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రామునగర జిల్లా జుబాయయతో
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## మూగేఁదేళేళశరు :

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## గేంజ゙న్మృల むంతే :









Theme base marks distribution : 2019-20

| $\begin{aligned} & \text { SL. } \\ & \text { No. } \end{aligned}$ | THEMES | UNITS | MARKS |
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| 7 | (MENSURATION) SURFACE AREA AND VOLUME | AREA RELATED TO CIRCLES | 10 |
|  |  | SURFACE AREAS AND VOLUMES |  |
|  |  | Total | 80 |
|  | * Internal choice questions can be asked from the unit or from the theme. |  |  |











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## 

## VERY IMPORTANT FORMULAE / STATEMENTS

## ARITHMETIC PROGRESSIONS

1) General form of arithmetic progression $a,(a+d),(a+2 d), \ldots \ldots . a+(n-1) d$
2) $n^{\text {th }}$ term of arithmetic progression $a_{n}=a+(n-1) d$
3) $\mathrm{n}^{\text {th }}$ term from last of an AP is $=l-(\mathrm{n}-1) \mathrm{d}$
4) The relation between any two terms of an AP is $a_{p}=a_{q}+(p-q) d$
5) Common difference of $A P, d=\frac{a_{p}-a_{q}}{p-q}$ (when any 2 terms are given)
6) Common difference of AP, $d=a_{2}-a_{1}$ or $d=\frac{a_{n}-a}{n-1}$
7) The sum of first $n$ positive integers $S_{n}=\frac{n(n+1)}{2}$
8) The sum of first $n$ odd natural numbers $S_{n}=n^{2}$
9) The sum of first $n$ even natural numbers $S_{n}=n(n+1)$
10) Sum of first $n$ terms of an AP $S_{n}=\frac{n}{2}[2 a+(n-1) d]$
11) Sum of first $n$ terms of an AP $S_{n}=\frac{n}{2}\left[a+a_{n}\right]$ or $S_{n}=\frac{n}{2}[a+l]$
12) In any progression $S_{n}-S_{n-1}=a_{n}$
13) If $a, b, c$ are in AP, then the arithmetic mean between $a$ and $c$ is given by $b=\frac{a+c}{2}$

## SIMILAR TRIANGLES

14) In $\triangle A B C$ if $D E \| B C$ then


| Thales theorem | corollary of <br> Thales theorem | corollary of <br> Thales theorem |
| :---: | :---: | :---: |
| $\frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$ | $\frac{\mathrm{AD}}{\mathrm{AB}}=\frac{\mathrm{AE}}{\mathrm{AC}}=\frac{\mathrm{DE}}{\mathrm{BC}}$ | $\frac{\mathrm{DB}}{\mathrm{AB}}=\frac{\mathrm{EC}}{\mathrm{AC}}$ |

15) In the given fig. if $\triangle A B C \sim \triangle P Q R$ then

$$
\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^{2}=\left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^{2}=\left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^{2}=\left(\frac{\mathrm{AM}}{\mathrm{PN}}\right)^{2}
$$


16) In right angled $\triangle \mathrm{ABC}$ if $\angle \mathrm{ABC}=90^{\circ}$ then $A C^{2}=A B^{2}+B C^{2} \quad$ (Pythagoras Theorem)

17)

| Pythagorean <br> Triplets | Details | Pythagorean <br> Triplets | Details |
| :---: | :---: | :---: | :---: |
| $3,4,5$ | $3^{2}+4^{2}=5^{2}$ | $8,15,17$ | $8^{2}+15^{2}=17^{2}$ |
| $6,8,10$ | $6^{2}+8^{2}=10^{2}$ | $12,16,20$ | $12^{2}+16^{2}=20^{2}$ |
| $\mathbf{t} 5,12,13$ | $5^{2}+12^{2}=13^{2}$ | $10,24,26$ | $10^{2}+24^{2}=26^{2}$ |

## PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

18) Standard form of linear equation in one variable $a x+b=0$ (here $a \neq 0$ )
19) Standard form of linear equation in two variables $a x+b y+c=0$ (here $a^{2}+b^{2} \neq 0$ )
20) The general form for a pair of linear equations in two variables $x$ and $y$ is
$a_{1} x+b_{1} y+c_{1}=0 \quad$ Here $a_{1}, b_{1}, c_{1}, a_{2}, b_{2}, c_{2}$ are all real numbers
$\left.a_{2} x+b_{2} y+c_{2}=0\right\}$ and $\quad a_{1}{ }^{2}+b_{1}{ }^{2} \neq 0, \quad a_{2}{ }^{2}+b_{2}{ }^{2} \neq 0$
21) $a_{1} x+b_{1} y+c_{1}=0$
$a_{2} x+b_{2} y+c_{2}=0$ The lines represented by these equations

* Intersect if $\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$ and pair of equations has a unique solution.
* Coincide if $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$ and pair of equations has infinitely many solutions.
* Are parallel if $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$ and pair of equations has no solution.

22) For the pair of linear equations $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$ the equation used to find solutions by Cross Multiplication Method is

$$
\begin{aligned}
& \frac{x}{b_{1} c_{2}-b_{2} c_{1}}=\frac{y}{c_{1} a_{2}-c_{2} a_{1}}=\frac{1}{a_{1} b_{2}-a_{2} b_{1}} \\
& \text { OR } \\
& x=\frac{b_{1} c_{2}-b_{2} c_{1}}{a_{1} b_{2}-a_{2} b_{1}}, \quad y=\frac{c_{1} a_{2}-c_{2} a_{1}}{a_{1} b_{2}-a_{2} b_{1}}
\end{aligned}
$$

## CIRCLES

23) A line that intersects a circle at two distinct points is called a secant.
24) A line that intersects a circle at only one point is called a tangent.
25) The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.
26) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
27) The length of tangents drawn from an external point to a circle are equal.
28) The length of the tangent drawn from an external point at a distance of ' $d$ ' units from the center of the circle of radius ' $r$ ' is given by $t=\sqrt{d^{2}-r^{2}}$

## AREAS RELATED TO CIRCLES

29) Length of the circumference of a circle of radius $r=2 \pi r$
30) Length of the circumference of a circle of diameter $d=\pi d$
31) Area of a circle of radius $r=\pi r^{2}$ sq. units
32) Area of the quadrant of a circle of radius $r=\frac{1}{4} \pi r^{2}$ sq.units
33) Area of the sector of angle $\theta=\frac{\theta}{360} \times \pi r^{2}$ sq. units
34) Length of an arc of a sector of angle $\theta=\frac{\theta}{360} \times 2 \pi r$ units
35) Area of the segment of a circle $=($ Area of the corresponding sector - Area of the corresponding triangle).

## CONSTRUCTIONS

36) In the given fig. the angle between the tangents

$$
\angle A P B=180^{\circ}-\angle A O B
$$


37) In a circle the point of intersection of the perpendicular bisectors of two non-parallel chords is the center of the circle.

## COORDINATE GEOMETRY

38) Distance between the points $P\left(x_{1}, y_{1}\right)$ and $\mathrm{Q}\left(x_{2}, y_{2}\right)$ is $\mathrm{PQ}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(\mathrm{y}_{2}-\mathrm{y}_{1}\right)^{2}}$
39) Distance of a point $\mathrm{P}(x, y)$ from the origin is given by $\sqrt{x^{2}+y^{2}}$
40) The coordinates of the point $\mathrm{P}(x, y)$ which divides the line segment joining the points $\mathrm{A}\left(x_{1}, y_{1}\right)$ and $\mathrm{B}\left(x_{2}, y_{2}\right)$ internally in the ratio $\mathrm{m}_{1}: \mathrm{m}_{2}$ are $\left(\frac{\mathrm{m}_{1} x_{2}+\mathrm{m}_{2} x_{1}}{\mathrm{~m}_{1}+\mathrm{m}_{2}}, \frac{\mathrm{~m}_{1} y_{2}+\mathrm{m}_{2} y_{1}}{\mathrm{~m}_{1}+\mathrm{m}_{2}}\right)$
41) The coordinates of the mid-point of the line segment joining the points $\mathrm{P}\left(x_{1}, y_{1}\right)$ and $\mathrm{Q}\left(x_{2}, y_{2}\right)$ are $\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
42) Area of the triangle whose vertices are $\left(x_{1}, y_{1}\right),\left(x_{2}, y_{2}\right)$ and $\left(x_{3}, y_{3}\right)$ is

$$
\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right] \text { sq.units }
$$

## REAL NUMBERS

43) Euclid's Division Lemma : Given positive integers a and $b$, there exist unique integers q and r satisfying $a=b q+r, \quad 0 \leq r<b$.
44) For any two positive integers a and $\mathrm{b}, \operatorname{HCF}(a, b) \times \operatorname{LCM}(a, b)=a \times b$.
45) If $x=\frac{p}{q}$ (here $\mathrm{p}, \mathrm{q}$ are co-primes) is a rational number which has a terminating decimal expansion then q is of the form $2^{n} \times 5^{m}(\mathrm{n}, \mathrm{m}$ are non-negative integers )

## POLYNOMIALS

46) A polynomial of degree 1 is called a linear polynomial.
47) A polynomial of degree 2 is called a quadratic polynomial.
48) A polynomial of degree 3 is called a cubic polynomial.
49) The general form of a linear polynomial in $x$ is of the form $a x+b$, where $\mathrm{a}, \mathrm{b}$ are real numbers and $a \neq 0$
50) The general form of a quadratic polynomial in $x$ is of the form $a x^{2}+b x+c$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are real numbers and $a \neq 0$
51) The general form of a cubic polynomial in $x$ is of the form $a x^{3}+b x^{2}+c x+d$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ are real numbers and $a \neq 0$
52) The zero of a linear polynomial $a x+b$ is $=\frac{-\mathrm{b}}{\mathrm{a}}$
53) If $\alpha$ and $\beta$ are the zeros of the quadratic polynomial $a x^{2}+b x+c, a \neq 0$ then

* Sum of zeros $\alpha+\beta=\frac{-\mathrm{b}}{\mathrm{a}}$
* Product of zeros $\alpha \beta=\frac{\mathrm{c}}{\mathrm{a}}$

54) If $\alpha, \beta$ and $\gamma$ are the zeros of a cubic polynomial $a x^{3}+b x^{2}+c x+d$ then,

$$
\begin{array}{l|l|l}
\hline \alpha+\beta+\gamma=\frac{-\mathrm{b}}{\mathrm{a}} & \alpha \beta+\beta \gamma+\gamma \alpha=\frac{\mathrm{c}}{\mathrm{a}} & \alpha \beta \gamma=\frac{-\mathrm{d}}{\mathrm{a}} \\
\hline
\end{array}
$$

55) Quadratic polynomial with $\alpha$ and $\beta$ as its zeros is $x^{2}-(\alpha+\beta) x+\alpha \beta$.
56) Cubic polynomial with $\alpha, \beta$ and $\gamma$ as its zeros is

$$
x^{3}-(\alpha+\beta+\gamma) x^{2}+(\alpha \beta+\beta \gamma+\gamma \alpha) x-\alpha \beta \gamma
$$

57) A linear polynomial has only one zero.
58) A quadratic polynomial can have atmost 2 zeros.
59) A cubic polynomial can have atmost 3 zeros.
60) A cubic polynomial has at least one zero.
61) A polynomial of degree ' $n$ ' has atmost $n$ zeros.
62) The graph of a linear polynomial is a straight line.
63) The graph of a quadratic polynomial is called parabola.
64) In the quadratic polynomial $a x^{2}+b x+c$ if $a>0$, then its graph opens upwards.
65) In the quadratic polynomial $a x^{2}+b x+c$ if $a<0$, then its graph opens upwards.
66) In the quadratic polynomial $a x^{2}+b x+c \quad$ if $a=c$, then the zeros are reciprocals to each other .
67) Division Algorithm for polynomials is given by $\mathrm{p}(x)=\mathrm{g}(x) \mathrm{q}(x)+\mathrm{r}(x)$
68) In polynomial division: Divisor $\mathrm{g}(x)=\frac{\mathrm{p}(x)-\mathrm{r}(x)}{\mathrm{q}(x)}$

## QUADRATIC EQUATIONS

69) The standard form of a quadratic equation in $x$ is $a x^{2}+b x+c=0$ here a, b, c are real numbers and $a \neq 0$
70) Formula used to find the roots of a quadratic equation is $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
71) Discriminant of a quadratic equation is $b^{2}-4 a c$
72) A quadratic equation $a x^{2}+b x+c=0$ has

* two distinct real roots, if $\mathrm{b}^{2}-4 \mathrm{ac}>0$,
* two equal real roots, if $b^{2}-4 a c=0$,
* no real roots, if $b^{2}-4 a c<0$.

73) In quadratic equation $\mathrm{a}^{2}+\mathrm{b} x+\mathrm{c}=0$ if $\mathrm{b}^{2}-4 \mathrm{ac}=0$
then the roots of the equation are $x=\frac{-\mathrm{b}}{2 \mathrm{a}} \quad$ or $x=\frac{-\mathrm{b}}{2 \mathrm{a}}$
74) In quadratic equation $\mathrm{a} x^{2}+\mathrm{b} x+\mathrm{c}=0$ if $\mathrm{b}=0$ then the roots are additive inverse.
75) In quadratic equation $\mathrm{a} x^{2}+\mathrm{b} x+\mathrm{c}=0$ if $\mathrm{a}=\mathrm{c}$ then the roots are reciprocals to each other.
76) In quadratic equation $\mathrm{a} x^{2}+\mathrm{b} x+\mathrm{c}=0$ if $c=0$ then one of the roots will be zero.
77) In quadratic equation $\mathrm{a}^{2}+\mathrm{b} x+\mathrm{c}=0$ if $\frac{1}{2} \mathrm{~b}=\sqrt{\mathrm{ac}}$ then the roots are equal.

## INTRODUCTION TO TRIGONOMETRY

78) The trigonometric ratios of acute angle $\angle \mathrm{A}$ in the given right triangles .


| Trigonometric ratios of acute angle $\angle \mathbf{A}$ |  |  |
| :---: | :---: | :---: |
| $\sin \mathrm{A}=\frac{\mathrm{opp}}{\mathrm{hyp}}$ | $\cos \mathrm{A}=\frac{\operatorname{adj}}{\text { hyp }}$ | $\tan \mathrm{A}=\frac{\mathrm{opp}}{\mathrm{adj}}$ |
| $\operatorname{cosec} \mathrm{A}=\frac{\text { hyp }}{\mathrm{opp}}$ | $\sec \mathrm{A}=\frac{\text { hyp }}{\mathrm{adj}}$ | $\cot \mathrm{A}=\frac{\operatorname{adj}}{\mathrm{opp}}$ |
| Reciprocals of trigonometric ratios |  |  |
| $\sin \mathrm{A}=\frac{1}{\operatorname{cosec} \mathrm{~A}}$ | $\cos \mathrm{~A}=\frac{1}{\sec \mathrm{~A}}$ | $\tan \mathrm{~A}=\frac{1}{\cot \mathrm{~A}}$ |
| $\operatorname{cosec} \mathrm{~A}=\frac{1}{\sin \mathrm{~A}}$ | $\sec \mathrm{~A}=\frac{1}{\cos \mathrm{~A}}$ | $\cot \mathrm{~A}=\frac{1}{\tan \mathrm{~A}}$ |

79) $\tan \mathrm{A}=\frac{\sin \mathrm{A}}{\cos \mathrm{A}}$ and $\cot \mathrm{A}=\frac{\cos \mathrm{A}}{\sin \mathrm{A}}$
80) Trigonometric ratios of some specific angles

| $\angle \mathrm{A}$ | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin \mathrm{A}$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \mathrm{~A}$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan \mathrm{~A}$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | N.D |
| $\operatorname{cosec} \mathrm{A}$ | N.D | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| $\sec \mathrm{~A}$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | N.D |
| $\cot \mathrm{A}$ | N.D | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

## 81) Trigonometric Ratios of Complementary angles

$$
\begin{array}{lll}
\sin \left(90^{\circ}-A\right)=\cos A & \text { or } & \cos \left(90^{\circ}-A\right)=\sin A \\
\tan \left(90^{\circ}-A\right)=\cot A & \text { or } & \cot \left(90^{\circ}-A\right)=\tan A \\
\operatorname{cosec}\left(90^{\circ}-A\right)=\sec A & \text { or } & \sec \left(90^{\circ}-A\right)=\operatorname{cosec} A
\end{array}
$$

## Trigonomtric Identities

82) $\sin ^{2} A+\cos ^{2} A=1$
83) $1+\tan ^{2} \mathrm{~A}=\sec ^{2} \mathrm{~A} \quad$ or $\quad \sec ^{2} \mathrm{~A}-\tan ^{2} \mathrm{~A}=1$
84) $1+\cot ^{2} \mathrm{~A}=\operatorname{cosec}^{2} \mathrm{~A}$ or $\operatorname{cosec}^{2} \mathrm{~A}-\cot ^{2} \mathrm{~A}=1$
85) $\sin ^{2} A=1-\cos ^{2} A=(1+\cos A)(1-\cos A)$
86) $\cos ^{2} A=1-\sin ^{2} A=(1+\sin A)(1-\sin A)$
87) $\sin A=\sqrt{1-\cos ^{2} A}$
88) $\cos \mathrm{A}=\sqrt{1-\sin ^{2} \mathrm{~A}}$

## SOME APPLICATIONS OF TRIGONOMETRY

89) In right triangle ABC if $\angle \mathrm{A}=\theta$ is an acute angle then

* Heighth $=\tan \theta \times \mathrm{d}$
* distance $d=\cot \theta \times h$
* slant height $x=\frac{h}{\sin \theta}$ or $x=\frac{d}{\cos \theta}$



## STATISTICS

90) Mid-point or Class mark $=\frac{\text { Upper class limit }+ \text { Lower class limit }}{2}$
91) Formula to find the Mean of Grouped data

* Direct Method: Mean $\bar{x}=\frac{\sum f_{i} x_{i}}{\sum f_{i}}$
* Assumed Mean Method: Mean $\bar{x}=a+\frac{\sum f_{i} d_{i}}{\Sigma f_{i}}$
* Step-deviation Method: Mean $\bar{x}=a+\left(\frac{\sum f_{i} u_{i}}{\Sigma f_{i}}\right) \times h$

92) Formula to find the Mode of Grouped data

$$
\not \quad \text { Mode }=l+\left[\frac{f_{1}-f_{0}}{2 f_{1}-f_{0}-f_{2}}\right] \times h
$$

93) Formula to find the Median of Grouped data

$$
\text { * Median }=l+\left[\frac{\frac{n}{2}-\mathrm{cf}}{f}\right] \times h
$$

94) Empirical relationship between the three measures of central tendency :

* 3 Median $=$ Mode +2 Mean.
* Mode $=3$ Median -2 Mean.
* 2 Mean $=3$ Median - Mode.


## PROBABILITY

95) Probability of an event $P(E)=\frac{n(E)}{n(S)}$
$>$ here $\mathrm{n}(\mathrm{E})$ is number of outcomes favourable to E and
$>\mathrm{n}(\mathrm{S})$ is number of all possible outcomes.
96) The probability of a sure event (or certain event) is 1 .
97) The probability of an impossible event is 0.
98) The probability of an event, $\mathrm{P}(\mathrm{E})$ is a number such that $0 \leq \mathrm{P}(\mathrm{E}) \leq 1$.
99) An event having only one outcome is called an elementary event.
100) The sum of the probabilities of all the elementary events of an experiment is 1.
101) If $E$ and $(\bar{E})$ are complementary events, then $P(E)+P(\bar{E})=1$.
102) If a coin is tossed once then the sample space $S=\{H, T\} \quad \therefore n(S)=2$
103) If two coins are tossed simultaneously then the sample space

$$
\mathrm{S}=\{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\} \quad \therefore \mathrm{n}(\mathrm{~S})=4
$$

104) If three coins are tossed simultaneously then the sample space

$$
\mathrm{S}=\{\text { HHH, HHT }, \text { HTH, HTT, THH, THT, TTH, TTT }\} \quad \therefore \mathrm{n}(\mathrm{~S})=8
$$

105) If a die is thrown once then the sample space $S=\{1,2,3,4,5,6\} \therefore n(S)=6$
106) If two unbiased dice are thrown simultaneously then the total outcomes $n(S)=36$.

## SURFACE AREAS AND VOLUMES

107) Table containing the formulae used to find the surface areas and volumes of solids

| SOLIDS | LSA | TSA | VOLUME |
| :--- | :---: | :---: | :---: |
| CUBE | $4 \mathrm{a}^{2}$ | $6 \mathrm{a}^{2}$ | $a^{3}$ |
| CUBOID | $2 \mathrm{~h}(l+\mathrm{b})$ | $2(l \mathrm{~b}+\mathrm{bh}+\mathrm{h} l)$ | $l \times \mathrm{b} \times \mathrm{h}$ |
| CYLINDER | $2 \pi r \mathrm{~h}$ | $2 \pi r(\mathrm{~h}+\mathrm{r})$ | $\pi r^{2} \mathrm{~h}$ |
| CONE | $\pi r l$ | $\pi r(l+r)$ | $\frac{1}{3} \pi r^{2} \mathrm{~h}$ |
| SPHERE | $4 \pi r^{2}$ | $4 \pi r^{2}$ | $\frac{4}{3} \pi r^{3}$ |
| HEMISPHERE | $2 \pi r^{2}$ | $3 \pi r^{2}$ | $\frac{2}{3} \pi r^{3}$ |
| FRUSTUM <br> of a CONE | $\pi l\left(r_{1}+r_{2}\right)$ | $\pi l\left(r_{1}+r_{2}\right)+\pi\left(r_{1}{ }^{2}+r_{2}{ }^{2}\right)$ | $\frac{1}{3} \pi h\left(r_{1}{ }^{2}+r_{2}{ }^{2}+r_{1} r_{2}\right)$ |

108) Perimeter of the base of cyclinder / cone / hemisphere $=2 \pi r$
109) Slant height of a Cone $l=\sqrt{r^{2}+h^{2}}$
110) Slant height of frustum of a cone $l=\sqrt{h^{2}+\left(\mathrm{r}_{1}-\mathrm{r}_{2}\right)^{2}}$

## UNIT - 1 : ARITHMETIC PROGRESSION

1 Mark Questions (MCQ)

1) Which of the following is an AP ?
A) $2,4,8,16 \ldots \ldots \ldots \ldots$
B) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots \ldots \ldots \ldots \ldots$
C) $1,3,9,27$
D) $1,3,4,6 \ldots \ldots \ldots \ldots \ldots$
2) If $\mathrm{a}, \mathrm{b}$ and c are in AP , then $\frac{b-a}{c-b}$ is
A) $\frac{b}{a}$
B) 0
C) 1
D) 2 a
3) $\frac{3}{2}, \frac{1}{2},-\frac{1}{2},-\frac{3}{2}, \ldots \ldots \ldots \ldots$ Common difference of this AP
A) -1
B) $\frac{1}{2}$
C) $-\frac{1}{2}$
D) $\frac{2}{3}$
4) $10,7,4$, $\qquad$ $30^{\text {th }}$ term of this AP
A) 97
B) 77
C) -77
D) -87
5) $-3,-\frac{1}{2}, 2, \ldots \ldots \ldots \ldots \ldots 11^{\text {th }}$ term of this AP is
A) 28
B) 22
C) -38
D) $-48 \frac{1}{2}$
6) In an AP if $a_{n}=3+4 n$ then the value of $a_{3}$ is
A) 15
B) 9
C) 12
D) 13
7) In an AP if $S_{n}=4 n-n^{2}$ then d is
A) 2
B) 1
C) -2
D) -1
8) In an AP if $S_{5}=30$ and $S_{4}=20$ then $a_{5}$ is
A) 10
B) 50
C) 20
D) 9
9) If $a_{7}=6$ in an AP of 13 terms then the value of $S_{13}$ is
A) 42
B) 24
C) 87
D) 78
10) The sum of first 50 odd natural numbers is
A) 250
B) 500
C) 2500
D) 5000

## 1 Mark Questions (VSA)

11) Write the $\mathrm{n}^{\text {th }}$ term of an AP whose first term is a and common difference is d .
12) In an AP if $a_{n}=3+2 n$ then find $a_{4}$.
13) If $-3, a, 2$ are the three consecutive terms of an AP then find a.
14) $\sqrt{2}, \sqrt{8}, \sqrt{18}$, $\qquad$ write the next term of this AP.
15) Who is the famous mathematician who easily calculated the sum of first 100 natural numbers?
16) If $a, b$ and $c$ are in AP , then find $(c-b)-(b-a)$.

| Ans | 1) B | 2) C | 3) A | 4) C | 5) B | 6) A | 7) C | 8) A | 9) D | 10) C |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 11) $a_{n}=a+(n-1) d$ | 12) $a_{4}=11$ | 13) $-\frac{1}{2}$ | 14) $\sqrt{32}$ | 15) Gauss | $16) 0$ |  |  |  |  |
|  | Marks Questions (SA) |  |  |  |  |  |  |  |  |  |  |

17) In an AP $2,7,12$, $\qquad$ find the $10^{\text {th }}$ term.
Solution: In this AP $a=2$ and $d=a_{2}-a_{1}=7-2=5, a_{10}=$ ?
$a_{n}=a+(n-1) d$
$a_{10}=2+(10-1) 5$
$a_{10}=2+9 \times 5$
$a_{10}=2+45$
$\therefore a_{10}=47$
18) In an AP $21,18,15$, $\qquad$ find $35^{\text {th }}$ term .
19) In an AP $3,8,13$, $\qquad$ .find $10^{\text {th }}$ term .
20) In an AP 10, 7, 4, $\qquad$ find $30^{\text {th }}$ term .
21) In an AP $-3,-\frac{1}{2}, 2$, $\qquad$ find $11^{\text {th }}$ term.
22) In an AP 10, $7,4, \ldots . . . . . . . . . .-62$ find $11^{\text {th }}$ term from last.

Solution : In this AP $d=a_{2}-a_{1}=7-10=-3$ and $l=-62$
$11^{\text {th }}$ term from last $=$ ?
$\mathrm{n}^{\text {th }}$ term of an AP from last $=l-(\mathrm{n}-1) \mathrm{d}$
$11^{\text {th }}$ term of this AP from last $=-62-(11-1)(-3)$
$11^{\text {th }}$ term of this AP from last $=-62-(10)(-3)$
$11^{\text {th }}$ term of this AP from last $=-62+30$
$\therefore 11$ th term of this AP from last $=-32$
23) In an AP $3,8,13, \ldots . . . . . . . . . . . . ~ 253$ find $20^{\text {th }}$ term from last.
24) In an AP $21,18,15, \ldots . . . . . . . .-81$ find $28^{\text {th }}$ term from last .
25) Which term of the AP $21,18,15, \ldots . . . . . . .$. is $\mathbf{- 8 1}$ ?

Solution : Here $a=21$ and $d=a_{2}-a_{1}=18-21=-3, a_{n}=-81, n=$ ?
$a_{n}=a+(n-1) d$
$-81=21+(n-1)(-3)$
$-81=21-3 n+3$
$3 n=24+81$
$3 n=105$
$n=\frac{105}{3}$
$\therefore n=35$
$\therefore 35^{\text {th }}$ term of the given AP is -81 .
26) Which term of the AP $3,8,13, \ldots . . . . . . .$. is 78 ?
27) Which term of the AP $7,13,19$, is 205 ?
28) Find the sum of the AP $8,3,-2$, $\qquad$ .upto 22 terms .
Solution : $a=8, \quad S_{22}=$ ?
$\mathrm{d}=a_{2}-a_{1}=3-8=-5$
$\mathrm{S}_{\mathrm{n}}=\frac{n}{2}[2 a+(n-1) d]$
$S_{20}=\frac{22}{2}[2(8)+(22-1)(-5)]$
$S_{20}=11[16+21(-5)]$
$S_{20}=11[16-105]$
$S_{20}=11 \times-89$
$S_{20}=-979$
29) Find the sum of the AP $2,7,12, \ldots . . . . .$. upto 10 terms .
30) Find the sum of the AP $-37,-33,-29$,.. $\qquad$ upto 12 terms .

## 3 Marks Questions (LA-1)

31) In an AP if the first term is 38 and $16^{\text {th }}$ term is 73 then find its $31^{\text {st }}$ term.

Solution : Here $a_{1}=38$ and $a_{16}=73$ then $a_{31}=$ ?
In an AP $d=\frac{a_{p}-a_{q}}{p-q}$
$d=\frac{a_{16}-a_{1}}{16-1}$
$d=\frac{73-38}{15}=\frac{35}{15}=\frac{7}{3}$
$a_{31}=a_{1}+30 d$
$a_{31}=38+30\left(\frac{7}{3}\right)$
$a_{31}=38+70$
$a_{31}=108$
32) In an $A P$ if the $3^{\text {rd }}$ term is 12 and $50^{\text {th }}$ term is 106 , then find its $29^{\text {th }}$ term .
33) In an $A P$ if the $3^{\text {rd }}$ term is 4 and $9^{\text {th }}$ term is -8 , then find its $5^{\text {th }}$ term .
34) Find the sum of first $\mathbf{4 0}$ positive integers divisible by 6

Solution: $6,12,18, \ldots . . . . . . . . .240\left(\because 40^{\text {th }}\right.$ term is $\left.40 \times 6=240\right)$
$S_{n}=6+12+18+$ $\qquad$ $+240$
$S_{n}=6(1+2+3+$ $\qquad$ +40 )
$S_{n}=6\left[\frac{40 \times(40+1)}{2}\right]\left(\because\right.$ sum of first n natural numbers $\left.\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right)$
$S_{n}=6 \times 20 \times 41$
$S_{n}=120 \times 41$
$S_{n}=4920$
$\therefore$ the sum of first 40 positive integers divisible by 6 is 4920 .

## Alternate Method

Solution : 6, 12, 18, ............. (upto 40 terms )

$$
\begin{aligned}
& a=6, \quad S_{40}=? \\
& d=a_{2}-a_{1}=12-6=6 \\
& S_{n}=\frac{n}{2}[2 a+(n-1) d] \\
& S_{40}=\frac{40}{2}[2(6)+(40-1)(6)] \\
& S_{20}=20[12+39(6)] \\
& S_{20}=20[12+234] \\
& S_{20}=20 \times 246 \\
& S_{20}=4920
\end{aligned}
$$

$\therefore$ the sum of first 40 positive integers divisible by 6 is 4920 .
35) Find the sum of first 15 multiples of 8 .
36) Find the sum of multiples of $\mathbf{7}$ between $\mathbf{1 0 0}$ and 200 .

Solution: $S_{n}=105+112+119+$. $\qquad$ + 196
$S_{n}=7(15+16+17+$. $\qquad$ +28 )
$S_{n}=7\left[\frac{28 \times(28+1)}{2}-\frac{14 \times(14+1)}{2}\right]\left(\because\right.$ sum of first n natural numbers $\left.\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}(\mathrm{n}+1)}{2}\right)$
$S_{n}=7[14 \times 29-7 \times 15]$
$S_{n}=7[406-105]$
$S_{n}=7 \times 301$
$S_{n}=2107$
$\therefore$ the sum of multiples of 7 between 100 and 200 is 2107 .

## Alternate Method

Solution : $105+112+119+$ $\qquad$ $+196$
$a=105, \quad l=a_{n}=196, \quad S_{n}=$ ?
$d=a_{2}-a_{1}=112-105=7$
$a_{n}=a+(n-1) d$
$196=105+(n-1) 7$
$196-105=7 n-7$
$91+7=7 n$
$98=7 n$
$n=14$
$S_{n}=\frac{n}{2}[a+l]$
$S_{14}=\frac{14}{2}[105+196]$
$S_{14}=7[301]$
$S_{14}=2107$
$\therefore$ the sum of multiples of 7 between 100 and 200 is 2107 .
37) Find the sum of odd numbers between 0 and 50 .
38) Find the sum of the terms from 10 th term to 20 th term of AP $3,7,11,15$
39) If 10 times the $10^{\text {th }}$ term of an AP is equal to 15 times the $15^{\text {th }}$ term, find the $25^{\text {th }}$ term.
Solution: In an AP $10 \times a_{10}=15 \times a_{15}$ (given)
$10[a+(10-1) d]=15[a+(15-1) d] \quad\left[\because a_{n}=a+(n-1) d\right]$
$10(a+9 d)=15(a+14 d)$
$10 a+90 d=15 a+210 d$
$10 a-15 a=210 d-90 d$

$$
\begin{aligned}
& -5 a=120 d \\
& a=-\frac{120}{5} d \\
& a=-24 d \\
& a_{25}=a+(25-1) d \\
& a_{25}=a+24 d \\
& a_{25}=-24 d+24 d \quad(\because \text { from eqn. }) \\
& a_{25}=0
\end{aligned}
$$

40) The $8^{\text {th }}$ term of an AP is zero. Prove that its $38^{\text {th }}$ term is triple its $18^{\text {th }}$ term.

Solution: In an AP $a_{8}=0$ ( $\because$ given)

$$
\begin{array}{ll}
a_{18}=a_{8}+10 d & \left(\because \mathrm{a}_{\mathrm{p}}=\mathrm{a}_{\mathrm{q}}+(\mathrm{p}-\mathrm{q}) \mathrm{d}\right) \\
a_{18}=0+10 d & \\
a_{18}=10 d & --->(1) \\
a_{38}=a_{18}+20 d & \left(\because \mathrm{a}_{\mathrm{p}}=\mathrm{a}_{\mathrm{q}}+(\mathrm{p}-\mathrm{q}) \mathrm{d}\right) \\
a_{38}=10 d+20 d & (\because \text { from eqn. } 1) \\
a_{38}=30 d & \\
a_{38}=3 \times 10 d & \\
a_{38}=3 \times a_{18} & (\because \text { from eqn. } 1)
\end{array}
$$

$\therefore$ Hence the $38^{\text {th }}$ term is triple its $18^{\text {th }}$ term
41) If $S_{n}$, the sum of first $n$ terms of an AP is given by $S_{n}=\left(3 n^{2}-4 n\right)$, then find its $25^{\text {th }}$ term.
Solution: In an AP $\mathrm{S}_{\mathrm{n}}=\left(3 n^{2}-4 n\right)$ and $\mathrm{a}_{25}=$ ?
$\mathrm{S}_{25}=\left(3 \times 25^{2}-4 \times 25\right)$
$\mathrm{S}_{25}=(1875-100)$
$\mathrm{S}_{25}=1775 \quad----->(1)$
$S_{24}=\left(3 \times 24^{2}-4 \times 24\right)$
$\mathrm{S}_{24}=(1728-96)$
$\mathrm{S}_{24}=1632 \quad----->(2)$
$\mathrm{S}_{25}-\mathrm{S}_{24}=\mathrm{a}_{25} \quad\left(\because \mathrm{~S}_{\mathrm{n}}-\mathrm{S}_{\mathrm{n}-1}=\mathrm{a}_{\mathrm{n}}\right)$
$\mathrm{S}_{25}-\mathrm{S}_{24}=1775-1632$
$\therefore \mathrm{a}_{25}=143$

## 4 Marks Questions (LA-2)

42) The $7^{\text {th }}$ term of an AP is four times the $2^{\text {nd }}$ term and the $12^{\text {th }}$ term exceeds thrice the $4^{\text {th }}$ term by 2 . Find the progression.
43) Four numbers are in AP. The sum of extremes is 10 and the product of the means is 24 . Find the numbers.
44) The sum of $4^{\text {th }}$ and $8^{\text {th }}$ terms of an AP is 24 and the sum of its $6^{\text {th }}$ and $10^{\text {th }}$ terms is 44. Find the sum of first three terms of the AP.
45) The sum of three positive integers in $A P$ is 24 and their product is 480 . Find the numbers.
46) Find three numbers in AP whose sum is 24 and the sum of whose squares is 224 .
47) Find the sum of first 10 terms of the AP whose $12^{\text {th }}$ term is -13 and the sum of first 4 terms is 24 .
48) Divide 32 in four parts such that they are in AP and the ratio of product of extremes to the product of means is $7: 15$. Find the numbers.

5 Marks Questions (LA-3)
49) Show that the sum of the first $n$ even natural numbers is equal to $\left(1+\frac{1}{n}\right)$ times the sum of the first $n$ odd natural numbers.
Solution: Let $\mathrm{S}_{1}$ denotes the sum of the first $n$ even natural numbers.
$\mathrm{S}_{1}=2+4+6+$ $\qquad$ up to n terms
$a=2, \quad d=4-2=2, \quad n=n$
$\mathrm{S}_{1}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\mathrm{S}_{1}=\frac{\mathrm{n}}{2}[2 \times 2+(\mathrm{n}-1) 2]$
$\mathrm{S}_{1}=\frac{2 \mathrm{n}}{2}[2+\mathrm{n}-1]$
$\mathrm{S}_{1}=\mathrm{n}(\mathrm{n}+1) \quad \quad--->(1)$
Let $S_{2}$ denotes the sum of the first $n$ odd natural numbers.
$\mathrm{S}_{2}=1+3+5+$ $\qquad$ up to n terms
$a=1, \quad d=3-1=2, \quad n=n$
$S_{2}=\frac{n}{2}[2 a+(n-1) d]$
$\mathrm{S}_{2}=\frac{\mathrm{n}}{2}[2 \times 1+(\mathrm{n}-1) 2]$
$S_{2}=\frac{2 n}{2}[1+n-1]$
$\mathrm{S}_{2}=\mathrm{n}^{2}$
$\frac{\mathrm{S}_{1}}{\mathrm{~S}_{2}}=\frac{\mathrm{n}(\mathrm{n}+1)}{\mathrm{n}^{2}} \quad[\because$ from eqn.(1) eqn. (2) $]$
$\frac{\mathrm{S}_{1}}{\mathrm{~S}_{2}}=\frac{(\mathrm{n}+1)}{\mathrm{n}}$
$\frac{\mathrm{S}_{1}}{\mathrm{~S}_{2}}=\left(\frac{\mathrm{n}}{\mathrm{n}}+\frac{1}{\mathrm{n}}\right)$
$\frac{\mathrm{S}_{1}}{\mathrm{~S}_{2}}=\left(1+\frac{1}{\mathrm{n}}\right)$
Hence, $\mathrm{S}_{1}=\left(1+\frac{1}{\mathrm{n}}\right) \mathrm{S}_{2}$
50) Find the sum of all natural numbers less than 1000 which are neither divisible by 2 nor by 5 .
51) If the ratio of the sum of $m$ terms and $n$ terms of an AP be $m^{2}: n^{2}$, prove that the ratio of its $m$ - th and $n$ - th term is $(2 m-1):(2 n-1)$.

## UNIT - 2 : TRIANGLES

1 Mark Questions (MCQ)

1) In the adjoining figure if $D E \| B C$, then the value of $x$ is
A) 3.36 cm
B) 4.34 cm
C) 7.41 cm
D) 4.66 cm

2) If perimeters of two similar triangles are in the ratio $5: 4$ then the ratio of their corresponding sides is ,
A) $5: 4$
B) $4: 5$
C) $10: 2$
D) $2: 10$
3) The sides of a triangle are $3,4,6$ units. The corresponding sides of another triangle similar to this are (in units )
A) $8,6,12$
B) $9,12,18$
C) $8,4,9$
D) $2,4 \frac{1}{2}, 4$
4) At a certain time of the day, a man 6 feet tall, casts his shadow 8 feet long. At the same time the length of the shadow cast by a building 45 feet high is
A) 90
B) 60
C) 48
D) 54
5) The ratio of corresponding sides of two similar triangles is 2: 1 . The ratio of their areas is
A) $2: 1$
B) $4: 2$
C) $4: 1$
D) $1: 4$
6) If the ratio of areas of two similar triangles is $16: 81$ then the ratio of their corresponding sides is
A) $2: 3$
B) $7: 9$
C) $4: 9$
D) $81: 61$
7) Among the following measures which one does not represent the sides of a right triangle?
A) $9,12,15$
B) $3,4,5$
C) $2,1, \sqrt{5}$
D) $5,7,9$
8) The side of a square is 12 cm .The length of its diagonal is
A) 12 cm
B) $12 \sqrt{2} \mathrm{~cm}$
C) $\sqrt{12} \mathrm{~cm}$
D) $\sqrt{2} \mathrm{~cm}$
9) The length of the diagonal of a square is $\sqrt{50} \mathrm{~m}$. Its side is
A) $\sqrt{10} \mathrm{~m}$
B) $5 \sqrt{2} \mathrm{~m}$
C) $2 \sqrt{5} \mathrm{~m}$
D) 5 m
10) In $\triangle \mathrm{ABC}$ if $\mathrm{AB}=6 \sqrt{3}, \quad \mathrm{AC}=12 \mathrm{~cm}$ and $\mathrm{BC}=6 \mathrm{~cm}$ then $\angle \mathrm{B}$ is
A) $120^{\circ}$
B) $60^{\circ}$
C) $90^{\circ}$
D) $45^{\circ}$
11) Height of an equilateral triangle whose side is 2 a units is
A) $\sqrt{3} a$ units
B) $\sqrt{3}$ units
C) $3 \sqrt{a}$ units
D) $\sqrt{2}$ a units 1 Mark Questions (VSA)
12) State Thales Theorem.
13) State Pythagoras Theorem.
14) State Converse of Pythagoras Theorem.
15) Write the two conditions for two polygons of same number of sides to be similar.
16) State Converse of Thales Theorem.
17) The areas of two similar triangles are in the ratio $4: 9$. Find the ratio of their corresponding medians.

| Ans | $1) \mathrm{D}$ | $2) \mathrm{A}$ | $3) \mathrm{B}$ | $4) \mathrm{B}$ | $5) \mathrm{C}$ | $6) \mathrm{C}$ | $7) \mathrm{D}$ | $8) \mathrm{B}$ | $9) \mathrm{D}$ | $10) \mathrm{C}$ | $11) \mathrm{A}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

12) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other sides are divided in the same ratio.
13) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
14) In a triangle if square of one side is equal to the sum of the squares of the other two sides , then the angle opposite to the first side is a right angle.
15) All the corresponding angles of polygons are equal and all the corresponding sides are in the same proportion.
16) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
17) $2: 3$

## 2 Marks Questions (SA)

18) In the given fig. if $D E \| B C$ then find EC.

Solution: In $\triangle A B C$ DE || $B C$
$\mathrm{AD}=1.5 \mathrm{~cm}, \quad \mathrm{DB}=3 \mathrm{~cm}$ and $\mathrm{AE}=1 \mathrm{~cm}$
$\frac{A D}{D B}=\frac{A E}{E C} \quad$ (Thales theorem)
$\frac{1.5}{3}=\frac{1}{\mathrm{EC}}$
$\frac{15}{30}=\frac{1}{\mathrm{EC}}$
$\frac{1}{2}=\frac{1}{\mathrm{EC}}$
$\mathrm{EC}=2 \mathrm{~cm}$
19) In the given fig. if $D E \| B C$, then find $A D$.

20) In $\triangle \mathrm{ABC} \quad \mathrm{DE} \| \mathrm{BC}, \mathrm{AD}=\mathrm{x}+1, \quad \mathrm{DB}=\mathrm{x}-1$
$A E=x+3$ and $E C=x$, find $x$.

21) In the given fig. if $A Q \perp A B, P B \perp A B, A D=20 \mathrm{~cm}$ $B D=12 \mathrm{~cm}$ and $P B=18 \mathrm{~cm}$ then find $A Q$.

22) In the fig. if $\frac{\mathrm{PS}}{\mathrm{SQ}}=\frac{\mathrm{PT}}{\mathrm{TR}}$ and $\angle \mathrm{PST}=\angle \mathrm{PRQ}$ then prove that $\triangle \mathrm{PQR}$ is an isosceles triangle.

23) A vertical pole of height 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.
24) Let $\triangle \mathrm{ABC} \sim \Delta \mathrm{DEF}$ and their areas be respectively, $64 \mathrm{~cm}^{2}$ and $121 \mathrm{~cm}^{2}$. If $E F=15.4 \mathrm{~cm}$, find $B C$.

Solution : $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$

$$
\begin{aligned}
& \frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{DEF})}=\left(\frac{\mathrm{BC}}{\mathrm{EF}}\right)^{2} \quad[\text { Theorem ] } \\
& \frac{64}{121}=\left(\frac{\mathrm{BC}}{15.4}\right)^{2} \\
& \left(\frac{8}{11}\right)^{2}=\left(\frac{\mathrm{BC}}{15.4}\right)^{2} \\
& \frac{8}{11}=\frac{\mathrm{BC}}{15.4} \\
& 11 \times \mathrm{BC}=15.4 \times 8 \\
& \mathrm{BC}=\frac{15.4 \times 8}{11} \\
& \mathrm{BC}=1.4 \times 8=11.2 \mathrm{~cm}
\end{aligned}
$$

25) In the fig. if $\mathrm{DE} \| \mathrm{BC}$ and $\mathrm{AD}: \mathrm{DB}=5: 4$, then find the ratios of areas of $\triangle \mathrm{ADE}$ and $\triangle \mathrm{ABC}$.

26) Diagonals of a trapezium $A B C D$ with $A B \| C D$ intersect each other at the point 0 . If $A B=2 C D$, find the ratio of the areas of $\triangle A O B$ and $\Delta$ COD .
Solution: In trapezium $A B C D, A B \| D C$
and diagonals intersect at 0 .
$\Rightarrow \mathrm{AB}=2 \mathrm{CD} \quad \frac{\mathrm{AB}}{\mathrm{CD}}=\frac{2}{1}--->$


In $\triangle$ AOB and $\triangle$ COD
$\angle \mathrm{A}=\angle \mathrm{C} \quad$ [alternate angles ]
$\angle \mathrm{B}=\angle \mathrm{D} \quad$ [alternate angles ]
$\Delta \mathrm{AOB} \sim \Delta \mathrm{COD}$ [AA similarity criterion]

$$
\begin{aligned}
& \frac{\operatorname{ar}(\triangle \mathrm{AOB})}{\operatorname{ar}(\triangle \mathrm{COD})}=\left(\frac{\mathrm{AB}}{\mathrm{CD}}\right)^{2}[\text { Theorem on areas }] \\
& \frac{\operatorname{ar}(\triangle \mathrm{AOB})}{\operatorname{ar}(\triangle \mathrm{COD})}=\left(\frac{2}{1}\right)^{2} \\
& \frac{\operatorname{ar}(\triangle \mathrm{AOB})}{\operatorname{ar}(\triangle \mathrm{COD})}=\frac{4}{1}
\end{aligned}
$$

27) Diagonals $A C$ and $B D$ of a trapezium $A B C D$ with $A B \| C D$ intersect each other at the point 0 .


Prove that $\frac{O A}{O C}=\frac{O B}{O D}$.
28) In $\triangle A B C, D$ is a point on $B C$ such that $\angle \mathrm{ADC}=\angle \mathrm{BAC}$. Prove that $\mathrm{CA}^{2}=\mathrm{CB} . \mathrm{CD}$


3 Marks Questions (LA -1)
29) Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.
Solution: In rhombus $A B C D, A B=B C=C D=A D[\because$ sides of rhombus are equal $]$ Diagonals AC and BD intersect perpendicularly at 0 .

$$
\angle \mathrm{AOB}=90^{\circ}
$$

In right $\triangle \mathrm{AOB}$
$\mathrm{AB}^{2}=\mathrm{AO}^{2}+\mathrm{BO}^{2} \quad[\because$ Pythagoras theorem $]$
$\mathrm{AB}^{2}=\left(\frac{1}{2} \mathrm{AC}\right)^{2}+\left(\frac{1}{2} \mathrm{BD}\right)^{2} \quad\left[\because \mathrm{AO}=\frac{1}{2} \mathrm{AC}, \quad \mathrm{BO}=\frac{1}{2} \mathrm{BD}\right]$

$A B^{2}=\frac{1}{4} A C^{2}+\frac{1}{4} \mathrm{BD}^{2}$
$4 \mathrm{AB}^{2}=\mathrm{AC}^{2}+\mathrm{BD}^{2} \quad[\because$ Multiplying by 4$]$
$\Rightarrow \mathrm{AB}^{2}+\mathrm{AB}^{2}+\mathrm{AB}^{2}+\mathrm{AB}^{2}=\mathrm{AC}^{2}+\mathrm{BD}^{2}$
$\therefore \mathrm{AB}^{2}+\mathrm{BC}^{2}+\mathrm{CD}^{2}+\mathrm{DA}^{2}=\mathrm{AC}^{2}+\mathrm{BD}^{2}[\because$ given $\mathrm{AB}=\mathrm{BC}=\mathrm{CD}=\mathrm{AD}]$.
30) Two poles of heights 6 m and 11 m stand on a play ground. If the distance between the feet of the poles is 12 m , find the distance between thier tops .
31) The perimeter of a right triangle is 20 cm and the sides containing the right angle are in the ratio $4: 3$ then find the sides .
32) $A B C$ is an isosceles triangle with $A C=B C$. If $A B^{2}=2 A C^{2}$, prove that $\triangle A B C$ is a right triangle.
33) In fig. $A B C$ and $D B C$ are two triangles on the same base $B C$.

If $A D$ intersects $B C$ at 0 , show that $\frac{\operatorname{ar}(\triangle A B C)}{\operatorname{ar}(\triangle D B C)}=\frac{A O}{D O}$

34) In a quadrilateral $A B C D$ the diagonals intersect at 0 .

If $\frac{A O}{B O}=\frac{C O}{D O}$ then prove that $A B C D$ is a trapezium.

35) In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitude.
Data : In equilateral triangle ABC ,
$\mathrm{AB}=\mathrm{BC}=\mathrm{AC}$ and AD is a perpendicular
height. $\angle \mathrm{ADB}=90^{\circ}$
To Prove : $3 \mathrm{AB}^{2}=4 \mathrm{AD}^{2}$
Proof: In $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ADC}$

$\angle \mathrm{D}=\angle \mathrm{D}=90^{\circ} \quad[\because$ given $]$
$\mathrm{AB}=\mathrm{AC} \quad[\because$ given $]$
$\mathrm{AD}=\mathrm{AD} \quad[\because$ common side $]$
$\Delta \mathrm{ABD} \cong \triangle \mathrm{ADC} \quad[\because$ RHS Postulate $]$
$\therefore \mathrm{BD}=\mathrm{CD}=\frac{1}{2} \mathrm{BC} \cdots--->(1)$
In right triangle $A B D$
$\mathrm{AB}^{2}=\mathrm{AD}^{2}+\mathrm{BD}^{2} \quad[\because$ Pythagoras theorem $]$
$A B^{2}=A D^{2}+\left(\frac{1}{2} B C\right)^{2} \quad\left[\because B D=\frac{1}{2} B C\right]$
$\mathrm{AB}^{2}=\mathrm{AD}^{2}+\frac{1}{4} \mathrm{AB}^{2} \quad[\because \mathrm{BC}=\mathrm{AB}]$
$4 \times \mathrm{AB}^{2}=4 \times \mathrm{AD}^{2}+\mathrm{AB}^{2} \quad[\because$ Multiplying by 4$]$
$4 \mathrm{AB}^{2}-\mathrm{AB}^{2}=4 \mathrm{AD}^{2}$
$3 \mathrm{AB}^{2}=4 \mathrm{AD}^{2}$. Hence Proved
36) BL and $C M$ are medians of a triangle ABC
right angled at A. Prove that
$4\left(\mathrm{BL}^{2}+\mathbf{C M}^{2}\right)=5 \mathrm{BC}^{2}$
Solution: in $\triangle A B C, \angle A=90^{\circ}$

$B L$ and $C M$ are medians of a triangle ABC
From $\triangle \mathrm{ABC}, \quad \mathrm{BC}^{2}=A \mathrm{~B}^{2}+\mathrm{AC}^{2}---->(1)(\because$ Pythagoras theorem $)$
From $\Delta \mathrm{ABL}, \quad \mathrm{BL}^{2}=\mathrm{AL}^{2}+\mathrm{AB}^{2} \quad(\because$ Pythagoras theorem $)$
$B L^{2}=\left(\frac{A C}{2}\right)^{2}+{A B^{2}}^{2}$
$(\because \mathrm{L}$ is the mid-of AC$)$
$\mathrm{BL}^{2}=\frac{\mathrm{AC}^{2}}{4}+\mathrm{AB}^{2}$
$4 \mathrm{BL}^{2}=\mathrm{AC}^{2}+4 \mathrm{AB}^{2}$
From $\triangle \mathrm{CMA}, \quad \mathrm{CM}^{2}=\mathrm{AC}^{2}+\mathrm{AM}^{2}$
$\begin{array}{ll}C M^{2}=A C^{2}+\left(\frac{A B}{2}\right)^{2} & (\because M \text { is the mid-of } A B) \\ C M^{2}=A C^{2}+\frac{A B^{2}}{4} & \\ 4 C M^{2}=4 A C^{2}+A B^{2} & ---->(3)\end{array}$
adding (2)and(3), we have
$4 \mathrm{BL}^{2}+4 \mathrm{CM}^{2}=\mathrm{AC}^{2}+4 \mathrm{AB}^{2}+4 \mathrm{AC}^{2}+\mathrm{AB}^{2}$
$4\left(\mathrm{BL}^{2}+\mathrm{CM}^{2}\right)=5\left(\mathrm{AB}^{2}+\mathrm{AC}^{2}\right)$
$\therefore 4\left(\mathrm{BL}^{2}+\mathrm{CM}^{2}\right)=5 \mathrm{BC}^{2} \quad[\because$ From (1) $]$
37) $D$ and $E$ are the points on the sides $C A$ and $C B$ respectively of a triangle $A B C$ right angled at $C$. Prove that $\mathbf{A E}^{2}+\mathbf{B D}^{\mathbf{2}}=\mathbf{A B}^{\mathbf{2}}+\mathbf{D E}^{2}$.
Solution: in $\triangle A B C, \angle C=90^{\circ}$

$D$ and $E$ are points on the sides $C A$ and $C B$.
From $\triangle \mathrm{ABC}, \quad \mathrm{AB}^{2}=\mathrm{AC}^{2}+\mathrm{BC}^{2}---->(1) \quad(\because$ Pythagoras theorem $)$
From $\triangle \mathrm{ACE}, \quad \mathrm{AE}^{2}=\mathrm{AC}^{2}+\mathrm{CE}^{2} \quad---->(2)(\because$ Pythagoras theorem $)$
From $\Delta B C D, \quad B D^{2}=D C^{2}+B C^{2} \quad---->(3)(\because$ Pythagoras theorem $)$
From $\triangle \mathrm{DCE}, \quad \mathrm{DE}^{2}=\mathrm{DC}^{2}+\mathrm{CE}^{2} \quad---->(4)(\because$ Pythagoras theorem)
$\mathrm{AE}^{2}+\mathrm{BD}^{2}=\mathrm{AC}^{2}+\mathrm{CE}^{2}+\mathrm{DC}^{2}+\mathrm{BC}^{2} \quad[\because$ adding (2)and(3) $]$
$\mathrm{AE}^{2}+\mathrm{BD}^{2}=\left(\mathrm{AC}^{2}+\mathrm{BC}^{2}\right)+\left(\mathrm{CE}^{2}+\mathrm{DC}^{2}\right)$
$\therefore \mathrm{AE}^{2}+\mathrm{BD}^{2}=\mathrm{AB}^{2}+\mathrm{DE}^{2} \quad[\because$ From (1) $\operatorname{and}(4)]$
38) A ladder reaches a window which is 12 m above the ground on one side of the street. Keeping its foot at at the same point, the ladder is turned to the other side of the street to reach another window 9 m high. Find the width of the street if the length of the ladder is 15 m .
Solution: Accodring to data $\mathrm{AB}=12 \mathrm{~m}, \mathrm{CD}=9 \mathrm{~m}$
Length of the ladder $\mathrm{BE}=\mathrm{DE}=15 \mathrm{~m}$.
From $\triangle \mathrm{ABE}$,
$\mathrm{BE}^{2}=\mathrm{AB}^{2}+\mathrm{AE}^{2} \quad(\because$ Pythagoras theorem $)$
$15^{2}=12^{2}+\mathrm{AE}^{2}$
$225-144=\mathrm{AE}^{2}$
$\mathrm{AE}^{2}=81$

$\mathrm{AE}^{2}=9^{2}$
$\therefore \mathrm{AE}=9 \mathrm{~m} \quad---->(1)$
From $\triangle C D E, \quad \mathrm{DE}^{2}=\mathrm{CD}^{2}+\mathrm{EC}^{2} \quad(\because$ Pythagoras theorem $)$
$15^{2}=9^{2}+\mathrm{EC}^{2}$
$225-81=\mathrm{EC}^{2}$
$\mathrm{EC}^{2}=144$
$\mathrm{EC}^{2}=12^{2}$
$\therefore \mathrm{EC}=12 \mathrm{~m}$ $\qquad$
$\mathrm{AC}=\mathrm{AE}+\mathrm{EC}=9+12=21 \mathrm{~m} \quad[\because$ From (1)and (2) $]$
$\therefore$ the width of the street is 21 m .

$$
4 \text { or } 5 \text { Marks Questions (LA-2/ LA-3) }
$$

39) State and prove Thales theorem (Basic Proportionality Theorem ).

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Data: In $\triangle \mathrm{ABC} \quad \mathrm{DE} \| \mathrm{BC}$.
To Prove : $\frac{A D}{D B}=\frac{A E}{E C}$

Construction: Draw DM $\perp$ AC and $\mathrm{EN} \perp \mathrm{AB}$. Join $B E$ and $C D$.


Proof:
$\frac{\operatorname{ar}(\triangle \mathrm{ADE})}{\operatorname{ar}(\triangle \mathrm{BDE})}=\frac{\frac{1}{2} \times \mathrm{AD} \times \mathrm{EN}}{\frac{1}{2} \times \mathrm{DB} \times \mathrm{EN}} \quad\left(\because\right.$ Area of $\Delta=\frac{1}{2} \times$ base $\times$ height $)$
$\frac{\operatorname{ar}(\triangle \mathrm{ADE})}{\operatorname{ar}(\triangle \mathrm{BDE})}=\frac{\mathrm{AD}}{\mathrm{DB}}$
$\frac{\operatorname{ar}(\triangle \mathrm{ADE})}{\operatorname{ar}(\triangle \mathrm{CED})}=\frac{\frac{1}{2} \times \mathrm{AE} \times \mathrm{DM}}{\frac{1}{2} \times \mathrm{EC} \times \mathrm{DM}} \quad\left(\because\right.$ Area of $\Delta=\frac{1}{2} \times$ base $\times$ height $)$
$\frac{\operatorname{ar}(\triangle \mathrm{ADE})}{\operatorname{ar}(\triangle \mathrm{CED})}=\frac{\mathrm{AE}}{\mathrm{EC}}$
But $\triangle \mathrm{BDE}$ and $\triangle \mathrm{CED}$ are standing on the same base DE and between $\mathrm{DE} \| \mathrm{BC}$.
$\operatorname{ar}(\triangle \mathrm{BDE})=\operatorname{ar}(\triangle \mathrm{CED})$ $\qquad$
$\therefore$ from equations (1), (2) and (3)
$\frac{\mathrm{AD}}{\mathrm{DB}}=\frac{\mathrm{AE}}{\mathrm{EC}}$

Hence the proof.
40) AAA Similarity Criterion :

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio ( or proportional ). Hence prove that the two triangles are similar.


Data: In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{DEF}$
$\angle A=\angle D$
$\angle \mathrm{B}=\angle \mathrm{E}$
$\angle \mathrm{C}=\angle \mathrm{F}$
To Prove : $\frac{A B}{D E}=\frac{B C}{E F}=\frac{A C}{D F}$
Construction : Mark points $P$ and $Q$ on $D E$ and $D F$ such that $D P=A B$ and $D Q=A C$. Join PQ.

Proof: In $\triangle A B C$ and $\triangle D P Q$

$$
\begin{aligned}
& \angle \mathrm{A}=\angle \mathrm{D} \quad(\because \text { Data }) \\
& \mathrm{AB}=\mathrm{DP} \quad(\because \text { Construction }) \\
& \mathrm{AC}=\mathrm{DQ} \quad(\because \text { Construction }) \\
& \Delta \mathrm{ABC} \cong \triangle \mathrm{DPQ} \quad(\because \text { SAS Postulate }) \\
& \therefore \mathrm{BC}=\mathrm{PQ} \quad(\because \mathrm{CPCT})-------->(1) \\
& \left.\begin{array}{l}
\angle \mathrm{B}=\angle \mathrm{P} \\
\angle \mathrm{~B}=\angle \mathrm{E}
\end{array}\right\} \begin{array}{l}
(\because \mathrm{CPCT}) \\
(\because \text { Data })
\end{array} \\
& \therefore \angle \mathrm{P}=\angle \mathrm{E} \quad(\because \text { Axiom }-1) \\
& \Rightarrow \mathrm{PQ} \| \mathrm{EF} \\
& \frac{D P}{D E}=\frac{P Q}{E F}=\frac{D Q}{D F} \quad(\because \text { Corollary of Thales theorem }) \\
& \frac{A B}{D E}=\frac{B C}{E F}=\frac{A C}{D F} \quad[\because \text { from eq. (1) and construction }]
\end{aligned}
$$

Hence the proof.

## 41) Areas of Similar Triangles:

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.


Data: $\triangle \mathrm{ABC} \sim \triangle \mathrm{PQR}$
$\Rightarrow \frac{\mathrm{AB}}{\mathrm{PQ}}=\frac{\mathrm{BC}}{\mathrm{QR}}=\frac{\mathrm{AC}}{\mathrm{PR}}$
To Prove : $\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^{2}=\left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^{2}=\left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^{2}$
Construction: Draw $\mathrm{AM} \perp \mathrm{BC}$ and $\mathrm{PN} \perp \mathrm{QR}$.
Proof : $\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\frac{\frac{1}{2} \times \mathrm{BC} \times \mathrm{AM}}{\frac{1}{2} \times \mathrm{QR} \times \mathrm{PN}} \quad\left(\because\right.$ Area of $\Delta=\frac{1}{2} \times$ base $\times$ height $)$
$\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\frac{\mathrm{BC} \times \mathrm{AM}}{\mathrm{QR} \times \mathrm{PN}} \cdots-\cdots---->(1)$
In $\triangle \mathrm{ABM}$ and $\triangle \mathrm{PQN}$
$\angle \mathrm{B}=\angle \mathrm{Q} \quad(\because \Delta \mathrm{ABC} \sim \Delta \mathrm{PQR})$
$\angle \mathrm{M}=\angle \mathrm{N}=90^{\circ} \quad(\because$ Construction $)$
$\therefore \triangle \mathrm{ABM} \sim \triangle \mathrm{PQN} \quad(\because$ AA Similarity criterion)
$\therefore \frac{\mathrm{AM}}{\mathrm{PN}}=\frac{\mathrm{AB}}{\mathrm{PQ}} \cdots----->(2)$
But $\frac{A B}{P Q}=\frac{B C}{Q R}=\frac{A C}{P R} \cdots-\cdots(3)(\because$ Data $)$
$\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\frac{\mathrm{AB}}{\mathrm{PQ}} \times \frac{\mathrm{AB}}{\mathrm{PQ}} \quad(\because$ substituting eqs.(2) and (3) in (1) )
$\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^{2}$
Now from eq.(3)
$\frac{\operatorname{ar}(\triangle \mathrm{ABC})}{\operatorname{ar}(\triangle \mathrm{PQR})}=\left(\frac{\mathrm{AB}}{\mathrm{PQ}}\right)^{2}=\left(\frac{\mathrm{BC}}{\mathrm{QR}}\right)^{2}=\left(\frac{\mathrm{AC}}{\mathrm{PR}}\right)^{2}$
Hence the proof.

## 42) State and and Prove Pythagoras Theorem.

" In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on other two sides".


Data : $\triangle \mathrm{ABC}$ is a right triangle and $\angle \mathrm{B}=90^{\circ}$
To Prove: $\mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}$

Construction: Draw BD $\perp$ AC

Proof: In $\triangle A D B$ and $\triangle A B C$
$\angle \mathrm{D}=\angle \mathrm{B}=90^{\circ} \quad(\because$ Data and Construction $)$
$\angle \mathrm{A}=\angle \mathrm{A} \quad(\because$ Common angle $)$
$\Delta \mathrm{ADB} \sim \Delta \mathrm{ABC} \quad(\because$ AAA Similarity Criterion $)$
$\therefore \frac{\mathrm{AD}}{\mathrm{AB}}=\frac{\mathrm{AB}}{\mathrm{AC}} \quad(\because$ Proportional sides $)$
AC. $A D=A B^{2}----->(1)$
Similarly
In $\triangle B D C$ and $\triangle A B C$
$\angle \mathrm{D}=\angle \mathrm{B}=90^{\circ} \quad(\because$ Data and Construction)
$\angle \mathrm{C}=\angle \mathrm{C} \quad(\because$ Common angle $)$
$\Delta \mathrm{BDC} \sim \triangle \mathrm{ABC} \quad(\because$ AAA Similarity Criterion $)$
$\therefore \frac{\mathrm{DC}}{\mathrm{BC}}=\frac{\mathrm{BC}}{\mathrm{AC}} \quad(\because$ Proportional sides $)$
$\mathrm{AC} . \mathrm{DC}=\mathrm{BC}^{2}----->(2)$
$\mathrm{AC} \cdot \mathrm{AD}+\mathrm{AC} \cdot \mathrm{DC}=\mathrm{AB}^{2}+\mathrm{BC}^{2}[\because$ By adding (1) and (2) $]$
$A C(A D+D C)=A B^{2}+B C^{2}$
$A C \times A C=A B^{2}+B C^{2} \quad(\because$ from fig. $A D+D C=A C)$
$A C^{2}=A B^{2}+B C^{2}$
Hence the proof.
43) State and and Prove the converse of Pythagoras Theorem.
" In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle".


Data: In $\triangle \mathrm{ABC}, \quad \mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}$
To Prove : $\angle \mathrm{B}=90^{\circ}$
Construction : construct another $\triangle \mathrm{PQR}$ right angled at Q such that
$P Q=A B$ and $Q R=B C$
Proof: In $\triangle \mathrm{PQR}$

$$
\begin{array}{ll}
\mathrm{PR}^{2}=\mathrm{PQ}^{2}+\mathrm{QR}^{2} & \left(\because \angle \mathrm{Q}=90^{\circ}, \text { Pythagoras theorem }\right) \\
\mathrm{PR}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}-->(1) & (\because \text { By Construction }) \\
\mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}-->(2) & (\because \text { Data }) \\
\therefore \mathrm{AC}=\mathrm{PR} \quad--->(3) \quad[\because \text { From }(1) \text { and }(2)]
\end{array}
$$

In $\triangle \mathrm{ABC}$ and $\triangle \mathrm{PQR}$
$A B=P Q$
$B C=Q R$
$\mathrm{AC}=\mathrm{PR}$
$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{PQR}$
$\therefore \angle \mathrm{B}=\angle \mathrm{Q}$
But, $\angle \mathrm{Q}=90^{\circ}$
$\therefore \angle \mathrm{B}=90^{\circ}$
Hence the proof.

## UNIT - 3 : PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1 Mark Questions (MCQ)

1) If the pair of linear equations $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$ has infinitely many solutions then which of the following condition is correct
A) $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
B) $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
C) $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}$
D) $\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
2) If the lines representing the equations $a_{1} x+b_{1} y+c_{1}=0$ and $a_{2} x+b_{2} y+c_{2}=0$ intersect at one point, then which of the following condition is correct
A) $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}$
B) $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
C) $\frac{a_{1}}{a_{2}} \neq \frac{b_{1}}{b_{2}}$
D) $\frac{b_{1}}{b_{2}} \neq \frac{c_{1}}{c_{2}}$
3) The straight lines representing the equations $x-2 y=0$ and $3 x+4 y-20=0$ are
A) Parallel
B) Intersect
C) Coincide
D) Does not intersect
4) Pair of the equations $2 x+3 y-9=0$ and $4 x+6 y-18=0$ has
A) No solution
B) Unique solution.
C) Only two solutions
D) Infinitely many solutions
5) The solutions of equations $x+3 y=6$ and $2 x-3 y=12$ are
A) $x=0, y=6$
B) $x=6, y=-6$
C) $x=6, y=0$
D) $x=0, y=0$
6) If the equations $2 x+y=3$ and $y=m x+3$ represent the same straight line then the value of $m$ is
A) -3
B) -2
C) 2
D) 3
7) If $x=-y$ and $y>0$, then which of the following statement in not correct?
A) $x^{2} y>0$
B) $x+y=0$
C) $x y<0$
D) $\frac{1}{x}-\frac{1}{y}=0$
8) If the equations $x+5 y=7$ and $4 x+20 y=-k$ represent coinciding straight lines then the value of $k$ is
A) -28
B) 24
C) 28
D) -24
9) The equations $x+2 y-4=0$ and $2 x+4 y-12=0$ represent parallel lines. So the equations has
A) No solution
B) Unique solution
C) Only two solutions
D) Infinitely many solutions
10) The intersecting point of the graphs of the equations $y=2 x-2$ and $y=4 x-4$ is
A) $(1,0)$
B) $(-1,0)$
C) $(0,1)$
D) $(0,-1)$

1 Mark Questions (VSA)
11) Write the coordinates of the origin.
12) Write the general form of a linear equation in one variable.
13) Write the general form of a pair of linear equations in two variables.

| Ans | 1) A | 2) C | 3) B | 4) D | 5) C | 6) B | 7) D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 8) A | 9) A | 10) A | 11) $(0,0)$ |  |  |  |
|  | 12) $a x+b=0$ (Here $a \neq 0$ and $a, b$ are real numbers ) |  |  |  |  |  |  |
|  | 13) $\left.\begin{array}{r}a_{1} x+b_{1} y+c_{1}=0 \\ a_{2} x+b_{2} y+c_{2}=0\end{array}\right\} \begin{gathered}\text { Here } a_{1}, b_{1}, c_{1} \& a_{2}, b_{2}, c_{2} \text { are real numbers } \\ \text { and } \quad a_{1}{ }^{2}+b_{1}{ }^{2} \neq 0, a_{2}{ }^{2}+b_{2}{ }^{2} \neq 0\end{gathered}$ |  |  |  |  |  |  |

2 Marks Questions (SA)
14) Solve the given pair of linear equations.
$x+y=14$
$x-y=4$
Solution :

$$
\begin{align*}
& x+y=14 \longrightarrow(1) \\
& x-y=4 \longrightarrow(2)  \tag{2}\\
& \begin{array}{l}
x=18
\end{array} \\
& \begin{array}{l}
x=\frac{18}{2} \\
\therefore x=9
\end{array} \quad[\because \text { Adding eq.(1) and eq.(2)] } \\
& \therefore x=18
\end{align*}
$$

Substituting the value of $x$ in eq. (1)
$9+y=14$
$y=14-9$
$\therefore y=5$
Solution is $x=9$ and $y=5$

## Alternate method:

Write the given equations in standard form and compare

$$
\begin{array}{lllll}
x+y-14=0 & --->(1) & \therefore & a_{1}=1, & b_{1}=1,
\end{array} \quad c_{1}=-14, ~ 子, ~ b_{2}=-1, \quad c_{2}=-4
$$

| By using cross multiplication formula |  |
| :--- | :--- |
| $x=\frac{b_{1} c_{2}-b_{2} c_{1}}{a_{1} b_{2}-a_{2} b_{1}}$ | $y=\frac{c_{1} a_{2}-c_{2} a_{1}}{a_{1} b_{2}-a_{2} b_{1}}$ |
| $x=\frac{1(-4)-(-1)(-14)}{1(-1)-1(1)}$ | $y=\frac{(-14) 1-(-4) 1}{1(-1)-1(1)}$ |
| $x=\frac{-4-14}{-1-1}$ | $y=\frac{-14+4}{-1-1}$ |
| $x=\frac{-18}{-2}$ | $y=\frac{-10}{-2}$ |
| $\therefore x=9$ | $\therefore y=5$ |

15) Solve the following pairs of linear equations.
(i) $2 x+3 y=11$
$2 x-4 y=-24$
(ii) $x-y=26$
(iii) $x+y=180$
$x-3 y=0$

$$
x-y=18
$$

16) The difference between two numbers is 26 and one number is three times the other Find them.
Solution : Let the two numbers be $x$ and $y$.
Given $x-y=26 \longrightarrow(1)$
and $x=3 y \longrightarrow(2)$
Substitute eq. (2) in eq.(1)
$3 y-y=26$
$2 y=26$
$y=\frac{26}{2}$
$y=13$
Substituting the value of $y$ in eq. (2)
$x=3(13)$
$x=39$
$\therefore$ The numbers are 39 and 13 .
17) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
18) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800 . Later she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.
19) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.
20) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?
21) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?
22) Find the value of $\boldsymbol{k}$ for which the system of equations has infinitely many solutions.
$2 x-3 y=7$
$(k+1) x+(1-2 k) y=(5 k-4)$
Solution: The given equations in standard form
$2 x-3 y-7=0 \quad$ here $a_{1}=2, \quad b_{1}=-3, \quad c_{1}=-7$,
$(k+1) x+(1-2 k) y+(4-5 k)=0$ here $a_{2}=k+1, b_{2}=1-2 k, c_{2}=4-5 k$
The equations has infinitely many solutions, so $\frac{a_{1}}{a_{2}}=\frac{b_{1}}{b_{2}}=\frac{c_{1}}{c_{2}}$
$\frac{2}{k+1}=\frac{-3}{1-2 k}=\frac{-7}{4-5 k}$
$\frac{2}{k+1}=\frac{-3}{1-2 k}$
and $\quad \frac{-3}{1-2 k}=\frac{-7}{4-5 k}$
$2-4 k=-3 k-3$
and $\quad-12+15 k=-7+14 k$
$2+3=4 k-3 k$
and $\quad 15 k-14 k=12-7$
$k=5$
and $\quad k=5$
23) Find the value of $k$ for which the system of equations has infinitely many solutions.
$k x+3 y-(k-3)=0$
$12 x+k y-k=0$

## 4 Marks Questions (LA-2)

24) Solve the given linear equation graphically.
$x-2 y=0$
$3 x+4 y=20$
Solution:

| $x-2 y=0$ |  |  |  |
| :---: | :---: | :---: | :---: |
| $x$ | 0 | 2 | 4 |
| $y=\frac{x}{2}$ | 0 | 1 | 2 |


| $x$ | 0 | 4 | -4 |
| :---: | :---: | :---: | :---: |
| $y=\frac{20-3 x}{4}$ | 5 | 2 | 8 |

Scale: X-axis: $1 \mathrm{~cm}=1$ unit
and $\quad Y$ - axis : $1 \mathrm{~cm}=1$ unit

25) Solve the following linear equations graphically.
(i) $2 x-y=2$
$4 x-y=4$
(ii) $x+3 y=6$
$2 x-3 y=12$
(iii) $x+y=5$
$2 x-y=4$
(iv) $2 x+y-6=0$
$2 x-y-2=0$
(v) $x=y-6$
$y=2-x$
(vi) $x+2 y=0$
$2 x+y+6=0$
(vii) $2 x+y=-7$
$x-y=1$
(viii) $x-2 y=2$
$2 x-y=-2$
(ix) $x-2 y=0$
$x+2 y=-8$
(x) $3 x+2 y=0$
$2 x-y=7$
(xi) $2 x+3 y=-2$
$3 x-y=8$
[xii] $x-y=4$
$2 x+y=5$

## 5 Marks Questions (LA-3)

26) Draw the graphs of the equations $x-y+1=0$ and $3 x+2 y-12=0$. Determine the coordinates of the vertices of the triangle formed by these lines and the $x$ - axis and shade the triangular region.

Solution: $\quad x-y+1=0$
$3 x+2 y-12=0$

| $x$ | 0 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| $y=x+1$ | 1 | 3 | 4 |


| $x$ | 0 | 2 | 4 |
| :---: | :---: | :---: | :---: |
| $y=\frac{12-3 x}{2}$ | 6 | 3 | 0 |

$$
\text { Scale : } \mathrm{X} \text { - axis: } 1 \mathrm{~cm}=1 \text { unit and } \mathrm{Y} \text { - axis : } 1 \mathrm{~cm}=1 \text { unit }
$$



## UNIT - 4: CIRCLES

1 Mark Questions (MCQ)

1) If the two end points of chord coincide, then the it is called
A) Secant
B) Tangent
C) line Segment
D) diameter
2) Number of tangent drawn at a point on a circle is
A) 2
B) 1
C) Infinite
D) 3
3) The straight line which intersects the circle at only one point is
A) Radius
B) tangent
C) Secant
D) line segment
4) A straight line which passes through any two distinct points of a circle
A) Tangent
B) diameter
C) secant
D) line segment
5) Maximum number of tangents drawn to a circle from an external point
A) 1
B) 3
C) 2
D) infinite
6) In the adjoining figure $O X$ is perpendicular in a circle of radius 5 cm . $O X=3 \mathrm{~cm}$ then the length of the chord $P Q$ is
A) 5 cm
B) 4 cm
C) 8 cm
D) 10 cm
7) In the adjoining figure if $\angle \mathrm{AOP}=60^{\circ}$ then $\angle \mathrm{APO}=$

A) $120^{\circ}$
B) $90^{\circ}$
C) $60^{\circ}$
D) $30^{\circ}$
8) In the figure $P Q$ and $R S$ is chord which are equidistant from the centre. If $P Q=6 \mathrm{~cm}$ then $R S=$
A) 5 cm
B) 6 cm
C) 8 cm
D) 3 cm

9) Tangents $P Q$ and $P R$ are drawn to a circle as shown in figure. if $\angle \mathrm{APO}=90^{\circ}$ and $\mathrm{PQ}=8 \mathrm{~cm}$ then radius of circle is

A) 5 cm
B) 6 cm
C) 8 cm
D) 3 cm
10) In the figure if PA and PB are tangents to a circle with centre 0. If $\angle \mathrm{APB}=40^{\circ}$ then, $\angle \mathrm{AOB}$ is,
A) $90^{\circ}$
B) $50^{\circ}$
C) $140^{\circ}$
D) $150^{\circ}$

11) In the figure if $B C$ is the diameter, the value of $X$ is
A) $90^{\circ}$
B) $50^{\circ}$
C) $180^{\circ}$
D) $160^{\circ}$

12) The length of the tangent from an external point at a distance of 5 cm from the centre of the circle of radius 3 cm is
A) 4 cm
B) 3.5 cm
C) 4.5 cm
D) 5.5 cm
13) The length of the tangent is 24 cm which is drawn to a circle of centre 0 at a distance 25 cm from its centre, then its radius is
A) 7 cm
B) 12 cm
C) 15 cm
D) 24.5 cm

## 1 Mark Questions (VSA)

14) In the figure if $\angle \mathrm{POQ}=130^{\circ} \mathrm{TP}$ and TQ are tangents to a circle of centre ' $o$ ' then what is the measure of $\angle \mathrm{PTQ}$ ?

15) In the figure name the tangents.


| Ans | 1) B | 2) B | 3) B | 4) C | 5) C |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6$) \mathrm{C}$ | $7) \mathrm{D}$ | 8) B | 9) C | 10) C | $11) \mathrm{A}$ |
| 12$) \mathrm{A}$ | $13) \mathrm{A}$ |  | 14) $50^{\circ}$ | 15) PQ and AB |  |

1 Marks Questions (SA)
16) In the adjoining figure quadrilateral $A B C D$ is inscribed in a circle.

Show that $A B+C D=A D+B C$.
Solution:- In fig AP = AS ----> (1) ( Theorem)
$B P=B Q \quad--->(2)$ (Theorem)
CQ = CR $---\gg(3)$ (Theorem)
DR = DS $--->(4)$ (Theorem)
LHS $=A B+C D$

$=(A P+B P)+(D R+C R)($ from fig $)$
$=(\mathrm{AS}+\mathrm{BQ})+(\mathrm{DS}+\mathrm{CQ})$
$=(\mathrm{AS}+\mathrm{DS})+(\mathrm{BQ}+\mathrm{CQ})$
$=A D+B C=R H S$

## 3Marks Questions (LA-1)

17) Theorem:- "The tangent at any point of a circle is perpendicular to the radius through the point of contact". Prove this.


Data: A circle with centre 0 and $X Y$ is a tangent to the circle at point 0 . $O P$ is the radius through the Point $P$.

To prove: $\mathrm{OP} \perp \mathrm{XY}$
Construction: Take a point Q other than P on XY and join OQ .
Let OQ intersct the circle at R.
Proof: OQ > OR ( $\because$ from the figure)
But $0 P=0 R \quad(\because$ radii of the same circle $)$
$\therefore \mathrm{OQ}>$ radius OP
Since this happens for every point on the line XY cxcept the point $P$.
$\therefore \mathrm{OP}$ is the shortest distance between the point O and tangent XY .
$\therefore \mathrm{OP} \perp \mathrm{XY}$
$\therefore$ Hence the proof.
18) Theorem :- "The tangents drawn to a circle from an external point are equal" prove this.
Data: A circle with centre 0 and $P$ is an external point.
$P Q$ and $P R$ are the tangents drawn from an external point $P$.
To prove: $\mathrm{PQ}=\mathrm{PR}$
Construction: Draw OP, OQ and OR.
Proof: In $\triangle \mathrm{OQP}$ and $\triangle \mathrm{ORP}$
$O Q=O R \quad(\because$ Radii of same circle $)$

$\mathrm{OP}=\mathrm{OP} \quad(\because$ Common side $)$
$\angle \mathrm{Q}=\angle \mathrm{R}=90^{\circ} \quad(\because$ tangent $\perp$ radius $)$
$\Delta \mathrm{OQP} \cong \triangle \mathrm{ORP} \quad(\because$ RHS criteria $)$
$P Q=P R \quad(\because \mathrm{CSCT})$
$\therefore$ Hence the proof.
19) In the figure, to a circle with centre ' 0 ', $T P$ and TQ are the tangents drawn from T , show that $\angle \mathrm{PTQ}=2 \angle \mathrm{OPQ}$

20) 3 cm and 5 cm are the radii of two concentric circles with centre ' 0 '. Find the length of the chord which touches the smaller circle.

21) PA and PB are the tangents to a circle with centre ' 0 ' drawn from an external point $P$ if $\angle \mathrm{APB}=120^{\circ}$ then show that $\mathrm{OP}=2 \mathrm{AP}$. (Hint: In $\triangle \mathrm{OAP}, \cos 60^{\circ}=\frac{\mathrm{AP}}{\mathrm{OP}}$ )

22) In the figure ' 0 ' is the Centre of the circle and $T$ is an external point, TP and TQ are tangents from T show that $\angle \mathrm{PTQ}+\angle \mathrm{POQ}=180^{\circ}$


## UNIT- 5: AREA RELATED TO CIRCLES

## 1 Mark Questions (MCQ)

1) The region bounded by two radii and corresponding arc of a circle is
A) Segment
B) Sector
C) Area
D) Perimeter
2) The region bounded by chord and corresponding arc of a circle is
A) Perimeter
B) Sector
C) Area
D) Segment
3) Area of sector with sector angle $x$ and radius of circle $P$ is
A) $\frac{\pi p x^{2}}{360}$
B) $\frac{\pi p x^{2}}{270}$
C) $\frac{\pi x p^{2}}{270}$
D) $\frac{\pi x p^{2}}{360}$
4) Angle of the sector is $90^{\circ}$ then the ratio of Area of circle to area of sector is
A) $1: 4$
B) $1: 2$
C) $4: 1$
D) $2: 3$
5) Area of square inscribed in a circle of unit radius is
A) $\frac{\pi}{2}$ Sq. units
B) $\pi$ Sq. units
C) $\sqrt{2}$ Sq. units
D) 2 Sq. units
6) Length of an arc of a sector of angle $\theta$ is,
A) $\frac{\theta}{360} \times \pi r$ Units
B) $\frac{\theta}{360} \times 2 \pi r$ Units
C) $\frac{\theta}{360} \times 2 \pi$ Units
D) $\frac{\theta}{360} \times \pi$ Units
7) If the perimeter and the area of a circle are numerically equal, then the radius of the circle is,
A) 2 Units
B) $\pi$ Units
C) 4 Units
D) 7 Units

## 1 Mark Questions (VSA)

8) In the figure, If $\theta$ is the sector angle and $r$ is the radius of the sector. The length of arc AXB is,

9) What is the area of the sector of circle with radius $r$ which makes angle $360^{\circ}$ ?
10) If the angle formed at the centre of a circle with radius $r$ is $1^{\circ}$. What is the area of sector?
11) If the angle formed at the centre of a circle with radius $r$ is $90^{\circ}$, what is the area of the sector?
12) Area of circle is $154 \mathrm{~cm}^{2}$, and Area of minor sector is $7.7 \mathrm{~cm}^{2}$, then calculate area of major sector of circle.
13) In the figure, area of two concentric circles is $154 \mathrm{~cm}^{2}$ and $1386 \mathrm{~cm}^{2}$ respectively. Find the area of shaded portion?
14) The radii of two concentric circle are 7 cm and 14 cm . What
 is the ratio of their circumference?
15) The radii of two concentric circle are 7 cm and 14 cm . What is the ratio of their areas?
16) The circumference of two concentric circles are $28 \pi$ and $42 \pi$ respectively. What is the numerical sum of their radii?
17) If the angle formed at the centre of a circle with radius $r$ is $\theta$. What is the perimeter of the sector?
18) The circumference of a circle is 44 cm . Find the circumference of a quadrant of the circle.

| Ans | 1) B | 2) D | 3) D | 4) C | 5) D |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6) B | 7) A | 8) $\frac{\theta}{360} \times 2 \pi r$ Units | 9) $\pi r^{2}$ | 10) $\frac{\pi r^{2}}{360}$ |  |  |  |
| 11) $\frac{\pi r^{2}}{4}$ | 12) $146.3 \mathrm{~cm}^{2}$ | 13) $1232 \mathrm{~cm}^{2}$ | 14) $1: 2$ | 15) $1: 4$ | 16) 35 |  |  |
| 17) $\left(\frac{\theta}{360} \times 2 \pi r+2 r\right)$ Units |  |  |  |  | 18) 11 cm |  |  |

2 Marks Questions (SA)
(Unless stated take $\pi=\frac{22}{7}$ )
19) The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
Solution:- Radii $\mathrm{r}_{1}=19 \mathrm{~cm}$ and $\mathrm{r}_{2}=9 \mathrm{~cm}$
$2 \pi R=2 \pi r_{1}+2 \pi r_{2}$
$2 \pi \mathrm{R}=2 \pi\left(\mathrm{r}_{1}+\mathrm{r}_{2}\right.$
$R=\left(r_{1}+r_{2}\right)$
$\mathrm{R}=19+9$
$\mathrm{R}=28 \mathrm{~cm}$
20) The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
Solution:- Radius $r_{1}=8 \mathrm{~cm}$ and $r_{2}=6 \mathrm{~cm}$
$\pi R^{2}=\pi r_{1}{ }^{2}+\pi r_{2}{ }^{2}$
$\pi R^{2}=\pi\left(r_{1}{ }^{2}+r_{2}{ }^{2}\right)$
$R^{2}=\left(r_{1}{ }^{2}+r_{2}{ }^{2}\right)$
$R^{2}=\left(r_{1}{ }^{2}+r_{2}{ }^{2}\right)$
$\mathrm{R}^{2}=\left(8^{2}+6^{2}\right)$
$\mathrm{R}^{2}=64+36=100$
$R^{2}=10^{2}$
$\mathrm{R}=10 \mathrm{~cm}$
21) Find the area of a sector of a circle with radius 4 cm if angle of the sector is $30^{\circ}$

Solution:- Radius $\quad \mathrm{r}=4 \mathrm{~cm}$
Angle of the sector $\theta=30^{\circ}$
Area of the sector $=\frac{\theta}{360} \times \pi r^{2}$
$=\frac{30^{\circ}}{360} \times \frac{22}{7} \times 4^{2}$
$=\frac{1}{12} \times \frac{22}{7} \times 16$
$=\frac{1}{3} \times \frac{22}{7} \times 4$
$=\frac{88}{21}$
$=4.19 \mathrm{~cm}^{2}$
22) If radius is 4 cm and angle of sector is $30^{\circ}$,find the area of corresponding major sector.
23) Find the area of a sector of a circle with radius 6 cm and if angle of the sector is $60^{\circ}$.
24) Find the area of a quadrant of a circle whose circumference is 22 cm .
25) In a circle of radius 21 cm , if an arc subtends an angle of $60^{\circ}$ at the centre, find the length of the arc.
Solution :- radiusr $=21 \mathrm{~cm}$
Angle of sector $\theta=60^{\circ}$
Length of an arc of a sector $=\frac{\theta}{360} \times 2 \pi r$
$=\frac{60^{\circ}}{360} \times 2 \times \frac{22}{7} \times 21$
$=\frac{1}{6} \times 2 \times 22 \times 3$
$\therefore$ Length of arc $=22 \mathrm{~cm}$
26) Find the area of the shaded region in the figure, if $A B C D$ is a square of side 14 cm and APD and BPC are semicircles.

Solution:- Length of side of square $=14 \mathrm{~cm}$
Area of square $=14 \times 14=196 \mathrm{~cm}^{2}$


Radius $\mathrm{r}=\frac{14}{2}=7 \mathrm{~cm}$
Area of semicircle $=\frac{1}{2} \times \pi r^{2}$
Area of semicircle $=\frac{1}{2} \times \frac{22}{7} \times 7^{2}$
Area of semicircle $=11 \times 7=77 \mathrm{~cm}^{2}----->(2)$
Area of shaded portion $=$ Area of square $-2 \times$ Area of semicircle

$$
\begin{aligned}
& =196-2 \times 77 \\
& =196-154=42 \mathrm{~cm}^{2}
\end{aligned}
$$

27) Find the area of the shaded region in figure where $A B C D$ is a square of side 14 cm .

28) Find the area of the shaded region in figure,
if $\mathrm{PQ}=24 \mathrm{~cm}, \mathrm{PR}=7 \mathrm{~cm}$ and ' 0 ' is the centre of the circle.


## 3 Marks Questions (LA-1)

29) The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour. Solution:- Diameter of wheel of a car $=\mathrm{d}=80 \mathrm{~cm}$
Radius $\mathrm{r}=\frac{80}{2}=40 \mathrm{~cm}$
Distance covered by the wheel in one rotation $=2 \pi r$
$=2 \times \frac{22}{7} \times 40 \mathrm{~cm} \cdots(1)$
Distance covered in 10 minutes $=\frac{66}{60} \times 10=11 \mathrm{~km}$ (according to data)
Distance covered by the wheel in $10 \mathrm{~min}=11 \times 1000 \times 100 \mathrm{~cm}--->(2)$
Number of rotations of wheels $=\frac{11 \times 1000 \times 100}{2 \times \frac{22}{7} \times 40}$
$=\frac{11 \times 1000 \times 100 \times 7}{2 \times 22 \times 40}$
$=\frac{11 \times 500 \times 10 \times 7}{2 \times 11 \times 4}$
$=\frac{500 \times 10 \times 7}{2 \times 4}=125 \times 5 \times 7$
Number of rotation of wheel of a car $=4375$
30) In the adjoining figure a square $A B C D$ of side 7 cm is inscribed in a circle. Find the area of the shaded region.
Solution: ABCD is a square
$\therefore \mathrm{DC}=\mathrm{BC}=7 \mathrm{~cm}$
Diagonal $\quad \mathrm{BD}=\sqrt{\mathrm{DC}^{2}+\mathrm{BC}^{2}} \quad$ ( $\because$ Pythagoras theroem $)$
$\mathrm{BD}=\sqrt{7^{2}+7^{2}}$
$\mathrm{BD}=\sqrt{49+49}$
$\mathrm{BD}=\sqrt{2 \times 49}$
$\mathrm{BD}=7 \sqrt{2} \mathrm{~cm}$
Radius $=\frac{\mathrm{BD}}{2}=\frac{7 \sqrt{2}}{2} \mathrm{~cm} \quad(\because \mathrm{BD}$ is a diameter $)$
Area of the square $=7 \times 7=49 \mathrm{~cm}^{2}--->(1)$
Area of the circle $=\pi r^{2}=\frac{22}{7} \times \frac{7 \sqrt{2}}{2} \times \frac{7 \sqrt{2}}{2}$
Area of the circle $=77 \mathrm{~cm}^{2}--->(2)$
Area of shaded region $=$ Area of the circle - Area of the square
Area of shaded region $=77-49=28 \mathrm{~cm}^{2}$ [From (1) and (2)]
31) In the adjoining figure a rectangle $A B C D$ is inscribed in a circle. Find the area of the shaded region.

Solution: ABCD is a rectangle
$A D=5 \mathrm{~cm}, \quad D C=10 \mathrm{~cm}$
Diagonal $\mathrm{AC}=\sqrt{\mathrm{AD}^{2}+\mathrm{DC}^{2}} \quad(\because$ Pythagoras theroem $)$
$\mathrm{AC}=\sqrt{5^{2}+10^{2}}$
$\mathrm{AC}=\sqrt{25+100}$

$\mathrm{AC}=\sqrt{125=25 \times 5}$
$\mathrm{AC}=5 \sqrt{5} \mathrm{~cm}$
Radius $=\frac{\mathrm{AC}}{2}=\frac{5 \sqrt{5}}{2} \mathrm{~cm}(\because \mathrm{AC}$ is a diameter. $)$
Area of the rectangle $=10 \times 5=50 \mathrm{~cm}^{2} \rightarrow-->$ (1)
Area of the circle $=\pi r^{2}=\frac{22}{7} \times \frac{5 \sqrt{5}}{2} \times \frac{5 \sqrt{5}}{2}$
Area of the circle $=98.2 \mathrm{~cm}^{2}--->$ (2)
Area of shaded region $=$ Area of the circle - Area of the rectangle
Area of shaded region $=98.2-50=48.2 \mathrm{~cm}^{2} \quad[\because$ From (1) and (2) $]$
32) Find the area of the shaded region in figure, if radii of the two concentric circle with centre 0 are 7 cm and 14 cm respectively and $\angle \mathrm{AOC}=40^{\circ}$.

33) In the adjoining figure a suare OQSP is inscribed in a quadrant of a circle. If $O P=15 \mathrm{~cm}$, find the area of the shaded region. $(\pi=3.14)$

34) In the adjoining figure each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in figure. Find the area of the remaining portion of the square.

35) In the adjoining figure $A B C D$ is a square of side 14 cm . with centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles find the area of the shaded region.

36) In the adjoining figure $O A C B$ is a quadrant of a circle with centre 0 and radius 3.5 cm , if $\mathrm{OD}=2 \mathrm{~cm}$, Find the area of the (i) quadrant OACB (ii) Shaded region.

37) In the adjoining figure a square $O A B C$ is inscribed in a quadrant $O P B Q$. If $O A=20 \mathrm{~cm}$, find the area of the shaded region.

38) Find the area of segment AMB, shown in figure if radius of the circle is 14 cm and $\angle A O B=120^{\circ}$.

39) In the adjoining figure, find the area of the shaded region which is not common between the two quadrants of circles of radius 16 cm each.
Solution: radius $r=16 \mathrm{~cm}$
Area of the shaded region =
\{[area of the the square ABCD - area of the the quadrant ABED]

+ [area of the the square ABCD - area of the the quadrant CBFD]\}

$=\left[a^{2}-\frac{1}{4} \pi r^{2}\right]+\left[a^{2}-\frac{1}{4} \pi r^{2}\right]$
$=\left[2 a^{2}-\frac{1}{2} \pi r^{2}\right]$
$=\left[2 \times 16^{2}-\frac{1}{2} \times \frac{22}{7} \times 16^{2}\right] \quad(\because$ side of the square $=$ radius of the quadrant of the circle $)$
$=\left[512-\frac{2816}{7}\right]$
$=[512-402.3]=109.7 \mathrm{~cm}^{2}$

40) Find the area of the shaded region in the following figure.


## UNIT- 6 :CONSTRUCTIONS <br> 2 Marks Questions (SA)

1) Draw a line segment of length 7.6 cm and divide it in the ratio $5: 8$. Measure the two parts.
Solution: $\mathrm{AB}=7.8 \mathrm{~cm}$ and $\mathrm{AC}: \mathrm{BC}=5: 8$

2) Draw a linesegment of length 8.5 cm and divide it in the ratio $3: 2$. Measure the two parts.
3) Draw a circle of radius 3 cm from a point 7 cm away from its centre. Contruct a pair of tangents to the circle and measure their lengths.
Solution: radius $\mathrm{r}=3 \mathrm{~cm}$
OP = 7 cm


Tangents $\mathrm{PQ}=\mathrm{PR}=6.3 \mathrm{~cm}$
4) Draw a circle of radius 3.5 cm from a point 7 cmaway from its centre. Contruct the pair of tangents to the circle and measure their lengths.
5) Draw a circle of radius 2.5 cm from a point 8 cm away from its centre, construction the pair of tangents to the circle and measure their length.
6) Draw a circle of diameter 5 cm from a point 9 cm away from its centre, construct the pair of tangents to the circle and measure their length.
7) Draw a circle of radius 3.5 cm from a point 4 cm away from the circle construct the pair of tangent to the circle and measure their length.
8) Draw a circle of radius 3 cm take two point $P$ and $Q$ on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points $P$ and $Q$ and measure their lengths.
9) Draw two concentric circle of radii 2 cm and 4 cm from a point 7 cm away from its centre. contruct the tangents to the circle.
10) Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of $60^{\circ}$.
Solution : radius $r=3 \mathrm{~cm}$
Angle between the tangents $=60^{\circ}$
Angle between the radii $=180^{\circ}-60^{\circ}=120^{\circ}$


Tangents are PQ and PR
11) Draw a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of $120^{\circ}$.
12) Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of $60^{\circ}$.
13) Draw a pair of tangents to a circle of radius 4 cm which are inclined to each other at an angle of $90^{\circ}$.
14) Draw a pair of tangents to a circle of diameter 5 cm such that the angle between the radii is $120^{\circ}$.
15) Draw a pair of tangents to a circle of radius 5 cm such that the angle between the radii is $45^{\circ}$

## 3 Marks Questions (LA -1)

16) Construct a triangle of sides $6 \mathrm{~cm}, 7 \mathrm{~cm}$ and 9 cm , then a triangle similar to its whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.
17) Draw a triangle $A B C$ with sides $A B=4 \mathrm{~cm}, A C=5 \mathrm{~cm}$ and $B C=6 \mathrm{~cm}$. then construct a triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle ABC.
$\mathrm{AB}=4 \mathrm{~cm}$
$\mathrm{AC}=5 \mathrm{~cm}$
$\mathrm{BC}=6 \mathrm{~cm}$
$\Delta \mathrm{A}^{\prime}{ }^{-1}{ }^{1} \sim \Delta \mathrm{ABC}$
$\frac{\mathrm{A}^{\mathrm{I}} \mathrm{B}}{\mathrm{AB}}=\frac{\mathrm{BC}^{\mathrm{l}}}{\mathrm{BC}}=\frac{\mathrm{A}^{\mathrm{l}} \mathrm{C}^{\mathrm{l}}}{\mathrm{AC}}=\frac{2}{3}$

18) Draw a triangle ABC with sides $\mathrm{BC}=6 \mathrm{~cm}, \mathrm{AB}=5 \mathrm{~cm}$ and $\angle \mathrm{ABC}=60^{\circ}$, then construct a triangle whose sides are $\frac{4}{5}$ of the corresponding sides of the $\triangle \mathrm{ABC}$.
19) Draw a triangle ABC with side $\mathrm{BC}=7 \mathrm{~cm}, \angle \mathrm{~A}=45^{\circ}$ and $\angle \mathrm{B}=105^{\circ}$, then construct a triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the $\triangle A B C$.
20) Draw a triangle ABC with sides( other than hypotenuse )are of lengths 3 cm and 4 cm then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the given triangle.
21) Construct a triangle with sides $5 \mathrm{~cm}, 6 \mathrm{~cm}$ and 7 cm . and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.
22) Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1 \frac{1}{2}$ times the corresponding sides of the isosceles triangle.

## UNIT - 7 : COORDINATE GEOMETRY

## 1 Mark Questions (MCQ)

1) Distance between point $P(x, y)$ and the origin is .
A) $\sqrt{x^{2}+y^{2}}$
B) $\sqrt{x+y}$
C) $\sqrt{x-y}$
D) $\sqrt{\left(x^{2}+y^{2}\right)^{2}}$
2) Distance between the points $P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$ is
A) $\sqrt{\left(x_{2}+x_{1}\right)^{2}-\left(y_{2}+y_{1}\right)^{2}}$
B) $\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
C) $\left(x_{2}+x_{1}\right)^{2}-\left(y_{2}+y_{1}\right)^{2}$
D) $\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$
3) Distance between the points $(4,6)$ and $(6,8)$ is
A) $\sqrt{2}$ units
B) 2 units
C) $2 \sqrt{2}$ units
D) 4 units
4) Distance between the points $(0,5)$ and $(-5,0)$ is
A) $5 \sqrt{2}$ units
B) 5 units
C) $2 \sqrt{5}$ units
D) $\sqrt{10}$ units
5) Distance between origin and the point $(4,-3)$ is
A) 1 unit
B) 5 units
C) 7 units
D) -1 units
6) Distance between the points $P(-6,8)$ and $Q(0,0)$ is
A) 2 units
B) 4 units
C) 10 units
D) 14 units
7) The distance of point $P(x, y)$ from the origin is 5 Units then the co-ordinates of point $P$ are.
A) $(-2,3)$
B) $(1,2)$
C) $(3,3)$
D) $(3,4)$
8) Co-ordinates of origin are.
A) $(1,1)$
B) $(-1,0)$
C) $(0,1)$
D) $(0,0)$
9) The coordinates of the mid point of of line joining the points (2,3)and (4, 7) are $(3, b)$ then the value of $b$ is
A) 2
B) 4
C) 5
D) 10
10) $\left(\frac{a}{3}, 4\right)$ are the co-ordinates of the midpoint of line joining the points $(-6,5)$ and $(-2,3)$ then the value of ' $a$ ' is
A) -4
B) -12
C) 12
D) -6
11) $(-1,1)$ are the co-ordinates of the mid point of line $A B$ joining the points $A(-3, b)$ and $B(1, b+4)$, then the value of $b$ is
A) 1
B) -1
C) 2
D) 0
12) The perpendicular distance of point $A(3,5)$ from the $x$-axis is
A) 3 units
B) 5 units
C) 6 units
D) 8 units

## 1 Mark Questions (VSA)

13) Write the coordinates of the midpoint of the line joining the points
$P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$.
14) Find the distance between points $(2,3)$ and $(4,1)$
15) Find the distance between the origin and $(12,-5)$.
16) Find the co-ordinates of the midpoint of the line joining the points $(2,3)$ and $(4,7)$

| Ans | 1) A | 2) B | 3) C | 4) A | 5) B | 6) C | 7) D | 8) D | 9) C |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10) В | 11) В | 12) В | 13) ( $\left.\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$ | 14) $2 \sqrt{2}$ Units | 15) 13 Units | 16)(3,5) |  |  |  |

17) Find the distance of the following points from the origin.
i) $(6,-8)$
ii) $(4,-3)$
iii) $(5,-5)$
iv) $(12,-5)$
v) $(-6,8)$

2 Marks Questions (SA)
18) Find the co-ordinates of the midpoint of the line segment joining the points $(8,5)$ and $(6,3)$.
Solution: $\left(x_{1}, y_{1}\right)=(8,5)$ and $\left(x_{2}, y_{2}\right)=(6,3)$
The co-ordinates of midpoint $(x, y)=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
$(x, y)=\left(\frac{8+6}{2}, \frac{5+3}{2}\right)$
$(x, y)=\left(\frac{14}{2}, \frac{8}{2}\right)$
$(x, y)=(7,4)$
19) Find the co-ordinates of the midpoint of the line segment joining by the following pairs of points.
i) $(8,3)(8,-7)$
ii) $(6,5)(4,4)$
iii) $(2,0)(0,3)$
iv) $(2,8)(6,8)$
v) $(4,6)(6,-3)$
20) Derive the formula to find the distance between the two points $P\left(x_{1}, y_{1}\right)$ and $Q\left(x_{2}, y_{2}\right)$ in the cartecian plane.

Solution: Given two points

$$
\mathrm{P}\left(x_{1}, y_{1}\right) \text { and } \mathrm{Q}\left(x_{2}, y_{2}\right)
$$

As shown in the figure, draw PR and QS perpendicular to X -axis. Also draw $\mathrm{PT} \perp \mathrm{QS}$.
$\mathrm{OR}=x_{1}, \quad \mathrm{OS}=x_{2}$
$\mathrm{RS}=\mathrm{PT}=x_{2}-x_{1}$
$\mathrm{RP}=\mathrm{ST}=y_{1}, \quad \mathrm{QS}=y_{2}$
$\mathrm{TQ}=y_{2}-y_{1}$


From the $\triangle \mathrm{PTQ}$
$\mathrm{PQ}^{2}=\mathrm{PT}^{2}+\mathrm{TQ}^{2}(\because$ Pythagoras theorem $)$
$\mathrm{PQ}^{2}=\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}$
$\therefore \mathrm{PQ}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$ units.
21) Find the distance between the points $(0,0)$ and $(36,15)$.

Solution: $\left(x_{1}, y_{1}\right)=(0,0)$ and $\left(x_{2}, y_{2}\right)=(36,15)$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$d=\sqrt{(36-0)^{2}+(15-0)^{2}}$
$d=\sqrt{(36)^{2}+(15)^{2}}$
$d=\sqrt{1296+225}$
$d=\sqrt{1521}$
$d=39$ units.
22) Find the distance between $(-5,-7)$ and ( $-1,3$ ).
23) Find the distance between ( $\mathrm{a}, \mathrm{b}$ ) and ( $-\mathrm{a},-\mathrm{b}$ ).
24) Find the distance between the following pair of points.
i) $(6,4) \&(3,1)$
ii) $(8,6) \&(3,1)$
iii) $(6,4) \&(3,1)$
iv) $(1,7) \&(4,2)$
v) $(-1,-1) \&(-4,4)$
25) If the distance between $P(2,-3)$ and $Q(10, y)$ is 10 units find the value of $y$.
26) Find the point on the $x$ - axis from which $(7,6)$ and $(-3,4)$ are equidistant.

Solution:- $(7,6)$ and $(-3,4)$
The point $(x, 0)$ is equidistant from them.
$(x-7)^{2}+(0-6)^{2}=\left[(x-(-3)]^{2}+(0-4)^{2}\right.$
$(x-7)^{2}+36=(x+3)^{2}+16$
$x^{2}-14 x+49+36=x^{2}+6 x+9+16$
$85-25=20 x$
$20 x=60$
$x=3$
The point on x - axis is $(x, 0)=(3,0)$
27) Find the point on the $x$-axis which is equidistant from points $(2,-5)$ and $(-2,9)$.
28) If the point $\mathrm{P}(x, y)$ is equidistant from points $(6,2)$ and $(-2,6)$, prove that $y=2 x$.
29) $\mathbf{A}(6,1), \mathbf{B}(8,2), \mathbf{C}(9,4)$ and $D(\mathbf{p}, 3)$ are the vertices of a parallelogram. Find the value of ' $p$ '.
Solution :-The diagonals of a parallelogram bisect each other.
The co-ordinates of the mid point of $\mathrm{AC}=$ The coordinates of the midpoint of BD
$\mathrm{M}(x, y)=\left(\frac{6+9}{2}, \frac{1+4}{2}\right)=\left(\frac{\mathrm{p}+8}{2}, \frac{3+2}{2}\right)$
$\mathrm{M}(x, y)=\left(\frac{15}{2}, \frac{5}{2}\right)=\left(\frac{\mathrm{p}+8}{2}, \frac{5}{2}\right)$
$\therefore \frac{\mathrm{p}+8}{2}=\frac{15}{2}$

$\mathrm{p}+8=15$
$\mathrm{p}=15-8$
$\mathrm{p}=7$

## 3 Marks Questions (LA -1)

30) Show that the following points form an isosceles triangle .
$\mathbf{A}(5,-2), \mathbf{B}(6,4)$ and $\mathbf{C}(7,-2)$
Solution : $\mathrm{d}=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$\mathrm{AB}=\sqrt{(6-5)^{2}+[4-(-2)]^{2}}$
$\mathrm{AB}=\sqrt{(1)^{2}+(6)^{2}}$
$B(6,4)$

$A B=\sqrt{1+36}=\sqrt{37}$ Units
$\mathrm{BC}=\sqrt{(6-7)^{2}+[4-(-2)]^{2}}$
$\mathrm{BC}=\sqrt{(-1)^{2}+(6)^{2}}$
$B C=\sqrt{1+36}=\sqrt{37}$ Units
$\mathrm{AC}=\sqrt{(7-5)^{2}+[-2-(-2)]^{2}}$
$\mathrm{AC}=\sqrt{(2)^{2}+(-2+2)^{2}}$
$\mathrm{AC}=\sqrt{4+0}=2$ units.
$\therefore \mathrm{AB}=\mathrm{BC}=\sqrt{37}$ units.
The given points form an isosceles triangle.
31) Show that $(3,0)(6,4)(-1,3)$ are the vertices of a right angled triangle.
32) Show that $(9,0)(9,6)(-9,6)$ and $(-9,0)$ are the vertices of a rectangle.
33) Find the area of a triangle whose vertices $\operatorname{are}(10,-6)(2,5)$ and $(-1,3)$.

Solution: $x_{1}=10, \quad x_{2}=2, \quad x_{3}=-1$,

$$
y_{1}=-6, \quad y_{2}=5, \quad y_{3}=3
$$

The area of a triangle $=\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$
$=\frac{1}{2}[10(5-3)+2(3-(-6)+-1(-6-5)]$
$=\frac{1}{2}[10(2)+2(9)+-1(-11)]$
$=\frac{1}{2}(20+18+11)$
$=\frac{1}{2}(49)$
$=\frac{49}{2}=24.5$ square units.
34) Find the area of the triangle having the following vertices.
i) $(2,-2),(-2,1),(5,2) \quad$ ii) $(2,3),(-1,0),(2,-4)$
iii) $(-5,7),(-4,-5),(4,5) \quad$ iv) $(-5,-1),(3,-5),(5,2)$
v) $\mathrm{A}(3,8) \mathrm{B}(-4,2), \mathrm{C}(5,-1)$ vi) $\mathrm{A}(1,-1), \mathrm{B}(-4,6), \mathrm{C}(-3,-5)$
35) If the points $(-3,12),(7,6)$ and $(x, 9)$ are collinear find the value of $\boldsymbol{x}$.

Solution:

$$
\begin{array}{ll}
x_{1}=-3, & x_{2}=7, \\
y_{3}=x \\
y_{1}=12, & y_{2}=6,
\end{array}
$$

If the points are collinear then the area of the triangle is zero.
$\therefore \frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]=0$
$\frac{1}{2}[-3(6-9)+7(9-12)+x(12-6)]=0$
$\frac{1}{2}[-3(-3)+7(-3)+x(6)]=0$
$\frac{1}{2}[9-21+6 x]=0$
$\frac{1}{2}[-12+6 x]=0$
$-12+6 x=0$
$6 x=12$
$\therefore x=2$
Alternate Method: $\quad$ Area of Triangle $=\frac{1}{2}$

$0=\frac{1}{2}[\{(-3) 6+7(9)+x(12)\}-\{12(7)+6(x)+9(-3)\}]$
$0=\frac{1}{2}[\{-18+63+12 x\}-\{84+6 x-27\}]$
$0=\frac{1}{2}[\{45+12 x\}-\{57+6 x\}]$
$0=\frac{1}{2}[45+12 x-57-6 x]$
$0=\frac{1}{2}[6 x-12]$
$6 x-12=0$
$\therefore x=2$
36) Find the value of ' $P$ ' if the following points are collinear.
i) $(3,2),(4, p),(5,3)$
ii) $(-3,9),(2, p),(4,-5)$
37) Show that the points $(1,-1),(5,2)$ and $(9,5)$ are collinear using distance formula.
38) Find the co-ordinates of the points which divides the line segment joining the points $(-5,11)$ and $(4,-7)$ in the ratio $7: 2$.

Solution: $-\left(x_{1}, y_{1}\right)=(-5,11), \quad\left(x_{2}, y_{2}\right)=(4,-7), \quad m_{1}: m_{2}=7: 2$
$\mathrm{P}(x, y)=\left(\frac{\mathrm{m}_{1} x_{2}+\mathrm{m}_{2} x_{1}}{\mathrm{~m}_{1}+\mathrm{m}_{2}}, \quad \frac{\mathrm{~m}_{1} y_{2}+\mathrm{m}_{2} y_{1}}{\mathrm{~m}_{1}+\mathrm{m}_{2}}\right)$
$\mathrm{P}(x, y)=\left(\frac{7(4)+2(-5)}{7+2}, \quad \frac{7(-7)+2(11)}{7+2}\right)$
$\mathrm{P}(x, y)=\left(\frac{28-10}{9}, \quad \frac{-49+22}{9}\right)$
$\mathrm{P}(x, y)=\left(\frac{18}{9}, \quad \frac{-27}{9}\right)$
$\mathrm{P}(x, y)=(2,-3)$
39) Find the ratio in which the point $(2,5)$ divides the line segment joining $(8,2)$ and ( $-6,9$ ).
40) Find the ratio in which $(-6$, a) divides the line segment joining $(-3,-1)$ and $(-8,9)$ and find the value of ' $a$ '.
41) $O$ is the origin. $B(-6,9)$ and $C(12,-3)$ are the vertices of the $\triangle A B C$. If the point $P$ divides $O B$ in the ratio $1: 2$ and the point $Q$ divides $O C$ in the ratio $1: 2$. show that $P Q=\frac{1}{3} B C$.

## 4 Marks Questions (LA-2)

42) In the adjoining figure $D, E$ and $F$ are the mid points of $A B, B C$ and $A C$ respectively. Find the area of the triangle DEF.
Solution: D, E and F are the mid points of $\mathrm{AB}, \mathrm{BC}$ and AC respectively.


The coordinates of mid point $\mathrm{D}=\left(\frac{4+2}{2}, \frac{4+2}{2}\right)=(3,3)=\left(x_{1}, y_{1}\right)$
The coordinates of mid point $\mathrm{E}=\left(\frac{4+2}{2}, \frac{4+6}{2}\right)=(3,5)=\left(x_{2}, y_{2}\right)$
The coordinates of mid point $\mathrm{F}=\left(\frac{2+2}{2}, \frac{6+2}{2}\right)=(2,4)=\left(x_{3}, y_{3}\right)$
$\therefore x_{1}=3, \quad x_{2}=3, \quad x_{3}=2 \quad$ and $\quad y_{1}=3, \quad y_{2}=5, \quad y_{3}=4$

Area of the $\Delta \mathrm{DEF}=\frac{1}{2}\left[x_{1}\left(y_{2}-y_{3}\right)+x_{2}\left(y_{3}-y_{1}\right)+x_{3}\left(y_{1}-y_{2}\right)\right]$
$=\frac{1}{2}[3(5-4)+3(4-3)+2(3-5)]$
$=\frac{1}{2}[3(1)+3(1)+2(-2)]$
$=\frac{1}{2}[3+3-4]$
$=\frac{1}{2}[2]$
$=1$ Square unit.

## 5 Marks Questions (LA -3)

43) In the adjoining figure $D(3,3), E(3,5)$ and $F(2,4)$ are the mid points of $A B, B C$ and $A C$ respectively. Find the coordinates of the vertices of the triangle $A B C$.


Solution: D, E and F are the mid points of $\mathrm{AB}, \mathrm{BC}$ and AC respectively.
The coordinates of mid point $\mathrm{D}(3,3)=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
The coordinates of mid point $\mathrm{D}\left(\frac{6}{2}, \frac{6}{2}\right)=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
$x_{1}+x_{2}=6 \quad--->(1)$ and $y_{1}+y_{2}=6 \quad---->(2)$
The coordinates of mid point $\mathrm{E}(3,5)=\left(\frac{x_{2}+x_{3}}{2}, \frac{y_{2}+y_{3}}{2}\right)$
The coordinates of mid point $\mathrm{E}\left(\frac{6}{2}, \frac{10}{2}\right)=\left(\frac{x_{2}+x_{3}}{2}, \frac{y_{2}+y_{3}}{2}\right)$
$x_{2}+x_{3}=6 \quad--->(3)$ and $y_{2}+y_{3}=10 \quad---->(4)$
The coordinates of mid point $\mathrm{F}(2,4)=\left(\frac{x_{1}+x_{3}}{2}, \frac{y_{1}+y_{3}}{2}\right)$
The coordinates of mid point $\mathrm{F}\left(\frac{4}{2}, \frac{8}{2}\right)=\left(\frac{x_{1}+x_{3}}{2}, \frac{y_{1}+y_{3}}{2}\right)$
$x_{1}+x_{3}=4---\gg(5)$ and $y_{1}+y_{3}=8$

| By adding (1), (3)\&(5) | By adding (2), (4) \& (6) |
| :--- | :--- |
| $2 x_{1}+2 x_{2}+2 x_{3}=16$ | $2 y_{1}+2 y_{2}+2 y_{3}=24$ |
| $x_{1}+x_{2}+x_{3}=8$ | $y_{1}+y_{2}+y_{3}=12$ |
| $\therefore x_{1}=8-6=2$ | $\therefore y_{1}=12-10=2$ |
| $\therefore x_{2}=8-4=4$ | $\therefore y_{2}=12-8=4$ |
| $\therefore x_{3}=8-6=2$ | $\therefore y_{3}=12-6=6$ |
| $\therefore \mathrm{~A}\left(x_{1}, y_{1}\right)=\mathrm{A}(2,2)$, | $\mathrm{B}\left(x_{2}, y_{2}\right)=\mathrm{B}(4,4)$, |

## Unit-8: REAL NUMBERS

## 1 MARKS QUESTIONS (MCQ)

1) 72 and 28 can be expressed using Euclid's division algorithm as
A) $28=(72-16) \times 2$
B) $72=(28 \times 2)+16$
C) $72=(28 \times 2)-16$
D) $16=72-(28+2)$
2) The HCF of 26 and 91 is
A) 7
B) 13
C) 20
D) 26
3) If the HCF of 6 and 20 is 2 , then the LCM is
A) 40
B) 120
C) 60
D) 240
4) Which of the following number is not a product of prime factors
A) 35
B) 26
C) 23
D) 15
5) If $x=\frac{p}{q}(q \neq 0)$ is a rational number having terminating decimal expression then the factor of ' $q$ ' are in the form
A) $2^{n} .5^{m}$ here $m, n$ are, non negative integers
B) $3^{\mathrm{n}} .5^{\mathrm{m}}$ here $\mathrm{m}, \mathrm{n}$ are non positive integers
C) $5^{\mathrm{n}} .7^{\mathrm{m}}$ here $\mathrm{m}, \mathrm{n}$ are non negative integers
D) $2^{\mathrm{n}} .7^{\mathrm{m}}$ here $\mathrm{m}, \mathrm{n}$ are non positive integers

$$
1 \text { Mark Questions (VSA) }
$$

6) If the HCF of 14 and 21 is 7 . Find their LCM.
7) If $\operatorname{HCF}(336,54)=6$, find $\operatorname{LCM}(336,54)$.
8) Find the LCM of 18 and 45.
9) Express 156 as a product of its prime factors.
10) The HCF of two numbers $a$ and $b$ is 5 and their LCM is 200 . Find the product ab.
11) State Euclid's division algorithm.
12) According to Euclid's division algorithm if $a=b q+r$, write all the possible values of $r$.

13) Express the following numbers as a product of prime factors
i) 140
ii) 120
iii) 1173
iv) 404
v) 210
vi) 715
vii) 336

2 Marks Questions (SA)
14) Find the HCF of 135 and 125 using Euclid's division.

Solution: - Applying Euclid's division algorithm
Step 1: $225=(135 \times 1)+90$
Step 2: $135=(90 \times 1)+45$
Step 3: $90=(45 \times 2)+0$
Now the remainder is 0
$\therefore$ The H.C.F $=45$
15) Find the H.C.F of the following numbers using Euclid's division algorithm.
(i) 255 and 867
(ii) 42 and 455
16) Find the H.C.F. and L.C.M. of 6,72 and 120 using prime factorization method.

Solution: $6=2 \times 3=2^{1} \times 3^{1}$
$72=2 \times 2 \times 2 \times 3 \times 3=2^{3} \times 3^{2}$
$120=2 \times 2 \times 2 \times 3 \times 5=2^{3} \times 3^{1} \times 5^{1}$
H.C.F $=2^{1} \times 3^{1}=6(\because \mathrm{HCF}=$ product of common prime factors $)$
L.C.M. $=2^{3} \times 3^{2} \times 5^{1}$
L.C.M. $=8 \times 9 \times 5$
L.C.M. $=360$
17) Find the H.C.F and L.C.M of the following numbers using prime factorization method.
(i) 12,15 and 21
(ii) 17,23 and 29
(iii) 8,9 and 25
18) Find the least number which is divisible by 306 and 657.

Solution: $306=2 \times 3 \times 3 \times 17=2^{1} \times 3^{2} \times 17$
$657=3 \times 3 \times 73=3^{2} \times 73$
L.C.M. $=2^{1} \times 3^{2} \times 17 \times 73$
L.C.M. $=18 \times 17 \times 73$
L.C.M. $=22338$

## Alternate Method:

Solution: - Applying Euclid's division algorithm
Step 1: $657=(306 \times 2)+45$
Step 2: $306=(45 \times 6)+36$
Step 3: $45=(36 \times 1)+9$
Step 4: $36=(9 \times 4)+0$ Now the remainder is 0
$\therefore$ The H.C.F $=9$
L.C.M. $=\frac{306 \times 657}{9} \quad\left[\because\right.$ L.C. $\left.M=\frac{\text { Product of numbers }}{\text { H.C.F }}\right]$
L.C.M. $=34 \times 657$
L.C.M. $=22338$
19) There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?
Solution: - they will meet again after a time equal to L.C.M of the time taken.
$18=2 \times 3 \times 3=2 \times 3^{2}$
$12=2 \times 2 \times 3=2^{2} \times 3$
L.C.M. $=2^{2} \times 3^{2}=36$
$\therefore$ They meet again at the starting point after 36 minutes.
20) Prove that $5-\sqrt{3}$ is an irrational number.

Solution: - Let us assume that $5-\sqrt{3}$ is a rational number.
That is $5-\sqrt{3}=\frac{p}{q}(p, q \in z$ and $\mathrm{q} \neq 0) p, q$ are co- primes.
$5-\frac{p}{q}=\sqrt{3}$
$\sqrt{3}=\frac{5 q-p}{q}$
$\Rightarrow \sqrt{3}$ is a rational number [ $\because \frac{5 q-p}{q}$ is a rational number.]
But this contradicts the fact that $\sqrt{3}$ is an irrational.
$\therefore$ our assumption is wrong.
Hence $5-\sqrt{3}$ is an irrational number.
21) If $\sqrt{2}$ is an irrational number, prove that $6+\sqrt{2}$ is an irrational number.
22) Prove that the following are irrational numbers.
i) $3 \sqrt{2}$
ii) $5+\sqrt{3}$
iii) $3+2 \sqrt{5}$
iv) $3-2 \sqrt{5}$
v) $\frac{2+\sqrt{3}}{5}$
vi) $\sqrt{2}+\sqrt{3}$
23) Show that $\frac{35}{50}$ has terminating decimal expension without long division.

Solution: $50=2 \times 5 \times 5=2^{1} \times 5^{2}$
$\frac{35}{50}=\frac{35}{2^{1} \times 5^{2}}$
The denominator is in the form $2^{n} \times 5^{m}$ and $n=1, m=2$ are non negative integers
$\therefore$ This is a terminating decimal expansion.
24) Without long method of division show that $\frac{77}{210}$ has non terminating recurring decimal expansion.
Solution: $210=2 \times 3 \times 5 \times 7$
$\frac{77}{210}=\frac{77}{2 \times 3 \times 5 \times 7}$
The denominator is not in the form $2^{n} \times 5^{m}$.
$\therefore$ This is a non terminating recurring decimal expansion.
25) Without long division method, find whether the following rational numbers have terminating decimal expansions.
i) $\frac{17}{8}$
ii) $\frac{64}{455}$
iii) $\frac{29}{343}$
iv) $\frac{23}{200}$
26) Find after how many places of decimal the decimal form of the number $\frac{27}{2^{3} \cdot 5^{4} \cdot 3^{2}}$ will terminate.
Solution: $\frac{27}{2^{3} \cdot 5^{4} \cdot 3^{2}}$
$=\frac{27}{8 \times 625 \times 9}$
$=\frac{3}{8 \times 625}$
$=\frac{3}{5000}$
$=\frac{3 \times 2}{5000 \times 2}$
$=\frac{6}{10000}$
$=0.0006$
$\therefore$ After 4 decimal places it terminates.

## UNIT - 9: POLYNOMIALS

1 Mark Questions (MCQ)

1) In the adjoining figure, graph of a polynomial is given. The number of zeros of the polynomial is,
A) 1
B) 2
C) 3
D) 4

2) Which of the following is a zero of the polynomial $x^{2}+4 x+4$ ?
A) 2
B) -2
C) 4
D) -4
3) The degree of the polynomial $-4 x^{2}+5 x^{3}+x-\sqrt{2}$ is
A) 0
B) 1
C) 2
D) 3
4) $\mathrm{f}(x)=x^{2}-9 x+20$ is a quadratic polynomial. The value of $f(0)$ is
A) 20
B) 11
C) -20
D) 29
5) If the zero of the polynomial $x^{2}+k x+4$ is -2 , then the value of ' $k$ ' is
A) 4
B) -2
C) -4
D) 2
6) Which of the following is a zero of the polynomial $x^{2}-3$ ?
A) 3
B) -3
C) $\sqrt{3}$
D) 9
7) If $x^{5}+a^{5}$ is divided by $(x+a)$, the remainder is
A) $a^{5}$
B) $2 a^{5}$
C) 0
D) 5
8) If $x^{5}+a^{5}$ is divided by $(x-a)$, the remainder is
A) $a^{5}$
B) $2 a^{5}$
C) 0
D) 5
9) If $(x-7)$ is the factor of $\left(x^{2}-k\right)$ then the value of ' $k$ ' is
A) 49
B) 7
C) -7
D) -49
10) If the zeros of the polynomial $3 x^{2}+8 x+k$ are reciprocal to each other, then the value of ' $k$ ' is
A) 3
B) -3
C) $\frac{1}{3}$
D) $-\frac{1}{3}$
11) If the sum of the zeros of the quadratic polynomial $k x^{2}+2 x+3 k$ is equal to the product of its zeros, then the value of $k$ is
A) $\frac{1}{3}$
B) $-\frac{1}{3}$
C) $\frac{2}{3}$
D) $-\frac{2}{3}$
12) The zeros of the quadratic polynomial $x^{2}+88 x+125$ are,
A) Both positive
B) Both negative
C) One positive and one negative
D) Both equal.

## 1 Mark Questions (VSA)

13) What is the maximum number of zeros of the polynomial $x^{3}+x+2+4 x^{5}$ ?
14) Write the degree of the polynomial $2-x^{3}$.
15) If $\mathrm{p}(x)=2 x^{2}+3 x+2$ then, find the value of $\mathrm{p}(2)$.
16) If $\mathrm{f}(x)=x^{2}-4$ then, find the value of $\mathrm{f}(4)$.
17) If $\mathrm{f}(x)=7 x^{2}+2 x+14$ then, find the value of $\mathrm{f}(-1)$.
18) Write the general form of a linear polynomial.
19) What is the degree of a linear polynomial?
20) Write the general form of a quadratic polynomial.
21) What is the degree of a quadratic polynomial?
22) Write the general form of a Cubic polynomial.
23) What is the degree of a cubic polynomial?
24) $\mathrm{f}(x)=10$, which type of a polynomial is this?
25) Find the zero of the polynomial $\mathrm{f}(x)=3 x+1$.


## 2 Marks Questions (SA)

26) If the sum of zeros is $\sqrt{2}$ and the product of zeros is $\frac{1}{3}$ find the polynomial.

Solution:- Let the zeros be $\alpha$ and $\beta$.
$\alpha+\beta=\sqrt{2}$ and $\alpha \beta=\frac{1}{3}$
the desired polynomial is $x^{2}-(\alpha+\beta) x+\alpha \beta$
$=x^{2}-(\sqrt{2}) x+\frac{1}{3}$
$=3 x^{2}-3 \sqrt{2} x+1$ (multiply each term by 3 )
$\therefore$ the required polynomial is $3 x^{2}-3 \sqrt{2} x+1$
27) Find the polynomial whose sum of zeros is $\frac{1}{4}$ and the product of the zeros is -1 .
28) Find the polynomial whose sum of zeros is $\frac{1}{4}$ and the product of the zeros is $-\frac{1}{4}$.
29) Find the polynomial whose sum of zeros is -3 and the product of the zeros is 2 .
30) Find a polynomial whose zeros are $\sqrt{3}$ and $-\sqrt{3}$.

Solution:- Let the zeros be $\alpha$ and $\beta$.
$\alpha=\sqrt{3}$
$\beta=-\sqrt{3}$
$\alpha+\beta=\sqrt{3}+(-\sqrt{3})=0$
$\alpha \beta=(\sqrt{3})(-\sqrt{3})=-(\sqrt{3})^{2}=-3$
The desired polynomial is $x^{2}-(\alpha+\beta) x+\alpha \beta$
$=x^{2}-(0) x+(-3)$
$=x^{2}-3$
$\therefore$ The required polynomial is $x^{2}-3$.
31) If the product of the zeros of the polynomial $k x^{2}-6 x-6$ is 4 , find the value of $k$.

Solution:- Let the zeros be $\alpha$ and $\beta$.
Comparing $k x^{2}-6 x-6$ with $a x^{2}+b x+c$
$a=k, \quad b=-6, \quad c=-6$
$\alpha \cdot \beta=\frac{c}{a}$
$4=\frac{-6}{k} \quad[\because$ product of the zeros $=4]$
$k=\frac{-6}{4}$
$\therefore k=\frac{-3}{2}$
32) If the difference of the zeros of the polynomial $3 x^{2}-x-4$ is $\frac{7}{3}$, find the difference of the squares of the zeros.
Solution:- Let the zeros be $\alpha$ and $\beta$.
Comparing $3 x^{2}-x-4$ with $a x^{2}+b x+c$
$a=3, \quad b=-1, \quad c=-4$
$\alpha-\beta=\frac{7}{3} \quad--->(1) \quad(\because$ data $)$
$\alpha+\beta=\frac{-b}{a}$
$\alpha+\beta=\frac{-(-1)}{3}=\frac{1}{3}$
$\alpha^{2}-\beta^{2}=(\alpha+\beta)(\alpha-\beta)[\because$ From (1) and (2)]
$\alpha^{2}-\beta^{2}=\frac{1}{3} \times \frac{7}{3}$
$\therefore \alpha^{2}-\beta^{2}=\frac{7}{9}$
3 Marks Questions (LA-1)
33) If one zero of the polynomial $\left(a^{2}+9\right) x^{2}+13 x+6 a$ is reciprocal of the other, find the value of $a$.
Solution: comparing $\left(a^{2}+9\right) x^{2}+13 x+6 a$ with $a x^{2}+b x+c$,
$a=\left(a^{2}+9\right), \quad b=13, \quad c=6 a$
Let the zeros be $\alpha$ and $\frac{1}{\alpha} \quad(\because$ zeros are reciprocal to each other $)$
$\alpha . \beta=\frac{c}{a}$
$\alpha \cdot \frac{1}{\alpha}=\frac{6 a}{\left(a^{2}+9\right)}$
$1=\frac{6 a}{\left(a^{2}+9\right)}$
$\therefore\left(a^{2}+9\right)=6 a$
$\therefore a^{2}-6 a+9=0$
$(a-3)^{2}=0$

$$
a-3=0
$$

$\therefore a=3$
34) Find the zeros of the polynomial $6 x^{2}-3-7 x$. Also verify the relation between zeros and the co-efficient.
Solution:- $6 x^{2}-3-7 \boldsymbol{x}$
$=6 x^{2}-7 x-3$
$=6 x^{2}-9 x+2 x-3$
$=3 x(2 x-3)+1(2 x-3)$

$$
\begin{gathered}
6 \times 3=18 \\
9 \times 2=18 \\
-9+2=-7
\end{gathered}
$$

$=(3 x+1)(2 x-3)$
$\Rightarrow$ The zeros, $x=-\frac{1}{3}$ and $x=\frac{3}{2}$
The sum of the zeros $=-\frac{1}{3}+\frac{3}{2}=\frac{-2+9}{6}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\operatorname{Co}-\mathrm{efficient} \text { of } x)}{\left(\text { Co-efficient of } x^{2}\right)}$
The product of the zeros $=-\frac{1}{3} \times \frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text { Constant }}{\text { Co-efficient of } x^{2}}$
35) Find the zeros of the polynomial $4 s^{2}-4 s+1$, and verify the relation between zeros and the co-efficient.
36) Find the zeros of the polynomial $x^{2}-\frac{x}{3}-\frac{4}{3}$, and verify the relation between zeros and the co-efficient.
37) Find the zeros of the polynomial $4 u^{2}-8 u$, and verify the relation between zeros and the co-efficient.
38) If $\alpha, \beta$ and $\gamma$ are zeros of the polynomial $3 x^{3}-5 x^{2}-11 x-3$, find the value of
(i) $\alpha+\beta+\gamma$
(ii) $\alpha \boldsymbol{\beta}+\boldsymbol{\beta} \boldsymbol{\gamma}+\boldsymbol{\gamma} \boldsymbol{\alpha}$ (iii) $\alpha \boldsymbol{\beta} \boldsymbol{\gamma}$

Solution: compare $3 x^{3}-5 x^{2} x-11 x-3$ with $a x^{3}+b x^{2}+c x+d$,
$a=3, \quad b=-5, \quad c=-11, \quad d=-3$
(i) $\alpha+\beta+\gamma=\frac{-b}{a}$
$\alpha+\beta+\gamma=\frac{-(-5)}{3}=\frac{5}{3}$
(ii) $\alpha \beta+\beta \gamma+\gamma \alpha=\frac{c}{a}$
$\alpha \beta+\beta \gamma+\gamma \alpha=\frac{-11}{3}$
(iii) $\alpha \beta \gamma=\frac{-d}{a}$
$\alpha \beta \gamma=\frac{-(-3)}{3}=1$
39) If $\alpha$ and $\beta$ are zeros of the polynomial $x^{2}-2 x-8$, find the value of $\alpha^{2} \beta+\alpha \beta^{2}$.

Solution: compare $x^{2}-2 x-8$ with $a x^{2}+b x+c$,
$a=1, \quad b=-2, \quad c=-8$.
Sum of the zeros $\alpha+\beta=\frac{-b}{a}$
$\alpha+\beta=\frac{-(-2)}{1}=2$
Product of the zeros $\alpha \beta=\frac{c}{a}$
$\alpha \beta=\frac{-8}{1}=-8$
$--->(1)$
$\alpha^{2} \beta+\alpha \beta^{2}=\alpha \beta(\alpha+\beta)$
$\alpha^{2} \beta+\alpha \beta^{2}=-8(2) \quad[\because$ From (1) and (2)]
$\alpha^{2} \beta+\alpha \beta^{2}=-16$
40) If $\alpha$ and $\beta$ are zeros of the polynomial $4 x^{2}-4 x-1$, find the value of the following.
(i) $\frac{1}{\alpha}+\frac{1}{\beta}$
(ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$
(iii) $\frac{1}{\alpha}-\frac{1}{\beta}$
(iv) $\alpha^{2}+\beta^{2}$
(v) $\alpha^{3}+\beta^{3}$
(vi) $\alpha^{3} \beta^{2}+\alpha^{2} \beta^{3}$
(vi) $\frac{1}{\alpha^{2}}+\frac{1}{\beta^{2}}$
(vi) $\frac{1}{\alpha^{3}}+\frac{1}{\beta^{3}}$
41) Divide the polynomial $x^{4}-3 x^{2}+4 x+5$ by the polynomial $x^{2}+1-x$ and find the quotient and remainder. And verify the result.
Solution :-

$$
\begin{aligned}
& x^{2}-x+1 \sqrt{x^{2}+x-3} \begin{array}{c}
x^{4}+0 x^{3}-3 x^{2}+4 x+5 \\
x^{4}-x^{3}+x^{2}
\end{array} \\
& x^{3}-4 x^{2}+4 x+5 \\
& \underset{(-))_{(+)}^{x^{3}}-x^{2}+x}{(-)} \\
& -3 x^{2}+3 x+5 \\
& \underset{(+)}{-3 x^{2}}+\underset{(-)}{ } 3 x-3 \\
& 8
\end{aligned}
$$

$\therefore$ Quotient $=x^{2}+x-3$ and remainder $=8$
Dividend $=$ Divisor $\times$ Quotient + Remainder
$x^{4}-3 x^{2}+4 x+5=\left(x^{2}-x+1\right)\left(x^{2}+x-3\right)+8$
$x^{4}-3 x^{2}+4 x+5=x^{4}+x^{3}-3 x^{2}-x^{3}-x^{2}+3 x+x^{2}+x-3+8$
$x^{4}-3 x^{2}+4 x+5=x^{4}-3 x^{2}+4 x+5$
LHS $=$ RHS
42) Dived the polynomial $x^{3}-3 x^{2}+5 x-3$ by the polynomial $x^{2}-2$ and find the quotient and remainder.
43) Dived the polynomial $x^{4}-5 x+6$ by the polynomial $2-x^{2}$ and find the quotient and remainder.
44) Divide the polynomial $3 x^{4}+5 x^{3}-7 x^{2}+2 x+2$ by the polynomial $x^{2}+3 x+1$. Verify if the second polynomial is a factor of the first polynomial.

Solution :-

$$
\begin{aligned}
& x^{2}+3 x+1 \sqrt{\frac{3 x^{2}-4 x+2}{3 x^{4}+5 x^{3}-7 x^{2}+2 x+2}} \\
& -4 x^{3}-10 x^{2}+2 x+2 \\
& -4 x_{(+)}^{3}-12 x^{2}-4 x \\
& 2 x^{2}+6 x+2 \\
& 2 x^{2}+6 x+2 \\
& 0
\end{aligned}
$$

$\therefore$ Quotient $=3 x^{2}-4 x+2$ and remainder $=0$
Since the remainder is zero, $x^{2}+3 x+1$ is factor of $3 x^{4}+5 x^{3}-7 x^{2}+2 x+2$.
45) Divide the polynomial $2 t^{4}+3 t^{3}-2 t^{2}-9 t-12$ by the polynomial $t^{2}-3$ and verify if the second polynomial is a factor of the first polynomial .
46) Divide the polynomial $x^{5}-4 x^{3}+x^{2}+3 x+1$ by the polynomial $x^{3}-3 x+1$ and verify if the second polynomial is a factor of the first polynomial.
47) Obtain all other zeros of $3 x^{4}-15 x^{3}+13 x^{2}+25 x-30$, If of its two zeros are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution: let $f(x)=3 x^{4}-15 x^{3}+13 x^{2}+25 x-30$
$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the two zeros of $f(x)$.
$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)$ are the factors of $f(x)$.
$\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^{2}-\frac{5}{3}\right)$
$=\frac{3 x^{2}-5}{3}=0 \quad \Rightarrow 3 x^{2}-5=0$
$\therefore\left(3 x^{2}-5\right)$ divides $f(x)$.

$$
\begin{aligned}
& 3 x ^ { 2 } - 5 \longdiv { x ^ { 2 } - 5 x + 6 } \begin{array} { c } 
{ 3 x ^ { 4 } - 1 5 x ^ { 3 } + 1 3 x ^ { 2 } + 2 5 x - 3 0 } \\
{ 3 x ^ { 4 } - 5 x ^ { 2 } }
\end{array} \\
& \text { (-) ( }+ \text { ) } \\
& -15 x^{3}+18 x^{2}+25 x-30 \\
& \underset{(+)}{-15 x^{3}} \underset{(-)}{+25 x} \\
& \begin{array}{ll}
+18 x^{2} & -30 \\
+18 x^{2} & -30 \\
(-) & (+)
\end{array} \\
& f(x)=\left(3 x^{2}-5\right)\left(x^{2}-5 x+6\right) \\
& =(\sqrt{3} x+\sqrt{5})(\sqrt{3} x-\sqrt{5})(x-2)(x-3) \\
& f(x)=0, \quad \Rightarrow(\sqrt{3} x+\sqrt{5})=0, \quad(\sqrt{3} x-\sqrt{5})=0, \quad(x-2)=0, \quad(x-3)=0 \\
& \therefore \text { zeros are } \quad x=\sqrt{\frac{5}{3}}, \quad x=-\sqrt{\frac{5}{3}} \quad x=2, \quad x=3 \text {. }
\end{aligned}
$$

48) If 3 and -3 are the zeros of the polynomial $x^{4}+2 x^{3}-8 x^{2}-18 x-9$, find all the zeros of the polynomial.
49) If the polynomial $x^{3}-3 x^{2}+x+2$ is divided by $\mathrm{g}(x)$, then the remainder is $-2 x+4$ and quotient is $\boldsymbol{x}-\mathbf{2}$ Find the divisor $\mathrm{g}(\boldsymbol{x})$.
Solution :- $p(x)=x^{3}-3 x^{2}+x+2, \quad q(x)=x-2$ and $r(x)=-2 x+4$
$p(x)-r(x)=\left(x^{3}-3 x^{2}+x+2\right)-(-2 x+4)$
$p(x)-r(x)=x^{3}-3 x^{2}+x+2+2 x-4$
$p(x)-r(x)=x^{3}-3 x^{2}+3 x-2$
$\mathrm{g}(x)=\frac{p(x)-r(x)}{q(x)}$
$\mathrm{g}(x)=\frac{x^{3}-3 x^{2}+3 x-2}{x-2}$
$\therefore \mathrm{g}(x)=x^{2}-x+1$

$$
\begin{array}{r}
x-2 \begin{array}{c}
x^{2}-x+1 \\
\frac{x^{3}-3 x^{2}+3 x-2}{x^{3}-2 x^{2}} \\
(-)(+)
\end{array} \\
\frac{\begin{array}{l}
-x^{2}+3 x-2 \\
-x^{2}+2 x \\
(-)
\end{array}}{\substack{x-2 \\
(-) \\
(-)}} \\
0
\end{array}
$$

## UNIT - 10: Quadratic Equations

1 Mark Questions (MCQ)

1) Which of the following is a quadratic equation?
A) $x^{2}+x^{3}=2$
B) $p(p-3)=0$
C) $x^{2}=6+x^{2}-x$
D) $x^{2}+\frac{1}{x}=5$
2) If $x^{2}+1=101$ then the value of $x$ is
A) $\pm 1$
B) $\pm 10$
C) $\pm 11$
D) $\pm \sqrt{10}$
3) The value of discriminant of the quadratic equation $2 x^{2}-5 x-1=0$ is
A) 33
B) 3
C) 0
D) 35
4) The discriminant of the quadratic equation $a x^{2}+b x+c=0$ is
A) $b^{2}-a c$
B) $b^{2}-4 a c$
C) $\sqrt{b^{2}-4 a c}$
D) $b^{2}+4 a c$
5) In the quadratic equation $a x^{2}+b x+c=0$, if $\frac{b^{2}}{4}=a c$ then the roots of the equation are
A) Equal
B) Distinct
C) Additive inverse
D) Reciprocals.
6) In the quadratic equation $a x^{2}+b x+c=0$, if $a=c$ then the roots are
A) Even numbers
B) Odd numbers
C) Negative numbers
D) Reciprocals
7) The roots of equations $x^{2}=49$ are
A) 7 and -7
B) 24 and 5
C) 8 and -8
D) 7 and 0
8) The roots of equations $x^{2}-4=0$ are
A) 2 and 0
B) 2 and -2
C) 4 and 5
D) 1 and -1
9) The roots of equations $x^{2}-4 x=0$ are
A) 0 and 2
B) -4 and 0
C) -2 and 0
D) 0 and 4

## 1 Mark Questions(VSA)

10) If the roots of equation $a x^{2}+b x+c=0$ are real and equal, what is the value of the discriminant?
11) If $143=t^{2}-1$ then solve for $t$.
12) Write the general form of a quadratic equation.

| Ans | 1) B | 2) B | 3) A | 4) B | 5) A | 6) D | 7) A | 8) B |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 9) D | 10) $b^{2}-4 a c=0$ | $11) \pm 12$ | 12) $a x^{2}+b x+c=0(a \neq 0)$ |  |  |  |  |

## 2 Marks Questions (SA)

13) Verify if $x^{2}-2 x=(-2)(3-x)$ is a quadratic equation.
14) Find the roots of the equation $2 x^{2}-x+\frac{1}{8}=0$ by factorization method

Solution :- $2 x^{2}-x+\frac{1}{8}=0$
$16 x^{2}-8 x+1=0$ (Multiplying each term by 8 ) $\mid$ First term $=+16 x^{2}$, Last term $=+1$
$16 x^{2}-4 x-4 x+1=0$
$4 x(4 x-1)-1(4 x-1)=0$
$(4 x-1)(4 x-1)=0$

Product $=16 x^{2}=-4 x \times-4 x$
Middle term $=-8 x=-4 x-4 x$
$4 x-1=0$, or $4 x-1=0$
$4 x=1$ or $4 x=1$
$\therefore$ Roots $x=\frac{1}{4} \quad$ or $\quad x=\frac{1}{4}$
15) Find the roots of the following quadratic equations by factor method.
(i) $16 x^{2}-3 x-10=0$ (ii) $2 x^{2}+x-6=0$ (iii) $100 x^{2}-20 x+1=0$
16) Solve the quadratic equation $2 x^{2}-5 x+2=0$ by completing the square .

Solution :- $2 x^{2}-5 x+2=0$ this is in the form $\mathrm{a} x^{2}+\mathrm{b} x+\mathrm{c}=0$.
$a=2, \quad b=-5, \quad c=2$
$\left(x+\frac{b}{2 a}\right)^{2}=\left(\frac{b}{2 a}\right)^{2}-\frac{c}{a}$
$\left(x+\frac{-5}{2(2)}\right)^{2}=\left(\frac{-5}{2(2)}\right)^{2}-\frac{2}{2}$
$\left(x-\frac{5}{4}\right)^{2}=\frac{25}{16}-1$
$\left(x-\frac{5}{4}\right)^{2}=\frac{9}{16}($ Taking square root on both side $)$
$x-\frac{5}{4}= \pm \frac{3}{4}$
$x= \pm \frac{3}{4}+\frac{5}{4}$
$x=+\frac{3}{4}+\frac{5}{4} \quad$ or $\quad x=-\frac{3}{4}+\frac{5}{4}$
$x=\frac{8}{4} \quad$ or $\quad x=\frac{2}{4}$
$x=2$ or $x=\frac{1}{2}$
17) Solve the following equations by completing the square.
(i) $5 x^{2}-6 x-2=0$
(ii) $9 x^{2}-15 x+6=0$
(iii) $2 x^{2}-5 x+3=0$
18) Solve $4 x^{2}+4 \sqrt{3} x+3=0$ by using formula.

Solution: - $\quad 4 x^{2}+4 \sqrt{3} x+3=0$ this is in the form $\mathrm{a} x^{2}+\mathrm{b} x+\mathrm{c}=0$.
$\mathrm{a}=4, \quad \mathrm{~b}=4 \sqrt{3}, \quad \mathrm{c}=3$
Roots $x=\frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^{2}-4 \mathrm{ac}}}{2 \mathrm{a}}$
$x=\frac{-(4 \sqrt{3}) \pm \sqrt{(4 \sqrt{3})^{2}-4(4)(3)}}{2(4)}$
$x=\frac{-4 \sqrt{3} \pm \sqrt{16 \times 3-48}}{8}$

$$
\begin{aligned}
& x=\frac{-4 \sqrt{3} \pm \sqrt{48-48}}{8} \\
& x=\frac{-4 \sqrt{3} \pm 0}{8} \\
& x=\frac{-4 \sqrt{3}}{8} \quad \text { or } \quad x=\frac{-4 \sqrt{3}}{8} \\
& x=\frac{-\sqrt{3}}{2} \quad \text { or } \quad x=\frac{-\sqrt{3}}{2}
\end{aligned}
$$

19) Find the roots of the following equations by formula method.
(i) $2 x^{2}+x-4=0$
(ii) $2 x^{2}-7 x+3=0$
(iii) $2 x^{2}-5 x+2=0$
(iv) $x^{2}+2 x-15=0$
(v) $x^{2}-11 x+30=0$
(vi) $x^{2}-2 x=8$
(vii) $x^{2}-7 x+12=0$
(viii) $x^{2}-4=3 x$
(ix) $2 x^{2}-2 \sqrt{2} x=-1$
20) Show that the equation, $3 x^{2}-4 \sqrt{3} x+4=0$ has real and equal roots.

Solution :- $3 x^{2}-4 \sqrt{3} x+4=0$
$a=3, \quad b=-4 \sqrt{3}, \quad c=4$
Discriminant $=b^{2}-4 \mathrm{ac}$
$=(-4 \sqrt{3})^{2}-4(3)(4)$
$=48-48$
$=0$
$\therefore \mathrm{b}^{2}-4 \mathrm{ac}=0$
$\Rightarrow$ The equation has real and equal roots.
21) Find the discriminant of the equation $3 x^{2}-2 x+\frac{1}{3}=0$. From this find the nature of the roots. Find the roots if they are real.
Solution:- $3 x^{2}-2 x+\frac{1}{3}=0 \quad \mathrm{a}=3, \quad \mathrm{~b}=-2, \quad \mathrm{c}=\frac{1}{3}$
Discriminant $=b^{2}-4 \mathrm{ac}$
$=(-2)^{2}-4(3)\left(\frac{1}{3}\right)$
$=4-4=0$
$\therefore \mathrm{b}^{2}-4 \mathrm{ac}=0$
$\Rightarrow$ The equation has two equal real roots .
$\therefore$ the roots are $\frac{-b}{2 a}=\frac{-(-2)}{2(3)}$ and $\frac{-b}{2 a}=\frac{-(-2)}{2(3)}$
The roots are $\frac{2}{2(3)}$ and $\frac{2}{2(3)}$
$\therefore$ The roots are $\frac{1}{3}$ and $\frac{1}{3}$.
22) Discuss the nature of the roots of the equation, $2 x^{2}-3 x+5=0$.
23) Discuss the nature of the roots of the equation, $x^{2}-6 x+3=0$.
24) Find the value of k so that the roots of the equation $2 x^{2}+\mathbf{k} \boldsymbol{x}+3=0$ are equal.

Solution: $2 x^{2}+\mathrm{k} x+3=0 \quad \mathrm{a}=2, \mathrm{~b}=\mathrm{k}, \mathrm{c}=3$
For equal roots, $b^{2}-4 a c=0$
$(\mathrm{k})^{2}-4(2)(3)=0$
$k^{2}-24=0$
$\mathrm{k}^{2}=24$
$\sqrt{\mathrm{k}^{2}}=\sqrt{4 \times 6}= \pm 2 \sqrt{6}$
25) Find the value of k so that the roots of the equation $k x(x-2)+6=0$ are equal.
26) A rectangular mango grove whose length is twice its breadth, and it's area is $800 \mathrm{~m}^{2}$. find it's length and breadth.
Solution :- Let breadth of a rectangular mango grove $=x \mathrm{~m}$ Length $=2 x \mathrm{~m}$
Area of Rectangular mango grove $=$ length $\times$ breadth
$(2 x)(x)=800$
$2 x^{2}=800$
$x^{2}=\frac{800}{2}=400$
$x= \pm \sqrt{400}= \pm 20$
Breadth of mango grove $=x=20 \mathrm{~m}$
length of mango grove $=2 x=2 \times 20=40 \mathrm{~m}$
27) The base of a rectangle is $(x+5) \mathrm{cm}$ and its height is $(x-5) \mathrm{cm}$. If the area of the rectangle is $56 \mathbf{c m}^{2}$, Find its dimensions.
Solution: base $\mathrm{b}=(x+5) \mathrm{cm}$ and height $\mathrm{h}=(x-5) \mathrm{cm}$.
Area of the rectangle $=56 \mathrm{~cm}^{2}$
$(x+5)(x-5)=56 \quad(\because$ Area of the rectangle $=$ base $\times$ height $)$
$x^{2}-5^{2}=56 \quad$ or $\quad$ (length $\times$ breadth)
$x^{2}-25=56$
$x^{2}=56+25$
$x^{2}=81$
$x^{2}=9^{2}$
$\therefore x=9$
$\mathrm{b}=(x+5)=9+5=14 \mathrm{~cm}$
$\mathrm{h}=(x-5)=9-5=4 \mathrm{~cm}$

$$
3 \text { or } 4 \text { Marks Questions (LA-1 / LA-2) }
$$

28) The present age of Kavya and Karthik is 11 and 14 years respectively. After how many years the product of their ages will become 304 ?

Solution: The present age of Kavya $=11$ years
The present age of Karthik $=14$ years
After $x$ years Kavya's age $=(11+x)$ years
After $x$ years Karthik's age $=(14+x)$ years
After $x$ years the product of their Ages $=304$
$(11+x)(14+x)=304$
$11 \times 14+11 x+14 x+x^{2}=304$
$154+25 x+x^{2}-304=0$
$x^{2}+25 x-150=0$
$x^{2}+30 x-5 x-150=0$
$\left(x^{2}+30 x\right)-(5 x+150)=0$
$30 \times 5=150$
$30-5=25$
$x(x+30)-5(x+30)=0$
$x+30=0$ or $x-5=0$
$\therefore x=-30$ or $x=5$
After 5 years the product of their ages will become 304.
29) Some children participated in a birthday party. In that birthday party each child gives 2 gifts to every other child. If the total number of gifts is 264 , find the number of children in that birthday party.
Solution: Let $x$ be the number of children participated in that birthday party.
Each child gives 2 gifts to $(x-1)$ children.
$x$ children gives 2 gifts to ( $x-1$ ) children.
Total Number of gifts $=x \times 2 \times(x-1)$
$x \times 2 \times(x-1)=264$
$2 x^{2}-2 x-264=0$
$x^{2}-x-132=0$ (Divide each term by 2)
$x^{2}-12 x+11 x-132=0$
$\left(x^{2}-12 x\right)+(11 x-132)=0$
$x(x-12)+11(x-12)=0$
$x-12=0$ or $x+11=0$
$\therefore x=12$ or $x=-11$
$\therefore 12$ children participated in that birthday party.
30) An express train takes 1 hour less than the passenger train to travel 132 km between Bengaluru and Mysuru. If the average speed of the express train is $11 \mathrm{~km} / \mathrm{h}$ more than that of the passenger train, what is the average speed of the two trains?
Solution : average speed of passenger train $=x \mathrm{~km} / \mathrm{h}$
Average speed of express train $=(x+11) \mathrm{km} / \mathrm{h}$
Total distance travelled $=132 \mathrm{~km}$
Time taken by passenger train $=\frac{132}{x} \mathrm{~h}$

Time taken by express train $=\frac{132}{x+11} \mathrm{~h}$
Time difference between these two journeys $=1 \mathrm{~h}$
$\therefore \frac{132}{x}-\frac{132}{x+11}=1$
$132(x+11)-132 x=x(x+11)$
$132 x+1452-132 x=x^{2}+11 x$
$x^{2}+11 x-1452=0$
$x^{2}+44 x-33 x-1452=0$
$x(x+44)-33(x+44)=0$
$(x+44)(x-33)=0$
$x+44=0, x-33=0$
$x=-44, \quad x=33$
Average speed of passenger train $=33 \mathrm{~km} / \mathrm{h}$
Average speed of express train $=(33+11)=44 \mathrm{~km} / \mathrm{h}$
31) A motor boat whose speed is $18 \mathrm{~km} / \mathrm{h}$ in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Determine the speed of the stream.

Solution : Let the speed of the stream $=x \mathrm{~km} / \mathrm{h}$
Speed of a motor boat in still water $=18 \mathrm{~km} / \mathrm{h}$
Then, the speed of the boat upstream $=(18-x) \mathrm{km} / \mathrm{h}$
The speed of the boat downstream $=(18+x) \mathrm{km} / \mathrm{h}$
Total distance travelled $=24 \mathrm{~km}$
Time taken to go upstream $=\frac{24}{18-x} \mathrm{~h}$
Time taken to go downstream $=\frac{24}{18+x} \mathrm{~h}$
Time difference between these two journeys $=1 \mathrm{~h}$
$\therefore \frac{24}{18-x}-\frac{24}{18+x}=1$
$\frac{24(18+x)-24(18-x)}{(18+x)(18-x)}=1$
$\frac{24 \times 18+24 x-24 \times 18+24 x}{18^{2}-x^{2}}=1$
$\frac{48 x}{18^{2}-x^{2}}=1$
$48 x=18^{2}-x^{2}$
$x^{2}+48 x-324=0$
$x^{2}+54 x-6 x-324=0$
$x(x+54)-6(x+54)=0$
$(x+54)(x-6)=0$
$x=-54$ or $x=6$
$\therefore$ the speed of the stream is $6 \mathrm{~km} / \mathrm{h}$.
32) Ravi buys a number of books for Rs 60 . If he had bought 5 more books for the same amount, each book would have cost him Re 1 less. How many books did he buy? Find the price of each book.
33) A train travels a distance of 300 km at a uniform speed. If the speed of the train is increased by 10 km an hour, the journey would have taken 1 hour less. Find the original speed of the train.
34) Two water taps together can fill a tank in $9 \frac{3}{8}$ hour. The tap of larger diameter taken 10 hour less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.
35) The difference of squares of two numbers is 180 . The square of the smaller number is 8 times the larger number. Find the two numbers.
36) Shwetha takes 6 days less than the time taken by Ankitha to finish a piece of work. If both Shwetha and Ankitha together can finish it in 4 days, Find the time taken by Ankitha to finish the work.
37) A merchant sells an article for Rs 24 and gains as much per cent as the cost price of the article. Find the cost price of the article.
38) A merchant sells an article for Rs 18.75 and loses as much per cent as the cost price of the article. Find the cost price of the article.
39) Find the length and breadth of a rectangular park whose perimeter is 80 m and it's area is $400 \mathrm{~m}^{2}$.
Solution:- let $l$ and $b$ are the length and breadth of a rectangular park
Perimeter $=2(l+b)=80$
$l+b=\frac{80}{2}=40$
$b=40-l$
Area $l \times b=400$
$l(40-l)=400$
$40 l-l^{2}=400$
$l^{2}-40 l+400=0$
$l^{2}-20 l-20 l+400=0$
$l(l-20)-20(l-20)=0$
$(l-20)(l-20)=0$

$$
\begin{aligned}
& 400 \times 1=400 \\
& 400=20 \times 20
\end{aligned}
$$

$l=20$ or $l=20$
Length $l=20 \mathrm{~m}$
Breadth $b=40-l=40-20=20 \mathrm{~m}$
40) The diagonal of a rectangular field is 60 meter more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.
41) The height of a triangle is 6 cm more than its base. If the area of the triangle is $108 \mathrm{~cm}^{2}$, find the length of the base and height of the triangle.
42) If the roots of the equation $(a-b) x^{2}+(b-c) x+(c-a)=0$ are equal, prove that $b+c=2 a$.
Solution: compare $(a-b) x^{2}+(b-c) x+(c-a)=0$ with $a x^{2}+b x+c=0$. $a=(a-b), \quad b=(b-c), \quad c=(c-a)$
Since roots are equal, $\quad b^{2}-4 a c=0$
$(b-c)^{2}-4(a-b)(c-a)=0$
$b^{2}+c^{2}-2 b c-4\left(c a-a^{2}-b c+a b\right)=0$
$b^{2}+c^{2}-2 b c-4 c a+4 a^{2}+4 b c-4 a b=0$
$4 a^{2}+b^{2}+c^{2}-4 a b+2 b c-4 c a=0$
$(-2 a)^{2}+b^{2}+c^{2}+2(-2 a) b+2 b c+2 c(-2 a)=0$
$(-2 a+b+c)^{2}=0$
$-2 a+b+c=0$
$\therefore b+c=2 a$
43) If the equation $\left(1+m^{2}\right) x^{2}+2 m c x+\left(c^{2}-a^{2}\right)=0$ has equal roots, prove that $c^{2}=a^{2}\left(1+m^{2}\right)$.
Solution: compare $\left(1+m^{2}\right) x^{2}+2 m c x+\left(c^{2}-a^{2}\right)=0$ with $a x^{2}+b x+c=0$. $a=\left(1+m^{2}\right), \quad b=2 m c, \quad c=\left(c^{2}-a^{2}\right)$

Since roots are equal, $b^{2}-4 a c=0$
$(2 m c)^{2}-4\left(1+m^{2}\right)\left(c^{2}-a^{2}\right)=0$
$4 m^{2} c^{2}-4\left(c^{2}-a^{2}+m^{2} c^{2}-m^{2} a^{2}\right)=0$
$4 m^{2} c^{2}-4 c^{2}+4 a^{2}-4 m^{2} c^{2}+4 m^{2} a^{2}=0$
$4 a^{2}\left(1+m^{2}\right)=4 c^{2}$
$a^{2}\left(1+m^{2}\right)=c^{2}$
$\therefore c^{2}=a^{2}\left(1+m^{2}\right)$
44) If the roots of the equation $\left(c^{2}-a b\right) x^{2}-2\left(a^{2}-b c\right) x+\left(b^{2}-a c\right)=0$ are real and equal, prove that either $a=0$ or $a^{3}+b^{3}+c^{3}=3 a b c$.
Solution: compare $\left(c^{2}-a b\right) x^{2}-2\left(a^{2}-b c\right) x+\left(b^{2}-a c\right)=0$ with $a x^{2}+b x+c=0$, $a=\left(c^{2}-a b\right), \quad b=-2\left(a^{2}-b c\right), \quad c=\left(b^{2}-a c\right)$

Since roots are real and equal, $b^{2}-4 a c=0$

$$
\begin{aligned}
& \left(-2\left(a^{2}-b c\right)\right)^{2}-4\left(c^{2}-a b\right)\left(b^{2}-a c\right)=0 \\
& 4\left(a^{2}-b c\right)^{2}-4\left(c^{2}-a b\right)\left(b^{2}-a c\right)=0 \\
& \left(a^{2}-b c\right)^{2}-\left(c^{2}-a b\right)\left(b^{2}-a c\right)=0
\end{aligned}
$$

$$
\begin{aligned}
& a^{4}+b^{2} c^{2}-2 a^{2} b c-b^{2} c^{2}+a c^{3}+a b^{3}-a^{2} b c=0 \\
& a^{4}+a b^{3}+a c^{3}-3 a^{2} b c=0 \\
& a\left(a^{3}+b^{3}+c^{3}-3 a b c\right)=0 \\
& \Rightarrow a=0 \text { or } a^{3}+b^{3}+c^{3}-3 a b c=0 \\
& \therefore a=0 \text { or } a^{3}+b^{3}+c^{3}=3 a b c
\end{aligned}
$$

## 5 Marks Questions (LA - 3)

45) In the adjoining figure , ABCD is a rhombus. The diagonals AC and BD intersect at Point E . If $\mathrm{BE}=\boldsymbol{x}+7, \mathrm{AE}=\boldsymbol{x}$ and $\mathrm{AB}=\boldsymbol{x}+8$, find the length of AC and BD .
Solution: $A E=x, \quad B E=x+7$ and $A B=x+8$
From the $\triangle A E B$
$A B^{2}=A E^{2}+B E^{2}(\because$ Pythagoras theorem $)$
$(x+8)^{2}=(x)^{2}+(x+7)^{2}$
$x^{2}+2(x)(8)+8^{2}=x^{2}+\left[x^{2}+2(x)(7)+7^{2}\right]$
$x^{2}+16 x+64=x^{2}+x^{2}+14 x+49$

$0=x^{2}+14 x-16 x+49-64$
$x^{2}-2 x-15=0$
$x^{2}-5 x+3 x-15=0$
$\left(x^{2}-5 x\right)+(3 x-15)=0$
$x(x-5)+3(x-5)=0$
$-5+3=-2$
$x-5=0$ or $x+3=0$
$\therefore x=5$ or $x=-3$
$\therefore A E=C E=x=5 \quad(\because$ The diagonals bisects each other $)$
diagonal $A C=A E+C E=5+5=10$
$\therefore B E=x+7=5+7=12$
$\therefore D E=B E=12 \quad(\because$ The diagonals bisects each other)
diagonal $B D=D E+B E=12+12=24$
$\therefore A C=10$ units and $B D=24$ units.

## UNIT- 11 : INTRODUCTION TO TRIGONOMETRY

1 Mark Questions (MCQ)

1) If $\tan \mathrm{A}=\frac{4}{3}$ then the value of $4 \cot \mathrm{~A}$ is
A) $\frac{1}{3}$
B) $\frac{3}{4}$
C) 4
D) 3
2) If $\cos \theta=\frac{12}{13}$ then the value of $\sec \theta$ is
A) $\frac{13}{12}$
B) $\frac{12}{25}$
C) $\frac{5}{13}$
D) $\frac{5}{12}$
3) If $\sin A=\frac{4}{5}$ then the value of $\operatorname{cosec} A$ is
A) $\frac{4}{5}$
B) $\frac{5}{4}$
C) $\frac{3}{4}$
D) $\frac{3}{5}$
4) If $\sqrt{3} \tan \mathrm{~A}=1$ then the value of $\angle \mathrm{A}$ is
A) $60^{\circ}$
B) $30^{\circ}$
C) $45^{\circ}$
D) $90^{\circ}$
5) The value of $\tan ^{2} 60^{\circ}$ is
A) $\sqrt{3}$
B) $\frac{1}{3}$
C) 3
D) $\frac{1}{\sqrt{3}}$
6) The value of $\operatorname{cosec}^{2} 45^{\circ}$ is
A) 2
B) $\sqrt{2}$
C) $\frac{1}{2}$
D) $\frac{1}{\sqrt{2}}$
7) The value of $1+\tan ^{2} 45^{\circ}$ is
A) 0
B) 2
C) 3
D) $\sqrt{2}$
8) The value of $1-\tan ^{2} 45^{\circ}$ is
A) 0
B) 2
C) 3
D) $\sqrt{2}$
9) The value of $\frac{\tan 65^{\circ}}{\cot 25^{\circ}}$ is
A) $\sqrt{2}$
B) 0
C) 1
D) $\frac{1}{\sqrt{2}}$
10) The value of $\cos 48^{\circ}-\sin 42^{\circ}$ is
A) $\frac{1}{2}$
B) 0
C) 1
D) $\frac{3}{2}$
11) If $\sin 2 \mathrm{~A}=2 \sin \mathrm{~A}$ is true when $\mathrm{A}=$ ?
A) $0^{\circ}$
B) $30^{\circ}$
C) $45^{\circ}$
D) $60^{\circ}$
12) The value $9 \sec ^{2} A-9 \tan ^{2} A$ is
A) 0
B) 1
C) 8
D) 9
13) The equal value of $\cos A$ is
A) $\frac{1}{\operatorname{cosec} A}$
B) $\frac{1}{\sec A}$
C) $\frac{1}{\sin A}$
D) $\frac{1}{\cot A}$
14) $(\sin A+\cos A)^{2}$ is equal to
A) $\sin ^{2} A+\cos ^{2} A$
B) $1-2 \sin A \cdot \cos A$
C) $\sin ^{2} A-\cos ^{2} A$
D) $1+2 \sin A \cdot \cos A$

## 1 Mark Questions (VSA)

15) If $\sin x=\frac{3}{5}$ then find the value of $3 \operatorname{cosec} x$.
16) If $\cot \theta=\frac{7}{8}$ then find the value of $\cot ^{2} \theta$.
17) If $2 \cos \theta=1$ then find the value of acute angle $\theta$.
18) If $\sqrt{3} \cot A=1$ then find the value of acute angle $A$.
19) Find the value of $\frac{\sin 18^{\circ}}{\cos 72^{\circ}}$
20) Find the value of $\operatorname{cosec} 31^{\circ}-\sec 59^{\circ}$.
21) Find the value of $\sin ^{2} 75^{\circ}+\cos ^{2} 75^{\circ}$.
22) Find the value of $\frac{\sin \theta}{\cos \left(90^{\circ}-\theta\right)}+\frac{\cos \theta}{\sin \left(90^{\circ}-\theta\right)}$
23) If $\tan x=3 \cot x$, find the value of $\tan ^{2} x$.

| Ans | 1) D | 2) A | 3) B | 4) B | 5) C | 6) A | 7) B |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 8)A | 9) C | 10) B | 11) A | 12) D | 13) B | 14) D | 15) 5 |
| 16) $\frac{49}{64}$ | 17) $60^{\circ}$ | 18) $60^{\circ}$ | 19) 1 | 20) 0 | 21) 1 | 22) 2 | 23) 3 |

2 Marks Questions (SA)
24) In $\triangle A B C$,find $\sin A$ and $\cos A$ If $\angle B=90^{\circ}, A B=24 \mathrm{~cm}$ and $B C=7 \mathbf{c m}$.

Solution: In $\triangle A B C, \angle B=90^{\circ}$
given that $A B=24 \mathrm{~cm}, B C=7 \mathrm{~cm}$
$\mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}(\because$ Pythagoras theorem $)$
$\mathrm{AC}^{2}=576+49$
$\mathrm{AC}^{2}=625$
$\mathrm{AC}^{2}=25^{2}$

$\mathrm{AC}=25 \mathrm{~cm}$
$\sin \mathrm{A}=\frac{\mathrm{opp}}{\text { hyp }}=\frac{7}{25}$
$\cos \mathrm{A}=\frac{\text { adj }}{\text { hyp }}=\frac{24}{25}$
25) If $\cot \theta=\frac{7}{8}$ find other 5 trigonometric ratios.
26) If $\sin A=\frac{3}{4}$ find other 5 trigonometric ratios.
27) If $\sec \theta=\frac{13}{12}$ find other 5 trigonometric ratios.
28) If $15 \cot A=8$, find $\sin A$ and $\sec A$.
29) If $2 \cos \theta=1$ find other 5 trigonometric ratios.
30) If $2 \sin \theta=\sqrt{3}$ find other 5 trigonometric ratios.
31) If $3 \tan A=\sqrt{3}$ find $\sin 3 A$ and $\cos 2 A$.

Solution: $3 \tan A=\sqrt{3}$
$\tan \mathrm{A}=\frac{\sqrt{3}}{3}$
$\tan A=\frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

$$
\begin{aligned}
& \tan A=\frac{1}{\sqrt{3}} \\
& \Rightarrow A=30^{\circ} \\
& \sin 3 A=\sin 3\left(30^{\circ}\right)=\sin 90^{\circ}=1 \\
& \cos 2 A=\cos 2\left(30^{\circ}\right)=\cos 60^{\circ}=\frac{1}{2}
\end{aligned}
$$

32) If $13 \sin \mathrm{~A}=5, \mathrm{~A}$ is a acute angle find $\frac{5 \sin \mathrm{~A}-2 \cos \mathrm{~A}}{\tan \mathrm{~A}}$.
33) If $A=60^{\circ}, B=30^{\circ}$ then prove $\cos (A+B)=\cos A \cdot \cos B-\sin A \cdot \sin B$.
34) If $A=60^{\circ}, B=30^{\circ}$ then prove that $\tan (A-B)=\frac{\tan A-\tan B}{1+\tan A \cdot \tan B}$.
35) If $B=15^{\circ}$, then prove that $4 \sin 2 B \cdot \cos 4 B \cdot \sin 6 B=1$.
36) Prove that $2 \cos ^{2} \theta-1=\cos ^{2} \theta-\sin ^{2} \theta$.

## 3 Marks Questions (LA-1)

37) Show that $\frac{1-\cos \theta}{1+\cos \theta}=(\boldsymbol{\operatorname { c o s e c } \theta} \boldsymbol{\theta}-\cot \theta)^{2}$.

Solution: LHS $=\frac{(1-\cos \theta)}{(1+\cos \theta)}$
$=\frac{(1-\cos \theta)(1-\cos \theta)}{(1+\cos \theta)(1-\cos \theta)}[$ Multiply numerator and denominator by $(1-\cos \theta)$.
$=\frac{(1-\cos \theta)^{2}}{1-\cos ^{2} \theta} \quad\left[\because 1-\cos ^{2} \theta=(1+\cos \theta)(1-\cos \theta)\right]$
$=\frac{1^{2}+\cos ^{2} \theta-2(1)(\cos \theta)}{\sin ^{2} \theta} \quad\left[\because \sin ^{2} \theta=1-\cos ^{2} \theta\right]$
$=\frac{1}{\sin ^{2} \theta}+\frac{\cos ^{2} \theta}{\sin ^{2} \theta}-\frac{2 \cos \theta}{\sin ^{2} \theta}$
$=\frac{1}{\sin ^{2} \theta}+\frac{\cos ^{2} \theta}{\sin ^{2} \theta}-\frac{2}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$
$\left[\because \frac{\cos ^{2} \theta}{\sin ^{2} \theta}=\cot ^{2} \theta \quad\right]$
$=\operatorname{cosec}^{2} \theta+\cot ^{2} \theta-2 \cdot \operatorname{cosec} \theta \cdot \cot \theta$
$\left[\because \frac{1}{\sin ^{2} \theta}=\operatorname{cosec}^{2} \theta\right]$
$=(\operatorname{cosec} \theta-\cot \theta)^{2}$
$=$ RHS
38) Prove that $\sqrt{\sec ^{2} \theta+\operatorname{cosec}^{2} \theta}=(\tan \theta+\cot \theta)$.

Solution: LHS $=\sqrt{\sec ^{2} \theta+\operatorname{cosec}^{2} \theta}$
$=\sqrt{\left(1+\tan ^{2} \theta\right)+\left(1+\cot ^{2} \theta\right)} \quad\left[\because 1+\tan ^{2} \theta=\sec ^{2} \theta, 1+\cot ^{2} \theta=\operatorname{cosec}^{2} \theta\right]$
$=\sqrt{\tan ^{2} \theta+\cot ^{2} \theta+2}$

$$
\begin{aligned}
& =\sqrt{\tan ^{2} \theta+\cot ^{2} \theta+2 \tan \theta \cdot \cot \theta} \quad[\because \tan \theta \cdot \cot \theta=1] \\
& =\sqrt{(\tan \theta+\cot \theta)^{2}} \\
& =(\tan \theta+\cot \theta)=\text { RHS }
\end{aligned}
$$

39) Prove that $\frac{\tan ^{2} \theta}{1+\tan ^{2} \theta}+\frac{\cot ^{2} \theta}{1+\cot ^{2} \theta}=1$.

Solution : LHS $=\frac{\tan ^{2} \theta}{1+\tan ^{2} \theta}+\frac{\cot ^{2} \theta}{1+\cot ^{2} \theta}$

$$
\begin{aligned}
& =\frac{\tan ^{2} \theta}{\sec ^{2} \theta}+\frac{\cot ^{2} \theta}{\operatorname{cosec}^{2} \theta} \quad\left[\because 1+\tan ^{2} \theta=\sec ^{2} \theta, \quad 1+\cot ^{2} \theta=\operatorname{cosec}^{2} \theta\right] \\
& =\frac{\sin ^{2} \theta}{\cos ^{2} \theta} \cdot \frac{1}{\sec ^{2} \theta}+\frac{\cos ^{2} \theta}{\sin ^{2} \theta} \cdot \frac{1}{\operatorname{cosec}^{2} \theta} \\
& =\frac{\sin ^{2} \theta}{\cos ^{2} \theta} \cdot \cos ^{2} \theta+\frac{\cos ^{2} \theta}{\sin ^{2} \theta} \cdot \sin ^{2} \theta \quad\left[\because \frac{1}{\sec ^{2} \theta}=\cos ^{2} \theta, \quad \frac{1}{\operatorname{cosec}^{2} \theta}=\sin ^{2} \theta\right] \\
& =\sin ^{2} \theta+\cos ^{2} \theta \\
& =1 \\
& =\text { RHS }
\end{aligned}
$$

40) Prove that $\frac{\sec \theta+\tan \theta}{\sec \theta-\tan \theta}=1+2 \tan ^{2} \theta+2 \sec \theta \cdot \tan \theta$

$$
\begin{aligned}
& \text { Solution : LHS }=\frac{\sec \theta+\tan \theta}{\sec \theta-\tan \theta} \\
& =\frac{\sec \theta+\tan \theta}{\sec \theta-\tan \theta} \times \frac{\sec \theta+\tan \theta}{\sec \theta+\tan \theta}[\text { Multiply numerator and denominator by sec } \theta+\tan \theta] \\
& =\frac{(\sec \theta+\tan \theta)^{2}}{\sec ^{2} \theta-\tan ^{2} \theta} \\
& =\frac{\sec ^{2} \theta+\tan ^{2} \theta+2 \sec \theta \cdot \tan \theta}{1+\tan ^{2} \theta-\tan ^{2} \theta} \\
& =1+\tan ^{2} \theta+\tan ^{2} \theta+2 \sec \theta \cdot \tan \theta\left[\because 1+\tan ^{2} \theta=\sec ^{2} \theta\right] \\
& =1+2 \tan ^{2} \theta+2 \sec \theta \cdot \tan \theta=\text { RHS }
\end{aligned}
$$

41) Prove that $\operatorname{cosec}^{4} \mathrm{~A}-\operatorname{cosec}^{2} \mathrm{~A}=\cot ^{4} \mathrm{~A}+\cot ^{2} \mathrm{~A}$.

Solution: LHS $=\operatorname{cosec}^{4} \mathrm{~A}-\operatorname{cosec}^{2} \mathrm{~A}$
$=\operatorname{cosec}^{2} \mathrm{~A}\left(\operatorname{cosec}^{2} \mathrm{~A}-1\right)$

$$
\begin{aligned}
& =\left(1+\cot ^{2} \mathrm{~A}\right)\left(1+\cot ^{2} \mathrm{~A}-1\right) \quad\left(\because \operatorname{cosec}^{2} \mathrm{~A}=1+\cot ^{2} \mathrm{~A}\right) \\
& =\left(1+\cot ^{2} \mathrm{~A}\right)\left(\cot ^{2} \mathrm{~A}\right) \\
& =\left(\cot ^{2} \mathrm{~A}+\cot ^{4} \mathrm{~A}\right) \\
& =\left(\cot ^{4} \mathrm{~A}+\cot ^{2} \mathrm{~A}\right)=\text { RHS }
\end{aligned}
$$

42) If $\pi=180^{\circ}$ and $A=\frac{\pi}{6}$, then prove that $\frac{(1+\cos A)(1-\cos A)}{(1-\sin A)(1+\sin A)}=\frac{1}{3}$.
43) If $\tan (A+B)=\sqrt{3}$ and $\tan (A-B)=\frac{1}{\sqrt{3}}$. Here $0^{\circ}<(A+B) \leq 90^{\circ}$;

Then find the value $A$ and $B$.
44) Prove that $\frac{\sin \theta}{1+\cos \theta}+\frac{1+\cos \theta}{\sin \theta}=2 \operatorname{cosec} \theta$.
45) Prove that $(1+\cot \theta-\operatorname{cosec} \theta)(1+\tan \theta+\sec \theta)=2$.

## 4 Marks Questions (LA-2)

46) Prove that $\frac{\cos \left(90^{\circ}-\theta\right)}{1+\cos \theta}+\frac{\sin \theta}{1-\sin \left(90^{\circ}-\theta\right)}=2 \operatorname{cosec} \theta$.

$$
\text { Solution: } \text { LHS }=\frac{\cos \left(90^{\circ}-\theta\right)}{1+\cos \theta}+\frac{\sin \theta}{1-\sin \left(90^{\circ}-\theta\right)}
$$

$$
\begin{aligned}
& =\frac{\sin \theta}{1+\cos \theta}+\frac{\sin \theta}{1-\cos \theta} \quad\left[\because \cos \left(90^{\circ}-\theta\right)=\sin \theta, \quad \sin \left(90^{\circ}-\theta\right)=\cos \theta\right] \\
& =\frac{(1-\cos \theta) \sin \theta+(1+\cos \theta) \sin \theta}{(1+\cos \theta)(1-\cos \theta)} \\
& =\frac{\sin \theta-\cos \theta \sin \theta+\sin \theta+\cos \theta \sin \theta}{(1+\cos \theta)(1-\cos \theta)} \\
& =\frac{2 \sin \theta}{1-\cos ^{2} \theta} \\
& =\frac{2 \sin \theta}{\sin ^{2} \theta} \\
& =\frac{2}{\sin \theta} \quad\left[\because 1-\cos ^{2} \theta=\sin ^{2} \theta\right] \\
& =2 \operatorname{cosec} \theta=\text { RHS } \quad\left[\because \frac{1}{\sin \theta}=\operatorname{cosec} \theta\right]
\end{aligned}
$$

47) Prove that $\frac{\sin \left(90^{\circ}-\theta\right)}{1+\sin \theta}+\frac{\cos \theta}{1-\cos \left(90^{\circ}-\theta\right)}=2 \sec \theta$.
48) Prove that $\frac{\cos \left(90^{\circ}-\theta\right)}{1+\sin \left(90^{\circ}-\theta\right)}+\frac{1+\sin \left(90^{\circ}-\theta\right)}{\cos \left(90^{\circ}-\theta\right)}=2 \operatorname{cosec} \theta$.
49) Prove that $\sin ^{6} A+\cos ^{6} A=1-3 \sin ^{2} A \cos ^{2} A$.

Solution: LHS $=\sin ^{6} \mathrm{~A}+\cos ^{6} \mathrm{~A}$
$=\left(\sin ^{2} \mathrm{~A}\right)^{3}+\left(\cos ^{2} \mathrm{~A}\right)^{3} \quad\left[a^{3}+b^{3}=(a+b)\left(a^{2}+b^{2}-a b\right)\right]$

$$
\begin{aligned}
& =\left(\sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}\right)\left[\left(\sin ^{2} \mathrm{~A}\right)^{2}+\left(\cos ^{2} \mathrm{~A}\right)^{2}-\sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}\right] \\
& =(1)\left[\left(\sin ^{2} \mathrm{~A}\right)^{2}+\left(\cos ^{2} \mathrm{~A}\right)^{2}+2 \sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}-2 \sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}-\sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}\right] \\
& =\left[\left(\sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}\right)^{2}-3 \sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}\right] \quad\left(\because \sin ^{2} \mathrm{~A}+\cos ^{2} \mathrm{~A}=1\right) \\
& =\left[(1)^{2}-3 \sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}\right] \\
& =1-3 \sin ^{2} \mathrm{~A} \cdot \cos ^{2} \mathrm{~A}=\text { RHS }
\end{aligned}
$$

## 5 Marks Questions (LA-3)

50) if $\cos \theta+\sin \theta=\sqrt{2} \cos \theta$, prove that $\cos \theta-\sin \theta=\sqrt{2} \sin \theta$.

Solution: $\cos \theta+\sin \theta=\sqrt{2} \cos \theta---->(1)$
$(\cos \theta+\sin \theta)^{2}=(\sqrt{2} \cos \theta)^{2} \quad(\because$ squaring on both sides. $)$
$\cos ^{2} \theta+\sin ^{2} \theta+2 \cos \theta \cdot \sin \theta=2 \cos ^{2} \theta$
$2 \cos \theta \cdot \sin \theta=2 \cos ^{2} \theta-\cos ^{2} \theta-\sin ^{2} \theta$
$2 \cos \theta \cdot \sin \theta=\cos ^{2} \theta-\sin ^{2} \theta$
$\cos ^{2} \theta-\sin ^{2} \theta=2 \cos \theta \cdot \sin \theta$
$(\cos \theta+\sin \theta)(\cos \theta-\sin \theta)=2 \cos \theta \cdot \sin \theta$
$\sqrt{2} \cos \theta(\cos \theta-\sin \theta)=2 \cos \theta \cdot \sin \theta \quad[\because$ From (1)]
$(\cos \theta-\sin \theta)=\frac{2 \cos \theta \cdot \sin \theta}{\sqrt{2} \cos \theta}$
$(\cos \theta-\sin \theta)=\frac{2 \sin \theta}{\sqrt{2}}$
$(\cos \theta-\sin \theta)=\frac{2 \times \sqrt{2} \sin \theta}{\sqrt{2} \times \sqrt{2}}$
$(\cos \theta-\sin \theta)=\frac{2 \times \sqrt{2} \sin \theta}{2}$
$\therefore(\cos \theta-\sin \theta)=\sqrt{2} \sin \theta$
51) Prove that $\frac{\sin \theta}{1-\cos \theta}+\frac{\tan \theta}{1+\cos \theta}=\sec \theta \operatorname{cosec} \theta+\cot \theta$.
52) Prove that $\frac{\tan \theta+\sec \theta-1}{\tan \theta-\sec \theta+1}=\frac{1+\sin \theta}{\cos \theta}$.
53) Prove that $\frac{\tan \mathrm{A}}{1-\cot \mathrm{A}}+\frac{\cot \mathrm{A}}{1-\tan \mathrm{A}}=1+\tan \mathrm{A}+\cot \mathrm{A}=1+\sec \mathrm{A} \operatorname{cosec} \mathrm{A}$.
54) Prove that $\frac{\tan \theta-\cot \theta}{\sin \theta \cos \theta}=\sec ^{2} \theta-\operatorname{cosec}^{2} \theta=\tan ^{2} \theta-\cot ^{2} \theta$.

## UNIT - 12 : SOME APPLICATIONS OF TRIGONOMETRY

## 1 Mark Questions (VSA)

1) Find the value of $x$ in the following figures.


| Ans | $1) 50 \mathrm{~m}$ | 2) $60^{\circ}$ | 3) 12 m |
| :--- | :--- | :--- | :--- |

2 / 3 Marks Questions (SA/LA-1)
2) Find the value of the unknown in the following figure.

3) The angle of elevation of ladder leaning against a wall is $\mathbf{6 0}$ and the foot of a ladder is 9.5 m away from the wall. find the length of ladder.

Solution : Here $A B=$ length of the wall
$B C=$ distance from wall to foot of ladder
$\mathrm{AC}=$ length of ladder
$\cos \theta=\frac{\mathrm{adj}}{\mathrm{hyp}}=\frac{9.5}{\mathrm{AC}}$
$\cos 60^{\circ}=\frac{9.5}{\mathrm{AC}}$

$\frac{1}{2}=\frac{9.5}{\mathrm{AC}}$
$A C=9.5 \times 2=19 \quad \therefore$ Length of ladder $=19 m$
4) The angles of elevation of the top of a tower fro two points on the ground at a distance $\boldsymbol{a}$ metres and $\boldsymbol{b}$ metres $(\boldsymbol{a}>b)$ from the base of the tower and in the same straight line are complementary. Prove that the height of the tower $\boldsymbol{h}=\sqrt{a b}$ metres.

Solution: The height of a tower $h=$ ?
According to data, let $\angle D=\theta$ and $\angle C=90^{\circ}-\theta$. $B D=a \mathrm{~m}$. and $B C=b \mathrm{~m}$.

From the $\triangle \mathrm{ABD}$,

$$
\begin{equation*}
\tan \theta=\frac{o p p}{a d j}=\frac{A B}{B D}=\frac{h}{a} \tag{1}
\end{equation*}
$$

From the $\Delta \mathrm{ABC}$,
$\cot \left(90^{\circ}-\theta\right)=\frac{a d j}{o p p}=\frac{B C}{A B}=\frac{b}{h}$

$\tan \theta=\frac{b}{h} \quad-\cdots(2) \quad\left[\because \cot \left(90^{\circ}-\theta\right)=\tan \theta\right]$
Equating (1) and (2)
$\frac{h}{a}=\frac{b}{h}$
$h^{2}=a b$
$h=\sqrt{a b} \mathrm{~m}$.
5) Two wind mills of height 50 m and 40 m are on either side of the field. A person observes the top of the wind mills from a point on the ground in between the towers. The angle of elevation was found to the $45^{\circ}$ in both the cases, find the distance between the wind mills.
6) A tower stands vertically on the ground from a point on the ground which is 50 m away from the foot of the tower the angle of elevation to top of the tower is $60^{\circ}$, find the height of the tower?
7) A tree is broken over by the wind forms a right angle triangle with the ground. If the broken part makes an angle of $60^{\circ}$ with ground and the top of the tree is now 20 m from its base. How tall was the tree?
8) From the top of a building 16 m high. The angular elevation of the top of a hill is $60^{\circ}$ and the angular depression of the foot of the hill is $30^{\circ}$. Find the height of the hill.

# UNIT - 13 : STATISTICS 

## 1 Mark Questions (MCQ)

1) In the following which is not a measure of central tendency?
A) Mode
B) Range
C) Median
D) Mean
2) The relationship between the measures of central tendency
A) Median $=$ Mode +2 Mean
B) Mode $=3$ Median -2 Mean
C) 3 Median $=2$ Mode +2 Mean
D) Mode $=3$ Median +2 Mean
3) The $x$ - co ordinate of the point of intersection of two ogives, which were drawn as "more than" type and "less than" type for same data, represents
A) Mean.
B) Median
C) Mode.
D) Cumulative frequency.
4) The midpoint of the CI $10-25$ is
A) 35
B) 15
C) 17.5
D) -7.5
5) Calculate mode if mean is 58 and median is 50 .
A) 34
B) 43
C) 108
D) 8
6) The point of intersection of two ogives, which were drawn as "more than" type and "less than" type for the some data is (66.4, 26.5 ). The Median of the same data is
A) 26.5
B) 39.9
C) 66.4
D) 33.2 1 Mark Questions (VSA)
7) Calculate median for the given scores $1,5,4,3,2$.
8) What is the other name of cumulative frequency curve?
9) Write the formula to find mean for grouped data.
10) Calculate median for the given scores $2,8,10,6,12,16$.

| Ans | 1) B | 2) B | 3) B | 4) C | 5) A |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6) C | 7) 3 | 8) Ogive | 9) $\bar{x}=\frac{\sum f_{i} x_{i}}{\sum f_{i}}$ | 10) 9 |

3 Marks Questions (LA-1)
11) Find the Mean of the following data.

| CI | $10-25$ | $25-40$ | $40-55$ | $55-70$ | $70-85$ | $85-100$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| f | 2 | 3 | 7 | 6 | 6 | 6 |

Solution: Direct method

| $\boldsymbol{C I}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | Mid point <br> $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{x}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: |
| $10-25$ | 2 | 17.5 | 35.0 |
| $25-40$ | 3 | 32.5 | 97.5 |
| $40-55$ | 7 | 47.5 | 332.5 |
| $55-70$ | 6 | 62.5 | 375.0 |
| $70-85$ | 6 | 77.5 | 465.0 |
| $85-100$ | 6 | 92.5 | 555.0 |
|  | $\Sigma \boldsymbol{f}_{\boldsymbol{i}}=\mathbf{3 0}$ |  | $\Sigma \boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{x}_{\boldsymbol{i}}=\mathbf{1 8 6 0 . 0}$ |

Mean $\bar{x}=\frac{\sum f_{i} x_{i}}{\sum f_{i}}$
Mean $\bar{x}=\frac{1860}{30}$
Mean $\bar{x}=62$
Solution: Assuumed mean method
Assumed Mean $\mathbf{a}=\mathbf{1 7 . 5}$

| $\boldsymbol{C I}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | Mid point <br> $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{d}_{\boldsymbol{i}}=\boldsymbol{x}_{\boldsymbol{i}}-\mathbf{a}$ | $\boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{d}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $10-25$ | 2 | 17.5 | 0 | 0 |
| $25-40$ | 3 | 32.5 | 15 | 45 |
| $40-55$ | 7 | 47.5 | 30 | 210 |
| $55-70$ | 6 | 62.5 | 45 | 270 |
| $70-85$ | 6 | 77.5 | 60 | 360 |
| $85-100$ | 6 | 92.5 | 75 | 450 |
|  | $\Sigma \boldsymbol{f}_{\boldsymbol{i}}=\mathbf{3 0}$ |  |  | $\sum \boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{d}_{\boldsymbol{i}}=\mathbf{1 3 3 5}$ |

Mean $\bar{x}=\mathrm{a}+\frac{\sum f_{i} d_{i}}{\sum f_{i}}$
Mean $\bar{x}=17.5+\frac{1335}{30}$
Mean $\bar{x}=17.5+44.5$
Mean $\bar{x}=62$

## Solution: Step deviation method

Assumed Mean $\mathbf{a = 1 7 . 5}, \quad$ size of calss interval $\mathbf{h}=\mathbf{1 5}$

| $\boldsymbol{C I}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | Mid point <br> $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{u}_{\boldsymbol{i}}=\frac{\boldsymbol{x}_{\boldsymbol{i}}-\mathbf{a}}{\boldsymbol{h}}$ | $\boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{u}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $10-25$ | 2 | 17.5 | 0 | 0 |
| $25-40$ | 3 | 32.5 | 1 | 3 |
| $40-55$ | 7 | 47.5 | 2 | 14 |
| $55-70$ | 6 | 62.5 | 3 | 18 |
| $70-85$ | 6 | 77.5 | 4 | 24 |
| $85-100$ | 6 | 92.5 | 5 | 30 |
|  | $\sum \boldsymbol{f}_{\boldsymbol{i}}=\mathbf{3 0}$ |  |  | $\sum \boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{u}_{\boldsymbol{i}}=\mathbf{8 9}$ |

$$
\begin{aligned}
& \text { Mean } \bar{x}=\mathrm{a}+\left[\frac{\sum f_{i} u_{i}}{\sum f_{i}}\right] \times \mathrm{h} \\
& \text { Mean } \bar{x}=17.5+\left[\frac{89}{30}\right] \times 15 \\
& \text { Mean } \bar{x}=17.5+\frac{89}{2} \\
& \text { Mean } \bar{x}=17.5+44.5 \\
& \text { Mean } \bar{x}=62
\end{aligned}
$$

12) Find the Mean of the following data.

| $C I$ | $15-25$ | $25-35$ | $35-45$ | $45-55$ | $55-65$ | $65-75$ | $75-85$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{i}$ | 6 | 11 | 7 | 4 | 4 | 2 | 1 |

13) Find the Mode of the following data.

| $C I$ | $5-15$ | $15-25$ | $25-35$ | $35-45$ | $45-55$ | $55-65$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{i}$ | 6 | 11 | 21 | 23 | 14 | 5 |

Solution:

| $\boldsymbol{C I}$ | $f_{i}$ |
| :---: | :---: |
| $5-15$ | 6 |
| $15-25$ | 11 |
| $25-35$ | $21 \mathrm{f}_{0}$ |
| $35-45$ | $\mathbf{2 3} \mathbf{f}_{\mathbf{1}}$ |
| $45-55$ | $14 \mathrm{f}_{2}$ |
| $55-65$ | 5 |
|  | $n=80$ |

Maximum frequency $=23$
Modal class $=35-45$
Lower limit of modal class $\boldsymbol{l}=35$
Class size $\boldsymbol{h}=10$
Frequency of the Modal class $\boldsymbol{f}_{\mathbf{1}}=23$
Frequency of the class preceeding the Modal class $f_{0}=21$
Frequency of the class suceeding the Modal class $f_{2}=14$

Mode $=l+\left[\frac{f_{1}-f_{0}}{2 f_{1}-f_{0}-f_{2}}\right] \times h$
Mode $=35+\left[\frac{23-21}{2 \times 23-21-14}\right] \times 10$
Mode $=35+\left[\frac{2}{46-35}\right] \times 10$
Mode $=35+\frac{20}{11}$
Mode $=35+1.82 \quad \therefore$ Mode $=36.82$
14) Find the Mode of the following data.

| $C I$ | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ | $60-70$ | $70-80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 7 | 12 | 13 | 14 | 20 | 15 | 11 | 8 |

15) Find the Mode of the following data.

| $C I$ | $15-20$ | $20-25$ | $25-30$ | $30-35$ | $35-40$ | $40-45$ | $45-50$ | $50-55$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 3 | 8 | 9 | 10 | 3 | 0 | 0 | 2 |

16) Find the Mode of the following data.

| $C I$ | $1-3$ | $3-5$ | $5-7$ | $7-9$ | $9-11$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 7 | 8 | 2 | 2 | 1 |

17) Find the Mode of the following data.

| $C I$ | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ | $100-120$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 10 | 35 | 52 | 61 | 38 | 29 |

18) Find the Median of the following data.

| $C I$ | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ | $100-120$ | $120-140$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 6 | 8 | 10 | 12 | 6 | 5 | 3 |

Solution:

| $\boldsymbol{C I}$ | $\boldsymbol{f}$ | $\boldsymbol{C f}$ |
| :---: | :---: | :---: |
| $0-20$ | 6 | 6 |
| $20-40$ | 8 | 14 |
| $40-60$ | 10 | $\mathbf{2 4} \boldsymbol{c f}$ |
| $\mathbf{6 0 - 8 0}$ | $\mathbf{1 2} \boldsymbol{f}$ | 36 |
| $80-100$ | 6 | 42 |
| $100-120$ | 5 | 47 |
| $120-140$ | 3 | 50 |
|  | $n=50$ |  |

Median $\frac{n}{2}=\frac{50}{2}=25^{\text {th }}$ value.
Median class $=60-80$
Lower limit of Median class $\boldsymbol{l}=\mathbf{6 0}$
Number of observation $\boldsymbol{n}=\mathbf{5 0}$
$\frac{n}{2}=\frac{50}{2}=25$
Cumulative frequency of class
preceeding the median class $\boldsymbol{c f}=24$
Frequency of median class $f=12$
Class size $\boldsymbol{h}=20$

Median $=l+\left[\frac{\frac{n}{2}-c f}{f}\right] \times h$
Median $=60+\left[\frac{25-24}{12}\right] \times 20$
Median $=60+\left[\frac{1}{3}\right] \times 5$
Median $=60+\frac{5}{3}$
Median $=60+1.67$
Median $=61.67$
19) Find the Median of the following data.

| $C I$ | $50-55$ | $55-60$ | $60-65$ | $65-70$ | $70-75$ | $75-80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 2 | 8 | 12 | 24 | 38 | 16 |

20) Find the Median of the following data.

| $C I$ | $15-25$ | $25-35$ | $35-45$ | $45-55$ | $55-65$ | $65-75$ | $75-85$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 6 | 11 | 7 | 4 | 4 | 2 | 1 |

21) Find the Median of the following data.

| $C I$ | $100-120$ | $120-140$ | $140-160$ | $160-180$ | $180-200$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 12 | 14 | 8 | 6 | 10 |

22) Find the Median of the following data.

| $C I$ | $135-140$ | $140-145$ | $145-150$ | $150-155$ | $155-160$ | $160-165$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 4 | 7 | 18 | 11 | 6 | 5 |

23) Find the Median of the following data.

| $C I$ | $f$ |
| :---: | :---: |
| $5-10$ | 2 |
| $10-15$ | 12 |
| $15-20$ | 2 |
| $20-25$ | 4 |
| $25-30$ | 3 |
| $30-35$ | 4 |
| $35-40$ | 3 |


| $C I$ | $f$ |
| :---: | :---: |
| $0-10$ | 12 |
| $10-20$ | 16 |
| $20-30$ | 6 |
| $30-40$ | 7 |
| $40-50$ | 9 |
|  | $\mathrm{n}=50$ |

24) During the medical check-up of 35 students of a class, their weights are recorded as follows:

| Weight (in kg) | Number of students |
| :---: | :---: |
| Less than 38 | 0 |
| Less than 40 | 3 |
| Less than 42 | 5 |
| Less than 44 | 9 |
| Less than 46 | 14 |
| Less than 48 | 28 |
| Less than 50 | 32 |
| Less than 52 | 35 |

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph.

Solution:

25) Change the following distribution to a less than type distribution, and draw its ogive.

| CI | $0-3$ | $3-6$ | $6-9$ | $9-12$ | $12-15$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f | 9 | 3 | 5 | 3 | 1 |

Solution:

| CI | f | Cumulative <br> frequency (cf ) |
| :---: | :---: | :---: |
| Less than 3 | 9 | 9 |
| Less than 6 | 3 | 12 |
| Less than 9 | 5 | 17 |
| Less than 12 | 3 | 20 |
| Less than 15 | 1 | 21 |


26) Change the following distribution to a less than type distribution, and draw its ogive.

| CI | $100-120$ | $120-140$ | $140-160$ | $160-180$ | $180-200$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f | 12 | 14 | 8 | 6 | 10 |

27) Change the following distribution to a more than type distribution, and draw its ogive.

| CI | $\mathbf{0 - 3}$ | $\mathbf{3 - 6}$ | $\mathbf{6 - 9}$ | $\mathbf{9 - 1 2}$ | $\mathbf{1 2 - 1 5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| f | $\mathbf{9}$ | $\mathbf{3}$ | $\mathbf{5}$ | $\mathbf{3}$ | $\mathbf{1}$ |

Solution:

| CI | f | Cumulative <br> frequency (cf ) |
| :---: | :---: | :---: |
| More than or equal to 0 | 9 | 21 |
| More than or equal to 3 | 3 | 12 |
| More than or equal to 6 | 5 | 9 |
| More than or equal to 9 | 3 | 4 |
| More than or equal to 12 | 1 | 1 |


28) Change the following distribution to a more than type distribution, and draw its ogive.

| CI | $50-55$ | $55-60$ | $60-65$ | $65-70$ | $70-75$ | $75-80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| f | 2 | 8 | 12 | 24 | 38 | 16 |

## 4Marks Questions (LA-2)

29) The following table shows a "less than" type distribution. Find the Mode of the distribution. If the median of the distribution is 56.43 , calculate its Mean.

| $\boldsymbol{C I}$ | Cumulative <br> frequency $(\boldsymbol{C} \boldsymbol{f})$ |
| :---: | :---: |
| Less than 20 | $\mathbf{8}$ |
| Less than 40 | $\mathbf{1 5}$ |
| Less than 60 | 29 |
| Less than 80 | $\mathbf{3 8}$ |
| Less than 100 | $\mathbf{5 3}$ |

Solution: Median $=56.43$

| $\boldsymbol{C I}$ |  |  |  |  | $\boldsymbol{f}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Less than 20 | 8 |  |  |  |  |
|  | $20-40$ | 7 |  |  |  |
| $\boldsymbol{f}_{\mathbf{0}}$ |  |  |  |  |  |
| $\boldsymbol{l} \quad 40-60$ | 14 | $\boldsymbol{f}_{\mathbf{1}}$ |  |  |  |
| $60-80$ | 9 | $\boldsymbol{f}_{\mathbf{2}}$ |  |  |  |
|  | $80-100$ | 15 |  |  |  |

Maximum frequency $=14$
Modal class $=40-60$
Lower limit of modal class $l=40$
Class size $\boldsymbol{h}=20$
Frequency of the Modal class $\boldsymbol{f}_{1}=14$
Frequency of the class preceeding the
Modal class $\quad f_{0}=7$
Frequency of the class suceeding the
Mode $=l+\left[\frac{f_{1}-f_{0}}{2 f_{1}-f_{0}-f_{2}}\right] \times h$
Modal class $\quad f_{2}=9$

Mode $=40+\left[\frac{14-7}{2(14)-7-9}\right] \times 20$
Mode $=40+\left[\frac{7}{28-16}\right] \times 20$
Mode $=40+\left[\frac{7}{12}\right] \times 20$
Mode $=40+\left[\frac{7}{3}\right] \times 5$
Mode $=40+\frac{35}{3}$
Mode $=40+11.67$
Mode $=51.67$
Mode = 3 Median - 2 Mean
$51.67=3 \times 56.43-2$ Mean
2 Mean $=169.29-51.67$
Mean $=\frac{117.62}{2}=58.81$
30) The following table gives the marks scored by the 50 students of a class in a Examination. If the arithmetic mean of the distribution is 25.2 , find the missing frequencies $\mathbf{p}$ and $\mathbf{q}$.

| Marks | $\mathbf{0 - 1 0}$ | $\mathbf{1 0 - 2 0}$ | $\mathbf{2 0 - 3 0}$ | $\mathbf{3 0 - 4 0}$ | $\mathbf{4 0 - 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> students | $\mathbf{8}$ | $\mathbf{p}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{q}$ |

Solution: Arithmetic mean $\bar{x}=25.2$ and $\sum f_{i}=50$

| $\boldsymbol{C I}$ | $\boldsymbol{f}_{\boldsymbol{i}}$ | Mid point <br> $\boldsymbol{x}_{\boldsymbol{i}}$ | $\boldsymbol{f}_{\boldsymbol{i}} \boldsymbol{x}_{\boldsymbol{i}}$ |
| :---: | :---: | :---: | :---: |
| $0-10$ | 8 | 5 | 40 |
| $10-20$ | p | 15 | 5 p |
| $20-30$ | 10 | 25 | 250 |
| $30-40$ | 11 | 35 | 385 |
| $40-50$ | q | 45 | 45 q |
|  | $\sum f_{i}=50$ |  | $\sum f_{i} x_{i}=675+5 p+45 q$ |

$p+q=50-(8+10+11)$
$p+q=50-29$
$p+q=21$
$p=21-q$
$\bar{x}=\frac{\sum f_{i} x_{i}}{\sum f_{i}}$
$25.2=\frac{675+5 p+45 q}{50}$
$1260.0=675+5 p+45 q$
$1260-675=5(p+9 q)$
$\frac{585}{5}=(p+9 q)$
$p+9 q=117$
$21-q+9 q=117[\because$ From (1)]
$8 q=117-21$
$8 q=96$
$\therefore \boldsymbol{q}=12$
$p=21-12[\because$ substituting for q in (1)]
$\therefore \boldsymbol{p}=\mathbf{9}$
31) The median of the following distribution is 28.5 , find the values of $x$ and $y$.

| Class <br> interval | $\mathbf{0 - 1 0}$ | $\mathbf{1 0 - 2 0}$ | $20-30$ | $\mathbf{3 0 - 4 0}$ | $40-50$ | $50-60$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| frequency | 5 | $x$ | 20 | 15 | $y$ | 5 | 60 |

solution: Median $=28.5$

| $C I$ | $f$ | $c f$ |
| :---: | :---: | :--- |
| $0-10$ | 5 | 5 |
| $10-20$ | $x$ | $\mathbf{5}+\boldsymbol{x} \quad \boldsymbol{c f}$ |
| $\mathbf{2 0} \mathbf{- 3 0}$ | $\mathbf{2 0} \boldsymbol{f}$ | $25+x$ |
| $30-40$ | 15 | $40+x$ |
| $40-50$ | y | $40+x+y$ |
| $50-60$ | 5 | $45+x+y$ |
|  | $n=60$ |  |

Median class $=20-\mathbf{3 0}$
Lower limit of Median class $\boldsymbol{l}=20$
Number of observation $\boldsymbol{n}=\mathbf{6 0}$
$\frac{n}{2}=\frac{60}{2}=30$
Cumulative frequency of class preceeding the median class $\boldsymbol{c f}=\mathbf{5 + \boldsymbol { x }}$

Frequency of median class $\boldsymbol{f}=\mathbf{2 0}$
Class size $\boldsymbol{h}=10$
From the table $45+x+y=60$

$$
\begin{aligned}
& x+y=60-45 \\
& x+y=15--->(1) \\
& \text { Median }=l+\left[\frac{\frac{n}{2}-c f}{f}\right] \times h \\
& 28.5=20+\left[\frac{30-5-x}{20}\right] \times 10 \\
& 28.5-20=\left[\frac{25-x}{2}\right] \\
& 8.5 \times 2=25-x \\
& 17=25-x \\
& x=25-17 \\
& \therefore x=\mathbf{8} \\
& x+y=15 \\
& 8+y=15[\because \text { substituting for } x \text { in (1) }] \\
& y=15-8 \\
& \therefore y=\mathbf{7}
\end{aligned}
$$

32) If the Mode of the following distribution is 36, Find the value of $x$.

| Marks | $\mathbf{0 - 1 0}$ | $\mathbf{1 0 - 2 0}$ | $20-30$ | $\mathbf{3 0 - 4 0}$ | $40-50$ | $50-60$ | $60-70$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> students | $\mathbf{8}$ | 10 | $x$ | 16 | 12 | 6 | 7 |

Solution: Mode $=36$

| CI | $f_{i}$ |
| :---: | :---: |
| 0-10 | 8 |
| 10-20 | 10 |
| 20-30 | $x \quad \boldsymbol{f}_{0}$ |
| $l$ - 30-40 | $16{ }^{\boldsymbol{f}}$ |
| 40-50 | $12 \mathrm{f}_{2}$ |
| 50-60 | 6 |
| 60-70 | 7 |

Mode $=36$ (given)
$\therefore$ Modal class $=30-40$
Lower limit of modal class $l=30$

Class size $\boldsymbol{h}=10$
Frequency of the Modal class $\boldsymbol{f}_{\mathbf{1}}=16$
Frequency of the class preceeding the Modal class $\quad \boldsymbol{f}_{\mathbf{0}}=\boldsymbol{x}$

$$
\begin{aligned}
& \text { Mode }=l+\left[\frac{f_{1}-f_{0}}{2 f_{1}-f_{0}-f_{2}}\right] \times h \\
& 36=30+\left[\frac{16-x}{2 \times 16-x-12}\right] \times 10 \\
& 36-30=\left[\frac{16-x}{20-x}\right] \times 10 \\
& \frac{6}{10}=\left[\frac{16-x}{20-x}\right] \\
& \frac{3}{5}=\left[\frac{16-x}{20-x}\right] \\
& 80-5 x=60-3 x \\
& 80-60=5 x-3 x \\
& 20=2 x \\
& \therefore x=10
\end{aligned}
$$

## 5 Marks Questions (LA-3)

33) The following table shows a "more than" type distribution. Find the Median of the distribution. If the Arithmetic mean of the distribution is 69.3, calculate its Mode.

| Marks scored | Number of <br> students |
| :---: | :---: |
| More than or equal to 50 | $\mathbf{1 0 0}$ |
| More than or equal to 55 | $\mathbf{9 8}$ |
| More than or equal to 60 | $\mathbf{9 0}$ |
| More than or equal to 65 | $\mathbf{7 8}$ |
| More than or equal to 70 | $\mathbf{5 4}$ |
| More than or equal to 75 | $\mathbf{1 6}$ |

Solution: Arithmetic mean $=69.3$

| $\boldsymbol{C I}$ | $\boldsymbol{f}$ | $\boldsymbol{C f}$ |
| :---: | :---: | :---: |
| $50-55$ | 2 | 2 |
| $55-60$ | 8 | 10 |
| $60-65$ | 12 | 22 |
| $65-70$ | 24 | $\mathbf{4 6} \boldsymbol{c f}$ |
| $\mathbf{7 0}-\mathbf{7 5}$ | $\mathbf{3 8} \boldsymbol{f}$ | 84 |
| $75-80$ | 16 | 100 |
|  | $n=100$ |  |

Median $\frac{n}{2}=\frac{100}{2}=50^{\text {th }}$ value.
Median class $=70-75$
Lower limit of Median class $\boldsymbol{l}=\mathbf{7 0}$
Number of observation $\boldsymbol{n}=\mathbf{1 0 0}$
$\frac{n}{2}=\frac{100}{2}=\mathbf{5 0}$
Cumulative frequency of class preceeding the median class $\boldsymbol{c f}=46$

Frequency of median class $\boldsymbol{f}=38$
Class size $\boldsymbol{h}=5$

$$
\begin{aligned}
& \text { Median }=l+\left[\frac{\frac{n}{2}-c f}{f}\right] \times h \\
& \text { Median }=70+\left[\frac{50-46}{38}\right] \times 5 \\
& \text { Median }=70+\left[\frac{4}{38}\right] \times 5 \\
& \text { Median }=70+\frac{20}{38} \\
& \text { Median }=70+\frac{10}{19} \\
& \text { Median }=70+0.5 \\
& \text { Median }=70.5 \\
& \text { Mode }=3 \text { Median }-2 \text { Mean } \\
& \text { Mode }=3 \times 70.5-2 \times 69.3 \\
& \text { Mode }=211.5-138.6 \\
& \text { Mode }=72.9
\end{aligned}
$$

## UNIT - 14: PROBABILITY

1 Mark Questions (MCQ)

1) Which of the following is the probability of an event?
A) $\frac{3}{2}$
B) -1.5
C) $15 \%$
D) -0.7
2) If $\mathrm{P}(\mathrm{E})=0.05$, what is the probability of 'not E '?
A) 0.05
B) 0.95
C) 0.005
D) 1.05
3) The probability of winning a game is $\frac{5}{6}$, then the probability of losing the game is
A) $\frac{-5}{6}$
B) $\frac{5}{6}$
C) $\frac{-1}{6}$
D) $\frac{1}{6}$
4) If the probability of an event is 1 , then the event is called
A) Complementary event
B) impossible event
C) mutually exclusive event
D) sure event
5) The probability of winning a game is 0.3 , then the probability of losing the game is
A) 0.1
B) 0.3
C) 0.7
D) 1.3
6) A die is thrown once, the probability of getting an odd number is.
A) $\frac{1}{6}$
B) $\frac{4}{6}$
C) $\frac{2}{6}$
D) $\frac{3}{6}$
7) The probability of winning a game is $60 \%$, then the probability of losing the game is
A) $40 \%$
B) $10 \%$
C) $60 \%$
D) $20 \%$
8) A die is thrown once, the probability of getting a number less than 6 is
A) $\frac{1}{6}$
B) $\frac{4}{6}$
C) $\frac{2}{6}$
D) $\frac{5}{6}$
9) Two fair coins are tossed once, the probability of getting head turns up is
A) $\frac{1}{4}$
B) $\frac{2}{4}$
C) $\frac{3}{4}$
D) $\frac{-1}{4}$
1 Mark Questions (VSA)
10) The chance of raining in a particular day is $35 \%$. then what is the chance of not raining on the same day?
11) If $E$ and $\bar{E}$ both are complementary events, then the value of $P(E)+P(\bar{E})$ is?
12) In a random experiment, what is the total probability of all the primary events?
13) The probability of losing the game is $\frac{1}{4}$. Find the probability of winning the same game.
14) The probability of an event is 0 . Name the type of event.

| Ans | 1) C | 2) B | 3) D | 4) D | 5) C | 6) D | 7) A | 8) D | 9) A |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10) $65 \%$ | $11) 1$ | 12) 1 | 13) $\frac{3}{4}$ | 14) Impossible Event |  |  |  |  |

## 2Marks Questions (SA)

15) A die is thrown once. what is the probability of getting a number less than or equal to 4

Solution: $-\mathrm{S}=\{1,2,3,4,5,6\} \quad \therefore \mathrm{n}(\mathrm{S})=6$
Let $E$ be the event getting a number less than equal to 4 .
$\mathrm{E}=\{1,2,3,4\} \quad \therefore \mathrm{n}(\mathrm{S})=4$
$P(E)=\frac{n(E)}{n(S)}$
$\therefore \mathrm{P}(\mathrm{E})=\frac{4}{6}=\frac{2}{3}$
16) A die is throw once. calculate the probability of getting
i) a prime number
ii) an odd number
iii) a number lying between 2 and 6 .
iv) a composite number
v) Even number
vi) a square number
17) A box contains 50 discs which are numbered from 1 to 50 . if one disc is drawn at random from the box, find the probability that it bears i) a perfect cube number ii) a number which is divisible by 2 and 3 .
18) Two dice are thrown. what is the probability of getting, a) two identical faces b) sum of two faces are equal to 8 .

Solution:- $S=\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$

$$
(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
$$

$$
(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)
$$

$$
(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
$$

$$
(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)
$$

$$
(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)\}
$$

Total outcomes $n(S)=36$
a) Let A be an event of getting two identical faces (Doublets)
$A=\{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$
$\therefore \mathrm{n}(\mathrm{A})=6$
$\mathrm{P}(\mathrm{A})=\frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{S})}$
$P(A)=\frac{6}{36}=\frac{1}{6}$
b) Let $B$ be an event of getting a sum 8 in two faces.
$B=\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$.
$\therefore \mathrm{n}(\mathrm{B})=5$
$P(B)=\frac{n(B)}{n(S)}$
$P(B)=\frac{5}{36}$
19) Two dice thrown simultaneously, find the probability of the following events.
i) Difference between the numbers in two faces is 2
ii) Sum between the numbers in two faces is 5
iii) Product of the numbers in two faces is 12
iv) Number except 5
v) Product of the numbers as a perfect square number in two faces.
vi) To get 5 at least once.
20) Three coins are tossed together, find the probability of getting (i) at least 2 tail (ii) at most 2 tail

Solution :- (i) three coins are tossed together, the outcomes $=\mathrm{n}(\mathrm{s})=8$
$\mathrm{S}=\{\mathrm{HHH}, \mathrm{HHT}, \mathrm{HTH}, \mathrm{THH}, \mathrm{HTT}, \mathrm{THT}, \mathrm{TTH}, \mathrm{TTT}\} \therefore \mathrm{n}(\mathrm{S})=8$
Let A is event of getting at least 2 tails.
$\mathrm{A}=\{$ HTT, THT, TTH, TTT $\} \therefore \mathrm{n}(\mathrm{A})=4$
$\mathrm{P}(\mathrm{A})=\frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{S})}$
$\therefore \mathrm{P}(\mathrm{A})=\frac{4}{8}=\frac{1}{2}$
(ii) Let $B$ is event of getting at most 2 tails.
$B=\{\mathrm{HHH}, \mathrm{HHT}, \mathrm{HTH}, \mathrm{THH}, \mathrm{HTT}, \mathrm{THT}, \mathrm{TTH}\} \therefore \mathrm{n}(\mathrm{B})=7$
$P(B)=\frac{n(B)}{n(S)}$
$\therefore \mathrm{P}(\mathrm{B})=\frac{7}{8}$
21) Three unbiased coins are tossed. Find the probability of getting
(i) no tail
(ii) at most one head.
22) There are three children in a family. Find the probability that there is one girl in the family.
23) A box contains 500 wrist watches, in which 50 are damaged. One wrist watch is selected randomly from the box. Find the probability of getting a damaged wrist watch.
Solution:- Total wrist watches $=500$
$\therefore \mathrm{n}(\mathrm{S})=500$
Let ' $A$ ' be an event of getting a damaged wrist watch
$\therefore \mathrm{n}(\mathrm{A})=50$
$\mathrm{P}(\mathrm{A})=\frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{S})}$
$P(A)=\frac{50}{500}=\frac{1}{10}$
24) A box contains 20 bulbs, in which 4 are damaged. One bulb is drawn out randomly from the box. Find the probability of getting (i) a damaged bulb (ii) a good bulb
25) From a bag containing 5 red, 8 white and 4 black marbles, one is drawn at random. Find the probability of getting,
i) a red marble
ii) a white marble
iii) not a black marble.
26) A bag containing 5 red balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.
Solution: Number of red balls $=5$
Let the number of blue balls in the bag be $x$
Then, total number of balls $=(5+x)$
The probability of drawing a red ball from the bag,
$\mathrm{P}(\mathrm{R})=\frac{\mathrm{n}(\mathrm{R})}{\mathrm{n}(\mathrm{S})}$
$\therefore \mathrm{P}(\mathrm{R})=\frac{5}{5+x}$


The probability of drawing a blue ball from the bag,
$\mathrm{P}(\mathrm{B})=\frac{\mathrm{n}(\mathrm{B})}{\mathrm{n}(\mathrm{S})}$
$\therefore \mathrm{P}(\mathrm{B})=\frac{x}{5+x}$
Given: $\mathrm{P}(\mathrm{B})=3 \times \mathrm{P}(\mathrm{R})$
$\frac{x}{5+x}=3 \times \frac{5}{5+x}$
$\frac{x}{5+x}=\frac{15}{5+x}$
$\therefore x=15$ Hence, the number of blue balls in the bag is 15 .
Note : In a random experiment of throwing two dice, to solve the problems related to addition the following table is very useful.

|  | Numbers on faces of $1^{\text {st }}$ die |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | + | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|  | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|  | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

Example : Probability of getting a sum 8 in two faces $\frac{5}{36}$

Note : In a random experiment of throwing two dice, to solve the problems related to multiplication the following table is very useful.

|  | Numbers on faces of 1st die |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\times$ | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 1 | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 2 | 2 | 4 | 6 | 8 | 10 | 12 |
|  | 3 | 3 | 6 | 9 | 12 | 15 | 18 |
|  | 4 | 4 | 8 | 12 | 16 | 20 | 24 |
|  | 5 | 5 | 10 | 15 | 20 | 25 | 30 |
|  | 6 | 6 | 12 | 18 | 24 | 30 | 36 |

Example : Probability of getting a Product 12 in two faces $\frac{4}{36}=\frac{1}{9}$

## UNIT - 15: SURFACE AREAS AND VOLUMES

1 Mark Questions (MCQ)

1) A solid has been melted and recast into a wire. Which of the following remains the same?
A) length
B) height
C) radius
D) volume
2) The curved surface area of a frustum of the cone is
A) $\pi\left(r_{1}+r_{2}\right) l$
B) $\pi\left(r_{1}+r_{2}\right) h$
C) $\pi\left(r_{1}-r_{2}\right) l$
D) $\pi\left(r_{1}-r_{2}\right) h$
3) A cylindrical Pencil, sharpened at one end is a combination of
A) Sphere and Cylinder
B) Cylinder and Cone
C) Cylinder and Hemi sphere
D) Cone and sphere
4) The perimeter of the base of a right circular cylinder is 44 cm . And the height of the cylinder is 10 cm . then it's curved surface area is
A) $440 \mathrm{~cm}^{2}$
B) $44 \mathrm{~cm}^{2}$
C) $880 \mathrm{~cm}^{2}$
D) $88 \mathrm{~cm}^{2}$
5) A cylinder of height 10 cm and the area of it's base is $154 \mathrm{~cm} .^{2}$ then the volume of the cylinder is
A) $1450 \mathrm{~cm}^{3}$
B) $1540 \mathrm{~cm}^{3}$
C) $4510 \mathrm{~cm}^{3}$
D) $154 \mathrm{~cm}^{3}$
6) A cone of slant height 10 cm and the perimeter of it's base is 44 cm . then the curved surface area of the cone is
A) $440 \mathrm{~cm}^{2}$
B) $220 \mathrm{~cm}^{2}$
C) $44.0 \mathrm{~cm}^{2}$
D) $4400 \mathrm{~cm}^{2}$
7) A cone of height 15 cm and the area of it's base is $154 \mathrm{~cm} .^{2}$ then the volume of the cone is
A) $770 \mathrm{~cm}^{3}$
B) $2013 \mathrm{~cm}^{3}$
C) $2310 \mathrm{~cm}^{3}$
D) $77 \mathrm{~cm}^{3}$
8) A cone and a cylinder have equal base and equal heights. If the volume of the cylinder is $300 \mathrm{~cm}^{3}$ then the volume of the cone is
A) $300 \mathrm{~cm}^{3}$
B) $900 \mathrm{~cm}^{3}$
C) $600 \mathrm{~cm}^{3}$
D) $100 \mathrm{~cm}^{3}$
9) A cone and a cylinder have equal base and equal heights. The volume of cone and cylinder are in the ratio,
A) $2: 1$
B) $3: 1$
C) $1: 4$
D) $\sqrt{2}: 3$
10) Formula used to find the total surface area of a solid hemi sphere is
A) $2 \pi r^{2}$
B) $3 \pi r^{2}$
C) $2 \pi r^{2}$
D) $3 \pi r^{2} h$
11) The surface area of a sphere of radius 7 cm is,
A) $616 \mathrm{~cm}^{2}$
B) $61.6 \mathrm{~cm}^{2}$
C) $313 \mathrm{~cm}^{2}$
D) $31.3 \mathrm{~cm}^{2}$
12) Formula used to find the total surface area of a cylinder is
A) $2 \pi r \mathrm{~h}$
B) $2 \pi r(h+r)$
C) $2 \pi r^{2} h$
D) $2 \pi r(l+r)$
13) Formula used to find the volume of a cone is
A) $\frac{1}{3} \pi r^{2} h$
B) $\frac{3}{2} \pi r^{2} h$
C) $\pi r^{2} h$
D) $\frac{4}{3} \pi r^{2} h$

1 Mark Questions (VSA)
14) Write the formula to find the volume of a hemi sphere.
15) Find the slant height of a cone of height 3 cm and the diameter of it's base is 8 cm .
16) Find the slant height of the frustum of a cone of height 5 cm and the difference between the radii of its two circular ends is 12 cm .
17) Name the solids in a petrol tanker.
18) Write the formula to find the curved surface area of a cylinder.
19) Find the volume of a cube whose side is 5 cm .
20) Find the curved surface area of a hemi sphere whose radius is 7 cm .

| Ans. | 1) D | 2) A | 3) B | 4) A | 5) B | 6) B | 7) A | 8) D | 9) B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10)B | 11)A | 12)B | 13)A | 14) $\frac{2}{3} \pi r^{3}$ cubic units | 15) 5 cm | 16) 13 cm |  |  |  |

2 Marks Questions (SA)
21) The slant height of a frustum of a cone is 4 cm . And the perimeters of its circular bases are 18 cm and 6 cm . Find it's curved surface area.

Solution :- $l=4 \mathrm{~cm}, \quad 2 \pi \mathrm{r}=6 \mathrm{~cm}, \quad 2 \pi \mathrm{R}=18 \mathrm{~cm}$.
$2 \pi \mathrm{r}=6, \quad \therefore \quad \pi \mathrm{r}=\frac{6}{2}=3$
$2 \pi \mathrm{R}=18, \quad \therefore \quad \pi \mathrm{R}=\frac{18}{2}=9$
$\therefore \pi \mathrm{r}+\pi \mathrm{R}=3+9$
$\therefore \pi(\mathrm{r}+\mathrm{R})=12---->(1)$
Curved surface area $=\pi(r+R) l$
$=12 \times 4=48 \mathrm{~cm}^{2}$ (from eqn.(1) and $l=4$ )
22) The slant height of a frustum of a cone is 10.5 cm . And the radii of its circular bases are 33 cm and 27 cm . Find the curved surface area.
23) A vessel is in the shape of a frustum of a cone. The radii of its circular bases are 28 cm and 7 cm and the height of the vessel is 45 cm . Find the volume of the vessel.

Solution :- $\mathrm{h}=45 \mathrm{~cm}, \mathrm{r}=7 \mathrm{~cm}, \mathrm{R}=28 \mathrm{~cm}$
Volume (V) $=\frac{1}{3} \pi h\left(r^{2}+R^{2}+R . r\right)$
$=\frac{1}{3} \times \frac{22}{7} \times 45\left(7^{2}+28^{2}+28 \times 7\right)$
$=\frac{1}{3} \times \frac{22}{7} \mathrm{X} 45(49+784+196)$
$=\frac{22}{7} \times 15 \times 1029$
$=48510 \mathrm{~cm}^{3}$
24) A drinking glass is in the shape of a frustum of a cone of height 14 cm . The diameters of its two circular ends are 4 cm and 2 cm . Find the capacity of the glass.
25) Combination of solids is given in the following table. Construct the formulae to find surface area and volume as shown in the table.

| Combination of solids | Surface area | Volume |
| :---: | :---: | :---: |
|  | $=(2 \times$ surfae area of a hemi sphere) + (surfae area of a cylinder ) $=2\left(2 \pi r^{2}\right)+2 \pi r h$ | $=(2 \times$ volume of a hemi sphere) + (volume of a cylinder ) $=2\left(\frac{2}{3} \pi r^{3}\right)+\pi r^{2} h$ |
|  |  |  |

Combination of solids $\quad$ Surface area | Volume |
| :---: | :---: |

## 3/4 Marks Questions (LA-1/ LA-2)

26) A cone of height 24 cm and radius of base 6 cm is made up of modelling clay.

A student reshapes it in the form of a sphere. Find the radius of the sphere.
Solution :- height of a cone $h=24 \mathrm{~cm}$, radius of base $\mathrm{r}=6 \mathrm{~cm}$,
Let R be the radius of a sphere.
Given, Volume of a cone = Volume of a sphere
$\frac{1}{3} \pi r^{2} h=\frac{4}{3} \pi R^{3}$
$\frac{1}{3} \pi\left(\mathrm{r}^{2} \mathrm{~h}\right)=\frac{1}{3} \pi\left(4 \mathrm{R}^{3}\right) \quad\left[\because\right.$ cancelling $\frac{1}{3} \pi$ on both sides $]$
$\mathrm{r}^{2} \mathrm{~h}=4 \mathrm{R}^{3}$
$6^{2} \times 24=4 R^{3}$
$6^{2} \times 6 \times 4=4 \mathrm{R}^{3} \quad[\because$ cancelling 4 on both sides]
$6^{3}=R^{3}$
$\mathrm{R}=6 \mathrm{~cm}$
$\therefore$ Radius of a sphere $\mathrm{R}=6 \mathrm{~cm}$
27) The length and breadth of the roof of an auditorium are 22 m and 20 m respectively. The rain water from this roof is drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m . If the vessel is just full, find the rain fall in $\mathbf{c m}$.
Solution: Let the rainfall be $x \mathrm{~cm}=\frac{x}{100} \mathrm{~m}$
Length of roof $l=22 \mathrm{~m}$ and breadth of roof $\mathrm{b}=20 \mathrm{~m}$ and $\mathrm{V}=l b h$
$\therefore$ Volume of water on the roof $=22 \times 20 \times \frac{x}{100}=\frac{22 x}{5} \mathrm{~m}^{3}-->(1)$
Radius of the base of cylindrical vessel $=1 \mathrm{~m} .(\because \mathrm{d}=2 \mathrm{~m})$
Height of the cylindrical vessel $=3.5 \mathrm{~m}$.
Volume of water in cylindrical vessel when just full $=\pi r^{2} h$
$=\frac{22}{7} \times 1^{2} \times 3.5=11 \mathrm{~m}^{3}$ $\qquad$
$\therefore$ Volume of water on the roof $=$ Volume of water in cylindrical vessel
$\Rightarrow \frac{22 x}{5}=11 \quad[\because$ from (1) and (2) $]$
$x=\frac{11 \times 5}{22}$
$x=\frac{5}{2}$
$x=2.5$
Hence, the rainfall is $=2.5 \mathrm{~cm}$
28) 2 cubes each of volume $64 \mathrm{~cm}^{3}$ are joined end to end. Find the surface area of the resulting cuboid.
29) Combination of some solids are given below. Find the surface area and the volume of each solid.

30) Combination of some solids are given below. Find the volume of each solid.


5 Marks Questions (LA-3)
31) A metalic right circular cone 20 cm high and whose vertical angle is $60^{\circ}$ is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{15} \mathrm{~cm}$, find the length of the wire.

Solution: as shown in the figure
vertical angle $\angle \mathrm{BAC}=60^{\circ}$
$\therefore \angle \mathrm{OAB}=30^{\circ}$
Height of cone $O A=20 \mathrm{~cm}$
DE intersects Cone such that
$\mathrm{AP}=\mathrm{OP}=10 \mathrm{~cm}$
$\therefore \mathrm{OP}=\mathrm{h}_{1}=10 \mathrm{~cm}$
Radii of frustum of cone
$\mathrm{DP}=\mathrm{r}_{1}=$ ?, $\mathrm{OB}=\mathrm{r}_{2}=$ ?
From the right triangle APD,
$\tan 30^{\circ}=\frac{\text { opp }}{\text { adj }}=\frac{\mathrm{DP}}{\mathrm{AP}}$

$\frac{1}{\sqrt{3}}=\frac{r_{2}}{10}$
$\mathrm{r}_{2}=\frac{10}{\sqrt{3}}--->(1)$
From the right triangle AOB,
$\tan 30^{\circ}=\frac{\mathrm{opp}}{\mathrm{adj}}=\frac{\mathrm{OB}}{\mathrm{OA}}$
$\frac{1}{\sqrt{3}}=\frac{\mathrm{r}_{1}}{20}$
$\mathrm{r}_{1}=\frac{20}{\sqrt{3}}$
The wire is in the form of a cylinder.
Length of the wire $l=$ height of the cylinder $h=$ ?
Radius of the wire $r=\frac{1}{2} \times \frac{1}{15} \mathrm{~cm}=\frac{1}{30} \mathrm{~cm}\left(\because \mathrm{r}=\frac{1}{2} \times d\right)$
Volume of the wire $=$ Volume of the frustum of the cone
$\pi \times r^{2} \times h=\frac{1}{3} \pi h_{1}\left(r_{1}{ }^{2}+r_{2}{ }^{2}+r_{1} \cdot r_{2}\right)$
$\pi \times\left(\frac{1}{30}\right)^{2} \times l=\frac{1}{3} \pi \times 10\left[\left(\frac{20}{\sqrt{3}}\right)^{2}+\left(\frac{10}{\sqrt{3}}\right)^{2}+\frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right]$
$\frac{1}{900} \times l=\frac{1}{3} \times 10\left(\frac{400}{3}+\frac{100}{3}+\frac{200}{3}\right)$
$\frac{1}{900} \times l=\frac{1}{3} \times 10\left(\frac{700}{3}\right)$
$l=\frac{7000}{9} \times 900 \mathrm{~cm}$
$l=7000 \times 100 \mathrm{~cm}$
$l=7000 \mathrm{~m} \quad \therefore$ Length of the wire $=7000 \mathrm{~m}$

## For more drill work

1) If the $p^{\text {th }}$ term of an $A P$ be $\frac{1}{q}$ and its $q^{\text {th }}$ term be $\frac{1}{p}$, then show that the sum of its $(p q)^{\text {th }}$ terms is $\frac{1}{2}(p q+1)$.
Solution: In an AP, $p^{\text {th }}$ term $a_{p}=\frac{1}{q}, \quad q^{\text {th }}$ term $a_{q}=\frac{1}{p}$
$\mathrm{d}=\frac{\mathrm{a}_{\mathrm{p}}-\mathrm{a}_{\mathrm{q}}}{\mathrm{p}-\mathrm{q}}$
$d=\frac{\frac{1}{q}-\frac{1}{p}}{p-q}$
$d=\frac{\frac{p-q}{p q}}{p-q}$
$d=\frac{(p-q)}{p q(p-q)}$
$\mathrm{d}=\frac{1}{\mathrm{pq}}$
$\mathrm{a}_{\mathrm{p}}=\frac{1}{\mathrm{q}}$
$\mathrm{a}+(\mathrm{p}-1) \mathrm{d}=\frac{1}{\mathrm{q}} \quad\left[\because a_{n}=a+(n-1) d\right]$
$a+(p-1) \frac{1}{p q}=\frac{1}{q} \quad[\because$ From (1) $]$
$\mathrm{a}+\frac{\mathrm{p}}{\mathrm{pq}}-\frac{1}{\mathrm{pq}}=\frac{1}{\mathrm{q}}$
$a+\frac{1}{q}-\frac{1}{p q}=\frac{1}{q}$
$\mathrm{a}=\frac{1}{\mathrm{pq}} \quad--->(2) \quad\left(\because\right.$ cancelling $\frac{1}{\mathrm{q}}$ on both sides)
$S_{n}=\frac{n}{2}[2 a+(n-1) d]$
Putn $=p q$,
$\mathrm{S}_{\mathrm{pq}}=\frac{\mathrm{pq}}{2}\left[2 \times \frac{1}{\mathrm{pq}}+(\mathrm{pq}-1) \frac{1}{\mathrm{pq}}\right] \quad[\because$ From (1)and (2) $]$
$S_{p q}=\frac{p q}{2} \times \frac{1}{p q}(2+p q-1)$
Hence, $\mathrm{S}_{\mathrm{pq}}=\frac{1}{2}(\mathrm{pq}+1)$
2) If the $p^{\text {th }}$ term of an $A P$ be $q$ and its $q^{\text {th }}$ term be $p$, then show that the sum of its $(p+q)^{\text {th }}$ terms is $\frac{1}{2}(p+q)(p+q-1)$.
3) The first, second and the last terms of an AP are $a, b$ and $2 a$ respectively. Show that the sum of the terms is $\frac{3 a b}{2(b-a)}$.

Solution: $\mathrm{a}_{1}=\mathrm{a}, \mathrm{a}_{2}=\mathrm{b}$ and $\mathrm{a}_{\mathrm{n}}=2 \mathrm{a}$
$\mathrm{d}=\mathrm{a}_{2}-\mathrm{a}_{1}$
$\mathrm{d}=\mathrm{b}-\mathrm{a}--->(1)$
$a+(n-1) d=a_{n}$
$(\mathrm{n}-1) \mathrm{d}=\mathrm{a}_{\mathrm{n}}-\mathrm{a}$
$(\mathrm{n}-1)=\frac{2 \mathrm{a}-\mathrm{a}}{\mathrm{d}} \quad\left(\because \mathrm{a}_{\mathrm{n}}=2 \mathrm{a}\right)$
$\mathrm{n}-1=\frac{\mathrm{a}}{\mathrm{d}}$
$n-1=\frac{a}{b-a} \quad--->(2) \quad[\because$ From (1) $]$
$\mathrm{n}=\frac{\mathrm{a}}{\mathrm{b}-\mathrm{a}}+1$
$\mathrm{n}=\frac{\mathrm{a}+\mathrm{b}-\mathrm{a}}{\mathrm{b}-\mathrm{a}}$
$\mathrm{n}=\frac{\mathrm{b}}{\mathrm{b}-\mathrm{a}}$
$\mathrm{S}_{\mathrm{n}}=\frac{\mathrm{n}}{2}[2 \mathrm{a}+(\mathrm{n}-1) \mathrm{d}]$
$\mathrm{S}_{\mathrm{n}}=\frac{1}{2} \times \frac{\mathrm{b}}{\mathrm{b}-\mathrm{a}}\left[2 \mathrm{a}+\left(\frac{\mathrm{a}}{\mathrm{b}-\mathrm{a}}\right)(\mathrm{b}-\mathrm{a})\right] \quad[\because$ From (1), (2) and (3) $]$
$S_{n}=\frac{b}{2(b-a)}(2 a+a)$
$S_{n}=\frac{b}{2(b-a)}(3 a)$
Hence, $S_{n}=\frac{3 a b}{2(b-a)}$
4) The sum of first 20 terms of an AP is 400 and that of the first 40 terms is 1600 . Find the sum of its first 10 terms.
5) The sum of first three terms of an AP is 21 and that of the last three terms is 276 . Find the sum of its first 20 terms.
6) If the middle term of an AP is 49 , find the sum of its first 25 terms.
7) The sum of midlle three terms of an AP is 225 and that of the last three terms is 429 . If there are 37 terms, find AP.
8) The interior angles of a quadrilateral are in AP. If the smallest angle of the quadrilateral is $27^{\circ}$, find the remaining angles.
9) The sum of all four numbers of an AP is 58 . If the difference between the extremes is 9 , find the numbers.
10) The sum of first six terms of an AP is 42 . If the ratio to the $10^{\text {th }}$ and $30^{\text {th }}$ terms is $1: 3$, find its $1^{\text {st }}$ and $13^{\text {th }}$ term.
11) Prove that $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}}+\sqrt{\frac{1-\sin \theta}{1+\sin \theta}}=2 \sec \theta$.
12) An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane, at an instant when the angles of the elevation of the two planes from the same point on the ground are $60^{\circ}$ and $45^{\circ}$ respectively. Find the vertical distance between the aeroplanes at that instant.
13) A man on a cliff observes a boat at an angle of depression of $30^{\circ}$ which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be $60^{\circ}$. Find the time taken by the boat to reach the shore.
14) If the angle of elevation of the stationary cloud from a point h metres above a lake is $\alpha$ and angle of depression of its reflection in the lake is $\beta$, prove that the height of the cloud is $\frac{\mathrm{h}(\tan \beta+\tan \alpha)}{\tan \beta-\tan \alpha}$.
15) If the angle of elevation of the stationary cloud from point 60 metres above a lake is $30^{\circ}$ and angle of depression of its reflection in the lake is $60^{\circ}$, find the height of the cloud.
16) A tent is made in the form of a conic frustum surmounted by a cone. The diameters of the base and the top of the frustum are 20 m and 6 m respectively and the height is 24 m . If the height of the tent is 28 m , find the quantity of canvas required.
17) Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute $50 \%$ of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but height 2.8 m , and the canvas to be used cost Rs. 100 per sq.m, find the amount the associations will have to pay. What values are shown by these associations. (use $\pi=\frac{22}{7}$ )
Solution: Tents are made in the form of a cylinder surmounted by a cone.
For cylinder and cone diameter $\mathrm{d}=4.2 \mathrm{~m}$
$\therefore \mathrm{r}=\frac{4.2}{2}=2.1 \mathrm{~m}$
Height of the cylinder $\mathrm{H}=4 \mathrm{~m}$
Height of the cone $\mathrm{h}=2.8 \mathrm{~m}$
Slant height of the cone $l=$ ?
$l^{2}=\mathrm{r}^{2}+\mathrm{h}^{2}$
$l^{2}=(2.1)^{2}+(2.8)^{2}$
$l^{2}=4.41+7.84$
$l=\sqrt{12.25}$
$l=3.5 \mathrm{~m}$


Curved surface area of the tent $=($ C.S.A. of the cylinder + C.S.A. of the cone)
Curved surface area of the tent $=2 \pi r H+\pi r l$
$=\pi r(2 \mathrm{H}+l)$
$=\frac{22}{7} \times 2.1(2 \times 4+3.5)$
$=22 \times 0.3 \times 11.5$
Curved surface area of the tent $=75.90 \mathrm{~m}^{2}$
$\therefore$ Curved surface area of 100 such tents $=75.90 \times 100=7590 \mathrm{~m}^{2}$
If the cost of $1 \mathrm{~m}^{2}$ canvas is Rs 100 ,
cost of $7590 \mathrm{~m}^{2}$ canvas $=7590 \times 100=$ Rs $7,59,000$
$50 \%$ of the cost $=$ Rs $7,59,000 \times \frac{50}{100}=$ Rs $3,79,500$
$\therefore$ Amount contributed by some welfare associations is Rs $3,79,500$
Values: * It is a symbol of humanity.

* It is a characteristic of good civilization.

18) Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m with conical upper part of same base radius but of height 2.1 m . If the canvas used to make the tents cost Rs 120 per square metre find the amount shared by each school to setup the tents. What value is generated by the above problem. (use $\pi=\frac{22}{7}$ )
19) A right circular cone is divided by a plane parallel to its base in two equal volumes. In what ratio will the plane divide the axis of the cone?
20) The height of a right circular cone is trisected by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1:7:19
21) A right triangle, whose sides are 15 cm and 20 cm , is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (use $\pi=3.14$ )
22) A vessel is in the form of an inverted cone. Its height is 11 cm and the diameter of its top, which is open, is 5 cm . It is filled with water up to the rim. When led shots, each of which is a sphere of radius $\frac{1}{4} \mathrm{~cm}$, are dropped into the vessel, $\frac{2}{5}$ of the water flows out. Find the number of lead shots dropped into the vessel.

23) In the adjoining figure, the slant height of a right circular conical vessel is 10 cm . And its height is 8 cm . It is filled with water up to the rim. A Iron sphere of radius equal to half of the radius of conical vessel, is then immersed in the water such that the sphere touches the water level as shown in the figure. Show that the volume of water displaced from the vessel and the volume of water remaining in the vessel are in the ratio 3: 5 .

24) Two solid cones are placed in a cylindrical tube as shown in the figure. If the capacities of the cones are in the ratio 2 : 3 , find the volume of the remaining portion of the cylinder.

25) A ladder 6.5 m long, placed against a wall reaches a window 6 m above the ground. If the foot of the ladder slips 3.5 m , how high above the ground will be the other end of the ladder.
26) In the adjoining figure, $B C$ is the diameter of the semi-circle with centre at point 0 . Centres of the three equal semi - circles lie on BC . ABC is an isosceles triangle with $A B=A C$. If $B C=84 \mathrm{~cm}$, find the area of the shaded region. (use $\pi=\frac{22}{7}$ )

