a^2 - b^2)$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{6a \pm \sqrt{(-6a)^2 - 4 \times 9k(a^2 - b^2)}}{2 \times 9}$$

$$= \frac{6a \pm \sqrt{36a^2 - 36a^2 + 36b^2}}{18}$$

$$= \frac{6a \pm \sqrt{36b^2}}{18} = \frac{6a \pm 6b}{18}$$

$$= \frac{a \pm b}{3}$$

$$x = \frac{(a+b)}{3}, \frac{(a-b)}{3}$$

Thus $x = \frac{a+b}{3}, x = \frac{a-b}{3}$

106. Solve the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$ for x.

Ans : [Board Term-2, 2012]

We have,
$$\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$(x-1)(x-2) = 0$$

Thus $x = 1, 2$.

107. Find the roots of the quadratic equation :

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

Ans : [Board Term-2, 2012]

We have $\quad a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$b^2x(a^2x+1) - 1(a^2x+1) = 0$$

$$(b^2x - 1)(a^2x + 1) = 0$$

$$x = \frac{1}{b^2} \text{ or } x = -\frac{1}{a^2}$$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

108. If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$, prove that $\frac{x}{a} = \frac{y}{b}$

Ans : [Board Term-2, 2014]

We have $\quad (x^2 + y^2)(a^2 + b^2) = (ax + by)^2$

$$x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$$

$$x^2b^2 + y^2a^2 - 2abxy = 0$$

$$(xb - ya)^2 = 0$$

$$xb = ya$$

Thus $\quad \frac{x}{a} = \frac{y}{b}$

Hence Proved.

109. Solve the following quadratic equation for x :

$$p^2x^2 + (p^2 - q^2)x - q^2 = 0$$

Ans : [Board Term-2, 2012]

We have $\quad p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p^2, b = p^2 - q^2, \ c = -q^2$$

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p^2 - q^2) - \sqrt{(p^2 - q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2 - q^2) - \sqrt{p^4 + q^4 - 2p^2q^2 + 4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2 - q^2) - \sqrt{p^4 + q^4 + 2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)-\sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2)\pm(p^2+q^2)}{2p^2}$$

Thus $x = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

and $x = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}$ and -1.

110. Solve the following quadratic equation for x :

$$9x^2-9(a+b)x+2a^2+5ab+2b^2 = 0$$

Ans : [Board Term-2, Foreign 2016]

We have $9x^2-9(a+b)x+2a^2+5ab+2b^2 = 0$

Now $2a^2+5ab+2b^2 = 2a^2+4ab+ab+2b^2$

$$= 2a[a+2b]+b[a+2b]$$

$$= (a+2b)(2a+b)$$

Hence the equation becomes

$$9x^2-9(a+b)x+(a+2b)(2a+b) = 0$$

$$9x^2-3[3a+3b]x+(a+2b)(2a+b) = 0$$

$$9x^2-3[(a+2b)+(2a+b)]x+(a+2b)(2a+b) = 0$$

$$9x^2-3(a+2b)x-3(2a+b)x+(a+2b)(2a+b) = 0$$

$$3x[3x-(a+2b)]-(2a+b)[3x-(a+2b)] = 0$$

$$[3x-(a+2b)][3x-(2a+b)] = 0$$

$$3x-(2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

d148

$$3x-(2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

111. Solve for x : $x^2+6x-(a^2+2a-8)$

Ans : [Board Term-2, Foreign 2015]

We have $x^2+6x-(a^2+2a-8) = 0$

Comparing with $Ax^2+Bx+C = 0$ we get

$$A = 1, B = 6, C = (a^2+2a-8)$$

The roots are given by the quadratic formula

$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

d149

$$= \frac{-6\pm\sqrt{36+4(a^2+2a-8)}}{2}$$

$$= \frac{-6\pm(2a+2)}{2}$$

Thus $x = \frac{-6+(2a+2)}{2} = a-2$

and $x = \frac{-6-(2a+2)}{2} = -a-4$

Thus $x = a-2, -a-4$

112. If the roots of the equation $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Ans : [Board Term-2 2016]

We have $(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2) = 0$

Comparing with $Ax^2+Bx+C = 0$ we get

$A = (a^2+b^2), B = -2(ac+bd), C = (c^2+d^2)$

If roots are equal, $D = B^2-4AC = 0$

d157

or $B^2 = 4AC$

Now $[-2(ac+bd)]^2 = 4(a^2+b^2)(c^2+d^2)$

$$4(a^2c^2+2abcd+b^2d^2) = 4(a^2c^2+a^2d^2+b^2c^2+b^2d^2)$$

$$a^2c^2+2abcd+b^2d^2 = a^2c^2+a^2d^2+b^2c^2+b^2d^2$$

$$2abcd = a^2d^2+b^2c^2$$

$$0 = a^2d^2-2abcd+b^2c^2$$

$$0 = (ad-bc)^2$$

$$0 = ad-bc$$

Thus $ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$

Hence Proved

113. If 2 is a root of the quadratic equation $3x^2+px-8 = 0$ and the quadratic equation $4x^2-2px+k = 0$ has equal roots, find k.

Ans : [Board Term-2 Foreign 2014]

We have $3x^2+px-8 = 0$

Since 2 is a root of above equation, it must satisfy it.

Substituting $x = 2$ in $3x^2+px-8 = 0$ we have

$$12+2p-8 = 0$$

$$p = -2$$

d200

Since $4x^2 - 2px + k = 0$ has equal roots,

or $4x^2 + 4x + k = 0$ has equal roots,

$$D = b^2 - 4ac = 0$$

$$4^2 - 4(4)(k) = 0$$

$$16 - 16k = 0$$

$$16k = 16$$

Thus $k = 1$

114. For what value of k, the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ are equal ?

Ans : [Board Term-2 Delhi 2014, 2013]

We have $kx(x - 2\sqrt{5}) + 10 = 0$

or, $kx^2 - 2\sqrt{5}\,kx + 10 = 0$

d201

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k, b = -2\sqrt{5}\,k \text{ and } c = 10$$

Since, roots are equal, $D = b^2 - 4ac = 0$

$$(-2\sqrt{5}\,k)^2 - 4 \times k \times 10 = 0$$

$$20k^2 - 40k = 0$$

$$20k(k - 2) = 0$$

$$k(k - 2) = 0$$

Since $k \neq 0$, we get $k = 2$

115. Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Ans : [Board Term-2, 2012]

We have $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

d202

$$a = 3,\ b = -4\sqrt{3},\ c = 4$$

$$b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$$

$$= 48 - 48 = 0$$

Thus roots are real and equal.

Roots are $\left(-\frac{b}{2a}\right), \left(-\frac{b}{2a}\right)$ or $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

116. Determine the positive value of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real and equal roots.

Ans : [Board Term-2, 2012, 2014]

We have $x^2 + kx + 64 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

d203

$$a = 1, b = k, c = 64$$

For real and equal roots, $b^2 - 4ac = 0$

Thus $k^2 - 4 \times 1 \times 64 = 0$

$$k^2 - 256 = 0$$

$$k = \pm 16 \quad (1)$$

Now for equation $x^2 - 8x + k = 0$ we have

$$b^2 - 4ac = 0$$

$$(-8)^2 - 4 \times 1 \times k = 0$$

$$64 = 4k$$

$$k = \frac{64}{4} = 16 \quad (2)$$

From (1) and (2), we get $k = 16$. Thus for $k = 16$, given equations have equal roots.

117. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k - 1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2 Delhi 2015]

We have $kx^2 + 1 - 2(k - 1)x + x^2 = 0$

$$(k + 1)x^2 - 2(k - 1)x + 1 = 0$$

d204

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k + 1, b = -2(k - 1), c = 1$$

For real and equal roots, $b^2 - 4ac = 0$

$$4(k - 1)^2 - 4(k + 1) \times 1 = 0$$

$$4k^2 - 8k + 4 - 4k - 4 = 0$$

$$4k^2 - 12k = 0$$

$$4k(k - 3) = 0$$

As k can't be zero, thus $k = 3$.

118. Find the value of k for which the quadratic equation $(k - 2)x^2 + 2(2k - 3)x + (5k - 6) = 0$ has equal roots.

Ans : [Board Term-2, 2015]

We have $(k - 2)x^2 + 2(2k - 3)x + (5k - 6) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k-2, b = 2(2k-3), c = (5k-6)$$

For real and equal roots, $b^2 - 4ac = 0$

$$\{2(2k-3)\}^2 - 4(k-2)(5k-6) = 0$$

d205

$$4(4k^2 - 12k + 9) - 4(k-2)(5k-6) = 0$$

$$4k^2 - 12k + 9 - 5k^2 + 6k + 10k - 12 = 0$$

$$k^2 - 4k + 3 = 0$$

$$k^2 - 3k - k + 3 = 0$$

$$k(k-3) - 1(k-3) = 0$$

$$(k-3)(k-1) = 0$$

Thus $\quad k = 1,3$

119. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b + c$.

Ans : [Board Term-2 Delhi 2016]

We have $(a-b)x^2 + (b-c)x + (c-a) = 0$

d206

Comparing with $ax^2 + bx + c = 0$ we get

$$a = (a-b,),\ b = (b-c),\ c = c-a$$

For real and equal roots, $b^2 - 4ac = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$

$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$

$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$

$$4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac = 0$$

Using $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$,

$$(-2a + b + c)^2 = 0$$

$$-2a + b + c = 0$$

Hence, $\quad b + c = 2a$

120. If the quadratic equation, $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$ in x has equal roots, prove that $c^2 = m^2(1+a^2)$

Ans : [Board Term-2, 2014]

We have $\quad (1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (1+a^2)b^2, B = 2abc, C = (c^2 - m^2)$$

If roots are equal, $B^2 - 4AC = 0$

d207

$$(2abc)^2 - 4(1+a^2)b^2(c^2 - m^2) = 0$$

$$4a^2b^2c^2 - (4b^2 + 4a^2b^2)(c^2 - m^2) = 0$$

$$4a^2b^2c^2 - [4b^2c^2 - 4b^2m^2 + 4a^2b^2c^2 - 4a^2b^2m^2] = 0$$

$$4a^2b^2c^2 - 4b^2c^2 + 4b^2m^2 - 4a^2b^2c^2 + 4a^2b^2m^2 = 0$$

$$4b^2[a^2m^2 + m^2 - c^2] = 0$$

$$c^2 = a^2m^2 + m^2$$

$$c^2 = m^2(1+a^2)$$

121. If -3 is a root of quadratic equation $2x^2 + px - 15 = 0$, while the quadratic equation $x^2 - 4px + k = 0$ has equal roots. Find the value of k.

Ans : [Board Term-2 OD Compt. 2017]

Given -3 is a root of quadratic equation.

We have $\quad 2x^2 + px - 15 = 0$

Since 3 is a root of above equation, it must satisfy it. Substituting $x = 3$ in above equation we have

$$2(-3)^2 + p(-3) - 15 = 0$$

$$2 \times 9 - 3p - 15 = 0 \Rightarrow p = 1$$

d208

Since $\quad x^2 - 4px + k = 0$ has equal roots,

or $\quad x^2 - 4x + k = 0$ has equal roots,

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4k = 0$$

$$16 - 4k = 0$$

$$4k = 16 \Rightarrow k = 4$$

122. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Ans : [Board Term-2 OD 2017]

We have $\quad (a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (a^2 + b^2), B = 2(ac + bd) \text{ and } C = (c^2 + d^2)$$

For no real roots, $D = B^2 - 4AC < 0$

d209

$$D = B^2 - 4AC$$

$$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$

$$= 4[a^2c^2 + 2abcd + b^2d^2] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$

$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$

$$= -4(ad - bc)^2$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative. Hence the equation has no real roots.

123. Find the value of c for which the quadratic equation $4x^2 - 2(c+1)x + (c+1) = 0$ has equal roots.

Ans : [Board Term-2 Delhi 2017]

We have $4x^2 - 2(c+1)x + (x+1) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 2(c+1), C = (c+1)$$

d210

If roots are equal, $B^2 - 4AC = 0$

$$[2(c+1)]^2 - 4 \times 4(c+1) = 0$$

$$4(c^2 + 2c + 1) - 4(4c + 4) = 0$$

$$4(c^2 + 2c + 1 - 4c - 4) = 0$$

$$c^2 - 2c - 3 = 0$$

$$c^2 - 3c + c - 3 = 0$$

$$c(c-3) + 1(c-3) = 0$$

$$(c-3)(c+1) = 0$$

$$c = 3, -1$$

Hence for equal roots $c = 3, -1$.

124. Show that if the roots of the following equation are equal then $ad = bc$ or $\frac{a}{b} = \frac{c}{d}$.

$$x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$$

Ans : [Board Term-2 OD Compt. 2017]

We have $x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = a^2 + b^2, B = 2(ac + bd), C = c^2 + d^2$$

d211

If roots are equal, $B^2 - 4AC = 0$

$$[2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$

$$4(a^2c^2 + 2abcd + b^2d^2) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$$

$$4(a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) = 0$$

$$-4(a^2d^2 + b^2c^2 - 2abcd) = 0$$

$$(ad - bc)^2 = 0$$

Thus $ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$ Hence Proved.

125. Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$.

Ans : [Board Term-2 SQP 2016]

We have $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

d228

$$\frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{x - a - b - x}{x(a+b+x)} = \frac{a+b}{ab}$$

$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$$x(a+b+x) = -ab$$

$$x^2 + (a+b)x + ab = 0$$

$$(x+a)(x+b) = 0$$

$$x = -a \text{ or } x = -b$$

FOUR MARKS QUESTIONS

126. Solve for x : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

Ans : [Board Term-2 2016]

We have $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$ then we have

$$y^2 + 5y - 24 = 0$$

d158

$$(y+8)(y-3) = 0$$

$$y = 3, -8$$

Taking $y = 3$ we have

$$\frac{2x}{x-5} = 3$$

$$2x = 3x - 15 \Rightarrow x = 15$$

Taking $y = -8$ we have

$$\frac{2x}{x-5} = -8$$

$$2x = -8x + 40$$

$$10x = 40 \Rightarrow x = 4$$

Hence, $x = 15, 4$

127. Solve for x : $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$ $\quad x \neq -1, -2, -4$

Ans : [Board Term-2 OD 2016]

We have
$$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$$

d159

$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$$
$$(3x+4)(x+4) = 4(x^2+3x+2)$$
$$3x^2+16x+16 = 4x^2+12x+8$$
$$x^2-4x-8 = 0$$

Now
$$x = \frac{-b\sqrt{b^2+4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2 \times 1}$$
$$= \frac{4 \pm \sqrt{16+32}}{2}$$
$$= \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$
$$= 2 \pm 2\sqrt{3}$$

Hence, $x = 2 + 2\sqrt{3}$ and $2 - 2\sqrt{3}$

128. Find x in terms of a, b and c :
$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

Ans : [Board Term-2, Delhi 2016]

d160

We have
$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}$$
$$a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$$
$$ax^2 - abx - acx + abc + bx^2 - bax - bcx + abc$$
$$= 2cx^2 - 2cxb - 2cxa + 2abc$$
$$ax^2 + bx^2 - 2cx^2 - abx - acx - bax - bcx + 2cbx + 2acx$$
$$= 0$$
$$x^2(a+b-2c) - 2abx + acx + bcx = 0$$
$$x^2(a+b-2c) + x(-2ab+ac+bc) = 0$$

Thus $x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

129. Solve for x : $\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$; $x \neq -1, 1, \frac{1}{4}$

Ans : [Board Term-2 Delhi 2015]

We have
$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$$

d161

$$\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$$
$$\frac{7x+1}{x^2-1} = \frac{29}{4x-1}$$
$$(7x+1)(4x-1) = 29x^2-29$$
$$28x^2 - 7x + 4x - 1 = 29x^2 - 29$$
$$-3x = x^2 - 28$$
$$x^2 + 3x - 28 = 0$$
$$x^2 + 7x - 4x - 28 = 0$$
$$x(x+7) - 4(x+7) = 0$$
$$(x+7)(x-4) = 0$$

Hence, $x = 4, -7$

130. Solve for x : $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$ where $x \neq -\frac{1}{2}, 1$

Ans : [Board Term-2, OD 2015]

We have
$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

d179

Substituting this value we obtain
$$y + \frac{1}{y} = 2$$
$$y^2 + 1 = 2y$$
$$y^2 - 2y + 1 = 0$$
$$(y-1)^2 = 0$$
$$y = 1$$

Substituting $y = \frac{x-1}{2x+1}$ we have
$$\frac{x-1}{2x+1} = 1 \text{ or } x - 1 = 2x + 1$$

or $\quad x = -2$

131. Find for x : $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$; $\quad x \neq 0, 1, 2$

Ans : [Board Term-2 OD 2017]

We have $\frac{1}{x-2}+\frac{2}{x-1} = \frac{6}{x}$

$$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$3x^2-5x = 6x^2-18x+12$$

$$3x^2-13x+12 = 0$$

$$3x^2-4x-9x+12 = 0$$

$$x(3x-4)-3(3x-4) = 0$$

$$(3x-4)(x-3) = 0$$

$$x = \frac{4}{3} \text{ and } 3$$

Hence, $x = 3, \frac{4}{3}$

d180

132. Solve, for x : $\sqrt{3}x^2+10x+7\sqrt{3} = 0$

Ans : [Board Term-2 Foreign 2017]

We have $\sqrt{3}x^2+10x+7\sqrt{3} = 0$

$$\sqrt{3}x^2+3x+7x+7\sqrt{3} = 0$$

$$(x+\sqrt{3})(\sqrt{3}x+7) = 0$$

$$(x+\sqrt{3})(\sqrt{3}x+7) = 0$$

$$x = -\sqrt{3} \text{ and } x = \frac{-7}{\sqrt{3}}$$

Hence roots $x = -\sqrt{3}$ and $x = \frac{-7}{\sqrt{3}}$

d181

133. Solve for x : $\frac{x+3}{x-2}-\frac{1-x}{x} = \frac{17}{4}$; $x \neq 0, 2$

Ans : [Board Term -2 Delhi Compt. 2017]

We have $\frac{x+3}{x-2}-\frac{1-x}{x} = \frac{17}{4}$

$$\frac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \frac{17}{4}$$

$$\frac{(x^2+3x)-(-x^2+3x-2)}{x^2-2x} = \frac{17}{4}$$

$$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$

$$8x^2+8 = 17x^2-34x$$

$$9x^2-34x-8 = 0$$

$$9x^2-36x+2x-8 = 0$$

$$9x(x-4)+2(x-4) = 0$$

$$(x-4)(9x+2) = 0$$

d192

$$x = 4 \text{ or } x = -\frac{2}{9}$$

Hence, $x = 4, -\frac{2}{9}$

134. Solve for x : $4x^2+4bx-(a^2-b^2) = 0$

Ans : [Board Term-2 Foreign 2017]

We have $4x^2+4bx-(a^2-b^2) = 0$

Comparing with $Ax^2+Bx+C = 0$ we get

$$A = 4, B = 4b \text{ and } C = b^2-a^2$$

$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$= \frac{-4b\pm\sqrt{(4b)^2-4.4(b^2-a^2)}}{2.4}$$

$$= \frac{-4b\pm\sqrt{16b^2-16b^2+16a^2}}{8}$$

$$= \frac{-4b\pm 4a}{8}$$

$$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$$

Hence the roots are $-\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$

d193

135. Find the zeroes of the quadratic polynomial $7y^2-\frac{11}{3}y-\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board 2019 OD]

We have $7y^2-\frac{11}{3}y-\frac{2}{3} = 0$

$$21y^2-11y-2 = 0 \quad ...(1)$$

$$21y^2-14y+3y-2 = 0$$

$$7y(3y-2)+(3y-2) = 0$$

$$(3y-2)(7y+1) = 0$$

$$y = \frac{2}{3}, \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

d312

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 21$, $b = -11$ and $c = -2$

Now, sum of roots, $\alpha + \beta = \frac{2}{3} + \left(-\frac{1}{7}\right)$

$$= \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$$

Thus $\alpha + \beta = -\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta = \frac{2}{3} \times \left(-\frac{1}{7}\right) = \frac{-2}{21}$

Thus $\alpha\beta = \frac{c}{a}$ Hence verified

136. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

Ans : [Board 2019 OD]

We have $x^2 + px + 16 = 0$ ...(1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$ ...(2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = p$ and $c = 16$

Substituting above in equation (2) we have

d313

$$p^2 - 4 \times 1 \times 16 = 0$$

$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p = 8$, from equation (1) we have

$$x^2 + 8x + 16 = 0$$

$$x^2 + 2 \times 4x + 4^2 = 0$$

$$(x + 4)^2 = 0 \Rightarrow x = -4, -4$$

Hence, roots are -4 and -4.

When $p = -8$ from equation (1) we have

$$x^2 - 8x + 16 = 0$$

$$x^2 - 2 \times 4x + 4^2 = 0$$

$$(x - 4)^2 = 0 \Rightarrow x = 4, 4$$

Hence, the required roots are either $-4, -4$ or $4, 4$

137. Solve for x : $x^2 + 5x - (a^2 + a - 6) = 0$

Ans : [Board 2019 OD]

We have $x^2 + 5x - (a^2 + a - 6) = 0$

d315

$$x^2 + 5x - [a^2 + 3a - 2a - 6] = 0$$

$$x^2 + 5x - [a(a + 3) - 2(a + 3)] = 0$$

$$x^2 + 5x - (a + 3)(a - 2) = 0$$

$$x^2 + [a + 3 - (a - 2)]x - (a + 3)(a - 2) = 0$$

$$x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$

$$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$$

$$[x + (a + 3)][x - (a - 2)] = 0$$

Thus $x = -(a + 3)$ and $x = (a - 2)$

Hence, roots of given equations are $x = -(a + 3)$ and $x = a - 2$.

138. Find the nature of the roots of the quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.

Ans : [Board 2019 OD]

We have $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$.

Now, $D = b^2 - 4ac$

d316

$$= (4\sqrt{3})^2 - 4 \times 4 \times 3$$

$$= 48 - 48 = 0$$

Since, $b^2 - 4ac = 0$, then roots of the given equation are real and equal.

139. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k.

Ans : [Board 2018]

If $x = 3$ is one root of the equation $x^2 - 2kx - 6 = 0$, it must satisfy it.

Thus substituting $x = 3$ in given equation we have

$$9 - 6k - 6 = 0$$

$$k = \frac{1}{2}$$

d317

140. Find the positive values of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have the real roots.

Ans : [Board Term-2 Foreign 2016]

(1) For $x^2 + kx + 64 = 0$ to have real roots

$$k^2 - 256 \geq 0$$

$$k^2 \geq 256$$

$$k \geq 16 \text{ or } k < -16$$

d213

(2) For $x^2 - 8x + k = 0$ to have real roots

$$64 - 4k \geq 0$$

$$16 - k \geq 0$$

$$16 \geq k$$

For (1) and (2) to hold simultaneously

$$k = 16$$

141. Find the values of k for which the equation $(3k+1)^2 + 2(k+1)x + 1$ has equal roots. Also find the roots.

Ans : [Board Term-2, 2014]

We have $(3k+1)^2 + 2(k+1)x + 1$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (3k+1), B = 2(k+1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$[2(k+1)]^2 - 4(3k+1)(1) = 0$$

$$4(k^2+2k+1) - (12k+4) = 0$$

$$4k^2 + 8k + 4 - 12k - 4 = 0$$

$$4k^2 - 4k = 0$$

$$4k(k-1) = 0$$

$$k = 0, 1.$$

Substituting $k = 0$, in the given equation,

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$x = -1$$

Again substituting $k = 1$, in the given equation,

$$4x^2 + 4x + 1 = 0$$

$$(2x+1)^2 = 0$$

or, $$x = -\frac{1}{2}$$

Hence, roots $= -1, -\frac{1}{2}$

142. Find the values of k for which the quadratic equations $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots. Also, find the roots.

Ans : [Board Term-2 Delhi 2014]

We have $(k+4)x^2 + (k+1)x + 1 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (k+4), B = (k+1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$(k+1)^2 - 4(k+4)(1) = 0$$

$$k^2 + 1 + 2k - 4k - 16 = 0$$

$$k^2 - 2k - 15 = 0$$

$$(k-5)(k+3) = 0$$

$$k = 5, -3$$

For $k = 5$, equation becomes

$$9x^2 + 6x + 1 = 0$$

$$(3x+1)^2 = 0$$

or $$x = -\frac{1}{3}$$

For $k = -3$, equation becomes

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

Hence roots are 1 and $-\frac{1}{3}$.

143. If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the value of k so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

Ans : [Board Term-2 Foreign 2015]

We have $3x^2 + 7x + p = 0$

Since $x = -2$ is the root of above equation, it must satisfy it.

Thus $$3(-2) + 7(-2) + p = 0$$

$$p = 2$$

Since roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal,

$$16k^2 - 4(k^2 - k + 2) = 0$$

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$3k^2 + k - 2 = 0$$

$$(3k-2)(k+1) = 0$$

$$k = \frac{2}{3}, -1$$

Hence, roots $= \frac{2}{3}, -1$

144. If $x = -4$ is a root of the equation $x^2 + 2x + 4p = 0$

, find the values of k for which the equation $x^2+px(1+3k)+7(3+2k)=0$ has equal roots.

Ans : [Board Term-2 Foreign 2015]

We have $x^2+2x+4p=0$

Since $x=-4$ is the root of above equation. It must satisfy it.

$$(-4)^2+(2\times-4)+4p=0$$

$$p=-2$$

Since equation $x^2-2(1+3k)x+7(3+2k)=0$ has equal roots.

$$4(1+3k)^2-28(3+2k)=0$$

d218

$$9k^2-8k-20=0$$

$$(9k+10)(k-2)=0$$

$$k\ \frac{-10}{9},2$$

Hence, the value of k are $-\frac{10}{9}$ and 2.

145. Find the value of p for which the quadratic equation $(p+1)x^2-6(p+1)x+3(p+9)=0$, $p\neq-1$ has equal roots. Hence find the roots of the equation.

Ans : [Board Term-2, 2015]

We have $(p+1)x^2-6(p+1)x+3(p+9)=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=p+1, b=-6(p+1), c=3(p+9)$$

For real and equal roots, $b^2-4ac=0$

$$36(p+1)^2-4(p+1)\times3(p+9)=0$$

d219

$$3(p^2+2p+1)-(p+1)(p+9)=0$$

$$3p^2+6p+3-(p^2+9p+p+9)=0$$

$$2p^2-4p-6=0$$

$$p^2-2p-3=0$$

$$p^2-3p+p-3=0$$

$$p(p-3)+1(p-3)=0$$

$$(p-3)(p+1)=0$$

$$p=-1,3$$

Neglecting $p\neq-1$ we get $p=3$

Now the equation becomes

$$4x^2-24x+36=0$$

or $$x^2-6x+9=0$$

or, $$(x-3)(x-3)=0$$

$$x=3,3$$

Thus roots are 3 and 3.

146. If the equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ has equal roots, prove that $c^2=a^2(1+m^2)$

Ans : [Board Term-2 Delhi 2015]

We have $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=1+m^2, B=2mc, C=(c^2-a^2)$$

If roots are equal, $B^2-4AC=0$

$$(2mc)^2-4(1+m^2)(c^2-a^2)=0$$

d220

$$4m^2c^2-4(1+m^2)(c^2-a^2)=0$$

$$m^2c^2-(c^2-a^2+m^2c^2-m^2a^2)=0$$

$$m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$$

$$-c^2+a^2+m^2a^2=0$$

, $$c^2=a^2(1+m^2)$$

Hence Proved.

147. If (-5) is a root of the quadratic equation $2x^2+px+15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, then find the values of p and k.

Ans : [Board Term-2 Delhi 2015]

We have $2x^2+px-15=0$

Since $x=-5$ is the root of above equation. It must satisfy it.

$$2(-5)^2+p(-5)-15=0$$

d221

$$50-5p-15=0$$

$$5p=35\Rightarrow p=7$$

Now $p(x^2+x)+k=0$ has equal roots

or $$7x^2+7x+k=0$$

Taking $b^2-4ac=0$ we have

$$7^2-4\times7\times k=0$$

$$7-4k=0$$

$$k=\frac{7}{4}$$

Hence $p=7$ and $k=\frac{7}{4}$.

148. If the roots of the quadratic equation

$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

Ans : [Board Term-2 Delhi 2015]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$$

$$x^2-ax-bx+ab+$$
$$+x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac = 0$$

d222

$$3x^2-2ac-2bx-2cx+ab+bc+ca = 0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times 3(ab+bc+ca) = 0$$
$$4(a+b+c)^2-12(ab+bc+ca) = 0$$
$$a^2+b^2+c^2-3(ab+bc+ca) = 0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac = 0$$
$$a^2+b^2+c^2-ab-ac-bc = 0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc] = 0$$
$$\tfrac{1}{2}\left[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)\right]=0$$
$$\tfrac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right] = 0$$

or, $(a-b)^2+(b-c)^2+(c-a)^2 = 0$

If $a\neq b\neq c$

$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$

If $(a-b)^2 = 0\Rightarrow a=b$

$(a-c)^2 = 0\Rightarrow b=c$

$(c-a)^2 = 0\Rightarrow c=a$

Thus $a=b=c$ Hence Proved

149. If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in x are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

Ans : [Board Term 2Outside Delhi 2017]

We have $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Comparing with $Ax^2+Bx+C = 0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$\left[2(a^2-bc)\right]^2-4(c^2-ab)(b^2-ac) = 0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc) = 0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc] = 0$$
$$4[a^4+ac^3+ab^3-3a^2bc] = 0$$
$$a(a^3+c^3+b^3-3abc) = 0$$
$$a=0 \text{ or } a^3+b^3+c^3 = 3abc$$

d223

150. Solve for x : $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$

where $a+b+x\neq 0$ and $a,b,x\neq 0$

Ans : [Board Term-2 Foreign 2017]

We have $\dfrac{1}{a+b+x}-\dfrac{1}{x} = \dfrac{1}{a}+\dfrac{1}{b}$

$$\frac{-(a+b)}{x^2+(a+b)x} = \frac{b+a}{ab}$$

d224

$$x^2+(a+b)x+ab = 0$$
$$(x+a)(x+b) = 0$$
$$x = -a, x=-b$$

Hence $x=-a,-b$

151. Check whether the equation $5x^2-6x-2=0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

Ans : [Board Term-2 SQP 2017]

We have $5x^2-6x-2=0$

Comparing with $ax^2+bx+c = 0$ we get

d227

$$a=5, b=(-6) \text{ and } c=(-2)$$
$$b^2-4ac = (-6)^2-4\times 5\times -2$$
$$= 36+40=76>0$$

So the equation has real and two distinct roots.

$$5x^2-6x = 2$$

Dividing both the sides by 5 we get

$$\frac{x^2}{5}-\frac{6}{5}x = \frac{2}{5}$$
$$x^2-2x\left(\frac{3}{5}\right) = \frac{2}{5}$$

Adding square of the half of coefficient of x

$$x^2-2x\left(\frac{3}{5}\right)+\frac{9}{25} = \frac{2}{5}+\frac{9}{25}$$
$$\left(x-\frac{3}{5}\right)^2 = \frac{19}{25}$$
$$x-\frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$$

$$= \frac{9+6\sqrt{19}+19}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$$

$$= \frac{28+6\sqrt{19}}{5} - \frac{18+6\sqrt{19}}{5} - 2$$

$$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$$

$$= 0$$

Similarly

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$$

Hence verified.

CHAPTER 5

ARITHMETIC PROGRESSION

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The n^{th} term of the AP $a, 3a, 5a, ...$ is

(a) na (b) $(2n-1)a$

(c) $(2n+1)a$ (d) $2na$

Ans : [Board 2020 OD Standard]

Given AP is $a, 3a, 5a, ...$

First term is a and $d = 3a - a = 2a$

e245

n^{th} term $\quad a_n = a + (n-1)d$

$$= a + (n-1)2a$$
$$= a + 2na - 2a$$
$$= 2na - a = (2n-1)a$$

Thus (b) is correct option.

2. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$, ... is

(a) 1 (b) $\frac{1}{p}$

(c) -1 (d) $-\frac{1}{p}$

Ans : [Board 2020 OD Standard]

Given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}$ ...

e246

Common difference

$$d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$$

Thus (c) is correct option.

3. The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is

(a) 6 (b) -6

(c) 18 (d) -18

Ans : [Board 2020 Delhi Standard]

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$(x+10) - 2x = (3x+2) - (x+10)$$
$$-x + 10 = 2x - 8$$
$$-x - 2x = -8 - 10$$
$$-3x = -18 \Rightarrow x = 6$$

e247

Thus (a) is correct option.

4. The first term of AP is p and the common difference is q, then its 10th term is

(a) $q+9p$ (b) $p-9q$

(c) $p+9q$ (d) $2p+9q$

Ans : [Board 2020 Delhi Standard]

We have $\quad a = p$ and $d = q$

$$a_{10} = a + (10-1)d$$
$$= p + 9q$$

e248

Thus (c) is correct option.

5. In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

Ans : (d) 28

In an AP, $\quad a_n = a + (n-1)d$

$$4 = a + (7-1)(-4)$$
$$4 = a + 6(-4)$$
$$4 + 24 = a \Rightarrow a = 28$$

e249

Thus (d) is correct option.

6. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

Ans : (b) 3.5

As, $d = 0$ all the terms are same whatever the value of n. So, $a_n = 3.5$.

Alternate Method :

In an AP, $a_n = a + (n-1)d$

$$a_n = 3.5 + (101-1) \times 0 = 3.5$$

Thus (b) is correct option.

e250

7. The 11th term of an AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots\ldots$, is

(a) -20 (b) 20

(c) -30 (d) 30

Ans : (b) 20

Here, $a = -5,\ d = \frac{-5}{2} - (-5) = \frac{5}{2}$

e251

nth term, $a_n = a + (n-1)d$

$$a_{11} = -5 + (11-1) \times \left(\frac{5}{2}\right)$$

$$a_{11} = -5 + 25 = 20$$

Thus (b) is correct option.

8. In an AP, if $a = 3.5$, $d = 0$ and $n = 101$, then a_n will be

(a) 0 (b) 3.5

(c) 103.5 (d) 104.5

e252

Ans : (b) 3.5

For an AP, $a_n = a + (n-1)d$

$$= 3.5 + (101-1) \times 0$$

$$= 3.5$$

Thus (b) is correct option.

9. Which term of an AP, 21, 42, 63, 84, ... is 210?

(a) 9th (b) 10th

(c) 11th (d) 12th

Ans : (b) 10th

e253

Let nth term of given AP be 210,

First term, $a = 21$

Common difference, $d = 42 - 21 = 21$

and $a_n = 210$

In an AP, $a_n = a + (n-1)d$

$$210 = 21 + (n-1)21$$

$$210 = 21 + 21n - 21$$

$$210 = 21n \Rightarrow n = 10$$

Hence, the 10th term of the given AP is 210.

Thus (b) is correct option.

10. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

e254

Ans : (c) 25

Given, the common difference of AP i.e, $d = 5$

Using, $a_n = a + (n-1)d$

We have, $a_{18} = a + (18-1)d$

and $a_{13} = a + (13-1)d$

Now, $a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$

$$= a + 17 \times 5 - a - 12 \times 5$$

$$= 85 - 60 = 25$$

Thus (c) is correct option.

11. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(a) 8 (b) -8

(c) -4 (d) 4

e255

Ans : (a) 8

We have $a_{18} - a_{14} = 32$

In an AP, $a_n = a + (n-1)d$

$$a + (18-1)d - [a + (14-1)d] = 32$$

$$a + 17d - a - 13d = 32$$

$$4d = 32 \Rightarrow d = 8$$

Hence, the required common difference of the given AP is 8.

Thus (a) is correct option.

12. The 4th term from the end of an AP $-11, -8, -5, \ldots\ldots, 49$ is

(a) 37 (b) 40

(c) 43 (d) 58

e256

Ans : (b) 40

Common difference,

$$d = -8-(-11) = -8+11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l-(n-1)\,d$$

, $$a_4 = 49-(4-1)\times 3$$

$$= 49-9 = 40$$

13. If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is

(a) 0 (b) 5

(c) 6 (d) 15

e258

Ans : (a) 0

We have $a = -5$ and $d = 2$

$$S_n = \frac{n}{2}\{2a+(n-1)\,d\}$$

$$S_6 = \frac{6}{2}[2a+(6-1)\,d]$$

$$= 3[2(-5)+5(2)]$$

$$= 3(-10+10) = 0$$

Thus (a) is correct option.

14. The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320 (b) 320

(c) -352 (d) -400

Ans : (a) -320

Given, AP, is 10, 6, 2 .....

We have $a = 10$ and $d = (6-10) = -4$

e259

$$S_n = \frac{n}{2}\{2a+(n-1)\,d\}$$

$$S_{16} = \frac{16}{2}[2a+(16-1)\,d]$$

$$= 8[2\times 10+15(-4)]$$

$$= 8(20-60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

15. In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19 (b) 21

(c) 38 (d) 42

e260

Ans : (c) 38

We have $a = 1$, $a_n = 20$ and $S_n = 399$

Now, $$S_n = \frac{n}{2}(a+a_n)$$

$$399 = \frac{n}{2}(1+20)$$

$$n = \frac{399\times 2}{21} = 38.$$

16. The sum of first five multiples of 3 is

(a) 45 (b) 55

(c) 65 (d) 75

e261

Ans : (a) 45

The first five multiples of 3 are 3, 6, 9, 12 and 15.
Here, first term, $a = 3$, $d = 6-3 = 3$ and $n = 5$

$$S_n = \frac{n}{2}\{2a+(n-1)\}\,d$$

$$S_5 = \frac{5}{2}[2a+(5-1)\,d]$$

$$= \frac{5}{2}[2\times 3+4\times 3]$$

$$= \frac{5}{2}(6+12) = \frac{5}{2}\times 18 = 45$$

Thus (a) is correct option.

17. If the sum of the series $2+5+8+11$ .... is 60100, then the number of terms are

(a) 100 (b) 200

(c) 150 (d) 250

e262

Ans : (b) 200

We have $a = 2$. $d = 5-2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a+(n-1)\,d] = S_n$$

$$\frac{n}{2}[4+(n-1)3] = 60100$$

$$n(3n+1) = 120200$$

$$3n^2+n-120200 = 0$$

$$(n-200)(3n+601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

18. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5 (b) 20

(c) 25 (d) 30

e263

Ans : (c) 25

Given, the common difference of AP i.e., $d = 5$

Now $a_n = a + (n-1)d$

$$\text{Now}, a_{18} - a_{13} = a + (18-1)d - [a + (13-1)d]$$

$$= a + 17 \times 5 - a - 12 \times 5$$

$$= 85 - 60 = 25$$

Thus (c) is correct option.

19. There are 60 terms is an AP of which the first term is 8 and the last term is 185. The 31[st] term is

(a) 56 (b) 94

(c) 85 (d) 98

e264

Ans : (d) 98

Let d be the common difference;

Now $a_n = a + (n-1)d$

Then 60[th] term, $a_{60} = 8 + (60-1)d$

$$185 = 8 + 59d$$

$$59d = 177 \Rightarrow d = 3$$

31[th] term $a_{31} = 8 + 30 \times 3 = 98$

Thus (d) is correct option.

20. The first and last term of an AP are a and ℓ respectively. If S is the sum of all the terms of the AP and the common difference is $\dfrac{\ell^2 - a^2}{k - (\ell + a)}$, then k is equal to

(a) S (b) $2S$

(c) $3S$ (d) None of these

Ans : (b) $2S$

We have, $S = \dfrac{n}{2}(a + \ell)$

$$\frac{2S}{a+\ell} = n \qquad (1)$$

Also, $\ell = a + (n-1)d$

$$d = \frac{\ell - a}{n-1} = \frac{\ell - a}{\frac{2S}{a+\ell} - 1}$$

e265

$$= \frac{\ell^2 - a^2}{2S - (\ell + a)}$$

Thus $k = 2S$

Thus (b) is correct option.

21. If the nth term of an AP is given by $a_n = 5n - 3$, then the sum of first 10 terms if

(a) 225 (b) 245

(c) 255 (d) 270

e266

Ans : (b) 245

We have $a_n = 5n - 3$

Substituting $n = 1$ and 10 we have

$$a = 2$$

$$a_{10} = 47$$

Thus $S_n = \dfrac{n}{2}(a + a_n)$

$$S_{10} = \frac{10}{2}(2 + 47)$$

$$= 5 \times 49 = 245$$

Thus (b) is correct option.

22. Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

(a) -1 (b) -8

(c) 7 (d) -9

e267

Ans : (c) 7

4th term of first AP,

$$a_4 = -1 + (4-1)d = -1 + 3d$$

and 4th term of second AP,

$$a'_4 = -8 + (4-1)d = -8 + 3d$$

Now, the difference between their 4th terms,

$$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$$

$$= -1 + 3d + 8 - 3d = 7$$

Hence, the required difference is 7.

Thus (c) is correct option.

23. An AP starts with a positive fraction and every alternate term is an integer. If the sum of the first 11 terms is 33, then the fourth term is

(a) 2 (b) 3

(c) 5 (d) 6

Ans : (a) 2

We have $S_{11} = 33$

e268

$$\frac{11}{2}[2a+10d] = 33$$

$$a+5d = 3$$

i.e. $a_6 = 3 \Rightarrow a_4 = 2$

Since, alternate terms are integers and the given sum is possible, $a_4 = 2$.

Thus (a) is correct option.

24. If the sum of the first $2n$ terms of 2, 5, 8, ... is equal to the sum of the first n terms of 57, 59, 61, ..., then n is equal to

(a) 10 (b) 12

(c) 11 (d) 13

e269

Ans : (c) 11

$$\frac{2n}{2}\{2\times 2+(2n-1)3\} = \frac{n}{2}\{2\times 57+(n-1)2\}$$

$$2(6n+1) = 112+2n$$

$$10n = 110 \Rightarrow n = 11$$

Thus (c) is correct option.

25. In an AP, if $d=-4$, $n=7$ and $a_n=4$, then a is equal to

(a) 6 (b) 7

(c) 20 (d) 28

e270

Ans : (d) 28

In an AP, $a_n = a+(n-1)d$

$$4 = a+(7-1)(-4)$$

$$4 = a+6(-4)$$

$$4+24 = a \Rightarrow a = 28$$

Thus (d) is correct option.

26. The first four terms of an AP whose first term is -2 and the common difference is -2 are

(a) $-2,0,2,4$ (b) $-2,4,-8,16$

(c) $-2,-4,-6,-8$ (d) $-2,-4,-8,-16$

Ans : (c) $-2,-4,-6,-8$

Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.

Given, that first term, $a=-2$ and common difference, $d=-2$, then we have an AP as follows

e271

$$-2,\quad -2-2,\quad -2+2(-2),\quad -2+3(-2)$$

$$=-2,-4,-6,-8$$

Thus (c) is correct option.

27. The 21[th] term of an AP whose first two terms are -3 and 4, is

(a) 17 (b) 137

(c) 143 (d) -143

e272

Ans : (b) 137

Given, first two terms of an AP are

$$a = -3$$

and $a+d = 4$

$$-3+d = 4 \Rightarrow d = 7$$

For an AP, $a_n = a+(n-1)d$

Thus $a_{21} = a+(21-1)d$

$$=-3+(20)7$$

$$=-3+140 = 137$$

Thus (b) is correct option.

28. The number of two digit numbers which are divisible by 3 is

(a) 33 (b) 31

(c) 30 (d) 29

Ans : (c) 30

Two digit numbers which are divisible by 3 are 12, 15, 18, ..., 99;

Here $a=12$, $d=3$ and $a_n=99$

e273

For an AP, $a_n = a+(n-1)d$

So, $99 = 12+(n-1)\times 3$

$$99-12 = 3n-3$$

$$99-12+3 = 3n$$

$$90 = 3n \Rightarrow n = 30$$

Thus (c) is correct option.

29. The list of numbers -10, -6, -2, 2, ... is

(a) an AP with $d=-16$ (b) an AP with $d=4$

(c) an AP with $d = -4$ (d) not an AP

Ans : (b) an AP with $d = 4$

The given numbers are $-10, -6, -2, 2, ...$

Here, $a_1 = 10,\ a_2 = -6,\ a_3 = -2$ and $a_4 = 2, ...$

Since, $d_1 = a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$$d_2 = a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$

$$d_3 = a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

Since, $d_1 = d_2 = d_3 = ... = 4$

i.e., each successive term of given list has same difference. So, the given list forms an AP with common difference, $d = 4$.

Thus (b) is correct option.

e274

30. If the nth term of an AP is $4n + 1$, then the common difference is

(a) 3 (b) 4

(c) 5 (d) 6

e275

Ans : (b) 4

Given that the n^{th} term of an AP is $4n + 1$.

$$a_n = 4n + 1$$

Substituting $n = 1, 2, 3, ...$ we have

$$a_1 = 4(1) + 1 = 5$$

$$a_2 = 4(2) + 1 = 9$$

Common difference,

$$d = a_2 - a_1 = 9 - 5 = 4$$

Thus (b) is correct option.

31. If a, b, c, d, e, f are in AP, then $e - c$ is equal to

(a) $2(c - a)$ (b) $2(d - c)$

(c) $2(f - d)$ (d) $(d - c)$

e276

Ans : (b) $2(d - c)$

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP, $a_n = a + (n - 1)d$

$$e = a + (5 - 1)x$$

$$e = a + 4x \qquad ...(1)$$

and $c = a + (3 - 1)x$

$$c = a + 2x \qquad ...(2)$$

Using equation (1) and (2), we get

$$e - c = a + 4x - a - 2x$$

$$= 2x = 2(d - c)$$

Thus (b) is correct option.

32. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be

(a) 7 (b) 11

(c) 18 (d) 0

e277

Ans : (d) 0

In an AP, $a_n = a + (n - 1)d$

Now, according to the question,

$$7a_7 = 11a_{11}$$

$$7[a + (7 - 1)d] = 11[a + (11 - 1)d]$$

$$7(a + 6d) = 11(a + 10d)$$

$$7a + 42d = 11a + 110d$$

$$4a + 68d = 0$$

$$4(a + 17d) = 0$$

$$a + 17d = 0 \qquad ...(1)$$

18th term of an AP,

$$a_{18} = a + (18 - 1)d = a + 17d$$

But from equation (1) this is zero.

33. The sum of 11 terms of an AP whose middle term is 30, is

(a) 320 (b) 330

(c) 340 (d) 350

Ans : (b) 330

Middle term is $\frac{11+1}{2} = 6$ th term.

e279

Now $a_n = a + (n - 1)d$

$$a_6 = a + 5d$$

$$30 = a + 5d$$

$$a = 30 - 5d$$

Now $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{11} = \frac{11}{2}(2a + 10d)$$

Substituting value of a we have

$$S_{11} = \frac{11}{2}[2(30-5d)+10d]$$

$$= \frac{11}{2}[60-10d+10d]$$

$$= 11 \times 30$$

$$S_{11} = 330$$

Thus (b) is correct option.

34. Five distinct positive integers are in a arithmetic progression with a positive common difference. If their sum is 10020, then the smallest possible value of the last term is

(a) 2002 (b) 2004

(c) 2006 (d) 2007

e280

Ans : (c) 2006

Let the five integers be $a-2d$, $a-d$, a, $a+d$, $a+2d$.

Then, we have,

$$(a-2d)+(a-d)+a+(a+d)+(a+2d) = 10020$$

$$5a = 10020 \Rightarrow a = 2004$$

Now, as smallest possible value of d is 1.

Hence, the smallest possible value of $a+2d$ is 2004+2 = 2006

Thus (c) is correct option.

35. If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33

(c) 37 (d) 38

e281

Ans : (b) 33

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a+(n-1)d$

$$a_2 = a+(2-1)d = 13$$

$$a+d = 13 \quad ...(1)$$

and $a_5 = a+(5-1)d = 25$

$$a+4d = 25 \quad ...(2)$$

Subtracting equation (1) from equation (2), we get

$$3d = 25-13 = 12 \Rightarrow d = 4$$

From equation (1), $a = 13-4 = 9$

Now, 7th term, $a_7 = a+(7-1)d$

$$= 9+6\times 4 = 33$$

Thus (b) is correct option.

36. Assertion : Common difference of the AP -5, -1, 3, 7, ... is 4.

Reason : Common difference of the AP $a, a+d, a+2d, ...$ is given by $d = a_2 - a_1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Common difference, $d = -1-(-5) = 4$

So, both A and R are correct and R explains A.

e282

Thus (c) is correct option.

37. Assertion : Sum of first 10 terms of the arithmetic progression $-0.5, -1.0, -1.5, ...$ is 31.

Reason : Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a+(n-1)d]$ where a is first term and d common difference.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Assertion, S_{10}

$$= \frac{10}{2}[2(-0.5)+(10-1)(-0.5)]$$

$$= 5[-1-4.5]$$

$$= 5(-5.5) = 27.5$$

Assertion (A) is false but reason (R) is true.

e283

Thus (d) is correct option.

38. Assertion : $a_n - a_{n-1}$ is not independent of n then the given sequence is an AP.

Reason : Common difference $d = a_n - a_{n-1}$ is constant or independent of n.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Common difference of an AP $d = a_n - a_{n-1}$ is independent of n or constant.

So, A is correct but R is incorrect.

Thus (d) is correct option.

e284

39. **Assertion :** If n^{th} term of an AP is $7-4n$, then its common differences is -4.

Reason : Common difference of an AP is given by $d = a_{n+1} - a_n$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

e285

Assertion, $a_n = 7-4n$

$$d = a_{n+1} - a_n$$

$$= 7-4(n+1)-(7-4n)$$

$$= 7-4n-4-7+4n = -4$$

Both are correct. Reason is the correct explanation.

Thus (a) is correct option.

40. **Assertion :** If sum of the first n terms of an AP is given by $S_n = 3n^2 - 4n$. Then its n^{th} term is $a_n = 6n-7$.

Reason : n^{th} term of an AP, whose sum to n terms is S_n, is given by $a_n = S_n - S_{n-1}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

nth term of an AP,

$$a_n = S_n - S_{n-1}$$

e286

$$= 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$

$$= 6n-7$$

So, both A and R are correct and R explains A.

Thus (a) is correct option.

Fill in the Blank Questions

41. In an AP, the letter d is generally used to denote the ....

Ans :

common difference

e287

42. If a and d are respectively the first term and the common difference of an AP, $a+10d$, denotes the .... term of the AP.

Ans :

eleventh

e288

43. An arithmetic progression is a list of numbers in which each term is obtained by .... a fixed number to the preceding term except the first term.

Ans :

adding

e289

44. If S_n denotes the sum of n term of an AP, then $S_{12} - S_{11}$ is the .... term of the AP.

Ans :

twelfth

e290

45. The nth term of an AP whose first term is a and common difference is d is ....

Ans :

$a+(n-1)d$

e291

46. The nth term of an AP is always a .... expression.

Ans :

linear

e292

47. The difference of corresponding terms of two AP's will be ....

Ans :

another AP

e293

48. Fill the two blanks in the sequence 2 ...., 26, ... so that the sequence forms an AP.

Ans : [Board 2020 SQP Standard]

Let a and b be the two numbers. AP will be 2, a, 26, b.

Now, $$26 - a = a - 2$$

$$2a = 28 \Rightarrow a = \frac{28}{2} = 14$$

e294

and $$b - 26 = 26 - a$$

$$a + b = 52$$

$$14 + b = 52 \Rightarrow b = 38$$

Thus $a = 14$ and $b = 38$.

Very Short Answer Questions

49. The sum of first 20 terms of the AP 1, 4, 7, 10 .... is

Ans : [Board 2020 Delhi Standard]

Given AP is 1, 4, 7, 10 ...

Here, $a = 1$, $d = 4 - 1 = 3$ and $n = 20$

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

e295

$$= \frac{20}{2}[2 \times 1 + (20-1)3]$$

$$= 10(2 + 57) = 10 \times 59 = 590$$

50. Show that $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

Ans : [Board 2020 Delhi Standard]

Given, $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$.

Common difference,

e307

$$d_1 = (a^2+b^2) - (a-b)^2$$

$$= (a^2+b^2) - (a^2+b^2-2ab)$$

$$= a^2 + b^2 - a^2 - b^2 + 2ab$$

$$= 2ab$$

and $$d_2 = (a+b)^2 - (a^2+b^2)$$

$$= a^2 + b^2 + 2ab - a^2 - b^2$$

$$= 2ab$$

Since, $d_1 = d_2$, thus, $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in AP.

51. Find the sum of all 11 terms of an AP whose middle term is 30.

Ans : [Board 2020 OD Standard]

In an AP with 11 terms, the middle term is $\frac{11+1}{2} = 6^{th}$ term.

Now, $$a_6 = a + 5d = 30$$

e312

Thus, $$S_{11} = \frac{11}{2}[2a + 10d]$$

$$= 11(a + 5d)$$

$$= 11 \times 30 = 330$$

52. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Ans : [Board 2020 Delhi Basic]

Let a be the first term and d be the common difference of the AP.

Now $$a_n = a + (n-1)d$$

e296

As per the information given in question

$$4 \times a_4 = 18 \times a_{18}$$

$$4(a+3d) = 18(a+17d)$$

$$2a + 6d = 9a + 153d$$

$$7a = -147d$$

$$a = -21d$$

$$a + 21d = 0$$

$$a + (22-1)d = 0$$

$$a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

53. If the first three terms of an AP are b, c and $2b$, then find the ratio of b and c.

Ans : [Board 2020 SQP Standard]

Given, b, c and $2b$ are in AP.

Thus $$c - b = 2b - c$$

e297

$$2c = 3b$$

$$\frac{2}{3} = \frac{b}{c}$$

$$\frac{b}{c} = \frac{2}{3} \Rightarrow b : c = 2 : 3$$

54. The n^{th} term of an AP is $(7 - 4n)$, then what is its common difference?

Ans : [Board 2020 Delhi Basic]

We have $a_n = 7 - 4n$

Putting $n = 1$, $a_1 = 7 - 4 = 3$

Putting $n = 2$, $a_2 = 7 - 8 = -1$

Common difference $d = a_2 - a_1$

$= -1 - 3 = -4$

e298

55. In an AP, if the common difference $d = -4$, and the seventh term a_7 is 4, then find the first term.

Ans : [Board 2018]

We have $d = -4$

and $a_7 = 4$

Now $a_n = a + (n-1)d$

$a_7 = a + (7-1)d$

$4 = a + (7-1)(-4)$

$4 = a - 24 \Rightarrow a = 4 + 24 = 28$

e324

First term of the AP is 28.

56. Find the sum of first 8 multiples of 3.

Ans : [Board 2018]

First 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 which are in AP where $a = 3$, $d = 3$ and $n = 8$.

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_8 = \frac{8}{2}[2 \times 3 + (8-1)3]$

$= 4[6 + 21]$

$S_8 = 4 \times 27 = 108$

e325

Thus, sum of first 8 multiples of 3 is 108.

57. Find, how many two digit natural numbers are divisible by 7.

Ans : [Board 2019 Delhi]

Two digits number which are divisible by 7 form an AP given by 14, 21, 28, ..., 98

Here, $a = 14$, $d = 21 - 14 = 7$ and $a_n = 98$

Now $a_n = a + (n-1)d$

$98 = 14 + (n-1)7$

$98 - 14 = 7n - 7$

$91 = 7n \Rightarrow n = 13$

e299

Hence, there are 13 numbers divisible by 7.

58. Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

Ans : [Board 2020 SQP Standard]

If any number is divisible by 2 and 5, it must be divisible by LCM of 2 and 5, i.e. 10.

Numbers between 102 .... 998 which are divisible by 2 and 5 are 110, 120, 130, ...990

Here $a = 110$, $d = 120 - 110 = 10$ and $a_n = 990$

$a_n = a + (n-1)d$

$990 = 110 + (n-1)10$

$880 = 10(n-1)$

$88 = n - 1$

$n = 88 + 1 = 89$

e300

59. Is -150 a term of the AP 11, 8, 5, 2, ...?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

We have $a = 11$, $d = -3$, $a_n = -150$

Now $a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 = 11 - 3n + 3$

$3n = 164$

or, $n = \frac{164}{3} = 54.66$

e101

Since, 54.66 is not a whole number, -150 is not a term of the given AP

60. Which of the term of AP 5, 2, -1,...... is -49?

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

We have $a = 5$, $d = -3$

Now $a_n = a + (n-1)d$

e102

Substituting all values we have

$$-49 = 5 + (n-1)(-3)$$
$$-49 = 5 - 3n + 3$$
$$3n = 49 + 5 + 3$$
$$n = \frac{57}{3} = 19^{th} \text{ term.}$$

61. Find the first four terms of an AP Whose first term is -2 and common difference is -2.

Ans : [Board Term-2 2012]

We have $a_1 = -2,$

$$a_2 = a_1 + d = -2 + (-2) = -4$$
$$a_3 = a_2 + d = -4 + (-2) = -6$$
$$a_4 = a_3 + d = -6 + (-2) = -8$$

Hence first four terms are $-2, -4, -6, -8$

62. Find the tenth term of the sequence $\sqrt{2}, \sqrt{8}, \sqrt{18}, ...$

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

Given AP is $\sqrt{2}, \sqrt{8}, \sqrt{18}$ or $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2}...$

where, $a = \sqrt{2}, d = \sqrt{2}, n = 10$

Now $a_n = a + (n-1)d$

$$a_{10} = \sqrt{2} + (10-1)\sqrt{2}$$
$$= \sqrt{2} + 9\sqrt{2}$$
$$= 10\sqrt{2}$$

Therefore tenth term of the given sequence $\sqrt{200}$.

63. Find the next term of the series $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ ....

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference d.

Here, $a = \sqrt{2},\ a + d = \sqrt{8} = 2\sqrt{2}$

$$d = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$
$$\text{Next term} = \sqrt{32} + \sqrt{2}$$
$$= 4\sqrt{2} + \sqrt{2}$$
$$= 5\sqrt{2}$$
$$= \sqrt{50}$$

64. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$ an AP? Give rea

Ans : [Board Te

Let common difference be d then we have

$$d = a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$$
$$d = a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$$
$$d = a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3$$

As common difference are not equal, the given series is not in AP

65. What is the next term of an AP $\sqrt{7},\ \sqrt{28},\ \sqrt{63}, ...?$

Ans : [Board Term-2 Foreign 2014]

Let the first term of an AP be a and common difference be d.

Here, $a = \sqrt{7},\ a + d = \sqrt{28}$

$$d = \sqrt{28} - \sqrt{7} = 2\sqrt{7} - \sqrt{7}$$
$$= \sqrt{7}$$
$$\text{Next term} = \sqrt{63} + \sqrt{7}$$
$$= 3\sqrt{7} + \sqrt{7} = 4\sqrt{7}$$
$$= \sqrt{7 \times 16}$$
$$= \sqrt{112}$$

66. If the common difference of an AP is -6, find $a_{16} - a_{12}$.

Ans : [Board Term-2 2014]

Let the first term of an AP be a and common difference be d.

Now $d = -6$

$$a_{16} = a + (16-1)(-6) = a - 90$$
$$a_{12} = a + (12-1)(-6) = a - 66$$
$$a_{16} - a_{12} = (a - 90) - (a - 66) = a - 90 - n + 66$$
$$= -24$$

67. For what value of k will the consecutive terms $2k+1$, $3k+3$ and $5k-1$ form an AP?

Ans : [Board Term-2 Foreign 2016]

If x, y and z are in AP then we have

$$y - x = z - y$$

Thus if $2k+1,\ 3k+3,\ 5k-1$ are in AP then

$$(5k-1) - 3k + 3 = (3k+3) - (2k+1)$$
$$5k - 1 - 3k - 3\ \ 3k + 3 - 2k - 1$$
$$2k - 4 = k + 2$$
$$2k - k = 4 + 2$$
$$k = 6$$

68. Find the 25^{th} term of the AP $-5, \frac{-5}{2}, \frac{5}{2}, ...$

Ans : [Board Term-2 Foreign 2015]

Let the first term of an AP be a and common difference be d.

Here, $a = -5,\ d = -\frac{5}{2} - (-5) = \frac{5}{2}$

$$a_n = a + (n-1)d`$$

$$a_{25} = 5 + (25-1) \times \left(\frac{5}{2}\right)$$

$$= -5 + 60 = 55$$

e110

69. The first three terms of an AP are $3y-1$, $3y+5$ and $5y+1$ respectively then find y.

Ans : [Board Term-2 Delhi 2015]

If x, y and z are in AP then we have

$$y - x = z - y$$

Therefore if $3y-1, 3y+5$ and $5y+1$ in AP

$$(3y+5) - (3y-1) = (5y+1) - (3y+5)$$

$$3y + 5 - 3y + 1 = 5y + 1 - 3y - 5$$

$$6 = 2y - 4$$

$$2y = 6 + 4$$

$$y = \frac{10}{2} = 5$$

e111

70. For what value of k, $k+9$, $2k-1$ and $2k+7$ are the consecutive terms of an AP

Ans : [Board Term-2 OD 2016]

If x, y and z are consecutive terms of an AP then we have

$$y - x = z - y$$

Thus if $k+9, 2k-1$, and $2k+7$ are consecutive terms of an AP then we have

$$(2k-1) - (k+9) = (2k+7) - (2k-1)$$

$$2k - 1 - k - 9 = 2k + 7 - 2k + 1$$

$$k - 10 = 8 \quad k \Rightarrow 18$$

e112

71. What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Ans : [Board Term-2 2016]

Let the first term of an AP be a and common difference be d.

$$a_{21} - a_7 = 84$$

e113

$$a + (21-1)d - [a + (7-1)d] = 84$$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = 6$$

72. In the AP $2, x, 26$ find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are in AP then we have

$$y - x = z - y$$

Since $2, x$ and 26 are in AP we have

$$x - 2 = 26 - x$$

$$2x = 26 + 2$$

$$x = \frac{28}{2} = 14$$

e114

73. For what value of k; $k+2$, $4k-6$, $3k-2$ are three consecutive terms of an AP.

Ans : [Board Term-2 Delhi 2014, 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

Since $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, we obtain

$$(4k-6)-(k+2) = (3k-2)-(4k-6)$$
$$4k-6-k-2 = 3k-2-4k+6$$
$$3k-8 = -k+4$$
$$4k = 4+8$$
$$k = \frac{12}{4} = 3$$

e115

74. If $18, a, b, -3$ are in AP, then find $a+b$.

Ans : [Board Term-2 2012]

If $18, a, b, -3$ are in AP, then,

$$a-18 = -3-b$$
$$a+b = -3+18$$
$$a+b = 15$$

e116

75. Find the common difference of the AP $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}, ...$

Ans : [Board Term-2 Delhi 2011]

Let common difference be d then we have

$$d = \frac{1-6q}{3q} - \frac{1}{3q}$$

e117

$$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

76. Find the first four terms of an AP whose first term is $3x+y$ and common difference is $x-y$.

Ans : [Board Term-2 2012]

Let the first term of an AP be a and common difference be d.

e118

Now
$$a_1 = 3x+y$$
$$a_2 = a_1+d = 3x+y+x-y = 4x$$
$$a_3 = a_2+d = 4x+x-y = 5x-y$$
$$a_4 = a_3+d = 5x-y+x-y$$
$$= 6x-2y$$

So, the four terms are $3x+y$, $4x, 5x-y$ and $6x-2y$.

77. Find the 37^{th} term of the AP $\sqrt{x}$, $3\sqrt{x}$, $5\sqrt{x}$.

Ans : [Board Term-2 2012]

Let the nth term of an AP be a_n and common difference be d.

Here, $a_1 = \sqrt{x}$

e119

$$a_2 = 3\sqrt{x}$$
$$d = a_2 - a_1 = 3\sqrt{x} - \sqrt{x} = 2\sqrt{x}$$
$$a_n = a+(n-1)d$$
$$a_{37} = \sqrt{x} + (37-1)2\sqrt{x}$$
$$= \sqrt{x} + 36 \times 2\sqrt{x} = 73\sqrt{x}$$

78. For an AP, if $a_{25} - a_{20} = 45$, then find the value of d.

Ans : [Board Term-2 2011]

Let the first term of an AP be a and common difference be d.

$$a_{25} - a_{20} = \{a+(25-1)d\} - \{a+(20-1)d\}$$
$$45 = a+24d-a-19d$$
$$45 = 5d$$
$$d \; \frac{45}{5} = 9$$

e120

79. Find the sum of first ten multiple of 5.

Ans : [Board Term-2 Delhi, 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e151

Here, $a = 5, n = 10, d = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{10} = \frac{10}{2}[2\times 5+(10-1)5]$$
$$= 5[10+9\times 5]$$
$$= 5[10+45]$$
$$= 5\times 55 = 275$$

Hence the sum of first ten multiple of 5 is 275.

80. Find the sum of first five multiples of 2.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

e152

Here, $a = 2, d = 2, n = 5$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_5 = \frac{5}{2}[2\times 2+(5-1)2]$$
$$= \frac{5}{2}[4+4\times 2] = \frac{5}{2}[4+8]$$
$$= \frac{5}{2}\times 12 = 5\times 6 = 30$$

81. Find the sum of first 16 terms of the AP 10, 6, 2, ...

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a = 10, d = 6 - 1 = -4, n = 16$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$$

$$= 8[20 + 15 \times (-4)]$$

$$= 8[20 - 60]$$

$$= 8 \times (-40)$$

$$= -320$$

82. What is the sum of five positive integer divisible by 6.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of $n$the term be S_n

Here, $a = 6, d = 6, n = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2 \times 6 + (5-1)(6)]$$

$$= \frac{5}{2}[12 + 4 \times 6]$$

$$= \frac{5}{2}[12 + 24] = \frac{5}{2}[36]$$

$$= 5 \times 18 = 90$$

83. If the sum of n terms of an AP is $2n^2 + 5n$, then find the 4^{th} term.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Now, $S_n = 2n^2 + 5n$

n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$

$$a_n = (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)]$$

$$= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5]$$

$$= 2n^2 + 5n - 2n^2 - n + 3$$

$$= 4n + 3$$

Thus 4^{th} term $a_4 = 4 \times 4 + 3 = 19$

84. If the sum of first k terms of an AP is $3k^2 - k$ and its common difference is 6. What is the first term?

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n. Let the sum of k terms of AP is S_k.

We have $S_k = 3k^2 - k$

Now k^{th} term of AP,

$$a_k = S_k - S_{k-1}$$

$$a_k = (3k^2 - k) - [3(k-1)^2 - (k-1)]$$

$$= 3k^2 - k - [3k^2 - 6k + 3 - k + 1]$$

$$= 3k^2 - k - 3k^2 + 7k - 4$$

$$= 6k - 4$$

First term $a = 6 \times 1 - 4 = 2$

85. Which term of the AP $8, 14, 20, 26, ...$ will be 72 more than its 41^{st} term.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 8, d = 6$.

Since n^{th} term is 72 more than 41^{st} term. we get

$$a_n = a_{41} + 72$$

$$8 + (n-1)6 = 8 + 40 \times 6 + 72$$

$$6n - 6 = 240 + 72$$

$$6n = 312 + 6 = 318$$

$$n = 53$$

86. If the n^{th} term of an AP $-1, 4, 9, 14, ...$ is 129. Find the value of n.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = -1$ and $d = 4 - (-1) = 5$

$$-1 + (n-1) \times 5 = a_n$$

$$-1 + 5n - 5 = 129$$

$$5n = 135$$

$$n = 27$$

Hence 27^{th} term is 129.

87. Write the n^{th} term of the AP $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}, ...$

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

e159

We have $a = \frac{1}{m}$

$$d = \frac{1+m}{m} - \frac{1}{m} = 1$$

$$a_n = \frac{1}{m} + (n-1)1$$

Hence , $a_n = \frac{1}{m} + n - 1$

88. What is the common difference of an AP which $a_{21} - a_7 = 84$.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

e160

We have $a_{21} - a_7 = 84$

$$a + 20d - a - 6d = 84$$

$$14d = 84$$

$$d = \frac{84}{14} = 6$$

Hence common difference is 6.

89. Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$.... is the first negative.

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 20$ and $d = -\frac{3}{4}$

Let the n^{th} term be first negative term, then

e161

$$a + (n-1)d < 0$$

$$20 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$20 - \frac{3}{4}n + \frac{3}{4} < 0$$

$$3n > 83$$

$$n > \frac{83}{3} = 27\frac{2}{3}$$

Hence 28^{th} term is first negative.

TWO MARKS QUESTIONS

90. If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first $(m+n)$ terms is zero.

Ans : [Board 2020 SQP Standard]

Let a be the first term and d be the common difference of the given AP. Then,

$$S_m = S_n$$

e303

$$\frac{m}{2}\{2a + (m-1)d\} = \frac{n}{2}\{2a + (n-1)d\}$$

$$2a(m-n) + \{m(m-1) - n(n-1)d\} = 0$$

$$2a(m-n) + [(m^2 - n^2) - (m-n)d] = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0$$

Now, $S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)d\}$

$$= \frac{m+n}{2} \times 0 = 0$$

91. If $3k-2$, $4k-6$ and $k+2$ are three consecutive terms of AP, then find the value of k.

Ans : [Board 2020 OD Basic]

To be term of an AP the difference between two consecutive terms must be the same.

If $3k-2$, $4k-6$ and $k+2$ are terms of an AP, then

$$4k - 6 - (3k-2) = k + 2 - (4k-6)$$

e304

$$4k - 6 - 3k + 2 = k + 2 - 4k + 6$$

$$k - 4 = 8 - 3k$$

$$4k = 12 \Rightarrow k = 3$$

Hence, the value of k is 3.

92. How many terms of AP 3, 5, 7, 9, ... must be taken to get the sum 120?

Ans : [Board 2020 OD Basic]

Given AP : 3, 5, 7, 9, ...

e305

We have $a = 3$, $d = 2$ and $S_n = 120$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$120 = \frac{n}{2}[2 \times 3 + (n-1)2]$$

$$120 = n(3 + n - 1)$$

$$120 = n(n+2)$$

$$n^2 + 2n - 120 = 0$$

$$n^2 + 12n - 10n - 120 = 0$$

$$(n+12)(n-10) = 0 \Rightarrow n = 10 \text{ or } n = -12$$

Neglecting $n = -12$ because n can't be negative we get $n = 10$. Hence, 10 terms must be taken to get the sum 120.

93. How many two digits numbers are divisible by 3?

Ans : [Board 2019 Delhi]

Numbers divisible by 3 are 3, 6, 9, 12, 15, ..., 96 and 99. Lowest two digit number divisible by 3 is 12 and highest two digit number divisible by 3 is 99.

Hence, the sequence start with 12, ends with 99 and common difference is 3.

So, the AP is 12, 15, 18, ..., 96, 99.

Here, $a = 12$, $d = 3$ and $a_n = 99$

$$a_n = a + (n-1)d$$

$$99 = 12 + (n-1)3$$

e315

$$99 - 12 = 3(n-1)$$

$$n - 1 = \frac{87}{3} = 29 \Rightarrow n = 30$$

Therefore, there are 30, two digit numbers divisible by 3.

94. Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Ans : [Board 2019 Delhi]

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$

$$a_{21} = 3 + (21-1) \times 12$$

$$= 3 + 20 \times 12 = 243$$

e316

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore, $a_n = a + (n-1)d$

$$363 = 3 + (n-1)12$$

$$360 = 12(n-1)$$

$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

95. If S_n the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the n^{th} term.

Ans : [Board 2019 Delhi]

We have $S_n = 3n^2 - 4n$

Substituting $n = 1$, we get

$$S_1 = 3 \times 1^2 - 4 \times 1 = -1$$

So, sum of first term of AP is -1, but sum of first term is the first term itself,

Thus first term $a_1 = -1$

e317

Now substituting $n = 2$ we have

$$S_2 = 3 \times 2^2 - 4 \times 2 = 4$$

Sum of first two terms is 4.

$$a_1 + a_2 = 4$$

$$-1 + a_2 = 4 \Rightarrow a_2 = 5$$

Hence, common difference,

$$d = a_2 - a_1 = 5 - (-1) = 6$$

Now n^{th} term, $a_n = a_1 + (n-1)d$

, $a_n = -1 + (n-1)6$

$$a_n = 6n - 7$$

Therefore, n^{th} term is $6n - 7$.

96. Find the 21^{st} term of the AP $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$

Ans : [Board 2019 OD]

Given AP is $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$ or $-\frac{9}{2}, -3, -\frac{3}{2}, ...$

First term, $a = \frac{-9}{2}$

Common difference,

$$d = -3 - \left(-\frac{9}{2}\right) = -3 + \frac{9}{2}$$

e323

$$= \frac{-6+9}{2} = \frac{3}{2}$$

Now $a_n = a + (n-1)d$

$$a_{21} = \left(-\frac{9}{2}\right) + (21-1)\left(\frac{3}{2}\right)$$

$$= -\frac{9}{2} + 20 \times \frac{3}{2} = -\frac{9}{2} + 30$$

$$= \frac{-9+30}{2} = \frac{51}{2} = 25\frac{1}{2}$$

Hence, 21^{st} term of given AP is $25\frac{1}{2}$.

97. If the sum of first n terms of an AP is n^2, then find its 10th term.

Ans : [Board 2019 Delhi]

We have $S_n = n^2$ ...(1)

Substituting $n = 1$ in equation (1), we have

$$S_1 = 1$$

Hence, sum of first term of AP is 1, but sum of first term is first term itself.

So, first term, $a = 1$ ...(2)

Substituting $n = 2$ in equation (1), we have

$$S_2 = (2)^2 = 4$$

Sum of first 2 terms is 4.

Now $a + a_2 = 4$ ...(3)

From equation (2) and (3) we have

$$a_2 = 3$$

e306

Now, common difference,

$$d = a_2 - a = 3 - 1 = 2$$

Now, 10^{th} term of AP,

$$a_{10} = a + (10-1)d$$

$$= 1 + 9 \times 2 = 19$$

Hence, the 10^{th} term of AP is 19.

98. Is 184 a term of the sequence $3, 7, 11, ...?$

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 184$

Here, $a = 3, d = 7 - 3 = 11 - 7 = 4$

e121

Now $a_n = a + (n-1)d,$

$$184 = 3 + (n-1)4$$

$$\frac{181}{4} = n - 1$$

$$45.25 = n - 1$$

$$46.25 = n$$

Since 46.25 is not an whole number, thus 184 is not a term of given AP

99. Find, 100 is a term of the AP $25, 28, 31, ...$ or not.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and number of terms be n.

Let $a_n = 100$

Here $a = 25, d = 28 - 25 = 31 - 28 = 3$

e122

Now $a_n = a + (n-1)d,$

$$100 = 25 + (n-1) \times 3$$

$$100 - 25 = 75 = (n-1) \times 3$$

$$25 = n - 1$$

$$n = 26$$

Since 26 is an whole number, thus 100 is a term of given AP.

100. Find the 7^{th} term from the end of AP $7, 10, 13, ... 184$.

Ans : [Board Term-2 2012]

Let us write AP in reverse order i.e., $184, ... 13, 10, 7$

Let the first term of an AP be a and common difference be d.

Now $d = 7 - 10 = -3$

$$a = 184, \; n = 7$$

e123

7^{th} term from the original end,

$$a_7 = a + 6d$$

$$a_7 = 184 + 6(-3)$$

$$= 184 - 18 = 166.$$

Hence, 166 is the 7^{th} term from the end.

101. Which term of an AP 150, 147, 144, ... is its first negative term?

Ans : [KVS 2014]

Let the first term of an AP be a, common difference be d and nth term be a_n.

For first negative term $a_n < 0$

$$a + (n-1)d < 0$$

$$150 + (n-1)(-3) < 0$$

e124

$$150 - 3n + 3 < 0$$

$$-3n < -153$$

$$n > 51$$

Therefore, the first negative term is 52^{nd} term.

102. In a certain AP 32^{th} term is twice the 12^{th} term. Prove that 70^{th} term is twice the 31^{st} term.

Ans : [Board Term-2 2015, 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now we have
$$a_{32} = 2a_{12}$$
$$a+31d = 2(a+11d)$$
$$a+31d = 2a+22d$$
$$a = 9d$$
$$a_{70} = a+69d$$
$$= 9d+69d = 78d$$
$$a_{31} = a+30d$$
$$= 9d+30d = 39d$$
$$a_{70} = 2a_{31}$$
Hence Proved.

e125

103. The 8^{th} term of an AP is zero. Prove that its 38^{th} term is triple of its 18^{th} term.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

We have, $a_8 = 0$ or, $a+7d = 0$ or, $a = -7d$

Now
$$a_{38} = a+37d$$
$$a_{38} = -7d+37d = 30d$$
$$a_{18} = a+17d$$
$$= -7d+17d = 10d$$
$$a_{38} = 30d = 3\times 10d = 3\times a_{18}$$
$$a_{38} = 3a_{18}$$
Hence Proved

e126

104. If five times the fifth term of an AP is equal to eight times its eighth term, show that its 13^{th} term is zero.

Ans : [Board Term-2 2012]

Let the first term of an AP be a, common difference be d and nth term be a_n.

Now
$$5a_5 = 8a_8$$
$$5(a+4d) = 8(a+7d)$$
$$5a+20d = 8a+56d$$
$$3a+36d = 0$$
$$3(a+12d) = 0$$
$$a+12d = 0$$
$$a_{13} = 0$$
Hence Proved

e127

105. The fifth term of an AP is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

e128

$$a+4d = 20 \quad ...(1)$$
$$a+6d+a+10d = 64$$
$$a+8d = 32 \quad ...(2)$$

Solving equations (1) and (2), we have
$$d = 3$$

106. The ninth term of an AP is -32 and the sum of its eleventh and thirteenth term is -94. Find the common difference of the AP

Ans : [Board Term-2 Foreign 2015]

Let the first term be a and common difference be d.

e129

Now
$$a+8d = a_9$$
$$a+8d = -32 \quad ...(1)$$
and
$$a_{11}+a_{13} = -94$$
$$a+10d+a+12d = -94$$
$$a+11d = -47 \quad ...(2)$$

Solving equation (1) and (2), we have
$$d = -5$$

107. The seventeenth term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Ans : [Board Term-2 2015, 2014]

Let the first term be a and common difference be d.

Now
$$a_{17} = a_{10}+7$$
$$a+16d = a+9d+7$$
$$16d-9d = 7$$
$$7d = 7$$
$$d = 1$$

e130

Thus common difference is 1.

108. The fourth term of an AP is 11. The sum of the fifth and seventh terms of the AP is 34. Find the c difference.

Ans : [Foreign

e131

Let the first term be a and common difference be d.

Now $a_4 = 11$

$a + 3d = 11$ ...(1)

and $a_5 + a_7 = 34$

$a + 4d + a + 6d = 34$

, $2a + 10d = 34$

, $a + 5d = 17$ ...(2)

Solving equations (1) and (2) we have

$$d = 3$$

109. Find the middle term of the AP 213, 205, 197, ... 37.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$a_m = a + (m-1)d$$

$$37 = 213 + (m-1)(-8)$$

$$37 - 213 = -8(m-1)$$

$$m - 1 = \frac{-176}{-8} = 22$$

$$m = 22 + 1 = 23$$

The middle term will be $= \frac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d$$

$$= 213 + (12-1)(-8)$$

$$= 213 - 88 = 125$$

Middle term will be 125.

110. Find the middle term of the AP 6, 13, 20, ... 216.

Ans : [Board Term-2 Delhi 2015]

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 6, a_m = 216, d = 13 - 6 = 7$

$$a_m = a + (m-1)d$$

$$216 = 6 + (m-1)(7)$$

$$216 - 6 = 7(m-1)$$

$$m - 1 = \frac{210}{7} = 30$$

$$m = 30 + 1 = 31$$

The middle term will be $= \frac{31+1}{2} = 16^{th}$

$$a_{16} = a + (16-1)d$$

$$= 6 + (16-1)(7)$$

$$= 6 + 15 \times 7$$

$$= 6 + 105 = 111$$

Middle term will be 111.

111. If the 2^{nd} term of an AP is 8 and the 5^{th} term is 17, find its 19^{th} term.

Ans : [Board Term-2 2016]

Let the first term be a and common difference be d.

Now $a_2 = a + d$

$8 = a + d$ (1)

and $a_5 = a + 4d$

$17 = a + 4d$ (2)

Solving (1) and (2), we have

$$a = 5, d = 3,$$

$$a_{19} = a + 18d$$

$$= 5 + 54 = 59$$

112. If the number $x + 3, 2x + 1$ and $x - 7$ are in AP find the value of x.

Ans : [Board Term-2 2012]

If x, y and z are three consecutive terms of an AP then we have

$$y - x = z - y$$

$$(2x+1) - (x+3) = (x-7) - (2x+1)$$

$$2x + 1 - x - 3 = x - 7 - 2x - 1$$

$$x - 2 = -x - 8$$

$$2x = -6$$

$$x = -3$$

113. Find the values of a, b and c, such that the numbers a, 10, b, c, 31 are in AP

Ans : [Board Term-2 2012]

Let the first term be a and common difference be d.

Since $a, 10, b, c, 31$ are in AP, then

$$a + d = 10 \quad (1)$$

$$a + 4d = a_5$$

$$a + 4d = 31 \quad (2)$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

e136

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

114. For AP show that $a_p + a_{p+2q} = 2a_{p+q}$.

Ans : [Board Term-2 2012]

Let the first term be a and the common difference be d. Let a_n be the nth term.

e137

$$a_p = a + (p-1)d$$

$$a_{p+2q} = a + (p + 2q - 1)d$$

$$a_p + a_{p+2q} = a + (p-1)d + a + (p + 2q - 1)d$$

$$= a + pd - d + a + pd + 2qd - d$$

$$= 2a + 2pd + 2qd - 2d$$

$$\text{or } a_p + a_{p+2q} = 2\left[a + (p + q - 1)d\right] \quad ...(1)$$

$$\text{But } \quad 2a_{p+q} = 2\left[a + (p + q - 1)d\right] \quad ...(2)$$

From (1) and (2), we get $a_p + a_{p+2q} = 2a_{p+q}$

115. The sum of first terms of an AP is given by $S_n = 2n^2 + 8n$. Find the sixteenth term of the AP.

Ans : [Board SQP 2017]

Let the first term be a, common difference be d and nth term be a_n.

e138

Now $\quad S_n = 2n^2 + 3n$

$$S_1 = 2 \times 1^2 + 3 \times 1 = 2 + 3 = 5$$

Since $S_1 = a_1$,

$$a_1 = 5$$

$$S_2 = 2 \times 2^2 + 3 \times 2 = 8 + 6 = 14$$

$$a_1 + a_2 = 14$$

$$a_2 = 14 - a_1 = 14 - 5 = 9$$

$$d = a_2 - a_1 = 9 - 5 = 4$$

$$a_{16} = a + (16 - 1)\,d$$

$$= 5 + 15 \times 4 = 65$$

116. The 4^{th} term of an AP is zero. Prove that the 25^{th} term of the AP is three times its 11^{th} term.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d and nth term be a_n.

e139

We have, $\quad a_4 = 0$

$$a + 3d = 0 \qquad [\,a + (n-1)d = a_n]$$

$$3d = -a$$

$$-3d = a \quad ...(1)$$

Now, $\quad a_{25} = a + 24d = -3d + 24d = 21d \quad ...(2)$

$$a_{11} = a + 10d = -3d + 10d = 7d \quad ...(3)$$

From equation (2) and (3) we have

$$a_{25} = 3a_{11}$$ Hence Proved.

117. How many terms of the AP $65, 60, 55, ...$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 65, d = -5,\ S_n = 0$

Now $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

e162

Let sum of n term be zero, then we have

$$\frac{n}{2}[130 + (n-1)(-5)] = 0$$

$$\frac{n}{2}[130 + 5n + 5] = 0$$

$$135n - 5n^2 = 0$$

$$n(135 - 5n) = 0$$

$$5n = 135$$

$$n = 27$$

118. How many terms of the AP $18, 16, 14 .....$ be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 18, d = -2, S_n = 0$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

e163

Let sum of n term be zero, then we have

$$\frac{n}{2}[36+(n-1)(-2)] = 0$$

$$n(38-2n) = 0$$

$$n = 19$$

119. How many terms of the AP $27, 24, 21....$ should be taken so that their sum is zero?

Ans : [Board Term-2 Delhi 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 27, d = -3, S_n = 0$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

e164

Let sum of n term be zero, then we have

$$\frac{n}{2}[54+(n-1)(-3)] = 0$$

$$n(-3n+57) = 0$$

$$n = 19$$

120. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of first n terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

e165

$$S_5 + S_7 = 167$$

$$\frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d) = 167$$

$$5a+10d+7a+21d = 167$$

$$12a+31d = 167 \quad ...(1)$$

Now we have $S_{10} = 235$, thus

$$\frac{10}{2}[2a+(10-1)d] = 235$$

$$5(2a+9d) = 235$$

$$2a+9d = 47 \quad (2)$$

Solving (1) and (2), we get

$$a = 1, d = 5$$

Thus AP is 1, 6, 11....

121. Find the sum of sixteen terms of an AP $-1, -5, -9, ...$

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_1 = -1, a_2 = -5$ and $d = -4$

e166

Now $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_{16} = \frac{16}{2}[2\times(-1)+(16-1)(-4)]$$

$$= 8[-2-60] = 8(-62)$$

$$= -496$$

122. If the n^{th} term of an AP is $7-3n$, find the sum of twenty five terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $n = 25, a_n = 7-3n$

Taking $n = 1,2,3,....$ we have

e167

$$a_1 = 7-3\times 1 = 4$$

$$a_2 = 7-3\times 2 = 1$$

$$a_3 = 7-3\times 3 = -2$$

Thus required AP is $4, 1, -2, ...$

Here, $a = 4, d = 1-4 = -3$

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$= \frac{25}{2}[2\times 4+(25-1)(-3)]$$

$$= \frac{25}{2}[8+24(-3)]$$

$$= \frac{25}{2}(8-72) = -800$$

123. If the 1^{st} term of a series is 7 and 13^{th} term is 35. Find the sum of 13 terms of the sequence.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a = 7, a_{13} = 35$

$$a_n = a+(n-1)d$$

$$a_{13} = a+12d$$

$$35 = 7+12d \Rightarrow d = \frac{7}{3}$$

e168

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{13} = \frac{13}{2}\left[2\times 7+12\times\left(\frac{7}{3}\right)\right]$$

$$= \frac{13}{2}[14+28]$$

$$= \frac{13}{2}\times 42 = 273$$

124. If the n^{th} term of a sequence is $3-2n$. Find the sum of fifteen terms.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here, $a_n = 3-2n$

Taking $n=1$, $a_1 = 3-2 = 1$

15th term, $a_{15} = 3-2\times 15 = 3-30 = -27$

Now $$S_n = \frac{n}{2}(a+1)$$

$$S_{15} = \frac{15}{2}[1+(-27)]$$

e169

$$= \frac{15}{2}[-26]$$

$$= 15\times(-13) = -195$$

125. If S_n denotes the sum of n terms of an AP whose common difference is d and first term is a, find $S_n - 2S_{n-1} + S_{n-2}$.

Ans : [Board Term-2 2011]

e170

We have $$a_n = S_n - S_{n-1}$$

$$a_{n-1} = S_{n-1} - S_{n-2}$$

$$S_n - 2S_{n-1} + S_{n-2} = S_n - S_{n-1} - S_{n-1} + S_{n-2}$$

$$= (S_n - S_{n-1}) - (S_{n-1} - S_{n-2})$$

$$= a_n - a_{n-1} = d$$

126. The sum of first n terms of an AP is $5n-n^2$. Find the n^{th} term of the AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have, $S_n = 5n - n^2$

e171

Now, n^{th} term of AP,

$$a_n = S_n - S_{n-1}$$

$$= (5n-n^2) - [5(n-1)-(n-1)^2]$$

$$= 5n-n^2 - [5n-5-(n^2+1-2n)]$$

$$= 5n-n^2-(5n-5-n^2-1+2n)$$

$$= 5n-n^2-7n+6+n^2$$

$$= -2n+6$$

$$a_n = -2(n-3)$$

Thus n^{th} term is $= -2(n-3)$

127. The first and last term of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5, a_n = 45$

Now $$45 = 5+(n-1)d$$

$$(n-1)d = 40 \quad ...(1)$$

Given, $S_n = 400$

Now $$S_n = \frac{n}{2}(a+a_n)$$

e172

$$400 = \frac{n}{2}(5+45)$$

$$800 = 50n$$

$$n = 16$$

Substituting this value of n in (1) we have

$$(n-1)d = 40$$

$$15d = 40$$

$$d = \frac{40}{15} = \frac{8}{3}$$

128. If the sum of the first 7 terms of an AP is 49 and that of the first 17 terms is 289, find the sum of its first n terms.

Ans : [Board Term-2 Foreign 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e173

Now $$S_7 = \frac{7}{2}(2a + 6d) = 49$$

$$a + 3d = 7 \quad ...(1)$$

and $$S_{17} = \frac{17}{2}(2a + 16d) = 289$$

$$a + 8d = 17$$

Subtracting (1) from (2), we get

$$5d = 10 \Rightarrow d = 2$$

Substituting this value of d in (1) we have

$$a = 1$$

Now $$S_n = \frac{n}{2}[2 \times 1 + (n-1)2]$$

$$= \frac{n}{2}[2 + 2n - 2] = n^2$$

Hence, sum of n terms is n^2.

129. How many terms of the AP $-6, \frac{-11}{2}, -5, -\frac{9}{2}....$ are needed to give their sum zero.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = -6, d = -\frac{11}{2} - (-6) = \frac{1}{2}$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e174

Let sum of n term be zero, then we have

$$\frac{n}{2}\left[2 \times -6 + (n-1)\frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[-12 + \frac{n}{2} - \frac{1}{2}\right] = 0$$

$$\frac{n}{2}\left[\frac{n}{2} - \frac{25}{2}\right] = 0$$

$$n^2 - 25n = 0$$

$$n(n-25) = 0$$

$$n = 25$$

Hence 25 terms are needed.

130. Which term of the AP $3, 12, 21, 30, ...$ will be 90 more than its 50^{th} term.

Ans : [Board Term-2 Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

We have $$a = 3, d = 9$$

e175

Now $$a_n = a + (n-1)d$$

$$a_{50} = 3 + 49 \times 9 = 444$$

Now, $$a_n - a_{50} = 90$$

$$3 + (n-1)9 - 444 = 90$$

$$(n-1)9 = 90 + 441$$

$$(n-1) = \frac{531}{9} = 49$$

$$n = 49 + 1 = 50$$

131. The 10^{th} term of an AP is -4 and its 22^{nd} term is -16. Find its 38^{th} term.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_{10} = a + 9d = -4 \quad (1)$$

and $$a_{22} = a + 21d = -16 \quad (2)$$

Subtracting (2) from (1) we have

$$12d = -12 \Rightarrow d = -16$$

e176

Substituting this value of d in (1) we get

$$a = 5$$

Thus $$a_{38} = 5 + 37 \times -1 = -32$$

Hence, $a_{38} = -32$

132. Find how many integers between 200 and 500 are divisible by 8.

Ans : [Board Term-2 Delhi Compt. 2017]

Number divisible by 8 are 208, 2016, 224, ... 496. It is an AP

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 208, d = 8$ and $a_n = 496$

e177

Now $$a + (n-1)d = a_n$$

$$208 + (n-1)d = 496$$

$$(n-1)8 = 496 - 208$$

$$n - 1 = \frac{288}{8} = 36$$

$$n = 36 + 1 = 37$$

Hence, required numbers divisible by 8 is 37.

133. The fifth term of an AP is 26 and its 10^{th} term is 51. Find the AP

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$a_5 = a+4d = 26 \quad ...(1)$$

$$a_{10} = a+9d = 51 \quad ...(2)$$

Subtracting (1) from (2) we have

$$5d = 25 \Rightarrow d = 5$$

e178

Substituting this value of d in equation (1) we get

$$a = 6$$

Hence, the AP is 6, 11, 16, ...

134. Find the AP whose third term is 5 and seventh term is 9.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $$a_3 = a+2d = 5 \quad ...(1)$$

and $$a_7 = a+6d = 9 \quad ...(2)$$

Subtracting (2) from (1) we have

$$4d = 4 \Rightarrow d = 1$$

e179

Substituting this value of d in (1) we get

$$a = 3$$

Hence AP is 3, 4, 5, 6, ...

135. Find whether -150 is a term of the AP $11, 8, 5, 2, ...$

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d and nth term be a_n.

Let the n^{th} term of given AP $11, 8, 5, 2, ...$ be -150

Hence $a = 11$, $d = 8-11 = -3$ and $a_n = -150$

$$a+(n-1)d = a_n$$

$$11+(n-1)(-3) = -150$$

$$(n-1)(-3) = -161$$

$$(n-1) = \frac{-161}{-3} = 53\frac{2}{3}$$

e180

which is not a whole number. Hence -150 is not a term of given AP.

136. If seven times the 7^{th} term of an AP is equal to eleven times the 11^{th} term, then what will be its 18^{th} term.

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d and nth term be a_n.

$$7a_7 = 11a_{11}$$

Now $$7(a+6d) = 11(a+10d)$$

e181

$$7a+42d = 11a+110d$$

$$11a-7a = 42d-110d$$

$$4a = -68d$$

$$4a+68d = 0$$

$$4(a+17d) = 0$$

$$a+17d = 0$$

Hence, $$a_{18} = 0$$

137. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP

Ans : [Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_{10} = 210$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

e182

$$210 = \frac{10}{2}(2a+9d)$$

$$42 = 2a+9d \quad (1)$$

Now $$a_{36} = a+35d$$

$$a_{50} = a+49d$$

Sum of last 15 terms,

$$S_{36-50} = \frac{n}{2}(a_{36}+a_{50})$$

$$2565 = \frac{15}{2}(a+35d+a+49d)$$

$$171 = \frac{1}{2}(2a+84d)$$

$$171 = a+42d \quad (2)$$

Solving (1) and (2) we get $a = 3$ and $d = 4$

Hence, AP is 3, 7, 11, ...

THREE MARKS QUESTIONS

138. The sum of four consecutive number in AP is 32 and the ratio of the product of the first and last term to the product of two middle terms is 7 : 15. Find the numbers.

Ans : [Board 2020 Delhi Standard, 2018]

Let the four consecutive terms of AP be $(a-3d)$, $(a-d)$, $(a+d)$ and $(a+3d)$.

As per question statement we have

$$a-3d+a-d+a+d+a+3d = 32$$

$$4a = 32 \Rightarrow a = 8$$

and $$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$$

$$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$$

e308

$$\frac{64-9d^2}{64-d^2} = \frac{7}{15}$$

$$960-135d^2 = 448-7d^2$$

$$7d^2-135d^2 = 448-960$$

$$-128d^2 = -512$$

$$d^2 = 4 \Rightarrow d = \pm 2$$

Hence, the number are 2, 6, 10 and 14 or 14, 10, 6 and 2.

139. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Ans : [Board 2020 Delhi Standard]

We have $S_7 = 63$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

e309

$$63 = \frac{7}{2}[2a+6d]$$

$$9 = a+3d \qquad ...(1)$$

Now, sum of next 7 terms,

$$S_{8-14} = 161$$

$$S_{8-14} = \frac{7}{2}(a_8+a_{14})$$

$$161 = \frac{7}{2}(a+7d+a+13d)$$

$$161 = \frac{7}{2}(2a+20d)$$

$$23 = a+10d \qquad ...(2)$$

Subtracting equation (1) from (2) we have

$$14 = 7d \Rightarrow d = 2$$

Substituting the value of d in (1), we get

$$a = 3$$

Hence, the AP is 3, 5, 7, 9, ...

140. Which term of the AP 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term.

Ans : [Board 2020 OD Standard]

Here, $a = 20$

and $$d = \frac{77}{4}-20 = -\frac{3}{4}$$

e313

Let a_n is the first negative term, thus $a_n < 0$.

Now $$a_n = a+(n-1)d$$

$$20+(n-1)\left(-\frac{3}{4}\right) < 0$$

$$80-3n+3 < 0$$

$$83-3n < 0$$

$$n > \frac{83}{3} \quad n > 27.6$$

$$n = 28$$

Hence, the first negative term is 28th term.

141. Find the middle term of the AP 7, 13, 19, ..., 247.

Ans : [Board 2020 OD Standard]

In this AP, $a = 7$

$$d = 13-7 = 6$$

e314

$$a_n = a+(n-1)d$$

$$247 = 7+(n-1)6$$

$$6(n-1) = 240$$

$$n-1 = 40 \Rightarrow n = 41$$

Hence, the middle term $= \frac{n+1}{2} = \frac{41+1}{2} = \frac{42}{2} = 21.$

$$a_{21} = 7+(21-1)6 = 127$$

142. Show that the sum of all terms of an AP whose first term is a, the second term is b and last term is c, is equal to $\dfrac{(a+c)(b+c-2a)}{2(b-a)}$

Ans : [Board 2020 OD Standard]

Given, first term, $A = a$

and second term $A_2 = b$

Common difference, $D = b-a$

Last term, $A_n = c$

$$A+(n-1)d = c$$

$$a+(n-1)(b-a) = c$$

$$(b-a)(n-1) = c-a$$

$$n-1 = \frac{c-a}{b-a}$$

$$n = \frac{c-a}{b-a}+1$$

$$= \frac{c-a+b-a}{b-a}$$

$$n = \frac{b+c-2a}{b-a}$$

Now sum of all terms

$$S_n = \frac{n}{2}[A+A_n] = \frac{(b+c-2a)}{2(b-a)}[a+c]$$

$$= \frac{(a+c)(b+c-2a)}{2(b-a)}$$ Hence Proved

143. If in an AP, the sum of first m terms is n and the sum of its first n terms is m, then prove that the sum of its first $(m+n)$ terms is $-(m+n)$.

Ans : [Board 2020 OD Standard]

Let 1[st] term of series be a and common difference be d, then we have

$$S_m = n$$

and $S_n = m$

$$\frac{m}{2}[2a+(m-1)d] = n \quad ...(1)$$

$$\frac{n}{2}[2a+(n-1)d] = m \quad ...(2)$$

Subtracting we have

$$a(m-n)+\frac{d}{2}[m(m-1)-n(n-1)] = n-m$$

$$2a(m-n)+d[m^2-n^2-(m-n)] = 2(n-m)$$

$$2a(m-n)+d(m-n)[(m+n)-1] = 2(n-m)$$

$$2a+d[(m+n)-1] = -2$$

Now, $$S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$= \frac{m+n}{2}(-2)$$

$$= -(m+n)$$

144. The 17[th] term of an AP is 5 more than twice its 8[th] term. If 11[th] term of AP is 43, then find its n^{th} term.

Ans : [Board 2020 OD Basic]

Let a be the first term and d be the common difference.

n^{th} term of an AP,

$$a_n = a+(n-1)d$$

Since 17[th] term of an AP is 5 more than twice of its 8[th] term, thus

$$a+(17-1)d = 5+2[a+(8-1)d]$$

$$a+16d = 5+2(a+7d)$$

$$a+16d = 5+2a+14d$$

$$2d-a = 5 \quad ...(1)$$

Since 11[th] term of AP is 43,

$$a+(11-1)d = 43$$

$$a+10d = 43 \quad ...(2)$$

Solving equation (1) and (2), we have

$$a = 3 \text{ and } d = 4$$

Hence, n^{th} term would be

$$a_n = 3+(n-1)4 = 4n-1$$

145. How many terms of the AP 24, 21, 18, ... must be taken so that their sum is 78?

Ans : [Board 2020 Delhi Basic]

Given : 24, 21, 18, ... are in AP.

Here, $a = 24,\ d = 21-24 = -3$

Sum of n term, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$78 = \frac{n}{2}[2\times 24+(n-1)(-3)]$$

$$156 = n(48-3n+3)$$

$$156 = n(51-3n)$$
$$3n^2 - 51n + 156 = 0$$
$$n^2 - 17n + 52 = 0$$
$$n^2 - 13n - 4n + 52 = 0$$
$$(n-4)(n-13) = 0 \Rightarrow n = 4, 13$$

When $n = 4$, $S_4 = \frac{4}{2}[2 \times 24 + (4-1)(-3)]$

$$= 2(48-9) = 2 \times 39 = 78$$

When $n = 13$, $S_{13} = \frac{13}{2}[2 \times 24 + (13-1)(-3)]$

$$= \frac{13}{2}[48 + (-36)] = 78$$

Hence, the number of terms $n = 4$ or $n = 13$.

146. Find the 20^{th} term of an AP whose 3^{rd} term is 7 and the seventh term exceeds three times the 3^{rd} term by 2. Also find its n^{th} term (a_n).

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_3 = a + 2d = 7$ (1)

$$a_7 = 3a_3 + 2$$
$$a + 6d = 3 \times 7 + 2 = 23 \quad (2)$$

Solving (1) and (2) we have

$$4d = 16 \Rightarrow d = 4$$
$$a + 8 = 7 \Rightarrow a = -1$$
$$a_{20} = a + 19d = -1 + 19 \times 4 = 75$$
$$a_n = a + (n-1)d$$
$$= -1 + 4n - 4$$
$$= 4n - 5.$$

e140

Hence n^{th} term is $4n - 5$.

147. If 7^{th} term of an AP is $\frac{1}{9}$ and 9^{th} term is $\frac{1}{7}$, find 63^{rd} term.

Ans : [Board Term-2 Delhi 2014]

Let the first term be a, common difference be d and nth term be a_n.

We have $a_7 = \frac{1}{9} \Rightarrow a + 6d = \frac{1}{9}$ (1)

$$a_9 = \frac{1}{7} \Rightarrow a + 8d = \frac{1}{7} \quad (2)$$

Subtracting equation (1) from (2) we get

$$2d = \frac{1}{7} - \frac{1}{9} = \frac{2}{63} \Rightarrow d = \frac{1}{63}$$

e141

Substituting the value of d in (2) we get

$$a + 8 \times \frac{1}{63} = \frac{1}{7}$$
$$a = \frac{1}{7} - \frac{8}{63} = \frac{9-8}{63} = \frac{1}{63}$$

Thus $a_{63} = a + (63-1)d$

$$= \frac{1}{63} + 62 \times \frac{1}{63} = \frac{1+62}{63}$$
$$= \frac{63}{63} = 1$$

Hence, $a_{63} = 1$.

148. The ninth term of an AP is equal to seven times the second term and twelfth term exceeds five times the third term by 2. Find the first term and the common difference.

Ans : [Board SQP 2016]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_9 = 7a_2$

e142

$$a + 8d = 7(a + d)$$
$$a + 8d = 7a + 7d$$
$$-6a + d = 0 \quad (1)$$

and $a_{12} = 5a_3 + 2$

$$a + 11d = 5(a + 2d) + 2$$
$$a + 11d = 5a + 10d + 2$$
$$-4a + d = 2 \quad ...(2)$$

Subtracting (2) from (1), we get

$$-2a = -2$$
$$a = 1$$

Substituting this value of a in equation (1) we get

$$-6 + d = 0$$
$$d = 6$$

Hence first term is 1 and common difference is 6.

149. Determine an AP whose third term is 9 and when fifth term is subtracted from 8^{th} term, we get 6.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d and

nth term be a_n.

We have $a_3 = 9$

$$a + 2d = 9 \quad ...(1)$$

and $a_8 - a_5 = 6$

$$(a+7d) - (a+4d) = 6$$

$$3d = 6$$

$$d = 2$$

e143

Substituting this value of d in (1), we get

$$a + 2(2) = 9$$

$$a = 5$$

So, AP is 5, 7, 9, 11, ...

150. Divide 56 in four parts in AP such that the ratio of the product of their extremes (1^{st} and 4^{rd}) to the product of means (2^{nd} and 3^{rd}) is 5:6.

Ans : [Board Term-2 Foreign 2016]

Let the four numbers be $a-3d, a-d, a+d, a+3d$

Now $a - 3d + a - d + a + d + a + 3d = 56$

$$4a = 56 \Rightarrow a = 14$$

Hence numbers are $14 - 3d, 14 - d, 14 + d, 14 + 3d$

Now, according to question, we have

$$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$$

e144

$$\frac{196 - 9d^2}{196 - d^2} = \frac{5}{6}$$

$$6(196 - 9d^2) = 5(196 - d^2)$$

$$6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$$

$$(6-5) \times 196 = 49d^2$$

$$d^2 = \frac{196}{49} = 4$$

$$d = \pm 2$$

Thus numbers are $a - 3d = 14 - 3 \times 2 = 8$

$$a - d = 14 - 2 = 12$$

$$a + d = 14 + 2 = 16$$

$$a + 3d = 14 + 3 \times 2 = 20$$

Thus required AP is $8, 12, 16, 20$.

151. [illegible] are a, b and c respectively, Show that $a(q-r) + b(r-p) + c(p-q) = 0$.

Ans : [Board Term-2 Foreign 2016]

Let the first term be A and the common difference be D.

$$a = A + (p-1)D$$

$$b = A + (q-1)D$$

$$c = A + (r-1)D$$

e145

Now $a(q-r) = [A + (p-1)D][q-r]$

$$b(r-p) = [A + (q-1)D][r-p]$$

and $c[p-q] = [A + (r-1)D][p-q]$

$$a(q-r) + b(r-p) + c(p-q)$$

$$= [A + (p-1)D][q-r] + \\ + [A + (q-1)D][r-p] + \\ + [A + (r-1)D][p-q] +$$

$$= A[p - q + q - p + q - r] + \\ + D(p-1)(q-r) + \\ + D(q-1)(r-p) + \\ + D(r-1)(p-q)$$

$$= A[0] + \\ + D[p(q-r) - (q-r)] \\ + D[q(r-p) - (r-p)] \\ + D[r(p-q) - (p-q)]$$

$$= D[p(q-r) + q(r-p) + r(p-q)] + \\ - D[(q-r) + (r-p) + (p-q)]$$

$$= D[pq - pr + qr - qp + rp - rq] + 0$$

$$= D[0] = 0$$

152. The sum of n terms of an AP is $3n^2 + 5n$. Find the AP Hence find its 15^{th} term.

Ans : [Board Term-2 2013, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e146

Now $S_n = 3n^2 + 5n$

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$= 3(n^2 + 1 - 2n) + 5n - 5$$

$$= 3n^2 + 3 - 6n + 5n - 5$$

$$= 3n^2 - n - 2$$

$$a_n = S_n - S_{n-1}$$

$$= 3n^2 + 5n - (3n^2 - n - 2)$$

$$= 6n + 2$$

Thus AP is 8, 14, 20, ...

Now $a_{15} = a + 14d = 8 + 14(6) = 92$

153. For what value of n, are the n^{th} terms of two APs 63, 65, 67, ... and 3, 10, 17, ... equal?

Ans :

Let a, d and A, D be the 1^{st} term and common difference of the 2 APs respectively.

n is same

For 1st AP, $a = 63,\ d = 2$

For 2nd AP, $A = 3, D = 7$

e148

Since nth term is same,

$$a_n = A_n$$

$$a + (n-1)d = A + (n-1)D$$

$$63 + (n-1)2 = 3 + (n-1)7$$

$$63 + 2n - 2 = 3 + 7n - 7$$

$$61 + 2n = 7n - 4$$

$$65 = 5n \Rightarrow n = 13$$

When n is 13, the n^{th} terms are equal i.e., $a_{13} = A_{13}$

154. In an AP the sum of first n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find the 25^{th} term.

Ans : [Board Term-2 SQP 2015]

We have $S_n = \dfrac{3n^2 + 13n}{2}$

e183

$$a_n = S_n - S_{n-1}$$

$$a_{25} = S_{25} - S_{24}$$

$$= \frac{3(25)^2 + 13(25)}{2} - \frac{3(24)^2 + 13(24)}{2}$$

$$= \frac{1}{2}\{3(25^2 - 24^2) + 13(25 - 24)\}$$

$$= \frac{1}{2}(3 \times 49 + 13) = 80$$

155. The sum of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each AP is 1 and common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$.

Ans : [Board Term-2 OD 2016]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $S_1 = 1 + 2 + 3 + \ldots.. n$

$S_2 = 1 + 3 + 5 + \ldots..$ up to n terms

$S_3 = 1 + 4 + 7 + \ldots..$ upto n terms

Now $S_n = \dfrac{n(n+1)}{2}$

e184

$$S_2 = \frac{n}{2}[2 + (n-1)2] = \frac{n}{2}[2n] = n^2$$

and $S_3 = \dfrac{n}{2}[2 + (n-1)3] = \dfrac{n(3n-1)}{2}$

Now, $S_1 + S_3 = \dfrac{n(n+1)}{2} + \dfrac{n(3n-1)}{2}$

$$= \frac{n[n + 1 + 3n - 1]}{2} = \frac{n[4n]}{2}$$

$= 2n^2 = 2s_2$ Hence Proved

156. If S_n denotes, the sum of the first n terms of an AP prove that $S_{12} = 3(S_8 - S_4)$.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e185

$$S_{12} = 6[2a + 11d] = 12a + 66d$$

$$S_8 = 4[2a + 7d] = 8a + 28d$$

$$S_4 = 2[2a + 3d] = 4a + 6d$$

$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$

$$= 3[4a + 22d] = 12a + 66d$$

$= 6[2a + 11d] = S_{12}$ Hence Proved

157. The 14^{th} term of an AP is twice its 8^{th} term. If the 6^{th} term is -8, then find the sum of its first 20 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here, $a_{14} = 2a_8$ and $a_6 = -8$

Now $a + 13d = 2(a + 7d)$

e186

$$a + 13d = 2a + 14d$$

$$a = -d \quad ...(1)$$

and $$a_6 = -8$$

$$a + 5d = -8 \quad ...(2)$$

Solving (1) and (2), we get

$$a = 2, d = -2$$

Now $$S_{20} = \frac{20}{2}[2 \times 2 + (20-1)(-2)]$$

$$= 10[4 + 19 \times (-2)]$$

$$= 10(4 - 38)$$

$$= 10 \times (-34) = -340$$

158. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

Ans : [Board Term-2 OD 2016]

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

e187

$$\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} = \frac{7n+1}{4n+27}$$

$$\frac{2a + (n-1)d}{2A + (n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a + \left(\frac{n-1}{2}\right)d}{A + \left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$$

Substituting $\frac{n-1}{2} = m - 1$ or $n = 2m - 1$ we get

$$\frac{a + (m-1)d}{A + (m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $$\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$$

159. If the sum of the first n terms of an AP is $\frac{1}{2}[3n^2 + 7n]$, then find its n^{th} term. Hence write its 20^{th} term.

Ans : [Board Term-2 Delhi 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Sum of n term, $$S_n = \frac{1}{2}[3n^2 + 7n]$$

e188

Sum of 1 term, $$S_1 = \frac{1}{2}[3 \times (1)^2 + 7(1)]$$

$$= \frac{1}{2}[3 + 7] = \frac{1}{2} \times 10 = 5$$

Sum of 2 term, $$S_2 = \frac{1}{2}[3(2)^2 + 7 \times 2]$$

$$= \frac{1}{2}[12 + 14] = \frac{1}{2} \times 26 = 13$$

Now $$a_1 = S_1 = 5$$

$$a_2 = S_2 - S_1 = 13 - 5 = 8$$

$$d = a_2 - a_1 = 8 - 5 = 3$$

Now, AP is $5, 8, 11, ...$

n^{th} term, $$a_n = a + (n-1)d$$

$$= 5 + (n-1)3$$

$$= 5 + (20-1)(3)$$

$$= 5 + 57$$

$$= 62$$

Hence, $$a_2 = 62$$

160. In an AP, if the 12^{th} term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_{12} = a + 11d = -13 \quad ...(1)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $$S_4 = 2[2a + 3d] = 24$$

$$2a + 3d = 12 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it we get

$$(2a + 22d) - (2a + 3d) = -26 - 12$$

$$19d = -38$$

$$d = -2$$

e189

Substituting the value of d in (1) we get

$$a + 11 \times -2 = -13$$

$$a = -13 + 22$$
$$a = 9$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$$
$$= 5 \times (18 - 18) = 0$$

Hence, $S_{10} = 0$

161. The tenth term of an AP, is -37 and the sum of its first six terms is -27. Find the sum of its first eight terms.

Ans : [Board Term-2 Foreign 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$a_n = a + (n-1)d$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e190

$$a + 9d = -37 \quad ...(1)$$
$$3(2a + 5d) = -27$$
$$2a + 5d = -9 \quad ...(2)$$

Multiplying (1) by 2 and subtracting (2) from it, we get

$$(2a + 18d) - (2a + 5d) = -74 + 9$$
$$13d = -65$$
$$d = -5$$

Substituting the value of d in (1) we get

$$a + 9 \times -5 = -37$$
$$a = -37 + 45$$
$$a = 8$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{8}{2}[2 \times 8 + (8-1)(-5)]$$
$$= 4[16 - 35]$$
$$= 4 \times -19 = -76$$

Hence, $S_n = -76$

162. Find the sum of first seventeen terms of AP whose 4^{th} and 9^{th} terms are -15 and -30 respectively.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d and nth term be a_n.

Now $a_4 = a + 3d = -15 \quad ...(1)$

$$a_9 = a + 8d = -30 \quad ...(2)$$

Subtracting eqn (1) from eqn (2), we obtain

$$(a + 8d) - (a + 3d) = -30 - (-15)$$
$$5d = -15 \Rightarrow d = \frac{-15}{5} = -3$$

Substituting the value of d in (1) we get

$$a + 3d = -15$$
$$a + 3(-3) = -15$$
$$a = -15 + 9 = -6$$

e191

Now $S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$$= \frac{17}{2}[-\,-12 + 16 \times (-3)]$$
$$= \frac{17}{2}[-12 - 48]$$
$$= \frac{17}{2}[-60] = 17 \times (-30)$$
$$= -510$$

Thus $S_{17} = -510$

163. The common difference of an AP is -2. Find its sum, if first term is 100 and last term is -10.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 100, d = -2, t_n = -10$

Now $a_n = a + (n-1)d$

$$-10 = 100 + (n-1)(-2)$$
$$-10 = 100 - 2n + 2$$
$$2n = 112$$
$$n = 56$$

e192

Thus 56^{th} term is -10 and number of terms in AP are 56.

Now $S_n = \frac{n}{2}(a + 1)$

$$S_{56} = \frac{56}{2}(100 - 10)$$

$$= \frac{56}{2}(90) = 56 \times 45 = 2520$$

Thus $S_n = 2520$

164. The 16^{th} term of an AP is five times its third term. If its 10^{th} term is 41, then find the sum of its first fifteen terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d. nth term be a_n and sum of n term be S_n.

e193

We have, $a_{16} = 5a_3$

$$a + 15d = 5(a + 2d)$$

$$4a = 5d \qquad ...(1)$$

and $a_{10} = 41$

$$a + 9d = 41 \qquad ...(2)$$

Solving (1) and (2), we get $a = 5, d = 4$

Now $S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 4]$

$$= \frac{15}{2}[10 + 56]$$

$$= \frac{15}{2} \times 66 = 15 \times 33 = 495$$

Thus $S_{15} = 495$

165. The 13^{th} term of an AP is four times its 3^{rd} term. If the fifth term is 16, then find the sum of its first ten terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Here $a_{13} = 4a_3$

$$a + 12d = 4(a + 2d)$$

$$3a = 4d \qquad ...(1)$$

and $a_5 = 16$

$$a + 4d = 16 \qquad ...(2)$$

Substituting the value of $a = \frac{4}{3}d$ in (2) we have

$$\frac{4}{3}d + 4d = 16$$

$$16d = 48 \Rightarrow d = 3$$

e194

Thus $a = 4$ and $d = 3$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3]$$

$$= 5[8 + 27] = 5 \times 35 = 175$$

Thus $S_{10} = 175$

166. The n^{th} term of an AP is given by $(-4n + 15)$. Find the sum of first 20 terms of this AP.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e195

We have $a_n = -4n + 15$

$$a_1 = -4 \times 1 + 15 = 11$$

$$a_2 = -4 \times 2 + 15 = 7$$

$$a_3 = -4 \times 3 + 15 = 3$$

$$d = a_2 - a_1 = 7 - 11 = -4$$

Now, we have $a = 11$, $d = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2 \times 11 + (20-1) \times (-4)]$$

$$= 10[22 - 76]$$

$$= 10 \times (-54) = -540$$

Thus $S_{20} = -540$

167. The sum of first 7 terms of an AP is 63 and sum of its next 7 terms is 161. Find 28^{th} term of AP

Ans : [Board Term-2 Foreign 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

e196

Now, $S_7 = 63$

$$\frac{7}{2}[2a + 6d] = 63$$

$$2a + 6d = 18 \qquad ...(1)$$

Also, sum of next 7 terms,

$$S_{14} = S_{first7} + S_{next7} = 63 + 161$$

$$\frac{14}{2}[2a + 13d] = 224$$

$$2a + 13d = 32 \qquad (2)$$

Subtracting equation (1) form (2) we get

$$7d = 14 \Rightarrow d = 2$$

Substituting the value of d in (1) we get

$$a = 3$$

Now $$a_n = a + (n-1)d$$

$$a_{28} = 3 + 2 \times (27)$$

$$= 57$$

Thus 28^{th} term is 57.

168. The sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$. Determine the AP and the 12^{th} term.

Ans : [Board Term-2 Delhi 2014, 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

$$S_n = 3n^2 - 4n$$

$$S_1 = 3(1)^2 - 4(1) = -1$$

$$S_2 = 3(2)^2 - 4(2) = 4$$

$$a_1 = S_1 = -1$$

$$a_2 = S_2 - S_1 = 4 - (-1) = 5$$

$$d = a_2 - a_1 = 5 - (-1) = 6$$

e197

Thus AP is $-1, 5, 11, ...$

Now $$a_{12} = a + 11d$$

$$= -1 + 11 \times 6 = 65$$

169. Find the sum of all two digit natural numbers which are divisible by 4.

Ans : [Board Term-2 Delhi Compt. 2017]

First two digit multiple of 4 is 12 and last is 96

So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

Now $$a + (n-1)d = a_n$$

$$12 + (n-1)4 = 96$$

$$(n-1)4 = 96 - 12 = 84$$

$$n - 1 = 21$$

$$n = 21 + 1 = 22$$

e198

Now, $$S_{22} = \frac{22}{2}[12 + 96]$$

$$= 11 \times 108$$

$$= 1188$$

170. Find the sum of the following series.

$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 + ... + (-5) + 81 + (-3)$

Ans : [Board Term-2 Foreign 2017]

The given series can be written as sum of two series

$$(5 + 9 + 13 + ... + 81) +$$
$$+ (-41) + (-39) + (-37) + (-35) ... (-5) + (-3)$$

For the series $(5 + 9 + 13 .....81)$

$$a = 5,\ d = 4 \text{ and } a_n = 81$$

Now $$a_n = a + (n-1)d$$

$$81 = 5 + (n-1)4$$

$$81 = 5 + (n-1)4$$

$$(n-1)4 = 76 \Rightarrow n = 20$$

$$S_n = \frac{20}{2}(5 + 81) = 860$$

e199

For series $(-41) + (-39) + (-37) + ... + (-5) + (-3)$

$$a_n = -3,\ a = -41 \text{ and } d = 2$$

$$a_n = -41 + (n-1)(2)$$

$$-3 = -41 + 2n - 2 \Rightarrow n = 20$$

Now $$S_n = \frac{20}{2}[-41 + -3] = -440$$

Sum of the series $= 860 - 440 = 420$

171. Find the sum of n terms of the series

$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ...$

Ans : [Board Term-2 Delhi 2017]

Let sum of n term be S_n

$$s_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + ... \text{ up to } n \text{term}$$

$$= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) +$$
$$+ \left(-\frac{1}{n} - \frac{2}{n} - \frac{3}{n} - ...... \text{ up to } n \text{ terms}\right)$$

$$= (4 + 4 + 4 + ... \text{ up to } n \text{ terms}) +$$
$$-\frac{1}{n}(1 + 2 + 3 + ... \text{ up to } n \text{ terms})$$

$$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= 4n - \frac{n+1}{2} = \frac{7n-1}{2}$$

e200

Hence, sum of n terms $= \frac{7n-1}{2}$

172. Find the number of multiple of 9 lying between 300 and 700.

Ans : [Board Term-2 OD Compt. 2017]

The numbers, multiple of 9 between 300 and 700 are 306, 315, 324, ... 693.

Let the first term be a, common difference be d and nth term be $a_n = 693$

$$a_n = 306 + (n-1)9$$

$$693 = 306 + (n-1)9$$

$$(n-1)9 = 693 - 306 = 387$$

$$n - 1 = \frac{387}{9} = 43$$

$$n = 43 + 1 = 44$$

e201

Hence there are 44 terms.

173. If the sum of the first 14 terms of an AP is 1050 and its first term is 10 find it 20^{th} term.

Ans : [Board Term-2 OD Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 10$, and $S_{14} = 1050$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$$

$$1050 = 7[20 + 13d]$$

$$20 + 13d = \frac{1050}{7} = 150$$

$$13d = 130 \Rightarrow d = 10$$

$$a_{20} = a + (n-1)d$$

$$= 10 + 19 \times 10 = 200$$

e202

Hence $a_{20} = 200$

174. If the tenth term of an AP is 52 and the 17^{th} term is 20 more than the 13^{th} term, find AP

Ans : [Board Term-2 OD 2017]

Let the first term be a, common difference be d and nth term be a_n.

Now $\quad a_{10} = 52$

$$a + 9d = 52 \qquad ...(1)$$

Also $\quad a_{17} - a_{13} = 20$

$$a + 16d - (a + 12d) = 20$$

$$4d = 20$$

$$d = 5$$

e203

Substituting this valued d in (1), we get

$$a = 7$$

Hence AP is 7, 12, 17, 22, ...

175. Find the sum of all odd number between 0 and 50.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Given AP is $1 + 3 + 5 + 7 + ... + 49$

Let total number of terms be n. Here $a = 1$, $d = 2$ and $a_n = 49$.

$$a_n = 1 + (n-1) \times 2$$

$$49 = 1 + 2n - 2$$

$$50 = 2n \Rightarrow n = 25$$

e204

Now $\quad S_{25} = \frac{n}{2}(a + a_n)$

$$= \frac{25}{2}(1 + 49)$$

$$= 25 \times 25 = 625$$

Hence, Sum of odd number is 625

176. Find the sum of first 15 multiples of 8.

Ans : [Board Term-2 Delhi Compt 2017]

Let the first term be $a = 8$, common difference be $d = 8$, nth term be a_n and sum of n term be S_n.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2 \times 8 + (15-1)8]$$

$$= \frac{15}{2}[16 + 112]$$

$$= \frac{15}{2} \times 128 = 996$$

e205

Hence, the sum of 15 terms is 960.

177. If m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ find the sum of first mn terms.

Ans : [Board Term-2 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

Now $$a_m = a+(m-1)d = \frac{1}{n} \quad ...(1)$$

$$a_n = a+(n-1)d = \frac{1}{m} \quad ...(2)$$

Subtracting (2) from (1) we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$$d = \frac{1}{mn}$$

Substituting this value of d in equation (1), we get

$$a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}\left(\frac{2}{mn} + (mn-1)\frac{1}{mn}\right)$$

$$= 1 + \frac{mn}{2} - \frac{1}{2} = \frac{1}{2} + \frac{mn}{2}$$

$$= \frac{1}{2}[mn+1]$$

Hence, the sum of mn term is $\frac{1}{2}[mn+1]$.

178. How many terms of an AP $9, 17, 25, ...$ must be taken to give a sum of 636?

Ans : [Board Term-2 Delhi Compt 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 9, d = 8, S_n = 636$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$636 = \frac{n}{2}[18+(n-1)8]$$

$$636 = n[9+(n-1)4]$$

$$636 = n(9+4n-4)$$

$$636 = n(5+4n)$$

$$636 = 5n+4n^2$$

$$4n^2+5n-636 = 0$$

$$4n^2-48n+53n-636 = 0$$

$$4n(n-12)+53(n-12) = 0$$

$$(4n+53)(n-12) = 0$$

Thus $$n = \frac{-53}{4} \text{ or } 12$$

As n is a natural number $n = 12$. Hence 12 terms are required to give sum 636.

179. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Ans : [Board Term-2 OD 2014]

The sequence goes like 110, 120, 130, ... 990

Since they have a common difference of 10, they form an AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 110,\ a_n = 990,\ d = 10$

$$a_n = a+(n-1)d$$

$$990 = 110+(n-1)\times 10$$

$$990-110 = 10(n-1)$$

$$880 = 10(n-1)$$

$$88 = n-1$$

$$n = 88+1 = 89$$

Hence, there are 89 terms between 101 and 999 divisible by both 2 and 5.

180. How many three digit natural numbers are divisible by 7?

Ans : [Board Term-2 2013]

Let AP is 105, 112, 119, ..., 994 which is divisible by 7.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 105, d = 112-105 = 7, a_n = 994$ then

$$a_n = a+(n-1)d$$

$$994 = 105+(n-1)\times 7$$

$$889 = (n-1)\times 7$$

$$n-1 = \frac{889}{7} = 127$$

$$n = 127+1 = 128$$

Hence, there are 128 terms divisible by 7 in AP.

181. How many two digit numbers are divisible by 7?

Ans : [Board Term-2 SQP 2016]

Two digit numbers which are divisible by 7 are 14, 21, 28, ... 98. It forms an AP

Let the first term be a, common difference b[illegible] th term be a_n.

Here $a = 14,\ d = 7, a_n = 98$

Now $a_n = a+(n-1)d$

$$98 = 14+(n-1)7$$

$$98-14 = 7n-7$$

$$84+7 = 7n$$

$$7n = 91 \Rightarrow n = 13$$

182. If the ratio of the 11^{th} term of an AP to its 18^{th} term is 2 : 3, find the ratio of the sum of the first five term of the sum of its first 10 terms.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e242

Now $$\frac{a_{11}}{a_{18}} = \frac{a+10d}{a+17d} = \frac{2}{3}$$

$$2(a+17d) = 3(a+10d)$$

$$a = 4d \quad ...(1)$$

Now, $$\frac{S_5}{S_{10}} = \frac{\frac{5}{2}(2a+4d)}{\frac{10}{2}[2a+9d]} = \frac{(a+2d)}{[2a+9d]}$$

Substituting the value $a = 4d$ we have

or, $$\frac{S_5}{S_{10}} = \frac{4d+2d}{8d+9d} = \frac{6}{17}$$

Hence $S_5 : S_{10} = 6:17$

183. How many three digit numbers are such that when divided by 7, leave a remainder 3 in each case?

Ans : [Board Term-2 2012]

When a three digit number divided by 7 and leave 3 as remainder are 101, 108, 115, ... 997

These are in AP. Let the first term be a, common difference be d, nth term be a_n.

Here $a = 101, d = 7, a_n = 997$

e230

Now $a_n = a+(n-1)d$

$$997 = 101+(n-1)7$$

$$997-101 = 896 = (n-1)7$$

$$\frac{896}{7} = n-1$$

$$n = 128+1 = 129$$

Hence, 129 numbers are divided by 7 which leaves remainder is 3.

184. How many multiples of 4 lie between 11 and 266?

Ans : [Board Term-2 2012]

First multiple of 4 is 12 and last multiple of 4 is 264. It forms a AP. Let multiples of 4 be n.

Let the first term be a, common difference be d, n th term be a_n.

Here, $a = 12, a_n = 264, d = 4$

e231

$$a_n = a+(n-1)d$$

$$264 = 12+(n-1)4$$

$$n = \frac{264-12}{4}+1$$

Hence, there are 64 multiples of 4 that lie between 11 and 266.

185. Prove that the n^{th} term of an AP can not be n^2+1. Justify your answer.

Ans : [Board Term-2 2015]

Let n^{th} term of AP,

$$a_n = n^2+1$$

Substituting the value of $n = 1, 2, 3, ...$ we get

$$a_1 = 1^2+1 = 2$$

$$a_2 = 2^2+1 = 5$$

$$a_3 = 3^2+1 = 10$$

e232

The obtained sequence is 2, 5, 10, 17,......

Its common difference

$$a_2-a_1 = a_3-a_2 = a_4-a_3$$

$$5-2 \neq 10-5 \neq 17-10$$

$$3 \neq 5 \neq 7$$

Since the sequence has no. common difference, n^2+1 is not a form of n^{th} term of an AP

186. If the p^{th} term of an AP is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$. Prove that the sum of first pq term of the AP is $\left[\frac{pq+1}{2}\right]$.

Ans : [Board Term-2 Delhi 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e241

$$a_p = a+(p-1)d = \frac{1}{q} \quad ...(1)$$

and $$a_q = a+(q-1)d = \frac{1}{p} \quad ...(2)$$

Solving (1) and (2) we get

$$a = \frac{1}{pq} \text{ and } d = \frac{1}{p}$$

$$S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1)\frac{1}{pq}\right] = \frac{pq+1}{2}$$

187. Find the sum of all two digits odd positive numbers.

Ans : [Board Term-2 2014]

The list of 2 digits odd positive numbers are 11, 13 ...... 99. It forms an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 11, d = 2, l = 99$

Now
$$a_n = a + (n-1)d$$
$$99 = 11 + (n-1)2$$
$$88 = (n-1)2$$
$$n = 44 + 1 = 45$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{45}{2}[11 + 99]$$
$$S_n = \frac{45 \times 110}{2} = 2475$$

e233

Hence the sum of given AP is $S_n = 2475$

188. Find the sum of the two digits numbers divisible by 6.

Ans : [Board Term-2 2013]

Series of two digits numbers divisible by 6 is 12, 18, 24, ...96. It forms an AP. Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 12, d = 18 - 12 = 6, a_n = 96$

$$a_n = a + (n-1)d$$
$$96 = 12 + (n-1) \times 6$$
$$84 = 6(n-1)$$
$$n = 14 + 1 = 15$$
$$S_n = \frac{n}{2}[a + a_n]$$
$$= \frac{15}{2}[12 + 96]$$
$$= \frac{15 \times 108}{2}$$
$$= 15 \times 54 = 810$$

e234

Hence the sum of given AP is 810.

189. Find the sum of the integers between 100 and 200 that are divisible by 6.

Ans : [Board Term-2 2012]

The series as per question is 102, 108, 114, ... 198. which is an AP.

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here $a = 102, d = 6$ and $l = 198$

Now
$$198 = 102 + (n-1)6$$
$$96 = (n-1)6$$
$$\frac{96}{6} = n - 1$$
$$n = 17$$

e235

Now
$$S_{17} = \frac{n}{2}(a + a_n)$$
$$= \frac{17}{2}[102 + 198]$$
$$= \frac{17}{2} \times 300 = 17 \times 150 = 2550$$

Hence the sum of given AP is 2550.

FOUR MARKS QUESTIONS

190. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Ans : [Board 2019 Delhi]

Let a be the first term and d be the common difference.

Sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 40$ and $S_{14} = 280$

$$\frac{4}{2}[2a + (4-1)d] = 40$$
$$2[2a + 3d] = 40$$
$$2a + 3d = 20 \quad (1)$$

e318

and
$$\frac{14}{2}[2a + (14-1)d] = 280$$
$$7[2a + 13d] = 280$$
$$2a + 13d = 40 \quad (2)$$

Solving equations (1) and (2), we get

$$a = 7 \text{ and } d = 2$$

Now $$S_n = \frac{n}{2}[2\times 7+(n-1)2]$$

$$= \frac{n}{2}[14+2n-2]$$

$$= \frac{n}{2}(12+2n) = 6n+n^2$$

Hence, sum of n terms is $6n+n^2$.

191. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Ans : [Board 2019 Delhi]

First term, $a = 3$

Last term, $a_n = 83$

Sum of n terms, $S_n = 903$

Since, $$S_n = \frac{n}{2}(a+a_n)$$

e319

$$903 = \frac{n}{2}(3+83)$$

$$1806 = 86n$$

$$n = \frac{1806}{86} \Rightarrow n = 21$$

Now $$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$903 = \frac{21}{2}[2\times 3+(21-1)d]$$

$$1806 = 21(6+20d)$$

$$6+20d = 86$$

$$20d = 80 \Rightarrow d = 4$$

Hence, the common difference is 4.

192. Find the common difference of the Arithmetic Progression (AP) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \dots (a \neq 0)$

Ans : [Board 2019 OD]

Given AP is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \dots \dots (a\neq 0)$

Here, first term, $a_1 = \frac{1}{a}$

Second term, $a_2 = \frac{3-a}{3a}$

e320

Third term, $a_3 = \frac{3-2a}{3a}$

Common difference,

$$d = a_2 - a_1$$

$$= \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a}$$

$$= \frac{-a}{3a} = \frac{-1}{3}$$

Here, common difference d of given AP is $\frac{-1}{3}$.

193. Which term of the Arithmetic Progression $-7, -12, -17, -22, \dots$ will be -82? Is -100 any term of the AP ? Given reason for your answer.

Ans : [Board 2019 OD]

Given AP is $-7, -12, -17, -22, \dots$

Here,

First term, $a_1 = -7$

e321

Second term $a_2 = -12$

Third term, $a_3 = -17$

Common difference,

$$d = a_2 - a_1 = -12-(-7)$$

$$= -12+7 = -5$$

$$d = -5$$

Let a_n be the n^{th} term of AP and it will be -82.

Since, $$a_n = a_1 + (n-1)d$$

$$-82 = -7+(n-1)(-5)$$

$$-82 = -7-5(n-1)$$

$$82 = 5n+2$$

$$5n = 80 \Rightarrow n = 16$$

Hence, 16^{th} term of AP is -82. Since, these numbers are not factor of 5, hence -100 will not be a term in the given AP.

194. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

Ans : [Board 2019 OD]

Given AP is 45, 39, 33, ...

Here, $a = 45,\ d = 39 - 45 = -6$ and $S_n = 180$

Now
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$
$$360 = n(90 - 6n + 6)$$
$$360 = n(96 - 6n)$$
$$60 = n(16 - n)$$
$$n^2 - 16n + 60 = 0$$
$$n^2 - 6n - 10n + 60 = 0$$
$$n(n-6) - 10(n-6) = 0$$
$$(n-10)(n-6) = 0$$
$$n = 10 \text{ or } n = 6$$

e322

Hence, 10 terms or 6 terms can be taken to get the sum of AP as 180.

Now, sum of 6 terms,
$$S_6 = \frac{6}{2}[2 \times 45 + (6-1)(-6)]$$
$$= 3(90 - 30)$$
$$= 3 \times 60 = 180$$
Hence, verified.

and sum of 10 terms,
$$S_{10} = \frac{10}{2}[2 \times 45 + (10-1)(-6)]$$
$$= 5(90 - 54)$$
$$= 5 \times 36 = 180$$
Hence, verified.

Here we have two values of n because d is negative. There will be negative terms after some positive terms. Thus first 6 term will give sum 180 and after 10 term it will be again 180 because negative term cancel positive term.

Series will be : 45, 39, 33, 27, 21, 15, 9, 3, -3, -9...

Here it may be easily seen that sum of initial 6 terms is 180. Sum of next 4 terms is zero. Thus sum of 10 terms is also 180.

195. The sum of three numbers in AP is 12 and sum of their cubes is 288. Find the numbers.

Ans : [Board Term-2 Delhi 2016]

Let the three numbers in AP be $a-d,\ a,\ a+d$.
$$a - d + a + a + d = 12$$
$$3a = 12$$
$$a = 4$$

e149

Also, $(4-d)^3 + 4^3 + (4+d)^3 = 288$
$$64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$$
$$24d^2 + 192 = 288$$
$$d^2 = 4$$
$$d = \pm 2$$

The numbers are 2, 4, 6 or 6, 4,2

196. Find the value of a, b and c such that the numbers $a, 7, b, 23$ and c are in AP

Ans : [Board Term-2 2015]

Let the common difference be d.

Since $a, 7, b, 23$ and c are in AP, we have

e150

$$a + d = 7$$
..(1)
$$a + 3d = 23 \quad ...(2)$$

Form equation (1) and (2), we get
$$a = -1, d = 8$$
$$b = a + 2d = -1 + 2 \times 8 = -1 + 16 = 15$$
$$c = a + 4d = -1 + 4 \times 8 = -1 + 32 = 31$$

Thus $a = -1, b = 15, c = 31$

197. If S_n denotes the sum of first n terms of an AP, prove that, $S_{30} = 3(S_{20} - S_{10})$

Ans : [Board Term-2 Delhi 2015, Foreign 2014]

Let the first term be a, and common difference be d.

e210

Now
$$S_{30} = \frac{30}{2}(2a + 29d) \quad ...(1)$$
$$= 15(2a + 29d)$$
$$3(S_{20} - S_{10}) = 3[10(2a + 19d) - 5(2a + 9d)]$$
$$= 3[20a + 190d - 10a - 45d]$$
$$= 3[10a + 145d]$$
$$= 15[2a + 29d] \quad ...(2)$$

Hence $S_{30} = 3(S_{20} - S_{10})$

198. The sum of first 20 terms of an AP is 400 and sum of first 40 terms is 1600. Find the sum of its first 10 terms.

Ans : [Board Term-2 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We know $S_n = \frac{n}{2}[2a + (n-1)d]$

e211

Now $S_{20} = \frac{20}{2}(2a + 19d)$

$$400 = \frac{20}{2}(2a + 19d)$$

$$400 = 10[2a + 19d]$$

$$2a + 19d = 40 \quad (1)$$

Also, $S_{40} = \frac{40}{2}(2a + 39d)$

$$1600 = 20[2a + 39d]$$

$$2a + 39d = 80 \quad (2)$$

Solving equation (1) and (2), we get $a = 1$ and $d = 2$.

Now $S_{10} = \frac{10}{2}[2 \times 1 + (10-1)(2)]$

$$= 5[2 + 9 \times 2]$$

$$= 5[2 + 18]$$

$$= 5 \times 20 = 100$$

199. Find $\left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(10 - \frac{3}{n}\right) + ...$ upto n terms.

Ans : [Board Term-2 2015]

Let sum of n term be S_n, then we have

$$s_n = \left(4 - \frac{1}{n}\right) + \left(7 - \frac{2}{n}\right) + \left(40 - \frac{3}{n}\right) + ... \text{ upto } n \text{ terms.}$$

$$= (4 + 7 + 10 + ... + n \text{ terms}) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} ..... + 1\right)$$

$$= (4 + 7 + 10 + ... + n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + ...n)$$

$$= \frac{n}{2}[2 \times 4 + (n-1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n-1)(1)]$$

$$= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1]$$

$$= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1)$$

e212

$$= \frac{3n^2 + 5n - n - 1}{2} = \frac{3n^2 + 4n - 1}{2}$$

200. Find the 60^{th} term of the AP $8, 10, 12, ...$, if it has a total of 60 terms and hence find the sum of its last 10 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e213

We have $a = 8, d = 10 - 8 = 2$

$$a_n = a + (n-1)d$$

Now $a_{60} = 8 + (60-1)2 = 8 + 59 \times 2 = 126$

and $a_{51} = 8 + 50 \times 2 = 8 + 100 = 108$

Sum of last 10 terms,

$$S_{51-60} = \frac{n}{2}(a_{51} + a_{60})$$

$$= \frac{10}{2}(108 + 126)$$

$$= 5 \times 234 = 1170$$

Hence sum of last 10 terms is 1170.

201. An arithmetic progression $5, 12, 19, ...$ has 50 terms. Find its last term. Hence find the sum of its last 15 terms.

Ans : [Board Term-2 OD 2015]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 5$, $d = 12 - 5 = 7$ and $n = 50$

$$a_{50} = 5 + (50-1)7$$

$$= 5 + 49 \times 7 = 348$$

Also the first term of the AP of last 15 terms be a_{36}

$$a_{36} = 5 + 35 \times 7$$

$$= 5 + 245 = 250$$

e214

Now, sum of last 15 terms,

$$S_{36-50} = \frac{15}{2}[a_{36} + a_{50}]$$

$$= \frac{15}{2}[250 + 348]$$

$$= \frac{15}{2} \times 598 = 4485$$

Hence, sum of last 15 terms is 4485.

202. If the sum of first n term of an an AP is given by

$S_n = 3n^2 + 4n$. Determine the AP and the n^{th} term.

Ans : [Board Term-2 2014]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

e218

We have $\quad S_n = 3n^2 + 4n.$

$$a_1 = 3(1)^2 + 4(1) = 7$$

$$a_1 + a_2 = S_2 = 3(2)^2 + 4(2)$$

$$= 12 + 8 = 20$$

$$a_2 = S_2 - S_1 = 20 - 7 = 13$$

$$a + d = 13$$

or, $\quad 7 + d = 13$

Thus $\quad d = 13 - 7 = 6$

Hence AP is $7, 13, 19, ...$

Now, $\quad a_n = a + (n-1)d$

$$= 7 + (n-1)(6)$$

$$= 7 + 6n - 6$$

$$= 6n + 1$$

$$a_n = 6n + 1$$

203. The sum of the 3^{rd} and 7^{th} terms of an AP is 6 and their product is 8. Find the sum of first 20 terms of the AP.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

e219

We have $\quad a_3 + a_7 = 6$

$$a + 2d + a + 6d = 6$$

$$a + 4d = 3 \qquad (1)$$

and $\quad a_3 \times a_7 = 8$

$$(a + 2d)(a + 6d) = 8 \qquad (2)$$

Substituting the value $a = (3 - 4d)$ in (2) we get

$$(3 - 4d + 2d)(3 - 4d + 6d) = 8$$

$$(3 + 2d)(3 - 2d) = 8$$

$$9 - 4d^2 = 8$$

$$4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$$

CASE 1 : Substituting $d = \frac{1}{2}$ in equation (1), $a = 1$.

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}\left[2 + \frac{19}{2}\right] = 115$$

Thus $d = \frac{1}{2}, a = 1$ and $S_{20} = 115$

CASE 2 : Substituting $d = -\frac{1}{2}$ in equation (1) $a = 5$

$$S_{20} = \frac{20}{2}\left[2 \times 5 + 19 \times \left(-\frac{1}{2}\right)\right]$$

$$= 10\left[10 - \frac{19}{2}\right] = 15$$

Thus $d = -\frac{1}{2}, a = 5$ and $S_{20} = 15$

204. If the sum of first m terms of an AP is same as the sum of its first n terms $(m \neq n)$, show that the sum of its first $(m+n)$ terms is zero.

Ans : [Board Term-2 2012]

Let the first term be a, common difference be d, nth term be a_n, and sum of n term be S_n

Now $\quad S_m = S_n$

e223

$$\frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$$

$$2ma + m(m-1)d = 2na + n(n-1)d$$

$$2a(m-n) + [(m^2 - n^2) - m + n]d = 0$$

$$2a(m-n) + [(m-n)(m+n) - (m-n)]d = 0$$

$$(m-n)[2a + (m+n-1)d] = 0$$

$$2a + (m+n-1)d = 0 \qquad [m - n \neq 0]$$

$$S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2} \times 0 = 0$$

205. If $1 + 4 + 7 + 10 ..... + n = 287$, Find the value of n.

Ans : [Board 2020 Std, Board Term-2 Foreign 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n.

We have $a = 1, d = 3$ and $S_n = 287$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{n}{2}[2 \times 1 + (n-1)3] = 287$$

$$\frac{n}{2}[2 + (3n - 3)] = 287$$

e225

$$3n^2 - n = 574$$

$$3n^2 - n - 574 = 0$$

$$3n^2 - 42n + 41n - 574 = 0$$

$$3n(n-14) + 41(n-14) = 0$$

$$(n-14)(3n+41) = 0$$

Since negative value is not possible, $n = 14$

$$a_{14} = a + (n-1)d$$

$$= 1 + 13 \times 3 = 40$$

206. Find the sum of first 24 terms of an AP whose n^{th} term is given by $a_n = 3 + 2n$.

Ans : [Board Term-2 OD Comptt. 2017]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a_n = 3 + 2n$

$$a_1 = 3 + 2 \times 1 = 5$$

e226

$$a_2 = 3 + 2 \times 2 = 7$$

$$a_3 = 3 + 2 \times 3 = 9$$

Thus the series is 5, 7, 9, ... in which

$$a = 5 \text{ and } d = 2$$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{24} = \frac{24}{2}(2 \times 5 + 23 \times 2)$$

$$= 12 \times 56$$

Hence, $S_{24} = 672$.

207. Find the number of terms of the AP $-12, -9, -6, ..., 21$. If 1 is added to each term of this AP, then find the sum of all the terms of the AP thus obtained.

Ans : [Board Term-2 2013]

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

We have $a = -12, d = -9 - (-12) = 3$

$$a_n = a + (n-1)d$$

$$21 = -12 + (n-1) \times 3$$

e236

$$21 + 12 = (n-1) \times 3$$

$$33 = (n-1) \times 3$$

$$n - 1 = 11$$

$$n = 11 + 1 = 12$$

Now, if 1 is added to each term we have a new AP with $-12+1, -a+1, -6+1......21+1$

Now we have $a = -11, d = 3$ and $a_n = 22$ and $n = 12$

Sum of this obtained AP,

$$S_{12} = \frac{12}{2}[-11 + 22]$$

$$= 6 \times 11 = 66$$

Hence the sum of new AP is 66.

208. How many terms of the AP $-6, \frac{11}{2}, -5, ...$ are needed to given the sum -25? Explain the double answer.

Ans : [Board Term-2 2012]

AP is $-6, -\frac{11}{2}, -5 ....$

Let the first term be a, common difference be d, nth term be a_n and sum of n term be S_n

Here we have $a = -6$

$$d = -\frac{11}{2} + \frac{6}{1} = \frac{1}{2}$$

e237

$$S_n = -25$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$-25 = \frac{n}{2}\left[-12 + (n-1) \times \frac{1}{2}\right]$$

$$-50 = n\left[\frac{-24 + (n-1)}{2}\right]$$

$$-100 = n[n - 25]$$

$$n^2 - 25n + 100 = 0$$

$$(n-20)(n-5) = 0$$

$$n = 20, 5$$

or, $S_{20} = S_5$

Here we have got two answers because two value of n sum of AP is same. Since a is negative and d is positive; the sum of the terms from 6^{th} to 20^{th} is zero.

209. If S_1, S_2, S_3 be the sum of $n, 2n, 3n$ terms respectively of an AP, prove that $S_3 = 3(S_2 - S_1)$.

Ans : [Board Term-2 2012]

Let the first term be a, and common difference be d.

Now $S_1 = \frac{n}{2}[2a+(n-1)d]$

$S_2 = \frac{2n}{2}[2a+(2n-1)d]$

e238

$S_3 = \frac{3n}{2}[2a+(3n-1)d]$

$$3(S_2 - S_1) = 3\left[\frac{2n}{2}[2a+(2n-1)d] - \frac{n}{2}[2a+(n-1)d]\right]$$

$$= 3\left[\frac{n}{2}[4a+2(2n-1)d] - [2a+(n-1)d]\right]$$

$$= 3\left[\frac{n}{2}(4a+4nd-2d-2a-nd+d)\right]$$

$$= 3\left[\frac{n}{2}(2a+3nd-d)\right]$$

$$= \frac{3n}{2}[2a+(3n-1)d] = S_3$$

210. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the AP.

Ans : [Board Term-2 SQP 2017]

Let the middle most terms of the AP be $(x-d)$, x and $(x+d)$.

We have $x-d+x+x+d = 225$

$3x = 225 \Rightarrow x = 75$

e243

and the middle term $= \frac{37+1}{2} = 19^{th}$ term

Thus AP is

$(x-18d), \ldots .(x-2d), (x-d), x, (x+d), (x+2d), \ldots \ldots .$

$(x-18d)$

Sum of last three terms,

$$(x+18d)+(x+17d)+(x+16d) = 429$$

$$3x+51d = 429$$

, $225+51d = 429 \Rightarrow d = 4$

First term $a_1 = x-18d = 75-18\times 4 = 3$

$$a_2 = 3+4 = 7$$

Hence AP = 3, 7, 11, ……., 147.

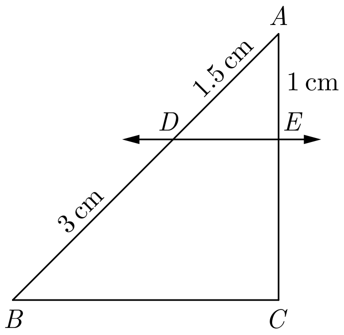
CHAPTER 6

TRIANGLES

ONE MARK QUESTIONS

Multiple Choice Questions

1. In the given figure, $DE \parallel BC$. The value of EC is



f196

(a) 1.5 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Ans :

Since, $DE \parallel BC$

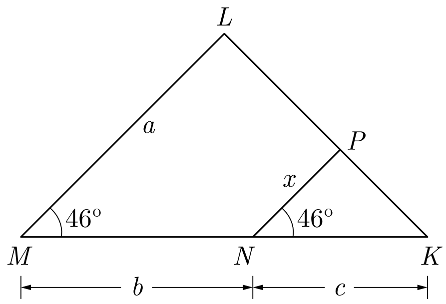
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2\,\text{cm}$$

Thus (c) is correct option.

2. In the given figure, x is

f197



(a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$
(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Ans :

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46°$$

Thus by $A-A$ criterion of similarity,

$$\Delta KNP \sim \Delta KML$$

Thus $$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

3. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

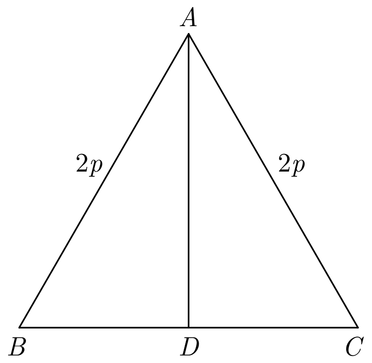
(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$
(c) $2p$ (d) $4p$

Ans :

f199

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In ΔADB, $AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

Thus (b) is correct option.

4. Which of the following statement is false?

(a) All isosceles triangles are similar.

(b) All quadrilateral are similar.

(c) All circles are similar.

(d) None of the above

f200

Ans :

Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false.

Thus (a) is correct option.

5. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is

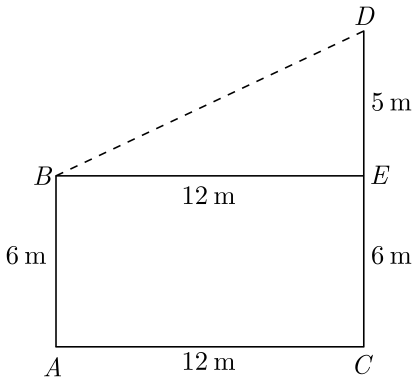
(a) 12 m (b) 14 m

(c) 13 m (d) 11 m

f201

Ans :

Let AB and CD be the vertical poles as shown below.



We have $AB = 6\text{ m},\ CD = 11\text{ m}$

and $AC = 12\text{ m}$

$$DE = CD - CE$$

$$= (11 - 6)\text{ m} = 5\text{ m}$$

In right angled, ΔBED,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$

$$BD = \sqrt{169}\text{ m} = 13\text{ m}$$

Hence, distance between their tops is 13 m.

Thus (c) is correct option.

6. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then

(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$

f204

(c) $AQ^2 + CP^2 = AC^2 + PQ^2$

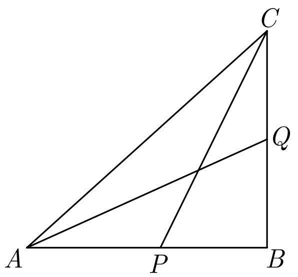
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

Ans :

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $$AQ^2 = AB^2 + BQ^2$$



$$CP^2 + AQ^2 = CB^2 + BP^2 + AB^2 + BQ^2$$

$$= CB^2 + AB^2 + BP^2 + BQ^2$$

$$= AC^2 + PQ^2$$

Thus (c) is correct option.

7. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm then the sum of the remaining sides of the triangles is

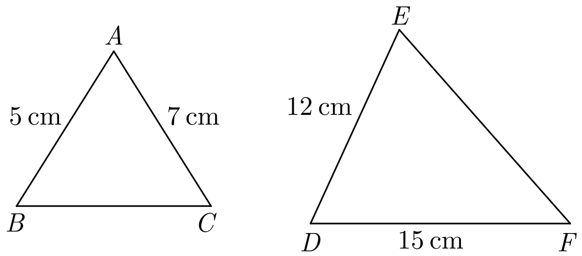
(a) 23.05 cm (b) 16.8 cm

(c) 6.25 cm (d) 24 cm

f205

Ans :

We have $\Delta ABC \sim \Delta EDF$



Now $$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$$

Thus (a) is correct option.

8. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm

(c) 17 cm (d) data insufficient

Ans : (b) 18 cm

Let c be the hypotenuse of the triangle, a and b be other sides.

Now $c = \sqrt{a^2 + b^2}$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40 \Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

f206

Squaring $c = 40 - (a + b)$ we have

$$c^2 = [40 - (a + b)]^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + (a + b)^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + a^2 + 2ab + b^2$$

$$0 = 1600 - 2 \times 40(a + b) + 2 \times 80$$

$$0 = 20 - (a + b) + 2$$

$$a + b = 22$$

$$c = 40 - (a + b) = 40 - 22 = 18 \text{ cm}$$

Thus (b) is correct option.

9. **Assertion :** In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

f207

We have,

$$AB^2 + BC^2 = (24)^2 + (10)^2$$

$$= 576 + 100 = 676 = AC^2$$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

FILL IN THE BLANK QUESTIONS

10. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the .... side.

Ans :

f210

third

11. .... theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Ans :

f211

Pythagoras

12. Line joining the mid-points of any two sides of a triangle is .... to the third side.

Ans :

f212

parallel

13. All squares are ....

Ans :

f213

similar

14. Two triangles are said to be .... if corresponding angles of two triangles are equal.

Ans :

f214

equiangular

15. All similar figures need not be ....

Ans :

congruent

f215

16. All circles are ....

Ans :

similar

f217

17. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the .... side.

Ans :

third

f218

18. If a line divides any two sides of a triangle in the same ratio, then the line is .... to the third side.

Ans :

parallel

f219

19. All congruent figures are similar but the similar figures need .... be congruent.

Ans :

not

f220

20. Two figures are said to be .... if they have same shape but not necessarily the same size.

Ans :

similar

f221

21. .... theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Ans :

Basic proportionality

f222

22. All .... triangles are similar.

Ans :

equilateral

f223

23. Two figures having the same shape and size are said to be ....

Ans :

congruent

f224

24. Two triangles are similar if their corresponding sides are ....

Ans : [Board 2020 OD Standard]

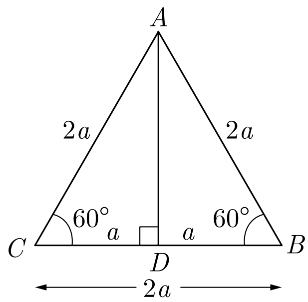
in the same ratio.

f225

25. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is ....

Ans : [Board 2020 Delhi Standard]

ΔABC is an equilateral triangle as shown below, in which $AD \perp BC$.



f227

Using Pythagoras theorem we have

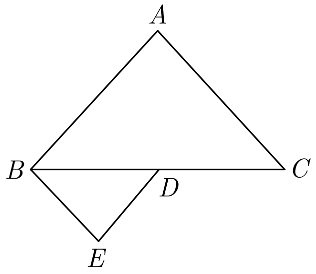
$$\begin{aligned} AB^2 &= (AD)^2 + (BD)^2 \\ (2a)^2 &= (AD)^2 + (a)^2 \\ 4a^2 - a^2 &= (AD)^2 \\ (AD)^2 &= 3a^2 \\ AD &= a\sqrt{3} \end{aligned}$$

Hence, the length of attitude is $a\sqrt{3}$.

26. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ....

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



f228

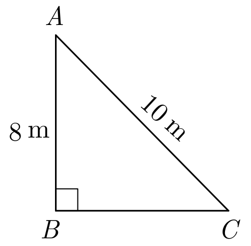
$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{(\frac{1}{2}BC)^2}$$

$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

27. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is .... m.

Ans : [Board 2020 Delhi Standard]

Let AB be the height of the window above the ground and BC be a ladder.



f229

Here, $AB = 8$ m

and $AC = 10$ m

In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 8^2 + BC^2$$

$$BC^2 = 100 - 64 = 36$$

$$BC = 6 \text{ m}$$

28. In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B =$ ....

Ans : [Board 2020 OD Standard]

We have $AB = 6\sqrt{3}$ cm,

$AC = 12$ cm and

$BC = 6$ cm

f231

Now $AB^2 = 36 \times 3 = 108$

$AC^2 = 144$

and $BC^2 = 36$

In can be easily observed that above values satisfy Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$108 + 36 = 144 \text{ cm}$$

Thus $\angle B = 90^\circ$

29. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ....

Ans : [Board 2020 Delhi Basic]

Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

f232

Thus $\dfrac{25}{15} = \dfrac{9}{\text{side}}$

$$\text{side} = \frac{9 \times 15}{25} = 5.4 \text{ cm}$$

Very Short Answer Questions

30. ΔABC is isosceles with $AC = BC$. If $AB^2 = 2AC^2$, then find the measure of $\angle C$.

Ans : [Board 2020 Delhi Basic]

We have $AB^2 = 2AC^2$

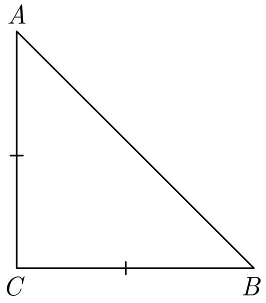
f236

$$AB^2 = AC^2 + AC^2$$

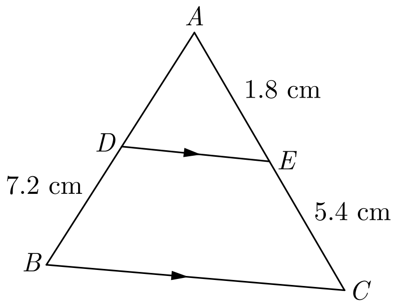
$$AB^2 = BC^2 + AC^2$$

$(BC = AC)$

It satisfies the Pythagoras theorem. Thus according to converse of Pythagoras theorem, ΔABC is a right angle triangle and $\angle C = 90^\circ$.



31. In Figure, $DE || BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm.



f237

Ans : [Board 2019 OD]

Since $DE || BC$ we have

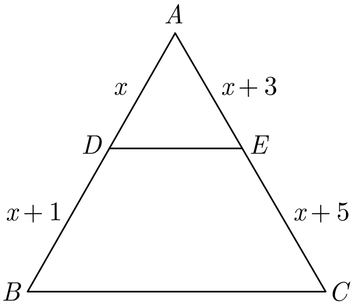
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4} = \frac{12.96}{5.4} = 2.4 \text{ cm}$$

32. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1 2016]

In the given figure $DE || BC$, thus

$$\frac{AD}{DB} = \frac{AE}{EC}$$

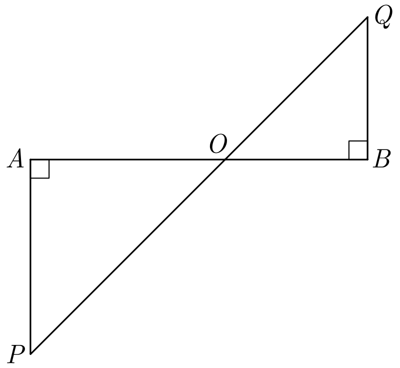
f101

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

33. In the given figure, if $\angle A = 90^0, \angle B = 90^0$, $OB = 4.5$ cm $OA = 6$ cm and $AP = 4$ cm then find QB.



Ans : [Board Term-1, 2015]

In ΔPAO and ΔQBO we have

f102

$$\angle A = \angle B = 90^0$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus $\Delta PAO \sim \Delta QBO$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

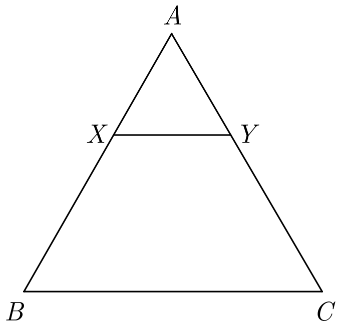
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

34. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

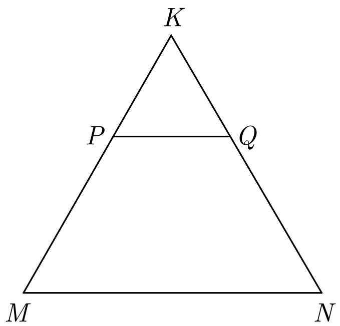
Now $$\frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

f103

Since $$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC.

35. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ.



Ans :

In the given figure $PQ || MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \quad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

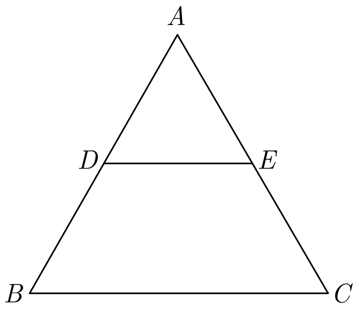
$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

36. In given figure $DE || BC$. If $AD = 3c, DB = 4c$ cm and $AE = 6$ cm then find EC.



Ans : [Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{BD} = \frac{AE}{EC}$$

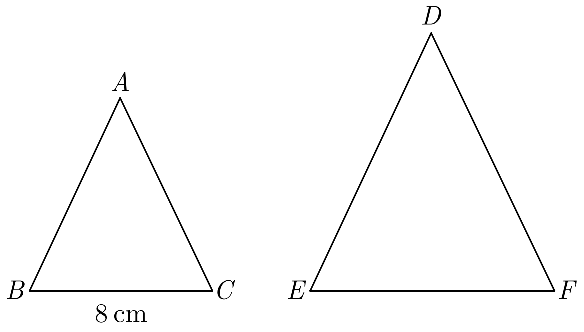
$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

37. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8$ cm then find EF.

Ans :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

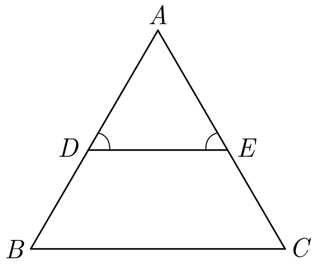
38. Are two triangles with equal corresponding sides always similar?

Ans : [Board Term-1 2015]

Yes, Two triangles having equal corresponding sides are are congruent and all congruent Δs have equal angles, hence they are similar too.

TWO MARKS QUESTIONS

39. In Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔBAC is an isosceles triangle.



Ans : [Board 2020 Delhi Standard]

We have, $\angle D = \angle E$

and $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$$\angle ADE = \angle ABC \text{ and}$$

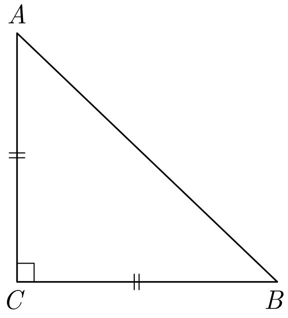
$$\angle AED = \angle ACB$$

Given $\angle ADE = \angle AED$

Thus $\angle ABC = \angle ACB$

Therefore BAC is an isosceles triangle.

40. In Figure, ABC is an isosceles triangle right angled at C with $AC = 4$ cm, Find the length of AB.



f241

Ans : [Board 2019 OD]

Since ABC is an isosceles triangle right angled at C,

$$AC = BC = 4 \text{ cm}$$

$$\angle C = 90^\circ$$

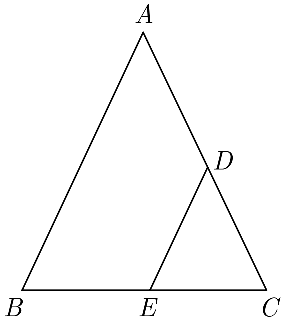
Using Pythagoras theorem in ΔABC we have,

$$AB^2 = BC^2 + AC^2$$

$$= 4^2 + 4^2 = 16 + 16 = 32$$

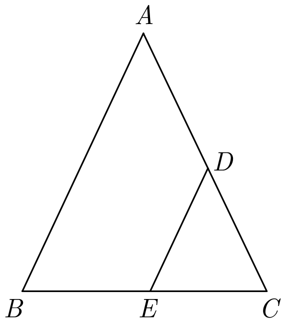
$$AB = 4\sqrt{2} \text{ cm.}$$

41. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of ΔABC, $DE || AB$. If $AD = 2x$, $DC = x + 3, BE = 2x - 1$ and $CE = x$, then find the value of x.



Ans : [Board Term-1 2015, 2016]

We have $\dfrac{CD}{AD} = \dfrac{CE}{BE}$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

f110

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA - CD} = \frac{CE}{CB - CE}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

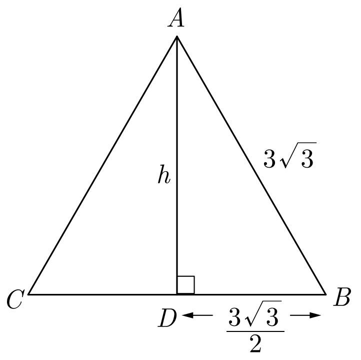
$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

42. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

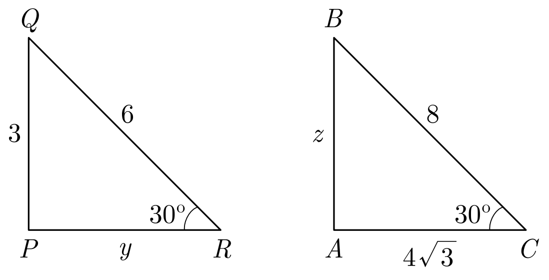
f111

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

43. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.



Ans : [Board Term-1 2010]

In the given figure $\Delta ABC \sim \Delta PQR$,

f112

Thus $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$

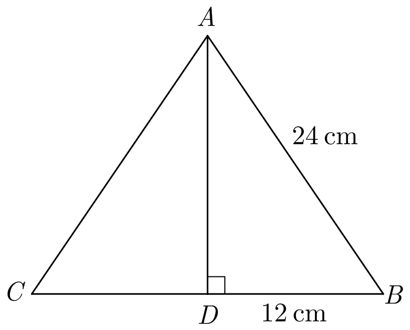
$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus $$y + z = 3\sqrt{3} + 4$$

44. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans : [Board Term-1 2015]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$

f113

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(24)^2 - (12)^2}$$

$$= \sqrt{576 - 144}$$

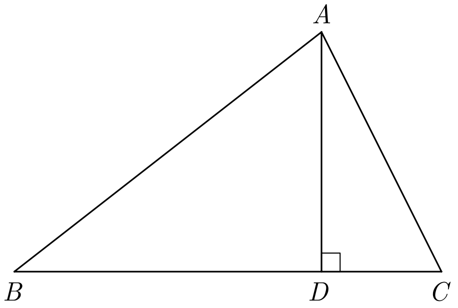
$$= \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm.

45. In $\Delta ABC, AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

f114

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\angle BAD = \angle ACD$;

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$

$$2\angle BAD + 2\angle DAC = 180^\circ$$

$$\angle BAD + \angle DAC = 90^\circ$$

$$\angle A = 90^\circ$$

Thus ΔABC is right angled at A.

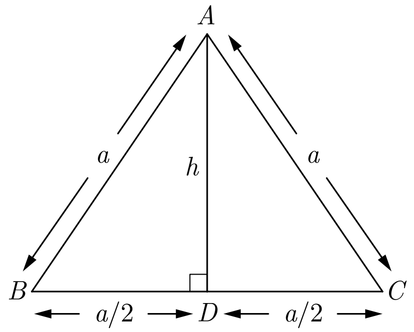
46. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. [Board 2020 SQP Standard]

or

Find the altitude of an equilateral triangle when each of its side is a cm.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



In ΔABD, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

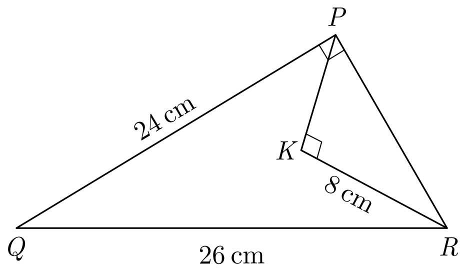
f115

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $h = \frac{\sqrt{3}a}{2}$

Thus $4h^2 = 3a^2$ Hence Proved

47. In the given triangle $PQR, \angle QPR = 90^\circ, PQ = 24$ cm and $QR = 26$ cm and in $\Delta PKR, \angle PKR = 90^\circ$ and $KR = 8$ cm, find PK.



Ans : [Board Term-1 2012]

In the given triangle we have

$$\angle QPR = 90^0$$

Thus $QR^2 = QP^2 + PR^2$

f118

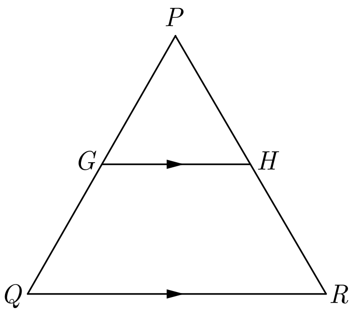
$$PR = \sqrt{26^2 - 24^2}$$
$$= \sqrt{100} = 10 \text{ cm}$$

Now $\angle PKR = 90^0$

Thus $PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$

$$= \sqrt{36} = 6 \text{ cm}$$

48. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Ans : [Board Term-1 2012]

Since G is the mid-point of PQ we have

$$PG = GQ$$
$$\frac{PG}{GQ} = 1$$

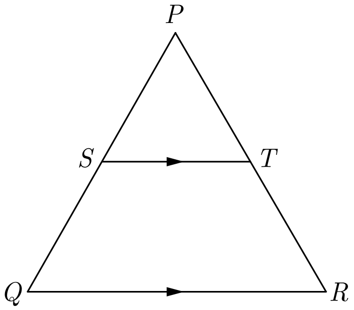
f121

We also have $GH || QR$, thus by BPT we get

$$\frac{PG}{GQ} = \frac{PH}{HR}$$
$$1 = \frac{PH}{HR}$$
$$PH = HR.$$ Hence proved.

Hence, H is the mid-point of PR.

49. In the given figure, in a triangle PQR, $ST || QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.



Ans : [Board Term-1 2011]

We have $\frac{PS}{SQ} = \frac{3}{5}$

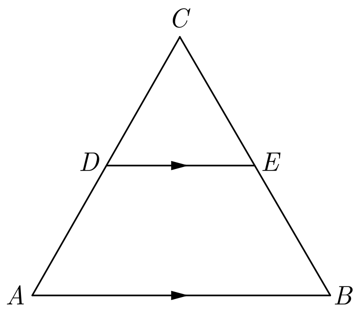
$$\frac{PS}{PS + SQ} = \frac{3}{3+5}$$
$$\frac{PS}{PQ} = \frac{3}{8}$$

f122

We also have, $ST || QR$, thus by BPT we get

$$\frac{PS}{PQ} = \frac{PT}{PR}$$
$$PT = \frac{PS}{PQ} \times PR$$
$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

50. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Ans : [Board Term-1, 2012, set-63]

In ΔCAB, we have

$$\angle A = \angle B \qquad (1)$$

By isosceles triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \qquad (2)$$

Dividing equation (2) by (1) we get,

$$\frac{CD}{AD} = \frac{CE}{BE}$$

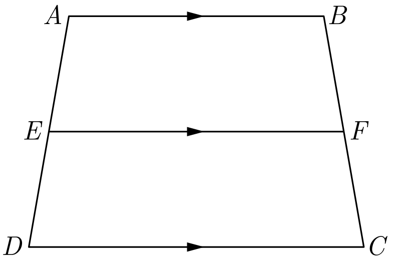
f123

By converse of BPT,

$$DE || AB.$$ Hence Proved

51. In the given figure, if $ABCD$ is a trapezium in which

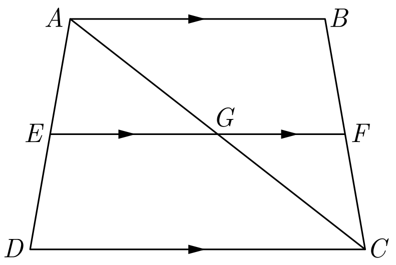
$AB \,||\, CD \,||\, EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans : [Board Term-1 2012]

We draw, AC intersecting EF at G as shown below.

f124



In $\Delta\, CAB$, $GF \,||\, AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \qquad ...(1)$$

In $\Delta\, ADC$, $EG \,||\, DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \qquad ...(2)$$

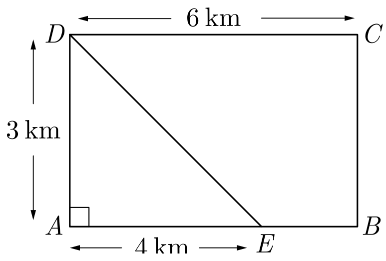
From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}.$$ Hence Proved.

52. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE,

f125

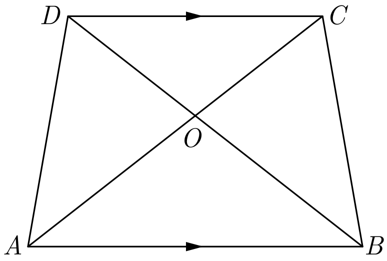
$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus $DE = 5$ km

53. $ABCD$ is a trapezium in which $AB \,||\, CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In $\Delta\, AOB$ and $\Delta\, COD$, $AB \,||\, CD$,

f126

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and $$\angle OBA = \angle ODC$$

By AA similarity we have

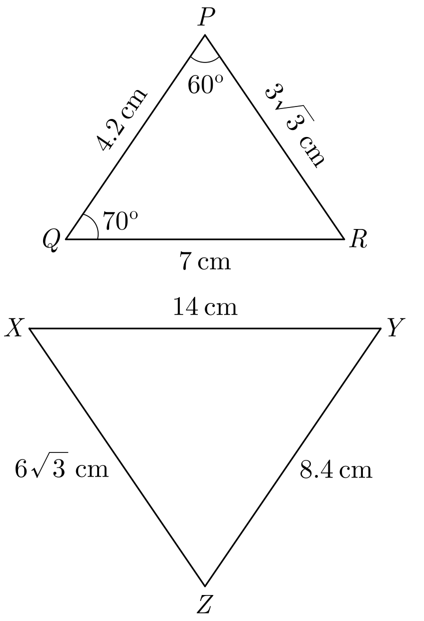
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}.$$ Hence Proved

54. In the given figures, find the measure of $\angle X$.



Ans : [Board Term-1 2012]

From given figures,

f127

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

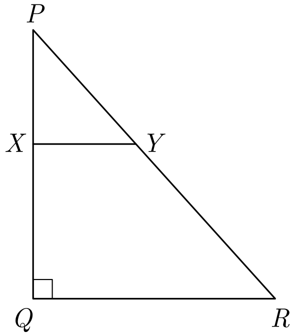
$$\Delta PQR \sim \Delta ZYX$$

Thus $$\angle X = \angle R$$

$$= 180^{\circ} - (60^{\circ} + 70^{\circ}) = 50^{\circ}$$

Thus $\angle X = 50^{\circ}$

55. In the given figure, PQR is a triangle right angled at Q and $XY || QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1:2$. Calculate the length of PR and QR.



Ans : [Board Term-1 2012]

f128

Since $XY \; || \; OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

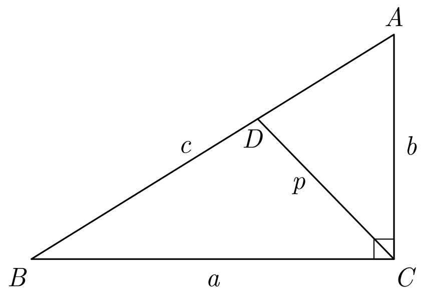
In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

56. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b, AB = c$ $PQR, ST || QR$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Ans : [Board Term-1 2012]

f129

In the given figure $CD \perp AB$, and $CD = p$

Area, $$\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} cp$$

Also,Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} ab$

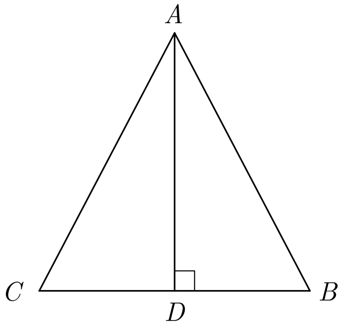
Thus $\frac{1}{2} cp = \frac{1}{2} ab$

$$cp = ab$$ Proved

57. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans : [Board Term-1 2012]

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

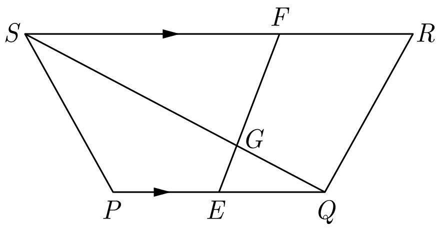
Since $\perp$ is the median in an equilateral Δ, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

58. In the figure, $PQRS$ is a trapezium in which $PQ || RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1 2016]

In ΔGEQ and ΔGFS,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

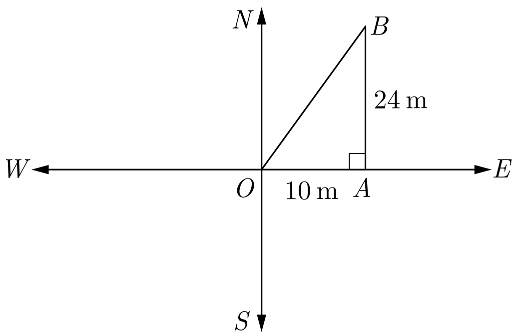
59. A man steadily goes 10 m due east and then 24 m due north.

(1) Find the distance from the starting point.

(2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B. The man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

$$= (10)^2 + (24)^2$$

$$= 100 + 576 = 676$$

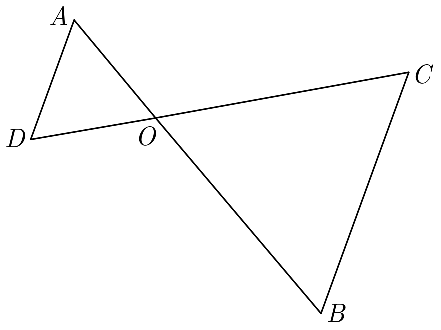
or, $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

(2) Pythagoras Theorem

60. In the given figure, $OA \times OB = OC \times OD$, show that

$\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

f133

Due to the vertically opposite angles,

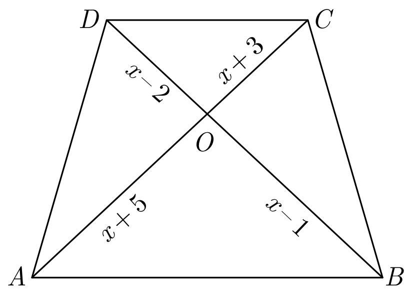
$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

61. In the given figure, if $AB || DC$, find the value of x.



Ans : [Board Term-1 2012]

We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$

f134

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

$$x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$$

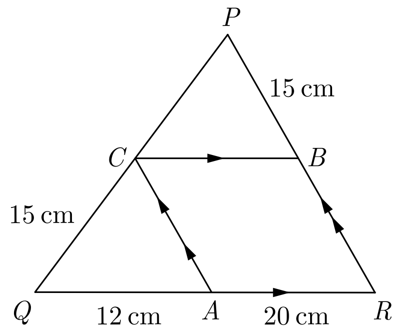
$$x^2 + 3x - 10 = x^2 + 2x - 3$$

$$3x - 10 = 2x - 3$$

$$3x - 2x = 10 - 3 \Rightarrow x = 7$$

Thus $x = 7$.

62. In the given figure, $CB || QR$ and $CA || PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Ans : [Board Term-1 2012]

In ΔPQR, $CA || PR$

By BPT similarity we have

f135

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $\quad CB \parallel QR$

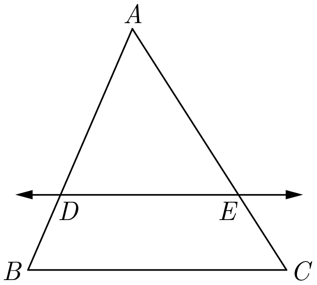
Thus $\quad \frac{PC}{CQ} = \frac{PR}{BR}$

$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

THREE MARKS QUESTIONS

63. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $\quad DE \parallel BC$

By BPT, $\quad \frac{AD}{DB} = \frac{AE}{EC}$

f242

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

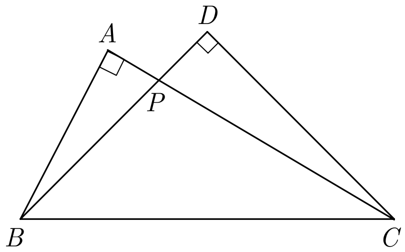
$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

64. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90^\circ$$

f243

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

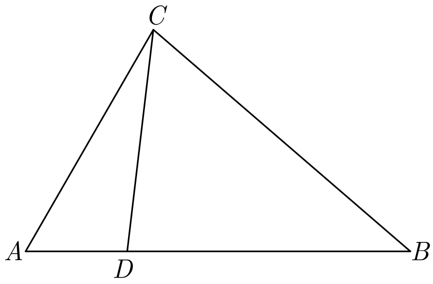
By AA similarity we have

$$\Delta BAP \sim \Delta CDP$$

Therefore $\quad \frac{BP}{PC} = \frac{AP}{PD}$

$$AP \times PC = BP \times PD \qquad \text{Hence Proved}$$

65. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \qquad \text{[given]}$$

$$\angle CAB = \angle CAD \qquad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $\quad \frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

f245

Now $\quad \frac{AB}{AC} = \frac{AC}{AD}$

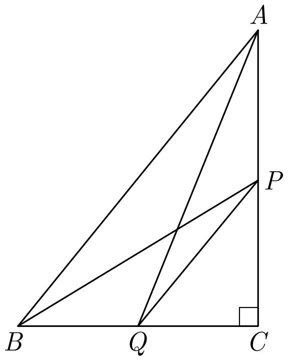
$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

66. If P and Q are the points on side CA and CB

respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



f246

Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \quad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$

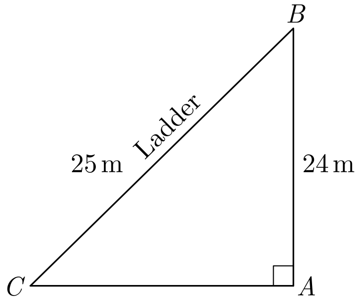
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $$AQ^2 + BP^2 = AB^2 + PQ^2$$ Hence Proved

67. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



f247

Here $$AB = 24 \text{ m}$$

$$CB = 25 \text{ m}$$

and $$\angle CAB = 90^\circ$$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or, $$CA^2 = CB^2 - AB^2$$

$$= 23^2 - 24^2$$

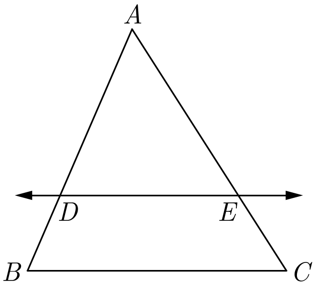
$$= 625 - 576 = 49$$

Thus $$CA = 7 \text{ m}$$

Hence, the distance of the foot of ladder from the building is 7 m.

THREE MARKS QUESTIONS

68. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $$DE \parallel BC$$

By BPT, $$\frac{AD}{DB} = \frac{AE}{EC}$$

f242

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

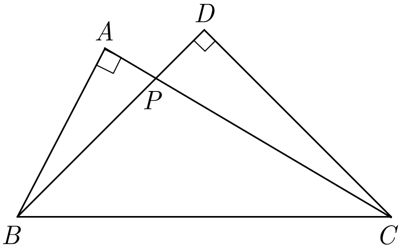
69. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that

$AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In $\triangle BAP$ and $\triangle CDP$ we have

$$\angle BAP = \angle CDP = 90^\circ$$

f243

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

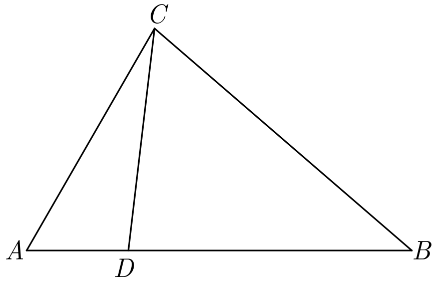
By AA similarity we have

$$\triangle BAP \sim \triangle CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD$$ Hence Proved

70. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

f245

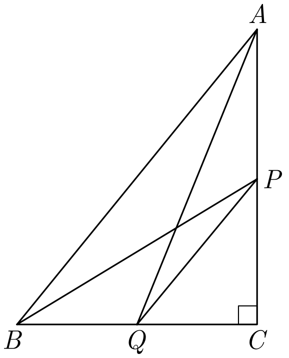
Now $$\frac{AB}{AC} = \frac{AC}{AD}$$

$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

71. If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



f246

Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \quad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$

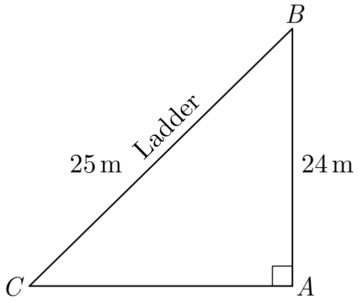
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $$AQ^2 + BP^2 = AB^2 + PQ^2$$ Hence Proved

72. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



f247

Here $$AB = 24 \text{ m}$$

$$CB = 25 \text{ m}$$

and $$\angle CAB = 90^\circ$$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or, $$CA^2 = CB^2 - AB^2$$

$$= 25^2 - 24^2$$

$$= 625 - 576 = 49$$

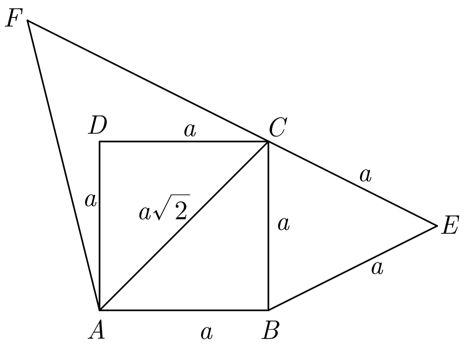
Thus $$CA = 7 \text{ m}$$

Hence, the distance of the foot of ladder from the building is 7 m.

73. Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board 2018, 2015]

As per given condition we have drawn the figure below. Let a be the side of square.



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= a^2 + a^2 = 2a^2$$

$$AC = \sqrt{2}\, a$$

f138

Area of equilateral triangle $\Delta\, BCE$,

$$\text{area}(\Delta\, BCE) = \frac{\sqrt{3}}{4} a^2$$

Area of equilateral triangle $\Delta\, ACF$,

$$\text{area}(\Delta\, ACF) = \frac{\sqrt{3}}{4}(\sqrt{2}\, a)^2 = \frac{\sqrt{3}}{2} a^2$$

Now, $$\frac{\text{area}(\Delta\, ACF)}{\text{area}(\Delta\, BCE)} = 2$$

$$\text{area}(\Delta\, ACF) = 2\text{area}(\Delta\, BEC)$$

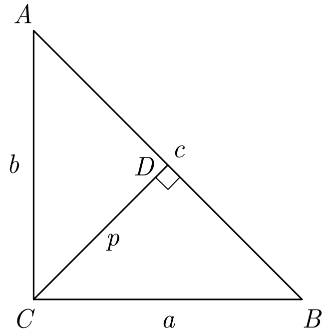
$$\text{area}(\Delta\, BEC) = \frac{1}{2}\text{area}(\Delta\, ACF)$$ Hence Proved.

74.

75. $\Delta\, ABC$ is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



In $\Delta\, ACB$ and $\Delta\, CDB$, $\angle B$ is common and

$$\angle ABC = \angle CDB = 90^0$$

Because of AA similarity we have

$$\Delta\, ABC \sim \Delta\, CDB$$

f140

Now $$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2 b^2}$$

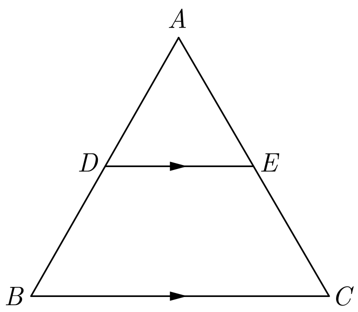
$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \qquad (c^2 = a^2 + b^2)$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence Proved

76. In $\Delta\, ABC, DE \,||\, BC$. If $AD = x + 2, DB = 3x + 16$, $AE = x$ and $EC = 3x + 5$, them find x.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$(x+2)(3x+5) = x(3x+16)$$

$$3x^2+5x+6x+10 = 3x^2+16x$$

$$11x+10 = 16x$$

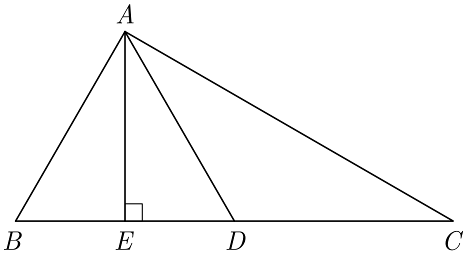
$$11x+10 = 10$$

$$5x = 10 \Rightarrow x = 2$$

77. If in $\Delta\, ABC$, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In $\Delta\, ABE$, using Pythagoras theorem we have

$$AB^2 = AE^2 + BE^2$$
$$= AD^2 - DE^2 + (BD - DE)^2$$
$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$
$$= AD^2 + BD^2 - 2BD \times DE \quad ...(1)$$

In $\Delta\, AEC$, we have

$$AC^2 = AE^2 + EC^2$$
$$= (AD^2 - ED^2) + (ED + DC)^2$$
$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$
$$= AD^2 + CD^2 + 2ED \times CD$$
$$= AD^2 + DC^2 + 2DC \times DE \quad ...(2)$$

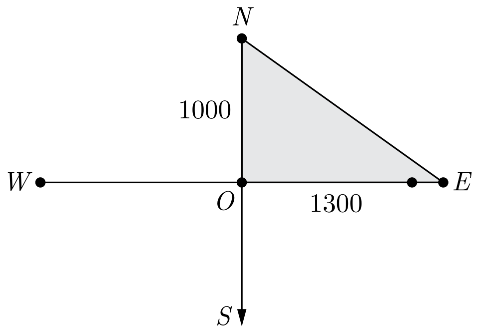
Adding equation (1) and (2) we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \qquad (BD = DC)$$
$$= 2AD^2 + 2\left(\tfrac{1}{2}BC\right)^2 \qquad (\ BD = \tfrac{1}{2}BC)$$
$$= 2AD^2 + \frac{1}{2}BC^2 \qquad \text{Hence Proves}$$

78. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



Distance covered by first aeroplane due North after two hours,

$$y = 500 \times 2 = 1{,}000 \text{ km.}$$

Distance covered by second aeroplane due East after two hours,

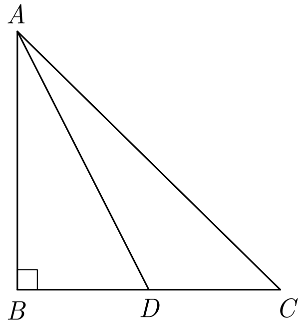
$$x = 650 \times 2 = 1{,}300 \text{ km.}$$

Distance between two aeroplane after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$
$$= \sqrt{(1000)^2 + (1300)^2}$$
$$= \sqrt{1000000 + 1690000}$$
$$= \sqrt{2690000}$$
$$= 1640.12 \text{ km}$$

79. In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC. Show that

$AC^2 = AD^2 + 3CD^2$.



Ans : [Board Term-1 2016]

We have $BD = CD = \frac{BC}{2}$

$BC = 2BD$

Using Pythagoras theorem in the right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$= AB^2 + (2BD)^2$$

$$= AB^2 + 4BD^2$$

$$= (AB^2 + BD^2) + 3BD^2$$

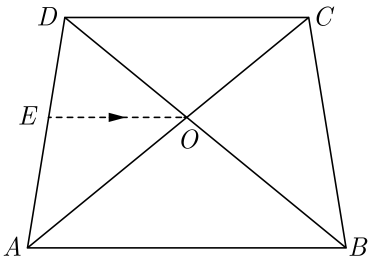
$$AC^2 = AD^2 + 3CD^2$$

f145

80. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans : [Board Term-1 2011]

As per given condition we have drawn quadrilateral $ABCD$, as shown below.



We have drawn $EO || AB$ on DA.

f146

In quadrilateral $ABCD$, we have

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\frac{AO}{CO} = \frac{BO}{DO} \quad ...(1)$$

In ΔABD, $EO || AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \quad ...(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $\frac{AE}{ED} = \frac{AO}{CO}$

$EO || DC$ (Converse of BPT)

$EO || AB$ (Construction)

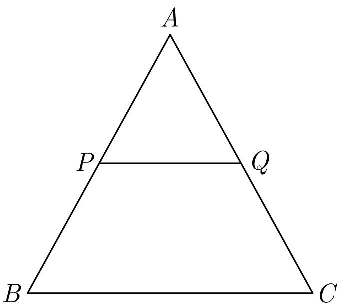
$AB || DC$

Thus in quadrilateral $ABCD$ we have

$AB \; AB \| CD$

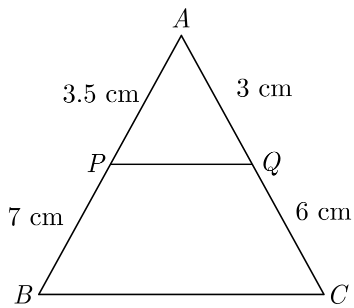
Thus $ABCD$ is a trapezium. Hence Proved

81. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Ans : [Board Term-1 2011]

We have redrawn the given figure as below.



We have $\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$

f147

and $\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

In ΔABC, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

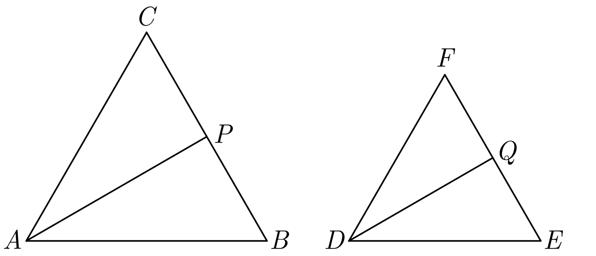
$$\Delta APQ \sim \Delta ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm.}$$

82. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



Prove that :

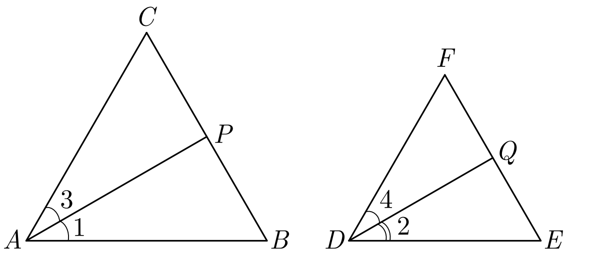
(1) $\frac{AP}{DQ} = \frac{AB}{DE}$

(2) $\Delta CAP \sim \Delta FDQ$.

f148

Ans : [Board Term-1 2016]

As per given condition we have redrawn the figure below.



(1) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE}$$ Hence Proved

(2) Since $\Delta ABC \sim \Delta DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

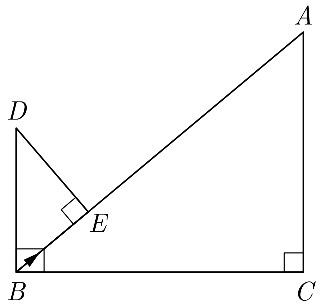
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

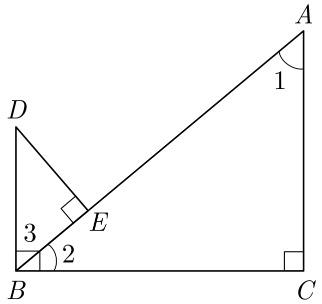
$$\Delta CAP \sim \Delta FDQ$$

83. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.



Ans : [Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In ΔABC, $\angle C = 90^\circ$, thus

$$\angle 1 + \angle 2 = 90^\circ$$

f149

But we have been given,

$$\angle 2 + \angle 3 = 90^{\circ}$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

$$\angle 1 = \angle 3$$

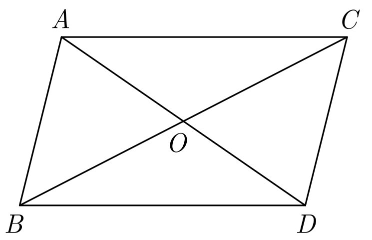
and $\angle ACB = \angle DEB = 90^{\circ}$

Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

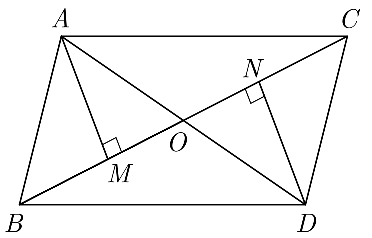
Thus $\dfrac{AC}{BC} = \dfrac{BE}{DE}$. Hence Proved

84. In the given figure, ΔABC and ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.



Ans : [Board 2020 OD Std, 2016, 2011]

As per given condition we have redrawn the figure below. Here we have drawn $AM \perp BC$ and $DN \perp BC$.



f150

In ΔAOM and ΔDON,

$$\angle AOM = \angle DON$$

(Vertically opposite angles)

$$\angle AMO = \angle DNO = 90^{\circ} \quad \text{(Construction)}$$

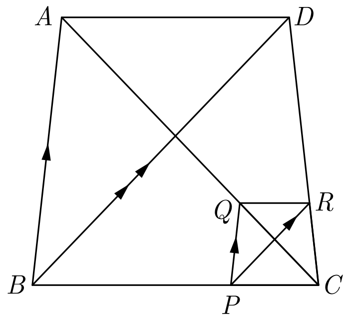
or, $\Delta AOM \sim \Delta DON$ (By AA similarity)

Thus $\dfrac{AO}{DO} = \dfrac{AM}{DN}$ ...(1)

Now, $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$

$$= \frac{AM}{DN} = \frac{AO}{DO} \text{From equation (1)}$$

85. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ || BA$ and $PR || BD$. Prove that $QR || AD$.



f151

Ans : [Board Term-1 2011]

In ΔABC, we have $PQ || AB$ and $PR || BD$.

By BPT we have

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad ...(1)$$

Again in ΔBCD, we have

$$PR \,||\, BD$$

By BPT we have

$$\frac{BP}{PC} = \frac{DR}{RC} \quad \text{(by BPT) ...(2)}$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

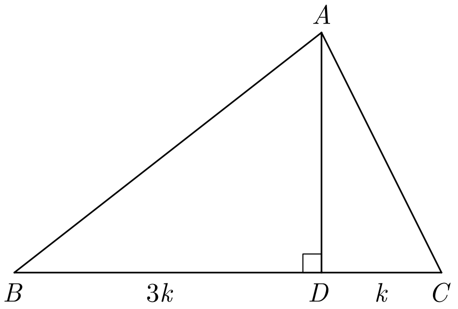
By converse of BPT,

$$PR \parallel AD \quad \text{Hence proved}$$

86. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans : [Board Term-1 2011, 2012, 2016]

As per given condition we have drawn the figure below.



Here $DB = 3CD$

$$BD = \frac{3}{4}BC$$

$$DC = \frac{1}{4}BC$$

f152

In ΔADB, we have

$$AB^2 = AD^2 + BD^2 \quad ...(1)$$

In ΔADC, $AC^2 = AD^2 + CD^2$ ...(2)

Subtracting equation (2) from (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

Since $DB = 3CD$ we get

$$AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$

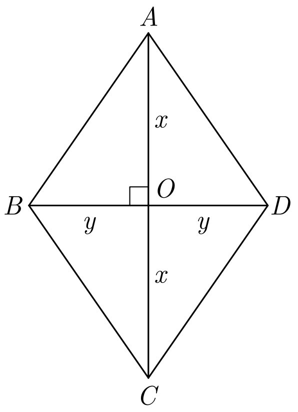
$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2 \quad \text{Hence Proved}$$

87. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals.

Ans : [Board Term-1 2011]

Let, $ABCD$ is a rhombus and we know that diagonals of a rhombus bisect each other at 90^o.



Now $AO = OC \Rightarrow AO^2 = OC^2$

$$BO = OD \Rightarrow BO^2 = OD^2$$

f155

and $\angle AOB = 90^o$

$$AB^2 = OA^2 + BO^2 = x^2 + y^2$$

Similarly, $AD^2 = OA^2 + OD^2 = x^2 + y^2$

$$CD^2 = OC^2 + OD^2 = x^2 + y^2$$

$$CB^2 = OC^2 + OB^2 = x^2 + y^2$$

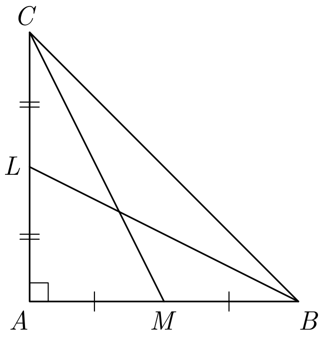
$$AB^2 + BC^2 + CD^2 + DA^2 = 4x^2 + 4y^2$$

$$= (2x)^2 + (2y)^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence Proved

88. In the given figure, BL and CM are medians of ΔABC, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



Ans : [Board Term-1 2011]

f156

We have a right angled triangle ΔABC at A where BL and CM are medians.

In ΔABL, $$BL^2 = AB^2 + AL^2$$

$$= AB^2 + \left(\frac{AC}{2}\right)^2 \quad (BL \text{ is median})$$

In ΔACM, $$CM^2 = AC^2 + AM^2$$

$$= AC^2 + \left(\frac{AB}{2}\right)^2 \quad (CM \text{ is median})$$

Now $$BL^2 + CM^2 = AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$$

$$4(BL^2 + CM^2) = 5AB^2 + 5AC^2$$

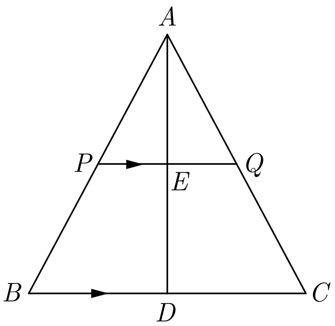
$$= 5(AB^2 + AC^2)$$

$$= 5BC^2$$ Hence Proved

89. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that the median AD bisects PQ.

Ans : [Board Term-1 2011]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E.

We have, PQ || BE

$\angle ApE = \angle B$ and $\angle AQE = \angle C$

(Corresponding angles)

Thus in ΔAPE and ΔABD we have

$$\angle APE = \angle ABD$$

$$\angle PAE = \angle BAD \quad \text{(common)}$$

Thus $$\Delta APE \sim \Delta ABD$$

$$\frac{PE}{BD} = \frac{AE}{AD} \quad ...(1)$$

Similarly, $$\Delta AQE \sim \Delta ACD$$

or, $$\frac{QE}{CD} = \frac{AE}{AD} \quad ...(2)$$

From equation (1) and (2) we have

$$\frac{PE}{BD} = \frac{QE}{CD}$$

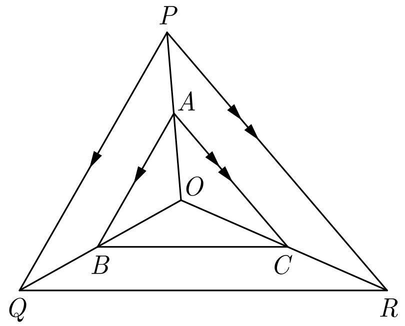
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

$$PE = QE$$

Hence, AD bisects PQ.

90. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB || PQ$ and $AC || PR$. Prove that $BC || QR$.



Ans : [Board Term-1 2012]

In ΔPOQ, $AB \parallel PQ$

By BPT $$\frac{AO}{AP} = \frac{OB}{BQ} \quad ...(1)$$

In ΔOPR, $AC \parallel PR$,

By BPT $$\frac{OA}{AP} = \frac{OC}{CR} \quad (2)$$

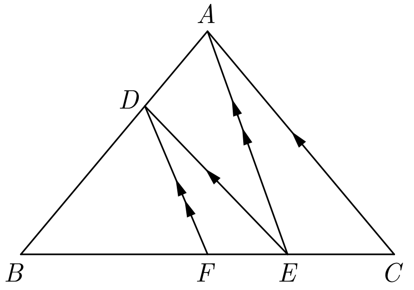
From equations (1) and (2), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

By converse of BPT we have

$$BC \parallel QR$$ Hence Proved

91. In the given figure, $DE || AC$ and $DF || AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Ans : [Board 2020 Delhi Std, 2012]

In ΔABC, $\quad DE \parallel AC$, (Given)

By BPT $\quad \frac{BD}{DA} = \frac{BE}{EC}$ ...(1)

In ΔABE, $\quad DF \parallel AE$, (Given)

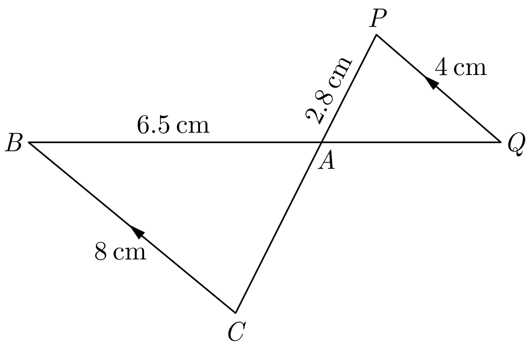
By BPT $\quad \frac{BD}{DA} = \frac{BF}{FE}$ ...(2)

From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

f160

92. In the given figure, $BC || PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm $AP = 2.8$ cm Find CA and AQ.



Ans : [Board Term-1 2012]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm, $PQ = 4$ cm and $AP = 2.8$ cm.

We have $\quad BC \parallel PQ$

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity,

$$\Delta ABC \sim \Delta AQP$$

$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

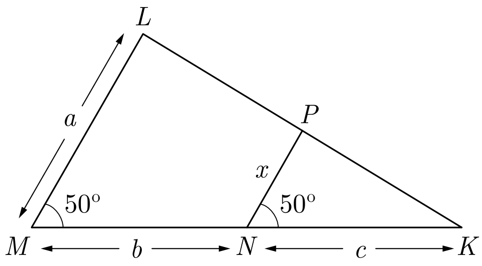
f161

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

93. In the given figure, find the value of x in terms of a, b and c.



Ans : [Board Term-1 2012]

In triangles LMK and PNK, $\angle K$ is common and

$$\angle M = \angle N = 50^{\circ}$$

Due to AA similarity,

f159

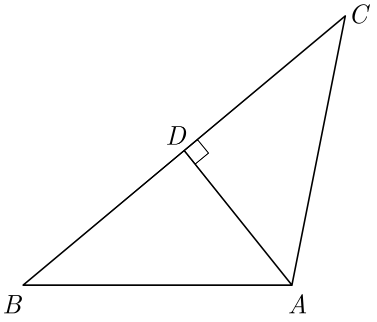
$$\Delta LMK \sim \Delta PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

94. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Ans : [Board 2020 OD Standard]

In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad ...(1)$$

In right ΔADB,

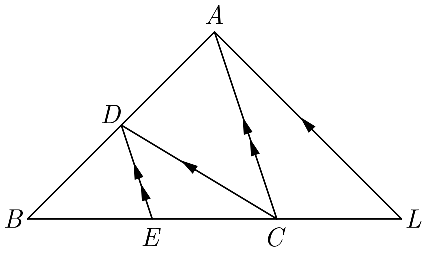
$$AB^2 = AD^2 + BD^2 \quad ...(2)$$

Subtracting equation (1) from (2) we have

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2.$$

95. In the given figure, $CD || LA$ and $DE || AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm.



Ans : [Board Term-1 2012]

In ΔABC, $DE || AC$, $BE = 4$ cm and $EC = 2$cm

By BPT $$\frac{BD}{DA} = \frac{BE}{EC} \quad ...(1)$$

In ΔABL, $DC || AL$

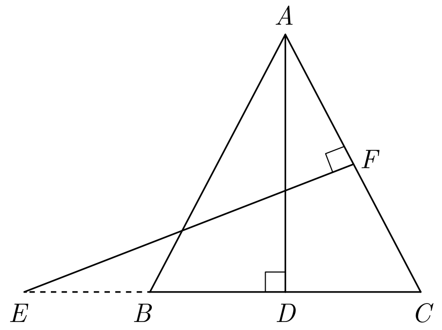
By BPT $$\frac{BD}{DA} = \frac{BC}{CL} \quad ...(2)$$

From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL} \Rightarrow CL = 3 \text{ cm}$$

96. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.



Ans : [Board Term-1 2012]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $$\angle ABC = \angle ACB$$

$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC \quad \text{(each } 90^\circ\text{)}$$

Due to AA similarity

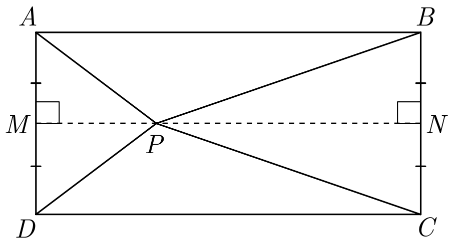
$$\Delta ABD \sim \Delta ECF \quad \text{Hence proved}$$

FOUR MARKS QUESTIONS

97. In a rectangle $ABCD$, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Ans : [Board 2020 OD Basic]

As per information given we have drawn figure below.



Here P is any point in the interior of rectangle $ABCD$. We have drawn a line MN through point P and parallel to AB and CD.

We have to prove $PA^2 + PC^2 = PB^2 + PD^2$

Since $AB \parallel MN$, $AM \parallel BN$ and $\angle A = 90°$, thus $ABNM$ is rectangle. $MNCD$ is also a rectangle.

Here, $PM \perp AD$ and $PN \perp BC$,

$$AM = BN \text{ and } MD = NC \quad ...(1)$$

Now, in ΔAMP, $\quad PA^2 = AM^2 + MP^2 \quad ...(2)$

In ΔPMD, $\quad PD^2 = MP^2 + MD^2 \quad ...(3)$

In ΔPNB, $\quad PB^2 = PN^2 + BN^2 \quad ...(4)$

In ΔPNC, $\quad PC^2 = PN^2 + NC^2 \quad ...(5)$

From equation (2) and (5) we obtain,

$$PA^2 + PC^2 = AM^2 + MP^2 + PN^2 + NC^2$$

Using equation (1) we have

$$PA^2 + PC^2 = BN^2 + MP^2 + PN^2 + MD^2$$

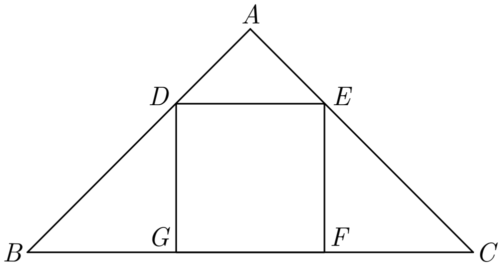
$$= (BN^2 + PN^2) + (MP^2 + MD^2)$$

Using equation (3) and (4) we have

$$PA^2 + PC^2 = PB^2 + PD^2$$

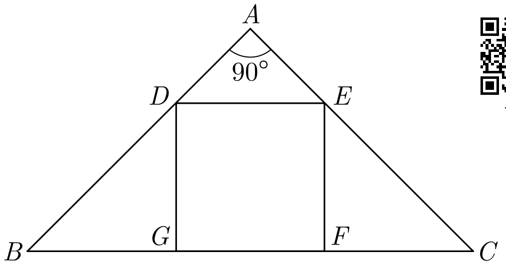
f248

98. In the given figure, $DEFG$ is a square and $\angle BAC = 90°$. Show that $FG^2 = BG \times FC$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



f249

In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \quad \text{[each } 90°]$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \quad \text{[each } 90°]$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

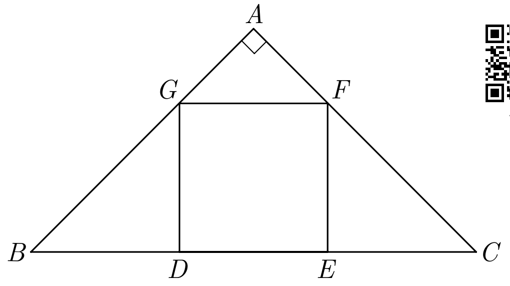
Thus $\quad \dfrac{GB}{FE} = \dfrac{GD}{FC}$

Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $\quad FG^2 = BG \times FC \quad$ Hence Proved

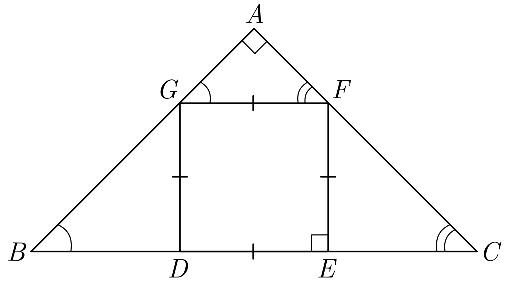
99. In Figure $DEFG$ is a square in a triangle ABC right angled at A. Prove that
(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$



f250

Ans : [Board 2020 Delhi, OD Basic]

We have redrawn the given figure as shown below.



Here ABC is a triangle in which $\angle BAC = 90°$ and $DEFG$ is a square.
(i) In ΔAGF and ΔDBG

$$\angle GAF = \angle BDG \quad (\text{each } 90°)$$

Due to corresponding angles,

$$\angle AGF = \angle GBD$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta DBG \qquad \text{Hence Proved}$$

(ii) In ΔAGF and ΔEFC,

$$\angle GAF = \angle CEF \qquad \text{(each } 90^\circ\text{)}$$

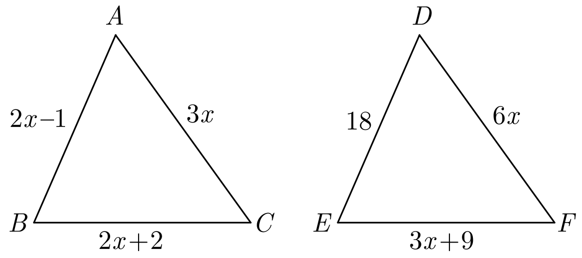
Due to corresponding angles,

$$\angle AFG = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta EFC \qquad \text{Hence Proved}$$

100. In Figure, if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.



Ans : [Board 2020 OD Standard]

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

f251

$$\frac{2x-1}{2x+2} = \frac{18}{3x+9}$$

$$(2x-1)(3x+9) = 18(2x+2)$$

$$(2x-1)(x+3) = 6(2x+2)$$

$$2x^2 - x + 6x - 3 = 12x + 12$$

$$2x^2 + 5x - 12x - 15 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(x-5)(2x+3) = 0 \Rightarrow x = 5 \text{ or } x = \frac{-3}{2}$$

But $x = \frac{-3}{2}$ is not possible, thus $x = 5$.

Now in ΔABC, we get

$$AB = 2x - 1 = 2 \times 5 - 1 = 9$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 12$$

$$AC = 3x = 3 \times 5 = 15$$

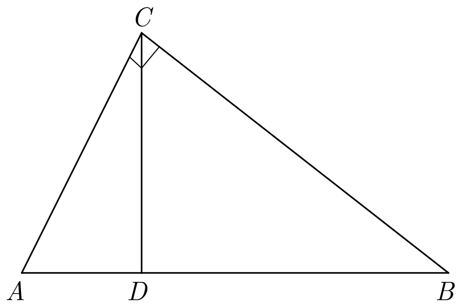
and in ΔDEF, we get

$$DE = 18$$

$$EF = 3x + 9 = 3 \times 5 + 9 = 24$$

$$DE = 6x = 6 \times 5 = 30.$$

101. In Figure, $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



Ans : [Board 2019 Delhi]

In ΔACB we have

$$\angle ACB = 90^\circ$$

and $\qquad CD \perp AB$

Thus $\qquad AB^2 = CA^2 + CB^2 \qquad ...(1)$

In ΔCAD, $\angle ADC = 90^\circ$, thus we have

$$CA^2 = CD^2 + AD^2 \qquad ...(2)$$

and in ΔCDB, $\angle CDB = 90^\circ$, thus we have

$$CB^2 = CD^2 + BD^2 \qquad ...(3)$$

Adding equation (2) and (3), we get

$$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

f254

Substituting AB^2 from equation (1) we have

$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) = BD^2 + 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD[(AB + AD) - BD] = 2CD^2$$

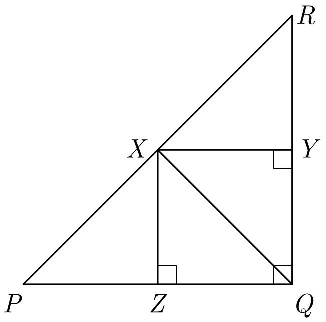
$$BD[AD + (AB - BD)] = 2CD^2$$

$$BD[AD + AD] = 2CD^2$$

$$BD \times 2AD = 2CD^2$$

$$CD^2 = BD \times AD \qquad \text{Hence Proved}$$

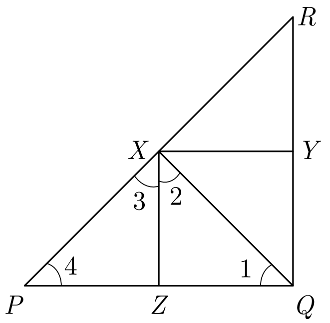
102. ΔPQR is right angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans : [Board Term-1 2015]

We have redrawn the given figure as below.

f165



It may be easily seen that $RQ \perp PQ$ and $XZ \perp PQ$ or $XZ \parallel YQ$.

Similarly $\qquad XY \parallel ZQ$

Since $\angle PQR = 90°$, thus $XYQZ$ is a rectangle.

In ΔXZQ, $\qquad \angle 1 + \angle 2 = 90^o \qquad ...(1)$

and in ΔPZX, $\qquad \angle 3 + \angle 4 = 90^o \qquad ...(2)$

$XQ \perp PR$ or, $\qquad \angle 2 + \angle 3 = 90^o \qquad ...(3)$

From eq. (1) and (3), $\qquad \angle 1 = \angle 3$

From eq. (2) and (3), $\qquad \angle 2 = \angle 4$

Due to AA similarity,

$$\Delta PZX \sim \Delta XZQ$$

$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

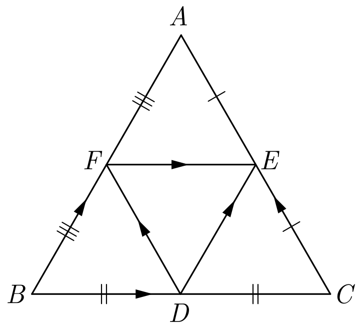
$$XZ^2 = PZ \times ZQ \qquad \text{Hence proved}$$

103. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC.)$

Ans : [Board Term-1 2015]

As per given condition we have given the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.

f166



Hence, $FE \parallel BC, DE \parallel AB$ and $DF \parallel AC$

By mid-point theorem,

If $\qquad DE \parallel BA$ then $DE \parallel BF$

and if $\qquad FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of parallelogram divides it into two equal Areas.

Hence $\qquad ar(\Delta BDF) = ar(\Delta DEF) \qquad ...(1)$

Similarly $ar(\Delta CDE) = ar(\Delta DEF) \qquad ...(2)$

$(\Delta AFE) = ar(\Delta DEF) \qquad ...(3)$

$(\Delta DEF) = ar(\Delta DEF) \qquad ...(4)$

Adding equation (1), (2), (3) and (4), we have

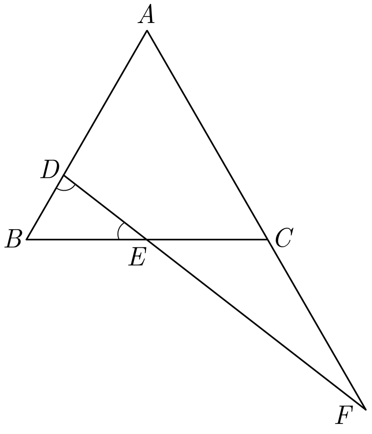
$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF)$$

$$= 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

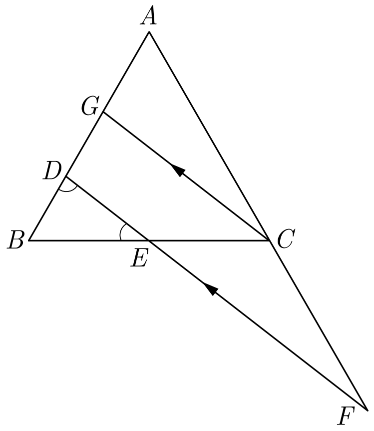
$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

104. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG \| FD$.



f168

We have $\angle BED = \angle BDE$

Since E is mid-point of BC,

$$BE = BD = EC \quad ...(1)$$

In ΔBCG, $DE \| FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In ΔADF, $CG \| FD$

By BPT $\frac{AG}{GD} = \frac{AC}{CF}$

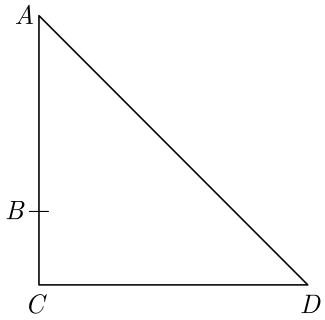
$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

, $\frac{AD}{GD} = \frac{AF}{CF}$

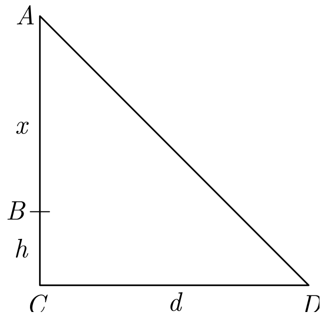
Thus $\frac{AF}{CF} = \frac{AD}{BE}$

105. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

Ans : [Board Term-1 2015]



We have redrawn the given figure as below.



We have $AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

f170

In right ΔACD, we have

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

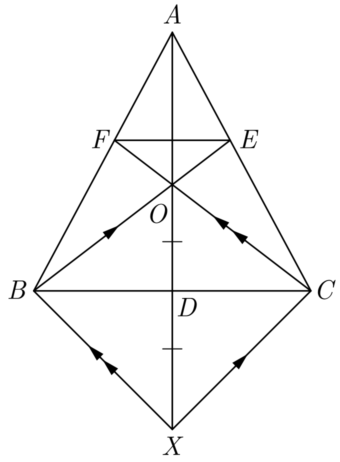
or, $$x = \frac{hd}{2h+d}$$

106. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(1) $EF \| BC$

(2) $AO : AX = AF : AB$



Ans : [Board Term-1 2015]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE \| XC$ and $BX \| CF$.

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \qquad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \qquad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

f169

By converse of BPT we have

$$EF \| BC$$

From (1) we get $$\frac{OX}{OA} = \frac{FB}{AF}$$

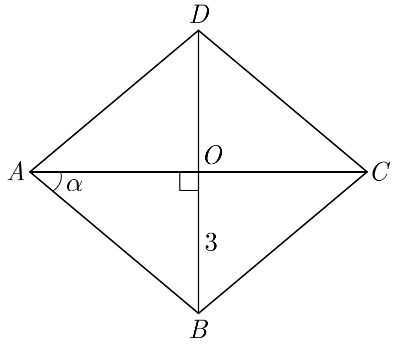
$$\frac{OX + OA}{OA} = \frac{FB + AF}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

107. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.



Ans : [Board Term-1 2013]

We have $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm

In ΔAOB, $\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$

f171

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$

$$(3x)^2 = (2x)^2 + (3)^2$$

$$9x^2 = 4x^2 + 9$$

$$5x^2 = 9$$

$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and $$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

Diagonal $$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

and $$AC = 2AO$$

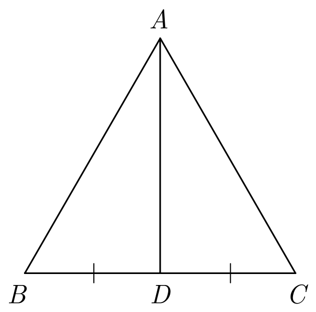
$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

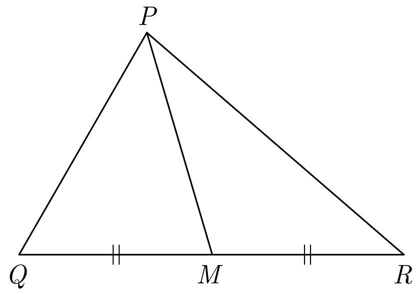
108. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012, 2013]

As per given condition we have drawn the figure below.

f173





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

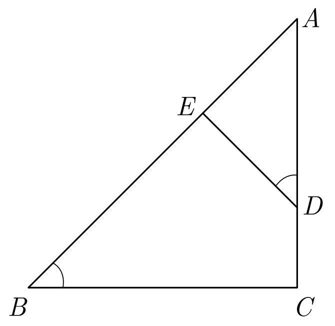
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

109. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.
Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.



Ans : [Board Term-1 2015]

In ΔADE and ΔABC, $\angle A$ is common.

and we have $\angle ADE = \angle ABC$

f167

Due to AA similarity,

$$\Delta ADE \sim \Delta ABC$$

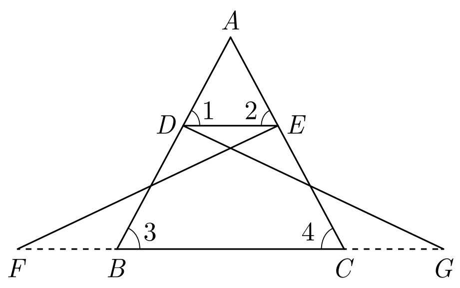
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

110. In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans : [Board Term-1 2012]

Since $\Delta FEC \cong \Delta GBD$

$$EC = BD \quad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \mid\mid BC, \qquad \text{(Converse of BPT)}$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 = \angle 4$$

f174

Thus in ΔADE and ΔABC,

$$\angle A = \angle A$$

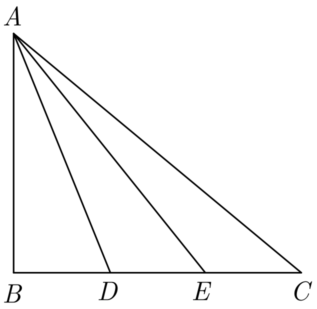
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

$$\Delta ADE \sim \Delta ABC$$ Hence proved

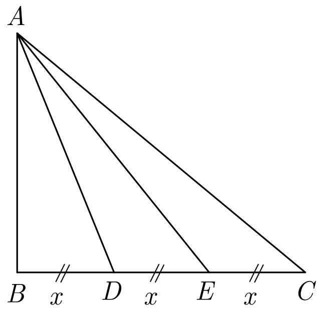
111. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Ans : [Board Term-1 2013]

As per given condition we have drawn the figure below.

f175



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $BE = 2x$ and $BC = 3x$

In ΔABE, $AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(1)

In ΔABC, $AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$ ...(2)

In ΔADB, $AD^2 = AB^2 + BD^2 = AB^2 + x^2$ ...(3)

Multiplying (2) by 3 and (3) by 5 and adding we have

$$\begin{aligned} 3AC^2 + 5AD^2 &= 3(AB^2 + 9x^2) + (AB^2 + x^2) \\ &= 3AB^2 + 27x^2 + 5AB^2 + 5x^2 \\ &= 8AB^2 + 32x^2 \\ &= 8(AB^2 + 4x^2) = 8AE^2 \end{aligned}$$

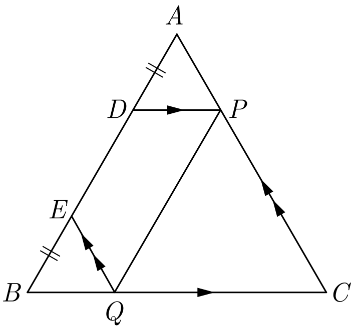
Thus $3AC^2 + 5AD^2 = 8AE^2$ Hence Proved

112. Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \mid\mid BC$ and $EQ \mid\mid AC$, then prove that $PQ \mid\mid AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.

f176



In ΔABC, $DP \mid\mid BC$

By BPT we have $\dfrac{AD}{DB} = \dfrac{AP}{PC}$, ...(1)

Similarly, in ΔABC, $EQ \mid\mid AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure, $EA = AD + DE$

$= BE + ED \quad (BE = AD)$

$= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we have

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

By converse of BPT,

$PQ \,||\, AB$ Hence Proved

113. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. [Board 2020 Delhi Basic, 2019 Delhi, 2018]

or

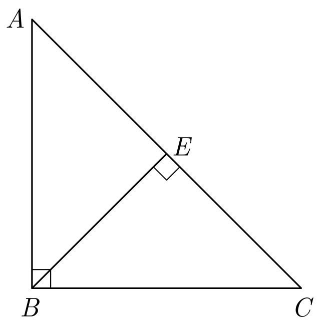
Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans : [Board Term -2 SQP 2017, 2015]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$

f178



In ΔAEB and ΔABC $\angle A$ common and

$\angle E = \angle B$ (each 90^o)

By AA similarity we have

$$\Delta AEB \sim \Delta ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, in ΔCEB and ΔCBA, $\angle C$ is common and

$\angle E = \angle B$ (each 90^o)

By AA similarity we have

$$\Delta AEB \sim \Delta CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \quad ...(2)$$

Adding equation (1) and (2) we have

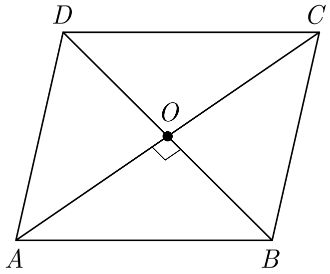
$AB^2 + BC^2 = AE \times AC + CE \times AC$

$= AC(AE + CE)$

$= AC \times AC$

Thus $AB^2 + BC^2 = AC^2$ Hence proved

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

$$AO = OC = \frac{1}{2}AC$$

and $$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$\angle AOB = 90^o$

$AB^2 = OA^2 + OB^2$

$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$

$= \frac{AC^2}{4} + \frac{BD^2}{4}$

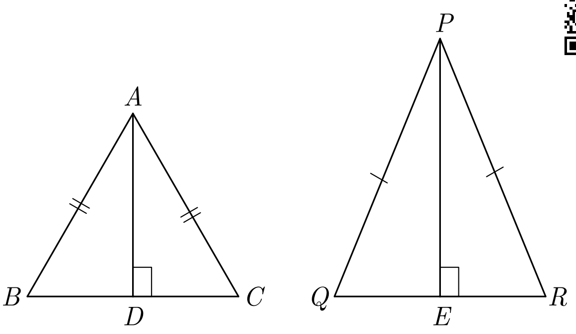
or $4AB^2 = AC^2 + BD^2$ Hence proved

114. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio $16:25$, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.

f179



Here $\angle A = \angle P$ $\angle B = \angle C$ and $\angle Q = \angle R$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^o$

$$x + \angle B + \angle B = 180^o \qquad (\angle B = \angle C)$$

$$2\angle B = 180^o - x$$

$$\angle B = \frac{180^o - x}{2} \qquad ...(1)$$

Now, in ΔPQR,

$$\angle P + \angle Q + \angle R = 180^o \qquad (\angle Q = \angle R)$$

$$x^2 + \angle Q + \angle Q = 180^o$$

$$2\angle Q = 180^o - x$$

$$\angle Q = \frac{180^o - x}{2}$$

In ΔABC and ΔPQR,

$$\angle A = \angle P \qquad \text{[Given]}$$

$$\angle B = \angle Q \qquad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\Delta ABC \sim \Delta PQR$$

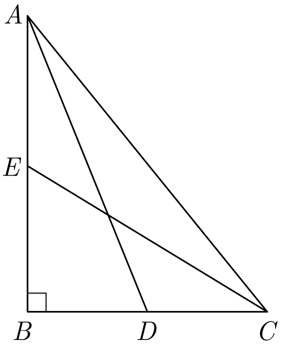
Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

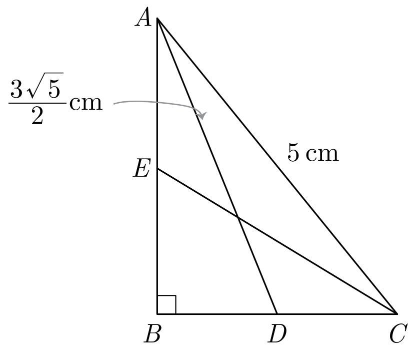
Thus $$\frac{AD}{PE} = \frac{4}{5}$$

115. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



Ans : [Board Term-1 2013]

We have redrawn the given figure as below.



Here in ΔABC, $\angle B = 90^o$, AD and CE are two medians.

Also we have $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \qquad ...(1)$$

In ΔABD, $$AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \qquad ...(2)$$

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4} \qquad ..(3)$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \quad ...(4)$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

f180

From equation (3) we get

$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

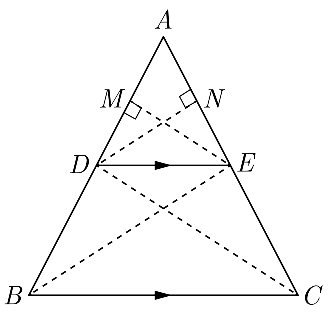
Thus $$CE = \sqrt{20} = 2\sqrt{5} \text{ cm.}$$

116. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board 2019 OD, SQP 2020 STD, 2012]

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.

f181



In ΔADE,

$$\text{area } (\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(1)$$

In ΔDEC,

$$\text{area } (\Delta DCE) = \frac{1}{2} \times CE \times DN \quad ...(2)$$

Dividing equation (1) by (2) we have,

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{AE}{CE} \quad ...(3)$$

Now in ΔADE,

$$\text{area}(\Delta ADE) = \frac{1}{2} \times AD \times EM \quad ...(4)$$

and in ΔDEB,

$$\text{area}(\Delta DEB) = \frac{1}{2} \times EM \times BD \quad ...(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD} \quad ...(6)$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area}(\Delta DEB) = \text{area}(\Delta DEC)$$

From equation (3) we have

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AE}{CE} \quad ...(7)$$

From equations (6) and (7) we get

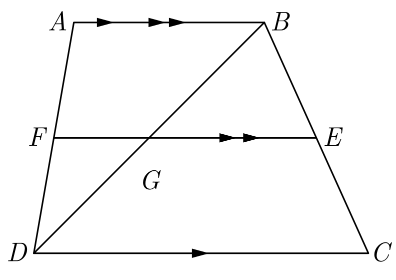
$$\frac{AE}{CE} = \frac{AD}{BD}.$$ Hence proved.

117. In a trapezium $ABCD, AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.

f182



In trapezium $ABCD$,

$$AB \,||\, DC \text{ and } DC = 2AB.$$

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

Thus $$EF \,||\, AB \,||\, CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \quad ...(1)$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \quad ...(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \quad ...(3)$$

Similarly, $$\Delta DGF \sim \Delta DBA$$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \quad ...(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7}DC + \frac{3}{7}AB$$

$$EF = \frac{4}{7}\times(2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$$

$$7EF = 11AB$$ Hence proved.

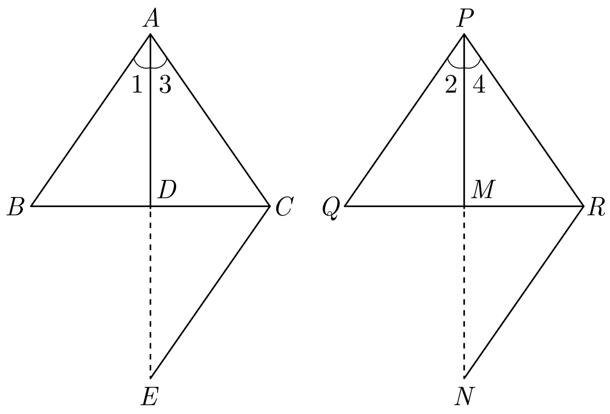
118. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN. As per given condition we have drawn the figure below.



In ΔABD and ΔEDC,

$$AD = DE \quad \text{(By construction)}$$

$$\angle ADB = \angle EDC \quad \text{(VOA)}$$

$$BD = DC \quad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \quad \text{(By CPCT)}$$

Similarly, $PQ = RN$ and $\angle A = \angle 2$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \quad \text{(Given)}$$

, $$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

f183

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

By SAS similarity, we have

$$\Delta ABC \sim \Delta PQR$$ Hence Proved

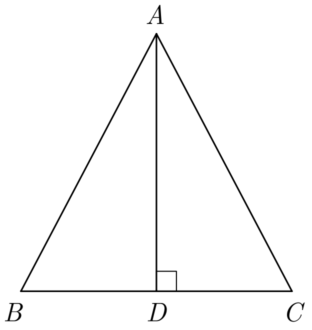
119. In $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans : [Board Term-1 2015]

It is given in a triangle $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.

As per given condition we have drawn the figure below.

f184



Since $\quad 2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$.

Since $\angle D = 90^\circ$ in ΔADB, we have

$$AB^2 = AD^2 + DB^2 = AD^2 + (3x)^2$$

$$= AD^2 + 9x^2$$

$$5AB^2 = 5AD^2 + 45x^2$$

$$5AD^2 = 5AB^2 - 45x^2 \quad ...(1)$$

and $\quad AC^2 = AD^2 + CD^2 = AD^2 + (2x)^2$

$$= AD^2 + 4x^2$$

$$5AC^2 = 5AD^2 + 20x^2$$

$$5AD^2 = 5AC^2 - 20x^2 \quad ...(2)$$

Comparing equation (1) and (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$

$$= 5AC^2 + 25x^2$$

$$= 5AC^2 + (5x)^2$$

$$= 5AC^2 + BC^2 \quad [BC = 5x]$$

Therefore $\quad 5AB^2 = 5AC^2 + BC^2$ Hence proved

120. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

Prove that : $\quad 9AQ^2 = 9AC^2 + 4BC^2$

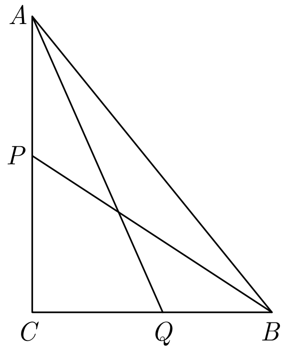
$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.

f185



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

and Q divides CB in the ratio $2:1$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

or, $\quad 9AQ^2 = 4BC^2 + 9AC^2 \quad ...(1)$

Similarly, we get

$$9BP^2 = 9BC^2 + 4AC^2 \quad ...(2)$$

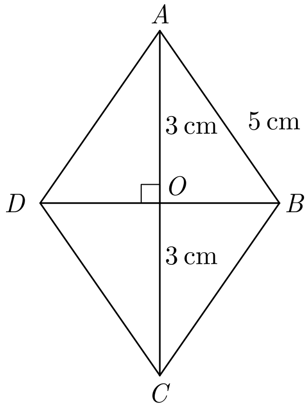
Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

121. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $\quad AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

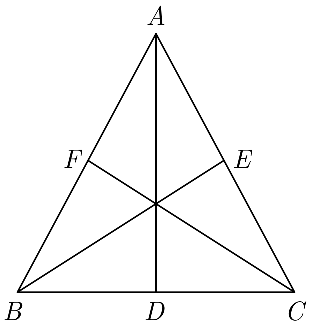
Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $\quad BD = 2 \times BO = 2 \times 4 = 8$ cm

122. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.

If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \qquad ...(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + BC^2) = 4BE^2 + AC^2 \qquad ...(2)$$

and $\quad 2(AC^2 + BC^2) = 4CF^2 + AB^2 \qquad ...(3)$

Adding, (1), (2) and (iii), we get

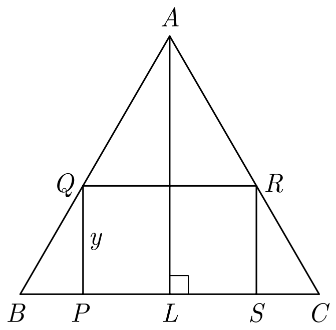
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

123. ABC is an isosceles triangle in which $AB = AC = 10$ cm $BC = 12$ cm $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = y$, $PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.

Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm.}$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA, angle $\angle B$ is common and

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

$$\frac{6 - x}{y} = \frac{6}{8}$$

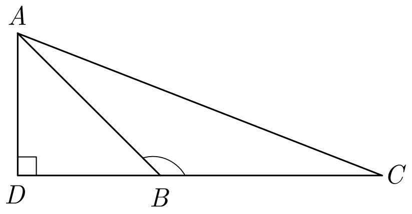
$$x = 6 - \frac{3y}{4} \qquad \text{Hence proved.}$$

124. If ΔABC is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :
$AC^2 = AB^2 + BC^2 + 2BC \times BD$

Ans : [Board 2020 Delhi Basic]

As per given condition we have drawn the figure below.

f189



In ΔADB, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \qquad ...(1)$$

In ΔADC, By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$

$$= AD^2 + (BC + BD)^2$$

$$= AD^2 + BC^2 + 2BC \times BD + BD^2$$

$$= (AD^2 + BD^2) + 2BC \times BD$$

Substituting $(AD^2 + BD^2) = AB^2$ we have

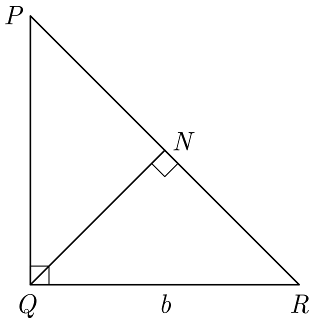
$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

125. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.

f190



Let $QR = b$, then we have

$$A = ar(\Delta PQR)$$

$$= \frac{1}{2} \times b \times PQ$$

$$PQ = \frac{2 \cdot A}{b} \qquad ...(1)$$

Due to AA similarity we have

$$\Delta PNQ \sim \Delta PQR$$

$$\frac{PQ}{PR} = \frac{NQ}{QR} \qquad ...(2)$$

From ΔPQR

$$PQ^2 + QR^2 = PR^2$$

$$\frac{4A^2}{b^2} + b^2 = PR^2$$

$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}}$$

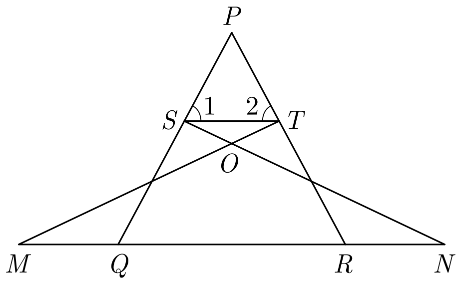
Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$

$$NQ = \frac{2A}{PR}$$

Altitude, $$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}} \qquad \text{Hence Proved.}$$

126. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Ans : [Board Term-1 SQP 2017]

We have $\Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

f191

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

Also $\angle 2 = \angle QPR$ (common)

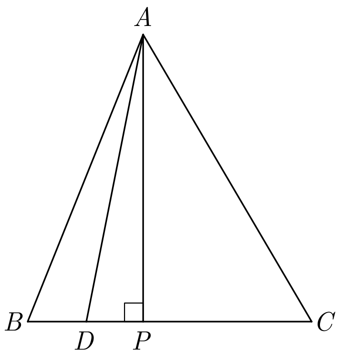
Thus by AAA similarity,

$$\Delta PTS \sim \Delta PRQ$$

127. In an equilateral triangle ABC, D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Ans : [Board 2018, SQP 2017]

As per given condition we have shown the figure below. Here we have drawn $AP \perp BC$.



Here $AB = BC = CA$ and $BD = \frac{1}{3}BC$.

f192

In ΔADP,

$$AD^2 = AP^2 + DP^2$$

$$= AP^2 + (BP - BD)^2$$

$$= AP^2 + BP^2 + BD^2 + 2BP \cdot BD$$

From ΔAPB using $AP^2 + BP^2 = AB^2$ we have

$$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$$

$$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2$$

$$9AD^2 = 7AB^2$$ Hence Proved

CHAPTER 7

COORDINATE GEOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. The point P on x-axis equidistant from the points $A(-1,0)$ and $B(5,0)$ is

(a) (2, 0) (b) (0, 2)

(c) (3, 0) (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

Let the position of the point P on x-axis be $(x,0)$, then

$$PA^2 = PB^2$$

$$(x+1)^2+(0)^2 = (5-x)^2+(0)^2$$

$$x^2+2x+1 = 25+x^2-10x$$

$$2x+10x = 25-1$$

$$12x = 24 \Rightarrow x = 2$$

Hence, the point $P(x,0)$ is (2, 0).

Thus (a) is correct option.

Alternative :

You may easily observe that both point $A(-1,0)$ and $B(5,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-1,0)$ and $B(5,0)$.

$$x = \frac{-1+5}{2} = 2$$

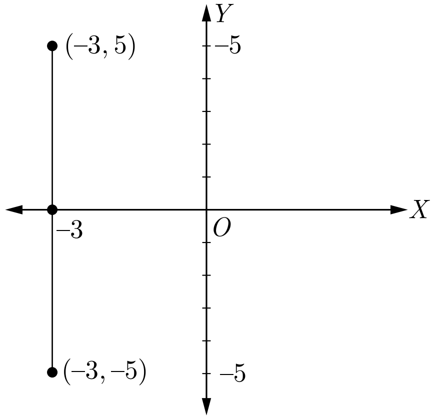
2. The co-ordinates of the point which is reflection of point $(-3,5)$ in x-axis are

(a) (3, 5) (b) $(3, -5)$

(c) $(-3,-5)$ (d) $(-3,5)$

Ans : [Board 2020 OD Standard]

The reflection of point $(-3,5)$ in x- axis is $(-3,-5)$.



Thus (c) is correct option.

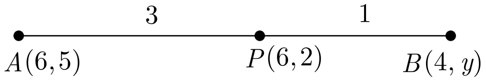
3. If the point P (6, 2) divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$ then the value of y is

(a) 4 (b) 3

(c) 2 (d) 1

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, $x_1 = 6,\ y_1 = 5$

and $x_2 = 4\ \ y_2 = y$

Now
$$y = \frac{my_2+ny_1}{m+n}$$

$$2 = \frac{3\times y+1\times 5}{3+1}$$

$$2 = \frac{3y+5}{4}$$

$$3y+5 = 8$$

$$3y = 8-5 = 3 \Rightarrow y = 1$$

Thus (d) is correct option.

4. The distance between the points $(a\cos\theta + b\sin\theta,\ 0)$, and $(0,\ a\sin\theta - b\cos\theta)$ is

(a) $a^2 + b^2$ (b) $a^2 - b^2$

(c) $\sqrt{a^2 + b^2}$ (d) $\sqrt{a^2 - b^2}$

Ans : [Board 2020 Delhi Standard]

We have $x_1 = a\cos\theta + b\sin\theta$ and $y_1 = 0$

and $x_2 = 0$ and $y_2 = a\sin\theta - b\cos\theta$

g218

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2$$
$$= (-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$
$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + $$
$$+ a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$
$$= a^2 \times 1 + b^2 \times 1 = a^2 + b^2$$

Thus $d^2 = a^2 + b^2$

$$d = \sqrt{a^2 + b^2}$$

Therefore (c) is correct option.

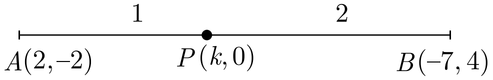
5. If the point $P(k,\ 0)$ divides the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ in the ratio 1 : 2, then the value of k is

(a) 1 (b) 2

(c) -2 (d) -1

Ans : [Board 2020 Delhi Standard]

As per question statement figure is shown below.



$$k = \frac{1(-7) + 2(2)}{1 + 2} \qquad \left(x = \frac{mx_2 + nx_1}{m + n}\right)$$
$$= \frac{-7 + 4}{3} = \frac{-3}{3} = -1$$

g219

Thus $k = -1$

Thus (d) is correct option.

6. The coordinates of a point A on y-axis, at a distance of 4 units from x-axis and below it are

(a) (4, 0) (b) (0, 4)

(c) $(-4, 0)$ (d) $(0, -4)$

g221

Ans :
[Board 2020 Delhi Basic]

Because the point is 4 units down the x-axis i.e., co-ordinate is -4 and on y-axis abscissa is 0. So, the coordinates of point A is $(0, -4)$.

Thus (d) is correct option.

7. The distance of the point $(-12,\ 5)$ from the origin is

(a) 12 (b) 5

(c) 13 (d) 169

g223

Ans :

The distance between the origin and the point (x, y) is $\sqrt{x^2 + y^2}$.

Therefore, the distance between the origin and point $(-12, 5)$

$$d = \sqrt{(-12 - 0)^2 + (5 - 0)^2}$$
$$= \sqrt{144 + 25} = \sqrt{169}$$
$$= 13 \text{ units}$$

Thus (c) is correct option.

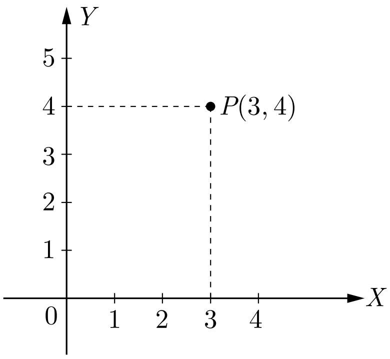
8. Distance of point $P(3, 4)$ from x-axis is

(a) 3 units (b) 4 units

(c) 5 units (d) 1 units

Ans : [Board 2020 Delhi Basic]

Point $P(3, 4)$ is 4 units from the x-axis and 3 units from the y-axis.



g224

Thus (b) is correct option.

9. The distance of the point $P(-3, -4)$ from the x-axis (in units) is

(a) 3 (b) -3

(c) 4 (d) 5

g225

Ans :
[Board 2020 SQP Standard]

Point $P(-3, -4)$ is 4 units from the x-axis and 3 units from the y-axis.

Thus (c) is correct option.

10. If $A(\frac{m}{3}, 5)$ is the mid-point of the line segment joining the points $Q(-6, 7)$ and $R(-2, 3)$, then the value of m is

(a) -12 (b) -4

(c) 12 (d) -6

g226

Ans : [Board 2020 SQP Standard]

Given points are $Q(-6, 7)$ and $R(-2, 3)$

$$\text{Mid point } A(\tfrac{m}{3}, 5) = \left(\frac{-6-2}{2}, \frac{7+3}{2}\right)$$

$$= (-4, 5)$$

$$\text{Equating,} \quad \frac{m}{3} = -4 \Rightarrow m = -12$$

Thus (a) is correct option.

11. The mid-point of the line-segment AB is $P(0, 4)$, if the coordinates of B are $(-2, 3)$ then the co-ordinates of A are

(a) $(2, 5)$ (b) $(-2, -5)$

(c) $(2, 9)$ (d) $(-2, 11)$

Ans : [Board 2020 OD Basic]

Let point A be (x, y).

Now using mid-point formula,

$$(0, 4) = \left(\frac{x-2}{2}, \frac{y+3}{2}\right)$$

g227

$$\text{Thus} \quad 0 = \frac{x-2}{2} \Rightarrow x = 2$$

$$\text{and} \quad 4 = \frac{y+3}{2} \Rightarrow y = 5$$

Hence point A is $(2, 5)$.

Thus (a) is correct option.

12. x-axis divides the line segment joining $A(2, -3)$ and $B(5, 6)$ in the ratio

(a) $2 : 3$ (b) $3 : 5$

(c) $1 : 2$ (d) $2 : 1$

Ans : [Board 2020 OD Basic]

Let point $P(x, 0)$ on x-axis divide the segment joining points $A(2, -3)$ and $B(5, 6)$ in ratio $k : 1$, then

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$0 = \frac{6k-3}{k+1}$$

g228

$$6k = 3 \Rightarrow k = \frac{1}{2}$$

Therefore ratio is $1 : 2$.

Thus (c) is correct option.

13. The point which divides the line segment joining the points $(8, -9)$ and $(2, 3)$ in the ratio $1 : 2$ internally lies in the

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Ans : [Board 2020 SQP Standard]

We have $x_1 = 8$, $y_1 = -9$, $x_2 = 2$ and $y_2 = 3$.

$$\text{and} \quad m_1 : m_2 = 1 : 2$$

Let the required point be $P(x, y)$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} = \frac{1 \times 2 + 2 \times 8}{1+2} = 6$$

$$\text{and} \quad y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} = \frac{1 \times 3 + 2(-9)}{1+2} = -5$$

Thus $(x, y) = (6, -5)$ and this point lies in IV quadrant.

Thus (d) is correct option.

g229

14. If the centre of a circle is $(3, 5)$ and end points of a diameter are $(4, 7)$ and $(2, y)$, then the value of y is

(a) 3 (b) -3

(c) 7 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, centre is the mid-point of end points of the diameter.

$$(3, 5) = \left(\frac{4+2}{2}, \frac{7+y}{2}\right)$$

g230

Comparing both the sides, we get

$$5 = \frac{7+y}{2}$$

$$7 + y = 10 \Rightarrow y = 3$$

Thus (a) is correct option.

15. If the distance between the points $A(4, p)$ and $B(1, 0)$ is 5 units then the value(s) of p is(are)

(a) 4 only (b) -4 only

(c) ± 4 (d) 0

Ans : [Board 2020 Delhi Basic]

Given, points are $A(4, p)$ and $B(1, 0)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y^2 - y_1)^2}$$

$$5 = \sqrt{(1-4)^2 + (0-p)^2}$$

$$25 = 9 + p^2$$

$$p^2 = 25 - 9 = 16$$

$$p = \pm 4$$

g231

Thus (c) is correct option.

16. If the points $(a, 0)$, $(0, b)$ and (1, 1) are collinear, then $\frac{1}{a} + \frac{1}{b}$ equals

(a) 1 (b) 2

(c) 0 (d) -1

g232

Ans :

Let the given points are $A(a, 0)$, $B(0, b)$ and $C(1, 1)$. Since, A, B, C are collinear.

Hence, $\text{ar}\,(\Delta ABC) = 0$

$$\frac{1}{2}[a(b-1) + 0(1-0) + 1(0-b)] = 0$$

$$ab - a - b = 0$$

$$a + b = ab$$

$$\frac{a+b}{ab} = 1$$

$$\frac{1}{a} + \frac{1}{b} = 1$$

Thus (a) is correct option.

17. If the points $A(4, 3)$ and $B(x, 5)$ are on the circle with centre $O(2, 3)$, then the value of x is

(a) 0 (b) 1

(c) 2 (d) 3

g233

Ans :

Since, A and B lie on the circle having centre O.

$$OA = OB$$

$$\sqrt{(4-2)^2 + (3-3)^2} = \sqrt{(x-2)^2 + (5-3)^2}$$

$$2 = \sqrt{(x-2)^2 + 4}$$

$$4 = (x-2)^2 + 4$$

$$(x-2)^2 = 0 \Rightarrow x = 2$$

Thus (c) is correct option.

18. The ratio in which the point $(2, y)$ divides the join of $(-4, 3)$ and (6, 3), hence the value of y is

(a) $2:3, y = 3$ (b) $3:2, y = 4$

(c) $3:2, y = 3$ (d) $3:2, y = 2$

Ans :

Let the required ratio be $k:1$

g234

Then, $2 = \frac{6k - 4(1)}{k+1}$

or $k = \frac{3}{2}$

The required ratio is $\frac{3}{2}:1$ or 3 : 2

Also, $y = \frac{3(3) + 2(3)}{3+2} = 3$

Thus (c) is correct option.

19. The point on the x-axis which is equidistant from the points $A(-2, 3)$ and $B(5, 4)$ is

(a) (0, 2) (b) (2, 0)

(c) (3, 0) (d) $(-2, 0)$

g235

Ans :

Let $P(x, 0)$ be a point on x-axis such that,

$$AP = BP$$

$$AP^2 = BP^2$$

$$(x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$$

$$x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$$

$$14x = 28$$

$$x = 2$$

Hence required point is (2, 0).

Thus (b) is correct option.

20. C is the mid-point of PQ, if P is $(4, x)$, C is $(y, -1)$ and Q is $(-2, 4)$, then x and y respectively are

(a) -6 and 1 (b) -6 and 2

(c) 6 and -1 (d) 6 and -2

Ans :

Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.

We have, $\frac{4-2}{2} = y \Rightarrow y = 1$

g236

and $\frac{4+x}{2} = -1 \Rightarrow x = -6$

Thus (a) is correct option.

21. If three points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ equals

(a) 2 (b) -3

(c) -4 (d) None of these

Ans :

Let the given points are $A(0,0)$, $B(3,\sqrt{3})$ and $C(3,\lambda)$.

Since, ΔABC is an equilateral triangle, therefore

$$AB = AC$$

$$\sqrt{(3-0)^2+(\sqrt{3}-0)^2} = \sqrt{(3-0)^2+(\lambda-0)^2}$$

$$9+3 = 9+\lambda^2$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$$

Thus (d) is correct option.

g237

22. If $x-2y+k=0$ is a median of the triangle whose vertices are at points $A(-1,3)$, $B(0,4)$ and $C(-5,2)$, then the value of k is

(a) 2 (b) 4

(c) 6 (d) 8

g239

Ans :

Coordinate of the centroid G of ΔABC

$$= \left(\frac{-1+0-5}{2}, \frac{3+4+2}{3}\right)$$

$$= (-2,3)$$

Since, G lies on the median, $x-2y+k=0$, it must satisfy the equation,

$$-2-6+k = 0 \Rightarrow k=8$$

Thus (d) is correct option.

23. The centroid of the triangle whose vertices are $(3,-7)$, $(-8,6)$ and $(5, 10)$ is

(a) (0, 9) (b) (0, 3)

(c) (1, 3) (d) (3, 5)

g240

Ans :

Centroid is $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

i.e. $$\left(\frac{3+(-8)+5}{3}, \frac{-7+6+10}{3}\right) = \left(\frac{0}{3}, \frac{9}{3}\right)$$

$$= (0,3)$$

Thus (b) is correct option.

24. The distance of the point $P(2,3)$ from the x-axis is

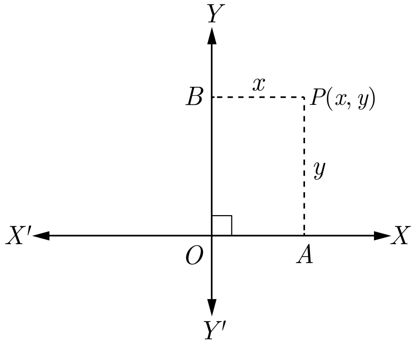
(a) 2 (b) 3

(c) 1 (d) 5

Ans :

We know that, if (x,y) is any point on the cartesian plane in first quadrant, then x is perpendicular distance from y-axis and y is perpendicular distance from x-axis.

Distance of the point $P(2,3)$ from the x-axis is 3.



g242

Thus (b) is correct option.

25. The distance between the points $A(0,6)$ and $B(0,-2)$ is

(a) 6 (b) 8

(c) 4 (d) 2

Ans :

Distance between the points (x_1,y_1) and (x_2,y_2) is given as,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 6$ and $x_2 = 0$, $y_2 = -2$

Distance between $A(0,6)$ and $B(0,-2)$

$$AB = \sqrt{(0-0)^2+(-2-6)^2}$$

$$= \sqrt{0+(-8)^2} = \sqrt{8^2} = 8$$

g243

Thus (b) is correct option.

26. The distance of the point $P(-6,8)$ from the origin is

(a) 8 (b) $2\sqrt{7}$

(c) 10 (d) 6

Ans :

Distance between the points (x,y) and origin is given as,

$$d = \sqrt{x^2+y^2}$$

Distance between $P(-6,8)$ and origin is,

$$PO = \sqrt{(6)^2+(-8)^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

g244

Thus (c) is correct option.

27. The distance between the points (0, 5) and $(-5,0)$ is

(a) 5 (b) $5\sqrt{2}$

(c) $2\sqrt{5}$ (d) 10

Ans :

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 5$ and $x_2 = -5,\ y_2 = 0$

Distance between the points (0, 5) and $(-5,0)$

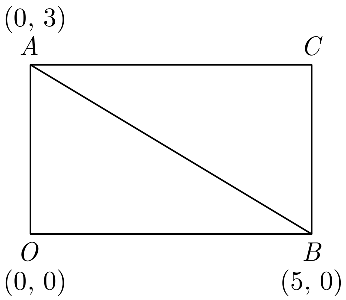
$$d = \sqrt{[-5-0]^2 + [0-(-5)]^2}$$

$$= \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

g245

Thus (b) is correct option.

28. If $AOBC$ is a rectangle whose three vertices are $A(0,3)$, $O(0,0)$ and $B(5,0)$, then the length of its diagonal is



(a) 5 (b) 3

(c) $\sqrt{34}$ (d) 4

g246

Ans :

Length of the diagonal is AB which is the distance between the points $A(0,3)$ and $B(5,0)$.

Distance between the points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0,\ y_1 = 3$, and $x_2 = 5,\ y_2 = 0$

Distance between the points $A(0,3)$ and $B(5,0)$

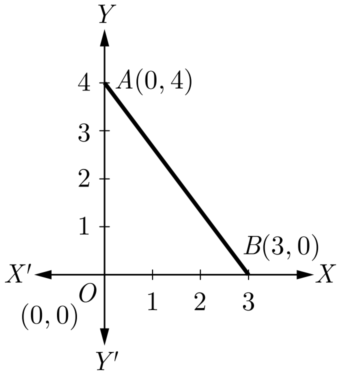
$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Thus (c) is correct option.

29. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is



g247

(a) 5 (b) 12

(c) 11 (d) $7+\sqrt{5}$

Ans :

We have $OA = 4$

$OB = 3$

and $AB = \sqrt{3^2 + 4^2} = 5$

Now, perimeter of ΔAOB is the sum of the length of all its sides.

$$p = OA + OB + AB = 4 + 3 + 5 = 12$$

Hence, the required perimeter of triangle is 12. However you can calculate perimeter direct from diagram.

Thus (b) is correct option.

30. The point which lies on the perpendicular bisector of the line segment joining the points $A(-2,-5)$ and $B(2,5)$ is

(a) (0, 0) (b) (0, 2)

(c) (2, 0) (d) $(-2,0)$

g249

Ans :

We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

Mid-point of the line segment joining the points $A(-2,-5)$ and $B(2,5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0,0)$$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Thus (a) is correct option.

31. If the point $P(2,1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then

(a) $AP = \frac{1}{3}AB$ (b) $AP = PB$

(c) $PB = \frac{1}{3}AB$ (d) $AP = \frac{1}{2}AB$

Ans :

Let, $AP:AB = m:n$

Using section formula, we have,

$$4 = \frac{8m+2n}{m+n}$$

and $$2 = \frac{4m+n}{m+n}$$

Solving these as linear equation, we get,

$$m = 1 \text{ and } n = 2$$

$$\frac{AP}{AB} = \frac{1}{2}$$

$$AP = \frac{1}{2}AB$$

Thus (d) is correct option.

32. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6,5)$ and $R(-2,3)$, then the value of a is

(a) -4 (b) -12

(c) 12 (d) -6

Ans :

Since $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the points $Q(-6,5)$ and $R(-2,3)$,

$$\left(\frac{a}{3}, 4\right) = \left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$$

$$\left(\frac{a}{3}, 4\right) = (-4, 4)$$

Now $$\frac{a}{3} = -4 \Rightarrow a = -12$$

Thus (b) is correct option.

33. The perpendicular bisector of the line segment joining the points $A(1,5)$ and $B(4,6)$ cuts the y-axis at

(a) (0, 13) (b) (0, $-$13)

(c) (0, 12) (d) (13, 0)

Ans :

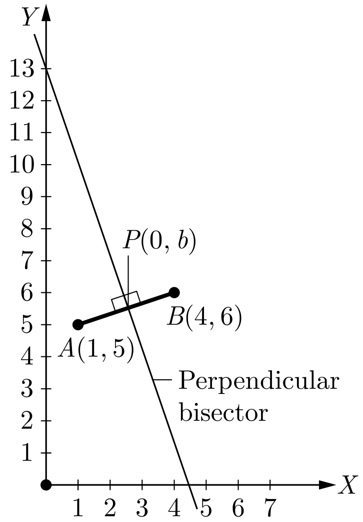
Let $P(0,b)$ be the required point. Since, any point on perpendicular bisector is equidistant from the end point of line segment.

i.e., $$PA = PB$$

$$\sqrt{(0-1)^2+(b-5)^2} = \sqrt{(0-4)^2+(b-6)^2}$$

$$1 + b^2 - 10b + 25 = 16 + b^2 - 12b + 36$$

$$2b = 26 \Rightarrow b = 13$$



Thus (a) is correct option.

34. If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is

(a) 4 only (b) ± 4

(c) -4 only (d) 0

Ans :

According to the question, the distance between the points $(4, p)$ and $(1, 0)$ is 5.

i.e., $$\sqrt{(1-4)^2+(0-p)^2} = 5$$

$$\sqrt{(-3)^2+p^2} = 5$$

$$\sqrt{9+p^2} = 5$$

Squaring both the sides, we get,

$$9 + p^2 = 25$$

$$p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

Thus (b) is correct option.

35. Assertion : The value of y is 6, for which the distance between the points $P(2,-3)$ and $Q(10, y)$ is 10.

Reason : Distance between two given points $A(x_1, y_1)$ *and* $B(x_2, y_2)$ is given,

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$PQ = 10$$
$$PQ^2 = 100$$
$$(10-2)^2+(y+3)^2 = 100$$
$$(y+3)^2 = 100-64 = 36$$
$$y+3 = \pm 6$$
$$y = -3\pm 6$$
$$y = 3, -9$$

Assertion (A) is false but reason (R) is true.
Thus (s) is correct option.

Fill in the Blank Questions

36. All the points equidistant from two given points A and B lie on the .... of the line segment AB.

Ans :

perpendicular bisector

37. The distance of a point from the y-axis is called its ....

Ans :

abscissa

38. The distance of a point from the x-axis is called its ....

Ans :

ordinate

39. The value of the expression $\sqrt{x^2+y^2}$ is the distance of the point $P(x,y)$from the ....

Ans :

origin

40. The distance of the point (p,q) from (a,b) is ....

Ans :

$\sqrt{(a-p)^2+(b-q)^2}$

41. If the area of the triangle formed by the vertices $A(x_1,y_1)\,B(x_2,y_2)$ and $C(x_3,y_3)$is zero, then the points A,B and C are ....

Ans :

collinear

42. A point of the form $(b,0)$ lies on ....

Ans :

x-axis

43. The distance of the point (x_1,y_1) from the origin is ....

Ans :

$\sqrt{x_1^2+y_1^2}$

44. A point of the form $(0,a)$ lies on ....

Ans :

y-axis

45. If the point $C(k,\ 4)$ divides the line segment joining two points $A(2,\ 6)$ and $B(5,1)$ in ratio 2 : 3, the value of k is ....

Ans : [Board 2020 Delhi Basic]

We have $m:n = 2:3$

By section formula,

$$\frac{mx_2+nx_1}{m+n} = x$$

Now, $$\frac{2\times 5+3\times 2}{2+3} = k \Rightarrow k = \frac{16}{5}$$

46. If points $A(-3,12)$, $B(7,6)$ and $C(x,9)$ are collinear, then the value of x is ....

Ans : [Board 2020 Delhi Basic]

If points are collinear, then area of triangle must be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$
$$\frac{1}{2}[-3(6-9)+7(9-12)+x(12-6)] = 0$$
$$\frac{1}{2}(9-21+6x) = 0$$
$$\frac{1}{2}(-12+6x) = 0$$
$$6x = 12 \Rightarrow x = 2$$

47. The co-ordinate of the point dividing the line segment joining the points $A(1,3)$ and $B(4,6)$ in the ratio 2 : 1 is ....

Ans : [Board 2020 OD Basic]

Let point $P(x,\ y)$ divides the line segment joir points $A(1,3)$ and $B(4,6)$ in the ratio 2 : 1.
Using section formula we have

$$(x,\ y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(x,\ y) = \left(\frac{2\times4+1\times1}{2+1}, \frac{2\times6+1\times3}{2+1}\right)$$

$$= \left(\frac{8+1}{3}, \frac{12+3}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3,5)$$

VERY SHORT ANSWER QUESTIONS

48. Find the distance of a point $P(x, y)$ from the origin.

Ans : [Board 2018]

Distance between origin (0, 0) and point $P(x, y)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x-0)^2 + (y-0)^2}$$

$$= \sqrt{x^2 + y^2}$$

g268

Distance between P and origin is $\sqrt{x^2 + y^2}$.

49. If the mid-point of the line segment joining the points $A(3,4)$ and $B(k,6)$ is $P(x,y)$ and $x + y - 10 = 0$, find the value of k.

Ans : [Board 2020 OD Standard]

If $P(x,y)$ is mid point of $A(3,4)$ and $B(k,6)$, then we have

$$\frac{3+k}{2} = x \text{ and } y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Substituting above value in $x + y - 10 = 0$ we have

$$\frac{3+k}{2} + 5 - 10 = 0$$

$$\frac{3+k}{2} = 5$$

g269

$$3 + k = 10 \Rightarrow k = 10 - 3 = 7$$

50. Write the coordinates of a point P on x-axis which is equidistant from the points $A(-2,0)$ and $B(6,0)$.

Ans : [Board 2019 OD]

Since it is equidistant from the points $A(-2,0)$ and $B(6,0)$ then

$$AP = BP$$

270

$$AP^2 = BP^2$$

Using distance formula we have

$$[(x-(-2)]^2 + (0-0)^2 = (x+6)^2 + (0-0)^2$$

$$(x+2)^2 = (x+6)^2$$

$$x^2 + 4x + 4 = x^2 + 12x + 36$$

$$8x = -32$$

$$x = -4$$

Hence, required point P is $(-4, 0)$.

Alternative :

You may easily observe that both point $A(-2,0)$ and $B(6,0)$ lies on x-axis because y ordinate is zero. Thus point P on x-axis equidistant from both point must be mid point of $A(-2,0)$ and $B(6,0)$.

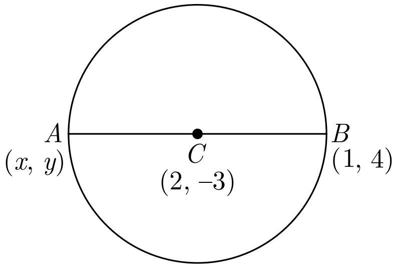
$$x = \frac{-2+6}{2} = 2$$

51. Find the coordinates of a point A, where AB is diameter of a circle whose centre is $(2,-3)$ and B is the point $(1,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.

g271



Let the co-ordinates of point A be (x, y).

x-coordinate of C,

$$\frac{x+1}{2} = 2$$

$$x + 1 = 4 \Rightarrow x = 3$$

and y-coordinate of C,

$$\frac{y+4}{2} = -3$$

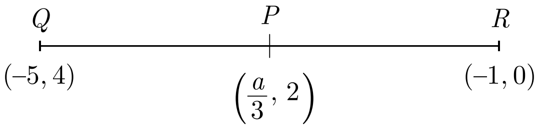
$$y + 4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A are $(3, -10)$.

52. Find the value of a, for which point $P(\frac{a}{3}, 2)$ is the midpoint of the line segment joining the Points $Q(-5,4)$ and $R(-1,0)$.

Ans : [Board Term-2 SQP 2016]

As per question, line diagram is shown below.



Since P is mid-point of QR, we have

$$\frac{a}{3} = \frac{-5+(-1)}{2} = \frac{-6}{2} = -3$$

Thus $\qquad a = -9$

53. The ordinate of a point A on y-axis is 5 and B has co-ordinates $(-3,1)$. Find the length of AB.

Ans : [Board Term-2 2014]

We have $A(0,5)$ and $B(-3,1)$.

Distance between A and B,

$$\begin{aligned} AB &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(-3-0)^2+(1-5)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \end{aligned}$$

54. Find the perpendicular distance of $A(5,12)$ from the y-axis.

Ans : [Board Term-2 2011]

Perpendicular from point $A(5,12)$ on y-axis touch it at $(0,12)$.

Distance between $(5,12)$ and $(0,12)$ is,

$$\begin{aligned} d &= \sqrt{(0-5)^2+(12-12)^2} \\ &= \sqrt{25} \\ &= 5 \text{ units.} \end{aligned}$$

55. If the centre and radius of circle is $(3,4)$ and 7 units respectively,, then what it the position of the point $A(5,8)$ with respect to circle?

Ans : [Board Term-2 2013]

Distance of the point, from the centre,

$$\begin{aligned} d &= \sqrt{(5-3)^2+(8-4)^2} \\ &= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

Since $2\sqrt{5}$ is less than 7, the point lies inside the circle.

56. Find the perimeter of a triangle with vertices $(0,4)$, $(0,0)$ and $(3,0)$.

Ans : [Board Term-2, 2011]

We have $A(0,4), B(0,0)$, and $C(3,0)$.

$$\begin{aligned} AB &= \sqrt{(0-2)^2+(0-4)^2} = \sqrt{16} = 4 \\ BC &= \sqrt{(3-0)^2+(0-0)^2} = \sqrt{9} = 3 \\ CA &= \sqrt{(0-3)^2+(4-0)^2} \\ &= \sqrt{9+16} = \sqrt{25} = 5 \end{aligned}$$

Thus perimeter of triangle is $4+3+5 = 12$

57. Locate a point Q on line segment AB such that $BQ = \frac{5}{7} \times AB$. What is the ratio of line segment in which AB is divided?

Ans : [Board Term-2 2013]

We have $\qquad BQ = \frac{5}{7}AB$

$$\frac{BQ}{AB} = \frac{5}{7} \Rightarrow \frac{AB}{BQ} = \frac{7}{5}$$

$$\frac{AB-BQ}{BQ} = \frac{7-5}{5}$$

$$\frac{AQ}{BQ} = \frac{2}{5}$$

Thus $\qquad AQ:BQ = 2:5$

58. Find the distance of the point $(-4,-7)$ from the y-axis.

Ans : [Board Term-2 2013]

Perpendicular from point $A(-4,-7)$ on y-axis touch it at $(0,-7)$.

Distance between $(-4,-7)$ and $(0,-7)$ is

$$\begin{aligned} d &= \sqrt{(0+4)^2+(-7+7)^2} \\ &= \sqrt{4^2+0} = \sqrt{16} = 4 \text{ units} \end{aligned}$$

59. If the distance between the points $(4,k)$ and $(1,0)$ is 5, then what can be the possible values of k.

Ans : [Board Term-2 2017]

Using distance formula we have

$$\begin{aligned} \sqrt{(4-1)^2+(k-0)^2} &= 5 \\ 3^2+k^2 &= 25 \\ k^2 &= 25-9 = 16 \\ k &= \pm 4 \end{aligned}$$

60. Find the coordinates of the point on y-axis which is nearest to the point $(-2,5)$.

Ans : [Board Term-2 SQP 2017]

Point $(0,5)$ on y-axis is nearest to the point $(-2,5)$.

g109

61. In what ratio does the x-axis divide the line segment joining the points $(-4,-6)$ and $(-1,7)$? Find the coordinates of the point of division.

Ans : [Board Term-2 SQP 2017]

Let x-axis divides the line-segment joining $(-4,-6)$ and $(-1,7)$ at the point $P(x,y)$ in the ratio $1:k$.

Now, the coordinates of point of division P,

$$(x,y) = \frac{1(-1)+k(-4)}{k+1}, \frac{1(7)+k(-6)}{k+1}$$

g110

$$= \frac{-1-4k}{k+1}, \frac{7-6k}{k+1}$$

Since P lies on x axis, therefore $y=0$, which gives

$$\frac{7-6k}{k+1} = 0$$

$$7-6k = 0$$

$$k = \frac{7}{6}$$

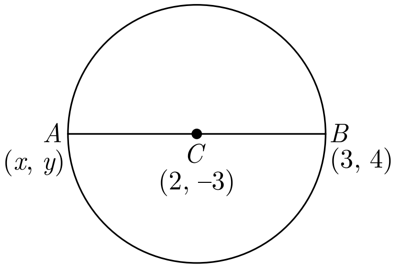
Hence, the ratio is $1:\frac{7}{6}$ or, $6:7$ and the coordinates of P are $\left(-\frac{34}{13}, 0\right)$.

TWO MARKS QUESTIONS

62. Find the coordinates of a point A, where AB is diameter of the circle whose centre is $(2,-3)$ and B is the point $(3,4)$.

Ans : [Board 2019 Delhi]

As per question we have shown the figure below. Since, AB is the diameter, centre C must be the mid point of the diameter of AB.



Let the co-ordinates of point A be (x,y).

x-coordinate of C,

g275

$$\frac{x+3}{2} = 2$$

$$x+3 = 4 \Rightarrow x = 1$$

and y-coordinate of C,

$$\frac{y+4}{2} = -3$$

$$y+4 = -6 \Rightarrow y = -10$$

Hence, coordinates of point A is $(1,-10)$.

63. Find a relation between x and y such that the point $P(x,y)$ is equidistant from the points $A(-5,3)$ and $B(7,2)$.

Ans : [Board Term-2 SQP 2016]

Let $P(x,y)$ is equidistant from $A(-5,3)$ and $B(7,2)$, then we have

$$AP = BP$$

$$\sqrt{(x+5)^2+(y-3)^2} = \sqrt{(x-7)^2+(y-2)^2}$$

g115

$$(x+5)^2+(y-3)^2 = (x-7)^2+(y-2)^2$$

$$10x+25-6y+9 = -14x+49-4y+4$$

$$24x+34 = 2y+53$$

$$24x-2y = 19$$

Thus $24x-2y-19 = 0$ is the required relation.

64. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2,-5)$ and $R(-3,6)$, find the co-ordinates of P.

Ans : [Board Term-2 2016]

Let the point P be $(2y,y)$. Since $PQ = PR$, we have

$$\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$$(2y-2)^2+(y+5)^2 = (2y+3)^2+(y-6)^2$$

g116

$$-8y+4+10y+25 = 12y+9-12y+36$$

$$2y+29 = 45$$

$$y = 8$$

Hence, coordinates of point P are $(16,8)$

65. Find the ratio in which y-axis divides the line segment joining the points $A(5,-6)$ and $B(-1,-4)$. Also find the co-ordinates of the point of division.

Ans : [Delhi Set I, II, III, 2016]

Let y -axis be divides the line-segment joining $A(5,-6)$ and $B(-1,-4)$ at the point $P(x,y)$ in the ratio $AP:PB = k:1$

Now, the coordinates of point of division P,

$$(x, y) = \left(\frac{k(-1)+1(5)}{k+1}, \frac{k(-4)+1(-6)}{k+1}\right)$$

$$= \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

Since P lies on y axis, therefore $x = 0$, which gives

$$\frac{5-k}{k+1} = 0 \Rightarrow k = 5$$

Hence required ratio is 5:1,

g117

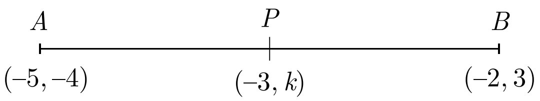
Now $$y = \frac{-4(5)-6}{6} = \frac{-13}{3}$$

Hence point on y-axis is $(0, -\frac{13}{3})$.

66. Find the ratio in which the point $(-3, k)$ divides the line segment joining the points $(-5, -4)$ and $(-2, 3)$. Also find the value of k.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



Let AB be divides by P in ratio $n:1$.

x co-ordinate for section formula

g118

$$-3 = \frac{(-2)n+1(-5)}{n+1}$$

$$-3(n+1) = -2n-5$$

$$-3n-3 = -2n-5$$

$$5-3 = 3n-2n$$

$$2 = n$$

Ratio $$\frac{n}{1} = \frac{2}{1} \text{ or } 2:1$$

Now, y co-ordinate,

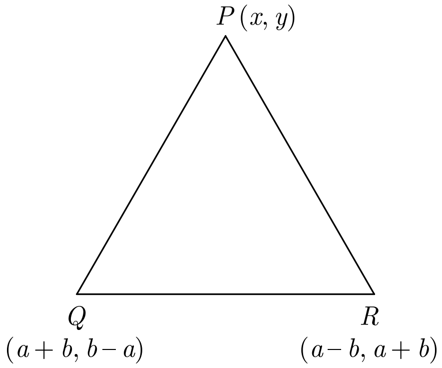
$$k = \frac{2(3)+1(-4)}{2+1} = \frac{6-4}{3} = \frac{2}{3}$$

67. If the point $P(x, y)$ is equidistant from the points $Q(a+b, b-a)$ and $R(a-b, a+b)$, then prove that $bx = ay$.

Ans : [Board Term-2 Delhi 2012, OD 2016]

We have $$|PQ| = |PR|$$

$$\sqrt{[x-(a+b)]^2+[y-(b-a)]^2} = \sqrt{[x-(a-b)]^2+[y-(b+a)]^2}$$



$$[x-(a+b)]^2+[y-(b-a)]^2 = [x-(a-b)]^2+[y-(a+b)]^2$$

$$-2x(a+b)-2y(b-a) = -2x(a-b)-2y(a+b)$$

$$2x(a+b)+2y(b-a) = 2x(a-b)+2y(a+b)$$

$$2x(a+b-a+b)+2y(b-a-a-b) = 0$$

$$2x(2b)+2y(-2a) = 0$$

$$xb-ay = 0$$

$$bx = ay$$ Hence Proved

g119

68. Prove that the point $(3,0)$, $(6,4)$ and $(-1,3)$ are the vertices of a right angled isosceles triangle.

Ans : [Board Term-2 OD 2016]

We have $A(3,0)$, $B(6,4)$ and $C(-1,3)$

Now $$AB^2 = (3-6)^2+(0-4)^2$$

g120

$$= 9 + 16 = 25$$

$$BC^2 = (6+1)^2 + (4-3)^2$$

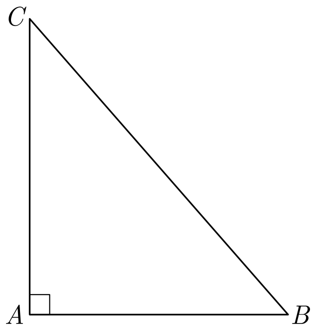
$$= 49 + 1 = 50$$

$$CA^2 = (-1-3)^2 + (3-0)^2$$

$$= 16 + 9 = 25$$

$$AB^2 = CA^2 \text{ or, } AB = CA$$

Hence triangle is isosceles.



Also, $25 + 25 = 50$

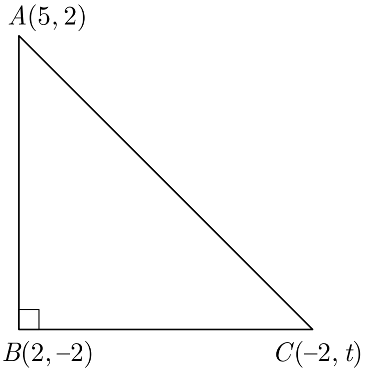
or, $AB^2 + CA^2 = BC^2$

Since Pythagoras theorem is verified, therefore triangle is a right angled triangle.

69. If $A(5,2)$, $B(2,-2)$ and $C(-2,t)$ are the vertices of a right angled triangle with $\angle B = 90^o$, then find the value of t.

Ans : [Board Term-2 Delhi 2015]

As per question, triangle is shown below.



Now $AB^2 = (2-5)^2 + (-2-2)^2 = 9 + 16 = 25$

$$BC^2 = (-2-2)^2 + (t+2)^2 = 16 + (t+2)^2$$

$$AC^2 = (5+2)^2 + (2-t)^2 = 49 + (2-t^2)$$

Since ΔABC is a right angled triangle

g121

$$AC^2 = AB^2 + BC^2$$

$$49 + (2-t)^2 = 25 + 16 + (t+2)^2$$

$$49 + 4 - 4t + t^2 = 41 + t^2 + 4t + 4$$

$$53 - 4t = 45 + 4t$$

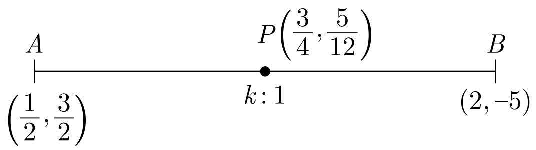
$$8t = 8$$

$$t = 1$$

70. Find the ratio in which the point $P(\frac{3}{4}, \frac{5}{12})$ divides the line segment joining the point A $(\frac{1}{2}, \frac{3}{2})$ and $(2,-5)$.

Ans : [Board Term-2 Delhi 2015]

Let P divides AB in the ratio $k{:}1$. Line diagram is shown below.



g122

Now $\dfrac{k(2) + 1(\frac{1}{2})}{k+1} = \dfrac{3}{4}$

$$8k + 2 = 3k + 3$$

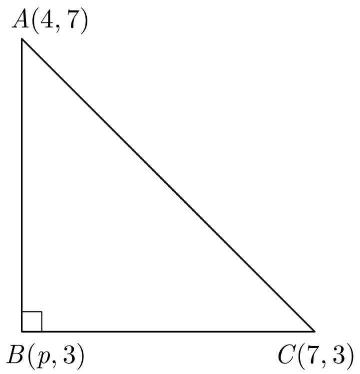
$$k = \frac{1}{5}$$

Thus required ratio is $\frac{1}{5}{:}1$ or 1:5.

71. The points $A(4,7)$, $B(p,3)$ and $C(7,3)$ are the vertices of a right triangle, right-angled at B. Find the value of p.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below. Here ΔABC is a right angle triangle,



$$AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + (3-7)^2 + (7-p)^2 + (3-3)^2 = (7-4)^2 + (3-4)^2$$

$$(p-4)^2 + (-4)^2 + (7-p)^2 + 0 = (3)^2 + (-4)^2$$

$$p^2 - 8p + 16 + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

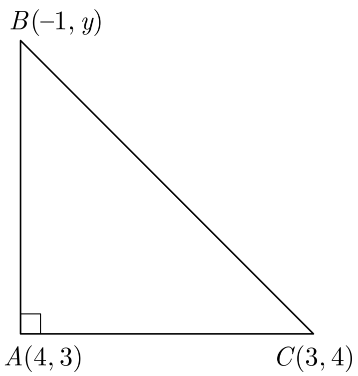
$$(p-4)(p-7) = 0$$

$$p = 7 \text{ or } 4$$

72. If $A(4,3)$, $B(-1,y)$, and $C(3,4)$ are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Ans : [Board Term-2 OD 2015]

As per question, triangle is shown below.



Now $AB^2 + AC^2 = BC^2$

$$(4+1)^2 + (3-y)^2 + (4-3)^2 = (3+1)^2 + (4-y)^2$$

$$(5)^2 + (3-y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4-y)^2$$

$$25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$$

$$36 + 2y - 32 = 0$$

$$2y + 4 = 0$$

$$y = -2$$

73. Show that the points (a,a), $(-a,-a)$ and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

Ans : [Board Term-2 Foreign 2015]

Let $A(a,a), B(-a,-a)$ and $C(-\sqrt{3}\,a, \sqrt{3}\,a)$.

Now $AB = \sqrt{(a+a)^2 + (a+a)^2}$

$$= \sqrt{4a^2 + 4a^2} = 2\sqrt{2}\,a$$

$$BC = \sqrt{(-a+\sqrt{3}\,a)^2 + (-a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 - 2\sqrt{3}\,a^2 + 3a^2 + a^2 + 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

$$AC = \sqrt{(a+\sqrt{3}\,a)^2 + (a-\sqrt{3}\,a)^2}$$

$$= \sqrt{a^2 + 2\sqrt{3}\,a^2 + 3a^2 + a^2 - 2\sqrt{3}\,a^2 + 3a^2}$$

$$= 2\sqrt{2}\,a$$

Since $AB = BC = AC$, therefore ABC is an equilateral triangle.

74. If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, find x, y.

Ans : [Board Term-2 OD 2012, Delhi 2014]

If the mid-point of the line segment joining $A\left[\frac{x}{2}, \frac{y+1}{2}\right]$ and $B(x+1, y-3)$ is $C(5,-2)$, then at mid point,

$$\frac{\frac{x}{2} + x + 1}{2} = 5$$

$$\frac{3x}{2} + 1 = 10$$

$$3x = 18 \Rightarrow x = 6$$

also $$\frac{\frac{y+1}{2} + y - 3}{2} = -2$$

$$\frac{y+1}{2} + y - 3 = -4$$

$$y + 1 + 2y - 6 = -8 \Rightarrow y = -1$$

75. Find the point on the x-axis which is equidistant from the points $(2,-5)$ and $(-2,9)$.

Ans : [Board Term-2 2012]

Let the point be $P(x,0)$ on the x-axis is equidistant from points $A(2,-5)$ and $B(-2,9)$.

Now $PA^2 = PB^2$

$$(2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$$

$$4 - 4x + x^2 + 25 = 4 + 4x + x^2 + 81$$

$$-8x = 56 \Rightarrow x = -7$$

Thus point is $(-7,0)$.

76. Show that $A(6,4)$, $B(5,-2)$ and $C(7,-2)$ are the vertices of an isosceles triangle.

Ans : [Board Term-2, 2012]

We have $A(6,4), B(5,-2), C(7,-2)$.

Now $AB = \sqrt{(6-5)^2+(4+2)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

g128

$BC = \sqrt{(5-7)^2+(-2+2)^2}$

$= \sqrt{(-2)^2+0^2} = 2$

$CA = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{1^2+6^2} = \sqrt{37}$

$AB = BC = \sqrt{37}$

Since two sides of a triangle are equal in length, triangle is an isosceles triangle.

77. If $P(2,-1)$, $Q(3,4)$, $R(-2,3)$ and $S(-3,-2)$ be four points in a plane, show that $PQRS$ is a rhombus but not a square.

Ans : [Board Term-2 OD 2012]

We have $P(2,\ -1)$, $Q(3,\ 4)$, $R(-2,\ 3)$, $S(-3,\ -2)$

$PQ = \sqrt{1^2+5^2} = \sqrt{26}$

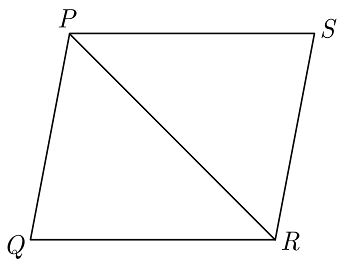
$QR = \sqrt{5^2+1^2} = \sqrt{26}$

$RS = \sqrt{1^2+5^2} = \sqrt{26}$

$PS = \sqrt{5^2+1^2} = \sqrt{26}$

g129

Since all the four sides are equal, $PQRS$ is a rhombus.



Now $PR = \sqrt{1^2+5^2} = \sqrt{26}$

$= \sqrt{4^2+4^2} = \sqrt{32}$

$PQ^2 + QR^2 = 2 \times 26 = 52 \neq (\sqrt{32})^2$

Since ΔPQR is not a right triangle, $PQRS$ is a rhombus but not a square.

78. Show that $A(-1,0)$, $B(3,1)$, $C(2,2)$ and $D(-2,1)$ are the vertices of a parallelogram $ABCD$.

Ans : [Board Term-2 2012]

Mid-point of AC,

$$\left(\frac{-1+2}{2},\ \frac{0+2}{2}\right) = \left(\frac{1}{2},1\right)$$

g130

Mid-point of BD,

$$\left(\frac{3-2}{2},\ \frac{1+1}{2}\right) = \left(\frac{1}{2},1\right)$$

Here Mid-point of AC = Mid-point of BD

Since diagonals of a quadrilateral bisect each other, $ABCD$ is a parallelogram.

79. If $(3,2)$ and $(-3,2)$ are two vertices of an equilateral triangle which contains the origin, find the third vertex.

Ans : [Board Term-2 OD 2012]

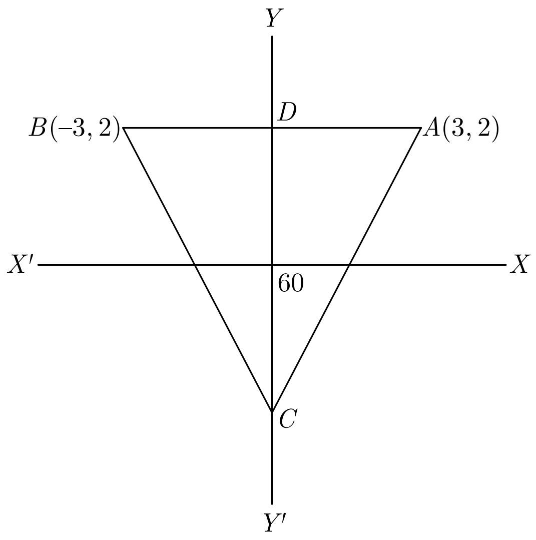
We have $A(3,2)$ and $B(-3,2)$.

It can be easily seen that mid-point of AB is lying on y-axis. Thus AB is equal distance from x-axis everywhere.

Also $OD \perp AB$

Hence 3^{rd} vertex of ΔABC is also lying on y-axis.

The digram of triangle should be as given below.



Let $C(x,y)$ be the coordinate of 3^{rd} vertex of ΔABC.

Now $AB^2 = (3+3)^2+(2-2)^2 = 36$

$BC^2 = (x+3)^2+(y-2)^2$

$AC^2 = (x-3)^2+(y-2)^2$

Since $AB^2 = AC^2 = BC^2$

$(x+3)^2+(y-2)^2 = 36$ ...(1)

$(x-3)^2+(y-2)^2 = 36$ ...(2)

Since $P(x,y)$ lie on y-axis, substituting $x=0$ in (1) we have

$$3^2+(y-2)^2 = 36-9 = 27$$

$$(y-2)^2 = 36-9 = 27$$

g131

Taking square root both side

$$y-2 = \pm 3\sqrt{3}$$

$$y = 2\pm 3\sqrt{3}$$

Since origin is inside the given triangle, coordinate of C below the origin,

$$y = 2-3\sqrt{3}$$

Hence Coordinate of C is $\left(0,\ 2-3\sqrt{3}\right)$

80. Find a so that $(3, a)$ lies on the line represented by $2x-3y-5=0$. Also, find the co-ordinates of the point where the line cuts the x-axis.

Ans : [Board Term-2 2012]

Since $(3, a)$ lies on $2x-3y-5=0$, it must satisfy this equation. Therefore

$$2\times 3-3a-5 = 0$$

$$6-3a-5 = 0$$

$$1 = 3a$$

$$a = \frac{1}{3}$$

g132

Line $2x-3y-5=0$ will cut the x-axis at $(x, 0)$. and it must satisfy the equation of line.

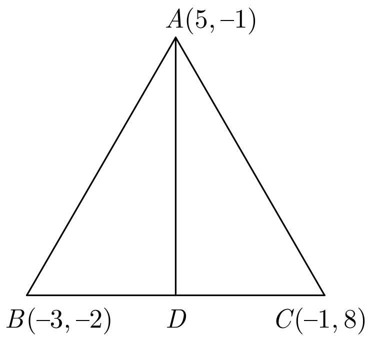
$$2x-5 = 0 \Rightarrow x = \frac{5}{2}$$

Hence point is $\left(\frac{5}{2}, 0\right)$.

81. If the vertices of ΔABC are $A(5,-1), B(-3,-2)$, $C(-1,8)$, Find the length of median through A.

Ans : [Board Term-2 2012]

Let AD be the median. As per question, triangle is shown below.



Since D is mid-point of BC, co-ordinates of D,

$$(x_1,\ y_2) = \left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$$

$$= (-2,\ 3)$$

$$AD = \sqrt{(5+2)^2+(-1-3)^2}$$

$$= \sqrt{(7)^2+(4)^2}$$

$$= \sqrt{49+16} = \sqrt{65} \text{ units}$$

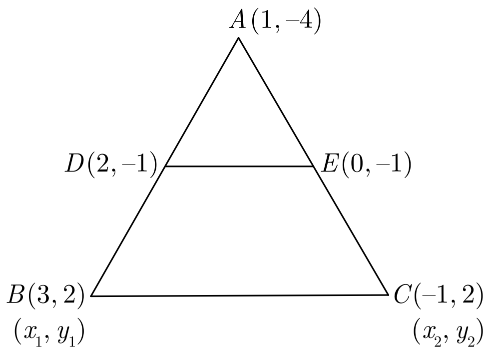
g133

Thus length of median is $\sqrt{65}$ units.

82. Find the mid-point of side BC of ΔABC, with $A(1,-4)$ and the mid-points of the sides through A being $(2,-1)$ and $(0,-1)$.

Ans : [Board Term-2 2012]

Assume co-ordinates of B and C are (x_1, y_1) and (x_2, y_2) respectively. As per question, triangle is shown below.



Now $$2 = \frac{1+x_1}{2} \Rightarrow x_1 = 3$$

and $$-1 = \frac{-4+y_1}{2} \Rightarrow y_1 = 2$$

$$0 = \frac{1+x_2}{2} \Rightarrow x = -1$$

$$-1 = \frac{-4+y_2}{2} \Rightarrow y_2 = 2$$

g134

Thus $$B(x_1, y_1) = (3, 2),$$

$$C(x_2, y_2) = (-1, 2)$$

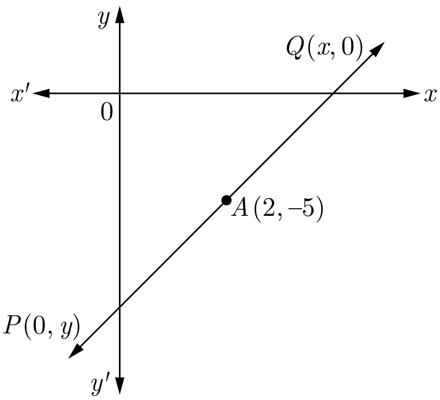
So, mid-point of BC is $\left(\frac{3-1}{2}, \frac{2+2}{2}\right) = (1, 2)$

83. A line intersects the y-axis and x-axis at the points P

and Q respectively. If $(2,-5)$ is the mid-point of PQ, then find the coordinates of P and Q.

Ans : [Board Term-2 OD 2017]

Let coordinates of P be $(0,y)$ and of Q be $(x,0)$.
$A(2,-5)$ is mid point of PQ.
As per question, line diagram is shown below.



Using section formula,

$$(2,-5) = \left(\frac{0+x}{2}+\frac{y+0}{2}\right)$$

g135

$$2 = \frac{x}{2} \Rightarrow x = 4$$

and $$-5 = \frac{y}{2} \Rightarrow y = -10$$

Thus P is $(0,-10)$ and Q is $(4,0)$

84. If $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, then show that the line $5x+3y+2=0$ passes through the point $(-1,3p)$.

Ans :

Since $(1,\frac{p}{3})$ is the mid point of the line segment joining the points $(2,0)$ and $(0,\frac{2}{9})$, we have

$$\frac{p}{3} = \frac{0+\frac{2}{9}}{2} = \frac{1}{9}$$

g136

$$p = \frac{1}{3}$$

Now the point $(-1,3p)$ is $(-1,1)$.
The line $5x+3y+2=0$, passes through the point $(-1,1)$ as $5(-5)+3(1)+2=0$

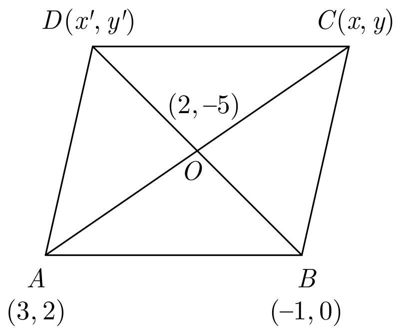
85. If two adjacent vertices of a parallelogram are $(3,2)$ and $(-1,0)$ and the diagonals intersect at $(2,-5)$ then find the co-ordinates of the other two vertices.

Ans : [Board Term-2 Foreign 2017]

Let two other co-ordinates be (x,y) and (x',y') respectively using mid-point formula.

As per question parallelogram is shown below.

g137



Now $$2 = \frac{x+3}{2} \Rightarrow x = 1$$

and $$-5 = \frac{2+y}{2} \Rightarrow y = -12$$

Again, $$\frac{-1+x'}{2} = 2 \Rightarrow x' = 5$$

and $$\frac{0+y'}{2} = -5 \Rightarrow y' = -10$$

Hence, coordinates of $C(1,-12)$ and $D(5,-10)$

86. In what ratio does the point $P(-4,6)$ divides the line segment joining the points $A(-6,10)$ and $B(3,-8)$?

Ans : [Board Term-2 Delhi Compt. 2017]

Let $$AP:PB = k:1$$

Now $$\frac{3k-6}{k+1} = -4$$

g138

$$3k-6 = -4k-4$$

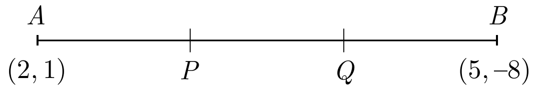
$$7k = 2 \Rightarrow k = \frac{2}{7}$$

Hence, $$AP:PB = 2:7$$

87. If the line segment joining the points $A(2,1)$ and $B(5,-8)$ is trisected at the points P and Q, find the coordinates P.

Ans : [Board Term-2 OD Compt. 2017]

As per question, line diagram is shown below.



Let $P(x,y)$ divides AB in the ratio $1:2$
Using section formula we get

g139

$$x = \frac{1 \times 5 + 2 \times 2}{1+2} = 3$$

$$y = \frac{1 \times -8 + 2 \times 1}{1+2} = -2$$

Hence coordinates of P are $(3, -2)$.

88. Prove that the points $(2, -2), (-2, 1)$ and $(5, 2)$ are the vertices of a right angled triangle. Also find the area of this triangle.

Ans : [Board Term-2 Foreign 2016]

We have $A(2, -2), B(-2, 1)$ and $(5, 2)$

Now using distance formula we get

g141

$$AB^2 = (2+2)^2 + (-2-1)^2$$

$$= 16 + 9 = 25$$

$$AB^2 = 25 \Rightarrow AB = 5.$$

Thus $AB = 5$.

Similarly $$BC^2 = (-2-5)^2 + (1-2)^2$$

$$= 49 + 1 = 50$$

$$BC^2 = 50 \Rightarrow BC = 5\sqrt{2}$$

$$AC^2 = (2-5)^2 + (-2-2)^2$$

$$= 9 + 16 = 25$$

$$AC^2 = 25 \Rightarrow AC = 5$$

Clearly $$AB^2 + AC^2 = BC^2$$

$$25 + 25 = 50$$

Hence the triangle is right angled,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

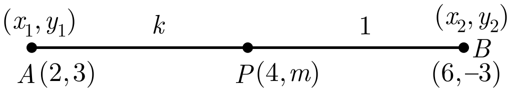
$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq unit.}$$

THREE MARKS QUESTIONS

89. Find the ratio in which $P(4, m)$ divides the segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.

Ans : [Board 2018]

Let $P(x, y)$ be the point which divide AB in $k:1$ ratio.



Now $$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

g276

$$4 = \frac{k(6) + 1(2)}{k+1}$$

$$4k + 4 = 6k + 2$$

$$6k - 4k = 4 - 2$$

$$2k = 2 \Rightarrow k = 1$$

Thus point P divides the line segment AB in $1:1$ ratio.

Now $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$m = \frac{1 \times (-3) + 1(3)}{1+1}$$

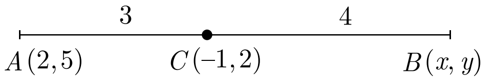
$$= \frac{-3+3}{2} = 0$$

Thus $m = 0$.

90. If the point $C(-1, 2)$ divides internally the line segment joining $A(2, 5)$ and $B(x, y)$ in the ratio $3:4$ find the coordinates of B.

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Using section formula,

$$-1 = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

$$-1 = \frac{3 \times x + 4 \times 2}{3+4} = \frac{3x+8}{7}$$

$$3x + 8 = -7$$

$$3x = -15 \Rightarrow x = -5$$

g277

and $$2 = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$2 = \frac{3y + 4 \times 5}{3+4} = \frac{3y+20}{7}$$

$$3y + 20 = 14$$

$$3y = 14 - 20 = -6$$

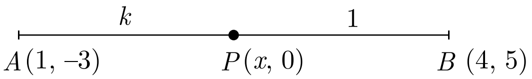
$$y = -2$$

Hence, the coordinates of $B(x, y)$ is $(-5, -2)$.

91. Find the ratio in which the segment joining the points $(1, -3)$ and $(4, 5)$ is divided by x-axis? Also find the coordinates of this point on x-axis.

Ans : [Board 2019 Delhi]

Let the required ratio be $k:1$ and the point on x-axis be $(x, 0)$.



Here, $(x_1, y_1) = (1, -3)$

and $(x_2, y_2) = (4, 5)$

Using section formula y coordinate, we obtain,

$$y = \frac{my_2 + ny_1}{m+n}$$

$$0 = \frac{k \times 5 + 1 \times 1(-3)}{k+1}$$

g278

$$0 = 5k - 3$$

$$5k = 3 \Rightarrow k = \frac{3}{5}$$

Hence, the required ratio is $\frac{3}{5}$ i.e $3:5$.

Now, again using section formula for x, we obtain

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$x = \frac{k \times (4) + 1 \times 1}{k+1}$$

$$= \frac{\frac{3}{5}(4) + 1}{\frac{3}{5} + 1} = \frac{12+5}{3+5} = \frac{17}{8}$$

Co-ordinate of P is $\left(\frac{17}{8}, 0\right)$.

92. Find the point on y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

Ans : [Board 2019 Delhi]

We have point $A = (5, -2)$ and $B = (-3, 2)$

Let $C(0, a)$ be point on y-axis.

According to question, point C is equidistant from A and B.

Thus $AC = BC$

Using distance formula we have

$$\sqrt{(0-5)^2 + (a+2)^2} = \sqrt{(0+3)^2 + (a-2)^2}$$

$$\sqrt{25 + a^2 + 4 + 4a} = \sqrt{9 + a^2 + 4 - 4a}$$

g280

$$25 + a^2 + 4 + 4a = 9 + a^2 + 4 - 4a$$

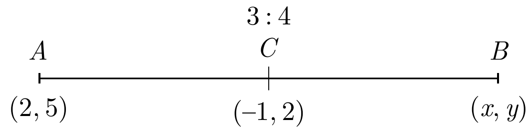
$$8a = -16 \Rightarrow a = -2$$

Hence, point on y-axis is $(0 - 2)$.

93. If the point $C(-1, 2)$ divides internally the line segment joining the points $A(2, 5)$ and $B(x, y)$ in the ratio $3:4$, find the value of $x^2 + y^2$.

Ans : [Board Term-2 Foreign 2016]

As per question, line diagram is shown below.



We have $\frac{AC}{BC} = \frac{3}{4}$

g144

Applying section formula for x co-ordinate,

$$-1 = \frac{3x + 4(2)}{3+4}$$

$$-7 = 3x + 8 \Rightarrow x = -5$$

Similarly applying section formula for y co-ordinate,

$$2 = \frac{3y + 4(5)}{3+4}$$

$$14 = 3y + 20 \Rightarrow y = -2$$

Thus (x, y) is $(-5, -2)$.

Now $x^2 + y^2 = (-5)^2 + (-2)^2$

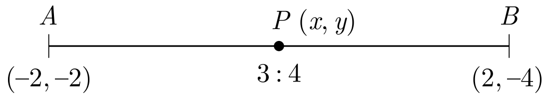
$$= 25 + 4 = 29$$

94. If the co-ordinates of points A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the co-ordinates of P such that $AP = \frac{3}{7}AB$, where P lies on the line segment AB.

Ans : [Board Term-2 OD 2017]

We have $AP = \frac{3}{7}AB \Rightarrow AP:PB = 3:4$

As per question, line diagram is shown below.



Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Applying section formula we get

$$x = \frac{3\times 2 + 4\times(-2)}{3+4} = -\frac{2}{7}$$

$$y = \frac{3\times(-4) + 4\times(-2)}{3+4} = -\frac{20}{7}$$

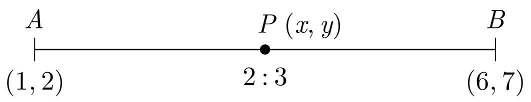
Hence P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$.

g145

95. Find the co-ordinate of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ such that $AP = \frac{2}{5}AB$.

Ans : [Board Term-2 OD 2015]

As per question, line diagram is shown below.



We have $AP = \frac{2}{5}AB \Rightarrow AP{:}PB = 2{:}3$

Section formula :

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + nx_1}{m+n}$$

Applying section formula we get

$$x = \frac{2\times 6 + 3\times 1}{2+3} = \frac{12+3}{5} = 3$$

g146

and $$y = \frac{2\times 7 + 3\times 2}{2+3} = \frac{14+6}{5} = 4$$

Thus $P(x,y) = (3,4)$

96. If the distance of $P(x,y)$ from $A(6,2)$ and $B(-2,6)$ are equal, prove that $y = 2x$.

Ans : [Board Term-2, 2015]

We have $P(x,y), A(6,2), B(-2,6)$

Now $PA = PB$

$$PA^2 = PB^2$$

g147

$$(x-6)^2 + (y-2)^2 = (x+2)^2 + (y-6)^2$$

$$-12x + 36 - 4y + 4 = 4x + 4 - 12y + 36$$

$$-12x - 4y = 4x - 12y$$

$$12y - 4y = 4x + 12x$$

$$8y = 16x$$

$$y = 2x$$

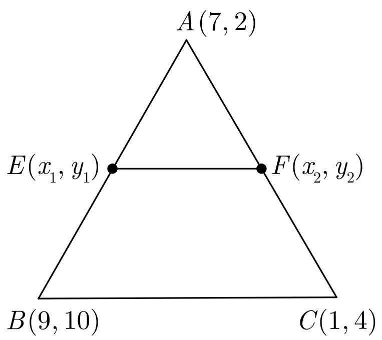
Hence Proved

97. The co-ordinates of the vertices of ΔABC are $A(7,2)$, $B(9,10)$ and $C(1,4)$. If E and F are the mid-points of AB and AC respectively, prove that $EF = \frac{1}{2}BC$.

Ans : [Board Term-2 2015]

Let the mid-points of AB and AC be $E(x_1, y_1)$ and $F(x_2, y_2)$. As per question, triangle is shown below.

g148



Co-ordinates of point E,

$$(x_1, y_1) = \left(\frac{9+7}{2}, \frac{10+2}{2}\right) = (8,6)$$

Co-ordinates of point F,

$$(x_2, y_2) = \left(\frac{7+1}{2}, \frac{2+4}{2}\right) = (4,3)$$

Length, $$EF = \sqrt{(8-4)^2 + (6-3)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5 \text{ units} \quad ...(1)$$

Length $$BC = \sqrt{(9-1)^2 + (10-4)^2}$$

$$= \sqrt{8^2 + 6^2}$$

$$= 10 \text{ units} \quad ...(2)$$

From equation (1) and (2) we get

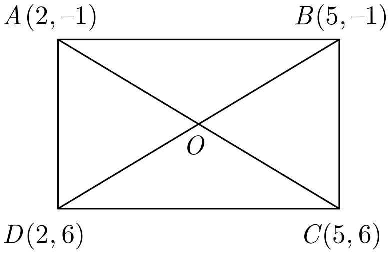
$$EF = \frac{1}{2}BC$$

Hence proved.

98. Prove that the diagonals of a rectangle $ABCD$, with vertices $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$ are equal and bisect each other.

Ans : [Board Term-2 2014]

As per question, rectangle $ABCD$, is shown below.



Now
$$AC = \sqrt{(5-2)^2+(6+1)^2}$$
$$= \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$$

g149

$$BD = \sqrt{(5-2)^2+(-1-6)^2}$$
$$= \sqrt{3^2+7^2} = \sqrt{9+49} = \sqrt{58}$$

Since $AC = BD = \sqrt{58}$ the diagonals of rectangle $ABCD$ are equal.

Mid-point of AC,
$$= \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

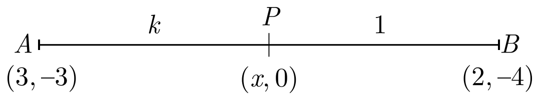
Mid-point of BD,
$$= \left(\frac{2+5}{2}, \frac{6-1}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$$

Since the mid-point of diagonal AC and mid-point of diagonal BD is same and equal to $\left(\frac{7}{5}, \frac{5}{2}\right)$. Hence they bisect each other.

99. Find the ratio in which the line segment joining the points $A(3,-3)$ and $B(-2,7)$ is divided by x-axis. Also find the co-ordinates of point of division.

Ans : [Board Term-2 Delhi 2014]

We know that y co-ordinate of any point on the $x-$ axis will be zero. Let $(x,\ 0)$ be point on x axis which cut the line. As per question, line diagram is shown below.



Let the ratio be $k:1$. Using section formula for y co-ordinate we have

$$0 = \frac{1(-3)+k(7)}{1+k}$$

g150

$$k = \frac{3}{7}$$

Using section formula for x co-ordinate we have

$$x = \frac{1(3)+k(-2)}{1+k} = \frac{3-2\times\frac{3}{7}}{1+\frac{3}{7}} = \frac{3}{2}$$

Thus co-ordinates of point are $\left(\frac{3}{2}, 0\right)$.

100. Find the ratio in which $(11,15)$ divides the line segment joining the points $(15,5)$ and $(9,20)$.

Ans : [Board Term-2 2014]

Let the two points $(15,5)$ and $(9,20)$ are divided in the ratio $k:1$ by point $P(11,15)$.

Using Section formula, we get

$$x = \frac{m_2x_1+m_1x_2}{m_2+m_1}$$

g151

$$11 = \frac{1(15)+k(9)}{1+k}$$

$$11+11k = 15+9k$$

$$k = 2$$

Thus ratio is 2 : 1.

101. Find the point on y-axis which is equidistant from the points $(5,-2)$ and $(-3,2)$.

Ans : [Board Term-2 2014, Delhi 2012]

Let point be $(0,y)$.

$$5^2+(y+2)^2 = (3)^2+(y-2)^2$$

g152

or, $$y^2+25+4y+4 = 9-4y+4$$

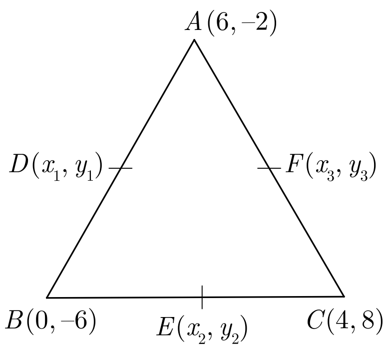
$$8y = -16 \text{ or, } y = -2$$

or, Point $(0,-2)$

102. The vertices of ΔABC are $A(6,-2), B(0,-6)$ and $C(4,8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Ans : [Board Term-2, 2014]

Let mid-point of AB, BC and AC be $D(x_1,y_1)$, $E(x_2,y_2)$ and $F(x_2,y_3)$. As per question, triangle is shown below.



Using section formula, the co-ordinates of the points D, E, F are

For D, $x_1 = \frac{6+0}{2} = 3$

$y_1 = \frac{-2-6}{2} = -4$

For E, $x_2 = \frac{0+4}{2} = 2$

$y_2 = \frac{-6+8}{2} = 1$

For F, $x_3 = \frac{4+6}{2} = 5$

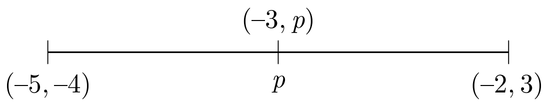
$y_3 = \frac{-2+8}{2} = 3$

The co-ordinates of the mid-points of AB, BC and AC are $D(3,-4)$, $E(2,1)$ and $F(5,3)$ respectively.

103. Find the ratio in which the point $(-3,p)$ divides the line segment joining the points $(-5,-4)$ and $(-2,3)$. Hence find the value of p.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let $X(-3,p)$ divides the line joining of $A(-5,-4)$ and $B(-2,3)$ in the ratio $k:1$.

The co-ordinates of p are $\left[\frac{-2k-5}{k+1}, \frac{3k-4}{k+1}\right]$

But co-ordinates of P are $(-3,p)$. Therefore we get

$$\frac{-2k-5}{k+1} = -3 \Rightarrow k = 2$$

and $$\frac{3k-4}{k+1} = p$$

Substituting $k = 2$ gives

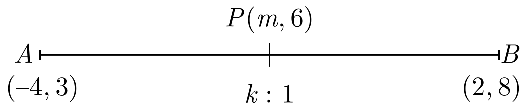
$$p = \frac{2}{3}$$

Hence ratio of division is 2:1 and $p = \frac{2}{3}$

104. Find the ratio in which the point $p(m,6)$ divides the line segment joining the points $A(-4,3)$ and $B(2,8)$. Also find the value of m.

Ans : [Board Term-2, 2012]

As per question, line diagram is shown below.



Let the ratio be $k:1$.

Using section formula, we have

$$m = \frac{2k+(-4)}{k+1} \quad (1)$$

$$6 = \frac{8k+3}{k+1} \quad (2)$$

$$8k+3 = 6k+6$$

$$2k = 3 \Rightarrow k = \frac{3}{2}$$

Thus ratio is $\frac{3}{2}:1$ or $3:2$.

Substituting value of k in (1) we have

$$m = \frac{2\left(\frac{3}{2}\right)+(-4)}{\frac{3}{2}+1} = \frac{3-4}{\frac{5}{2}} = \frac{-1}{\frac{5}{2}} = \frac{-2}{5}$$

105. If $A(4,-1), B(5,3), C(2,y)$ and $D(1,1)$ are the vertices of a parallelogram $ABCD$, find y.

Ans : [Board Term-2, 2012]

Diagonals of a parallelogram bisect each other.

Mid-points of AC and BD are same.

Thus $$\left(3, \frac{-1+y}{2}\right) = (3,2)$$

$$\frac{-1+y}{2} = 2 \Rightarrow y = 5$$

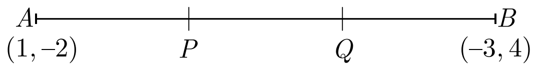
106. Find the co-ordinates of the points of trisection of the line segment joining the points $A(1,-2)$ and $B(-3,4)$.

Ans : [Board Term-2, 2012]

Let $P(x_1,y_1), Q(x_2,y_2)$ divides AB into 3 equal parts.

Thus P divides AB in the ratio of 1:2.

As per question, line diagram is shown below.



Now $$x_1 = \frac{1(-3)+2(1)}{1+3} = \frac{-3+2}{3} = \frac{-1}{3}$$

$$y_1 = \frac{1(4)+2(-2)}{1+2} = \frac{4-4}{3} = 0$$

Co-ordinates of P is $\left(-\frac{1}{3}, 0\right)$.

Here Q is mid-point of PB.

Thus $$x_2 = \frac{-\frac{1}{3}+(-3)}{2} = \frac{-10}{6} = \frac{-5}{3}$$

$$y_2 = \frac{0+4}{2} = 2$$

Thus co-ordinates of Q is $\left(-\frac{5}{2}, 2\right)$.

107. If (a, b) is the mid-point of the segment joining the points $A(10, -6)$ and $B(k, 4)$ and $a - 2b = 18$, find the value of k and the distance AB.

Ans : [Board Term-2, 2012]

We have $A(10, -6)$ and $B(k, 4)$.
If $P(a, b)$ is mid-point of AB, then we have

$$(a, b) = \left(\frac{k+10}{2}, \frac{-6+4}{2}\right)$$

g158

$$a = \frac{k+10}{2} \text{ and } b = -1$$

From given condition we have

$$a - 2b = 18$$

Substituting value $b = -1$ we obtain

$$a + 2 = 18 \Rightarrow a = 16$$

$$a = \frac{k+10}{2} = 16 \Rightarrow k = 22$$

$$P(a, b) = (16, 1)$$

$$AB = \sqrt{(22-10)^2 + (4+6)^2}$$

$$= 2\sqrt{61} \text{ units}$$

108. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the co-ordinates of the point of division.

Ans : [Board Term-2, 2012]

Let a point $P(x, y)$ on line $2x + 3y - 5 = 0$ divides AB in the ratio $k:1$.

Now $$x = \frac{2k+8}{k+1}$$

and $$y = \frac{k-9}{k+1}$$

Substituting above value in line $2x + 3y - 5 = 0$ we have

$$2\left(\frac{2k+8}{k+1}\right) + 3\left(\frac{k-9}{k+1}\right) - 5 = 0$$

g159

$$4k + 16 + 3k - 27 - 5k - 5 = 0$$

$$2k - 16 = 0$$

$$k = 8$$

Thus ratio is 8 : 1.

Substituting the value $k = 8$ we get

$$x = \left(\frac{2 \times 8 + 8}{8+1}\right) = \frac{8}{3}$$

$$y = \left(\frac{8-9}{8+1}\right) = -\frac{1}{9}$$

Thus $$P(x, y) = \left(\frac{8}{3}, -\frac{1}{9}\right)$$

109. Find the area of the rhombus of vertices $(3, 0), (4, 5), (-1, 4)$ and $(-2, -1)$ taken in order.

Ans : [Board Term-2, 2012]

We have $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$, $D(-2, -1)$

Diagonal AC, $$d_1 = \sqrt{(3+1)^2 + (0-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32}$$

g160

$$= \sqrt{16 \times 2} = 4\sqrt{2}$$

Diagonal BD, $$d_2 = \sqrt{(4+2)^2 + (5+1)^2}$$

$$= \sqrt{36+36} = \sqrt{72}$$

$$= \sqrt{36 \times 2} = 6\sqrt{2}$$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} 4\sqrt{2} \times 6\sqrt{2}$$

$$= 24 \text{ sq. unit.}$$

110. Find the ratio in which the line joining points $(a+b, b+a)$ and $(a-b, b-a)$ is divided by the point (a, b).

Ans : [Board Term-2, 2013]

Let $A(a+b, b+a)$, $B(a-b, b-a)$ and $P(a, b)$ and P divides AB in $k:1$, then we have

$$a = \frac{k(a-b) + 1(a+b)}{k+1}$$

g161

$$a(k+1) = k(a-b) + a + b$$

$$ak + a = ak - bk + a + b$$

$$bk = b$$

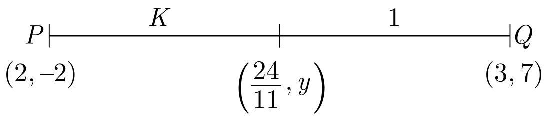
$$k = 1$$

Thus (a,b) divides $A(a+b, b+a)$ and $B(a-b, b-a)$ in 1:1 internally.

111. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divides the line segment joining the points $P(2,-2)$ and $Q(3,7)$? Also find the value of y.

Ans : [Board Term-2 SQP 2012]

As per question, line diagram is shown below.



Let $P\left(\frac{24}{11}, y\right)$ divides the segment joining the points $P(2,-2)$ and $Q(3,7)$ in ratio $k:1$.

Using intersection formula $x = \dfrac{mx_2 + nx_1}{m+n}$ we have

$$\frac{3k+2}{k+1} = \frac{24}{11}$$

g162

$$33k + 22 = 24k + 24$$

$$9k = 2 \Rightarrow k = \frac{2}{9}$$

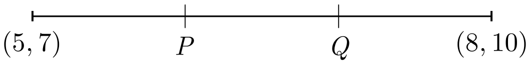
Hence, $$y = \frac{-18+14}{11} = -\frac{4}{11}$$

112. Find the co-ordinates of the points which divide the line segment joining the points $(5,7)$ and $(8,10)$ in 3 equal parts.

Ans : [Board Term-2 OD Compt. 2017]

Let $P(x_1, y_2)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2

As per question, line diagram is shown below.



Using section formula we have,

Now $$x = \frac{1(8)+2(5)}{3} = 6$$

g163

$$y = \frac{1(10)+2(7)}{3} = 8$$

Thus $P(x_1, y_1)$ is $P(6,8)$. Since Q is the mid point of PB, we have

$$x_1 = \frac{6+8}{2} = 7$$

$$y_1 = \frac{8+10}{2} = 9$$

Thus $Q(x_2, y_2)$ is $Q(7,9)$

113. Find the co-ordinates of a point on the $x-$axis which is equidistant from the points $A(2,-5)$ and $B(-2,9)$.

Ans : [Board Term-2 Delhi Compt. 2017]

Let the point P on the x axis be $(x,0)$. Since it is equidistant from the given points $A(2,-5)$ and $B(-2,9)$

$$PA = PB$$

g164

$$PA^2 = PB^2$$

$$(x-2)^2 + [0-(-5)]^2 = (x-(-2))^2 + (0-9)^2$$

$$x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$-4x + 29 = 4x + 85$$

$$x = -\frac{56}{8} = -7$$

Hence the point on x axis is $(-7,0)$

114. The line segment joining the points $A(3,-4)$ and $B(1,2)$ is trisected at the points P and Q. Find the coordinate of the PQ.

Ans : [Delhi Compt. Set-II, 2017]

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ trisect AB. Thus P divides AB in the ratio 1:2.

As per question, line diagram is shown below.

Using intersection formula

$$x = \frac{1\times 1 + 2\times 3}{1+2} = \frac{7}{3}$$

g165

$$y = \frac{1\times 2 + 2\times -4}{1+2} = -2$$

Hence point P is $\left(\frac{7}{3}, -2\right)$

115. Show that ΔABC with vertices $A(-2,0)$, $B(0,2)$ and $C(2,0)$ is similar to ΔDEF with vertices

$D(-4,0)$, $F(4,0)$ and $E(0,4)$.

Ans : [Board Term-2 Delhi 2017, Foreign 2017]

Using distance formula

$$AB = \sqrt{(0+2)^2+(2-0)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$

g166

$$BC = \sqrt{(2-0)^2+(0-2)^2} = \sqrt{4+4}$$
$$= 2\sqrt{2} \text{ units}$$
$$CA = \sqrt{(-2-2)^2+(0-0)^2} = \sqrt{16}$$
$$= 4 \text{ units}$$

and
$$DE = \sqrt{(0+4)^2+(4-0)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$EF = \sqrt{(4-0)^2+(0-4)^2} = \sqrt{32}$$
$$= 4\sqrt{2} \text{ units}$$
$$FD = \sqrt{(-4-4)^2+(0-0)^2} = \sqrt{64}$$
$$= 8 \text{ units}$$
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$

Since ratio of the corresponding sides of two similar $\Delta\, s$ is equal, we have

$$\Delta ABC \sim \Delta DEF$$ Hence Proved.

116. Find the co-ordinates of the point on the $y-$ axis which is equidistant from the points $A(5,3)$ and $B(1,-5)$

Ans : [Board Term-2 OD Compt. 2017]

Let the points on y-axis be $P(0,y)$

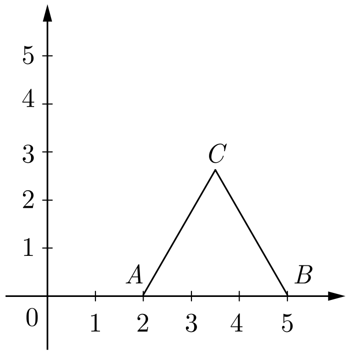
Now
$$PA = PB$$
$$PA^2 = PB^2$$

g167

$$(0-5)^2+(y-3)^2 = (0-1)^2+(y+5)^2$$
$$5^2+y^2-6y+9 = 1+y^2+10y+25$$
$$16y = 8 \Rightarrow y = \frac{1}{2}$$

Hence point on y-axis is $(0,\frac{1}{2})$.

117. In the given figure ΔABC is an equilateral triangle of side 3 units. Find the co-ordinates of the other two vertices.



Ans : [Board Term-2 Foreign 2017]

The co-ordinates of B will be $(2+3,0)$ or $.(5,0)$

Let co-ordinates of C be (x,y). Since triangle is equilateral, we have

$$AC^2 = BC^2$$
$$(x-2)^2(y-0)^2 = (x-5)^2+(y-0)^2$$
$$x^2+4-4x+y^2 = x^2+25-10x+y^2$$

g168

$$6x = 21$$
$$x = \frac{7}{2}$$

and
$$(x-2)^2+(y-0)^2 = 9$$
$$\left(\frac{7}{2}-2\right)^2+y^2 = 9$$
$$\frac{9}{4}+y^2 = 9 \text{ or, } y^2 = 9-\frac{9}{4}$$
$$y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$$

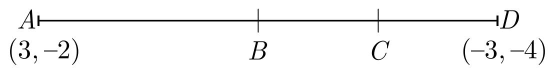
Hence C is $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$.

118. Find the co-ordinates of the points of trisection of the line segment joining the points $(3,-2)$ and $(-3,-4)$.

Ans : [Board Term-2 Foreign 2017]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.

As per question, line diagram is shown below.



Thus P divides AB in the ratio 1:2.

Using intersection formula $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$

$$x_1 = \frac{1(-3)+2(3)}{1+2} = 1$$

g169

and $$y_1 = \frac{1(-4)+2(-2)}{1+2} = -\frac{8}{3}$$

Thus we have $x = 1$ and $y = -\frac{8}{3}$

Since Q is at the mid-point of PB, using mid-point formula

$$x_2 = \frac{1-3}{2} = -1$$

and $$y_2 = \frac{-\frac{8}{3}+(-4)}{2} = -\frac{10}{3}$$

Hence the co-ordinates of P and Q are $(1, -\frac{8}{3})$ and $(-1, -\frac{10}{3})$

119. If the distances of $P(x, y)$ from $A(5,1)$ and $B(-1,5)$ are equal, then prove that $3x = 2y$.

Ans : [Board Term-2 OD 2016]

Since $P(x, y)$ is equidistant from the given points $A(5,1)$ and $B(-1,5)$,

$$PA = PB$$

g170

$$PA^2 = PB^2$$

Using distance formula,

$$(5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$$

$$(5-x)^2 + (1-y)^2 = (1+x)^2 + (5-y)^2$$

$$25 - 10x + 1 - 2y = 1 + 2x + 25 - 10y$$

$$-10x - 2y = 2x - 10y$$

$$8y = 12x$$

$$3x = 2y$$

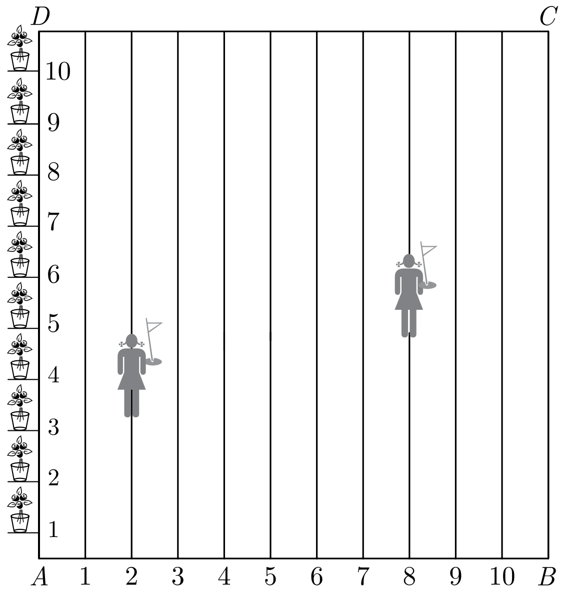
Hence proved.

FOUR MARKS QUESTIONS

120. To conduct Sports Day activities, in your rectangular school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Figure. Niharika runs ¼th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th distance AD on the eighth line and posts a red flag.

(i) What is the distance between the two flags?

(ii) If Rashmi has to post a blue flag exactly half way between the line segment joining the two flags, where should she post the blue flag?



Ans : [Board 2020 Delhi Basic]

We assume A as origin (0, 0), AB as x-axis and AD as y-axis.

Niharika runs in the 2^{nd} line with green flag and distance covered (parallel to AD),

$$= \frac{1}{4} \times 100 = 25 \text{ m}$$

Thus co-ordinates of green flag are (2, 25) and we label it as P i.e., $P(2, 25)$.

Similarly, Preet runs in the eighth line with red flag and distance covered (parallel to AD),

$$= \frac{1}{5} \times 100 = 20 \text{ m}$$

g281

Co-ordinates of red flag are (8, 20) and we label it as Q i.e., $Q(8, 20)$

(i) Now, using distance formula, distance between green flag and red flag,

$$PQ = \sqrt{(8-2)^2 + (20-25)^2}$$

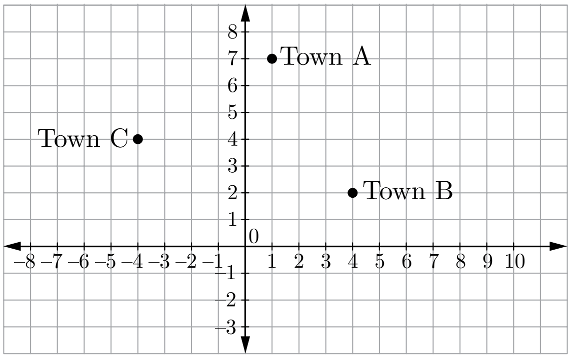
$$= \sqrt{6^2 + (-5)^2} = \sqrt{36+25}$$

$$= \sqrt{61} \text{ m}$$

(ii) Also, Rashmi has to post a blue flag the mid-point of PQ, therefore by using mid-point formula, we obtain $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ i.e. $\left(5, \frac{45}{2}\right)$

Hence, the blue flag is in the fifth line, at a distance of $\frac{45}{2}$ i.e., 22.5 m along the direction parallel to AD.

121. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometown represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation answer the following questions:



(i) Who will travel more distance, Seema or Aditya, to reach to their hometown?
(ii) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.
(iii) Find the area of the triangle formed by joining the points represented by A, B and C.

Ans : [Board 2020 SQP Standard]

From the given figure, the coordinates of points A, B and C are $(1,7),(4,2)$ and $(-4,4)$ respectively.

(i) Distance travelled by seema

$$CA = \sqrt{(-4-1)^2+(4-7)^2}$$
$$= \sqrt{(-5)^2+(-3)^2}$$
$$= \sqrt{25+9} \qquad = \sqrt{34}$$

g282

units

Thus distance travelled by seema is $\sqrt{34}$ units.
Similarly, distance travelled by Aditya

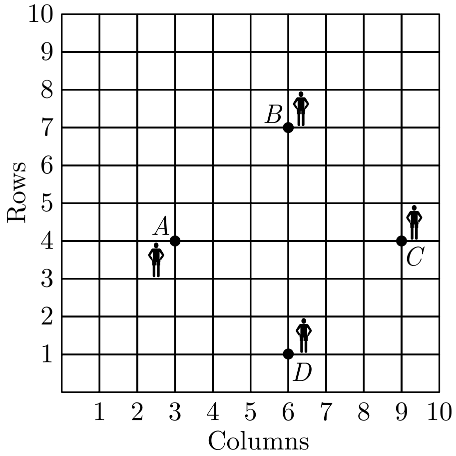
$$CB = \sqrt{(4+4)^2+(4-2)^2}$$
$$= \sqrt{8^2+2^2} = \sqrt{64+4}$$
$$= \sqrt{68} \text{ units}$$

Distance travelled by Aditya is $\sqrt{68}$ units and Aditya travels more distance.
(ii) Since, D is mid-point of town A and town B

$$D = \left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

(iii) Removed from syllabus

122. In a classroom, 4 friends are seated at the points A, B, C, and D as shown in Figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, Don't you think $ABCD$ is a square? Chameli disagrees. Using distance formula, find which of them is correct.



Ans : [Board 2020 Delhi Basic]

Coordinates of points A, B, C, D are $A(3,4)$, $B(6,7)$, $C(9,4)$ and $D(6,1)$.

Distance formula, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Now $$AB = \sqrt{(3-6)^2+(4-7)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

g283

$$BC = \sqrt{(6-9)^2+(7-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$CD = \sqrt{(9-6)^2+(4-1)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$
$$DA = \sqrt{(6-3)^2+(1-4)^2}$$
$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

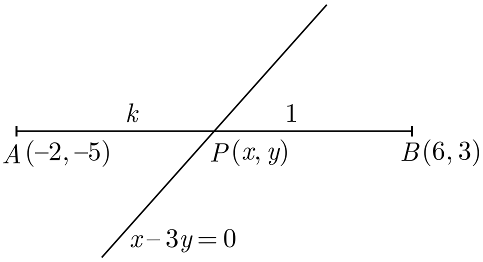
Now $$AC = \sqrt{(3-9)^2+(4-4)^2}$$
$$= \sqrt{36+0} = 6 \text{ units}$$
$$DB = \sqrt{(6-6)^2+(1-7)^2}$$
$$= \sqrt{0+36} = 6 \text{ units}$$

Since, $AB=BC=CD=DA$ and $AC=DB$, $ABCD$ is a square and Champa is right.

123. Find the ratio in which the line $x-3y=0$ divides the line segment joining the points $(-2,-5)$ and $(6,3)$. Find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let $k:1$ be the ratio in which line $x-3y=0$ divides the line segment at $p(x, y)$.



g285

Using section formula, we get

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{k \times 6 + 1 \times (-2)}{k+1}$$

$$x = \frac{6k-2}{k+1} \quad ...(1)$$

and $$y = \frac{my_2 + ny_1}{m+n} = \frac{k \times 3 + 1 \times (-5)}{k+1}$$

$$y = \frac{3k-5}{k+1} \quad ...(2)$$

The point $P(x, y)$ lies on the line, hence it satisfies the equation of the given line.

$$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$$

$$6k - 2 - 3(3k-5) = 0$$

$$6k - 2 - 9k + 15 = 0$$

$$-3k + 13 = 0 \Rightarrow k = \frac{13}{3}$$

Hence, the required ratio is 13 : 3.

Now, substituting value of k in x and y, we get

$$x = \frac{6 \times \frac{13}{3} - 2}{\frac{13}{3} + 1} = \frac{78-6}{16} = \frac{72}{16} = \frac{9}{2}$$

$$y = \frac{3 \times \frac{13}{3} - 5}{\frac{13}{3} + 1} = \frac{8 \times 3}{16} = \frac{24}{16} = \frac{3}{2}$$

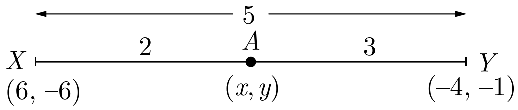
Hence, the co-ordinates of point of intersection

$$P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$$

124. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y+1) = 0$, find the value of k.

Ans : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



We use section formula for point $A(x, y)$.

Here, $m_1 = 2$, $m_2 = 3$, $x_1 = 6$, $x_2 = -4$, $y_1 = -6$ and $y_2 = -1$

Now $$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2 \times (-4) + 3(6)}{2+3}$$

$$= \frac{-8+18}{5} = \frac{10}{5} = 2$$

and $$y = \frac{m_1y_2 + m_2y_2}{m_1 + m_2} = \frac{2 \times (-1) + 3(-6)}{2+3}$$

$$= \frac{-2-18}{5} = \frac{-20}{5} = -4$$

Hence, coordinates of point A is $(2, -4)$.

Since point A also lies on the line $3x + k(y+1) = 0$, its coordinates must satisfies this line.

Thus $$3(2) + k(-4+1) = 0$$

$$6 + (-3k) = 0$$

$$3k = 6 \Rightarrow k = 2$$

g286

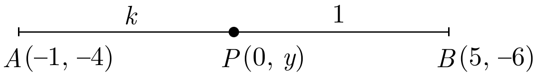
Hence, value of k is 2.

125. Find the ratio in which the y-axis divides the line segment joining the points $(-1, -4)$ and $(5, -6)$. Also find the coordinates of the point of intersection.

Ans : [Board 2019 OD]

Let points $P(0, y)$ divides the line joining the point $A(-1, -4)$ and $B(5, -6)$ in ratios $k:1$.

As per given information in question we have drawn figure below.



Section formula is given by

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} \quad ...(1)$$

and $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \quad ...(2)$$

Here, $m_1 = k$ and $m_2 = 1$,

$x_1 = -1$ and $x_2 = 5$

$y_1 = -4$ and $y_2 = -6$

g287

Now $$0 = \frac{k \times 5 + 1 \times (-1)}{k+1}$$

$$5k-1=0 \Rightarrow k=\frac{1}{5}$$

Substitute value of k in eq (2), we get

$$y=\frac{k(-6)+1(-4)}{k+1}$$

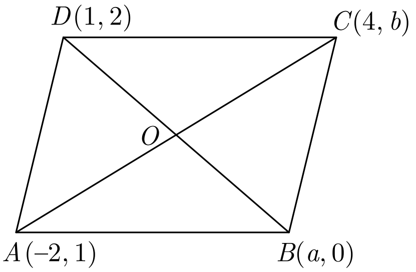
$$=\frac{\frac{1}{5}(-6)+1(-4)}{\frac{1}{5}+1}=\frac{-26}{6}=\frac{-13}{3}$$

Hence, value of k is $\frac{1}{5}$ and required point is $\left(0,-\frac{13}{3}\right)$

126. If $A(-2,1)$, $B(a,0)$, $C(4,b)$ and $D(1,2)$ are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides.

Ans : [Board 2018]

As per information given in question we have drawn the figure below.



Here $ABCD$ is a parallelogram and diagonals AC and BD bisect each other. Therefore mid point of BD is same as mid point of AC.

$$\left(\frac{a+1}{2},\frac{2}{2}\right)=\left(\frac{-2+4}{2},\frac{b+1}{2}\right)$$

g288

$$\frac{a+1}{2}=1 \Rightarrow a=1$$

and
$$\frac{b+1}{2}=1 \Rightarrow b=1$$

Now
$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(1+2)^2+(0-1)^2}$$
$$=\sqrt{9+1}=\sqrt{10} \text{ unit}$$
$$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$=\sqrt{(4-1)^2+(1-0)^2}$$
$$=\sqrt{9+1}=\sqrt{10} \text{ unit}$$

Since $ABCD$ is a parallelogram,

$$AB=CD=\sqrt{10} \text{ unit}$$
$$BC=AD=\sqrt{10} \text{ unit}$$

Therefore length of sides are $\sqrt{10}$ units each.

127. If $P(9a-2,-b)$ divides the line segment joining $A(3a+1,-3)$ and $B(8a,5)$ in the ratio 3:1. find the values of a and b.

Ans : [Board Term-2 SQP 2016]

Using section formula we have

$$9a-2=\frac{3(8a)+1+(3a+1)}{3+1} \quad ...(1)$$

$$-b=\frac{3(5)+1(-3)}{3+1} \quad ...(2)$$

Form (2)
$$-b=\frac{15-3}{4}=3 \Rightarrow b=-3$$

From (1),
$$9a-2=\frac{24a+3a+1}{4}$$

g180

$$4(9a-2)=27a+1$$
$$36a-8=27a+1$$
$$9a=9 \Rightarrow a=1$$

128. Find the coordinates of the point which divide the line segment joining $A(2,-3)$ and $B(-4,-6)$ into three equal parts.

Ans : [Board Term-2 SQP 2016]

Let $P(x_1,y_1)$ and $Q(x_2,y_2)$ trisect the line joining $A(3,-2)$ and $B(-3,-4)$.

As per question, line diagram is shown below.

P divides AB in the ratio of 1:2 and Q divides AB in the ratio 2:1.

g181

By section formula

$$x_1=\frac{mx_2+nx_1}{1+2} \text{ and } y=\frac{my_2+ny_1}{m+n}$$

$$P(x_1,y_1)=\left(\frac{1(-4)+2(2)}{2+1},\frac{2(-6)+1(-3)}{2+1}\right)$$

$$=\left(\frac{-4+4}{3},\frac{-6-(-6)}{3}\right)=(0,-4)$$

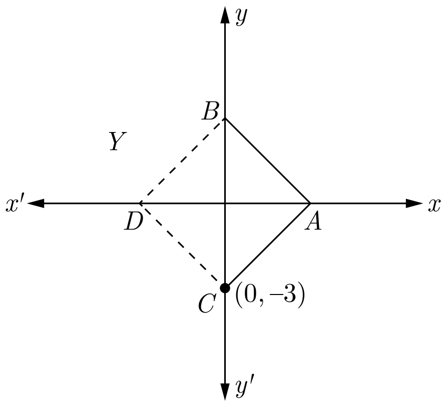
$$Q(x_2,y_2)=\left(\frac{2(-4)+1(2)}{2+1},\frac{2(-6)+1(-3)}{2+1}\right)$$

$$=\left(\frac{-8+2}{3},-\frac{12+(-3)}{3}\right)=(-2,-5)$$

129. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are $(0,3)$. The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that $BACD$ is a rhombus.

Ans : [Board Term-2 Foreign 2015]

As per question, diagram of rhombus is shown below.



Co-ordinates of point B are $(0,3)$.

g182

Thus $BC = 6$ unit

Let the co-ordinates of point A be $(x,0)$

Now $AB = \sqrt{x^2+9}$

Since $AB = BC$, thus we have

$$x^2+9 = 36$$

$$x^2 = 27 \Rightarrow x = \pm 3\sqrt{3}$$

Co-ordinates of point A is $(3\sqrt{3},0)$.

Since $ABCD$ is a rhombus,

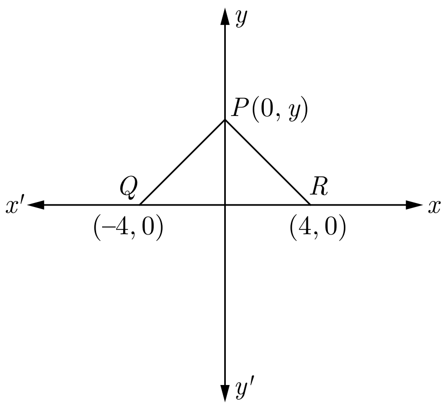
$$AB = AC = CD = DB$$

Thus co-ordinate of point D is $(-3\sqrt{3},0)$.

130. The base QR of an equilateral triangle PQR lies on x-axis. The co-ordinates of point Q are $(-4,0)$ and the origin is the mid-point of the base. find the co-ordinates of the point P and R.

Ans : [Board Term-2 Delhi 2017, Foreign 2015]

As per question, line diagram is shown below.



Co-ordinates of point R is $(4,0)$.

g183

Thus $QR = 8$ units

Let the co-ordinates of point P be $(0,y)$

Since $PQ = QR$

$$(-4-0)^2+(0-y)^2 = 64$$

$$16+y^2 = 64$$

$$y = \pm 4\sqrt{3}$$

Coordinates of P are $(0,4\sqrt{3})$ or $(0,-4\sqrt{3})$

131. The vertices of quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ and $D(1,-4)$. Prove that $ABCD$ is a rhombus.

Ans : [Board Term-2 Delhi 2015]

The vertices of the quadrilateral $ABCD$ are $A(5,-1)$, $B(8,3)$, $C(4,0)$ $D(1,-4)$.

g191

Now $AB = \sqrt{(8-5)^2+(3+1)^2}$

$= \sqrt{3^2+4^2} = 5$ units

$BC = \sqrt{(8-4)^2+(3-0)^2}$

$= \sqrt{4^2+3^2} = 5$ units

$CD = \sqrt{(4-1)^2+(0+4)^2}$

$= \sqrt{(3)^2+(4)^2} = 5$ units

$AD = \sqrt{(5-1)^2+(-1+4)^2}$

$= \sqrt{(4)^2+(3)^2} = 5$ units

Diagonal, $AC = \sqrt{(5-4)^2+(-1-0)^2}$

$= \sqrt{1^2+1^2} = \sqrt{2}$ units

Diagonal $BD = \sqrt{(8-1)^2+(3+4)^2}$

$= \sqrt{(7)^2+(7)^2} = 7\sqrt{2}$ units

As the length of all the sides are equal but the length of the diagonals are not equal. Thus $ABCD$ is not square but a rhombus.

132. The co-ordinates of vertices of ΔABC are $A(0,0)$, $B(0,2)$ and $C(2,0)$. Prove that ΔABC is an isosceles triangle. Also find its area.

Ans : [Board Term-2 Delhi 2014]

Using distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ we have

$$AB = \sqrt{(0-0)^2 + (0-2)^2} = \sqrt{4} = 2$$

$$AC = \sqrt{(0-2)^2 + (0-0)^2} = \sqrt{4} = 2$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Clearly, $AB = AC \neq BC$

Thus ΔABC is an isosceles triangle.

Now, $AB^2 + AC^2 = 2^2 + 2^2 = 4 + 4 = 8$

also, $BC^2 = (2\sqrt{2})^2 = 8$

$$AB^2 + AC^2 = BC^2$$

Thus ΔABC is an isosceles right angled triangle.

Now, area of ΔABC

$$\Delta_{ABC} = \frac{1}{2}\text{base} \times \text{height}$$

g193

$$= \frac{1}{2} \times 2 \times 2$$

$$= 2 \text{ sq. units.}$$

$$= \frac{1}{2}[3 \times (-1) + 7 \times 2 + 5 \times (-1)]$$

$$= \frac{1}{2}[-3 + 14 - 5]$$

$$= 3 \text{ units}$$

Area $\square_{ABCD} = \frac{5}{2} + 3 = \frac{11}{2}$ sq. units.

133. Find the ratio is which the line segment joining the points $A(3, -3)$ and $B(-2, 7)$ is divided by x-axis. Also find the co-ordinates of the point of division.

Ans : [Board Term-2 OD 2014]

We have $A(3, -3)$ and $B(-2, 7)$.
At any point on x-axis y-coordinate is always zero.
So, let the point be $(x, 0)$ that divides line segment AB in ratio $k:1$.

Now $$(x,\ 0) = \left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$$

g200

$$\frac{7k-3}{k+1} = 0$$

$$7k - 3 = 0 \Rightarrow k = \frac{3}{7}$$

The line is divided in the ratio of 3 : 7.

Now $$\frac{-2k+3}{k+1} = x$$

$$\frac{-2 \times \frac{3}{7} + 3}{\frac{3}{7} + 1} = x$$

$$\frac{-6+21}{3+7} = x$$

$$\frac{15}{10} = x \Rightarrow x = \frac{3}{2}$$

The coordinates of the point is $\left(\frac{3}{2}, 0\right)$.

134. Determine the ratio in which the straight line $x - y - 2 = 0$ divides the line segment joining $(3, -1)$ and $(8, 9)$.

Ans : [Board Term-2, 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $k:1$.

Now $$x_1 = \frac{8k+3}{k+1}$$

g201

$$y_1 = \frac{9k-1}{k+1}$$

Since point $P(x_1, y_1)$ lies on line $x - y - 2 = 0$, so co-ordinates of P must satisfy the equation of line.

Thus $$\frac{8k+3}{k+1} - \frac{9k-1}{k+1} - 2 = 0$$

$$8k + 3 - 9k + 1 - 2k - 2 = 0$$

$$-3k + 2 = 0 \Rightarrow k = \frac{2}{3}$$

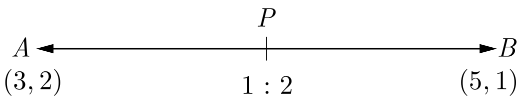
So, line $x - y - 2 = 0$ divides AB in the ratio 2:3

135. The line segment joining the points $A(3, 2)$ and $B(5, 1)$ is divided at the point P in the ratio 1:2 and P lies on the line $3x - 18y + k = 0$. Find the value of k.

Ans : [Board Term-2 Delhi 2012]

Let co-ordinates of P be (x_1, y_1) and it divides line AB in the ratio $1:2$.

g202



$$x_1 = \frac{mx_2 + nx_1}{m+n} = \frac{1 \times 5 + 2 \times 3}{1+2} = \frac{11}{3}$$

$$y_2 = \frac{my_2 + ny_1}{m+n} = \frac{1 \times 2 + 2 \times 2}{1+2} = \frac{5}{3}$$

Since point $P(x_1, y_1)$ lies on line ,$3x - 18y + k = 0$, so co-ordinates of P must satisfy the equation of line.

$$3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0$$

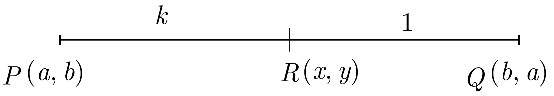
$$k = 19$$

136. If $R(x, y)$ is a point on the line segment joining the points $P(a, b)$ and $Q(b, a)$, then prove that $x + y = a + b$.

Ans : [Board Term-2, 2012 Set (28)]

As per question line is shown below.

g203



Let point $R(x, y)$ divides the line joining P and Q in the ratio $k:1$, then we have

$$x = \frac{kb + a}{k + 1}$$

and $$y = \frac{ka + b}{k + 1}$$

Adding, $$x + y = \frac{kb + a + ka + b}{k + 1}$$

$$= \frac{k(a + b) + (a + b)}{k + 1}$$

$$= \frac{(k + 1)(a + b)}{k + 1} = a + b$$

$$x + y = a + b$$ Hence Proved

137. (i) Derive section formula.

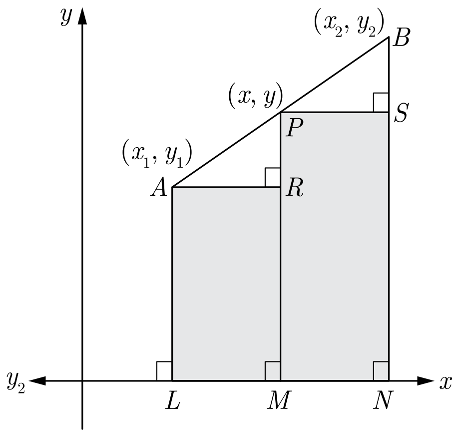
(ii) In what ratio does $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$

Ans : [Board Term-2 Delhi 2014]

(i) Section Formula : Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points. Let $P(x, y)$ be a point on line, joining A and B, such that P divides it in the ratio $m_1 : m_2$.

Now $$(x, y) = \left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$$

g204



Proof : Let AB be a line segment joining the points. $A(x_1, y_1)$, $B(x_2, y_2)$.

Let P divides AB in the ratio $m_1 : m_2$. Let P have co-ordinates (x, y).

Draw $AL, PM, PN, \perp$ to x-axis

It is clear form figure, that

$$AR = LM = OM - OL = x - x_1$$

$$PR = PM - RM = y - y_1.$$

also, $$PS = ON - OM = x_2 - x$$

$$BS = BN - SN = y_2 - y$$

Now $$\Delta APR \sim \Delta PBS \quad [AAA]$$

Thus $$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$$

and $$\frac{AR}{PS} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$

$$m_2 x - m_2 x_1 = m_1 x_2 - m_1 x$$

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

Now $$\frac{PR}{BS} = \frac{AP}{PB}$$

$$\frac{y - y_2}{y_2 - y} = \frac{m_1}{m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Thus co-ordinates of P are $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$

(ii) Assume that $(-4, 6)$ divides the line segment joining the point $A(-6, 4)$ and $B(3, -8)$ in ratio $k:1$

Using section formula for x co-ordinate we have

$$-4 = \frac{k(3) - 6}{k + 1}$$

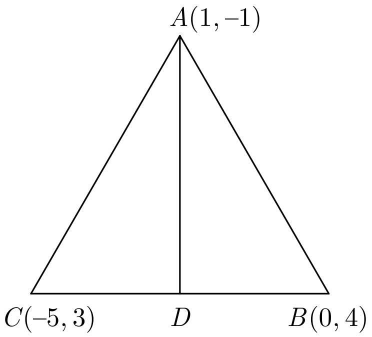
$$-4k - 4 = 3k - 6 \Rightarrow k = \tfrac{2}{7}$$

138. $(1, -1)$, $(0, 4)$ and $(-5, 3)$ are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex $(1, -1)$ the mid-point of the opposite side.

Ans : [Board Term-2, 2015]

Let the vertices of ΔABC be $A(1, -1)$, $B(0, 4)$ and $C(-5, 3)$. Let $D(x, y)$ be mid point of BC. N triangle is shown below.

g210



Using distance formula, we get

$$AB = \sqrt{(1-0)^2+(-1-4)^2} = \sqrt{1+5^2} = \sqrt{26}$$

$$BC = \sqrt{(-5-0)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(-5-1)^2+(3+1)^2} = \sqrt{36+16} = 2\sqrt{13}$$

Since $AB = BC \neq AC$, triangle ΔABC is isosceles.

Now, using mid-section formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x,y) = \left(-\frac{5}{2},\frac{7}{2}\right)$$

Length of median AD,

$$AD = \sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}+1\right)^2}$$

$$= \sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ square unit}$$

Thus length of median AD is $\frac{\sqrt{130}}{2}$ units.

139. Point $(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$. Find the values of y. Hence find the radius of the circle.

Ans : [Board Term-2 Delhi 2014]

Since, $A(-1,y)$ and $B(5,7)$ lie on a circle with centre $O(2,-3y)$, OA and OB are the radius of circle and are equal. Thus

g214

$$OA = OB$$

$$\sqrt{(-1-2)^2+(y+3y)^2} = \sqrt{(5-2)^2+(7+3y)^2}$$

$$9+16y^2 = 9y^2+42y+58$$

$$y^2-6y-7 = 0$$

$$(y+1)(y-7) = 0$$

$$y = -1, 7$$

When $y=-1$, centre is $O(2,-3y) = (2,3)$ and radius

$$OB = \left|\sqrt{(5-2)^2+(7-3)^2}\right|$$

$$= \sqrt{9+16} = 5 \text{ unit}$$

When $y=7$, centre is $O(2,-3y) = (2,-21)$ and radius

$$OB = \left|\sqrt{(2-5)^2+(-21-7)^2}\right|$$

$$= \left|\sqrt{9+784}\right| = \sqrt{793} \text{ unit}$$

CHAPTER 8

INTRODUCTION OF TRIGONOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

(a) 0° (b) 90°

(c) 60° (d) 30°

h215

Ans :

[Board 2020 SQP Standard]

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ$ ...(1)

and $\cos\beta = 0$

$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ$ ...(2)

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

2. If ΔABC is right angled at C, then the value of $\sec(A+B)$ is

(a) 0 (b) 1

(c) $\frac{2}{\sqrt{3}}$ (d) not defined

Ans : [Board 2020 SQP Standard]

h216

We have $\angle C = 90^\circ$

Since, $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B = 180^\circ - \angle C$

$= 180^\circ - 90^\circ = 90^\circ$

Now, $\sec(A+B) = \sec 90^\circ$ not defined

Thus (d) is correct option.

3. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, $(\theta \neq 90^\circ)$ then the value of $\tan\theta$ is

(a) $\sqrt{2} - 1$ (b) $\sqrt{2} + 1$

(c) $\sqrt{2}$ (d) $-\sqrt{2}$

Ans : [Board 2020 SQP Standard]

h217

We have $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Dividing both sides by $\cos\theta$, we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sqrt{2}\frac{\cos\theta}{\cos\theta}$$

$$\tan\theta + 1 = \sqrt{2}$$

$$\tan\theta = \sqrt{2} - 1$$

Thus (a) is correct option.

4. If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{5}{3}$

h218

Ans :

We have $\cos A = \frac{4}{5}$

We know that, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$$\text{Perpendicular} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = 3$$

Now, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Thus (b) is correct option.

5. If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

h219

Ans :

We have $\sin A = \frac{1}{2}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{2}$$

Now, $\text{Base} = \sqrt{2^2 - 1^2} = \sqrt{3}$

So, $\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Hence, the required value of $\cot A$ is $\sqrt{3}$.
Thus (a) is correct option.

6. If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$

(c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

h221

Ans :

We have $\sin\theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Base} = \sqrt{b^2-a^2}$$

So, $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2-a^2}}{b}$

Thus (c) is correct option.

7. If $\cos(\alpha+\beta) = 0$, then $\sin(\alpha-\beta)$ can be reduced to

(a) $\cos\beta$ (b) $\cos 2\beta$

(c) $\sin\alpha$ (d) $\sin 2\alpha$

Ans :

Given, $\cos(\alpha+\beta) = 0 = \cos 90^\circ$ $[\cos 90^\circ = 0]$

$$\alpha+\beta = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

h222

Now, $\sin(\alpha-\beta) = \sin(90^\circ-\beta-\beta)$

$$= \sin(90^\circ - 2\beta)$$

$= \cos 2\beta$

Thus (b) is correct option.

8. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) 1 (d) 0

h223

Ans :

We have $\cos 9\alpha = \sin\alpha$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin\alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$

$$\tan 5\alpha = \tan(5\times 9^\circ)$$

$$= \tan 45^\circ = 1 \quad [\tan 45^\circ = 1]$$

Thus (c) is correct option.

9. If ΔABC is right angled at C, then the value of $\cos(A+B)$ is

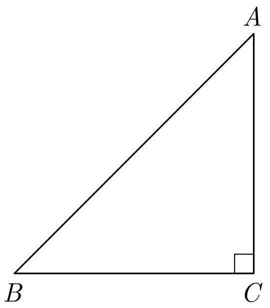
(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

h224

Ans :

We know that in ΔABC,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A+B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

10. If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha+\beta)$ is

(a) 0° (b) 30°

(c) 60° (d) 90°

h225

Ans :

Given, $\sin\alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos\beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$$\alpha+\beta = 30^\circ + 60^\circ = 90^\circ$$

Thus (d) is correct option.

11. If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

h226

Ans :

Given, $4\tan\theta = 3$

$$\tan\theta = \frac{3}{4} \quad ...(i)$$

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

12. If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

h227

Ans :

Given, $\sin\theta - \cos\theta = 0$

$$\sin\theta = \cos\theta$$

$$\sin\theta = \sin(90^\circ - \theta)$$

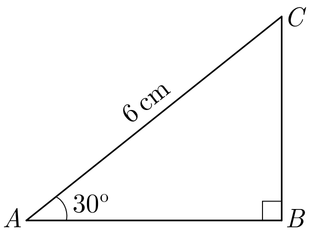
$$\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Thus (c) is correct option.

13. In the adjoining figure, the length of BC is



h228

(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3 cm

Ans :

In ΔABC, $\sin 30^\circ = \frac{BC}{AC}$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3\,\text{cm}$$

Thus (d) is correct option.

14. If $x = p\sec\theta$ and $y = q\tan\theta$, then

(a) $x^2 - y^2 = p^2 q^2$ (b) $x^2 q^2 - y^2 p^2 = pq$

(c) $x^2 q^2 - y^2 p^2 = \frac{1}{p^2 q^2}$ (d) $x^2 q^2 - y^2 p^2 = p^2 q^2$

Ans :

We know, $\sec^2\theta - \tan^2\theta = 1$

Substituting $\sec\theta = \frac{x}{p}$ and $\tan\theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$

h229

$$x^2 q^2 - y^2 p^2 = p^2 q^2$$

Thus (d) is correct option.

15. If $b\tan\theta = a$, the value of $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ is

(a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$

(c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans :

h230

We have $\tan\theta = \frac{a}{b}$

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} - b}{a\frac{\sin\theta}{\cos\theta} + b} = \frac{a\tan\theta - b}{a\tan\theta + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Thus (d) is correct option.

16. $(\cos^4 A - \sin^4 A)$ is equal to

(a) $1 - 2\cos^2 A$ (b) $2\sin^2 A - 1$

(c) $\sin^2 A - \cos^2 A$ (d) $2\cos^2 A - 1$

Ans :

$$\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

$$= (\cos^2 A - \sin^2 A)(1)$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= 2\cos^2 A - 1$$

h231

Thus (d) is correct option.

17. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

(a) 15° (b) 5°

(c) 20° (d) 10°

h232

Ans :

We have, $\sec 5A = \text{cosec}(A + 30^\circ)$

$$\sec 5A = \sec[90^\circ - (A - 30^\circ)]$$

$$\sec 5A = \sec(60^\circ - A)$$

$$5A = 60^\circ - A$$

$$6A = 60^\circ \Rightarrow A = 10^\circ$$

Thus (d) is correct option.

18. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

h233

Ans :

We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$$(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$$

$$x\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$x\sin\theta = \sin\theta\cos\theta \Rightarrow x = \cos\theta$$

Now, $x\sin\theta = y\cos\theta$

$$\cos\theta\sin\theta = y\cos\theta$$

$$y = \sin\theta$$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

Thus (c) is correct option.

19. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then $m^2 - n^2$ is equal to

(a) $\sqrt{mn}$ (b) $\sqrt{\frac{m}{n}}$

(c) $4\sqrt{mn}$ (d) None of these

h235

Ans :

Given, $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$

$$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$$

$$= 4\tan\theta\sin\theta$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\sqrt{\sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{mn}$$

Thus (c) is correct option.

20. If $0 < \theta < \frac{\pi}{4}$, then the simplest form of $\sqrt{1 - 2\sin\theta\cos\theta}$ is

(a) $\sin\theta - \cos\theta$ (b) $\cos\theta - \sin\theta$

(c) $\cos\theta + \sin\theta$ (d) $\sin\theta\cos\theta$

h236

Ans :

$$\sqrt{1 - 2\sin\theta\cos\theta} = \sqrt{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}$$

$$= \sqrt{(\cos\theta - \sin\theta)^2}$$

$$= \cos\theta - \sin\theta$$

For $0^\circ < \theta < 45^\circ$

	0	$\pi/6$	$\pi/4$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$
$\sin\theta$	0	1/2	$1/\sqrt{2}$

Here, we see that $\cos\theta > \sin\theta$, when $0 < \theta < \frac{\pi}{4}$, that's why we take $(\cos\theta - \sin\theta)^2$ instead of taking $(\sin\theta - \cos\theta)^2$.

Thus (b) is correct option.

21. If $f(x) = \cos^2 x + \sec^2 x$, then $f(x)$

(a) ≥ 1 (b) ≤ 1

(c) ≥ 2 (d) ≤ 2

h237

Ans : (c) ≥ 2

Given, $f(x) = \cos^2 x + \sec^2 x$

$$= \cos^2 x + \sec^2 x - 2 + 2$$

$$= \cos^2 x + \sec^2 x - 2\cos x \cdot \sec x + 2$$

$$= (\cos x - \sec x)^2 + 2$$

We know that, square of any expression is always greater than equal to zero.

$f(x) \geq 2$ Hence proved.

Thus (c) is correct option.

22. Assertion : The value of $\sin\theta = \frac{4}{3}$ is not possible.

Reason : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

h238

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

23. Assertion : $\sin^2 67° + \cos^2 67° = 1$

Reason : For any value of $\theta, \sin^2\theta + \cos^2\theta = 1$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $\sin^2\theta + \cos^2\theta = 1$

h239

$$\sin^2 67° + \cos^2 67° = 1$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

1. FILL IN THE BLANK

1. Maximum value for sine of any angle is ....

Ans :

1

h241

2. Triangle in which we study trigonometric ratios is called ....

Ans :

Right Triangle

h242

3. Cosine of 90° is ....

Ans :

Zero

h243

4. Sum of .... of sine and cosine of angle is one.

Ans :

Square

h244

5. Reciprocal of $\sin\theta$ is ....

Ans :

$\operatorname{cosec}\theta$

h245

6. The value of $\sin A$ or $\cos A$ never exceeds ....

Ans :

1

h246

7. sine of $(90° - \theta)$ is ....

Ans :

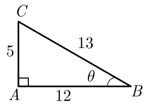
$\cos\theta$

h247

8. If $\sin\theta = \frac{5}{13}$, then the value of $\tan\theta$ is ....

Ans : [Board 2020 OD Basic]

From $\sin\theta = \frac{5}{13}$ we can draw the figure as given below.



h248

Now, $\tan\theta = \frac{AC}{BC} = \frac{5}{12}$

9. The value of the $(\tan^2 60° + \sin^2 45°)$ is ....

Ans : [Board 2020 OD Basic]

$$\tan^2 60° + \sin^2 45° = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3 + \frac{1}{2} = \frac{7}{2}$$

h250

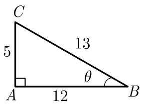
10. If $\cot\theta = \frac{12}{5}$, then the value of $\sin\theta$ is ....

Ans : [Board 2020 Delhi Basic]

Given, $\cot\theta = \frac{12}{5} \Rightarrow \tan\theta = \frac{5}{12}$

From $\tan\theta = \frac{5}{12}$ we can draw the figure as given below.

h252



So, $\sin\theta = \frac{AC}{CB} = \frac{5}{13}$

11. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is ....

Ans : [Board 2020 Delhi Basic]

h253

We have $\tan(A+B) = \sqrt{3}$

$= \tan 60°$

Hence, $A + B = 60°$

...(1)

Again, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$= \tan 30°$

$A - B = 30°$ ...(2)

Adding equation (1) and (2) we get

$2A = 90° \Rightarrow A = 45°$

12. The value of $\left(\sin^2\theta + \frac{1}{1+\tan^2\theta}\right) =$ ....

Ans : [Board 2020 Delhi Standard]

h262

$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1$

13. The value of $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) =$ ....

Ans : [Board 2020 Delhi Standard]

h263

$(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$

$= \sec^2\theta(1-\sin^2\theta)$

$= \sec^2\theta \times \cos^2\theta$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$

VERY SHORT ANSWER QUESTIONS

14. Prove that

$(1+\tan A - \sec A) \times (1 + \tan A + \sec A) = 2\tan A$

Ans : [Board 2020 Delhi Basic]

h266

LHS $= (1+\tan A - \sec A) \times (1+\tan A + \sec A)$

$= (1+\tan A)^2 - \sec^2 A$

$= 1 + \tan^2 A + 2\tan A - \sec^2 A$

$= \sec^2 A + 2\tan A - \sec^2 A$

$= 2\tan A =$ RHS

15. If $\tan A = \cot B$, then find the value of $(A+B)$.

Ans : [Board 2020 OD Standard]

h267

We have $\tan A = \cot B$

$\tan A = \tan(90° - B)$

$A = 90° - B$

Thus $A + B = 90°$

16. If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

Ans : [Board 2020 OD Basic]

h269

We have $x = 3\sin\theta + 4\cos\theta$

and $y = 3\cos\theta - 4\sin\theta$

$x^2 + y^2$

$= (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2$

$= (9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta) +$

$+ (9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta)$

$= 9(\sin^2\theta + \cos^2\theta) + 16(\sin^2\theta + \cos^2\theta)$

$= 9 + 16 = 25$

17. Evaluate $\sin^2 60° - 2\tan 45° - \cos^2 30°$

Ans : [Board 2019 OD]

h270

$\sin^2 60° - 2\tan 45° - \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} - 2 - \frac{3}{4} = -2$

18. If $\sin\theta + \sin^2\theta = 1$ then prove that $\cos^2\theta + \cos^4\theta = 1$.

Ans : [Board 2020 OD Basic]

We have $\sin\theta + \sin^2\theta = 1$

$$\sin\theta + (1-\cos^2\theta) = 1$$

$$\sin\theta - \cos^2\theta = 0$$

$$\sin\theta = \cos^2\theta$$

Squaring both sides, we get

$$\sin^2\theta = \cos^4\theta$$

$$1-\cos^2\theta = \cos^4\theta$$

$$\cos^4\theta + \cos^2\theta = 1$$

Hence Proved

h272

19. In a triangle ABC, write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A.

Ans : [Board Term-1 2016]

h101

In a triangle $A+B+C = 180^{\circ}$

$$B+C = 180^{\circ} - A$$

Thus $\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^{\circ}-A}{2}\right]$

$$= \cos\left[90 - \frac{A}{2}\right]$$

$$= \sin\frac{A}{2}$$

20. If $\sec\theta \cdot \sin\theta = 0$, then find the value of θ.

Ans : [Board Term-1 2016]

h102

We have $\sec\theta\cdot\sin\theta = 0$

$$\frac{1}{\cos\theta}\cdot\sin\theta = 0$$

$$\frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = 0 = \tan 0^{\circ}$$

Thus $\theta = 0^{\circ}$

21. If $\tan 2A = \cot(A+60^{\circ})$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1 2016]

h104

We have $\tan 2A = \cot(A+60^{\circ})$

$$\cot(90^{\circ}-2A) = \cot(A+60^{\circ})$$

$$90^{\circ}-2A = A+60^{\circ}$$

$$3A = 30^{\circ} \Rightarrow A = 10^{\circ}$$

22. If $\tan(3x+30^{\circ}) = 1$ then find the value of x.

Ans : [Board Te

h107

We have $\tan(3x+30^{\circ}) = 1 = \tan 45^{\circ}$

$$3x+30^{\circ} = 45^{\circ}$$

$$x = 5^{\circ}$$

23. What happens to value of $\cos\theta$ when θ increases from 0° to 90°.

Ans : [Board Term-1 2015]

$\cos\theta$ decreases from 1 to θ.

h108

24. If A and B are acute angles and $\sin A = \cos B$, then find the value of $A+B$.

Ans : [Board Term-1 2016]

h110

We have $\sin A = \cos B$

$$\sin A = \sin(90^{\circ}-B)$$

$$A = 90^{\circ}-B$$

$$A+B = 90^{\circ}$$

25. If $\cos A = \frac{2}{5}$, find the value of $4+4\tan^2 A$.

Ans : [Board SQP 2018]

h113

$$4+4\tan^2 A = 4(1+\tan^2 A)$$

$$4\sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4\times\frac{25}{4} = 25$$

26. If $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$, then find the value of k.

Ans : [Board Term-1 2015]

We have $k+1 = \sec^2\theta(1+\sin\theta)(1-\sin\theta)$

$$= \sec^2\theta(1-\sin^2\theta)$$

$$= \sec^2\theta.\cos^2\theta$$

$$= \sec^2\theta\times\frac{1}{\sec^2\theta}$$

h154

$$k+1 = 1 \Rightarrow k = 1-1 = 0$$

Thus $k=0$

27. Find the value of $\sin^2 41^{\circ} + \sin^2 49^{\circ}$

Ans : [Board Term-1 2012, NCERT]

We have

$$\sin^2 41 + \sin^2 49 = \sin^2(90^{\circ}-49^{\circ}) + \sin^2 49^{\circ}$$

$$= \cos^2 49 + \sin^2 49^o$$

$$= 1$$

h155

TWO MARKS QUESTIONS

28. Prove that $1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$

Ans : [Board 2020 OD Standard]

h273

$$1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\operatorname{cosec}^2\alpha - 1}{1+\operatorname{cosec}\alpha}$$

$$= 1 + \frac{(1+\operatorname{cosec}\alpha)(\operatorname{cosec}\alpha - 1)}{1+\operatorname{cosec}\alpha}$$

$$= 1 + \operatorname{cosec}\alpha - 1$$

$$= \operatorname{cosec}\alpha$$ Hence Proved

29. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Ans : [Board 2018]

h275

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \tan A\frac{[1-2(1-\cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A\frac{[1-2+2\cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A\frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A$$ Hence Proved

30. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

Ans : [Board 2020 OD Standard]

h276

$$\tan^4\theta + \tan^2\theta = \tan^2\theta(1+\tan^2\theta)$$

$$= \tan^2\theta \times \sec^2\theta$$

$$= (\sec^2\theta - 1)\sec^2\theta$$

$$= \sec^4\theta - \sec^2\theta$$ Hence Proved

31. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$.

Ans :

h278

$$\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{RHS}$$ Hence Proved

32. Prove that : $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$

Ans : [Board 2020 OD Basic]

h279

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2\theta}{\sec^2\theta}$$

$$= \frac{1}{\sec^2\theta} - \frac{\tan^2\theta}{\sec^2\theta}$$

$$= \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta$$

$$= \cos^2\theta - \sin^2\theta$$ Hence Proved

33. Prove that $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta} = 1$.

Ans : [Board 2020 Delhi Basic]

h280

$$\text{LHS} = \frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$$

$$= \frac{\tan^2\theta}{\sec^2\theta} + \frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{RHS}$$

34. Prove that : $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

Ans : [Board 2020 Delhi Basic]

h281

$$\text{LHS} = \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$

$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = 2\sec^2\theta = \text{RHS}$$

35. Prove that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}\theta\left[\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right]$$

$$= \operatorname{cosec}\theta\left[\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right]$$

$$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$$

h283

$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$

$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{RHS}$$ Hence Proved

36. If $5\tan\theta = 3$, then what is the value of $\left(\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}\right)$?

Ans : [Board 2020 Delhi Basic]

We have $5\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{5}$

h285

Dividing numerator and denominator by $\cos\theta$ we have

$$\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} + 3} = \frac{5\tan\theta - 3}{4\tan\theta + 3}$$

$$= \frac{5 \times \frac{3}{5} - 3}{4 \times \frac{3}{5} + 3} = \frac{3 - 3}{\frac{12}{5} + 3} = 0$$

37. Evaluate :

$$\frac{3\tan^2 30^o + \tan^2 60^o + \operatorname{cosec} 30^o - \tan 45^o}{\cot^2 45^o}$$

Ans : [Board Term-1 2016]

$$\frac{3\tan^2 30^o + \tan^2 60^o + \operatorname{cosec} 30^o - \tan 45^o}{\cot^2 45^o}$$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

h114

$$= 1 + 3 + 2 - 1 = 5$$

38. If $\sin(A+B) = 1$ and $\sin(A-B) = \frac{1}{2}$, $0 \leq A + B < 90^o$ and $A > B$, then find A and B.

Ans : [Board Term-1 2016]

h115

We have $\sin(A+B) = 1 = \sin 90^o$

$$A + B = 90^o \quad ...(1)$$

and $\sin(A-B) = \frac{1}{2} = \sin 30^o$

$$A - B = 30^o \quad ...(2)$$

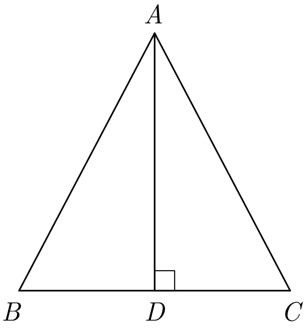
Solving eq. (1) and (2), we obtain

$$A = 60^o \text{ and } B = 30^o$$

39. Find $\operatorname{cosec} 30^o$ and $\cos 60^o$ geometrically.

Ans : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



h116

In ΔABC, $\angle A = \angle B = \angle C = 60^o$

Now we draw AD perpendicular to BC, then

$$\Delta BDA \cong \Delta CDA$$

$$BD = CD$$

$$\angle BAD = CAD = 30^o \quad \text{by } CPCT$$

$$AD = \sqrt{3}a$$

In ΔBDA, $\operatorname{cosec} 30^o = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^o = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

40. Evaluate : $\frac{\sin 90^o}{\cos 45^o} + \frac{1}{\operatorname{cosec} 30^o}$

Ans : [Board Term-1 2013]

h118

We have $\frac{\sin 90^o}{\cos 45^o} + \frac{1}{\operatorname{cosec} 30^o} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2}+1}{2}$$

41. If $\sqrt{2}\sin\theta = 1$, find the value of $\sec^2\theta - \csc^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{2}\sin\theta = 1$

$$\sin\theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

h120

Thus $\theta = 45^\circ$

Now $\sec^2\theta - \csc^2\theta = \sec^2 45^\circ - \csc^2 45^\circ$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

42. If $4\cos\theta = 11\sin\theta$, find the value of $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta}$.

Ans : [Board Term-1 2012]

We have $4\cos\theta = 11\sin\theta$

or, $\cos\theta = \frac{11}{4}\sin\theta$

h121

Now $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$

$$= \frac{\sin\theta(\frac{121}{4} - 7)}{\sin\theta(\frac{121}{4} + 7)}$$

$$= \frac{121-28}{121+28} = \frac{93}{149}$$

43. If $\tan(A+B) = \sqrt{3}$, $\tan(A-B) = \frac{1}{\sqrt{3}}$ $0^\circ < A+B \leq 90^\circ$, then find A and B.

Ans : [Board Term-1 2012]

We have $\tan(A+B) = \sqrt{3} = \tan 60^\circ$

$$A + B = 60^\circ \quad ...(1)$$

Also $\tan(A-B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$$A - B = 30^\circ \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2} = 45^\circ$$

h122

Substituting this value of A in equation (1), we get

$$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$

44. If $\cos(A-B) = \frac{\sqrt{3}}{2}$ and $\sin(A+B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B, where $(A+B)$ and $(A-B)$ are acute angles.

Ans : [Board Term-1 2012]

We have $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$$A - B = 30^\circ \quad ...(1)$$

Also $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$$A + B = 60^\circ \quad ...(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ$$

$$A = 45^\circ$$

h123

Substituting this value of A in equation (1), we get $B = 15^\circ$

45. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$.

Ans : [Board Term-1 2012, Set-25]

We have $2\sin 2\theta = \sqrt{3}$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

h125

$$2\theta = 60^\circ$$

Hence, $\cos 2\theta = \cos 60^\circ = \frac{1}{2}$.

46. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ is it equal to $\sin 90^\circ$ or $\cos 90^\circ$?

Ans : [Board Term-1 2016]

$$\sin 30^\circ\cos 60^\circ + \cos 30^\circ \sin 60^\circ = \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^\circ = 1$ but not equal to $\cos 90^\circ$ as $\cos 90^\circ = 0$.

h126

47. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^\circ < \theta < 90^\circ$, find the value of θ.

Ans : [Boar Term-1, 2012]

We have

h128

$$\sqrt{3}\sin\theta - \cos\theta = 0 \text{ and } 0^\circ < \theta < 90^\circ$$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \left[\tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\theta = 30^\circ$$

48. Evaluate : $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ}$

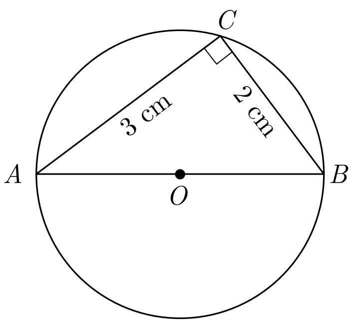
Ans : [Board Term-1 2012]

We have $\dfrac{\cos 45^\circ}{\sec 30^\circ} + \dfrac{1}{\sec 60^\circ} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \dfrac{1}{2}$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6}+2}{4}$$

49. In the given figure, AOB is a diameter of a circle with centre O, find $\tan A \tan B$.



Ans : [Board Term-1 2012]

In ΔABC, $\angle C$ is a angle in a semi-circle, thus

$$\angle C = 90^\circ$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and $\tan B = \dfrac{AC}{BC} = \dfrac{3}{2}$

$$\tan A \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

50. If $\sin\phi = \frac{1}{2}$, show that $3\cos\phi - 4\cos^3\phi = 0$.

Ans :

We have $\sin\phi = \dfrac{1}{2}$

$$\phi = 30^\circ$$

Now substituting this value of θ in LHS we have

$$3\cos\phi - 4\cos^3\phi = 3\cos 30^\circ - 4\cos^3 30^\circ$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$= 0$ Hence Proved

51. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans : [Board Term-1 2015]

We have $\sec A = \dfrac{1}{\cos A} = \dfrac{1}{\sqrt{1-\sin^2 A}}$

and $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sin A}{\sqrt{1-\sin^2 A}}$

52. Prove that : $\dfrac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$

Ans : [Board Term-1 2015]

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\cos^2\theta}$$

$$= 1$$

53. Prove that : $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Ans : [Board Term-1 2015]

We have

$$\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1)$$

$[[1 + \tan^2\theta = \sec^2\theta]]$

$$= \sec^2\theta(\tan^2\theta)$$

$$= (1 + \tan^2\theta)\tan^2\theta$$

$$= \tan^2\theta + \tan^4\theta$$

Hence Proved.

54. Find the value of θ, if,

$\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4; \theta \le 90^\circ$

Ans : [Board Term-1 2015]

We have $\dfrac{\cos\theta}{1-\sin\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 4$

$$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

Thus $\theta = 60^\circ$.

55. Prove that : $-1+\dfrac{\sin A\sin(90^\circ-A)}{\cot(90^\circ-A)} = -\sin^2 A$

Ans : [Board Term-1 2012]

$$-1+\frac{\sin A\sin(90^\circ-A)}{\cot(90^\circ-A)} = -\sin^2 A$$

$$\frac{\sin A\sin(90^\circ-A)}{\cot(90^\circ-A)} = 1-\sin^2 A$$

h160

$$\frac{\sin A\cos A}{\tan A} = \cos^2 A$$

$$\frac{\sin A\cos A}{\frac{\sin A}{\cos A}} = \cos^2 A$$

$$\frac{\cos A}{\sin A}\sin A\cos A = \cos^2 A$$

$$\cos^2 A = \cos^2 A \quad \text{Hence Proved.}$$

56. Prove that : $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A-\cot A$

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A}\times\frac{1-\cos A}{1-\cos A}}$$

$$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}$$

h161

$$= \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$

$$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A}-\frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A-\cot A \qquad \text{Hence Proved.}$$

57. If $\sin\theta-\cos\theta = \frac{1}{2}$, then find the value of $\sin\theta+\cos\theta$.

Ans : [Board Term-1 2013]

We have $\sin\theta-\cos\theta = \frac{1}{2}$

Squaring both sides, we get

h162

$$(\sin\theta-\cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta = \frac{1}{4}$$

$$1-2\sin\theta\cos\theta = \frac{1}{4}$$

$$2\sin\theta\cos\theta = 1-\frac{1}{4} = \frac{3}{4}$$

Again, $(\sin\theta+\cos\theta)^2 = \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$$= 1+2\sin\theta\cos\theta$$

$$= 1+\frac{3}{4} = \frac{7}{4}$$

Thus $\sin\theta+\cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

58. If θ be an acute angle and $5\operatorname{cosec}\theta = 7$, then evaluate $\sin\theta+\cos^2\theta-1$.

Ans : [Board Term-1 2012]

We have $5\operatorname{cosec}\theta = 7$

h163

$$\operatorname{cosec}\theta = \frac{7}{5}$$

$$\sin\theta = \frac{5}{7} \qquad \left[\operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$\sin\theta+\cos^2\theta-1 = \sin\theta-(1-\cos^2\theta)$$

$$= \sin\theta-\sin^2\theta \quad [\sin^2\theta+\cos^2\theta = 1]$$

$$= \frac{5}{7}-\left(\frac{5}{7}\right)^2 = \frac{35-25}{49} = \frac{10}{49}$$

59. If $\sin A = \dfrac{\sqrt{3}}{2}$, find the value of $2\cot^2 A-1$.

Ans : [Board Term-1 2012]

Using $\cot^2\theta = -1+\operatorname{cosec}^2\theta$ we have

h164

$$2\cot^2 A-1 = 2(\operatorname{cosec}^2 A-1)-1$$

$$= \frac{2}{\sin^2 A}-3$$

$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2}-3 = \frac{8}{3}-3 = \frac{-1}{3}$$

Thus $2\cot^2 A-1 = \dfrac{-1}{3}$

THREE MARKS QUESTIONS

60. Show that : $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}=1$

Ans : [Board 2020 OD Standard]

$$\text{LHS} = \frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$$

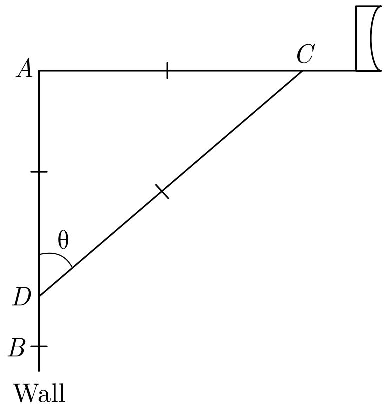
$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$$

$$= \frac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$$

$$= \frac{1}{1} = 1 = \text{RHS}$$

h286

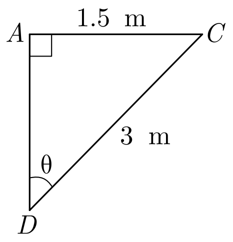
61. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If $AC = 1.5$ m long and $CD = 3$ m, find (i) $\tan\theta$ (ii) $\sec\theta + \operatorname{cosec}\theta$.



h287

Ans : [Board 2020 Delhi Standard]

From the given information we draw the figure as below



In right angle triangle ΔCAD, applying Pythagoras theorem,

$$AD^2 + AC^2 = DC^2$$

$$AD^2 + (1.5)^2 = (3)^2$$

$$AD^2 = 9 - 2.25 = 6.75$$

$$AD = \sqrt{6.75} = 2.6 \text{ m (Approx)}$$

(i) $$\tan\theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$$

(ii) $$\sec\theta + \operatorname{cosec}\theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$$

62. Prove that : $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1} = \dfrac{1+\cot\theta}{\sin\theta}$

Ans : [Board 2020 Delhi Standard]

h288

$$\text{LHS} = \frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}+1}$$

$$= \frac{\sin\theta(\cos\theta+1-\sin\theta)}{\sin\theta(\cos\theta-1+\sin\theta)}$$

$$= \frac{\sin\theta\cos\theta+\sin\theta-\sin^2\theta}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{\sin\theta\cos\theta+\sin\theta-(1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{\sin\theta(\cos\theta+1)-(1-\cos^2\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta-1+\cos\theta)}{\sin\theta(\cos\theta+\sin\theta-1)}$$

$$= \frac{1+\cos\theta}{\sin\theta} = \text{RHS}$$

63. If $\sin\theta + \cos\theta = \sqrt{2}$ prove that $\tan\theta + \cot\theta = 2$

Ans : [Board 2020 OD Standard]

We have $$\sin\theta + \cos\theta = \sqrt{2}$$

Squaring both the sides, we get

$$(\sin\theta+\cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

$$2\sin\theta\cos\theta = 1$$

$$\sin\theta\cos\theta = \frac{1}{2} \quad \text{...(1)}$$

h289

Now $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\frac{1}{2}} = 2 = \text{RHS}$$

64. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 SQP Standard]

Given, $\sin\theta + \cos\theta = \sqrt{3}$

h290

Squaring above equation, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$
$$1 + 2\sin\theta\cos\theta = 3$$
$$2\sin\theta\cos\theta = 3 - 1 = 2$$
$$\sin\theta\cos\theta = 1$$

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

65. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or ½.

Ans : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

h291

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$
$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$
$$\text{cosec}^2\theta + 1 = 3\cot\theta$$
$$1 + \cot^2\theta + 1 = 3\cot\theta$$
$$\cot^2\theta - 3\cot\theta + 2 = 0$$
$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$
$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$
$$(\cot\theta - 2)(\cot\theta - 1) = 0$$
$$\cot\theta = 1 \text{ or } 2$$
$$\tan\theta = 1 \text{ or } ½.$$

66. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

Ans : [Board 2019 Delhi Standard]

h292

$$\text{LHS} = (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$$
$$= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\,\theta) +$$
$$+(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$
$$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta)$$
$$+2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$
$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$
$$= 7 + \tan^2\theta + \cot^2\theta$$
$$= \text{RHS}$$

67. Prove that $(1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A) = 2$

Ans : [Board 2019 Delhi]

h293

$$\text{LHS} = (1 + \cot A - \text{cosec}\,A)(1 + \tan A + \sec A)$$
$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$
$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$
$$= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A\cos A}$$
$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A\cos A}$$
$$= \frac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$$
$$= \frac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$$
$$= 2 = \text{RHS}$$

68. Prove that $\frac{\sin A - \cos A - 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

Ans : [Board 2019 Delhi]

h294

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$
$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A\sin A - \sin A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A\cos A}$$
$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)}$$
$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$
$$= \sec A + \tan A$$
$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$

69. Prove that : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Ans : [Board 2020 Delhi Standard]

$$\text{LHS} = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta] +$$
$$- 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= - \sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$$

$$= -(\sin^2\theta + \cos^2\theta)^2 + 1$$

$$= -1 + 1 = 0 = \text{RHS}$$

h295

70. Prove that $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A\sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

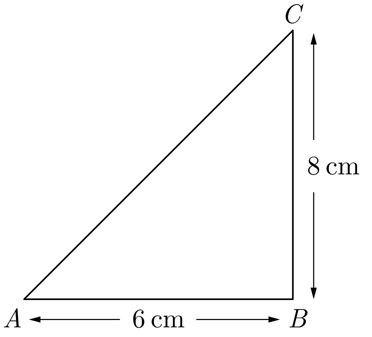
h296

71. If in a triangle ABC right angled at B, $AB = 6$ units and $BC = 8$ units, then find the value of $\sin A\cos C + \cos A\sin C$.

Ans : [Board Term-1 2016]

As per question statement figure is shown below.

h132



We have $AC^2 = 8^2 + 6^2 = 100$

$AC = 10$ cm

Now $\sin A = \frac{BC}{AC} = \frac{8}{10}$;

$\cos A = \frac{AB}{AC} = \frac{6}{10}$

and $\sin C = \frac{AB}{AC} = \frac{6}{10}$;

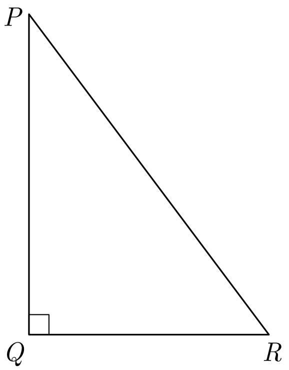
$\cos C = \frac{BC}{AC} = \frac{8}{10}$

Thus $\sin A\cos C + \cos A\sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$

$$= \frac{64}{100} + \frac{36}{100}$$

$$= \frac{100}{100} = 1$$

72. In the given $\angle PQR$, right-angled at Q, $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans : [Board Term-1 2015]

Using Pythagoras theorem we have

$$PQ^2 + QR^2 = PR^2$$

$$PQ^2 + 9^2 = (PQ+1)^2$$

h133

$$PQ^2 + 81 = (PQ+1)^2$$

$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$

$$PQ = 40$$

Since $PR - PQ = 1$, thus,

$$PR = 1 + 40 = 41$$

$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

73. If $\cos(40^o + x) = \sin 30^o$, find the value of x.

Ans : [Board Term-1 2015]

We have

$$\cos(40^o - x) = \sin 30^o$$

$$\cos(40^o + x) = \sin(90^o - 60^o)$$

$$\cos(40^o + x) = \cos 60^o$$

$$40^o + x = 60^o$$

$$x = 60^o - 40^o = 20^o$$

h135

Thus $x = 20^o$.

74. Evaluate : $\dfrac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$

Ans : [Board Term-1 2013]

$$\frac{5\cos^2 60^o + 4\cos^2 30^o - \tan^2 45^o}{\sin^2 30^o + \cos^2 60^o}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

h136

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

75. Verify : $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta}$, for $\theta = 60^o$

Ans :

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos 60^o}{1+\cos 60^o}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \qquad \left(\cos 60^o = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin\theta}{1+\cos\theta} = \frac{\sin 60^o}{1+\cos 60^o}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

h139

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^o$.

76. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans : [Board Term-1 2015]

We have $\tan A + \cot A = 2$

Squaring both sides, we have

$$(\tan A + \cot A)^2 = (2)^2$$

h140

$$\tan^2 A + \cot^2 A + 2\tan A \cot A = 4$$

$$\tan^2 A + \cot^2 A + 2\tan A \times \frac{1}{\tan A} = 4$$

$$\tan^2 A + \cot^2 A + 2 = 4$$

$$\tan^2 A + \cot^2 A = 4 - 2$$

$$\tan^2 A + \cot^2 A = 2$$

77. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that $\cos\theta - \sin\theta = \sqrt{2}\cos\theta$.

Ans : [Board Term-1 2011]

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

h141

We have
$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$= (\sqrt{2} - 1)\cos\theta$$

$$= \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}+1)}\cos\theta$$

Thus
$$\sin\theta = \frac{1}{\sqrt{2}+1}\cos\theta$$

$$(\sqrt{2}+1)\sin\theta = \cos\theta$$

$$\sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Hence proved.

78. Prove that : $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$.

Ans : [Board Term-1 2013, 2011]

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

h142

$$= \frac{\cos A}{1-\left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1-\left(\frac{\cos A}{\sin A}\right)}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)}$$

$$= \cos A + \sin A$$

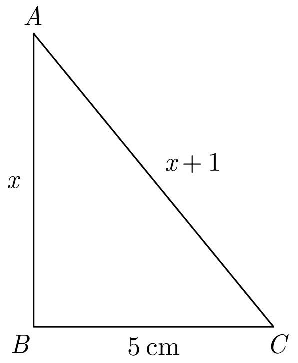
$$= \sin A + \cos A$$

$= \text{RHS}$ Hence proved.

79. In ΔABC, $\angle B = 90°$, $BC = 5$ cm, $AC - AB = 1$, Evaluate : $\frac{1+\sin C}{1+\cos C}$.

Ans : [Board Term-1 2011]

As per question we have drawn the figure given below.



h143

We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now
$$AC^2 = AB^2 + BC^2$$

$$(x+1)^2 = x^2 + 5^2$$

$$x^2 + 2x + 1 = x^2 + 25$$

$$2x = 24$$

$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now
$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now
$$\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$

80. Prove that : $\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A} = \cos A - \sin A$

Ans : [Board Term-1 2016]

h165

$$\frac{\cos A}{1+\tan A} - \frac{\sin A}{1+\cot A}$$

$$= \frac{\cos A}{1+\frac{\sin A}{\cos A}} - \frac{\sin A}{1+\frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A}$$

$= \cos A - \sin A$ Hence Proved.

81. If $b\cos\theta = a$, then prove that $\operatorname{cosec}\theta + \cot\theta = \sqrt{\frac{b+a}{b-a}}$.

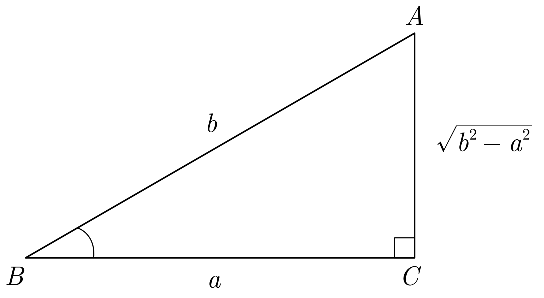
Ans : [Board Term-1 2015]

h166

We have $b\cos\theta = a$

or,
$$\cos\theta = \frac{a}{b}$$

Now consider the triangle shown below.



$$AC^2 = AB^2 - BC^2$$

or, $$\cos\theta = \frac{a}{b}$$

$$AC = \sqrt{b^2-a^2}$$

Now $$\operatorname{cosec}\theta = \frac{b}{\sqrt{b^2-a^2}}, \cot\theta = \frac{a}{\sqrt{b^2-a^2}}$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{b+a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

82. Prove that : $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

Ans : [Bard Term-1 2015]

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

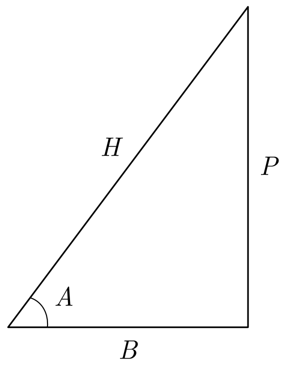
$$= \tan\theta$$

h167

83. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans : [Board Term-1 2015, NCERT]

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement. Consider the triangle shown below.



Let $\tan A = \dfrac{P}{B}$ and $\sec A = \dfrac{H}{B}$

h168

$$H^2 = P^2 + B^2$$

Now $$1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$$

$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$

$$= \left(\frac{H}{B}\right)^2$$

$$= \sec^2 A$$ Hence Proved.

84. Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

Ans : [Board Term-1 2015]

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

h169

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$ Hence Proved.

85. Prove that :

$(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Ans : [Board Term-1 2015]

$$\text{LHS} = (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta)$$

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta\cos\theta}\right) \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \cos\theta\sin\theta \times \frac{1}{\sin\theta\cos\theta} = 1$$

h170

86. Show that :

$\text{cosec}^2\theta - \tan^2(90^o - \theta) = \sin^2\theta + \sin(90^o - \theta)$

Ans : [Board Term-1 2013]

$$\text{cosec}^2\theta - \tan^2(90^o - \theta)$$

$$= \text{cosec}^2\theta - \cot^2\theta$$

h171

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta}$$

$$= 1$$

$$= \sin^2\theta + \cos^2\theta$$

$$= \sin^2\theta + \sin^2(90^o - \theta)$$

Hence Proved

87. Prove that : $\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1 2013]

We have

$$\frac{\text{cosec}^2\theta}{\text{cosec}\,\theta - 1} - \frac{\text{cosec}^2\theta}{\text{cosec}\,\theta + 1} = \text{cosec}^2\theta\left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1}\right]$$

$$= \text{cosec}^2\theta\left[\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta}\right]$$

$$= \frac{1}{\sin^2\theta}\sin\theta\left[\frac{(1+\sin\theta)-(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}\right]$$

h172

$$= \frac{1}{\sin\theta}\left[\frac{2\sin\theta}{1-\sin^2\theta}\right]$$

$$= \frac{2}{\cos^2\theta} = 2\sec^2\theta$$

Hence Proved

88. Prove that :

$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}.$$

Ans : [Board Term-1 2011]

$$\frac{1}{\text{cosec}\,A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\text{cosec}\,A + \cot A}$$

$$\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$$

$$\frac{1}{\text{cosec}\,A - \cot A} + \frac{1}{\text{cosec}\,A + \cot A} = \frac{2}{\sin A}$$

$$\frac{\text{cosec}\,A + \cot A + \text{cosec}\,A - \cot A}{(\text{cosec}\,A - \cot A)(\text{cosec}\,A + \cot a)} = \frac{2}{\sin A}$$

h173

$$\frac{2\,\text{cosec}\,A}{\text{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A}$$

$$\frac{2\frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A} \quad \text{Hence Proved.}$$

89. If $\sec\theta = x + \frac{1}{4x}$ prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Ans : [Board Term-1 2011]

We have $\sec\theta = x + \frac{1}{4x}$ (1)

Squaring both side we have

h174

$$\sec^2\theta = x^2 + 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$\tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2} - 1$$

$$= x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$= x^2 - 2x\frac{1}{4x} + \frac{1}{16x^2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

Now $\tan\theta = x - \frac{1}{4x}$ (2)

or $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ (3)

Adding (1) and (2) we have

$$\tan\theta + \sec\theta = 2x$$

Adding (1) and (3) we have

$$\sec\theta + \tan\theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x}$$ Hence proved.

90. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$

Ans : [Board Term-1 2011]

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta}$$

$$= \frac{2}{2\sin^2\theta - 1} = \text{RHS}$$

h175

Hence Proved.

91. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1 2011]

We have $\quad x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta \quad (1)$

and $\quad x\sin\theta = y\cos\theta$

or, $\quad x = \frac{y\cos\theta}{\sin\theta} \quad (2)$

Eliminating x from equation (1) and (2) we obtain,

$$\frac{y\cos\theta}{\sin\theta}\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$y\cos\theta[\sin^2\theta + \cos^2\theta] = \sin\theta\cos\theta$$

$$y(\sin^2\theta + \cos^2\theta) = \sin\theta$$

$$y = \sin\theta \quad ...(3)$$

h176

Substituting this value of y in equation (2) we have,

$$x = \cos\theta \quad (4)$$

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$ Hence Proved.

92. Prove that $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

Ans : [Board Term-1 2011]

$$X = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$$

$$= (1 - \sin\theta\cos\theta)$$

$$Y = \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$$= (1 + \sin\theta\cos\theta)$$

h177

Now given expression

$$X + Y = \frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$$

$$= (1 - \sin\theta\cos\theta) + (1 + \sin\theta\cos\theta)$$

$$= 2 - \sin\theta\cos\theta + \sin\theta\cos\theta$$

$$= 2 = \text{RHS}$$ Hence Proved.

93. Express : $\sin A, \tan A$ and $\operatorname{cosec} A$ in terms of $\sec A$.

Ans : [Board Term-1 2011]

(1) $\quad \sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

h181

(2) $\quad \tan A = \frac{\sin A}{\cos A} = \sin A\sec A$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(3) $\quad \operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

94. If $\sin\theta + \cos\theta = \sqrt{2}$, then evaluate $\tan\theta + \cot\theta$.

Ans : [Board SQP 2018]

We have $\quad \sin\theta + \cos\theta = \sqrt{2}$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta = 2$$

h184

$$2\sin\theta\cos\theta \quad 2-1 = 1$$

$$\frac{1}{\sin\theta\cos\theta} = 2$$

Now, $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} = 2$$

FOUR MARKS QUESTIONS

95. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Ans : [Board 2020 Delhi Standard]

h297

We have $$\sin\theta + \cos\theta = \sqrt{3}$$

Squaring both the sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 3 - 1 = 2$$

$$\sin\theta\cos\theta = 1 \qquad ...(1)$$

Now $$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

or $$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

Substituting the value of $\sin\theta\cos\theta$ from equation (1) we have

$$\tan\theta + \cot\theta = \frac{1}{1} = 1$$

Hence, $$\tan\theta + \cot\theta = 1$$

96. If $\sec\theta = x + \frac{1}{4x}$, $x \neq 0$ find $(\sec\theta + \tan\theta)$.

Ans : [Board 2019 Delhi]

h298

We have $$\sec\theta = x + \frac{1}{4x} \qquad ...(1)$$

Since, $$\tan^2\theta = \sec^2\theta - 1$$

Substituting value of $\sec\theta$ we have

$$\tan^2\theta$$

$$= \left(x + \frac{1}{4x}\right)^2 - 1$$

$$= x^2 + \frac{2x}{4x} + \frac{1}{16x^2} - 1$$

$$= x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = x - \frac{1}{4x}$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right) = 2x$$

When $\sec\theta = x + \frac{1}{4x}$ and $\tan\theta = -\left(x - \frac{1}{4x}\right)$ we have

$$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left\{-\left(x - \frac{1}{4x}\right)\right\}$$

$$= x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$= \frac{2}{4x} = \frac{1}{2x}$$

97. If $\sin A = \frac{3}{4}$ calculate $\sec A$.

Ans : [Board 2019 OD]

h299

We have $$\sin A = \frac{3}{4}$$

Now $$\cos^2 A = 1 - \sin^2 A$$

$$\cos^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

Thus $$\sec A = \frac{1}{\cos A} = \frac{4}{\sqrt{7}}$$

98. Prove that: $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\,\theta$

Ans : [Board 2019 OD]

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$= \frac{(\tan\theta-1)(\tan^2\theta+1+\tan\theta)}{\tan\theta(\tan\theta-1)}$$

$$= \frac{\tan^2\theta+1+\tan\theta}{\tan\theta}$$

h301

$$= \tan\theta+\cot\theta+1$$

$$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1$$

$$= \frac{1}{\sin\theta\cos\theta}+1$$

$$= \operatorname{cosec}\theta\sec\theta+1$$

$= 1+\sec\theta\operatorname{cosec}\theta$ Hence Proved

99. Prove that: $\dfrac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta} = 2+\dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

Ans : [Board 2019 OD]

$$\text{LHS} = \frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta+1}$$

$$= \frac{1-\cos^2\theta}{\cos\theta+1} = \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta+1}$$

$$= 1-\cos\theta \qquad ...(1)$$

Now, $\text{RHS} = 2+\dfrac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

$$= 2+\frac{\sin\theta}{\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}} = 2+\frac{\sin^2\theta}{\cos\theta-1}$$

h302

$$2+\frac{1-\cos^2\theta}{\cos\theta-1} = 2-\frac{(\cos^2\theta-1)}{(\cos\theta-1)}$$

$$= 2-\frac{(\cos\theta-1)(\cos\theta+1)}{\cos\theta-1}$$

$$= 2-(\cos\theta+1) = 1-\cos\theta$$

$= \text{LHS}$ Hence Proved

100. Find A and B if $\sin(A+2B) = \frac{\sqrt{3}}{2}$ and $\cos(A+4B) = 0$, where A and B are acute angles.

Ans : [Board 2019 OD]

We have $\sin(A+2B) = \dfrac{\sqrt{3}}{2}$

$$\sin(A+2B) = \sin 60^\circ \quad \left(\sin 60^\circ = \tfrac{\sqrt{3}}{2}\right)$$

$$A+2B = 60^\circ \qquad ...(1)$$

Also, given $\cos(A+4B) = 0$

$$\cos(A+4B) = \cos 90^\circ \quad (\cos 90^\circ = 0)$$

$$A+4B = 90^\circ \qquad ...(2)$$

Subtracting equation (2) from equation (1) we get

$$-2B = -30^\circ \Rightarrow B = 15^\circ$$

From equation (1) we have

h303

$$A+2(15^\circ) = 60^\circ$$

$$A = 60^\circ-30^\circ$$

$= 30^\circ$

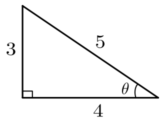
Hence angle $A = 30^\circ$ and angle $B = 15^\circ$.

101. If $4\tan\theta = 3$, evaluate $\left(\dfrac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1}\right)$

Ans : [Board 2018]

We have $4\tan\theta = 3 \Rightarrow \tan\theta = \dfrac{3}{4}$

h304



We know very well that if $\tan\theta = \frac{3}{4}$, then

$$\sin\theta = \frac{3}{5} \text{ and } \cos\theta = \frac{4}{5}$$

Substituting above values in given expression,

$$\frac{4\sin\theta-\cos\theta+1}{4\sin\theta+\cos\theta-1} = \frac{4\times\frac{3}{5}-\frac{4}{5}+1}{4\times\frac{3}{5}+\frac{4}{5}-1} = \frac{13}{11}$$

102. Evaluate :

$\tan^2 30^\circ\sin 30^\circ+\cos 60^\circ\sin^2 90^\circ\tan^2 60^\circ-2\tan 45^\circ\cos^2 0^\circ\sin 90^\circ$

Ans : [Board Term-1 2015]

$\tan^2 30^\circ\sin 30^\circ+\cos 60^\circ\sin^2 90^\circ\tan^2 60^\circ-2\tan 45^\circ\cos^2 0^\circ\sin 90^\circ$

$$= \left(\frac{1}{\sqrt{3}}\right)^2\times\frac{1}{2}+\frac{1}{2}\times(1)^2\times(\sqrt{3})^2-2\times1\times1^2\times1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1 + 9 - 12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

h144

103. Given that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

find the values of $\tan 75^o$ and $\tan 90^o$ by taking suitable values of A and B.

Ans : [Board Term-1 2012, NCERT]

We have $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

h145

(i)

$$\tan 75^o = \tan(45^o + 30^o)$$

$$= \frac{\tan 45^o + \tan 30^o}{1 - \tan 45^o \tan 30^o}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3}$$

Hence $\tan 75^o = 2 + \sqrt{3}$

(ii)

$$\tan 90^o = \tan(60^o + 30^o)$$

$$= \frac{\tan 60^o + \tan 30^o}{1 - \tan 60^o \tan 30^o}$$

$$= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3+1}{\sqrt{3}}}{0}$$

Hence, $\tan 90^o = \infty$

104. Evaluate :

$$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$$

Ans : [Board Term-1 2013]

h146

$$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin 90^o - 2\cos^2 90^o + \frac{1}{24}$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$$

$$= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$$

$$= \frac{3 + 32 + 12 + 1}{24} = \frac{48}{24} = 2$$

105. Evaluate : $4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$

Ans : [Board Term-1 2013]

$$4(\sin^4 30^o + \cos^4 60^o) - 3(\cos^2 45 - \sin^2 90^o)$$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

h147

106. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$.

Ans : [Board Term-1 2012]

We have

$$15\tan^2\theta + 4\sec^2\theta = 23$$

$$15\tan^2\theta + 4(\tan^2\theta + 1) = 23$$

$$15\tan^2\theta + 4\tan^2\theta + 4 = 23$$

$$19\tan^2\theta = 19$$

$$\tan\theta = 1 = \tan 45^o$$

h148

Thus $\theta = 45^o$

Now, $(\sec\theta + \operatorname{cosec}\theta)^2 - \sin^2\theta$

$$= (\sec 45^o + \operatorname{cosec} 45^o)^2 - \sin^2 45^o$$

$$= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2}$$

107. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$, then we have

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$

$$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$

$$x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

h149

Thus $\cot\theta = \sqrt{3}$ or $\cot\theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^o$ or $\theta = 60^o$

If $\theta = 30^o$, then

$$\cot^2 30^o + \tan^2 30^o = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

If $\theta = 60^o$, then

$$\cot^2 60^o + \tan^2 60^o = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}.$$

108. Evaluate the following :

$$\frac{2\cos^2 60^o + 3\sec^2 30^o - 2\tan^2 45^o}{\sin^2 30^o + \cos^2 45^o}$$

Ans : [Board Term-1 2012]

$$\frac{2\cos^2 60^o + 3\sec^2 30^o - 2\tan^2 45^o}{\sin^2 30^o + \cos^2 45^o} = \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{2(\frac{1}{2})^2 + 3(\frac{2}{\sqrt{3}})^2 - 2(1)^2}{(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2}$$

$$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}$$

h150

109. Prove that : $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$.

Ans : [Board Term-1 2012]

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1-\tan\theta)\tan\theta}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)}$$

$$= \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta}$$

h152

$$= \tan\theta + 1 + \cot\theta$$

Hence Proved.

110. In an acute angled triangle ABC if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Ans : [Board Term-1 2012]

We have $\sin(A+B-C) = \frac{1}{2} = \sin 30^o$

$$A + B - C = 30^o \quad ...(1)$$

and $\cos(B+C-A) = \frac{1}{\sqrt{2}} = \cos 45^o$

$$B + C - A = 45^o \quad ...(2)$$

Adding equation (1) and (2), we get

$$2B = 75^o \Rightarrow B = 37.5^o$$

Subtracting equation (2) from equation (1) we get,

$$2(A-C) = -15^o$$

$$A - C = -7.5^o \quad ...(3)$$

Now $A + B + C = 180^o$

$$A + C = 180^\circ - 37.5^\circ = 142.5^o \quad ...(4)$$

Adding equation (3) and (4), we have

$$2A = 135^o \Rightarrow A = 67.5^o$$

h153

and, $C = 75^o$

Hence, $\angle A = 67.5^o, \angle B = 37.5^o, \angle C = 75^o$

111. Prove that $b^2x^2 - a^2y^2 = a^2b^2$, if :

(1) $x = a\sec\theta, y = b\tan\theta$, or

(2) $x = a\,\mathrm{cosec}\,\theta, y = b\cot\theta$

Ans : [Board Term-1 2015]

(1) We have $x = a\sec\theta, y = b\tan\theta$,

$$\frac{x^2}{a^2} = \sec^2\theta,\ \frac{y^2}{b^2} = \tan^2\theta$$

h185

or, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

(ii) We have $x = a\,\mathrm{cosec}\,\theta, y = b\cot\theta$

$$\frac{x^2}{a^2} = \mathrm{cosec}^2\theta,\ \frac{y^2}{b^2} = \cot^2\theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \mathrm{cosec}^2\theta - \cot\theta = 1$$

Thus $b^2x^2 - a^2y^2 = a^2b^2$ Hence Proved

112. If $\mathrm{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$, then prove that $\mathrm{cosec}\,\theta + \cot\theta = \sqrt{2}\,\mathrm{cosec}\,\theta$.

Ans : [Board Term-1 2015]

We have $\mathrm{cosec}\,\theta - \cot\theta = \sqrt{2}\cot\theta$

h186

Squaring both sides we have

$$\mathrm{cosec}^2\theta + \cot^2\theta - 2\,\mathrm{cosec}\,\theta\cot\theta = 2\cot^2\theta$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = 2\operatorname{cosec}\theta\cot\theta$$

$$(\operatorname{cosec}\theta - \cot\theta = \sqrt{2}\cot\theta)$$

$$(\operatorname{cosec}\theta + \cot\theta)\sqrt{2}\cot\theta = 2\operatorname{cosec}\theta\cot\theta$$

$$\operatorname{cosec}\theta + \cot\theta = \sqrt{2}\operatorname{cosec}\theta$$

Hence Proved.

113. Prove that :

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2} = \frac{\sec\theta\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Board Term-1 2015]

$$\frac{\cot^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\tan^3\theta\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

h187

$$= \frac{\frac{\cos^3\theta}{\sin^3\theta}\times\sin^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta}\times\cos^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{\cos^3\theta}{(\cos\theta+\sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)^2}$$

$$= \frac{1-\sin\theta\cos\theta}{\cos\theta+\sin\theta} = \frac{\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}}{\frac{\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\operatorname{cosec}\theta\sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Hence Proved

114. Prove that : $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = 2\operatorname{cosec}\theta.$

Ans : [Board Terim-1, 2012, Set-9]

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}} + \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} = \frac{(\sec\theta-1)+(\sec\theta+1)}{\sqrt{(\sec\theta+1)(\sec\theta-1)}}$$

$$= \frac{2\sec\theta}{\sqrt{\sec^2\theta-1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$$

$$= 2\times\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta}$$

h188

$$= 2\times\frac{1}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta$$

Hence Proved

115. Prove that : $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}.$

Ans : [Board Term-1 2012]

We have $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$

h189

$$= \frac{\sin\theta(\frac{1}{\cos\theta}+1)}{\sin\theta(\frac{1}{\cos\theta}-1)}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Proved.

116. Prove that : $\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1} + \frac{\operatorname{cosec}A}{\operatorname{cosec}A+1} = 2\sec^2 A$

Ans : [Board Term-1 2012]

$$\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1} + \frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}$$

$$= \frac{\operatorname{cosec}^2A+\operatorname{cosec}A+\operatorname{cosec}^2A-\operatorname{cosec}A}{(\operatorname{cosec}A-1)(\operatorname{cosec}A+1)}$$

$$= \frac{2\operatorname{cosec}^2A}{\operatorname{cosec}^2A-1} = \frac{2\operatorname{cosec}^2A}{\cot^2A}$$

h190

$$= \frac{\frac{2}{\sin^2A}}{\frac{\cos^2A}{\sin^2A}} = \frac{2}{\sin^2A}\times\frac{\sin^2A}{\cos^2A}$$

$$= \frac{2}{\cos^2A} = 2\sec^2A$$

Hence Proved.

117. If $\operatorname{cosec}\theta+\cot\theta = p$, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}.$

Ans : [Board Term-1 2016

$$\frac{p^2-1}{p^2+1} = \frac{(\operatorname{cosec}\theta+\cot\theta)^2-1}{(\operatorname{cosec}\theta+\cot\theta)^2+1}$$

$$= \frac{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{1+\cot^2\theta+\cot^2\theta+2\operatorname{cosec}\theta\cot\theta-1}{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta-1+2\operatorname{cosec}\theta\cot\theta+1}$$

$$= \frac{2\cot\theta(\cot\theta+\operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+\cot\theta)}$$

h191

$$= \frac{\cos\theta}{\sin\theta}\times\sin\theta = \cos\theta$$

118. If $a\cos\theta+b\sin\theta = m$ and $a\sin\theta-b\cos\theta = n$, prove that $m^2+n^2 = a^2+b^2$

Ans : [Board Term-1 2012]

We have

$$m^2 = a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta \quad ...(1)$$

and, $\quad n^2 = a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta \quad ...(2)$

Adding equations (1) and (2) we get

$$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$
$$= a^2(1) + b^2(1)$$
$$= a^2 + b^2$$

h192

119. Prove that : $\dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta} = 1 + \sin\theta\cos\theta$.

Ans : [Board Term-1 2012]

$$\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta-\cos\theta}$$
$$= \frac{\cos^3\theta}{\cos\theta-\sin\theta} - \frac{\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$$
$$= \frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$
$$= 1+\sin\theta\cos\theta$$

Hence Proved

h193

120. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \mathrm{cosec}\,\theta = q$, prove that $q(p^2-1) = 2p$

Ans : [Board Term-1 2012]

We have $\cos\theta + \sin\theta = p$ and $\sec\theta + \mathrm{cosec}\,\theta = q$

$$q(p^2-1) = (\sec\theta+\mathrm{cosec}\,\theta)[(\cos\theta+\sin\theta)^2-1]$$
$$= (\sec\theta+\mathrm{cosec}\,\theta)(\cos^2\theta+\sin^2\theta+2\sin\theta\cos\theta-1)$$
$$= (\sec\theta+\mathrm{cosec}\,\theta)[1+2\sin\theta\cos\theta-1]$$
$$= \left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$
$$= \left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)2\sin\theta\cos\theta$$
$$= 2(\sin\theta+\cos\theta) = 2p$$

Hence Proved.

h194

121. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, then prove that $x^2+y^2+z^2 = r^2$

Ans : [Board Term-1 2012, Set-50]

Since, $\quad x^2 = r^2\sin^2 A\cos^2 C$

$\quad y^2 = r^2\sin^2 A\sin^2 C$

and $\quad z^2 = r^2\cos^2 A$

h195

$$x^2+y^2+z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$$
$$= r^2\sin^2 A(\cos^2 C+\sin^2 C) + r^2\cos^2 A$$
$$= r^2\sin^2 A + r^2\cos^2 A$$
$$= r^2(\sin^2 A+\cos^2 A)$$
$$= r^2$$

Hence Proved.

122. Prove that: $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$.

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$
$$= \frac{2}{\cos\theta} = 2\sec\theta$$

Hence Proved

h196

123. Prove that

$(1-\sin\theta+\cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$.

Ans : [Board Term-1 2012]

$$(1-\sin\theta+\cos\theta)^2$$
$$= 1+\sin^2\theta+\cos^2\theta-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 1+1-2\sin\theta-2\sin\theta\cos\theta+2\cos\theta$$
$$= 2+2\cos\theta-2\sin\theta-2\sin\theta\cos\theta$$
$$= 2(1+\cos\theta)-2\sin\theta(1+\cos\theta)$$
$$= (1+\cos\theta)(2-2\sin\theta)$$
$$= 2(1+\cos\theta)(1-\sin\theta)$$

Hence Proved

h197

124. Prove that : $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta-1} = \sec\theta+\tan\theta$

Ans : [Board Term-1 2012]

h198

$$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$
$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$
$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta$$ Hence Proved

125. Prove that :

$(\sin\theta + \mathrm{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta\cot^2\theta$

Ans : [Board Term-1 2012]

h199

$(\sin\theta + \mathrm{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + \mathrm{cosec}^2\theta + 2\sin\theta\,\mathrm{cosec}\,\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$= (\sin^2\theta + \cos^2\theta) + \mathrm{cosec}^2\theta + 2 + \sec^2\theta + 2$

$= 1 + (1 + \cot^2\theta) + 2 + (1 + \tan^2\theta) + 2$

$= 7 + \tan^2\theta + \cot^2\theta$ Hence Proved

126. If $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$ and $d > 0$, find the value of $\cos\theta$ and $\tan\theta$.

Ans : [Board Term-1 2013]

We have $\sin\theta = \frac{c}{\sqrt{c^2 + d^2}}$

Now
$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)^2$$
$$= 1 - \frac{c^2}{c^2 + d^2}$$
$$= \frac{c^2 + d^2 - c^2}{c^2 + d^2} = \frac{d^2}{c^2 + d^2}$$

h200

Thus $\cos\theta = \frac{d}{\sqrt{c^2 + d^2}}$

Again, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{c}{\sqrt{c^2 + d^2}}}{\frac{d}{\sqrt{c^2 + d^2}}} = \frac{c}{d}$

Thus $\tan\theta = \frac{c}{d}$

127. If $\tan\theta = \frac{1}{\sqrt{5}}$,

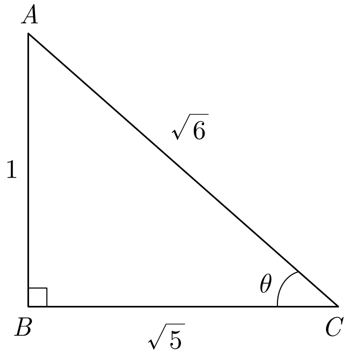
(1) Evaluate : $\frac{\mathrm{cosec}^2\theta - \sec^2\theta}{\mathrm{cosec}^2\theta + \sec^2\theta}$

(2) Verify the identity : $\sin^2\theta + \cos^2\theta = 1$

Ans : [Board Term-1 2012]

We have $\tan\theta = \frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all dimensions.



Now
$$\cot\theta = \frac{1}{\tan\theta} = \sqrt{5}$$
$$\sin\theta = \frac{1}{\sqrt{6}}$$
$$\cos\theta = \frac{\sqrt{5}}{\sqrt{6}}$$

h201

(1)
$$\frac{\mathrm{cosec}^2\theta - \sec^2\theta}{\mathrm{cosec}^2\theta + \sec^2\theta} = \frac{(1 + \cot^2\theta) - (1 + \tan^2\theta)}{(1 + \cot^2\theta) + (1 + \tan^2\theta)}$$
$$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$$
$$= \frac{(\sqrt{5})^2 - (\frac{1}{5})^2}{2 + (\sqrt{5})^2 + (\frac{1}{\sqrt{5}})^2}$$
$$= \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} = \frac{25 - 1}{35 + 1} = \frac{24}{36} = \frac{2}{3}$$

(2)
$$\sin^2\theta + \cos^2\theta = \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$
$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6}$$
$$= 1$$ Hence proved.

128. If $\sec\theta + \tan\theta = p$, show that $\sec\theta - \tan\theta = \frac{1}{p}$, Hence, find the values of $\cos\theta$ and $\sin\theta$.

Ans : [Board Term-1 2015]

We have $\sec\theta + \tan\theta = p$ (1)

Now
$$\frac{1}{p} = \frac{1}{\sec\theta + \tan\theta} \times \frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$$
$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \sec\theta - \tan\theta$$

h204

or $\quad \frac{1}{p} = \sec\theta - \tan\theta \qquad (2)$

Solving $\sec\theta + \tan\theta = p$ and $\sec\theta - \tan\theta = \frac{1}{p}$,

$$\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right) = \frac{p^2+1}{2p}$$

Thus $\quad \cos\theta = \frac{2p}{p^2+1}$

and $\quad \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right) = \frac{p^2-1}{2p}$

and $\quad \sin\theta = \tan\theta\cos\theta = \frac{p^2-1}{p^2+1}$

129. Prove that : $(\operatorname{cosec}\theta + \cot\theta)^2 = \frac{\sec\theta+1}{\sec\theta-1}$

Ans :

$$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta.\cot\theta$$

$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 + \frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta + 2\cos\theta}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

h205

$$= \frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

Hence Prove.

130. Prove that :

$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A\operatorname{cosec} A)^2$

Ans : [Board Term-1 2012]

$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$$

$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + $$

$$+ \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + $$

$$+2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\ldots}\right)$$

h206

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A\cos A}\right)$$

$$= 1 + \frac{1}{\sin^2 A\cos^2 A} + \frac{2}{\sin A\cos A}$$

$$= \left(1 + \frac{1}{\sin A\cos A}\right)^2$$

$$= (1 + \sec A\operatorname{cosec} A)^2$$

Hence Proved

131. If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$
$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$
Prove that each of the side is equal to ± 1.

Ans : [Board Term-1 2012]

We have

h207

$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)$

$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

Multiply both sides by

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

or, $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)\times$

$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$

$= (\sec A - \tan A)^2(\sec B - \tan B)^2(\sec C - \tan C)^2$

or, $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$

$= (\sec A - \tan A)^2(\sec A - \tan B)^2(\sec C - \tan C)^2$

or, $1 = \left[(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)\right]^2$

or, $(\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$

132. If $4\sin\theta = 3$, find the value of x if
$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$

Ans : [Board Term-1 2012]

We have $\quad \sin\theta = \frac{3}{4}$

or, $\quad \sin^2\theta = \frac{9}{16}$

h208

Since $\sin^2\theta + \cos^2 = 1$, we have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

and $\quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$

Thus $\quad \sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$

$$\sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

Thus $x = \frac{4}{3}$

133. Prove that $\sec^2\theta + \mathrm{cosec}^2\theta$ can never be less than 2.

Ans : [Board-Term 1 2011]

Let $\sec^2\theta + \mathrm{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

h209

$\tan^2\theta \geq 0$ and $\cot^2\theta \geq 0$

Thus $x > 2$

Thus $\sec^2\theta + \mathrm{cosec}^2\theta > 2$

Hence $\sec^2\theta + \mathrm{cosec}^2\theta$ can never be less than 2.

134. (a) Solve for ϕ, if $\tan 5\phi = 1$

(b) Solve for ϕ, if $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

Ans :

(a) $\tan 5\phi = 1$

$$\tan 5\phi = \tan 45^\circ$$

$$5\phi = 45^\circ$$

h210

Thus $\phi = 9^\circ$

(b) $$\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$$

$$\frac{\sin^2\phi + (1+\cos\theta)^2}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{\sin^2\phi + 1 + 2\cos\phi + \cos^2\phi}{\sin\phi + \sin\phi\cos\phi} = 4$$

$$\frac{\sin^2\phi + \cos^2\phi + 1 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2}{\sin\phi} = 4$$

$$\sin\phi = \frac{1}{2}$$

$$\sin\phi = \sin 30^\circ$$

Thus $\phi = 30^\circ$

135. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Ans : [Board-Term 1 2009

h211

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$m^2 - n^2 = (\tan A + \sin A)^2 - (\tan A - \sin A)^2$$

$$= (\tan^2 A + \sin^2 A + 2\sin A\tan A)$$
$$-(\tan^2 A + \sin^2 A - 2\sin A\tan A)$$

$$= \tan^2 A + \sin^2 A + 2\sin A\tan A$$
$$-\tan^2 A - \sin^2 A + 2\sin A\tan A$$

$$= 4\sin A\tan A$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A - \sin^2 A\cos^2 A}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}}$$

$$= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}}$$

$$= 4\frac{\sin A \times \sin A}{\cos A}$$

$$= 4\sin A \times \frac{\sin A}{\cos A}$$

$$= 4\sin A\tan A$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

136. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that

$(m^2+n^2)\cos^2\beta = n^2$.

Ans : [Board-Term 1 2010]

h212

We have $\dfrac{\cos\alpha}{\cos\beta} = m$ and $\dfrac{\cos\alpha}{\sin\beta} = n$

$$m^2 = \frac{\cos^2\alpha}{\cos^2\beta} \text{ and } n^2 = \frac{\cos^2\alpha}{\sin^2\beta}$$

$$(m^2+n^2)\cos^2\beta = \left[\frac{\cos^2\alpha}{\cos^2\beta}+\frac{\cos^2\alpha}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\left[\frac{1}{\cos^2\beta}+\frac{1}{\sin^2\beta}\right]\cos^2\beta$$

$$= \cos^2\alpha\frac{\sin^2\beta+\cos^2\beta}{\cos^2\beta\sin^2\beta}\cos^2\beta$$

$$= \cos^2\alpha\left(\frac{1}{\cos^2\beta\sin^2\beta}\right)\cos^2\beta$$

$$= \frac{\cos^2\alpha}{\sin^2\beta}$$

$$= n^2 \qquad \text{Hence Proved.}$$

137. If $7\,\text{cosec}\,\phi - 3\cot\phi = 7$, prove that $7\cot\phi - 3\,\text{cosec}\,\phi = 3$.

Ans :

h213

We have $7\,\text{cosec}\,\phi - 3\cot\phi = 7$

$$7\,\text{cosec}\,\phi - 7 = 3\cot\phi$$

$$7(\text{cosec}\,\phi - 1) = 3\cot\phi$$

$$7(\text{cosec}\,\phi - 1)(\text{cosec}\,\phi + 1) = 3\cot\phi(\text{cosec}\,\phi + 1)$$

$$7(\text{cosec}^2\phi - 1) = 3\cot\phi(\text{cosec}\,\phi + 1)$$

$$7\cot^2\phi = 3\cot\phi(\text{cosec}\,\phi + 1)$$

$$7\cot\phi = 3(\text{cosec}\,\phi + 1)$$

$$7\cot\phi - 3\,\text{cosec}\,\phi = 3 \qquad \text{Hence Proved}$$

138. Prove that : $\dfrac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1} = \text{cosec}\,\theta + \cot\theta$

Ans : [Board SQP 2018]

h214

$$\text{LHS} = \frac{\cos\theta - \sin\theta + 1}{\cos\theta + \sin\theta - 1}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1-\cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1-\cos\theta)(1+\cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{(1+\cos\theta)(\cos\theta + \sin\theta - 1)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \text{cosec}\,\theta + \cot\theta \qquad \text{Hence Proved}$$

CHAPTER 9

SOME APPLICATIONS OF TRIGONOMETRY

ONE MARK QUESTIONS

Multiple Choice Questions

1. If the angle of depression of an object from a 75 m high tower is 30°, then the distance of the object from the tower is

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m

(c) $75\sqrt{3}$ m (d) 150 m

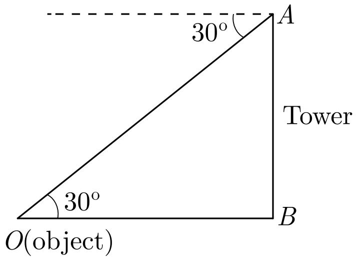
Ans :

i183

We have $\tan 30^\circ = \frac{AB}{OB}$

$$\frac{1}{\sqrt{3}} = \frac{75}{OB}$$

$$OB = 75\sqrt{3} \text{ m}$$



Thus (c) is correct option.

2. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45°. The height of a tree is

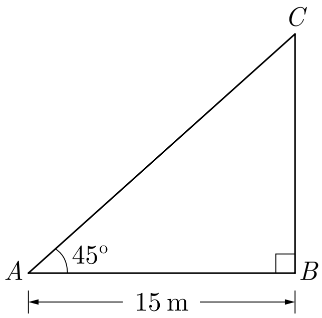
(a) 10 m (b) 14 m

(c) 8 m (d) 15 m

i184

Ans : (d) 15 m

Let BC be the tree of height h meter. Let AB be the shadow of tree.



In ΔABC, $CB = 90^\circ$

$$\frac{BC}{BA} = \tan 45^\circ$$

$$BC = AB = 15 \text{ m}$$

Thus (d) is correct option.

3. If the height and length of the shadow of a man are equal, then the angle of elevation of the sun is,

(a) 45° (b) 60°

(c) 90° (d) 120°

i185

Ans :

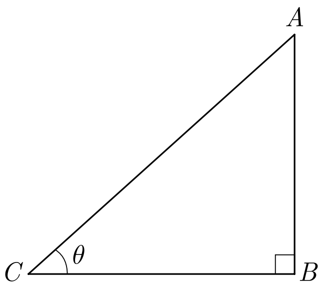
Let AB be the height of a man and BC be the shadow of a man.

$$AB = BC$$

In ΔABC, $\tan\theta = \frac{AB}{BC}$

$$\frac{AB}{AB} = \tan\theta$$

$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$



Thus (a) is correct option.

4. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$ then the angle of elevation of the sun is

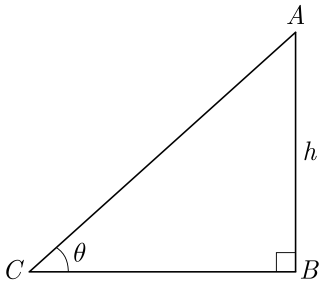
(a) 90° (b) 45°

(c) 30° (d) 75°

i186

Ans :

Let AB be the rod of length h, BC be its shadow of length $\sqrt{3}\,h$, θ be the angle of elevation of the sun.

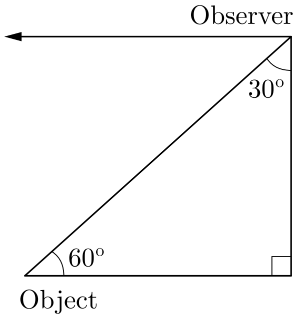


In $\Delta\,ABC$, $\dfrac{h}{\sqrt{3}\,h} = \tan\theta$

$$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

Thus (c) is correct option.

5. In the given figure, the positions of the observer and the object are mentioned, the angle of depression is



i187

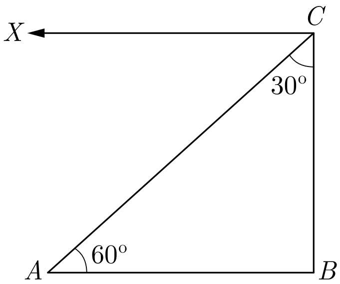
(a) 30° (b) 90°

(c) 60° (d) 45°

Ans :

$$\angle XCA = \angle CAB = 60^\circ$$

Hence, angle of depression $= 60^\circ$



Thus (c) is correct option.

6. A tree is broken by the wind. The top struck the ground at an angle of 30° and at distance of 10 m from its root. The whole height of the tree is $(\sqrt{3} = 1.732)$

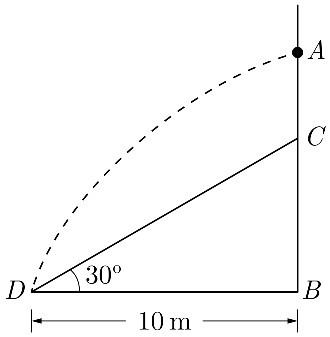
(a) $10\sqrt{3}$ m (b) $3\sqrt{10}$ m

(c) $20\sqrt{3}$ m (d) $3\sqrt{20}$ m

i188

Ans :

Let AB be the tree of height x, and AC be the broken part of tree.



Now $AC = CD$

$\angle CDB = 30^\circ$

$BD = 10$ m

In $\Delta\,CDB$, $\tan 30^\circ = \dfrac{CB}{DB} = \dfrac{CB}{10}$

$$\frac{1}{\sqrt{3}} = \frac{CB}{10}$$

$$CB = \frac{10}{\sqrt{3}}$$

Also, $\cos 30^\circ = \dfrac{DB}{DC} = \dfrac{10}{DC}$

$$DC = \frac{20}{\sqrt{3}} = AC$$

Height of tree,

$$AC + CB = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}} = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3} \text{ m}$$

Thus (a) is correct option.

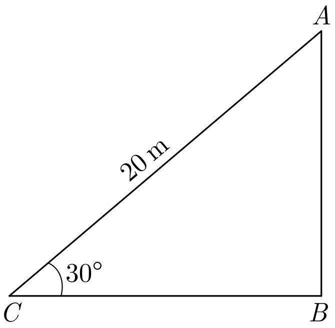
7. A circle artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground, then the height of pole, if the angle made by the rope with the ground level is 30°, is

(a) 5 m (b) 10 m

(c) 15 m (d) 20 m

Ans :

Let AB be the vertical pole and CA be the 20 m long rope such that its one end A is tied from the top of the vertical pole AB and the other end C is tied to a point C on the ground.



i189

In $\triangle ABC$, we have

$$\sin 30° = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{AC}$$

$$\frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

Thus (b) is correct option.

8. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, then the height of kite is

(a) 75 m (b) 78.05 m

(c) 226 m (d) None of these

Ans :

Length of the string of the kite,

$$AB = 85 \text{ m}$$

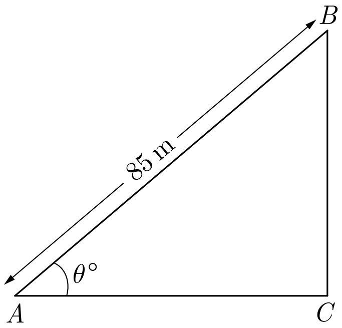
i190

and $$\tan\theta = \frac{15}{8}$$

$$\cot\theta = \frac{8}{15}$$

$$\operatorname{cosec}^2\theta - 1 = \frac{64}{225}$$

$$\operatorname{cosec}^2\theta = 1 + \frac{64}{225} = \frac{289}{225}$$



$$\operatorname{cosec}\theta = \sqrt{\frac{289}{225}} = \frac{17}{15}$$

$$\sin\theta = \frac{15}{17}$$

In $\triangle ABC$, $$\sin\theta = \frac{BC}{AB}$$

$$\frac{15}{17} = \frac{BC}{85} \Rightarrow BC = 75 \text{ m}$$

Thus height of kite is 75 m.

Thus (a) is correct option.

9. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

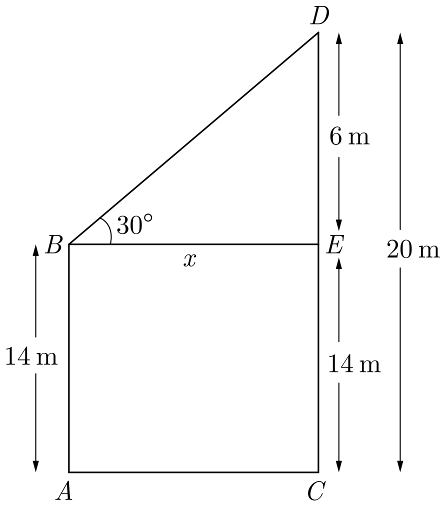
(a) 12 m (b) 10 m

(c) 8 m (d) 6 m

Ans :

Height of big pole $CD = 20$ m

Height of small pole $AB = 14$ m



$$DE = CD - CE$$

$$= CD - AB \qquad [AB = CE]$$

$$= 20 - 14 = 6 \text{ m}$$

i191

In $\triangle BDE$, $\sin 30^\circ = \frac{DE}{BD}$

$$\frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12 \text{ m}$$

Thus length of wire is 12 m.

Thus (a) is correct option.

10. An observer, 1.5 m tall is 20.5 away from a tower 22 m high, then the angle of elevation of the top of the tower from the eye of observer is

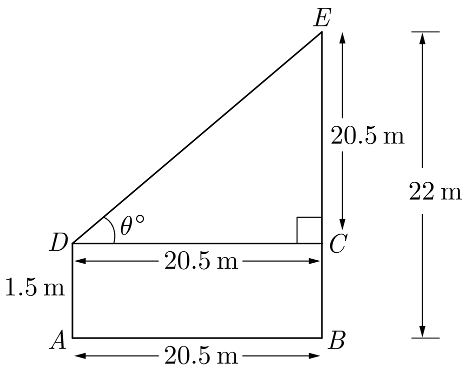
(a) 30° (b) 45°

(c) 60° (d) 90°

i192

Ans :

Let $BE = 22$ m be the height of the tower and $AD = 1.5$ m be the height of the observer. The point D be the observer's eye. We draw $DC \parallel AB$ as shown below.



Then, $AB = 20.5 \text{ m} = DC$

and $EC = BE - BC = BE - AD$

$$= 22 - 1.5 = 20.5 \text{ m} \qquad [BC = AD]$$

Let θ be the angle of elevation make by observer's eye to the top of the tower i.e. $\angle DCE$,

$$\tan\theta = \frac{P}{B} = \frac{CE}{DC} = \frac{20.5}{20.5}$$

$$\tan\theta = 1$$

$$\tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

Thus (b) is correct option.

11. A 6 m high tree cast a 4 m long shadow. At the same time, a flag pole cast a shadow 50 m long. How long is the flag pole?

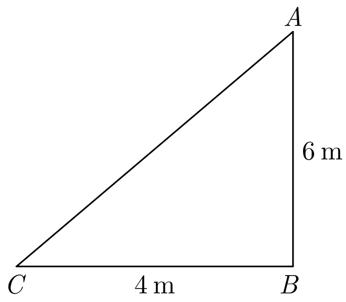
(a) 75 m (b) 100 m

(c) 150 m (d) 50 m

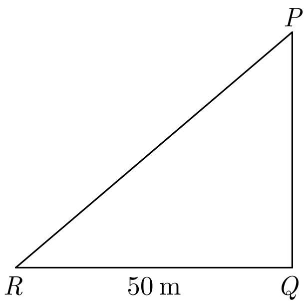
i193

Ans : (a) 75 m

Let AB be height of tree and BC its shadow.



Again, let PQ be height of pole and QR be its shadow. At the same time, the angle of elevation of tree and poles are equal i.e $\triangle ABC \sim PQR$



Thus $$\frac{AB}{BC} = \frac{PQ}{QR}$$

$$\frac{6}{4} = \frac{PQ}{50}$$

$$PQ = \frac{50 \times 6}{4} = 75 \text{ m}$$

Thus (a) is correct option.

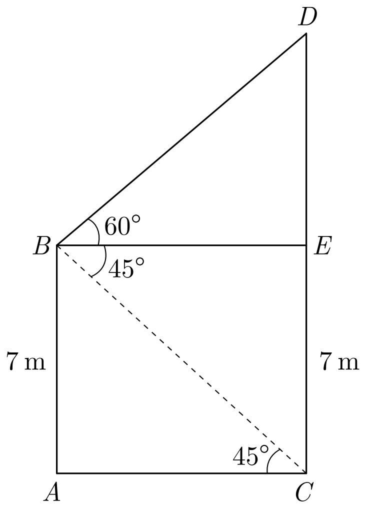
12. From the top of a 7 m high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°, then the height of the tower is

(a) 14.124 m (b) 17.124 m

(c) 19.124 m (d) 15.124 m

Ans :

Let AB be the building and CD be the tower. We draw $BE \perp CD$ as shown below.



Here $CE = AB = 7 \text{ m}$

$\angle EBD = 60^\circ$

and $\angle ACB = \angle CBE = 45^\circ$

From $\triangle ACB$, we have

$$\cot 45^\circ = \frac{AC}{AB}$$

$$\frac{AC}{7} = 1 \Rightarrow AC = 7 \text{ m}$$

$$BE = AC = 7 \text{ m}$$

From $\triangle EBD$, we have

$$\tan 60^\circ = \frac{DE}{BE}$$

$$\frac{DE}{7} = \sqrt{3} \Rightarrow DE = 7\sqrt{3} \text{ m}$$

Height of the tower $= (7 + 7\sqrt{3}) = 7(\sqrt{3} + 1)$

$= 7(1.732 + 1) = 7 \times 2.732$

$= 19.124 \text{ m}$

Thus (c) is correct option.

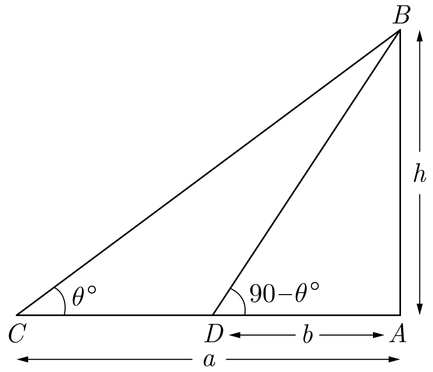
13. The angles of elevation of the top of a tower from the points P and Q at distance of a and b respectively from the base and in the same straight line with it, are complementary. The height of the tower is

(a) ab (b) $\sqrt{ab}$

(c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

Ans :

Let AB be the tower. Let C and D be two points at distance a and b respectively from the base of the tower.



In $\triangle ABC$, $\tan\theta = \frac{AB}{AC}$

$$\tan\theta = \frac{h}{a} \qquad ...(1)$$

In $\triangle ABD$, $\tan(90^\circ - \theta) = \frac{AB}{AD}$

$$\cot\theta = \frac{h}{b} \qquad ...(2)$$

From equation (1) and (2), we have

$$\tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b}$$

$$1 = \frac{h^2}{ab} \Rightarrow h = \sqrt{ab}$$

Thus (b) is correct option.

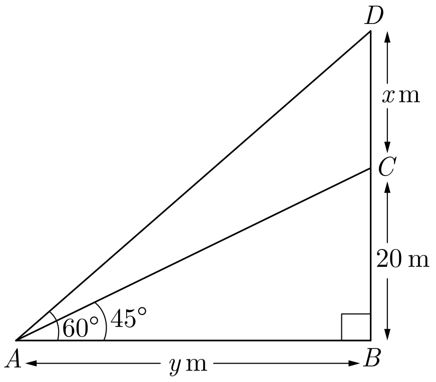
14. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively, then the height of the tower is

(a) 14.64 m (b) 28.64 m

(c) 38.64 m (d) 19.64 m

Ans :

Let the height of the building be BC, $BC = 20 \text{ m}$ and height of the tower be CD Let the point A be at a distance y from the foot of the building.



Now, in $\triangle ABC$, $\frac{BC}{AB} = \tan 45^\circ = 1$

$$\frac{20}{y} = 1 \Rightarrow y = 20 \text{ m}$$

i.e. $AB = 20$ m

Now, in $\triangle ABC$, $\frac{BD}{AB} = \tan 60^\circ = \sqrt{3}$

$$\frac{BD}{AB} = \sqrt{3}$$

$$\frac{20+x}{20} = \sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

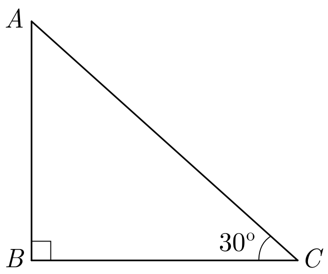
$$x = 20\sqrt{3} - 20$$

$$= 20 \times 0.732$$

$$= 14.64 \text{ m}$$

Thus (a) is correct option.

15. Assertion : In the figure, if $BC = 20$ m, then height AB is 11.56 m.



i197

Reason : $\tan\theta = \frac{AB}{BC} = \frac{\text{perpendicular}}{\text{base}}$ where θ is the angle $\angle ACB$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $\tan 30^\circ = \frac{AB}{BC} = \frac{AB}{20}$

$$AB = \frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73} = 11.56 \text{ m}$$

Both the assertion and reason are correct, reason is the correct explanation of the assertion.

Thus (a) is correct option.

Fill in the Blank Questions

16. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

i198

Ans :

angle of elevation

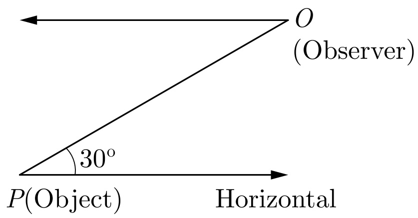
17. The .... of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

i199

Ans :

angle of depression

18. In the adjoining figure, the positions of observer and object are marked. The angle of depression is ....



i200

Ans :

30°

19. The .... is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Ans :

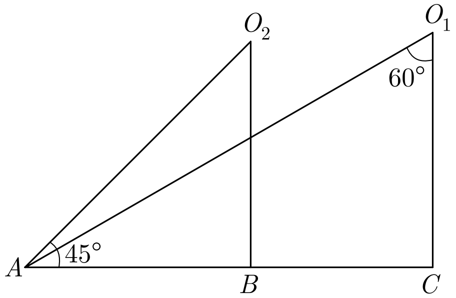
line of sight

20. .... are used to find height or length of an object or distance between two distant objects.

Ans :

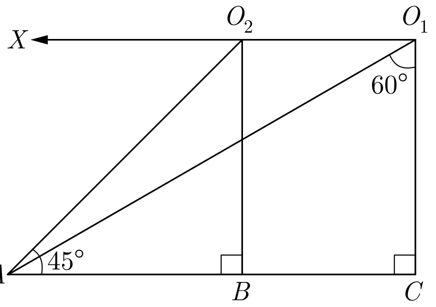
Trigonometric ratios

21. In Figure, the angles of depressions from the observing positions O_1 and O_2 respectively of the object A are ....



Ans : [Board 2020 OD Standard]

Here we have drawn O_1X parallel to AC.



$$\angle AO_1X = 90^\circ - 60^\circ = 30^\circ$$

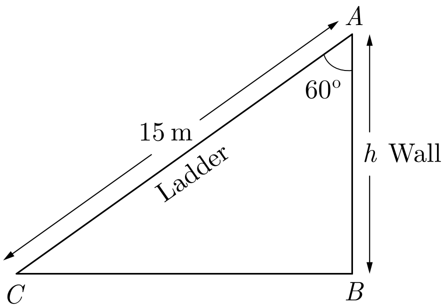
$$\angle AO_2X = \angle BAO_2 = 45^\circ$$

Very Short Answer Questions

22. A ladder 15 m long leans against a wall making an angle of 60° with the wall. Find the height of the point where the ladder touches the wall.

Ans : [Board Term-2 2014]

Let the height of wall be h. As per given in c we have drawn figure below.



$$\frac{h}{15} = \cos 60^\circ$$

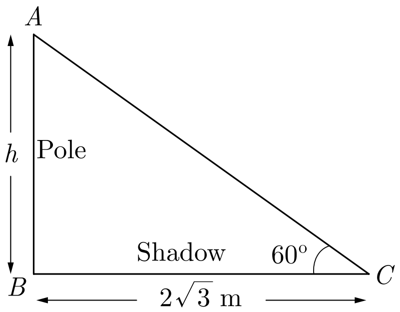
$$h = 15 \times \cos 60^\circ$$

$$= 15 \times \frac{1}{2} = 7.5 \text{ m}$$

23. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the Sun's elevation is 60°. Find the height of the pole.

Ans : [Board Term-2 Foreign 2015]

Let the height of pole be h. As per given in question we have drawn figure below.



Now
$$\frac{h}{2\sqrt{3}} = \tan 60^\circ$$

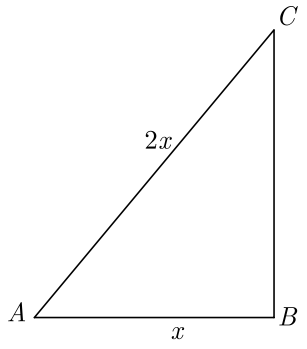
$$h = 2\sqrt{3}\tan 60^\circ$$

$$= 2\sqrt{3} \times \sqrt{3} = 6 \text{ m}$$

24. If the length of the ladder placed against a wall is twice the distance between the foot of the ladder and the wall. Find the angle made by the ladder with the horizontal.

Ans : [Board Term-2 2015]

Let the distance between the foot of the ladder and the wall is x, then length of the ladder will be $2x$. As per given in question we have drawn figure be



In ΔABC, $\angle B = 90^{o}$

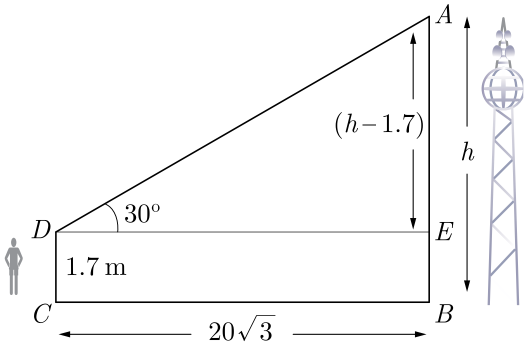
$$\cos A = \frac{x}{2x} = \frac{1}{2} = \cos 60^{o}$$

$$A = 60^{o}$$

25. An observer, 1.7 m tall, is $20\sqrt{3}$ m away from a tower. The angle of elevation from the eye of observer to the top of tower is 30^{o}. Find the height of tower.

Ans : [Board Term-2 Foreign 2016]

Let height of the tower AB be h. As per given in question we have drawn figure below.



i104

Here $AE = h - 1.7$

and $BC = DE = 20\sqrt{3}$

In ΔADE, $\angle E = 90^{o}$

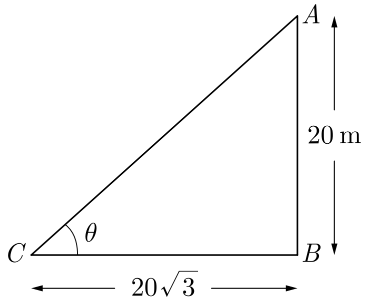
$$\tan 30^{o} = \frac{h-1.7}{20\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{h-1.7}{20\sqrt{3}}$$

$$h - 1.7 = 20$$

or $h = 20 + 1.7 = 21.7$ m

26. In figure, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. find the Sun's altitude.



i105

Ans : [Board Term-2 OD 2015]

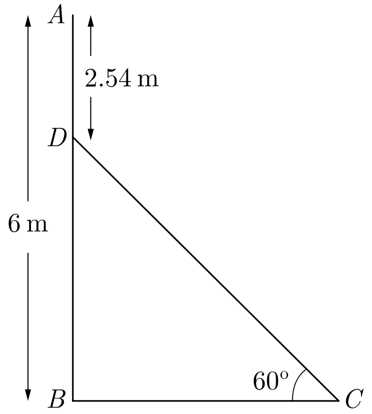
Let the $\angle ACB$ be θ.

$$\tan\theta = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^{\circ}$$

Thus $\theta = 30^{\circ}$

27. In the given figure, AB is a 6 m high pole and DC is a ladder inclined at an angle of 60° to the horizontal and reaches up to point D of pole. If $AD = 2.54$ m, find the length of ladder. (use $\sqrt{3} = 1.73$)

i106



Ans : [Board Term-2 Delhi 2016]

We have $AD = 2.54$ m

$DB = 6 - 2.54 = 3.46$ m

In ΔBCD, $\angle B = 90^{\circ}$

$$\sin 60^{\circ} = \frac{BD}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

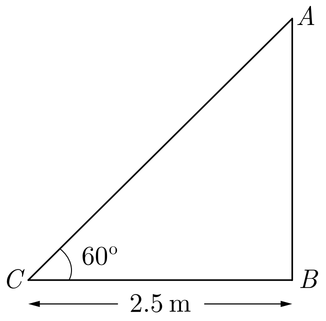
$$DC = \frac{3.46 \times 2}{\sqrt{3}} = \frac{3.46}{1.73} = 4$$

Thus length of ladder is 4 m.

28. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

Ans : [Board Term-2 2011]

As per given in question we have drawn figure below.



In $\Delta\, ACB$ with $\angle C = 60^\circ$, we get

$$\cos 60^\circ = \frac{2.5}{AC}$$

$$\frac{1}{2} = \frac{2.5}{AC}$$

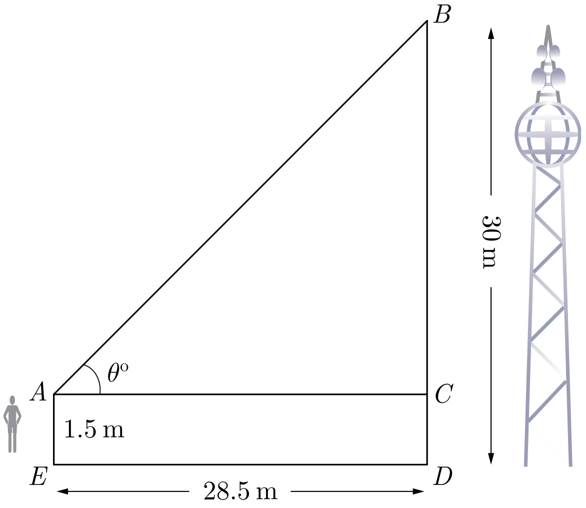
$$AC = 2 \times 2.5 = 5 \text{ m}$$

i107

29. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

Ans : [Board Term-2 2012]

As per given in question we have drawn figure below.



Here $AE = 1.5$ m is height of observer and $BD = 30$ m m is tower.

Now $BC = 30 - 1.5 = 28.5$ m

In $\Delta\, BAC$, $\tan\theta = \frac{BC}{AC}$

$$\tan\theta = \frac{28.5}{28.5} = 1 = \tan 45^\circ$$

$$\theta = 45^\circ$$

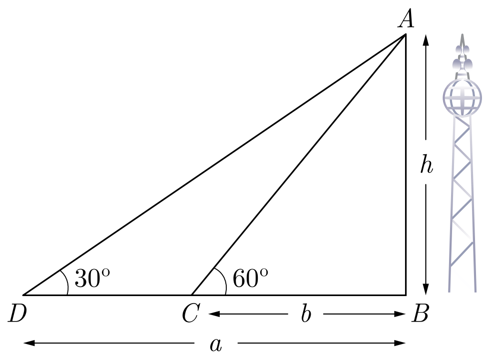
i108

Hence angle of elevation is 45°.

30. If the angles of elevation of the top of a tower from two points distant a and $b(a > b)$ from its foot and in the same straight line from it are respectively 30° and 60°, then find the height of the tower.

Ans : [Board Term-2 2014]

Let the height of tower be h. As per given in question we have drawn figure below.



From $\Delta\, ABD$, $\frac{h}{a} = \tan 30^\circ$

$$h = a \times \frac{1}{\sqrt{3}} = \frac{a}{\sqrt{3}} \quad ...(1)$$

From $\Delta\, ABC$, $\frac{h}{b} = \tan 60^\circ$

$$h = b \times \sqrt{3} = b\sqrt{3} \quad ...(2)$$

From (1) we get $a = \sqrt{3}\,h$

From (2) get $b = \frac{h}{\sqrt{3}}$

i109

Thus $a \times b = \sqrt{3}\,h \times \frac{h}{\sqrt{3}}$

$$ab = h^2$$

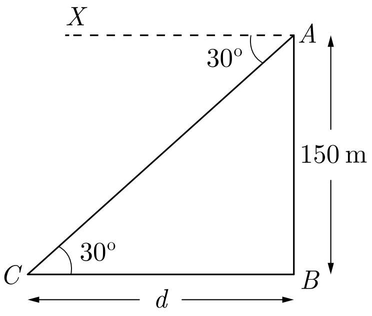
$$h = \sqrt{ab}$$

Hence, the height of the tower is $\sqrt{ab}$.

31. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. Find the distance of the car from the tower (in m).

Ans : [Board Term-2, 2014]

Let the distance of the car from the tower be d. As per given in question we have drawn figure below.



Due to alternate angles we have

$$\angle CAX = \angle ACB = 30^\circ$$

i110

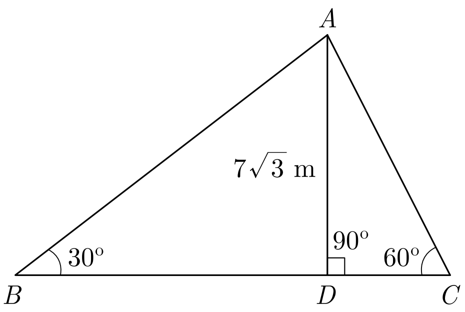
In ΔABC, $\angle B = 90^\circ$

$$\tan 30^\circ = \frac{150}{d}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{d}$$

Thus $d = 150\sqrt{3}$ m.

32. In the given figure, if $AD = 7\sqrt{3}$ m, then find the value of BC.



Ans : [Board Term-2 2012]

Let $BD = x$ and $DC = y$

From ΔADB we get

$$\tan 30^\circ = \frac{7\sqrt{3}}{x}$$

i111

$$\frac{1}{\sqrt{3}} = \frac{7\sqrt{3}}{x}$$

$$x = 7\sqrt{3} \times \sqrt{3} = 21 \text{ m}$$

From ΔADC,

$$\tan 60^\circ = \frac{7\sqrt{3}}{y}$$

$$\sqrt{3} = \frac{7\sqrt{3}}{y}$$

$$y = 7 \text{ m.}$$

Now $BC = BD + DC$

$$= 21 + 7 = 28 \text{ m.}$$

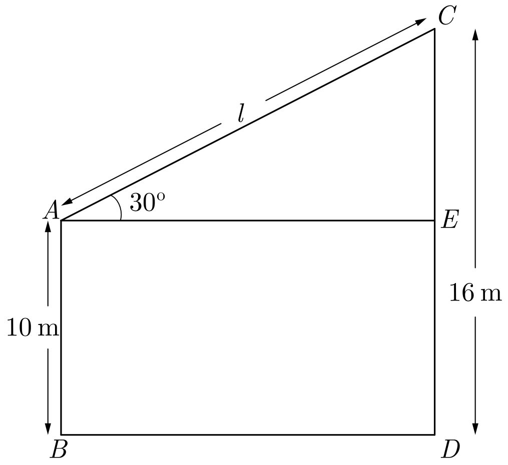
Hence, the value of BC is 28 m.

33. The top of two poles of height 16 m and 10 m are connected by a length l meter. If wire makes an angle of 30° with the horizontal, then find l.

Ans : [Board Term-2, 2012]

Let AB and CD be two poles, where $AB = 10$ m, $CD = 16$ m.

As per given in question we have drawn figure below.



Length $CE = CD - CE = CD - AB$

$$= 16 - 10 = 6 \text{ m.}$$

i112

From ΔAEC, $\sin 30^\circ = \frac{CE}{l}$

$$\frac{1}{2} = \frac{CE}{l}$$

$$l = 2CE = 6 \times 2 = 12 \text{ m.}$$

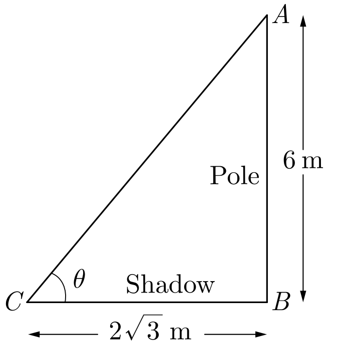
Hence, the value of l is 12 m.

34. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then find the Sun's elevation.

Ans : [Board Term-2 2012]

Let the Sun's elevation be θ. As per given in question we have drawn figure below.

i113



Length of pole is 6 m and length of shadow is $2\sqrt{3}$ m.

From $\Delta\, ABC$, we have

$$\tan\theta = \frac{AB}{BC} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

$$\theta = 60^\circ$$

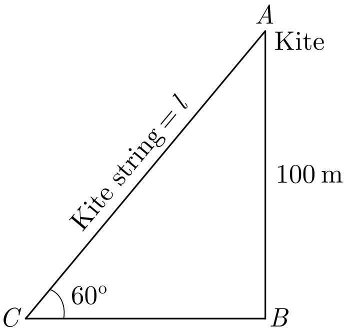
Hence sun's elevation is 60°.

35. Find the length of kite string flying at 100 m above the ground with the elevation of 60°.

Ans : [Board Term-2, 2012]

Let the length of kite string $AC = l$. As per given in question we have drawn figure below.

i114



Here $\angle ACB = 60^\circ$, height of kite $AB = 100$ m.

From $\Delta\, ABC$, we have

$$\sin 60^\circ = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{2} = \frac{100}{l}$$

$$l = \frac{2\times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ m}$$

$$= \frac{200}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{200\sqrt{3}}{3} \text{ m}$$

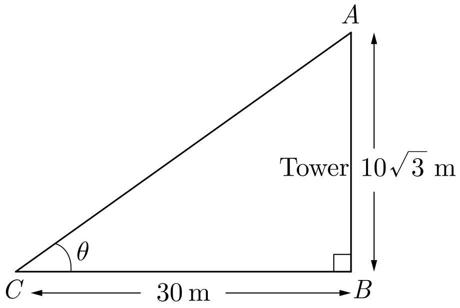
Hence length the kite string is $\frac{200\sqrt{3}}{3}$

36. Find the angle of elevation of the top of the tower from the point on the ground which is 30 m away from the foot of the tower of height $10\sqrt{3}$ m.

Ans : [Board Term-2 2012]

Let the angle of elevation of top of the tower be θ. As per given in question we have drawn figure below.

i115



From $\Delta\, ABC$,

$$\tan\theta = \frac{AB}{BC} = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

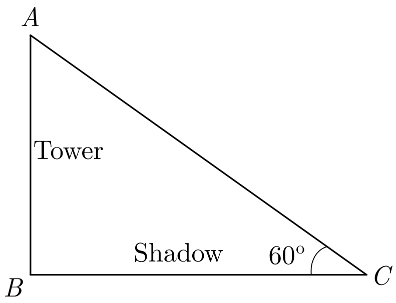
Thus $\theta = 30^\circ$

Hence angle of elevation is 30°.

37. If the altitude of the sun is 60°, what is the height of a tower which casts a shadow of length 30 m ?

Ans : [Board Term-2, 2011]

Let AB be the tower whose height be h. As per given in question we have drawn figure below.



Here shadow is $BC = 30$ m.

From ΔABC, we get

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{h}{30} = \sqrt{3}$$

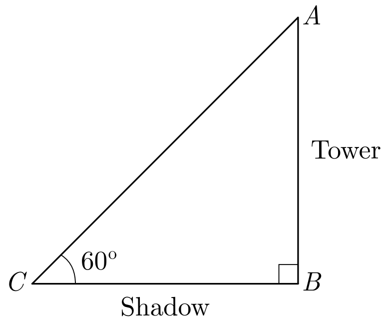
$$h = 30\sqrt{3} \text{ m}$$

Hence, height of tower is $30\sqrt{3}$ m.

38. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3} : 1$. What is the angle of elevation of the sun ?

Ans : [Board Term-2, 2016]

Let height of tower be AB and its shadow be BC. As per given in question we have drawn figure below.



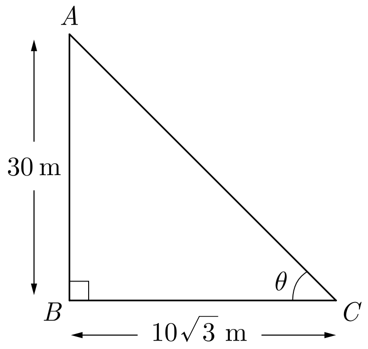
$$\frac{AB}{BC} = \tan\theta = \frac{\sqrt{3}}{1} = \tan 60^\circ$$

Hence, angle of elevation of sun is 60°.

39. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ?

Ans : [Board Term-2 OD 2017]

Tower AB is 30 m and shadow BC is $10\sqrt{3}$. As per given in question we have drawn figure below.



In right ΔABC we have,

$$\tan\theta = \frac{AB}{BC} = \frac{30}{10\sqrt{3}} = \sqrt{3} = \tan 60^\circ$$

Thus $\theta = 60^\circ$

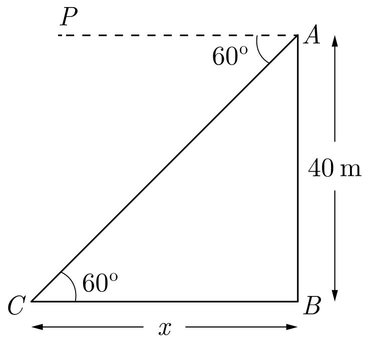
so, angle of elevation of sun is 60°.

TWO MARKS QUESTIONS

40. From the top of light house, 40 m above the water, the angle of depression of a small boat is 60°. Find how far the boat is from the base of the light house.

Ans : [Board Term-2 2015]

Let AB be the light house and C be the position of the boat. As per given in question we have drawn figure below.



Since $\angle PAC = 60^\circ \Rightarrow \angle ACB = 60^\circ$

Let $CB = x$. Now in ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{40}{x}$$

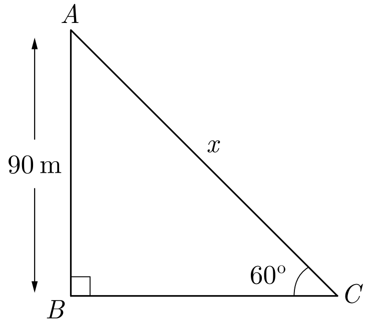
$$x = \frac{40}{\sqrt{3}} = \frac{40 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{40\sqrt{3}}{3} \text{ m}$$

Hence, the boat is $\frac{40\sqrt{3}}{3}$ m away from the foot of light house.

41. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string assuming that there is no slack in the string.

Ans : [Board Term-2 2011, 2014]

As per given in question we have drawn figure below.



In right ΔABC, we have

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{90}{x}$$

$$x = \frac{90 \times 2}{\sqrt{3}} = \frac{180}{\sqrt{3}} = \frac{3 \times 60}{\sqrt{3}}$$

$$= 60\sqrt{3}$$

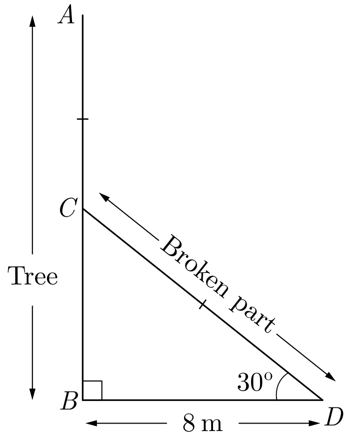
$$= 60 \times 1.732$$

Hence length of string is 103.92 m.

42. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Ans : [Board Term-2 2011]

Let the tree be AC and is broken at B. The broken part touches at the point D on the ground. As per given in question we have drawn figure below.



In right ΔCBD, $\cos 30^\circ = \frac{BD}{CD}$

$$\frac{\sqrt{3}}{2} = \frac{8}{CD}$$

$$CD = \frac{16}{\sqrt{3}}$$

and $\tan 30^\circ = \frac{BC}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$BC = \frac{8}{\sqrt{3}}$$

Height of tree,

$$BC + CD = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

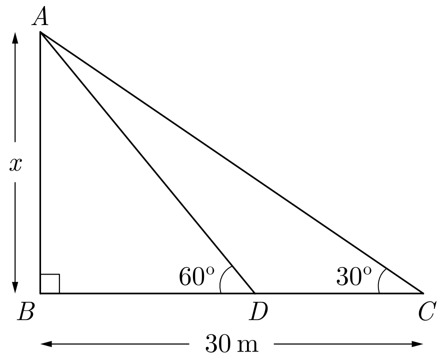
$$= \frac{24}{\sqrt{3}} = 8\sqrt{3}$$

Hence the height of the tree is $8\sqrt{3}$ m.

43. If the shadow of a tower is 30 m long, when the Sun's elevation is 30°. What is the length of the shadow, when Sun's elevation is 60° ?

Ans : [Board Term-2 2011]

As per given in question we have drawn figure below. Here AB is tower and BD is shadow at 60° and BC is shadow at 30° elevation.



In ΔABC, $\frac{AB}{BC} = \tan 30^\circ$

$$\frac{AB}{30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

In ΔABD, $\frac{AB}{BD} = \tan 60^\circ$

$$\frac{10\sqrt{3}}{BD} = \tan 60^\circ = \sqrt{3}$$

$$BD = 10 \text{ m}$$

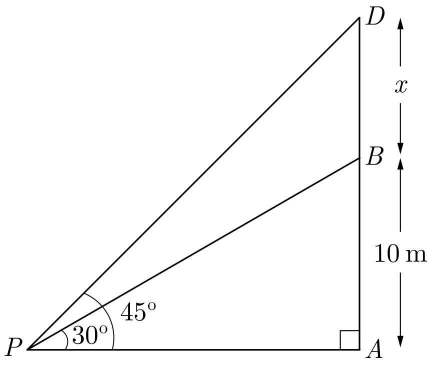
Hence the length of shadow is 10 m.

44. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top the of the building and the angle of elevation of the length of the flagstaff from P is 45°. Find the length of the flagstaff and distance of building from point P. [Take $\sqrt{3} = 1.732$]

Ans : [Board Term-2 2011, Delhi 2012, 2013]

Let height of flagstaff be $BD = x$. As per given in question we have drawn figure below.

i123



$$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{AP}$$

$$AP = 10\sqrt{3}$$

Distance of the building from P,

$$= 10 \times 1.732 = 17.32 \text{ m}$$

Now $\tan 45^\circ = \frac{AD}{AP}$

$$1 = \frac{10 + x}{17.32}$$

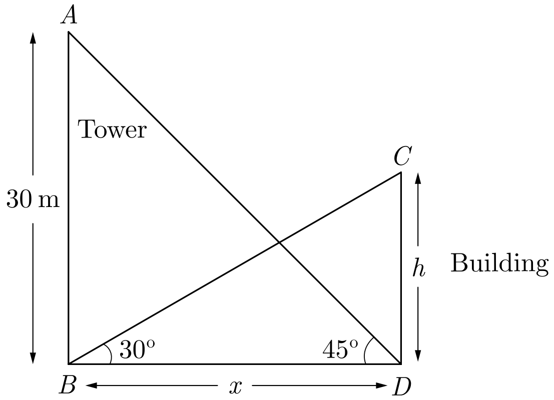
$$x = 17.32 - 10.00 = 7.32 \text{ m}$$

Hence, length of flagstaff is 7.32 m.

45. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building.

Ans : [Board Term-2 Delhi 2015]

Let the height of the building be $AB = h$. and distant between tower and building be, $BD = x$. As per given in question we have drawn figure below.



In ΔABD $\tan 45^\circ = \frac{AB}{BD}$

i124

$$1 = \frac{30}{x}$$

$$x = 30 \quad \text{...(1)}$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\sqrt{3}\,h = x \Rightarrow h = \frac{x}{\sqrt{3}} \quad \text{...(2)}$$

From (1) and (2), we get

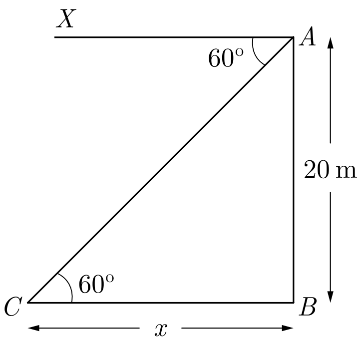
$$h = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

Therefore height of the building is $10\sqrt{3}$ m

46. A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. Take $\sqrt{3} = 1.732$

Ans : [Board Term-2 2011]

Let C be the point where the ball is lying. As per given in question we have drawn figure below.



Due to alternate angles we obtain

$$\angle XAC = \angle ACB = 60^\circ$$

i125

In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{x}$$

$$x = \frac{20}{\sqrt{3}} = 20\left(\frac{\sqrt{3}}{3}\right)$$

Hence, distance between ball and foot of tower is 11.53 m.

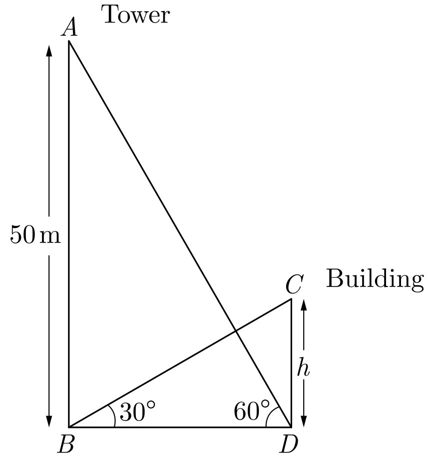
THREE MARKS QUESTIONS

47. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of a tower from the foot of the building is 60°. If the tower is 50 m high, then find the height of the building.

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below.

i204



In ΔABD, $\tan 60^\circ = \frac{AB}{BD}$

$$\sqrt{3} = \frac{50}{BD}$$

$$BD = \frac{50}{\sqrt{3}}$$

Now in ΔBDC,

$$\tan 30^\circ = \frac{CD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}} = \frac{h\sqrt{3}}{50}$$

$$3h = 50$$

$$h = \frac{50}{3} = 16.67$$

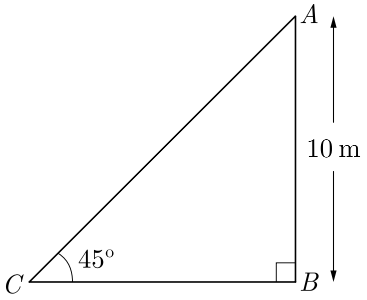
Hence, the height of the building is 16.67 m.

48. An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45^{o} with the horizontal through the foot of the pole, find the length of the wire. [Use $\sqrt{2} = 1.414$]

Ans : [Board Term-2 2016]

Let OA be the electric pole and B be the point on the ground to fix the pole. Let BA be x.

As per given in question we have drawn figure below.



In $\Delta\,ABC$ we have,

$$\sin 45^{o} = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{2}} = \frac{10}{AC}$$

$$AC = 10\sqrt{2} = 10 \times 1.414 = 14.14 \text{ m}$$

i126

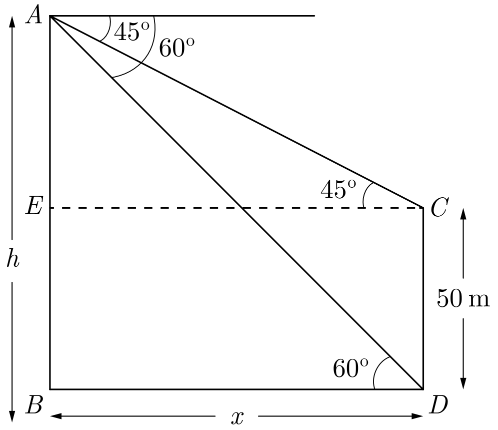
Hence, the length of wire is 14.14 m

49. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45^{o} and 60^{o} respectively. Find the height of the tower and the horizontal distance between the tower and the building. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Delhi 2016]

As per given in question we have drawn figure below. Here AC is tower and DC is building.

i127



We have
$$\tan 45^{o} = \frac{h-50}{x}$$

$$x = h - 50 \quad ...(1)$$

and
$$\tan 60^{o} = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad ...(2)$$

From (1) and (2) we have

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$\sqrt{3}\,h - 50\sqrt{3} = h$$

$$\sqrt{3}\,h - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50(3+\sqrt{3})}{2}$$

$$= 25\,(3+\sqrt{3})$$

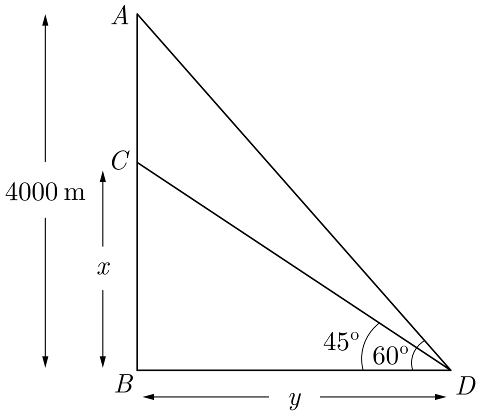
$$= 75 + 25\sqrt{3} = 118.25 \text{ m}$$

Thus $h = 118.25$ m.

50. An aeroplane, when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60^{o} and 45^{o} respectively. Find the vertical distance between the aeroplanes at that instant. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Foreign 2016]

Let the height first plane be $AB = 4000$ m and the height of second plane be $BC = x$ m. As per given in question we have drawn figure below.



Here $\angle BDC = \angle 45^o$ and $\angle BDA = 60^o$

In $\Delta\, CBD$, $\dfrac{x}{y} = \tan 45^o = 1 \Rightarrow x = y$

and in $\Delta\, ABD$, $\dfrac{4000}{y} = \tan 60^o = \sqrt{3}$

$$y = \frac{4000\sqrt{3}}{3}$$

$$= 2306.67 \text{ m}$$

i128

Thus vertical distance between two,

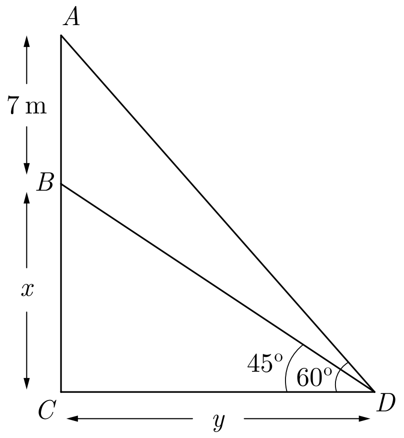
$$4000 - y = 4000 - 2306.67$$

$$= 1693.33 \text{ m}$$

51. A 7 m long flagstaff is fixed on the top of a tower standing on the horizontal plane. From point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60^o and 45^o respectively. Find the height of the tower correct to one place of decimal. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Foreign 2016]

As per given in question we have drawn figure below. Here AB is flagstaff and BC is tower.



$$\frac{x}{y} = \tan 45^o = 1 \Rightarrow x = y$$

i129

$$\frac{x+7}{x} = \tan 60^o = \sqrt{3}$$

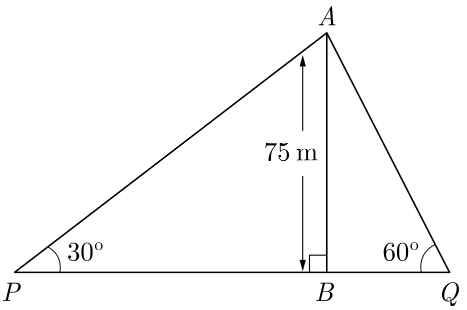
$$7 = (\sqrt{3} - 1)x$$

$$x = \frac{7(\sqrt{3}+1)}{2} = \frac{7(2.73)}{2} = 9.6 \text{ m}$$

52. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30^o and 60^o. Find the distance between the two men. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 Foreign 2016]

Let AB be the building and the two men are at P and Q. As per given in question we have drawn figure below.



In $\Delta\, ABP$, $\tan 30^o = \dfrac{AB}{BP}$

i130

$$\frac{1}{\sqrt{3}} = \frac{75}{BP}$$

$$BP = 75\sqrt{3} \text{ m}$$

In $\Delta\, ABQ$, $\tan 60^o = \dfrac{AB}{BQ}$

$$\sqrt{3} = \frac{75}{BQ}$$

$$BQ = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

Distance between the two men,

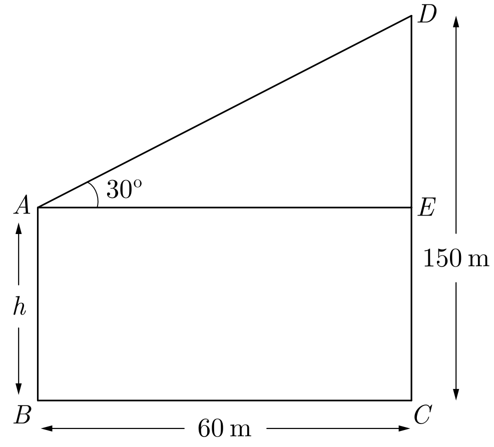
$$PQ = BP + BQ = 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3} = 100 \times 1.73 = 173$$

53. The horizontal distance between two towers is 60 m. The angle of elevation of the top of the taller tower as seen from the top of the shorter one is 30^o. If the height of the taller tower is 150 m, then find the height of the shorter tower.

Ans : [Board Term-2 2015]

Let AB and CD be two towers. Let the height of the shorter tower $AB = h$. As per given in question we have drawn figure below.



Here $BC = AE = 60$ m, $DE = DC - EC = (150 - h)$

In ΔAED, $\dfrac{DE}{AE} = \tan 30^{\circ}$

$$\frac{150 - h}{60} = \tan 30^{\circ} = \frac{1}{\sqrt{3}}$$

i131

$$150\sqrt{3} - h\sqrt{3} = 60$$

$$\sqrt{3}\,h = 150\sqrt{3} - 60$$

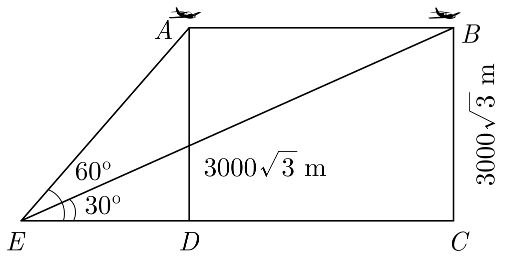
$$\sqrt{3}\,h = 150\sqrt{3} - 20\sqrt{3} \times \sqrt{3}$$

or $$h = (150 - 20\sqrt{3})\text{ m}$$

54. The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 30 seconds the angle of elevation becomes 30°. If the aeroplane is flying at a constant height of $3000\sqrt{3}$ m, find the speed of the aeroplane.

Ans : [Board 2020 SQP Standard, 2014]

As per given in question we have drawn figure below. Here



$$\angle AED = 60^{\circ}, \angle BED = 30^{\circ}$$

$$AD = BC = 3000\sqrt{3}\text{ m}$$

Let the speed of the aeroplane be x.

$$AB = DC \times 30 \times x = 30x\text{ m} \quad ...(1)$$

In right ΔAED, we have

$$\tan 60^{\circ} = \frac{AD}{DE}$$

$$\sqrt{3} = \frac{3000\sqrt{3}}{DE}$$

$$DE = 3000\text{ m} \quad ...(2)$$

In right ΔBEC,

$$\tan 30^{\circ} = \frac{BC}{EC}$$

i132

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{DE + CD}$$

$$DE + CD = 3000 \times 3$$

$$3000 + 30x = 9000$$

$$30x = 6000$$

$$x = 200\text{ m/s}$$

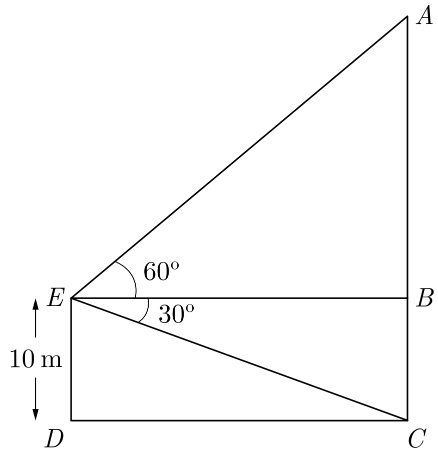
Hence, speed of plane is 200 m/s

$$= 200 \times \frac{18}{5} = 720\text{ km/hr}$$

55. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below. Here AC is height of hill and man is at E. $ED = 10$ is height of ship from water level.



In ΔBCE, $BC = EC = 10$ m and

$$\angle BEC = 30^o$$

i133

Now $\tan 30^o = \frac{BC}{BE}$

$$\frac{1}{\sqrt{3}} = \frac{10}{BE}$$

$$BE = 10\sqrt{3}$$

Since $BE = CD$, distance of hill from ship

$$CD = 10\sqrt{3}\,\text{m} = 10 \times 1.732 \text{ m}$$

$$= 17.32 \text{ m}$$

Now in ΔABE, $\angle AEB = 60^o$

where $AB = h$, $BE = 10\sqrt{3}\,\text{m}$

and $\angle AEB = 60^o$

Thus $\tan 60^o = \frac{AB}{BE}$

$$\sqrt{3} = \frac{AB}{10\sqrt{3}}$$

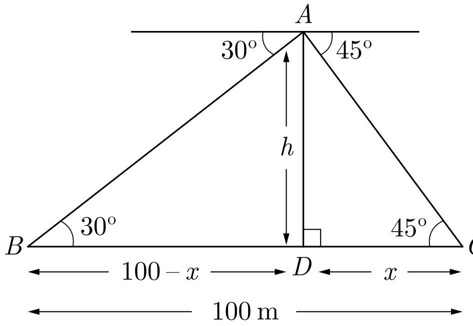
$$AB = 10\sqrt{3} \times \sqrt{3} = 30\,m$$

Thus height of hill $AB + 10 = 40$ m

56. Two ships are approaching a light house from opposite directions. The angle of depression of two ships from top of the light house are 30^o and 45^o. If the distance between two ships is 100 m, Find the height of light-house.

Ans : [Board Term-2 Foreign 2014]

As per given in question we have drawn figure below. Here AD is light house of height h and BC is the distance between two ships.



We have $BC = 100$ m

In ΔADC, $\tan 45^o = \frac{h}{x} \Rightarrow h = x$

i134

In ΔABD, $\tan 30^o = \frac{h}{100 - x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$100 - x = h\sqrt{3}$$

$$100 - h = h\sqrt{3} \qquad h = x$$

$$100 = h + h\sqrt{3}$$

$$= h(1 + \sqrt{3})$$

$$h = \frac{100}{1 + \sqrt{3}}$$

$$= \frac{100}{(\sqrt{3} + 1)} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)}$$

$$= \frac{100(\sqrt{3} - 1)}{3 - 1}$$

$$= 50(\sqrt{3} - 1)$$

$$= 50(1.732 - 1)$$

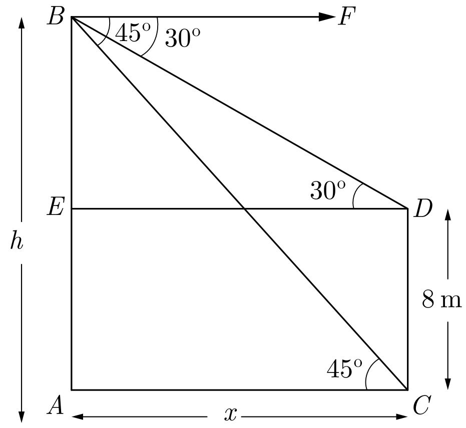
$$= 50 \times 0.732$$

Thus height of light house is 36.60 m.

57. The angles of depression of the top and bottom of an 8 m tall building from top of a multi-storeyed building are 30^o and 45^o, respectively. Find the height of multi-storey building and distance between two buildings.

Ans : [Board Term-2 OD 2014]

As per given in question we have drawn figure below.



Here $AE = CD = 8$ m

$$BE = AB - AE = (h - 8)$$

and $AC = DE = x$

i135

Also, $\angle FBD = \angle BDE = 30^\circ$

$\angle FBC = \angle BCA = 45^\circ$

In right angled $\Delta\,CAB$ we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$1 = \frac{h}{x} \Rightarrow x = h \qquad ...(1)$$

In right angled $\Delta\,EDB$

$$\tan 30^\circ = \frac{BE}{ED}$$

$$\frac{1}{\sqrt{3}} = \frac{h-8}{x}$$

$$x = \sqrt{3}(h-8) \qquad ...(2)$$

From (1) and (2), we get

$$h = \sqrt{3}\,h - 8\sqrt{3}$$

$$8\sqrt{3} = \sqrt{3}\,h - h$$

$$h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 4\sqrt{3}(\sqrt{3}+1) = (12+4\sqrt{3})\text{ m}$$

Since, $x = h$, $\quad x = (12+4\sqrt{3})$

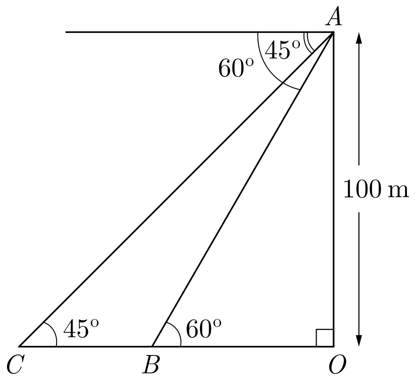
$$\text{Distance} = (12+4\sqrt{3})\text{ m}$$

Hence the height of multi storey building is $(4\sqrt{3}+12)$ m.

58. From a top of a building 100 m high the angle of depression of two objects are on the same side observed to be 45° and 60°. Find the distance between the objects.

Ans : [Board Term-2 OD 2014]

Let A be a point on top of building and B, C be two objects. As per given in question we have drawn figure below.



Here $\quad \angle ACO = \angle CAX = 45^\circ$

and $\quad \angle ABO = \angle XAB = 60^\circ$

i136

In right $\Delta\,AOC$, $\dfrac{AO}{CO} = \tan 45^\circ$

$$\frac{100}{CO} = 1$$

$$CO = 100 \text{ m}$$

Also in right $\Delta\,AOB$, we have

$$\frac{AO}{OB} = \tan 60^\circ$$

$$\frac{100}{OB} = \sqrt{3}$$

$$OB = \frac{100}{\sqrt{3}}$$

Thus $\quad BC = CO - OB = 100 - \dfrac{100}{\sqrt{3}}$

$$= 100\left(1 - \frac{1}{\sqrt{3}}\right) = 100\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

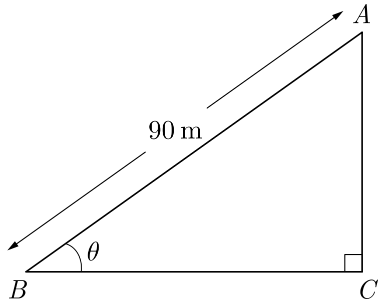
$$= 100\frac{(\sqrt{3}-1)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{100(3-\sqrt{3})}{3}\text{ m}$$

59. A boy, flying a kite with a string of 90 m long, which is making an angle θ with the ground. Find the height of the kite. (Given $\tan\theta = \frac{45}{8}$)

Ans : [Board Term-2 OD 2014]

Let A be the position of kite and AB be the string. As per given in question we have drawn figure below.



i137

Since $\quad \tan\theta = \frac{15}{8} = \frac{AC}{BC} = k$

Let AC be $15k$ and BC be $8k$. Now using Pythagoras Theorem

$$AB = \sqrt{BC^2 + AC^2}$$

$$= \sqrt{(15k)^2 + (8k)^2} = 17k$$

In ΔACB, $\quad \frac{AC}{AB} = \sin\theta$

$$\frac{AC}{90} = \frac{15k}{17k} = \frac{15}{17}$$

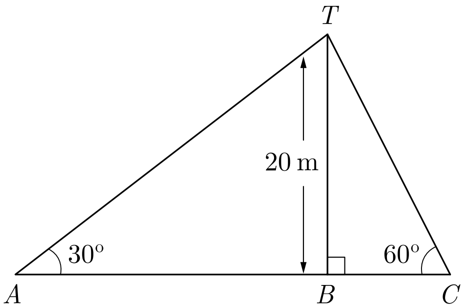
$$AC = \frac{15 \times 90}{17} = 79.41 \text{ m}$$

Hence, height of kite is 79.41 m.

60. Two men standing on opposite sides of a tower measure the angles of elevation of he top of the tower as 30° and 60° respectively. If the height of the tower in 20 m, then find the distance between the two men.

Ans : [Board Term-2 OD 2013]

Let two men are standing at A and C and BT is the tower. As per given in question we have drawn figure below.



i138

In right angle triangle ΔABT,

$$\tan 30^\circ = \frac{BT}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{20}{AB}$$

$$AB = \sqrt{3}\,20$$

In right angle triangle ΔTBC,

$$\tan 60^\circ = \frac{BT}{BC}$$

$$\sqrt{3} = \frac{20}{BC}$$

$$BC = \frac{20}{\sqrt{3}}$$

Thus distance between two men,

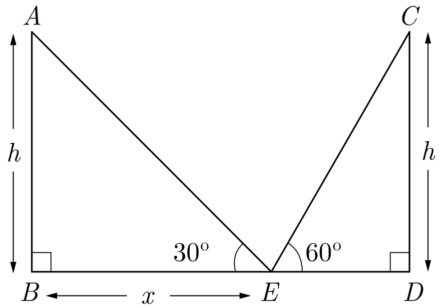
$$AB + BC = 20\sqrt{3} + \frac{20}{\sqrt{3}} = \frac{60+20}{\sqrt{3}} = \frac{80\sqrt{3}}{3} \text{ m.}$$

Hence, distance between the men is $\frac{80\sqrt{3}}{3}$ m.

61. Two poles of equal heights are standing opposite to each other on either side of a road, which is 80 m wide. From a point between them on the road, angles of elevation of their top are 30° and 60°. Find the height of the poles and distance of point from poles.

Ans : [Board 2019 Delhi Std, OD 2011]

Let the distance between pole AB and man E be x. As per given in question we have drawn figure below.



Here distance between pole CD and man is $80 - x$.

In right angle triangle ΔABE,

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \qquad ...(1)$$

In angle triangle ΔCDE,

$$\tan 60° = \frac{h}{80 - x}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \qquad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

i139

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

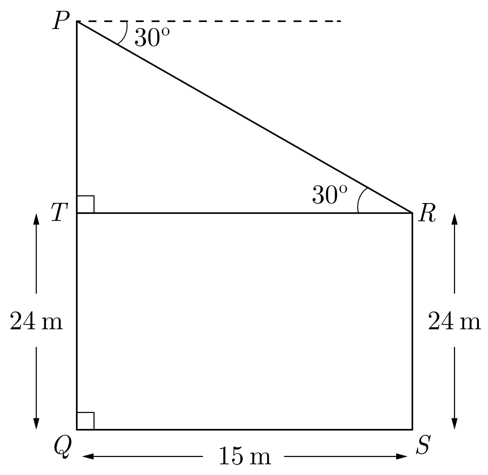
$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Hence, height of the pole is 34.64 m

62. The horizontal distance between two poles is 15 m. The angle of depression of the top of first pole as seen from the top of second pole is 30°. If the height of the first of the pole is 24 m, find the height of the second pole. [Use $\sqrt{3} = 1.732$]

Ans : [Board Term-2 2013]

Let RS be first pole and PQ be second pole. As per given in question we have drawn figure below.



In right ΔPTR,

$$\tan 30° = \frac{PT}{TR}$$

i140

$$\frac{1}{\sqrt{3}} = \frac{PT}{15}$$

$$PT = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$= 5 \times 1.732 = 8.66$$

$$PQ = PT + TQ$$

$$= 8.66 + 24$$

$$= 32.66 \text{ m}$$

Thus height of the second pole is 32.66 m.

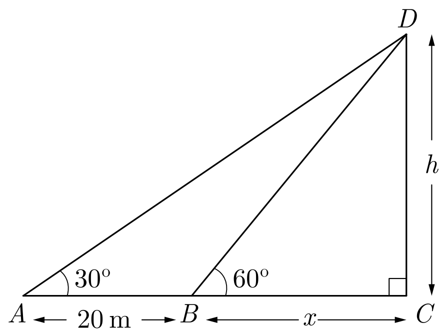
63. The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increase to 60°. Find the height of the tower and the distance of the tower from the point A.

Ans : [Board Term-2 2012]

Let height of tower CD be h and distance BC be x.

As per given in question we have drawn figure below.

i141



In right ΔDBC, $\frac{h}{x} = \tan 60°$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+20} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 20 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x + 20$$

$$x = 10 \text{ m} \qquad ...(3)$$

Thus $AC = 20 + x = = 30$ m.

and $h = \sqrt{3} \times 10 = 10\sqrt{3}$

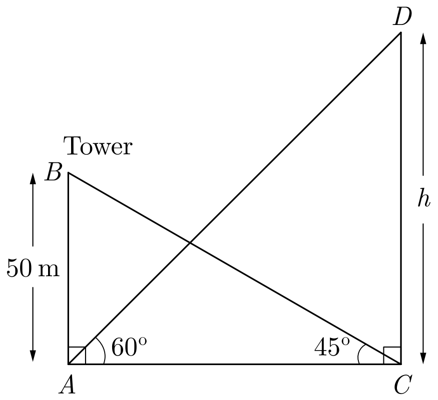
$$= 10 \times 1.732 = 17.32 \text{ m}$$

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m.

64. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, find the height of the hill.

Ans : [Board Term-2 2012]

Let AB be tower of height of 50 m and DC be hill of height h. As per given in question we have drawn figure below.



In right ΔBAC,

$$\cos 30^\circ = \frac{AC}{50}$$

i142

$$\sqrt{3} = \frac{AC}{50}$$

$$AC = 50\sqrt{3}$$

In right ΔACD,

$$\tan 60^\circ = \frac{CD}{50\sqrt{3}}$$

$$\sqrt{3} = \frac{CD}{50\sqrt{3}}$$

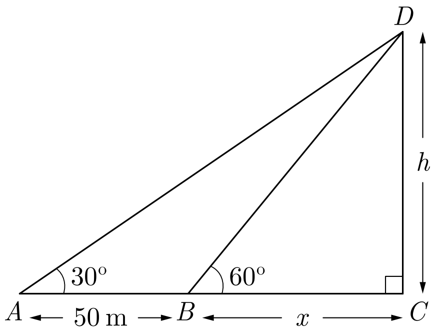
$$CD = 50\sqrt{3} \times \sqrt{3} = 150 \text{ m}$$

Thus height of the hill $CD = 150$ m

65. A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Ans : [Board Term-2 2012]

Let DC be tower of height h. As per given in question we have drawn figure below.



Here A is the point at elevation 30° and B is the point of elevation at 60°.

Let BC be x.

Now $$AC = (50 + x) \text{ m}$$

i143

In right ΔDCB, $$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔDCA,

$$\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 50 \qquad (1)$$

Substituting the value of h from (1) in (2), we have

$$3x = x + 50$$

$$2x = 50 \Rightarrow x = 25 \text{ m}$$

$$h = 25\sqrt{3}$$

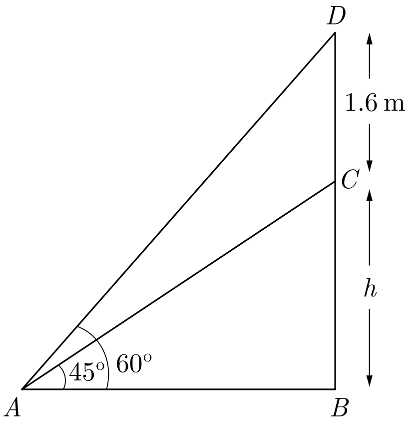
$$= 25 \times 1.732 = 43.3 \text{ m}$$

Hence height of tower is 43.3 m.

66. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Ans : [Board Term-2 OD 2012]

Let CD be statue of 1.6 m and pedestal BC of height h. Let A be point on ground. As per given in question we have drawn figure below.



In right $\Delta\, ABD$,

$$\cot 60^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{h+1.6}$$

$$AB = \frac{h+1.6}{\sqrt{3}} \qquad ...(1)$$

In right $\Delta\, ABC$,

$$\frac{AB}{BC} = \cot 45^\circ$$

$$1 = \frac{AB}{h}$$

$$AB = h \qquad ...(2)$$

From (1) and (2), we get

$$h = \frac{h+1.6}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1.6$$

$$h\sqrt{3} - h = 1.6$$

$$h(\sqrt{3}-1) = 1.6$$

$$h = \frac{1.6}{\sqrt{3}-1} = \frac{1.6}{1.732-1}$$

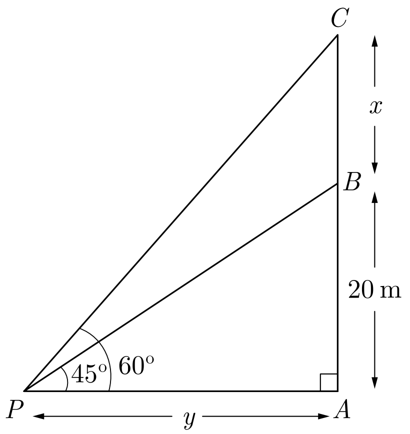
$$= \frac{1.6}{0.732} = 2.185 \text{ m}$$

Height of pedestal h is 2.2 m.

67. From a point on a ground, the angle of elevation of bottom and top a transmission tower fixed on the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans : [Board Term-2 OD Compt. 2017]

Let P be the point on ground, AB be the building of height 20 m and BC be the tower of height x. As per given in question we have drawn figure below.



In right $\Delta\, BAP$ we have

$$\frac{BA}{PA} = \tan 45^\circ$$

$$\frac{20}{y} = 1$$

$$y = 20$$

In right $\Delta\, CAP$,

$$\frac{CA}{PA} = \tan 60^\circ$$

$$\frac{20+x}{y} = \sqrt{3}$$

$$20 + x = y\sqrt{3}$$

$$20 + x = 20\sqrt{3}$$

$$x = 20\sqrt{3} - 20$$

$$= 20(\sqrt{3}-1)$$

$$= 20 \times (1.732 - 1)$$

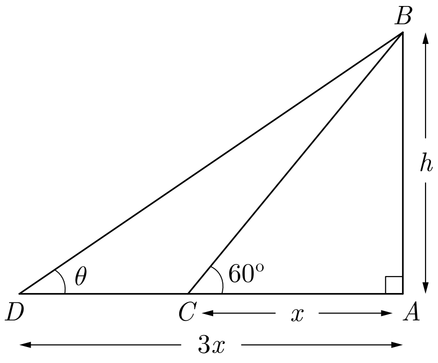
$$= 20 \times 0.73 = 14.64$$

Hence, height of the tower is 14.64 m.

68. The shadow of a tower at a time is three times as long as its shadow when the angle of elevation of the sun is 60°. Find the angle of elevation of the sun at the of the longer shadow.

Ans : [Board Term-2 Foreign 2017]

Let AB be tower of height h, AC be the shadow at elevation of sun of 60°. As per given in question we have drawn figure below.



In right $\Delta\, BAC$,

$$\frac{AB}{AC} = \tan 60^\circ$$

$$\frac{h}{x} = \sqrt{3}$$

$$h = x\sqrt{3}$$

i146

In right $\Delta\, BAD$,

$$\frac{AB}{AD} = \tan\theta$$

$$\frac{h}{3x} = \tan\theta$$

$$\frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

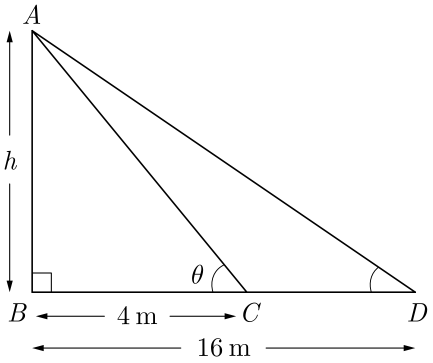
Thus $\theta = 30^\circ$.

69. On a straight line passing through the foot of a tower, two C and D are at distance of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

Ans : [Board Term-2 OD 2017]

Let AB be tower of height h, C and D be the two point. As per given in question we have drawn figure below.

i147



Since $\angle ACB$ and $\angle ADB$ are complementary,

$$\angle ACB = \theta \text{ and } \angle ADB = 90^\circ - \theta$$

Now, in right $\Delta\, ABC$,

$$\tan\theta = \frac{AB}{BC} = \frac{h}{4} \quad ...(1)$$

In right $\Delta\, ABD$,

$$\tan(90 - \theta) = \frac{AB}{BD} = \frac{h}{16}$$

$$\cot\theta = \frac{h}{16}$$

$$\tan\theta = \frac{16}{h} \quad ...(2)$$

From (1) and (2) we have

$$\frac{h}{4} = \frac{16}{h}$$

$$h^2 = 4 \times 16 = 64 = 8^2 \Rightarrow h = 8 \text{ m}$$

Thus height of tower is 8 m.

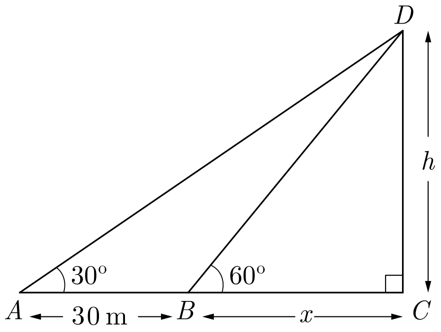
FOUR MARKS QUESTIONS

70. The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is 60°. When he moves 30 m away from the bank, he finds the angle of elevation to be 30°. Find the height of tree and width of the river.

Ans : [Board 2020 OD Basic]

Let CD be the tree of height h. Let A be the position of person after moving 30 m away from point B on bank of river. Let $BC = x$ be the width of the river.

As per given in question we have drawn figure below.



i205

In right ΔDBC, $\frac{h}{x} = \tan 60^\circ$

$$h = \sqrt{3}\,x \qquad ...(1)$$

In right ΔADC,

$$\frac{h}{x+30} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = x + 30 \qquad ...(2)$$

Substituting the value of h from eq. (1) in eq. (2), we get

$$3x = x + 30$$

$$x = 15 \text{ m} \qquad ...(3)$$

Thus $h = \sqrt{3} \times 15 = 15\sqrt{3}$

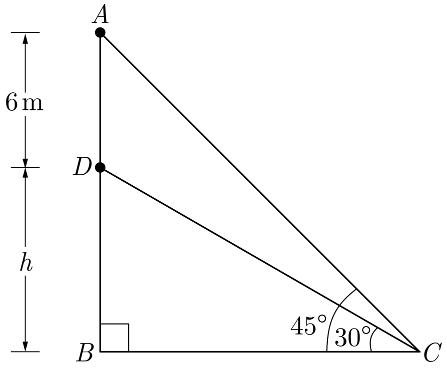
$$= 15 \times 1.732 = 25.98 \text{ m}$$

Hence, height of tree is 25.98 m and width of river is 15 m.

71. A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the ground, angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3} = 1.73$)

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



i206

Here AD is a flagstaff and BD is a tower.

In ΔABC $\tan 45^\circ = \frac{AB}{BC}$

$$1 = \frac{h+6}{BC}$$

$$BC = h + 6 \qquad ...(1)$$

In ΔDBC, $\tan 30^\circ = \frac{DB}{BC}$ from (1)

$$\frac{1}{\sqrt{3}} = \frac{h}{h+6}$$

$$h\sqrt{3} = h + 6$$

$$h(\sqrt{3} - 1) = 6$$

$$h = \frac{6}{\sqrt{3} - 1}$$

$$= \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{6(\sqrt{3}+1)}{2}$$

$$= 3(\sqrt{3}+1)$$

$$= 3(1.73 + 1)$$

$$= 3 \times 2.73$$

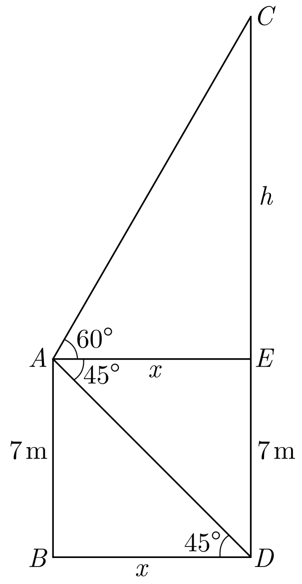
$$= 8.19 \text{ m}$$

Thus height of tower is 8.19 m.

72. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Ans : [Board 2020 Delhi Standard]

Let AB be a building of height 7 m and CD be tower of height CD. From the given information we have drawn the figure as below.



i207

Now $CD = (7+h)$

$BD = AE = x$

In ΔABD, $\tan 45^\circ = \dfrac{AB}{BD}$

$$1 = \frac{7}{x} \Rightarrow x = 7 \text{ cm}$$

In ΔCEA, $\tan 60^\circ = \dfrac{CE}{AE}$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3}$$

Substituting the value of x, we get

$$h = 7\sqrt{3}$$

Now, $CD = CE + ED$

$$= (7+7\sqrt{3}) \text{ m}$$

$$= 7(1+\sqrt{3}) \text{ m}$$

$$= 7(1+1.732) \text{ m}$$

$$= 7 \times 2.732 \text{ m}$$

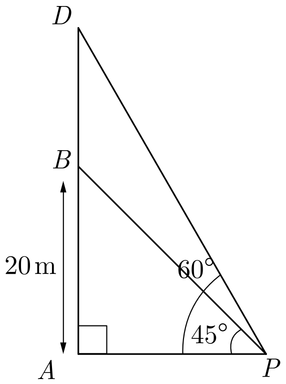
$$= 19.124 \text{ m}$$

Hence height of tower is 19.12 m approximately.

73. From a point on the ground, the angles of elevation of the bottom and the top of a tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below. Here AB is the building and BD is tower on building.



i208

In ΔPAB, $\tan 45^\circ = \dfrac{AB}{AP}$

$$1 = \frac{20}{AP} \Rightarrow AP = 20 \text{ m}$$

In ΔPAD, $\tan 60^\circ = \dfrac{AD}{AP} = \dfrac{20+BD}{20}$

$$\sqrt{3} = \frac{20+BD}{20}$$

$$20 + BD = 20\sqrt{3}$$

$$BD = 20\sqrt{3} - 20 = 20(\sqrt{3}-1)$$

$$= 20(1.732-1)$$

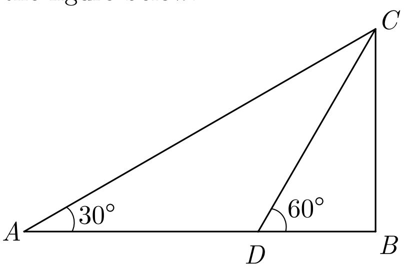
$$= 20 \times 0.732$$

$$= 14.64 \text{ cm}.$$

74. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

Ans : [Board 2019 Delhi Standard]

As per given information in question we have drawn the figure below.



Here D is first position and A is position after 2 minutes.

Height of the light house,

$$BC = 100 \text{ m}$$

From ΔDBC, $\angle B = 90^\circ$

i209

So, $\tan 60° = \frac{BC}{BD}$

$$\sqrt{3} = \frac{100}{BD}$$

$$BD = \frac{100}{\sqrt{3}} \text{m}$$

Now, after time 2 minute boat is at A. New distance from light house is AB and angle is 30°.

From ΔABC, $\angle B = 90°$

So, $\tan 30° = \frac{BC}{AB}$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3}$$

Therefore, distance d travelled in 2 min,

$$AD = AB - DB = 100\sqrt{3} - \frac{100}{3}$$

$$= 173.2 - \frac{100}{3}\sqrt{3}$$

$$= 173.2 - 57.73 = 115.47 \text{ m}$$

Speed $s = \frac{d}{t} = \frac{115.47 \text{ m}}{2 \text{ min}}$

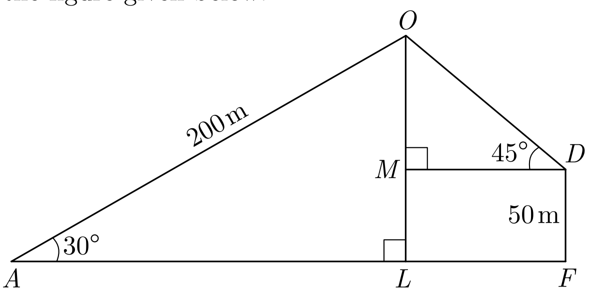
$$= 57.74 \text{ m/min}$$

Hence, going away from the light house with a speed of 57.74 m/ min.

75. Amit, standing on a horizontal plane, find a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, find the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

Ans : [Board 2019 OD Standard]

As per given information in question we have drawn the figure given below.



Let O be the position of the bird, A be the position for Amit, D be the position for Deepak and FD be the building at which Deepak is standing at height 50 m.

In ΔOLA, $\angle L = 90°$

$$\sin 30° = \frac{OL}{OA}$$

$$\frac{1}{2} = \frac{OL}{200} \Rightarrow OL = \frac{200}{2} = 100 \text{ m}$$

$$OM = OL - LM$$

$$= OL - FD$$

$$= (100 - 50) \text{ m} = 50 \text{ m}$$

i210

In ΔOMD, $\angle M = 90°$

$$\sin 45° = \frac{OM}{OD}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{OD}$$

$$OD = 50\sqrt{2}$$

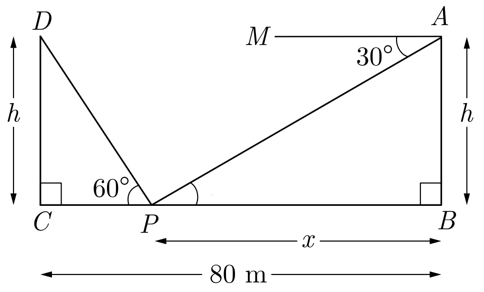
$$= 50 \times 1.414 = 70.7 \text{ m}$$

Thus, the distance of the bird from the Deepak is 70.7 m.

76. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole of a pole is 60° and the angle of depression from the top of the other pole of point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

Ans : [Board 2019 OD Standard]

Let the distance between pole AB and point P be x. As per given in question we have drawn figure below.



Here distance between pole CD and P is $80 - x$.

In right angle triangle ΔABP, $\angle APB = 30°$

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \qquad ...(1)$$

In angle triangle $\Delta\, CDP$,

$$\tan 60^\circ = \frac{CD}{CP} = \frac{CD}{CB - PB}$$

$$\sqrt{3} = \frac{h}{80 - x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \qquad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$$

Hence, height of the pole AB and CD is 34.64 m

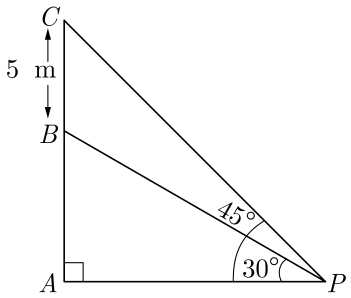
Distance of point P from pole AB is 20 m.

Distance of point P from pole CD is 60 m.

77. From a point P on the ground, the angle of elevation of the top of a tower is 30° and that of the top of the flagstaff is 45°. If height of flagstaff is 5 m, find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans : [Board 2019 OD Standard]

Let AB denotes the height of the tower and BC denotes the height of the flag. As per given information in question we have drawn the figure as given below.



From $\Delta\, BAP$, $\angle A = 90^\circ$

Now, $$\tan 30^\circ = \frac{AB}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AP}$$

$$AP = \sqrt{3}\, AB \qquad ...(1)$$

Again from $\Delta\, CAP$,

$$\angle A = 90^\circ$$

and $$\tan 45^\circ = \frac{AC}{AP}$$

$$1 = \frac{AC}{AP}$$

$$AP = AC = (AB + BC)$$

$$AP = (AB + 5) \qquad ...(2)$$

From equation (1) and (2), we obtain,

$$(AB + 5) = \sqrt{3}\, AB$$

$$5 = \sqrt{3}\, AB - AB$$

$$AB = \frac{5}{(\sqrt{3} - 1)} = \frac{5}{(1.732 - 1)}$$

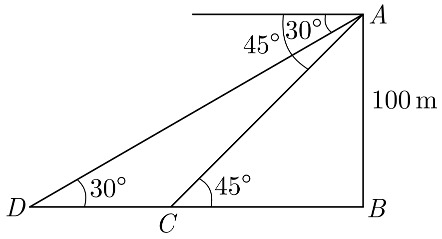
$$= \frac{5}{0.732} = 6.8306 \text{ m.}$$

Hence, height of the tower, $AB = 6.8306$ m.

78. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use $\sqrt{3} = 1.732$]

Ans : [Board 2018]

Let AB be the tower and ships are at points C and D. As per question statement we have shown digram below.



Now in ΔABC we have

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\frac{AB}{AC} = 1 \Rightarrow AB = BC$$

Now in ΔABD we have

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AB + CD}$$

$$AB + CD = \sqrt{3}\,AB$$

$$CD = AB(\sqrt{3} - 1)$$

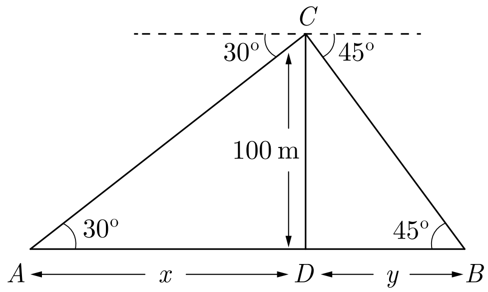
$$= 100 \times (1.732 - 1)$$

$$= 73.2\text{ m}$$

79. Distance between two ships is 73.2 m. From the top of tower, 100 m high, a man observes two cars on the opposite sides of the tower with the angles of depression 30° and 45° respectively. Find the distance between the cars. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 SQP 2016]

Let DC be tower of height 100 m. A and B be two car on the opposite side of tower. As per given in question we have drawn figure below.



In right $\Delta\,ADC$,

$$\tan 30° = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x}$$

$$x = 100\sqrt{3} \qquad ...(1)$$

In right $\Delta\,BDC$,

$$\tan 45° = \frac{CD}{DB}$$

$$1 = \frac{100}{y} \Rightarrow y = 100\text{ m}$$

Distance between two cars

$$AB = AD + DB = x + y$$

$$= (100\sqrt{3} + 100)$$

$$= (100 \times 1.73 + 100)\text{ m}$$

$$= (173 + 100)\text{ m}$$

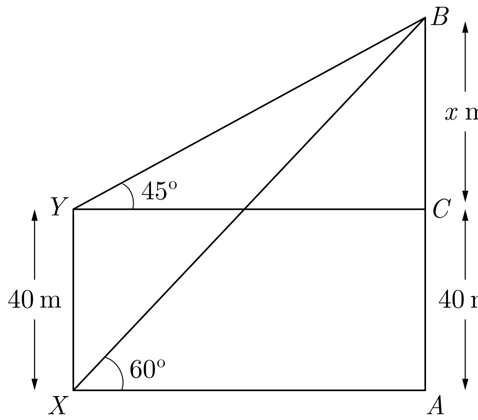
$$= 273\text{ m}$$

Hence, distance between two cars is 273 m.

80. The angle of elevation of the top B of a tower AB from a point X on the ground is 60°. At point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Find the height of the tower AB and the distance XB.

Ans : [Board Term-2 OD 2016]

As per given in question we have drawn figure below.



In right $\Delta\,YCB$, we have

$$\tan 45° = \frac{BC}{YC}$$

$$1 = \frac{x}{YC}$$

$$YC = x = XA$$

In right $\Delta\,XAB$ we have

$$\tan 60° = \frac{AB}{XA}$$

$$\sqrt{3} = \frac{x + 40}{x}$$

$$\sqrt{3}\,x = x + 40$$

$$x\sqrt{3} - x = 40$$

$$x = \frac{40}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= 20(\sqrt{3} + 1)$$

$$= 20\sqrt{3} + 20$$

Thus height of the tower,

$$AB = x + 40$$
$$= 20\sqrt{3} + 20 + 40$$
$$= 20\sqrt{3} + 60$$
$$= 20(\sqrt{3} + 3)$$

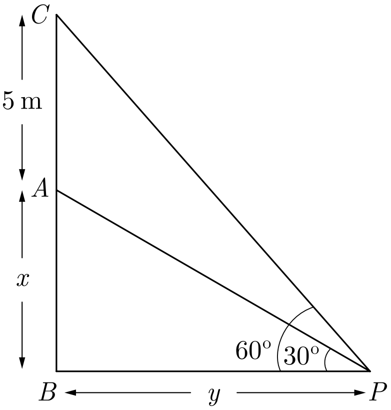
In right ΔXAB we have,

$$\sin 60^\circ = \frac{AB}{BX}$$
$$\frac{\sqrt{3}}{2} = \frac{AB}{BX}$$
$$BX = \frac{2AB}{\sqrt{3}} = \frac{20 \times 2(\sqrt{3}+3)}{\sqrt{3}}$$
$$= 40(1 + \sqrt{3})$$
$$= 40 \times 2.73 = 109.20$$

81. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (take $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign Set I, 2016]

Let AB be tower of height x and AC be flag staff of height 5 m. As per given in question we have drawn figure below.



In right ΔABP,

$$\frac{AB}{BP} = \tan 30^\circ$$
$$\frac{x}{y} = \frac{1}{\sqrt{3}}$$
$$y = \sqrt{3}\,x \quad ...(1)$$

In right ΔCBP

$$\frac{x+5}{y} = \tan 60^\circ = \sqrt{3} \quad ...(2)$$

Substituting the value of y from (1) we have

$$\frac{x+5}{\sqrt{3}\,x} = \sqrt{3}$$
$$x + 5 = 3x \Rightarrow x = 2.5 \text{ m}$$

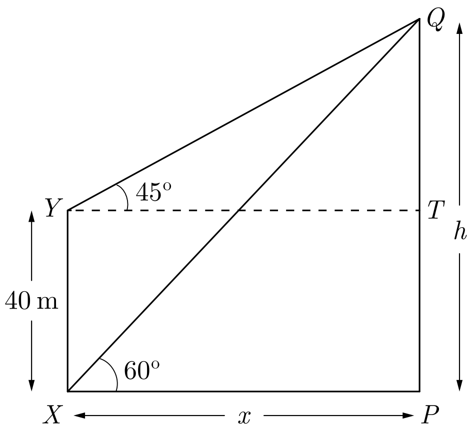
Height of tower is $= 2.5$ m

Distance of P from tower $= (2.5 \times 1.732)$ or 4.33 m.

82. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the PQ and the distance PX. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 OD 2015]

Let PX be x and PQ be h. As per given in question we have drawn figure below.



Now $\quad QT = (h - 40)$ m

In right ΔPQX we have,

$$\tan 60^\circ = \frac{h}{x}$$
$$\sqrt{3} = \frac{h}{x}$$
$$h = \sqrt{3}\,x \quad ...(1)$$

In right ΔQTY we have

$$\tan 45^\circ = \frac{h-40}{x}$$

$$1 = \frac{h-40}{x}$$

$$x = h - 40 \qquad ...(2)$$

Solving (1) and (2), we get

$$x = \sqrt{3}\,x - 40$$

$$\sqrt{3}\,x - x = 40$$

$$(\sqrt{3}-1)x = 40$$

$$x = \frac{40}{\sqrt{3}-1} = 20(\sqrt{3}+1) \text{ m}$$

Thus $h = \sqrt{3} \times 20(\sqrt{3}+1)$

$$= 20(3+\sqrt{3}) \text{ m}$$

$$= 20(3+1.73)$$

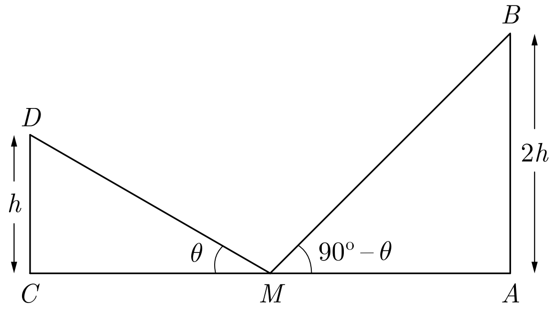
$$= 20 \times 4.73$$

Hence, height of tower is 94.6 m.

83. Two post are k metre apart and the height of one is double that of the other. If from the mid-point of the line segment joining their feet, an observer finds the angles of elevation of their tops to be complementary, then find the height of the shorted post.

Ans : [Board Term-2 Foreign 2015]

Let AB and CD be the two posts such that $AB = 2CD$. Let M be the mid-point of CA. As per given in question we have drawn figure below.



Here $CA = k$, $\angle CMD = \theta$ and $\angle AMB = 90^\circ - \theta$

Clearly, $CM = MA = \frac{1}{2}k$

Let $CD = h$. then $AB = 2h$

i152

Now, $\frac{AB}{AM} = \tan(90^\circ - \theta)$

$$\frac{2h}{\frac{k}{2}} = \cot\theta$$

$$\frac{4h}{k} = \cot\theta \qquad ...(1)$$

Also in right $\Delta\, CMD$,

$$\frac{CD}{CM} = \tan\theta$$

$$\frac{h}{\frac{k}{2}} = \tan\theta$$

$$\frac{2h}{k} = \tan\theta \qquad ...(2)$$

Multiplying (1) and (2), we have

$$\frac{4h}{k} \times \frac{2h}{k} = \tan\theta \times \cot\theta = 1$$

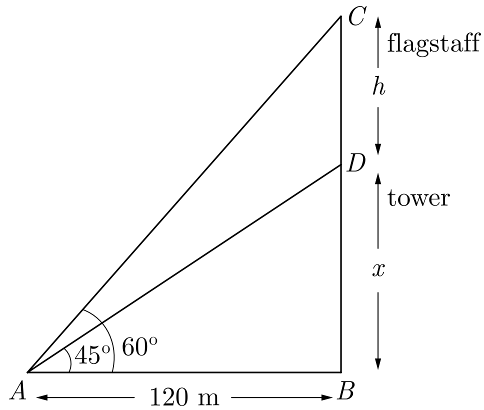
$$h^2 = \frac{k^2}{8}$$

$$h = \frac{k}{2\sqrt{2}} = \frac{k\sqrt{2}}{4}$$

84. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff. [Use $\sqrt{3} = 1.73$]

Ans : [Board Term-2 OD 2014]

Let BD be the tower of height x and CD be flagstaff of height h. As per given in question we have drawn figure below.



Here $\angle DAB = 45^\circ, \angle CAB = 60^\circ$

and $AB = 120$ m

In right angled $\Delta\, ABD$ we have

i153

$$\frac{x}{AB} = \tan 45^\circ = 1$$

$$x = AB = 120 \text{ m}$$

In right angled $\Delta\, ACB$ we have

$$\frac{h+x}{120} = \tan 60^\circ = \sqrt{3}$$

$$h + 120 = 120\sqrt{3}$$

$$h = 120\sqrt{3} - 120$$

$$= 120(\sqrt{3} - 1)$$

$$= 120(1.73 - 1)$$

$$= 120 \times 0.73$$

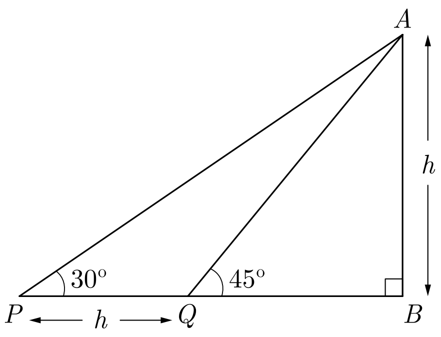
$$h = 87.6 \text{ m}$$

Hence, height of the flagstaff is 87.6 m.

85. A man on the top of a vertical tower observes a car moving at a uniform speed towards him. If it takes 12 min. for the angle of depression to change from 30° to 45°, how soon after this, the car will reach the tower ?

Ans : [Board Term-2 OD 2014]

Let AB be the tower of height h. As per given in question we have drawn figure below.



Car is at P at 30° and is at Q at 45° elevation.

Here $\angle AQB = 45^\circ$

Now, in right ΔABQ we have,

i154

$$\tan 45^\circ = \frac{AB}{BQ}$$

$$1 = \frac{h}{BQ}$$

$$BQ = h$$

In right ΔAPB we have,

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+h}$$

$$x + h = h\sqrt{3}$$

$$x = h(\sqrt{3} - 1)$$

Thus, $\text{Speed} = \frac{h(\sqrt{3}-1)}{12}$ m/min

Time for remaining distance,

$$t = \frac{h}{\frac{h(\sqrt{3}-1)}{12}} = \frac{12}{(\sqrt{3}-1)}$$

$$= \frac{12(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{12(\sqrt{3}+1)}{3-1}$$

$$= \frac{12}{2}(\sqrt{3}+1)$$

$$= 6(\sqrt{3}+1)$$

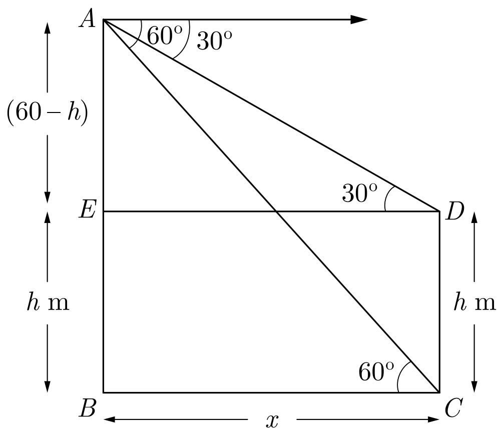
$$t = 6 \times 2.73 = 16.38$$

Hence, time taken by car is 16.38 minutes.

86. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.

Ans : [Board Term-2 2011, 2012, OD 2014]

Let AB be the building of height 60 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $DC = EB = h$ and let $BC = x$

$$AE = (60 - h) \text{ m}$$

In right angled ΔAED we have

i155

$$\frac{60-h}{ED} = \tan 30^\circ$$

$$\frac{60-h}{x} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}(60-h) = x \qquad ...(1)$$

In right $\Delta\,ABC$ we have

$$\frac{60}{x} = \tan 60^\circ$$

$$60 = \sqrt{3}\,x \qquad ...(2)$$

Substituting the value of x from equation (1) in equation (2), we have

$$60 = \sqrt{3} \times \sqrt{3}(60-h)$$

$$60 = 3 \times (60-h)$$

$$20 = 60-h$$

$$h = 40 \text{ m}$$

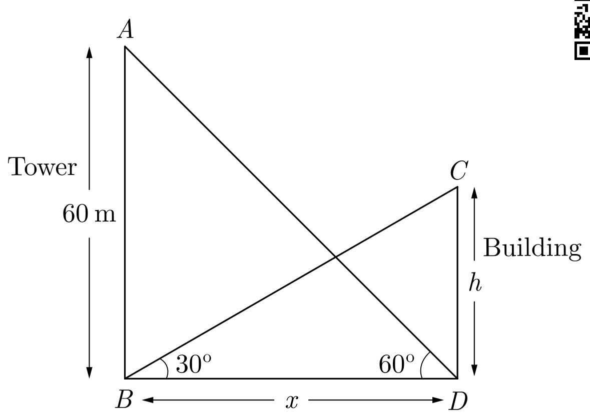
Hence, height of tower is 40 m.

87. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

Ans : [Board 2020 Delhi Basic, Delhi 2013]

Let AB be the tower of 60 m height and CD be the building of h height. As per given in question we have drawn figure below.

i156



In right $\Delta\,ABD$ we have

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{60}{x}$$

$$x = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

Now, in right $\Delta\,BCD$ we have

$$\tan 30^\circ = \frac{CD}{BD} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{20\sqrt{3}}$$

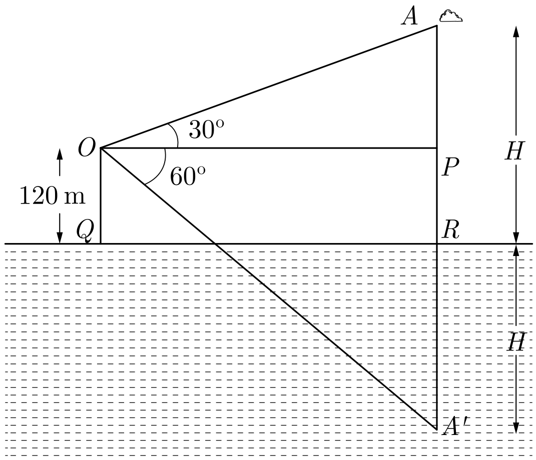
$$h = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Hence height of the building is 20 m.

88. The angle of elevation of a cloud from a point 120 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the height of the cloud.

Ans : [Board Term-2 OD 2012]

As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right $\Delta\,AOP$ we have

i157

$$\tan 30^\circ = \frac{PA}{OP}$$

$$\frac{1}{\sqrt{3}} = \frac{H-120}{OP}$$

$$OP = (H-120)\sqrt{3} \qquad ...(1)$$

In right $\Delta\,OPA'$ we have

$$\tan 60^\circ = \frac{PA'}{OP}$$

$$\sqrt{3} = \frac{H+120}{OP}$$

$$OP = \frac{H+120}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{H+120}{\sqrt{3}} = \sqrt{3}(H-120)$$

$$H+120 = 3(H-120)$$

$$H+120 = 3H-360$$

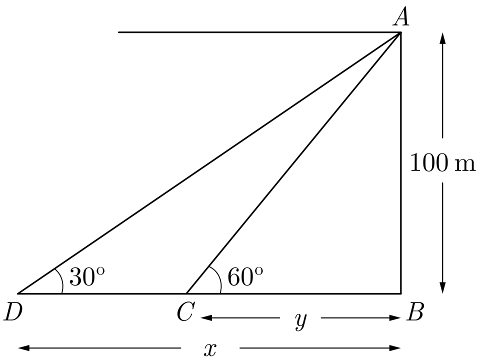
$$2H = 480 \Rightarrow H = 240$$

Thus height of cloud is 240 m.

89. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$)

Ans : [Board Term-2 OD 2016]

Let AB be the light house of height 100 m. Let C and D be the position of ship at elevation 60° and 30°. As per given in question we have drawn figure below.



In right $\Delta\,ABC$ we have

i158

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{100}{y} = \sqrt{3}$$

$$y = \frac{100}{\sqrt{3}}$$

In right $\Delta\,ABD$, we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$

$$x = 100\sqrt{3}$$

Distance CD travelled by ship,

$$x-y = 100\sqrt{3} - \frac{100}{\sqrt{3}} \text{ m}$$

$$= 100\left[\frac{3-1}{\sqrt{3}}\right]$$

$$= \frac{100 \times 2}{\sqrt{3}}$$

$$= \frac{200}{\sqrt{3}} = \frac{200\sqrt{3}}{3}$$

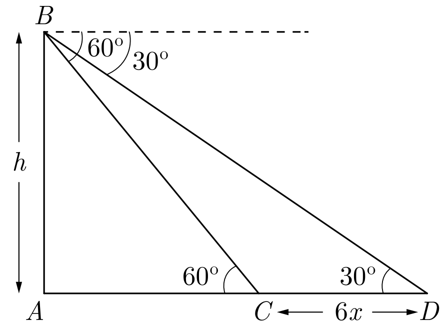
$$= \frac{200 \times 1.73}{3} = \frac{3.46}{3} \text{ m}$$

$$= 115.33 \text{ m}$$

90. A straight highway leads to the foot of a tower. A man standing on its top observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. 6 seconds later, the angle of depression of the car becomes 60°. Find the time taken by the car to reach the foot of tower from this point.

Ans : [Board Term-2 Delhi Compt. 2017]

Let AB be the tower of height h. Let point C and D be location of car. As per given in question we have drawn figure below.



Let the speed of car be x.
Thus distance covered in 6 sec $= 6x$.

i159

Hence $DC = 6x$

Let distance (remaining) CA covered in t sec.

$$CA = tx$$

Now in right $\Delta\,ADB$,

$$AD = AC + CD = 6x + tx$$

$$\tan 30^\circ = \frac{h}{6x + tx}$$

$$\frac{h}{x} = \frac{6+t}{\sqrt{3}} \quad ...(1)$$

In right ΔACB we have,

$$\tan 60^\circ = \frac{h}{tx}$$

$$\sqrt{3}\,t = \frac{h}{x} \quad ...(2)$$

From eqn. (1) and (2) we get

$$\sqrt{3}\,t = \frac{6+t}{\sqrt{3}}$$

$$3t = 6 + t$$

$$2t = 6$$

$$t = 3$$

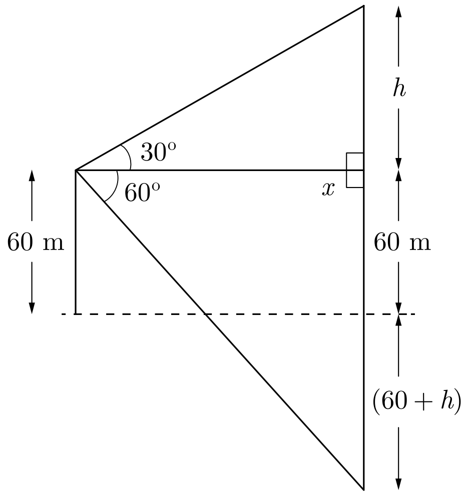
Hence, car takes 3 seconds.

91. An angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water is 60°. Find the height of the cloud from the surface of water.

Ans : [Board Term-2 Delhi 2017]

As per given in question we have drawn figure below.

i160



Here $\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$$x = h\sqrt{3} \quad ...(1)$$

and $\frac{h + 60 + 60}{x} = \tan 60^\circ$

$$\frac{h+120}{x} = \sqrt{3}$$

$$h + 120 = x\sqrt{3} \quad ...(2)$$

From (1) and (2) we get

$$h + 120 = \sqrt{3}\,h \times \sqrt{3}$$

$$h + 120 = 3h$$

$$h = \frac{120}{2} = 60 \text{ m}$$

Hence height of cloud from surface of water

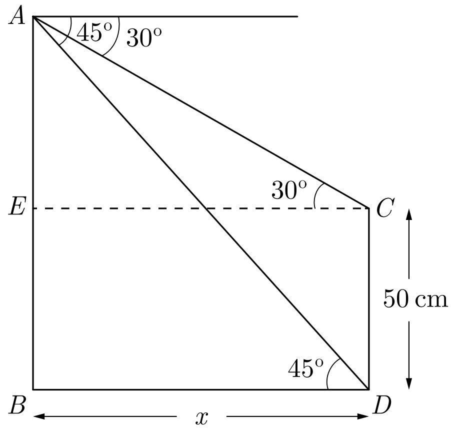
$$= 60 + 60 = 120 \text{ m}$$

92. The angle of depression of the top and bottom of a building 50 metres high as observed from the top of a tower are 30° and 45° respectively. Find the height of the tower and also the horizontal distance between the building and the tower.

Ans : [Board Term-2 SQP 2018]

Let CD be the building of height 50 m and AB be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.

i161



Let distance between BD be x.

Now, in right ΔABD we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x \quad ...(1)$$

In right ΔAEC we have

, $$\frac{AE}{EC} = \tan 30^\circ$$

$$\frac{h-50}{x} = \frac{1}{\sqrt{3}}$$

$$x = h\sqrt{3} - 50\sqrt{3} \quad ...(2)$$

From (1) and (2) we get

$$h = h\sqrt{3} - 50\sqrt{3}$$

$$h\sqrt{3} - h = 50\sqrt{3}$$

$$h(\sqrt{3}-1) = 50\sqrt{3}$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{50(3+\sqrt{3})}{3-1}$$

$$h = 25(3+\sqrt{3})$$

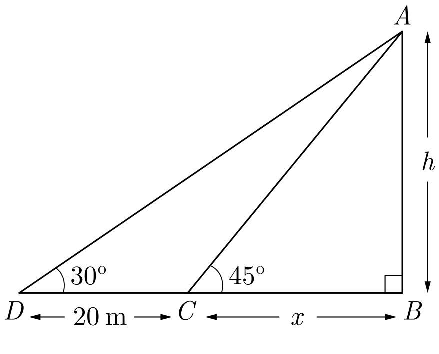
$$= 25 \times 4.732 = 118.3 \text{ m}$$

Hence, the height of tower = distance between building and tower = 118.3 m

93. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m, towards the base of the tower, the angle of elevation of the top increase by 15°, find the height of the tower.

Ans : [Board Term-2 Delhi 2017]

Let AB be the tower of height h. Angle of elevation from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



Here $CB = x$ and $DC = 20$ m

i162

Now in right ΔABC,

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{h}{x} = 1$$

$$h = x$$

In right ΔABD we have

$$\frac{AB}{DB} = \tan 30^\circ$$

$$\frac{h}{(20+x)} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = 20 + x$$

Substituting the value of x from (1) in (2)

$$h\sqrt{3} = 20 + h$$

$$h\sqrt{3} - h = 20$$

$$h(\sqrt{3}-1) = 20$$

$$h = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{20(\sqrt{3}+1)}{3-1}$$

$$= 10(\sqrt{3}+1)$$

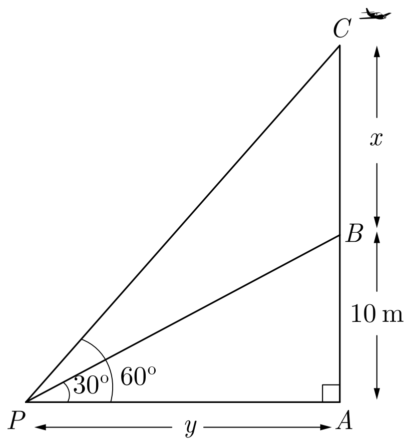
Hence, the height of tower $= 10(\sqrt{3}+1)$ m

94. From a point P on the ground, the angles of elevation of the top of a 10 m tall building and a helicopter, hovering at some height vertically over the top of the building are 30° and 60° respectively. Find the height of the helicopter above the ground.

Ans : [Board Term-2 OD Compt. 2017]

Let AB be the building of height 10 m and the height of the helicopter from top the building be x. As per given in question we have drawn figure below.

i163



Let the distance between point and building be y.

Height of the helicopter from ground

$$= (10 + x) \text{ m}$$

In right $\Delta\, BAP$ we have

$$\frac{AB}{BP} = \tan 30^\circ$$

$$\frac{10}{y} = \frac{1}{\sqrt{3}}$$

$$y = 10\sqrt{3} \qquad ...(1)$$

In right $\Delta\, CAP$,

$$\frac{AC}{PA} = \tan 60^\circ$$

$$\frac{10 + x}{y} = \sqrt{3}$$

$$10 + x = y\sqrt{3} \qquad ...(2)$$

From (1) and (2) we have

$$10 + x = 10\sqrt{3} \times \sqrt{3} = 30$$

$$x = 20$$

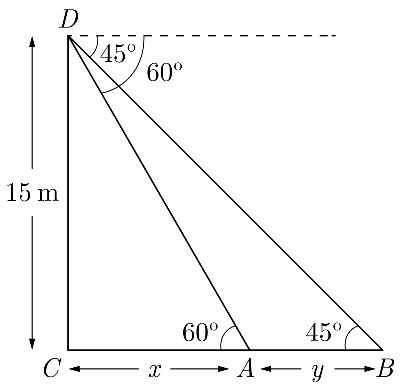
Hence height of the helicopter is 20 m.

95. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

Ans : [Board Term-2 OD 2017]

Let CD be the tower of height 15 m. Let A and B point on same side of tower As per given in question we have drawn figure below.

i164



In right $\Delta\, DCA$ we have

$$\frac{DC}{CA} = \tan 60^\circ$$

$$\frac{15}{x} = \sqrt{3}$$

$$x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

In right $\Delta\, DCB$ we have

$$\frac{DC}{CB} = \tan 45^\circ$$

$$\frac{15}{x + y} = 1$$

$$x + y = 15$$

$$5\sqrt{3} + y = 15$$

$$y = 15 - 5\sqrt{3}$$

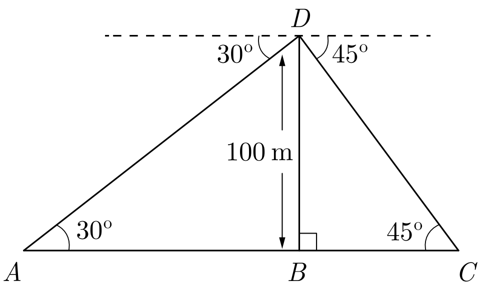
$$= 5(3 - \sqrt{3}) \text{ m}$$

Hence, the distance between points $= 5(3 - \sqrt{3})$ m

96. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3} = 1.732$]

Ans : [Board Term-2 OD Compt. 2017]

Let BD be the tower of height 100 m. Let A and C be location of car on opposite side of tower. As per given in question we have drawn figure below.



i165

In right $\Delta\, ABD$,

$$\angle DAB = 30^\circ$$

In $\Delta\, BDC$, $\angle BCD = 45^\circ$

also, $BD = 100$ m

In right ΔABD we have,

$$\tan 30^\circ = \frac{DB}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AB}$$

$$AB = 100\sqrt{3} \text{ m}$$

In right ΔDBC we have,

$$\tan 45^\circ = \frac{DB}{BC}$$

$$1 = \frac{100}{BC}$$

$$BC = 100 \text{ m}$$

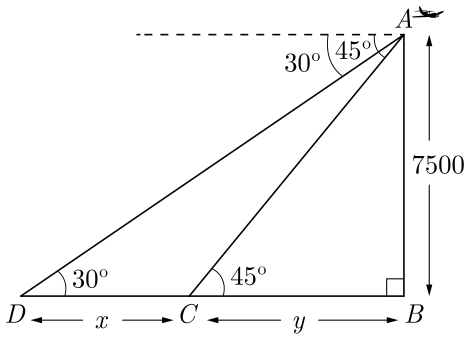
Now, $$AB + BC = 100 + 100\sqrt{3} = 100(\sqrt{3}+1)$$

$$= 100 + 173.2 = 273.2 \text{ m}$$

97. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. if both the ships are in the same that one ship is exactly behind the other, find the distance between the ships.

Ans : [Board Term-2 Foreign 2017]

Let A, C and D be the position of aeroplane and two ship respectively. Aeroplane is flying at 7500 m height from point B. As per given in question we have drawn figure below.



In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7500}{y} = 1$$

$$y = 7500 \quad \text{...(1)}$$

In right ΔABD we have

$$\frac{AB}{BD} = \tan 30^\circ$$

$$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$$

$$x + y = 7500\sqrt{3} \quad \text{...(2)}$$

Substituting the value of y from (1) in (2) we have

$$x + 7500 = 7500\sqrt{3}$$

$$x = 7500\sqrt{3} - 7500$$

$$= 7500(\sqrt{3} - 1)$$

$$= 7500(1.73 - 1)$$

$$= 7500 \times 0.73$$

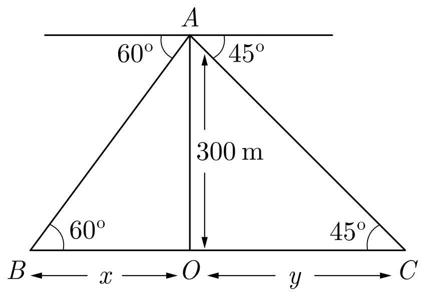
$$= 5475 \text{ m}$$

Hence, the distance between two ships is 5475 m.

98. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a respectively. Find the width of the river. River in opposite direction are 45° and 60°.

Ans : [Board Term-2 OD 2017]

Let A be helicopter flying at a height of 300 m above the point O on ground. Let B and C be the bank of river. As per given in question we have drawn figure below.



Let BO be x and OC be y.

In right ΔAOC we have

$$\frac{AO}{OC} = \tan 45^\circ$$

$$\frac{300}{y} = 1 \Rightarrow y = 300$$

In right ΔAOB we have

$$\frac{AO}{BO} = \tan 60^\circ$$

$$\frac{300}{x} = \sqrt{3}$$

$$x\sqrt{3} = 300 \Rightarrow x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

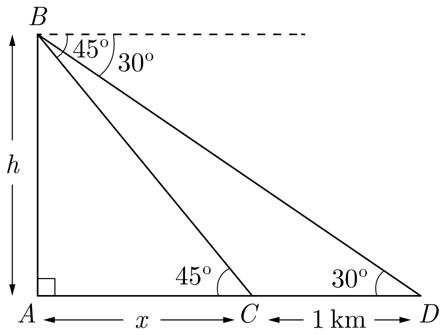
$$BC = y + x = 300 + 100\sqrt{3}$$
$$= 300 + 100 \times 1.732 = 473.2 \text{ m}$$

Hence, the width of river is 473.2 m.

99. From the top of a hill, the angle of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. [Use $\sqrt{3} = 1.73$]

Ans : [Board Term-2 OD 2016]

Let AB be the hill of height h. Angle of depression from point D and C are given 30° and 45° respectively. As per given in question we have drawn figure below.



In right ΔABC we have

i168

$$\frac{AB}{AC} = \tan 45^\circ$$

$$\frac{h}{x} = 1 \Rightarrow h = x$$

In right ΔABD we have

$$\frac{AB}{AC + CD} = \tan 30^\circ$$

$$\frac{h}{x + 1000} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = h + 1000$$

$$h(\sqrt{3} - 1) = 1000$$

$$h = \frac{1000}{\sqrt{3} - 1} = \frac{1000(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{1000(\sqrt{3} + 1)}{3 - 1}$$

$$= 500(\sqrt{3} + 1) = 500(1.73 + 1)$$

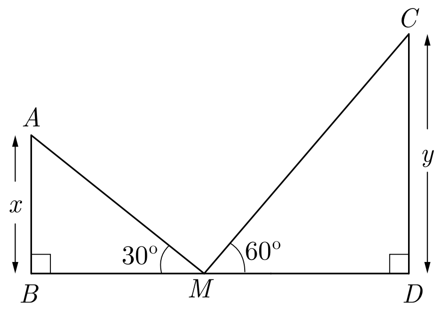
$$= 500 \times 2.73 = 1365$$

Hence height of the hill is 1365 m.

100. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$.

Ans : [Board Term-2 OD 2015]

Let AB be the tower of height x and CD be the tower of height y. Angle of depressions of both tower at centre point M are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here M is the centre of the line joining their feet.

Let $BM = MD = z$

In right ΔABM we have,

i169

$$\frac{x}{z} = \tan 30^\circ$$

$$x = z \times \frac{1}{\sqrt{3}}$$

In right ΔCDM we have,

$$\frac{y}{z} = \tan 60^\circ$$

$$y = z \times \sqrt{3}$$

From (1) and (2), we get

$$\frac{x}{y} = \frac{z \times \frac{1}{\sqrt{3}}}{z \times \sqrt{3}}$$

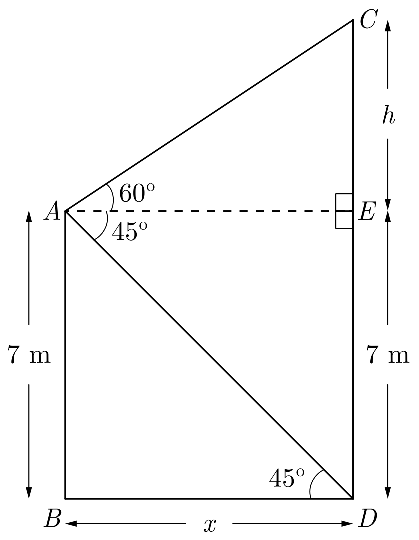
$$\frac{x}{y} = \frac{1}{3}$$

Thus $\qquad x : y = 1 : 3$

101. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. (Use $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign 2013]

Let AB be the building of height 7 m and CD be the tower of height h. Angle of depressions of top and bottom are given 30° and 60° respectively. As per given in question we have drawn figure below.



Here $\angle CBD = \angle ECB = 45^\circ$ due to alternate angles.

In right ΔABC we have

$$\frac{AB}{BC} = \tan 45^\circ$$

$$\frac{7}{x} = 1 \Rightarrow x = 7$$

i170

In right ΔAEC we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3} = 7\sqrt{3}$$

$$h = 7\sqrt{3} + 7$$

$$= 7(\sqrt{3} + 1)$$

$$= 7(1.732 + 1)$$

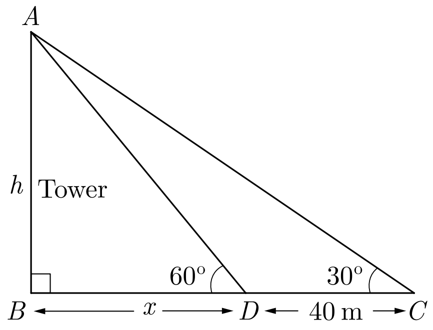
Hence, height of tower $= 19.124$ m

102. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30°, then when it is 60°. Find the height of the tower.

Ans : [Board Term-2 OD 2011]

Let AB be the tower of height h. Let BC be the shadow at 60° and BD be shadow at 30°.

As per given in question we have drawn figure below.



In right ΔABC we get,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}\,x$$

i171

In right ΔABD we have,

$$\tan 30^\circ = \frac{AB}{BC + 40}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x + 40}$$

$$x + 40 = \sqrt{3}\,h = \sqrt{3} \times \sqrt{3}\,x = 3x$$

$$40 = 2x \Rightarrow x = 20 \text{ m}$$

$$h = \sqrt{3} \times 20 = 20\sqrt{3} \text{ m}$$

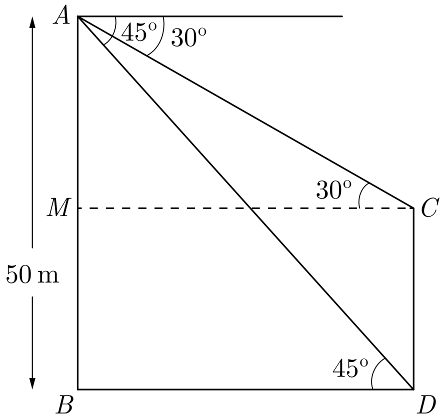
Thus height of tower is $20\sqrt{3}$ m.

103. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30°

and 45° respectively. Find :
(1) How far the pole is from the bottom of the tower,
(2) The height of the pole. (Use $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Foreign 2015]

Let AB be the tower of height 50 m and CD be the pole of height h. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. As per given in question we have drawn figure below.



In right $\Delta\, ABD$ we have,

$$\tan 45^\circ = \frac{AB}{BD} = 1$$

$$1 = \frac{50}{x} \Rightarrow x = 50 \text{ m}$$

(1) Thus distance of pole from bottom of tower is 50 m.

Now in $\Delta\, AMC$ we have

$$\tan 30^\circ = \frac{AM}{MC} = \frac{AM}{x}$$

$$AM = \frac{50}{\sqrt{3}} \text{ or } 28.87 \text{ m.}$$

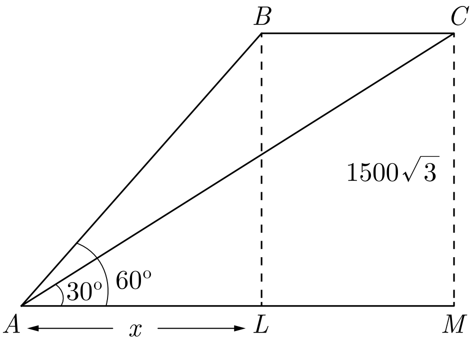
(2) Height pole $h = CD = BM$

$$= 50 - 28.87 = 21.13 \text{ m.}$$

104. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changed to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr.

Ans : [Board Term-2 OD 2015]

Let A be the point on ground, B and C be the point of location of aeroplane at height of $1500\sqrt{3}$ per given in question we have drawn figure be



In right $\Delta\, BAL$

$$\frac{BL}{AL} = \tan 60^\circ$$

$$\frac{1500\sqrt{3}}{x} = \sqrt{3} \qquad BL = CM = 1500\sqrt{3}$$

$$x = 1500 \text{ m.}$$

In right $\Delta\, CAM$ we have

$$\frac{CM}{AL + LM} = \tan 30^\circ$$

$$\frac{1500\sqrt{3}}{x + y} = \frac{1}{\sqrt{3}}$$

$$x + y = 1500 \times 3$$

$$1500 + y = 4500 \Rightarrow y = 3000 \text{ m.}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{y}{t}$$

$$= \frac{3000}{15} = 200 \text{ m/s}$$

$$= \frac{200}{1000} \times 60 \times 60$$

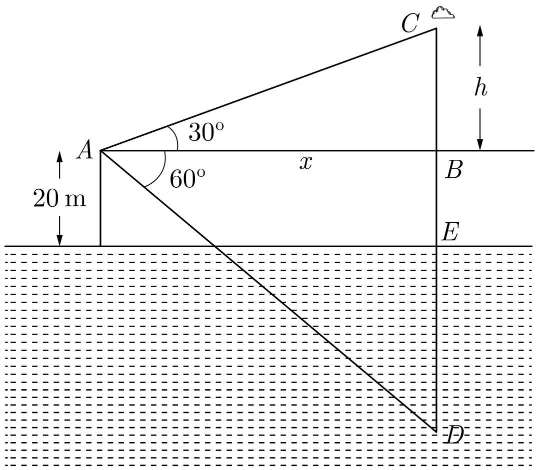
$$= 720 \text{ km/hr.}$$

Hence, the speed of the aeroplane is 720 km/hr.

105. At a point A, 20 metre above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A ?

Ans : [Board Term-2 OD 2015]

As per given in question we have drawn figure below. Here cloud is at C, D is reflection of cloud in water.



In right ΔABC we have

$$\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$x = \sqrt{3}\,h \qquad ...(1)$$

Here $DE = EC$ because D is reflection of cloud and E is at water level.

In right ΔABD we have

i174

$$\frac{BD}{BA} = \tan 60^\circ$$

$$\frac{DE + EB}{x} = \sqrt{3}$$

$$\frac{EC + EB}{x} = \sqrt{3}$$

$$\frac{h + 20 + 20}{x} = \sqrt{3}$$

$$h + 40 = \sqrt{3}\,x \qquad ...(2)$$

From (1) and (2),

$$h + 40 = \sqrt{3} \times \sqrt{3}\,h = 3h$$

$$h = 20 \text{ m}$$

$$x = \sqrt{3}\,h = 20\sqrt{3}$$

Now $$AC = \sqrt{h^2 + x^2}$$

$$= \sqrt{(20)^2 + (20\sqrt{3})^2}$$

$$= \sqrt{400 + 1200}$$

$$= 40 \text{ m.}$$

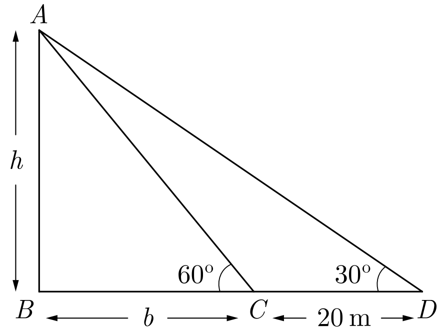
Hence distance of the cloud is 40 m.

106. A person standing on the bank of a river, observes that the angle of elevation of the top of the tree standing on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the breadth of the river.

Ans : [Board Term-2 OD 2012]

Let AB be the tree of height h and breadth of river be b. As per given in question we have drawn figure below. Here point C and D are the location of person .

i175



In right ΔABC we have,

$$\frac{h}{b} = \tan 60^\circ = \sqrt{3}$$

$$h = \sqrt{3}\,b \qquad ...(1)$$

In right ΔABD we have

$$\frac{h}{b + 20} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{b + 20}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2) we have

$$b\sqrt{3} = \frac{b + 20}{\sqrt{3}}$$

$$3b = b + 20 \Rightarrow b = 10 \text{ m}$$

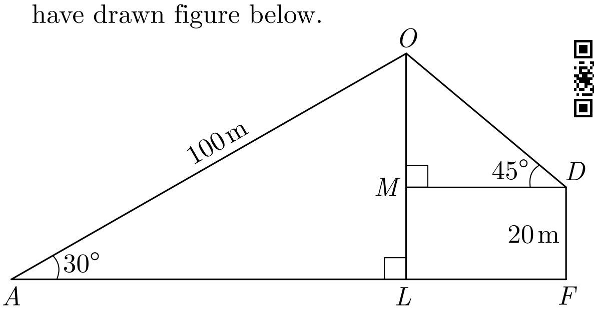
$$h = b\sqrt{3} = 10 \times 1.73 = 17.3 \text{ m}$$

Thus height of tree is 17.3 m and breadth of river is 10 m.

107. A boy observes that the angle of elevation of a bird flying at a distance of 100 m is 30°. At the same distance from the boy, a girl finds the angle of elevation of the same bird from a building 20 m high is 45°. Find the distance of the bird from the girl.

Ans : [Board Term-2 OD 2014]

Let O be the position of the bird and B be the position of the boy. Let FG be the building and G be the position of the girl. As per given in question we have drawn figure below.



i176

In right $\Delta\, OLB$ we have

$$\frac{OL}{BO} = \sin 30^\circ$$

$$\frac{OL}{100} = \frac{1}{2} \Rightarrow OL = 50 \text{ m}$$

$$OM = OL - ML$$

$$= OL - FG = 50 - 20 = 30 \text{ m}$$

In right $\Delta\, OMG$ we have

$$\frac{OM}{OG} = \sin 45^\circ$$

$$\frac{OM}{OG} = \frac{1}{\sqrt{2}}$$

$$\frac{30}{OG} = \frac{1}{\sqrt{2}}$$

$$OG = 30\sqrt{2} \text{ m}$$

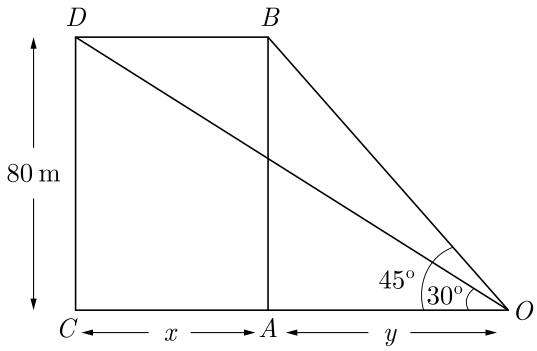
Hence, distance of the bird from the girl is $30\sqrt{2}$ m.

108. A bird sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$)

Ans : [Board Term-2 Delhi 2016]

Let CD be the tree of height 80 m and bird is sitting at D. Point O on ground is reference point from where we observe bird. As per given in question we have drawn figure below.

i177



In right AOB we have

$$\tan 45^\circ = \frac{80}{y}$$

$$y = 80$$

In right DOC we have

$$\tan 30^\circ = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+y}$$

$$x + y = 80\sqrt{3}$$

$$x = 80\sqrt{3} - y = 80\sqrt{3} - 80$$

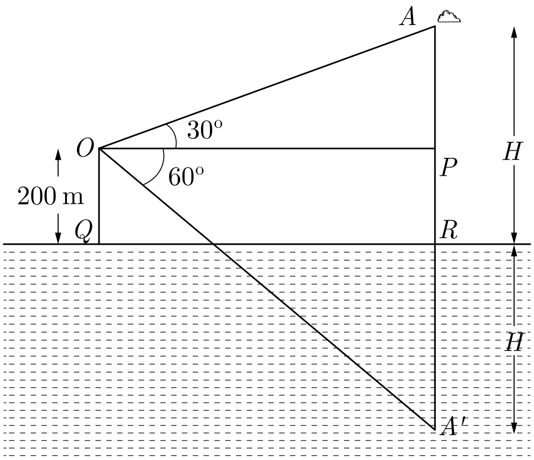
$$= 80(\sqrt{3} - 1) = 58.4 \text{ m.}$$

Hence, speed of bird $= \frac{58.4}{2} = 29.2$ m

109. The angle of elevation of a cloud from a point 200 m above the lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud above the lake.

Ans : [Board Term-2 OD 2012, 2011]

Let H be the height of cloud at A from lake. As per given in question we have drawn figure below.



Here A is cloud and A' is refection of cloud.

In right $\Delta\, AOP$ we have

$$\tan 30^\circ = \frac{PA}{OP}$$

i178

$$\frac{1}{\sqrt{3}} = \frac{H-200}{OP}$$

$$OP = (H-120)\sqrt{3} \qquad ...(1)$$

In right $\Delta\, OPA'$ we have

$$\tan 60^\circ = \frac{PA'}{OP}$$

$$\sqrt{3} = \frac{H+200}{OP}$$

$$OP = \frac{H+200}{\sqrt{3}} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{H+200}{\sqrt{3}} = \sqrt{3}(H-200)$$

$$H+200 = 3(H-200)$$

$$H+200 = 3H-600$$

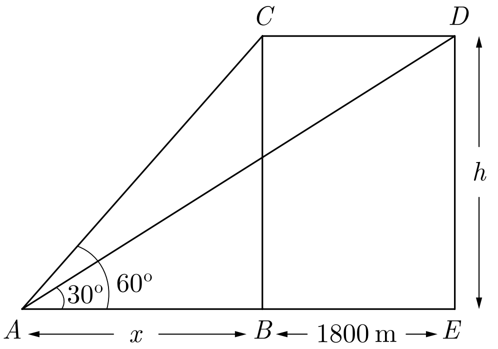
$$2H = 800 \Rightarrow H = 400$$

Thus height of cloud is 400 m.

110. The angle of elevation of a jet fighter point A on ground is 60°. After flying 10 seconds, the angle changes to 30°. If the jet is flying at a speed of 648 km/hour, find the constant height at which the jet is flying.

Ans : [Board Term-2 Delhi 2012]

Let C and D are the point of location of jet at height h. Point B and E are foot print on ground of get at thee location. As per given in question we have drawn figure below.



In 3600 sec distance travelled by plane = 648000 m

In 10 sec distance travelled by plane $= \frac{648000}{3600} \times 10$

$$= 1800 \text{ m}$$

In right $\Delta\, ABC$, we have

$$\frac{h}{x} = \tan 60^\circ = \sqrt{3}$$

$$h = x\sqrt{3} \qquad ...(1)$$

In right $\Delta\, ADE$ we have

$$\frac{h}{x+1800} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$h = \frac{x+1800}{\sqrt{3}} \qquad ...(2)$$

From equations (1) and (2), we get

$$x\sqrt{3} = \frac{x+1800}{\sqrt{3}}$$

i179

$$3x = x+1800$$

$$2x = 1800$$

$$x = 900 \text{ m}$$

$$h = x\sqrt{3}$$

$$= 900 \times 1.732$$

$$= 1558.5 \text{ m}$$

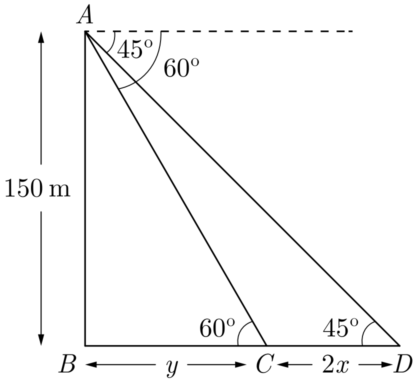
Thus height of jet is 1558.8 m.

111. A moving boat observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

Ans : [Board Term-2 Delhi 2017]

Let AB be the cliff of height 150 m. Let C and D be the point of boat at 60° and 45°. Let the speed of the boat be x m/min. Let BC be y

As per given in question we have drawn figure below.



Here distance covered in 2 minutes is $2x$.

Thus $\quad CD = 2x$

In right $\Delta\, ABD$ we have

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{150}{y} = \sqrt{3}$$

$$y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \qquad ...(1)$$

In right $\Delta\, ABD$ we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{150}{y+2x} = 1$$

$$y + 2x = 150 \qquad ...(2)$$

From equations (1) and (2), we get

$$50\sqrt{3} + 2x = 150$$

$$2x = 150 - 50\sqrt{3}$$

$$2x = 50(3-\sqrt{3})$$

$$x = 25(3-\sqrt{3})$$

$$\text{Speed of the boat} = 25(3-\sqrt{3}) \text{ m/min.}$$

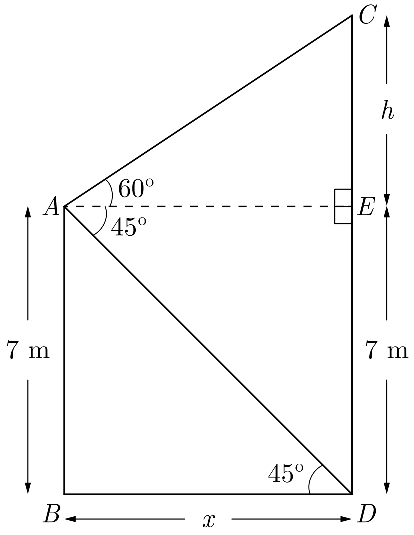
$$= \frac{25(3-\sqrt{3})\times 60}{1000}$$

$$= \frac{3}{2}(3-\sqrt{3}) \text{ km/hr.}$$

112. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Ans : [Board Term-2 Delhi 2017]

Let AB be the building of height 7 m and CD be the tower. Let distance between two be x. Angle of depressions of top and bottom of tower are given 60° and 45° respectively. As per given in question we have drawn figure below.



CD be the height of tower $= (7+h)$ m

$$BD = AE = x \text{ m}$$

In right $\Delta\, ABD$ we have

$$\frac{AB}{BD} = \tan 45^\circ$$

$$\frac{7}{x} = 1 \Rightarrow x = 7 \text{ m} \qquad ...(1)$$

In right $\Delta\, CEA$ we have

$$\frac{CE}{AE} = \tan 60^\circ$$

$$\frac{h-7}{x} = \sqrt{3}$$

$$h - 7 = x\sqrt{3} \qquad ...(2)$$

Substituting values of x we have

$$h - 7 = 7\sqrt{3}$$

$$h = 7 + 7\sqrt{3} = 7(1+\sqrt{3}) \text{ m}$$

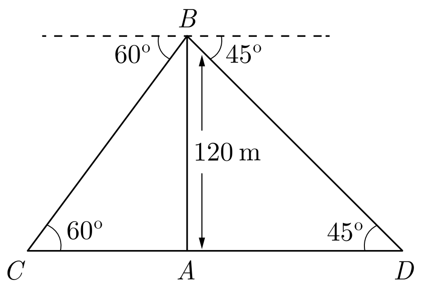
Hence, the height of tower is $7(1+\sqrt{3})$ m

113. From the top of a 120 m high tower, a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of repression as 60° and 45°. Find the distance between two cars.

Ans : [Delhi Compt. 2017]

Let AB be the tower of height 120 m. Let C and D be location of car on opposite side of tower. As per given in question we have drawn figure below.

i182



In right $\Delta\, BAD$ we have

$$\frac{AB}{AD} = \tan 45^\circ$$

$$\frac{120}{AB} = 1$$

$$AB = 120$$

In right $\Delta\, BAC$ we have

$$\frac{AB}{CA} = \tan 60^\circ$$

$$\frac{120}{CA} = \sqrt{3}$$

$$CA = \frac{120}{\sqrt{3}} = 40\sqrt{3}$$

$$\begin{aligned} CD &= CA + AD \\ &= 120 + 40\sqrt{3} \\ &= 120 + 40 \times 1.732 \\ &= 189.28 \text{ m} \end{aligned}$$

Hence the distance between two men is 189.28 m.

CHAPTER 10

CIRCLE

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

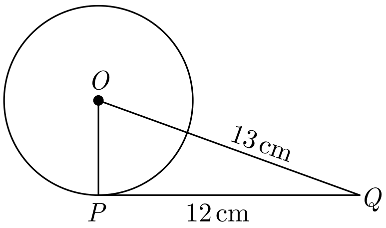
1. From an external point Q, the length of tangent to a circle is 12 cm and the distance of Q from the centre of circle is 13 cm. The radius of circle (in cm) is

(a) 10 (b) 5

(c) 12 (d) 7

Ans : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



j101

We have $OQ = 13$ cm

and $PQ = 12$ cm

Radius is perpendicular to the tangent at the point of contact.

Thus $OP \perp PQ$

In ΔOPQ, using Pythagoras theorem,

$$OP^2 + PQ^2 = OQ^2$$

$$OP^2 + 12^2 = 13^2$$

$$OP^2 = 13^2 - 12^2$$

$$= 169 - 144$$

$$= 25$$

Thus $OP = 5$ cm

Thus (b) is correct option.

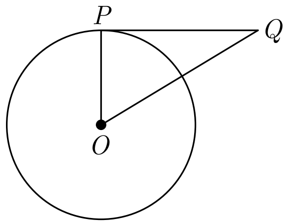
2. QP is a tangent to a circle with centre O at a point P on the circle. If ΔOPQ is isosceles, then $\angle OQR$ equals.

(a) 30° (b) 45°

(c) 60° (d) 90°

Ans : [Board 2020 Delhi Basic]

Let O be the centre of the circle. As per given information we have drawn the figure below.



j102

We know that, the radius and tangent are perpendicular at their point of contact.

Now, in isosceles triangle POQ we have

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$

Equal sides subtend equal angles in isosceles triangle.

Thus $2\angle OQP + 90^\circ = 180^\circ$

$$\angle OQP = 45^\circ$$

Thus (b) is correct option.

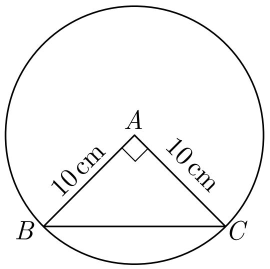
3. A chord of a circle of radius 10 cm, subtends a right angle at its centre. The length of the chord (in cm) is

(a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$

(c) $10\sqrt{2}$ (d) $10\sqrt{3}$

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.



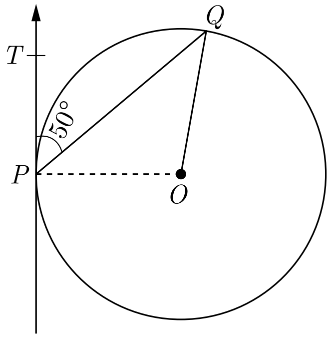
j103

Using Pythagoras theorem in ΔABC, we get

$$BC^2 = AB^2 + AC^2$$
$$= 10^2 + 10^2$$
$$= 100 + 100 = 200$$
$$BC = 10\sqrt{2} \text{ cm}$$

Thus (c) is correct option.

4. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with $PQ\angle POQ$ is



j104

(a) 130° (b) 90°
(c) 100° (d) 75°

Ans : [Board 2020 OD Basic]

Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$
$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

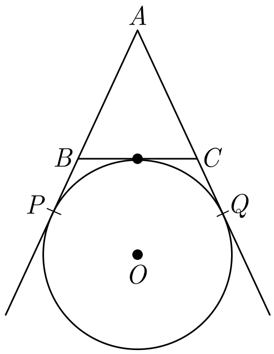
Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

$$\angle OPQ = \angle OQP = 40^\circ$$
$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$
$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

Thus (c) is correct option.

5. In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



(a) 15 (b) 10
(c) 9 (d) 7.5

Ans : [Board 2020 Delhi Basic]

j105

Due to tangents from external points,

$BP = BR, CR = CQ$, and $AP = AQ$

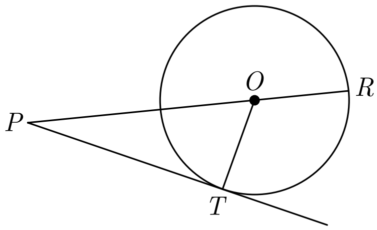
Perimeter of ΔABC,

$$AB + BC + AC$$
$$= AB + BR + RC + AC$$
$$5 + 4 + 6 = AB + BP + CQ + AC$$
$$15 = AP + AQ$$
$$15 = 2AP$$

Thus $AP = \frac{15}{2} = 7.5$ cm

Thus (d) is correct option.

6. In figure, on a circle of radius 7 cm, tangent PT is drawn from a point P such that $PT = 24$ cm. If O is the centre of the circle, then the length of PR is



(a) 30 cm (b) 28 cm
(c) 32 cm (d) 25 cm

Ans : [Board 2020 Delhi Basic]

Tangent at any point of a circle is perpendicular to the radius at the point of contact.

Thus $OT \perp PT$

j106

Now in right-angled triangle PTO

$$OP^2 = OT^2 + PT^2$$

$$= (7)^2 + (24)^2$$

$$= 49 + 576$$

$$= 625$$

Thus $OP = 25$ cm

Since $OR = OT$ because of radii of circle,

$$PR = OP + OR = 25 + 7 = 32 \text{ cm}$$

Thus (c) is correct option.

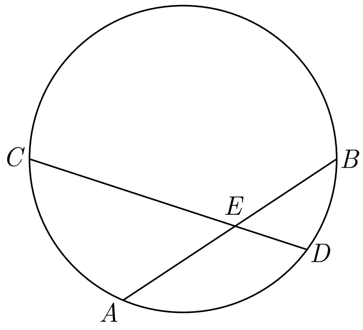
7. Two chords AB and CD of a circle intersect at E such that $AE = 2.4$ cm, $BE = 3.2$ cm and $CE = 1.6$ cm. The length of DE is

(a) 1.6 cm (b) 3.2 cm

(c) 4.8 cm (d) 6.4 cm

Ans : (c) 4.8 cm

j107



Applying the rule, $AE \times EB = CE \times ED$

$$2.4 \times 3.2 = 1.6 \times ED$$

$$ED = 4.8 \text{ cm}$$

Thus (c) is correct option.

8. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is

(a) $3r$ (b) $6r$

(c) $9r$ (d) $12r$

Ans :

Side of the regular hexagon inscribed in a circle of radius r is also r, the perimeter is $6r$.

Thus (b) is correct option.

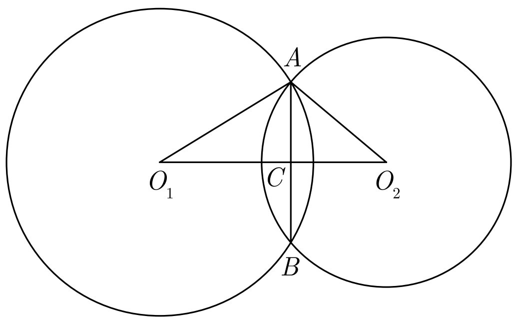
j108

9. Two circles of radii 20 cm and 37 cm intersect in A and B. If O_1 and O_2 are their centres and $AB = 24$ cm, then the distance O_1O_2 is equal to

(a) 44 cm (b) 51 cm

(c) 40.5 cm (d) 45 cm

Ans :



Since C is the mid-point of AB,

j109

$$AC = 12$$

$$AO_1 = 37$$

and $AO_2 = 20$

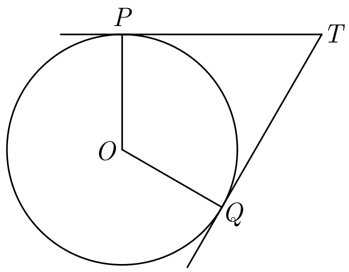
$$CO_1 = \sqrt{37^2 - 12^2} = 35$$

$$CO_2 = \sqrt{20^2 - 12^2} = 16$$

$$O_1O_2 = 35 + 16 = 51$$

Thus (b) is correct option.

10. In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If $\angle POQ = 110^\circ$, then $\angle PTQ$ is



(a) 60° (b) 70°

(c) 80° (d) 90°

Ans :

Here $OP \perp PT$ and $OQ \perp QT$,

In quadrilateral $OPTQ$, we have

$$\angle POQ + \angle OPT + \angle PTQ + \angle OQT = 360^\circ$$

$$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$$

$$\angle PTQ = 70^\circ$$

j110

Thus (b) is correct option.

11. AB and CD are two common tangents to circles which touch each other at a point C. If D lies on AB such that $CD = 4$ cm then AB is

(a) 12 cm (b) 8 cm

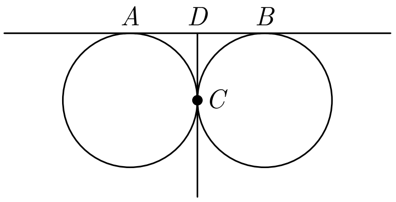
(c) 4 cm (d) 6 cm

Ans :

$$AD = CD \text{ and } BD = CD$$

$$AB = AD + BD = CD + CD$$

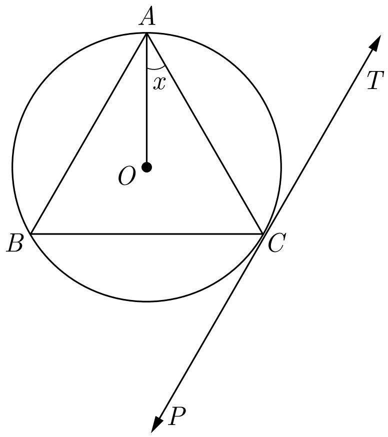
$$= 2CD = 2 \times 4 = 8 \text{ cm}$$



j111

Thus (b) is correct option.

12. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118^\circ$, then the measure of $\angle x$ is

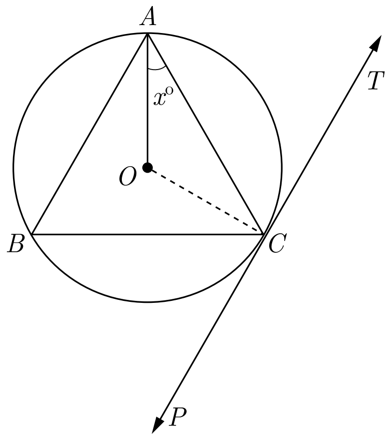


(a) 28° (b) 32°

(c) 42° (d) 38°

Ans :

We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $OC \perp PT$

j112

$$\angle OCP = 90^\circ$$

Given, $\angle ACP = 118^\circ$

$$\angle ACO = \angle ACP - \angle OCP$$

$$= 118^\circ - 90^\circ = 28^\circ$$

$$\angle ACO = 28^\circ$$

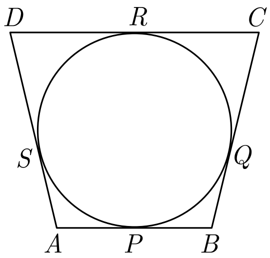
Since O is the circumcentre, thus $OA = OC$ (radius)

$$\angle OAC = \angle ACO$$

$$x = 28^\circ$$

Thus (a) is correct option.

13. In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm, then length of AD is



(a) 3 cm (b) 4 cm

(c) 5 cm (d) 6 cm

Ans :

Four sides of a quadrilateral $ABCD$ are tangent to a circle.

$$AB + CD = BC + AD$$

$$6 + 4 = 7 + AD$$

j114

$$AD = 10 - 7 = 3 \text{ cm}$$

Thus (a) is correct option.

14. Two concentric circles of radii a and b where $a > b$, The length of a chord of the larger circle which touches the other circle is

(a) $\sqrt{a^2 + b^2}$ (b) $2\sqrt{a^2 + b^2}$

(c) $\sqrt{a^2 - b^2}$ (d) $2\sqrt{a^2 - b^2}$

Ans :

In $\Delta\, OAL$,

$$OA^2 = OL^2 + AL^2$$

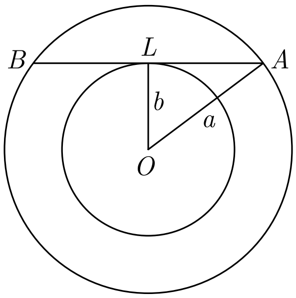
$$a^2 = OL^2 + b^2$$

$$OL = \sqrt{a^2 - b^2}$$

j113

Length of chord,

$$2AL = 2\sqrt{a^2 - b^2}$$



Thus (d) is correct option.

15. Two concentric circles are of radii 10 cm and 8 cm, then the length of the chord of the larger circle which touches the smaller circle is

(a) 6 cm (b) 12 cm

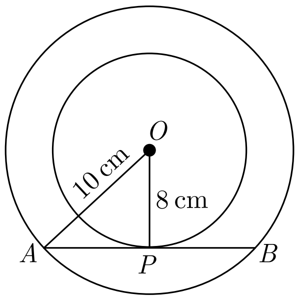
(c) 18 cm (d) 9 cm

Ans :

Let O be the centre of the concentric circles of radii 10 cm and 8 cm, respectively. Let AB be a chord of the larger circle touching the smaller circles at P.

Then, $$AP = PB \text{ and } OP \perp AB$$

j115



Applying Pythagoras theorem in $\Delta\, OPA$, we have

$$OA^2 = OP^2 + AP^2$$

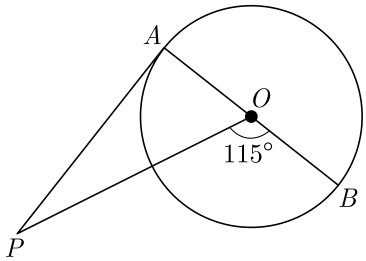
$$100 = 64 + AP^2$$

$$AP^2 = 100 - 64 = 36 \Rightarrow AP = 6 \text{ cm}$$

$$AB = 2AP = 2 \times 6 = 12 \text{ cm}$$

Thus (b) is correct option.

16. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115°$, then perimeter of $\angle APO$ is



(a) 25° (b) 20°

(c) 30° (d) 65°

Ans :

Since tangent at a point to a circle is perper to the radius,

$$\angle OAP = 90°$$

j116

Now, $\angle AOP + \angle BOP = 180^\circ$

$$\angle AOP + 115^\circ = 180^\circ$$

$$\angle AOP = (180^\circ - 115^\circ) = 65^\circ$$

From angle sum property of triangle we have

$$\angle OAP + \angle AOP + \angle APO = 180^\circ$$

$$90^\circ + 65^\circ + \angle APO = 180^\circ$$

$$155^\circ + \angle APO = 180^\circ$$

$$\angle APO = 180^\circ - 155^\circ = 25^\circ$$

Thus (a) is correct option.

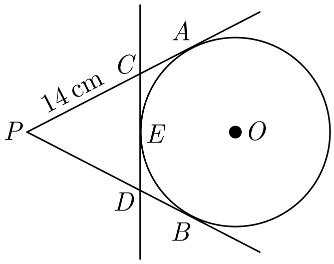
17. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14$ cm. The perimeter of ΔPCD is

(a) 14 cm (b) 21 cm

(c) 28 cm (d) 35 cm

Ans :

As per information given in question we have drawn figure below.



j117

Here $PA = PB = 14$ cm

Also, CD is tangent at point E on the circle.

So, CA and CE are tangent to the circle from point C.

Therefore, $CA = CE$,

Similarly, $DB = DE$

Now, perimeter of ΔPCD,

$$PC + CD + PD = PC + CE + ED + PD$$

$$= PC + CA + PD + DB$$

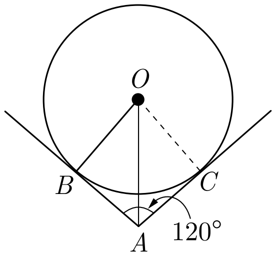
$$= PA + PB$$

$$= 14 + 14$$

$$= 28 \text{ cm}$$

Thus (c) is correct option.

18. In the given figure, two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$, then OA is equal to that



(a) $2AB$ (b) $3AB$

(c) $4AB$ (d) $5AB$

Ans :

In ΔOAB and ΔOAC, we have,

$$\angle OBA = \angle OCA = 90^\circ$$

$$OA = OA \quad \text{[common]}$$

and $OB = OC$ [radii of circle]

So, by RHS congruence criterion,

$$\Delta OBA \cong \Delta OCA$$

$$\angle OAB = \angle OAC = \frac{1}{2} \times 120^\circ = 60^\circ$$

In ΔOBA, we have,

$$\cos 60^\circ = \frac{AB}{OA}$$

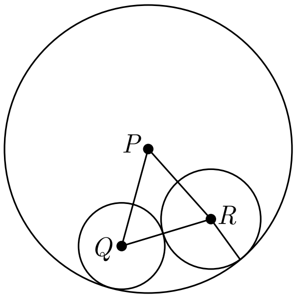
$$\frac{1}{2} = \frac{AB}{OA}$$

$$OA = 2AB$$

j118

Thus (a) is correct option.

19. In the given figure, three circles with centres P, Q and R are drawn, such that the circles with centres Q and R touch each other externally and they touch the circle with centre P, internally. If $PQ = 10$ cm, $PR = 8$ cm and $QR = 12$ cm, then the diameter of the largest circle is:



(a) 30 cm (b) 20 cm
(c) 10 cm (d) None of these

Ans :

Let radii of the circles with centres P, Q and R are p, q and r, respectively.

Then, $PQ = p - q = 10$ ...(1)

$PR = p - r = 8$ ...(2)

and $QR = q + r = 12$ ...(3)

Adding equation (2) and (3), we get,

$p + q = 20$ ...(4)

Adding equation (1) and (4), we get,

$2p = 30$

Hence, diameter of the largest circle $2p = 30$.

Thus (a) is correct option.

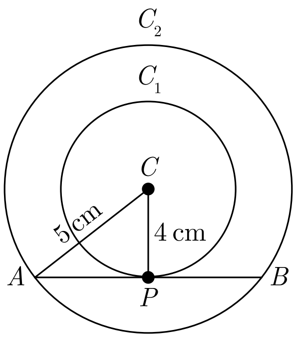
20. If radii of two concentric circles are 4 cm and 5 cm, then the length of each of one circle which is tangent to the other circle, is

(a) 3 cm (b) 6 cm
(c) 9 cm (d) 1 cm

Ans :

Let C be the centre of two concentric circles C_1 and C_2, whose radii are $r_1 = 4\text{ cm}$ and $r_2 = 5\text{ cm}$.

Now, we draw a chord $AB$of circle C_2, which touches C_1 at P.



AB is tangent at P and CP is radius at P. Tangent at any point of circle is perpendicular to the radius through the point of contact.

Thus $CP \perp AB$

Now, in right triangle PAC

By Pythagoras theorem we have

$$AP^2 = AC^2 - PC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

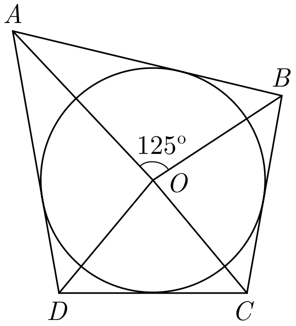
$$AP = 3\text{ cm}$$

So, length of chord,

$$AB = 2AP = 2 \times 3 = 6\text{ cm}$$

Thus (b) is correct option.

21. In figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to



(a) 62.5° (b) 45°
(c) 35° (d) 55°

Ans :

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

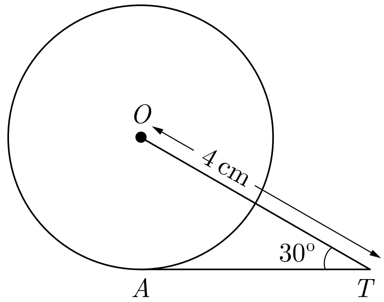
i.e. $\angle AOB + \angle COD = 180^\circ$

$125^\circ + \angle COD = 180^\circ$

$\angle COD = 180^\circ - 125^\circ = 55^\circ$

Thus (d) is correct option.

22. In figure, AT is a tangent to the circle with centre O such that $OT = 4\text{ cm}$ and $\angle OTA = 30^\circ$. Then, AT is equal to



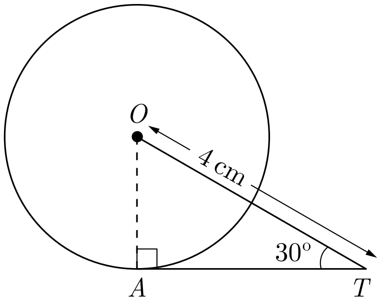
(a) 4 cm (b) 2 cm

(c) $2\sqrt{3}$ cm (d) $4\sqrt{3}$ cm

Ans :

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OAT = 90^\circ \text{ and } OT = 4 \text{ cm (given)}$$



In ΔOAT, $\cos 30^\circ = \dfrac{AT}{OT}$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$

$$AT = 2\sqrt{3} \text{ cm}$$

Thus (c) is correct option.

23. **Assertion :** If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

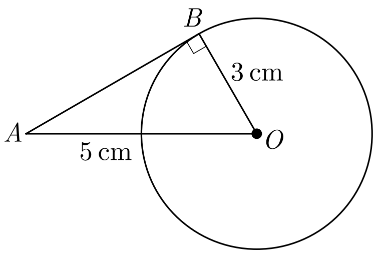
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :



$$OA^2 = AB^2 + OB^2$$

$$5^2 = AB^2 + 3^2$$

$$AB = \sqrt{25-9} = 4 \text{ cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

24. **Assertion :** The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Reason : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

From an external point the two tangents drawn subtend equal angles at the centre.

So assertion is true.

Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

25. **Assertion :** PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.

Reason : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

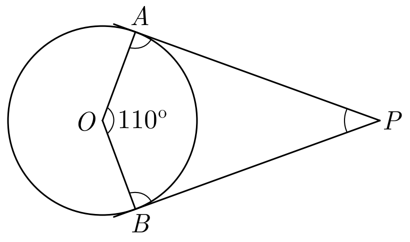
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Ans : (d) Assertion (A) is false but reason (R) is true.

As per information given in question we have drawn figure below.



Radius is perpendicular to the tangent at point of contact.

Thus, $OA \perp AP$ and $OB \perp PB$.

In quadrilateral, $OAPB$, we have

$$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$$

$$90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$$

$$\angle APB = 70^\circ$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

26. The lengths of the two tangents from an external point to a circle are ....

Ans :

parallel

j126

27. A line that intersects a circle in one point only is called ....

Ans :

tangent

j127

28. The tangents drawn at the ends of a diameter of a circle are ....

Ans :

two

j128

29. A tangent of a circle touches it at .... point(s).

Ans :

one

j129

30. Tangent is perpendicular to the .... through the point of contact.

Ans :

radius

j130

31. A line intersecting a circle at two points is called a ....

Ans :

secant

j131

32. A circle can have .... parallel tangents at the most.

Ans :

two

j132

33. The common point of a tangent to a circle and the circle is called ....

Ans :

point of contact

j133

34. There is no tangent to a circle passing through a point lying .... the circle.

Ans :

inside

j134

35. The tangent to a circle is .... to the radius through the point of contact.

Ans :

perpendicular

j135

36. There are exactly two tangents to a circle passing through a point lying .... the circle.

Ans :

outside equal

j136

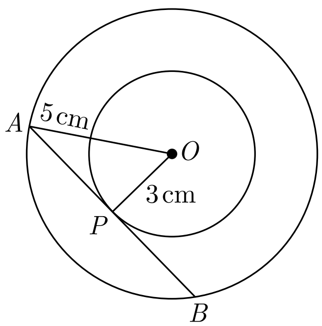
37. Length of two tangents drawn from an external point are ....

Ans :

equal

j137

38. In given figure, the length PB = ... cm.



Ans : [Board 2020 OD Standard]

We have $AO = 5\text{ cm}$

and $OP = 3\text{ cm}$

Since AB is a tangent at P and OP is radius, we have

$$\angle APO = 90^\circ$$

j138

In right angled $\Delta\, OPA$,

$$AP^2 = AO^2 - OP^2$$

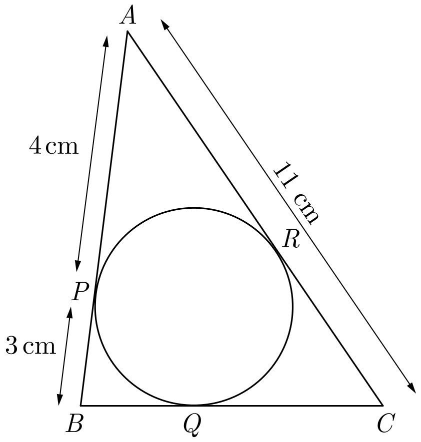
$$= (5)^2 - (3)^2 = 25 - 9 = 16$$

$$AP = 4\text{ cm}$$

Perpendicular from centre to chord bisect the chord. Thus

$$AP = BP = 4\text{ cm}$$

39. In figure, $\Delta\, ABC$ is circumscribing a circle, the length of BC is .... cm.



Ans : [Board 2020 Delhi Standard]

Since AP and AR are tangents to the circle from external point A, we have

$$AP = AR = 4\text{ cm}$$

j139

Similarly, PB and BQ are tangents.

Therefore $BP = BQ = 3\text{ cm}$

Now, $CR = AC - AR = 11 - 4 = 7\text{ cm}$

Similarly, CR and CQ are tangents.

Therefore $CR = CQ = 7\text{ cm}$

Now, $BC = BQ + CQ = 3 + 7 = 10\text{ cm}$

Hence, the length of BC is 10 cm.

Very Short Answer Questions

40. If the angle between two radii of a circle is 130°, then what is the angle between the tangents at the end points of radii at their point of intersection ?

Ans : [Board Te

Sum of the angles between radii and between

j140

intersection point of tangent is always 180°.

Thus angle at the point of intersection of tangents

$$= 180^\circ - 130^\circ = 50^\circ$$

41. To draw a pair of tangents to a circle which are inclined to each other at an angle of 30°, it is required to draw tangents at end points of two radii of the circle, what will be the angle between them ?

Ans : [Board Term-2 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

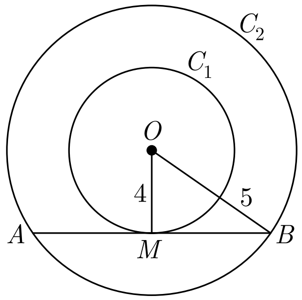
j141

Angle between the radii $= 180^\circ - 30^\circ = 150^\circ$

42. If the radii of two concentric circle are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

Ans :

As per given information we have drawn the figure below.



Since chord AB is tangent to circle C_1 at point M,

$$OM \perp AB$$

In $\Delta\, OMB$,

$$OB^2 = OM^2 + MB^2$$

j142

$$25 = 4^2 + MB^2$$

$$MB^2 = 25 - 16 = 9$$

$$MB = 3$$

Since, $OM \perp AB$, we obtain $AM = MB$

Now, $AB = 2MB = 2 \times 3 = 6\text{ cm}$

Hence, length of chord is 6 cm.

43. If a circle can be inscribed in a parallelogram how will the parallelogram change?

Ans : [Board Term-2, 2014]

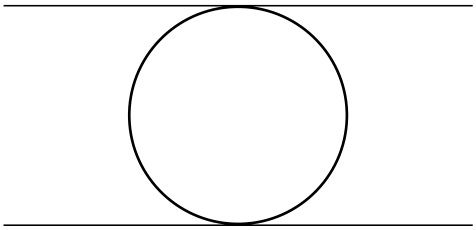
It changes into a rectangle or a square.

j143

44. What is the maximum number of parallel tangents a circle can have on a diameter?

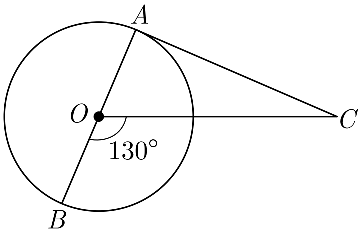
Ans : [Board Term-2 2012]

Tangent touches a circle on a distinct point. Only two parallel tangents can be drawn on the diameter of a circle. It has been shown in figure given below.



j144

45. In the given figure, AOB is a diameter of the circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, the find $\angle ACO$.



Ans : [Board Term-2 Foreign 2016]

Here OA is radius and AC is tangent at A, since radius is always perpendicular to tangent, we have

$$\angle OAC = 90^o$$

From exterior angle property,

$$\angle BOC = OAC + \angle ACO$$

$$130^o = 90^o + \angle ACO$$

$$\angle ACO = 130^o - 90^o = 40^o$$

j145

46. If a line intersects a circle in two distinct points, what is it called ?

Ans : [Board Term-2, 2012]

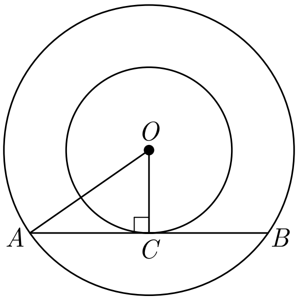
The line which intersects a circle in two distinct points is called secant.

j146

47. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of larger circle (in cm) which touches the smaller circle.

Ans : [Board 2020 OD Basic, Foreign 2014]

As per the given question we draw the figure as below.



Here AB is the chord of large circle which touch the smaller circle at point C. We can see easily that ΔAOC is right angled triangle.

Here, $AO = 5$ cm, $OC = 3$ cm

$$AC = \sqrt{AO^2 - OC^2}$$

$$= \sqrt{5^2 - 3^2}$$

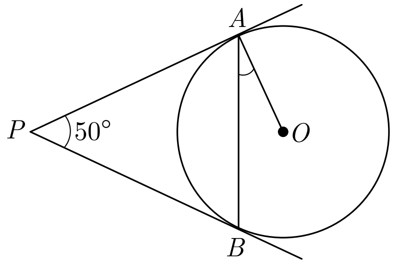
$$= \sqrt{25 - 9} = \sqrt{16} = 4$$

cm

j147

Length of chord, $AB = 8$ cm.

48. In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^o$. Write the measure of $\angle OAB$.



Ans : [[Board Term-2 Delhi 2015]

We have $\angle APB = 50^o$

$$\angle PAB = \angle PBA = \frac{180^o - 50^o}{2} = 65^o$$

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

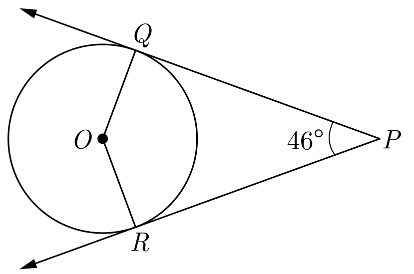
$$\angle OAP = 90^o$$

Now $\angle OAB = \angle OAP - \angle PAB$

$$= 90^o - 65^o = 25^o$$

j148

49. If PQ and PR are two tangents to a circle with centre O. If $\angle QPR = 46^o$ then find $\angle QOR$.



Ans : [Board Term-2 Delhi 2014]

We have $\angle QPR = 46^o$

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^o$$

$$\angle QOR + 46^o = 180^o$$

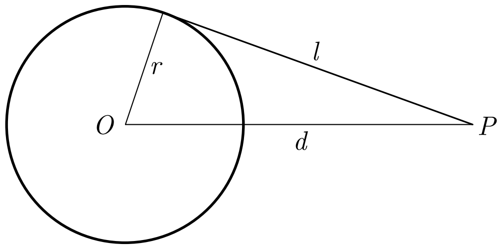
$$\angle QOR = 180^o - 46^o = 134^o$$

j149

50. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm ?

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



Length of the tangent, $l = \sqrt{d^2 - r^2}$

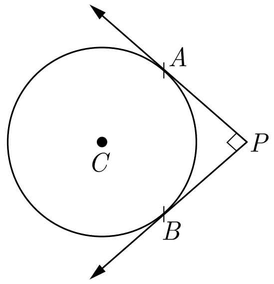
$= \sqrt{8^2 - 6^2}$

$= \sqrt{64 - 36}$

$= \sqrt{28} = 2\sqrt{3}$ cm.

j150

51. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If $PA \perp PB$, then find the length of each tangent.



Ans : [Board Term-2, 2013]

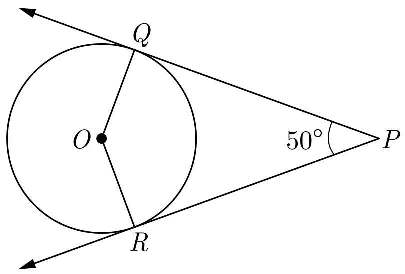
Here tangent drawn on circle from external point P are at aright angle, $CAPB$ will be a square.

Thus $CA = AP = PB = BC = 4$ cm

Thus length of tangent is 4 cm.

j151

52. In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50^o$, Then find $\angle OQR$.



Ans : [Board Term-2 Delhi 2012, 2015]

We have $\angle QPR = \angle 50^o$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^o$$

$$\angle QOR = 180^o - \angle QPR$$

$$= 180^o - 50^o = 130^o$$

j152

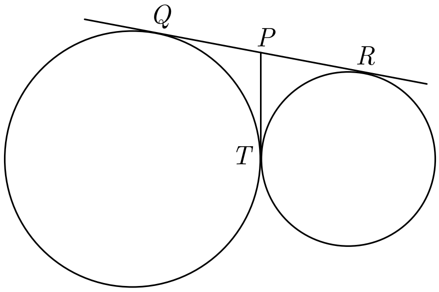
From $\Delta\, OQR$ we have

$$\angle OQR = \angle ORQ = \frac{180^o - 130^o}{2}$$

$$= \frac{50^o}{2} = 25^o$$

53. In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If

$QP = 3.8$ cm, then find length of QR.



Ans : [Board Term-2 Delhi 2012, 2014]

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \quad ....(1)$$

Now consider the small circle. For this circle we can also write using same logic,

$$PR = PT$$

But we have $PT = 3.8$ cm

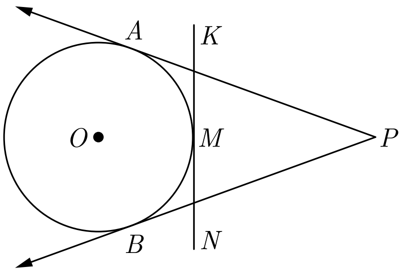
j153

Thus $$PR = PT = 3.8 \text{ cm}$$

Now $$QR = QP + PR$$

$$= 3.8 + 3.8 = 7.6 \text{ cm.}$$

54. PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$



Ans : [Board Te

j154

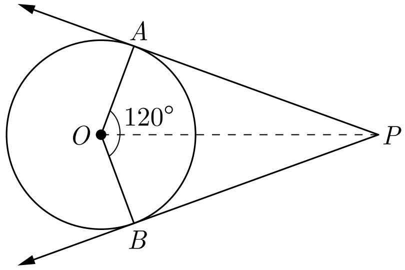
Since length of tangents from an external point to a circle are equal,

$$PA = PB, KA = KM, NB = NM,$$

$$KA + NB = KM + NM$$

$$AK + BN = KN. \qquad \text{Hence Proved}$$

55. In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120^\circ$, then find $\angle OPA$.



Ans : [Board Term-2 Delhi 2012, 2014]

Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^\circ$$

j155

Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120^\circ}{2} = 60^\circ$$

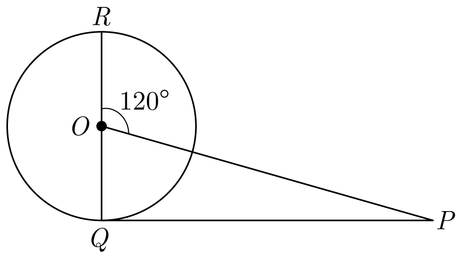
Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180^\circ$$

$$\angle APO + 90^\circ + 60^\circ = 180^\circ$$

$$\angle APO = 180^\circ - 150^\circ = 30^\circ.$$

56. PQ is a tangent drawn from an external point P to a circle with centre O, QOR is the diameter of the circle. If $\angle POR = 120^\circ$, What is the measure of $\angle OPQ$?



Ans : [Board Term-2 Foreign 2017]

Since PQ is a tangent to the circle, ΔOQP is right angle triangle

In $\Delta\, OQP$ because of exterior angle,

$$\angle POR = \angle OQP + \angle OPQ$$

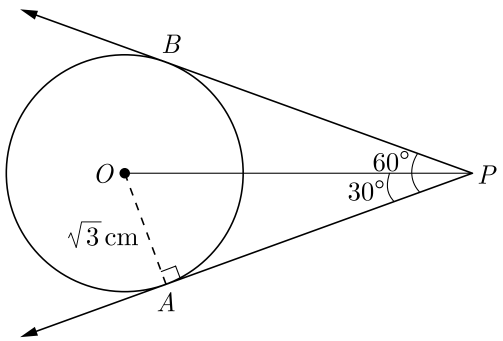
j156

Thus
$$\angle OPQ = \angle POR - \angle OQP$$
$$= 120^\circ - 90^\circ = 30^\circ$$

57. Two tangents making an angle of 60° between them are drawn to a circle of radius $\sqrt{3}$ cm, then find the length of each tangent.

Ans : [Board, Term-2, 2013]

As per the given question we draw the figure as below.

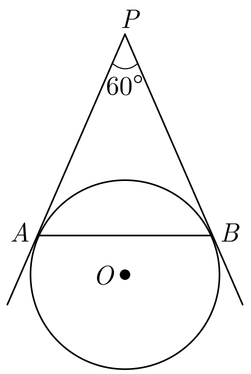


Since,
$$\tan\theta = \frac{OA}{AP}$$

j157

So.
$$\tan 30^\circ = \frac{OA}{AP}$$
$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{AP}$$
$$AP = \sqrt{3}\times\sqrt{3} = 3 \text{ cm.}$$

58. In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Ans : [Board Term-2 Delhi 2016]

Since length of 2 tangents drawn from an external point to a circle are equal, we have

$$PA = PB$$

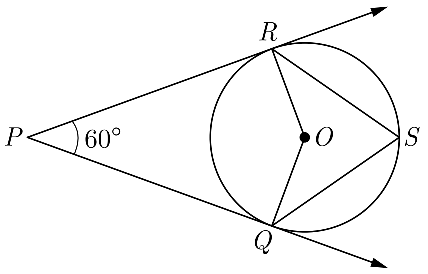
Thus
$$\angle PAB = \angle PBA = 60^\circ$$

j158

Hence $\Delta\, PAB$ is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

59. In the given figure, find $\angle QSR$.



Ans : [Board Term-2, 2012]

Sum of the angles between radii and between intersection point of tangent is always 180°.

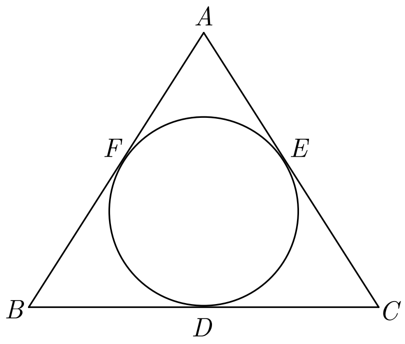
j159

Thus
$$\angle ROQ + \angle RPQ = 180^\circ$$
$$\angle ROQ = 180^\circ - 60^\circ = 120^\circ$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus
$$\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2}\times 120^\circ$$
$$= 60^\circ$$

60. A triangle ABC is drawn to circumscribe a circle. If $AB = 13$ cm, $BC = 14$ cm and $AE = 7$ cm, then find AC.



Ans : [Board Term-2 Delhi 2012]

Since AF and AE are tangent of the circle, $AF = AE$

Thus $AF = AE = 7$ cm

Now $BF = AB - AF = 13 - 7 = 6$ cm

Since BF and BD are tangent of the circle, $BF = BD$

Thus $BD = BF = 6$ cm

Now $CD = BC - BD = 14 - 6 = 8$ cm

Since CD and CE are tangent of the circle, $CD = CE$

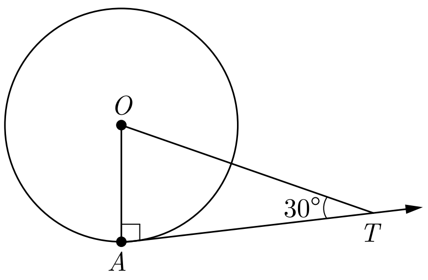
Thus $CE = CD = 8$ cm

Now $AC = AE + EC$

$= 7 + 8 = 15$ cm.

j160

61. In given figure, if AT is a tangent to the circle with centre O, such that $OT = 4$ cm and $\angle OTA = 30°$, then find the length of AT (in cm).



Ans : [Board Term-2, 2012 Set (13)]

Since AT is a tangent to the circle, ΔOAT is right angle triangle

Now $\cos 30° = \frac{AT}{OT}$

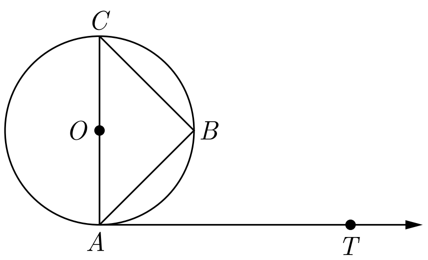
j161

$AT = OT \cos 30°$

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

Thus the length of AT is $2\sqrt{3}$ cm.

62. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, find $\angle BAT$.



Ans : [Board Term-2 2012]

We have $\angle ACB = 50°$

Since $\angle CBA$ is angle in semi-circle,

j162

$\angle CBA = 90°$

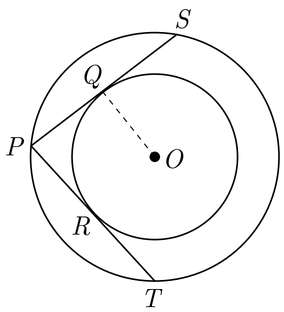
Now $\angle OAB = 180° - 90° - 50°$

$= 40°$

$\angle BAT = 90° - \angle OAB$

$= 90° - 40° = 50°$

63. In the figure there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If $PR = 5$ cm find the length of PS.



Ans : [Board Term-2 Delhi Compt. 2017]

Since PQ and PR are tangent of the circle, $PQ = PR$

$PQ = PR = 5$ cm

Since PS is chord of circle and point Q bisect it, thus

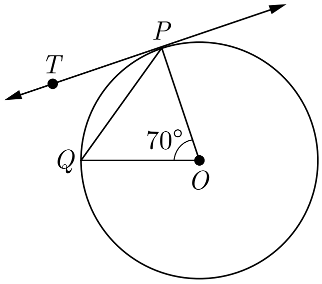
$PQ = QS$

$PS = 2PQ$

$= 2 \times 5 = 10$ cm

j163

64. In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P.



Ans : [Board Term-2 OD 2017]

We have $\angle OPQ = \angle OQP$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

j164

Thus $TPQ = 90^\circ - 55^\circ = 35^\circ$

TWO MARKS QUESTIONS

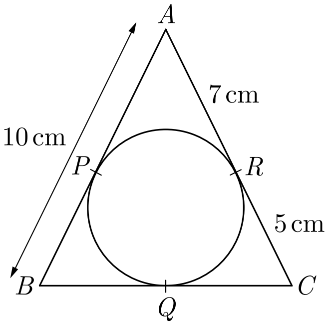
65. A circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively. If $AB = 10$ cm $AR = 7$ cm and $CR = 5$ cm, then find the length of BC

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.

Here a circle is inscribed in a ΔABC touching AB, BC and AC at P, Q and R respectively.

j165



Since, tangents drawn to a circle from an external point are equal,

$$AP = AR = 7 \text{ cm}$$

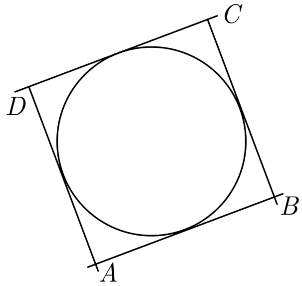
$$CQ = CR = 5 \text{ cm}$$

Now, $BP = (AB - AP) = 10 - 7 = 3$ cm

$$BP = BQ = 3 \text{ cm}$$

$$BC = BQ + QC = 3 + 5 = 8 \text{ cm}$$

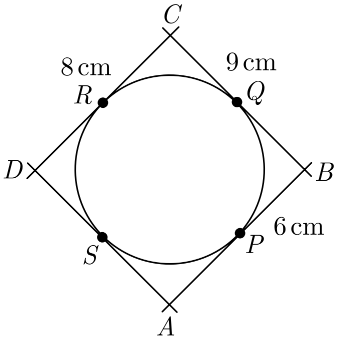
66. In figure, a circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm, then find the length of AD.



Ans : [Board 2020 Delhi Basic]

As per given information we have redrawn the figure below.

j166



Tangents drawn from an external point to a circle are equal in length.

Thus $AP = AS$ and let it be x.

Similarly, $BP = BQ$, $CQ = CR$ and $RD = DS$

Now $BP = AB - AP = 6 - x$

$$BP = BQ = 6 - x$$

$$CQ = BC - BQ = 9 - (6 - x) = 3 + x$$

Now, $CQ = CR = 3 + x$

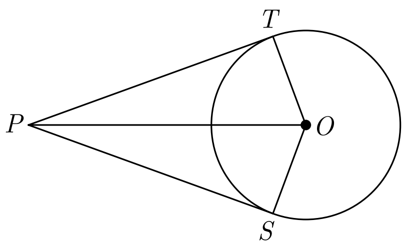
$$RD = CD - CR = 8 - (3 + x) = 5 - x$$

Now, $RD = DS = 5 - x$

$$AD = AS + SD = x + 5 - x = 5$$

Thus AD is 5 cm.

67. In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.



Ans : [Board Term-2 Foreign 2016]

We have $\angle SPT = 120^\circ$

j167

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

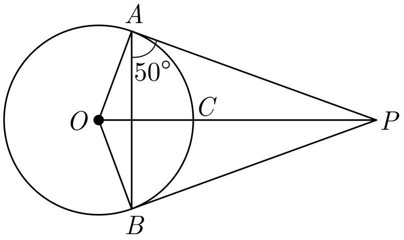
$$\frac{1}{2} = \frac{PS}{OP}$$

$$OP = 2PS$$ Hence proved.

68. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^o$, then find $\angle AOB$.

Ans : [Board Term-2 Delhi 2016]

As per the given question we draw the figure as below.



Since $PA \perp OA$, $\angle OAP = 90^\circ$

j168

$$\angle OAB = \angle OAP - \angle BAP$$

$$= 90^o - 50^o = 40^o$$

Since OA and OB are radii, we have

$$\angle OAB = \angle OBA = 40^o$$

Now

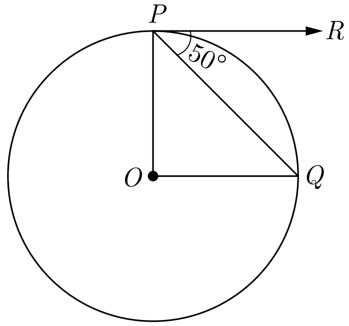
$$\angle AOB + \angle OAB + \angle OBA = 180^o$$

$$\angle AOB + 40^o + 40^o = 180^o$$

$$\angle AOB = 180^o - 80^o = 100^o$$

Hence $\angle AOB = 100^o$

69. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, find $\angle POQ$.



Ans : [Board Term-2, 2012]

We have $\angle RPQ = 50^\circ$

Since $\angle OPQ + \angle QPR$ is right angle triangle,

j169

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Since, $OP = OQ$ because of radii of circle, we have

$$\angle OPQ = \angle OQR = 40^\circ$$

In ΔPOQ we have

$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$

$$= 180^\circ - (40^\circ + 40^\circ)$$

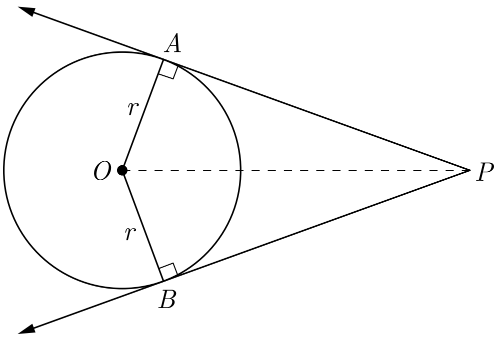
$$= 100^\circ$$

70. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Ans : [Board 2020 OD Basic, 2018]

Consider a circle of radius r and centre at O as shown in figure below. Here we have drawn two tangent from P at A and B. We have to prove that

$$AP = PB$$



We join OA, OB and OP. In ΔPAO and ΔPBO, OP

is common and $OA = OB$ radius of same circle.

Since radius is always perpendicular to tangent, at point of contact,

$$\angle OAP = \angle OBP = 90^\circ$$

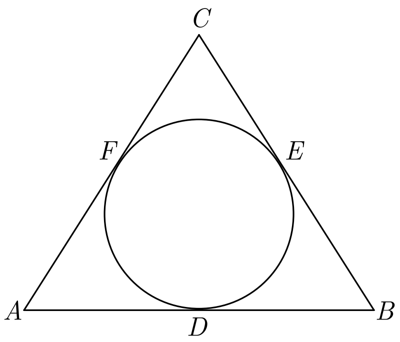
j170

Thus $\quad \Delta PAO \cong \Delta PBO.$

and hence, $\quad AP = BP$

Thus length of 2 tangents drawn from an external point to a circle are equal.

71. In the given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF.



Ans : [Board Term-2 Delhi 2016]

Since AF and AD are tangent of the circle, $AF = AD$

Let $\quad AF = AD = x$

Now $\quad DB = AB - AD = 12 - x$

Since BD and BE are tangent of the circle, $BD = BE$

Thus $\quad BE = BD = 12 - x$

Now $\quad CE = CB - BE = 8 - (12 - x)$

Since CF and CE are tangent of the circle, $CF = CE$

Thus $\quad CF = CE = 8 - (12 - x)$ cm

But $\quad AC = CF + FA$

Substituting values we have

$$10 = 8 - (12 - x) + x$$

$$10 = 2x - 4$$

$$2x = 10 + 4 = 14$$

$$x = 7$$

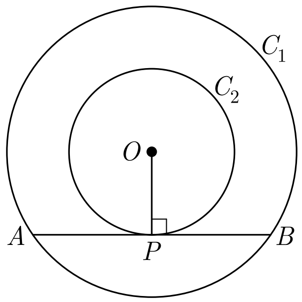
j171

Thus $AD = 7$ cm, $BE = 5$ cm, $CF = 3$ cm

72. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle is bisected at the point of contact.

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



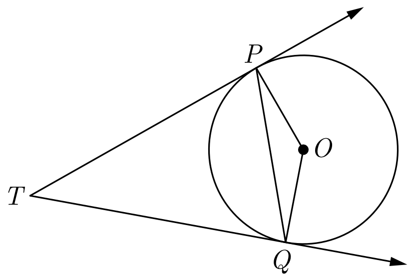
j172

Since OP is radius and APB is tangent, $OP \perp AB$.

Now for bigger circle, O is centre and AB is chord such that $OP \perp AB$.

Thus OP bisects AB.

73. In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



Ans : [Board Term-2 Delhi 2016]

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

Thus $\quad \angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

$$\angle POQ + \angle PTQ = 180^\circ$$

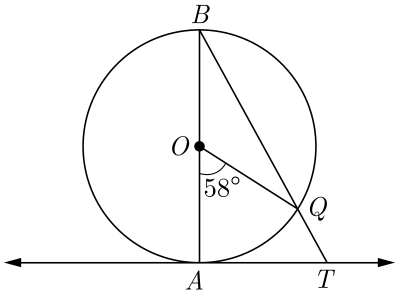
$$\angle PTQ = 180^\circ - \angle POQ$$

$$= 180^\circ - 60^\circ = 120^\circ$$

j173

Thus $\angle PTQ = 120^\circ$

74. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$.



Ans : [Board Term-2, 2015]

We have $\angle AOQ = 58°$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58° = 29°$$

j174

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90°$

$$\angle ABQ = \angle ABT$$

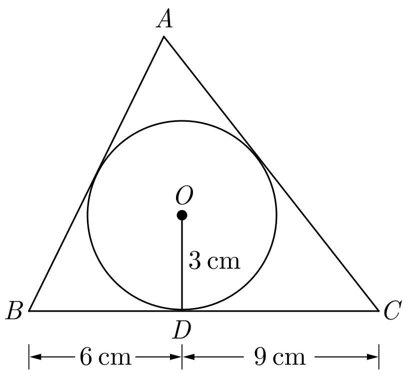
Now in ΔBAT,

$$\angle ATB = 90° - \angle ABT$$

$$= 90° - 29° = 61°$$

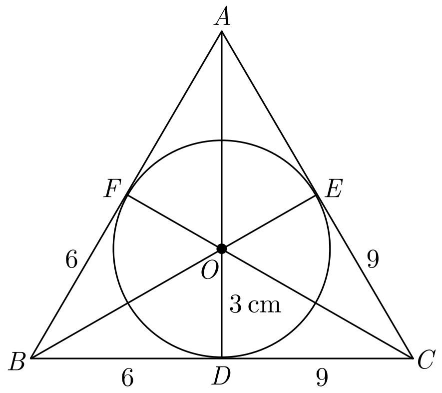
Thus $\angle ATQ = \angle ATB = 61°$

75. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.



Ans : [Board Term-2 OD 2015]

We redraw the given circle as shown below.



Since tangents from an external point to a circle are equal,

$$AF = AE$$

$$BF = BD = 6 \text{ cm}$$

$$CE = CD = 9 \text{ cm}$$

j175

Let $AF = AE = x$

Now $AB = AF + FB = 6 + x$

$$AC = AE + EC = x + 9$$

$$BC = 6 + 9 = 15 \text{ cm}$$

Perimeter of $\Delta\,ABC$,

$$p = 15 + 6 + x + 9 + x$$

$$= 30 + 2x$$

Now area, $\Delta\,ABC = \frac{1}{2}rp$

Here $r = 3$ is the radius of circle. Substituting all values we have

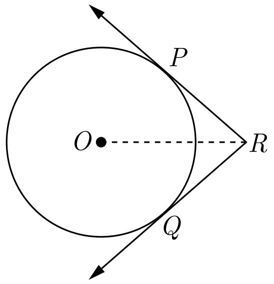
$$54 = \tfrac{1}{2} \times 3 \times (30 + 2x)$$

$$54 = 45 + 3x$$

or $$x = 3$$

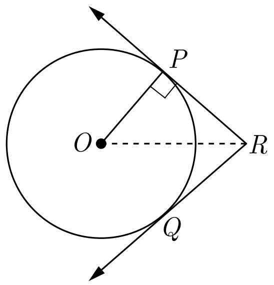
Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

76. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans : [Board Term-2 OD 2015]

We redraw the given figure by joining O to P as shown below.



$$\angle PRO = \tfrac{1}{2}\angle PRQ$$

$$= \frac{120^\circ}{2} = 60^\circ$$

Here $\Delta\,OPR$ is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO$$

$$= 90^\circ - 60^\circ = 30^\circ$$

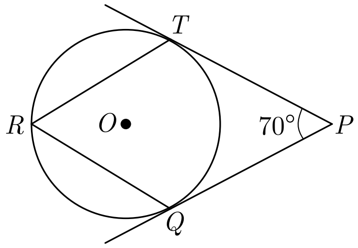
Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

$$OR = 2PR = PR + PR$$

Since $PR = QR$,

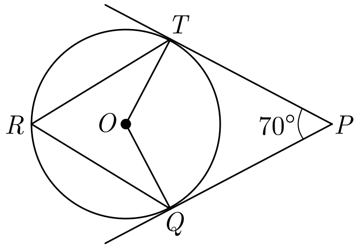
$$OR = PR + QR$$ Hence Proved

77. In figure, O is the centre of a circle. PT are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$.



Ans : [Board Term-2 Foreign 2015]

We redraw the given figure by joining O to T and Q as shown below.



Here angle $\angle TOQ$ and $\angle TPQ$ are supplementary angle.

Thus $$\angle TOQ = 180^\circ - \angle TPQ$$

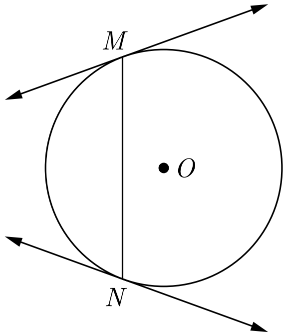
$$= 180^\circ - 70^\circ = 110^\circ$$

Since angle $\angle TRQ$ and $\angle TOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle TRQ = \frac{1}{2}\angle TOQ$$

$$= \frac{1}{2} \times 110^\circ = 55^\circ$$

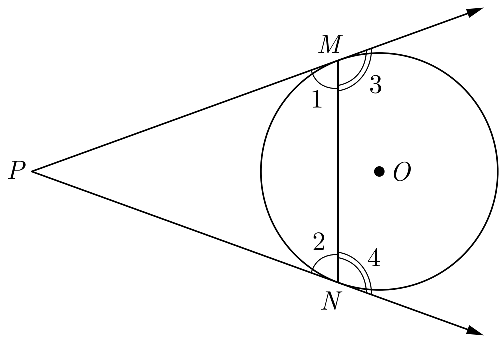
78. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Ans : [Board Term-2 Delhi 2015]

We redraw the given figure by joining M and N to P as shown below.

j178



Since length of tangents from an external point to a circle are equal,

$$PM = PN$$

Since angles opposite to equal sides are equal,

$$\angle 1 = \angle 2$$

Now using property of linear pair we have

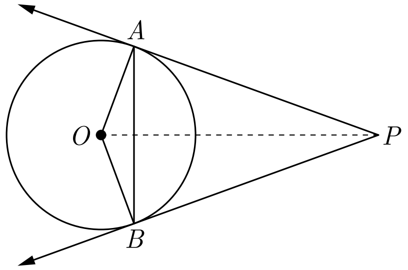
$$180^o - \angle 1 = 180^o - \angle 2$$

$$\angle 3 = \angle 4$$ Hence Proved

79. Two tangents PA and PB are drawn from an external point P to a circle inclined to each other at an angle of 70^o, then what is the value of $\angle PAB$?

Ans : [Board Term-2, 2012]

As per question we draw the given circle as shown below.



j179

Here angle $\angle AOB$ and $\angle APB$ are supplementary angle.

Thus $\angle AOB = 180^\circ - \angle APB$

$$= 180^\circ - 70^\circ = 110^\circ$$

OA and OB are radius of circle and equal in length, thus angle $\angle OAB$ and $\angle OBA$ are also equal. Thus in triangle ΔOAB we have

$$\angle OBA + \angle OAB + \angle AOB = 180^\circ$$

$$\angle OAB + \angle OBA = 180^\circ - \angle AOB$$

$$2\angle OAB = 180^\circ - 110^\circ = 70^\circ$$

$$\angle OAB = 35^\circ$$

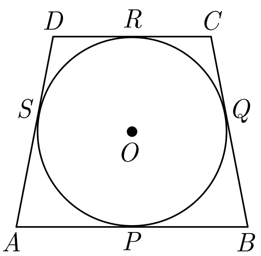
Since OA is radius and AP is tangent at A, $OA \perp AP$

$$\angle OAP = 90^o$$

Now $\angle PAB = \angle OAP - \angle OAB$

$$= 90^o - 35^o = 55^o$$

80. In Figure a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD, and DA touch the circle at the points P, Q, R and S respectively. Prove that. $AB + CD = BC + DA$.



Ans : [Board Term-2 OD 2016]

Since length of tangents from an external point to a circle are equal,

j180

At A, $AP = AS$ (1)

At B $BP = BQ$ (2)

At C $CR = CQ$ (3)

At D $DR = DS$ (4)

Adding eqn. (1), (2), (3), (4)

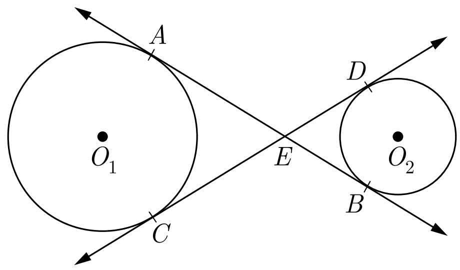
$$AP + BP + DR + CR = AS + DS + BQ + CQ$$

$$AP + BP + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Hence Proved

81. In Figure, common tangents AB and CD to the two circle with centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Ans : [Board Term-2 OD 2014]

Since EA and EC are tangents from point E to the circle with centre Q_1

$$EA = EC \quad ...(1)$$

and EB and ED are tangents from point E to the circle with centre O_2

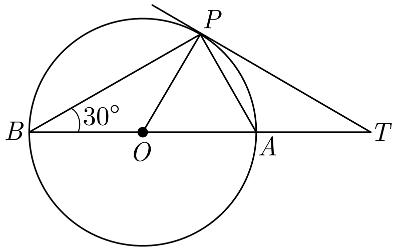
$$EB = ED \quad (2)$$

Adding eq (1) and (2) we have

$$EA + BE = CE + ED$$

$$AB = CD$$ Hence Proved

82. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^0$, what is the measure of $\angle PTA$?



Ans : [Board Term-2, 2012]

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^0$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^0$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^0$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

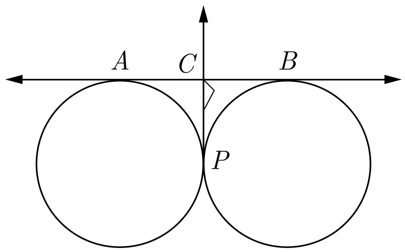
Substituting $\angle OPT = 90^0$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$

$$= 180^\circ - 150^\circ = 30^\circ$$

Thus $\angle PTA = \angle PTO = 30^\circ$

83. In the given figure, if $BC = 4.5$ cm, find the length of AB.



Ans : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

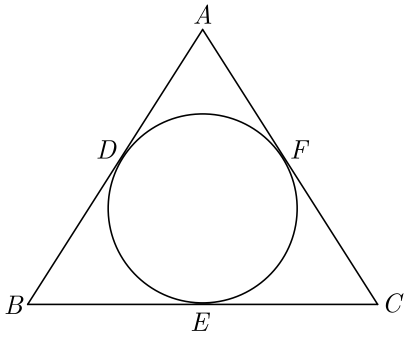
$$CB = CP = 4.5 \text{ cm}$$

and $CA = CP$

Now
$$AB = AC + CB$$
$$= CP + CP = 2CP$$
$$= 2 \times 4.5 = 9 \text{ cm}$$

j183

84. In the given figure, if $AB = AC$, prove that $BE = CE$.



Ans : [Board Term-2 OD 2017]

Since tangents from an external point to a circle are equal,

j184

$$AD = AF \quad (1)$$
$$BD = BE \quad (2)$$
$$CE = CF \quad (3)$$

From $AB = AC$ we have

$$AD + DB = AF + FC$$

or $DB = FC$ $(AD = AF)$

From eq (2) and (3) we have

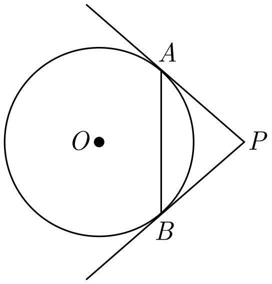
$$BE = EC$$ Hence Proved

85. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Ans : [Board Term-2 OD 2017]

As per question we draw figure shown below.

j185



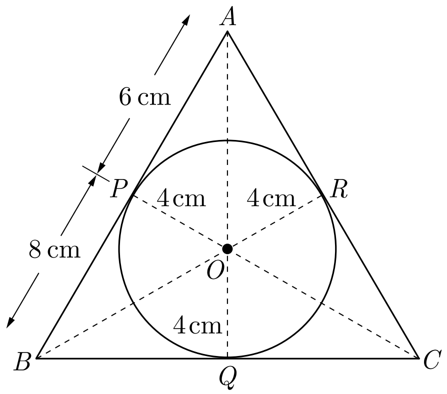
Since length of tangents from an external point to a circle are equal,

$$PA = PB$$

Since angles opposite to equal sides are equal,

$$\angle PAB = \angle PBA$$

86. In Figure the radius of incircle of ΔABC of area 84 cm^2 and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Ans : [Board Term-2 Delhi 2012, 2014, OD Compt. 2017]

Since length of tangents from an external point to a circle are equal,

At A, $AP = AR = 6$ cm (1)

At B, $,BP = BQ = 8$ cm (2)

At C, $CR = CQ = x$ (3)

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$
$$= 6 + 8 + 8 + x + x + 6$$
$$= 28 + 2x$$

Now area $\Delta ABC = \frac{1}{2} rp$

j186

Here $r = 4$ is the radius of circle. Substituting all values we have

$$84 = \tfrac{1}{2} \times 4 \times (28 + 2x)$$

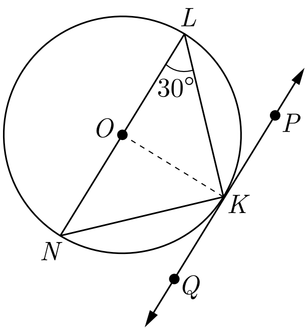
$$84 = 56 + 4x$$

$$21 = 14 + x \Rightarrow x = 7$$

Thus $AC = AR + RC = 6 + 7 = 13$ cm

$BC = BQ + QC = 8 + 7 = 15$ cm

87. In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^o$, find $\angle PKL$.



Ans : [Board Term-2 OD Compt 2017]

Since OK and OL are radius of circle, thus

j187

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^o$$

Tangent is perpendicular to the end point of radius,

$$\angle OKP = 90^o \qquad \text{(Tangent)}$$

Now $\angle PKL = \angle OKP - \angle OKL$

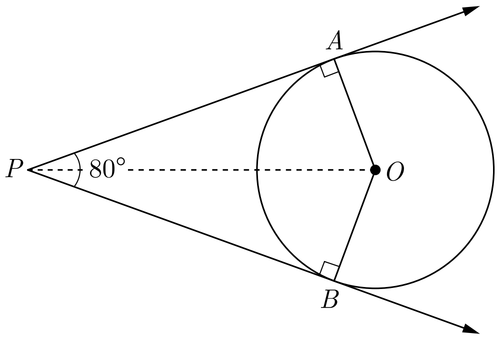
$$= 90^o - 30^o = 60^o$$

THREE MARKS QUESTIONS

88. If tangents PA and PB drawn from an external point P to a circle with centre O are inclined to each other at an angle of 80°, then find $\angle POA$.

Ans : [Board 2020 Delhi Basic]

As per given information we have drawn the figure below.



Since PA and PB are the tangents, PO will be angle bisector of $\angle P$

Hence, $\angle APO = 40^\circ$

Now, in ΔAPO, $\angle PAO$ is 90° because this is angle between radius and tangent.

Now $\angle PAO + \angle APO + \angle POA = 180^\circ$

$$90^\circ + 40^\circ + \angle POA = 180^\circ$$

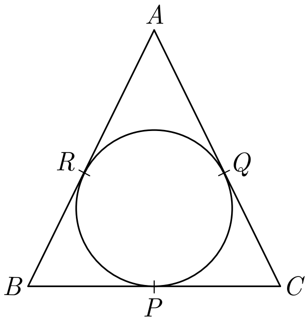
$$\angle POA = 50^\circ$$

j188

89. An isosceles triangle ABC, with $AB = AC$, circumscribes a circle, touching BC at P, AC at Q and AB at R. Prove that the contact point P bisects BC.

Ans : [Board 2020 OD Basic]

As per given information we have drawn the figure below.



j189

Since, the tangents drawn from externals points are equal,

$$AR = AQ$$

$$BR = BP$$

$$CP = CQ$$

Now we have, $AB = AC$

$$AR + BR = AQ + CQ$$

$$AR + BP = AQ + CP$$

$$AQ + BP = AQ + CP$$

$$BP = CP$$

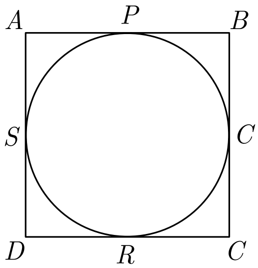
Hence, the point of contact P bisects BC.

90. Prove that the rectangle circumscribing a circle is a square.

Ans : [Board 2020 SQP Standard]

We have a rectangle $ABCD$ circumscribe a circle which touches the circle at P, Q, R, S. We have to prove that $ABCD$ is a square.

As per given information we have drawn the figure below.



Since tangent drawn from an external point to a circle are equals,

$$AP = AS$$

$$PB = BQ$$

j190

$$DR = DS$$

$$RC = QC$$

Adding all above equation we have

$$AP + PB + DR + RC = AS + SD + BQ + QC$$

$$AB + CD = AD + BC$$

Since $ABCD$ is rectangle, $AB = CD$ and $AD = BC$,

Thus $$2AB = 2BC$$

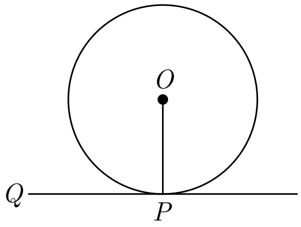
$$AB = BC$$

Since adjacent sides are equal are equal. So, $ABCD$ is a square.

91. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Ans : [Board 2020 Delhi Basic]

Given, a circle with centre O and tangent AB at P. We take a point Q on the tangent AB and join OQ meeting the circle at R.



To prove that $OP \perp AB$, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly $$OP = OR \quad \text{(radius)}$$

$$OQ = OR + RQ$$

$$OQ > OR$$

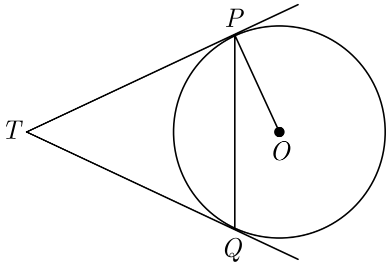
j191

$$OQ > OP$$

Thus OP is shorter than any other segment joining O to any other point of AB and shortest line is perpendicular.

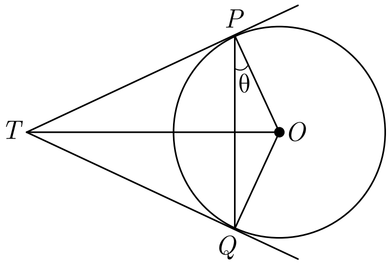
Thus $OP \perp AB$ Hence Proved

92. In figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Ans : [Board 2020 Delhi Standard]

We redraw the given figure as shown below.



j192

Let $\angle OPQ$ be θ, then

$$\angle TPQ = 90^\circ - \theta$$

Since, $TP = TQ$, due to opposite angles of equal sides we have

$$\angle TQP = 90^\circ - \theta$$

From angle sum property of a triangle we can write,

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$

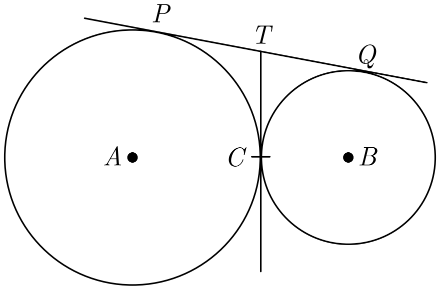
$$90^\circ - \theta + 90^\circ - \theta + \angle PTQ = 180^\circ$$

$$\angle PTQ = 180^\circ - 180^\circ + 2\theta$$

$$\angle PTQ = 2\theta$$

Hence, $\angle PTQ = 2\angle OPQ$

93. In given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Ans : [Board 2020 OD Basic, 2020 Delhi Standard]

Here PT and TC are the tangents of circle A from extended point, thus

$$PT = TC$$

Here TQ and TC are the tangents of circle B from extended point, thus

$$QT = TC$$

j193

Thus, $PT = QT$

Now, $PQ = PT + TQ$

$$= PT + PT$$

$$= 2PT$$

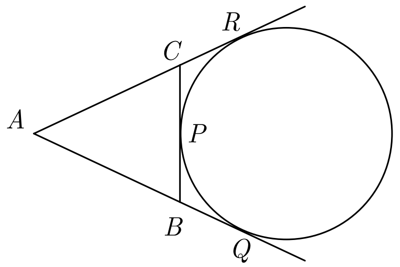
Thus $\frac{1}{2}PQ = PT$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q.

94. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$

Ans : [Board 2020 OD Standard, 2016]

As per given information in question we have drawn the figure below,



j194

From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $BQ = BP$

From the point A, $AQ = AR$

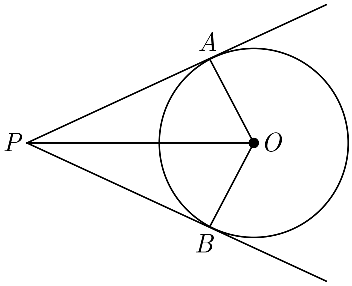
From the point C, $CP = CR$

Now

$$AB + BC + CA = (AQ - BQ) + (BP + PC) + (AR - CR)$$
$$= (AQ - BQ) + (BQ + CR) + (AQ - CR)$$
$$= 2AQ$$

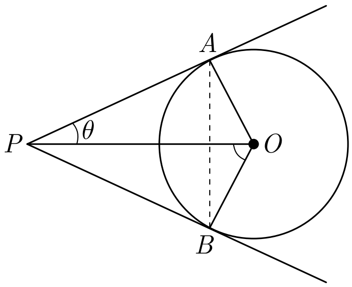
$$AQ = \frac{1}{2}(BC + CA + AB)$$ Hence proved.

95. In the given figure, OP is equal to the diameter of a circle with centre O and PA and PB are tangents. Prove that ABP is an equilateral triangle.



Ans : [Board Term-2, 2014]

We redraw the given figure by joining A to B as shown below.



Since OA is radius and PA is tangent at A, $OA \perp AP$.

Now in right angle triangle ΔOAP, OP is equal to diameter of circle, thus

$$OP = 2OA$$

$$\frac{OA}{OP} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

j195

Since PO bisect the angle $\angle APB$,

Hence, $\angle APB = 2 \times 30^\circ = 60^\circ$

Now, in ΔAPB,

$$AP = AB$$

$$\angle PAB = \angle PBA$$

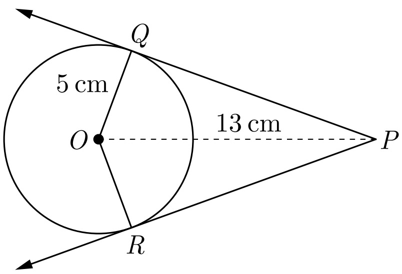
$$= \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

Thus ΔAPB is an equilateral triangle.

96. From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Ans : [Board Term-2, 2012]

As per the given question we draw the figure as below.



Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, ΔOQP is right angle triangle.

Now
$$PQ = \sqrt{OP^2 - OR^2}$$
$$= \sqrt{13^2 - 5^2}$$
$$= \sqrt{169 - 25}$$
$$= \sqrt{144} = 12 \text{ cm}$$

j196

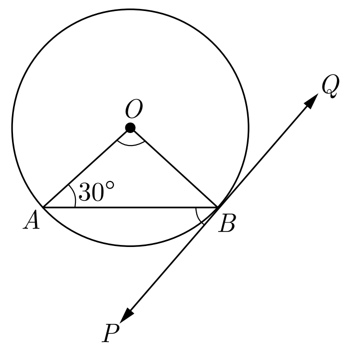
Area of triangle ΔOQP,

$$\Delta = \tfrac{1}{2}(OQ)(QP)$$
$$= \tfrac{1}{2} \times 12 \times 5 = 30$$

Area of quadrilateral $PQOR$,

$$2 \times \Delta POQ = 2 \times 30 = 60 \text{ cm}^2$$

97. In the figure, PQ is a tangent to a circle with centre O. If $\angle OAB = 30^\circ$, find $\angle ABP$ and $\angle AOB$.



Ans : [Board Term-2 Delhi 2014]

Here OB is radius and QT is tangent at B, $OB \perp PQ$

$$\angle OBP = 90^\circ$$

Here OA and OB are radius of circle and equal. Since angles opposite to equal sides are equal,

$$\angle OAB = \angle OBA = 30^\circ$$

Now $\angle AOB = 180^\circ - (30^\circ + 30^\circ)$

$$= 120^\circ$$

$$\angle ABP = \angle OBP - \angle OBA$$

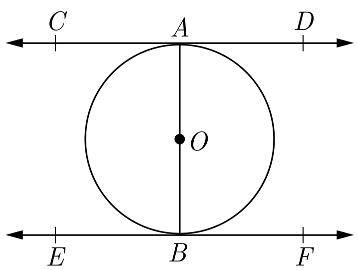
$$= 90^\circ - 30^\circ = 60^\circ$$

j197

98. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Ans : [Board 2020 Delhi Basic, 2017, 2014]

Let AB be a diameter of a given circle and let CD and RF be the tangents drawn to the circle at A and B respectively as shown in figure below.



j198

Here $AB \perp CD$ and $AB \perp EF$

Thus $\angle CAB = 90^\circ$ and $\angle ABF = 90^\circ$

Hence $\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$

Hence $\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles.

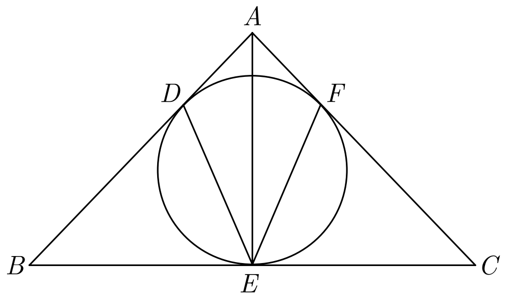
$CD || EF$ Hence Proved

99. In $\Delta ABD, AB = AC$. If the interior circle of ΔABC touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC.

Ans : [Board Term-2 Delhi 2014, 2012]

As per question we draw figure shown below.

j199



Since length of tangents from an external point to a circle are equal,

At A, $AF = AD$ (1)

At B $BE = BD$ (2)

At C $CE = CF$ (3)

Now we have $AB = AC$

$AD + DB = AF + FC$

$BD = FC$ $(AD = AF)$

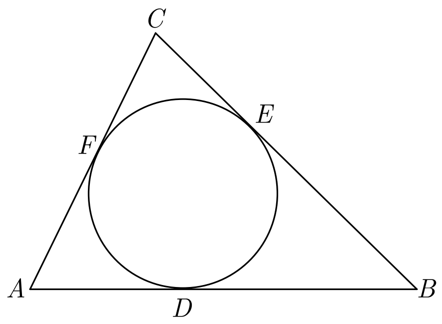
$BE = EC$ $(BD = BE, CE = CF)$

Thus E bisects BC.

100. A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Ans : [Board Term-2 Delhi 2013, 2012]

As per question we draw figure shown below.



j200

We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

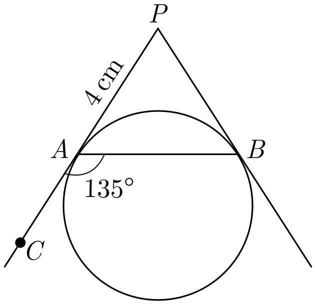
or $x = 3$

Now $AD = 3$ cm,

$BE = 10 - 3 = 7$ cm

and $CF = 8 - 3 = 5$

101. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135^0$. Find the length of chord AB.



Ans : [Board Term-2 OD 2017]

Since length of tangents from an external point to a circle are equal,

$$PA = PB = 4 \text{ cm}$$

Here $\angle PAB$ and $\angle BAC$ are supplementary angles,

$$\angle PAB = 180^\circ - 135^\circ = 45^\circ$$

Angle $\angle ABP$ and $= \angle PAB = 45^\circ$ opposite angles of equal sides, thus

$$\angle ABP = \angle PAB = 45^\circ$$

j201

In triangle ΔAPB we have

$$\angle APB$$

$$= 180^\circ - \angle ABP - \angle BAP$$

$$= 180^\circ - 45^\circ - 45^\circ = 90^\circ$$

Thus ΔAPB is a isosceles right angled triangle

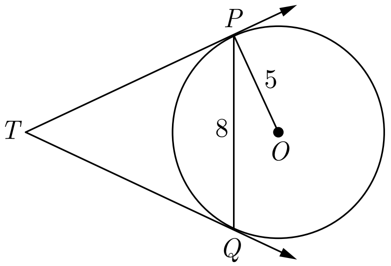
Now $$AB^2 = AP^2 + BP^2 = 2AP^2$$

$$= 2 \times 4^2 = 32$$

Hence $$AB = \sqrt{32} = 4\sqrt{2} \text{ cm}$$

FOUR MARKS QUESTIONS

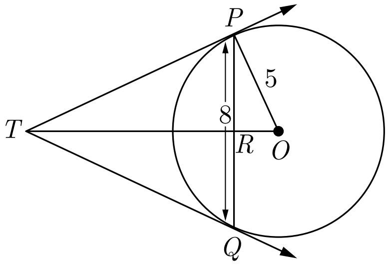
102. In Figure, PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Ans : [Board 2019 Delhi Standard]

We redraw the given figure as shown below. Here OT is perpendicular bisector of PQ,

j202



Since, OT is perpendicular bisector of PQ,

, $$PR = QR = 4 \text{ cm}$$

In right angle triangle ΔOTP and ΔPTR, we have

$$TP^2 = TR^2 + PR^2 \quad ...(1)$$

Also, $$OT^2 = TP^2 + OP^2$$

Substituting TP^2 from equation (1) we have

$$OT^2 = (TR^2 + PR^2) + OP^2$$

$$(TR + OR)^2 = TR^2 + PR^2 + OR^2$$

Now $$OR^2 = OP^2 - PR^2$$

$$= 5^2 - 4^2 = 3^2$$

Thus $$OR = 3 \text{ cm}$$

Thus substituting $OR = 3$ cm we have

$$(TR + 3)^2 = TR^2 + 4^2 + 5^2$$

$$TR^2 + 9 + 6TR = TR^2 + 16 + 25$$

$$6TR = 32$$

$$TR = \frac{16}{3}$$

Now, from (1), $$TP^2 = TR^2 + PR^2$$

$$= \left(\frac{16}{3}\right)^2 + 4^2$$

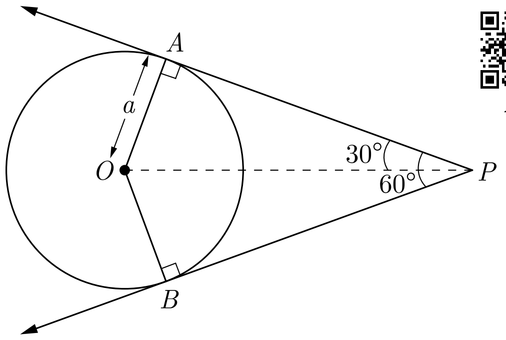
$$= \frac{256}{9} + 16 = \frac{400}{9}$$

$$TP = \frac{20}{3} \text{ cm}$$

103. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60°, then find the length of OP.

Ans : [Board 2020 SQP STD]

As per the given question we draw the figure as below.



j203

Tangents are always equally inclined to line joining the external point P to centre O.

$$\angle APO = \angle BPO = \frac{60^{\circ}}{2} = 30^{\circ}$$

Also radius is also perpendicular to tangent at point of contact.

In right $\Delta\, OAP$ we have,

$$\angle APO = 30^{\circ}$$

Now, $$\sin 30^{\circ} = \frac{OA}{OP}$$

Here OA is radius whose length is a, thus

$$\frac{1}{2} = \frac{a}{OP}$$

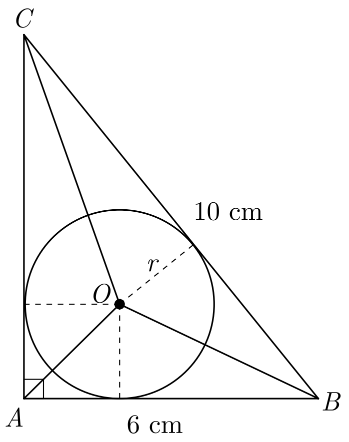
or $$OP = 2a$$

104. A right triangle ABC, right angled at A is circumscribing a circle. If $AB = 6$ cm and $BC = 10$ cm, find the radius r of the circle.

Ans : [Board 2020 Delhi Basic]

As per question we draw figure shown below.

j204



In triangle $\Delta\, ABC$,

$$AC = \sqrt{10^2 - 36^2} = 8 \text{ cm}$$

Area of triangle $\Delta\, ABC$,

$$\Delta\, ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

Here we have joined AO, BO and CO.

For area of triangle we have

$$\Delta\, ABC = \Delta\, OBC + \Delta\, OCA + \Delta\, OAB$$

$$24 = \tfrac{1}{2} rBC + \tfrac{1}{2} rAC + \tfrac{1}{2} rAB$$

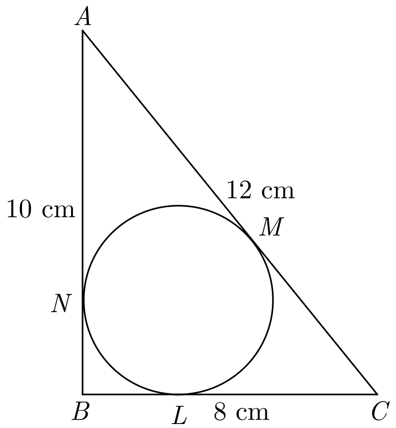
$$= \tfrac{1}{2} r(BC + AC + AB)$$

$$= \tfrac{1}{2} r(6 + 10 + 8) = 12r$$

or $$12r = 24$$

Thus $r = 2$ cm.

105. In figure, a circle is inscribed in a $\Delta\, ABC$ having sides $BC = 8$ cm, $AB = 10$ cm and $AC = 12$ cm. Find the length BL, CM and AN.



Ans : [Board 2019 Delhi Standard]

Tangents from external a point on a circle are always equal in length.

Let x be length of BL, then we have

j205

$$BL = x = BN$$

So, $$LC = MC = (8 - x)$$

and $$AN = AM = (10 - x)$$

Since, $$AC = 12$$

$$AM + MC = 12$$

$$(10 - x) + (8 - x) = 12$$

$$18 - 2x = 12 \Rightarrow x = 3$$

Hence, $BL = 3\text{ cm}$

$CM = 8 - 3 = 5\text{ cm}$

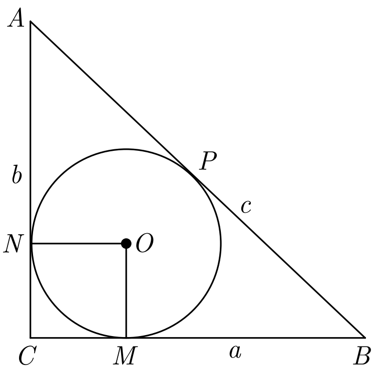
and $AN = 10 - 3 = 7\text{ cm}$

106. a, b and c are the sides of a right triangle, where c is the hypotenuse. A circle, of radius r, touches the sides of the triangle. Prove that $r = \frac{a+b-c}{2}$.

Ans : [Board Term-2 Delhi 2016]

As per question we draw figure shown below.

j206



Let the circle touches CB at M, CA at N and AB at P.

Now $OM \perp CB$ and $ON \perp AC$ because radius is always perpendicular to tangent

OM and ON are radius of circle, thus

$$OM = ON$$

CM and CN are tangent from C, thus

$$CM = CN$$

Therefore $OMCN$ is a square. Let

Let $OM = r = CM = CN = ON$

Since length of tangents from an external point to a circle are equal,

$$AN = AP, CN = CM \text{ and } BM = BP$$

Now taking $AN = AP$

$$AC - CN = AB - BP$$

$$b - r = c - BM$$

$$b - r = c - (a - r)$$

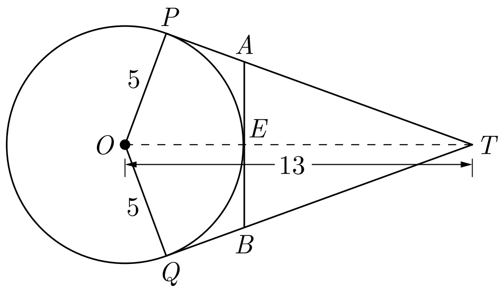
$$b - r = c - a + r$$

$$2r = a + b - c$$

$$r = \frac{a+b-c}{2}$$ Hence Proved.

107. In figure O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.

j207



Ans : [Board Term-2 Delhi 2016]

Here ΔOPT is right angled triangle because PT is tangent on radius OP.

Thus $PT = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE$

$= 13 - 5 = 8$ cm

Since length of tangents from an external point to a circle are equal,

Let $PA = AE = x$

Here ΔAET is right angled triangle because AB is tangent on radius OE.

In ΔAET, $TA^2 = TE^2 + EA^2$

$$(TP - PA)^2 = 8^2 + x^2$$

$$(12 - x)^2 = 64 + x^2$$

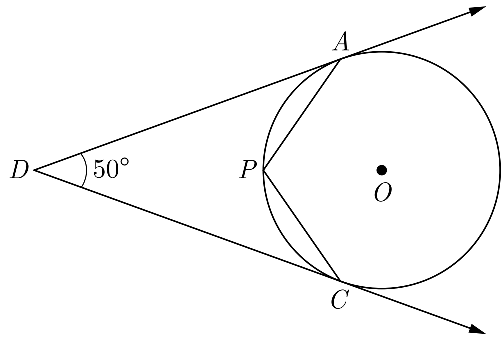
$$144 - 24x + x^2 = 64 + x^2$$

$$24x = 144 - 64 = 80$$

or, $x = 3.3$ cm.

Thus $AB = 2 \times x = 2 \times 3.3 = 6.6$ cm.

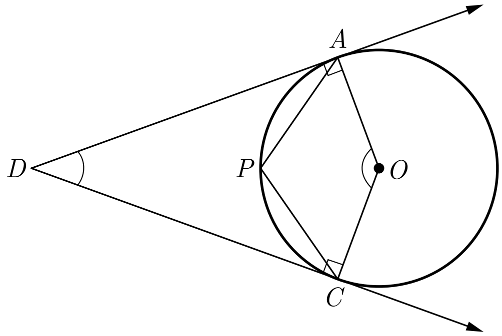
108. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50°$.



Ans : [Board Term-2, 2015]

We redraw the given figure by joining A and C to O as shown below.

j208



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$

$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$

$$230^\circ + \angle AOC = 360^\circ$$

$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

Now $\quad$ Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$

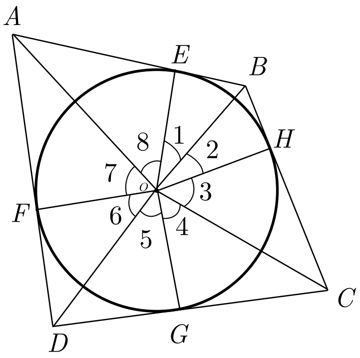
$$= \frac{1}{2} \times 230^\circ = 115^\circ$$

109. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans : [Board Term-2 Foreign 2017]

A circle centre O is inscribed in a quadrilateral as shown in figure given below.

j209



Since OE and OF are radius of circle,

$$OE = OF$$

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\quad \angle OEA = \angle OFA = 90^\circ$

Now in ΔAEO and ΔAFO,

$$OE = OF$$

$$\angle OEA = \angle OFA = 90^\circ$$

$$OA = OA \quad \text{(Common side)}$$

Thus $\quad \Delta AEO \cong \Delta AFO \quad$ (SAS congruency)

$$\angle 7 = \angle 8$$

Similarly, $\quad \angle 1 = \angle 2$

$$\angle 3 = \angle 4$$

$$\angle 5 = \angle 6$$

Since angle around a point is 360°,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

$$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360^\circ$$

$$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180^\circ$$

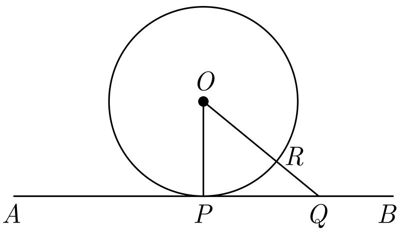
$$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$$

$$\angle AOB + \angle COD = 180^\circ$$ Hence Proved.

110. Prove that tangent drawn at any point of a circle perpendicular to the radius through the point contact.

Ans : [Board Term-2 OD 2016]

Consider a circle with centre O with tangent AB at point of contact P as shown in figure below



j210

Let Q be point on AB and we join OQ. Suppose it touch the circle at R.

We $\quad OP = OR \quad$ (Radius)

Clearly $\quad OQ > OR$

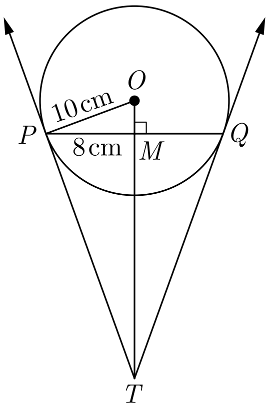
$$OQ > OP$$

Same will be the case with all other points on circle. Hence OP is the smallest line that connect AB and smallest line is perpendicular.

Thus $\quad OP \perp AB$

or, $\quad OP \perp PQ \quad$ Hence Proved

111. In figure, PQ, is a chord of length 16 cm, of a circle of radius 10 cm. the tangents at P and Q intersect at a point T. Find the length of TP.



Ans : [Board Term-2 Delhi 2014]

Here PQ is chord of circle and OM will be perpendicular on it and it bisect PQ. Thus $\Delta\, OMP$ is a right angled triangle.

We have $\quad OP = 10$ cm $\quad$ (Radius)

$\quad PM = 8$ cm $\quad (PQ = 16$ cm$)$

Now in $\Delta\, OMP, OM = \sqrt{10^2 - 8^2}$

$$= \sqrt{100-64} = \sqrt{36}$$

$$= 6 \text{ cm}$$

j211

Now $\quad \angle TPM + \angle MPO = 90^\circ$

Also, $\quad \angle TPM + \angle PTM = 90^\circ$

$$\angle MPO = \angle PTM$$

$$\angle TMP = \angle OMP = 90^\circ$$

Thus due to AA symmetry we have

$$\Delta\, TMP \sim \Delta\, PMO$$

Now $\quad \dfrac{TP}{PO} = \dfrac{MP}{MO}$

$$\frac{TP}{10} = \frac{8}{6}$$

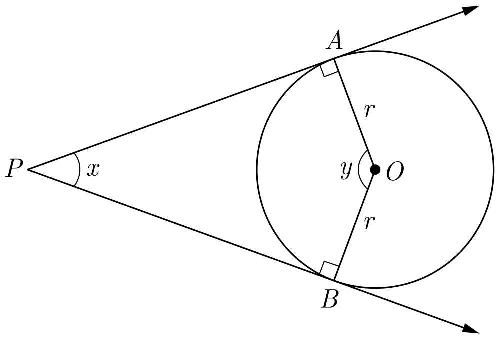
$$TP = \frac{80}{6} = \frac{40}{3}$$

Hence length of TP is $\frac{40}{3}$ cm.

112. Two tangents PA and PB are drawn from an external point P to a circle with centre O, such that $\angle APB = \angle x$ and $\angle AOB = y$. Prove that opposite angles are supplementary.

Ans : [Board Term-2, 2011]

As per question we draw figure shown below.



Now $OA \perp AP$ and $OB \perp BP$ because tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Thus $\quad \angle A = \angle B = 90^\circ$

Since, $AOBP$ is a quadrilateral,

j212

$$\angle A + \angle B + x + y = 360^\circ$$

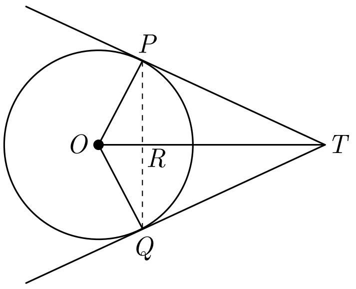
$$90^\circ + 90^\circ + x + y = 360^\circ$$

$$180 + x + y = 360^\circ$$

$$x + y = 180^\circ$$

Therefore opposite angle are supplementary.

113. In figure PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents drawn at P and Q intersect at T. Find the length of TP.



Ans : [Board Term-2 OD Compt 2017]

Since length of tangents from an external point to a circle are equal,

$$PT = QT$$

Thus ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTQ$.

Thus $OT \perp PQ$ and OT also bisects PQ.

j213

Thus $PR = PQ = \frac{8}{2} = 4$ cm

Since $\Delta\, OPR$ is right angled isosceles triangle,

$$OR = \sqrt{OP^2 - PR^2}$$

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= 3 \text{ cm}$$

Now, Let $TP = x$ and $TR = y$ then we have

$$x^2 = y^2 + 16 \qquad (1)$$

Also in $\Delta\, OPT$,

$$x^2 + (5)^2 = (y+3)^2 \qquad (2)$$

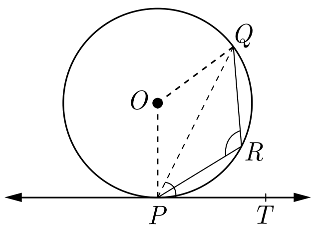
Solving (1) and (2) we get

$$y = \frac{16}{3} \text{ and } x = \frac{20}{3}$$

Hence, $TP = \frac{20}{3}$

114. In figure, PQ is a chord of a circle O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.

Ans : [Board Term-2 OD 2015, 2017]



j214

We have $\angle QPT = 60^\circ$

Here $\angle OPT = 90^\circ$ because of tangent at radius.

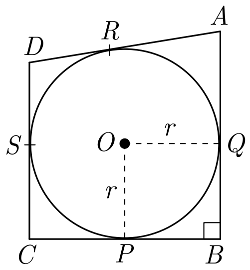
Now
$$\angle OPQ = \angle OQP$$
$$= \angle OPT - \angle QTP$$
$$= 90^\circ - 60^\circ = 30^\circ$$
$$\angle POQ = 180^\circ - (\angle OPQ + \angle OQP)$$
$$= 180^\circ - (30^\circ + 30^\circ)$$
$$= 180^\circ - 60^\circ = 120^\circ$$

Now Reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$$\angle PRQ = \frac{1}{2} \text{ Reflex } \angle POQ$$

$$= \frac{1}{2} \times 240^\circ = 120^\circ$$

115. In figure, a circle with centre O is inscribed in a quadrilateral $ABCD$ such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If $AB = 29$ cm, $AD = 23$ cm, $\angle B = 90^\circ$ and $DS = 5$ cm, then find the radius of the circle (in cm).



Ans : [Board Term-2, 2013]

Since length of tangents from an external point to a circle are equal,

j215

$$DR = DS = 5 \text{ cm}$$
$$AR = AQ$$
$$BQ = BP$$

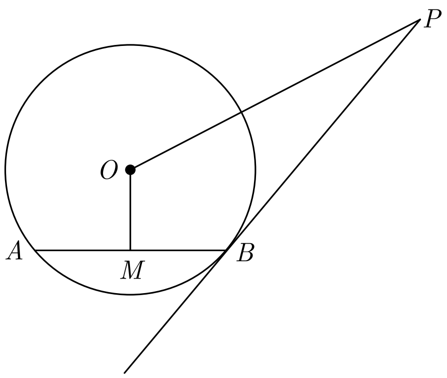
Now
$$AR = AD - DR$$
$$= 23 - 5 = 18 \text{ cm}$$
$$AQ = AR = 18 \text{ cm}$$
$$QB = AB - AQ$$
$$= 29 - 18 = 11 \text{ cm}$$
$$PB = QB = 11$$

Now $\angle OQB = \angle OPB = 90^\circ$ because radius is always perpendicular to tangent.

Thus $$OP = OQ = PB = BQ$$

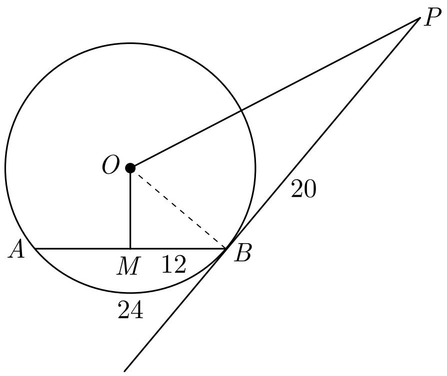
So, $POQB$ is a square. Hence, $r = OP = PB = 11$ cm

116. PB is a tangent to the circle with centre O to $B.AB$ is a chord of length 24 cm at a distance of 5 cm from the centre. It the tangent is length 20 cm, find the length of PO.



Ans : [Board Term-2 Delhi 2015]

We redraw the given figure by joining O to B as shown below.



Here ΔOMB right angled triangle because AB is chord and OM is perpendicular on it.

In right angled triangle ΔOMB we have,

j216

$$OB^2 = OM^2 + MB^2$$
$$= 5^2 + 12^2 = 13^2$$

Thus $$OB = 13$$

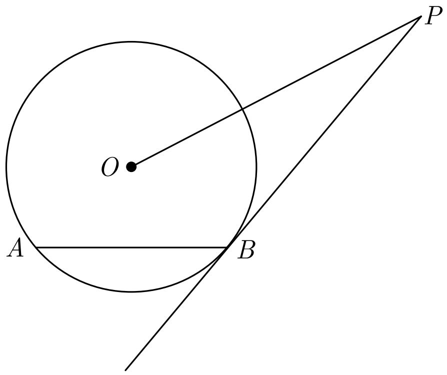
Here ΔOBP right angled triangle because PB is tangent on radius OB.

This in right angled triangle ΔOBP we have,

$$OP^2 = OB^2 + BP^2$$
$$= 13^2 + 20^2 = 569$$

Thus $$OP = \sqrt{569} = 23.85 \text{ cm}$$

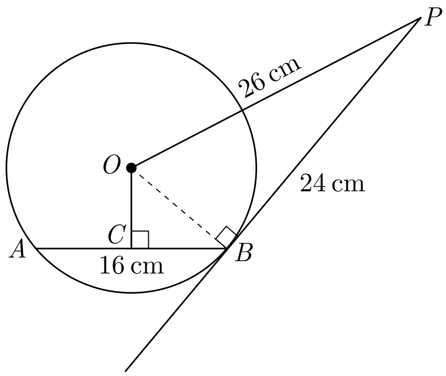
117. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find its distance from the centre.



Ans : [Board Term-2 Delhi 2014, 2012]

We redraw the given figure by joining O to B as shown below.

j217



Here we have drawn perpendicular OC on chord AB. Thus Triangle ΔOCB is also right angled triangle,

We have $PB = 24$ cm, $OP = 26$ cm.

Triangle ΔOPB is right angled triangle because PB is tangent at radius OB and $\angle OPB = 90^\circ$.

In right angled ΔOPB, we have

$$OB = \sqrt{OP^2 - BP^2}$$
$$= \sqrt{26^2 - 24^2}$$
$$= \sqrt{676 - 576} = \sqrt{100}$$
$$= 10 \text{ cm}$$

Since perpendicular drawn from the centre to a chord bisect it, we have

$$BC = \frac{1}{2}AB = \frac{16}{2} = 8 \text{ cm}$$

Now in $\Delta\, OBC$, $OC^2 = OB^2 - BC^2$

$$= 10^2 - 8^2 = 36$$

$$OC = 6 \text{ cm}$$

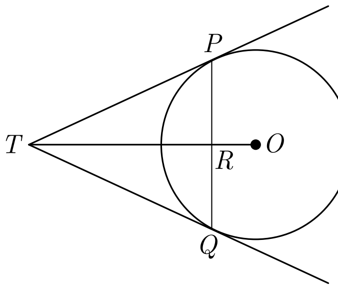
Thus distance of the chord from the centre is 6 cm.

118. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Ans : [Board Term-2 Delhi 2015]

A circle with centre O. Tangents TP and TQ are drawn from a point T outside a circle as shown in figure below.

j218



Since length of tangents from an external point to a circle are equal,

$$TP = TQ$$

Angle $\angle TPR$ and $\angle TQR$ are opposite angle of equal sides, thus

$$\angle TPR = \angle TQR$$

Now in $\Delta\, PTR$ and $\Delta\, QTR$

$$TP = TQ$$

$$TR = TR \quad \text{(Common)}$$

$$\angle TPR = \angle TQR$$

Thus $\Delta\, PTR \cong \Delta\, QTR$

and $PR = QR$

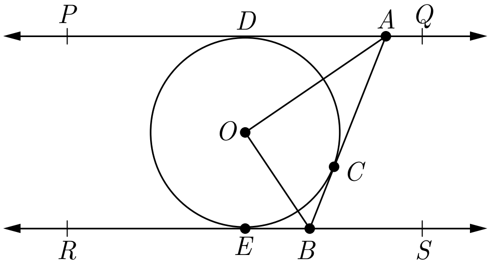
and $\angle PRT = \angle QRT$

But $\angle PRT + QRT = 180^\circ$ as PQ is line segment,

$$\angle PRT = \angle QRT = 90^\circ$$

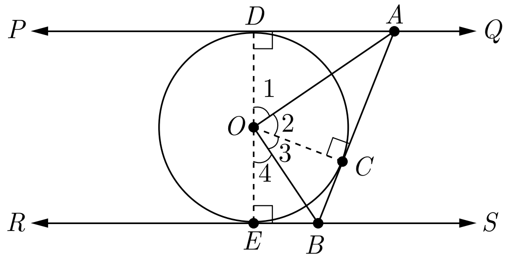
Therefore TR or OT is the right bisector of line segment PQ. Hence proved.

119. In Figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that $\angle AOB = 90^\circ$.



Ans : [Board 2019 OD STD, 2014, 2012]

We redraw the given figure as shown below.



In $\Delta\, DOA$ and $\Delta\, COA$, DA and AC are tangents drawn from common point,

Thus $DA = AC$

Due to angle between tangent and radius,

$$\angle ODA = \angle OCA = 90^\circ$$

j219

Due to radius of circle,

$$OD = OC$$

By SAS symmetry we have

$$\Delta\, DOA \cong \Delta\, COA$$

Hence, by CPCT, $\angle 1 = \angle 2$

i.e., $\angle DOA = \angle COA$ ...(1)

Similarly, by SAS

$$\Delta\, BOC = \Delta\, BOE$$

and by CPCT $\angle 3 = \angle 4$

i.e., $\angle COB = \angle BOE$ ...(2)

Now, angles on a straight line,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

From equation (1) and (2) we have

$$2\angle 2 + 2\angle 3 = 180^\circ$$

$$\angle 2 + \angle 3 = 90^\circ$$

i.e., $\angle AOC + \angle BOC = 90^\circ$

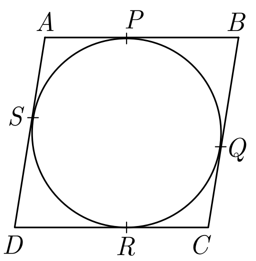
or $\angle AOB = 90^\circ$ Hence Proved

120. Prove that the parallelogram circumscribing a circle is a rhombus.

Ans : [Board 2020 Delhi STD, 2013, 2014]

Let $ABCD$ be the parallelogram.

$$AB = CD, AD = BC \quad (1)$$



j220

Since length of tangents from an external point to a circle are equal,

At A, $AP = AS$ (2)

At B $BP = BQ$ (3)

At C $CR = CQ$ (4)

At D $DR = DS$ (5)

Adding above 4 equation we have

$$AP + PB + CR + DR = AS + BQ + CQ + DS$$

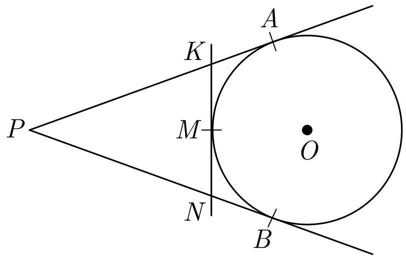
or, $AB + CD = AD + BC$

From (1) $2AB = 2AD$

or $AB = AD$

Thus $ABCD$ is a rhombus.

121. In given figure, PA and PB are tangents from a point P to the circle with centre O. At the point M, other tangent to the circle is drawn cutting PA and PB at K and N. Prove that the perimeter of $\Delta PNK = 2PB$.



Ans : [Board Term-2, 2012]

Since length of tangents from an external point to a circle are equal,

j221

$PA = PB$

$KM = KA$

$MN = BN$

Now $KN = KM + MN$

$= KA + BN$

Now perimeter of ΔPNK

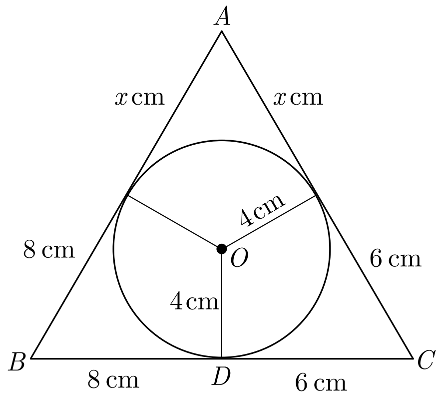
$p = PN + KN + PK$

$= PN + BN + KA + PK$

$= PB + PA$

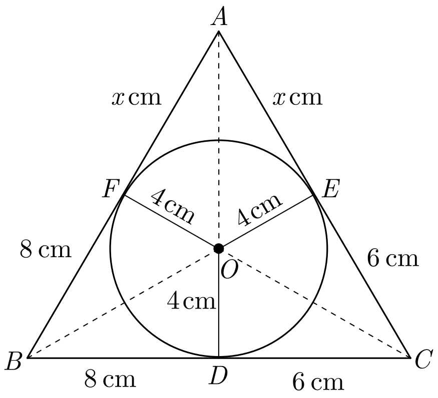
$= 2PB$ $(PA = PB)$

122. In the figure, the ΔABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find AB and AC.



Ans : [Board Term-2 Delhi 2014, 2012]

We redraw the given circle by joining AO, BO and CO shown in figure below. Let length of AF be x.



Since length of tangents from an external point to a circle are equal,

At A, $AF = AE = x$ (2)

At B $BF = BD = 8$ cm (3)

At C $CD = CE = 6$ cm (4)

Now $AB = x + 8$

$AC = x + 6$

$BC = 8 + 6 = 14$ cm

j222

Perimeter of circle

$$p = AB + BC + CA$$
$$= x + 8 + 14 + x + 6$$
$$= 2(x + 14)$$

Semi-perimeter of circle

$$s = \tfrac{1}{2}p = x + 14$$

Area or triangle ΔABC

$$\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48x^2 + 672x} \quad (1)$$

Area or triangle ΔABC,

$$\Delta ABC = \tfrac{1}{2}rp$$
$$= \tfrac{1}{2} \times 4 \times 2(x + 14)$$
$$= 4(x + 14) \quad (2)$$

From equation (1) and (2) we have

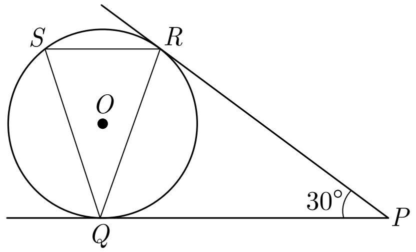
$$48x^2 + 672x = 16(x + 14)^2$$
$$48x(x + 14) = 16(x + 14)^2$$
$$3x = x + 14$$

or, $x = 7$

Thus $AC = 6 + 7 = 13$ cm

and AB $8 + 7 = 15$ cm.

123. In the figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$.



Ans : [Board Term-2 Delhi 2015]

Since length of tangents from an external point to a circle are equal,

$$PR = PQ$$

Now $\angle PRQ = \angle PQR = \dfrac{180° - 30°}{2}$

$$= \frac{150°}{2} = 75°$$

Since $SR \parallel QP$, $\angle SRQ$ and $\angle RQP$ are alternate angle,

$$\angle SRQ = \angle RQP = 75°$$

j223

Thus $SQ = RQ$

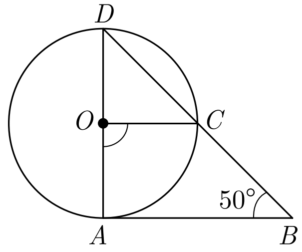
and $\angle RSQ = \angle SRQ = 75°$

In triangle ΔAQR,

$$\angle SQR + \angle QSR + \angle QRS = 180°$$
$$\angle SQR + 75° + 75° = 180°$$
$$\angle SQR = 180° - 150° = 30°$$

Thus $\angle SQR = 30°$.

124. In the given figure, AD is a diameter of a circle with centre O and AB is a tangent at A. C is a point on the circle such that DC produced intersects the tangent at B and $\angle ABC = 50^\circ$. Find $\angle AOC$.



Ans : [Board Term-2 2015]

Tangent drawn at any point of a circle is perpendicular to the radius through the point contact.

Therefore $\angle A = 90^\circ$

j224

Now in ΔDAB we have

$$\angle D + \angle A + \angle B = 180^\circ$$

$$\angle D + 90^\circ + 50^\circ = 180^\circ$$

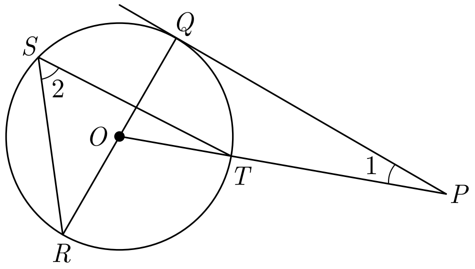
$$\angle D = 40^\circ$$

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle AOC = 2\angle ADC = 2\angle D$$

$$= 2 \times 40^\circ = 80^\circ$$

125. In figure PQ is a tangent from an external point P to a circle with centre O and OP cuts the circle at T and $\angle QOR$ is a diameter. If $\angle POR = 130^\circ$ and S is a point on the circle, find $\angle 1 + \angle 2$.



Here $\angle OQP = 90^\circ$ because radius is always perpendicular to tangent at point of contact.

Angle subtended at the centre is always 2 time of angle subtended at circumference by same arc. Thus

$$\angle 2 = \frac{1}{2}\angle TOR = \frac{1}{2}\angle POR$$

$$= \frac{1}{2} \times 130^\circ = 65^\circ$$

j225

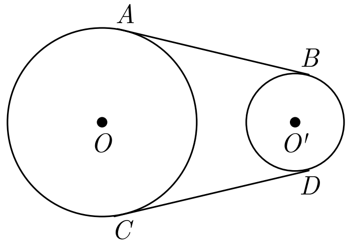
Now $\angle POQ = 180^\circ - 130^\circ = 50^\circ$

$$\angle 1 = 180^\circ - \angle OQP - \angle POQ$$

$$= 180^\circ - 90^\circ - 50^\circ = 40^\circ$$

Now $\angle 2 + \angle 1 = 65^\circ + 40^\circ = 105^\circ$

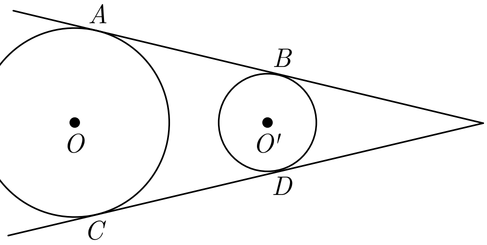
126. In the figure AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



Ans : [Board Term-2 Delhi Compt. 2017]

We redraw the given figure by extending AB and BD which intersect at P as shown in figure below

j226



Since length of tangents from an external point to a circle are equal,

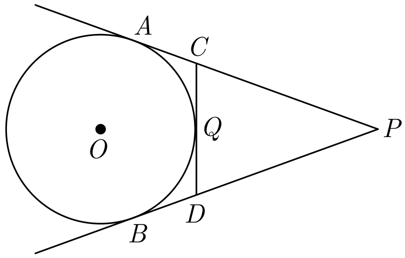
$$PA = PC$$

and $$PB = PD$$

Now, $$PA - PB = PC - PD$$

$$AB = CD$$ Hence Proved

127. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If $PA = 12$ cm, $QC = QD = 3$ cm, then find $PC + PD$.



Ans : [Board Term-2 Delhi Compt. 2017]

Since length of tangents from an external point to a circle are equal,

j227

$$CA = CQ = 3 \text{ cm}$$

$$DQ = DB = 3 \text{ cm}$$

and $$PB = PA = 12 \text{ cm}$$

$$PA + PB = PC + CA + PD + DB$$

$$PC + PD = PA - CA + PB - DB$$

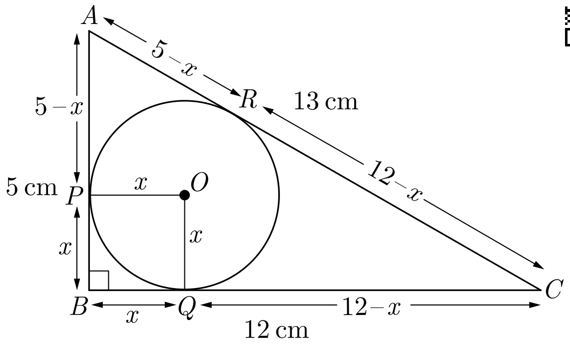
$$= 12 - 3 + 12 - 3 = 18 \text{ cm}$$

128. In a right angle $\Delta ABC, BC = 12$ cm and $AB = 5$ cm. Find the radius of the circle inscribed in this triangle.

Ans : [Board Term-2 Delhi 2014]

Let the radius of circle be x. As per given in question we draw the figure shown below.

j228



Since length of tangents from an external point to a circle are equal,

At A, $$AP = AR = 5 - x \quad (1)$$

At B $$BP = BQ = x \quad (2)$$

At C $$CR = CQ = 12 - x \quad (3)$$

Here, $AB = 5$ cm, $BC = 12$ cm and $\Delta B = 90^\circ$

Now $$AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

Now $$AC = AR + RC$$

$$13 = 5 - x + 12 - x$$

$$2x = 17 - 13 = 4$$

$$x = \frac{4}{2} = 2 \text{ cm}$$

Hence, radius of the circle is 2 cm.

CHAPTER 11

Constructions

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. To divide a line segment AB is the ratio $6:7$, a ray AX is drawn first such that $\angle BAX$ is an acute angle and then points $A_1, A_2, A_3, \ldots\ldots\ldots$ are located equal distances on the ray AX and the point B is joined with

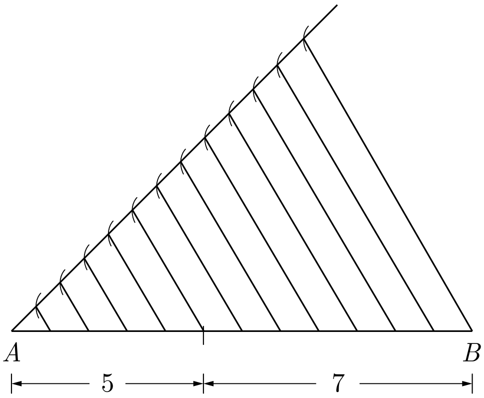
(a) A_{12} (b) A_{13}

(c) A_{10} (d) A_{11}

Ans : (b) A_{13}

The maximum number of points $= 5 + 7 = 12$

In this process, once line AX is drawn, it is divided into 12 equal parts using a pair of compasses. The points are marked from point a towards X. The last point is then joined to point B to form line XB. Lines are then drawn parallel to XB and passing through the points that were marked on AX. These lines can be drawn using set squares to ensure they are parallel. These parallel lines will divide line AB into 12 equal parts. So, to divide the line in the ratio 5:7, the first five portions will be taken and the last 7 left as shown in the attached figure.



2. The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as

(a) scale factors (b) length factor

(c) side factor (d) K-factor

Ans : (a) scale factors

The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.

3. To divide a line segment AB in the ratio $3:5$ first a ray AX is drawn so that $\angle BAX$ is an acute angle and then at equal distances points are marked on the ray AX such that the minimum number of these points is

(a) 8 (b) 9

(c) 10 (d) 11

Ans : (a) 8

Minimum number of points $= 3 + 5 = 8$

4. Given a triangle with side $AB = 8$ cm. To get a line segment $AB' = \frac{3}{4}$ of AB, it required to divide the line segment AB in the ratio.

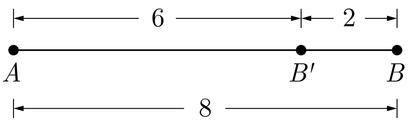
(a) 3 : 4 (b) 4 : 3

(c) 1 : 3 (d) 3 : 1

Ans : (d) 3 : 1

We have $AB = 8$ cm

$$AB' = \frac{3}{4} \text{ of } AB$$



$$= \frac{3}{4} \times 8 = 3 \times 2 = 6 \text{ cm}$$

and $BB' = AB - AB' = 8 - 6 = 2$

$$AB' : BB' = 6:2 = 3:1$$

Hence, the line segment AB should be divided in 3 : 1.

5. To divide a line segment AB in the ratio 3 : 4, we draw a ray AX, so that $\angle BAX$ is an acute angle and then mark the points on ray AX at equal distances such that the minimum number of these points is

(a) 3 (b) 4

(c) 7 (d) 10

Ans : (c) 7

Minimum number of these points $= 3 + 4 = 7$

6. To divide a line segment AB in the ratio 2 : 5, first a ray AX is drawn, so that $\angle BAX$ is an acute angle and then at equal distance points are marked on the ray AX such that the minimum number of these point is

(a) 2 (b) 5

(c) 4 (d) 7

Ans : (d) 7

We know that, to divide a line segment AB in the ratio $m:n$, first draw a ray AX which makes an acute $\angle BAX$ then, marked $m+n$ points at equal distance.

Here, $m = 2,\ n = 5$

Minimum number of these points $= 2 + 5 = 7$

7. To divide a line segment AB in ratio $m:n$ (m, n are positive integers), draw a ray AX to that $\angle BAX$ is an acute angle and the mark point on ray AX at equal distances such that the minimum number of these points is

(a) greater of m and n (b) $m+n$

(c) $m+n-1$ (d) mn

Ans : (b) $m+n$

To divide a line segment in the ratio $m:n$, the maximum number of the points to mark are $m+n$.

8. The sides of a triangle (in cm) are given below. In which case, the construction of triangle is not possible.

(a) 8, 7, 3 (b) 8, 6, 4

(c) 8, 4, 4 (d) 7, 6, 5

Ans : (c) 8, 4, 4

We know that, in a triangle sum of two sides of triangle is greater than the third side. Here, the sides of triangle given in option (c) does not satisfy this condition. So, with these sides the construction of a triangle is not possible.

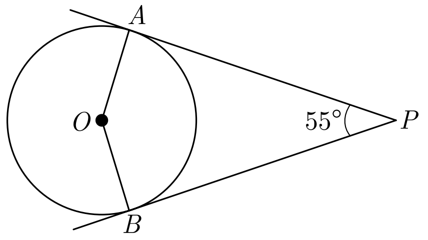
9. To draw a pair of tangents to a circle which are inclined to each other at an angle of 55°, it is required to draw tangents at the end points of these two radii of the circle, the angle between two radii is

(a) 105° (b) 70°

(c) 125° (d) 135°

Ans : (c) 125°

According to the question we can draw the following diagram.



From figure,

$$\angle AOB + \angle APB = 180^\circ$$

$$\angle AOB = 180^\circ - \angle APB$$

$$= 180^\circ - 55^\circ = 125^\circ$$

10. From the following ratios, a line segment cannot be devided into A ratio.

(a) $A \to \sqrt{5} : \frac{1}{\sqrt{5}}$ (b) $A \to \frac{1}{\sqrt{5}} : \frac{1}{\sqrt{5}}$

(c) $A \to \frac{2}{\sqrt{5}} : \frac{\sqrt{5}}{\sqrt{2}}$ (d) $A \to \frac{1}{5} : 1$

Ans : (c) $A \to \frac{2}{\sqrt{5}} : \frac{\sqrt{5}}{\sqrt{2}}$

Since,

a. (a) $\sqrt{5} : \frac{1}{\sqrt{5}} = 5:1$

b. (b) $\frac{1}{\sqrt{5}} : \frac{1}{\sqrt{5}} = 1:1$

c. (c) $\frac{2}{\sqrt{5}} : \frac{\sqrt{5}}{\sqrt{2}} = 2\sqrt{2}:5$

d. (d) $\frac{1}{5}:1 = 1:5$

Since, (a), (b) and (d) are the ratio of 2 integers. So, it is possible to divide a line segment into these points.

Fill in the Blank Questions

11. Two points on a line segment are marked such that the three parts they make are equal then we say that the two points .... the line segment.

Ans :

Trisect

12. Two circles are drawn with same centre then the .... circle have bigger radius.

Ans :

Outer

13. Only two .... can be drawn to a circle from an external point.

Ans : Tangents

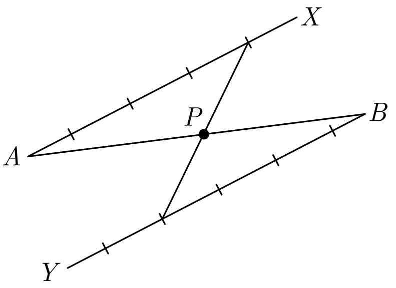
14. A curve made by moving one point at a fixed distance from another is called ....

Ans :

Circle

VERY SHORT ANSWER QUESTIONS

15. In given figure, in what ratio does P divides AB internally ?



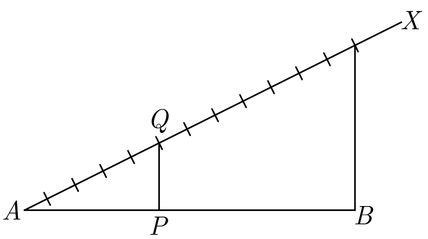
Ans : [Board Term-2, 2012]

Point P divide AB internally in the ratio 4:4 i.e. 1:1.

16. To divide a line segment AB in the ratio 5:7, first AX is drawn, so that $\angle BAX$ is an acute angle and then at equal distance, points are marked on the ray AX, find the minimum number of these points.

Ans : [Board Term-2 2012]

Minimum number of points marked on AX are $5+7=12$



17. To divide a line segment AB in the ratio 2:5, a ray AX is drawn such that $\angle BAX$ is acute. Then points are marked at equal intervals on AX. What is the minimum number of these points ?

Ans : [Board Term-2, 2012]

Minimum number of points marked on AX are $2+5=7$.

18. To divide the line segment AB in the ratio $2:3$, a ray AX is drawn such that $\angle BAX$ is acute, AX is then marked at equal intervals. Find minimum number of these marks.

Ans : [Board Term-2 2012]

Minimum number of points marked on AX are $2+3=5$.

19. To find a point P on the line segment $AB=6$ cm, such that $\frac{AP}{AB}=\frac{2}{5}$, in which ratio the line segment AB is divided.

Ans : [Board Term-2 2012]

The line segment AB is divided in the ratio $AP:PB=2:(5-2)=2:3$

20. A line Segment AB is divided at point P such that $\frac{PB}{AB}=\frac{3}{7}$, then find the ratio $AP:PB$.

Ans : [Board Term-2, 2012 Set (44)]

Here, $AB=7, PB=3$

Thus $AP = AB-PB=7-3=4$

$$AP:PB = 4:3$$

21. What is the ratio of division of the line segment AB by the point P from A ?

Ans : [Board Term-2 2012]

The ratio of division of the line segment AB by the point P from A is $AP:AB=3:5$.

22. In drawing a triangle, if $AB=3$ cm, $BC=2$ cm and $AC=6$ cm. What is the possibility that a triangle cannot be drawn.

Ans : [Board Term-2 2014]

When $AB+BC<AC$ triangle cannot be drawn.

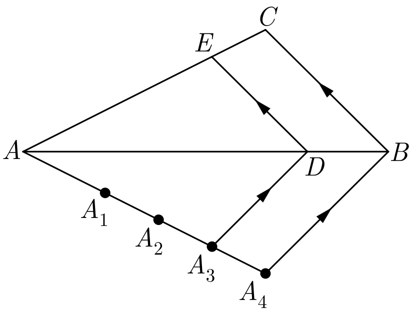
Here $3\text{ cm}+2\text{ cm}<6\text{ cm}$. Hence ΔABC can not be drawn.

23. When construction of a triangle similar to a given triangle in the scale factor $\frac{5}{3}$, then what is the nature of given triangle ?

Ans : [Board Term-2 2014]

Triangle is bigger than to original Δ.

24. In figure, ΔADE is constructed similar to ΔABC, write down the scale factor.



Ans : [Board Term-2 2012]

Scale factor is $\frac{3}{4}$.

25. Triangle PQR is constructed similar to triangle ABC with scale factor $\frac{2}{3}$. Find triangle PQR.

Ans : [Board Term-2 2011]

Triangle PQR is smaller to triangle ABC. Reduced scale factor figures are smaller in size.

26. Give three sides such that construction of a triangle is possible.

Ans : [Board Term-2 2011]

To construct a triangle sum of two sides of a triangle must be greater than largest side. Let the sides are 3 cm, 4 cm and 5 cm.

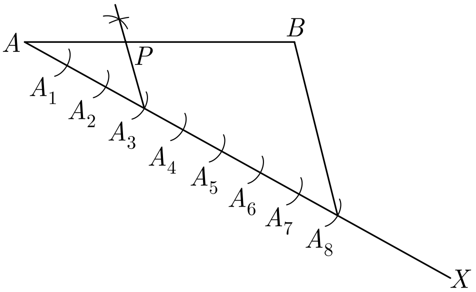
TWO MARKS QUESTIONS

27. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio $3:5$.

Ans : [Board Term-2, 2015]

Steps of Construction :

1. Draw a line segment AB of length 7 cm.
2. Draw any ray AX making an acute angle with AB.
3. Mark eight point $A_1, A_2, A_3, ..., A_8$ on AX such that $AA_1 = A_1A_2 = A_2A_3 = ..., A_7A_8$.
4. Join BA_8.
5. At point A_3, draw a line PA_3 parallel to BA_8. Hence $AP : PB = 3 : 5$

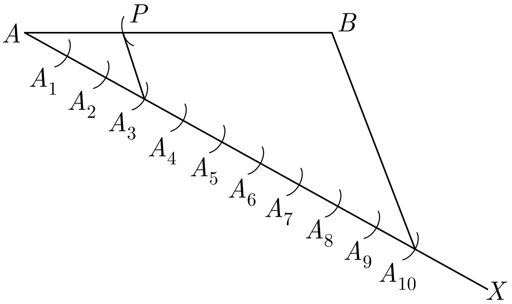


28. Draw a line segment of length 5 cm and divide it in the ratio $3:7$.

Ans : [Board Term-2 2015]

Step of Construction :

1. Draw a line segment AB of length 5 cm.
2. Draw any ray AX making on acute angle with AB.
3. Mark ten points $A_1, A_2, A_3, ...A_{10}$ on AX such that $AA_1 = A_1A_2 = ... = A_9A_{10}$.
4. Join BA_{10}.
5. At point A_3 draw a line PA_3 parallel to BA_{10}. Hence $AP : PB = 3 : 7$



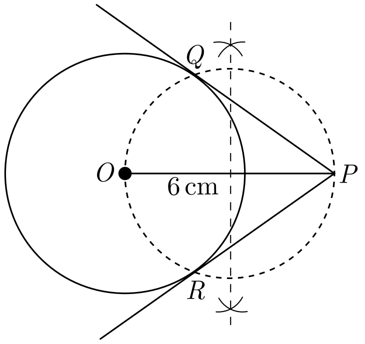
THREE MARKS QUESTIONS

29. Draw a circle of radius 3.5 cm. From a point P, 6 cm from its centre, draw two tangents to the circle.

Ans : [Board 2020 OD Standard]

Step of construction :

1. Draw a line segment OP of length 6 cm.
2. From the point O, draw a circle of radius = 3.5 cm.
3. Draw a perpendicular bisector of OP. Let M be the mid point of OP.
4. Taking M as centre and OM as radius draw a circle.
5. This circle intersects the given circle at Q and R.
6. Join PQ and PR, which are tangents to the circles.

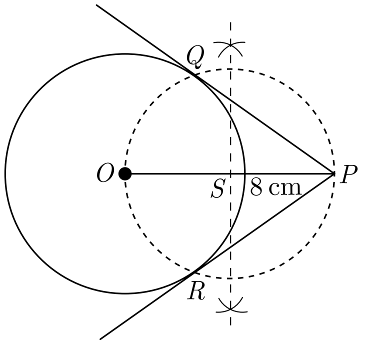


30. Construct a pair tangents PQ and PR to a circle of radius 4 cm from a point P outside the circle 8 cm away from the centre. Measure PQ and PR.

Ans : [Board Term-2 2014]

Steps of Construction :

1. Draw a line segment OP of length 8 cm.
2. Draw a circle with centre O and radius 4 cm.
3. Taking OP as diameter draw another circle which intersects the first circle at Q and R.
4. Join P to Q and P to R. On measuring, we get $PQ = PR = 5$ cm

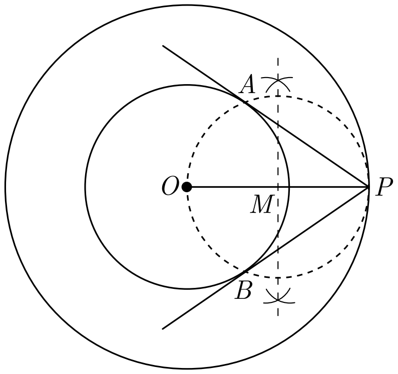


31. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm.

Ans : [Board Term-2, 2013]

Steps of Construction :

1. Draw a circle with centre O and radius 4 cm.
2. Draw another circle with centre O and radius 6 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.

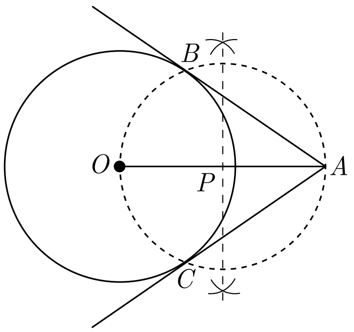


32. Draw a circle of radius 5 cm. Marks a point A which is 8 cm away from its centre O, construct the tangents AB and AC. Measure the lengths of AB and AC.

Ans :

Steps of Construction :

1. Draw a line segment OA of length 8 cm.
1. Draw a circle with centre O and radius 5 cm.
3. Taking OA as diameter draw another circle which intersects the given circle at B and C.
4. Join A to B and A to C. Thus AB and AC are required tangents.
5. $AB = AC = 6.2$ cm.



$\therefore$ AB and AC are required tangents.
$AB = AC = 6.2$ cm.

FOUR MARKS QUESTIONS

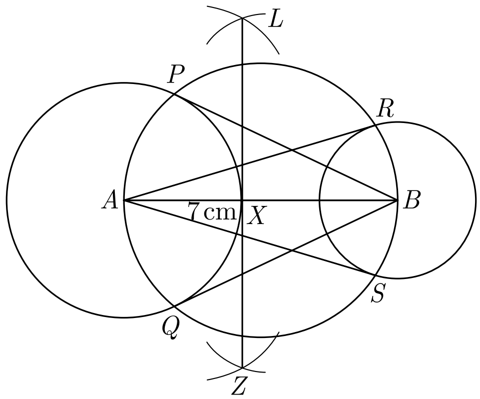
33. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Ans : [Board 2020 Delhi Standard]

Steps of construction :

1. Draw a line segment AB of length 7 cm.

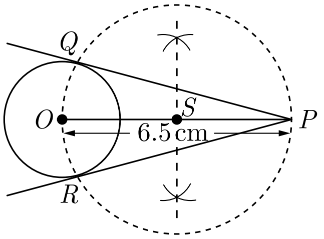
2. Draw a circle with A as centre and radius 3 cm.
3. Draw another circle with B as centre and radius 2 cm.
4. Draw another circle taking AB as diameter circle, which intersects first two circles at P and Q, R and S.
5. Join B to P, B to Q A to R and A to S. Hence, BP, BQ, AR and AS are the required tangents.



34. Draw a circle of radius 2 cm with centre O and take a point P outside the circle such that $OP = 6.5$ cm. From P, draw two tangents to the circle.

Ans : [Board 2020 OD Standard]

1. Draw a line segment OP of length 6.5 cm.
2. Draw a circle taking O as centre and radius 2 cm.
3. Taking OP as diameter draw another circle which intersects the first circle at Q and R.
4. Join P to Q and P to R. Hence PQ and PR are two tangents.

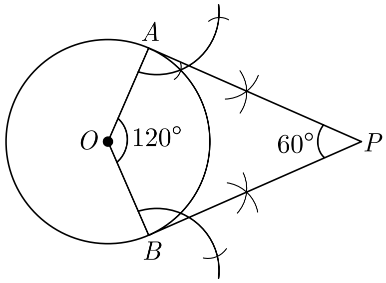


35. Draw two tangents to a circle of radius 4 cm, which are inclined to each other at an angle of 60°.

Ans : [Board 2020 OD Standard]

Step of construction :

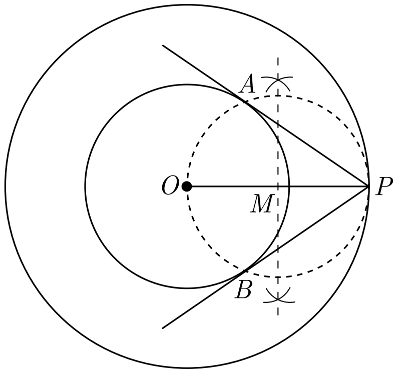
1. Draw a circle of radius 4 cm with O as centre.
2. Draw two radii OA and OB inclined to each other at an angle of 120°.
3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B. which meet at P.
4. PA and PB are the required tangents inclined to each other an angle of 60°.



36. Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

Ans : [Board 2019 OD Standard]

1. Draw a circle with centre O and radius 2 cm.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.



$$PA = \sqrt{5^2 - 2^2}$$

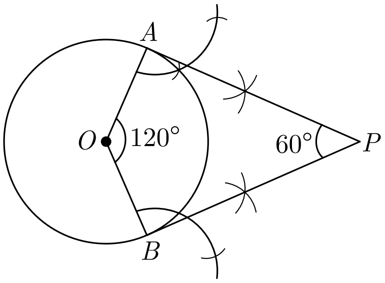
$$= \sqrt{21} = 4.6 \text{ cm}$$

37. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Ans : [Board Term-2 Foreign 2015, OD 2016]

Steps of Construction :

1. Draw a circle with centre O and radius 6 cm.
2. Draw two radii OA and OB inclined to each other at an angle of 120°.
3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B, which meet at P.
4. PA and PB are the required tangents inclined to each other an angle of 60°.



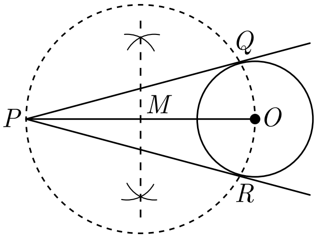
38. Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

Ans : [Board Term-2 Foreign 2015]

Steps of Construction :

1. Draw a line segment PO of length 7 cm.
2. Draw a circle with centre O and radius 3 cm.
3. Draw a perpendicular bisector of PO. Let M be the mid-point of PO.
4. Taking M as centre and OM as radius draw a circle. Let this circle intersects the given circle at the point Q and R.
5. Join PQ and PR. On measuring we get

$$PQ = PR = 6.3 \text{ cm.}$$

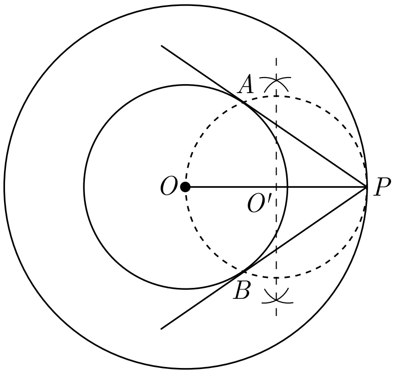


39. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Ans : [Foreign Set I 2017]

Steps of Construction :

1. Draw a circle with radius 3 cm and centre O.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on the circumference of outer circle and join O to P.
4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B.
5. Join A to P and B to P. AP and BP are the required tangents.

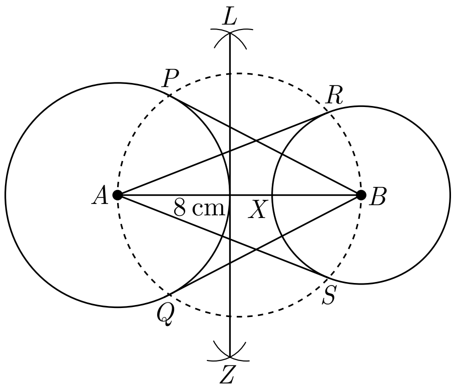


40. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

Ans : [Board Term-2 Foreign 2017, OD 2014]

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q, and R and S.
5. Join B to P, B to Q, A to R and A to S. Thus BP, BQ, AR and AS are the required tangents.



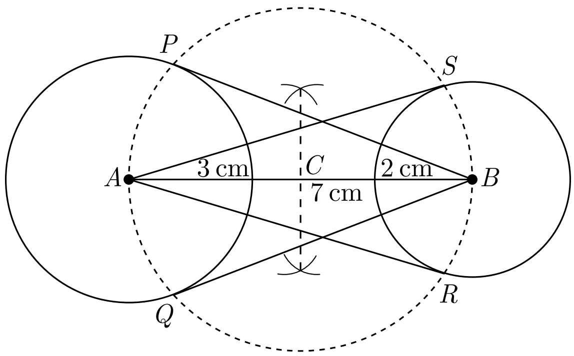
41. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as center, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Ans : [Board Term-2 Delhi 2015]

Steps of Construction :

1. Draw a line segment AB of 7 cm.
2. Taking A and B as centre draw two circle of 3 cm and 2 cm radius respectively.

3. Bisect the line AB. Let mid-point of AB be C.
4. Taking C as centre draw a circle of radius AC with intersects the two circles at point P, Q, R and S.
5. Join BP, BQ, AS and AR. BP, BQ and AR, AS are the required tangents.

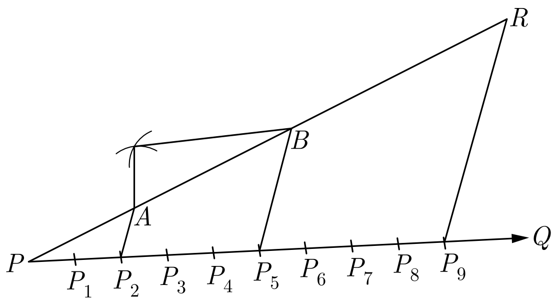


42. Construct a triangle whose perimeter is 13.5 cm and the ratio of the three sides is 2:3:4.

Ans : [Board Term-2 2011, 2012]

Steps of Construction :

1. Draw a line segment PR of length 13.5 cm.
2. At the point P draw a ray PQ making an acute angle RPQ with PR.
3. On PQ mark $(2+3+4)$ a points $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9$ such that $PP_1 = P_1P_2 = P_2P_3 = P_3P_4 = P_4P_5 = P_5P_6 = P_6P_7 = P_7P_8 = P_8P_9$.
4. Join P_9R
5. Through P_2 and P_5 draw lines P_2A and P_5B respectively parallel to P_9R intersecting PR at A and B respectively.
6. With A as centre and radius AP draw and arc. ABC is the required triangle.
7. With B as centre and radius BR draw another arc to intersect first arc.
8. Join A to C and B to C.

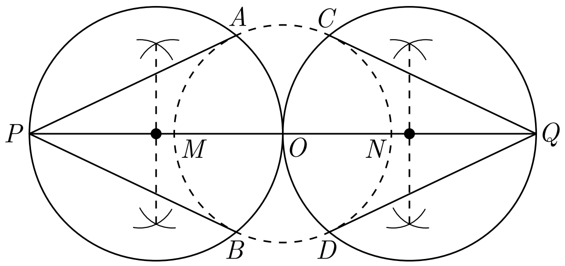


43. Draw a circle of radius of 3 cm. Take two points P and Q one of its diameter extended on both sides, each at a distance of 7 cm on opposite sides of its centre. Draw tangents to the circle from these two points.

Ans : [Board Term-2 Foreign 2017]

Steps of Construction :

1. Draw a circle with centre O and radius 3 cm.
2. Draw its diameter MON and extend it to both the sides to P and Q. Such that $OP = OQ = 7$ cm.
3. Taking diameters as OP and OQ draw two circles each of which intersects the first circle at the points A, B and C, D respectively.
4. Join PA, PB, QC and QO to get the required tangents

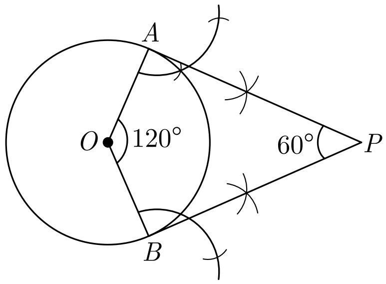


44. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Ans : [Board Term-2 Foreign 2015, OD 2016]

Steps of Construction :

1. Draw a circle with centre O and radius 6 cm.
2. Draw two radii OA and OB inclined to each other at an angle of 120°.
3. Draw $AP \perp OA$ at A and $BP \perp OB$ at B, which meet at P.
4. PA and PB are the required tangents inclined to each other an angle of 60°.



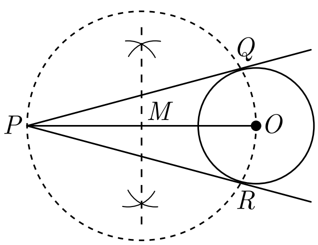
45. Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

Ans : [Board Term-2 Foreign 2015]

Steps of Construction :

1. Draw a line segment PO of length 7 cm.
2. Draw a circle with centre O and radius 3 cm.
3. Draw a perpendicular bisector of PO. Let M be the mid-point of PO.
4. Taking M as centre and OM as radius draw a circle. Let this circle intersects the given circle at the point Q and R.
5. Join PQ and PR. On measuring we get

$PQ = PR = 6.3$ cm.

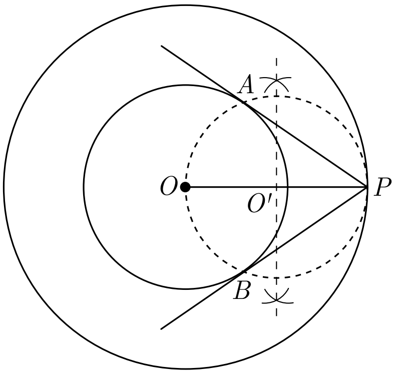


46. Draw two concentric circle of radii 3 cm and 5 cm. Taking a point on the outer circle, construct the pair of tangents to the inner circle.

Ans : [Foreign Set I 2017]

Steps of Construction :

1. Draw a circle with radius 3 cm and centre O.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on the circumference of outer circle and join O to P.
4. Taking OP as diameter draw another circle which intersect the smallest circle at A and B.
5. Join A to P and B to P. AP and BP are the required tangents.

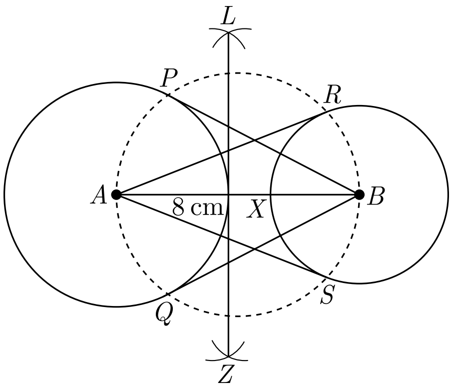


47. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm, and taking B as centre draw another circle of radius 3 cm. Construct tangents to each circle of radius centre of the other circle.

Ans : [Board Term-2 Foreign 2017, OD 2014]

Steps of Construction :

1. Draw a line segment AB of length 8 cm.
2. Draw a circle with centre A and radius 4 cm.
3. Draw another circle with centre B and radius 3 cm.
4. Taking AB as diameter draw another circle, which intersects first two circles at P and Q, and R and S.
5. Join B to P, B to Q, A to R and A to S. Thus BP, BQ, AR and AS are the required tangents.



CHAPTER 12

AREAS RELATED TO CIRCLES

ONE MARK QUESTIONS

Multiple Choice Questions

1. The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$)

(a) 44 sq. cm. (b) 66 sq. cm.
(c) 22 sq. cm. (d) 33 sq. cm.

I187

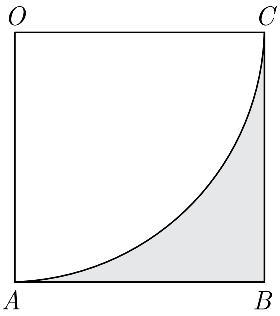
Ans :

Let the radii of the outer and inner circles be r_1 and r_2 respectively, we have

$$\begin{aligned} \text{Area} &= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2) \\ &= \pi(r_1 - r_2)(r_1 + r_2) \\ &= \frac{22}{7} \times (5.7 - 4.3)(5.7 + 4.3) \\ &= \frac{22}{7} \times 1.4 \times 10 \text{ sq. cm} \\ &= 44 \text{ sq cm} \end{aligned}$$

Thus (a) is correct option.

2. In the adjoining figure, $OABC$ is a square of side 7 cm. OAC is a quadrant of a circle with O as centre. The area of the shaded region is



I188

(a) 10.5 cm^2 (b) 38.5 cm^2
(c) 49 cm^2 (d) 11.5 cm^2

Ans :

$$\begin{aligned} \text{Required area} &= \left(r^2 - \frac{1}{4} \times \frac{22}{7} \times r^2\right) \text{cm}^2 \\ &= \left(7^2 - \frac{1}{4} \times \frac{22}{7} \times 7^2\right) \text{cm}^2 \\ &= (49 - 38.5) \text{cm}^2 \end{aligned}$$

Thus (a) is correct option.

3. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240°. If another circle of the area same as the sector is formed, then radius of the new circle is

(a) 79.5 cm (b) 81.5 cm
(c) 83.4 cm (d) 88.5 cm

I189

Ans :

$$\text{Area of sector} = \frac{240^\circ}{360^\circ} \times \pi(100)^2 = 20933 \text{ cm}^2$$

Let r be the radius of the new circle, then

$$\begin{aligned} 20933 &= \pi r^2 \\ r &= \sqrt{\frac{20933}{\pi}} = 81.6 \text{ cm} \end{aligned}$$

Thus (b) is correct option.

4. If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

(a) 1450 m^2 (b) 1576 m^2
(c) 1694 m^2 (d) 3368 m^2

I190

Ans :

Radius of outer concentric circle,

$$= (35 + 7) \text{ m} = 42 \text{ m}.$$

$$\begin{aligned} \text{Area of path} &= \pi(42^2 - 35^2) \text{ m}^2 \\ &= \frac{22}{7}(42^2 - 35^2) \text{ m}^2 \end{aligned}$$

Thus (c) is correct option.

5. If the area of a semi-circular field is 15400 sq m, then perimeter of the field is

(a) $160\sqrt{2}$ m (b) $260\sqrt{2}$ m

(c) $360\sqrt{2}$ m (d) $460\sqrt{2}$ m

I191

Ans :

Let the radius of the field be r.

Then, $$\frac{\pi r^2}{2} = 15400$$

$$\frac{1}{2} \times \frac{22}{7} \times r^2 = 15400$$

$$r^2 = 15400 \times 2 \times \frac{7}{22} = 9800$$

$$r = 70\sqrt{2}\text{ m}$$

Thus, perimeter of the field

$$\pi r + 2r = \frac{22}{7} \times 70\sqrt{2} + 2 \times 70\sqrt{2}$$

$$= 220\sqrt{2} + 140\sqrt{2}$$

$$= \sqrt{2}(220 + 140)$$

$$= 360\sqrt{2}\text{ m}$$

Thus (c) is correct option.

6. The area of the circle that can be inscribed in a square of side 6 cm is

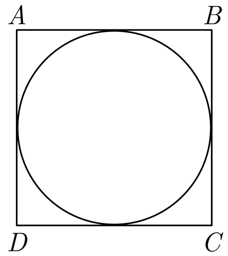
(a) $36\pi\text{ cm}^2$ (b) $18\pi\text{ cm}^2$

(c) $12\pi\text{ cm}^2$ (d) $9\pi\text{ cm}^2$

I192

Ans :

Given, side of square $= 6$ cm



Diameter of circle is equal to the side of square.

Diameter of a circle, $d = = 6$ cm

Radius of a circle, $r = \frac{d}{2} = \frac{6}{2} = 3$ cm

Area of circle, $\pi r^2 = \pi 3^2 = 9\pi\text{ cm}^2$

Thus (d) is correct option.

7. The sum of the areas of two circle, which touch each other externally, is $153\,\pi$. If the sum of their radii is 15, then the ratio of the larger to the smaller radius is

(a) 4 : 1 (b) 2 : 1

(c) 3 : 1 (d) None of these

Ans :

Let the radii of the two circles be r_1 and r_2, then

$$r_1 + r_2 = 15 \qquad ...(1)$$

and $$\pi r_1^2 + \pi r_2^2 = 153\,\pi \qquad ...(2)$$

$$r_1^2 + r_2^2 = 153$$

I193

$$r_1^2 + (15 - r_1)^2 = 153$$

$$r_1^2 + 225 - 30r_1 + r_1^2 = 153$$

$$2r_1^2 - 30r_1 + 72 = 0$$

$$r_1^2 - 15r_1 + 36 = 0$$

Solving, we get $r_1 = 12$ and $r_2 = 3$.

Thus required ratio is 12 : 3 or 4 : 1.

Thus (a) is correct option.

8. A race track is in the form of a ring whose inner and outer circumference are 437 m and 503 m respectively. The area of the track is

(a) 66 sq. cm. (b) 4935 sq. cm.

(c) 9870 sq. cm (d) None of these

Ans :

I194

We have $$2\pi r_1 = 503 \Rightarrow r_1 = \frac{503}{2\pi}$$

and $$2\pi r_2 = 437 \Rightarrow r_2 = \frac{437}{2\pi}$$

Area of ring

$$\pi(r_1^2 - r_2^2) = \pi(r_1 + r_2)(r_1 - r_2)$$

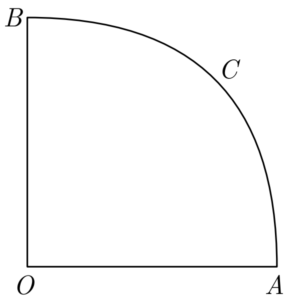
$$= \pi\left(\frac{503 + 437}{2\pi}\right)\left(\frac{503 - 437}{2\pi}\right)$$

$$= \frac{940}{2}\left(\frac{66}{2\pi}\right) = \frac{940}{2} \times \frac{66}{2} \times \frac{7}{22}$$

$$= 235 \times 21 = 4935\text{ sq. cm.}$$

Thus (b) is correct option.

9. In the given figure, $OACB$ is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is



(a) 11 cm (b) 18 cm

(c) 25 cm (d) 36 cm

Ans :

$$\text{Perimeter} = \frac{1}{4}\times 2\pi r + 2r$$

$$= \left(\frac{1}{2}\times\frac{22}{7}\times 7 + 2\times 7\right)\text{cm}$$

$$= 25\,\text{cm}$$

Thus (c) is correct option.

10. If the circumference of a circle increases from 4π to 8π, then its area is

(a) halved (b) doubled

(c) tripled (d) quadrupled

Ans :

$$2\pi r = 4\pi \Rightarrow r = 2$$

$$\text{Area} = \pi(2)^2 = 4\pi$$

When, $2\pi r = 8\pi \Rightarrow r = 4$

$$\text{Area} = 16\,\pi$$

Thus area is quadrupled.

Thus (d) is correct option.

11. If the radius of a circle is diminished by 10%, then its area is diminished by

(a) 10% (b) 19%

(c) 36% (d) 20%

Ans :

Let r be the radius of circle, then area $= \pi r^2$

When r is diminished by 10%

Then, $\text{area} = \pi\left(r - \frac{r}{10}\right)^2 = \pi r^2\left(\frac{81}{100}\right)$

Thus area is diminished by

$$\left(1 - \frac{81}{100}\right)\% = 19\,\%$$

Thus (b) is correct option.

12. If the perimeter of a semi-circular protractor is 36 cm, then its diameter is

(a) 10 cm (b) 14 cm

(c) 12 cm (d) 16 cm

Ans :

$$\text{Perimeter} = \frac{2\pi r}{2} + 2r = \pi r + 2r$$

$$(\pi + 2)\,r = 36$$

$$\left(\frac{36}{7}\right) - r = 36 \Rightarrow r = 7\,\text{cm}$$

Hence, diameter $2r = 7\times 2 = 14\,\text{cm}$

Thus (b) is correct option.

13. The area of a circular path of uniform width d surrounding a circular region of radius r is

(a) $\pi d(2r + d)$ (b) $\pi(2r + d)r$

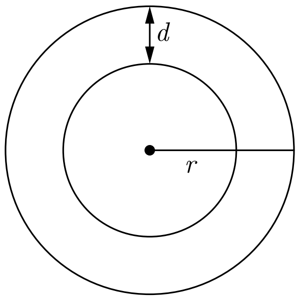
(c) $\pi(d + r)r$ (d) $\pi(d + r)d$

Ans :

$$\text{Required area} = \pi\left[(r + d)^2 - r^2\right]$$

$$= \pi\left[r^2 + d^2 + 2rd - r^2\right]$$

$$= \pi\left[d^2 + 2rd\right] = \pi d\left[d + 2r\right]$$



Thus (a) is correct option.

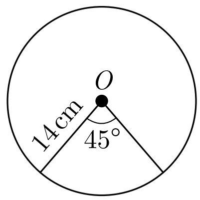
14. In a circle of radius 14 cm, an arc subtends an angle of $45°$ at the centre, then the area of the sector is

(a) $71\,\text{cm}^2$ (b) $76\,\text{cm}^2$

(c) $77\,\text{cm}^2$ (d) $154\,\text{cm}^2$

Ans :

Given, $r = 14\,\text{cm}$ and $\theta = 45°$



$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{8} \times 22 \times 2 \times 14 = 77 \text{ cm}^2$$

Thus (c) is correct option.

15. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

Ans :

According to the given condition,

$$\text{Area of circle} = \text{Area of first circle} + \text{Area of second circle}$$

$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$

$$R^2 = R_1^2 + R_2^2$$

I201

Thus (b) is correct option.

16. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$

(c) $R_1 + R_2 > R$ (d) $R_1 + R_2 < R$

Ans :

According to the given condition,

$$2\pi R = 2\pi R_1 + 2\pi R_2$$

$$R = R_1 + R_2$$

I202

Thus (a) is correct option.

17. If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square

I203

Ans :

Let r and a be the radius of circle and side of square respectively.

$$2\pi r = 4a$$

$$\frac{22}{7} r = 2a$$

$$11 r = 7a$$

$$r = \frac{7a}{11} \quad \text{...(1)}$$

Now, area of circle, $A_1 = \pi r^2$

From equation (1), we get

$$A_1 = \pi\left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121}$$

$$A_1 = \frac{14a^2}{11} \quad \text{...(2)}$$

and area of square, $A_2 = (a)^2$ ...(3)

From equations (2) and (3),

$$A_1 = \frac{14}{11} A_2$$

$$A_1 > A_2$$

Hence, Area of the circle >Area of the square.

Thus (b) is correct option.

18. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22:7 (b) 14:11

(c) 7:22 (d) 11:14

I204

Ans :

Let radius of circle be r and side of a square be a.

According to the given condition,

$$\text{Perimeter of a circle} = \text{Perimeter of a square}$$

$$2\pi r = 4a$$

$$a = \frac{\pi r}{2} \quad \text{...(1)}$$

Now, $\dfrac{\text{Area of circle}}{\text{Area of square}} = \dfrac{\pi r^2}{(a)^2} = \dfrac{\pi r^2}{(\pi \frac{r}{2})^2}$ [from Eq. (1)]

$$= \frac{\pi r^2}{\pi^2 \frac{r^2}{4}} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$$

$$= \frac{28}{22} = \frac{14}{11}$$

Hence, the required ratio is 14:11.

Thus (b) is correct option.

19. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b) 15 m

(c) 20 m (d) 24 m

I205

Ans :

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$

$$= \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = \left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2$$

$$= (8)^2 + (6)^2 = 100$$

$$R = \sqrt{100} = 10$$

Thus (a) is correct option.

20. The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm^2 (b) 128 cm^2

(c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

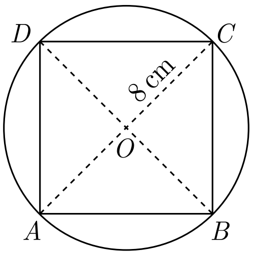
I206

Ans :

Radius of circle, $OC = 8$ cm,

Diameter of the circle, $AC = 2 \times OC$

$$= 2 \times 8 = 16 \text{ cm}$$



which is equal to the diagonal of a square.

Let side of square be x.

In right angled ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$(16)^2 = x^2 + x^2$$

$$256 = 2x^2$$

$$x^2 = 128$$

Area of square, $x^2 = 128$ cm^2

Alternate Method :

Radius of circle, $r = 8$ cm

Diameter of circle, $d = 2r = 2 \times 8 = 16$ cm

Since, square inscribed in circle.

Diagonal of square = Diameter of circle

Now, Area of square $= \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2}$

$$= 128 \text{ cm}^2$$

Thus (b) is correct option.

21. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm

(c) 28 cm (d) 16 cm

I207

Ans :

We have $2\pi r = 2\pi r_1 + 2\pi r_2$

$$2\pi r = \pi d_1 + \pi d_2$$

$$2r = d_1 + d_2 = 36 + 20$$

$$2r = 56 \Rightarrow r = 28 \text{ cm}$$

Thus (c) is correct option.

22. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(a) 31 cm (b) 25 cm

(c) 62 cm (d) 50 cm

I208

Ans :

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$

$$= 24^2 + 7^2 = 625$$

$$R = \sqrt{625} = 25 \text{ cm}$$

Diameter of a circle

$$2R = 2 \times 25 = 50 \text{ cm}$$

Thus (d) is correct option.

23. Assertion : In a circle of radius 6 cm, the angle of a sector 60°. Then the area of the sector is $18\frac{6}{7}\text{ cm}^2$.

Reason : Area of the circle with radius r is πr^2.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$\text{Area of the sector } = \frac{\theta}{360} \times \pi r^2$$

I209

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} = 18\frac{6}{7}\text{cm}^2.$$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

24. Assertion : If the circumference of a circle is 176 cm, then its radius is 28 cm.

Reason : Circumference $= 2\pi \times$ radius

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $C = 2 \times \frac{22}{7} \times r = 176$

I210

$$r = \frac{176 \times 7}{2 \times 22} = 28\text{ cm}$$

Both assertion and reason are correct. Also Reason is the correct explanation of the assertion.

Thus (a) is correct option.

25. Assertion : If the outer and inner diameter of a circular path is 10 m and 6 m then area of the path is $16\pi\, m^2$.

Reason : If R and r be the radius of outer and inner circular path, then area of path is $\pi(R^2 - r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

$$\text{Area of the path } = \pi\left[\left(\frac{10}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right]$$

I211

$$= \pi(25 - 9) = 16\,\pi$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

26. Assertion : If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is 40 cm^2.

Reason : Circumference of the circle = length of the wire.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have $2\pi r = 22$

I212

$$r = 3.5\text{ cm}$$

$$\text{Area of the circle } = \frac{22}{7} \times 3.5 \times 3.5$$

$$= 38.5\text{ cm}^2$$

Assertion is not correct, but reason is true.

Thus (d) is correct option.

Fill in the Blank Questions

27. The boundary of a sector consists of an arc of the circle and the two ....

Ans :

radii

I213

28. Angle formed by two radii at the centre is known as ....

Ans :

central angle

I214

29. Concentric circles are circles having same...........

Ans :

centre

I215

30. The area of a circle is the measurement of the region enclosed by its ....

Ans :

boundary

I216

31. Segment is the region enclosed between chord and ....

Ans :

arc

I217

32. Pie (π) is the ratio between circumference and .... of the circle.

Ans :

diameter

I218

33. The region enclosed by an arc and a chord is called the .... of the circle.

Ans :

segment

I219

34. Perimeter of a semi circle ....

Ans :

$(\pi r + d)$ units

I220

35. Circumference of a circle is ....

Ans :

$2\pi r$

I221

36. Area of a circle is ....

Ans :

πr^2

I222

37. Measure of angle in a semi circle is ....

Ans :

90°

I223

38. Length of an arc of a sector of a circle with radius r and angle with degree measure θ is ....

Ans :

$\frac{\theta}{360} \times 2\pi r$

I224

39. A sector of a circle is called a .... sector if the minor arc of the circle is a part of its boundary.

Ans :

minor

I225

VERY SHORT ANSWER QUESTIONS

40. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of a circle of a circle which has circumference equal to sum of their circumferences.

Ans : [Board 2020 Delhi Basic]

Radius of 1[st] circle $r_1 = 9$ cm

Radius of 2[nd] circle $r_2 = 19$ cm

I226

Let r the radius of required circle. According to question, circumference of required circle is sum of circumference of two circles.

$$2\pi r = 2\pi r_1 + 2\pi r_2$$

$$2\pi r = 2\pi(r_1 + r_2)$$

$$r = r_1 + r_2 = 9 + 19 = 28 \text{ cm.}$$

Hence, radius of required circle is 28 cm

41. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Ans : [Board 2020 Delhi Standard]

Angle subtended in 1 minutes $= 6^\circ$

Angle subtended in 35 minutes $= 6^\circ \times 35 = 210^\circ$

Area of the face of the clock by the minute hand, i.e. area of sector,

$$\frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{12 \times 12 \times 210^\circ}{360^\circ}$$

$$= \frac{22}{7} \times \frac{12 \times 12 \times 7 \times 30^\circ}{12 \times 30^\circ}$$

$$= 22 \times 12 = 264 \text{ cm}^2$$

I227

42. The radius of a circle is 17.5 cm. find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

Ans : [Board 2020 OD Basic]

Given, arc length $= 44$ cm

Radius of circle, $r = 17.5$ cm

I228

So, area of sector $= \frac{\text{arc length}}{2\pi r} \times \pi r^2$

$$= \frac{\text{arc length} \times r}{2} = \frac{44 \times 17.5}{2}$$

$= 22 \times 17.5 = 385$ sq. cm.

43. Find the area of the sector of a circle of radius 6 cm whose central angle is 30°. (Take $\pi = 3.14$)

Ans : [Board 2020 OD Standard]

I229

Radius, $r = 6$ cm

Central angle, $\theta = 30^\circ$

Area of the sector,

$$\frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 6 \times 6 \times 30^\circ}{360^\circ}$$

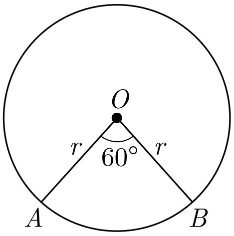
$$= 9.42 \text{ cm}^2$$

44. What is the perimeter of the sector with radius 10.5 cm and sector angle 60°.

Ans : [Board Term-2 2012]

As per question the digram is shown below.

I101



Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

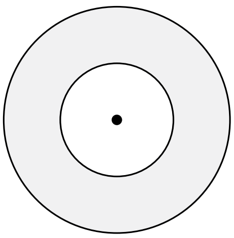
$$= 21 + 11 = 32 \text{ cm}$$

45. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

Ans : [Board Term-2 Delhi 2013]

As per question statement figure is shown below.

I102



Circumference of the outer circle, $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

Circumference of the outer circle, $2\pi r_2 = 66$ cm

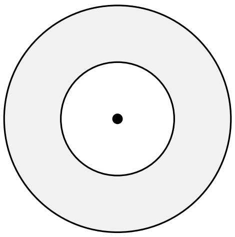
$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

Width of the ring,

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 3.5 \text{ cm}$$

46. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.

I103



Ans : [Board Term-2 2012]

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of the shaded region} = \pi(2)^2 - \pi(1)^2$$

$$= 4\pi - \pi = 3\pi \text{ sq cm}$$

47. The diameter of two circle with centre A and B are 16 cm and 30 cm respectively. If area of another circle with centre C is equal to the sum of areas of these two circles, then find the circumference of the circle with centre C.

Ans : [Board Term-2 2012]

Let the radius of circle with centre C be r.
According to question we have,

$$\pi(8)^2 + \pi(15)^2 = \pi r^2$$

$$64\pi + 225\pi = \pi r^2$$

$$289\pi = \pi r^2$$

$$r^2 = 289 \text{ or } R = 17 \text{ cm}$$

Circumference of circle

$$2\pi r = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

48. The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions.

Ans : [Board Term-2 2012]

Distance covered in 1 revolution is equal to circumference of wheel and that is πd.

Distance covered in 500 revolutions

$$= 500 \times \pi \times 1.26$$

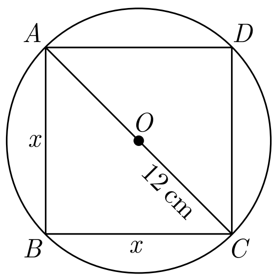
$$= 500 \times \frac{22}{7} \times 1.26$$

$$= 1980 \text{ m.} = 1.98 \text{ km}$$

49. What is the area of the largest square that can be inscribed in a circle of radius 12 cm.?

Ans : [Board Term-2 2012]

As per question the digram is shown below.



$$\text{Radius of the circle} = 12 \text{ cm}$$

$$\text{Diameter of circle} = 24 \text{ cm}$$

$$\text{Diagonal of square} = 24 \text{ cm}$$

Let the side of square be x.
From Pythagoras theorem we have

$$x^2 + x^2 = (24)^2$$

$$2x^2 = 24 \times 24$$

$$x^2 = \frac{24 \times 24}{2} = 288$$

Thus area of square,

$$x^2 = 288 \text{ cm}^2$$

50. What is the name of a line which intersects a circle at two distinct points?

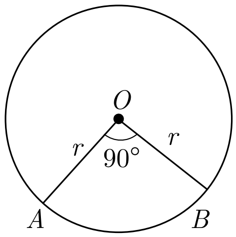
Ans : [Board Term-2 2012]

A line intersecting the circle at two distinct points is called a secant.

51. What is the perimeter of a sector of a circle whose central angle is 90° and radius is 7 cm?

Ans : [Board Term-2 2012]

As per question the digram is shown below.



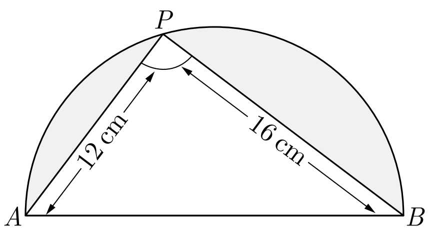
Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 2 \times 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90}{360}$$

$$14 + 11 = 25 \text{ cm}$$

52. In the given figure, AB is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of π as 3, find the perimeter of the shaded region.



Ans : [Board Term-2 2012]

From Pythagoras theorem we have

$$AB = \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400} = 20 \text{ cm}$$

Radius of circle $= 10$ cm.

Perimeter of shaded region

$$\pi r + AP + PB = 3 \times 10 + 12 + 16$$

$$= 30 + 12 + 16 = 58 \text{ cm}$$

53. Find the area of circle that can be inscribed in a square of side 10 cm.

Ans : [Board Term-2 2012]

I110

Radius of the circle $= \frac{10}{2} = 5$ cm

Area of the circle,

$$\pi r^2 = \pi \times (5)^2 = 25\pi \, cm^2$$

54. A thin wire is in the shape of a circle of radius 77 cm. It is bent into a square. Find the side of the square (Taking, $\pi = \frac{22}{7}$).

Ans : [Board Term-2 2012]

I111

Let side of square be x.

Perimeter of the circle = Perimeter of square

$$2\pi r = 4x$$

$$2 \times \frac{22}{7} \times 77 = 4x$$

$$x = \frac{2 \times 22 \times 11}{4} = 121$$

Thus side of the square is 121 cm.

55. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

Ans : [Board Term-2 2012]

I112

Area of the circle = sum of areas of two circles

$$\pi R^2 = \pi \times (40)^2 + \pi (9)^2$$

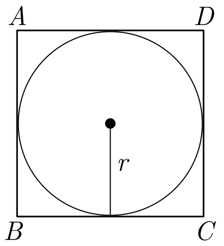
$$R^2 = 1600 + 81$$

$$R = \sqrt{1681} = 41 \text{ cm}$$

Thus diameter of given circle $= 41 \times 2 = 82$ cm

56. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm.

Ans : [board Term-2, 2012 Set (28, 32, 33)]



I113

Side of square = diameter of circle = 8 cm

Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle, $\pi r^2 = \pi \times 4 \times 4 = 16\pi \text{ cm}^2$

57. If the radius of a circle is doubled, what about its area?

Ans : [Board Term-2 2012]

I114

Let the radius of the circle be r, then area will be πr^2

Now if radius is doubled,

Area $= \pi (2r)^2 = 4\pi r^2 = 4 \times \pi r^2$

The area will be 4 times the area of the first circle.

58. If the perimeter and the area of the circle are numerically equal, then find the radius of the circle.

Ans : [Board Term-2, 2012 Set(13)]

I115

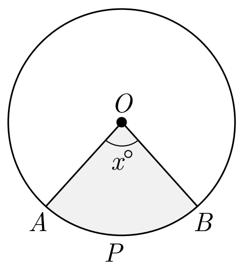
Perimeter of the circle = area of the circle.

$$2\pi r = \pi r^2$$

$$r = 2 \text{ units}$$

59. In given fig., O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.

I116



Ans : [Board Term-2 2012]

Area of the sector,

$$A_{\frown} = \frac{\pi r^2 \theta}{360^\circ}$$

Area of sector $OAPB$ is $\frac{5}{36}$ times the area of circle.

Thus $$\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$$

$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50^{\circ}$$

60. If circumference of a circle is 44 cm, then what will be the area of the circle?

Ans : [Board Term-2 2012]

I117

Circumference of a circle $= 44$ cm

Radius of the circle $= \frac{22}{2 \times \frac{22}{7}} = 7$ cm

Area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ cm^2$

61. A steel wire when bent in the form of a square encloses an area or 121 cm^2. If the same wire is bent in the form of a circle, then find the circumference of the circle.

Ans : [Board Term-2 2012]

I118

Area of square $= (\text{side})^2 = 121\text{ cm}^2$

Side of square $= \sqrt{121} = 11$ cm

Parameter of square$= 4 \times 11 = 44$ cm

Circumference of the circle = Perimeter of the square

$= 44$ cm

62. Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm

Ans : [Board Term-2 2012]

I119

Circumference of the circle,

$$2\pi r = 2\pi \times 18 + 2\pi \times 10$$

$$r = 18 + 10 = 28 \text{ cm}$$

Hence radius of given circle is 28 cm.

63. Find the diameter of a circle whose area is equal to the sum of areas of two circles of diameter 16
12 cm.

Ans : [Board Te

I120

Let r be the radius of the circle. Since area of the circle is equal to the sum of areas of two circles,

$$\pi r^2 = \pi \times (8)^2 + \pi(6)^2$$

$$\pi r^2 = \pi(64 + 36)$$

$$r^2 = 100 \text{ or, } r = 10 \text{ cm}$$

Diameter of the circle $= 2 \times 10 = 20$ cm.

64. If the circumference of a circle increases from 4π to 8π, then what about its area ?

Ans : [Board Term-2 Delhi 2013]

Circumference of the circle

$$2\pi r = 4\pi \text{ cm or } r = 2 \text{ cm.}$$

I121

Increased circumference

$$2\pi R = 8\pi \text{ cm or } R = 4 \text{ cm.}$$

Area of the 1^{st} circle

$$\pi r^2 = \pi \times (2)^2 = 4\pi \text{ cm}$$

Area of the new circle

$$\pi R^2 = \pi(4)^2 = 16\pi = 4 \times 4\pi$$

Area of the new circle= 4 times the area of first circle.

65. If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

Ans : [Board Term-2 2014]

Area of the sector$= \frac{1}{2} \times$(length of the corresponding arc)$\times$ radius

$$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$$

$$= 36 \text{ cm}^2$$

I122

66. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. $(\pi = 3.14)$

Ans : [Board Term-2 2012]

Radius of circle $r = 10$ cm, central angle $= 90^{\circ}$

Area of minor segment,

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^{\circ}\right]$$

I123

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2$$

67. If the perimeter of a semi-circular protractor is 36 cm, find its diameter. (Use $\pi = \frac{22}{7}$).

Ans : [Board Term-2 2012]

Perimeter $\pi r + 2r = (\pi + 2)r = 36$

or, $\left(\frac{22}{7} + 2\right)r = 36$ or, $r = 7$

Diameter $= 14$ cm.

I124

TWO MARKS QUESTIONS

68. The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences?

Ans : [Board 2020 Delhi Basic]

Given,

$$\frac{\text{Area of } 1^{st} \text{circle}}{\text{Area of } 2^{nd} \text{circle}} = \frac{9}{4}$$

i.e.,
$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{4}$$

$$\frac{r_1^2}{r_2^2} = \frac{9}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{2}$$

I230

Ratio of their circumference

$$\frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{3}{2}$$

Hence, the ratio of their circumference is 3 : 2.

69. The length of the minute hand of clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

Ans : [Board 2020 OD Basic]

Minute hand completes full circle degree in 1 hour. So, degree swept by minute hand in 1 hour (60 minutes) is $360°$.

Degree swept by minute hand in 1 minute is $\frac{360°}{60} = 6°$ and degree swept by minute hand in 15 minutes,

$$\theta = 6° \times 15 = 90°$$

Hence, $\theta = 90°$

and $r = 14$ cm

I231

Area swept by minute hand

$$\frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times (14)^2$$

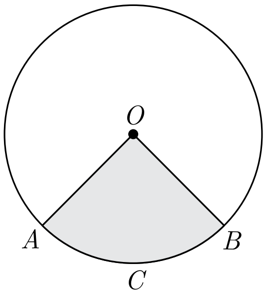
$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

70. The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the area of the sector.

Ans : [Board 2020 Delhi Basic]

Given, Radius $= 6.5$ cm

Let O be the centre of a circle with radius 6.5 cm and $OACBO$ be its sector with perimeter 31 cm.



I232

Thus, we have

$$OA + OB + \overparen{ACB} = 31 \text{ cm}$$

$$6.5 + 6.5 + \overparen{ACB} = 31 \text{ cm}$$

$$\overparen{ACB} = 18 \text{ cm}$$

Now, area of sector $OACBO$

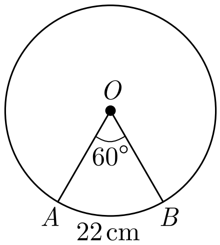
$$= \frac{1}{2} \times \text{radius} \times \overparen{ACB}$$

$$= \frac{1}{2} \times 6.5 \times 18 = 58.5 \text{ cm}^2$$

71. A piece of wire 22 cm long is bent into the form an arc of a circle subtending an angle of $60°$ at its centre. Find the radius of the circle. [Use $\pi = \frac{22}{7}$]

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



I233

Here AB is an arc of a circle of radius r.

$$\text{Length of arc} = \frac{2\pi r\theta}{360°}$$

$$22 = \frac{2 \times 22 \times r \times 60^\circ}{7 \times 360^\circ}$$

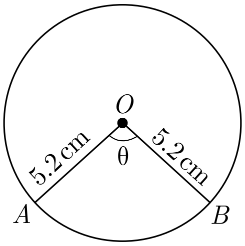
$$22 = \frac{22 \times r}{21} \Rightarrow r = 21$$

Hence, the radius of the circle is 21 cm.

72. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



Perimeter of the sector

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$16.4 = 10.4 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$6 = \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

I234

$$\frac{3}{5.2} = \frac{\theta \times \pi}{360^\circ}$$

Now, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \left(\frac{\theta \times \pi}{360^\circ}\right) r^2$

$$= \frac{3}{5.2} \times (5.2)^2 = 15.6 \text{ sq. units.}$$

73. The area of a circular play ground is 22176 cm^2. Find the cost of fencing this ground at the rate of 50 per metre.

Ans : [Board 2020 OD Standard]

Area of a circular play ground,

$$A = 22176 \text{ cm}^2$$

i.e., $\pi r^2 = 22176 \text{ cm}^2$

I235

$$r^2 = 22176 \times \frac{7}{22}$$

$$= 7056$$

$$r = 84 \text{ cm} = 0.84 \text{ m}$$

Perimeter of ground,

$$p = 2\pi r$$

Cost of fencing this ground,

$$= ₹\, 50 \times 2\pi r$$

$$= ₹\, 50 \times 2 \times \frac{22}{7} \times 0.84 = ₹\, 264$$

74. The wheel of a motorcycle is of radius 35 cm. How many revolutions are required to travel a distance of 11 m?

Ans : [Board 2020 OD Basic]

Given, radius of wheel, $r = 35$ cm

Circumference of the wheel,

$$2\pi r = 2 \times \frac{22}{7} \times 35 = 220 \text{ cm}$$

Number of revolutions required to cover 11 m or 1100 cm,

$$= \frac{1100}{220} = 5 \text{ revolutions}$$

I236

75. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m.

Ans : [Board Term-2 2012]

Angle subtended by minute hand in 60 minute = 360 ∘

Angle subtended in 1 minute $= \frac{360^\circ}{60} = 6^\circ$

Angle subtended in 35 minutes,

$$\theta = 35 \times 6^\circ = 210^\circ$$

Area swept by the minute hand

I179

$$= \text{Area of a sector}$$

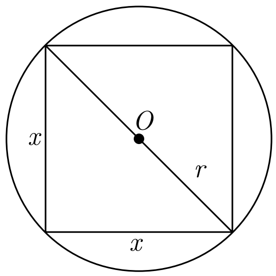
$$= \pi r^2 \frac{\theta}{360^\circ} = \frac{22}{7} \times 14 \times 14 \times \frac{210^\circ}{360^\circ}$$

$$= \frac{1078}{3} = 259.33 \text{ cm}^2$$

76. Find the area of the square that can be inscribed in a circle of radius 8 cm.

Ans : [Board Term-2 2015]

As per question the digram is shown below.



I125

Let the side of square be x and radius of circle be r.

Radius of the circle, $r = 8$ cm

Diameter of circle, $2r = 16$ cm

Diagonal of square $2r = 16$ cm

From Pythagoras theorem we have

$$x^2 + x^2 = (2r)^2$$

$$x^2 + x^2 = (16)^2$$

$$2x^2 = 16 \times 16$$

$$x^2 = \frac{16 \times 16}{2} = 128$$

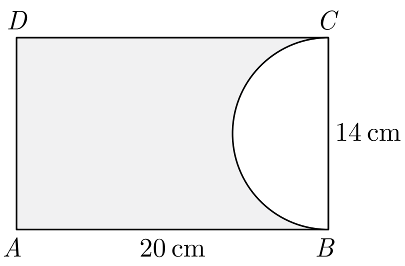
Thus area of square,

$$x^2 = 128 \text{ cm}^2$$

77. A paper is in the form of a rectangle $ABCD$ in which $AB = 20$ cm, $BC = 14$ cm. A semi-circular portion with BC as diameter is cut off. Find the area of the part. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 2012, Foreign 2014]

As per question the digram is shown below.



I126

Area of remaining part,

= Area of rectangle − Area of semi-circle

$$= 20 \times 14 - \frac{1}{2}\pi 7^2$$

$$= 280 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

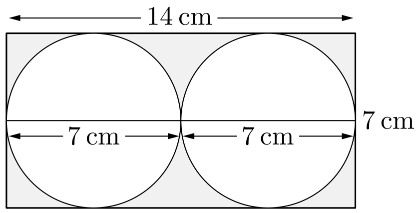
$$= 280 - 77 = 203 \text{ cm}$$

Hence, area of remaining part is 203 cm.

78. Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm × 7 cm. find the area of the remaining cardboard. (Use $\pi = \frac{22}{7}$)

Ans : [Board Term-2 Delhi 2013]

As per question the digram is shown below.



I127

Area of the remaining cardboard

= Area of rectangular cardboard − 2 × Area of circle

$$= 14 \times 7 - 2\pi\left(\frac{7}{2}\right)^2$$

$$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 98 - \frac{44}{7} \times \frac{49}{4} = 98 - 77 = 21$$

Hence, area of remaining card board is 21 cm²

79. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

Ans : [Board Term-2 Delhi 2012]

I128

Let r be the radius of the circle.

Now, circumference − radius = 37

$$2\pi r - r = 37$$

$$2 \times \frac{22}{7} r - r = 37$$

$$r\left(\frac{22-7}{7}\right) = 37$$

$$r \times \frac{37}{7} = 37$$

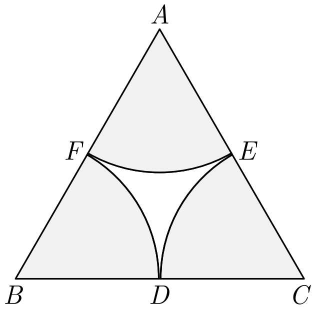
$$r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

Circumference of the circle,

$$2\pi r = 2 \times \frac{22}{7} \times 7$$

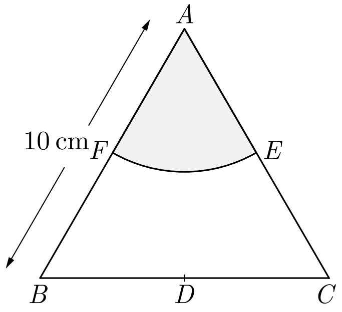
$$= 44 \text{ cm.}$$

80. In fig. arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the side BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region. (Use $\pi = 3.14$).



Ans : [Board Term-2 2011]

Figure given below shows the single sector.



Since ΔABC is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

Here we have 3 sector and area of all three sector is equal.

Area of sector $AFEA$,

$$\text{Area}_{AFEA} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \pi (5)^2 = \frac{25}{6}\pi \text{ cm}^2$$

Thus total area of shaded region

$$\text{Area} = 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$

$$= 39.25 \text{ cm}^2$$

81. If the perimeter of a protractor is 72 cm, calculate its area. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2, 2012 Set (22)]

Perimeter of semi-circle

$$\pi r + 2r = 72 \text{ cm}$$

$$(\pi + 2)r = 72 \text{ cm}$$

$$\left(\frac{22}{7} + 2\right)r = 72 \text{ cm}$$

$$r\left(\frac{22 + 14}{7}\right) = 72 \text{ cm}$$

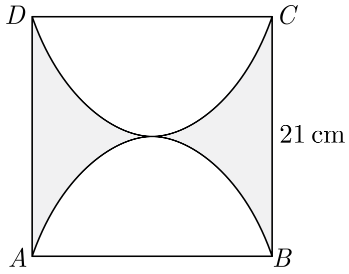
$$\frac{36}{7}r = 72 \Rightarrow r = 14 \text{ cm}$$

Area of protractor,

$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 308 \text{ cm}^2$$

82. Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and APB and CPD are semicircle. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 SQP 2016]

It may be seen easily that perimeter of the shaded region include AD, BC and two semi circle arc.

Thus perimeter of the shaded region, $= AD + BC +$

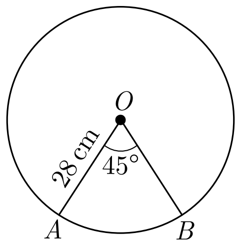
$+$ lengths of the arcs of semi circles APB and CPD

$$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) = 42 + 66 = 108 \text{ cm.}$$

83. Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle 45°.

Ans : [Board Term-2 2015]

As per question statement figure is shown bel



Area of major sector,

$$= \text{area of circle} - \text{area of minor sector}$$

$$= \pi r^2\left(1 - \frac{\theta}{360^\circ}\right)$$

$$= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45^\circ}{360^\circ}\right)$$

$$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$$

$$= 2156 \text{ cm}^2$$

84. The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

Ans : [Board Term-2 Delhi 2013]

Circumference of front wheel

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

I132

$$= \frac{22}{7} \times 200 = \frac{4400}{7} \text{ cm}$$

Revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}} = \frac{1760 \times 800}{4400} = 320 \text{ revolutions}$$

THREE MARKS QUESTIONS

85. A road which is 7 m wide surrounds a circular park whose circumference is 88 m. Find the area of the road.

Ans : [Board 2020 Delhi Basic]

Let $w = 7$ m be the width of road.
Circumference of a circular park,

I237

$$2\pi r = 88 \text{ m}$$

Inner radius of park,

$$r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22}$$

$$= 2 \times 7 = 14 \text{ m}$$

Outer radius of park including road width,

$$R = r + w$$

$$= 14 + 7 = 21 \text{ m}$$

Area of the road,

$$\pi(R^2 - r^2) = \pi(R + r)(R - r)$$

$$= \frac{22}{7}(21 + 14)(21 - 14)$$

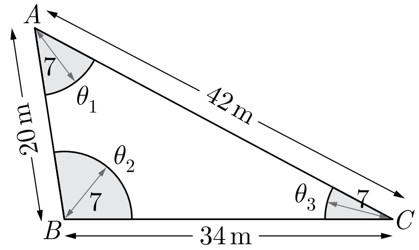
$$= \frac{22}{7} \times 35 \times 7 = 770 \text{ m}^2$$

Hence, the area of the road is 770 m^2.

86. Three horses are tied each with 7 m long rope at three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses.

Ans : [Board 2020 Delhi Basic]

As per information given in question we have drawn the figure below.



I238

Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the horses is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 7$ m.

$$\frac{\pi r^2\theta_1}{360^\circ} + \frac{\pi r^2\theta_2}{360^\circ} + \frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3) \quad ...(1)$$

From angle sum property of a triangle we have

$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}\times 180^\circ = \frac{\pi r^2}{2}$$

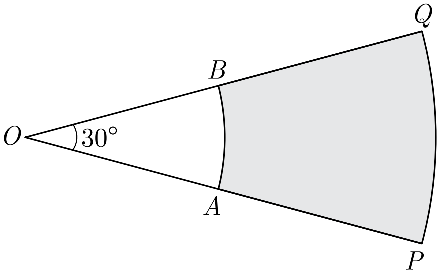
$$= \frac{22}{7}\times\frac{1}{2}\times(7)^2$$

$$= \frac{22}{7}\times\frac{1}{2}\times 7\times 7$$

$$= 77 \text{ m}^2$$

Hence, the area grazed by the horses is 77 m^2

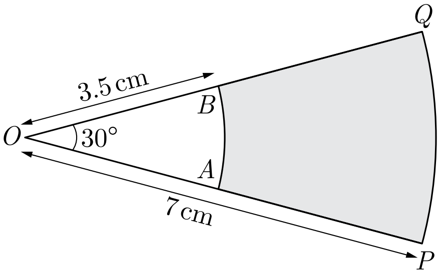
87. In Figure, PQ and AB are two arcs of concentric circles of radii 7 cm and 3.5 cm respectively, with centre O. If $\angle POQ = 30^\circ$, then find the area of shaded region.



I240

Ans : [Board 2020 OD Basic]

We redraw the given figure as below.



Area of shaded region

$$\pi[R^2-r^2]\frac{\theta}{360^\circ} = \frac{22}{7}[7^2-(3.5)^2]\frac{30^\circ}{360^\circ}$$

$$= \frac{22}{7}(7+3.5)(7-3.5)\times\frac{1}{12}$$

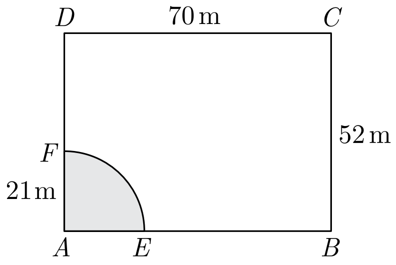
$$= \frac{22}{7}\times 10.5\times 3.5\times\frac{1}{12}$$

$$= 9.625 \text{ cm}^2$$

88. A horse is tethered to one corner of a rectangular field of dimensions 70 m $\times$ 52 m, by a rope of length 21 m. How much area of the field can it graze?

Ans : [Board 2020 OD Basic]

As per information given in question we have drawn the figure below.



I241

Length of the rope is 21 cm.

Shaded portion $AEFA$ indicates the area in which the horse can graze. Clearly it is the area of a quadrant of a circle of radius, $r = 21$ m.

Area of quadrant,

$$\frac{1}{4}\pi r^2 = \frac{1}{4}\times\frac{22}{7}\times(21)^2$$

$$= \frac{1}{4}\times\frac{22}{7}\times 21\times 21$$

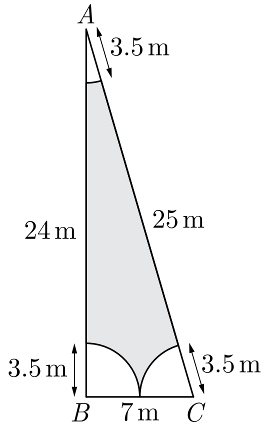
$$= 346.5 \text{ m}^2$$

Hence, the graze area is 346.5 m^2

89. Sides of a right triangular field are 25 m, 24 m and 7 m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals.

Ans : [Board 2020 SQP Standard]

As per information given in question we have drawn the figure below.



I242

Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the animals is the sum of the areas of three sectors with central angles θ_1, θ_2 and θ_3 each with radius $r = 3.5$ m.

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1+\theta_2+\theta_3) \quad ...(1)$$

From angle sum property of a triangle we have

$$\theta_1+\theta_2+\theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}\times 180^\circ = \frac{\pi r^2}{2}$$

$$= \frac{22}{7}\times\frac{1}{2}\times(3.5)^2$$

$$= 19.25$$

Hence, the area grazed by the horses is $19.25\,\text{m}^2$.

$$\text{Area of }\Delta\,ABC = \frac{1}{2}\times AB\times BC$$

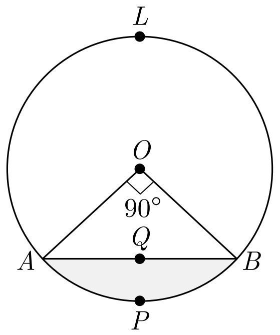
$$= \frac{1}{2}\times 24\times 7 = 84\,\text{m}^2$$

Area of the field that cannot be grazed by these animals = Area of triangle − Area of three sectors

$$= 84 - 1925 = 64.75\,\text{m}^2$$

90. In the given figure, a chord AB of the circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $ALBQA$. (Use $\pi = 3.14$)

I133



Ans : [Board Term-2 Foreign 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi\text{ cm}^2$$

Area of ΔAOB,

$$= \frac{1}{2}\times 10\times 10 = 50\text{ cm}^2$$

Area of minor segment $AQBP$,

$$= (25\pi - 50)\text{ cm}^2$$

$$= 25\times 3.14 - 50$$

$$= 78.5 - 50$$

$$= 28.5\text{ cm}^2$$

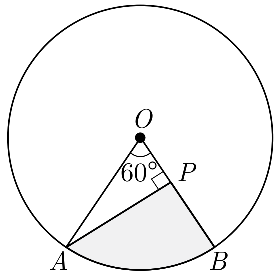
Also area of circle $= \pi(10)^2$

$$= 3.14\times 100 = 314\text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5\text{ cm}^2$$

91. In the given figure, AOB is a sector of angle 60° of a circle with centre O and radius 17 cm. If $AP\perp OB$ and $AP = 15$ cm, find the area of the shaded region.



Ans : [Board Term-2 2016]

Here $OA = 17$ cm $AP = 15$ cm and ΔOPA is right triangle

Using Pythagoras theorem, we have

I134

$$OP = \sqrt{17^2 - 15^2} = 8\text{ cm}$$

Area of the shaded region

= Area of the sector $\Delta\,OAB$ − Area of $\Delta\,OPA$

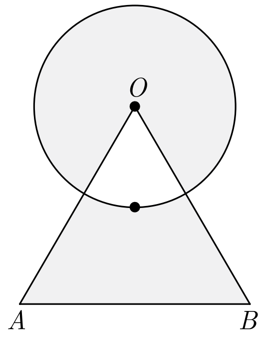
$$= \frac{60}{360}\times\pi r^2 - \frac{1}{2}\times b\times h$$

$$= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times 17\times 17 - \frac{1}{2}\times 8\times 15$$

$$= 151.38 - 60 = 91.38\text{ cm}^2$$

92. Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.

I135



Ans : [Board Term-2 Foreign SQP 2016]

Since OAB is an an equilateral triangle,we have

$$\angle AOB = 60°$$

Area of shaded region = Area of major sector + (Area of $\Delta AOB -$ Area of minor sector)

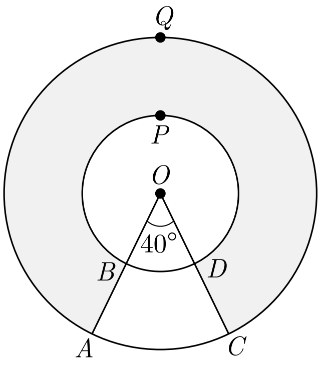
$$= \frac{300°}{360°} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7}$$

$$= 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

93. In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$. Use $\pi = \frac{22}{7}$.

I136



Ans : [Board Term-2 OD 2016]

Radii of two concentric circle is 7 cm and 14 cm.
Angle $\angle AOC = 40°$,
Angle $\angle AOC = 360° - 40° = 320°$
Area of shaded region,

$$\frac{\theta}{360°}\pi[R^2 - r^2] = \frac{320°}{360°} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21 = \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

Required area $= \frac{1232}{3} \text{ cm}^2$

$$= 410.67 \text{ cm}^2$$

94. Find the area of minor segment of a circle of radius 14 cm, when its centre angle is $60°$. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 OD 2015]

Here, $r = 14$ cm, $\theta = 60°$

I137

Area of minor segment,

$$\pi r^2 \frac{\theta}{360°} - \frac{1}{2}r^2 \sin\theta = \pi(14)^2 \frac{60°}{360°} - \frac{1}{2} \times (14)^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60°}{360°} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

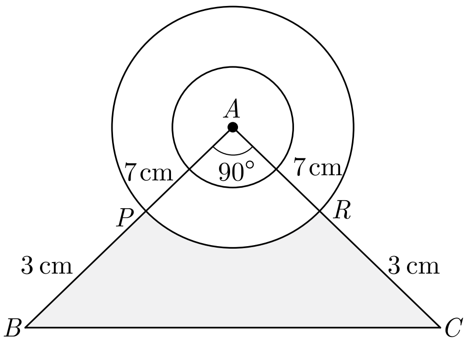
$$= \frac{22}{7} \times 14 \times 14 - \frac{308}{3} + 49\sqrt{3}$$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

$$= 598 \text{ cm}^2 \text{ approx.}$$

95. A momento is made as shown in the figure. Its base $PBCR$ is silver plate from the front side. Find the area which is silver plated. Use $\pi = \frac{22}{7}$.

I138



Ans : [Board Term-2 2015]

From the given figure area of right-angled ΔABC,

$$\frac{1}{2}AC \times AB = \frac{1}{2} \times 10 \times 10 = 50$$

Area of quadrant APR is the $\frac{1}{4}$ of the circle of radii 7 cm.

Thus area of quadrant APR of the circle of radii 7 cm

$$\frac{1}{4}\pi(7)^2 = \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5 \text{ cm}^2$$

Area of base $PBCR$

$$= \text{Area of } \Delta ABC - \text{Area of quadrant } APR$$

$$= 50 - 38.5 = 11.5 \text{ cm}^2$$

96. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 2015]

Let radius of the circle be r.

Now as per question statement we have

I139

$$\text{Circumference} = \text{Diameter} + 16.8 \text{ cm}$$

$$2\pi r = 2r + 16.8 \text{ cm}$$

$$2\left(\frac{22}{7}\right)r = 2r + 16.8$$

$$\frac{44}{7}r = 2r + 16.8$$

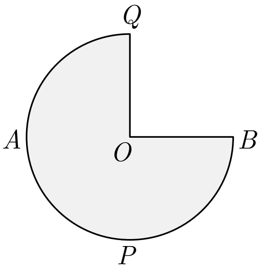
$$44r = 14r + 16.8 \times 7$$

$$30r = 177.6$$

$$r = \frac{117.6}{30} = 3.92$$

Thus $r = 3.92$ cm

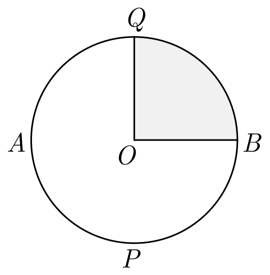
97. In fig. APB and AQP are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 Delhi 2015]

We have redrawn the given figure as shown below;

I140



Let r be the radius of given circle. It is given that perimeter of given figure is 47 cm.

$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$

$$\frac{3\pi r}{2} + 2r = 47$$

$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$

$$r\left(\frac{33}{7} + 2\right) = 47$$

$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

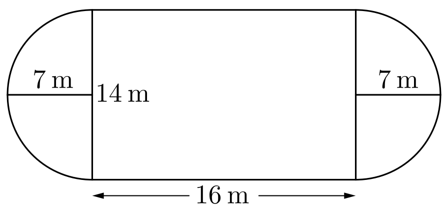
$$A = \text{area of circle} - \frac{1}{4} \text{ area of circle}$$

$$= \frac{3}{4} \text{ area of circle}$$

$$= \frac{3}{4}\pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

98. Find the area of the adjoining diagram.



Ans : [Board Term-2, 2014]

The given figure is combination of one rectangle and two semicircle of same radii .

Required area,

I141

$= $ area of two semi-circles + area of rectangle

$= $ area of one circle + area of rectangle

$= \pi r^2 + (l \times b)$

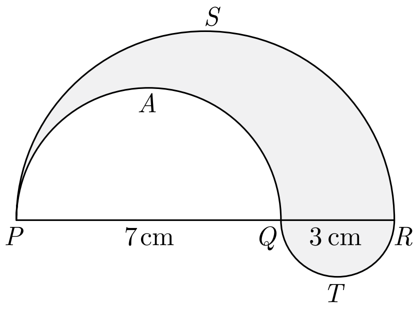
(where r is radius of circle and l and b are length and breadth of rectangle)

$= \frac{22}{7} \times 7^2 + (16 \times 14)$

$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$

$= 154 + 224 = 378 \text{ m}^2$

99. In the fig., PSR, RTQ and PAQ are three semi-circles of diameters 10 cm, 3 cm and 7 cm region respectively. Find the perimeter of shaded region. Use $\pi = \frac{22}{7}$.



I142

Ans : [Board Term-2 Delhi 2014]

Perimeter of shaded region

$= $ Perimeter of semi-circles $PSR + RTQ + PAQ$

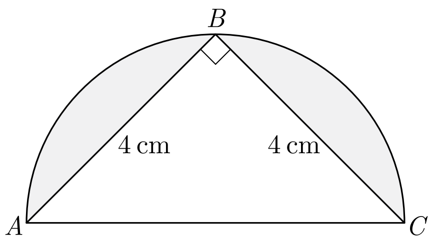
$= \pi(5) + \pi(1.5) + \pi(3.5)$

$= \pi(10)$

$= \frac{22}{7} \times 10 = \frac{220}{7} = 31.4$ cm

Perimeter of shaded region is 31.4 cm approx.

100. In the figure, ΔABC is in the semi-circle, find the area of the shaded region given that $AB = BC = 4$ cm.(Use $\pi = 3.14$)



Ans : [Board Term-2 Delhi 2014]

As ΔABC is a triangle in semi-circle, $\angle B$ is right angle,

$$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm}$$

I143

Radius of circle $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm

Area of shaded portion,

$= $ Area of the semi-circle $-$ (Area of ΔABC)

$= \left\{\frac{1}{2}\pi \times (2\sqrt{2})^2\right\} - \left\{\frac{1}{2} \times 4 \times 4\right\}$

$= \left\{\frac{1}{2} \times 3.14 \times 8\right\} - 8$

$= 12.56 - 8 = 4.56 \text{ cm}^2$

101. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of sector formed by the arc.

Ans : [Board Term-2 Delhi Compt. 2017]

I144

We have $r = 21$ cm and $\theta = 60^\circ$

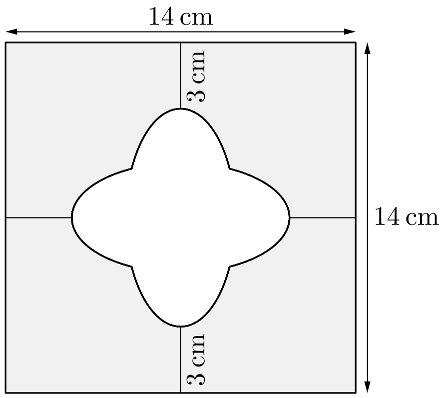
Area formed the sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times 22 \times 3 \times 21$

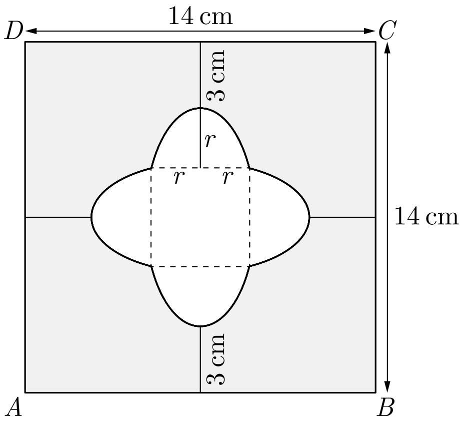
$= 11 \times 21 = 231 \text{ cm}^2$

102. In fig., find the area of the shaded region ($\pi = 3.14$)



Ans : [Board Term-2 2011, Delhi 2015]

We have redrawn the given figure as shown below.



$$3 + r + 2r + r + 3 = 14$$

$$4r + 6 = 14 \Rightarrow r = 2$$

Thus radius of the semi-circle formed inside is 2 cm and length of the side of square formed inside the semi-circle is 4 cm.

Area of square $ABCD$

$$= 14 \times 14 = 196 \text{ cm}^2$$

I145

Thus area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$

$$= 2 \times 3.14 \times 2 \times 2 = 25.12 \text{ cm}^2$$

Area of the square formed inside the semi-circle

$$(2r)^2 = 4 \times 4 = 16 \text{ cm}^2$$

Area of the shaded region,

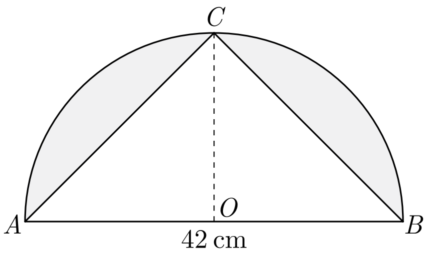
$=$ area of square $ABCD$

$-$ (Area of 4 semi-circle $+$Area of square)

$$= 196 - (25.12 + 16)$$

$$= 196 - 41.12 = 154.88 \text{ cm}^2$$

103. In the figure, ΔACB is in the semi-circle. Find the area of shaded region given that $AB = 42$ cm.



Ans : [Board Term-2 2014]

Here base of triangle is equal to the diameter of semicircle which is 42 cm.

Base of triangle $=$ diameter of semicircle

$$= 42 \text{ cm}$$

and its heigh $=$ radius of semicircle

$$= \frac{42}{2} = 21 \text{ cm}$$

I146

Area of shaded portion,

$= $ Area of semicircle$-$area of ΔABC

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \frac{22}{7} \times (21)^2 - \frac{1}{2} \times 42 \times 21$$

$$= \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{2} \times 42 \times 21$$

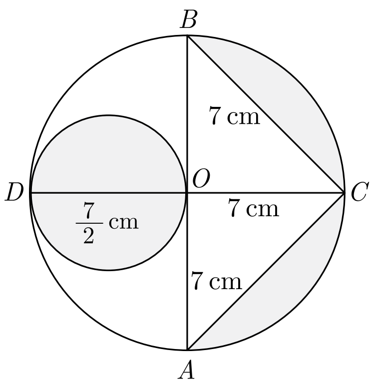
$$= 11 \times 3 \times 21 - 21 \times 21$$

$$= 693 - 441 = 252$$

Hence, the area of shaded portion $= 252 \text{ cm}^2$

104. AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region.

I147



Ans : [Board Term-2, 2012]

Area of a circle with DO as diameter

$$\pi r^2 = \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ sq.cm}$$

Area of semi-circle with AB as diameter

$$\frac{\pi r_1^2}{2} = \frac{1}{2}\pi(7)^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ sq.cm}$$

Area of ΔABC $= \frac{1}{2} \times 14 \times 7 = 49$ sq.cm

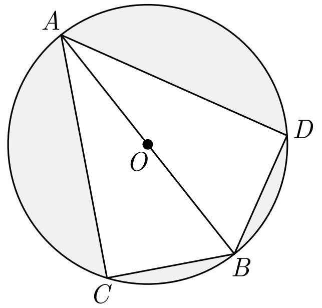
Area of shaded region

$= \text{Area of circle} + \text{Area of semi-circle} - \text{Area of } \Delta ABC$

$= \frac{77}{2} + 77 - 49 = 66.5 \text{ cm}^2$

105. Find the area of the shaded region in figure, if $BC = BD = 8$ cm, $AC = AD = 15$ cm and O is the centre of the circle. (Take $\pi = 3.14$)

I148



Ans : [Board Term-2 2012]

Since $\angle ADB$ and $\angle ACB$ are angle in a semicircle,

$$\angle ADB = \angle ACB = 90^\circ$$

Since $\Delta ADB \cong \Delta ACB$

Thus $\text{ar}\Delta ADB = \text{ar}\Delta ACB$

$$= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$$

and $\text{ar}\Delta ADB + \text{ar}\Delta ACB = 2 \times 60 = 120 \text{ cm}^2$

Now in ΔABC, we have

$$AB = \sqrt{AC^2 + BC^2}$$

$$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

$$= 17 \text{ cm}$$

Area of circle $\pi r^2 = \frac{22}{7} \times \frac{17}{2} \times \frac{17}{2}$

$$= 226.87 \text{ cm}^2$$

Area of shaded portion,

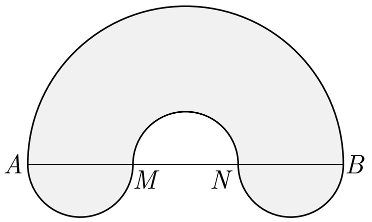
$= \text{area of circle} - \text{area of sum of } \Delta ACB \text{ and } \Delta ADB.$

$= 226.87 - 120 = 106.87 \text{ cm}^2$

Hence, area of shaded region

$$= 106.87 \text{ cm}^2$$

106. In the given figure, AB is the diameter of the largest semi-circle. $AB = 21$ cm, $AM = MN = NB$. Semi-circle are drawn with AM, MN and NB as shown. Using $\pi = \frac{22}{7}$, calculate the area of the shaded region.



Ans : [Board Term-2 2012]

We have $AB = 21$ cm

Radius of semi-circle with diameter AB,

I149

$$R = \frac{21}{2}$$

Here $AM = MN = NB = \frac{21}{3} = 7$ cm

Thus radii of smaller semi circle $r = \frac{7}{2}$ cm

Area of semi-circle with radius R

$$\frac{1}{2}\pi R^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{4} \text{ cm}^2$$

Area of semi-circle with diameter AM, MN and NB are equal

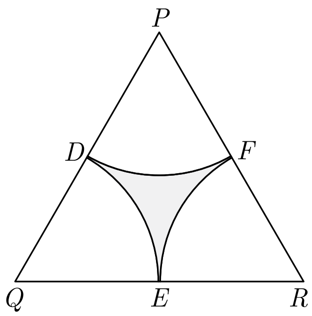
$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4} \text{ cm}^2$$

Area of shaded region

$= \text{Area largest semicircle} + \text{smallest semicircle}$

$= \frac{693}{4} + \frac{77}{4} = \frac{770}{4} = 192.5 \text{ cm}^2$

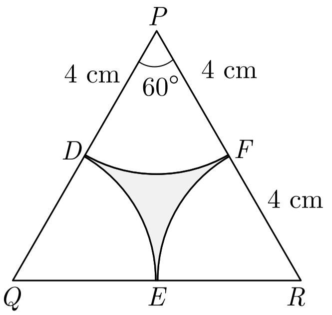
107. In the given figure, ΔPQR is an equilateral triangle of side 8 cm and D, E, F are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$) and $\sqrt{3} = 1.732$



Ans : [Board Term-2, 2012]

Here angle $\angle P = \angle Q = \angle R = 60^\circ$ because triangle is equilateral. side of triangle is 8 cm.

Consider circular section PDE. Radius of circular arc is 4 cm.



I150

Area of sector PDF,

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times 3.14 \times 4 \times 4$$

$$= \frac{1}{6} \times 3.14 \times 16 = 8.373$$

Area of shaded region

$$= \text{Area of } \Delta PQR - 3(\text{area of sector})$$

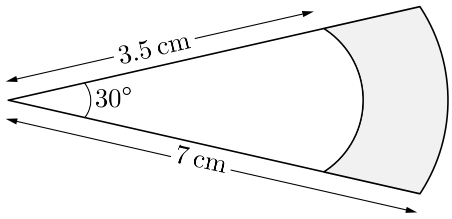
$$= \frac{\sqrt{3}}{4}(8)^2 - 3 \times 8.373$$

$$= 16\sqrt{3} - 3 \times 8.373$$

$$= 16 \times 1.732 - 25.12$$

$$= 27.712 - 25.12 = 2.59 \text{ cm}^2$$

108. In fig., sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 2012]

Area of shaded region,

$$\frac{\theta}{360^\circ}\pi(R^2 - r^2) = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times (7^2 - 3.5^2)$$

$$= \frac{1}{12} \times \frac{22}{7} \times (7 + 3.5)(7 - 3.5)$$

I151

$$= \frac{1}{12} \times \frac{22}{7} \times 10.5 \times 3.5$$

$$= \frac{1}{12} \times \frac{22}{7} \times \frac{21}{2} \times \frac{7}{2} = \frac{77}{8} = 9.62 \text{ cm}^2$$

109. A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm². If the wire is bent in the form of a circle, find the area enclosed by the circle. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 OD 2017]

Let l be length of wire. If it is bent in the form of an equilateral triangle, side of triangle will be $\frac{l}{3}$.

Area enclosed by the triangle,

$$\frac{\sqrt{3}}{4} \times \left(\frac{l}{3}\right)^2 = 121\sqrt{3}$$

I152

$$\frac{1}{4} \times \left(\frac{l}{3}\right)^2 = 121$$

$$\frac{1}{2} \times \frac{l}{3} = 11$$

$$l = 66 \text{ cm}$$

Same wire is bent in the form of circle. Thus circumference of circle will be 66.

$$2\pi r = 66$$

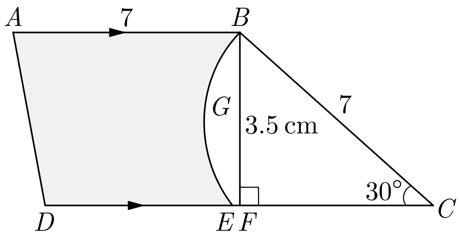
$$r = \frac{66}{2\pi} = \frac{66}{2 \times \frac{22}{7}} = \frac{21}{2}$$

Area enclosed by the circle

$$\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$$

110. Adjoining fig, $ABCD$ is a trapezium with $AB || DC$ and $\angle BCD = 30^\circ$. Fig. $BGEC$ is a sector of a circle with centre C and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the area of the shaded region. Use $\pi = \frac{22}{7}$.

I153



Ans : [Board Term-2 OD Compt. 2017]

We have $AB = 7$ cm

$DE = 4$ cm, and

$BF = 3.5$ cm

Now $DC = DE + EC = 4 + 7 = 11$ cm

Area of Trapezium $ABCD$

$$\text{Area}_{\square} = \frac{1}{2}(DC+AB)(BF)$$

$$= \frac{1}{2}(11+7)\times 3.5 = \frac{1}{2}\times 18\times 3.5$$

$$= 31.5 \text{ cm}^2$$

Area of circular sector,

$$\text{Area}_{\frown} = \frac{30^\circ}{360^\circ}\times\frac{22}{7}\times 7\times 7$$

$$= \frac{1}{12}\times 22\times 7$$

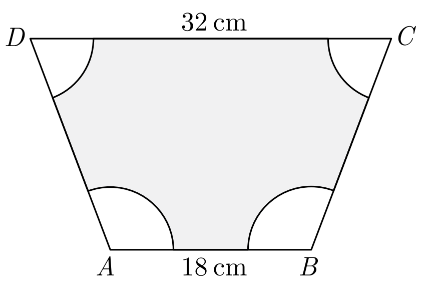
$$= 12.83 \text{ cm}^2$$

Area of shaded region,

$$= \text{Area}_{\square} - \text{Area}_{\frown}$$

$$= 31.5 - 12.83 = 18.67 \text{ cm}^2$$

111. In the given figure $ABCD$ is a trapezium with $AB || DC, AB = 18$ cm and $DC = 32$ cm and the distance between AB and AC is 14 cm. If arcs of equal radii 7 cm taking A, B, C and D have been drawn, then find the area of the shaded region.



I154

Ans : [Board Term-2 Foreign 2017]

In trapezium $ABCD$, we have $AB = 18$ cm, $CD = 32$ cm $AB || CD$ and distance between || lines $= 14$ cm and the radius of each sector $= 7$ cm.

Area of trapezium $ABCD$,

$$\text{Area}_{\square} = \frac{1}{2}(18+32)\times 14 = \frac{1}{2}\times 50\times 14$$

$$= 350 \text{ cm}^2$$

Let, $\angle A = \theta, \angle B = \theta_2, \angle C = \theta_3$ and $\angle D = \theta_4$

Area of sector A,

$$\frac{\theta_1}{360^\circ}\pi r^2 = \frac{\theta_1}{360^\circ}\times\frac{22}{7}\times 7\times 7$$

$$= \frac{\theta_1}{360^\circ}\times 154 \text{ cm}^2$$

$$\text{area of sector } B = \frac{\theta_2}{360^\circ}\times 154 \text{ cm}^2$$

$$\text{area of sector } C = \frac{\theta_3}{360^\circ}\times 154 \text{ cm}^2$$

$$\text{area of sector } D = \frac{\theta_4}{360^\circ}\times 154 \text{ cm}^2$$

$$\text{area of 4 sectors} = \frac{\theta_1+\theta_2+\theta_3+\theta_4}{360^\circ}\times 154$$

$$\text{Area}_{4\frown} = \frac{360^\circ}{360^\circ}\times 154 = 154 \text{ cm}^2$$

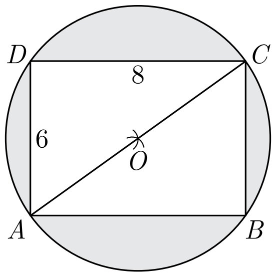
Thus area of shaded region,

$$= \text{Area}_{\square} - \text{Area}_{4\frown}$$

$$= 350 - 154 = 196 \text{ cm}^2$$

FOUR MARKS QUESTIONS

112. Find the area of the shaded region in Figure, if $ABCD$ is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take $\pi = 3.14$)



I243

Ans : [Board 2019 Delhi]

In ΔABC, $\angle B = 90^\circ$

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 6^2$$

$$= 64 + 36 = 100$$

$$AC = 10 \text{ cm}$$

Since, AC is the diameter of circle,

Radius of circle, $r = 5$ cm

Area of the shaded region

$= \text{(area of the circle)} - \text{(area of the rectangle)}$

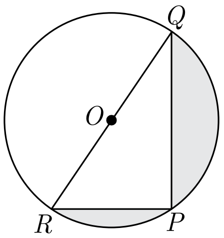
$= \pi r^2 - (AB \times BC)$

$= 3.14 \times 5^2 - (8 \times 6)$

$= 78.5 - 48$

$= 30.5 \text{ cm}^2$

113. Find the area of the shaded region in Figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle.



I245

Ans : [Board 2020 OD Standard]

We have $PQ = 24$ cm

$PR = 7$ cm

The angle in the semicircle is right angle, therefore

$\angle RPQ = 90^\circ$

In ΔRPQ, $RQ^2 = PR^2 + PQ^2$

$RQ^2 = (7)^2 + (24)^2$

$= 49 + 576 = 625$

$RQ = 25 \text{ cm}$

Area of $\Delta RPQ = \frac{1}{2} \times RP \times PQ$

$= \frac{1}{2} \times 7 \times 24$

$= 84 \text{ cm}^2$

area of semi-circle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2$

$= \frac{11 \times 625}{7 \times 4} = \frac{6875}{28}$ cm

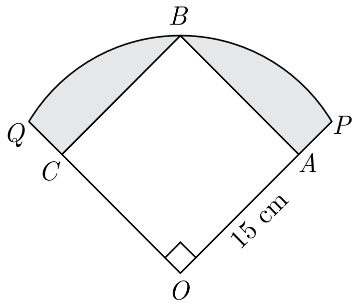
Now, area of shaded region

$= \text{area of semi-circle} - \text{area of } \Delta RPQ$

$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$

$= \frac{4523}{28} = 161.54 \text{ cm}^2$

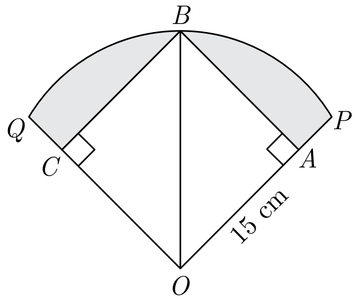
114. In Figure, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 15$ cm, find the area of the shaded region. (Use $\pi = 3.14$).



I246

Ans : [Board 2019 OD]

We have redrawn the figure given below.



Using Pythagoras theorem in ΔBAO,

$OB^2 = OA^2 + AB^2 = 15^2 + 15^2$

$= 225 + 225 = 450$

$OB = \sqrt{450} = 15\sqrt{2}$

Thus radius $OB = 15\sqrt{2}$ cm.

Area of square $= (OA)^2 = (15)^2 = 225 \text{ cm}^2$

Now, area of quadrant,

$\frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times (15\sqrt{2})^2$

$= \frac{1}{4} \times 3.14 \times 225 \times 2$

$= \frac{3.14 \times 225}{2}$

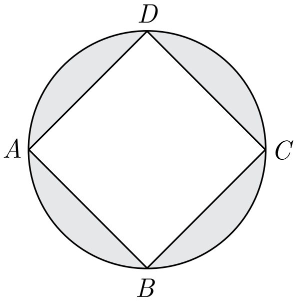
$= 353.25 \text{ cm}^2$

Therefore, area of shaded region

= Area of quadrant $OPBQ$ − area of square $OABC$

$= 353.25 - 225 = 128.25 \text{ cm}^2$

115. In Figure, $ABCD$ is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region.(Use $\pi = 3.14$).



I247

Ans : [Board 2019 OD]

Side of square, $a = 2\sqrt{2}$ cm.

Area of square $a^2 = (2\sqrt{2})^2 = 8 \text{ cm}^2$

Length of the diagonal of a square is given by,

$$d = a\sqrt{2}$$

$$= 2\sqrt{2} \times \sqrt{2} = 4 \text{ cm}$$

Since, the square is inscribed in a circle, hence the diagonal of square will be the diameter of the circle,

Radius, $r = \frac{d}{2} = \frac{4}{2} = 2 \text{ cm}$

Area of the circle,

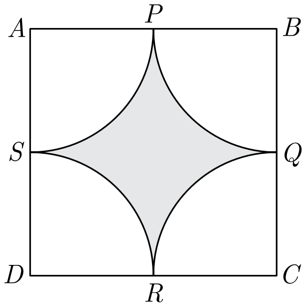
$$\pi r^2 = 3.14 \times (2)^2 = 12.56 \text{ cm}^2$$

Therefore, area of shaded region

= Area of circle − Area of the square

$= (12.56 - 8) = 4.56 \text{ cm}^2$

116. Find the area of the shaded region in Figure, where arcs drawn with centres A, B, C and D intersect in pairs at midpoint P, Q, R and S of the sides AB, BC, CD and DA respectively of a square $ABCD$ of side 12 cm. [Use $\pi = 3.14$]



I248

Ans : [Board 2018]

Radius of each arc drawn is $r = \frac{12}{2} = 6$ cm.

Area of one quadrant is $\frac{1}{4}\pi r^2$, thus area of four quadrants,

$$4 \times \tfrac{1}{4}\pi r^2 = \pi \times 6^2 = 3.14 \times 36$$

$$= 113.04 \text{ cm}^2$$

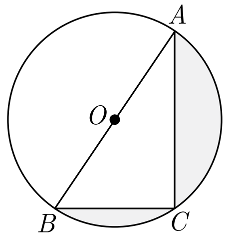
Area of square $ABCD$,

$$= 12 \times 12 = 144 \text{ cm}^2$$

Hence Area of shaded region

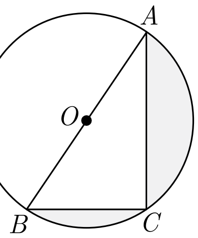
$$= 144 - 113.04 = 30.96 \text{ cm}^2$$

117. In the figure, O is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$cm. BC is joined. Find the area of the shaded region. ($\pi = 3.14$)



Ans : [Board Term-2 OD 2016]

We redraw the given figure as below.



I155

Radius of semi circle ACB,

$$r = \frac{13}{2} \text{ cm}$$

Area of semicircle,

$$\frac{\pi}{2}r^2 = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2}$$

$$= \frac{3.14 \times 169}{8} = \frac{530.66}{8} \text{ cm}^2$$

The angle subtended on a semicircle is a right angle, thus $\angle ACB = 90°$

In ΔABC,

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area of triangle ΔABC,

$$\Delta = \frac{1}{2} \times \text{Base} \times \text{Hight}$$

$$= \frac{1}{2} \times AC \times BC$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Area of shaded region,

$$\frac{\pi}{2} r^2 - \Delta = \frac{530.66}{8} - 30$$

$$= (66.3325 - 30) \text{ cm}^2$$

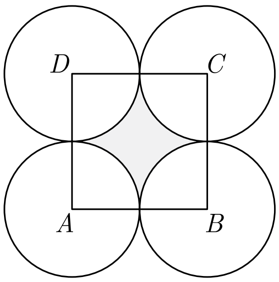
$$= 36.3325 \text{ cm}^2$$

118. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm². Find the radius of each circle.

Ans : [Board Term-2 SQP 2017]

As per question statement the figure is shown below.

I156



Let r be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm²

$$(2r)^2 - 4\left(\pi r^2 \times \frac{90^\circ}{360^\circ}\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7} r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

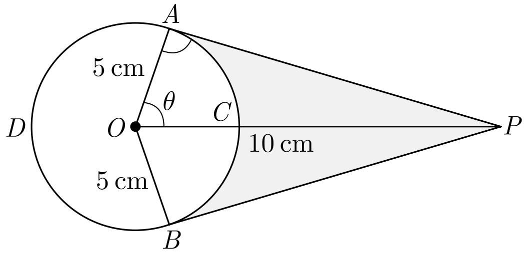
$$6r^2 = 24$$

$$r^2 = 4 \Rightarrow r = \pm 2$$

Thus radius of each circle is 2 cm.

119. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Ans : [Board Term-2 Delhi 2016]

I157

Here AP is tangent at point A on circle.

Thus $\angle OAP = 90^\circ$

Now $$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} = \cos 60^\circ$$

Thus $$\theta = 60^\circ$$

Reflex $$\angle AOB = 360^\circ - 60^\circ - 60^\circ = 240^\circ$$

Now $$\text{arc } ADB = \frac{2 \times 3.14 \times 5 \times 120^\circ}{360^\circ}$$

$$= 20.93 \text{ cm}$$

Hence length of elastic in contact is 20.93 cm.

Now, $$AP = 5\sqrt{3} \text{ dm}$$

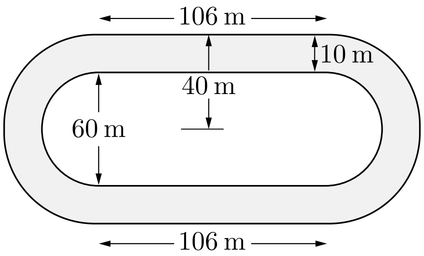
Area $(\Delta\, OAP + \Delta\, OBP) = 25\sqrt{3} = 43.25$ cm²

Area of sector $OACB$,

$$= 25 \times 3.14 \times \frac{120^\circ}{360^\circ} = 26.16 \text{ cm}^2.$$

Shaded Area $= 43.25 - 26.16 = 17.09$ cm²

120. Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.



Ans : [Board Term-2 2011]

I158

Width of the inner parallel lines $= 60$ m

Width of the outer lines $= 40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the outer semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180$ m^2

Area of outer rectangle $= 106 \times 80 = 4240$ m^2.

Area of the inner semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7} \text{ m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7} \text{ m}^2$$

Area of racing track

= (area of outer rectangle + area of outer semicircles)
− (area of inner rectangle + area of inner semicircles)

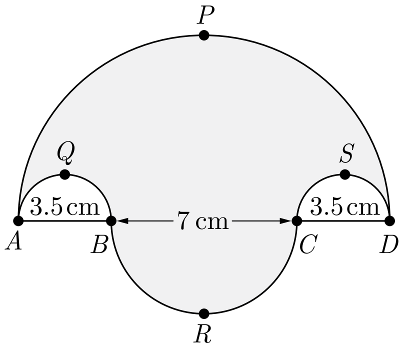
$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$

$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$

$$= \frac{22820}{7} = 3260 \text{ m}^2$$

Hence, area of track is 3260 m^2

121. Find the area of the shaded region in Figure, $\overparen{APD}, \overparen{AQB}, \overparen{BRC}$ and $\overparen{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 Foreign 2016]

Diameter of the largest semi circle $= 14$ cm

$$\text{Radius } = \frac{14}{2} = 7 \text{ cm}$$

Diameter of two equal unshaded semicircle $= 3.5$ cm

$$\text{Radius of each circle } = \frac{3.5}{2} \text{ cm}$$

Diameter of smaller shaded semi-circle $= 7$ cm

$$\text{Radius } = 3.5 \text{ cm}$$

I159

Area of shaded portion

= area of largest semi-circle+

+ area of smaller shaded semicircle+

− area of two unshaded semicircles

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$-2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

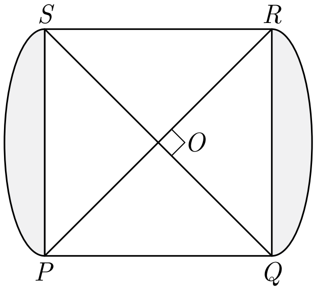
$$= \frac{1}{2} \times \frac{22}{7}\left[7^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{7}{4}\right)^2\right] \text{cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2\left[1 + \frac{1}{4} - \frac{1}{8}\right]$$

$$= 11 \times 7\left[\frac{9}{8}\right]$$

$$= \frac{693}{8} \text{ sq. cm or } 86.625 \text{ cm}^2$$

122. In figure, $PQRS$ is square lawn with side $PQ = 42$ metre. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



Ans : [Board Term-2 OD 2015]

Radius of circle with centre O is OR.

Let OR be x then using Pythagoras theorem we have

$$x^2 + x^2 = (42)^2 \Rightarrow x = 21\sqrt{2} \text{ m}$$

Area of segment of circle with centre angle 90°

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (21\sqrt{2})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2$$

$$= 11 \times 3 \times 21 = 693$$

I160

Area of triangle ΔROQ,

$$= \frac{1}{2} \times (21\sqrt{2})^2 = 21 \times 21 = 441$$

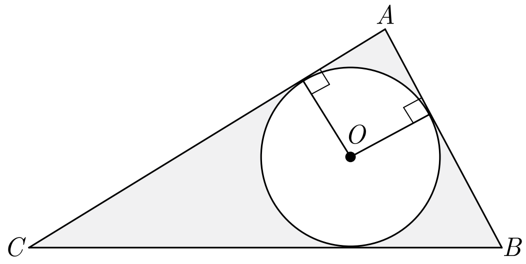
Area of the one side flower bed

$$= 693 - 441 = 252 \text{ m}^2$$

Area of flower bed of both

$$= 2 \times 252 = 504 \text{ m}^2$$

123. In the figure, ABC is a right angled triangle right angled at $\angle A$. Find the area of the shaded region, if $AB = 6$ cm, $BC = 10$ cm and O is the centre of the circle of the triangle ABC.



Ans : [Board Term-2 2015]

Let r be the radius of incircle.

Using the tangent properties we have

I161

$$BC = 8 - r + 6 - r$$

$$10 = 14 - 2r$$

,

$$2r = 4 \Rightarrow r = 2 \text{ cm}$$

Area of circle $\pi r^2 = \frac{22}{7} \times 2 \times 2 = \frac{88}{7} = 12.57 \text{ cm}^2$

Now,area of ΔABC,

$$\Delta_{ABC} = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Area of shaded region

$$= \text{Area of } \Delta ABC - \text{ Area of the circle}$$

$$= 24 - 12.57 \text{ cm}^2 = 11.43 \text{ cm}^2$$

124. Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is 130π cm^2. Find the radius of each bead.

Ans : [Board Term-2, 2015]

Let the radii of the circles are r_1 and r_2.

$$r_1 + r_2 = 14 \quad ...(1)$$

Sum, of their areas,

$$\pi(r_1^2 + r_2^2) = 130\pi$$

$$r_1^2 + r_2^2 = 130 \quad ...(2)$$

Now $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1 r_2$

$$(14)^2 = 130 + 2r_1 r_2$$

$$2r_1 r_2 = 196 - 130 = 66$$

I162

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1 r_2$$

$$= 130 - 66 = 64$$

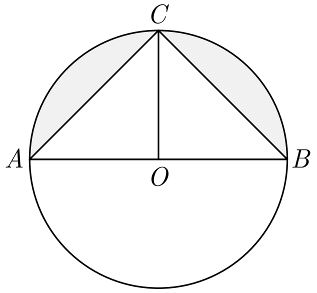
Thus $r_1 - r_2 = 8 \quad ...(3)$

From (1) and (3), we get

$$2r_1 = 22 \Rightarrow r_1 = 11 \text{ cm}$$

$$r_2 = 14 - 11 = 3 \text{ cm.}$$

125. A round thali has 2 inbuilt triangular for serving vegetables and a separate semi-circular area for keeping rice or chapati. If radius of thali is 21 cm, find the area of the thali that is shaded in the figure.



Ans : [Board Term-2 2014]

Since AOB is the diameter of the circle, area of shaded region,

I163

$= (\text{Area of semi-circle} - \text{Area of } \Delta ABC)$

Area of semi-circle

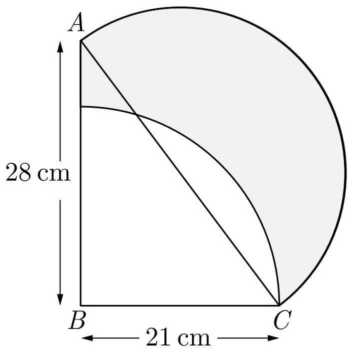
$$\frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{1386}{2} = 693 \text{ cm}^2$$

$$\text{Area of triangle} = \frac{1}{2} \times 21 \times 42 = 441 \text{ cm}^2$$

$$\text{Area of shaded region} = 693 - 441 = 252 \text{ cm}^2$$

126. In the fig., ABC is a right-angle triangle, $\angle B = 90^\circ$, $AB = 28$ cm and $BC = 21$ cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region.



Ans : [Board Term -2 2011, Foreign 2014]

In right angled triangle ΔABC using Pythagoras theorem we have

I164

$$AC^2 = AB^2 + BC^2$$

$$= 28^2 + 21^2$$

$$= 784 + 441$$

or $AC^2 = 1225 \Rightarrow AC = 35$ cm

Area of shaded region,

$= \text{area of } \Delta ABC +$

$+ \text{ area of semi-circle with diameter } AC +$

$- \text{ area of quadrant with radius } BC$

$$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 - \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

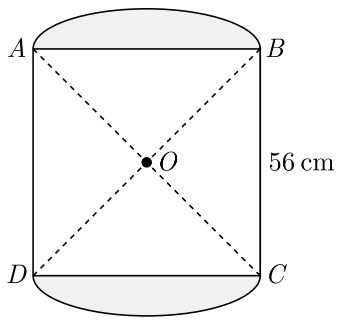
$$= 21 \times 14 + \frac{11}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$

$$= 21 \times 14 + \frac{55}{2} \times \frac{35}{2} - \frac{11}{2} \times 3 \times 21$$

$$= 294 + 481.25 - 346.5$$

$$= 775.25 - 346.5 = 428.75 \text{ cm}^2.$$

127. In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.



Ans : [Board Term-2 2011]

Side of square $= 56$

Diagonal of square $= 56\sqrt{2}$

I165

Radius of circle $= \frac{1}{2} \times 56\sqrt{2} = 28\sqrt{2}$

Total area $=$ Area of sector $OAB +$

$+$ Area of sector $ODC +$

$+$ Area of $\Delta OAD +$

$+$ Area of ΔOBC

$$= \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} + \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} +$$

$$+\frac{1}{4}\times 56\times 56+\frac{1}{4}\times 56\times 56$$

$$=\frac{1}{4}\times\frac{22}{7}\times(28\sqrt{2})^2+\frac{1}{4}\times\frac{22}{7}\times(28\sqrt{2})^2+$$

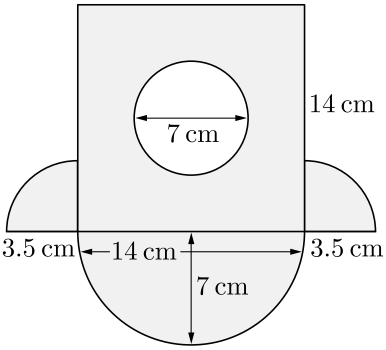
$$+\frac{1}{4}\times 56\times 56+\frac{1}{4}\times 56\times 56$$

$$=\frac{1}{4}\times 28\times 56\left(\frac{22}{7}+\frac{22}{7}+2+2\right)\ \text{m}^2$$

$$=7\times 56\left(\frac{22+22+14+14}{7}\right)\ \text{m}^2$$

$$=56\times 72=4032\ \text{m}^2.$$

128. In fig., find the area of the shaded region. Use $\pi=\frac{22}{7}$.



Ans : [Board Term-2 2011]

Area of square $=(14)^2=196\ \text{cm}^2$

Area of internal circle $=\frac{22}{7}\times\left(\frac{7}{2}\right)^2\ \text{cm}^2$

$$=\frac{77}{2}=38.5\ \text{cm}^2$$

Area of semi-circle with 14 cm diameter

$$=\frac{1}{2}\times\frac{22}{7}\times 7^2\ \text{cm}^2$$

$$=77\ \text{cm}^2$$

Area of two quarter circles of radius $\frac{7}{2}$ cm

$$=2\times\frac{1}{4}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2=\frac{77}{4}=19.25\ \text{cm}^2$$

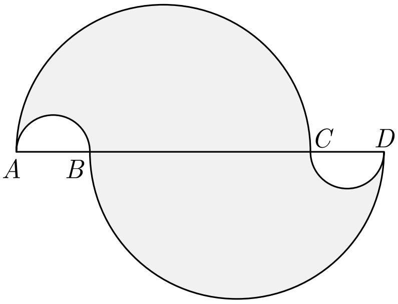
Shaded area $=196-38.5+77+19.25$

$$=292.25-38.5$$

$$=253.75\ \text{cm}^2.$$

I166

129. In fig., $AC=BD=7$ cm and $AB=CD=1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. Use $\pi=\frac{22}{7}$.



Ans : [Board Term-2 2011]

Area of shaded region

$=2$(Area of semi-circle of radius $\frac{7}{2}$ cm)

-2(Area of semi-circle of radius $\frac{7}{8}$ cm)

$$=2\left[\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\right]-2\left[\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{8}\right)^2\right]$$

$$=2\times\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\left[1-\left(\frac{1}{4}\right)^2\right]$$

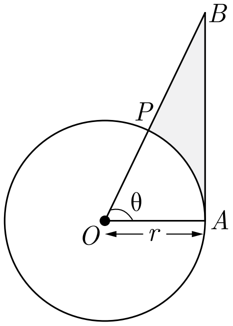
$$=\frac{77}{2}\left[1-\frac{1}{16}\right]=\frac{77}{2}\times\frac{15}{16}=\frac{1155}{32}\ \text{cm}^2$$

$$=36.09\ \text{cm}^2$$

I167

130. The given fig. is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is

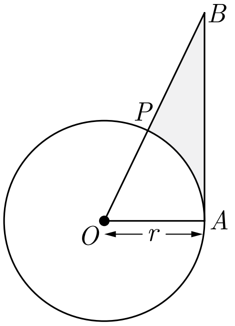
$r = \left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$



Ans : [Board Term-2 OD 2015, 2016]

As per question statement we have redrawn this figure as given below.

I168



Here OAP is sectors of circle with centre O, $\angle POA = \theta$ and $OA \perp AB$.

Perimeter of shaded region $= BP + AB + \widehat{AP}$ (1)

Now $\tan\theta = \frac{AB}{r} \Rightarrow r\tan\theta = AB$ ...(2)

$$\sec\theta = \frac{OB}{r} \Rightarrow r\sec\theta = OB$$

$$OB - OP = BP \Rightarrow r\sec\theta - r = OP \quad ...(3)$$

Length of arc AP,

$$\widehat{AP} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{\theta}{360} \times 2\pi r = \frac{\theta\pi r}{180} \quad ...(4)$$

Putting value from equation (2), (3), (4) in equation (1) we get perimeter of shaded region as

$$= r\tan\theta + r\sec\theta - r + \frac{\theta\pi r}{180}$$

$$= r\left[\tan\theta + \sec\theta + \frac{\theta\pi}{180} - 1\right]$$

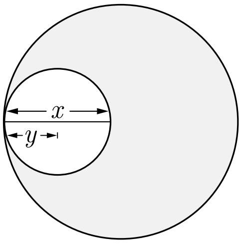
Hence, Proved.

131. Two circles touch internally. The sum of their areas is 116π and the difference between their centres is 6 cm. Find the radii of the circles.

Ans : [Board Term-2 Foreign 2017]

Let the radius of larger circle be x and the radius of smaller circle be y. As per question statement we have shown diagram below.

I169



Now $x - y = 6$ ...(1)

and $\pi x^2 + \pi y^2 = 116\pi$

$$\pi(x^2 + y^2) = 116\pi$$

$$x^2 + y^2 = 116 \quad ...(2)$$

From (1) and (2) we have

$$x^2 + (x-6)^2 = 116$$

$$x^2 + x^2 - 12x + 36 = 116$$

$$x^2 - 6x - 40 = 0$$

$$x^2 - 10x + 4x - 40 = 0$$

$$x(x-10) + 4(x+10) = 0$$

$$x = 10, \text{ and } y = 10 - 6 = 4$$

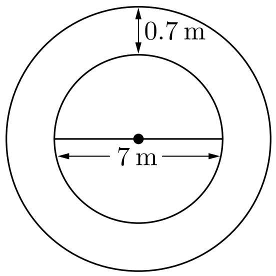
Hence, radii of the circles are 10 cm and 4 cm.

132. A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is Rs.110 per sq. m.

Ans : [Board Term-2 Foreign 2017]

As per question statement we have shown diagram below.

I170



The inner diameter of park $= 7$ m

$$\text{radius} = \frac{7}{2} = 3.5 \text{ m}$$

$$\text{Width of path} = 0.7 \text{ m}$$

Radius of park with path

$$= 3.5 + 0.7 = 4.2 \text{ m}$$

$$\text{Area of the path} = \pi(4.2)^2 - \pi(3.5)^2$$

$$= \frac{22}{7}(17.64 - 12.25)$$

$$= \frac{22}{7} \times 5.39 = 22 \times 0.77$$

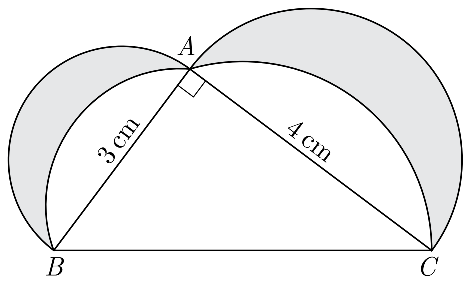
$$= 16.94 \text{ m}^2$$

Cost of the cementing the path

$$= 16.94 \times 110$$

$$= Rs. 1863.40$$

133. In the given figure, ΔABC is a right angled triangle in which $\angle A = 90^\circ$. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Ans : [Board Term-2 OD 2017]

I171

In ΔABC we have

$\angle A = 90^\circ$, $AB = 3 = 3$ cm, and $AC = 4$ cm

Now $\quad BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2} = 5$ cm.

Area of shaded Area

$= $ Area of semicircle with radius $\frac{3}{2}$ cm

$+$ area of semi circle with radius $\frac{4}{2}$ cm

$+$ Area of triangle ΔABC)

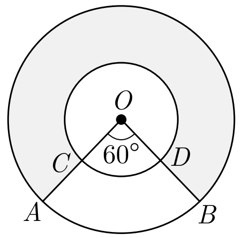
$-$ Area of semicircle with radius $\frac{5}{2}$ cm

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 + \frac{1}{2} \times 3 \times 4 - \frac{\pi}{2}\left(\frac{5}{2}\right)^2$$

$$= \frac{9\pi}{8} + 2\pi + 6 - \frac{25\pi}{8} = \frac{9\pi + 16\pi - 25\pi}{8} + 6$$

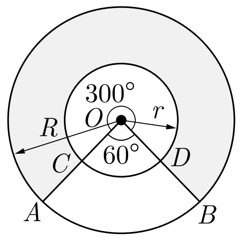
$$= 6 \text{ cm}^2$$

134. In the given figure, two concentric circle with centre O have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans :

We have redrawn the given figure as shown below.



I172

Here $\angle AOB = 60^\circ$ and $\angle COD = 60^\circ$

$$R = 42 \text{ cm}, \; r = 21 \text{ cm}$$

Reflex of $\angle AOB$,

$$\theta = (360^\circ - 60^\circ) = 300^\circ$$

Now, area of shaded region

$$\pi R^2 \frac{\theta}{360^\circ} - \pi r^2 \frac{\theta}{360^\circ} = \frac{\theta\pi}{360^\circ}(R^2 - r^2)$$

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (42^2 - 21^2)$$

$$= \frac{5}{6} \times \frac{22}{7} \times (42-21)(42+21)$$

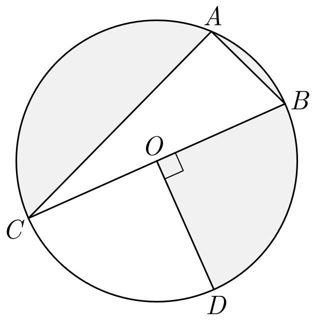
$$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$$

$$= 5 \times 11 \times 63$$

$$= 3465 \text{ cm}^2$$

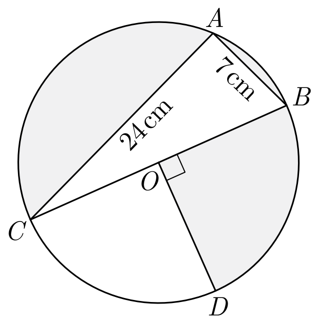
Thus area of shaded region is 3465 cm^2.

135. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of the shaded region.



Ans :

We have redrawn the given figure as shown below.



I173

Here $\Delta\, CAB$ is right angle triangle with $\angle CAB = 90°$

In right $\Delta\, CAB$, by Pythagoras theorem, we have

$$BC^2 = AC^2 + AB^2$$

$$= 24^2 + 7^2$$

$$= 576 + 49 = 625$$

Thus $BC = 25$ cm which is diameter. Now radius is $\frac{25}{2}$ or 12.5 cm.

Area of shaded region,

= area of semicircle + area of quadrant − area of $\Delta\, ACB$

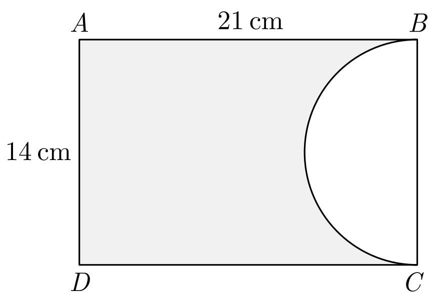
$$= \frac{1}{2}\pi r^2 + \frac{1}{4}\pi r^2 - \frac{1}{2} \times AB \times AC$$

$$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times 7 \times 24 = \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$$

$$= 368.3035 - 84 = 284.3 \text{ cm}^2$$

Thus area of shaded region $= 284.3035$ cm^2

136. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Ans : [Board Term-2 OD 2017]

Area of shaded region,

= Area of rectangle $ABCD$ − area of semicircle

$$= 21 \times 14 - \frac{\pi}{2} \times 7^2$$

$$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

I174

$= 294 - 77 = 217 \text{ cm}^2$

Perimeter of shaded area

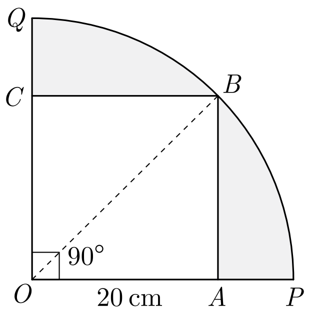
$$= AB + AD + CD + \widehat{CB}$$

$$= 21 + 14 + 21 + \frac{22}{7} \times 7$$

$$= 21 + 14 + 21 + 22 = 78 \text{ cm}$$

Hence, area of shaded region is 217 cm^2 and perimeter is 78 cm.

137. A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$]



Ans : [Board Term-2 Delhi 2014]

We have $OB = \sqrt{OA^2 + AB^2}$

$$= \sqrt{20^2 + 20^2} = \sqrt{800}$$

I175

Thus $OB = 20\sqrt{2}$ cm

radius $r = 20\sqrt{2}$

Area of shaded region

= Area of sector $OQBPO -$ Area of square $OABC$

$$= \pi r^2 \frac{90^\circ}{360^\circ} - (20)^2$$

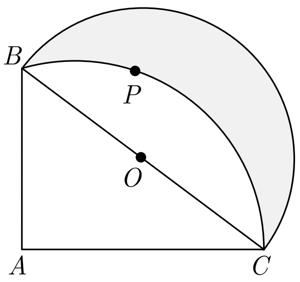
$$= 3.14 \times (20\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} - (20)^2$$

$$= 3.14 \times 200 - 400$$

$$= 628 - 400 = 228$$

Required area is 228 cm^2.

138. In given figure $ABPC$ is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the are of the shaded region.



Ans : [Board Term-2 SQP 2017]

Radius of the quadrant $AB = AC = 14$ cm

$$BC = \sqrt{14^2 + 14^2} = 14\sqrt{2} \text{ cm}$$

Radius of semicircle $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

I176

Area of semicircle $= \frac{1}{2}\pi(7\sqrt{2})^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 98$$

$$= 154 \text{ cm}^2$$

Area of segment $BPCO$

$$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2}r^2 = r^2\left(\frac{\pi\theta}{360^\circ} - \frac{1}{2}\right)$$

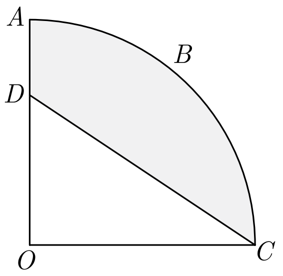
$$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\right)$$

$$= 14 \times 14\left(\frac{11}{14} - \frac{1}{2}\right)$$

$$= 14 \times 14 \times \frac{2}{7} = 56 \text{ cm}^2$$

Hence, area of shaded region is 56 cm^2.

139. In the figure $OABC$ is a quadrant of a circle of radius 7 cm. If $OD = 4$ cm, find the area of shaded region.



Ans : [Board Term-2 Foreign 2014]

Area of shaded region,

$= \text{Area of sector } OCBAD - \text{ Area of } \Delta\, ODC$

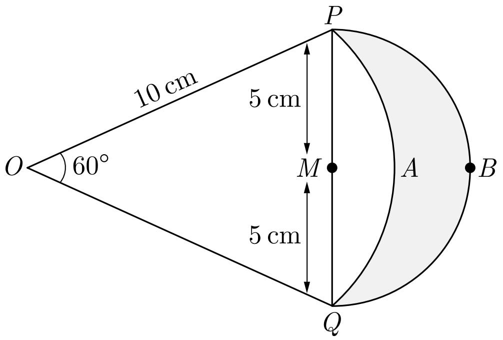
$= \pi \times 7^2 \times \frac{90^\circ}{360^\circ} - \frac{1}{2} \times 7 \times 4$

$= \pi \times 49 \times \frac{1}{4} - 14$

I178

$= \frac{49\pi}{4} - 14 = 24.5 \text{ cm}^2$

140. Figure shows two arcs PAQ and PQB. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If $OP = PQ = 10$ cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm^2.



Ans : [Board Term-2 Delhi 2016]

We have $\angle POQ = 60^\circ$

and $OP = OQ = PQ = 10$

Area of segment $PAQM$,

I180

$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)$

cm^2

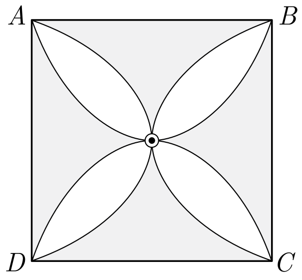
Area of semicircle $= \frac{\pi 5^2}{2} = \frac{25\pi}{2}$ cm^2

Area of shaded region,

$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$

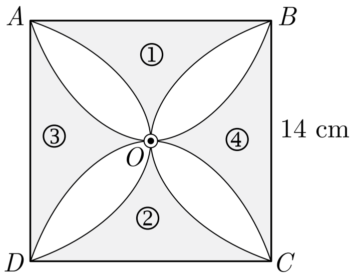
$= 25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm^2.

141. In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 Delhi 2016]

We have redrawn the given figure as shown below.



I181

If we subtract area of two semicircle AOD and COB , from square $ABCD$ we will get area of part 1 and part 2.

Area of square $= 14 \times 14 = 196$ cm^2

Radius of semicircle $= \frac{14}{2} = 7$ cm

Area of semicircle $AOB + DOC$

$= \frac{22}{7} \times 7^2 = 154$ cm^2

So, area of each of two shaded part

$196 - 154 = 42$ cm^2

Hence, area of four shaded parts is 84 cm^2.

142. The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

Ans : [Board Term-2 Foreign 2015]

I182

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours. Distance travelled by tips of hands in one round is equal to the circumference of circle.

Radius of the circle formed by long hand = 6 cm. and radius of the circle formed by short hand = 4 cm.

Distance travelled by long hand in one round

$=$ circumference of the circle $2 \times 6 \times \pi$

Distance travelled by long hand in 24 rounds

$$= 24 \times 12\pi = 288\pi$$

Distance travelled by short hand in a round $= 2 \times 4\pi$

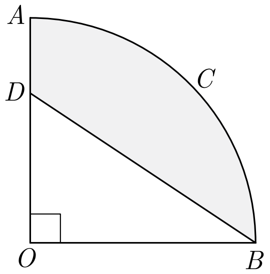
Distance travelled by short hand in 2 round

$$= 2 \times 8\pi = 16\pi$$

Sum of the distance $= 288\pi + 16\pi = 304\pi$

$$= 304 \times 3.14 = 954.56 \text{ cm}$$

143. In the given figure $DACB$ is a quadrant of a circle with centre O and radius 3.5 cm. If $OD = 2$ find the area of the region.



Ans : [Board Term-2 Delhi 2017]

Area of shaded region,

$= $ area of quadrant $OACB -$ area $\Delta\, DOB$

$$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5^2 - \frac{1}{2} \times 2 \times 3.5$$

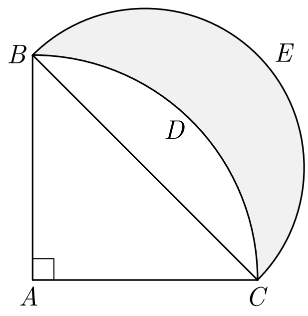
$$= 3.5\left(\frac{1}{4} \times \frac{22}{7} \times 3.5 - 1\right)$$

$$= 3.5\left(\frac{11}{4} - 1\right) = 3.5 \times \frac{7}{4} = 6.125$$

I183

Hence the area of shaded region is 6.125 cm.

144. As $ABDC$ is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.



Ans : [Board Term-2 SQP 2017]

As ABC is a quadrant of the circle, $\angle BAC$ will be 90°.

In $\Delta\, ABC$, $BC^2 = AC^2 + AB^2$

$$= (28)^2 + (28)^2 = 2 \times (28)^2$$

$$BC = 28\sqrt{2} \text{ cm}$$

Radius of semi-circle drawn on BC,

$$= \frac{28\sqrt{2}}{2} = 14\sqrt{2}$$

I184

Area of semi-circle $= \frac{1}{2}\pi(14\sqrt{2})^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \times 2$$

$$= 616 \text{ cm}^2$$

Area of $\Delta\, ABC$ $= \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$

Area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

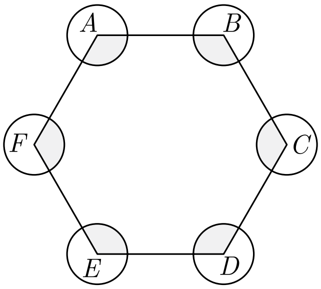
$$= 616 \text{ cm}^2$$

Area of the shaded region

$=$ Area of semi-circle $+$area of Δ $-$ Area of quadrant

$= 616 + 392 - 616 = 392 \text{ cm}^2$.

145. In fig., $ABCDEF$ is any regular hexagon with different vertices A, B, C, D, E and F as the centres of circle with same radius r are drawn. Find the area of the shaded portion.



Ans : [Board Term-2 2011]

Let n be number of sides.

Now $n \times$ each angle $= (n-2) \times 180^\circ$

$6 \times$ each angle $= 4 \times 180^\circ$

each angle $= 120^\circ$

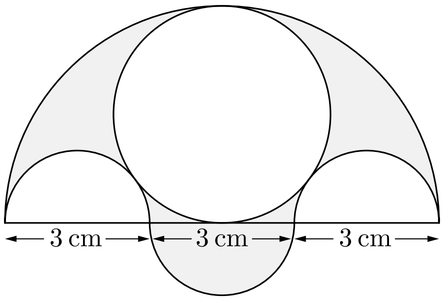
Area of a sector $= \pi r^2 \times \dfrac{120^\circ}{360^\circ}$

I185

Area of 6 shaded regions $= 6\pi r^2 \times \dfrac{120^\circ}{360^\circ}$

$= 2\pi r^2$

146. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Ans :

Area of shaded region

$=$ Area of semicircle with $d = 9$ cm

$+$ Area of semicircle with $d = 3$ cm

$-2 \times$ area of semicircle with $d = 3$ cm

$-$ area of circle with $d = 4.5$ cm

I186

$$= \frac{1}{2} \times \pi \times \left(\frac{9}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - \pi \times \left(\frac{4.5}{2}\right)^2$$

$$= \frac{\pi}{8}[(9)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[4(4.5)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[2(4.5)^2 - (3)^2] = \frac{\pi}{8}[2(3 \times 1.5)^2 - (3)^2]$$

$$= \frac{\pi(3)^2}{8}[2(1.5)^2 - 1] = \frac{9\pi}{8}[4.5 - 1]$$

$$= \frac{9 \times 22}{8 \times 7} \times 3.5 = \frac{99}{8} = 12.375 \text{ cm}^2$$

Thus area of shaded region is 12.375 cm^2

CHAPTER 13

SURFACE AREAS AND VOLUMES

ONE MARK QUESTIONS

Multiple Choice Questions

1. If the radius of the sphere is increased by 100%, the volume of the corresponding sphere is increased by

(a) 200% (b) 500%

(c) 700% (d) 800%

m221

Ans :

Let r be the original radius of sphere. If we increased radius by 100 %. it will be $2r$.

$$V_r = \frac{4}{3}\pi r^3$$

Now $$V_{2r} = \frac{4}{3}\pi \times (2r)^3 = \frac{4}{3}\pi \times 8r^2$$

Thus new volume is 8 times of original volume.

Hence when the radius is increased by 100%, the corresponding volume becomes 800% and thus increase is 700%.

Thus (c) is correct option.

2. A sphere is melted and half of the melted liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is

(a) $\left(\frac{4}{3}\right)^{1/3}$ (b) $\left(\frac{8}{3}\right)^{1/3}$

(c) $(3)^{1/3}$ (d) 2

m222

Ans :

As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$\frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

Thus (b) is correct option.

3. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is

(a) 2 : 1 (b) 1 : 2

(c) 1 : 3 (d) 3 : 1

m223

Ans :

$$\pi rl = 2\pi rh$$

$$\frac{l}{h} = \frac{2}{1}$$

Thus (a) is correct option.

4. If the perimeter of one face of a cube is 20 cm, then its surface area is

(a) 120 cm^2 (b) 150 cm^2

(c) 125 cm^2 (d) 400 cm^2

m224

Ans :

Edge of cube, $$a = \frac{20}{4}\text{ cm} = 5\text{ cm}$$

Surface area $$6a^2 = 6 \times 5^2\text{ cm}^2 = 150\text{ cm}^2$$

Thus (b) is correct option.

5. Ratio of lateral surface areas of two cylinders with equal height is

(a) 1 : 2 (b) $H : h$

(c) $R : r$ (d) None of these

Ans :

$$2\pi Rh : 2\pi rh = R : r$$

Thus (c) is correct option.

m225

6. Ratio of volumes of two cylinders with equal height is

(a) $H : h$ (b) $R : r$

(c) $R^2 : r^2$ (d) None of these

Ans :

$$\pi R^2 h : \pi r^2 h = R^2 : r^2$$

Thus (c) is correct option.

m226

7. Ratio of volumes of two cones with same radii is

(a) $h_1 : h_2$ (b) $s_1 : s_2$

(c) $r_1 : r_2$ (d) None of these

Ans :

m227

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$$

$$\frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_1^2 h_2$$

$(r_1 = r_2)$

$$h_1 : h_2$$

Thus (a) is correct option.

8. The diameter of hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

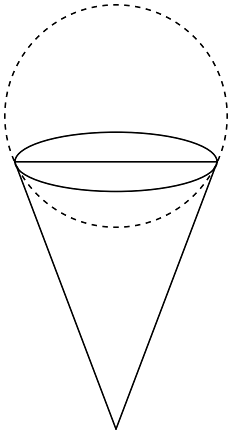
(a) 50% (b) less than 50%

(c) more then 50% (d) 100%

Ans :

Though it is given that diameter of the cone is equal to the diameter of the spherical ball. But the ball will not fit into the cone because of its slant shape. Hence more than 50% of the portion of the ball will be outside the cone.

m228



Thus (c) is correct option.

9. Volume of a spherical shell is given by

(a) $4\pi(R^2 - r^2)$ (b) $\pi(R^3 - r^3)$

(c) $4\pi(R^3 - r^3)$ (d) $\frac{4}{3}\pi(R^3 - r^3)$

m229

Ans :

$$\text{Volume of spherical shell} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(R^3 - r^3)$$

Thus (d) is correct option.

10. The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 2 mm. The length of the wire is

(a) 12 m (b) 18 m

(c) 36 m (d) 66 m

m230

Ans :

Let the length of the wire be l. Since, metallic sphere is converted into a cylindrical shaped wire of length l, Volume of the metal used in wire is equal to the volume of the sphere.

$$\pi r^2 l = \frac{4}{3}\pi R^3$$

$$\pi \times \left(\frac{2}{2} \times \frac{1}{10}\right)^2 \times l = \frac{4}{3} \times \pi \times \left(\frac{6}{2}\right)^3$$

$$\pi \times \frac{1}{100} \times h = \frac{4}{3} \times \pi \times 3^3$$

$$\frac{l}{100} = 4 \times 3^2 = 36$$

$$l = 3600 \text{ cm} = 36 \text{ m}$$

Thus (c) is correct option.

11. A 20 m deep well, with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. The height of the platform is

(a) 2.5 m (b) 3.5 m

(c) 3 m (d) 2 m

m231

Ans : (a) 2.5 m

$$\text{Radius of the well} = \frac{7}{2}\text{m} = 3.5 \text{ m}$$

$$\text{Volume of the earth dug out} = \frac{22}{7} \times (3.5)^2 \times 20$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 20$$

$$= 770 \text{ m}^3$$

$$\text{Area of platform} = (22 \times 14)\text{ m}^2$$

$$= 308 \text{ m}^2$$

$$\text{Height} = \frac{770}{308} = 2.5 \text{ m}$$

12. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed, then the volume of remaining solid is

(a) $280\,\pi \text{cm}^3$ (b) $330\,\pi \text{cm}^3$

(c) $240\,\pi \text{cm}^3$ (d) $440\,\pi\,\text{cm}^3$

m232

Ans :

Volume of the remaining solid

=Volume of the cylinder − Volume of the cone

$$= \pi \times 6^2 \times 10 - \frac{1}{3} \times \pi \times 6^2 \times 10$$

$$= (360\pi - 120\pi) = 240\pi \text{ cm}^3$$

Thus (c) is correct option.

13. If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(a) $4\pi r^2$ (b) $6\pi r^2$

(c) $3\pi r^2$ (d) $8\pi r^2$

m233

Ans :

Because curved surface area of a hemisphere is $2\pi r^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid $= 2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Thus (a) is correct option.

14. A right circular cylinder of radius r and height h (where, $h > 2r$) just encloses a sphere of diameter

(a) r (b) $2r$

(c) h (d) $2h$

m234

Ans :

Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is $2r$.

Thus (b) is correct option.

15. During conversion of a solid from one shape to another, the volume of the new shape will

(a) increase (b) decrease

(c) remain unaltered (d) be doubled

m235

Ans :

During conversion of a solid from one shape to another, the volume of the new shape will remain unaltered.

Thus (c) is correct option.

16. A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm} \times 33\,\text{cm} \times 24\,\text{cm}$, is moul[...] form a solid sphere. The radius of the sphere i[...]

(a) 21 cm (b) 23 cm

(c) 25 cm (d) 19 cm

m236

Ans :

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\,\text{cm}^3$$

$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$

$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 441 \times 21$$

$$r^3 = 21 \times 21 \times 21$$

$$r = 21\,\text{cm}$$

Thus (a) is correct option.

17. Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. The diameter of each sphere is

(a) 4 cm (b) 3 cm

(c) 2 cm (d) 6 cm

Ans :

Volume of the twelve solid sphere is equal to the volume of cylinder.

m237

$$V_{12\,\text{sphere}} = V_{\text{cylinder}}$$

$$12 \times \frac{4}{3}\pi r^3 = \pi\left(\frac{2}{1}\right)^2 \times 16$$

$$16\pi r^3 = 16\pi$$

$$r^3 = 1 \Rightarrow r = 1\,\text{cm}$$

Diameter of each sphere, $d = 2r = 2 \times 1 = 2\,\text{cm}$

Thus (c) is correct option.

18. In a right circular cone, the cross-section made by a plane parallel to the base is a

(a) circle (b) frustum of a cone

(c) sphere (d) hemisphere

Ans :

In a right circular cone, if any cut is made parallel to its base, the result would be the base of the cone, which in cross-section is a circle.

Thus (a) is correct option.

m240

19. Volumes of two spheres are in the ratio 64 : 27. The the ratio of their surface areas is

(a) 3 : 4 (b) 4 : 3

(c) 9 : 16 (d) 16 : 9

Ans :

Let the radii of the two spheres are r_1 and r_2, respectively.

Given, ratio of their volumes,

$$V_1 : V_2 = 64 : 27$$

$$\frac{V_1}{V_2} = \frac{64}{27}$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\frac{r_1^3}{r_2^3} = \frac{64}{27}$$

$$\frac{r_1}{r_2} = \frac{4}{3}$$

m241

Now, ratio of their surface area,

$$\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Hence, the required ratio of their surface area is 16 :9.

Thus (d) is correct option.

20. Assertion : Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.

Reason : If r be the radius and h be the height of the cylinder, then total surface area $= (2\pi rh + 2\pi r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

m242

Total surface area,

$$2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r)$$

$$= 2 \times \frac{22}{7} \times 14(30 + 14) = 88(44)$$

$$= 3872\,\text{cm}^2$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

21. Assertion : The slant height of the frustum of a cone is 5 cm and the difference between the radii of its two circular ends is 4 cm. Then the height of the frustum is 3 cm.

Reason : Slant height of the frustum of the cone is given by $l = \sqrt{(R - r)^2 + h^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have, $l = 5\,\text{cm},\ R - r = 4\,\text{cm}$

$$5 = \sqrt{(4)^2 + h^2}$$

$$16 + h^2 = 25$$

$$h^2 = 25 - 16 = 9$$

$$h = 3\,\text{cm}$$

m243

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

22. Assertion : If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.

Reason : If r be the radius and h be the slant height of the cone, then slant height $= \sqrt{h^2 + r^2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Slant height $l = \sqrt{\left(\frac{14}{2}\right)^2 + (24)^2}$

$$= \sqrt{49 + 576}$$

$$= \sqrt{625} = 25$$

m244

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

23. The volume of a hemisphere is .... the volume of a cylinder if its height and radius is same as that of the cylinder.

Ans :

two-third

m246

24. If a solid of one shape is converted to another, then the volume of the new solid..........

Ans :

remains same

m247

25. A sharpened pencil is a combination of .... and .... shapes.

Ans :

cylinder, cone

m248

26. If we cut a cone by a plane parallel to its base, we obtain a .... and ....

Ans :

cone, frustum of a cone

m249

27. If the radius of a sphere is halved, its volume becomes .... time the volume of original sphere.

Ans :

one-eighth

m250

28. Surahi is the combination of .... and ....

Ans :

sphere, cylinder

m251

29. The volume of a solid is the measurement of the portion of the .... occupied by it.

Ans :

Space

m252

30. In a right circular cone, the cross-section made by a plane parallel to the base is a ....

Ans :

Circle

m253

31. Total curved surface area of the frustum is ....

Ans :

$\pi(r_1+r_2)l+\pi r_1^2+\pi r_2^2$

m254

32. The TSA, CSA stand for .... and .... respectively.

Ans :

m255

Total surface area, Curved surface area.

33. A shuttle cock used for playing badminton has the shape of the combination of .... of cone and hemisphere.

Ans :

Frustum

m256

34. .... is measured in square units.

Ans :

Area

m257

35. In the gilli-danda game, the shape of a gilli is a combination of two cones and ....

Ans :

Cylinder

m258

36. .... is measured in cubic units.

Ans :

Volume

m259

37. A cube is a special type of ....

Ans :

Cuboid

m260

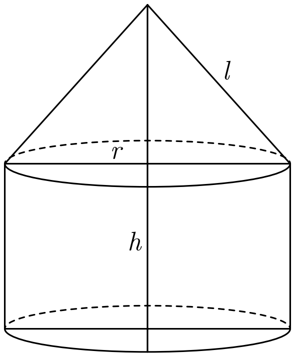
38. The total surface area of a solid hemisphere having radius r is ....

$3\pi r^2$

m261

39. The total surface area of the given solid figure is ....

m262



Ans : [Board 2020 SQP Standard]

Given figure is combination of right circular cone and cylinder.

Total surface area

$= $ Area of base of cylinder+

$+$ Curved surface area of cylinder+

$+$ Curved surface area of cone

$$= \pi r^2 + 2\pi rh + \pi rl$$

$$= \pi r(r+2h+l)$$

Very Short Answer Questions

40. A solid metallic cuboid 24 cm × 11 cm × 7 cm is melted and recast and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Ans :

Let n be the number of cones formed.
Now, according to question,

Volume of n cones = Volume of cuboid

$$n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 = 24 \times 11 \times 7$$

$$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$$

Thus $n = 24$.

m263

41. The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. Find the ratio of its height to its diameter.

Ans : [Board Term-2 2014]

Curved Surface area of cylinder is $2\pi rh$ and volume of cylinder $\pi r^2 h$.

Now $$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$$

$$\frac{r}{2} = \frac{7}{2} \Rightarrow r = 7$$

m101

Substituting $r = 7$ in $2\pi rh = 264$ we have

$$2 \times \frac{22}{7} \times 7 \times h = 264$$

$$h = 6 \text{ m}$$

Now $$\frac{h}{2r} = \frac{6}{14} = \frac{3}{6}$$

Hence, $h : d = 3 : 7$

42. A rectangular sheet paper 40 cm × 22 cm is rolled to form a hollow cylinder of height 40 cm. Find the radius of the cylinder.

Ans : [Board Term-2 Foreign 2014]

Here, $h = 40$ cm, circumference = 22 cm

$$2\pi r = 22$$

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} = 3.5 \text{ cm}$$

m102

43. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Ans : [Board Term-2 Delhi 2014]

$$V_{cylinder} : V_{cone} : V_{hemisphere} = \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$$

$$= \pi r^2 r : \frac{1}{3}\pi r^2 r : \frac{2}{3}\pi r^3 \qquad (h = r)$$

$$= 1 : \frac{1}{3} : \frac{2}{3}$$

$$= 3 : 1 : 2$$

m103

44. What is the ratio of the total surface area of the solid hemisphere to the square of its radius.

Ans : [Board Term-2, 2012]

$$\frac{\text{Total surface area of hemisphere}}{\text{Square of its radius}} = \frac{3\pi r^2}{r^2} = \frac{3\pi}{1}$$

m104

Thus required ratio is $3\pi : 1$.

45. Two cubes each of volume 8 cm^3 are joined end to end, then what is the surface area of resulting cuboid.

Ans : [Board Term-2 2012]

Side of the cube, $a = \sqrt[3]{8} = \sqrt{2}$ cm

Length of cuboid, $l = 4$ cm

Breadth, $b = 2$ cm

Height, $h = 2$ cm

m105

Surface area of cuboid $= 2(l \times b + b \times h + h \times l)$

$$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$$

$$= 2 \times 20 = 40 \text{ cm}^2$$

46. The radius of sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts.

Ans : [Board Term-2 2012]

Whole surface of each part

$$= 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Total surface of two parts

$$= 2 \times 3\pi r^2 = 6\pi r^2$$

m106

47. What is the volume of a right circular cylinder of base radius 7 cm and height 10 cm ? Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

We have $r = 7$ cm, $h = 10$ cm,
Volume of cylinder,

$$\pi r^2 h = \frac{22}{7} \times (7)^2 \times 10$$

$$= 1540 \text{ cm}^3$$

m107

48. If the radius of the base of a right circular cylinder is halved, keeping the height same, find the ratio of the volume of the reduced cylinder to that of original cylinder.

Ans : [Board Term-2 2012]

$$\frac{\text{Volume of reduced cylinder}}{\text{Volume of original cylinder}} = \frac{\pi \times \left(\frac{r}{2}\right)^2 h}{\pi r^2 h} = \frac{1}{4}$$

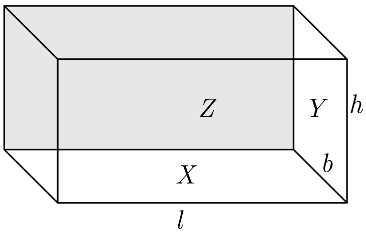
$$= 1 : 4$$

m108

49. If the area of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid.

Ans : [Board Term-2 2012]

Let the length, breadth and height of the cuboid be l, b and h respectively.



m109

Now
$$X = l \times b$$
$$Y = b \times h$$
$$Z = l \times h$$
$$XYZ = l^2 \times b^2 \times h^2$$

Volume of cuboid,
$$V = lbh = \sqrt{XYZ}$$

50. The radii of two cylinders are in the ratio $2 : 3$ and their heights are in the ratio $5 : 3$, find the ratio of their volumes.

Ans : [Board Term-2 2012]

$$\frac{\text{Volume of 1}^{\text{st}}\text{cylinder}}{\text{Volume of 2}^{\text{nd}}\text{cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$
$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$
$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$

m110

$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$
$$= 20 : 27$$

51. Volume of two spheres are in the ratio $64 : 27$, find the ratio of their surface areas.

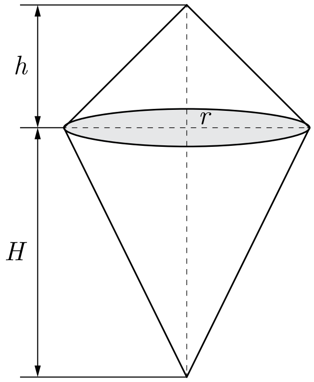
Ans : [Board Term-2 2012]

$$\frac{\text{Volume of I}^{\text{st}}\text{sphere}}{\text{Volume of II}^{\text{nd}}} = \frac{64}{27}$$
$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$
$$\frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$
$$\frac{r_1}{r_2} = \frac{4}{3}$$

m111

Ratio of their surface areas,
$$\frac{2\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

52. A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace.



m112

Ans : [Board Term-2, 2012]

Volume of the upper cone $= \frac{1}{3}\pi r^2 h$

Volume of the lower cone $= \frac{1}{3}\pi r^2 H$

Total volume of both the cones $= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$
$$= \frac{1}{3}\pi r^2 (h + H)$$

The quantity of water displaced will be $\frac{1}{3}\pi r^2 (h + H)$ cube units.

53. Find the volume (in cm³) of the largest right circular cone that can be cut off from a cube of edge 4.2 cm.

Ans : [Board Term-2 2012]

$$\text{Edge of the cube} = 4.2 \text{ cm.}$$

$$\text{Height of the cone} = 4.2 \text{ cm.}$$

$$\text{Radius of the cone} = \frac{4.2}{2} = 2.1 \text{ cm.}$$

m113

Volume of the cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 4.2$$

$$= 19.4 \text{ cm}^3$$

54. The circumference of the edge of a hemisphere bowl is 132 cm. When π is taken as $\frac{22}{7}$, find the capacity of the bowl in cm³.

Ans : [Board Term-2 2012]

Let r be the radius of bowl, then circumference of bowl,

$$2\pi r = 132$$

$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

m114

Capacity i.e volume of the bowl,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19404 \text{ cm}^3$$

55. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

Ans : [Board Term-2 Delhi 2017]

Let radius of sphere be r.

Now Volume of sphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

m115

$$r = \frac{9}{2} \text{ units}$$

Diameter $d = \frac{9}{2} \times 2 = 9$ units

56. Find the number of solid sphere of diameter 6 cm can be made by melting a solid metallic cylinder of height 45 cm and diameter 4 cm.

Ans : [Board Term-2 Delhi 2014]

Let the number of sphere be n.

Radius of sphere = 3 cm,

m116

Radius of cylinder = 2 cm

Volume of spheres = Volume of cylinder

$$n \times \frac{4}{3}\pi r^3 = \pi r_1^2 h$$

$$n \times \frac{4}{3} \times \frac{22}{7} \times (3)^3 = \frac{22}{7} \times (2)^2 \times 45$$

$$36n = 180$$

$$n = \frac{180}{36} = 5$$

Number of solid sphere is 5.

57. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball, find its radius.

Ans : [Board Term-2, 2014]

Let the radius of spherical ball be r.

Volume of spherical ball = Volume of three balls

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi[3^3 + 4^3 + 5^3]$$

$$r^3 = 27 + 64 + 125 = 216$$

, $r = 6$ cm

m117

58. 12 solid spheres of the same size are made by melting a solid metallic cone of base radius 1 cm and height of 48 cm. Find the radius of each sphere.

Ans : [Board Term-2, 2014]

No. of spheres = 12

Radius of cone, $r = 1$ cm

Height of the cone = 48 cm

m118

Volume of 12 spheres = Volume of cone

Let the radius of sphere be R. Let r and h be radius and height of cone.

Now $$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$12 \times \frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (1)^2 \times 48$$

$$R^3 = 1$$

$$R = 1 \text{ cm}$$

59. Three cubes of iron whose edges are 3 cm, 4 cm and 5 cm respectively are melted and formed into a single cube, what will be the edge of the new cube formed ?

Ans : [Board Term-2 Delhi 2012]

Let the edge of single cube be x.

Volume of single cube= Volume of three cubes

$$x^3 = 3^3 + 4^3 + 5^3$$

$$= 27 + 64 + 125 = 216$$

$$x = 6 \text{ cm}$$

m119

60. A solid sphere of radius r melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

Ans : [Board Term-2 Delhi 2012]

Let the radius of cone be R cm.

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$$

$$4r^3 = R^2 r$$

$$R^2 = 4r^2$$

$$R = 2r$$

m120

61. A cylinder and a cone have base radii 5 cm and 3 cm respectively and their respective heights are 4 cm and 8 cm. Find the ratio of their volumes.

Ans : [Board Term-2 2012]

Volume of cylinder,

$$\pi r^2 h = \pi(5)^2 \times 4 \text{ cm}^3$$

$$= 100\pi \text{ cm}^3$$

m124

Volume of cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \times 8$$

$$= 24\pi$$

$$\text{Required ratio} = 100\pi : 24\pi$$

$$= 25 : 6.$$

TWO MARKS QUESTIONS

62. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Ans : [Board 2020 Delhi Standard]

Let h_1 and h_2 be height and r_1 and r_2 be radii of two cones.

Now $$\frac{h_1}{h_2} = \frac{1}{3} \text{ and } \frac{r_1}{r_2} = \frac{3}{1}$$

m264

Ratio of their volumes,

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right) = \left(\frac{3}{1}\right)^2\left(\frac{1}{3}\right) = \frac{3}{1}$$

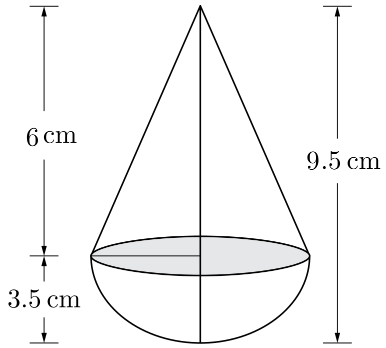
Hence, ratio of their volumes is 3 :1.

63. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Ans :

As per question the figure is shown below. Here total volume of the toy is equal to the sum of volume of hemisphere and cone.

m265



Volume of toy,

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times (6 + 2 \times 3.5)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times (6 + 7)$$

$$= \frac{1}{3} \times \frac{22}{2} \times 3.5 \times 13$$

$$= \frac{1}{3} \times 11 \times 3.5 \times 13$$

$$= \frac{500.5}{3} = 166.83 \text{ cm}^3 \quad \text{(Approx)}$$

Hence, the volume of the solid is 166.83 cm^3.

64. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in the pond rise by 21 cm?

Ans :

Let t be the time in which the level of the water in the tank will rise by 21 cm.

Length of water that flows in 1 hour is 15 km or 15000 m.

Radius of pipe is $\frac{14}{2} = 7$ cm or 0.07 m.

Volume of water in 1 hour,

$$= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000$$

m266

$$= 231 \text{ m}^3$$

Volume of water in time t,

$$= 231t \text{ m}^3$$

This volume of water is equal to the water flowed into the cuboidal pond which is 50 m long, 44 m wide and 0.21 m high.

Thus $231t = 50 \times 44 \times 0.21$

$$t = \frac{50 \times 44 \times 0.21}{231} = 2 \text{ Hours}$$

65. An open metal bucket is in the shape of a frustum of cone of height 21 cm with radii of its lower and upper ends are 10 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre.

Ans :

m267

Height of a frustum of a cone,

$$h = 21 \text{ cm}$$

Radius $r_1 = 10$ cm

and $r_2 = 20$ cm

Volume of frustum is the capacity of bucket.

Volume of frustum,

$$V = \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 21[(10)^2 + (20)^2 + 10 \times 20]$$

$$= 22[100 + 400 + 200]$$

$$= 22 \times 700 = 15400 \text{ cm}^3$$

Quantity of milk,

$$= \frac{15400}{1000} \text{ litres} \qquad (1000 \text{ cm}^3 = 1 \text{ liter})$$

$$= 154 \text{ litres}$$

Total cost of milk $= 15.4 \times$ ₹ $40 =$ ₹ 616

Hence, the cost of milk which can completely fill the bucket at the rate of ₹ 40 per liter is ₹ 616.

66. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Ans :

m268

Radius of conical heap $r = 12$ m

Height of heap, $h = 3.5$ m

Volume of rice,

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ m}^3$$

$$= 528 \text{ m}^3$$

Slanted height,

$$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m}$$

Area of canvas cloth required,

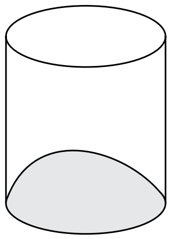
$$\pi rl = \frac{22}{7} \times 12 \times 12.5 = 471.4 \text{ m}^2$$

67. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

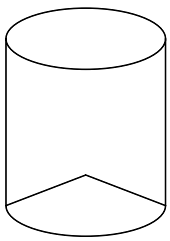
Aisha, a juice seller, was serving juice to her customers in two types of glasses.

Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.

First type : A glass with hemispherical raised bottom.



Second type : A glass with conical raised bottom of height 1.5 cm.



Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much?

Ans :

Let H and h be the height of cylinder and height of cone. Let r be the common radius of cone and cylinder and hemisphere.

Capacity of first glass,

$$= \text{Volume of cylinder} - \text{Volume of hemisphere}$$

$$= \pi r^2 H - \frac{2}{3}\pi r^3$$

m269

$$= \pi r^2\left[H - \frac{2}{3}r\right]$$

$$= \pi \times (3)^2\left[10 - \frac{2}{3} \times 3\right]$$

$$= 9\pi \times 8 = 72\pi \text{ cm}^2$$

Capacity of second glass,

$$= \text{Volume of cylinder} - \text{Volume of cone}$$

$$= \pi r^2 H - \frac{1}{3}\pi r^2 h$$

$$= \pi r^2\left[H - \frac{1}{3}h\right]$$

$$= \pi(3)^2\left[10 - \frac{1}{3} \times 15\right]$$

$$= 9\pi \times 9.5 = 85.5\pi \text{ cm}^2$$

Therefore Suresh got more juice of quantity,

$$= 85.5\pi - 72\pi \text{ cm}^2 = 13.5\pi \text{ cm}^3$$

68. A sphere of maximum volume is cut out from a solid hemisphere of radius 6 cm. Find the volume of the cut out sphere.

Ans : [Board Term-2 2012]

Here diameter of sphere is equal to the radius of hemisphere which is 6 cm.

m125

$$\text{Diameter of sphere} = \text{Radius of hemisphere}$$

$$= 6 \text{ cm}$$

$$\text{Radius of sphere} = 3 \text{ cm}$$

$$\text{Volume,} \quad V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 3^3 \text{ cm}^3.$$

$$= 113.14 \text{ cm}^3.$$

69. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.

Ans : [Board Term-2 2012]

Here diameter of hemisphere is equal to the side of cubical block which is 7 cm.

Diameter of hemisphere = Side of cubical block

$$2r = 7 \Rightarrow r = \frac{7}{2}$$

Surface area of solid

m126

= Surface area of the cube

− Area of base of hemisphere

+ curved surface area of hemisphere

$$= 6l^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 + \pi r^2$$

$$= 6 \times 7^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 6 \times 49 + \frac{77}{2} = 332.5 \text{ cm}^2$$

70. A glass cylinder with diameter 20 cm has water to a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. Calculate the height by which water will rise in the cylinder. Use $\pi = \frac{22}{7}$

OR

A cylinder glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed in it completely. By how much the water will rise in the glass tube. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

Let h be the height of water raised measured.

Volume of water displaced in cylinder $= \pi(10)^2 h$

Volume of cube,

$$\pi(10)^2 h = 8 \times 8 \times 8$$

$$h = \frac{8 \times 8 \times 8 \times 7}{22 \times 10 \times 10}$$

m127

$$= 1.629 \text{ cm.}$$

71. Two cubes of 5 cm each are kept together joining edge to edge to form a cuboid. Find the surface area of the cuboid so formed.

Ans : [Board Term-2, 2015]

Let l be the length of the cuboid so formed.

Now $l = 5+5 = 10$ cm, $b = 5$ cm; $h = 5$ cm.

$$\begin{aligned}\text{Surface area} &= 2(l \times b + b \times h + h \times l)\\ &= 2(10 \times 5 + 5 \times 5 + 5 \times 10)\\ &= 2(50 + 25 + 50)\\ &= 2 \times 125\\ &= 250 \text{ cm}^2.\end{aligned}$$

m128

72. If the total surface area of a solid hemisphere is 462 cm^2, find its volume. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 OD 2014]

Total surface area of hemisphere,

$$3\pi r^2 = 462 \text{ cm}^2$$

m129

$$\frac{22r^2}{7} = \frac{462}{3}$$

$$r^2 = \frac{462 \times 7}{22 \times 3} = 49$$

$$r = 7 \text{ cm.}$$

Volume of hemisphere,

$$\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{2156}{3} = 718.67 \text{ cm}^3.$$

73. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs.25 per meter.

Ans : [Board Term-2 Foreign 2014, Delhi 2014]

We have radius $r = 7$ m and height $h = 24$ m

Slant height of tent,

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$

$$= \sqrt{625} = 25 \text{ m.}$$

m130

Curved surface area of cone,

$$\pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Curves surface area of tent will be required area of cloth. Let x meter of cloth is required

$$5x = 550 \text{ or, } x = \frac{550}{5} = 110 \text{ m.}$$

Thus 110 m of cloth is required.

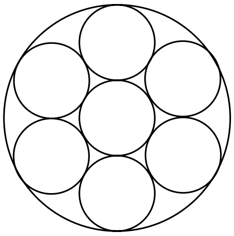
$$\text{Cost of cloth} = 25 \times 110 = Rs.2750.$$

74. Find the number of plates, 1.5 cm in diameter and 0.2 cm thick, that can be fitted completely inside a right circular of height 10 cm and diameter 4.5 cm.

Ans : [Board Term-2 2014]

As per question we can arrange circular plate in right circular as follows. Here smaller circle is plate of 1.5 cm diameter and large circle is cylinder of 4.5 cm diameter.

m131



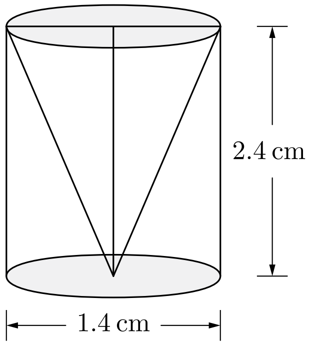
From figure it may be easily seen that 6 plate will be fitted in cylinder in one layer.

Height of six plate is 0.2 cm. Total height of cylinder is 10 cm. Thus layer of plate in cylinder is $\frac{10}{0.2} = 50$ layer. Thus total plate $50 \times 6 = 300$

75. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the volume of the remaining solid to the nearest cm^3. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

As per question the figure is shown below.



m132

Volume of remaining solid is difference of volume of cylinder and volume of cone.

$$\pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times (0.7)^2 \times 2.4$$

$$= 44 \times 0.1 \times 0.7 \times 0.8$$

$$= 4.4 \times .56 = 2.464 \text{ cm}^3.$$

76. A solid metallic of dimensions 9m $\times$ 8m $\times$ 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

Ans : [Board Term-2 Foreign 2017]

Volume of cuboid $= 9 \times 8 \times 2 \text{ cm}^3$

Volume of cube $= 2^3 \text{ cm}^3$

Let number of recast cubes be n.

Volume of n cubes = Volume of cuboid

$$n2^3 = 9 \times 8 \times 2$$

$$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$$

$$n = \frac{9 \times 8 \times 2}{2 \times 2 \times 2} = 18$$

Hence, number of cubes recast is 18.

77. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

Ans : [Board Term-2 2016]

Let the number of recasted balls be N.

Radius of cylinder $R = 3.5$ cm

Height of cylinder $h = 14$cm

Radius of recasted ball $r = \frac{7}{12}$

Volume of balls = Volume of cylinder

$$n\frac{4}{3}\pi r^3 = \pi R^2 h$$

$$n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$

$$n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$

$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$

$$= 648$$

Hence, number of recasted balls is 648.

78. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel ?

Ans : [Board Sample Paper, 2016]

Radius of sphere $\frac{6}{2} = 3$ cm

Radius of cylinder vessel $\frac{12}{2} = 6$ cm

Let the level of water rise in cylinder be h.

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4\pi 3^3}{3}$

$$= 4\pi 3^2 = 36\pi \text{ cm}^3$$

Volume of sphere = Increase volume in cylinder

$$36\pi = \pi(6)^2 h$$

$$36\pi = \pi \times 6 \times 6 \times h$$

$$h = 1 \text{ cm}$$

Thus level of water rise in vessel is 1 cm.

79. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Ans : [Board Term-2 SQP 2016]

Volume of of any cylinder shape is $\pi r^2 h$.

Volume of coin $= \pi(0.75)^2 \times 0.2 \text{ cm}^3$

Volume of cylinder $= \pi(2.25)^2 \times 10 \text{ cm}^3$

$$\text{No. of coins} = \frac{\text{Volume of cylinder}}{\text{Volume of coin}}$$

$$= \frac{\pi(2.25)^2 \times 10}{\pi(0.75)^2 \times 0.2} = \frac{(3)^2 \times 10}{0.2}$$

$$= 450$$

80. A cone of height 24 cm and radius of base 6 cm is made up of clay. If we reshape it into a sphere, find the radius of sphere.

Ans : [Board Term-2 2014]

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r_2^2 h$$

$$\frac{4}{3} \times r_1^3 = (6)^2 \times \frac{24}{3}$$

$$4r_1^3 = 36 \times 24$$

$$r_1^3 = 6^3 \Rightarrow r_1 = 6 \text{ cm}$$

Hence, radius of sphere is 6 cm.

81. A metallic sphere of total volume π is melted and recast into the shape of a right circular cylinder of radius 0.5 cm. What is the height of cylinder ?

Ans : [Board Term-2 2012]

Volume of cylinder = Volume of sphere,

$$\pi r^2 h = \pi$$

m138

where r and h are radius of base and height of cylinder

$$(0.5)^2 h = 1$$

$$0.25h = 1 \Rightarrow h = 4 \text{ cm.}$$

82. A metallic solid sphere of radius 4.2 cm is melted and recast into the shape of a solid cylinder of radius 6 cm. Find the height of the cylinder.

Ans : [Board Term-2, 2012]

Volume of sphere = Volume of cylinder

$$\frac{4\pi R^3}{3} = \pi r^2 h$$

m139

$$\frac{4\pi}{3} \times (4.2)^3 = \pi 6^2 \times h$$

$$h = \frac{4 \times 4.2 \times 4.2 \times 4.2}{3 \times 6 \times 6}$$

Hence, height of cylinder is $h = 2.744$ cm.

THREE MARKS QUESTIONS

83. From a solid cylinder whose height is 15 cm and the diameter is 16 cm, a conical cavity of the same height and same diameter is hollowed out, Find the total surface area of remaining solid. (Given your answer in terms of π).

Ans :

m270

Height of cylinder, $h = 15$ cm

Radius of cylinder, $r = \frac{16}{2} = 8$ cm

Radius of base of cone, $r = 8$ cm

Let slant height of cone be l, then we have

$$l = \sqrt{r^2 + h^2} = \sqrt{8^2 + 15^2}$$

$$= \sqrt{64 + 225} = \sqrt{289}$$

Thus $l = 17$ cm

TSA of reaming solid

$$= \text{Top area of cylinder} +$$
$$+ \text{ CSA of cylinder} + \text{CSA of conical vanity}$$

$$= \pi r^2 + 2\pi rh + \pi rl$$

$$= \pi r(r + 2h + l)$$

$$= \pi \times 8(3 + 2 \times 15 + 17)$$

$$= \pi \times 8 \times 55 = 440\pi$$

TSA of reaming solid is 440π.

84. The volume of a right circular with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. (Use $\pi = \frac{22}{7}$)

Ans : [Board 2020 OD Standard]

Let r be the radius of base of cylinder and h be height.

$$\text{Volume of a right circular cylinder} = 25\frac{1}{7} \text{ cm}$$

$$\pi r^2 h = \frac{176}{7}$$

m273

$$\frac{22}{7} \times h^2 \times h = \frac{176}{7}$$

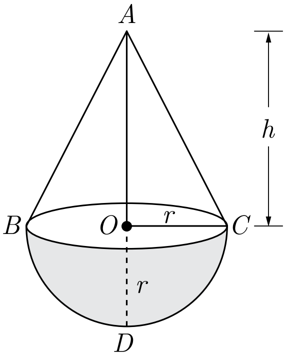
$$h^3 = \frac{176}{22} = 8 = 2^3.$$

Hence, height of the cylinder = 2 cm.

85. A solid is in the shape of a cone mounted on a hemisphere of same base radius. If the curved surface areas of the hemispherical part and the conical part are equal, then find the ratio of the radius and the height of the conical part.

Ans : [Board 2020 OD Standard]

Let ABC be a cone, which is mounted on a hemisphere.



m275

We have $OC = OD = r$

Curved surface area of the hemispherical part

$$= \frac{1}{2}(4\pi r^2) = 2\pi r^2$$

Slant height of a cone,

$$l = \sqrt{r^2 + h^2}$$

Curved surface area of a cone $= \pi rl$

$$= \pi r\sqrt{h^2 + r^2}$$

Since curved surface areas of the hemispherical part and the conical part are equal,

$$2\pi r^2 = \pi r\sqrt{h^2 + r^2}$$

$$2r = \sqrt{h^2 + r^2}$$

Squaring both of the sides, we have

$$4r^2 = h^2 + r^2$$

$$4r^2 - r^2 = h^2$$

$$3r^2 = h^2$$

$$\frac{r^2}{h^2} = \frac{1}{3}$$

$$\frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio of the radius and the height is $1:\sqrt{3}$

86. From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

Ans : [Board 2020 OD Standard]

Let h and r be the height and radius of cylinder and cone.

Height, $h = 14$ cm

and radius, $r = 6$ cm

Volume of the remaining solid,

$$V_{remain} = V_{cylinder} - V_{cone}$$

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

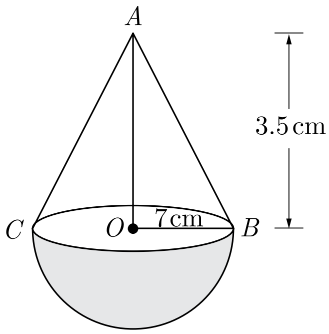
$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$

$$= 1056 \text{ cm}^2$$

87. A solid is in the shape of a hemisphere surmounted by a cone. If the radius of hemisphere and base radius of cone find the volume of the solid. (Take $\pi = \frac{22}{7}$)

Ans : [Board 2020 OD Standard]

As per given information in question we have drawn the figure below,



Here, radius $r = 7$ cm

and height of a cone $= 3.5$ cm

Volume of the solid,

$$= \text{Volume of hemisphere} + \text{volume of a cone}$$

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

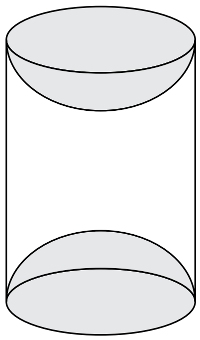
$$= \frac{2}{3} \times \frac{22}{7} \times 7^3 + \frac{1}{3} \times \frac{22}{7} \times 7^3 \times 3.5$$

$$= \frac{1}{3}(2156 + 539)$$

$$= \frac{1}{3} \times 2695$$

$$= 898.33 \text{ cm}^3.$$

88. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Ans : [Board 2018]

Total surface Area of article

$= $ CSA of cylinder + CSA of 2 hemispheres

$$\text{CSA of cylinder} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 3.5 \times 10$$

$$= 220 \text{ cm}^2$$

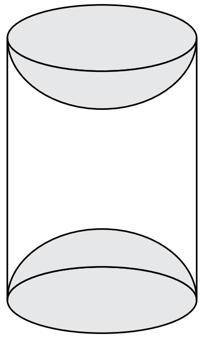
Surface area of two hemispherical scoops

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 154 \text{ cm}^2$$

Total surface area of article$= 220 + 154 = 374 \text{ cm}^2$

89. wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



m279

Ans : [Board 2018]

Total surface Area of article

$=$ CSA of cylinder + CSA of 2 hemispheres

CSA of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 10$$

$$= 220 \text{ cm}^2$$

Surface area of two hemispherical scoops

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 154 \text{ cm}^2$$

Total surface area of article $= 220 + 154 = 374 \text{ cm}^2$

90. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed?

Ans : [Board 2020 OD STD, 2019 Delhi]

Canal is the shape of cuboid where

m280

Breadth $= 6$ m

Depth $= 1.5$ m

and speed of water $= 10$ km/hr

Length of water moved in 60 minutes i.e. 1 hour

$$= 10 \text{ km}$$

Length of water moved in 30 minutes i.e. $\frac{1}{2}$ hours,

$$= \frac{1}{2} \times 10 = 5 \text{ km} = 5000 \text{ m}$$

Now, volume of water moved from canal in 30 minutes

$$= \text{Length} \times \text{Breadth} \times \text{Depth}$$

$$= 5000 \times 6 \times 1.5 \text{ m}^3$$

Volume of flowing water in canal

$=$ volume of water in area irrigated

$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times 8 \text{ cm}$$

$$5000 \times 6 \times 1.5\text{m}^3 = \text{Area Irrigated} \times \frac{8}{100} \text{ m}$$

$$\text{Area Irrigated} = \frac{5000 \times 6 \times 1.5 \times 100}{8} \text{ m}^2$$

$$= 5.625 \times 10^5 \text{ m}^2$$

91. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm². Find the volume of the cone. (Use $\pi = 3.14$)

Ans : [Board Term-2 Delhi 2016]

We have $r = 3, \pi rl = 47.1$

Thus $$l = \frac{47.1}{3 \times 3.14} = 5$$

m140

$$h = \sqrt{5^2 - 3^2} = 4 \text{ cm}$$

Volume of cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4$$

$$= 37.68 \text{ cm}^3$$

92. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2016]

We have $r + h = 37$ (1)

and $2\pi r(r + h) = 1628$ (2)

Thus $2\pi r \times 37 = 1628$

$$2\pi r = \frac{1628}{37} \Rightarrow r = 7 \text{ cm}$$

Substituting $r = 7$ in (1) we have

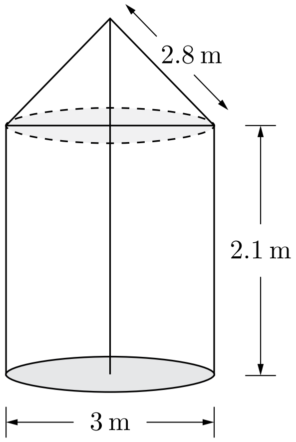
$$h = 30 \text{ cm.}$$

m141

Here volume of cylinder

$$\pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cm}^3$$

93. A tent is in the shape of cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs.500 per square meter. Use $\pi = \frac{22}{7}$.



m142

Ans : [Board Term-2 OD 2016]

Area of canvas required will be surface area of tent.

Height of cylinder $= 2.1$ m

Radius of cylinder $=$ radius of cone $= \frac{3}{2}$ m

Slant height of cone $= 2.8$ m

Surface area of tent,

$$= C.S.A \text{ of cone} + C.S.A \text{ of cylinder.}$$

$$= \pi r l + 2\pi r h = \pi r(l + 2h)$$

Thus $\quad \pi r(l+2h) = \frac{22}{7} \times \frac{3}{2}(2.8 + 2 \times 2.1)$

$$= \frac{33}{7} \times 7 = 33 \text{ m}^2$$

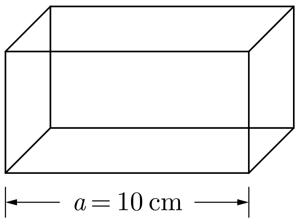
Total Cost $= 33 \times 500 = 16{,}500 Rs$

94. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs.5 per 100 sq. cm. Use $\pi = 3.14$.

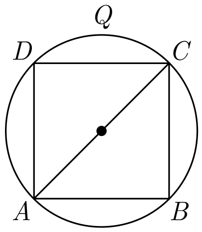
Ans : [Board Term-2 OD 2015]

As per question the figure is shown below.

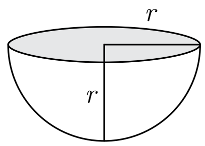
m143



(i)



(ii)



(iii)

Side of given cube $\quad a = 10$ cm

Area of cube(excluding base)

$$A_1 = \text{area of 4 walls} + \text{area of Top}$$

$$= 4a^2 + a^2 = 5a^2 = 5(10)^2 = 500 \text{ cm}^2$$

Let r be the largest radius of hemisphere. From fig. (ii) we have

$\square ABCD$, in the square of side 10 cm.

In ΔABC, $\quad \angle B = 90$

From Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$

$$(2r)^2 = (10)^2 + (10)^2$$

$$4r^2 = 200 \text{ cm}^2$$

$$r = \sqrt{\frac{200}{4}} = 5\sqrt{2} \text{ cm}$$

Hence, the required diameter of hemisphere

$$d = 2r = 2 \times 5\sqrt{2} = 10\sqrt{2} \text{ cm}$$

Now, area of unshaded part in fig (ii)

$$A_2 = \text{area of circle} - \text{area of square } ABCD$$

$$= \pi r^2 - (a)^2 = \left[\pi \times 50 - (10)^2\right]$$

$$= (157 - 100) = 57 \text{ cm}^2$$

Now, Total surface area of solid

$$A = A_1 + A_2 + 2\pi r^2$$
$$= [500 + 57 + 2 \times 3.14 \times 50]$$
$$= 871 \text{ cm}^2$$

The cost of painting of solid

$$= \left(871 \times \frac{5}{100}\right) = 43.55 \text{ Rs}$$

95. A hemispherical bowl of internal diameter 36 cm contains liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of the each bottle, if 10% liquid is wasted in this transfer.

Ans : [Board Term-2 OD 2015]

Volume of the hemispherical bowl of internal diameter 36 cm will be eual to the 72 cylindrical bottles of diameter 6 cm.

$$\text{Volume of bowl} = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\pi \times (18)^3 \text{ cm}^3$$

Volume of liquid in bowl is equal to the volume of bowl.

$$\text{Volume of liquid after wastage} = \frac{2}{3}\pi(18)^3 \times \frac{90}{100} \text{ cm}^3$$

Volume of one bottle $= \pi r^2 h$

Volume of liquid in 72 bottles

$$= \pi \times (3)^2 \times h \times 72 \text{ cm}^2$$

Volume of bottles = volume in liquid after wastage

$$\pi \times (3)^2 \times h \times 72 = \frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}$$

$$h = \frac{\frac{2}{3}\pi \times (18)^2 \times \frac{90}{100}}{\pi \times (3)^2 \times 72}$$

m144

Hence, the height of bottle = 5.4cm

96. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

Ans : [Board Term-2 Foreign 2015]

Volume of cylinder,

$$\pi r^2 h = \pi(3)^2 \times 5$$

m145

$$= 45\pi \text{ cm}^3$$

Volume of conical hole,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi \text{ cm}^3$$

$$\text{Metal left in cylinder} = 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$$

$$\frac{\text{Volume of metal left}}{\text{Volume of metal taken out}} = \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$$

Hence required ratio is 133 : 2

97. A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder in cubic metre. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 Foreign 2015]

Volume of water in cylinder is equal to the volume of cylinder. Thus

Volume of water in cylinder = Volume of cylinder

$$\pi r^2 h = \pi(60)^2 \times 180$$
$$= 648000\pi \text{ cm}^3$$

Water displaced on dropping cone is equal to the volume of solid cone, which is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (30)^2 \times 60$$
$$= 18000\pi \text{ cm}^3$$

m146

Volume of water left in cylinder

= Volume of cylinder − Volume of cone

$$= 648000\pi - 18000\pi = 630000\pi \text{ cm}^3$$
$$= \frac{630000 \times 22}{1000000 \times 7} \text{ m}^3 = 1.98 \text{ m}^3$$

98. The rain water from 22m × 20m roof drains into cylindrical vessel of diameter 2 m and height 3.5 m. If the rain water collected from the roof fills $\frac{4th}{5}$ of cylindrical vessel then find the rainfall in cm.

Ans : [Board Term-2 Foreign 2015]

Let h be the rainfall.

Volume of water collected in cylindrical vessel,

$$\frac{4}{5}\pi r^2 h = \frac{4}{5} \times \pi \times (1)^2 \times \left(\frac{7}{2}\right) \text{ m}^3$$
$$= \frac{44}{5} \text{ m}^3$$

m147

Rain water from roof $= 22 \times 20 \times h$ m^3

Now $\quad 22\times 20\times h = \frac{44}{5}$

$$h = \frac{44}{5}\times\frac{1}{22\times 20} = \frac{1}{50}\text{ m}^3$$

$$= \frac{1}{50}\times 100 = 2\text{ cm}$$

99. A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Ans : [Board Term-2, 2015]

Volume of copper used in making the pipe is equal to the difference of volume of external cylinder and volume of internal cylinder.

Height of cylindrical pipe,

$$h = 21\text{ dm}$$

$$= 210\text{ cm}$$

m148

External Radius, $\quad R = \frac{10}{2} = 5\text{ cm}$

Internal Radius, $\quad r = \frac{6}{2} = 3\text{ cm}$

Volume of copper used in making the pipe

= (Volume of External Cylinder)

− (Volume of Internal Cylinder)

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h(R^2 - h^2)$$

$$= \frac{22}{7}\times 210\times(5^2 - 3^2)$$

$$= \frac{22}{7}\times 210\times(25-9)$$

$$= \frac{22}{7}\times 210\times 16$$

$$= 10560\text{ cm}^3.$$

100. A glass is in the shape of a cylinder of radius 7 cm and height 10 cm. Find the volume of juice in litre required to fill 6 such glasses. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, 2015]

Radius of the glass $\quad r = 7$ cm

Height of the glass $\quad h = 10$ cm

m149

Volume of 1 glass,

$$\pi r^2 h = \frac{22}{7}\times 7\times 7\times 10$$

$$= 1540\text{ cm}^3$$

Volume of juice to fill 6 glasses,

$$6\pi r^2 h = 6\times 1540 = 9240\text{ cm}^3$$

$$\text{Volume in litre} = \frac{9240}{1000} = 9.240\text{ litre.}$$

101. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2, OD 2014]

The diameter of the largest possible sphere is the side of the cube.

Side of cube $\quad a = 7$ cm

Thus radius of sphere $\quad r = \frac{7}{2}$ cm.

m150

Volume of the wood left,

$$V_{cube} - V_{sphere} = a^3 - \frac{4}{3}\pi r^3$$

$$= 7^3 - \frac{4}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3$$

$$= 7^3\left[1 - \frac{4}{3}\times\frac{22}{7}\times\left(\frac{1}{2}\right)^3\right]$$

$$= 7^3\left[1 - \frac{4}{3}\times\frac{22}{7}\times\frac{1}{8}\right]$$

$$= 7^3\left[1 - \frac{11}{21}\right] = 7^3\times\frac{10}{21} = \frac{490}{3}$$

Hence, volume of wood $= 163.3\text{ cm}^3$.

102. A girl empties a cylindrical bucket, full of sand, of radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

Ans : [Board Term-2 Foreign 2014]

Here volume of cone is equal to the volume of cylinder.

Let r_1 and r_2 be the radii of the cylinder and cone respectively.

m152

$$\text{Volume of cone} = \text{Volume of Cylinder}$$

$$\frac{1}{3}\pi r_2^2 h = \pi r_1 h^2$$

$$\frac{1}{3} \times r_2^2 \times 24 = 18 \times 18 \times 32$$

$$r_2^2 = \frac{3 \times 18 \times 18 \times 32}{24}$$

$$r_2^2 = 1296 \Rightarrow r_2 = 36 \text{ cm}$$

Radius of cone $r_2 = 36$ cm

Now, slant height of cone

$$l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2}$$

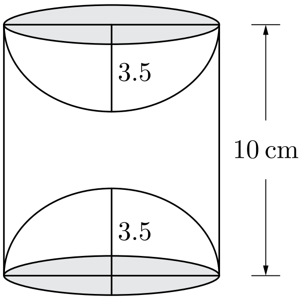
$$= \sqrt{576 + 1296} = 43.2 \text{ cm.}$$

103. A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2013]

As per question the figure is shown below.

m153



Here radius of toy is equal to the radius of cylinder which is 3.5 cm.

Radius of toy = radius of cylinder = 3.5 cm

Vol. of toy = Vol. of cylinder $- 2 \times$ Vol. of hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2\left[h - \frac{4r}{3}\right]$$

$$= \frac{22}{7} \times (3.5)^2\left[10 - \frac{4 \times 3.5}{3}\right]$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times \left[\frac{30 - 4 \times 3.5}{3}\right]$$

$$= \frac{22}{3} \times 0.5 \times 3.5 \times 16$$

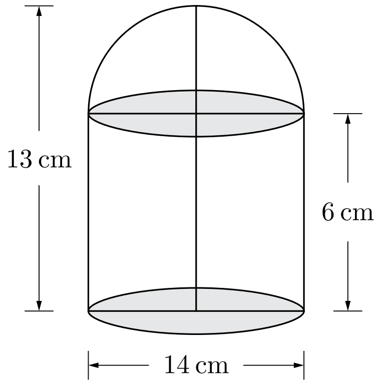
$$= 204.05 \text{ cm}^3.$$

104. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2013]

As per question the figure is shown below.

m154



Radius of hemisphere $r = \frac{14}{2} = 7$ cm

Height of cylinder $h = 13 - 7 = 6$ cm

Total slanted area of cylinder,

$$= \text{S.A of hemisphere} + \text{S.A. of cylinder}$$

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r(r + h)$$

$$= \frac{2 \times 22 \times 7}{7} \times (7 + 6)$$

$$= 44 \times 13 = 572 \text{ cm}^2$$

105. The radii of two right circular cylinders are in the ratio of 2 : 3 and their height are in the ratio of 5 : 4. Calculate the ratio of their curved surface area and radio of their volumes.

Ans : [Board Term-2 2012]

Let the radii of two cylinders be $2r$ and $3r$ and their heights be $5h$ and $4h$ respectively.

Ratio of their curved surface areas,

$$= \frac{2\pi \times 2r \times 5h}{2\pi \times 3r \times 4h} = \frac{5}{6}$$

Thus their curved surface areas are in the ratio of 5 : 6.

Ratio of their volumes,

$$= \frac{\pi \times (2r)^2 \times 5h}{2\pi \times 3r \times 4h} = \frac{5 \times 4}{4 \times 9} = \frac{5}{9}$$

Hence, their volumes are in the ratio of 5 : 9 and their $C.SA$ are in the ratio of 5 : 6.

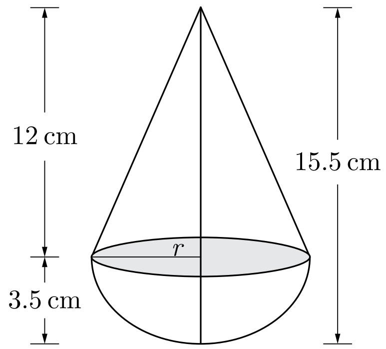
106. A toy is in the form of a cone radius 3.5 cm mounted on a hemisphere of same radius. If the total height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$ [Board OD 2020 Basic]

or

A toy is in the form of a cone surmounted on a hemisphere of common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

As per question the figure is shown below. Here total surface area of the toy is equal to the sum of surface area of hemisphere and curved surface area of cone.



Radius $r = \frac{7}{2} = 3.5$ cm

and height $h = 12$ cm

Slant height of cone,

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{3.5^2 + 12^2} = 12.5$$

Total surface area of the toy

= Surface area of hemisphere +

+ Curved surface area of cone

$$= 2\pi r^2 + \pi r l$$

$$= \pi r(2r + l)$$

$$= \frac{22}{7} \times 3.5 \times (2 \times 3.5 + 12.5)$$

$$= 11 \times 19.5 = 214.5 \text{ cm}^2$$

107. Water is flowing at 7 m/s through a circular pipe of internal diameter of 4 cm into a cylindrical tank, the radius of whose base is 40 cm. Find the increase in water level in 30 minutes.

Ans : [Board Term-2 2012]

Length of water that flows in 1 sec is 7 m or 700 cm.

Radius of pipe is $\frac{4}{2} = 2$ cm.

Thus volume of water in 1 second,

$$= \pi \times (2)^2 \times 700 \text{ cm}^3$$

Volume of water in 30 minutes,

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Radius of tank is 40 cm.

Volume of water in the tank,

$$\pi 40^2 \times h = \pi \times 4 \times 700 \times 60 \times 30$$

$$h = \frac{700 \times 60 \times 30 \times 4}{40 \times 40} = 3150 \text{ cm}$$

Hence, water level increased is 3150 cm or 31.5 m.

108. A metallic solid sphere of radius 10.5 cm melted and recasted into smaller solid cones each of radius 3.5 cm and height 3 cm. How may cones will be made ?

Ans : [Board Term-2 Delhi 2017]

Radius of given sphere $R = 10.5$ cm

Volume of sphere,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (10.5)^3 \text{ cm}^3$$

Radius of one recasted cone,

$$r = 3.5\text{cm}$$

Height $h = 3$ cm

Volume $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (3.5)^2 \times 3$

$$= \pi(3.5)^2 \text{ cm}^3$$

Let the number of recasted cones be n. Volume of sphere is equal to the n recasted cone.

$$n\pi(3.5)^2 = \frac{4}{3}\pi(10.5)^3$$

$$n = \frac{4}{3}\frac{(10.5)^3}{(3.5)^2}$$

$$= \frac{4}{3} \times 10.5 \times \left(\frac{10.5}{3.5}\right)^2$$

$$= \frac{4}{3} \times 10.5 \times (3)^3$$

$$= 4 \times 10.5 \times 3 = 126$$

Hence, number of recasted cones is 126.

109. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Ans : [Board Term-2 Delhi 2017]

Radius of given sphere $R = \frac{16}{2} = 8$ cm

Volume of sphere,

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(8)^3 \text{ cm}^3$$

m160

Radius of one recasted cone,

$$r = 4 \text{ cm}$$

Height $h = 8$ cm

Volume $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2 \times 8$

Let the number of recasted cones be n. Volume of sphere is equal to the n recasted cone.

$$n \times \frac{1}{3}\pi(4)^2 \times 8 = \frac{4}{3}\pi(8)^3$$

$$n = \frac{4 \times (8)^3}{(4)^2 \times 8} = \frac{8^2}{4} = \frac{64}{4} = 16$$

Hence, number of recasted cones is 16.

110. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged into water, by how much will the level of water rise in the cylindrical vessel ?

Ans :

m161

Let h be the rise in level of water.

Radius of sphere $= 3$ cm.

Radius of cylinder $= \frac{12}{2} = 6$ cm

Volume of water displaced in cylinder will be equal to the volume of sphere.

$$\pi(6)^2 h = \frac{4\pi}{3}(3)^3$$

$$6 \times 6 \times h = \frac{4}{3} \times 3 \times 3 \times 3$$

$$6 \times 6 \times h = 4 \times 3 \times 3$$

$$h = \frac{4 \times 3 \times 3}{6 \times 6} = 1 \text{ cm}$$

Hence the water level rises is 1 cm.

111. A conical vessel, with base radius 5 cm height 24 cm, is full of water. This water emptied into a cylindrical vessel, of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 OD 2016]

Here radius and height of conical vessel are 5 cm and 24 cm.

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

m163

$$= \frac{1}{3}\pi \times 2.5 \times 24$$

When water is emptied into cylindrical vessel, water will rise in cylindrical vessel. Let rise in height be h. Volume of water raised $= \pi r^2 h$. This volume is equal to the volume of cone.

Thus $\pi \times (10)^2 \times h = \frac{1}{3}\pi \times 25 \times 24$

$$100h = 25 \times 8$$

$$h = 2 \text{ cm}$$

112. Water is flowing at the rate of 0.7 m/sec through a circular pipe whose internal diameter is 2 cm into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the level of water in half hour.

Ans : [Board Term-2 SQP 2016]

Length of water that flows in 1 sec is 0.7 m or 70 cm.

Radius of pipe is $\frac{2}{2} = 1$ cm.

Volume of water in 1 second,

$$= \pi \times (1)^2 \times 70 = 70\pi \text{cm}^3$$

m164

Volume of water in 30 minutes,

$$= 70\pi \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Radius of tank is 40 cm.

Volume of water in the tank,

$$\pi 40^2 \times h = 70\pi \times 60 \times 30$$

$$h = \frac{70 \times 60 \times 30}{40 \times 40} = 78.75 \text{ cm}$$

113. A well of diameter 4 m dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

Ans : [Board Term-2 Delhi 2016]

Radius of earth dug out $r = \frac{4}{2} = 2$ m

Depth of the earth $d = 21,$

Volume of earth $\pi r^2 d = \frac{22}{7} \times (2)^2 \times 21$

$$= 22 \times 4 \times 3 = 264 \text{ m}^2$$

Width of embankment $= 3$ m

Outer radius of ring $= 2 + 3 = 5$ m

Let the height of embankment be h.

Volume of embankment,

$$\pi(R - r)^2 h = 264$$

m165

$$\frac{22}{7} \times (5^2 - 2^2) \times h = 264$$

$$\frac{22}{7} \times (25 - 4) \times h = 264$$

$$\frac{22}{7} \times 21 \times h = 264$$

$$22 \times 3 \times h = 264$$

$$h = \frac{264 \times 7}{22 \times 21} = 4$$

Height of embankment is 4 m.

114. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cones.

Ans : [Board Term-2 Foreign 2016]

For cylindrical tub,

Radius $R = \frac{12}{2} = 6$ cm

m166

Height $H = 15$ cm.

Volume $\pi R^2 H = \pi(6)^2 \times 15 = 540\pi \text{ cm}^3$

Each child will get the ice-cream $\frac{540\pi}{10}$ cm^3

$$= 54\pi \text{ cm}^3$$

For cone, height $h = 2 \times d = 2 \times 2r = 4r$

Volume of cone,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 4r = \frac{4}{3}\pi r^3$$

Volume of hemisphere$= \frac{2}{3}\pi r^3$

Total volume of cone and hemisphere

$$= \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{6}{3}\pi r^3 = 2\pi r^3$$

According to question,

$$2\pi r^3 = 54\pi$$

$$r^3 = 27 \Rightarrow r = 3$$

Hence diameter of conical part of ice-cream cones,

$$= 2r = 2 \times 3 = 6 \text{ cm.}$$

115. A hemispherical tank, of diameter 3 m, is full of water. It is being emptied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty ? Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Foreign 2016]

Radius $r = \frac{3}{2}$ m

Volume of hemispherical tank,

m167

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\frac{3}{2}\right)^3 \text{ m}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{27}{8} \text{ m}^3$$

$$= \frac{11}{7} \times \frac{9}{2} = \frac{99}{14} \text{ m}^3$$

Since 1 m^3 = 1000 litre, we have

$$V = \frac{99}{14} \times 1000 \text{ litre}$$

Volume of half of the hemisphere

$$\frac{V}{2} = \frac{1}{2} \times \frac{99}{14} \times 1000 \text{ Litres}$$

Let time taken for this volume to flow out be t. Then according to question,

$$3\frac{4}{7}t = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$\frac{25t}{7} = \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$t = \frac{7}{25} \times \frac{1}{2} \times \frac{99}{14} \times 1000$$

$$= 990 \text{ sec}$$

$$= 16 \text{ minutes } 30 \text{ sec.}$$

116. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2015]

Volume of single cone,

$$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{\pi}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3$$

Volume of recast sphere,

$$V_{sphere} = \frac{4}{3}\pi r^3$$

Volume of sphere is equal to the volume of 504 cones.

Thus $$V_{sphere} = 504\, V_{cone}$$

$$\frac{4\pi}{3} \times r^3 = 504 \times \frac{\pi}{3} \times \left(\frac{3.5}{2}\right)^2 \times 3$$

$$4r^3 = 504 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

m168

$$r^3 = 126 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 7 \times 9 \times 2 \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$= 3 \times 3 \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \times 3$$

$$r = 3 \times \frac{7}{2} = 10.5 \text{ cm}$$

Thus diameter is 21 cm.

Surface area $$4\pi r^2 = 4 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1386 \text{ cm}^2$$

117. A solid metallic cone of radius 2 cm and height 8 cm is melted into a sphere. Find the radius of sphere.

Ans : [Board Term-2 2014]

Let R be the radius of sphere.

Volume of sphere = Volume of cone

m169

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi \times (2)^2 \times 8$$

$$4R^3 = 4 \times 8$$

$$R^3 = 8 \Rightarrow R = 2 \text{ cm}$$

118. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level into the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

Ans : [Board Term-2 OD 2016]

Radius of sphere $$= \frac{12}{2} = 6 \text{ cm}$$

m170

Volume $$= \frac{4}{3}\pi \times 6^3 \text{ cm}^3$$

It is submerged into water, in cylindrical vessel, then water level rise by $3\frac{5}{9} = \frac{32}{9}$ cm. Volume of submerged sphere is equal to the volume of water rise in cylinder.

Volume submerged = Volume rise

Let r be radius of cylinder. Therefore

$$\pi \times r^2 \times \frac{32}{9} = \frac{4}{3}\pi \times 6^3 \text{ cm}$$

$$r^2 = \frac{216 \times 3 \times 4}{32} = \frac{27 \times 3 \times 4}{4}$$

$$r^2 = 27 \times 3 = 81 \Rightarrow r = 9 \text{ cm}$$

Diameter $$2r = 2 \times 9 = 18 \text{ cm.}$$

119. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Ans : [Board Term-2 Delhi 2017]

Radius of conical vessel $= 5$ cm

Height of conical vessel $= 24$ cm

Volume of this vessel, $= \frac{\pi}{3} \times (5)^2 \times 24$

$= 200\pi \text{ cm}^3$

Internal radius of cylindrical vessel$= 10$

m171

Let the h be the height of emptied water.

Volume of water in cylinder,

$$\pi r^2 h = \frac{3}{4} \times \text{Volume of cone}$$

$$\pi \times 10 \times 10 \times h = \frac{3}{4} \times 200\pi$$

$$100h = 150 \Rightarrow h = 1.5 \text{ cm}$$

Hence the height of water is 1.5 cm.

120. Rampal decided to donate canvas for 10 tents conical in shape with base diameter 14 m and height 24 m to a centre for handicapped person's welfare. If the cost of 2 m wide canvas is Rs. 40 per meter, find the amount by which Rampal helped the money.

Ans : [Board Term-2 OD Compt. 2017]

Radius of tent $r = \frac{14}{2} = 7$ m

Height $h = 24$ m

m172

Slant height $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576} = 25$ m

Surface area of the tent,

$$\pi rl = \pi rl$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

Surface area of 10 tents,

$$= 550 \times 10 = 5500$$

$$\text{Total cost} = 5500 \times \frac{40}{2} = 110000$$

Hence, Rampal helped the centre of 110000 Rs.

121. A cone of maximum size is curved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is curved out.

Ans : [Board Term-2 SQP 2017]

If a cone of maximum size is curved out from a cube edge a, diameter and height of cone will be a

Side of cube $a = 14$ cm.

If cone of maximum size is curved out,

m173

Radius of cone $r = \frac{14}{2} = 7$ cm

Height of cone $h = 7$ cm

Slant height $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 14^2}$

$= \sqrt{49 + 196} = \sqrt{245}$

$= 15.65$ cm.

Total surface area,

$=$ Surface area cube $+$ curved Surface area of cone

$-$ Circular area of base of cone

$= 6a^2 + \pi rl - \pi r^2$

$$= 6 \times 14 \times 14 + \frac{22}{7} \times 7 \times 15.65 - \frac{22}{7} \times 7 \times 7$$

$= 1176 + [22(15.65 - 7)]$

$= 1176 \times 22 \times 8.65$

$= 223792.8 \text{ cm}^2$

122. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

Ans :

Water flow in 1 hour,

$=$ Area of cross-section $\times$ Speed of water

$= 5.4 \times 1.8 \times 25000 \text{ m}^3$

$= 54 \times 18 \times 250 \text{ m}^3$

Water flow in 40 minutes,

m174

$$= 54 \times 18 \times 250 \times \frac{40}{60} \text{ m}^3$$

$$= 54 \times 6 \times 500 \text{ m}^3$$

Let A be the irrigated area then volume of water in irrigated area is equal to the water flow.

Thus $A \times 0.1 = 54 \times 6 \times 500$

$$A = 54 \times 6 \times 500 \times 10$$

$$= 1620000 \text{ m}^3$$

123. From a solid cylinder whose height is 8 cm and radius 6 cm, a conical cavity of same height and same base radius is hollowed out. Find the total surface area of the remaining solid. (Take $\pi = 3.14$)

Ans : [Board Term-2 OD Compt. 2017]

Height and radius of cylinder are equal to the height and radius of cone.

Height of cylinder = height of cone = 8 cm

radius of cylinder = radius of cone = 6 cm

$$\text{Slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$= 10 \text{ cm}$$

Total surface area of remaining solid,

= CSA of cylinder+

+ CSA of cone + area of top

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 6(2 \times 8 + 10 + 6)$$

$$= \frac{22}{7} \times 6 \times 32$$

$$= 603.43$$

Hence total surface area is 603.43 cm^2.

124. From a solid cylinder of height 24 cm and diameter 14 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

Ans : [Board Term-2 Delhi Compt. 2017]

Height and radius of cylinder are equal to the height and radius of cone.

Height of cylinder = height of the cone = 24 cm

$$\text{radius of cylinder} = \text{radius of cone} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{Slant height of cone} = \sqrt{r^2 + h^2}$$

$$= \sqrt{7^2 + 24^2}$$

$$= \sqrt{49 + 576} = 25 \text{ cm}$$

Total surface area of remaining part

= Surface area of cylinder+

+ Surface area of cone + area of top

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 7(2 \times 24 + 25 + 7)$$

$$= 22 \times 80$$

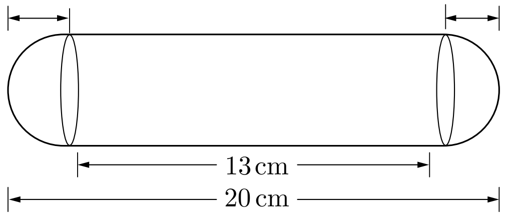
$$= 1760 \text{ cm}^2$$

FOUR MARKS QUESTIONS

125. A solid is in the form of a cylinder with hemispherical end. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. (Use $\pi = \frac{22}{7}$)

Ans : [Board 2019 OD]

As per given information in question we have drawn the figure given below.



Height of the cylinder,

$$h = (20 - 7) \text{ cm} = 13 \text{ cm}$$

Radius of circular part,

$$r = \frac{7}{2} \text{ cm}$$

Volume of solid,

= Volume of cylinder +2 × Volume of hemisphere

$$V = \pi r^2 h + 2 \times \left(\frac{2\pi}{3} r^3\right)$$

$$= \pi r^2 \left(h + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left[13 + \frac{4}{3} \times \frac{7}{2}\right]$$

$$= \frac{77}{2}\left(\frac{53}{3}\right)\text{cm}^3$$

$$= 680.2\ \text{cm}^3$$

126. The weight of two spheres of same metal are 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

Ans : [Board 2019 OD Standard]

m285

Weight of smaller sphere, $W_1 = 1\ \text{kg}$

Weight of larger sphere, $W_2 = 7\ \text{kg}$

Radius of smaller sphere, $r_1 = 3\ \text{cm}$

Volume of smaller sphere, $V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3$

$$= \frac{4}{3}\pi(27) = 36\pi\ \text{cm}^3$$

Now weight of recanted metal sphere

$$= (1+7)\ \text{kg} = 8\ \text{kg}$$

Since, 1 kg metal sphere occupies $36\pi\ \text{cm}^3$ space.

Thus 8 kg metal sphere occupies $8 \times 36\pi\ \text{cm}^3$ space.

Let R be the radius of new sphere, then volume of new 8 kg sphere is $\frac{4}{3}\pi R^3$.

Thus $\frac{4}{3}\pi R^3 = 36 \times 8\pi\ \text{cm}^3$

$$R^3 = 36 \times 2 \times 3$$

$$R^3 = 9 \times 4 \times 2 \times 3 = 3^3 \times 2^3$$

$$R = 2 \times 3 = 6\ \text{cm}$$

Diameter of new sphere

$$2R = 2 \times 6 = 12\ \text{cm}$$

127. A right cylindrical container of radius 6 cm and height 15 cm if full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

Ans : [Board 2019 OD Standard]

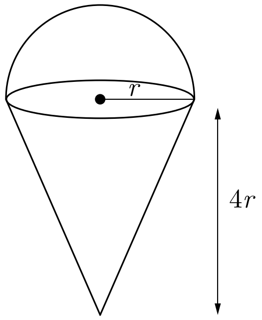
For cylindrical container $R = 6\ \text{cm}$ and $H = 15\ \text{cm}$.

Volume of ice cream in the cylindrical container

$$\pi R^2 H = \pi(6)^2 \times 15 = 36 \times 15\pi$$

As per given information in question we have drawn the figure of cone as given below. Here r is the common radius of cone and hemisphere.

m286



Volume of each cone with hemispherical top

$$\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$$

$$= \frac{1}{3}\pi r^2(4r + 2r)$$

$$= 2\pi r^3$$

Now, volume of ice-cream in container is equal to 10 cone of ice-cream.

$$36 \times 15\pi = 10 \times 2\pi x^3$$

$$r^3 = \frac{36 \times 15}{20} = 27$$

$$r = 3\ \text{cm}$$

128. Hence, radius of the ice-cream cone is 3 cm A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 m high embankment. Find the width of the embankment.

Ans : [Board Term-2 2012]

m181

Depth of well, $d = 14\ \text{m}$,

Radius, $r = 12\ \text{m}$.

Volume of earth taken out,

$$\pi r^2 h = \frac{22}{7} \times (2)^2 \times 14$$

$$= \frac{22}{7} \times 2 \times 2 \times 14$$

$$= 176\ \text{m}^3$$

Let r be the width of embankment. The radius of outer circle of embankment

$$= 2 + r$$

Area of upper surface of embankment

$$= \pi\left[(2+r)^2 - (2)^2\right]$$

Volume of embankment = Volume of earth taken out

$$\pi\left[(2+r)^2-(2)^2\right]\times 0.4 = 176$$

$$\pi[4+r^2+4r-4]\times 0.4 = 176$$

$$\frac{0.4\times 22}{7}(r^2+4r) = 176$$

$$r^2+4r = \frac{176\times 7}{0.4\times 22} = 140$$

$$r^2+4r-140 = 0$$

$$(r+14)(r-10) = 0 \Rightarrow r = 10$$

Hence width of embankment is 10 m.

129. A hemispherical depression is cut from one face of a cubical block, such that diameter l of hemisphere is equal to the edge of cube. find the surface area of the remaining solid.

Ans : [Board Term-2 2014]

Let r be the radius of hemisphere.

Now $r = \frac{l}{2}$

m182

Now, the required surface area

= Surface area of cubical block +

− Area of base of hemisphere +

+ Curved surface area of hemisphere.

$$= 6(l)^2-\pi r^2+2\pi r^2$$

$$= 6l^2-\pi\left(\frac{l}{2}\right)^2+2\pi\left(\frac{l}{2}\right)^2$$

$$= l^2\left(6-\frac{\pi}{4}+\frac{2\pi}{4}\right)$$

$$= l^2\left(6+\frac{\pi}{4}\right)$$

$$= l^2\left(6+\frac{22}{7\times 4}\right)$$

$$= l^2\left(6+\frac{11}{14}\right) = \frac{95l^2}{14}$$

130. Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area in hectare will it irrigate in 30 minutes if 8 cm of standing water is needed ?

Ans : [Board Term-2 2012, Delhi 2014]

Water flow in 1 hour,

= Area of cross-section × Speed c

$= 6\times 1.5\times 10000$ m^3

m183

$= 90000$ m^3

Water flow in 40 minutes,

$$= 90000\times\frac{30}{60}\text{ m}^3$$

$= 45000$ m^3

Let A be the irrigated area then volume of water in irrigated area is equal to the water flow.

Thus $A\times 0.08 = 45000$

$$A = \frac{45000}{0.08} = 562500\text{ m}^3$$

$= 56.25$ hectare.

131. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, in how much time will the tank be filled ?

Ans : [Board Term-2 2012, Delhi 2015]

Radius of the tank $R = \frac{10}{2} = 5$ m

m184

Depth of tank $D = 2$ m

Volume of tank $V = \pi R^2 D$

$$= \pi(5)^2\times 2 = 50\pi$$

Radius of pipe $r = \frac{20}{2} = 10$ cm $= 0.10$ m

Speed of the water is 3 km/hr.

Speed of water in minute,

$$= \frac{3000}{60} = 50\text{ m/min}$$

Volume of water supplied in one minute

$$\pi r^2 h = \pi\times 0.10\times 0.10\times 50$$

Time taken to fill the tank,

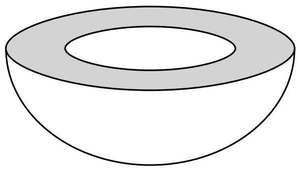
$$t = \frac{50\pi}{\pi\times 0.10\times 0.10\times 50} = 100$$

Hence time taken to fill the tank is 100 minutes.

132. The internal and external diameters of a hollow hemispherical vessel are 16 cm and 12 cm respectively. If the cost of painting 1 cm^2 of the surface area is Rs. 5.00, find the total cost of painting the vessel all over. (Use $\pi = 3.14$)

Ans :

As per question the figure is shown below.



m185

Here $R = 8$ cm, $r = 6$ cm

Surface area $= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi[2 \times 8^2 + 2 \times 6^2 + (8^2 - 6^2)]$

$= \pi[2 \times 64 + 2 \times 36 + (64 - 36)]$

$= \pi[128 + 72 + 28]$

$= 228 \times 3.14 = 715.92$ cm^2

Total cost $= 715.92 \times 5 = 3579.60$ Rs

133. Water is flowing through a cylindrical pipe, of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm, at the rate of 0.4 m/s. Determine the rise in level of water in the tank in half an hour.

Ans : [Board Term-2 Delhi 2013]

Radius of pipe $r = \frac{2}{2} = 1$

Water flow rate $= 0.4$ m/s $= 40$ cm/s

Volume of water flowing through pipe in 1 sec.

$$\pi r^2 h = \pi \times (1)^2 \times 40 = 40\pi \text{ cm}^3$$

Volume of water flowing in 30 min $(30 \times 60 \text{ sec})$

$$= 40\pi \times 30 \times 60 = 72000\pi$$

Volume of water in cylindrical tank in 30 min,

Now $\pi R^2 H = \pi(40)^2 \times H$

$\pi(40)^2 \times H = 72000\pi$

$40 \times 40 \times H = 72000\pi$

m186

Rise in water level

$$H = \frac{72000}{40 \times 40} = 45 \text{ cm.}$$

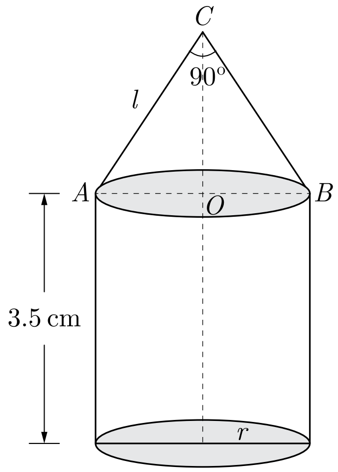
Thus level of water in the tank is 45 cm.

134. A toy is in the form of a cylinder of diameter $2\sqrt{2}$ m and height 3.5 m surmounted by a cone whose vertical angle is 90°. Find total surface area of the toy.

Ans : [Board Term-2 2012]

As per question the figure is shown below.

m187



Here $\angle C = 90°$ and $AC = BC = l$

Thus $AB^2 = AC^2 + BC^2$

$= l^2 + l^2 = 2l^2$

Now $(2\sqrt{2})^2 = 2l^2$

Thus $l = 2$ and $r = \sqrt{2}$ m

Slant height of conical portion, $l = 2$ m

Total surface area of toy

$2\pi rh + \pi r^2 + \pi rl = \pi r[7 + \sqrt{2} + 2]$ m^2

$= \pi\sqrt{2}[9 + \sqrt{2}]$ m^2

$= \pi[2 + 9\sqrt{2}]$ m^2

135. Find the volume of the largest solid right circular cone that can be cut out off a solid cube of side 14 cm.

Ans : [Board Term-2 2012]

The base of cone is the largest circle that can be inscribed in the face of the cube and the height will be equal to edge of the cube.

Radius of cone, $r = \frac{14}{2} = 7$ cm

m188

Height of cone, $h = 14$ cm

Volume of cone, $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 14$$

$$= \frac{2156}{3} = 718.67.$$

136. Water is flowing at the rate of 15 km/hr through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm ?

Ans : [Board Term-2 2012]

Radius of pipe, $r = \frac{14}{2} = 7$ cm

m189

Cross section area of pipe,

$$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{100}\right)^2$$

Speed of water flowing through the pipe

$$= 15 \text{ km/hr} = 15000 \text{ m/hr}$$

In an hour length of water = 15000 m

Volume of water flowing from pipe in 1 hr,

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000 \text{ m}^3$$

Let t be time taken to fill the tank. Now total volume of water flowing in time t,

$$\pi r^2 ht = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 15000t$$

Volume of water flown= Volume of water in tank

$$\pi r^2 ht = l \times b \times y$$

$$\frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 1500t = 50 \times 44 \times \frac{21}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000t = 50 \times 44 \times \frac{21}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000t = 50 \times 44 \times \frac{21}{100}$$

$$22 \times 7 \times 150t = 50 \times 44 \times 21$$

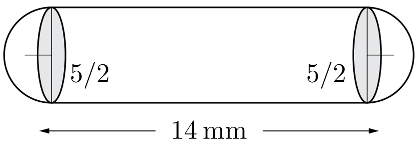
$$t = \frac{50 \times 44 \times 21}{22 \times 150 \times 7} = 2$$

Hence, time taken to fill the tank is 2 hours.

137. A medicine capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends, the length of the entire capsule is 15 mm and the diameter of the capsule is 5 mm. Find the Volume of the capsule.

Ans : [Board Term-2 2012]

As per question the figure is shown below.



m190

Total height = 14 mm

Height of cylinder $= 14 - 2 \times 2.5 = 9$ mm

Radius of cylinder = 2.5mm

Radius of hemisphere = 2.5 mm

Volume of capsule = Volume of two hemispheres + Volume of cylinder

$$= 2 \times \frac{2\pi r^3}{3} + \pi r^2 h$$

$$= \frac{4}{3}\pi\left(\frac{5}{2}\right)^3 + \pi\left(\frac{5}{2}\right)^2 \times 9$$

$$= \pi\left(\frac{5}{2}\right)^2\left(\frac{4}{3} \times \frac{5}{2} + 9\right)$$

$$= \frac{25\pi}{4}\left(\frac{10}{3} + 9\right)$$

$$= \frac{25}{4}\pi\left(\frac{10+27}{3}\right) = \frac{25}{4}\pi\left[\frac{37}{3}\right]$$

$$= \frac{25}{4} \times \frac{22}{7} \times \frac{37}{3} = \frac{10175}{42} \text{ mm}^3$$

$$= 242.26 \text{ mm}^3.$$

138. A milk tanker cylindrical in shape having diameter 2 m and length 4.2 m supplies milk to the two booths in the ratio of 3 : 2. One of the milk booths has cuboidal vessel having base area 3.96 sq. m. and the other has a cylindrical vessel having radius 1 m. Find the level of milk in each of the vessels. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

Radius of milk tanker $R = \frac{2}{2} = 1$ m

m191

Length of mil tanker L 4.2 m

Volume of milk tanker,

$$\pi R^2 L = \frac{22}{7} \times 1 \times 4.2 = 13.2 \text{ m}^3$$

Supply of milk to booth I,

$$= 13.2 \times \frac{3}{5} = 2.64 \times 3 = 7.92 \text{ m}^3$$

Supply of milk to booth II,

$$= 13.2 \times \frac{2}{5} = 2.64 \times 2 = 5.28 \text{ m}^3$$

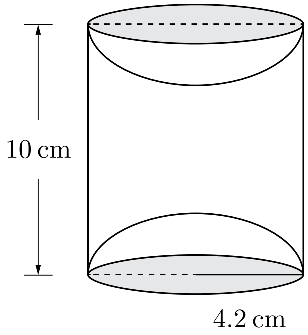
$$\text{Height in } 1^{sh} \text{ vessel} = \frac{7.92}{3.96} = 2 \text{ m}$$

$$\text{Height in } 2^{nd} \text{ vessel} = \frac{5.28}{\frac{22}{7} \times 1} = \frac{5.28 \times 7}{22} = 1.68 \text{ m}$$

139. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 OD 2015]

As per question the figure is shown below.



m193

Volume of cylinder,

$$\pi R^2 H = \pi(4.2)^2 \times 10 \text{ cm}^3$$

Volume of metal scooped out ,

$$= 2 \times \text{Volume of hemisphere}$$

$$= 2 \times \frac{2}{3} \times \pi r^3 = \frac{4}{3}\pi r^3$$

$$= \frac{4\pi}{3}(4.2)^3$$

Volume of rest of cylinder,

$$= \pi(4.2)^2 \times 10 - \frac{4\pi}{3}(4.2)^3 \text{ cm}^3$$

$$= \pi(4.2)^2\left(10 - \frac{4}{3} \times 4.2\right) \text{ cm}^3$$

$$= \pi(4.2)^2(10 - 5.6) \text{ cm}^3$$

$$= \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

Now from rest volume a wire of thickness 1.4 cm i.e radius 0.7 cm is formed. Let l be length of wire. Volume of wire and rest cylinder will be equal.

Volume of wire,

$$\pi r^2 l = \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

$$\pi(0.7)^2 l = \pi(4.2)^2 \times 4.4 \text{ cm}^3$$

$$l = \frac{4.2 \times 4.2 \times 4.4}{07 \times 0.7} \text{ cm}^3$$

$$= 6 \times 6 \times 4.4 = 158.4 \text{ cm}$$

140. 150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Ans : [Board Term-2 OD 2014]

Radius of spherical marble $r_1 = \frac{1.4}{2} = 0.7$ cm

Radius of cylindrical vessel $R = \frac{7}{2} = 3.5$ cm

Let h be the rise in water level then,

Volume of 150 spherical marbles = Volume of water rise

$$150 \times \frac{4\pi}{3} \times \left(\frac{7}{10}\right)^3 = \pi \times \left(\frac{7}{2}\right)^2 \times h$$

$$150 \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{7}{2} \times \frac{7}{2} \times h$$

$$h = \frac{4 \times 7}{5}$$

m194

$$\frac{28}{5} = h \Rightarrow h = 5.6 \text{ cm}$$

Thus 5.6 cm will be rise in the level of water.

141. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a cone of radius 3 cm and height 9 cm. Find the number of toys formed so.

Ans : [Board Term-2 OD Compt. 2017]

Height of cylinder, $H = 15$ cm

Radius of cylinder, $R = \frac{12}{2} = 6$ cm

m195

Radius of cone $r = 3$ cm

Height $h = 9$ cm

Let the number of toys recast be n.

Volume of n conical toys = Volume of cylinder

$$n \times \frac{1}{3}\pi r^2 h = \pi R^2 H$$

$$n \times \frac{1}{3} \times 3 \times 3 \times 9 = 6 \times 6 \times 15$$

$$n = \frac{6 \times 6 \times 15}{3 \times 9} = 20$$

Hence the number of toys is 20.

142. A well diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly around it to a width of 5 m. to form a embankment. Find the height of the embankment.

Ans : [Board Term-2 Foreign 2017]

The volume of soil taken out from the well,

$$\pi^2 rh = \pi \times \left(\frac{3}{2}\right)^2 \times 14 \text{ m}^3$$

m196

The radius of embankment with well

$$= \frac{3}{5} + 5 = \frac{13}{2} \text{ m}$$

Let the y be height of embankment. Then the volume of soil used in embankment,

$$\pi(R^2 - r^2)y = \pi r^2 h$$

$$\pi\left[\left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right]y = \pi \times \left(\frac{3}{2}\right)^2 \times 14$$

$$\frac{160}{4}y = \frac{3}{2} \times \frac{3}{2} \times 14$$

$$y = \frac{3 \times 3 \times 14}{160} = 0.7875 \text{ m}$$

Hence the height of embankment is 78.75 cm.

143. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a rectangular tank of dimensions 50 m $\times$ 44 m. Find the time in which the level of water in the tank will rise by 7 cm.

Ans : [Board Term-2 Delhi Compt. 2017]

Radius of pipe, $r = \frac{14}{2} = 7$ cm

Cross section area of pipe,

$$\pi r^2 = \frac{22}{7} \times \left(\frac{7}{100}\right)^2$$

m197

Speed of water flowing through the pipe

$$= 5 \text{ km/hr} = 15000 \text{ m/hr}$$

In an hour length of water $= 5000$ m

Volume of water flowing from pipe in 1 hr,

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000 \text{ m}^3$$

Let t be time taken to fill the tank. Now total volume of water flowing in time t,

$$\pi r^2 ht = \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000t$$

Volume of water flown= Volume of water in tank

$$\pi r^2 ht = l \times b \times y$$

$$\frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 500t = 50 \times 44 \times \frac{7}{100}$$

$$\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000t = 50 \times 44 \times \frac{7}{100}$$

$$22 \times 50t = 50 \times 44$$

$$t = \frac{50 \times 44}{22 \times 50} = 2$$

Hence, Time taken to fill the tank is 2 hours.

144. From a rectangular block of wood, having dimensions 15 cm $\times$ 10 cm $\times$ 3.5 cm, a pen stand is made by making four conical depressions. The radius of each one of the depression is 0.5 cm and the depth 2.1 cm. Find the volume of wood left in the pen stand.

Ans : [Board Term-2 Delhi Compt. 2017]

Volume of cuboidal block

$$l \times b \times h = 15 \times 10 \times 3.5 = 525 \text{ cm}^3$$

m204

Volume of one cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \text{ cm}^3$$

$$= 0.55 \text{ cm}^3$$

Volume of 4 cones

$$4 \times \frac{\pi r^2 h}{3} = 0.55 \times 4 = 2.2 \text{ cm}^3$$

Volume of wood remaining in pen stand

$$= 525 - 2.2 = 522.80 \text{ cm}^3$$

145. The ratio of the volumes of two spheres is 8 : 27. If r and R are the radii of sphere respectively, then find the $(R - r) : r$.

Ans : [Board Term-2 2012]

Ratio of volumes

$$\frac{\text{Volume of 1}^{\text{st}}\text{sphere}}{\text{Volume of 2}^{\text{nd}}\text{sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{8}{27}$$

m206

$$\frac{r^3}{R^3} = \frac{8}{27}$$

$$\frac{r}{R} = \frac{2}{3}$$

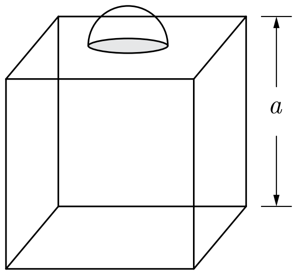
$$\frac{r}{R - r} = \frac{2}{3 - 2} = \frac{2}{1}$$

$$\frac{R - r}{r} = \frac{1}{2}$$

146. A decorative block, made up of two solids - a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. Use $\pi = \frac{22}{7}$.

Ans : [Board Term-2 Delhi 2016]

Let a be the side of cube and r be the radius of hemisphere. As per question the figure is shown below.



Surface area of block

$$= 6a^2 - \pi r^2 + 2\pi r^2$$

$$= 6a^2 + \pi r^2$$

$$= 6 \times (6)^2 + \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

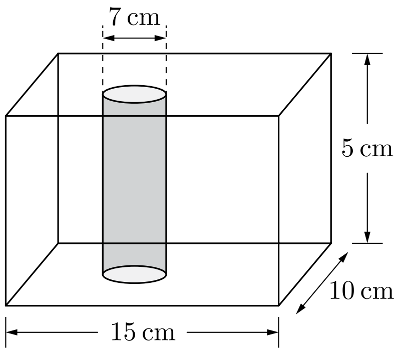
$$= 225.625 \text{ cm}^2.$$

m207

147. In fig., from a cuboidal solid metallic block of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2015]

As per question the figure is shown below.



m208

We have $l = 15$ cm, $b = 10$ cm, $h = 5$cm, $r = \frac{7}{2}$ cm

$$\text{Total Surface area} = 2(lb + bh + hl) + 2\pi rh - 2\pi r^2$$

TSA of cuboidal block

$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$

$$= 550 \text{ cm}^2.$$

Area of curved surface cylinder,

$$2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 = 110 \text{ cm}^2$$

$$\text{Area of two circular bases} = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

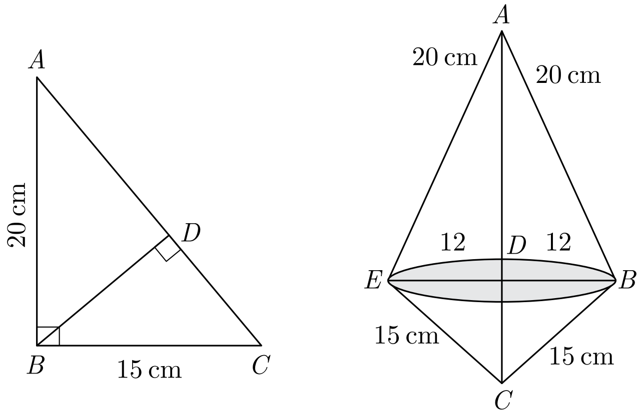
$$= 77 \text{ cm}^2$$

$$\text{Required area} = 550 + 110 - 77 = 583 \text{ cm}^3.$$

148. A right triangle whose sides are 15 cm is made to revolve about its hypotenuse. Find the volume and the surface area of the double cone so formed. (Use $\pi = 3.14$)

Ans : [Board Term-2 2012]

As per question the figure is shown below.



We have $AC^2 = 20^2 + 15^2 = 625$

$$AC = 25 \text{ cm}$$

$$\text{area}(\Delta ABC) = \text{area}(\Delta ABC)$$

$$\frac{1}{2} \times AC \times BD = \frac{1}{2} \times BC \times AB$$

$$25 \times BD = 15 \times 20 = 300$$

$$BD = 12 \text{ cm}$$

m209

Volume of double cone,

$$= \text{Volume of upper cone} + \text{Volume of lower cone}$$

$$= \frac{1}{3}\pi(BD)^2 \times AD + \frac{1}{3}\pi(BD)^2 \times CD$$

$$= \frac{1}{3}\pi(BD)^2(AD + CD)$$

$$= \frac{1}{3}\pi(BD)^2(AC)$$

$$= \frac{1}{3} \times 3.14 \times (12)^2 \times 25$$

$$= \frac{1}{3} \times 3.14 \times 144 \times 25 = 3768 \text{ cm}^2$$

Surface area= CSA of upper cone + CSA of lower cone

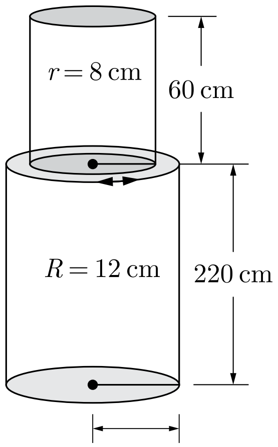
$$= \pi(12)(20) + \pi(12)(15)$$
$$= 12\pi\{20 + 15\}$$
$$= 12 \times 3.14 \times 35$$
$$= 1318.8 \text{ cm}^2$$

149. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pipe, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Ans : [Board 2019 OD, 2012]

As per question the figure is shown below.

m210



Radius of lower cylinder, $R = 12$ cm

Height of lower cylinder, $H = 220$ cm

Radius of upper cylinder, $r = 8$ cm

Height of upper cylinder, $h = 60$ cm

Volume of solid iron pole,

$$\pi R^2 H + \pi r^2 h = 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$$
$$= 111532.8 \text{ cm}^3$$
$$\text{Mass of pole} = 111532.8 \times 8 \text{ g}$$
$$= 892262.4 \text{ g}$$
$$= 892.2624 \text{ kg.}$$

150. A heap of wheat is in the form of cone of diameter 6 m and height 3.5 m. Find its volume . How much canvas cloth is required to just cover the heap ? Use $\pi = \frac{22}{7}$

Ans :

Radius of cone, $r = \frac{6}{2} = 3$ cm

m211

Height of cone, $h = 3.5$ cm

Volume of wheat in the form of cone

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 3.5$$
$$= 11 \times 3 = 33 \text{ m}^3$$
$$l = \sqrt{3^2 + 3.5} = 4.609 \text{ m}$$

Canvas required to cover the heap,

$$\pi rl = \frac{22}{7} \times 3 \times 4.609$$
$$= 43.45 \text{ m}^2.$$

151. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. 100 spherical lead balls are dropped into vessel. One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

Ans : [Board Term-2 Foreign 2015]

Volume of water in cone

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (5)^3 \times 8 = \frac{200}{3}\pi \text{ cm}^3$$

Volume of water flows out

$$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi \text{ cm}^3$$

Let r be the radius of one spherical ball.

m212

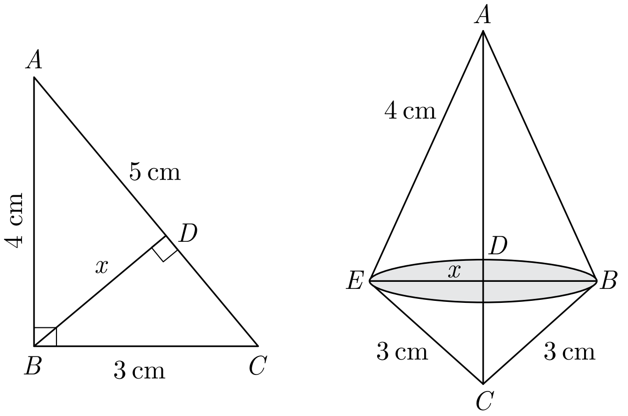
Volume of 100 spherical ball,

$$\frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$$
$$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$$
$$r = \frac{1}{2} = 0.5 \text{ cm}$$

152. A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the longest side. Find the surface area of figure obtained. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 2012]

As per question the figure is shown below.



By revolving right triangle about longest side double cone is generated. Let x be radius of double cone.

$$\text{area}(\Delta\, ABC) = \text{area}(\Delta\, ABC)$$

$$\frac{1}{2} \times 5 \times x = \frac{1}{2} \times 3 \times 4$$

m214

$$x = \frac{12}{5} = 2.4 \text{ cm}$$

Surface area of double cone,

$$\pi r l_1 + \pi r l_2 = \pi x (l_1 + l_2)$$

$$= \frac{22}{7} \times 2.4 \times (3+4)$$

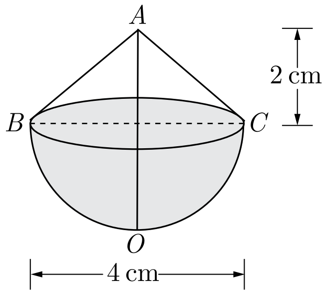
$$= 22 \times 2.4 = 52.8 \text{ cm}^2.$$

153. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volume of the cylinder and toy. (Use $\pi = 3.14$)

Ans : [Board Term-2 2012]

Let BOC is a hemisphere and ABC is a cone. As per question the figure is shown below.

m215



Radius of hemisphere is equal to the radius of cone which is $\frac{4}{2} = 2$ cm.

Height of cone, $h = 2$ cm

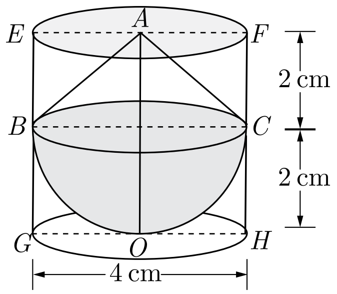
$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1}{3}\pi r^2 (2r + h) = \frac{1}{3} \times 3.14 \times 2 \times 2(2 \times 2 + 2)$$

$$= \frac{1}{3} \times 3.14 \times 4 \times 6$$

$$= 25.12 \text{ cm}^3$$

Let right circular cylinder $EFGH$ circumscribe the given solid toy.



$$\text{Radius of cylinder} = 2 \text{ cm}$$

$$\text{Height of cylinder} = 4 \text{ cm}$$

Volume of right circular cylinder

$$\pi r^2 h = 3.14 \times (2)^2 \times 4 \text{ cm}^3$$

$$= 50.24 \text{ cm}^3$$

Difference of two volume

$$= \text{Volume of cylinder} - \text{Volume of toy}$$

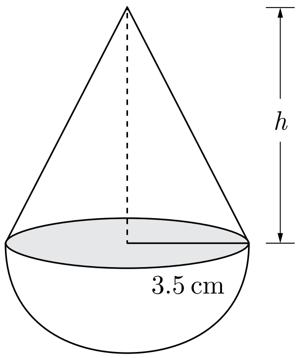
$$= 50.24 - 25.12 = 25.12 \text{ cm}^3.$$

154. A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also find the cost of painting the hemisphere part of the toy at the rate of Rs. 10 per cm^2. Use $\pi = \frac{22}{7}$

Ans : [Board Term-2 Delhi 2015]

As per question the figure is shown below.

m216



Radius of hemisphere is equal to the radius of cone which is 3.5 cm.

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$166\frac{5}{6} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$\frac{1001}{6} = \frac{\pi r^2}{3}(2r+h)$$

$$1001 = 2\pi r^2(2r+h)$$

$$1001 = 2 \times \frac{22}{7} \times (3.5)^2(2 \times 3.5 + h)$$

$$1001 = 22 \times 3.5 \times (7+h)$$

$$91 = 2 \times 3.5 \times (7+h)$$

$$13 = 7 + h \Rightarrow h = 6$$

Height of the toy $= 6 + 3.5 = 9.5$ cm.

CSA of hemisphere,

$$2\pi r^2 = 2 \times \frac{22}{7} \times 3.5 \times 3.5 = 77 \text{ cm}^2$$

$$\text{Cost of painting} = 10 \times 77 = 770 \text{ Rs}$$

155. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm. If the increase in the level of the water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.

Ans : [Board Term-2 Delhi 2015]

Let r be the internal radius of the pipe, then cross section area of pipe is πr^2.

Speed of water flowing through the pipe

$$= 2.52 \text{ km/hr} = 2520 \text{ m/hr}$$

In an hour length of water $= 2520$ m

Volume of water flowing from pipe in 1 hr,

$$\pi r^2 h = \pi r^2 2520 \text{ m}^3$$

m217

In 30 minute or in 0.5 hour,

Volume of water flown= Volume of water in tank

$$\pi r^2 2520 \times 0.5 = \pi \times (0.4)^2 \times 3.15$$

$$1260 r^2 = 0.4 \times 0.4 \times 3.15$$

$$400 r^2 = 0.4 \times 0.4$$

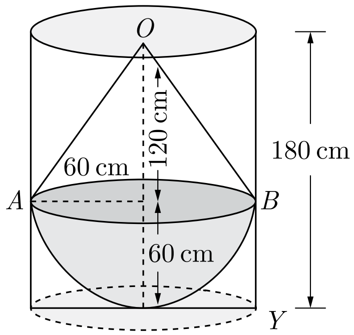
$$20 r = 0.4 \Rightarrow r = \frac{0.4}{20} = 0.02 \text{ m}$$

Internal radius is 2 cm and diameter of pipe is 4 cm.

156. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Ans : [Board Term-2, 2015]

As per question the figure is shown below.



m218

Height of cone, $h = 120$ cm,

Radius of cone, $r = 60$ cm

Radius of hemisphere, $r = 60$ cm.

Height of cylinder, $H = 180$ cm,

Radius of cylinder, $R = 60$ cm

Radius of cone, hemisphere and cylinder is equal to $r = 60$ cm

Volume of solid,

$$V_{solid} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}(h + 2r)$$

$$= \frac{\pi r^2}{3} \times 240 = 80\pi r^2$$

Volume of water in the cylinder is equal to the volume of cylinder.

$$V_{cylinder} = \pi r^2 h$$

$$= \pi \times r^2 \times 180 = 180\pi r^2$$

Water left in the cylinder is equal to the difference of the volume of water in cylinder and volume of solid.

Water left in the cylinder,

$$= V_{cylinder} - V_{solid}$$

$$= 180\pi r^2 - 80\pi r^2$$

$$= 100\pi r^2$$

$$= 100 \times \frac{22}{7} \times (60)^2$$

$$= \frac{100 \times 22 \times 60 \times 60}{7}$$

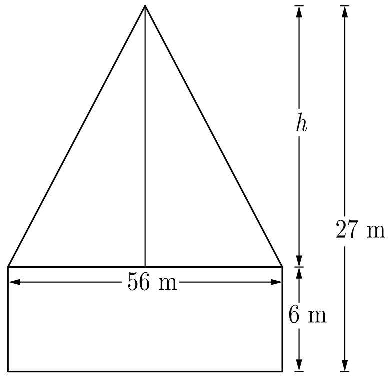
$$= 1131428 \text{ cm}^3$$

157. A circus tent is in the shape of a cylinder surmounted by a conical top of same diameter. If their common diameter is 56 m, the height of cylindrical part is 6 m and the total height of the tent above the ground is 27 m, find the area of canvas used in the tent.

Ans : [Board Term-2 Delhi Compt. 2017]

m219

As per question the figure is shown below.



Total height of tent $H_{Total} = 27$ m

Height of cylindrical part $h = 6$ m

Height of conical part $H = 27 - 6 = 21$ m

Radius of cone $R = \frac{56}{2} = 28$ m

Radius of cylinder $R = \frac{56}{2} = 28$ m

Slant height of cone $L = \sqrt{R^2 + H^2}$

$$= \sqrt{28^2 + 21^2}$$

$$= \sqrt{784 + 441} = \sqrt{1225}$$

$$= 35 \text{ m}$$

Area of canvas used,

$$2\pi rh + \pi rl = \pi r(2h + l)$$

$$= \frac{22}{7} \times 28(2 \times 6 + 35)$$

$$= 22 \times 4 \times 47$$

$$= 4136 \text{ m}^2$$

158. From a right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same radius is cut-out. Find the total surface area of the remaining solid.

Ans : [Board Term-2 OD 2017]

m220

Radius of cylinder and cone,

$$r = 0.7 \text{ cm}$$

Height of cylinder and cone,

$$h = 2.4 \text{ cm}$$

Slant height of cone

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{0.7^2 + 2.4^2} = 2.5 \text{ m}$$

Total surface area of remaining solid,

= CSA of cylinder + CSA of cone + Area of top.

$$= 2\pi rh + \pi rl + \pi r^2$$

$$= \pi r(2h + l + r)$$

$$= \frac{22}{7} \times 0.7(2 \times 2.4 + 2.5 + 0.7)$$

$$= \frac{22}{7} \times 0.7 \times 8 = \frac{176}{10}$$

Hence total surface area is 17.6 cm^2

CHAPTER 14

STATISTICS

ONE MARK QUESTIONS

Multiple Choice Questions

1. The median and mode respectively of a frequency distribution are 26 and 29, Then its mean is

(a) 27.5 (b) 24.5

(c) 28.4 (d) 25.8

Ans : [Board 2020 Delhi Basic]

We have $M_o = 3M_d - 2M$

$$29 = 3 \times 26 - 2M$$

$$2M = 78 - 29 = 49$$

$$M = \frac{49}{2} = 24.5$$

n101

Thus (b) is correct option.

2. The cumulative frequency table is useful in determining

(a) Mean (b) Median

(c) Mode (d) All of these

Ans : [Board 2020 OD Basic]

Cumulative frequency is defined as a running total of frequencies. It is helpful in finding the mean, median and mode.

n102

Thus (d) is correct option.

3. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

(a) 6 (b) 7

(c) 8 (d) 12

n103

Ans :

Let x be the upper limit and y be the lower limit.

Since the mid value of the class is 10.

Hence, $$\frac{x+y}{2} = 10$$

$$x + y = 20 \quad ...(1)$$

Since width of the class is 6,

$$x - y = 6 \quad ...(2)$$

Solving (1) and (2), we get $y = 7$

Hence, lower limit of the class is 7.

Thus (b) is correct option.

4. Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

The upper limit of the median class in the given data is

(a) 165 (b) 155

(c) 160 (d) 170

n104

Ans : [Board 2020 SQP Standard]

We prepare the following cumulative table

Height x (in cm)	Number of Students (f)	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	08	46
170-175	09	55
175-180	08	63
	$N = 63$	

We have, $$N = 63; \frac{N}{2} = \frac{63}{2} = 31.5$$

The cumulative frequency just greater than $\frac{N}{2}$ is 38 and the corresponding class is 160-165. Thus upper limit is 165.

Thus (a) is correct option.

5. For finding the popular size of readymade garments, which central tendency is used?

(a) Mean

(b) Median

(c) Mode

(d) Both Mean and Mode

n105

Ans :

For finding the popular size of ready made garments, mode is the best measure of central tendency.

Thus (c) is correct option.

6. If the difference of mode and median of a data is 24, then the difference of median and mean is

(a) 12 (b) 24

(c) 08 (d) 36

Ans :

n106

We have, $M_o - M_d = 24$

We know $M_o = 3M_d - 2M$

Now $M_o - M_d = 2M_d - 2M$

$24 = 2(M_d - M)$

$M_d - M = 12$

Thus (a) is correct option.

7. If the mean of the numbers $27+x$, $31+x$, $89+x$ $107+x, 156+x$ is 82, then the mean of $130+x$, $126+x$, $68+x$, $50+x$, and $1+x$ is

(a) 75 (b) 157

(c) 82 (d) 80

n107

Ans :

Given,

$$82 = \frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$$

$$82 \times 5 = 410 + 5x$$

$$410 - 410 = 5x \Rightarrow x = 0$$

Required mean is,

$$\overline{x} = \frac{130+x+126+x+68+x+50+x+1+x}{5}$$

$$= \frac{375+5x}{5} = \frac{375+0}{5}$$

$$= \frac{375}{5} = 75$$

Thus (a) is correct option.

8. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observation of the set is increased by 2, then the median of the new set

(a) Is increased by 2

(b) Is decreased by 2

(c) Is two times the original median

(d) Remains the same as that of the original set

Ans :

n108

Since, $n = 9$

then, median term $= \left(\frac{9+1}{2}\right)^{th} = 5^{th}$ item.

Now, last four observations are increased by 2, but median is 5^{th} observation, which is remaining unchanged. Hence there will be no change in median.

Thus (d) is correct option.

9. If the coordinates of the point of intersection of less than ogive and more than ogive is (13.5,20), then the value of median is

(a) 13.5 (b) 20

(c) 33.5 (d) 7.5

n109

Ans :

The abscissa of point of intersection gives the median of the data. So, median is 13.5.

Thus (a) is correct option.

10. A set of numbers consists of three 4's, five 5's, six 6's, eight 8's and seven 10's. The mode of this set of numbers is

(a) 6 (b) 7

(c) 8 (d) 10

n110

Ans :

Mode of the data is 8 as it is repeated maximum number of times.

Thus (c) is correct option.

11. If the mean of the observation $x, x+3, x+5, x+7$ and $x+10$ is 9, the mean of the last three observation is

(a) $10\frac{1}{3}$ (b) $10\frac{2}{3}$

(c) $11\frac{1}{3}$ (d) $11\frac{2}{3}$

n111

Ans :

$$\text{Mean} = \frac{\text{Sum of all the observations}}{\text{Total no. of observation}}$$

$$9 = \frac{x+x+3+x+5+x+7+x+10}{5}$$

$$9 = \frac{5x+25}{5}$$

$$x = 4$$

So, mean of last three observation,

$$= \frac{x+5+x+7+x+10}{3} = \frac{5x+22}{3}$$

$$\frac{3x+22}{3} = \frac{3 \times 4 + 22}{3}$$

$$= \frac{12+22}{3} = \frac{34}{3} = 11\frac{1}{3}$$

Thus (c) is correct option.

12. The mean weight of 9 students is 25 kg. If one more student is joined in the group the mean is unaltered, then the weight of the 10th student is

(a) 25 kg (b) 24 kg

(c) 26 kg (d) 23 kg

n112

Ans :

The sum of the weights of the 9 students $= 25 \times 9 = 225$. If one more student is joined in the group, then total number of students is 10 and mean is 25.

Hence, the sum of the weights of the 10th students $= 25 \times 10 = 250$.

Hence, the weight of the 10th student is $250 - 225 = 25$ kg.

However we can answer this question without any calculation. If mean is not altered on adding more data, then added data must be of mean value.

Thus (a) is correct option.

13. The mean and median of the data a, b and c are 50 and 35 respectively, where $a < b < c$. If $c - a = 55$, then $(b - a)$ is

(a) 8 (b) 7

(c) 3 (d) 5

n113

Ans :

Since, a, b and c and are in ascending order, therefore median is b i.e $b = 35$.

Mean $$\frac{a+b+c}{3} = 50$$

$$a+b+c = 150$$

$$a+c = 150-35 = 115 \quad ...(1)$$

Also, it is given that $c - a = 55$ ...(2)

Subtracting equation (2) and (1), we get

$$a = 30$$

Hence, $b - a = 35 - 30 = 5$

Thus (d) is correct option.

14. Observations of some data are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. If the median of the data is 4. then the value of x is

(a) 5 (b) 15

(c) 9 (d) 10

n114

Ans :

Given observations are $\frac{x}{5}$, x, $\frac{x}{3}$, $\frac{2x}{3}$, $\frac{x}{4}$, $\frac{2x}{5}$ and $\frac{3x}{4}$ where $x > 0$. On arranging the above observations in ascending order, we get

$$\frac{x}{5}, \frac{x}{4}, \frac{x}{3}, \frac{2x}{5}, \frac{2x}{3}, \frac{3x}{4}, x$$

Here, total number of observations are 7, which is odd.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

$$= \left(\frac{7+1}{2}\right)^{th} \text{observation}$$

$$= 4^{th} \text{ observation} = \frac{2x}{5}$$

$$\text{Median} = \frac{2x}{5} = 4$$

$$x = \frac{4 \times 5}{2} = 10$$

Thus (d) is correct option.

15. If the mean of the squares of first n natural numbers is 105, then the first n natural numbers is

(a) 8 (b) 9

(c) 10 (d) 11

n115

Ans :

Sum of square, $\sum x^2 = \frac{n(n+1)(2n+1)}{6}$

Mean of squares of first n natural numbers,

$$105 = \frac{(n+1)(2n+1)}{6}$$

$$2n^2 + 3n + 1 = 630$$

$$2n^2 + 3n - 629 = 0$$

$$2n^2 + 37n - 34n - 629 = 0$$

$$n(2n+37) - 17(2n+37) = 0$$

$$(2n+37)(n-17) = 0 \Rightarrow n = 17$$

Since, n is odd, therefore median is $= \left(\frac{17+1}{2}\right)^{\text{th}} = 9^{\text{th}}$ observation.

Thus (b) is correct option.

16. Mode of the following grouped frequency distribution is

Class	**Frequency**
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

(a) 13.6 (b) 15.6

(c) 14.6 (d) 16.6

n116

Ans :

We observe that the class 12-15 has maximum frequency 23. Therefore, this is the modal class.

We have, $l = 12$, $h = 3$, $f_1 = 23$, $f_0 = 10$ and $f_2 = 21$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + \frac{13}{5} = 14.6$$

Thus (c) is correct option.

17. While computing the mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes

(b) centred at the class marks of the classes

(c) centred at the upper limits of the classes

(d) centred at the lower limits of the classes

Ans :

n117

While computing mean of ground data, we assume that the frequencies distribution table.

Thus (b) is correct option.

18. If median is 137 and mean is 137.05, then the value of mode is

(a) 156.90 (b) 136.90

(c) 186.90 (d) 206.90

n118

Ans :

$$M_o = 3M_d - 2M$$

$$= 3(137) - 2(137.05)$$

$$= 411 - 274.10 = 136.90$$

Thus (b) is correct option.

19. The following data gives the distribution of total household expenditure (in $<$) of manual workers in a city.

Expenditure (in $<$)	**Frequency**
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	07

Then, find the average expenditure which is done by the maximum number of manual workers.

(a) 1747.26 (b) 1847.26

(c) 1947.26 (d) 2047.26

n119

Ans :

We observe that the class 1500-2000 has maximum frequency 40. Therefore, this is the modal class.

We have $l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$ and $f_2 = 23$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500$$

$$= 1500 + \frac{16}{23} \times 500$$

$$= 1847.26$$

Thus (b) is correct option.

20. For the following distribution

Marks	Number of Students	Marks	Number of students
Below 10	3	Below 40	57
Below 20	12	Below 50	75
Below 30	28	Below 60	80

The modal class is

(a) 0-20 (b) 20-30

(c) 30-40 (d) 50-60

n120

Ans :

Let us first construct the following frequency distribution table.

Marks	Number of Students
0-10	3
10-20	9
20-30	16
30-40	29
40-50	18
50-60	5

Since, the maximum frequency is 29 and the class corresponding to this frequency is 30-40. So, the modal class is 30-40.

Thus (c) is correct option.

21. If X, M and Z are denoting mean, median and mode of a data and $X:M = 9:8$, then the ratio $M:Z$ is

(a) 3 : 4 (b) 4 : 9

(c) 4 : 3 (d) 2 : 5

Ans :

Since, $M_o = 3M_d - 2M$

$$Z = 3M - 2X \quad ...(1)$$

Now $X:M = 9:8$

$$\frac{X}{M} = \frac{9}{8}$$

n121

$$X = \frac{9M}{8}$$

Substituting the value of X in equation (1), we get

$$Z = 3M - 2 \times \frac{9M}{8} = 3M - \frac{9M}{4}$$

$$Z = \frac{3M}{4}$$

$$\frac{M}{Z} = \frac{4}{3}$$

or $M:Z = 4:3$

Thus (c) is correct option.

22. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 min and summarised in the table give below.

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15
70-80	08

Then, the mode of the data is

(a) 34.7 (b) 44.7

(c) 54.7 (d) 64.7

n122

Ans :

Here, modal class is 40-50. Since, it has maximum frequency which is 20.

So, $l = 40$, $f_1 = 20$, $f_0 = 12$, $f_2 = 11$ and $h = 10$

$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{40 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{80}{17}$$

$$= 40 + 4.7 = 44.7 \text{ cars}$$

Thus (b) is correct option.

23. If the mean of a, b, c is M and $ab + bc + ca = 0$, the mean of a^2, b^2 and c^2 is KM^2, then K is equal to

(a) 3 (b) 9

(c) 6 (d) 4

Ans :

We have $\frac{a + b + c}{3} = M$

n123

$$a + b + c = 3M$$

and $\frac{a^2 + b^2 + c^2}{3} = KM^2$

Now, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$(a+b+c)^2 = a^2+b^2+c^2 \quad [ab+bc+ca=0]$$

$$(3M)^2 = 3KM^2$$

$$9M^2 = 3KM^2 \Rightarrow K = 3$$

Thus (a) is correct option.

24. In the formula $\overline{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data d_i's are deviation from a of

(a) lower limits of the classes

(b) upper limits of the classes

(c) mid-points of the classes

(d) frequencies of the class marks

n124

Ans :

Mid-point of the classes $= (x_i - a)$,

where, $$x_i = \frac{\text{upper limit} + \text{lower limit}}{2}$$

So, the option (c) is correct, which is the required answer.

Thus (c) is correct option.

25. While computing mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes

(b) centred at the class marks of the classes

(c) centred at the upper limits of the classes

(d) centred at the lower limits of the classes

Ans : (b) centred at the class marks of the classes

Frequencies are centred at the class-marks of the classes.

So, the option (b) is correct, which is the required answer.

Thus (b) is correct option.

n125

26. If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\overline{x}$ is the mean, then $\sum(f_i x_i - \overline{x})$ is equal to

(a) 0 (b) -1

(c) 1 (d) 2

n126

Ans :

$$\sum(f_i x_i - \overline{x}) = \sum f_i x_i - \sum \overline{x} = \sum f_i x_i - n\overline{x}$$

$$= \sum f_i x_i - \sum f_i x_i = 0 \qquad \left(\overline{x} = \frac{\sum f_i x_i}{n}\right)$$

So, the option (a) is correct, which is the required answer.

27. In the formula $\overline{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped frequency distribution, u_i is equal to

(a) $\frac{x_i + a}{h}$ (b) $h(x_i - a)$

(c) $\frac{x_i - a}{h}$ (d) $\frac{a - x_i}{h}$

n127

Ans :

We know that, $$u_i = \frac{x_i - a}{h}$$

So, the option (c) is correct, which is the required answer.

Thus (c) is correct option.

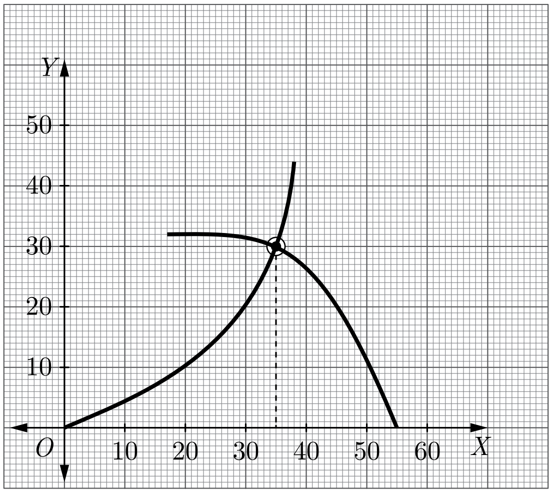
28. The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its

(a) mean (b) median

(c) mode (d) All of these

n128

Ans :



It gives median of the grouped-data.

So, the option (b) is correct, which is the required answer.

Thus (b) is correct option.

29. For the following distribution.

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is

(a) 15 (b) 25

(c) 30 (d) 35

n129

Ans :

Here,

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Now, $\frac{N}{2} = \frac{33}{2} = 33$, which lies in the interval 10-15. Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval 15-20. Therefore, lower limit of modal class is 15. Hence, required sum is $10 + 15 = 25$.

Thus (b) is correct option.

30. Consider the following frequency distribution

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

(a) 17 (b) 17.5

(c) 18 (d) 18.5

n130

Ans :

Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

Class	Frequency	Cumulative frequency
−0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval $11.5 - 17.5$. Hence, the upper limit is 17.5.

Thus (b) is correct option.

31. For the following distribution:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

The modal class is

n131

(a) 10-20 (b) 20-30

(c) 30-40 (d) 50-60

Ans :

Marks	Number of students
0-10	$3 - 0 = 3$
10-20	$12 - 3 = 9$
20-30	$27 - 12 = 15$
30-40	$57 - 27 = 30$
40-50	$75 - 57 = 18$
50-60	$80 - 75 = 5$

Class 30-40 has the maximum frequency 30, therefore this is model class.

Thus (c) is correct option.

32. Consider the data:

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

(a) 0 (b) 19

(c) 20 (d) 38

n132

Ans :

Class	Frequency	Cumulative frequency
65-85	4	7
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2} = \frac{67}{2} = 33.5$, which lies in the interval $125 - 145$. Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145. Hence, the lower limit of modal class is 125.

Required difference

$$= \text{Upper limit of median class} - \text{Lower limit of modal class}$$

$$= 145 - 125 = 20$$

Thus (c) is correct option.

33. The times, in seconds, taken by 150 athletes to run a 110 m hurdle race are tabulated below

Class	Frequency
13.8-14	2
14-14.2	4
14.2-14.4	5
14.4-14.6	71
14.6-14.8	48
14.8-15	20

The number of athletes who completed the race in less than 14.6 second is :

(a) 11 (b) 71

(c) 82 (d) 130

n133

Ans :

The number of athletes who completed the race in less than 14.6

$$= 2 + 4 + 5 + 71 = 82$$

Thus (c) is correct option.

34. Consider the following distribution :

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is :

(a) 3 (b) 4

(c) 48 (d) 51

n134

Ans :

Marks obtained	Number of students
0-10	$(63-58) = 5$
10-20	$(58-55) = 3$
20-30	$(55-51) = 4$
30-40	$(51-48) = 3$
40-50	$(48-42) = 6$
50-60	$42 = 42$

Hence, frequency in the class interval 30-40 is 3.

Thus (a) is correct option.

35. **Assertion :** If the number of runs scored by 11 players of a cricket team of India are 5, 19, 42, 11, 50, 30, 21, 0, 52, 36, 27 then median is 30.

Reason : Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ value, if n is odd.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Arranging the terms in ascending order,

0, 5, 11, 19, 21, 27, 30, 36, 42, 50, 52

n135

$$\text{Median value } = \left(\frac{11+1}{2}\right)^{\text{th}}$$

$$= 6^{\text{th}} \text{ value } = 27$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

36. .... is mid value of class interval.

Ans :

Class mark

n136

37. .... is the value of the observation having the maximum frequency.

Ans :

Mode

n137

38. The mid-point of a class interval is called its ....

Ans :

class-mark

n138

39. The .... is the most frequently occurring observation.

Ans :

mode

n139

40. Facts or figures, collected with a definite purpose, are called ....

Ans :

data

n140

41. To find the mode of a grouped data, the size of the classes is ....

Ans :

uniform

n141

42. .... is graphical representation of cumulative frequency distribution.

Ans :

Ogive

n142

43. Median divides the total frequency into .... equal parts.

Ans :

two

n143

44. Average of a data is called ....

Ans :

Mean

n144

45. On an ogive, point A (say), whose Co-ordinated is $\frac{n}{2}$ (half of the total observation), has its X-coordinate equal to .... of the data.

Ans :

Median

n145

46. Value of the middle-most observation (s) is called ....

Ans :

median

n146

47. Two ogive, for the same data intersect at the point P. Then Y-coordinate of P represents ....

Ans :

cumulative

n147

48. The algebraic sum of the deviations from arithmetic mean is always ....

Ans :

zero

n148

VERY SHORT ANSWER QUESTIONS

49. Find the class-marks of the classes 10-25 and 35-66.

Ans : [Board 2020 OD Standard]

Class mark of 10 – 25, $= \frac{10+25}{2} = \frac{35}{2} = 17.5$

and class mark of 35 – 55, $= \frac{35+55}{2} = \frac{90}{2} = 45$

n149

50. Find the class marks of the classes 15-35 and 45-60.

Ans : [Board 2020 OD Standard]

Class mark of 15 – 35 $= \frac{15+35}{2} = \frac{50}{2} = 25$

n150

and class mark of 45 – 60 $= \frac{45+60}{2} = \frac{105}{2} = 52.5$

51. If the mean of the first n natural number is 15, then find n.

Ans : [Board 2020 Delhi Standard]

Given : 1, 2, 3, 4, ... to n terms.

The sum of first n natural numbers

n151

$$S_n = \frac{n(n+1)}{2}$$

Mean, $$M = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n+1}{2}$$

$$n+1 = 30 \Rightarrow n = 29$$

52. Find the class marks of the classes 20-50 and 35-60.

Ans : [Board 2020 OD Standard]

n152

Class mark of 20 – 50 $= \frac{20+50}{2} = \frac{70}{2} = 35$ and

Class mark of 35 – 60 $= \frac{35+60}{2} = \frac{95}{2} = 47.5.$

53. If the median of a series exceeds the mean by 3, find by what number the mode exceeds its mean?

Ans : [Board Term-1, 2015]

We have $$M_d = M + 3$$

Now $$M_o = 3M_d - 2M$$

$$= 3(M+3) - 2M$$

$$= 3M + 9 - 2M = M + 9$$

n153

Hence mode exceeds mean by 9.

54. From the following frequency distribution, find the median class :

Cost of living index	1400-1500	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

Ans : [Board Term-1, 2015]

We prepare following cumulative frequency table to find median class.

Cost of living index	Number of weeks f	c.f.
1400-1500	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52

We have $N = 52$; $\frac{N}{2} = 26$

Cumulative frequency just greater than $\frac{N}{2}$ is 44 and the corresponding class is 1700-1850. Thus median class is 1700-1850.

n154

55. In the following frequency distribution, find the median class.

Height (in cm)	104-145	145-150	150-155	155-160	160-165	165-170
Frequency	5	15	25	30	15	10

Ans : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Height	Frequency	c.f.
140-145	5	5
145-150	15	20
150-155	25	45
155-160	30	75
160-165	15	90
165-170	10	100
	$N = 100$	

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 75 and the corresponding class is 155-160. Thus median class is 155-160.

n155

56. Find median of the data, using an empirical relation when it is given that Mode = 12.4 and Mean = 10.5.

Ans : [Board Term-1, 2015]

Mode, $M_o = 12.4$

Mean, $M = 10.5$

Median, $M_d = \frac{1}{3}M + \frac{2}{3}M_o$

$$= \frac{1}{3}(12.4) + \frac{2}{3}(10.5)$$

$$= \frac{12.4}{3} + \frac{21}{3}$$

$$= \frac{12.4 + 21}{3} = \frac{33.4}{3}$$

$$= \frac{33.4}{3} = 11.13$$

n156

57. Consider the following distribution :

Marks Obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
Number of students	63	58	55	51	48	42

(i) Calculate the frequency of the class 30 - 40.

(ii) Calculate the class mark of the class 10 - 25.

Ans : [Board Term-1, 2014]

Class Interval	c.f.	f
0-10	63	5
10-20	58	3
20-30	55	4
30-40	51	3
40-50	48	6
50-60	42	42

(i) Frequency of the class 30 - 40 is 3.

n157

(ii) Class mark of the class : $10 - 25 = \frac{10 + 25}{2}$

$$= \frac{35}{2} = 17.5$$

58. Which central tendency is obtained by the abscissa of point of intersection of less than type and more than type ogives ?

Ans :

Median.

n158

59. What is abscissa of the point of intersection of the "Less than type" and of the "More than type" cumulative frequency curve of a grouped data ?

Ans :

The abscissa of the point of intersection of the "Less than type" and "More than type" cumulative frequency curve of a grouped data is median.

n159

60. Find the mean of the data using an empirical formula when it is given that mode is 50.5 and median in 45.5.

Ans : [Board Term-1 2015]

Mode, $M = 50.5$

Median, $M_d = 45.5$

n160

Now $3M_d = M_o + 2M$

$3 \times 45.5 = 50.5 + 2M$

Mean, $M = \dfrac{136.5 - 50.5}{2} = 43$

Hence mean is 43.

61. Find the mean of first odd multiples of 5.

Ans : [Board Term-1 2012]

The first five odd multiples of 5, according to the problem are : 5, 15, 25, 35, 45

n161

Mean $= \dfrac{5 + 15 + 25 + 35 + 45}{5} = \dfrac{125}{5} = 25$

62. Median of a data is 52.5 and its mean is 54, use empirical relationship between three measure of central tendency to find its mode.

Ans : [Board Term-1 2012]

Median $M_d = 52.5$

and mean $M = 54$

n162

Now $3M_d = M_o + 2M$

$3 \times 52.5 = M_o + 2 \times 54$

Mode $M_o = 157.5 - 108 = 49.5$

TWO MARKS QUESTIONS

63. Find the mean the following distribution :

Class	3-5	5-7	7-9	9-11	11-13
Frequency	5	10	10	7	8

Ans : [Board 2020 Delhi Standard]

Class	Frequency (f_i)	Mid-Value (x_i)	$f_i x_i$
3-5	5	4	20
5-7	10	6	60
7-9	10	8	80
9-11	7	10	70
11-13	8	12	96
	$\sum f = 40$		$\sum f_i x_i = 326$

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{326}{40} = 8.15$

n163

64. Find the mode of the following data :

Class :	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

Ans : [Board 2020 Delhi Standard]

Class 60-80 has the maximum frequency 12, therefore this is model class.

Hence, $l = 60,\ f_1 = 12,\ f_0 = 6,\ f_2 = 6$ and $h = 20$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 60 + \dfrac{12 - 10}{2 \times 12 - 10 - 6} \times 20$

$= 60 + \dfrac{2 \times 20}{24 - 16}$

n164

$= 60 + \dfrac{40}{8} = 60 + 5$

$= 65$

65. The mode of the following frequency distribution is 36. Find the missing frequency f.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	8	10	f	16	12	6	7

Ans : [Board 2020 OD Basic]

Mode is 36 which lies in class 30-40, therefore this is model class.

Here, $f_0 = f,\ f_1 = 16,\ f_2 = 12,\ l = 30$ and $h = 10$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$36 = 30 + \dfrac{16 - f}{2 \times 16 - f - 12} \times 10$

$$6 = \frac{16-f}{20-f} \times 10$$

$$120 - 6f = 160 - 10f$$

$$4f = 40 \Rightarrow f = 10$$

n165

Thus (d) is correct option.

66. Find the median for the given frequency distribution :

Class	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Frequency	2	3	8	6	6	3	2

Ans : [Board 2020 OD Basic]

Class	Frequency	*c.f.*
40-45	2	2
45-50	3	5
50-55	8	13
55-60	6	19
60-65	6	25
65-70	3	28
70-75	2	30
	$N = 30$	

We have $N = 30$; $\frac{N}{2} = 15$

Cumulative frequency just greater than $\frac{N}{2}$ is 19 and the corresponding class is 55-60. Thus median class is 55-60.

Now $l = 55,\ f = 6,\ F = 13,\ h = 5$

n166

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

$$= 55 + \left(\frac{15-13}{6}\right) \times 5$$

$$= 55 + \frac{5}{3} = 55 + 1.67$$

$$= 56.67$$

67. Find the mean of the following distribution :

Class	10-25	25-40	40-55	55-70	70-85	85-100
Frequency	2	3	7	6	6	6

Ans : [Board 2020 Delhi Basic]

Let $a = 62.5$ be assumed mean.

Class Interval	Frequency (f_i)	*c.f.*	x_i	$U_i = \frac{x-a}{h}$	$f_i u_i$
10-25	2	2	17.5	-3	-6
25-40	3	5	32.5	-2	-6
40-55	7	12	47.5	-1	-7
55-70	6	18	62.50= a	0	0
70-85	6	24	77.5	1	6
85-100	6	30	92.5	2	12
	$\sum f_i = 30$				$\sum f_i u_i = -1$

Mean, $$\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

n167

$$= 62.5 + \frac{-1}{30} \times 15$$

$$= 62.5 - \frac{1}{2} = 62.5 - 0.5 = 62$$

68. Find the mean of the following data :

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	20	35	52	44	38	31

Ans :

Let $a = 70$ be assumed mean.

C.I.	Frequency f	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-20	20	10	-3	-60
20-40	35	30	-2	-70
40-60	52	50	-1	-52
60-80	44	70	0	0
80-100	38	90	1	38
100-120	31	110	2	62
	$\sum f_i = 220$			$\sum f_i u_i = -82$

Mean, $$\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

n168

$$= 70 + \frac{(-82)}{220} \times 20$$

$$= 70 - \frac{82}{11} = 70 - 7.45 = 62.55$$

69. Find the mode of the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70

Frequency	8	10	10	16	12	6	7

Ans : [Board 2019 Delhi]

Class 30-40 has the maximum frequency 16, therefore this is model class.

We have $l = 30, f_0 = 10, f_1 = 16, f_2 = 12, h = 10$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 30 + \left(\frac{16 - 10}{2 \times 16 - 10 - 12}\right) \times 10$$

$$= 30 + \left(\frac{6}{32 - 22}\right) \times 10$$

$$= 30 + \left(\frac{6}{10}\right) \times 10$$

$$= 30 + 6 = 36$$

n169

70. The data regarding marks obtained by 48 students of a class in a class test is given below. Calculate the modal marks of students.

Marks obtained	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
Number of students	1	0	2	0	0	10	25	7	2	1

Ans : [Board Term-1, 2015]

Class 30-35 has the maximum frequency 25, therefore this is model class.

Now $l = 30,\ f_1 = 25\ \ f_0 = 10\ ,\ f_2 = 7,\ h = 5$

Mode $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

n170

$$= 30 + \frac{25 - 10}{50 - 10 - 7} \times 5$$

$$= 30 + 2.27 \text{ or } 32.27 \text{ approx.}$$

71. Find the value of λ, if the mode of the following data is 20 :

15, 20, 25, 18, 13, 15, 25, 15, 18, 17, 20, 25, 20, λ, 18.

Ans : [Board Term-1, 2015]

First we prepare the following table as discrete frequency distribution.

n171

x_i	f_i
13	1
15	3
17	1
18	3
20	3
λ	1
25	3

Frequency of 20 must be highest to be mode of the frequency distribution, $\lambda = 20$.

72. The mean and median of 100 observation are 50 and 52 respectively. The value of the largest observation is 100. It was later found that it is 110. Find the true mean and median.

Ans : [Board Term-1 2016]

n172

Mean, $$M = \frac{\sum fx}{\sum f}$$

$$50 = \frac{\sum fx}{100}$$

$$\sum fx = 5000$$

Correct, $$\sum fx' = 5000 - 100 + 110$$

$$= 5010$$

Correct Mean $$= \frac{5010}{100}$$

$$= 50.1$$

Median will remain same i.e median is 52.

73. Find the arithmetic mean of the following frequency distribution :

x_i	3	4	5	7	10
f_i	3	4	8	5	10

Ans : [Board Term-1, 2015]

We prepare the following table to fine mean.

x_i	f_i	$f_i x_i$
3	3	9
4	4	16
5	8	40
7	5	35
10	10	100
Total	$\sum f_i = 30$	$\sum f_i x_i = 200$

Mean, $$M = \frac{\sum f_i x_i}{\sum f_i} = \frac{200}{30} = 6.67$$

n173

74. Given below is the distribution of weekly pocket money received by students of a class. Calculate the pocket money that is received by most of the students.

Pocket Money (in Rs.)	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Number of students.	2	2	3	12	18	5	2

Ans : [Board Term-1 2015]

Class Interval	Frequency
0-20	2
20-40	2
40-60	3
60-80	12
80-100	18
100-120	5
120-140	2
Total	44

Class 80-100 has the maximum frequency 18, therefore this is model class.

We have $l = 80,\ f_1 = 18,\ f_2 = 5,\ f_0 = 12\ ,\ h = 20$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 80 + \left(\frac{18-12}{36-12-5}\right) \times 20$

$= 80 + \frac{6}{19} \times 20$

n174

$= 80 + 6.31$

$= 86.31$

75. Find the mean of the following distribution :

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	5	4	1	6	4

Ans : [Board Term-1 2015]

x_i	f_i	$f_i x_i$
3	5	15
9	4	36
15	1	15
21	6	126
27	4	108
Total	$\sum f_i = 20$	$\sum f_i x_i = 300$

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{300}{20} = 15$

n175

76. The following table gives the life time in days of 100 bulbs :

Life time in days	Less than 50	Less than 100	Less than 150	Less than 200	Less than 250	Less than 300
Number of Bulbs	8	23	55	81	93	100

Change the above distribution as frequency distribution.

n176

Ans : [Board Term-1 2012]

Life time in days (Class Interval)	Number of Bulbs (Frequency)
0-50	8
50-100	15
100-150	32
150-200	26
150-200	12
150-200	7
Total	100

77. Find the unknown values in the following table :

Class Interval	Frequency	Cumulative Frequency
0-10	5	5
10-20	7	x_1
20-30	x_2	18
30-40	5	x_3
40-50	x_4	30

Ans : [Board Term-1 2016]

We have $x_1 = 5 + 7 = 12$

n177

$x_2 = 18 - x_1 = 18 - 12 = 6$

$x_3 = 18 + 5 = 23$

and $\quad x_4 = 30 - x_3 = 30 - 23 = 7$

78. Calculate the median from the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	15	30	8	2

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Marks	No. of students	c.f.
0-10	5	5
10-20	15	20
20-30	30	50
30-40	8	58
40-50	2	60
	$N = 60$	

We have $\quad N = 60 \;;\; \frac{N}{2} = 30$

Cumulative frequency just greater than $\frac{N}{2}$ is 50 and the corresponding class is 20-30. Thus median class is 20-20.

Now $\quad l = 20,\ f = 30,\ F = 20,\ h = 10$

n178

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$$

$$= 20 + \left(\frac{30 - 20}{30}\right) \times 10$$

$$= 20 + \frac{100}{30} = 20 + \frac{10}{3}$$

$$= 20 + 3.33$$

Thus $\quad Md = 23.33$

79. Find the sum of the lower limit of the median class and the upper limit of the modal class :

Classes	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	3	5	9	7	3
Cumulative Frequency	1	4	9	18	25	28

We have $\quad N = 28 \;;\; \frac{N}{2} = \frac{28}{2} = 14$

Cumulative frequency just greater than $\frac{N}{2}$ is 18 and the corresponding class is 40 - 50. Thus median class is 40-50.

n179

Lower limit is 40 and upper limit is 5. Their sum is $= 40 + 50 = 90$

80. Write the relationship connecting three measures of central tendencies. Hence find the median of the give data if mode is 24.5 and mean is 29.75.

Ans : [Board Term-1 2012]

Mode, $\quad M_o = 24.5$

and mean, $\quad M = 29.75$

The relationship connecting measures of central tendencies is,

$$3M_d = M_o + 2M$$

n180

Thus $\quad 3M_d = 24.5 + 2 \times 59.50$

$$= 24.5 + 59.50 = 84.0$$

Median $\quad M_d = \frac{84}{3} = 28$

81. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Ans : [Board Term-1 2012]

Class 20-30 has the maximum frequency 40, therefore this is model class.

n181

Here, $\quad l = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10$

$$\text{Mode,} \quad M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 20 + \frac{(40 - 24)}{80 - 24 - 36} \times 10$$

$$= 20 + \frac{16 \times 10}{20} = 28$$

82. Find the unknown entries a, b, c, d in the following distribution of heights of students in a class :

Height (in cm)	Frequency	Cumulative Frequency
150-155	12	12
155-160	a	25
160-165	10	b
165-170	c	43
170-175	5	48
175-180	2	d

Ans : [Board Term-1 2012]

From the table,

$$12 + a = 25 \Rightarrow a = 25 - 12 = 13$$

$$25 + 10 = b \Rightarrow b = 35,$$

$$b + c = 43 \Rightarrow c = 43 - b = 13 - 35 = 8$$

and $48 + 2 = d \Rightarrow d = 50$

n182

83. Find the mode of the following distribution :

Classes	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Ans :

Class 35-40 has the maximum frequency 50, therefore this is model class.

n183

Now $l = 35,\ f_1 = 50,\ f_2 = 42,\ f_0 = 34,\ h = 5$

Mode,
$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 35 + \frac{50 - 34}{100 - 34 - 42} \times 5$$

$$= 35 + \frac{16 \times 5}{24} = 38.33$$

84. Find x and y from the following cumulative frequency distribution :

Classes	Frequency	c.f.
0-8	15	15
8-16	x	28
16-24	15	43
24-32	18	y
32-40	09	70

Ans : [Board Term-1 2012]

From the cumulative frequency distribution,

$$15 + x = 28 \Rightarrow x = 28 - 15 = 13$$

and $43 + 18 = y \Rightarrow y = 61$

Hence, $x = 13$ and $y = 61$

n184

85. The frequency distribution of agricultural holdings in a village below :

Area of land (in hectare)	1-3	3-5	5-7	7-9	9-11	11-13
Number of families	20	45	80	55	40	12

Find the modal agricultural holding of the village.

Ans : [Board Term-1 2012]

Class 5-7 has the maximum frequency 80, therefore this is model class.

n185

Here $l = 5, f_1 = 80, f_0 = 45,\ h = 2,\ f_2 = 55$

Mode,
$$M_o = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 5 + \frac{80 - 45}{160 - 45 - 55} \times 2 = 5 + \frac{35 \times 2}{60}$$

$$= 6.17$$

86. Write the median class of the following distribution :

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

n186

Classes	Frequency	Less than c.f.
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
	$N = 50$	

We have $N = 50$; $\frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 30-40. Thus median class is 20-20.

87. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day :

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	60	42	55	70	53	20

Form the "less than type" cumulative frequency distribution table.

n187

Ans : [Board Term-1 2012]

Age	Number of Patients
Less then 20	60
Less then 30	102
Less then 40	157
Less then 50	227
Less then 60	280
Less then 70	300

88. Find the mean of the following data :

Class	Frequency
0.5-5.5	13
5.5-10.5	16
10.5-15.5	22
15.5-20.5	18
20.5-25.5	11

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0.5-5.5	3	13	39
5.5-10.5	8	16	128
10.5-15.5	13	22	286
15.5-20.5	18	18	324
20.5-25.5	23	11	253
	Total	$\sum f_i = 80$	1,030

Mean $\overline{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1,030}{80} = 12.9$

n188

89. Find the mean number of plants per house from the following data :

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
	Total	20	162

Mean $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$

n189

Mean number of plants per house = 8.1.

90. Given below is a frequency distribution showing the marks by 50 students of a class :

Marks	Number of students
Below 20	17
Below 40	22
Below 60	29
Below 80	37
Below 100	50

Form the distribution table for the above data.

Ans : [Board Term-1 2012]

n190

Class	Frequency
0-20	17
20-40	5
40-60	7
60-80	8
80-100	13
Total	50

91. Find the mode of the following frequency distribution :

Classes	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Ans : [Board Term-1 2012]

Class 18-24 has the maximum frequency 12, therefore this is model class.

Now $l = 18,\ f_1 = 12,\ f_0 = 10,\ f_2 = 6,\ h = 6$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 18 + \dfrac{12 - 10}{24 - 10 - 6} \times 6$

n191

$= 18 + 1.5 = 19.5$

92. Find the mean of the following frequency distribution :

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-6	3	7	21
6-12	9	5	45
12-18	15	10	150
18-24	21	12	252
24-30	27	6	162
		$\sum f_i = 40$	$\sum f_i x_i = 630$

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{630}{40} = 15.75$

n192

93. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

Ans : [Board Term-1 2015]

We prepare following table to find mean.

Class-Interval	Mid-Point x_i	f_i	$f_i x_i$
0-10	5	4	20
10-20	15	6	90
20-30	25	10	250
30-40	35	6	210
40-50	45	p	45 p
		$26 + p$	$570 + 45p$

We have $M = \dfrac{\sum f_i x_i}{\sum f_i}$

n193

$25 = \dfrac{570 + 45p}{26 + p}$

$650 + 25p = 570 + 45p$

$650 - 570 = 45p - 25p$

Thus $p = 4$

94. The data regarding the height of 50 girls of class X of a school is given below :

Height (in cm)	120-130	130-140	140-150	150-160	160-170	Total

Number of girls	2	8	12	20	8	50

Change the above distribution to 'more than type' distribution.

n194

Ans : [Board Term-1 2012]

Heights	No. of girls
more than 120	50
more than 130	48
more than 140	40
more than 150	28
more than 160	6

95. Convert the following distribution to more than type, cumulative frequency distribution :

Class	50-60	60-70	70-80	80-90	90-100
Frequency	12	18	10	15	5

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table.

n195

Class	Cumulative Frequency
More than 50	60
More than 60	48
More than 70	30
More than 80	20
More than 90	5

96. Convert the following cumulative distribution to a frequency distribution :

Height (in cm)	less than 140	less than 145	less than 150	less than 155	less than 160	less than 165
Number of students	4	11	29	40	46	51

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table.

n196

Class	Frequency	Cumulative Frequency
135-140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

97. Prepare a cumulative frequency distribution of 'more than type' for the following data :

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	3	8	15	7	5

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table.

n197

Marks	Cumulative Frequency
More than 0	38
More than 10	35
More than 20	27
More than 30	12
More than 40	5

98. Change the following distribution to 'more than type' of distribution :

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of students	12	14	8	6	10

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table.

n198

Daily income	No. of workers
More than 100	50
More than 120	38
More than 140	24
More than 160	16
More than 180	10

99. Convert the following data into 'more than type' distribution :

Class	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	2	8	12	24	38	16

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table.

n199

Class	Frequency
More than 50	100
More than 55	98
More than 60	90
More than 65	78
More than 70	54
More than 75	16

100. Given below is a frequency distribution table showing daily income of 100 workers of a factory :

Daily income of workers (in Rs.)	200-300	300-400	400-500	500-600	600-700
Number of workers	12	18	35	20	15

Convert this table to a cumulative frequency distribution table of 'more than type'.

Ans : [Board Term-1 2016]

Cumulative frequency distribution table (more than type)

n200

Daily income of workers (in Rs.)	Number of workers
More than 200	100
More than 300	88
More than 400	70
More than 500	35
More than 600	15
More than 700	0

101. The given distribution shows the number of runs scored by the batsmen in inter-school cricket matches :

Runs scored	0-50	50-100	100-150	150-200	200-250
Number of batsmen	4	6	9	7	5

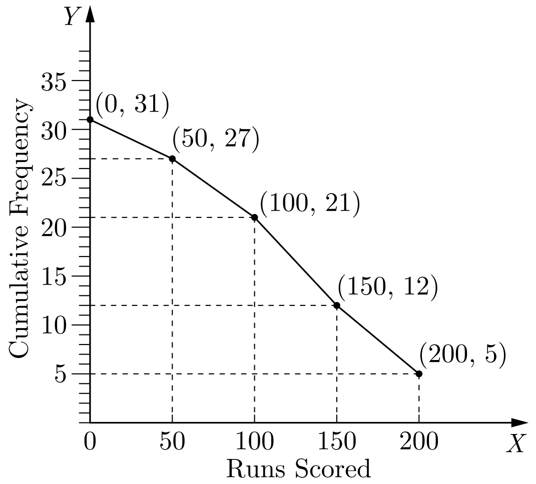
Draw a 'more than type' ogive for the above data .

Ans : [Board Term-1 2015]

Units on $x-$ axis 1 cm $= 50$, y-axis 1cm $= 5$

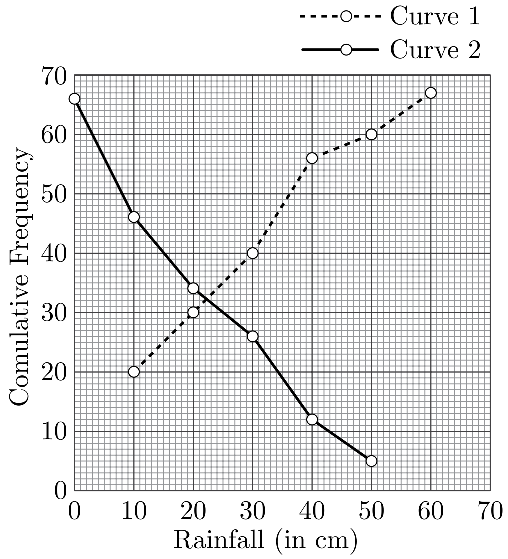
n201

More than	$c.f.$
0	31
50	27
100	21
150	12
200	5



THREE MARKS QUESTIONS

102. A TV reporter was given a task to prepare a report on the rainfall of the city Dispur of Indian in a particular year. After collecting the data, he analysed the data and prepared a report on the rainfall of the city, Using this report, he drew the following graph of a particular time period of 66 days



Based on the above graph, answer the following questions :

(i) Identify less than type ogive and more than type ogive from the given graph.
(ii) Find the median rainfall of Dispur.
(iii) Obtain the Mode of the data if mean rainfall is 23.4 cm

Ans : [Board 2020 SQP Standard]

(i) Curve-1 shows less than ogive and curve-2 shows more than ogive.

(ii) The abscissa of intersecting point of less than and more than ogive give the median. Thus median is 21 cm.

(iii) Mode of data,

n202

$$M_o = 3M_d - 2M$$
$$= 3 \times 21 - 2 \times 23.4$$
$$= 63 - 46.8 = 16.2 \text{ cm}$$

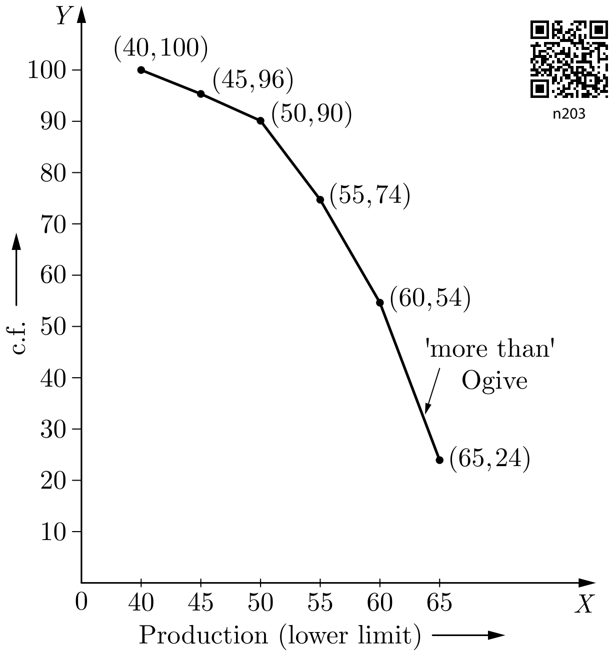
103. The following table gives production yield per hectare (in quintal) of wheat of 100 farms of a village :

Production yield/hect.	40-45	45-50	50-55	55-60	60-65	65-70
No. of farms	4	6	16	20	30	24

Change the distribution to a more than type distribution and draw its ogive.

Ans : [Board 2020 Delhi STD, OD STD]

Production yield/hectare	*c.f.*
more than 40	100
more than 45	96
more than 50	90
more than 55	74
more than 60	54
more than 65	24



n203

104. Compute the mode for the following frequency distribution:

Size of items (in cm)	0-4	4-8	8-12	12-16	16-20	20-24	24-28
Frequency	5	7	9	17	12	10	6

Ans : [Board 2020 OD Standard]

Class 12-16 has the maximum frequency 17, therefore this is model class.

We have $l = 12$, $f_1 = 17$, $f_0 = 9$, $f_2 = 12$ and $h = 4$

Mode $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 12 + \left(\frac{17 - 9}{2 \times 17 - 9 - 12}\right) \times 4$$
$$= 12 + \frac{8 \times 4}{13}$$
$$= 12 + 2.46 = 14.46$$

n204

105. The mean of the following frequency distribution is 18. The frequency f in the class interval 19-21 is missing. Determine f.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Ans : [Board 2020 OD Standard]

Class	Class Mark	Frequency	$f_i x_i$
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
	Total	$40+f$	$704+20f$

We have $\quad \sum f_i = 40 + f$

$$\sum f_i x_i = 704 + 20f$$

n205

Mean, $$M = \frac{\sum f_i x_i}{\sum f_i}$$

$$18 = \frac{704 \times 20f}{40+f}$$

$$720 + 18f = 704 + 20f$$

$$f = 8$$

106. Find the mode of the following frequency distribution :

Class	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	8	9	10	3	2

Ans : [Board 2020 OD Standard]

Class 30-35 has the maximum frequency 10, therefore this is model class.

Now $\quad l = 30,\ f_0 = 9,\ f_1 = 10,\ f_2 = 3$ and $h = 5$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 30 + \left(\frac{10-9}{2\times 10 - 9 - 3}\right)\times 5$$

$$= 30 + \frac{5}{8}$$

n206

$$= 30 + 0.625 = 30.625$$

107. The marks obtained by 110 students in an examination are given below

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Number of Students	14	16	28	23	18	8	3

Find the mean marks of the students.

Ans : [Board 2019 OD Standard]

Marks	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
30-35	14	32.5	-3	-42
35-40	16	37.5	-2	-32
40-45	28	42.5	-1	-28
45-50	23	47.5	0	0
50-55	18	52.5	1	18
55-60	8	57.5	2	16
60-65	3	62.5	3	9
	$\sum f_i = 110$			$\sum f_i u_i = -59$

Let a be assumed mean,

$$a = 47.5$$

Mean $$M = a + \frac{\sum f_i u_i}{N} \times h$$

n207

$$= 47.5 + \frac{(-59)}{110} \times 5$$

$$= 47.5 - 2.682 = 44.818$$

108. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Daily expenditure (in ₹)	100-150	150-200	200-250	250-300	300-350
Number of households	4	5	12	2	2

Ans : [Board 2019 Delhi]

Let $a = 225$ be assumed mean,

Daily Expenditure (in ₹)	No. of household (f_i)	(x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-150	4	125	-2	-8
150-200	5	175	-1	-5

200-250	12	225	0	0
250-300	2	275	1	2
300-350	2	325	2	4
	$\sum f_i = 25$			$\sum f_i u_i = -7$

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

n208

$= 225 + \frac{(-7)}{25} \times 50$

$= 225 - 14 = 211$

Hence, mean of daily expenditure on food is ₹211.

109. The mean of the following distribution is 48 and sum of all the frequency is 50. Find the missing frequencies x and y.

Class	20-30	30-40	40-50	50-60	60-70
Frequency	8	6	x	11	y

Ans : [Board Term-1 2015, 2016]

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
20-30	8	25	-2	-16
30-40	6	35	-1	-6
40-50	x	$45 = a$	0	0
50-60	11	55	1	11
60-70	y	65	2	$2y$
Total	$\sum f_i = 25 + x + y$			$\sum f_i u_i = 2y - 11$

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$48 = 45 + \frac{2y - 11}{50} \times 10$

n209

$48 - 45 = \frac{2y - 11}{5}$

$3 \times 5 = 2y - 11$

$15 = 2y - 11 \Rightarrow y = 13$

Also $\sum f_i = 25 + x + y = 50$

$x + y = 25$

$x = 25 - 13 = 12$

Thus $x = 12$ and $y = 13$

110. Find the mean of the following distribution :

Height (in cm)	Less than 75	Less than 100	Less than 125	Less than 150	Less than 175	Less than 200
No. of students	5	11	14	18	21	28
Height (in cm)	Less than 225	Less than 250	Less than 275	Less than 300		
No. of students	33	37	45	50		

Ans : [Board Term-1 2016]

We prepare following table to find mean.

Class Interval Height (in cm)	Frequency f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
50-75	5	62.5	-5	-25
75-100	6	87.5	-4	-24
100-125	3	112.5	-3	-9
125-150	4	137.5	-2	-8
150-175	3	162.5	-1	-3
175-200	7		0	0
200-225	5	212.5	1	5
225-250	4	237.5	2	8
250-275	8	262.5	3	24
275-300	5	287.5	4	20
	$\sum f_i = 50$			$\sum f_i y_i = -12$

Here, $\sum f_i u_i = -12$; $\sum f_i = 50$, $h = 25$

Mean $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

n210

$= 187.5 + \frac{-12}{50} \times 25$

$= 187.5 - 6 = 181.5$

111. Following frequency distribution shows the expenditure on milk of 30 households in a locality :

Daily expenditure on milk (Rs.)	0-30	30-60	60-90	90-120	120-150
Number of households	5	6	9	6	4

Find the mode for the above data.

Ans : [Board Term-1 2016]

Class 60-90 has the maximum frequency 9, therefore this is model class.

n211

Here, $l_1 = 60$, $f_1 = 9$, $f_0 = 6$, $f_2 = 6$ and $h = 30$.

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 60 + \left(\frac{9-6}{2\times 9 - 6 - 6}\right)\times 30$$

$$= 60 + \frac{30\times 3}{6} = 60 + 15 = 75$$

112. The weekly expenditure of 500 families is tabulated below :

Weekly Expenditure(Rs.)	Number of families
0-1000	150
1000-2000	200
2000-3000	75
3000-4000	60
4000-5000	15

Find the median expenditure.

Ans : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Expenditure	f (families)	c.f.
0-1000	150	150
1000-2000	200	350
2000-3000	75	425
3000-4000	60	485
4000-5000	15	500
	$\sum f = 500$	

We have $N = 500$; $\frac{N}{2} = 250$

Cumulative frequency just greater than $\frac{N}{2}$ is 350 and the corresponding class is 1000-2000. Thus median class is 1000-2000.

n212

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 1000 + \frac{250 - 150}{200}\times 1000$$

$$= 1000 + 500 = 1,500$$

Thus median expenditure is Rs. 1500 per week.

113. Find the median of the following data :

Height (in cm)	Less than 120	Less than 140	Less than 160	Less than 180	Less than 200
Number of students	12	26	34	40	50

Ans : [Board Term-1 2015]

We prepare following cumulative frequency table to find median class.

Height	Frequency	$c.f.$
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50
Total	$N = 50$	

We have $N = 50$; $\frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 120-140. Thus median class is 120-140.

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 120 + \left(\frac{25-12}{14}\right)\times 20$$

$$= 120 + \frac{260}{14}$$

$$= 120 + 18.57$$

$$= 138.57$$

n213

114. The mean of the following distribution is 314. Determine the missing frequency x.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	10	12	7	8

Ans : [Board Term-1 2016]

We prepare following table to find mean.

C.I.	x	f	$u_i = \frac{x - f}{h}$	$f_i u_i$
1-10	5	5	-3	-15
10-20	15	x	-2	$-2x$
20-30	25	10	-1	-10
30-40	35	12	0	0
40-50	45	7	1	7
50-60	55	8	2	16
Total		$42+x$		$-2x-2$

Let mid point of class 30-40 be assumed mean a.

$$a = 35$$

Mean
$$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

n214

$$31.4 = 35 + \frac{-2x-2}{42+x} \times 10$$

$$(2x+2)10 = (42+x)(3.6)$$

$$20x + 20 = 151.2 + 3.6x$$

$$16.4x = 131.2 \Rightarrow x = 8$$

115. Calculate the mean of the following frequency distribution :

Class	10-30	30-50	50-70	70-90	90-110
Frequency	15	18	25	10	2

Ans :

We prepare following table to find mean.

C.I.	f_i	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
10-30	15	20	-2	-30
30-50	18	40	-1	-18
50-70	25	$60 = a$	0	0
70-90	10	80	1	10
90-110	2	100	2	4
Total	$\sum f = 70$			$\sum f_i u_i = -34$

Let mid point of class 50-60 be assumed mean a.

$$a = 60$$

Mean
$$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

n215

$$= 60 + \frac{-34}{70} \times 20$$

$$= 60 - 9.71 = 50.29$$

116. Heights of students of class X are given in the following distribution :

Heights (in cm)	150-155	155-160	160-165	165-170	170-175
Number of students	15	8	20	12	5

Find the modal height.

Ans : [Board Term-1 2015]

Class 160-165 has the maximum frequency 20, therefore this is model class.

Now $l = 160,\ f_1 = 20,\ f_0 = 8,\ f_2 = 12,\ h = 5$

Mode,
$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 160 + \left(\frac{20-8}{40-8-12}\right) \times 5$$

$$= 160 + \left(\frac{12}{20}\right) \times 5$$

n216

$$= 163$$

Thus modal height is 163 cm.

117. A school conducted a test (of 100 marks) in English for students of Class X. The marks obtained by students are shown in the following table :

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	1	2	4	15	15	25	15	10	2	1

Find the modal marks.

Ans : [Board Term-1 2015]

Class 50-60 has the maximum frequency 25, therefore this is model class.

Here $l = 50,\ f_1 = 25,\ f_0 = 15,\ f_2 = 15,\ h = 10$

Mode,
$$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 50 + \frac{25-15}{50-15-15} \times 10$$

$$= 50 + \frac{10}{20} \times 10$$

$$= 50 + 10 = 55$$

n217

118. The following frequency distribution shows the number of runs scored by some batsman of India in one-day cricket matches :

Run scored	2000-4000	4000-6000	6000-8000	8000-10000	10000-12000
Number of batsmen	9	8	10	2	1

Find the mode for the above data.

Ans : [Board Term-1 2015]

Class 6000-8000 has the maximum frequency 10, therefore this is model class.

Here $f_0 = 8,\ f_1 = 10,\ f_2 = 2,\ h = 2000$, and $l = 6000$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

$$= 6000 + \left(\frac{10-8}{20-8-2}\right) \times 2000$$

$$= 6000 + \frac{2}{10} \times 2000$$

$$= 6000 + 400$$

$$= 6400$$

n218

119. A group of students conducted a survey of their locality to collect the data regarding number of plants and recorded it in the following table :

Number of plants	0-3	3-6	6-9	9-12	12-15
Number of houses	2	4	5	1	2

Find the mode for the above data.

Ans : [Board Term-1 2015]

Class 6-9 has the maximum frequency 5, therefore this is model class.

Now $l_1 = 6,\ f_1 = 5,\ f_0 = 4,\ f_2 = 1,\ h = 3$

Mode, $$M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$$

n219

$$= 6 + \frac{5-4}{10-4-1} \times 3$$

$$= 6 + \frac{1}{5} \times 3$$

$$= 6 + 0.6 = 6.6$$

120. If the median for the following frequency distribution is 28.5, find the value of x and y :

Class	Frequencies
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5
Total	60

Ans : [Board Term-1 2013]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-10	5	5
10-20	x	$x+5$
20-30	20	$x+25$
30-40	15	$x+40$
40-50	y	$x+y+40$
50-60	5	$x+y+45$
	$\sum f = 60$	

Since, median is 28.5 which lies between 20-30. Thus model class is 20-30.

From table $$N = x + y + 45$$

$$60 = x + y + 45$$

$$x + y = 60 - 45 = 15 \quad ...(1)$$

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$28.5 = 20 + \frac{[30 - (x+5)]}{20} \times 10$$

$$8.5 = \frac{25 - x}{2}$$

$$25 - x = 17 \Rightarrow x = 25 - 17 = 8$$

From (1), $$y = 15 - 8 = 7$$

Hence, $x = 8$ and $y = 7$

n220

121. If the mean of the following data is 14.7, find the values of p and q.

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
Frequency	10	p	4	7	q	4	1	40

Ans : [Board Term-1 2013]

Class	x_i	f_i	$f_i x_i$
0-6	3	10	30
6-12	9	p	$9p$
12-18	15	4	60
18-24	21	7	147
24-30	27	q	$27q$
30-36	33	4	132
36-42	39	1	39
	Total	$\sum f_i =$ $26+p+q=40$	$\sum f_i x_i =$ $408+9p+27q$

We have $\sum f_i = 40,$

$$26+p+q = 40$$

$$p+q = 14 \qquad ...(1)$$

Mean $M = \dfrac{\sum x_i f_i}{\sum f_i}$

$$14.7 = \frac{408+9p+27q}{40}$$

n221

$$588 = 408+9p+27q$$

$$180 = 9p+27q$$

$$p+3q = 20 \qquad ...(2)$$

Subtracting equation (1) from (2) we have,

$$2q = 6 \Rightarrow q = 3$$

Substituting this value of q in equation (2) we get

$$p = 14-q = 14-3 = 11$$

Hence, $p = 11,\ q = 3$

122. Find the mean and mode of the following frequency distribution :

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	3	8	10	15	7	4	3

Ans : [Board Term-1 2013]

We prepare following table to find mean.

Classes	x_i	f_i	$f_i x_i$
0-10	5	3	15
10-20	15	8	120
20-30	25	10	250
30-40	35	15	525
40-50	45	7	315
50-60	55	4	220
60-70	65	3	195
		$\sum f_i = 50$	$\sum f_i x_i = 1640$

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{1640}{50} = 32.8$

Class 30-40 has the maximum frequency 35, therefore this is model class.

Here $l = 30,\ f_1 = 15,\ f_2 = 7,\ f_0 = 10,\ h = 10$

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 30 + \frac{15-10}{30-10-7} \times 10$$

$$= 30 + \frac{5}{13} \times 10$$

$$= 30 + \frac{50}{13}$$

n222

$$= 30 + 3.85 = 33.85$$

123. Find the mean and median for the following data :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	8	16	36	34	6

Ans : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

Class	x_i (class marks)	f_i	$f_i x_i$	c.f.
0-10	5	8	40	8
10-20	15	16	240	24
20-30	25	36	900	60
30-40	35	34	1190	94
40-50	45	6	270	100
		$\sum f_i = 100$	$\sum f_i x_i = 2640$	

Mean $M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{2640}{100} = 26.4$

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 20-30. Thus median class is 20-30.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$= 20 + \frac{50 - 24}{36} \times 10$

$= 20 + 7.22 = 27.22$

n223

124. If the median of the following data is 240, then find the value of f :

Classes	0-100	100-200	200-300	300-400	400-500	500-600	600-700
Frequency	15	17	f	12	9	5	2

Ans : [Board Term-1 2011]

We prepare following cumulative frequency table to find median class.

Classes	f_i	$c.f.$
0-100	15	15
100-200	17	32
200-300	f	32+f
300-400	12	44+f
400-500	9	53+f
500-600	5	58+f
600-700	2	60+f

From table, $N = 60 + f \Rightarrow \frac{N}{2} = \frac{60 + f}{2}$

Since median is 240 which lies between class 200-300. Thus median class is 200-300.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$240 = 200 + \left[\frac{\frac{60+f}{2} - 32}{f}\right] \times 100$

$40 = \left[\frac{60 + f - 64}{2f}\right] \times 100$

n224

$8f = 10f - 40$

$2f = 40 \Rightarrow f = 20$

125. The following table shows the weights (in gms) of a sample of 100 apples, taken from a large consignment :

Weight (in gms)	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130
No. of Apples	8	10	12	16	18	14	12	10

Find the median weight of apples.

Ans : [Board Term-1 2011]

C.I.	50-60	60-70	70-80	80-90	90-100	100-110	110-120	120-130
f	8	10	12	16	18	14	12	10
$c.f.$	8	18	30	46	64	78	90	100

We have $N = 100$; $\frac{N}{2} = 50$

Cumulative frequency just greater than $\frac{N}{2}$ is 64 and the corresponding class is 90-100. Thus median class is 90-100.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$= 90 + \left(\frac{50 - 46}{18}\right) \times 10$

$= 90 + \frac{40}{18} = 92.2$

$= 92.2$ gm.

n225

Thus median weight is 92.2.

126. Weekly income of 600 families is given below :

Income (in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Find the median.

Ans :

We prepare following cumulative frequency table to find median class.

Income	No. of Families	$c.f.$
0-1000	250	250
1000-2000	190	440
2000-3000	100	540

3000-4000	40	580
4000-5000	15	595
5000-6000	5	600
	$N = 600$	

We have $N = 600 \;;\; \frac{N}{2} = 300$

Cumulative frequency just greater than $\frac{N}{2}$ is 440 and the corresponding class is 1000-2000. Thus median class is 1000-2000.

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$\text{Median} = 1000 + \left(\frac{300 - 250}{190}\right) \times 1000$$

$$= 1000 + \frac{50}{190} \times 1000$$

$$= 1000 + \frac{5000}{19}$$

$$= 1000 + 263.16$$

$$= 1263.16$$

Median = Rs. 1263.16

n226

127. Find the mean of the following distribution by step deviation method :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	13	20	15	7	5

Ans : [Board Term-1 2011]

Class	x_i (Class Marks)	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
0-10	5	-3	5	-15
10-20	15	-2	13	-26
20-30	25	-1	20	-20
30-40	35	0	15	0
40-50	45	1	7	7
50-60	55	2	5	10
	Total		$\sum f_i = 65$	$\sum f_i u_i = -44$

Let assumed mean, $a = 35$ and given $h = 10$.

Mean, $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$$= 35 + \frac{-44}{65} \times 10$$

$$= 35 - 6.76 = 28.24$$

n227

128. The mean of the following distribution is 53. Find the missing frequency p :

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

Ans : [Board Term-1 2011]

Class	x_i (Class marks)	f_i	$f_i x_i$
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	p	$70p$
80-100	90	13	1170
	Total	$\sum f_i = 72 + p$	$\sum f_i x_i = 3340 + 70p$

Mean, $\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$

$$53 = \frac{3340 + 70p}{72 + p}$$

n228

$$3340 + 70p = 53(72 + p)$$

$$3340 + 70p = 3816 + 53p$$

$$70p - 53p = 3816 - 3340$$

$$17p = 476$$

$$p = \frac{476}{17} = 28$$

129. Find the mean for the following data :

Class	24.5-29.5	29.5-34.5	34.5-39.5	39.5-44.5	44.5-49.5	49.5-54.5	54.5-59.5
Frequency	4	14	22	16	6	5	3

Ans : [Board Term-1 2011]

We prepare following table to find mean.

Class	Class marks (x_i)	f_i	$f_i x_i$
24.5-29.5	27	4	108
29.5-34.5	32	14	448
34.5-39.5	37	22	814
39.5-44.5	42	16	672
44.5-49.5	47	6	282
49.5-54.5	52	5	260

54.5-59.5	57	3	171
		$\sum f_i = 70$	$\sum f_i x_i = 2,755$

Mean $\quad M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{2755}{70} = 39.36$

n229

130. Find the mode of following data :

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
Number of students	8	20	45	58	70

Ans :

Class-Interval	Frequency
0-10	8
10-20	12
20-30	25
30-40	13
40-50	12
Total	70

Class 20-30 has the maximum frequency, therefore this is model class.

Now $\quad l = 20,\ f_1 = 25,\ f_2 = 13,\ f_0 = 12,\ h = 10$

Mode, $\quad M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 20 + \left(\frac{25-12}{50-12-13}\right) \times 10$$

$$= 20 + \frac{13}{25} \times 10$$

$$= 20 + 5.2 = 25.2$$

n230

131. Find the mean of the following data :

Class	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Frequency	15	37	74	99	120

Ans : [Board Term-1 2011]

We prepare following table to find mean.

C.I.	f_i	x_i	$x_i f_i$
0-20	15	10	150
20-40	22	30	660
40-60	37	50	1850
60-80	25	70	1750
80-100	21	90	1890
	$\sum f_i = 120$		$\sum x_i f_i = 6,300$

Mean $\quad M = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{6300}{120} = 52.5$

n231

132. Find the mean of the following data :

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	6	8	10	12	8	6

Ans : [Board Term-1, 2011, Set-66]

We prepare following table to find mean.

Classes	Frequency	Mid points	$f_i x_i$
0-20	6	10	60
20-40	8	30	240
40-60	10	50	500
60-80	12	70	840
80-100	8	90	720
100-120	6	110	660
	$\sum f_i = 50$		$\sum f_i x_i = 3020$

Mean, $\quad M = \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{3020}{50} = 60.4$

n232

133. The sum of deviations of a set of values x_1, x_2, x_3, ..., x_n, measured from 50 is -10 and the sum of deviations of the values from 46 is 70. Find the value of n and the mean.

Ans :

We have,

$$\sum_{i=1}^{n}(x_i - 50) = -10 \text{ and } \sum_{i=1}^{n}(x_i - 46) = 70$$

$$\sum_{i=1}^{n} x_i - 50n = -10 \qquad ...(1)$$

and $$\sum_{i=1}^{n} x_i - 46n = 70 \qquad ...(2)$$

Subtracting (2) from (1) we get,

$$-4n = -80 \Rightarrow n = 20$$

Substituting this value of n in equation (1) we have

$$\sum_{i=1}^{n} x_i - 50 \times 20 = -10$$

n233

$$\sum_{i=1}^{n} x_i = 990$$

Mean $\quad M = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{990}{20} = 49.5$

Hence, $\quad n = 20$ and mean $= 49.5$

134. Prove that $\sum (x_i - \overline{x}) = 0$

Ans :

n234

We have $\quad \overline{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$

$$n\overline{x} = \sum_{i=1}^{n} x_i$$

Now, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 - \overline{x}) + (x_2 - \overline{x}) + \ldots + (x_n - \overline{x})$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 + x_2 + \ldots + x_n) - n\overline{x}$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = \sum_{i=1}^{n} x_i - n\overline{x} = \sum_{i=1}^{n}(x_i - \overline{x})$$

Hence, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = 0$

135. Compute the median from the following data :

Mid-values	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

Ans :

Here, the mid-values are given So, we should first find the upper and lower limits of the various classes. The difference between two consecutive values is $h = 125 - 115 = 10$

$$\text{Lower limit of a class} = \text{Mid-value} - \frac{h}{2}$$

$$\text{Upper limit} = \text{Mid- value} + \frac{h}{2}$$

Mid-value	Class Groups	Frequency	Cumulative Frequency
115	110-120	6	6
125	120-130	25	31
135	130-140	48	79
145	140-150	72	151
155	150-160	116	267
165	106-170	60	327
175	170-180	38	365
185	180-190	22	387
195	190-200	3	390

Now $\quad N = 390;\ \frac{N}{2} = 195$

Cumulative frequency just greater than $\frac{N}{2}$ is 36 and the corresponding class is 150-160. Thus median class is 150-160.

Here, $\quad l = 150,\ f = 116,\ h = 10,\ F = 151$

n235

Median, $\quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 150 + \frac{195 - 151}{116} \times 10$$

$$= 153.8$$

136. The mean of n observations is $\overline{x}$, if the first term is increased by 1, second by 2 and so on. What will be the new mean ?

Ans :

n236

I term $+1$
II term $+2$
III term $+3$
n terms $+n$

The mean of the new numbers is

$$\overline{x} + \frac{\frac{n(n+1)}{2}}{n} = \overline{x} = \frac{(n+1)}{2}$$

137. The mode of a distribution is 55 and the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class.

Ans :

Mode, $\quad M_o = 55$

$$\text{Modal class} = 45 - 60$$

n237

Frequency of the class preceding,

$$f = 5$$

Frequency of the class succeeding the modal class,

$$f_2 = 10$$

Let the frequency of modal class be f.

Mode $$M_o = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$55 = 45 + \frac{f - 5}{2f - 5 - 10} \times 15$$

$$10 = \left(\frac{f - 5}{2f - 15}\right) \times 15$$

$$\frac{10}{15} = \frac{f - 5}{2f - 15}$$

$$\frac{2}{3} = \frac{f - 15}{2f - 15}$$

$$4f - 30 = 3f - 15$$

$$4f - 3f = -15 + 30 \Rightarrow f = 15$$

FOUR MARKS QUESTIONS

138. The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Ans : [Board 2020 Delhi OD STD]

We prepare cumulative frequency table as given below.

Class Interval	Frequency (f)	Cumulative frequency $c.f.$
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
	$N = 100$	

From table we have

$$76 + x + y = 100$$

$$x + y = 100 - 76 = 24 \qquad ...(1)$$

Here median is 525 which lies between class $500 - 600$. Thus median class is 500-600.

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$

$$25 = (50 - 36 - x)5$$

$$14 - x = \frac{25}{5} = 5$$

n238

$$x = 14 - 5 = 9$$

Substituting the value of x is equation (1), we get

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$

139. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency	f_1	5	9	12	f_2	3	2	40

Ans : [Board 2019 Delhi]

Class	Frequency (f)	Cumulative Frequency (cf)
0-10	f_1	f_1
10-20	5	f_1+5
20-30	9	f_1+14
30-40	12	f_1+26
40-50	f_2	f_1+f_2+26
50-60	3	f_1+f_2+29
60-70	2	f_1+f_2+31
	$N=\sum f=40$	

Now, $$f_1+f_2+31 = 40$$
$$f_1+f_2 = 9$$
$$f_2 = 9-f_1 \qquad ...(1)$$

Since median is 32.5, which lies in 30-40, median class is 30-40.

Here $l=30$, $\frac{N}{2}=\frac{40}{2}=20$, $f=12$ and $F=14+f_1$

Now, median $= 3.25$

$$l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h = 32.5$$

$$30+\left(\frac{20-(14+f_1)}{12}\right)\times 10 = 32.5$$

$$\left(\frac{6-f_1}{12}\right)\times 10 = 2.5$$

$$\frac{60-10f_1}{12} = 2.5$$

$$60-10f_1 = 30$$

$$10f_1 = 30 \Rightarrow f_1 = 3$$

From equation (1), we get $f_2 = 9-3 = 6$

Hence, $f_1=3$ and $f_2=6$

n240

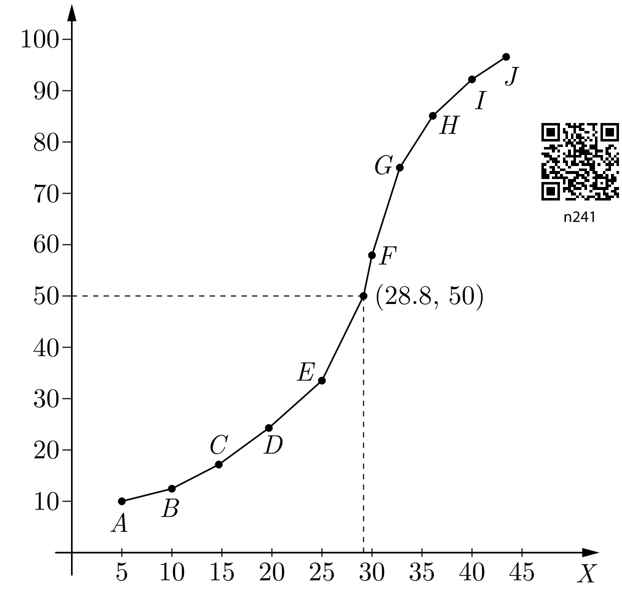
140. The marks obtained by 100 students of a class is an examination are given below:

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45	45-50
No. of Students	2	5	6	8	10	25	20	18	4	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median.

Ans : [Board 2019 Delhi]

Marks	No of students	cf
Less than 5	2	2
Less than 10	5	7
Less than 15	6	13
Less than 20	8	21
Less than 25	10	31
Less than 30	25	56
Less than 35	20	76
Less than 40	18	94
Less than 45	4	98
Less than 50	2	100



n241

From graph, $\frac{N}{2}=\frac{100}{2}=50$

Now, locate the point on the ogive where ordinate is 50. The x-coordinate corresponding to this ordinate is 28.8. Therefore, the required median on the graph is 28.8.

141. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	k	13

Ans : [Board 2019 Delhi]

Class Interval	Class Marks (x_i)	Frequency (f_i)	$f_i x_i$
0-20	10	12	120
20-40	30	15	450
40-60	50	32	1600
60-80	70	k	$70k$
80-100	90	13	1170
	Total	$72 + k$	$3340 + 70k$

Mean, $M = \dfrac{\sum f_i x_i}{\sum f_i}$

$$53 = \frac{3340 + 70k}{72 + k}$$

n242

$$53(72 + k) = 3340 + 70k$$

$$3816 + 53k = 3340 + 70k$$

$$70k - 53k = 3816 - 3340$$

$$17k = 476 \Rightarrow k = 28$$

Hence, value of k is 28.

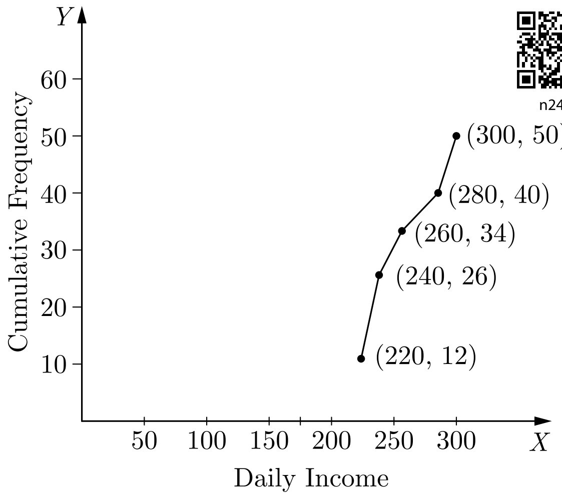
142. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in ₹)	200-220	220-240	240-260	260-280	280-300
Number of workers	12	14	8	6	10

Convert the distribution above to a 'less than type' cumulative frequency distribution and draw its ogive.

Ans : [Board 2019 Delhi]

Daily Income (in ₹)	Cumulative Frequency
Less than 220	12
Less than 240	26
Less than 260	34
Less than 280	40
Less than 300	50



n243

143. Find the mode of the following frequency distribution

Class Interval	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Ans : [Board 2019 OD Standard]

25-30	25
30-35	34
35-40	50
40-45	42
45-50	38
50-55	14

Class 35-40 has the maximum frequency 50, therefore this is model class.

Now, $l = 35$ $f_1 = 50$, $f_0 = 34$, $f_2 = 42$, $h = 5$

n244

Mode, $M_o = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 35 + \left(\frac{50 - 34}{2 \times 50 - 34 - 42}\right) \times 5$$

$$= 35 + \frac{16 \times 5}{24} = 35 + \frac{10}{3}$$

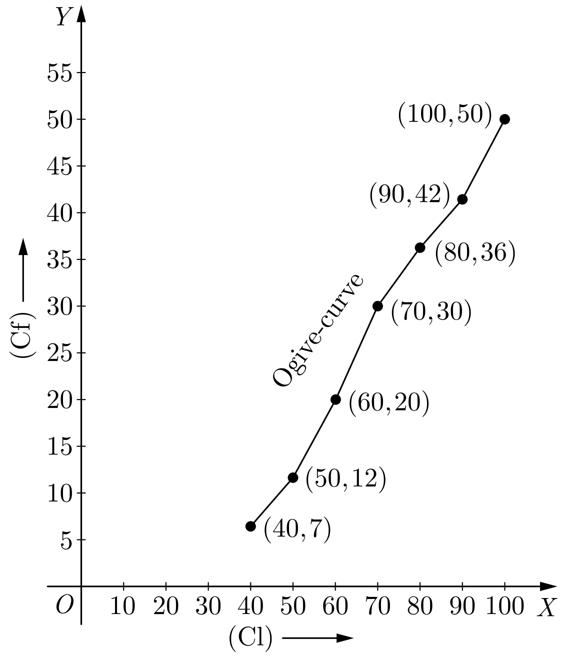
$$= \frac{105 + 10}{3} = \frac{115}{3} = 38.33$$

144. Change the following data into 'less than type' distribution and draw its ogive:

Class Interval	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	7	5	8	10	6	6	8

Ans : [Board 2019 OD Standard]

Classes	Cumulative frequency
less than 40	7
less than 50	$7+5=12$
less than 60	$12+8=20$
less than 70	$20+10=30$
less than 80	$30+6=36$
less than 90	$36+6=42$
less than 100	$42+8=50$



Scale : at x-axis,1 small division $=10$ units

at y-axis, 1 small division $=5$ units

n245

145. The table below show the salaries of 280 persons:

Salary (In thousand ₹)	No. of Persons
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

Calculate the median salary of the data.

Ans : [Board 2018]

Salary (In thousand ₹)	No. of Persons (f)	c.f.
5-10	49	49
10-15	133	182
15-20	63	245
20-25	15	260
25-30	6	266
30-35	7	273
35-40	4	277
40-45	2	279
45-50	1	280

We have $$\frac{N}{2}=\frac{280}{2}=140$$

Commutative frequency greater than just greater than $\frac{N}{2}=140$ is 182 and the corresponding class is 10-15. Thus median class is 10-15.

Median $$M_d=l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

n246

$$=10+\frac{(140-49)}{133}\times 5$$

$$=10+\frac{91\times 5}{133}=13.42$$

Median salary is ₹ 13.42 thousand or ₹ 13420 (approx)

146. The mean of the following distribution is 18. Find the frequency of the class 19-21.

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

Ans : [Board 2018]

Class	Class mark	Frequency (f)	fx_i
11-13	12	3	36
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	f	$20f$
21-23	22	5	110
23-25	24	4	96
		$40+f$	$704+20f$

Mean, $$M=\frac{\Sigma f x_i}{\Sigma f_i}$$

$$18 = \frac{704+20f}{40+f}$$

$$720+18f = 704+20f \Rightarrow f = 8$$

n247

147. The following distribution gives the daily income of 50 workers of a factory:

Daily Income (in ₹)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

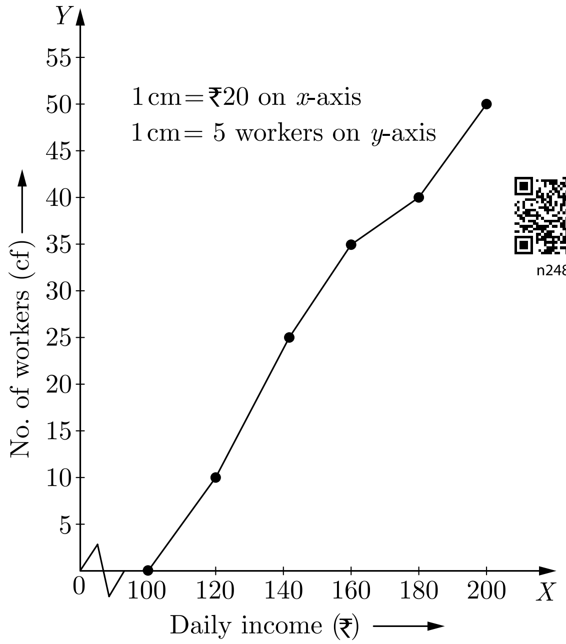
Ans : [Board 2018]

Cumulative frequency distribution table is given below.

Daily Income (in ₹)	Number of Workers (f)	Cumulative Frequency ($c.f.$)
100-120	12	12
120-140	14	26
140-160	8	34
160-180	6	40
180-200	10	50

Cumulative frequency distribution table less than type is

Less than Daily income in (₹)	Number of Workers (f)
100	0
120	12
140	26
160	34
180	40
200	50



n248

148. Literacy rates of 40 cities are given in the following table. It is given that mean literacy rate is 63.5, then find the missing frequencies x and y.

Literacy rate (in %)	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Number of cities	1	2	3	x	y	6	8	4	2	3	2

Ans : [Board Term-1 2016]

We prepare following table to find mean.

C.I.	x_i	u_i	f_i	f_iu_i
35-40	37.5	-5	1	-5
40-45	42.5	-4	2	-8
45-50	75.5	-3	3	-9
50-55	52.5	-2	x	-2x
55-60	57.5	-1	y	-y
60-65	$62.5 = a$	0	6	0
65-70	67.5	1	8	8
70-75	72.5	2	4	8
75-80	77.5	3	2	6
80-85	82.5	4	3	12
85-90	87.5	5	2	10
Total			$\sum f_i = 31+x+y$	$\sum f_iu_i = 22-2x-y$

Here, $\sum f_i = 31 + x + y = 40$

$x + y = 9$ ...(1)

$\sum f_i u_i = 22 - 2x - y$

Mean $M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

n249

$63.5 = 62.5 + \frac{(22 - 2x - y)}{40} \times 5$

$2x + y = 14$ ...(2)

Solving equation (1) and (2) we have $x = 5$ and $y = 4$

149. Find the mode of the following frequency distribution :

Class-Interval	f
25-35	7
35-45	31
45-55	33
55-65	17
65-75	11
75-85	1

Ans : [Board Term-1 2015]

Class 44-45 has the maximum frequency 33, therefore this is model class.

Now $l_1 = 45,\ f_0 = 31,\ f_1 = 33\ f_2 = 17,\ h = 10$

n252

Mode, $M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$

$= 45 + \frac{33 - 31}{66 - 31 - 17} \times 10$

$= 45 + \frac{2}{18} \times 10 = 46.1$

150. On the sports day of a school, 300 students participated. Their ages are given in the following distribution :

Age (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of students	67	33	41	95	36	13	15

Find the mean and mode of the data.

Ans : [Board Term-1 2015]

We prepare following table to find mean.

Age	x_i	f_i	$f_i x_i$
5-7	6	67	402
7-9	8	33	264
9-11	10	41	410
11-13	12	95	1140
13-15	14	36	504
15-17	16	13	208
17-19	18	15	270
		$\sum f_i = 300$	$\sum f_i x_i = 3,198$

Mean, $M = \frac{\sum f_i x_i}{\sum f_i} = \frac{3,198}{300} = 10.66$

Class 11-13 has the maximum frequency 95, therefore this is model class.

Now $l = 11,\ f_1 = 95,\ f_0 = 41,\ f_2 = 36,\ h = 2$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$= 11 + \frac{95 - 41}{190 - 41 - 36} \times 2$

$= 11 + \frac{54}{113} \times 2$

$= 11 + 0.95 = 11.95$

n253

151. The median of the following data is 525. Find the values of x and y if the total frequency is 100.

Class Interval	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4
	$N = 100$

Ans : [Board Term-1 2013]

We prepare following cumulative frequency table to find median class.

Class Interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7

200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$
	$N=100$	

Here median is 525, which lies between class 500-600. Thus median class is 500-600.

Now, $$76+x+y = 100$$

$$x+y = 100-76 = 24 \qquad ...(1)$$

Median, $$M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

$$525 = 500+\left[\frac{\frac{100}{2}-(36+x)}{20}\right]\times 100$$

$$25 = (50-36-x)5$$

$$(14-x) = \frac{25}{5} = 5$$

$$x = 14-5 = 9$$

n254

Substituting the value of x in equation (1),

$$y = 24-9 = 15$$

Hence, $x = 9$ and $y = 15$

152. Monthly expenditures on milk in 100 families of a housing society are given in the following frequency distribution :

Monthly expenditure (in Rs.)	0-175	175-350	350-525	525-700	700-875	875-1050	1050-1125
Number of families	10	14	15	21	28	7	5

Find the mode and median for the distribution.

Ans : [Board Term-1 2016]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-175	10	10
157-350	14	24
350-525	15	39
525-700	21	60
700-875	28	88
875-1050	7	95
1050-1225	5	100
	$N=100$	

We have $N=100\ ;\ \frac{N}{2}=50$

Cumulative frequency just greater than $\frac{N}{2}$ is 60 and the corresponding class is 525-700. Thus median class is 525-700.

Median, $$M_d = l+\left(\frac{\frac{N}{2}-F}{f}\right)h$$

$$= 525+\frac{50-39}{21}\times 175$$

$$= 525+\frac{11}{21}\times 175$$

n255

$$= 525+91.6$$

$$= 616.6$$

Class 700-875 has the maximum frequency 28, therefore this is model class.

Here $l = 700,\ f_0 = 21,\ f_1 = 28\ f_2 = 7,\ h = 175$

Mode, $$M_o = l+h\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)$$

$$= 700+\left(\frac{28-21}{2\times 28-21-7}\right)\times 175$$

$$= 700+\frac{7}{28}\times 175$$

$$= 700+43.75$$

$$= 743.75$$

153. Calculate the average daily income (in Rs.) of the following data about men working in a company :

Daily income (Rs.)	< 100	< 200	< 300	< 400	< 500
Number of men	12	28	34	41	50

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Class	x_i (class mark)	f_i	$f_i x_i$
0-100	50	12	600
100-200	150	16	2400
20-300	250	6	1500
300-400	350	7	2450
400-500	450	9	4050
		$\sum f_i = 50$	$\sum f_i x_i = 11,000$

Mean $M = \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{11000}{50}$

$= 200$

n256

Average daily income is Rs. 220.

154. If the mean of the following frequency distribution is 91, and sum of frequency is 150, find the missing frequency x and y :

Class	0- 30	30-60	60-90	90-120	120-150	150-180
Frequency	12	21	x	52	y	11

Ans : [Board Term-1 2012]

We prepare following table to find mean.

Class	x_i (Class marks)	f_i	$f_i x_i$
0-30	15	12	180
30-60	45	21	945
60-90	75	x	$75x$
90-120	105	52	5460
120-150	135	y	$135y$
150-180	165	11	1815
	Total	$\sum f_i = x + y + 96 = 150$	$\sum f_i x_i = 8400 + 75x + 135y$

$$96 + x + y = 150$$

$$x + y = 54 \quad ...(1)$$

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

$$91 = \frac{8400 + 75x + 135y}{150}$$

n258

$$13650 = 8,400 + 75x + 135y$$

$$75x + 135y = 5250$$

$$5x + 9y = 350 \quad ...(2)$$

Solving equation (1) and (2) we get $x = 34$ and $y = 20$

155. Find the median of the following data :

Profit (in lakh of rupee)	Number of shops
More than of equal to 5	30
More than of equal to 10	28
More than of equal to 15	16
More than of equal to 20	14
More than of equal to 25	10
More than of equal to 30	7
More than of equal to 35	3

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

Class	f	$c.f.$
5-10	2	2
10-15	12	14
15-20	2	16
20-25	4	20
25-30	3	23
30-35	4	27
35-40	3	30
Total	$\sum f = 30 = N$	

We have $N = 30$; $\dfrac{N}{2} = 15$

Cumulative frequency just greater than $\frac{N}{2}$ is 16 and the corresponding class is 15-20. Thus median class is 15-20.

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

Now, $l = 15,\ N = 30,\ F = 14,\ f = 2\ h = 5$

n259

Median, $M_d = l + \left(\dfrac{\frac{N}{2} - F}{f}\right)h$

$$= 15 + \left(\frac{15 - 14}{2}\right) \times 5$$

$$= 15 + 2.5 = 17.5$$

156. Find the value of x and y, if the median for the following data is 31.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	6	y	6	5	40

Ans : [Board Term-1 2012]

We prepare following cumulative frequency table to find median class.

C.I.	f	$c.f.$
0-10	5	5
10-20	x	$5+x$
20-30	6	$11+x$
30-40	y	$11+x+y$
40-50	6	$17+x+y$
50-60	5	$22+x+y$
	Total 40	

Since median is 31, which lies between 30-40. Thus median class is 30-40.

Here from table, $N = 22 + x + y$

$$40 = 22 + x + y$$

$$x + y = 18 \qquad ...(1)$$

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$31 = 30 + \left[\frac{20 - (11 + x)}{y}\right] \times 10$$

$$1 = \frac{(9 - x) \times 10}{y}$$

n260

$$y = 90 - 10x$$

$$10x + y = 90 \qquad ...(2)$$

Solving equation (1) and (2) we get $x = 8$ and $y = 10$

157. The following table gives the daily income of 50 workers of a factory.

Daily income (in Rs.)	100-120	120-140	140-160	160-180	180-200
Number of Workers	12	14	8	6	10

Find the mean, mode and median of the above data.

Ans : [Board Term-1 2009]

We prepare following table to find mean.

C.I.	f_i	$c.f.$	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-120	12	12	110	-2	-24
120-140	14	26	130	-1	-14
140-160	8	34	150	0	0
160-180	6	40	170	1	6
180-200	10	50	190	2	20
	$\sum f = 50$				$\sum f_i u_i = -12$

Let a be assumed mean be $a = 150$

Mean $$M = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

n261

$$= 150 + \frac{-12}{50} \times 20$$

$$= 150 - 4.8 = 145.2$$

Now $N = 50\,;\ \frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 120-140. Thus median class is 120-140.

Now $l = 120,\ f = 14,\ F = 12$ and $h = 20$

Median, $$M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + 18.57 \times 138.57$$

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

$$= 3 \times 138.57 - 2 \times 145.2$$

$$= 415.71 - 290.4 = 125.31$$

Hence, mean is 145.2, median is 138.57 and mode is 125.31.

158. Find the mode of the following distribution of marks obtained by the students in an examination :

Marks obtained	0-20	20-40	40-60	60-80	80-100
Number of students	15	18	21	29	17

Given the mean of the above distribution is 53, using empirical relationship estimate the value of its median.

Ans : [Board Term-1 SQP 2017]

Class 60-80 has the maximum frequency 29, therefore this is model class.

Here, $l = 60,\ f_1 = 29,\ f_0 = 21,\ f_2 = 17$ and $h = 20$

Mode, $$M_o = l + h\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 60 + \frac{8}{58 - 38} \times 20$$

$$= 60 + 8 = 68$$

n262

Now $$3M_d = M_o + 2M$$

$$= 68 + 2 \times 53$$

$$M_d = \frac{174}{3} = 58$$

Hence median is 58.

159. On the annual day of school, age-wise participation of students is given in the following frequency distribution table :

Age (in years)	Number of students
Less than 6	2
Less than 8	6
Less than 10	12
Less than 12	22
Less than 14	42
Less than 16	67
Less than 18	76

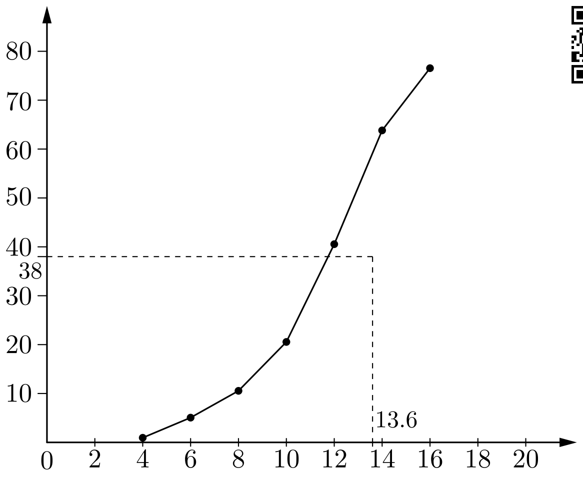
Find the median of the students and how can get the median graphically ?

Ans : [Board Term-1 2016]

Age of students	C.I.	$c.f.$	f
Less than 6	4-6	2	2
Less than 8	6-8	6	4
Less than 10	8-10	12	6
Less than 12	10-12	22	10
Less than 14	12-14	42	20
Less than 16	14-16	67	25
Less than18	16-18	76	9

Now $$N = 76;\ \frac{N}{2} = 38$$

Cumulative frequency just greater than $\frac{N}{2}$ is 42 and the corresponding class is 12-14. Thus median class is 12-14.



n263

160. Find the median of the following data :

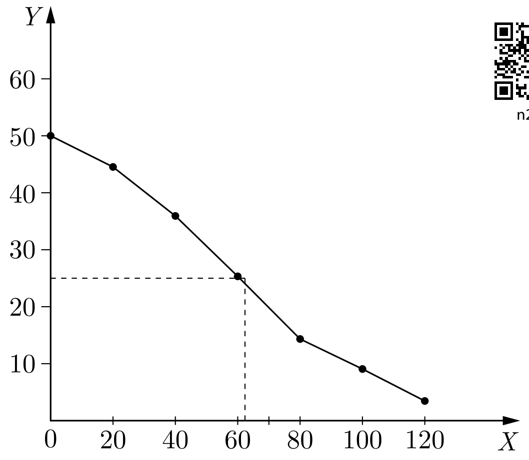
Class Interval	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency	6	8	10	12	6	5	3

How can we find the median graphically ?

Ans : [Board Term-1 2015]

Classes	$c.f.$
More than 0	50
More than 20	44
More than 40	36
More than 60	26
More than 80	14
More than 100	8
More than 120	3

To draw an ogive we take the indices : (0, 50), (20, 44), (40, 36), (60, 26), (80, 14), (100, 8) and (120, 3).



n264

From graph, $\frac{N}{2} = \frac{50}{2} = 25$

Median, $M_d = 61.6$

By Formula Method :

Classes	f	$c.f.$	
0-20	6	6	
20-40	8	14	
40-60	10	24	
60-80	12	36	Median Class
80-100	6	42	
100-120	5	47	
120-140	3	50	

Now $N = 50;\ \frac{N}{2} = 25$

Cumulative frequency just greater than $\frac{N}{2}$ is 36 and the corresponding class is 60-80. Thus median class is 60-80.

Now $l = 60,\ f = 12,\ F = 24,\ h = 20$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 60 + \frac{(25-24)}{12} \times 20$$

$$= 60 + \frac{1}{12} \times 20 = 60 + \frac{5}{3}$$

$$= \frac{185}{3}$$

$$= 61.67$$

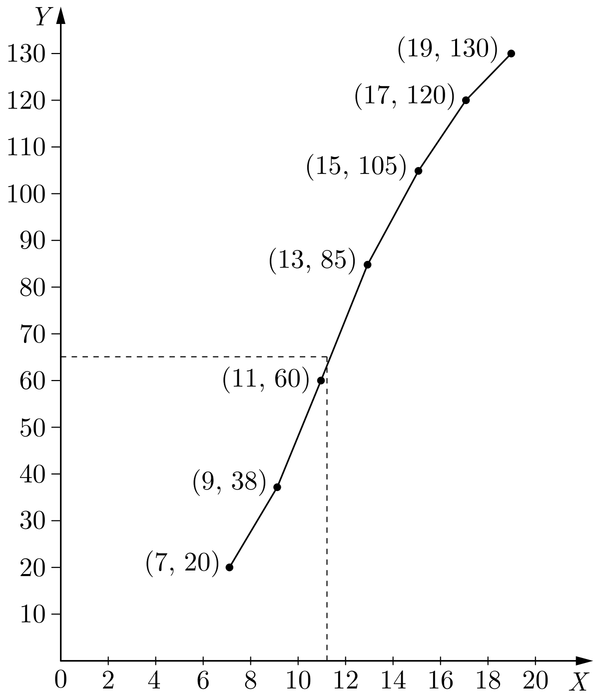
161. In annual day of a school, age-wise participation of students is shown in the following frequency distribution :

Age of students (in years)	5- 7	7- 9	9-11	11-13	13-15	15-17	17-19
Number of students	20	18	22	25	20	15	10

Draw a 'less than type' ogive for the above data and from it find the median age.

Ans : [Board Term-1 2015]

Students	$c.f.$
Less than 7	20
Less than 9	38
Less than 11	60
Less than 13	85
Less than 15	105
Less than 17	120
Less than 19	130



This curve is the required cumulative frequency curve or an ogive of the less than type.

n265

Here, $N = 130,$

So, $\frac{N}{2} = \frac{130}{2} = 65$

Now, we locate the point on the ogive whose ordinate is 65. The x co-ordinate corresponding to this ordinate is 11.4. Hence. the required median on the graph is 11.4.

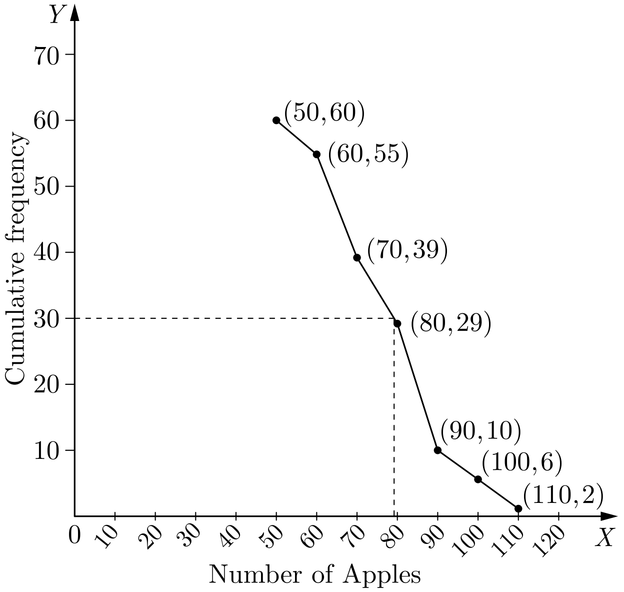
162. In an orchard, the number of apples on trees are below :

Number of apples	more than of equal to 50	more than of equal to 60	more than of equal to 70	more than of equal to 80	more than of equal to 90	more than of equal to 100	more than of equal to 110
Number of trees	60	55	39	29	10	6	2

Draw a 'more than type' ogive and hence obtain median from the curve.

Ans : [Board Term-1 2015]

Apples	*c.f.*
More than 50	60
More than 60	55
More than 70	39
More than 80	29
More than 90	10
More than 100	6
More than 110	2



This curve shows cumulative frequency on an ogive of the 'more than type'.

Here $N = 60,$

So $\frac{N}{2} = \frac{60}{2} = 30$

n266

Now, we locate the point on the ogive whose ordinate is 30. The x-co-ordinate corresponding to this ordinate is 79. Hence, the required median on the graph is 79.

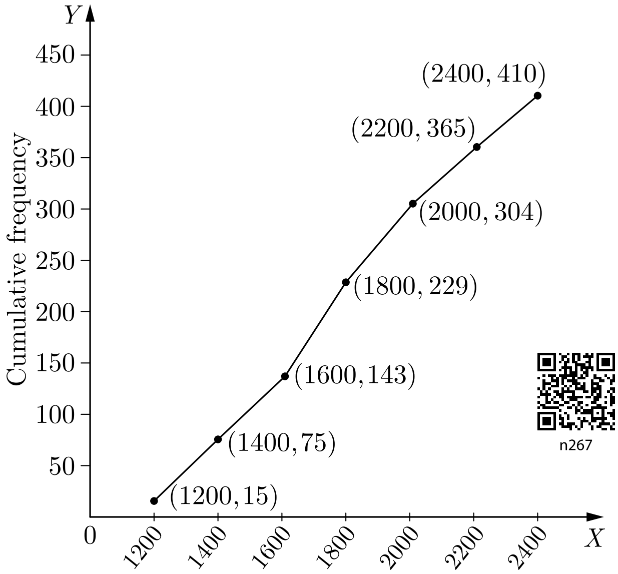
163. The following distribution gives the distribution of life times of washing machines of a certain company :

Life time (in hours)	1000-1200	1200-1400	1400-1600	1600-1800	1800-2000	2000-2200	2200-2400
Number of washing machines	15	60	68	86	75	61	45

Convert the above distribution into 'less than type' and draw its ogive.

Ans : [Board Term-1 2015]

Life time	c.f.
Less than 1200	15
Less than 1400	75
Less than 1600	143
Less than 1800	229
Less than 2000	304
Less than 2200	365
Less than 2400	410



n267

164. Following distribution shows the marks obtained by a class of 100 students :

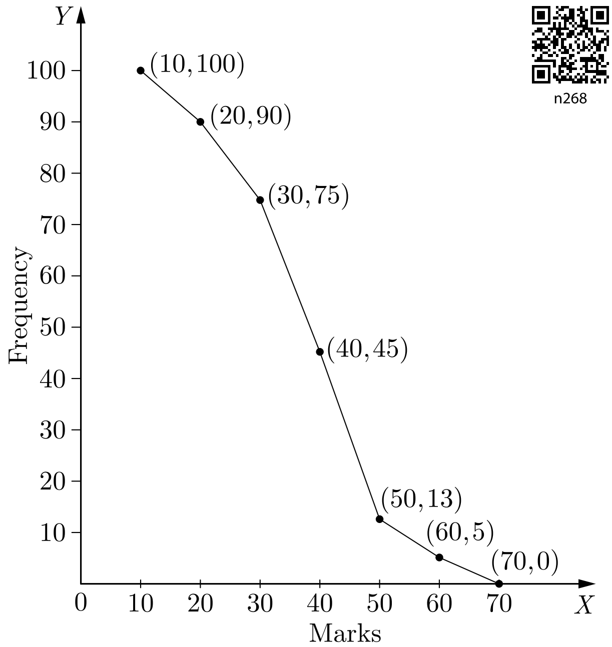
Marks	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	15	30	32	8	5

Draw a 'more than' ogive for the above data

Ans : [Board Term-1, 2012, Set-48]

Marks	Frequency
More than 10	100
More than 20	90
More than 30	75
More than 40	45
More than 50	13
More than 60	5
More than 70	0

'More than' ogive is shown below :



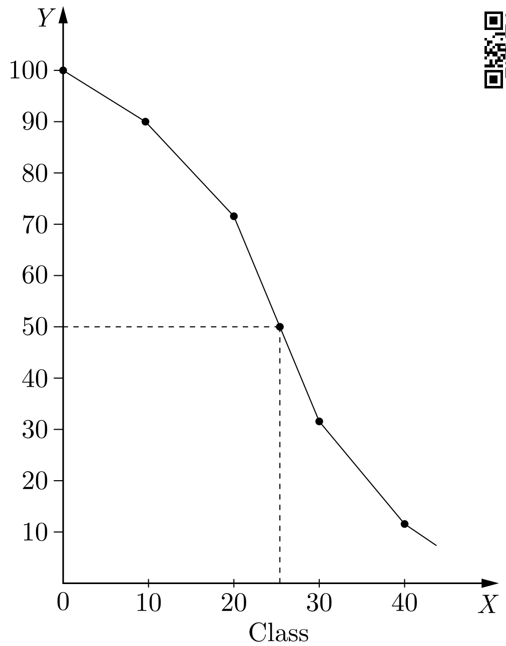
n268

165. Draw more than ogive for the following distribution. Find the median from the curve.

Classes	0-10	10-20	20-30	30-40	40-50
Frequency	10	18	40	20	12

Ans : [Board Term-1, 2012, Set-48]

More than	c.f.
0	100
10	90
20	72
30	32
40	12



n269

From graph, $\frac{N}{2} = \frac{100}{2} = 50$

Hence, Median $= 25$

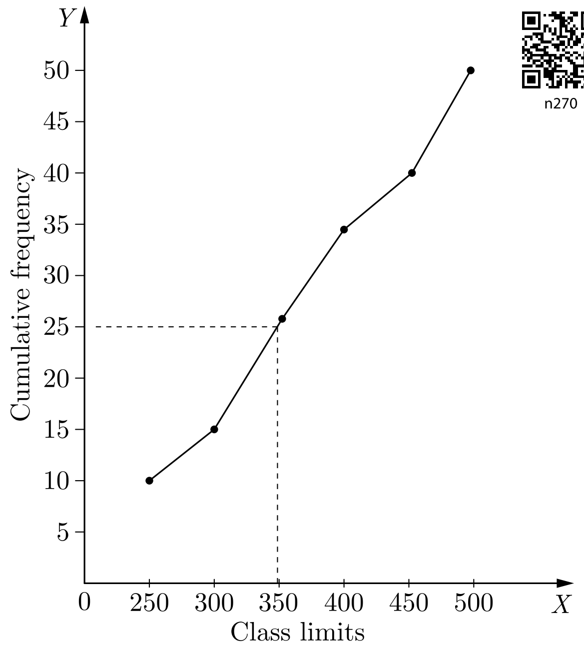
166. The following distribution gives the daily income of 50 workers of a factory :

Daily income(In Rs.)	200-250	250-300	300-350	350-400	400-450	450-500
Number of workers	10	5	11	8	6	10

Convert the distribution to a 'less than type' cumulative frequency distribution and draw its ogive. Hence obtain the median of daily income.

Ans : [Board Term-2, 2012, Set-55]

Daily income (Classes)	No. of workers ($c.f.$)
Less than 250	10
Less than 300	15
Less than 350	26
Less than 400	34
Less than 450	40
Less than 500	50



n270

From graph. $\frac{N}{2} = \frac{50}{2} = 25$

Hence, Median daily income = Rs. 345.

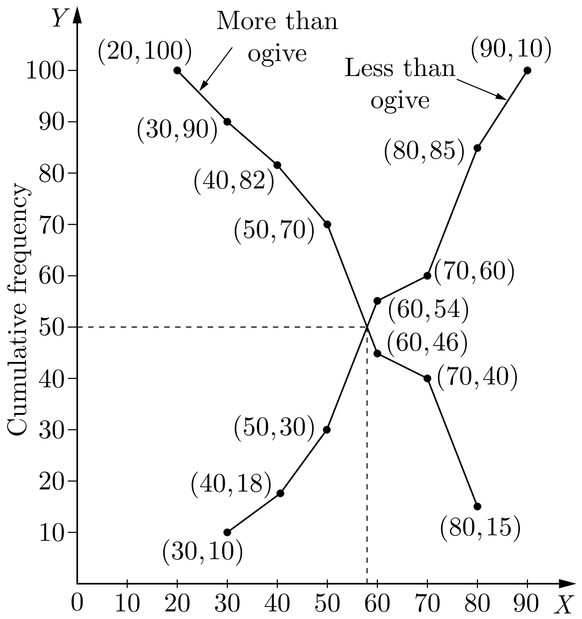
167. Draw "less than ogive" and more than ogive" for the following distribution and hence find its median :

Class	30-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	10	8	12	24	6	25	15

Ans : [Board Term-1, 2012, Set-39, 48, 50]

Less than	*c.f.*	More than	*c.f.*
30	10	20	100
40	18	30	90
50	30	40	82
60	54	50	70
70	60	60	46
80	85	70	40
90	100	80	15

n271



168. The following table gives the weight of 120 articles :

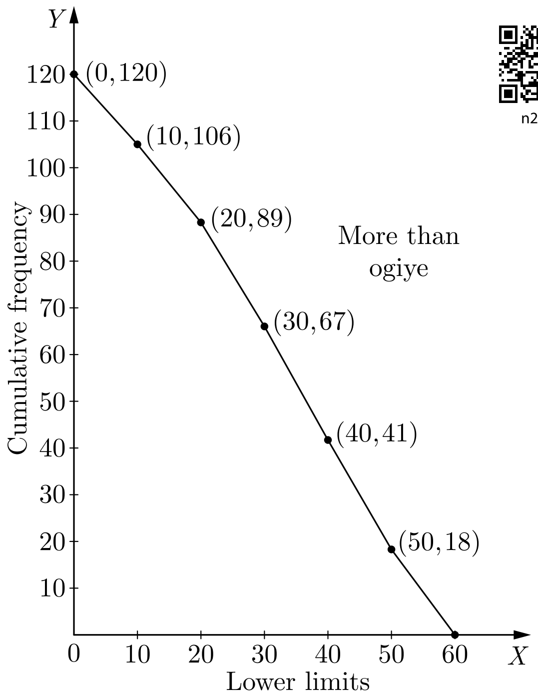
Weight (in kg)	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	14	17	22	26	23	18

Change the distribution to a 'more than type' distribution and draw its ogive.

Ans : [Board Term-1, 2012, Set-48]

Weight (in kg) 0	Cumulative Frequency
More than to 10	120
More than to 20	106
More than to 30	89
More than to 40	67
More than to 40	41
More than to 50	18
More than to 60	0

Plotting the points :



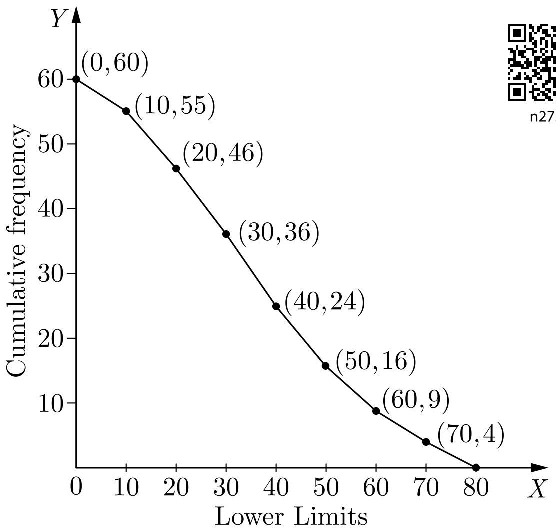
n272

169. Draw a 'more than ogive' for the following data :

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	9	10	12	8	7	5	4

Ans : [Board Term-1, 2012, Set-48]

More than	c.f.
0	60
10	55
20	46
30	36
40	24
50	16
60	9
70	4
80	0



n273

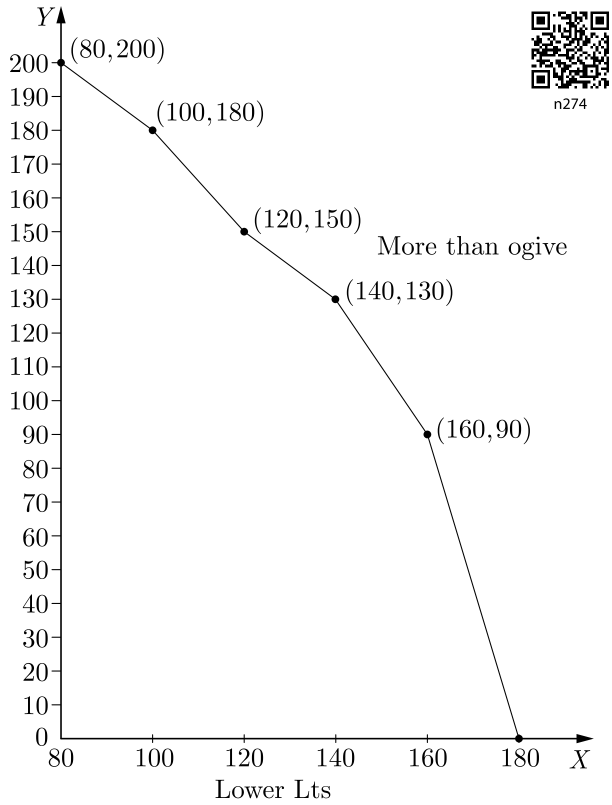
170. The distribution of monthly wages of 200 workers of a certain factory is as given below :

Monthly wages (in Rs.)	80-100	100-120	120-140	140-160	160-180
Number of workers	20	30	20	40	90

Change the above distribution to a 'more than type' distribution and draw its ogive.

Ans : [Board Term-1, 2012, Set-60]

Wages	*c.f.*
More than 80	200
More than 100	180
More than 120	150
More than 140	130
More than 160	90
More than 180	0



171. The following are the ages of 200 patients getting medical treatment in a hospital on a particular day :

Age (In years)	10-20	20-30	30-40	40-50	50-60	60-70
Number of Patients	40	22	35	50	23	30

Write the above distribution as 'less than type' cumulative frequency distribution and also draw an ogive to find the median.

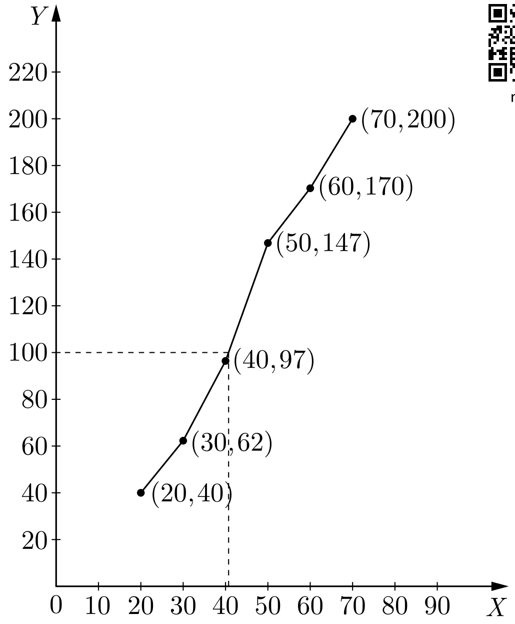
Ans : [Board Term-1 2015]

Less than	*c.f.*
10	0
20	40
30	62
40	97
50	147
60	170
70	200

We have $N = 200$

So, $\frac{N}{2} = 100$

Plotting the obtained coordinates (20,40), (30, 62), (40, 97), (50, 147), (60, 170) and (70, 200) and draw 'less than type' curve.



Median $= 40.6$

172. The following frequency distribution shows the distance (in meters) thrown by 68 students in a Javelin throw competition.

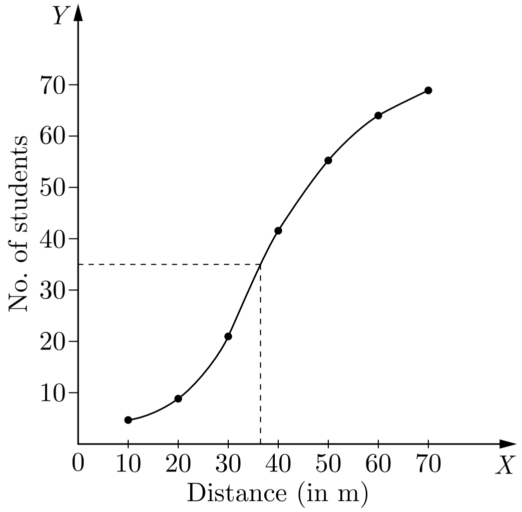
Distance (in m)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	4	5	13	20	14	8	4

Draw a less than Ogive for the given data and find the median distance through using this curve.

Ans :

Distance (in m)	Number of Students	Less than distance (in m)	*c.f.*
0-10	4	Less than 0	0
10-20	5	Less than 10	4
20-30	13	Less than 20	9
30-40	20	Less than 30	22
40-50	14	Less than 40	42
50-60	8	Less than 50	56
60-70	4	Less than 60	64
		Less than 70	68

The co-ordinates for drawing an ogive are (0, 0), (10, 4), (20, 9), (30, 22), (40, 42), (50, 56), (60, 64), (70, 68).



Less than Ogive

CHAPTER 15

PROBABILITY

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. The probability that a number selected at random from the numbers 1, 2, 3, ..., 15 is a multiple of 4 is

(a) $\frac{4}{15}$ (b) $\frac{2}{15}$

(c) $\frac{1}{15}$ (d) $\frac{1}{5}$

Ans : [Board 2020 Delhi Basic]

Total possible outcome, $n(S) = 15$

Number of multiples of 4 between 1 to 15 are 4. 8. 12 i.e. 3 favourable outcome.

o101

$$n(E) = 3$$

Required Probability, $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{3}{15} = \frac{1}{5}$$

Thus (d) is correct option.

2. Two coins are tossed simultaneously. The probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Ans : [Board 2020 OD Basic]

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favourable outcomes are {HT, TH, TT}.

o102

$$n(E) = 3$$

Probability of getting at most one head,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

Thus (d) is correct option.

3. If an event cannot occur, then its probability is

(a) 1 (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) 0

o103

Ans :

The event which cannot occur is said to be impossible event and probability of impossible event is zero.

Thus (d) is correct option.

4. Which of the following cannot be the probability of an event?

(a) $\frac{1}{3}$ (b) 0.1

(c) 3% (d) $\frac{17}{16}$

o104

Ans :

Probability of an event always lies between 0 and 1.

Thus (d) is correct option.

5. An event is very unlikely to happen. Its probability is closest to

(a) 0.0001 (b) 0.001

(c) 0.01 (d) 0.1

o105

Ans :

The probability of an event which is very unlikely to happen is closest to zero and from the given options 0.0001 is closest to zero.

Thus (a) is correct option.

6. If the probability of an event is p, then the probability of its complementary event will be

(a) $p-1$ (b) p

(c) $1-p$ (d) $1-\frac{1}{p}$

Ans :

Since,

$$P(E) + P(\overline{E}) = 1$$

$$P(E) = 1 - P(\overline{E})$$

$$= 1 - p$$

o106

Thus (c) is correct option.

7. The probability expressed as a percentage of a particular occurrence can never be

(a) less than 100

(b) less than 0

(c) greater than 1

(d) anything but a whole number

Ans :

We know that the probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

o107

Thus (b) is correct option.

8. The $P(A)$ denotes the probability of an event A, then

(a) $P(A) < 0$

(b) $P(A) > 1$

(c) $0 \leq P(A) \leq 1$

(d) $-1 \leq P(A) \leq 1$

Ans :

Probability of an event always lies between 0 and 1.

o108

Thus (c) is correct option.

9. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(a) $\frac{3}{26}$

(b) $\frac{3}{13}$

(c) $\frac{2}{13}$

(d) $\frac{1}{2}$

Ans :

In a deck of 52 cards, there are 12 face cards i.e., 6 red and 6 black cards.

$$n(S) = 52$$

$$n(E) = 6$$

So, probability of getting a red face card,

o109

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

Thus (a) is correct option.

10. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(a) 4

(b) 13

(c) 48

(d) 51

Ans :

In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.

Hence, the number of outcomes favourable

$$n(E) = 52 - 1 = 51$$

o110

Thus (d) is correct option.

11. When a die is thrown, the probability of getting an odd number less than 3 is

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 0

Ans :

Odd number less than 3 is 1 only.

$$n(S) = 6$$

o111

$$n(E) = 1$$

So, probability of getting an odd number less than 3,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Thus (a) is correct option.

12. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(a) 7

(b) 14

(c) 21

(d) 28

Ans :

We have $n(S) = 400$

o112

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.035 = \frac{x}{400}$$

$$x = 0.035 \times 400 = 14$$

Thus (b) is correct option.

13. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40

(b) 240

(c) 480

(d) 750

Ans :

Total number of sold tickets are 6000. Let she bought x tickets.

Now $n(S) = 6000$

o113

$$n(E) = x$$

$$P(E) = \frac{n(E)}{n(S)}$$

$$0.08 = \frac{x}{6000}$$

$$x = 0.08 \times 6000 = 480$$

Hence, she bought 480 tickets.

Thus (c) is correct option.

14. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{3}$

Ans :

Multiples of 5 are 5, 10, 15, 20, 25, 30, 35 and 40 thus 8 outcome.

$$n(S) = 40$$

$$n(E) = 8$$

o114

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{40} = \frac{1}{5}$$

Thus (a) is correct option.

15. Someone is asked to take a number from 1 to 100. The probability that it is a prime, is

(a) $\frac{8}{25}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{13}{50}$

Ans :

Prime numbers between 1 to 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97, i.e 25 outcome.

$$n(S) = 100$$

$$n(E) = 25$$

o115

$$P(E) = \frac{n(E)}{n(S)} = \frac{25}{100} = \frac{1}{4}$$

Thus (c) is correct option.

16. The probability of getting a number greater then 3 in throwing a die is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Ans :

$$n(S) = 6$$

o116

$$n(E) = 2$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

Thus (d) is correct option.

17. Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{3}{4}$ (d) $\frac{2}{3}$

Ans :

One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.

$$n(S) = 4$$

$$n(E) = 1$$

o117

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$

Thus (b) is correct option.

18. A bag contains 3 red and 2 blue marbles. If a marble is drawn at random, then the probability of drawing a blue marble is:

(a) $\frac{2}{5}$ (b) $\frac{1}{4}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

Ans :

There are 5 marbles in the bag. Out of these 5 marbles one can be choose in 5 ways. Since, the bag contains 2 blue marbles. Therefore, one blue marble can be drawn in 2 ways.

$$n(S) = 5$$

$$n(E) = 2$$

o118

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{5}$$

Thus (b) is correct option.

19. A single letter is selected at random from the word

PROBABILITY. The probability that the selected letter is a vowel is

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$

(c) $\frac{4}{11}$ (d) 0

Ans :

There are 11 letter in word PROBABILITY. Out of these 11 letter, 4 letter are vowels.

$$n(S) = 11$$

$$n(E) = 4$$

o119

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{11}$$

Thus (c) is correct option.

20. A fair die is thrown once. The probability of getting a composite number less than 5 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $\frac{2}{3}$ (d) 0

Ans :

The outcomes are 1, 2, 3, 4, 5, 6. Out of these, 4 is the only composite number which is less than 5.

$$n(S) = 6$$

$$n(E) = 1$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

o120

Thus (b) is correct option.

21. If a letter is chosen at random from the letter of English alphabet, then the probability that it is a letter of the word DELHI is

(a) $\frac{1}{5}$ (b) $\frac{1}{26}$

(c) $\frac{5}{26}$ (d) $\frac{21}{26}$

Ans :

The English alphabet has 26 letters in all. The word DELHI has 5 letter, so the number of favourable outcomes is 5.

$$n(S) = 26$$

$$n(E) = 5$$

o121

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{26}$$

Thus (c) is correct option.

22. The probability that a two digit number selected at random will be a multiple of 3 and not a multiple of 5 is

(a) $\frac{2}{15}$ (b) $\frac{4}{15}$

(c) $\frac{1}{15}$ (d) $\frac{4}{90}$

Ans :

24 out of the 90 two digit numbers are divisible by 3 and not by 5.

$$n(S) = 90$$

$$n(E) = 24$$

o122

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{24}{90} = \frac{4}{15}$$

Thus (b) is correct option.

23. If in a lottery, there are 5 prizes and 20 blanks, then the probability of getting a prize is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) 1

Ans :

We have $n(S) = 20 + 5 = 25$

$$n(E) = 5$$

o123

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

Thus (c) is correct option.

24. If a number x is chosen at random from the numbers $-2, -1, 0, 1, 2$. Then, the probability that $x^2 < 2$ is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$

(c) $\frac{1}{5}$ (d) $\frac{3}{5}$

Ans :

Total number of possible outcomes are 5.

We observe that $x^2 < 2$ when x takes anyone of the following three values -1, 0 and 1.

We have $n(S) = 5$

$$n(E) = 3$$

o124

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{5}$$

Thus (d) is correct option.

25. Which of the following relationship is the correct?

(a) $P(E) + P(\overline{E}) = 1$ (b) $P(\overline{E}) - P(E) = 1$

(c) $P(E) = 1 + P(\overline{E})$ (d) None of these

Ans :

$$P(E) + P(\overline{E}) = 1$$

Thus (a) is correct option.

o125

26. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4, is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Ans :

Total number of outcomes is 36.

Here, all possible outcome is (1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2) and (6, 6),

$$n(S) = 36$$

$$n(E) = 9$$

o126

P(sum of two numbers will be multiple of 4)

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

Thus (d) is correct option.

27. A letter is chosen at random from the letters of the word ASSASSINATION, then the probability that the letter chosen is a vowel is in the form of $\frac{6}{2x+1}$, then x is equal to

(a) 5 (b) 6

(c) 7 (d) 8

Ans :

There are 13 letters in the word ASSASSINATION out of which one letter can be chosen in 13 ways. Hence, total number of outcomes are 13. There are 6 vowels in the word ASSISSINATION. So, there are 6 ways of selecting a vowel.

$$n(S) = 13$$

$$n(E) = 6$$

o127

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{13}$$

But given that, $\frac{6}{2x+1} = \frac{6}{13}$

$$2x + 1 = 13 \Rightarrow x = 6$$

Thus (b) is correct option.

28. Ramesh buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random a tank containing 5 male fish and 9 female fish. Then, the probability that the fish taken out is a male fish, is

(a) $\frac{5}{13}$ (b) $\frac{5}{14}$

(c) $\frac{6}{13}$ (d) $\frac{7}{13}$

Ans :

There are $14 = (5+9)$ fish out of which one can be chosen in 14 ways.

There are 5 male fish out of which one male fish can be chosen in 5 ways.

$$n(S) = 14$$

$$n(E) = 5$$

o128

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{14}$$

Thus (b) is correct option.

29. A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9 then the probability that the product xy of the two numbers will be less than 9 is

(a) $\frac{3}{7}$ (b) $\frac{4}{9}$

(c) $\frac{5}{9}$ (d) $\frac{7}{9}$

Ans :

Number x can be selected in three ways and corresponding to each such way there are three ways of selecting number y.

Therefore two numbers can be selected in 9 ways as (1, 1), (1, 4), (1, 9), (2, 1), (2, 4), (2, 9), (3, 1), (3, 4), (3, 9) So, total numbers of possible outcomes are 9.

The product xy will be less than 9, if x and y are chosen in one of the following ways: (1, 1), (1, 4), (2, 1), (2, 4), (3, 1)

$$n(S) = 9$$

o129

$$n(E) = 5$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

Thus (c) is correct option.

30. There are 1000 sealed envelopes in a box. 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well-shuffled and an envelope is picked up out, then the probability that is contains no cash prize is

(a) 0.65 (b) 0.69

(c) 0.54 (d) 0.57

Ans :

Total number of envelopes in the box = 1000

Number of envelopes containing cash prize

$$= 10 + 100 + 200 = 310$$

Number of envelopes containing no cash

$$= 1000 - 310 = 690$$

Now $n(S) = 1000$

$$n(E) = 690$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{690}{1000} = 0.69$$

Thus (b) is correct option.

Fill in the Blank Questions

31. The probability of an event that is certain to happen is .... Such an event is called ....

Ans :

1, sure or certain event

32. The sum of the probabilities of all the elementary events of an experiment is ....

Ans :

1

33. On a single roll of a die, the probability of getting a number 8 is ....

Ans :

zero

34. The probability of an event is greater than or equal to .... and less than or equal to ....

Ans :

0, 1

35. On a single roll of a die, the probability of getting a number less than 7 is ....

Ans :

one

36. A number is chosen at random from the numbers $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. Then the probability that square of this number is less than or equal to 1 is ....

Ans : [Board 2020 SQP Standard]

Given numbers are $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ and their squares are 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25.

Total number of outcomes $n(S) = 11$.

Favourable outcome are -1, 0, 1, thus number of favourable outcomes is $n(E) = 3$.

Required probability, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{11}$

Very Short Answer Questions

37. Find the probability of an impossible event.

Ans : [Board Term-2, 2012]

Probability of impossible event is 0.

38. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting a red king.

Ans : [Board 2020 OD Basic]

Total no. of cards, $n(S) = 52$

Number of red kings, $n(E) = 2$

P(a red king), $P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

39. A card drawn at random from a well shuffled deck of 52 playing cards. What is the probability of getting a black king?

Ans : [Board 2020 OD Basic]

Total no. of cards, $n(S) = 52$

Number of black kings, $n(E) = 2$

P(black king), $P(E) = \frac{n(E)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

40. A die is thrown once. What is the probability of getting a number less than 3?

Ans : [Board 2020 Delhi Standard]

There are 6 possible outcome for a die.

$$n(S) = 6$$

o140

Favourable outcome are 1 and 2 i.e. two outcomes.

$$n(E) = 2$$

P(number less than 3)

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

41. If the probability of wining a game is 0.07, what is the probability of losing it?

Ans : [Board 2020 Delhi Standard]

P(winning the game), $P(E) = 0.07$

o141

P(number less game), $P(\overline{E}) = 1 - P(E)$

$$= 1 - 0.07$$

$= 0.93$

42. A die is thrown once. Find the probability of getting "at most 2."

Ans : [Board Term-2 OD Compt 2017]

All possible outcome i.e. sample space,

$$S = \{1, 2, 3, 4, 5, 6\}$$

o142

Number of all possible outcome,

$$n(S) = 6$$

Favourable outcomes,

$$E = \{1,\ 2\}$$

Number of favourable outcome,

$$n(E) = 2$$

Thus $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

43. If $P(E) = 0.20$, then what is the probability of 'not E'?

Ans : [Board Term-2, 2012]

We have $P(E) = 0.20$

o143

$$P(\text{not}E) = 1 - P(E)$$

$$= 1 - 0.20 = 0.80$$

44. If the probability of winning a game is $\frac{5}{11}$, find the probability of losing the game.

Ans : [Board Term-2, 2014]

Probability of winning the game,

$$P(E) = \frac{5}{11}$$

o144

Probability of losing the game

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{5}{11} = \frac{6}{11}$$

45. If E be an event such that $P(E) = \frac{3}{7}$, what is P(not E) equal to?

Ans : [Board Term-2, 2014]

We have $P(E) = \frac{3}{7}$

o145

$$P(\text{not } E) = 1 - P(E)$$

$$= 1 - \frac{3}{7} = \frac{4}{7}$$

46. A bag contains lemon flavoured candies only. Shalini takes out one candy without looking into the bag. What is the probability that she takes out an orange flavoured candy?

Ans : [Board Term-2, 2012]

Bag contains only lemon flavoured candies. So, getting an orange flavoured candy is an impossible.

o146

$$P(E) = 0$$

47. If a number x is chosen a random from the number $-3, -2, -1, 0, 1, 2, 3$. What is probability that $x^2 \leq 4$?

Ans : [Board 2020 Delhi Standard]

We have 7 possible outcome. Thus

$$n(S) = 7$$

Favourable outcomes are $-2, -1, 0, 1, 2$ i.e. 5.

$$n(E) = 5$$

o147

$P(x^2 \leq 4)$, $P(E) = \frac{n(E)}{n(S)} = \frac{5}{7}$

48. Out of 200 bulbs in a box, 12 bulbs are defective. One bulb is taken out at random from the box. What is the probability that the drawn bulb is not defective?

Ans : [Board Term-2 SQP 2016]

Total number of bulbs,

$$n(S) = 200$$

Number of favourable cases,

o148

$$n(E) = 200 - 12 = 188$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{188}{200} = \frac{47}{50}$$

49. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting neither a red card nor a queen.

Ans : [Board Term-2 OD 2016]

There are 26 red cards out of total 52 cards and 2 black queen also.

Total number of cards, $n(S) = 52$

o149

Cards which are neither red nor queen,

$$n(E) = 52 - (26 + 2) = 24$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{24}{52} = \frac{6}{13}$$

50. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant.

Ans : [Board Term-2 Delhi 2015, 2020 Delhi STD]

In the English language there are 26 alphabets. Consonant are 21. The probability of chosen a consonant

$$n(S) = 26$$

$$n(E) = 21$$

o150

$$P(E) = \frac{n(E)}{n(S)} = \frac{21}{26}$$

51. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number.

Ans : [Board Term-2 2016]

Total number of outcomes,

$$n(S) = 48$$

Favourable outcomes are 4, 9, 16, 25, 36 and 49.

No. of favourable outcomes,

$$n(E) = 6$$

o151

P(perfect square number),

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{48} \text{ or } = \frac{1}{8}$$

52. 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is a multiple of 3 or 7.

Ans : [Board Term-2 Foreign 2016]

Total number of cases,

o152

$$n(S) = 20$$

Favourable outcome,

$$E = \{3, 6, 7, 9, 12, 14, 15, 18\}$$

Number of favourable cases,

$$n(E) = 8$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

53. What is the probability that a non-leap year has 53 Mondays ?

Ans : [Board Term-2, 2015]

There are 365 days in a non-leap year.

365 days = 52 weeks + 1 day

One day can be M, T, W, Th, F, S, S i.e. total 7 possible outcomes and only one favourable outcome.

Thus $n(S) = 7$ and $n(E) = 1$

o153

P(53 Mondays in non-leap year)

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$$

54. Two different dice are tossed together. Find the probability that the product of the number on the top of the dice is 6.

Ans : [Board Term-2 OD 2015]

Total number of possible outcomes,

$$n(S) = 6 \times 6 = 36$$

Product of 6 are (1, 6), (2, 3), (6, 1) and (3, 2).

Number of favourable outcomes,

$$n(E) = 4$$

o154

Total number of chances

$$n(S) = 6 \times 6 = 36$$

P(Product of 6)

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

55. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. Find the probability that the arrow will point at any factor of 8 ?

Ans : [Board Term-2 Foreign 2015]

Total number of points are 8. Thus, total number of possible outcomes

$$n(S) = 8$$

Favourable outcomes are 1, 2, 4, and 8

No. of favourable outcomes,

o155

$$n(E) = 3$$

P(factor of 8)

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{2}$$

56. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that number is divisible by both 2 and 3.

Ans : [Board Term-2 Foreign 2014]

Since bag contains 25 cards,

$$n(S) = 25$$

The numbers divisible by 2 and 3 both are 6, 12, 18, 24 which are 4 numbers.

Thus $n(E) = 4$

P(number divisible by 2 and 3)

o156

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{25}$$

57. A number is selected at random from 1 to 30. Find the probability that it is a prime number.

Ans : [Board Term-2, 2014]

Number of possible outcomes,

$$n(S) = 30$$

Prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

o157

Number of favourable outcomes, $n(E) = 10$

P(prime), $\quad P(E) = \frac{n(E)}{n(S)} = \frac{10}{30} = \frac{1}{3}$

58. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from box, find the probability that it bears a prime number less than 23.

Ans : [Board Term-2, 2012]

Number of possible outcomes,

$$n(S) = 90$$

Prime numbers less than 23 are 2, 3, 5, 7, 11, 13

Number of favourable outcomes

o158

$$n(E) = 8$$

P(prime no. less than 23)

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{90} = \frac{4}{45}$$

59. From the number 3, 5, 5, 7, 7, 7, 9, 9, 9, 9, one number is selected at random, what is the probability that the selected number is mean?

Ans : [Board Term-2, 2012]

Total outcomes, $\quad n(S) = 10$

Mean,

$$M = \frac{3+5+5+7+7+7+9+9+9+9}{10} = \frac{70}{10} = 7$$

Thus 7 is the mean of given numbers and frequency of 7 is 3 in given data.

Number of favourable outcomes,

$$n(E) = 3$$

o159

P(mean), $\quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{10}$

60. A die is thrown once. Find the probability of getting a prime number.

Ans : [Board Term-2, 2012]

Total outcomes, $\quad n(S) = 6$

Prime numbers are 2, 3, 5.

o160

$$n(E) = 3$$

P(prime no.), $\quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

61. A girl calculates the probability of her winning the game in a match and find it 0.08. What is the probability of her losing the game?

Ans : [Board Term-2, 2012]

P(winning the game), $\quad P(E) = 0.08$

o161

P(losing the game), $\quad P(\overline{E}) = 1 - P(E)$

$$= 1 - 0.08 = 0.92$$

62. The probability of getting a bad egg in a lot of 400 eggs is 0.035. Find the number of bad eggs in the lot.

Ans : [Board Term-2, 2012]

Number x be number of bad eggs.

$$n(E) = x$$

Total eggs, $n(S) = 400$

P(bad eggs) $P(E) = 0.035$

P(bad eggs), $P(E) = \frac{n(E)}{n(S)}$

$$0.035 = \frac{x}{400}$$

$$x = 400 \times 0.035 = 14$$

Thus there are 14 bad eggs in lot.

63. In tossing a die, what is the probability of getting an odd number or number less than 4 ?

Ans : [Board Term-2, 2012]

Total outcome, $n(S) = 6$

Odd numbers are 1, 3, 5 and number less than 4 are 1, 2, 3. Thus there are 4 favourable outcome.

$$n(E) = 4$$

P(an odd no. or a no. <4),

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

64. A card is drawn from a well shuffled deck of playing cards. Find the probability of drawing a red face card.

Ans : [Board Term-2, 2012]

Total outcomes, $n(S) = 52$

Red face card, $n(E) = 6$

P(red face card), $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{6}{52} = \frac{3}{26}$$

65. Find the probability of getting a sum of 9, when two dice are thrown simultaneously.

Ans : [Board Term-2, 2012]

If two dice are thrown there are $6 \times 6 = 36$ possible outcomes. Thus there are 4 favourable outcome (3, 6), (6, 3), (4, 5) and (5, 4). In these case sum of both faces are 9.

Number of total outcomes,

$$n(S) = 36$$

Number of favourable outcomes

$$n(E) = 4$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

66. Can 1.1 be probability of an event ?

Ans : [Board Term-2, 2012]

No. Since the probability of an event cannot be more than 1.

67. A bag contains 3 red and 5 black balls. A ball is drawn at random from the bag. What is the probability that the drawn ball is not red ?

Ans : [Board Term-2 Delhi 2017]

There are total $3 + 5 = 8$ balls in bag. Thus total possible outcomes,

$$n(S) = 8$$

5 black balls are not red. Thus favourable outcome

$$n(E) = 5$$

P(drawn ball is not red),

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{8}$$

68. If three different coins are tossed together, then find the probability of getting two heads.

Ans : [Board Term-2 OD Compt. 2017]

All possible outcomes are {HHH, THH, HTH, HHT, TTT, TTH, THT, HTT}.

Number of possible outcomes,

$$n(S) = 8$$

Number of favourable outcomes,

$$n(E) = 3$$

P(getting two heads),

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

69. A number is chosen at random from the numbers $-3, -2, -1, 0, 1, 2, 3$. What will be the probability that square of this number is less than or equal to 1.

Ans : [Board Term-2 Delhi 2017]

No. of all possible outcomes,

$$n(S) = 7$$

No. of favourable outcomes are $-1, 0, 1$.

$$n(E) = 3$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$$

70. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap ?

Ans :

Let E be the event of getting a rotten apple.

Total apples, $n(S) = 900$

Probability of selecting a rotten apple,

$$n(E) = 0.18$$

o170

Let $n(E)$ be number of rotten apples,

Then, $P(E) = \dfrac{n(E)}{n(S)}$

$$0.18 = \frac{n(E)}{900}$$

$$0.18 \times 900 = n(E)$$

$$n(E) = 162$$

So, there are 162 rotten apples in the heap.

TWO MARKS QUESTIONS

71. A number x is chosen from 25, 24, 23, $-2, -1$, 0, 1, 2, 3. Find the probability that $|x| < 3$.

Ans : [Board Term-2, 2015]

Total possible outcomes,

$$n(S) = 9$$

Favourable outcome are $-2, -1$, 0, 1, and 2.

Favourable outcomes $n(E) = 5$

o171

$$P(|x| < 3) = \frac{n(E)}{n(S)} = \frac{5}{9}$$

72. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball at random from the bag is three times that of a red ball, find the number of blue balls in the bag.

Ans : [Board 2020 Delhi Basic]

Let x be blue balls.

o172

Total balls, $n(S) = 5 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

P (red ball), $P(R) = \dfrac{n(R)}{n(S)} = \dfrac{5}{5+x}$

P (blue ball), $P(R) = \dfrac{n(B)}{n(S)} = \dfrac{x}{5+x}$

As per question we have

$$\frac{x}{5+x} = \frac{3 \times 5}{5+x}$$

Thus $x = 15$

Hence, bag contains 15 blue balls.

73. If a pair of dice is thrown once, then what is the probability of getting a sum of 8?

Ans : [Board 2020 Delhi Basic]

Number of possible outcomes,

$$n(S) = 6^2 = 36$$

The favourable outcomes are (sum of getting 8) $\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ i.e. 5 outcomes.

Number of favourable outcome,

$$n(E) = 5$$

Probability (getting sum of 8),

o173

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

74. A die thrown once. What is the probability of getting an even prime number?

Ans : [Board 2020 Delhi Standard]

Total possible outcomes of die is 6.

$$n(S) = 6$$

Favourable outcomes is only 2 i.e. there is one possible outcome.

$$n(E) = 1$$

P (getting an even prime number),

o174

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

75. Two different dice are thrown together, find the probability that the sum of the numbers appeared is less than 5.

Ans : [Board 2020 OD Basic]

Number of possible outcomes,

$$n(S) = 6^2 = 36$$

The favourable outcomes are (sum less than 5) $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2) \text{ and } (3, 1)\}$ i.e. 6 outcomes.

Number of favourable outcome,

$$n(E) = 6$$

P (have sum less than 5)

o175

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

76. In a lottery, there are 10 prizes and 25 blanks. What is the probability of getting a prize?

Ans : [Board 2020 OD Basic]

Total number of possible outcomes,

$$n(S) = 10 + 25 = 35$$

o176

Total number of prizes,

$$n(E) = 10$$

Probability of getting a prize,

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$$

77. Two different coins are tossed simultaneously, What is the probability of getting at least one head?

Ans : [Board 2020 Delhi OD Basic]

All possible outcomes are {HH, HT, TH, TT}.

Thus $n(S) = 4$

Favourable outcomes are {HT, TH, HH}.

o177

$$n(E) = 3$$

Probability of getting at least one head,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

78. A pair of dice is thrown once. What is the probability of getting a doublet?

Ans : [Board 2020 Delhi Standard]

There are total $6^2 = 36$ possible outcomes. Thus

$$n(S) = 36$$

Favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

Number of favourable outcomes,

o178

$$n(E) = 6$$

P(getting doublet),

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

79. A die is thrown once. What is the probability of getting a prime number.

Ans : [Board 2020 OD Standard]

There are 6 possible outcome for a die.

$$n(S) = 6$$

Favourable outcome are 1 and 2 i.e. two outcome.

$$n(E) = 2$$

P (number less than 3),

o179

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

80. If a number x is chosen at random from the numbers -3, -2, -1. 0, 1, 2, 3, then find the probability of $x^2 < 4$.

Ans : [Board 2020 OD Standard]

Possible outcome are -3, -2, -1. 0, 1, 2, 3 i.e 7 outcomes.

Thus $n(S) = 7$

Favourable outcomes are $x^2 < 4$ i.e. $= -1, 0, 1$.

$$n(E) = 3$$

o180

$P(x^2 < 4)$, $\quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$

81. A bag contains cards with numbers written on it from 1–80. A card is pulled out at random. Find the probability that the card shows a perfect square.

Ans : [Board Term-2 2016]

We have $S = \{1, 2..........80\}$

Number of possible outcomes,

$$n(S) = 80$$

Favourable outcome are $\{1, 4, 9, 16, 25, 36, 49, 64\}$

Number of favourable outcomes,

$$n(E) = 8$$

o181

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{80} = \frac{1}{10}$$

82. A bag contains 6 red and 5 blue balls. Find the probability that the ball drawn is not red.

Ans : [Board Term-2, 2014, 2015]

No. of possible outcomes,

$$n(S) = 6 + 5 = 11$$

o182

Since 5 blue balls are favourable outcome,

$$n(E) = 5$$

P(not red), $\quad P(E) = \frac{n(E)}{n(S)} = \frac{5}{11}$

83. There are 30 cards of the same size in a bag in which the numbers 1 to 30 are written. One card is taken out of the bag at random. Find the probability that the number on the selected card is not divisible by 3.

Ans : [Board Term-2 Foreign 2014]

Total cards $n(S) = 30$

Number divisible by 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27 and 30 i.e 10 numbers.

Number of favourable outcomes,

$$n(E) = 30 - 10 = 20$$

Required probability,

o183

$$P(E) = \frac{n(E)}{n(S)} = \frac{20}{30} = \frac{2}{3}$$

84. Two different dice are tossed together. Find the probability :

(i) that the number on each die is even.

(ii) that the sum of numbers appearing on the two dice is 5.

Ans : [Board Term-2 OD 2014]

In both case, $n(S) = 36$

(i) Even numbers events are (2, 2) (2, 4) (2, 6) (4, 2) (4, 4) (4, 6) (6, 2), (6,4) and (6, 6) which are 9 event.

$$n(E_1) = 9$$

P(number of each die is even),

o184

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) Sum of numbers is 5 in (1, 4) (2, 3) (3, 2) (4, 1)

$$n(E_2) = 4$$

P(sum of numbers appearing on two dice is 5)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

85. A letter of English alphabet is chosen at random, find the probability that the letter so chosen is :

(i) a vowel,

(ii) a consonant.

Ans : [Board Term-2 Delhi 2014]

Since total number in English alphabet is 26, in which 5 vowels and 21 consonants.

In both case total possible outcome

$$n(S) = 26$$

(i) a vowel,

$$n(E_1) = 5$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{5}{26}$$

o185

(ii) a consonant.

$$n(E_2) = 21$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{21}{26}$$

86. Harpreet tosses two different coins simultaneously. What is the probability that she gets :

(i). at least one head ?

(ii) one head and one tail ?

Ans : [Board Term-2 Foreign 2014]

All possible outcomes are {HH, TT, TH, HT}

$$n(S) = 4$$

(i) At least one head,

All favourable outcome are {HH, TH, HT}

o186

$$n(E_1) = 3$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{4}$$

(ii) One head and one tail

All favourable outcome are {TH, HT}

$$n(E_2) = 2$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

87. A bag contains cards bearing numbers from 11 to 30. A card is taken out from the bag at random. Find the probability that the selected card has multiple of 5 on it.

Ans : [Board Term-2 Delhi 2014, 2012]

No. of cards $n(S) = 20$

Multiples of 5 from 11 to 30 are 15, 20, 25 and 30 i.e 4 numbers .

Thus number of favourable outcomes,

o187

$$n(E) = 4$$

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{20} = \frac{1}{5}$$

88. A bag contains 5 red, 8 green and 7 white balls. One ball is drawn at random from the bag, find the probability of getting :
(i) not a white ball,
(ii) neither a green nor a red ball.

Ans : [Board Term-2, 2012, 2014]

Bag contains 5 red, 8 green and 7 white balls i.e. total 20 ball.

Total number of possible outcomes,

$$n(S) = 20$$

o188

(i) not a white ball,

There are 5 red and 8 green balls which are not white.

Thus number of favourable outcome,

$$n(E_1) = 13$$

P (not a white ball),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{20}$$

(ii) neither a green nor a red ball.

There are 7 white balls which are neither a green nor a red ball.

$$n(E_2) = 7$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{20}$$

89. Two dice are rolled simultaneously. Find the probability that the sum of numbers appearing is 10.

Ans : [Board Term-2 Foreign 2014]

When two dice are thrown, we have $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

Favourable outcomes are (4, 6), (6, 4) and (5, 5). In these outcomes, sum of numbers appearing is 10.

No. of favourable outcomes

$$n(E) = 3$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

o189

90. A bag contains 3 red, 4 green and 5 white candles, one candle is drawn at random from the bag, find the probability that candle is not red.

Ans : [Board Term-2 2014]

Total number of possible outcomes are $3 + 4 + 5 = 12$.

$$n(S) = 12$$

When candles not red, there are 9 possibilities,

$$n(E) = 9$$

P(candle is not red),

$$P(E) = \frac{n(E)}{n(S)} = \frac{9}{12} = \frac{3}{4}$$

o190

91. In a family of two children find the probability of having at least one girl.

Ans : [Board Term-2, 2012]

All possible outcomes,

$$S = \{GG, GB, BG, BB\}$$

o191

Total number of possible outcomes,

$$n(S) = 4$$

Favourable outcomes are GG, GB and BG.

Thus $n(E) = 3$

P(at least one girl),

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

92. Find the probability that a leap year has 53 Sundays

Ans : [Board Term-2, 2012]

366 days = 52 weeks + 2 days

2 days can be MT, TW, WTh, ThF, FS, SS, SM out of which SS and SM are favourable outcome.

Total number of possible outcomes,

$$n(S) = 7$$

Thus number of favourable outcome,

$$n(E) = 2$$

o192

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

93. Two dice, one blue and one grey, are thrown at the same time. What is the probability that the sum of the two numbers appearing on the top of the dice is

8 ?

Ans : [Board Term-2, 2012]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

We have 5 favourable outcomes are (2, 6), (3, 5), (4, 4), (5, 3), (6, 2).

$$n(E) = 5$$

o193

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

94. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of the red ball, find the number of blue balls in the bag.

Ans : [Board Term-2, 2012]

Let x be blue balls in bag.

Total balls $n(S) = 5 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

o194

P(red ball), $P(R) = \frac{n(R)}{n(S)} = \frac{5}{5+x}$

P(blue ball), $P(B) = \frac{n(B)}{n(S)} = \frac{x}{5+x}$

As per question we have

$$\frac{x}{5+x} = \frac{3 \times 5}{5+x}$$

Thus $x = 15$

95. Two coins are tossed together. Find the probability of getting both heads or both tails.

Ans : [Board Term-2, 2012]

Possibilities are HH, HT TH, TT out of which HH and TT are favourable.

$$n(S) = 4$$

$$n(E) = 2$$

o195

P(HH or TT), $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

96. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting :

(i) a non face card,

(ii) a black king.

Ans : [Board Term-2, 2012]

Total cards, $n(S) = 52$

o196

(i) There are 12 face cards and thus 40 non-face cards.

$$n(E_1) = 40$$

P(non-faces), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{40}{52} = \frac{10}{13}$

(ii) There are 2 black king

$$n(E_2) = 2$$

P(black king), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{52} = \frac{1}{26}$

97. Two dice are thrown together. What is the probability of getting a doublet ?

Ans : [Board Term-2, 2012]

When two dice are thrown, we have $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6). Thus we have 6 favourable outcomes.

$$n(E) = 6$$

P(a doublet), $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

o197

98. A lot consists of 144 ball pens of which 20 are defective and others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that :

(i) she will buy it ?

(ii) she will not buy it ?

Ans : [Board Term-2, 2012]

Total no. of pens, $n(S) = 144$

No. of good pen, $n(E) = 144 - 20 = 124$

Probability of purchasing pen,

$$P(E) = \frac{n(E)}{n(S)} = \frac{124}{144} = \frac{31}{36}$$

o198

Probability of not purchasing pen,

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{5}{36} = \frac{31}{36}$$

99. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of red ball, determine the number of blue balls in the bag.

Ans : [Board Term-2, 2012]

Let x be blue balls in bag.

Total balls, $n(S) = 5 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

o199

P(red ball), $P(R) = \frac{n(R)}{n(S)} = \frac{5}{5+x}$

P(blue ball), $P(R) = \frac{n(B)}{n(S)} = \frac{x}{5+x}$

As per question we have

$$\frac{x}{5+x} = \frac{2 \times 5}{5+x}$$

Thus $x = 10$

100. Two different dice are thrown together. Find the probability that the product of the number appeared is less than 18.

Ans : [Board Term-2 Foreign 2017]

There are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

o200

Favourable outcomes are (4, 2), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 3), (6, 1), (6, 2), (1, 1), (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (2, 5) and (4, 1).

No. of favourable outcomes,

$$n(E) = 26$$

P(Product appears is less than 18)

$$P(E) = \frac{n(E)}{n(S)} = \frac{26}{36} = \frac{13}{18}$$

101. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number of the drawn card is

(i) A perfect square number

(ii) A multiple of 7.

Ans : [Board Term-2 SQP 2017]

Total number of all possible outcomes,

$$n(S) = 113$$

(i) Perfect square numbers between 11 to 123 are 16, 25, 36, 49, 64, 81, 100 and 121.

No. of all favourable outcomes

$$n(E_1) = 8$$

o201

P(Number drawn is perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{8}{113}$$

(ii) No. of multiples of 7 from 11 to 123 is 16 i.e 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112 and 119.

No. of all favourable outcomes.

$$n(E_2) = 16$$

P(number drawn card is multiple of 7)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{113}$$

102. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag.

Ans : [Board Term-2 SQP 2017]

Let x be red balls in the box out of 12 balls.

$$P(R) = \frac{x}{12}$$

After putting 6 red balls in the bag, total numbers of balls in box is $12 + 6 = 18$ and red ball are $x + 6$.

$$P'(R) = \frac{x+6}{18}$$

According to the problem

o202

$$2 \times \frac{x}{12} = \frac{x+6}{18}$$

$$18x = 6x + 36 \Rightarrow x = 3$$

Hence there were 8 red ball.

103. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of blackballs in the bag.

Ans :

Let x be black balls and 15 white balls.

Total balls, $n(S) = 15 + x$

o203

Let $P(B)$ be the probability of drawing black ball and

$P(W)$ be the probability of drawing white ball.

Now $P(B) = 3 \times P(W)$

$$\frac{x}{(15+x)} = 3 \times \frac{15}{(15+x)}$$

$$x = 45$$

Thus there are 45 black balls in the bag.

THREE MARKS QUESTIONS

104. An integer is chosen between 70 and 100. Find the probability that it is
(i) a prime number (ii) divisible by 7

Ans : [Board 2020 SQP Standard]

There are 29 integer from 70 to 100. Total number of outcomes are 29 in both case.

$$n(S) = 29$$

(i) There are 6 prime numbers between 70 and 100 as 71, 73, 79, 83, 89 and 97 i.e. 6 favourable outcome.

$$n(E_1) = 6$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{29}$$

o204

(ii) There are 4 numbers between 70 and 100 which are divisible by 7 as 77, 84, 91 and 98 i.e. 4 favourable outcome.

$$n(B) = 4$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{29}$$

105. Find the probability that 5 Sundays occur in the month of November of a randomly selected year.

Ans : [Board 2020 Delhi Basic]

Total no. of days in November = 30

So, it has 4 weeks and 2 days. 4 weeks have 4 Sundays. The two remaining days should be

1. Sunday, Monday
2. Monday, Tuesday
3. Tuesday, Wednesday
4. Wednesday, Thursday
5. Thursday, Friday
6. Friday, Saturday
7. Saturday, Sunday

o205

Thus number of possible outcomes,

$$n(S) = 7$$

Number of favourable outcome,

$$n(E) = 2$$

So, the probability of getting 5 Sunday in the month of November,

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$$

106. Two dice are tossed simultaneously. Find the probability of getting
(i) an even number on both dice.
(ii) the sum of two numbers more than 9.

Ans : [Board 2020 OD Basic]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) an even number on both dice.

Favourable outcome are (2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4) and (6, 6).

Number of favourable outcomes

o206

$$n(E_1) = 9$$

P(an even number on both dice),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) sum of two numbers more than 9

Favourable outcome are (4, 6), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

Number of favourable outcomes

$$n(E_2) = 6$$

P (sum of two numbers more than 9),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

107. In a family of three children, find the probability of having at least two boys.

Ans : [Board 2020 OD Basic]

If there are three children in family all possible outcome are {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}.

So, the total number of outcomes,

$$n(S) = 2^3 = 8$$

At-least two of them are boys means all those cases in which we have either 2 or 3 boys. Thus favourable outcome are {BBB, BBG, BGB, GBB}

Number of favourable outcome,

o207

$$n(E) = 4$$

The probability of having at least two boys

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

108. A child has a die whose six faces show the letters as shown below:

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{D}$ $\boxed{E}$ $\boxed{A}$

The die is thrown once. What is the probability of getting (i) A, (ii) D?

Ans : [Board 2020 OD Standard]

Total possible outcomes, $n(S) = 6$

(i) Probability of getting letter A,

$$n(E_1) = 2.$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

o208

(ii) Probability of getting letter D,

$$n(E_2) = 1$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

109. A child has a die whose six faces show the letters as shown below:

$\boxed{A}$ $\boxed{A}$ $\boxed{B}$ $\boxed{C}$ $\boxed{C}$ $\boxed{C}$

The die is thrown once. What is the probability of getting (i) A, (ii) C?

Ans : [Board 2020 OD Standard]

o209

Total possible outcomes, $n(S) = 6$

(i) Probability of getting letter A,

$$n(E_1) = 2.$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

(ii) Probability of getting letter C,

$$n(E_2) = 3$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

110. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Ans : [Board 2019 Delhi Standard]

Possible outcomes are {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

Total possible outcomes,

$$n(S) = 2^3 = 8$$

o210

Number of outcomes where the game lost,

$$n(E) = 8 - 2 = 6$$

Probability of losing the game,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

111. A die is thrown once. Find the probability of getting a number which (i) is a prime number (ii) lies between 2 and 6.

Ans : [Board 2019 Delhi Standard]

Total outcomes $n(S) = 6$

(i) is a prime number

Prime numbers are 2, 3 and 5.

o211

$$n(E_1) = 3$$

P(prime no.),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) lies between 2 and 6

$$n(E_2) = 3$$

P(lies between 2 and 6),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

112. A die is thrown twice. Find the probability that
(i) 5 will come up at least once.
(ii) 5 will not come up either time.

Ans : [Board 2019 OD Standard]

There are $6 \times 6 = 36$ possible outcome. Thus sample space for two die is

$$n(S) = 36$$

(i) 5 will come up at least once

Favourable case are $(5,1), (5,2), (5,3), (5,4), (5,5), (5,6) (1,5), (2,5), (3,5), (4,5)$ and (6, 5)thus 11 case. Number of favourable outcome,

$$n(E_1) = 11$$

Probability that 5 will come up at least once,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{11}{36}$$

o212

(ii) 5 will not come up either time

Probability that 5 will come up either time

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{11}{36} = \frac{36 - 11}{36} = \frac{25}{36}$$

113. Two different dice are tossed together. Find the probability:
(i) of getting a doublet
(ii) of getting a sum 10, of the numbers on the two dice.

Ans : [Board 2018]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) of getting a doublet

Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6) which are 6 doublets.

Number of favourable outcomes,

o213

$$n(E_1) = 6$$

$$P(\text{doublet}), \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) of getting a sum 10, of the numbers on the two dice.

Favourable outcomes are (4, 6), (5, 5), (6, 4) i.e., 3.

Number of favourable outcomes,

$$n(E_2) = 3$$

$$P(\text{sum } 10), \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

114. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8.

(ii) not divisible by 8.

Ans : [Board 2018]

Total number of outcomes,

$$n(S) = 100 - 2 = 98$$

(i) divisible by 8.

Favourable outcomes are 8, 16, 24, ..., 96, i.e., 12.

Number of favourable outcomes,

$$n(E_1) = 12$$

$P(\text{Divisible by } 8)$,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{98} = \frac{6}{49}$$

o214

(ii) not divisible by 8.

$P(\text{not divisible by } 8)$,

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{6}{49} = \frac{43}{49}$$

115. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is

(i) a black king,

(ii) a card of red colour,

(iii) a card of black colour.

Ans : [Board Term-2 OD 2015]

There are total 52 cards out of which 6 cards are removed.

Total number of all possible outcomes,

$$n(S) = 52 - 6 = 46$$

o215

Number of black king,

$$n(E_1) = 2$$

(i) a black king,

Probability of drawing black king

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{46} = \frac{1}{23}$$

(ii) a card of red colour,

Total red card,

$$n(E_2) = 26 - 6 = 20$$

Probability of drawing red colour card

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{20}{46} = \frac{10}{23}$$

(iii) a card of black colour.

Total card of black colour,

$$n(E_3) = 26$$

Probability of drawing black colour card

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{26}{46} = \frac{13}{23}$$

116. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

Ans : [Board Term-2 2012]

Let x be blue balls.

Total balls $\quad n(S) = 6 + x$

$$n(R) = 5 \text{ and } n(B) = x$$

o216

$$P(\text{red ball}) \quad P(R) = \frac{n(R)}{n(S)} = \frac{6}{6+x}$$

$$P(\text{blue ball}) \quad P(R) = \frac{n(B)}{n(S)} = \frac{x}{6+x}$$

As per question we have

$$\frac{x}{6+x} = \frac{2 \times 6}{6+x} \Rightarrow x = 12$$

Thus there are 12 blue balls.

117. A bag contains cards numbered 1 to 49. Find the probability that the number on the drawn card is :

(i) an odd number

(ii) a multiple of 5

(iii) Even prime

Ans : [Board Term-2 2014]

Total cards, $n(S) = 49$

o217

(i) an odd number

Odd number are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total odd number, $n(E_1) = 25$

P(odd number), $P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{n(O)}{n(S)} = \dfrac{25}{49}$

(ii) a multiple of 5

Multiple of 5 are 5, 10, 15, 20, 25, 30, 35, 40 and 45.

Total multiple of 5 number,

$$n(E_2) = 5$$

P(multiple of 5), $P(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{9}{49}$

(iii) Even prime

Only 2 is even prime number. Therefore

$$n(E_3) = 1$$

P(even prime), $P(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{49}$

118. Two unbiased coins are tossed simultaneously. Find the probability of getting :

(i) at least one head,

(ii) almost one head,

(iii) no head.

Ans : [Board Term-2, 2012, 2014]

There are 4 possible outcome when two unbiased coins are tossed simultaneously.

Sample space $S = \{\text{HH, HT, TH, TT}\}$

o218

$$n(S) = 4$$

(i) at least one head,

Favourable outcomes are {HT, TH, HH}.

$$n(E_1) = 3$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{4}$$

(ii) almost one head,

Favourable outcomes are {HT, TH, HH}.

$$n(E_2) = 3$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{4}$$

(iii) no head.

Favourable outcomes is {TT} only.

$$n(E_3) = 1$$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{4}$$

119. Three different coins are tossed together. Find the probability of getting

(i) exactly two heads.

(ii) at least two heads

(iii) at least two tails.

Ans : [Board Term-2 OD 2016]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$$n(S) = 8$$

(i) Exactly two heads

Sample space $E_1 = \{\text{HHT, HTH, THH}\}$

o219

$$n(E_1) = 3$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{8}$$

(ii) At least two heads.

Sample space $E_2 = \{\text{HHT, HTH, THH, HHH}\}$

$$n(E_2) = 4$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii) At least two tails,

Sample space $E_3 = \{\text{TTH, THT, HTT, TTT}\}$

$$n(E_3) = 4$$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

120. A game consists of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e either all three heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

Ans : [Board Term-2 Foreign 2016, Delhi 2017]

There are 8 possible outcome when one coin is tossed three times : {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$$n(S) = 8$$

In the case of same result on all the tosses,

$$E = \{HHH, TTT\}$$

$$n(E) = 2$$

P(Ramesh will win the game)

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{8} = \frac{1}{4}$$

P(Ramesh will loose the game)

$$P(\overline{E}) = 1 - P(E)$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

121. In a single throw of a pair of different dice, what is the probability of getting
(i) a prime number on each dice ?
(ii) a total of 9 or 11 ?

Ans : [Board Term-2 Delhi 2016]

When two dice are thrown there are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

(i) a prime number on each dice ?

Favourable outcomes are (2, 2) (2, 3) (2, 5) (3, 2) (3, 3) (3, 5) (5, 2) (5, 3) and (5, 5) i.e. 9 outcomes.

$$n(E_1) = 9$$

P(a prime number on each die)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) a total of 9 or 11 ?

Favourable outcomes are (3, 6) (4, 5) (5, 4) (6, 3) (5, 6) and (6, 5) i.e. 6 outcomes.

$$n(E_1) = 6$$

P(a total of 9 or 11)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

122. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but 'Kewal' another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that :
(i) Ramesh will buy the selected shirt ?
(ii) Kewal will buy the selected shirt ?

Ans : [Board Term-2 Delhi 2016]

Since box consists of 100 shirts, there are 100 possible outcomes.

$$n(S) = 100$$

(i) Ramesh will buy the selected shirt ?

Number of good shirts

$$n(E_1) = 88$$

P(Ramesh buys the shirt)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{88}{100} = \frac{22}{25}$$

(ii) Kewal will buy the selected shirt ?

Number of shirts without major defect,

$$n(E_2) = 88 + 8 = 96$$

P(Kewal buys a shirt)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{96}{100} = \frac{24}{25}$$

123. A box contains 100 cards marked from 1 to 100. If one card is drawn at random from the box, find the probability that it bears :
(i) a single digit number
(ii) a number which is a perfect square
(iii) a number which is divisible by 7

Ans : [Board Term-2 2016]

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) a single digit number

Number of favourable outcomes,

$$n(E_1) = 9$$

P(single digit number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) a number which is a perfect square

Perfect square number are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

No. of favourable outcomes,

$$n(E_2) = 10$$

P(perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(iii) a number which is divisible by 7

Number divisible by 7 are 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98 i.e. 14 numbers.

No. of favourable outcomes,

$$n(E_3) = 14$$

P(a number divisible by 7),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{14}{100}$$

124. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card.

(i) is divisible by 9 and is a perfect square.

(ii) is a prime number greater than 80.

Ans : [Board Term-2 OD 2016]

Since bag consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) is divisible by 9 and is a perfect square.

Number divisible by 9 and perfect square are 9, 36 and 81 i.e. 3 numbers.

$$n(E_1) = 3$$

o224

Required probability,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{100}$$

(ii) is a prime number greater than 80.

Prime numbers greater than 80 and less than 100 are 83, 89 and 97 i.e 3 numbers.

$$n(E_2) = 3$$

Required probability,

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{100}$$

125. Cards numbered 2 to 101 are placed in a box. A card is selected at random from the box, find the probability that the card selected :

(i) has a number which is a perfect square.

(ii) has an odd number which is not less than 70.

Ans : [Board Term-2 2012]

Since box consists of 100 cards, there are 100 possible outcomes.

$$n(S) = 100$$

(i) has a number which is a perfect square.

Perfect squares are 4, 9, 16, 25, 36, 49, 64, 81 and 100.

Number of favourable outcomes,

$$n(E_1) = 9$$

o225

P(Perfect square),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{100}$$

(ii) has an odd number which is not less than 70.

Favourable outcomes are 71, 73, 75, ...101.

Number of favourable outcomes,

$$n(E_2) = 16$$

P(odd number not less than 70),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{16}{100} = \frac{4}{25}$$

126. All red face cards are removed from a pack of playing cards. The remaining cards are well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is :

(i) a red card

(ii) a face card

(iii) a card of clubs

Ans : [Board Term-2 Delhi 2015]

Since red face cards are removed, number of all possible outcomes are $52 - 6 = 46$

$$n(S) = 46$$

(i) a red card

No. of remaining red cards,

o226

$$n(E_1) = 26 - 6 = 20$$

$$P(\text{red card}), \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{46} = \frac{10}{26}$$

(ii) a face card

Number of remaining face cards,

$$n(E_2) = 12 - 6 = 6$$

P(a face card),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{46} = \frac{3}{23}$$

(iii) a card of clubs

Number of cards clubs

$$n(E_3) = 13$$

P(a card of clubs),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{13}{46}$$

127. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of ball in the jar.

Ans : [Board Term-2 OD 2015]

Probability of red ball, $P(R) = \frac{1}{4}$

Probability of blue ball, $n(B) = \frac{1}{3}$

o227

Probability of orange,

$$P(O) = 1 - [P(R) + P(B)]$$

$$= 1 - \left(\frac{1}{4} + \frac{1}{3}\right) = \frac{5}{12}$$

Now $$P(O) = \frac{n(O)}{n(S)}$$

$$\frac{5}{12} = \frac{10}{n(S)}$$

Total numbers of balls,

$$n(S) = \frac{10 \times 12}{5} = 24$$

128. Two different dice are thrown together. Find the probability of :
(i) getting a number greater than 3 on each die.
(ii) getting a total of 6 or 7 of the numbers on two dice.

Ans : [Board Term-2 Delhi 2016]

When two dice are thrown there are $6 \times 6 = 36$ possible outcomes.

$$n(S) = 36$$

(i) getting a number greater than 3 on each die.
Favourable outcomes are (4, 5), (4, 4), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5) and (6, 6).

No. of favourable outcomes,

o228

$$n(E_1) = 9$$

P(a number $>$ 3 on each die)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

(ii) getting a total of 6 or 7 of the numbers on two dice.
Favourable outcomes are (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).

No. of favourable outcomes $n(B) = 11$

P(a total of 6 to 7),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{11}{36}$$

129. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting
(i) Non face card,
(ii) Black king or a Red queen,
(iii) Spade card.

Ans : [Board Term-2 SQP 2016]

o229

Total cards $n(S) = 52$

(i) Non face card
Total number of non-face card,

$$n(E_1) = 52 - 12 = 40$$

P(non-face cards),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{40}{52} = \frac{10}{13}$$

(ii) Black king or a red queen,

Number of black kings = 2

Number of red queens = 2

Thus there are 4 favourable outcome.

$$n(E_2) = 4$$

P(a black Kind or a red queen),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(iii) Spade card
Number of spade cards,

$$n(E_3) = 13$$

P(Spade cards),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

130. Three coins are tossed simultaneously once. Find the probability of getting :
(i) at least one tail,
(ii) no tail.

Ans : [Board Term-2 2012]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$$n(S) = 2^3 = 8$$

(i) at least one tail,
Number of favourable outcomes,

o230

$$n(E_1) = 7$$

P(at least one tail),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{7}{8}$$

(ii) no tail.
Number of favourable outcomes,

$$n(E_2) = 1$$

P(no tail), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{8}$

131. A game consists of tossing a one-rupee coin three times and noting its outcome each time. Find the probability of getting :

(i) three heads,

(ii) at least two tails.

Ans : [Board Term-2 Foreign 2015]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

o231

$$n(S) = 2^3 = 8$$

(i) three heads,

Favourable outcome is {HHH} i.e. only one outcome.

Thus $n(E_1) = 1$

P(three heads), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$

(ii) at least two tails.

Favourable outcome are {TTT, TTH, THT, HTT}.

Number of favourable outcomes,

$$n(E_2) = 4$$

P (at least two tails), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

132. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :

(i) a red face card,

(ii) a spade,

(iii) either a king or a black cards.

Ans : [Board Term-2 2012, 2015]

Total cards, $n(S) = 52$

(i) Red face card

Total number of red-face card,

o232

$$n(E_1) = 6$$

P(red face cards)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{52} = \frac{3}{26}$$

(ii) Spade card

Number of spade cards

$$n(E_2) = 13$$

P(Spade cards),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(iii) Black king or a red queen,

Number of kings $= 4$

Number of black cards $= 26 - 2 = 24$

Thus there are 4 favourable outcome.

$$n(E_3) = 24 + 4 = 28$$

P(a black Kind or a red queen)

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{28}{52} = \frac{7}{13}$$

133. Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3 respectively. They are thrown and the sum of the numbers on them is noted. What is the probability of getting even sum :

Ans :

Total number of outcomes $= 6 \times 6 = 36$

Possible sum of two numbers on the two dice are 2, 3, 4, 5, 6, 7, 8, 9. i.e. outcomes favourable to event are (1, 1), (1, 1), (2, 2), (3, 1), (3, 1), (1, 3), (1, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1), (5, 3), (5, 3), (6, 2), (6, 2)

Hence, number of outcomes favourable to E is 18.

$$n(S) = 36$$

$$n(E) = 18$$

o233

Required probability,

$$P(E) = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

134. Three unbiased coins are tossed together. Find the probability of getting :

(i) at least two heads,

(ii) almost two heads.

Ans : [Board Term-2 2015]

Sample space for three coins tossed is {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

$$n(S) = 8$$

(i) Sample space for at least 2 heads is {HHH, HHT, HTH, THH}

Number of favourable outcomes,

o234

$$n(E_1) = 4$$

P(at least two heads).

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(ii) Sample space for almost two heads is {HHT, HTH, TTT, THH, THT, TTH, HTT}

Number of favourable outcomes,

$$n(E_2) = 7$$

P(almost 2 heads), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{8}$

135. A bag contains, white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag.

Ans : [Board Term-2 OD 2015]

We have $P(W) = \frac{3}{10}$

$$P(B) = \frac{2}{5}$$

o235

$$P(R) = 1 - \left(\frac{3}{10} + \frac{2}{5}\right) = \frac{3}{10}$$

Now $P(B) = \frac{n(B)}{n(S)}$

Substituting $P(B) = \frac{2}{5}$ and $n(B) = 20$ in above equation we have

$$\frac{2}{5} = \frac{20}{n(S)} \Rightarrow n(S) = \frac{20 \times 5}{2} = 50$$

Thus there are 50 total balls.

136. A bag contains 18 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, what is the probability that it is not red ?

(ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be $\frac{9}{8}$ times the probability of drawing a red ball in the first case. Find the value of x.

Ans : [Board Term-2 Foreign 2015]

Total ball, $n(S) = 18$

o236

Red ball $n(R) = x$

(i) not red

P(red ball), $P(R) = \frac{n(R)}{n(S)} = \frac{x}{18}$

P(no red ball),

$$P(\overline{R}) = 1 - \frac{x}{18} = \frac{18 - x}{18}$$

(ii) Now two more red balls are added.

Now total ball $n'(S) = 18 + 2 = 20$

There are total $x + 2$ red ball.

$$n'(R) = x + 2$$

P(red balls), $P'(R) = \frac{n'(R)}{n'(S)} = \frac{x+2}{20}$

Now, according to the question,

$$\frac{x+2}{20} = \frac{9}{8} \times \frac{x}{18}$$

$$180x = 144x + 288$$

$$36x = 288$$

$$x = \frac{288}{36} = 8$$

Now substituting $x = 8$ we have

$$P(\overline{R}) = \frac{18-8}{18} = \frac{10}{18} = \frac{5}{9}$$

137. Cards numbered 1 to 30 are put in a bag. A card is drawn at random. Find the probability that the drawn card is

(i) prime number > 7

(ii) not a perfect square

Ans : [Board Term-2, 2014]

We have 30 cards and thus there are 30 possible outcomes.

o237

$$n(S) = 30$$

(i) prime number > 7

Favourable outcomes are 11, 13, 17, 19, 23, 29. Thus number of favourable outcomes,

$$n(E_1) = 6$$

P(prime no. > 7) $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{30} = \frac{1}{5}$

(ii) not a perfect square

Favourable outcomes are 1, 4, 9, 16, 25. Thus number of favourable outcomes,

P(not a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{25}{30} = \frac{5}{6}$$

138. Two dice are thrown at the same time. Find the probability of getting :

(i) same number on both dice

(ii) sum of two numbers appearing on both the dice is 8.

Ans : [Board Term-2 2012]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) same number on both dice

Favourable outcome are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).

Thus number of favourable outcome

$$n(E_1) = 6$$

o238

P(Same number on both dice)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) sum of two numbers appearing on both the dice is 8.

Favourable outcome are (2, 6), (3, 5), (4, 4), (6, 2) and (5, 3). Thus number of favourable outcomes,

$$n(E_2) = 5$$

P(Sum is 8),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{36}$$

139. Five cards, ten, Jack, Queen, King and Ace of diamonds are well shuffled. One card is picked up from them.

(i) Find the probability that the drawn card is Queen.

(ii) If Queen is put aside, then find the probability that the second card drawn is an ace.

Ans : [Board Term-2 2014]

We have 5 cards and thus there are 5 possible outcomes.

$$n(S) = 5$$

(i) drawn card is queen

No. of favourable outcomes,

$$n(E_1) = 1$$

o239

P(queen), $\quad P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{1}{5}$

(ii) second card drawn is an ace

Since, queen was kept, number of all possible outcomes

$$n(S) = 5 - 1 = 4$$

Number of favourable outcomes

$$n(E_2) = 1$$

P(second card drawn is an ace),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{4}$$

140. From all the two digit numbers a number is chosen at random. Find the probability that the chosen number is a multiple of 7.

Ans : [Board Term-2 OD Compt. 2017]

All possible outcomes are 10, 11, 12 ...., 98 and 99.

No. of all possible outcomes

$$n(S) = 90$$

All favourable outcomes are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98 i.e 13 outcome.

No. of favourable outcomes

$$n(E) = 13$$

o240

P(getting a number multiple of 7),

$$P(E) = \frac{n(E)}{n(S)} = \frac{13}{90}$$

141. A box contains cards, number 1 to 90. A card is drawn at random from the box. Find the probability that the selected card bears a :

(i) Two digit number.

(ii) Perfect square number

Ans : [Board Term-2 Delhi Compt. 2017]

We have 90 cards and thus there are 90 possible outcomes.

$$n(S) = 90$$

(i) No. of cards having 2 digit number $90 - 9 = 81$.

Number of favourable outcomes,

$$n(E_1) = 81$$

P(selected card bears two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) Perfect square number between 1 to 90 are 1, 4, 9, 16, 25, 36, 49, 64 and 81 i.e. 9 numbers.

No. of favourable outcomes,

$$n(E_2) = 9$$

o241

P(perfect square numbers)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

142. Two different dice are thrown together. Find the

probability that the number obtained :
(i) have a sum less than 7.
(ii) have a product less than 16.
(iii) is a doublet of odd numbers.

Ans : [Board Term-2 Delhi 2017]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) have a sum less than 7.

Favourable outcome are (1, 1), (1, 2), (1, 3), (1, 4), (1,5) (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2) and (5, 1).

Number of favourable outcomes

o242

$$n(E_1) = 15$$

P(have sum less than 7),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(ii) have a product less than 16.

Favourable outcome are (1, 2), (1, 3), (1, 4), (1, 5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1) and (6, 2).

No. of favourable outcomes,

$$n(E_2) = 24$$

P(have a product less than 16),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{24}{36} = \frac{2}{3}$$

(iii) is a doublet of odd numbers.

Favourable outcome are (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3) and (5, 5).

No. of favourable outcomes,

$$n(E_3) = 9$$

P(a doublet of odd number),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

FOUR MARKS QUESTIONS

143. What is the probability that a randomly taken leap year has 52 Sundays?

Ans : [Board 2020 OD Standard]

Number of days in a leap year = 366

$$\text{Number of weeks} = \frac{366}{7} = 52.28$$

So, there will be 52 weeks and 2 days
So, every leap year has 52 Sundays
Now, the probability depends on remaining 2 days
The Possible pairing of days are
Sunday – Monday
Monday – Tuesday
Tuesday – Wednesday
Wednesday – Thursday
Thursday – Friday
Friday – Saturday
Saturday – Sunday

There are total 7 pairs and out of 7 pairs, only 2 pairs have Sunday. The remaining 5 pairs does not include Sunday.

$$n(S) = 7$$

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{7}$$

Therefore, the probability of only 52 Sunday in a Leap year is $\frac{5}{7}$.

144. Jayanti throws a pair of dice and records the product of the numbers appearing on the dice. Pihu throws 1 dice and records the squares the number that appears on it. Who has the better chance of getting the number 36? Justify?

Ans : [Board 2020 SQP Standard]

Jayanti throws two dice together. There are $6^2 = 36$ total number of possible outcomes.

$$n(S) = 36$$

She get 36 only when she gets (6, 6),

No. of favourable outcomes,

o243

$$n(E_1) = 1$$

P(getting the numbers of product 25)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{36}$$

Pihu throws one dice. There are 6 total number of all possible outcomes.

$$n(S) = 6$$

The number where square is 36 is 6.

No. of favourable outcomes,

$$n(E_2) = 1$$

P(getting a number whose square is 36)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

$$P(E_2) > P(E_1)$$

Hence, Pihu has better chances to the number square 36.

145. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the numbers 25.

Ans : [Board Term-2 Delhi 2017]

Peter throws two dice together. There are $6^2 = 36$ total number of possible outcomes.

$$n(S) = 36$$

He get 25 only when he gets (5, 5),

No. of favourable outcomes,

o244

$$n(E_1) = 1$$

P(getting the numbers of product 25),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Rina throws one dice. There are 6 total number of all possible outcomes.

$$n(S) = 6$$

The number where square is 25 is 5.

No. of favourable outcomes,

$$n(E_2) = 1$$

P(getting a number whose square is 25)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{6}$$

$$P(E_2) > P(E_1)$$

Hence, Rina has better chances to the number square 25.

146. The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is $\frac{1}{5}$. The probability of selecting a black marble at random from the same jar is $\frac{1}{4}$. If the jar contains 11 green marbles, find the total number of marbles in the jar.

Ans : [Board 2019 OD]

Let x and y be the number of blue and black marbles.

No of green marbles = 11

Total number of marbles $= x + y + 11$

According to the problem,

$$P(\text{black marbles}) = \frac{1}{4}$$

o245

$$\frac{y}{x+y+11} = \frac{1}{4}$$

$$x = 3y - 11 \qquad ...(1)$$

Again, $P(\text{blue marble}) = \frac{1}{5}$

$$\frac{x}{x+y+11} = \frac{1}{5}$$

$$5x = x + y + 11$$

$$x = \frac{y+11}{4} \qquad ...(2)$$

From equation (1) and (2), we have

$$3y - 11 = \frac{y+11}{4}$$

$$12y - 44 = y + 11$$

$$12y - y = 11 + 44$$

$$11y = 55 \Rightarrow y = 5$$

From equation (1) we have

$$x = 3 \times 5 - 11 = 4$$

Hence, total number of marbles in the jar,

$$x + y + 11 = 4 + 5 + 11 = 20$$

147. Cards marked with numbers 3, 4, 5, ...50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is :

(i) Divisible by 7.

(ii) A perfect square.

(iii) A multiple of 6.

Ans : [Board Term-2 SQP 2016]

We have 48 cards and thus there are 48 possible outcomes.

$$n(S) = 48$$

(i) Divisible by 7.

Number of cards divisible by 7 are 7, 14, 21, 35, 42 and 49.

No. of favourable outcomes,

o246

$$n(E_1) = 7$$

P(cards divisible by 7),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{7}{48}$$

(ii) A perfect square.

Number of cards having a perfect square are 4, 9, 16, 25, 36 and 49.

No. of favourable outcomes,

$$n(E_2) = 6$$

P(cards having a perfect square),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{48} = \frac{1}{8}$$

(iii) A multiple of 6.

Number of multiples of 6 from 3 to 50 are 6, 12, 24, 30, 36, 42, and 48.

No. of favourable outcomes,

$$n(E_3) = 6$$

P(multiple of 6 from 3 to 50),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{8}{48} = \frac{1}{6}$$

148. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards, after reshuffling them. Find the probability that the drawn card is

(i) of red colour
(ii) a queen
(iii) an ace
(iv) a face card.

Ans : [Board Term-2 OD 2015]

There are $52 - 6 = 46$ cards after removing black face cards. We have 46 cards and thus there are 48 possible outcomes.

$$n(S) = 46$$

o247

(i) red colour

Number of red cards, $n(E_1) = 26 - 6 = 20$

P(red colour), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{46} = \frac{10}{23}$

(ii) a queen

No. of queen, $n(E_2) = 4 - 2 = 2$

P(a queen), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{46} = \frac{1}{23}$

(iii) an ace

No. of ace, $n(E_3) = 4$

P(an ace), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{46} = \frac{2}{23}$

(iv) a face card

Number of face cards, $n(E_4) = 12 - 6 = 6$

P(a face card) $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{46} = \frac{3}{23}$

149. All the black face cards are removed from a pack of 52 cards. Find the probability of getting a

(i) face card
(ii) red card
(iii) black card
(iv) king

Ans : [Board Term-2 2014]

There are $52 - 6 = 46$ cards after removing black face cards. We have 46 cards and thus there are 48 possible outcomes.

$$n(S) = 46$$

o248

(i) face card

Number of red cards, $n(E_1) = 12 - 6 = 6$

P(face card), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{46} = \frac{3}{23}$

(ii) red card

Number of red card, $n(E_2) = 26$

P(red card), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{26}{46} = \frac{13}{23}$

(iii) black card

Number of black card, $n(E_3) = 26 - 6 = 20$

P(black card), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{20}{46} = \frac{10}{23}$

(iv) king

Number of king, $n(E_4) = 4 - 2 = 2$

P(king), $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{2}{46} = \frac{1}{23}$

150. A box contains 20 cards from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3

(ii) a prime number

Ans : [Board Term-2, 2015]

We have 20 cards and thus there are 20 possible outcomes.

$$n(S) = 20$$

o249

(i) divisible by 2 or 3

Number divisible by 2 or 3 are 6, 12, 18.

Number of favourable outcomes,

$$n(E_1) = 3$$

P(divisible by 2 or 3),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{20}$$

(ii) a prime number

Prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19 i.e 8 numbers.

Number of favourable outcomes,

$$n(E_2) = 8$$

P(a prime no.),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{8}{20} = \frac{2}{5}$$

151. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears,

(i) a one digit number.

(ii) a number divisible by 5.

(iii) an odd number less than 30.

(iv) a composite number between 50 and 70.

Ans : [Board Term-2 Foreign 2015]

We have $70 - 5 = 65$ cards and thus there are 65 possible outcomes.

$$n(S) = 65$$

o250

(i) a one digit number.

One digit numbers are 6, 7, 8 and 9.

Number of favourable outcomes

$$n(E_1) = 4$$

P(one digit number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{65}$$

(ii) a number divisible by 5.

Number divisible by 5 are 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65 and 70 i.e. 13 numbers.

Number of favourable outcomes,

$$n(E_2) = 13$$

P(a number divisible by 5),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{65} = \frac{1}{5}$$

(iii) an odd number less than 30.

Odd number less than 30 are 7, 9, 11, 13, 15, 17, 19 23, 25, 27 and 29.

Number of favourable outcomes,

$$n(E_3) = 12$$

P(a odd number less than 30),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{12}{65}$$

(iv) a composite number between 50 and 70

Composite number between 50 and 70 are 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68 and 69.

Number of favourable outcomes,

$$n(E_4) = 15$$

P(a composite number between 50 and 70)

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{15}{65} = \frac{3}{13}$$

152. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is :

(i) a card of spade or an ace.

(ii) a black king.

(iii) neither a jack nor a king.

(iv) either a king or a queen.

Ans : [Board Term-2 OD 2015]

We have 52 cards and thus there are 52 possible outcomes.

$$n(S) = 52$$

o251

(i) a card of spade or an ace

Cards of spade or an ace,

$$n(E_1) = 13 + 3 = 16$$

P(spade or an ace),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

(ii) a black king

Number of black kings,

$$n(E_2) = 2$$

P(a black king),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(iii) neither a jack nor a king

There are $4 + 4 = 8$ Jack or king.

Number of neither jack nor a king,

$$n(E_3) = 52 - 8 = 44$$

P(neither jack nor a king),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{44}{52} = \frac{11}{13}$$

(iv) either a king or a queen

There are $4 + 4 = 8$ king or queen.

$$n(E_4) = 8$$

P(either a king or a queen),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

153. A bag contains 15 balls of which x are blue and the remaining are red. If the number of red balls are increased by 5, the probability of drawing the red balls doubles. Find :

(i) P(red ball)

(ii) P(blue ball)

(iii) P(blue ball if of 5 extra red balls are actually added)

Ans : [Board Term-2, 2015]

Total ball, $n(S) = 15$

Blue ball $n(B) = x$

Red ball $n(R) = 15 - x$

o252

Now probability of drawing red ball,

$$P(R) = \frac{n(R)}{n(S)} = \frac{15 - x}{15} \quad ...(1)$$

If the number of red balls are increased by 5, i.e. total the probability of drawing the red balls doubles.

In this case, number of total ball,

$$n(S') = 15 + 5 = 20$$

and number of red ball,

$$n(R') = 15 - x + 5 = 20 - x.$$

Now in this case probability of drawing red ball,

$$P(R') = \frac{n'(R)}{n'(S)} = \frac{20 - x}{20}$$

According to the question, we have

$$P(R') = 2P(R)$$

$$\frac{20 - x}{20} = 2\left(\frac{15 - x}{15}\right)$$

$$1 - \frac{x}{20} = 2 - \frac{2x}{15}$$

$$\frac{2x}{15} - \frac{x}{20} = 2 - 1$$

$$\frac{8x - 3x}{60} = 1$$

$$5x = 60 \Rightarrow x = 12$$

(i) P(red ball)

$$P(R) = \frac{n(R)}{n(S)} = \frac{15 - 12}{15} = \frac{3}{15} = \frac{1}{5}$$

(ii) P(blue ball)

$$P(R) = \frac{n(B)}{n(S)} = \frac{12}{15} = \frac{4}{5}$$

(iii) P(blue ball if of 5 extra red balls are actually added)

$$P'(R) = \frac{n'(R)}{n'(S)} = \frac{3 + 5}{15 + 5} = \frac{8}{20} = \frac{2}{5}$$

154. Three digit number are made using the digits 4, 5, 9 (without repetition). If a number among them is selected at random, what is the probability that the number will :

(i) be a multiple of 5 ?

(ii) be a multiple of 9 ?

(iii) will end with 9 ?

Ans : [Board Term-2, 2014]

Total number of three digit numbers are 459, 495, 549, 594, 945 and 954. Thus we have 6 possible outcomes.

$$n(S) = 6$$

(i) be a multiple of 5

Multiple of 5 are 495 and 945.

$$n(E_1) = 2$$

P(multiple of 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

o253

(ii) be a multiple of 9

All are multiple of 9.

$$n(E_2) = 6$$

P(multiple of 9),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{6} = 1$$

(iii) will end with 9

Numbers 459 and 549 ends with 9.

$$n(E_3) = 2$$

P(ending with 9),

$$P(E_3) = \frac{n(E_2)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

155. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16.

Ans : [Board Term-2 OD 2016]

We have,

Total possible outcome are 1, 2, 3, 4, 4, 8, 9, 12, 16, 16, 18, 27, 32, 36 48 and 64 which are shown in following table.

$\times$	1	2	3	4
1	1	2	3	4
4	4	8	12	16
9	9	18	27	36
16	16	32	48	64

There are 16 possible outcomes,

$$n(S) = 16$$

Total favourable number having product less than 16 are 1, 2, 3, 4, 4, 8, 9 and 12.

Number of favourable outcomes

o254

$$n(E) = 8$$

P(product of x and y is less than 16),

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{16} = \frac{1}{2}$$

156. Two different dice are rolled together once. Find the probability of numbers coming on the tops whose product is a perfect square.

Ans : [Board Term-2 OD Compt. 2017]

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

Favourable outcome are (2, 2), (3, 3), (4, 4) (5, 5), (6, 6), (1, 1), (4, 1) and (1, 4).

Number of favourable outcomes

o255

$$n(E) = 8$$

P(product is a prefect square),

$$P(E) = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$$

157. A box contains 125 shirts of which 110 are good 12 have minor defects and 3 have major defects. Ram Lal will buy only those shirts which are good while Naveen will reject only those which have major defects. A shirt is taken out at random from the box. Find the probability that :

(i) Ram Lal will buy it

(ii) Naveen will buy it

Ans : [Board Term-2 OD 2017]

For both case total shirt,

o256

$$n(S) = 125$$

(i) Ram Lal will buy it

Ramlal will buy only a good shirt.

No. of all possible outcomes,

$$n(E_1) = 110$$

P(Ramlal will buy a shirt),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{110}{125} = \frac{22}{25}$$

(ii) Naveen will buy it

Naveen will reject the shirt which have major defects and will buy all other shirts.

No. of favourable outcomes,

$$n(E_2) = 125 - 3 = 122$$

P(Naveen will buy the shirt)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{122}{125}$$

158. The king, queen and jack of clubs are removed from a deck of 52 cards. The remaining cards are mixed together and then a card is drawn at random from it. Find the probability of getting

(i) a face card,

(ii) a card of heart,

(iii) a card of clubs

(iv) a queen of diamond

Ans : [Board Term-2 Delhi Compt. 2017]

There are $52 - 3 = 49$ cards in deck. Thus we have 44 possible outcomes.

$$n(S) = 49$$

o257

(i) a face card,

Number of face cards, $n(E_1) = 12 - 3 = 9$

P(a face card), $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{49}$

(ii) a card of heart,

No. of card of heart in the deck

$$n(E_2) = 13$$

P(a card of heart), $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{49}$

(iii) a card of clubs

Number of cards of clubs

$$n(E_3) = 13 - 3 = 10$$

P(a card of clubs), $P(E_3) = \frac{n(E_3)}{n(S)} = \frac{10}{49}$

(iv) a queen of diamond.

There is only one queen of diamond.

$$n(E_4) = 1$$

P(queen of diamond), $P(E_4) = \frac{n(E_4)}{n(S)} = \frac{1}{49}$

159. A box contains 90 discs which are numbered 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number,

(ii) number divisible by 5.

Ans : [Board Term-2 Foreign 2017]

Total number of discs in the box are 90.

Thus we have 90 possible outcomes.

$$n(S) = 90$$

(i) a two digit number,

Discs with two digit number are 10, 11, ...89 and 90 which are 81 numbers.

No. of favourable outcomes,

o258

$$n(E_1) = 81$$

P(a disc with two digit number)

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) number divisible by 5

The numbers divisible by 5 between 1 to 90 are 5, 10, 15 ..... 85 and 90 which are 18 numbers.

No. of favourable outcomes,

$$n(E_2) = 18$$

P (a disc with a number divisible by 5)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

160. Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product.

Ans :

There are 36 possible outcomes of rolling two dices.

$$n(S) = 36$$

(i) even sum

Favourable outcome are (1, 3), (1, 5), (1, 1), (2, 2), (2, 4), (2, 6), (3, 1) (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3) (5, 5) (6, 2), (6, 4) and (6, 6).

Number of favourable outcomes,

o259

$$n(E_1) = 18$$

P(even sum),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{18}{36} = \frac{1}{2} \text{ or } 0.5$$

(ii) even product

Favourable outcome are (1, 2), (1, 4), (1, 6), (2, 1), (2,2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2) (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5) and (6, 6).

Number of favourable outcomes

$$n(E_2) = 27$$

P(have a product less than 16),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{27}{36} = \frac{3}{4} = 0.75$$

Probability of getting even product is $\frac{3}{4}$ or 0.75.

161. From a deck of 52 playing cards, Jacks and kings of red colour and Queen and Aces of black colour are removed. The remaining cards are mixed and a card is drown at random. Find the probability that the drawn card is

(i) a black queen

(ii) a card of red colour

(iii) a Jack of black colour

(iv) a face card

Ans : [Board Term-2 OD Compt 2017]

There are $52 - (2 + 2 + 2 + 2) = 44$ cards in deck.

Thus we have 44 possible outcomes.

$$n(S) = 44$$

(i) a black queen

Number of black Queens in the deck,

$$n(E_1) = 0$$

P(getting a black queen),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{0}{44} = 0$$

Hence it is an impossible event

(ii) a card of red colour

Number of red cards,

$$n(E_2) = 26 - 4 = 22$$

P(getting a red card),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{22}{44} = \frac{1}{2}$$

(iii) a Jack of black colour

Number of Jacks (black),

$$n(E_3) = 2$$

P(getting a black coloured Jack),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{2}{44} = \frac{1}{22}$$

(iv) a face card

Number of face cards in the deck,

$$n(E_4) = 12 - 6 = 6$$

P(getting a face card),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{44} = \frac{3}{22}$$

162. Cards on which numbers 1, 2, 3 .... 100 are written (one number on one card and no number is repeated), put in a bag and are mixed thoroughly. A card is drawn at random from the bag. Find the probability that card taken out has

(i) an even number

(ii) a number which is a multiple of 13.

(iii) a perfect square number.

(iv) a prime number less than 20.

Ans : [Board Term-2 Delhi Compt 2017]

There are 100 cards in bags. Thus we have 100 possible outcomes.

$$n(S) = 100$$

(i) an even number

Even numbers 1 to 100 are 50.

Number of favourable outcomes,

$$n(E_1) = 50$$

P(an even number),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{50}{100} = \frac{1}{2}$$

(ii) A number which is a multiple of 13

Numbers multiples of 13, 26, 39, 52, 65, 78 and 91.

No. of favourable outcomes,

$$n(E_2) = 7$$

P(card taken out has multiple of 13),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{7}{100}$$

(iii) a perfect square number

Perfect square number in 1 to 100 are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.

No. of all favourable outcomes,

$$n(E_3) = 10$$

P(perfect square number),

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

(iv) a prime number less than 20

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

No. of all favourable outcomes,

$$n(E_4) = 8$$

P(prime number less than 20),

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{100} = \frac{2}{25}$$

163. A bag contains 20 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, find the probability that it is not red.

(ii) If 4 more red balls are out into the bag, the probability of drawing a red ball will be $\frac{5}{4}$ times the probability of drawing a red ball in the first case. Find the value of x.

Ans : [Board Term-2 Foreign 2015]

Total ball, $n(S) = 20$

Red ball $n(R) = x$

(i) not red

P(red ball), $P(R) = \frac{n(R)}{n(S)} = \frac{x}{20}$

P(no red ball),

$$P(\overline{R}) = 1 - \frac{x}{20} = \frac{20 - x}{20} \quad ...(1)$$

(ii) Now two more red balls are added.

Total ball $n'(S) = 20 + 4 = 24$

There are total $x+4$ red ball.

$$n'(R) = x+4$$

P(red balls), $$P'(R) = \frac{n'(R)}{n'(S)} = \frac{x+4}{24}$$

Now, according to the question,

$$\frac{x+4}{24} = \frac{5}{4} \times \frac{x}{20}$$

$$\frac{x+4}{24} = \frac{x}{16}$$

$$16x + 64 = 24x$$

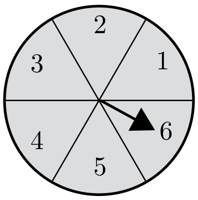
$$64 = 8x \Rightarrow x = 8$$

For first case, substituting $x = 8$ in equation (1) we have

$$P(\overline{R}) = \frac{20-8}{20} = \frac{12}{20} = \frac{3}{5}$$

164. In Figure a disc on which a player spins an arrow twice. The fraction $\frac{a}{b}$ is formed, where a is the number of sector on which arrow stops on the first spin and 'b' is the number of the sector in which the arrow stops on second spin, On each spin, each sector has equal chance of selection by the arrow.

Find the probability that the fraction $\frac{a}{b} > 1$



Ans : [Board Term-2 Foreign 2016]

For $\frac{a}{b} > 1$, when $a = 1$, b can not take any value.
For $a = 2$, b can take 1 value i.e. 1.
For $a = 3$, b can take 2 values, i.e. 1 and 2.
For $a = 4, b$ can take 3 values i.e. 1, 2, and 3.
For $a = 5, b$ can take 4 values i.e. 1, 2, 3 and 4.
For $a = 6, b$ can take 5 values i.e. 1, 2, 3, 4 and 5

Total possible outcomes,

$$n(S) = 36$$

o263

Favourable outcomes,

$$n(E) = 0+1+2+3+4+5 = 15$$

$p\left(\frac{a}{b} > 1\right)$, $$P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

165. A bag contains 25 cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is :

(i) divisible by 3 or 5

(ii) a perfect square number.

Ans : [Board Term-2, 2015)

Total cards $n(S) = 25$

(i) divisible by 3 or 5

Number divisible by 3 are 3, 6, 9, 12, 15, 16, 21, 24, and number divisible by 5 are 5, 10, 15, 20 and 25.

Thus number divisible by 3 or 5,

$$n(E_1) = 12$$

o264

P(divisible by 3 or 5),

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{12}{25}$$

(ii) a perfect square number.

Perfect square number are 1, 4, 9, 16 and 25.

$$n(E_2) = 5$$

P(a perfect square no.),

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{25} = \frac{1}{5}$$

166. A dice is rolled twice. Find the probability that :

(i) 5 will not come up either time.

(ii) 5 will come up exactly one time.

Ans : [Board Term-2 Delhi 2014]

When a dice is rolled twice, total number of outcomes,

$$n(S) = 6^2 = 36$$

There are 25 outcomes when 5 not come up either time.

Thus $n(E_1) = 25$

P(5 will not come up either time),

o265

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{25}{36}$$

(ii) 5 will come up exactly one time.

Possible outcomes are (1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6) and (6, 5).

$$n(E_2) = 10$$

P(5 will come up exactly one time)

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$