# 10th MATHEMATICS

## ENGLISH MEDIUM

## Contents

# Pair of Linear Equations in two Variables 3

**Linear equation with one variable:** **The algebraic equation of the type $ax + b = 0$ ($a \neq 0$ and $b$ are real numbers, $x -$ variable is called linear equation of one variable.**
**These type of equations having only one solution.**
**Example : $2x + 5 = 0 \Rightarrow 2x = -5 \Rightarrow x = \frac{-5}{2}$**

## 3.2 Pair of Linear Equations in Two Variables

$2x + 3y = 5$ ;
$x - 2y - 3 = 0$ and
$x - 0y = 2, \Rightarrow x = 2$

An equation which can be put in the form $ax + by + c = 0$, where a, b and c are real numbers, and a and b are not both zero, is called a linear equation in two variables x and y. A solution of such an equation is a pair of values, one for x and the other for y, which makes the two sides of the equation equal.
In fact, this is true for any linear equation, that is, each solution (x, y) of a linear equation in two variables, $ax + by + c = 0$, corresponds to a point on the line representing the equation, and vice versa.

$2x + 3y=5$; $x - 2y - 3 = 0$

$a_1x+ b_1x + c_1 = 0$ and $a_2x+ b_2x + c_2 = 0$
Here, $a_1$, $b_1$, $c_1$, $a_2$, $b_2$, $c_2$ are real numbers

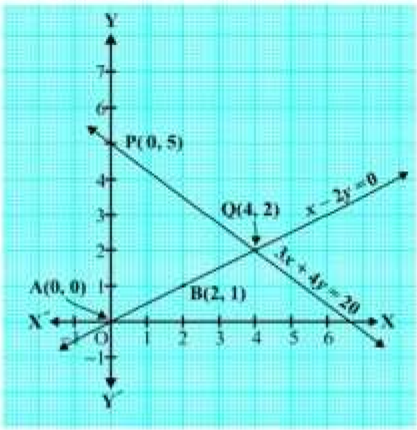
Example 1: **Akhila goes to a fair with Rs 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically).**

**Solution:** The pair of equations formed is :
$y = \frac{1}{2}x \Rightarrow 2y = x$
$\Rightarrow x - 2y = 0$ (1) and $3x + 4y = 20$ (2)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

| x | 0 | 2 |
|---|---|---|
| $y = \frac{x}{2}$ | 2 | 1 |

| x | 0 | 4 | 8 |
|---|---|---|---|
| $y=\frac{20-3x}{4}$ | 5 | 2 | -1 |




Example 2 : Romila went to a stationery shop and purchased 2 pencils and 3 erasers for Rs 9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for Rs 18. Represent this situation algebraically and graphically.

**Solution** : Let us denote the cost of 1 pencil by x' and one eraser by y'. Then the algebraic representation is given by the following equations:

$2x + 3y = 9$ -------------(1)

$4x + 6y = 18$ -----------(2)

(1) $\Rightarrow 3y = 9 - 2x$

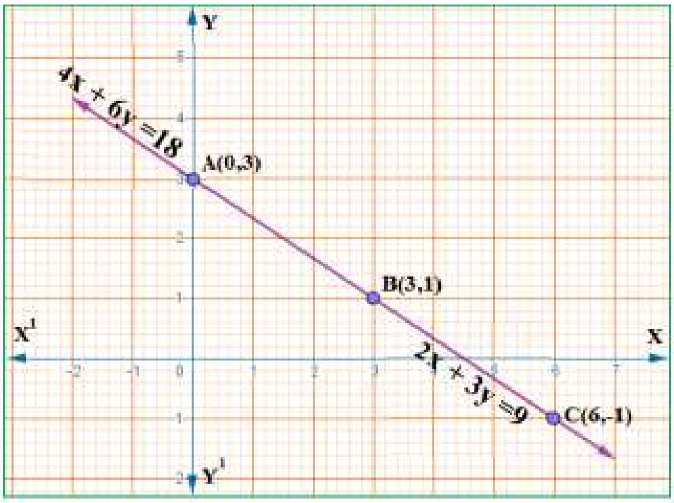
$y = \frac{9-2x}{3}$

| x | 0 | 3 | 6 |
|---|---|---|---|
| $y = \frac{9-2x}{3}$ | 3 | 1 | -1 |

(2) $\Rightarrow 6y = 18 - 4x$

$y = \frac{18-4x}{6}$

| x | 0 | 3 | 6 |
|---|---|---|---|
| $y = \frac{18-4x}{6}$ | 3 | 1 | -1 |




**Example 3 : Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ Represent this situation geometrically.**

**Solution :** Two solutions of each of the equations :

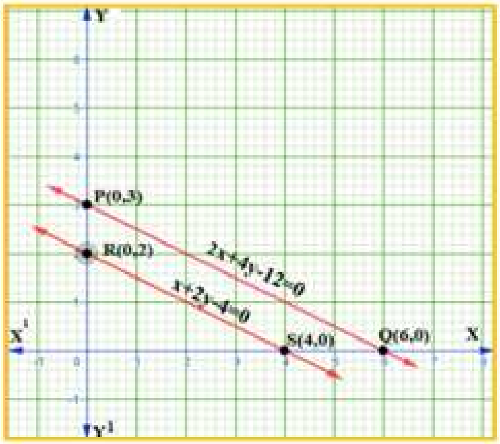
$x + 2y = 4$ and $2x + 4y = 12$

$x + 2y = 4 \Rightarrow 2y = 4 - x \quad \Rightarrow y = \frac{4-x}{2}$

| x | 0 | 4 |
|---|---|---|
| $y = \frac{4-x}{2}$ | 2 | 0 |

$2x + 4y = 12 \Rightarrow y = \frac{12-2x}{4}$

| x | 0 | 6 |
|---|---|---|
| $y = \frac{12-2x}{4}$ | 3 | 0 |



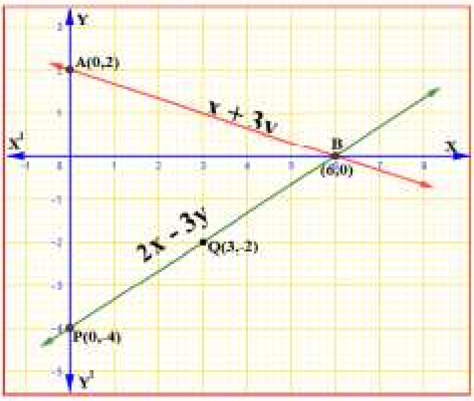

| Comparing the ratios | Representing on graph | Algebraic solution | Consistency |
|---|---|---|---|
| $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ | Intersecting each other | Unique solution | Consistant |
| $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ | coincident lines. | Infinite sulutions | dependent |
| $\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ | Parallel lines | No solutions | Inconsistent |

**Example 4 : Check graphically whether the pair of equations**

**1) $x + 3y = 6$ (1) and**

**$2x - 3y = 12$ (2)**

**is consistent. If so, solve them graphically.**




$x + 3y = 6 \Rightarrow 3y = 6 - x \Rightarrow y = \frac{6-x}{3}$

$x = 0 \Rightarrow y = \frac{6-0}{3} = \frac{6}{3} = 2$

$x = 6 \Rightarrow y = \frac{6-6}{3} = \frac{0}{3} = 0$

| $x$ | 0 | 6 |
|---|---|---|
| $y = \frac{6-x}{3}$ | 2 | 0 |

$2x - 3y = 12 \Rightarrow 3y = 2x - 12 \quad \Rightarrow y = \frac{2x-12}{3}$

$x = 0 \Rightarrow y = \frac{2(0)-12}{3} = \frac{-12}{3} = -4$

$x = 3 \Rightarrow y = \frac{2(3)-12}{3} = \frac{-6}{3} = -2$

| $x$ | 0 | 3 |
|---|---|---|
| $y = \frac{2x-12}{3}$ | $-4$ | $-2$ |

Both lines are intersecting at the point (6,0). Therefore the solution of the equation is $x = 6$ and $y = 0 \Rightarrow$ The equations are consistant pair.

**Example 6 : Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.**

Let us denote the number of pants by x and the number of skirts by y.

Then the equations are:

$y = 2x - 2$ (1)

$y = 4x - 4$ (2)

$y = 2x - 2$

$x = 2 \Rightarrow y = 2(2) - 2 = 4 - 2 = 2$

$x = 1 \Rightarrow y = 2(1) - 2 = 2 - 2 = 0$

$x = 0 \Rightarrow y = 2(0) - 2 = 0 - 2 = -2$

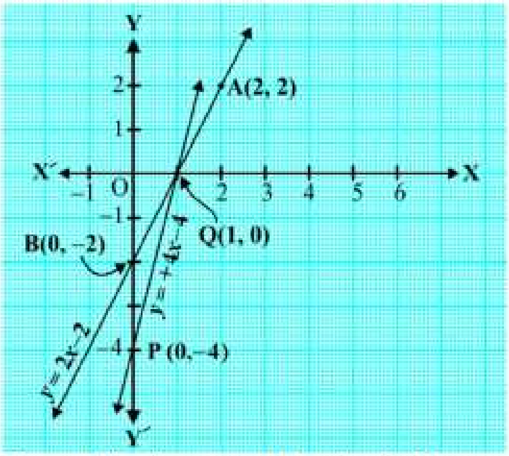
| $x$ | 2 | 1 | 0 |
|---|---|---|---|
| $y = 2x - 2$ | 2 | 0 | $-2$ |

$y = 4x - 4$

$x = 0 \Rightarrow y = 4(0) - 4 = 0 - 4 = -4$

$x = 1 \Rightarrow y = 4(1) - 4 = 4 - 4 = 0$

| $x$ | 0 | 1 |
|---|---|---|
| $y = 4x - 4$ | $-4$ | 0 |




The two lines intersect at the point (1, 0). So, x = 1, y = 0 is the required solution of the pair of linear equations, i.e., the number of pants she purchased is 1 and she did not buy any skirt.

**Example:10 Let us consider the Example 3 of Section 3.2. Will the rails cross each other?**

$x + 2y - 4 = 0$ (1)

$2x + 4y - 12 = 0$ (2)

Equation (1) $\Rightarrow x = 4 - 2y$ (3)

Substituting $x$ in equation (2) we get,

$2(4 - 2y) + 4y - 12 = 0$

$8 - 4y + 4y - 12 = 0$

$8 - 12 = 0$

$-4 = 0$

which is a false statement. Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

# Exercise 3.3

1) **Solve the following pair of linear equations by substitution method.**

**(i)** $x + y = 14$ **(1)**

$x - y = 4$ **(2)**

Equation (1) $\Longrightarrow x = 14 - y$ (3)

Substituting $x$ in equation (2) we get,

$14 - y - y = 4 \Rightarrow 14 - 2y = 4$

$-2y = 4 - 14 \Rightarrow -2y = -10 \Rightarrow y = \frac{-10}{-2} = 5$

Substituting $y = 5$ in equation (3)

$x = 14 - y = 14 - 5 \Rightarrow x = 9$

$\therefore x = 9,\ y = 5$

**(ii)** $s - t = 3$ **(1)**

$\frac{s}{3} + \frac{t}{2} = 6$ **(2)**

Equation (1) $\Longrightarrow s = 3 + t$ (3)

Substituting $s$ in equation (2) we get,

$\frac{3+t}{3} + \frac{t}{2} = 6 \Rightarrow \frac{6+2t+3t}{6} = 6$

$\Rightarrow 6 + 5t = 36 \Rightarrow 5t = 36 - 6 \Rightarrow t = \frac{30}{5}$

Substituting $t = 6$ in equation (3)

$s = 3 + t \Rightarrow s = 3 + 6 \Rightarrow s = 9$

$\therefore s = 9,\ t = 6$

**(iii)** $3x - y = 3$ **(1)**

$9x - 3y = 9$ **(2)**

Equation (1) $\Longrightarrow y = 3x - 3$ Substituting $y$ in equation (2) we get,

$9x - 3(3x - 3) = 9 \Rightarrow 9x - 9x + 9 = 9$

$9 = 9$

This statement is true for all values of y. However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x. This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

**(iv)** $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

$0.2x + 0.3y = 1.3$ $(1) \times 10$

$0.4x + 0.5y = 2.3$ $(2) \times 10$

$2x + 3y = 13$ (3)

$4x + 5y = 23$ (4)

Equation (3) $\Longrightarrow 2x = 13 - 3y \Longrightarrow x = \frac{13-3y}{2}$ (5)

Substituting $x$ in equation (4) we get,

$4\left(\frac{13-3y}{2}\right) + 5y = 23$

$26 - 6y + 5y = 23$

$26 - 23 = y \Rightarrow = 3$ . Substituting $y = 3$ in equation (5)

$x = \frac{13-3(3)}{2} = \frac{13-9}{2} = \frac{4}{2} = 2$

$\therefore x = 2,\ y = 3$

**(v)** $\sqrt{2}x + \sqrt{3}y = 0$ **(1)**

$\sqrt{3}x - \sqrt{8}y = 0$ **(2)**

Equation (1) $\Rightarrow \sqrt{2}x = -\sqrt{3}y \Rightarrow x = -\frac{\sqrt{3}\,y}{\sqrt{2}}$ (3)

Substituting $x$ in equation (2) we get,

$\sqrt{3}\left(-\frac{\sqrt{3}\,y}{\sqrt{2}}\right) - \sqrt{8}y = 0 \Rightarrow -\frac{3y}{\sqrt{2}} - \sqrt{4 \times 2}\,y = 0$

$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}\,y = 0 \Rightarrow y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$

$y = 0$ . Substituting $y = 0$ in equation (3)

$x = -\frac{\sqrt{3}\,(0)}{\sqrt{2}} = 0$

$\therefore x = 0,\ y = 0$

# Exercise 3.4

1. **Solve the following pair of linear equations by the elimination method and the substitution method :**

**(i) $x + y = 5$ ಮತ್ತು $2x - 3y = 4$**

**Eliminating method:**

$x + y = 5$ (1)

$2x - 3y = 4$ (2)

Multiply Equation (1) by 2 to make the coefficients of x equal.

$2x + 2y = 10$ (3)

Substracting (2) from (1).

| | |
|---|---|
| $2x + 2y = 10$ | (3) |
| $2x - 3y = 4$ | (2) |
| $5y = 6$ | |

$\Rightarrow y = \frac{6}{5}$

Substitute $y = \frac{6}{5}$ in equation (1).

$x + \frac{6}{5} = 5 \Rightarrow 5x + 6 = 25 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

**Substituting Method:**

$x + y = 5$ (1)

$2x - 3y = 4$ (2)

(1) $\Rightarrow y = 5 - x$

Substitute $y = 5 - x$ in (2)

$\Rightarrow 2x - 3(5 - x) = 4$ (3)

$\Rightarrow 2x - 15 + 3x = 4 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5}$

Substitute $x = \frac{19}{5}$ in (1)

$\frac{19}{5} + y = 5 \Rightarrow 19 + 5y = 25 \Rightarrow 5y = 25 - 19 \Rightarrow y = \frac{6}{5}$

$\therefore x = \frac{19}{5}$ and $y = \frac{6}{5}$

**(ii) $3x + 4y = 10$ and $2x - 2y = 2$**

**Eliminating Method:**

$3x + 4y = 10$ (1)

$2x - 2y = 2$ (2)

Multiply Equation (2) by 2 to make the coefficients of y equal.

$2x - 2y = 2$ (2) x 2

$4x - 4y = 4$ (3)

Adding equation (1) and (2)

| | |
|---|---|
| $3x + 4y = 10$ | (1) |
| $4x - 4y = 4$ | (3) |
| $7x = 14$ | |

$\Rightarrow x = 2$

Substitute $x = 2$ in (1)

$3(2) + 4y = 10 \Rightarrow 6 + 4y = 10 \Rightarrow 4y = 10 - 6 \Rightarrow 4y = 4 \Rightarrow y = 1$

$\therefore x = 2, y = 1$

**Substituting Method:**

$3x + 4y = -6 \qquad (1)$

$3x - y = 9 \qquad (2)$

$(2) \Rightarrow -y = 9 - 3x \Rightarrow y = 3x - 9 \qquad (3)$

Substitute $y = 3x - 9$ in (1)

$3x + 4(3x - 9) = -6 \Rightarrow 3x + 12x - 36 = -6 \Rightarrow 15x = 30 \Rightarrow x = 2$

Substitute $x = 2$ in (3)

$y = 3(2) - 9 \Rightarrow y = 6 - 9 \Rightarrow y = -3$

$\therefore x = 2$ and $y = -3$

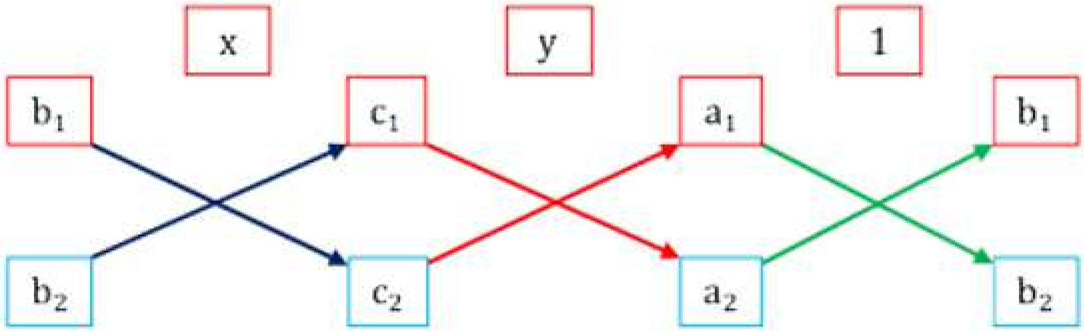
## 3.4.3 Cross - Multiplication Method

**Equations are:**

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \qquad [a_1b_2 - a_2b_1 \neq 0]$$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$




## Exercise 3.5

**1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method**

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -3$ and $a_2 = 3$, $b_2 = -9$, $c_2 = -2$

$\frac{a_1}{a_2} = \frac{1}{3}$; $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$; $\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore the given pair of linear equations are parallel and not intersecting each other.

Hence the pair has no solution.

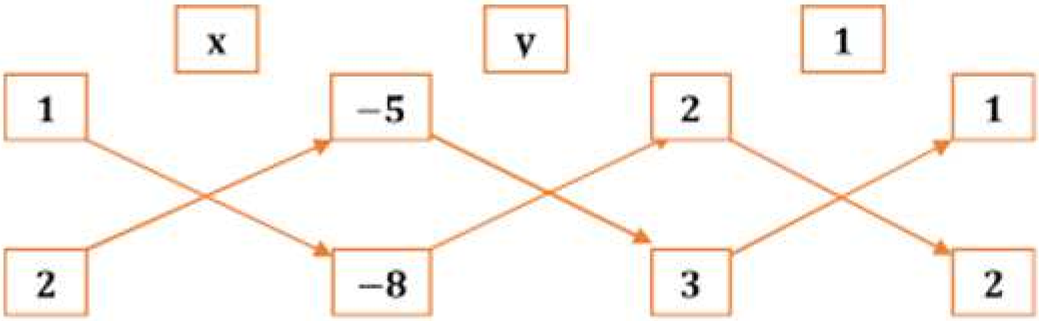
(ii) $2x + y = 5 \Rightarrow 2x + y - 5 = 0$

$3x + 2y = 8 \Rightarrow 3x + 2y - 8 = 0$

Here $a_1 = 2$, $b_1 = 1$, $c_1 = -5$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -8$

$\frac{a_1}{a_2} = \frac{2}{3}$; $\frac{b_1}{b_2} = \frac{1}{2}$; $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8} \quad \Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore the pair of linear equations has unique solution




$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$
$$\Rightarrow\frac{x}{1(-8)-2(-5)}=\frac{y}{(-5)3-(-8)2}=\frac{1}{2(2)-3(1)}$$
$$\Rightarrow\frac{x}{-8+10}=\frac{y}{-15+16}=\frac{1}{4-3}\Rightarrow\frac{x}{2}=\frac{y}{1}=\frac{1}{1}$$
$$\Rightarrow\frac{x}{2}=\frac{1}{1}\Rightarrow x=2$$
$$\frac{y}{1}=\frac{1}{1}\Rightarrow y=1$$

Therefore $x = 2$ and $y = 1$

**iii)** $3x - 5y = 20 \Rightarrow 3x - 5y - 20 = 0$

$6x - 10y = 40 \Rightarrow 6x - 10y - 40 = 0$

Here, $a_1 = 3$, $b_1 = -5$, $c_1 = -20$ and $a_2 = 6$, $b_2 = -10$, $c_2 = -40$

$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2}$; $\frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2}$; $\frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$

$\Rightarrow\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Therefore the given pair of linear equations are coincident and the pair has infinite solutions.

**(iv)** $x - 3y - 7 = 0$

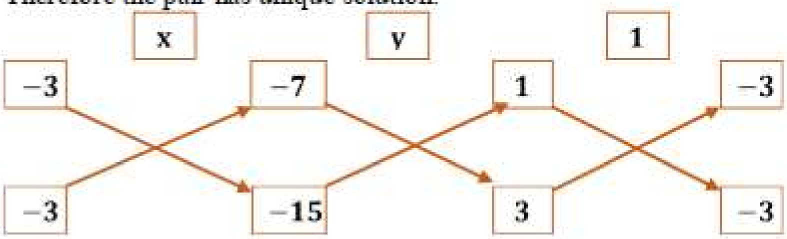
$3x - 3y - 15 = 0$

Here, $a_1 = 1$, $b_1 = -3$, $c_1 = -7$ and $a_2 = 3$, $b_2 = -3$, $c_2 = -15$

$\frac{a_1}{a_2}=\frac{1}{3}$ ; $\frac{b_1}{b_2}=\frac{-3}{-3}=1$; $\frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$

$\Rightarrow\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$

Therefore the pair has unique solution.




$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$
$$\Rightarrow\frac{x}{(-3)(-15)-(-3)(-7)}=\frac{y}{(-7)3-(-15)1}=\frac{1}{1(-3)-3(-3)}$$
$$\Rightarrow\frac{x}{45-21}=\frac{y}{-21+15}=\frac{1}{-3+9}$$
$$\Rightarrow\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}\quad\Rightarrow\frac{x}{24}=\frac{1}{6}\Rightarrow 6x=24\Rightarrow x=4$$
$$\frac{y}{-6}=\frac{1}{6}\Rightarrow 6y=-6\Rightarrow y=-1$$

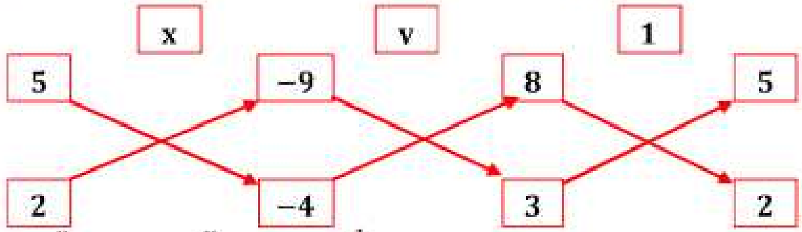
There fore $x = 4$ and $y = -1$

**Cross multiplication Method:**

$8x + 5y = 9 \Rightarrow 8x + 5y - 9 = 0$ (1)

$3x + 2y = 4 \Rightarrow 3x + 2y - 4 = 0$ (2)

Here, $a_1 = 8$, $b_1 = 5$, $c_1 = -9$ and $a_2 = 3$, $b_2 = 2$, $c_2 = -4$




$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \frac{x}{(5)(-4)-(2)(-9)} = \frac{y}{(-9)3-(-4)8} = \frac{1}{8(2)-3(5)}$$

$$\Rightarrow \frac{x}{-20+18} = \frac{y}{-27+32} = \frac{1}{16-15}$$

$$\Rightarrow \frac{x}{-2} = \frac{y}{5} = 1 \Rightarrow \frac{x}{-2} = 1 \Rightarrow x = -2$$

$$\frac{y}{5} = 1 \Rightarrow y = 5$$

Therefore the solution: $x = -2$ ಮತ್ತು $y = 5$

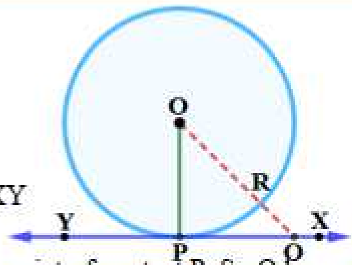
# 4 Circles

## 4.2 Tangent to a Circle

Tangent to a circle is a line that intersects the circle at only one point. There is only one tangent to a circle at a point. The common point of the tangent and the circle is called the point of contact.

**Theorem 4.1**

The tangent at any point of a circle is perpendicular to the radius through the point of contact.




**Given:** A circle with centre O and tangent XY at a point P.

**To Prove:** $OP \perp XY$

**Consruction:** Take any point Q, other than P on the tangent XY and join OQ

**Proof:** Hence, Q is a point on the tangent XY, other than the point of contact P. So Q lies outside the circle..

[∵There is only one point of contact to a tangent]

Let OQ intersect the circle at R

∴ OP = OR [∵ Radius of the same circle ]

Now, OQ = OR + RQ

⇒OQ > OR

⇒OQ> OP [∵OP = OR]

Therefore, OP is the shortest distance to the tangent from the center O

∴ $OP \perp XY$ [∵ Perpendicular distance is always the shortest distance]

**Remarks :**

1. By theorem above, we can also conclude that at any point on a circle there can be one and only one tangent.

2. The line containing the radius through the point of contact is also sometimes called the 'normal' to the circle at the point.

# Exercise 4.1

1. **How many tangents can a circle have?**

   Answer: Infinite

2. **Fill in the blanks :**

   **i) A tangent to a circle intersects it in __________ point (s).**
   Answer: One

   **(ii) A line intersecting a circle in two points is called a __________.**
   **Answer: Secant**

   **iii) A circle can have __________ parallel tangents at the most.**
   Answer: Two [Note: we can draw only two(pair) parallel tangents each other. But we can draw infinite parallel pair of tangents]

   **iv) The common point of a tangent to a circle and the circle is called ________.**
   Answer: Point of cantact

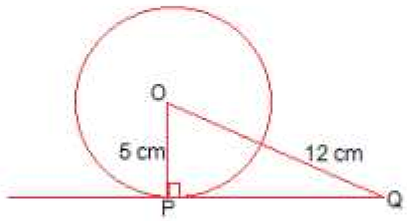
3. **A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :**

   **a) 12 cm  b) 13 cm  c) 8.5 cm  d) $\sqrt{119}$ cm**

   Answer:

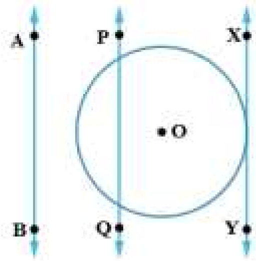
   The line drawn from the point of contact to the center of the circle is perpendicular to the tangent.

   $\Rightarrow OP \perp PQ$
   In $\Delta OPQ$,
   $OQ^2 = OP^2 + PQ^2$ [Pythagoras Theorem]
   $\Rightarrow (12)^2 = 5^2 + PQ^2$
   $\Rightarrow PQ^2 = 144 - 25$
   $\Rightarrow PQ^2 = 119$
   $\Rightarrow PQ = \sqrt{119}$ cm
   **(d) $\sqrt{119}$ cm**




4. **Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.**

   AB – A line
   PQ – A secant
   XY – A tangent




## 4.3 Number of Tangents from a Point on a Circle

**Case 1 : There is no tangent to a circle passing through a point lying inside the circle.**

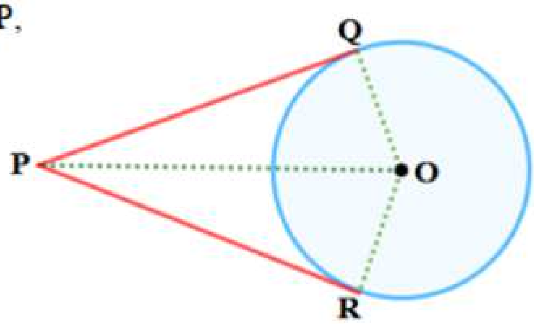
**Case 2 : There is one and only one tangent to a circle passing through a point lying on the circle.**

**Case 3 : There are exactly two tangents to a circle through a point lying outside the circle.**

**Theorem 4.2**

**The lengths of tangents drawn from an external point to a circle are equal.**

ದತ್ತ: PQ and PR are the two tangents drawn from an external point P to a circle of center O. JoinOP, OQ, OR




**T Prove:** PQ = PR

**Proof:** In right angle triangle OQP and ORP.

OQ = OR [Radius of the same circle]

OP = OP [Common side]

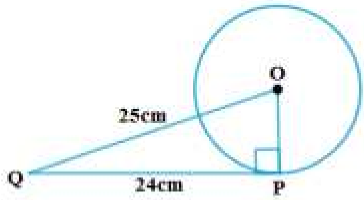
$\therefore$, $\Delta OQP \cong \Delta ORP$ [RHS]

$\therefore$, PQ = PR [CPCT]

## EXERCISE 4.2

**In Q.1 to 3, choose the correct option and give justification.**

1. **From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is**




$OP \perp PQ$ and $\Delta OPQ$ is a right angle triangle.

OQ = 25 cm and PQ = 24 cm

In $\Delta OPQ$, By Pythagoras theorem,

$\Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576 \Rightarrow OP^2 = 49 \Rightarrow OP = 7$ cm

ಉತ್ತರ: **(A) 7 cm.**

A) 7 cm B) 12 cm C) 15 cm D) 24.5 cm

2. **In Fig. 4.11, if TP and TQ are the two tangents to a circle with centre O so that POQ = 110°, then PTQ is equal to**

TP and TQ are the tangents to a circle at P and Q OP and OQ are radius of the circle at point of contacts P and Q

$\therefore OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = \angle OQT = 90°$

In Quadrilateral POQT,

$\angle PTQ + \angle OPT + \angle POQ + \angle OQT = 360°$

$\Rightarrow \angle PTQ + 90° + 110° + 90° = 360°$

$\Rightarrow \angle PTQ = 70°$

**⇒Ans (B) 70°.**

A) 60 B) 70 C) 80 D) 90

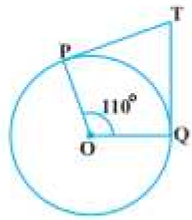



Fig 4.11

6. **The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle**

AB is the tangent to the circle at point B. OB is the radius at point of contact

$\therefore OB \perp AB$

OA = 5cm and AB = 4 cm [Given]

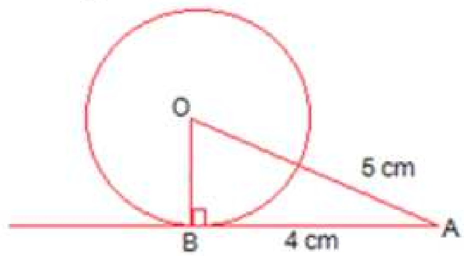
In $\Delta ABO$,

$OA^2 = AB^2 + BO^2$ [Pythagoras theorem]

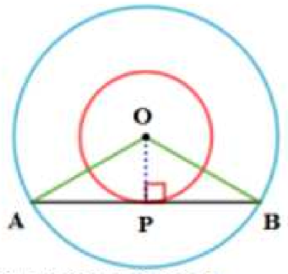
$\Rightarrow 5^2 = 4^2 + BO^2 \Rightarrow BO^2 = 25 - 16$

$\Rightarrow BO^2 = 9 \Rightarrow BO = 3$

$\therefore$ Radius = 3 cm.



**7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**




Two concentric circles of radius 5cm and 3cm drawn with common center O

AB is the chord of circle with radius 5cm such that it touches the circle of radius 3cm at P

$\therefore$ AB is the tangent to the smaller circle at P

$\Rightarrow OP \perp AB$

$\therefore AP = PB$ [The perpendicular drawn from the center to the chord bisects the chord]

$OA^2 = AP^2 + OP^2$ [By Pythagoras theorem]

$\Rightarrow 5^2 = AP^2 + 3^2 \Rightarrow AP^2 = 25 - 9 \Rightarrow AP = 4,$

$AB = 2AP = 2 \times 4 = 8$ cm

$\therefore$ The length of the chord = 8 cm.

**8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 4.12). Prove that AB + CD = AD + BC**

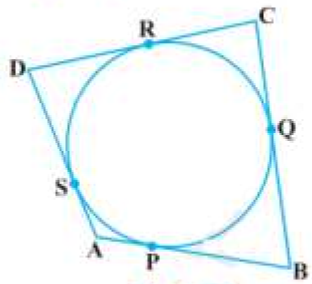



Fig 4.12

From the figure,

DR = DS [Tangents from the external point D ] (1)

AP = AS [Tangents from the external point A ] (2)

BP = BQ [Tangents from the external point B ] (3)

CR = CQ [Tangents from the external point C ] (4)

(1) + (2) + (3) + (4)

DR + AP + BP + CR = DS + AS + BQ + CQ

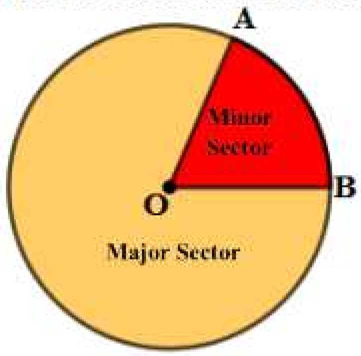
$\Rightarrow$ (BP + AP) + (DR + CR) = (DS + AS) + (CQ + BQ)

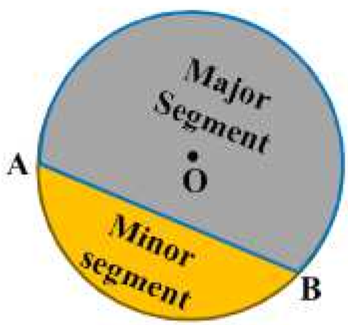
$\Rightarrow$ AB + CD = AD + BC

# 5 Area Related to circles

## 5.3 Areas of Sector and Segment of a Circle

The portion (or part) of the circular region enclosed by two radii and the corresponding arc is called a sector of the circle and the portion (or part) of the circular region enclosed between a chord and the corresponding arc is called a segment of the circle.



**some relations (or formulae) to calculate their areas.**

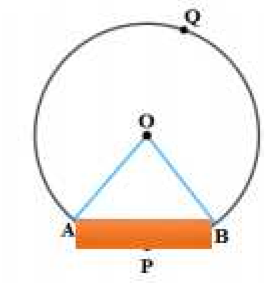
Let OAPB be a sector of a circle with centre O and radius r (see Fig. 5.6). Let the degree measure of $\angle AOB$ be $\theta$ ,

If the angle at the center is $360^0$, then the area of the sector = $\pi r^2$

So, when the degree measure of the angle at the Centre is 1, area of the sector = $\frac{\pi r^2}{360}$

Therefore, when the degree measure of the angle at the centre is $\theta$,

Area of the sector = $\frac{\pi r^2}{360}$ x $\theta \Rightarrow \frac{\theta}{360}$ x $\pi r^2$




**Area of the sector of angle $\theta$ = $\frac{\theta}{360}$ x $\pi r^2$**

**Length of the arc of a sector of angle $\theta$ = $\frac{\theta}{360}$ x $2\pi r$**

**Areaof SegmentAPB** → **Area of OAPB Sector – Area of $\Delta$OAB**

**Area of OAQB Major sector** → **$\pi r^2$ - Area of Minor sectorOAPB**

**Area of AQB Major segment** → **$\pi r^2$ – Area of APB Minor segment**

**Example 2 : Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use = 3.14)**

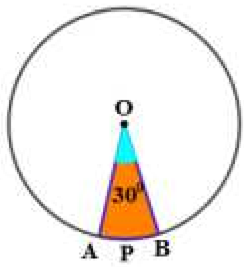
**Solution:** Given sector is OAPB.

Area of the sector OAPB = $\frac{\theta}{360}$ x $\pi r^2$

$\Rightarrow \frac{30}{360}$ x 3.14 x 4 x 4 = $\frac{12.56}{3}$ $\approx$ 4.19 cm$^2$

Area of the corresponding major sector

= $\pi r^2$ - Area of sector OAPB = (3.14 x 16 - 4.19) cm$^2$ $\approx$ 46.1 cm$^2$




**Alternate Method:**

Area of the corresponding major sector = $\frac{360-\theta}{360}$ x $\pi r^2$

= $\frac{360-30}{360}$ x 3.14 x 4 x 4= 46.05 $\approx$ 46.1 cm$^2$

# Exercise 5.2

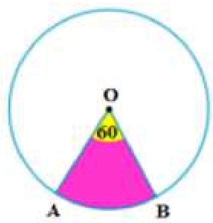
**[Unless stated, otherwise use $\pi = \frac{22}{7}$]**

1. **Find the area of a sector of a circle with radius 6 cm if angle** of the sector is 60°.

   Area of the sector of angle $\theta$ = $\frac{\theta}{360°} \times \pi r^2$

   Area of the sector of angle $60^0$ = $\frac{60}{360°} \times \pi r^2$ cm$^2$

   = $\frac{1}{6} \times 6 \text{x} 6 \text{ x } \frac{22}{7}$ = $\frac{132}{7}$ cm$^2$




2. **Find the area of a quadrant of a circle whose circumference is 22 cm**

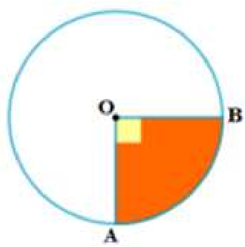
   Quadrant of a circle = Angle of sector $90^0$

   Circumference C = $2\pi r$ = 22 cm

   Radius r = $\frac{22}{2\pi}$ cm = $\frac{22 \text{x} 7}{2 \text{x} 22}$ = $\frac{7}{2}$ cm

   Area of the sector of angle $\theta$ = $\frac{\theta}{360°} \times \pi r^2$

   Area of the sector of angle 90° = $\frac{90}{360°} \times \pi r^2 = \frac{1}{4} \times \frac{22}{7} \text{x} \frac{7}{2} \text{x} \frac{7}{2} = \frac{77}{8}$ cm$^2$



3. **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

The minute hand is the radius of the circle. $\Rightarrow$ Radius(r) = 14 cm

The angle of rotation formed by minute hand in 1 hour $= 360^\circ$

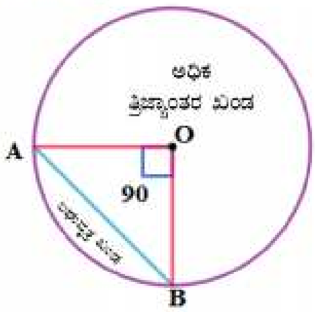
$\therefore$ The angle of rotation in 5 minutes $= \frac{360^\circ}{60} \times 5 = 30^\circ$

Area of the sector of angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$

$\therefore$ Area of the sector of angle $30^0 = \frac{30}{360^\circ} \times \frac{22}{7}$ x 14 x 14 $= \frac{1}{12} \times \frac{22}{7}$ x 14 x 14

$= \frac{1}{3} \times 22$ x 7 $= \frac{154}{3}$ cm$^2$

4. **A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use =3.14)**




Radius of the circle = 10 cm

The angle of the Major sector = $360^\circ - 90^\circ = 270^\circ$

Area of the major sector $= \frac{270}{360^\circ} \times \pi r^2$ cm$^2$

$= \frac{3}{4} \times 3.14$ x10 x 10 = 75 x 3.14 cm$^2$ = 235.5 cm$^2$

In right angle $\Delta AOB$ , OA = 10 cm, OB = 10 cm

Area of $\Delta AOB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 10 \times 10 = 50$ cm$^2$ --- (1)

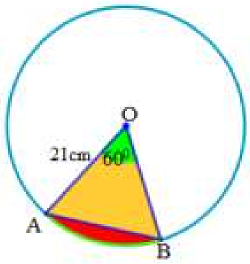
The angle of the Minor sector = $90^\circ$

Area of the minor sector $= \frac{90}{360^\circ} \times \pi r^2$ cm$^2$

$= \frac{1}{4} \times 3.14$ x10 x 10 = 25 x 3.14 cm$^2$ = 25 $\times$ 3.14 cm$^2$ = 78.5 cm$^2$ -------- (2)

Area of minor segment = (2) − (1) = 78.5 cm$^2$ - 50 cm$^2$ = 28.5 cm$^2$

5. **In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:**
**(i) the length of the arc (ii) area of the sector formed by the arc**
**(iii) area of the segment formed by the corresponding chord**




Radius of the circle = 21 cm

(i) The length of the Arc AB $= \frac{\theta}{360^\circ} \times 2\pi r$

Arc AB $= \frac{60}{360^\circ} \times 2$ x $\frac{22}{7}$ x 21 $= \frac{1}{6} \times 2$ x 22 x 3 = **22cm**

(ii) The angle formed by arc AB = $60^\circ$

Area of the sector of angle $60^0 = \frac{60}{360^\circ} \times \pi r^2$ cm$^2$

$= \frac{60}{360^\circ}$ x $\frac{22}{7}$ x 21x21 cm$^2$ $= \frac{1}{6}$ x 22x 3 x21 cm$^2$ $= \frac{1}{2}$ x 22x21 cm$^2$

= 11x21 cm$^2$ = **231 cm$^2$**

(iii) The area of the equilateral $\Delta AOB = \frac{\sqrt{3}}{4}$ x $(OA)^2 = \frac{\sqrt{3}}{4}$ x $(21)^2 = \frac{441\sqrt{3}}{4}$ cm$^2$

Hence the required area = Area of the sector formed by the Arc − area of $\Delta AOB$

$= \left(\mathbf{231} - \frac{441\sqrt{3}}{4}\right)$**cm$^2$**

8. **A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 5.11). Find**
**(i) The area of that part of the field in which the horse can graze**
**(ii) The increase in the grazing area if the rope were10 m long instead of 5 m. (Use = 3.14)**

Given, the side of the square = 15 m

The length of the rope [Radius of the arc(r)] = 5 m

The radius of the field in which the horse can graze = 5 m.

(i) Area of the field graze by the horse

[Horse is tied at the corner of the square.So, it graze only quadrant of the circle of radius 5m]

$= \frac{\pi r^2}{4} = \frac{3.14 \times 5^2}{4} = \frac{78.5}{4} = \mathbf{19.625\ m^2}$

(ii) The length of the rope is 10m then, the area graze by the horse $= \frac{\pi r^2}{4} = \frac{3.14 \times 10^2}{4} = \frac{314}{4} = \mathbf{78.5\ m^2}$

Therefore increase in grazing area

= 78.5 m$^2$ - 19.625 m$^2$ = **58.875 m$^2$**

Fig. 12.11

**An umbrella has 8 ribs which are equally spaced (see Fig. 5.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.**

Fig 5.13

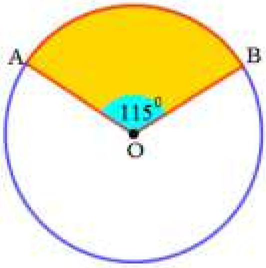
Total ribs in the umbrella = 8

The radius of the umbrella when it to be flat = 45 cm

The area between the two consecutive ribs = $\frac{\text{Total Area}}{\text{number of ribs}}$

$= \frac{\pi r^2}{8} = \frac{\frac{22}{7} \times 45^2}{8} = \frac{44550}{56} = \frac{22275}{28}\ \text{cm}^2$ **= 795.5 cm²**

11. **A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.**




The angle of the sector formed by the wiper = 115°

Radius of the sector = length of the wiper = 25 cm

Area of the sector formed by the wiper = $\frac{115°}{360°} \times \pi r^2\ \text{cm}^2$

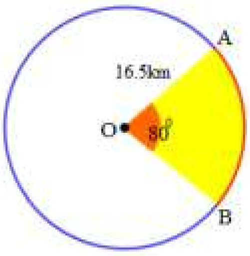
$= \frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25\ \text{cm}^2$

$= \frac{23}{72} \times \frac{22}{7} \times 625\ \text{cm}^2 = \frac{23}{36} \times \frac{11}{7} \times 625\ \text{cm}^2 = \frac{158125}{252}\ \text{cm}^2$

The total area coveed by blades of two wipers

$= 2 \times \frac{158125}{252}\ \text{cm}^2 = \frac{158125}{126}$ **= 1254.96 cm²**

12. **To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use =3.14)**




Let the lighthouse be at O

Radius of the sector = length of the beam r = 16.5 km

Angle of the sector formed by the beam = 80°

The area of the sector which light spreads = Area of the sector = $\frac{80°}{360°} \times \pi r^2\ \text{km}^2$

$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5\ \text{km}^2 = \frac{2}{9} \times 3.14 \times 272.25\ \text{km}^2$ **= 189.97 km²**

**Example 5: Find the area of the shaded region in Fig. 5.16, where ABCD is a square of side 14 cm**

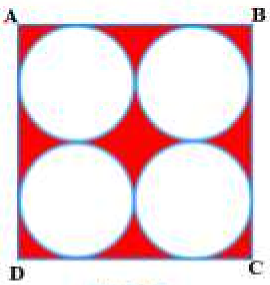



Fig 5.16

**Solution :** Area of square ABCD = 14 x 14 cm² = 196 cm

Diaameter of each circle = $\frac{14}{2}$ = 7cm

So, radius of the circle = $\frac{7}{2}$ cm

So, area of each circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}\ \text{cm}^2$

Therefore area of four circles = $4 \times \frac{77}{2} = 154\text{cm}^2$

Therefore area of shaded region = (196 − 154)= 42cm²

ಉದಾಹರಣೆ 6: ABCD ಯು 10 cm ಬಾಹುವುಳ್ಳ ಚೌಕವಾಗಿದೆ ಮತ್ತು ಪ್ರತಿ ಚೌಕದ ಬಾಹುವು ವ್ಯಾಸವಾಗಿರುವಂತೆ ಅರ್ಧವೃತ್ತವನ್ನು ಎಳೆದಿದೆ. ಚಿತ್ರ 5.17 ರಲ್ಲಿ ಛಾಯೆಗೊಳಿಸಿದ ವಿನ್ಯಾಸದ ವಿಸ್ತೀರ್ಣವನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ.
($\pi$ = 3.14 ಎಂದು ಬಳಸಿ)

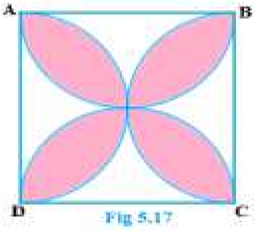


Fig 5.17

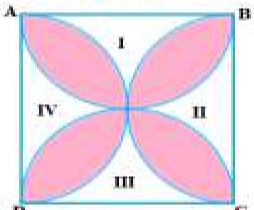


Fig 5.18

Area I + Area II = Area ABCD – Area of the two semicircles circle of radius 5 cm

$\Rightarrow$Area of ABCD – Area of the circle of radius5 cm = $a^2 - \pi r^2$

$\Rightarrow 10 \times 10 - 3.14 \times 5^2 = 100 - 3.14 \times 25 = 100 - 78.5 = 21.5\text{cm}^2$

Area III + Area IV = $21.5\text{cm}^2$

Therefore, Area of shaded region = Area ABCD – Area [ I + II + III + IV]

$= 100 - 2\times(21.5) = 100 - 43 = 57\text{cm}^2$

# Exercise 5.3

[ Unless stated otherwise, use $\pi = \frac{22}{7}$ ]

1. **Find the area of the shaded region in Fig. 5.19, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.**

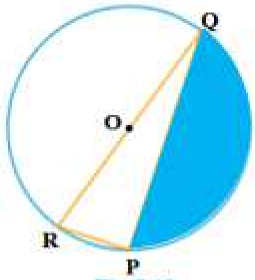


Fig 5.19

PQ = 24 cm and PR = 7 cm

$\angle P = 90°$ [Angle of semi circle ]

$\therefore$ Hypotenuse QR = Diameter of the circle

$QR^2 = PR^2 + PQ^2$ [Pythagoras theorem in $\Delta$ PRQ]

$\Rightarrow QR^2 = 7^2 + 24^2 \Rightarrow QR^2 = 49 + 576$

$\Rightarrow QR^2 = 625 \Rightarrow QR = 25$ cm

$\therefore$ Radius of the circle = $\frac{25}{2}$ cm

Area of semi circle $= \frac{\pi R^2}{2} = \frac{\frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}}{2} = \frac{13750}{56}\text{ cm}^2 = \mathbf{\frac{6875}{28}\text{ cm}^2 = 245.54\text{ cm}^2}$

Area of $\Delta PQR = \frac{1}{2} \times PR \times PQ$

$= \frac{1}{2} \times 7 \times 24\text{ cm}^2 = \mathbf{84\text{ cm}^2}$

$\therefore$ **Area of shaded region** $= 245.54\text{ cm}^2 - 84\text{ cm}^2 = \mathbf{161.54\text{ cm}^2}$

[ Or $\frac{6875}{28} - 84 = \frac{6875-2352}{28} = \frac{4523}{28}\text{cm}^2$ ]

2. **Find the area of the shaded region in Fig. 5.20, if radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and AOC = 40°.**

Radius of the inner circle = 7 cm

Radius of the outer circle = 14 cm

The angle of the sector = 40°

Area of the sector OAC $= \frac{40°}{360°} \times \pi r^2\text{ cm}^2$

$= \frac{1}{9} \times \frac{22}{7} \times 14^2\text{ cm}^2 = \frac{1}{9} \times 22 \times 2 \times 14\text{ cm}^2 = \mathbf{\frac{616}{9}\text{ cm}^2}$

Area OBD = Area of the sector $= \frac{40°}{360°} \times \pi r^2\text{ cm}^2$

$= \frac{1}{9} \times \frac{22}{7} \times 7^2\text{ cm}^2 = \frac{1}{9} \times 22 \times 7\text{ cm}^2 = \mathbf{\frac{154}{9}\text{ cm}^2}$

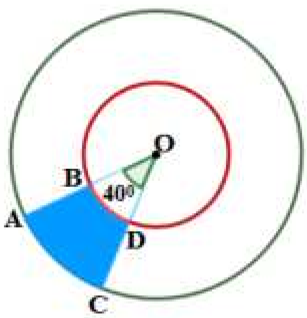


Fig 5.20

$\therefore$ **Area of shaded region**

= Area of the sector OAC - Area of sector OBD

$= \left(\frac{616}{9} - \frac{154}{9}\right)\text{cm}^2 = \frac{462}{9}\text{ cm}^2 = \mathbf{\frac{154}{3}\text{ cm}^2}$

3. **Find the area of the shaded region in Fig. 5.21, if ABCD is a square of side 14 cm and APD and BPC are semicircles.**

Side of the square = 14 cm

Diameter of the semi circle = 14 cm

∴ Radius of the semi circle = 7 cm

Area of the square = 14 × 14 = 196 cm$^2$

Area of the semi circle $= \frac{\pi R^2}{2} = \frac{\frac{22}{7} \times 7 \times 7}{2} = \frac{154}{2} = 77$ cm$^2$

Area of two semicircle = 2 × 77 cm$^2$ = 154 cm$^2$

∴ **Area of shaded region** = 196 cm$^2$ - 154 cm$^2$ = **42 cm$^2$**

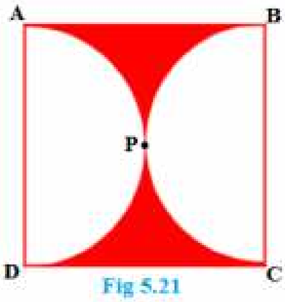


Fig 5.21

5. **From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in Fig. 5.23. Find the area of the remaining portion of the square.**

The side of the square = 4 cm; Radius of the circle = 1 cm

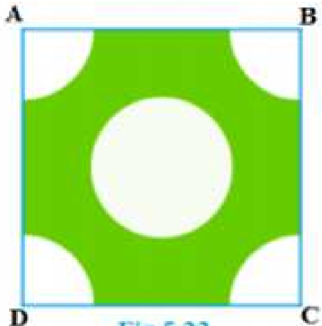


Fig 5.23

Area of the square = (Side)$^2$ = 4$^2$ = 16 cm$^2$

Area of each quadrant $= \frac{\pi R^2}{4}$ cm$^2$ $= \frac{\frac{22}{7} \times 1^2}{4} = \frac{11}{14}$ cm$^2$

∴ Area of four quadrant $= 4 \times \frac{11}{14}$ cm$^2$ $= \frac{22}{7}$ **cm$^2$**

Area of the circle = $\pi$ R$^2$ cm$^2$ $= \frac{22}{7} \times 1^2 = \frac{22}{7}$ cm$^2$

Area of the square = Side$^2$ = 4$^2$ = 16cm$^2$

$\left(\frac{22}{7} + \frac{22}{7}\right)$ cm$^2$ = $\frac{44}{7}$ **cm$^2$**

∴ **Area of shaded region** = Area of square – [Area of four quadrants+ area of circle]

$= 16 - \left(\frac{22}{7} + \frac{22}{7}\right)$cm$^2$ $= \left(\frac{112-44}{7}\right)$cm$^2$ $= \frac{68}{7}$ **cm$^2$**

7. **In Fig. 5.25, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.**

Side of the Square = 14 cm

∴ Radius of each circle $= \frac{14}{2} = 7$ cm

Area of square ABCD = 14$^2$ = 196 cm$^2$

Area of the quadrant $= \frac{\pi R^2}{4}$ cm$^2$ $= \frac{\frac{22}{7} \times 7^2}{4} = \frac{154}{4}$ cm$^2$ $= \frac{77}{2}$ cm$^2$

∴ Area of four quadrant $= 4 \times \frac{77}{2}$ cm$^2$ = 154 cm$^2$

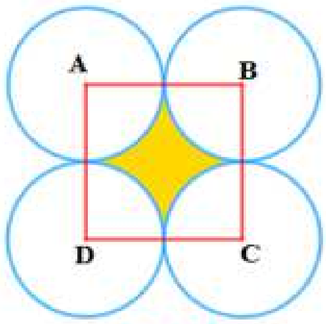


Fig 5.25

∴ **Area of shaded region**

= Area of the square ABCD - 4 Area of four quadrant

= 196 cm$^2$ - 154 cm$^2$= **42 cm$^2$**

9. **In Fig. 5.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.**

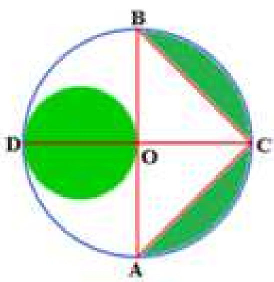


Fig 5.27

Radius of the greater circle R = 7 cm

Radius of the smaller circle r $= \frac{7}{2}$ cm

Height of $\Delta$BCA = OC = 7 cm; Base of $\Delta$BCA = AB = 14 cm

Area of $\Delta$BCA $= \frac{1}{2} \times$ AB × OC $= \frac{1}{2} \times 7 \times 14 =$ **49 cm$^2$**

Area of greater circle $= \pi R^2 = \frac{22}{7} \times 7^2 =$ **154 cm$^2$**

Area of greater semi-circle = $\frac{154}{2}$ cm$^2$ = **77 cm$^2$**

Area of smaller circle = $\pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2}$**cm$^2$**

**Area of shaded aregion**

= Area of greater semi-circle - Area of $\Delta$BCA + Area of smaller circle

$= \left(77 - 49 + \frac{77}{2}\right)$ cm$^2$ = $\left(\frac{154-98+77}{2}\right)$ cm$^2$ = $\left(\frac{133}{2}\right)$ cm$^2$ = **66.5 cm$^2$**

**11. On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 5.29). Find the area of the remaining portion of the handkerchief.**

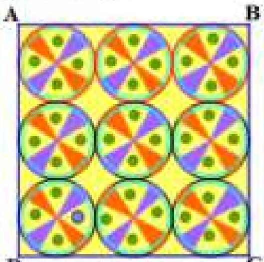


Fig 5.29

Number of circles = 9; Radius of each circle = 7 cm

There are three circles touch each other

∴ Side of the square = 3 × diameter of the circle = 3 × 14 = 42 cm

Area of the square = 42 × 42 cm$^2$ = 1764 cm$^2$

Area of 9 equal circle = $9\pi r^2 = 9\text{x } \frac{22}{7} \times 7 \times 7 = 1386$ cm$^2$

**The area of remaing part of the handkerchief**

= Area of the square - Area of 9 equal circle = 1764 - 1386 = **37**

**12. In Fig. 5.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the**

**i) quadrant OACB ii) shaded region.**

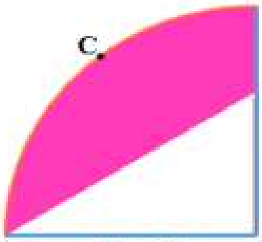


Fig 5.30

Radius of the quadrant of the circle = 3.5 cm = $\frac{7}{2}$ cm

(i) Area of OACB quadrant = $\frac{\pi R^2}{4}$ cm$^2$ = $\frac{\frac{22}{7}\text{x}\frac{7}{2}\text{x}\frac{7}{2}}{4}$ cm$^2$= $\frac{77}{8}$ **cm$^2$**

(ii) Area of $\Delta$BOD = $\frac{1}{2} \times \frac{7}{2} \times 2$ cm$^2$= $\frac{7}{2}$ **cm$^2$**

**Area of shaded region**

= Area of OACB – Area of $\Delta$ BOD

$= \left(\frac{77}{8} - \frac{7}{2}\right)$ cm$^2$= $\left(\frac{77}{8} - \frac{28}{8}\right)$ cm$^2$ = $\left(\frac{49}{8}\right)$ cm$^2$ = **6.125 cm$^2$**

**15. In Fig. 5.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.**

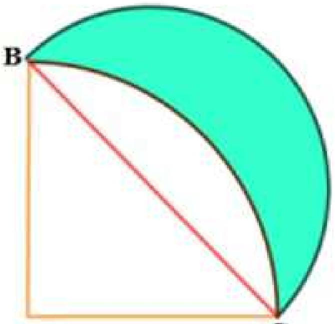


Fig 5.33

The radius of quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is the diameter of semi circle

Now, ABC is a right angle triangle

∴BC$^2$ = AB$^2$ + AC$^2$ [Pythgoras theorem]

⇒ BC$^2$ = 14$^2$ + 14$^2$ ⇒ BC = $14\sqrt{2}$ cm

Radius semi circle = $\frac{14\sqrt{2}}{2}$cm = $7\sqrt{2}$ cm

Area of $\Delta$ABC = $\frac{1}{2} \times 14 \times 14$ cm$^2$ = 7 × 14 × 14 = **98 cm$^2$**

The area of the quadrant of circle = $\frac{\pi R^2}{4}$ cm$^2$ = $\frac{\frac{22}{7}\text{x } 14\text{x}14}{4}$ cm$^2$ = **154 cm$^2$**

Area of semi circle = $\frac{\pi R^2}{2}$ = $\frac{\frac{22}{7}\text{x } 7\sqrt{2} \text{ x } 7\sqrt{2}}{2}$ = **154 cm$^2$**

**Area of shaded region**

= Area of semi circle + Area of $\Delta$ABC- Area of quadrant of circle

= 154 + 98 - 154 cm$^2$ = **98 cm$^2$**

# 7 Coordinate Geometry

## Coordinate axes:

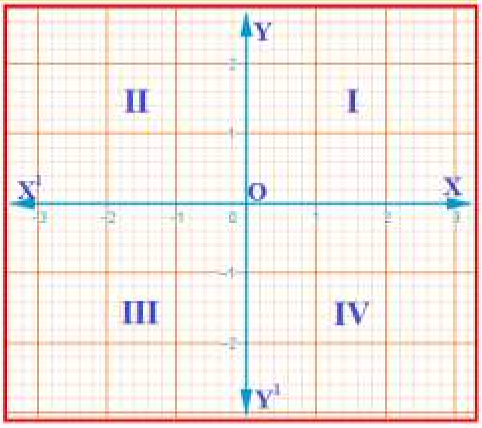
**A set of a pair of perpendicular axes $X^lOX$ and $YOY^l$**

Horizontal line

$X^1OX$ — X - Axis

Vertical line

$YOY^1$ — Y - Axis




The intersection point of X and Y axes is called the Orgin 'O'

The distance of a point from the y-axis is called its x-coordinate, or abscissa. The distance of a point from the x-axis is called its y-coordinate, or ordinate. The coordinates of a point on the x-axis are of the form (x, 0), and of a point on the y-axis are of the form (0, y).

**The Coordinate axes divides the plane in to four parts. They are called quadrants.**

**The coordinaes of the orgin is $(0, 0)$**

## 7.2 Distance Formula

The distance between two points on X-axis or on the straight line paralle to X-axis is

**Distance** $= x_2 - x_1$

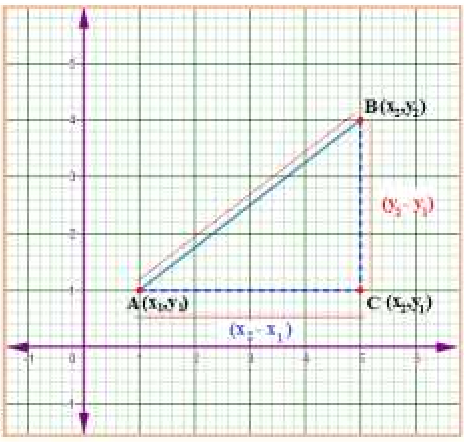
The distance between two points on Y-axis or on the straight line paralle to Y-axis is

**Distance** $= y_2 - y_1$

$AB^2 = AC^2 + BC^2$

The distance between two points which are neither on X or Y axis nor on the line paralle to X orY axis

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$




The distance between the point P(x,y) and the orgin

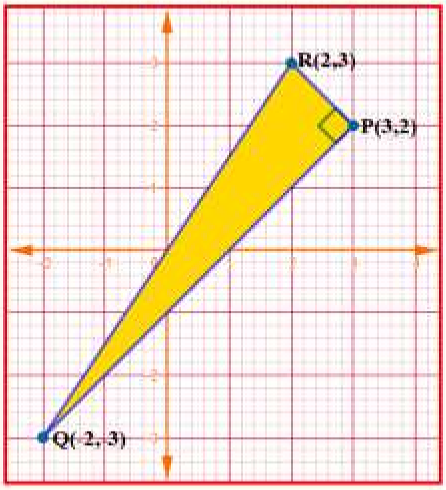
$$d = \sqrt{x^2 + y^2}$$

**Example 1:Do the points (3, 2), (–2, –3) and (2, 3) form a triangle? If so, name the type of triangle formed.**

$P(3,2),\quad Q(-2,-3),\quad R(2,3)$

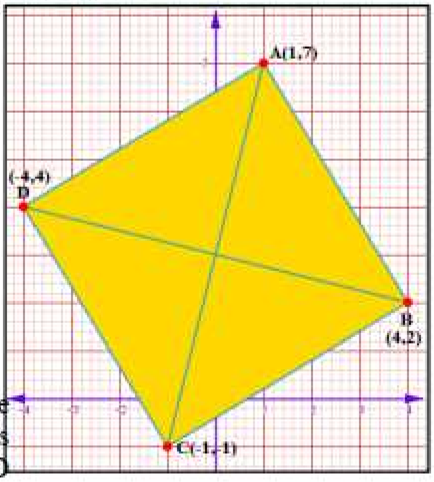
**Formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(3-(-2))^2+(2-(-3))^2}$
$= \sqrt{(3+2)^2+(2+3)^2}$
$= \sqrt{(5)^2+(5)^2} = \sqrt{25+25} = \sqrt{50} = 7.07$
$QR = \sqrt{(-2-2)^2+(-3-3)^2}$
$= \sqrt{(-4)^2+(-6)^2} = \sqrt{16+36} = \sqrt{52} = 7.21$
$PR = \sqrt{(3-2)^2+(2-3)^2} = \sqrt{(1)^2+(-1)^2}$
$= \sqrt{1+1} = \sqrt{2} = 1.41$
Since the sum of any two of these distances is greater than the third distance, therefore the point P,Q and R form a triangle.
Also, $PQ^2 + PR^2 = QR^2$ by the converse of Pythagoras theorem $\angle P = 90^0$ we have
Therefore, PQR is a right triangle.

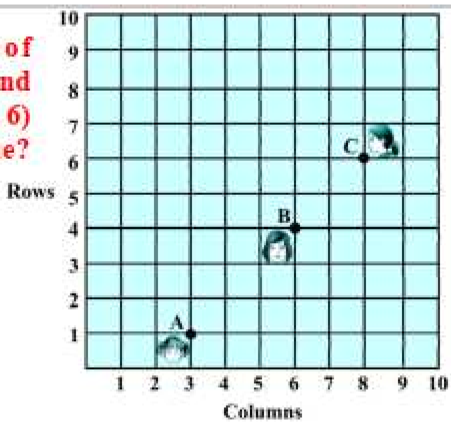



**Example2: Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.**

**Solution:** A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4)
$AB = \sqrt{(4-1)^2+(2-7)^2} = \sqrt{(3)^2+(-5)^2} = \sqrt{9+25} = \sqrt{34}$
$BC = \sqrt{(-1-4)^2+(-1-2)^2} = \sqrt{(-5)^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$
$CD = \sqrt{(-4-(-1))^2+(4-(-1))^2}$
$= \sqrt{(-4+1)^2+(4+1)^2}$
$= \sqrt{(-3)^2+(5)^2} = \sqrt{9+25} = \sqrt{34}$
$DA = \sqrt{(1-(-4))^2+(7-4)^2}$
$= \sqrt{(1+4)^2+(3)^2} = \sqrt{(5)^2+(3)^2} = \sqrt{25+9} = \sqrt{34}$
$AC = \sqrt{(-1-1)^2+(-1-7)^2}$
$= \sqrt{(-2)^2+(-8)^2} = \sqrt{4+64} = \sqrt{68}$
$BD = \sqrt{(-4-4)^2+(4-2)^2}$
$= \sqrt{(-8)^2+(2)^2} = \sqrt{64+4} = \sqrt{68}$
Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Thereore, ABCD is a square




**Example 3 : Fig. 7.6 shows the arrangement of desks in a classroom. Ashima, Bharti and Camella are seated at A (3, 1), B(6, 4) and C(8, 6) respectively. Do you think they are seated in a line? Give reasons for your answer.**




$AB = \sqrt{(6-3)^2+(4-1)^2} = \sqrt{(3)^2+(3)^2}$
$= \sqrt{9+9} = \sqrt{18} = \sqrt{9\times 2} = \mathbf{3\sqrt{2}}$
$BC = \sqrt{(8-6)^2+(6-4)^2} = \sqrt{(2)^2+(2)^2}$
$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = \mathbf{2\sqrt{2}}$
$AC = \sqrt{(8-3)^2+(6-1)^2} = \sqrt{(5)^2+(5)^2}$
$= \sqrt{25+25} = \sqrt{50} = \sqrt{25\times 2} = \mathbf{5\sqrt{2}}$

$AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$
Since., **AB** + **BC** = **AC** we can say that the points A, B and C are collinear.
Therefore, they are seated in a line

# Exercise 7.1

1. **Find the distance between the following pairs of points :**
**i) (2, 3), (4, 1) ii) (-5, 7), (-1, 3) iii) (a, b), (-a, -b)**

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| 2 | 3 | 4 | 1 |

i) $(x_1, y_1) = (2, 3)$, $(x_2, y_2) = (4, 1)$

**Formula** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2}$

$d = \sqrt{4+4} = \sqrt{2\times 4} = 2\sqrt{2}$ Units

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -5 | 7 | -1 | 3 |

ii) $(x_1, y_1) = (-5, 7)$, $(x_2, y_2) = (-1, 3)$

$d = \sqrt{(-1-[-5])^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2}$

$d = \sqrt{16+16} = \sqrt{2\times 16} = 4\sqrt{2}$ Units

iii) $(x_1, y_1) = (a, b)$, $(x_2, y_2) = (-a, -b)$

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| a | b | -a | -b |

$d = \sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2}$

$d = \sqrt{4a^2+4b^2} = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$ Units

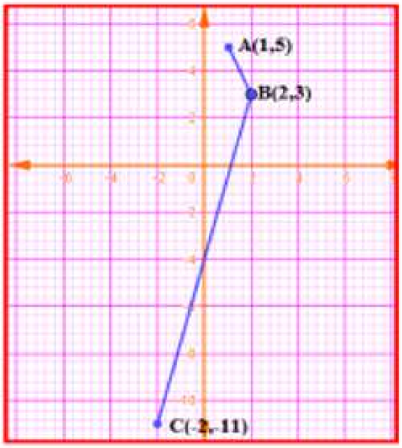
2. **Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.**

$(x, y) = (36, 15)$

$d = \sqrt{x^2+y^2} = \sqrt{36^2+15^2} = \sqrt{1296+225} = \sqrt{1521} = \mathbf{39}$ **Units**

We can find the distance between the two towns A and B. Suppose town A is at the Orgin, then the town has to be at (36,15). The distance between these two towns is 39km (1, 5).

3. **Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.**




A (1, 5), B (2, 3) ಮತ್ತು C (-2, -11)

$AB = \sqrt{(2-1)^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$BC = \sqrt{(-2-2)^2+(-11-3)^2}$

$= \sqrt{(-4)^2+(-14)^2}$

$= \sqrt{16+196} = \sqrt{212}$

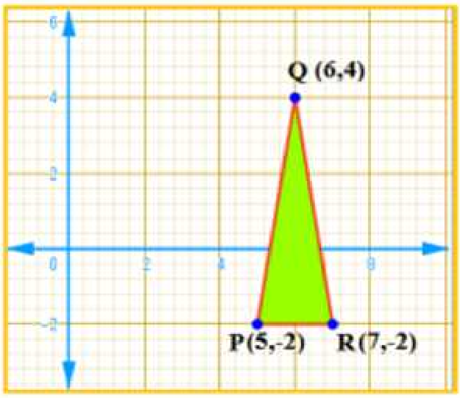
$AC = \sqrt{(-2-1)^2+(-11-5)^2}$

$= \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$

$AB + BC \neq AC$

$\therefore$ **These are non-collinear points**

4. **Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.**




Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(6-5)^2+(4-(-2))^2}$

$= \sqrt{(1)^2+(6)^2} = \sqrt{1+36} = \sqrt{37}$ (i)

$QR = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{(1)^2+(-6)^2} = \sqrt{1+36} = \sqrt{37}$ (ii)

$PR = \sqrt{(7-5)^2+(-2-[-2])^2}$

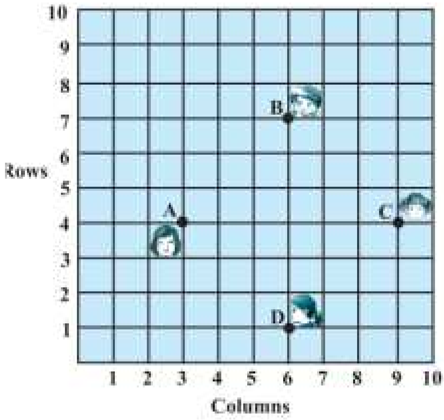
$= \sqrt{(2)^2+(0)^2} = \sqrt{4} = 2$ (iii)

(i), (ii), (iii) $\Rightarrow$ PQ = QR.

Since, Two sides of the tringle are equal.

Hence, $\Delta PQR$ is an isosceles triangle.

5. **In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.**




The coordinates of the points A,B,C and D are, A(3,4) , B(6,7) , C(9,4), D(6,1)

$AB = \sqrt{(6-3)^2+(7-4)^2} = \sqrt{(3)^2+(3)^2}$

$= \sqrt{9+8} = \sqrt{18} = 3\sqrt{2}$ **(i)**

$BC = \sqrt{(9-6)^2+(4-7)^2} = \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(ii)**

$CD = \sqrt{(6-9)^2+(1-4)^2} = \sqrt{(-3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(iii)**

$DA = \sqrt{(6-3)^2+(1-4)^2} = \sqrt{(3)^2+(-3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ **(iv)**

$AB = BC = CD = DA$

Diagonal $AC = \sqrt{(9-3)^2+(4-4)^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36} = 6$ **(v)**

Diagonal $BD = \sqrt{(6-6])^2+(7-1)^2} = \sqrt{(0)^2+(6)^2} = \sqrt{36} = 6$ **(vi)**

$AC = BD$

Thus, $AB = BC = CD = DA$ , diagonals: $AC = DB$

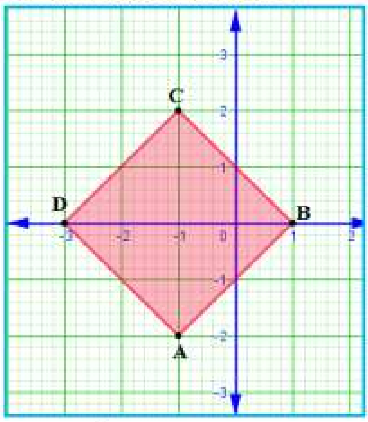
Since all the four sides and diagonals are equal.

Hence, ABCD is a square. So, Champa is correct.

6. **Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:**

**i) (-1, -2), (1, 0), (-1, 2), (-3, 0) ii) (-3, 5), (3, 1), (0, 3), (-1, -4) iii) (4, 5), (7, 6), (4, 3), (1, 2)**

i) $\mathbf{A(-1,-2)}$, $\mathbf{B(1,0)}$, $\mathbf{C(-1,2)}$, $\mathbf{D(-3,0)}$




$AB = \sqrt{\left(1-(-1)\right)^2+\left(0-(-2)\right)^2}$

$= \sqrt{(1+1)^2+(0+2)^2} = \sqrt{(2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2+(2-0)^2} = \sqrt{(-2)^2+(2)^2}$

$= \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$CD = \sqrt{\left(-3-(-1)\right)^2+(0-2)^2} = \sqrt{(-3+1)^2+(-2)^2}$

$= \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$DA = \sqrt{\left(-3-(-1)\right)^2+\left(0-(-2)\right)^2}$

$= \sqrt{(-3+1)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$AB = BC = CD = DA$

$AC = \sqrt{\left(-1-(-1)\right)^2+\left(2-(-2)\right)^2} = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{(0)^2+(4)^2} = \sqrt{16} = 4$

$BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16} = 4$

Thus, $AC = BD$

Since, the four sides AB, BC, CD and DA are equal and the diagonals $AC = DB$ are equal.

So the quadrilateral ABCD is a square.

**ii) A(-3, 5), B(3, 1), C(0, 3), D(-1, -4)**

$AB = \sqrt{(3-(-3))^2+(1-(-3))^2}$

$= \sqrt{(3+3)^2+(1+3)^2} = \sqrt{(6)^2+(4)^2}$

$= \sqrt{36+16} = \sqrt{52}$

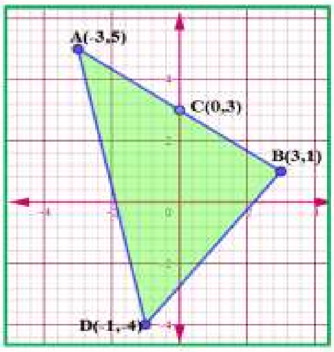
$BC = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{(-3)^2+(2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(-1-0)^2+(-4-3)^2}$

$= \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49}$

$= \sqrt{50} = \sqrt{25\times 2} = 5\sqrt{2}$




$DA = \sqrt{(-3-(-1))^2+(-4-5)^2}$

$= \sqrt{(-3+1)^2+(-9)^2} = \sqrt{(-2)^2+(-9)^2}$

$= \sqrt{4+81} = \sqrt{85}$

$AB \neq BC \neq CD \neq DA$

Since, the four sides AB, BC, CD and DA are not equal.Hence these poist does not form a quadrilateral.

**iii) A(4, 5), B(7, 6), C(4, 3), D(1, 2)**

Formula: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB = \sqrt{(7-4)^2+(6-5)^2}$

$= \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2+(3-6)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$

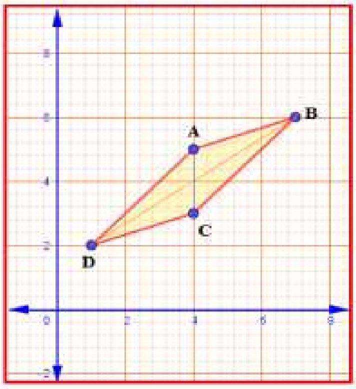
$CD = \sqrt{(1-4)^2+(2-3)^2}$

$= \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(1-4)^2+(2-5)^2}$

$= \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$




$AB = CD, \; BC = DA$

$AC = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

$BD = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = \sqrt{4\times 13} = 2\sqrt{13}$

$AC \neq DB$

Thus opposite sides are equal. $AB = CD$, & $BC = DA$

But diagonals are not equal. $AC \neq DB$ ∴ The given points are forming a parallelogram.

7. **Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).**

   We know that a point on the X – axis is of the form (x, 0).

   Let the point P (x, 0) is equi distance from the points A(2, -5) and B(-2, 9)

   AP = BP

   $(x-2)^2+(0-(-5))^2 = (x-(-2))^2+(0-9)^2$

   $(x-2)^2+5^2 = (x+2)^2+(-9)^2$

   $x^2+2^2-2(x)(2)+25 = x^2+2^2+2(x)(2)+81$

   $-4x+25 = 4x+81$

   $-4x-4x = 81-25 \Rightarrow -8x = 56 \Rightarrow x = \frac{56}{-8} = -7$

   Thus, the required point is $(-7, 0)$

8. **Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.**

   $(x_1, y_1) = (2, -3)$, $(x_2, y_2) = (10, y)$, $d = 10$

   Formula: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

   $10 = \sqrt{(10-2)^2+(y-(-3))^2} = \sqrt{(8)^2+(y+3)^2}$

   $10^2 = 64+(y+3)^2 \Rightarrow 100-64 = (y+3)^2$

   $\Rightarrow (y+3)^2 = 36 \Rightarrow y+3 = \pm\sqrt{36} \Rightarrow y+3 = \pm 6$

   $\Rightarrow y = 6-3 = 3$ or $x = -6-3 = -9$

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| 2 | -3 | 10 | y |

**9. If Q(0, 1) is equidistant from P(5, –3) and R(x, 6), find the values of x. Also find the distances QR and PR.**

The point Q (0, 1) is equi distance from the points P (5, -3) and R ($x$, 6)

$PQ = QR \;\; \Rightarrow PQ^2 = PR^2$

$PQ = \sqrt{(5\;-0)^2+(-3-1)^2} = \sqrt{(5\,)^2+(-4)^2} = \sqrt{25+16} = \sqrt{41}$

$QR = \sqrt{(x\;-0)^2+(6-1)^2} = \sqrt{(x)^2+(5)^2} = \sqrt{x^2+25}$

$PQ^2 = PR^2 \;\Rightarrow \left(\sqrt{x^2+25}\right)^2 = \left(\sqrt{41}\right)^2$

$x^2 + 25 = 41 \Rightarrow\; x^2 = 41 - 25 \Rightarrow\; x^2 = 16 \;\;\Rightarrow\; x = \pm\sqrt{16} \;\;\Rightarrow\; x = \pm 4$

The coordinate of the point R is $(4{,}6)$ or $(-4{,}6)$

If the coordinates of R is $(4{,}6)$ then,

$QR = \sqrt{(4\;-0)^2+(6-1)^2} = \sqrt{(4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(4-5)^2+(6-(-3))^2} = \sqrt{(-1)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$

If the coordinates of R is $(-4{,}6)$ then,

$QR = \sqrt{(-4\;-0)^2+(6-1)^2} = \sqrt{(-4)^2+(5)^2} = \sqrt{16+25} = \sqrt{41}$

$PR = \sqrt{(-4-5)^2+(6-(-3))^2} = \sqrt{(-9)^2+(6+3)^2} = \sqrt{81+81} = \sqrt{81\times 2} = 9\sqrt{2}$

**10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (– 3, 4).**

The point P ($x$, y) is equidistance from the points A (3, 6) and B (-3, 4).

$PA = PB \;\; \Rightarrow PA^2 = PB^2$

$PA = \sqrt{(x\;-\;3)^2+(y-6)^2}$

$PB = \sqrt{(x-(-3))^2+(y-4)^2}$

$AP^2 = BP^2 \Rightarrow \left(\sqrt{(x\;-\;3)^2+(y-6)^2}\right)^2 = \left(\sqrt{(x-(-3))^2+(y-4)^2}\right)^2$

$(x\;-\;3)^2 + (y\;-\;6)^2 = (x+\;3)^2 + (y\;-\;4)^2$

$x^2 + 3^2 - 2(x)(3) + y^2 + 6^2 - 2(y)(6) = x^2 + 3^2 + 2(x)(3) + y^2 + 4^2 - 2(y)(4)$

$x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$x^2 - x^2 - 6x - 6x + y^2 - y^2 - 12y + 8y = 25 - 45$

$-12x - 4y = -20 \qquad \div -4$

$3x + y - 5 = 0$ This is the required relation

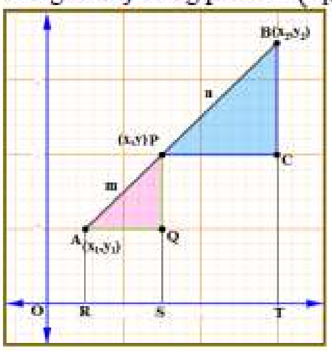
$3x + y - 5 = 0$ is representing a straight line

Thus the point equidistance from the point A and B on the perpendicular bisector of AB

## 7.3 Section Formula

The coordinates of the point P(x, y) which divides the line segment joining points $\mathbf{A}(x_1, y_1)$ and $\mathbf{B}(x_2,\; y_2)$, internally, in the ratio $m_1 : m_2$ are

$$P(x,y) = \left(\frac{m_1x_2\;+\;m_2\;x_1}{m_1+m_2}\;,\;\frac{m_1y_2\;+\;m_2\;y_1}{m_1+m_2}\right)$$




The mid-point of a line segment divides the line segment in the ratio 1: 1. Then the coordinates of the midpoint of the line segment,

$$P(x,y) = \left(\frac{x_2\;+\;x_1}{2}\;,\;\frac{y_2\;+\;y_1}{2}\right)$$

**Example 6 : Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.**

$(x_1, y_1) = (4, -3),\ (x_2, y_2) = (8,5), m_1 : m_2 = 3:1$

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| 4 | -3 | 8 | 5 |

$x = \frac{m_1x_2 + m_2\ x_1}{m_1+m_2} = \frac{3(8)+1(4)}{3+1} = \frac{24+4}{4} = \frac{28}{4} = 7$

$y = \frac{m_1y_2 + m_2\ y_1}{m_1+m_2} = \frac{3(5)+1(-3)}{3+1} = \frac{15-3}{4} = \frac{12}{4} = 3$

Therefore the required point is (7,3)

**Example 7 : In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?**

$P(x,y) = (-4,6),\ A(x_1,\ y_1) = (-6,10)\ , B(x_2,\ y_2) = (3,-8), m_1 = ?, m_2 = ?$

$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$(-4,6) = \left(\frac{m_1(3) + m_2\ (-6)}{m_1+m_2}\ ,\ \frac{m_1(-8) + m_2\ (10)}{m_1+m_2}\right)$

$-4 = \frac{3m_1 - 6\ m_2}{m_1+m_2}$ Or $6 = \frac{-8m_1 + 10\ m_2}{m_1+m_2}$

$-4m_1 - 4m_2 = 3m_1 - 6\ m_2$

$-4m_1 - 3m_1 = -6\ m_2 + 4m_2$

$-7m_1 = -2m_2$

$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7} \Rightarrow m_1 : m_2 = 2:7$

We should verify that the ratio satisfies the y-coordinate also.

$\frac{-8m_1 + 10\ m_2}{m_1+m_2} = \frac{-8(2)+10\ (7)}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7

**Example: Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).**

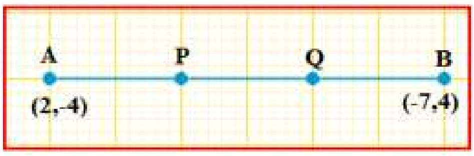
Let P and Q be the trisection points of AB. ⇒ AP = PQ = QB

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula,

$A(x_1, y_1) = (2, -2)\ , B(x_2, y_2) = (-7, 4)$

$m_1 = 1,\ \ m_2 = 2$

$P(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$




$= \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right)$

$= \left(\frac{-3}{3}, \frac{0}{3}\right) = (-1, 0)$

Now, Q also divides AB internally in the ratio 2 : 1. So, the coordinates of Q are

$A(x_1, y_1) = (2, -2)\ , B(x_2, y_2) = (-7, 4)$

$m_1 = 2,\ \ m_2 = 1$

$Q(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$= \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3},\ \ \frac{6}{3}\right) = (-4,\ \ 2)$

Therefore, the coordinates of the points of trisection of the line segment joining A and B are (–1, 0) and (– 4, 2).

# Exercise 7.2

**1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.**

Let the Coordinates of the Points be(x,y)

$m_1 : m_2 = 2:3 \quad (x_1, y_1) = (-1, 7), (x_2, y_2) = (4, -3)$,

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -1 | 7 | 4 | -3 |

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{2(4)+3(-1)}{2+3}, \frac{2(-3)+3(7)}{2+3}\right) = \left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = \left(\frac{5}{5}, \frac{15}{5}\right) \Rightarrow (x, y) = (1, 3)$$

**2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).**

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -1 | 7 | 4 | -3 |

Let P and Q are the trisection points of AB

$\Rightarrow$ AP = PQ = QB

$\therefore$ The point P divides AB internally in the ratio 1 : 2

$A(x_1, y_1) = (4, -1)$, $B(x_2, y_2) = (-2, -3)$,

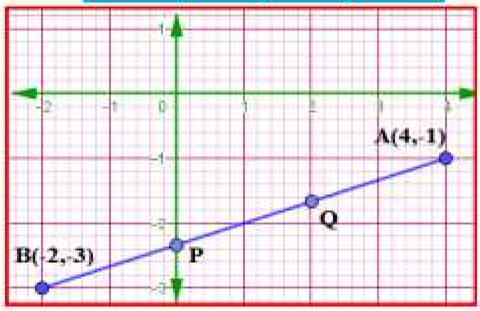
$m_1 = 1, \quad m_2 = 2$

$\therefore$ The coordinates of P is,

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{1(-2)+2(4)}{1+2}, \frac{1(-3)+2(-1)}{1+2}\right)$$

$$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right) = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$$




The point Q divides AB internally in the ratio 2 : 1

$A(x_1, y_1) = (4, -1)$, $\quad B(x_2, y_2) = (-2, -3)$ ; $m_1 = 2, \quad m_2 = 1$

$$Q(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right) \text{ [Using section formula]}$$

$$= \left(\frac{2(-2)+1(4)}{2+1}, \frac{2(-3)+1(-1)}{2+1}\right) = \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right) = \left(\frac{0}{3}, \frac{-7}{3}\right) = \left(0, \frac{-7}{3}\right)$$

**4. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).**

$P(x, y) = (-1, 6)$, $A(x_1, y_1) = (-3, 10)$, $B(x_2, y_2) = (6, -8)$, $m_1 = ?$, $m_2 = ?$

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -3 | 10 | 6 | -8 |

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

$$(-1, 6) = \left(\frac{m_1 6) + m_2(-3)}{m_1+m_2}, \frac{m_1(-8) + m_2(10)}{m_1+m_2}\right)$$

$$-1 = \frac{6m_1 - 3m_2}{m_1+m_2} \quad \text{Or} \quad 6 = \frac{-8m_1 + 10m_2}{m_1+m_2}$$

$$-m_1 - m_2 = 6m_1 - 3m_2$$

$$-m_1 - 6m_1 = -3m_2 + m_2$$

$$-7m_1 = -2m_2$$

$$\frac{m_1}{m_1} = \frac{-2}{-7} = \frac{2}{7}$$

$m_1 : m_2 = 2:7$ We should verify that the ratio satisfies the y-coordinate also

$$\frac{-8m_1 + 10m_2}{m_1+m_2} = \frac{-8(2)+10(7)}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$$

Therefore, the point (– 1, 6) divides the line segment joining the points A(– 3, 10) and B(6, – 8) in the ratio 2 : 7

5. **Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.**

We know that a point on the X – axis is of the form $(x, 0)$ Let the ratio be $k : 1$

$A(x_1, y_1) = (1, -5)$ , $B(x_2, y_2) = (-4, 5)$ $m_1 = k,\ m_2 = 1$

$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$(x, 0) = \left(\frac{k(-4) + 1(1)}{k+1}\ ,\ \frac{k(5) + 1\ (-5)}{k+1}\right)$

$0 = \frac{5k-5}{k+1}$

$5k - 5 = 0 \Rightarrow 5k = 5$

$k = 1$, the ratio is $1 : 1$

$x = \frac{1(-4) + 1(1)}{1+1} = \frac{-4+1}{2} = \frac{-3}{2}$

$\therefore$ The coordinates of the point of division = $\left(\frac{-3}{2},\ 0\right)$

6. **If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.**

**Solution:** Let A(1,2), B(4,y), C(x,6) and D(3,5) are the vertices of the parallelogram.

Since ABCD is a parallelogram

Therefore diagonals AC and BD bisects each other.

So, the coordinates of both AC and BD are same.

$\therefore$ Mid point of AC = Mid point of BD = $\left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$

$\left(\frac{x+1}{2}\ ,\ \frac{6+2}{2}\right) = \left(\frac{3+4}{2}\ ,\ \frac{5+y}{2}\right)$

$\left(\frac{x+1}{2}\ ,\ \frac{8}{2}\right) = \left(\frac{7}{2}\ ,\ \frac{5+y}{2}\right)$

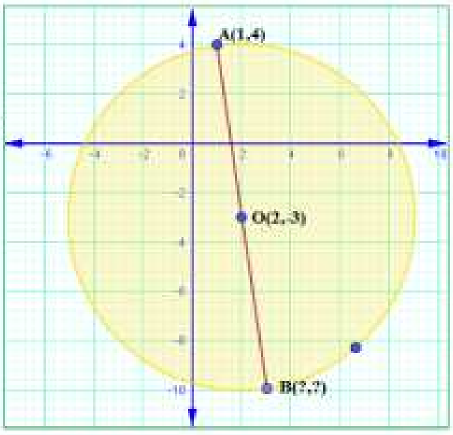
$\frac{x+1}{2} = \frac{7}{2}\ ,\ \frac{5+y}{2} = \frac{8}{2}$

$x + 1 = 7,\ \ 5 + y = 8$

$x = 7 - 1,\ \ y = 8 - 5$

$x = 6,\ \ y = 3$

7. **Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).**




The center of the Circle is the mid-point of the diameter

$\therefore (x, y) = (2, -3),\ A(x_1, y_1) = ?\ , B(x_2, y_2) = (1, 4)$

$(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$

$(2, -3) = \left(\frac{1 + x_1}{2}\ ,\ \frac{4 + y_1}{2}\right)$

$\frac{1 + x_1}{2} = 2,\ \frac{4 + y_1}{2} = -3$

$1 + x_1 = 4,\ 4 + y_1 = -6$

$x_1 = 4 - 1,\ \ y_1 = -6 - 4$

$x_1 = 3,\ y_1 = -10$

$\therefore$ The coordinates of a point A is $(3, -10)$

8. **If A and B are (-2, -2) and (2, -4) respectively, find the coordinates of P such that AP $= \frac{3}{7}$ AB and P lies on the line segment AB**

Given AP $= \frac{3}{7}$ AB


P divides AB in the ratio 3:4

$\Rightarrow$ AP: PB = 3: 4

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -2 | -2 | 2 | -4 |

$Q(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$

$= \left(\frac{3(2)+4(-2)}{3+4},\ \frac{3(-4)+4(-2)}{3+4}\right) = \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

9. **Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts**

The point X divides AB in the ratio 1: 3

The coordinates of X is,

| $x_1$ | $y_1$ | $x_2$ | $y_2$ |
|---|---|---|---|
| -2 | 2 | 2 | 8 |

$$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$$

$$= \left(\frac{1(2)+3(-2)}{1+3},\ \frac{1(8)+3(2)}{1+3}\right) = \left(\frac{2-6}{4}, \frac{8+6}{4}\right) = \left(\frac{-4}{4},\ \frac{14}{4}\right) = \left(-1,\ \frac{7}{2}\right)$$

The point Y is the mid-point of AB

The coordinates of Y

$$(x, y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right) = \left(\frac{2-2}{2},\ \frac{8+2}{2}\right) = \left(\frac{0}{2}, \frac{10}{2}\right) = (0,\ 5)$$

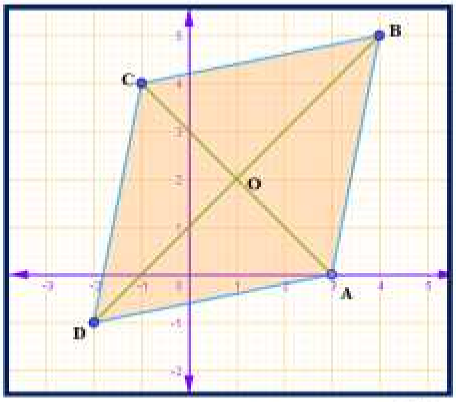
The point Z divides AB in the ratio 3: 1

The coordinates of Z is,

$$(x, y) = \left(\frac{m_1x_2 + m_2\ x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\ y_1}{m_1+m_2}\right)$$

$$= \left(\frac{3(2)+1(-2)}{3+1},\ \frac{3(8)+1(2)}{3+1}\right) = \left(\frac{6-2}{4}, \frac{24+2}{4}\right) = \left(\frac{4}{4},\ \frac{26}{4}\right) = \left(1,\ \frac{13}{2}\right)$$

10. **Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order[Hint: Area of rhombus = $\frac{1}{2}$(product of its diagonals)]**




$AC = \sqrt{(-1-3)^2+(4-0)^2} = \sqrt{(-4)^2+(4)^2}$

$= \sqrt{16+16} = \sqrt{16\times 2} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2+(-1-5)^2} = \sqrt{(-6)^2+(-6)^2}$

$= \sqrt{36+36} = \sqrt{36\times 2} = 6\sqrt{2}$

The area of the rhombus $= \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$

$= \frac{24(\sqrt{2})^2}{2} = 12(2) = 24$ square units.

## 7.4 Area of a Triangle

Area of triangle $= \frac{1}{2}\ \times$ base $\times$ height

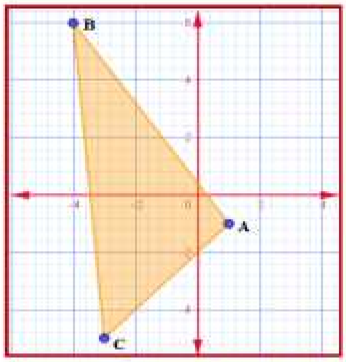
By Heron's Formula Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ . Here, $s = \frac{a+b+c}{2}$

a, b and c are the sides of the triangle.

We could find the lengths of the three sides of the triangle using distance formula. But this could be tedious, particularly if the lengths of the sides are irrational number. Then we can use the following formula to find the area of the triangle.

$$\textbf{Area of the triangle } = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

**Example 11:Find the area of a triangle whose vertices are (1, –1),(– 4, 6) and (–3, –5).**




A (1, -1), B(-4, 6) ಮತ್ತು C (-3, -5)

Area $= \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}\big[1(6-(-5)) + (-4)\big(-5-(-1)\big) + (-3)(-1-6)\big]$

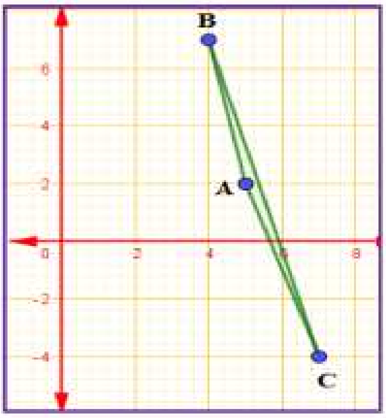
$= \frac{1}{2}[1(6+5) + (-4)(-5+1) + (-3)(-7)]$

$= \frac{1}{2}[11+16+21]$

$= \frac{1}{2}(48) = 24$

Area of the triangle is $= 24$ Square units

**Example 12 : Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C (7, – 4).**




A (5, 2), B (4, 7) and C (7, -4)

Area $=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}[5(7-(-4))+4(-4-2)+7(2-7)]$

$=\frac{1}{2}[5(7+4)+4(-6)+7(-5)]$

$=\frac{1}{2}[55-24-35]$

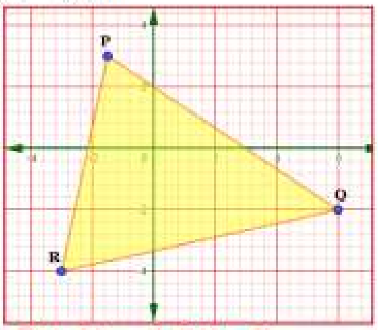
$=\frac{1}{2}(55-59)$

$=\frac{1}{2}(-4)=-2$

Since area is a measure, which cannot be negative, we will take the numerical value of – 2, i.e., 2. Therefore, the area of the triangle = 2 square units.

**Example 13 : Find the area of the triangle formed by the points P(–1.5, 3), Q(6, –2) and R(–3, 4).**

Area of the triangle$=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$




$=\frac{1}{2}[(-1.5)(-2-4)+6(4-3)+(-3)(3-(-2))]$

$=\frac{1}{2}[(-1.5)(-6)+6(1)+(-3)(3+2)]$

$=\frac{1}{2}[9+6-15]\ =\frac{1}{2}(15-15)$

$=\frac{1}{2}(0)=0$

If the area of a triangle is 0 square units, then its vertices will be collinear.

**Example 14 : Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.**

Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$

$\frac{1}{2}[2(k-(-3))+4(-3-3)+6(3-k)]=0$

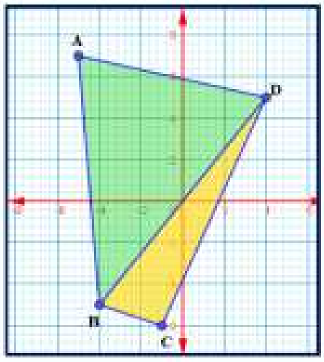
$\frac{1}{2}[2(k+3)+4(-6)+6(3-k)]=0$

$\frac{1}{2}[2k+6-24+18-6k]=0$

$\frac{1}{2}(-4k)=0\ \Rightarrow\ k=0$

**Example 15 : If A(–5, 7), B(– 4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.**

By joining B to D, we will get two triangles ABD and BCD




$\therefore$ Area $\Delta ABD\ =\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}[(-5)(-5-5)+(-4)(5-7)+4(7-(-5))]$

$=\frac{1}{2}[(-5)(-10)+(-4)(-2)+4(7+5)]\ =\frac{1}{2}[50+8+48]$

$=\frac{1}{2}(106)\ =53$ Sq.units

$\therefore$ Area $\Delta BCD=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$=\frac{1}{2}[(-4)(-6-5)+(-1)(5-(-5))+4(-5-(-6))]$

$=\frac{1}{2}[(-4)(-11)+(-1)(5+5)+4(-5+6)]$

$=\frac{1}{2}[44-10+4]\ =\frac{1}{2}(38)\ =19$ Sq.units

Therefore the area of quadrilateral ABCD = 53 + 19 = 72 Sq.units

# Exercise 7.3

1. **Find the area of the triangle whose vertices are :**
   **i) (2, 3), (-1, 0), (2, -4)** **ii) (-5, -1), (3, -5) (5, 2)**

   **i) (2, 3), (-1, 0), (2, -4)**

   Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

   $= \frac{1}{2}[2(0-(-4)) + (-1)(-4-3) + 2(3-0)]$

   $= \frac{1}{2}[2(4) + (-1)(-7) + 2(3)] = \frac{1}{2}[8+7+6] = \frac{1}{2}(21) = \frac{21}{2}$ Sq.units

   **ii) (-5, -1), (3, -5) (5, 2)**

   Area $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

   $= \frac{1}{2}[(-5)(-5-2) + 3(2-(-1)) + 5(-1-(-5))]$

   $= \frac{1}{2}[(-5)(-7) + 3(2+1) + 5(-1+5)] = \frac{1}{2}[35+9+20] = \frac{1}{2}(64)$

   $= 32$ Sq.units.

2. **In each of the following find the value of 'k', for which the points are collinear.**
   **i) (7, -2), (5, 1), (3, k)** **ii) (8, 1), (k, -4) (2, -5)**

   **i) (7, -2), (5, 1), (3, k)**

   Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

   $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

   $\frac{1}{2}[7(1-k) + 5(k-(-2)) + 3(-2-1)] = 0$

   $\frac{1}{2}[7(1-k) + 5(k+2) + 3(-3)] = 0$

   $\frac{1}{2}[7 - 7k + 5k + 10 - 9] = 0$

   $\frac{1}{2}(-2k+8) = 0$

   $-2k = -8 \Rightarrow k = \frac{-8}{-2} = 4$

**ii) (8, 1), (k, -4) (2, -5)**

Since the given points are collinear, the area of the triangle formed by them must be 0, i.e.,

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2}[8(-4-(-5)) + k(-5-1) + 2(1-(-4))] = 0$

$\frac{1}{2}[8(-4+5) + k(-6) + 2(1+4)] = 0$

$\frac{1}{2}[8(1) + k(-6) + 2(5)] = 0$

$\frac{1}{2}[8 - 6k + 10] = 0$

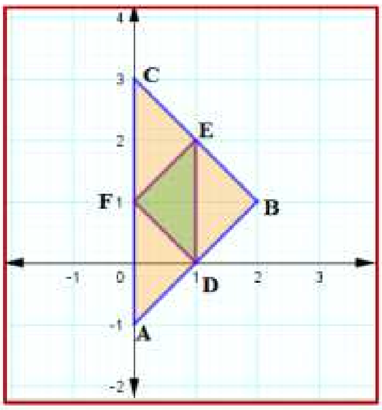
$\frac{1}{2}(-6k+18) = 0$

$-6k = -18 \Rightarrow k = \frac{-18}{-6} = 3$

3. **Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.**

   Let A(0, -1), B(2, 1) and C(0, 3) be the vertices of the triangle ABC

   D, E and F are the mid-point of AB, BC and AC

   The coordinates of D

   $(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{2+0}{2}, \frac{1-1}{2}\right)$

   $= \left(\frac{2}{2}, \frac{0}{2}\right) = (1, 0)$

   The coordinates of E

   $(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{0+2}{2}, \frac{3+1}{2}\right)$

   $= \left(\frac{2}{2}, \frac{4}{2}\right) = (1, 2)$

   The coordinates of F

   $(x, y) = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right) = \left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$

   $= \left(0, \frac{2}{2}\right) = (0, 1)$

The area of $\Delta DEF$ with vertices $D(1,\ 0)$, $E(1,\ 2)$ and $F(0,\ 1)$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[1(2-1) + 1(1-0) + 0(0-2)] = \frac{1}{2}[1(1) + 1 + 0]$

$= \frac{1}{2}[1+1] = \frac{1}{2}(2) = 1$ Sq.units

The area of given triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[0(1-3) + 2(3-(-1)) + 0(-1-1)] = \frac{1}{2}[0 + 2(3+1)) + 0]$

$= \frac{1}{2}[0+8+0] = \frac{1}{2}(8) = 4$ Sq.units

The ratio of the $\Delta ABC$ and $\Delta DEF = 4:1$

4. **Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).**

A(-4, -2), B(-3, -5), C(3, -2) and D(2, 3)

By joining B to D, we will get two triangles ABD and BCD

$\therefore$ Area ABD

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-4)(-5-3) + (-3)(3-(-2)) + 2(-2-(-5))]$

$= \frac{1}{2}[(-4)(-8) + (-3)(3+2) + 2(-2+5)]$

$= \frac{1}{2}[32 - 15 + 6] = \frac{1}{2}(23) = \frac{23}{2}$ Sq.units

$\therefore$ AreaBCD

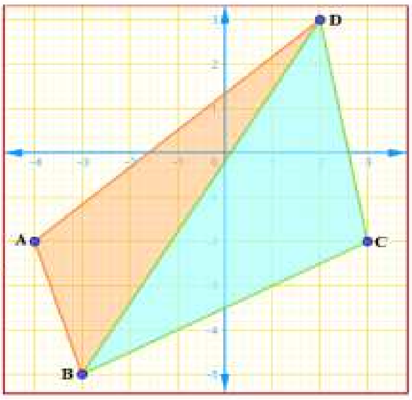
$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[(-3)(-2-3) + 3(3-(-5)) + 2(-5-(-2))]$

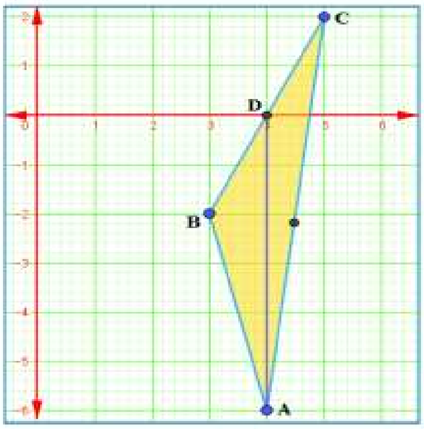
$= \frac{1}{2}[(-3)(-5) + 3(3+5) + 2(-5+2)]$

$= \frac{1}{2}[15 + 24 - 6] = \frac{1}{2}(33) = \frac{33}{2}$ Sq.units

$\therefore$ Area of ABCD $= \frac{23}{2} + \frac{33}{2} = \frac{56}{2} = 28$ Sq.units




5. **You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for ABC whose vertices area (4 – 6), B(3, –2) and C(5, 2).**




Coordinates of D, the midpoint of BC

$(\mathbf{x}, \mathbf{y}) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right) = \left(\frac{5+3}{2}\ ,\ \frac{2-2}{2}\right)$

$= \left(\frac{8}{2}\ ,\ \frac{0}{2}\right) = (\mathbf{4},\ \mathbf{0})$

Area$\Delta ABD$

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4(-2-0) + 3(0-(-6)) + 4(-6-(-2))]$

$= \frac{1}{2}[4(-2) + 3(6) + 4(-6+2)]$

$= \frac{1}{2}[-8 + 18 - 16] = \frac{1}{2}(18 - 24)$

$= \frac{1}{2}(-6) = -3$ Sq.units

Since area is a measure, which cannot be negative, we will take the numerical value of – 3, i.e., 3. Therefore, the area of the triangle = 3square units.

Area $\Delta ADC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[4(0-2) + 4(2-(-6)) + 5(-6-0)]$

$= \frac{1}{2}[4(-2) + 4(2+6) + 5(-6)] = \frac{1}{2}[-8 + 32 - 30] = \frac{1}{2}(-6) = -3$ ಚದರಮಾನಗಳು

Since area is a measure, which cannot be negative, we will take the numerical value of – 3, i.e., 3. Therefore, the area of the triangle = 3square units.

Hence, the mid-point of a triangle divides it into two triangles of equal areas.

## 7.5 Summary

1. The distance between two given points $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. The distance from the orgin to the given points $d = \sqrt{x^2 + y^2}$

3. Section formula :P is the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$
   If the point P divides the line in the ratio $m:n$ then the coordinates of P
   $P(x,y) = \left(\frac{m_1x_2 + m_2\,x_1}{m_1+m_2}\ ,\ \frac{m_1y_2 + m_2\,y_1}{m_1+m_2}\right)$
4. If P is the midpoint of AB, it divides in the ratio 1:1
   P £À ¤zÉÃð±ÀAPÀUÀ¼ÀÄ $P(x,y) = \left(\frac{x_2 + x_1}{2}\ ,\ \frac{y_2 + y_1}{2}\right)$
5. Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

# 8 Real Numbers

Euclid's division algorithm, as the name suggests, has to do with divisibility of integers. Stated simply, it says any positive integer a can be divided by another positive integer b in such a way that it leaves a remainder r that is smaller than b.

## 8.2 Euclid's Division Lemma

### Theorem 8.1

**(Euclid's Division Lemma) : Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, 0 r < b.**

A lemma is a proven statement used for proving another statement

**Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576.**

| | | |
|---|---|---|
| 4052 | 12576 | 3 |
| | 12156 | |
| | 420 | |

12576 = 4052 x 3 + 420

| | | |
|---|---|---|
| 420 | 4052 | 9 |
| | 3780 | |
| | 272 | |

4052 = 420 x 9 + 272

| | | |
|---|---|---|
| 272 | 420 | 1 |
| | 272 | |
| | 148 | |

420 = 272 x 1 + 148

| | | |
|---|---|---|
| 148 | 272 | 1 |
| | 148 | |
| | 124 | |

272 = 148 x 1 + 124

| | | |
|---|---|---|
| 124 | 148 | 1 |
| | 124 | |
| | 24 | |

148 = 124 x 1 + 24

| | | |
|---|---|---|
| 24 | 124 | 5 |
| | 120 | |
| | 4 | |

124 = 24 x 5 + 4

| | | |
|---|---|---|
| 4 | 24 | 6 |
| | 24 | |
| | 0 | |

24 = 4 x 6 + 0 ∴ **The HCF of 4052 and 12576 is 4**

**Example 4 : A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?**

420 = 130 x 3 + 30

130 = 30 x 4 + 10

30 = 10 x 3 + 0

So, the HCF of 420 and 130 is 10.

Therefore the sweetseller can make stacks of 10 for both kinds of burfi

# Exercise 8.1

1. Use Euclid's division algorithm to find the HCF of :
   (i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

(i) 135 ಮತ್ತು 225

| | | |
|---|---|---|
| 135 | 225 | 1 |
| | 135 | |
| | 90 | |

$225 = 135 \times 1 + 90$

| | | |
|---|---|---|
| 90 | 135 | 1 |
| | 90 | |
| | 45 | |

$135 = 90 \times 1 + 45$

| | | |
|---|---|---|
| 45 | 90 | 2 |
| | 90 | |
| | 0 | |

$90 = 45 \times 2 + 0$

$\therefore$ HCF = 45

(ii) 196 ಮತ್ತು 38220

| | | |
|---|---|---|
| 196 | 38220 | 195 |
| | 38220 | |
| | 0 | |

$38220 = 196 \times 195 + 0$

$\therefore$ HCF = 196

(iii) 867 ಮತ್ತು 255

| | | |
|---|---|---|
| 255 | 867 | 3 |
| | 765 | |
| | 102 | |

$867 = 255 \times 3 + 102$

| | | |
|---|---|---|
| 102 | 255 | 2 |
| | 204 | |
| | 51 | |

$255 = 102 \times 2 + 51$

| | | |
|---|---|---|
| 51 | 102 | 2 |
| | 102 | |
| | 0 | |

$102 = 51 \times 2 + 0$

$\therefore$ HCF = 51

3. **An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?**

(iii) 867 ಮತ್ತು 255

| | | |
|---|---|---|
| 32 | 616 | 19 |
| | 608 | |
| | 8 | |

$616 = 32 \times 19 + 8$

| | | |
|---|---|---|
| 8 | 32 | 4 |
| | 32 | |
| | 0 | |

$32 = 8 \times 4 + 0$

$\therefore$ HCF = 8

**They can march maximum 8 columns.**

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.
   [Hint : Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Any positive integer divisible by 3, we get the remainder 0,1 or 2

$\Rightarrow$ a is of the form 3q, 3q + 1 or 3q + 2

i) if a = 3q.

$a^2 = (3q)^2 = 9q^2 = 3(3q^2) = 3m$ $\quad (m = 3q^2)$

ii) if a = 3q + 1.

$a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2) + 1 = 3m + 1$ $\quad (m = 3q^2 + 2)$

iii) if a = 3q + 2 .

$a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 \Rightarrow a^2 = 9q^2 + 12q + 3 + 1$

$\Rightarrow 3(3q^2 + 4q + 1) + 1 = 3m + 1$ $\quad (m = 3q^2 + 4q + 1)$

From (i) (ii) and (iii)

We say, square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

## 8.3 The Fundamental Theorem of Arithmetic

**Example 6 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.**

**Solution:** $6 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

HCF (6,20) = 2 and LCM (6, 20) = $2 \times 2 \times 3 \times 5 = 60$

**Any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.**

We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.

**Example 7 : Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.**

**Solution:** We can write the prime factors of 96 and 404 are as follows

$96 = 2^5 \times 3$ ; $404 = 2^2 \times 101$

HCF(96,404) $= 2^2 = 4$

$\therefore$ LCM (96, 404) $= \frac{96 \times 404}{4} = 9696$

**Example 8 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.**

$6 = 2 \times 3$ ; $72 = 2^3 \times 3^2$; $120 = 2^3 \times 3 \times 5$

$\therefore$ HCF (6, 72, 120) $= 2^1 \times 3^1 = 2 \times 3 = 6$

$\therefore$ LCM (6, 72, 120) $= 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

## Exercise 8.2

1. **Express each number as a product of its prime factors:**
   **(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429**

   (i) $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

   (ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

   (iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

   (iv) $5005 = 5 \times 7 \times 11 \times 13$

   (v) $7429 = 17 \times 19 \times 23$

2. **Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.**
   **(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54.**

   (i) $26 = 2 \times 13$

   $91 = 7 \times 13$

   HCF = 13; LCM $= 2 \times 7 \times 13 = 182$

   Product of two numbers $= 26 \times 91 = 2366$

   LCM x HCF $= 13 \times 182 = 2366$

   $\therefore$ LCM x HCF = Product of two numbers

   (ii) $510 = 2 \times 3 \times 5 \times 17$

   $92 = 2 \times 2 \times 23$

   HCF = 2; LCM $= 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

   Product of two numbers $= 510 \times 92 = 46920$

   LCM x HCF $= 2 \times 23460 = 46920$

   $\therefore$ LCM x HCF = Product of two numbers

   (iii) $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

   $54 = 2 \times 3 \times 3 \times 3$

   HCF $= 2 \times 3 = 6$; LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$

   Product of two numbers $= 336 \times 54 = 18144$

   LCM x HCF $= 6 \times 3024 = 18144$

   $\therefore$ LCM x HCF = Product of two numbers

3. **Find the LCM and HCF of the following integers by applying the prime factorisation method (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25**

(i) $12 = 2 \times 2 \times 3$; $15 = 3 \times 5$; $21 = 3 \times 7$

HCF = 3; LCM $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

(ii) $17 = 1 \times 17$; $23 = 1 \times 23$; $29 = 1 \times 29$

HCF = 1; LCM $= 1 \times 17 \times 19 \times 23 = 11339$

(iii) $8 = 1 \times 2 \times 2 \times 2$; $9 = 1 \times 3 \times 3$; $25 = 1 \times 5 \times 5$

HCF = 1; LCM $= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

7. **There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?**

To find the time they meet again in the same point, we have to find the LCM of time

$18 = 2 \times 3 \times 3$; $12 = 2 \times 2 \times 3$

LCM $= 2 \times 2 \times 3 \times 3 = 36$

Therefore after 36 minutes they meet again at the starting point.

## 8.4 Revisiting Irrational Numbers

A number which can not be expressed in the form of $\frac{p}{q}$ is called irrational number.Here, $p, q \in Z, q \neq 0$

**Theorem 8.3: Let p be a prime number. If p divides $a^2$, then p divides a, where a is a positive integer.**

**Theorm 8.4: $\sqrt{2}$ is irrational.**

Proof: Let us assume, to the contrary, that $\sqrt{2}$ is rational.

$\Rightarrow \sqrt{2} = \frac{p}{q}$ [ $p, q \in Z$, $q \neq 0$ and $(p,q)=1$ ]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2}q = p$ Squaring on both sides we get,

$(\sqrt{2}q)^2 = p^2 \Rightarrow 2q^2 = p^2$ (1)

$\Rightarrow 2$ divides $p^2 \Rightarrow 2$, divides $p$. [ By theorem]

$\therefore$ Let $p = 2m$.

(1) $\Rightarrow 2q^2 = (2m)^2 \Rightarrow q^2 = 2m^2$

$\Rightarrow 2$, divides $q^2 \Rightarrow 2$, divides $q$ [By theorem]

$\therefore$ 2 is the common factor for both p and q

This contradicts that there is no common factor of p and q .

Therefore our assumption is wrong. So, $\sqrt{2}$ is a an irrational number.

**Example 9 : Prove that $\sqrt{3}$ is irrational.**

Proof: Let us assume, to the contrary, that $\sqrt{3}$ is rational.

$\Rightarrow \sqrt{3} = \frac{p}{q}$ [ p,q∈ Z, q ≠ 0 and (p,q)=1 ]

So, there is no other common factors for p and q other than 1

Now, $\sqrt{3} = \frac{p}{q} \Rightarrow \sqrt{3}q = p$ Squaring on both sides we get,

$(\sqrt{3}q)^2 = p^2 \Rightarrow 3q^2 = p^2$ (1)

$\Rightarrow$ 3 divides $p^2 \Rightarrow$ 3 divides p [ By theorem]

$\therefore$ Let $p = 3m$,

(1) $\Rightarrow 3q^2 = (3m)^2 \Rightarrow q^2 = 3m^2$

$\Rightarrow$3 divides $q^2 \Rightarrow$ 3 divides q [ By theorem]

$\therefore$ 3 is the common factor for both p and q , This is not possible.

Therefore our assumption is wrong. So, $\sqrt{3}$ is a an irrational number.

- **The sum or difference of a rational and an irrational number is irrational and**
- **The product and quotient of a non-zero rational and irrational number is irrational.**

**Example 10 : Show that $5 - \sqrt{3}$ is irrational**

Proof: Assume that $5 - \sqrt{3}$ is a rational number.

$\Rightarrow 5 - \sqrt{3} = \frac{p}{q}$ [ p,q∈ Z, q ≠ 0 and (p,q)=1 ]

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \quad \Rightarrow \frac{5q-p}{q} = \sqrt{3}$

Here, $\frac{5q-p}{q}$ is a rational number but $\sqrt{3}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $5 - \sqrt{3}$ is an irrational number.


- **The sum or difference of a rational and an irrational number is irrational and**
- **The product and quotient of a non-zero rational and irrational number is irrational.**

**Example 10 : Show that $5 - \sqrt{3}$ is irrational**

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$\Rightarrow 5 - \sqrt{3} = \frac{p}{q}$ [ p,q∈ Z, q ≠ 0 and (p,q)=1 ]

$\Rightarrow 5 - \frac{p}{q} = \sqrt{3} \quad \Rightarrow \frac{5q-p}{q} = \sqrt{3}$

Here, $\frac{5q-p}{q}$ is a rational number but $\sqrt{3}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $5 - \sqrt{3}$ is an irrational number.


**Example 11 : Show that $3\sqrt{2}$ is irrational.**

Proof: Assume that $3\sqrt{2}$ is a rational number.

$\Rightarrow 3\sqrt{2} = \frac{p}{q}$ [ p,q∈ Z, q ≠ 0 and (p,q)=1 ]

$\Rightarrow \sqrt{2} = \frac{p}{3q}$

Here, $\frac{p}{3q}$ s a rational number but $\sqrt{2}$ is an irrational number. This is not possible

So, our assumption is wrong. Therefore $3\sqrt{2}$ is an irrational number ಆದ್ದರಿಂದ $3\sqrt{2}$ ಒಂದು

# Exercise 8.3

1. **Prove that $\sqrt{5}$ is irrational.**

   Proof: Let us assume, to the contrary, that $\sqrt{5}$ is rational.

   $\Rightarrow \sqrt{5} = \frac{p}{q}$ [ p,q$\in$ Z, q $\neq$ 0 and (p,q)=1 ]

   So, there is no other common factors for p and q other than 1

   Now, $\sqrt{5} = \frac{p}{q} \Rightarrow \sqrt{5}q = p$ , squaring on both sides we get,

   $(\sqrt{5}q)^2 = p^2 \Rightarrow 5q^2 = p^2$ (1)

   $\Rightarrow$ 5 divides $p^2 \Rightarrow$ 5 divide s p [ By theorem]

   $\therefore$ Let p = 5m,

   (1) $\Rightarrow 5q^2 = (3m)^2 \Rightarrow q^2 = 5m^2$

   $\Rightarrow$5 divides $q^2 \Rightarrow$ 5 divides q [ By theorem]

   $\therefore$ 5 is the common factor for both p and q ; this is not possible

   Therefore our assumption is wrong. So, $\sqrt{5}$ is a an irrational number.

2. **Prove that $3 + 2\sqrt{5}$ is irrational.**

   **Proof:** Assume that $3 + 2\sqrt{5}$ is a rational number.

   $\Rightarrow 3 + 2\sqrt{5} = \frac{p}{q}$ [ p,q$\in$ Z, q $\neq$ 0 and (p,q)=1 ]

   $\Rightarrow 2\sqrt{5} = \frac{p}{q} - 3 \Rightarrow \sqrt{5} = \frac{p-3q}{2q}$

   Here, $\frac{p-3q}{2q}$ is a rational number but $\sqrt{5}$ is an irrational number. This is not possible

   So, our assumption is wrong. Therefore $3 + 2\sqrt{5}$ is an irrational number.

1. Prove that the following are irrationals:

   (i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

   **(i) $\frac{1}{\sqrt{2}}$**

   **Proof**: Assume that $\frac{1}{\sqrt{2}}$ is a rational number.

   $\Rightarrow \frac{1}{\sqrt{2}} = \frac{p}{q}$ [ p,q$\in$ Z, q $\neq$ 0 and (p,q)=1 ]

   $\Rightarrow \frac{\sqrt{2}}{2} = \frac{p}{q} \Rightarrow \sqrt{2} = \frac{2p}{q}$

   Here, $\frac{2p}{q}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

   Therefore our assumption is wrong. $\therefore \frac{1}{\sqrt{2}}$ is an irrational number.

**(ii) $7\sqrt{5}$**

**Proof:** Assume that $7\sqrt{5}$ is a rational number.

$7\sqrt{5} = \frac{p}{q}$ [ p,q$\in$ Z, q $\neq$ 0 and (p,q)=1 ]

$\Rightarrow \sqrt{5} = \frac{p}{7q}$

Here, $\frac{p}{7q}$ is a rational number, but $\sqrt{5}$ is an irrational. This is impossible.

Therefore our assumption is wrong. $\therefore 7\sqrt{5}$ is an irrational number.

**(iii) $6 + \sqrt{2}$**

**Proof:** Assume that $6 + \sqrt{2}$ is a rational number

$\Rightarrow 6 + \sqrt{2} = \frac{p}{q}$ [ p,q$\in$ Z, q $\neq$ 0 and (p,q)=1 ]

$\Rightarrow \sqrt{2} = \frac{p}{q} - 6 \quad \Rightarrow \sqrt{2} = \frac{p-6q}{2}$

Here, $\frac{p-6q}{2}$ is a rational number, but $\sqrt{2}$ is an irrational. This is impossible.

Therefore our assumption is wrong. $\therefore 6 + \sqrt{2}$ is an irrational number.

# Exercise 8.4

**1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion**

**(i)** $\frac{13}{3125}$ **(ii)** $\frac{17}{8}$ **(iii)** $\frac{64}{455}$ **(iv)** $\frac{15}{1600}$ **(v)** $\frac{29}{343}$

**(vi)** $\frac{23}{2^3 5^3}$ **(vii)** $\frac{23}{2^2 5^7 7^5}$ **(viii)** $\frac{6}{15}$ **(ix)** $\frac{35}{50}$ **(x)** $\frac{77}{210}$

(i) $\frac{13}{3125}$ - Factorising the denominator $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 2^0 \times 5^5$

Here, The factors of 3125 is of the form $2^n 5^m$ So, **this** has a terminating decimal expansion.

(ii) $\frac{17}{8}$ - Factorising the denominator $8 = 2 \times 2 \times 2 = 2^3 x 5^0$

Here, The factors of 8 is of the form $2^n 5^m$. So, **this** has a terminating decimal expansion.

(iii) $\frac{64}{455}$ - Factorising the denominator $455 = 5 \times 7 \times 13$

Here, The factors of 455 is 5x7x13 is not in the form $2^n \times 5^m$

So, **this** has non-terminating repeating decimal expansion.

**(iv)** $\frac{15}{1600}$ - Factorising the denominator $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^6 \times 5^2$

Here, The factors of 1600 is of the form $2^n 5^m$.

So, **this** has a terminating decimal expansion.

**(v)** $\frac{29}{343}$ - Factorising the denominator $343 = 7 \times 7 \times 7 = 7^3$

Here, The factors of 343 is not in the form $2^n \times 5^m$

So, **this** has non-terminating repeating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$ - denominator is of the form $2^n \times 5^m$

So, **this** has a terminating decimal expansion.

(vii) $\frac{23}{2^2 5^7 7^5}$ denominator is not in the form $2^n \times 5^m$

So, this has non-terminating repeating decimal expansion.

(viii) $\frac{6}{15} \Rightarrow \frac{6}{15} = \frac{2}{5}$ dinominator $2^0 \times 5^1$ is of the form $2^n \times 5^m$

So, **this** has a terminating decimal expansion.

(ix) $\frac{35}{50} \Rightarrow \frac{35}{50} = \frac{7}{10} = \frac{7}{2x5}$ dinominator $2^1 \times 5^1$ is of the form $2^n \times 5^m$

So, **this** has a terminating decimal expansion.

x) $\frac{77}{210} \Rightarrow \frac{77}{210} = \frac{11}{30} = \frac{11}{2x3x5}$ dinominator not in the form $2^n \times 5^m$

So, **this** has non-terminating repeating decimal expansion.

**2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.**

(i) $\frac{13}{5^5} \Rightarrow \frac{13}{5^5} \times \frac{2^5}{2^5} = \frac{15x32}{105} = \frac{416}{100000} = 0.00416$

(ii) $\frac{17}{8} \Rightarrow \frac{17}{8} = \frac{17}{2^3} \times \frac{5^3}{5^3} = \frac{17x125}{1000} = 2.125$

# 9 Polynomials

## Degree of the polynomial:

**p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x).**

**Examples:**

$4x + 2$ is a polynomial in the variable x ofdegree 1.

A polynomial of degree 1 is called a **linear polynomial.**

$2y^2 - 3y + 4$ is a polynomial in the variable y of degree

A polynomial of degree 2 is called a **quadratic polynomial.**

quadratic polynomial in x is of the form $\mathbf{ax^2 + bx + c}$, where a, b, c are real numbers $a \neq 0$ .

is a polynomial in the variable x of degree 3

$5x^3 - 4x^2 + 2 - \sqrt{2}$ is a polynomial in the variable x of degree 3

A polynomial of degree 3 is called a **cubic polynomial**. General form of a cubic polynomial is

$\mathbf{ax^3 + bx^2 + cx + d}$

Where a, b, c, d are real numbers and $a \neq 0$

[$7u^6 - \frac{3}{2}u^4 + 4u^2 + u - 8$ is a polynomial of variable x and the degree of this polynomial is 6]

Example:: $\sqrt{x} + 1$ , $\frac{2}{x}$ , $\frac{1}{x^3+x^2-1}$

If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

**What is the value of $p(x) = x^2 - 3x - 4$ when $x = -1$ ?**

$p(-1) = (-1)^2 - 3(-1) - 4 = 0$

Similarlly, $p(4) = (4)2 - 3(4) - 4 = 0$

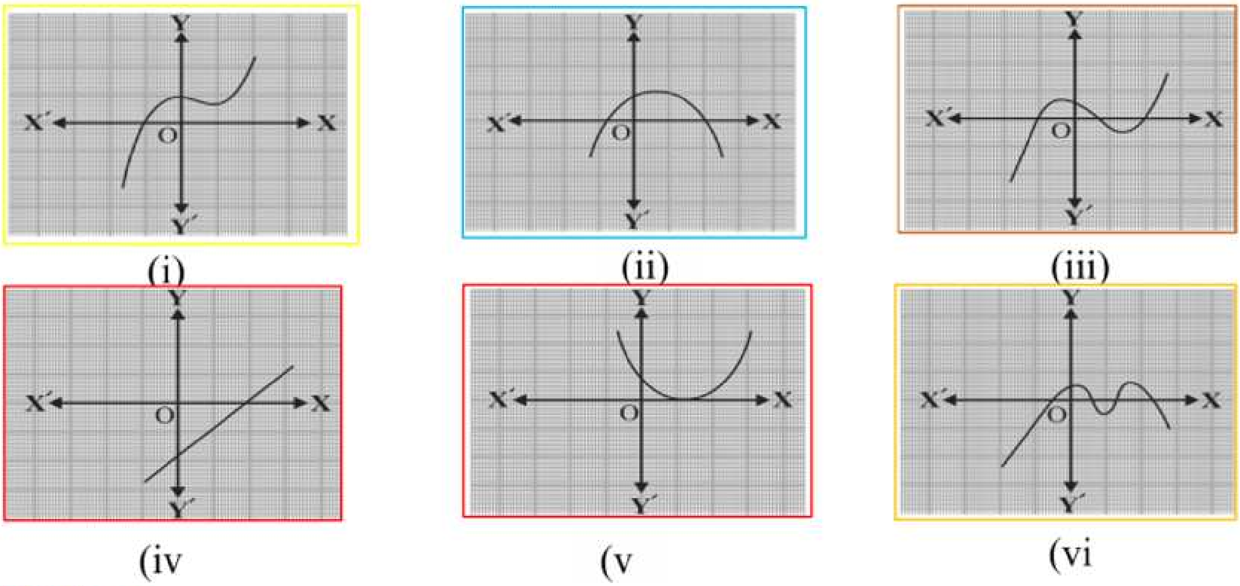
As $p(-1) = 0$ and $p(4) = 0$ $-1$ and $4$ are called the zeros of the polynomial $x^2 - 3x - 4$

If k is a real number such that $p(k) = 0$ then k is called the Zeros of the polynomial p(x)

If k is the zero of the polynomial $p(x) = ax + b$ then $p(k) = ak + b = 0 \Rightarrow k = -\frac{b}{a}$

The zero of the lenear equation $ax + b$ is $-\frac{b}{a}$

**Example 1 : Look at the graphs in Fig. 9.9 given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).**




**Solution :**

(i) The number of zeroes is 1 as the graph intersects the x-axis at one point only.

(ii) The number of zeroes is 2 as the graph intersects the x-axis at two points.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at three points

(iv) The number of zeroes is 1 as the graph intersects the x-axis at one point only.

(v) The number of zeroes is 1. as the graph intersects the x-axis at one point only.

(vi) The number of zeroes is 4. as the graph intersects the x-axis at four points

## Exercise 9.1

**1. The graphs of y = p(x) are given in Fig. 9.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.**

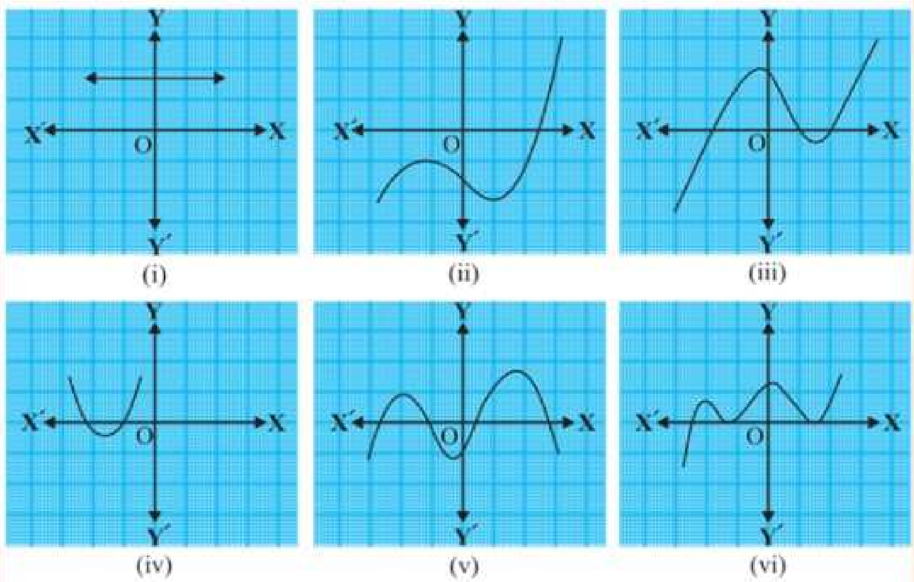



Fig. 9.10

(i) The number of zeroes is 0 as the graph not intersects the x-axis

(ii) The number of zeroes is 1 as the graph intersects the x-axis at one point only.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at three points .

(iv) The number of zeroes is 2 as the graph intersects the x-axis at two points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at four points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at three points.

## 9.3 Relationship between Zeroes and Coefficients of a Polynomial

$\alpha$ and $\beta$ are the zeros of the polynomial $p(x) = ax^2 + bx + c, \quad a \neq 0$
$(x - \alpha)$ and $(x - \beta)$ are the factors of $p(x)$.

Sum of Zeros $\boxed{\alpha + \beta = \frac{-b}{a}}$ Product of Zeros $\boxed{\alpha\beta = \frac{c}{a}}$

**Example 2 : Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients**

**Solution:** $x^2 + 7x + 10 = x^2 + 5x + 2x + 10$

$= x(x+5) + 2(x+5) = (x+2)(x+5)$

$\therefore$ The value of $x^2 + 7x + 10$ is zero when $x = -2$ or $x = -5$

$\therefore -2$ and $-5$ are the zeros of $x^2 + 7x + 10$

Sum of the zeros $= (-2) + (-5) = -7 = \frac{-7}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients**

**Solution:** $a^2 - b^2 = (a - b)(a + b)$

$\therefore x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

$\therefore \sqrt{3}$ and $-\sqrt{3}$ are the zeros of $x^2 - 3$

Sum of the zeros $= \sqrt{3} + -\sqrt{3} = 0 = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are $-3$ and 2, respectively.**

**Solution:** Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\therefore \alpha + \beta = -3 = \frac{-b}{a}$ and $\alpha\beta = 2 = \frac{c}{a}$

$\Rightarrow$ If $a = 1$ then $b = 3$ and $c = 2$

$\therefore$ Quadratic polynomial $= x^2 + 3x + 2$

## The relation between the zeros and the coefficients of Cubic polynomials:

If $\alpha$, $\beta$, $\gamma$ are the zeros of the cubic polynomia $ax^3 + bx^2 + cx + d$ then

$$\alpha + \beta + \gamma = \frac{-b}{a}; \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}; \quad \alpha\beta\gamma = \frac{-d}{a}$$

# Exercise 9.2

1. **Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.**

**(i)** $x^2 - 2x - 8$ **(ii)** $4s^2 - 4s - 1$ **(iii)** $6x^2 - 3 - 7x$

**(iv)** $4u^2 - 8u$ **(v)** $t^2 - 15$ **(vi)** $3x^2 - x - 4$

**(i)** $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = (x - 4) + 2(x - 4) = (x - 4)(x + 2)$

$\Rightarrow x = 4$ and $x = -2$ are the zeros of polynomial $x^2 - 2x - 8$

Sum of the zeros $= 4 + (-2) = 2 = \frac{-(-2)}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= (4))(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**(ii)** $4s^2 - 4s + 1 = 4s^2 - 2s - 2s + 1 = 2s(s - 1) - 1(2s - 1) = (2s - 1)(2s - 1)$

$\Rightarrow s = \frac{1}{2}$ and $s = \frac{1}{2}$ are the zeros of the polynomial $4s^2 - 4s + 1$

Sum of the zeros $= \frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**(iii)** $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (3x+1)(2x-3)$

$\Rightarrow x = -\frac{1}{3}$ and $x = \frac{3}{2}$ are the zeros of the polynomial $6x^2 - 3 - 7x$

Sum of the zeros $= -\frac{1}{3} + \frac{3}{2} = 1 = \frac{-2+9}{6} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= -\frac{1}{3} \times \frac{3}{2} = \frac{-3}{6} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**(iv)** $4u^2 + 8u = 4u^2 + 8u + 0 = 4u(u + 2)$

$\Rightarrow u = 0$ and $u = -2$ are the zeros of the polynomial $4u^2 + 8u$

Sum of the zeros $= 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= 0 \times -2 = 0 = \frac{0}{4} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**(v)** $t^2 - 15 = t^2 - 0.t - 15 = (t - \sqrt{15})(t + \sqrt{15})$

$\Rightarrow t = \sqrt{15}$ and $t = -\sqrt{15}$ are the zeros of the polynomial $t^2 - 15$

Sum of the zeros $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{0}{1} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \sqrt{15} \times (-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

**(vi)** $3x^2 - x - 4 = 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$

$\Rightarrow x = \frac{4}{3}$ and $x = -1$ are the zeros of the polynomial $3x^2 - x - 4$

Sum of the zeros $= \frac{4}{3} + (-1) = \frac{4-3}{3} = \frac{1}{3} = \frac{-(-1)}{3}\ \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{4}{3} + (-1) = -\frac{4}{3} = \frac{-4}{1} = \frac{\text{Constant}}{\text{Coefficient of } x^2}$

1. **Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively. (i) $\frac{1}{4}$, $-1$ (ii) $\sqrt{2}$, $\frac{1}{3}$ (iii) $0, \sqrt{5}$ (iv) $1, 1$ (v) $-\frac{1}{4}$, $\frac{1}{4}$ (vi) $4, 1$**

**(i)** $\frac{1}{4}, -1$ - Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\alpha + \beta = \frac{1}{4} = \frac{-(-1)}{4} = \frac{-b}{a}$ and $\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$

$\Rightarrow a = 4, b = -1$ and $c = -4$

$\therefore$ The required polynomial is $4x^2 - x - 4$

**(ii) $\sqrt{2}$ , $\frac{1}{3}$** - Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\alpha + \beta = \sqrt{2} = \frac{-(3\sqrt{2})}{3} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$

$\Rightarrow a = 3, b = -3\sqrt{2}$ and $c = 1$

$\therefore$ The required polynomial is $3x^2 - 3\sqrt{2} + 1$

**(iii) 0, $\sqrt{5}$ –** Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$ and $\alpha\beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = 0$ and $c = \sqrt{5}$

$\therefore$ The required polynomial is $x^2 + \sqrt{5}$

**(iv) 1, 1** - Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\alpha + \beta = 1 = \frac{-(-1)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = -1$ and $c = 1$

$\therefore$ The required polynomial is $x^2 - x + 1$

**(v) $-\frac{1}{4}$, $\frac{1}{4}$** - Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$.

$\alpha + \beta = -\frac{1}{4} = \frac{-1}{4} = \frac{-b}{a}$ and $\alpha\beta = \frac{1}{4} = \frac{1}{4} = \frac{c}{a}$

$\Rightarrow a = 4, b = 1$ and $c = 1$

$\therefore$ The required polynomial is $4x^2 + x + 1$

**(vi) 4,1** - Let the required polynomial be $ax^2 + bx + c$ and its zeros are $\alpha$ and $\beta$

$\alpha + \beta = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$ and $\alpha\beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\Rightarrow a = 1, b = -4$ and $c = 1$

$\therefore$ The required polynomial is $x^2 - 4x + 1$

## 9.4 Division Algorithm for Polynomials:

Let the zero of $x^3 - 3x^2 - x + 3$ is 1, then the factor is $(x - 1)$

Now, divide $x^3 - 3x^2 - x + 3$ by the factor $(x - 1)$ then the quotient is $x^2 - 2x - 3$.

By factorising $x^3 - 3x^2 - x + 3$ we get the factors $= (x - 1)(x + 1)(x - 3)$

$\therefore$ the zeros of the polynomial $x^3 - 3x^2 - x + 3$ is $1, -1$ and $3$

**Example 6 : Divide $2x^2 + 3x + 1$ by $x + 2$.**

| $x + 2$ | $2x^2 + 3x + 1$ | $2x - 1$ |
|---|---|---|
| | $2x^2 + 4x$ | |
| | $-x + 1$<br>$-x - 2$ | |
| | $+3$ | |

**Solution:** Note that we stop the division process when either the remainder is zero or its degree is less than the degree of the divisor. So, here the quotient is $2x - 1$ and the remainder is 3. Also, $(2x - 1)(x + 2) = 2x^2 + 3x - 2 + 3 = 2x^2 + 3x + 1$

**$\Rightarrow$Dividend= Divisor x Quotient + Remainder**

**Example: Divide 7: $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$**

| $1 + 2x + x^2$ | $3x^3 + x^2 + 2x + 5$ | $3x - 5$ |
|---|---|---|
| | $3x^3 + 6x^2 + 3x$ | |
| | $-5x^2 - x + 5$<br>$-5x^2 - 10x - 5$ | |
| | $9x + 10$ | |

We first arrange the terms of the dividend and the divisor in the decreasing order of their degrees. Recall that arranging the terms in this order is called writing the polynomials in standard form. In this example, the dividend is already in standard form, and the divisor, in standard form, is $x^2 + 2x + 1$

$(x^2 + 2x + 1)(3x - 5) + (9x + 10) = 3x^3 + x^2 + 2x + 5$

$\Rightarrow$Dividend= Divisor x Quotient + Remainder

**If $p(x)$ and $g(x)$ are any two polynomials and $g(x) \neq 0$ then,**

$$\boxed{p(x) = g(x) \cdot q(x) + r(x)}$$

$q(x)$ - Quotient and $r(x)$ - remainder

Here, $r(x) = 0$ or the degree of $r(x) <$ the degree of $g(x)$

**This is known as The Division Algorithm for polynomials**

**Example 8: divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$ and verify the division algorithm.**

Note that To carry out division, we first write both the dividend and divisor in decreasing orders of their degrees. So, dividend $= -x^3 + 3x^2 - 3x + 5$ and divisor $= -x^2 + x - 1$.

| $-x^2 + x - 1$ | $-x^3 + 3x^2 - 3x + 5$ | $x - 2$ |
|---|---|---|
| | $-x3 + x^2 - x$ | |
| | $2x^2 - 2x + 5$<br>$2x^2 - 2x + 2$ | |
| | $3$ | |

$\therefore$ Quotient $= x - 2$, Remainder $= 3$

Divisor x Quotient + Remainder $= (-x^2 + x - 1)(x - 2) + 3 = -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$ $= -x^3 + 3x^2 - 3x + 5$ = Dividend. Hence, the division algorithm is verifyied.

**Example 9 : Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$**

**Solution:** Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$

$(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ are the factors of the polynomial

| $x^2 - 2$ | $2x^4 - 3x^3 - 3x^2 + 6x - 2$ | $2x^2 - 3x + 1$ |
|---|---|---|
| | $-2x^4 + 4x^2$ | |
| | $-3x^3 + x^2 + 6x - 2$<br>$-3x^3 + 6x$ | |
| | $+ x^2 - 2$<br>$+ x^2 - 2$ | |
| | $0$ | |

Now, divide the polynomial by $x^2 - 2$

Factorise the Quotient $= 2x^2 - 3x + 1$

$2x^2 - 2x - x + 1 = 2x(x - 1) - 1(2x - 1)$

$= (2x - 1)(x - 1)$

$\Rightarrow x = \frac{1}{2}, x = 1$ are the zeros

$\therefore$ All 4 zeros are $\sqrt{2}$, $-\sqrt{2}$, $\frac{1}{2}$, and 1

# Exercise 9.3

1. **Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :**

   **(i) $p(x) = x^3 - 3x^2 + 5x - 3$ $g(x) = x^2 - 2$ (ii) $p(x) = x^4 - 3x^2 + 4x + 5$ $g(x) = x^2 + 1 - x$**

   **(iii) $p(x) = x^4 - 5x + 6$ $g(x) = 2 - x^2$**

**(i) $p(x) = x^3 - 3x^2 + 5x - 3$ $g(x) = x^2 - 2$**

| | | |
|---|---|---|
| $x^2 - 2$ | $x^3 - 3x^2 + 5x - 3$ | $x - 3$ |
| | $x^3 - 0 - 2x$ | |
| | $- 3x^2 + 7x - 3$ | |
| | $- 3x^2 + 0 + 6$ | |
| | $+ 7x - 9$ | |

Quotient = $x - 3$ ; remainder = $7x - 9$

**(ii) $p(x) = x^4 - 3x^2 + 4x + 5$ $g(x) = x^2 + 1 - x$**

| | | |
|---|---|---|
| $x^2 - x + 1$ | $x^4 + 0.x^3 - 3x^2 + 4x + 5$ | $x^2 + x - 3$ |
| | $x^4 - x^3 + x^2$ | |
| | $x^3 - 4x^2 + 4x$ | |
| | $x^3 - x^2 + x$ | |
| | $- 3x^2 + 3x + 5$ | |
| | $- 3x^2 + 3x - 3$ | |
| | $8$ | |

Quotient = $x^2 + x - 3$ ;remainder = 8

**(iii) $p(x) = x^4 - 5x + 6$ $g(x) = 2 - x^2$**

| | | |
|---|---|---|
| $- x^2 + 2$ | $x^4 + 0x^3 + 0x^2 - 5x + 6$ | $-x^2 - 2$ |
| | $x^4 + 0 - 2x^2$ | |
| | $2x^2 - 5x + 6$ | |
| | $2x^2 + 0 - 4$ | |
| | $- 5x + 10$ | |

Quotient $= -x^2 - 2$ ;remainder = $-5x + 10$

**2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:**

**(i)** $t^2 - 3,\quad 2t^4 + 3t^3 - 2t^2 - 9t - 12$ **(ii)** $x^2 + 3x + 1,\quad 3x^4 + 5x^3 - 7x^2 + 2x + 2$

**(iii)** $x^3 - 3x + 1,\quad x^5 - 4x^3 + x^2 + 3x + 1$

**(i)** $t^2 - 3 \quad 2t^4 + 3t^3 - 2t^2 - 9t - 12$

| | | |
|---|---|---|
| $t^2 - 3$ | $2t^4 + 3t^3 - 2t^2 - 9t - 12$ | $2t^2 + 3t + 4$ |
| | $2t^4 + 0 - 6t^2$ | |
| | $+ 3t^3 + 4t^2 - 9t$ | |
| | $+ 3t^3 + 0 - 9t$ | |
| | $+4t^2 + 0 - 12$ | |
| | $+4t^2 + 0 - 12$ | |
| | 0 | |

Remainder is Zero. Therefore first polynomial is the factor of the second polynomial.

**(ii)** $x^2 + 3x + 1 \quad 3x^4 + 5x^3 - 7x^2 + 2x + 2$

| | | |
|---|---|---|
| $x^2 + 3x + 1$ | $3x^4 + 5x^3 - 7x^2 + 2x + 2$ | $3x^2 - 4x + 2$ |
| | $3x^4 + 9x^3 + 3x^2$ | |
| | $- 4x^3 - 10x^2 + 2x$ | |
| | $- 4x^3 - 12x^2 - 4x$ | |
| | $+2x^2 + 6x + 2$ | |
| | $+2x^2 + 6x + 2$ | |
| | 0 | |

Remainder is Zero. Therefore first polynomial is the factor of the second polynomial.

**(iii)** $x^3 - 3x + 1 \quad x^5 - 4x^3 + x^2 + 3x + 1$

| | | |
|---|---|---|
| $x^3 - 3x + 1$ | $x^5 - 4x^3 + x^2 + 3x + 1$ | $x^2 - 1$ |
| | $x^5 - 3x^3 + x^2$ | |
| | $- x^3 + 0 + 3x + 1$ | |
| | $- x^3 + 0 + 3x - 1$ | |
| | 2 | |

Remainder is 2 Therefore first polynomial is not the factor of the second polynomial.

3. **Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$**

$\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ are the factors of the polynomial.

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$. Dividing the polynomial by $x^2 - \frac{5}{3}$

| $x^2 - \frac{5}{3}$ | $3x^4 + 6x^3 - 2x^2 - 10x - 5$ | $3x^2 + 6x + 3$ |
|---|---|---|
| | $3x^4 + 0 - 5x^2$ | |
| | $+ 6x^3 + 3x^2 - 10x$ | |
| | $+ 6x^3 + 0 - 10x$ | |
| | $+3x^2 + 0 - 5$ | |
| | $+3x^2 + 0 - 5$ | |
| | 0 | |

$3x^2 + 6x + 3 = 3(x^2 + 2x + 1)$

By Factorising $(x^2 + 2x + 1)$

$\Rightarrow x(x + 1) + 1(x + 1)$

$= (x + 1)(x + 1)$

Therefore The factors of $3x^4+6x^3-2x^2-10x-5$ are

$3\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)(x + 1)(x + 1)$

Therefore All the Zeros of the polynomials are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and $-1$

4. **On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).**

Dividend $P(x) = x^3 - 3x^2 + x + 2$; Divisor $g(x) = ?$;

Quotient $q(x) = x - 2$; Remainder $r(x) = -2x + 4$

$P(x) = g(x).q(x) + r(x)$

$x^3 - 3x^2 + x + 2 = g(x).(x - 2) + (-2x + 4)$

$\Rightarrow g(x).(x - 2) = x^3 - 3x^2 + x + 2 - (-2x + 4) \Rightarrow g(x).(x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$\Rightarrow g(x).(x - 2) = x^3 - 3x^2 + 3x - 2 \Rightarrow g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x - 2}$

| $x - 2$ | $x^3 - 3x^2 + 3x - 2$ | $x^2 - x + 1$ |
|---|---|---|
| | $x^3 - 2x^2$ | |
| | $-x^2 + 3x$ | |
| | $-x^2 + 2x$ | |
| | $+ x - 2$ | |
| | $+ x - 2$ | |
| | 0 | |

$\therefore g(x) = x^2 - x + 1$

5. **Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and (i) degp(x) = degq(x) (ii) deg(x) = degr(x) (iii) deg r(x) = 0**

**(i) $p(x) = 6x^2 + 2x + 2$**

$g(x) = 2$; $q(x) = 3x^2 + x + 1$

$r(x) = 0 \Rightarrow \deg p(x) = \deg q(x) = 2$

Verifying Division Algorithm,

$g(x) \times q(x) + r(x) = 2(3x^2 + x + 1) + 0$

$g(x) \times q(x) + r(x) = 6x^2 + 2x + 2 = P(x)$

$\Rightarrow p(x) = g(x) \times q(x) + r(x)$

$\therefore$ It is satisfying divison algorithm is verified.

**(ii) $p(x) = x^3 + x$**

$g(x) = x^2$; $q(x) = x$ and $r(x) = x$; deg $g(x)$ = deg $r(x)$ = 1

Verifying Division Algorithm,

$g(x) \times q(x) + r(x) = (x^2) \times x + x$

$g(x) \times q(x) + r(x) = x^3 + x = p(x) \quad \Rightarrow p(x) = g(x) \times q(x) + r(x)$

$\therefore$ It is satisfying divison algorithm is verified.

**(iii) $p(x) = x^3 + 1$**

$g(x) = x^2$ ; $q(x) = x$ and $r(x) = 1$; deg $r(x)$ = 0

Verifying Division Algorithm

$g(x) \times q(x) + r(x) = (x^2) \times x + 1$

$\Rightarrow g(x) \times q(x) + r(x) = x^3 + 1 = P(x) \quad \Rightarrow p(x) = g(x) \times q(x) + r(x)$

$\therefore$ It is satisfying divison algorithm is verified.

# 10 Quadratic Equations

When we equate this polynomial to zero, we get a quadratic equation.

**Any equation of the form $p(x) = 0$, where $p(x)$ is a polynomial of degree 2, is a quadratic equation.**

**Standard form of quadratic equations:**

$\boxed{ax^2 + bx + c = 0,}$ Where $a \neq 0$

**The features of quadratic equations:**

- The quadratic equations has one variable
- The hieghest power of the variable is 2
- Standard form of quadratic equation: $ax^2 + bx + c = 0$,

**Affected quadratic equations :** In a quadratic equation $ax^2 + bx + c = 0$ , $a \neq 0$, $b \neq 0$ then it is called affected quadratic equations.

Then, $x^2 - 3x - 5 = 0$ , $x^2 + 5x + 6 = 0$, $x + \frac{1}{x} = 5$, $(2x - 5)^2 = 81$

**Pure Quadratic equations :** The quadrtic equations where $a \neq 0$, $b = 0$ is called pure quadratic equations.

The standard form of pure quadratic equation: $ax^2 + c = 0$ $[a \neq 0]$

(i) Let the number of marbles with Jhon be 'x'

Then the number of marbles with Jivanthi = $45 - x$ [$\because$Total number of marbles 45]

The number of marbles left with John, when he lost 5 marbles = $x - 5$

The number of marbles left with Jivanti, when she lost 5 marbles = $45 - x - 5 = 40 - x$

$\therefore$ Their products = 124

$(x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124 \Rightarrow -x^2 + 45x - 200 = 124$

$\Rightarrow -x^2 + 45x - 324 = 0 \Rightarrow x^2 - 45x + 324 = 0$

Therefore, the number of marbles John had, satisfies the quadratic equation $x^2 - 45x + 324 = 0$ which is the required representation of the problem mathematically.

(ii) Let the number of toys produced on that day be x.
Therefore, the cost of production (in rupees) of each toy that day = 55 – x
So, the total cost of production (in rupees) that day = x (55 – x)
$\therefore$ x (55 – x ) = 750
$\Rightarrow 55x - x^2 = 750 \Rightarrow -x^2 + 55x - 750 = 0 \Rightarrow x^2 - 55x + 750 = 0$
$\therefore$ the number of toys produced that day satisfies the quadratic equation $x^2 - 55x - 750 = 0$ which is the required representation of the problem mathematically.

**Example 2 : Check whether the following are quadratic equations:**

**(i)** $(x - 2)^2 + 1 = 2x - 3$ **(ii)** $x(x + 1) + 8 = (x + 2)(x - 2)$
**(iii)** $x(2x + 3) = x^2 + 1$ **(iv)** $(x + 2)^3 = x^3 - 4$

(i) $(x - 2)^2 + 1 = 2x - 3$

$x^2 - 4x + 4 + 1 = 2x - 3 \Rightarrow x^2 - 4x - 2x + 5 + 3 = 0 \Rightarrow x^2 - 6x + 8 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(ii)** $\mathbf{x(x + 1) + 8 = (x + 2)(x - 2)}$

$x^2 + x + 8 = x^2 - 4 \Rightarrow x^2 - x^2 + x + 8 + 4 = 0 \Rightarrow x + 12 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

**(iii)** $\mathbf{x(2x + 3) = x^2 + 1}$

$2x^2 + 3x = x^2 + 1 \Rightarrow 2x^2 - x^2 + 3x - 1 = 0 \Rightarrow x^2 + 3x - 1 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(iv)** $\mathbf{(x + 2)^3 = x^3 - 4}$

$x^3 + 2^3 + 3(x)(2)^2 + 3x^2(2) = x^3 - 4$

$x^3 + 8 + 12x + 6x^2 = x^3 - 4 \Rightarrow x^3 - x^3 + 6x^2 + 12x + 8 + 4 = 0$

$\Rightarrow 6x^2 + 12x + 12 = 0 \div 6 \Rightarrow x^2 + 2x + 2 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

# Exercise 10.1

**1. Check whether the following are quadratic equations :**

**(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$ (iii) $(x-2)(x+1) = (x-1)(x+3)$**
**(iv) $(x-3)(2x+1) = x(x+5)$ (v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$**
**(vii) $(x+2)^3 = 2x(x^2 - 1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$**

**(i) $(x+1)^2 = 2(x-3)$**

$x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 2x - 2x + 1 + 6 = 0 \Rightarrow x^2 + 7 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(ii) $x^2 - 2x = (-2)(3-x)$**

$x^2 - 2x = -6 + 2x \Rightarrow x^2 - 2x - 2x + 6 = 0 \Rightarrow x^2 - 4x + 6 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(iii) $(x-2)(x+1) = (x-1)(x+3)$**

$x^2 + x - 2x - 2 = x^2 + 3x - x - 3 \Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x^2 - x - 2x - 2 + 3 = 0 \Rightarrow -3x + 3 = 0 \times -1 \Rightarrow 3x - 1 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

**(iv) $(x-3)(2x+1) = x(x+5)$**

$2x^2 + x - 6x - 3 = x^2 + 5x \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - x^2 - 5x - 5x - 3 = 0 \Rightarrow x^2 - 10x - 3 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(v) $(2x-1)(x-3) = (x+5)(x-1)$**

$2x^2 - 6x - x + 3 = x^2 - x + 5x - 5 \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - x^2 - 7x - 4x + 3 + 5 = 0 \Rightarrow x^2 - 11x + 8 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

**(vi) $x^2 + 3x + 1 = (x-2)^2$**

$x^2 + 3x + 1 = x^2 - 2(x)(2) + 2^2 \Rightarrow x^2 - x^2 + 3x + 4x + 1 - 4 = 0$

$7x - 3 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

**(vii) $(x+2)^3 = 2x(x^2 - 1)$**

$x^3 + 2^3 + 3(x)(2)^2 + 3x^2(2) = 2x^3 - 2x \Rightarrow x^3 + 8 + 12x + 6x^2 = 2x^3 - 2x$

$\Rightarrow x^3 - 2x^3 + 6x^2 + 12x + 2x + 8 = 0 \Rightarrow -x^3 + 6x^2 + 14x + 8 = 0 \times -1$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

This is not in the form of $ax^2 + bx + c = 0$

Therefore the given equation is not a quadratic equation.

**(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$**

$x^3 - 4x^2 - x + 1 = x^3 - 2^3 + 3(x)(2)^2 - 3x^2(2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 + 12x - 6x^2$

$\Rightarrow x^3 - x^3 - 4x^2 + 6x^2 - x - 12x + 1 + 8 = 0 \Rightarrow 2x^2 - 13x + 9 = 0$

This is in the form of $ax^2 + bx + c = 0$

Therefore the given equation is quadratic equation.

2. **Represent the following situations in the form of quadratic equations :**

**(i) The area of a rectangular plot is 528 $m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.**

Let breadth $b = x$ m $\Rightarrow$ Length $l = (2x+1)$m

Area of the rectangle $= l \times b \Rightarrow 528 = x(2x+1) \Rightarrow 528 = 2x^2 + x$

$\Rightarrow 2x^2 + x - 528 = 0$

**(ii) The product of two consecutive positive integers is 306. We need to find the integers.**

Let two consecutive integers be $x$ and $(x+1)$; Their products $= 306$

$\Rightarrow x(x+1) = 306 \Rightarrow x^2 + x - 306 = 0$

**(iii)Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.**

Let the present age of Rohan $= x$ ; The present age of his mother $= x + 26$

After 3 Rohan's age $= x + 3$

After 3 years his mothers age $= x + 26 + 3 = x + 29$

Product of their ages after 3 years $= 360$

$\therefore (x+3)(x+29) = 360 \Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

**(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.**

Let the speed of the train $= x\ km/h$

The time taken to travel $480\ km = \frac{480}{x}\ hrs$

Reducing speed by 8 km/h, the speed of the train $= (x-8)km/h$

Therefore the time taken to travel $480\ km = \left(\frac{480}{x-8}\right) hrs$

$\Rightarrow \frac{480}{x} + 3 = \frac{480}{x-8} \Rightarrow 480(x-8) + 3x(x-8) = 480x$

$\Rightarrow 480x - 3840 + 3x^2 - 24x = 480x \Rightarrow 3840 + 3x^2 - 24x = 0$

$\Rightarrow 3x^2 - 24x + 3840 = 0$

$\Rightarrow x^2 - 8x + 1280 = 0$

## 10.3 Solution of a Quadratic Equation by Factorisation

**Note: The zeros of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation are the same.**

**Example 3 : Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorization.**

$2x^2 - 5x + 3 = 0$
$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$
$\Rightarrow 2x(x-1) - 3(x-1) =$
$\Rightarrow (x-1)(2x-3) = 0$
$\Rightarrow x - 1 = 0, \quad 2x - 3 = 0$
$\Rightarrow x = 1,\ 2x = 3$
$x = 1,\ x = \frac{3}{2}$

First term = $2x^2$, Last term = $+3$
Their product = $+6x^2$
The middle term = $-5x$
Divide middle term such that product = $+6x^2$ and their sum $-5x \Rightarrow -5x = -2x - 3x$

**Example 4: Find the roots the equation $6x^2 - x - 2 = 0$**

$6x^2 - x - 2 = 0$
$6x^2 - 4x + 3x - 2 = 0$
$2x(3x-2) + 1(3x-2) = 0$
$(2x+1)(3x-2) = 0$
$2x + 1 = 0, 3x - 2 =$
$2x = -1, 3x = 2$
$\Rightarrow x = \frac{-1}{2}, x = \frac{2}{3}$

First term = $6x^2$, Last term = $-2$
Their product = $-12x^2$
The middle term = $-x$
Divide middle term such that product = $-12x^2$ and sum $-x$
$\Rightarrow -x = -4x + 3x$

**Example 5: Find the roots the equation $3x^2 - 2\sqrt{6}\,x + 2 = 0$**

$\mathbf{3x^2 - 2\sqrt{6}\,x + 2 = 0}$
$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$
$(\sqrt{3})^2 x^2 - \sqrt{2}.\sqrt{3}\,x - \sqrt{2}.\sqrt{3}\,x + (\sqrt{2})^2 = 0$
$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$
$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$
$(\sqrt{3}\,x - \sqrt{2}) = 0,\ (\sqrt{3}\,x - \sqrt{2}) = 0$
$\sqrt{3}\,x = \sqrt{2},\ \sqrt{3}\,x = \sqrt{2} \Rightarrow x = \sqrt{\frac{2}{3}},\ x = \sqrt{\frac{2}{3}}$

First term = $3x^2$, Last term = $+2$
Their product = $6x^2$
The middle term = $-2\sqrt{6}\,x$
Divide middle term such that product = $6x^2$ and sum $-2\sqrt{6}\,x$
$\Rightarrow -2\sqrt{6}\,x = \sqrt{6}\,x - \sqrt{6}\,x$

**Example 6 : Find the dimensions of the prayer hall discussed in Section 10.1.**

$2x^2 + x - 300 = 0$
$2x^2 - 24x + 25x - 300 = 0$
$2x(x-12) + 25(x-12) = 0$
$(x-12)(2x+25) =$
$x - 12 = 0, \quad 2x + 25 = 0$
$x = 12,\ 2x = -25 \Rightarrow x = \frac{-25}{2} = -12.5$
Breadth $= x = 12\,m$
Length $= 2x + 1 = 2(12) + 1 = 24 + 1 = 25\,m$

First term $= 2x^2$, Last term $= -300$
Their produ= $-600x^2$
The middle term = $+x$
Divide middle term such that product = $-600x^2$ and sum x
$\Rightarrow +x = -24x + 25x$

# Exercise 10.2

1. **Find the roots of the following quadratic equations by factorisation:**

   **(i)** $x^2 - 3x - 10 = 0$ **(ii)** $2x^2 + x - 6 = 0$ **(iii)** $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

   **(iv)** $2x^2 - x + \frac{1}{8} = 0$ **(v)** $100x^2 - 20x + 1 = 0$

(i) $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0 \quad \Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0 \quad \Rightarrow (x-5) = 0, (x+2) = 0$

$\Rightarrow x = 5, x = -2$

(ii) $2x^2 + x - 6 = 0$

$2x^2 + x - 6 = 0 \quad \Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0 \quad \Rightarrow (x+2)(2x-3) = 0$

$\Rightarrow x + 2 = 0, 2x - 3 = 0$

$\Rightarrow x = -2, 2x = 3 \Rightarrow x = -2, x = \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0 \quad \Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow \sqrt{2}x + 5 = 0, \; x + \sqrt{2} = 0 \Rightarrow \sqrt{2}x = -5, \; x = -\sqrt{2} \quad \Rightarrow x = \frac{-5}{\sqrt{2}}, \quad x = -\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \quad \Rightarrow 4x(4x-1) - 1(4x-1) = 0$

$\Rightarrow (4x-1)(4x-1) = 0 \Rightarrow 4x - 1 = 0, \; 4x - 1 = 0$

$\Rightarrow 4x = 1, 4x = 1 \Rightarrow x = \frac{1}{4}, \; x = \frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$

$100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0 \Rightarrow 10x(10x-1) - 1(10x-1) = 0$

$\Rightarrow (10x-1)(10x-1) = 0 \quad \Rightarrow 10x - 1 = 0, 10x - 1 = 0$

$\Rightarrow 10x = 1, 10x = 1 \Rightarrow x = \frac{1}{10}, \; x = \frac{1}{10}$

## Solving the quadratic equations using formula:

Find the roots of the quadratic equation $ax^2 + bx + c = 0$ by completing the square.

$ax^2 + bx = -c$ [ multiply the equation by 4a ]

$4a^2x^2 + 4abx = -4ac$ [Add $b^2$ to both the sides]

$4a^2x^2 + 4abx + b^2 = -4ac + b^2$

$\Rightarrow (2ax)^2 + 2(2ax)(b) + b^2 = b^2 - 4ac \Rightarrow (2ax + b)^2 = b^2 - 4ac$

$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac} \quad \Rightarrow 2ax = -b \pm \sqrt{b^2 - 4ac} \quad \Rightarrow x = \frac{-b+\sqrt{b^2-4ac}}{2a}$

Roots are: $x = \frac{-b + \sqrt{b^2-4ac}}{2a}$, $x = \frac{-b - \sqrt{b^2-4ac}}{2a}$

**Example 13 : Find the roots of the following quadratic equations, if they exist, using the quadratic formula.(i) $3x^2 - 5x + 2 = 0$ (ii) $x^2 + 4x + 5 = 0$ (iii) $2x^2 - 2\sqrt{2}x + 1$**

**(i) $3x^2 - 5x + 2 = 0$** this is in the form of $ax^2 + bx + c = 0$

$a = 3,\ \ b = -5,\ \ c = +2$

Roots are: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(2)}}{2(3)} \Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{6}$

$\Rightarrow x = \frac{5 \pm \sqrt{1}}{6} \Rightarrow x = \frac{5 \pm 1}{6}$

$x = \frac{6}{6}$ or $x = \frac{4}{6} \Rightarrow x = 1$ or $x = \frac{2}{3}$

**(ii) $x^2 + 4x + 5 = 0$** this is in the form of $ax^2 + bx + c = 0$

$a = 1,\ \ b = 4,\ \ c = +5$

Roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} \Rightarrow x = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$\Rightarrow x = \frac{-4 \pm \sqrt{-4}}{2} \Rightarrow x = \frac{-4 \pm 2\sqrt{-1}}{2} \Rightarrow$ Roots are not Real numbers.

**(iii) $2x^2 - 2\sqrt{2}x + 1$** this is in the form of $ax^2 + bx + c = 0$

$a = 2,\ \ b = -2\sqrt{2},\ \ c = +1$

roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(2)(1)}}{2(2)}$

$\Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{8 - 8}}{4} \Rightarrow x = \frac{2\sqrt{2} \pm \sqrt{0}}{4} \Rightarrow x = \frac{2\sqrt{2}}{4} \Rightarrow x = \frac{\sqrt{2}}{2} \Rightarrow \frac{1}{\sqrt{2}}$

**Example 14:Find the roots of the following equations**

**(i) $x + \frac{1}{x} = 3, x \neq 0$ (ii) $\frac{1}{x} + \frac{1}{x-2} = 3 \quad x \neq 0, x \neq 2$**

**(i) $x + \frac{1}{x} = 3, x \neq 0$**

$x + \frac{1}{x} = 3$ – Multiply both sides bu x

$x^2 + 1 = 3x \Rightarrow x^2 - 3x + 1 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 1,\ \ b = -3,\ \ c = 1$

Roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)} \Rightarrow x = \frac{3 \pm \sqrt{9 - 4}}{2} \Rightarrow x = \frac{3 \pm \sqrt{5}}{2} \Rightarrow x = \frac{3 + \sqrt{5}}{2},\ x = \frac{3 - \sqrt{5}}{2}$

**(ii) $\frac{1}{x} - \frac{1}{x-2} = 3 \Rightarrow$** $\frac{x - 2 - x}{x(x-2)} = 3$

$\Rightarrow \frac{-2}{x^2 - 2x} = 3 \Rightarrow -2 = 3x^2 - 6x$

$3x^2 - 6x + 2 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 3,\ \ b = -6,\ \ c = 2$

roots are, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$

$\Rightarrow x = \frac{6 \pm \sqrt{36 - 24}}{6} \Rightarrow x = \frac{6 \pm \sqrt{12}}{6} \Rightarrow x = \frac{6 \pm \sqrt{4 \times 3}}{6} \Rightarrow x = \frac{6 \pm 2\sqrt{3}}{6}$

$\Rightarrow x = \frac{2(3 \pm \sqrt{3})}{6} \Rightarrow x = \frac{3 \pm \sqrt{3}}{3} \Rightarrow x = \frac{3 + \sqrt{3}}{3},\ \ x = \frac{3 - \sqrt{3}}{3}$

# Exercise 10.3

1. **Find the roots of the following quadratic equations, if they exist, by the method of completing the square:.**

   (i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$ (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

**(i) $2x^2 - 7x + 3 = 0$**

$2x^2 - 7x = -3 \times 2$

$4x^2 - 14x = -6$

$4x^2 - 14x + \left(\frac{7}{2}\right)^2 = -6 + \left(\frac{7}{2}\right)^2$ [Add $\left(\frac{7}{2}\right)^2$ to both the sides]

$(2x)^2 - 2(2x)(\frac{7}{2}) + \left(\frac{7}{2}\right)^2 = -6 + \frac{49}{4}$

$\left(2x - \frac{7}{2}\right)^2 = \frac{-24+49}{4} \Rightarrow \left(2x - \frac{7}{2}\right) = \pm\sqrt{\frac{25}{4}}$

$\Rightarrow 2x - \frac{7}{2} = \pm\frac{5}{2} \Rightarrow 2x = \pm\frac{5}{2} + \frac{7}{2} \Rightarrow 2x = \frac{\pm 5+7}{2} \Rightarrow x = \frac{\pm 5+7}{4}$ , $2x = 1$

$\Rightarrow x = \frac{5+7}{4}$ . $x = \frac{-5+7}{4} \Rightarrow x = \frac{12}{4}$ . $x = \frac{2}{4}$

$\Rightarrow x = 3$ . $x = \frac{1}{2}$

2ab = 14x

$2(2x)b = 14x$

$b = \frac{14x}{4x} = \frac{7}{2}$

$b^2 = \left(\frac{7}{2}\right)^2$

**(ii) $2x^2 + x - 4 = 0$**

$2x^2 + x = 4 \qquad \times 2$

$4x^2 + 2x = 8$

$4x^2 + 2x + \left(\frac{1}{2}\right)^2 = 8 + \left(\frac{1}{2}\right)^2$ [Add $\left(\frac{1}{2}\right)^2$ to both the sides]

$(2x)^2 + 2(2x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 = 8 + \frac{1}{4}$

$\left(2x + \frac{1}{2}\right)^2 = \frac{32+1}{4} \Rightarrow \left(2x + \frac{1}{2}\right) = \pm\sqrt{\frac{33}{4}}$

$\Rightarrow 2x + \frac{1}{2} = \pm\frac{\sqrt{33}}{2} \Rightarrow 2x = \pm\frac{\sqrt{33}}{2} + \frac{1}{2} \Rightarrow 2x = \frac{\pm\sqrt{33}-1}{2} \quad \Rightarrow x = \frac{\pm\sqrt{33}-1}{4}$

$\Rightarrow x = \frac{\sqrt{33}-1}{4}$, $x = \frac{-\sqrt{33}-1}{4}$

2ab = 2x

$2(2x)b = 2x$

$b = \frac{2x}{4x} = \frac{1}{2}$

$b^2 = \left(\frac{1}{2}\right)^2$

**(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$**

$4x^2 + 4\sqrt{3}x = -3$

$4x^2 + 4\sqrt{3}x + (\sqrt{3})^2 = -3 + (\sqrt{3})^2$[Add $(\sqrt{3})^2$ to both the sides]

$(2x)^2 + 2(2x)(\sqrt{3}) + (\sqrt{3})^2 = -3 + 3$

$(2x + \sqrt{3})^2 = 0$

$(2x + \sqrt{3}) = 0, \quad (2x + \sqrt{3}) = 0$

$2x = -\sqrt{3}.\ 2x = -\sqrt{3} \quad \Rightarrow x = \frac{-\sqrt{3}}{2}$ . $x = \frac{-\sqrt{3}}{2}$

$2ab = 4\sqrt{3}x$

$2(2x)b = 4\sqrt{3}x$

$b = \frac{4\sqrt{3}x}{4x} = \sqrt{3}$

$b^2 = (\sqrt{3})^2$

**(iv) $2x^2 + x + 4 = 0$**

$2x^2 + x = -4 \qquad \times 2$

$4x^2 + 2x = -8$

$(2x)^2 - 2(2x)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 = -8 + \left(\frac{1}{4}\right)^2$[Add $\left(\frac{1}{4}\right)^2$ to both the sides]

$\left(2x - \frac{1}{4}\right)^2 = -8 + \frac{1}{16} \Rightarrow \left(2x - \frac{1}{4}\right)^2 = \frac{-128+1}{16}$

$\Rightarrow \left(2x - \frac{1}{4}\right)^2 = \frac{-127}{16} < 0$ There are no roots.The roots are imaginary

$2ab = 2x$

$2(2x)b = 2x$

$b = \frac{2x}{4x} = \frac{1}{2}$

$b^2 = \left(\frac{1}{2}\right)^2$

**2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula**

(i) $2x^2-7x+3=0$ this is in the form of $ax^2+bx+c=0$

$a=2,\quad b=-7,\quad c=3$

Roots are, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(3)}}{2(2)}\Rightarrow x=\frac{7\pm\sqrt{49-24}}{4}$

$\Rightarrow x=\frac{7\pm\sqrt{25}}{4}\Rightarrow x=\frac{7\pm5}{4}$

$\Rightarrow x=\frac{7+5}{4},\quad x=\frac{7-5}{4}\quad\Rightarrow x=\frac{12}{4}\ ,\ x=\frac{2}{4}\ \Rightarrow x=3\ ,\ x=\frac{1}{2}$

$2x^2+x-4=0$ this is in the form of $ax^2+bx+c=0$

(ii) $a=2,\quad b=1,\quad c=-4$

Roots are, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(-4)}}{2(2)}\Rightarrow x=\frac{-1\pm\sqrt{1+32}}{2}\Rightarrow x=\frac{-1\pm\sqrt{33}}{2}$

$x=\frac{-1+\sqrt{33}}{2},\quad x=\frac{-1-\sqrt{33}}{2}$

(iii) $4x^2+4\sqrt{3}x+3=0$ this is in the form of $ax^2+bx+c=0$

$a=4,\quad b=4\sqrt{3},\quad c=+3$

Roots are, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(4\sqrt{3})\pm\sqrt{(4\sqrt{3})^2-4(4)(3)}}{2(4)}$

$x=\frac{-4\sqrt{3}\pm\sqrt{16\times3-48}}{8}\Rightarrow x=\frac{-4\sqrt{3}\pm\sqrt{48-48}}{8}$

$\Rightarrow x=\frac{-4\sqrt{3}\pm0}{8},\ \Rightarrow x=\frac{-4\sqrt{3}}{8},x=\frac{-4\sqrt{3}}{8}\quad\Rightarrow x=\frac{-\sqrt{3}}{2},\ x=\frac{-\sqrt{3}}{2}$

(iv) $2x^2+x+4=0$ this is in the form of $ax^2+bx+c=0$

$a=2,\quad b=1,\quad c=4$

Roots are, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(4)}}{2(2)}\Rightarrow x=\frac{-1\pm\sqrt{1-32}}{4}\Rightarrow x=\frac{-1\pm\sqrt{-31}}{4}$

There is no real root for this equation.

**3. Find the roots of the following equations:**

**(i)** $x+\frac{1}{x}=3,\quad x\neq0$ **(ii)** $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30},\quad x\neq-4,\ 7$

**(i)** $x-\frac{1}{x}=3,\quad x\neq0$

$x-\frac{1}{x}=3$ Multiply the equation by x

$x^2-1=3x\ \Rightarrow x^2-3x-1=0$ this is in the form of $ax^2+bx+c=0$

$a=1,\quad b=-3,\quad c=-1$

Roots are, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-1)}}{2(1)}$

$x=\frac{3\pm\sqrt{9+4}}{2}\Rightarrow x=\frac{3\pm\sqrt{13}}{2}\ \Rightarrow x=\frac{3+\sqrt{13}}{2},\ \ x=\frac{3-\sqrt{13}}{2}$

## 10.5 Nature of Roots

The value of $b^2 - 4ac$ decides the roots of quadratic equation $ax^2 + bx + c = 0$ has real or not, therefore

**$b^2 - 4ac$ is called the discriminant of this quadratic equation.and denoted by $\Delta$ [delta]**

So, the quadratic equation $ax^2 + bx + c = 0$ has

| Discriminant | Nature of the roots |
|---|---|
| $\Delta = 0$ | Two equal real roots |
| $\Delta > 0$ | Two distinct real roots |
| $\Delta < 0$ | No real roots |

**Example 16 : Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots**

$a = 2, \quad b = -4, \quad c = 3$

$\Delta = b^2 - 4ac \Rightarrow \Delta = (-4)^2 - 4(2)(3)$

$\Rightarrow \Delta = 16 - 24 \Rightarrow \Delta = -8 < 0$ Roots are imaginary

## Exercise: 10.4

1. **Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:.(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$**

**(i) $2x^2 - 3x + 5 = 0$**

$a = 2, \quad b = -3, \quad c = 5$

$\Delta = b^2 - 4ac$

$\Delta = (-3)^2 - 4(2)(5) \Rightarrow \Delta = 9 - 40$

$\Rightarrow \Delta = -31 \Rightarrow \Delta < 0 \Rightarrow$ Roots are imaginary

**(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$**

$a = 3, \quad b = -4\sqrt{3}, \quad c = 4$

$\Delta = b^2 - 4ac$

$\Delta = (-4\sqrt{3})^2 - 4(3)(4) \Rightarrow \Delta = 48 - 48$

$\Delta = 0 \Rightarrow$ Roots are real and equal

The roots are: $\frac{-b}{2a}, \frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2(3)}, \frac{-(-4\sqrt{3})}{2(3)} = \frac{4\sqrt{3}}{6}, \frac{4\sqrt{3}}{6}$

$= \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3} \Rightarrow \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

**(iii) $2x^2 - 6x + 3 = 0$**

$a = 2, \quad b = -6, \quad c = 3$

$\Delta = b^2 - 4ac$

$\Delta = (-6)^2 - 4(2)(3) \Rightarrow \Delta = 36 - 24$

$\Rightarrow \Delta = 12 \Rightarrow \Delta > 0 \Rightarrow$ Roots are real and distinct

The roots $= \frac{-b+\sqrt{\Delta}}{2a}, \frac{-b-\sqrt{\Delta}}{2a}$

$= \frac{-(-6)+\sqrt{12}}{2(2)}, \frac{-(-6)-\sqrt{12}}{2(2)} = \frac{6+\sqrt{12}}{4}, \frac{6-\sqrt{12}}{4}$

$= \frac{6+2\sqrt{3}}{4}, \frac{6-2\sqrt{3}}{4} = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2. **Find the values of k for each of the following quadratic equations, so that they have two equal roots (i) $2x^2 + kx + 3 = 0$ (ii) $kx(k - 2) + 6 = 0$**

**(i) $2x^2 + kx + 3 = 0$** $a = 2,\ b = k,\quad c = 3$

$b^2 - 4ac = 0$

$(k)^2 - 4(2)(3) = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k^2 = 24$

$k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

**(ii) $kx(x - 2) + 6 = 0$**

$kx^2 - 2kx + 6 = 0 \Rightarrow a = k,\quad b = -2k,\quad c = 6$

$b^2 - 4ac = 0$

$\Rightarrow (-2k)^2 - 4(k)(6) = 0 \Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k - 6) = 0 \Rightarrow 4k = 0, k - 6 = 0$

$\Rightarrow k = 0,\quad k = 6$

3. **Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 $m^2$? If so, find its length and breadth**

The breadth of the mango grove $= l$ ;The length $= 2l$

The area of the grove = Length x breadth

$\Rightarrow (l)(2l) = 800 \Rightarrow 2l^2 = 800 \quad \Rightarrow l^2 = \frac{800}{2} = 400 \Rightarrow l = \pm\sqrt{400} = \pm 20$

$\therefore$ The breadth of the mango grove $= l = 20$ m

$\therefore$ The breadth of the mango grove $= 2l = 2 \times 20 = 40$ m

4. **Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.**

Let the age of A friend $= x$ Years

The age of B friend $= (20 - x)$ years

The age of friend A before 4 $= (x - 4)$

The age of B friend before 4 years $= (20 - x - 4) = 16 - x$

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$

$-x^2 + 20x - 64 - 48 = 0$

$x^2 - 20x + 112 = 0$

$a = 1,\quad b = -20,\quad c = 112$

$b^2 - 4ac = (-20)^2 - 4(1)(112)$

$= 400 - 448 = -48$

The equation has no real roots. Therefore this situation is not possible

5. **Is it possible to design a rectangular park of perimeter 80 m and area 400 $m^2$? If so, find its length and breadth.**

Let the length and breadth of the rectangle be $l$ and $b$; The perimeter $= 2(l + b) = 80$

$l + b = \frac{80}{2} = 40 \Rightarrow l = 40 - b$

Area $l \times b = 400 \Rightarrow l(40 - l) = 400$

$\Rightarrow 40l - l^2 = 400 \Rightarrow l^2 - 40l + 400 = 0$

$a = 1,\quad b = -40,\quad c = 400$

$b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

$b^2 - 4ac = 0$ Roots are rea and equal

Roots are: $\frac{-b}{2a}, \frac{-b}{2a} = \frac{-(-40)}{2(1)}, \frac{-(-40)}{2(1)} = \frac{40}{2}, \frac{40}{26} = 20, \ 20$

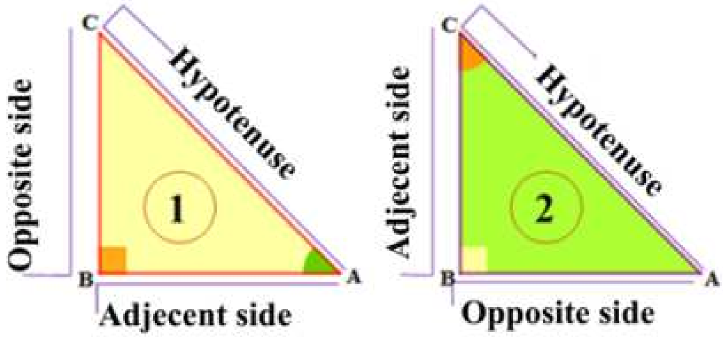
Length $= 20\,m$ ; Breadth $b = 40 - l = 40 - 20 = 20m$

# 11 ITRODUCTION TO TRIGONOMETRY

Trigonometry is the study of relationships between the sides and angles of a triangle.

## 11.2 Trigonometric Ratios:

To know the trigonometric ratio we have to consider right angle triangle.




**There are six trigonometric ratios:**

| Trigonometric ratios | | Triangle 1 | Triangle 2 |
|---|---|---|---|
| SinA | $\frac{\text{Opposite}}{\text{Hypotenuse}}$ | $\frac{BC}{AC}$ | $\frac{AB}{AC}$ |
| CosA | $\frac{\text{Adjecent}}{\text{Hypotenuse}}$ | $\frac{AB}{AC}$ | $\frac{BC}{AB}$ |
| Tan A | $\frac{\text{Opposite}}{\text{Adjecent}}$ | $\frac{BC}{AB}$ | $\frac{AB}{BC}$ |
| CosecA | $\frac{\text{Hypotenuse}}{\text{Opposite}}$ | $\frac{AC}{BC}$ | $\frac{AC}{AB}$ |
| SecA | $\frac{\text{Hypotenuse}}{\text{Adjecent}}$ | $\frac{AC}{AB}$ | $\frac{AC}{BC}$ |
| CotA | $\frac{\text{Adjecent}}{\text{Opposite}}$ | $\frac{AB}{BC}$ | $\frac{BC}{AB}$ |

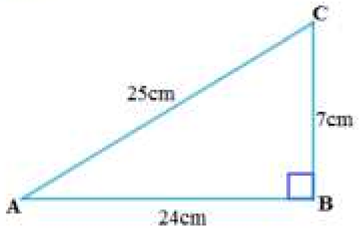
| Inverse of trigonometric values | | |
|---|---|---|
| $\frac{1}{\text{SinA}}$ | $\frac{\text{Hypotenuse}}{\text{Opposite}}$ | CosecA |
| $\frac{1}{\text{CosA}}$ | $\frac{\text{Hypotenuse}}{\text{Adjecent}}$ | SecA |
| $\frac{1}{\text{Tan A}}$ | $\frac{\text{Adjecent}}{\text{Opposite}}$ | CotA |
| $\frac{1}{\text{CosecA}}$ | $\frac{\text{Opposite}}{\text{Hypotenuse}}$ | SinA |
| $\frac{1}{\text{SecA}}$ | $\frac{\text{Adjecent}}{\text{Hypotenuse}}$ | SecA |
| $\frac{1}{\text{CotA}}$ | $\frac{\text{Opposite}}{\text{Adjecent}}$ | CotA |

# Exercise 11.1

[ for solving problems, the value of constant k is taken as 1]

**1. In $\Delta ABC$, right-angled at B, AB = 24 cm, BC = 7 cm. Determine**

**i) sin A, cos A (ii) sin C, cos C**




In $\Delta$ ABC, $\angle B = 90^0$ $\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2 = (24)^2 + 7^2$

$= (576+49)\ cm^2 = 625\ cm^2 \Rightarrow AC = 25$

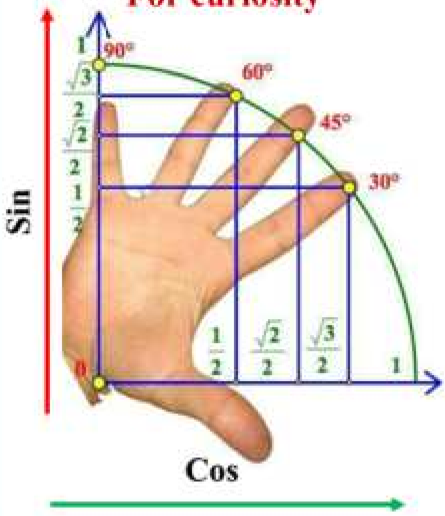
(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$ ; $\cos C = \frac{BC}{AC} = \frac{7}{25}$

## Table 11.1

| $\angle A$ | $0^0$ | $30^0$ | $45^0$ | $60^0$ | $90^0$ |
|---|---|---|---|---|---|
| Sin | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| Cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| Tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ND |
| osec | ND | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| Sec | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | ND |
| Cot | ND | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

**For curiosity**




**Example 6 : In $\Delta ABC$, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ (see Fig. 11.19). Determine the lengths of the sides BC and AC.**

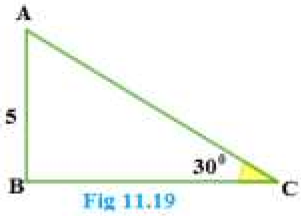



Fig 11.19

$Tan30^0 = \frac{5}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC} \Rightarrow 5\sqrt{3}cm$

$Sin30^0 = \frac{5}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC} \Rightarrow AC = 10cm$

**Example 7 : In $\Delta PQR$, right-angled at Q (see Fig. 11.20), PQ = 3 cm and PR = 6 cm. Determine $\angle QPR$ and $\angle PRQ$.**


Fig 11.20

$SinR = \frac{3}{6} = \frac{1}{2}$

$\Rightarrow \angle R = 30^0 \Rightarrow \angle PRQ = 30^0$

$\therefore \angle QPR = 60^0$

**Example 8: If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, $0 < A + B \leq 90$, $A > B$ find A and B**

If $\sin(A - B) = \frac{1}{2}$ then $\sin 30^0 = \frac{1}{2} \Rightarrow A - B = 30^0$ (1)

If $\cos(A + B) = \frac{1}{2}$ then $\cos 60^0 = \frac{1}{2} \Rightarrow A + B = 60^0$ (2)

$(1) + (2) = 2A = 90^0 \Rightarrow A = 45^0$

From (2) $\Rightarrow 45^0 - B = 30^0 \Rightarrow B = 15^0$

# Exercise 11.2

1. **Evaluate the following:**

   i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$   ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

   iii) $\frac{\cos 45^0}{\sec 30^0 + \operatorname{cosec} 30^0}$   iv) $\frac{\operatorname{Sin} 30^0 + \tan 45^0 - \operatorname{Cosec} 45^0}{\operatorname{Sec} 30^0 + \cos 60^0 + \cot 45^0}$   iv) $\frac{5\operatorname{Cos}^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\operatorname{Sin}^2 30^0 + \cos^2 30^0}$

i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$

ii) $2\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

$= 2\,(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2$

iii) $\frac{\cos 45^0}{\operatorname{Sec} 30^0 + \operatorname{cosec} 30^0}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}} = \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2}$

$= \frac{2\sqrt{6}-6\sqrt{2}}{4\text{x}2 - 4\text{x}6} = \frac{2(\sqrt{6}-3\sqrt{2})}{8-24} = \frac{2(\sqrt{6}-3\sqrt{2})}{-16} = \frac{\sqrt{6}-3\sqrt{2}}{-8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$

iv) $\frac{\operatorname{Sin} 30^0 + \tan 45^0 - \operatorname{Cosec} 60^0}{\operatorname{Sec} 30^0 + \cos 60^0 + \cot 45^0}$

$= \frac{\left(\frac{1}{2}\right)+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+3\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{4+3\sqrt{3}}$

$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \frac{4-3\sqrt{3}}{4-3\sqrt{3}} = \frac{12\sqrt{3}-16-9\sqrt{9}+12\sqrt{3}}{(4)^2-(3\sqrt{3})^2}$

$= \frac{12\sqrt{3}-16-27+12\sqrt{3}}{16-27} = \frac{24\sqrt{3}-43}{-11} = \frac{43-24\sqrt{3}}{11}$

iv) $\frac{5\operatorname{Cos}^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\operatorname{Sin}^2 30^0 + \cos^2 30^0}$

$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{1}{4}+\frac{3}{4}} = \frac{\frac{15+64-12}{12}}{1} = \frac{67}{12}$

2. **Choose the correct option and justify your choice:**

   i) $\frac{2\tan 30^0}{1+\tan^2 30^0}$

   A) $\sin 60^0$   B) $\cos 60^0$   C) $\tan 60^0$   D) $\sin 30^0$

   $= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}$

   **Ans: A) $\sin 60^0$**

ii) $\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$

A) $\tan 90^0$   B) 1   C) $\sin 45^0$   D) 0

$\frac{1-1}{1+1} = \frac{0}{2} = 0$

**Ans: D) 0**

iii) $\sin 2A = 2 \sin A$ is true when $A =$

A) 0   B) 30   C) 45   D) 60

$\sin 2\text{x}0 = 2 \sin 0 \Rightarrow \sin 0 = 2 \sin 0 \Rightarrow 0 = 0$

**Ans: A) 0**

iv) $\frac{2\tan 30^0}{1-\tan^2 30^0}$

A) $\cos 60^0$   B) $\sin 60^0$   C) $\tan 60^0$   D) $\sin 30^0$

$\frac{2\text{x}\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

**Ans: C) $\tan 60^0$**

3. **If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0 < A+B \leq 90$ ; $A > B$ find A and B**

$\tan(A+B) = \sqrt{3} \Rightarrow A+B = 60^0$ (1)

$\tan(A-B) = \frac{1}{\sqrt{3}} \Rightarrow A-B = 30^0$ (2)

$(2)-(1) \Rightarrow 2B = 30^0 \Rightarrow B = 15^0 \Rightarrow (1)$ ರಿಂದ $A = 60-15 = 45^0$

**4. State whether the following are true or false. Justify your answer.**

**i) sin (A + B) = sin A + sin B**

Let $A = 30^0$ and $B = 90^0$

$\sin(30^0+60^0) = \sin 90^0 = 1 \Rightarrow \sin 30^0 + \sin 60^0 = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

$\therefore$ The statement is false

**ii) The value of sin θ increases as θ increases**

$\sin 0^0 = 0$ , $\sin 90^0 = 1$

$\therefore$ The statement is true

**iii)The value of cos θ increases as θ increases.**

$\cos 0^0 = 1$ , $\cos 90^0 = 0$

Here, we observe that as θ increases the value of cos θ dicreases

$\therefore$ The statement is false

**iv)sinθ = cosθ for all values of θ**

$\sin 30^0 = \frac{1}{2}$ ; $\cos 30^0 = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta \neq \cos\theta$ for all values of θ

$\therefore$ The statement is false

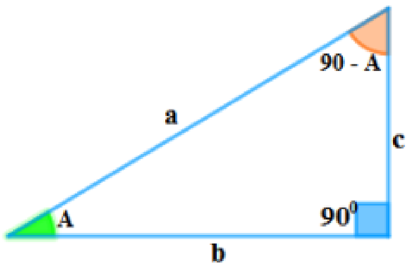
**v) cot A is not defined for $A = 0^0$**

The statement is true

## 11.4 Trigonometric Ratios of Complementary Angles

**Two angles are given and if their sum is equal to $90^0$ then angles are called complementary Angles**

| Trigonometric ratios | | Trigonometric ratios of complementary angles |
|---|---|---|
| SinA | $\frac{c}{a}$ | Cos(90-A) |
| CosA | $\frac{b}{a}$ | Sin(90-A) |
| TanA | $\frac{c}{b}$ | Cot(90-A) |
| CosecA | $\frac{a}{c}$ | Sec(90-A) |
| SecA | $\frac{a}{b}$ | Cosec(90-A) |
| CotA | $\frac{b}{c}$ | Tan(90-A) |




**Example 9 : Evaluate $\frac{\tan 65^0}{\cot 25^0}$**

$\frac{\tan 65^0}{\cot 25^0} = \frac{\tan(90-25)^0}{\cot 25^0} = \frac{\cot 25^0}{\cot 25^0} = 1$

**Example 10 : If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value**

Given $\sin 3A = \cos(A-26)$

$\Rightarrow \cos(90-3A) = \cos(A-26^0) \Rightarrow 90-3A = A-26^0$

$\Rightarrow 90+26 = A+3A \Rightarrow 116 = 4A \Rightarrow A = 29^0$

**Example 11 : Express cot 85° + cos 75° in terms of trigonometric ratios of angles between $0^0$ and $45^0$**

$\cot 85^0 = \cot(90-5^0) = \tan 5^0$

$\cos 75^0 = \cos(90-15^0) = \sin 15^0$

# Exercise 11.3

**1. Evaluate: i) $\frac{\sin 18^0}{\cos 72^0}$ ii) $\frac{\sin 26^0}{\cos 64^0}$ iii) $\cos 48^0 - \sin 42^0$ vi) $\operatorname{cosec} 31^0 - \sec 59^0$**

**i) $\frac{\sin 18^0}{\cos 72^0}$**

$\frac{\sin 18^0}{\cos 72^0} = \frac{\sin(90-72^0)}{\cos 72^0} = \frac{\cos 72^0}{\cos 72^0} = 1$

**ii) $\frac{\sin 26^0}{\cos 64^0}$**

$\frac{\sin 26^0}{\cos 64^0} = \frac{\sin(90-64^0)}{\cos 64^0} = \frac{\cos 64^0}{\cos 64^0} = 1$

**iii) $\cos 48^0 - \sin 42^0$**

$\cos 48^0 - \sin(90\text{-}48^0) = \cos 48^0 - \cos 48^0 = 0$

**vi) $\operatorname{cosec} 31^0 - \sec 59^0$**

$\operatorname{cosec} 31^0 - \sec 59^0 = \operatorname{cosec} 31^0 - \sec(90 - 31^0) = \operatorname{cosec} 31^0 - \operatorname{cosec} 31^0 = 0$

**2. Show that i) $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$ ii) $\cos 38^0 \cos 52^0 - \sin 38=^0 \sin 52^0 = 0$ i)**

**$\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$**

LHS = $\tan 48^0 \tan 23^0 \tan(90\text{-}48^0) \tan(90\text{-}23^0)$

$= \tan 48^0 \tan 23^0 \cot 48^0 \cot 23^0 = \tan 48^0 \text{ x } \tan 23^0 \text{ x } \frac{1}{\tan 48^0} \text{x} \frac{1}{\tan 23^0} = 1$

**ii) $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0 = 0$**

LHS = $\cos 38^0 \cos 52^0 - \sin 38^0 \sin 52^0$

$= \cos 38^0 \cos 52^0 - \sin(90 - 52^0) \sin(90\text{-}38)^0 = \cos 38^0 \cos 52^0 - \cos 52^0 \cos 38^0$

$= \cos 38^0 \cos 52^0 - \cos 52^0 \cos 38^0 = 0$ **RHS**

**3. If $\tan 2A = \cot (A - 180)$ and 2A is an acute angle find the value of A**

$\tan 2A = \cot (A - 18^0)$

$\Rightarrow \cot(90\text{-}2A) = \cot(A\text{-}18^0) \Rightarrow 90^0 - 2A = A - 18^0 \Rightarrow 3A = 108^0 \Rightarrow A = 36^0$

4. **If $\tan A = \cot B$, Prove that $A + B = 90^0$**

LHS = $\tan A = \cot B$

$\Rightarrow \cot(90\text{-}A) = \cot B \Rightarrow 90 - A = B \Rightarrow A + B = 90^0$

5. **If $\sec 4A = \operatorname{cosec} (A - 20^0)$ and 4A is an acute angle find the value of A**

$\sec 4A = \operatorname{cosec} (A - 20^0)$

$\Rightarrow \operatorname{cosec}(90 - 4A) = \operatorname{cosec}(A - 20^0) \Rightarrow 90 - 4A = A - 20^0 \Rightarrow 5A = 110 \Rightarrow A = 22^0$

**6. If A, B and C are the interior angles of $\Delta ABC$ then show that $\sin\frac{(B+C)}{2} = \cos\frac{A}{2}$**

Let A , B and C are the interior angles of $\Delta ABC$

$\Rightarrow A + B + C = 180^0 \Rightarrow B + C = 180 - A$

$\Rightarrow \frac{B+C}{2} = \frac{180-A}{2} \quad \Rightarrow \frac{B+C}{2} = 90 - \frac{A}{2}$

$\Rightarrow \sin\frac{(B+C)}{2} = \sin\left(90 - \frac{A}{2}\right) \quad \Rightarrow \sin\frac{(B+C)}{2} = \cos\frac{A}{2}$

7. **Express $\text{Sin}67^0 + \cos 75^0$ in terms of the trigonometric ratios in between $0^0$ and $45^0$**

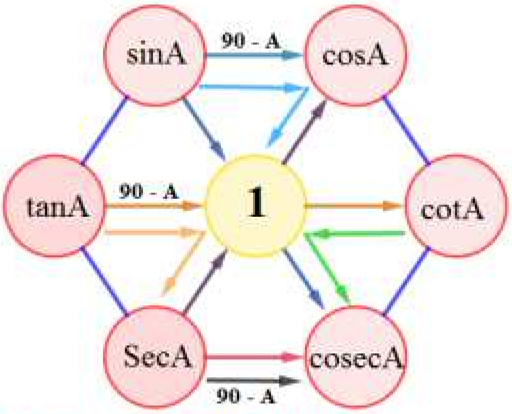
$\text{Sin}67^0 + \cos 75^0$

$= \sin(90\text{-}23^0) + \cos(90\text{-}15^0) = \cos 23^0 + \sin 15^0$

## 11.5 ತ್ರಿಕೋನಮಿತಿ ನಿತ್ಯಸಮೀಕರಣಗಳು

You may recall that an equation is called an identity w
involved. Similarly, an equation involving trigonor
trigonometric identity, if it is true for all values of th

**For curiosity**




**$\sin^2 A + \cos^2 A = 1$**

**$\tan^2 + 1 = \sec^2 A$**

**$1 + \cot^2 A = \mathrm{cosec}^2 A$**

> Note: $\frac{\sin A}{\cos A} = \tan A$
> $\frac{\cos A}{\sin A} = \cot A$

**Example 12 : Express the ratios cos A, tan A and sec A in terms of sin A.**

$\cos^2 A + \sin^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A$

$\cos A = \sqrt{1 - \sin 2A}$

$\therefore \tan A = \frac{\sin A}{\cos A} \Rightarrow \frac{\sin A}{\sqrt{1 - \sin 2A}} \Rightarrow \sec A = \frac{1}{\cos A} \Rightarrow \frac{1}{\sqrt{1 - \sin 2A}}$

**Example 13 : Prove that sec A (1 – sin A)(sec A + tan A) = 1.**

LHS $= \sec A\,(1 - \sin A)(\sec A + \tan A)$

$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) = \left(\frac{1 - \sin A}{\cos A}\right)\left(\frac{1 + \sin A}{\cos A}\right) = \frac{1 - \sin^2 A}{\cos^2 A} = \frac{\cos^2 A}{\cos^2 A} = 1$

**Example 14: Prove that $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\mathrm{cosec}\, A - 1}{\mathrm{cosec}\, A + 1}$**

$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} = \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)} = = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\mathrm{cosec}\, A - 1}{\mathrm{cosec}\, A + 1}$

**ಉದಾಹರಣೆ 15:Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using the identity $\sec^2\theta = 1 + \tan^2\theta$**

$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$

$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} \times \frac{\tan\theta - \sec\theta}{\tan\theta - \sec\theta} = \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{(\tan^2\theta - \sec^2\theta) - (\tan\theta - \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-1 - \tan\theta + \sec\theta)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$

$= \frac{-1}{(\tan\theta - \sec\theta)} = \frac{1}{\sec\theta - \tan\theta}$

## Exercise 11.4

1. **Express the trigonometric ratios sinA, secA ಮತ್ತು tanA in terms of cotA**

   $\mathrm{cosec}^2 A - \cot^2 A = 1$

   $\Rightarrow \mathrm{cosec}^2 A = 1 + \cot^2 A \Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A \Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A} \Rightarrow \sin A = \frac{\pm 1}{\sqrt{1 + \cot^2 A}}$

   $\sin^2 A = \frac{1}{1 + \cot^2 A} \Rightarrow 1 - \cos^2 A = \frac{1}{1 + \cot^2 A} \Rightarrow \cos^2 A = 1 - \frac{1}{1 + \cot^2 A} \Rightarrow \cos^2 A = \frac{1 + \cot^2 A - 1}{1 + \cot^2 A}$

   $\Rightarrow \frac{1}{\sec^2 A} = \frac{\cot^2 A}{1 + \cot^2 A} \Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A} \Rightarrow \sec A = \frac{\pm\sqrt{1 + \cot^2 A}}{\cot A} \Rightarrow \tan A = \frac{1}{\cot A}$

2. Write all the trigonometric ratios $\angle A$ in terms of sec A

   $\sec A = \frac{1}{\cos A} \Rightarrow \cos A = \frac{1}{\sec A}$

   $\cos^2 A + \sin^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A \Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$

   $\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A} \Rightarrow \sin A = \frac{\pm\sqrt{\sec^2 A - 1}}{\sec A}$

$\sin A = \frac{1}{\operatorname{cosec} A} \Rightarrow \operatorname{cosec} A = \frac{1}{\sin A}$ $\Rightarrow$ **cosec A** $= \frac{\pm \sec A}{\sqrt{\sec^2 A - 1}}$

$\sec^2 A - \tan^2 A = 1 \Rightarrow \tan^2 A = \sec^2 A + 1$

$\Rightarrow \tan A = \sqrt{\sec^2 A + 1}$

$\tan A = \frac{1}{\cot A} \Rightarrow \cot A = \frac{1}{\tan A}$ $\Rightarrow$ **cot A** $= \frac{1}{\sqrt{\sec^2 A + 1}}$

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**3. Evaluate:**

i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$ ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

i) $\frac{\sin^2 63^0 + \sin^2 27^0}{\cos^2 17^0 + \cos^2 73^0}$

$= \frac{\sin^2(90-27^0) + \sin^2 27^0}{\cos^2(90-73^0) + \cos^2 73^0} = \frac{\cos^2 27^0 + \sin^2 27^0}{\sin^2 73^0 + \cos^2 73^0} = \frac{1}{1} = 1$

ii) $\sin 25^0 \cos 65^0 + \cos 25^0 \sin 65^0$

$\sin 25° \cos 65° + \cos 25° \sin 65°$

$= \sin(90°-25°) \cos 65° + \cos(90°-65°) \sin 65°$

$= \cos 65° \cos 65° + \sin 65° \sin 65° = \cos^2 65° + \sin^2 65° = 1$

**4. Choose the correct option and justifyyour choice**

i) $9 \sec^2 A - 9 \tan^2 A$

A) 1 B) 9 C) 8 D) 0

$9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$ $[\because \sec^2 A - \tan^2 A = 1]$

**Ans: B) 9**

ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

A) 0 B) 1 C) 2 D) -1

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)$

$= \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \times \frac{\sin\theta + \cos\theta - 1}{\sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\cos\theta . \sin\theta} = \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta} = \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta . \sin\theta} = \frac{2\cos\theta\sin\theta}{\cos\theta . \sin\theta} = 2$

**Ans C) 2**

**iii) (secA + tanA) (1 - sinA) =**

**A) secA B) sinA C) cosecA D) cosA**

$(\sec A + \tan A)(1 - \sin A)$

$(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin A) = \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin A) = \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$

**Ans: D) cosA**

iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$

A) $\sec^2 A$ B) -1 C) $\cot^2 A$ D) $\tan^2 A$

$\frac{1 + \tan^2 A}{1 + \cot^2 A}$

$= \frac{1 + \frac{1}{\cot^2 A}}{1 + \cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A} \times \frac{1}{1 + \cot^2 A} = \frac{1}{\cot^2 A} = \tan^2 A$

**Ans: D) $\tan^2 A$**

**5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined.**

**i) $(\text{cosec } \theta - \cot \theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$**

L.H.S. = $(\text{cosec } \theta - \cot \theta)^2$

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec } \theta \cot \theta) = \left(\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}\right)$

$= \left(\frac{1+\cos^2\theta-2\cos\theta}{1-\cos^2\theta}\right) = \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$ RHS

**ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$**

L.H.S. = $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} = \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A} = \frac{2}{\cos A}$ $= 2\sec A$ = R.H.S.

**iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1+\sec\theta.\cos\theta$**

[Hint : Write the expression in terms of sin and cos ]

L.H.S. = $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^3\theta-\cos^3\theta}{\cos\theta.\sin\theta}\right]$

$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\cos\theta.\sin\theta}\right] = \left[\frac{(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\cos\theta.\sin\theta}\right]$

$= \left[\frac{1+\sin\theta\cos\theta}{\cos\theta.\sin\theta}\right] = \left[\frac{1}{\cos\theta.\sin\theta}+1\right]$ $= 1 + \sec\theta \text{ cosec } \theta$ = R.H.S.

**iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A} = 2\sec A$**

**[Hint: simplify LHS and RHS separately]**

L.H.S. = $\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \text{x} \frac{\cos A}{1} = \cos A + 1$

R.H.S. = $\frac{\sin^2 A}{1-\cos A} = \frac{(1+\cos A)(1-\cos A)}{1-\cos A} = \cos A + 1$

L.H.S. = R.H.S.

**v) Prove that $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \text{cosec } A + \cot A$ using the identity $\text{cosec}^2 A = 1 + \cot^2 A$**

L.H.S. = $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

$= \frac{\frac{\cos A-\sin A+1}{\sin A}}{\frac{\cos A+\sin A-1}{\sin A}} = \frac{\cot A-1+\text{cosec}A}{\cot A+1-\text{cosec}A}$ [ Divide both denominator and numerator by sin A]

$= \frac{\cot A-\text{cosec}^2 A+\cot^2 A+\text{cosec}A}{\cot A+1-\text{cosec}A}$ (using $\text{cosec}^2 A - \cot^2 A = 1$)

$= \frac{\cot A+\text{cosec}A - (\text{cosec}^2 A-\cot^2 A)}{\cot A+1-\text{cosec}A} = \frac{(\cot A+\text{cosec}A)(1-\text{cosec}A-\cot A)}{1-\text{cosec}A+\cot A}$

$= \cot A + \text{cosec } A$ = R.H.S.

**vi)** $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

$= \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$ = secA + tanA=RHS

**vii)** $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

L.H.S. $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta-1)}$

$= \frac{\sin\theta[1-2+2\cos^2\theta]}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta[2\cos^2\theta-1]}{\cos\theta(2\cos^2\theta-1)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ = R.H.S.

**viii)** $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S. $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right) + 1 + \tan^2 A + 1 + \cot^2 A$

$= 1 + 2 + 2 + 2 + \tan^2 A + \cot^2 A = 7 + \tan^2 A + \cot^2 A$ = R.H.S.

**ix)** $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

**[Hint: simplify LHS and RHS separately]**

L.H.S. $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right) = \cos A \sin A$

R.H.S. $= \frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\cos A . \sin A}}$

$= \frac{1}{\frac{1}{\cos A . \sin A}} = \cos A . \sin A$

L.H.S. = R.H.S.

**x)** $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

L.H.S. $= \frac{1+\tan^2 A}{1+\cot^2 A}$

$= \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{1+\tan^2 A}{\tan^2 A}} = \tan^2 A$

$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)^2 = (-\tan A)^2 = \tan^2 A$

# Summery:

1. In right angle triangle ABC. $\angle B = 90^0$

| | |
|---|---|
| **SinA** | $\frac{\textbf{Opposite side}}{\textbf{Hypotenuse}}$ |
| **CosA** | $\frac{\text{Adjacent side}}{\text{Hypotenuse}}$ |
| **Tan A** | $\frac{\text{Opposite side}}{\text{Adjacent}}$ |

| | | |
|---|---|---|
| **$\frac{1}{SinA}$** | **$\frac{Hypotenuse}{oppsite\ side}$** | **CosecA** |
| **$\frac{1}{CosA}$** | $\frac{Hypotenuse}{Adjacent\ side}$ | **SecA** |
| **$\frac{1}{Tan\ A}$** | $\frac{Adjacent\ side}{Opposite\ side}$ | **CotA** |

3. If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.
4. The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.
5. $\sin(90° - A) = \cos A$, $\cos(90° - A) = \sin A$;
   $\tan(90° - A) = \cot A$, $\cot(90° - A) = \tan A$; s
   ec $(90° - A) = \text{cosec } A$, $\text{cosec}(90° - A) = \sec A$
6. $\sec^2 A - \tan^2 A = 1$, $0^0 \le A < 90^0$
   $\text{cosec}^2 A = 1 + \cot^2 A$, $0^0 \le A < 90^0$
   $\sin^2 A + \cos^2 A = 1$,

# 12 Some Applications of Trigonometry

## Exercise 12.1

**A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see Fig. 9.11).**


Height of the pole BC

$\text{Sin}30^0 = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{20} \Rightarrow BC = 10m$

$\therefore$ Height BC = 10m

2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

Let BC is the broken part of the tree

$\therefore$ Total height of the tree = AB+BC

$\cos 30° = \frac{AC}{BC}$

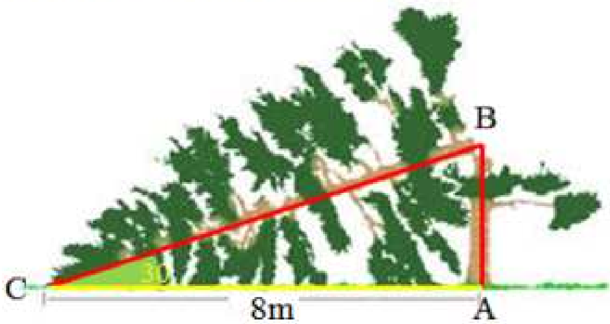
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{BC} \Rightarrow BC = \frac{16}{\sqrt{3}}$

$\tan 30° = \frac{AB}{AC}$

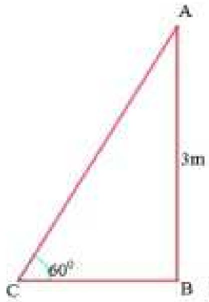
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8} \Rightarrow AB = \frac{8}{\sqrt{3}}$ m

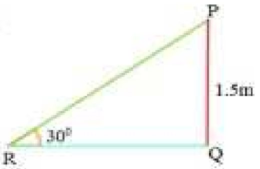
$\therefore$ Height of the tree

$= AB + BC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$m



3. **A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?**




Let the length of the side making inclination $60^0$ = AC and

Length of the slide making inclination 30° = PR

According to question,

In right angle triangle $\Delta ABC$,

$\sin 30° = \frac{PQ}{PR} \Rightarrow \frac{1}{2} = \frac{1.5}{PR} \Rightarrow PR = 3m$

In right angle triangle $\Delta PQR$,

$\sin 60° = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{AC} \Rightarrow AC = \frac{6}{\sqrt{3}} m = 2\sqrt{3}\ m$

$\therefore$ Length of the slides 3m and $2\sqrt{3}$ m .

4. **The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower**

Let height of the tower = AB

Distance from the foot of the tower to the point

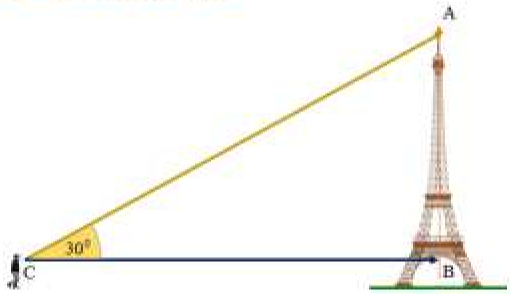
BC = 30m

In right angle triangle $\Delta ABC$ ,

$\tan 30^0 = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}m$




5. **A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.**

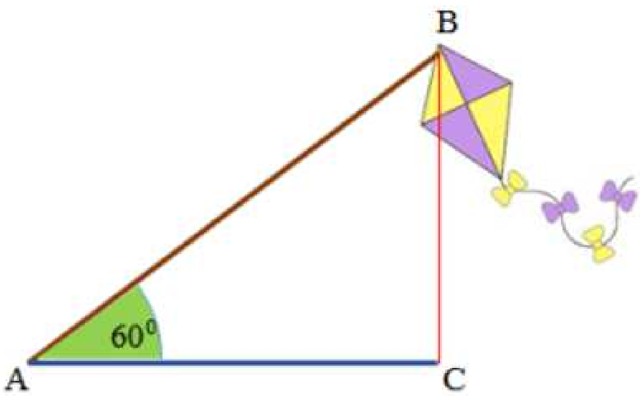
Height of the kite BC = 60m

Length of the tread = AB,

In right angle triangle $\Delta ABC$ ,

$\mathrm{Sin}60^0 = \frac{BC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AB}$

$\Rightarrow AB = \frac{120}{\sqrt{3}} = 40\sqrt{3}m$



13

# Statistics

## 13.2 Mean of Grouped data

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$ [ $i = 1$ to $n$ ]

**Example 1 : The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below. Find the mean of the marks obtained by the students**

| $x$ | $y$ |
|---|---|
| 10 | 1 |
| 20 | 1 |
| 36 | 3 |
| 40 | 4 |
| 50 | 3 |
| 56 | 2 |
| 60 | 4 |
| 70 | 4 |
| 72 | 1 |
| 80 | 1 |
| 88 | 2 |
| 92 | 3 |
| 95 | 1 |

=

| $x_i$ | $f_i$ | $x_i f_i$ |
|---|---|---|
| 10 | 1 | 10 |
| 20 | 1 | 20 |
| 36 | 3 | 108 |
| 40 | 4 | 160 |
| 50 | 3 | 150 |
| 56 | 2 | 112 |
| 60 | 4 | 240 |
| 70 | 4 | 280 |
| 72 | 1 | 72 |
| 80 | 1 | 80 |
| 88 | 2 | 176 |
| 92 | 3 | 276 |
| 95 | 1 | 96 |
| | $\sum f_i = 30$ | $\sum x_i f_i = 1779$ |

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$\frac{1779}{30} = 59.53$

## Direct Method to find average:

| C.I. | No.of students |
|---|---|
| 10–25 | 2 |
| 25–40 | 3 |
| 40–55 | 7 |
| 55–70 | 6 |
| 70–85 | 6 |
| 85–100 | 6 |

| Class Interval | $(f_i)$ | Mid-point $(x_i)$ | $f_i x_i$ |
|---|---|---|---|
| 10–25 | 2 | 17.5 | 35.0 |
| 25–40 | 3 | 32.5 | 97.5 |
| 40–55 | 7 | 47.5 | 332.5 |
| 55–70 | 6 | 62.5 | 375.0 |
| 70–85 | 6 | 77.5 | 465.0 |
| 85–100 | 6 | 92.5 | 555.0 |
| | $\sum f_i = 30$ | | $\sum f_i x_i = 1860$ |

Average $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1860}{30} = 62$

## Assumed Mean Method:

$d_i = x_i - a$ [ Here, a = 47.5 ]

| Class Interval | $(f_i)$ | Mid-point $(x_i)$ | $d_i = x_i - 47.5$ | $f_i d_i$ |
|---|---|---|---|---|
| 10–25 | 2 | 17.5 | −30 | −60 |
| 25–40 | 3 | 32.5 | −15 | −45 |
| 40–55 | 7 | 47.5 | 0 | 0 |
| 55–70 | 6 | 62.5 | 15 | 90 |
| 70–85 | 6 | 77.5 | 30 | 182 |
| 85–100 | 6 | 92.5 | 45 | 270 |
| | $\sum f_i = 30$ | | | $\sum f_i d_i = 435$ |

Average $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62$

## Exercise – 13.1

**1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house**

**Which method did you use for finding the mean, and why?**

a = 7 , h = 2

| No.of Plants | No.of Houses |
|---|---|
| 0–2 | 1 |
| 2–4 | 2 |
| 4–6 | 1 |
| 6–8 | 5 |
| 8–10 | 6 |
| 10–12 | 2 |
| 12–14 | 3 |

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 7$ | $u_i = \frac{x_i - 20}{2}$ | $f_i x_i$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|---|
| 0–2 | 1 | 1 | −6 | −3 | 1 | −6 | −3 |
| 2–4 | 2 | 3 | −4 | −2 | 6 | −8 | −4 |
| 4–6 | 1 | 5 | −2 | −1 | 5 | −2 | −1 |
| 6–8 | 5 | 7 | 0 | 0 | 35 | 0 | 0 |
| 8–10 | 6 | 9 | 2 | 1 | 54 | 12 | 6 |
| 10–12 | 2 | 11 | 4 | 2 | 22 | 8 | 4 |
| 12–14 | 3 | 13 | 6 | 3 | 39 | 18 | 9 |
| | $\sum f_i = 20$ | | | 0 | 162 | 22 | 11 |

From the above table $\sum f_i = 35$. $\sum f_i x_i = 162$. $\sum f_i d_i = 20$. $\sum f_i u_i = 11$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1620}{20} = 8.1$

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 7 + \frac{22}{20} = 7 + 1.1 = 8.1$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 7 + \frac{11}{20} \times 2 = 7 + 1.1 = 8.1$

**[You can use any method. Because of simple tabulation we can use direct method here]**

**2. Consider the following distribution of daily wages of 50 workers of a factory. Find the mean daily wages of the workers of the factory by using an appropriate method**

a = 75.5, h = 3

| Daily wages (Rs) | No.of workers |
|---|---|
| 100–120 | 12 |
| 120–140 | 14 |
| 140–160 | 8 |
| 160–180 | 6 |
| 180–200 | 10 |

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 150$ | $u_i = \frac{x_i - 150}{20}$ | $f_i x_i$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|---|
| 100–120 | 12 | 110 | −40 | −2 | 1320 | −480 | −24 |
| 120–140 | 14 | 130 | −20 | −1 | 1820 | −280 | −14 |
| 140–160 | 8 | 150 | 0 | 0 | 1200 | 0 | 0 |
| 160–180 | 6 | 170 | 20 | 1 | 1020 | 120 | 6 |
| 180–200 | 10 | 190 | 40 | 2 | 1900 | 400 | 20 |
| | 50 | | | | 7260 | −240 | −12 |

From the above table $\sum f_i = 50$. $\sum f_i x_i = 7260$. $\sum f_i d_i = -240$. $\sum f_i u_i = -12$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{7260}{50} = 145.2$

Average from assumed Mean Method $\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i} = 150 + \frac{-240}{50} = 150 - 4.8 = 145.2$

Average from step deviation method $\bar{x}$=a +$\frac{\sum f_i u_i}{\sum f_i}$ x h =150+ $\frac{-12}{50}$ x 20 =150 – 4.8 = 145.2

**[Can use any method. But Assumed mean method is more suitable here]**

**3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ` 18. Find the missing frequency f**

| Daily Pocket allowences(Rs) | 11–13 | 13–15 | 15–17 | 17–19 | 19–21 | 21–23 | 23–25 |
|---|---|---|---|---|---|---|---|
| No.of Children | 7 | 6 | 9 | 13 | f | 5 | 4 |

a = 18 , h = 2

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 18$ | $u_i = \frac{x_i - 18}{2}$ | $f_i x_i$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|---|
| 11–13 | 7 | 12 | −6 | −3 | 84 | −42 | −21 |
| 13–15 | 6 | 14 | −4 | −2 | 84 | −24 | −12 |
| 15–17 | 9 | 16 | −2 | −1 | 144 | −18 | −9 |
| 17–19 | 13 | 18 | 0 | 0 | 234 | 0 | 0 |
| 19–21 | f | 20 | 2 | 1 | 20f | 2f | 1f |
| 21–23 | 5 | 22 | 4 | 2 | 110 | 20 | 10 |
| 23–25 | 4 | 24 | 6 | 3 | 96 | 24 | 12 |
| | $\sum f_i$ =44+ f | | | | 752+20f | −40+2f | −20+ f |

From the above table $\sum f_i$ =44+ f. $\sum f_i x_i = 752 + 20f$. $\sum f_i d_i = -40 + 2f$. $\sum f_i u_i = -20 + f$

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{752+20f}{44+f} \Rightarrow 18(44 + f) = 752 + 20f$

$\Rightarrow 792 + 18f = 752 + 20f \Rightarrow 40 = 2f \Rightarrow f = 20$

Average from assumed Mean Method $\bar{x}$ =a + $\frac{\sum f_i d_i}{\sum f_i}$

$18 = 18 + \frac{-40+2f}{44+f} \Rightarrow 0 = (-40 + 2f) \Rightarrow 2f = 40 \Rightarrow f = 20$

Average from step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i}$ x h

$\Rightarrow 18 = 18 + \frac{-20+f}{44+f}$ x 20 $\Rightarrow -20 + f = 0 \Rightarrow f = 20$

[Wecan use any method here]

**4. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method**

| No.of Heart beats/Minute | 65–68 | 68–71 | 71–74 | 74–77 | 77–80 | 80–83 | 83–86 |
|---|---|---|---|---|---|---|---|
| No.of women | 2 | 4 | 3 | 8 | 7 | 4 | 2 |

a = 75.5 , h = 3

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 75.5$ | $u_i = \frac{x_i - 75.5}{3}$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|
| 65–68 | 2 | 66.5 | −9 | −3 | −18 | −6 |
| 68–71 | 4 | 69.5 | −6 | −2 | −24 | −8 |
| 71–74 | 3 | 72.5 | −3 | −1 | −9 | −3 |
| 74–77 | 8 | 75.5 | 0 | 0 | 0 | 0 |
| 77–80 | 7 | 78.5 | 3 | 1 | 21 | 7 |
| 80–83 | 4 | 81.5 | 6 | 2 | 24 | 8 |
| 83–86 | 2 | 84.5 | 9 | 3 | 18 | 6 |
| | $\sum f_i = 30$ | | | | 12 | 4 |

From the above table $\sum f_i = 30$, $\sum f_i d_i = 12$, $\sum f_i u_i = 4$

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} = 75.5 + \frac{12}{30} = 75.5 + 0.4 = 75.9$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 75.5 + \frac{4}{30} \times 3 = 75.5 + 0.4 = 75.9$

[Direct method is not suitable here]

**5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.**

| No.of Mangoes | 50–52 | 53–55 | 56–58 | 59–61 | 62–64 |
|---|---|---|---|---|---|
| No.of boxes | 15 | 110 | 135 | 115 | 25 |

**Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?**

a = 57 , h = 3

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 150$ | $u_i = \frac{x_i - 75.5}{3}$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|
| 50–52 | 15 | 51 | −6 | −2 | −90 | −30 |
| 53–55 | 110 | 54 | −3 | −1 | −330 | −110 |
| 56–58 | 135 | 57 | 0 | 0 | 0 | 0 |
| 59–61 | 115 | 60 | 3 | 1 | 345 | 115 |
| 62–64 | 25 | 63 | 6 | 2 | 150 | 50 |
| | $\sum f_i = 400$ | | | | 75 | 25 |

Average from assumed Mean Method $\bar{x} = a + \frac{\sum d_i x_i}{\sum f_i}$

$= 57 + \frac{75}{400} = 57 + 0.1875 = 57.1875 \approx 57.19$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 57 + \frac{25}{400} \times 3 = 57 + 0.1875 = 57.1875 \approx 57.19$

Here, Assumed mean method is more suitable

**6. The table below shows the daily expenditure on food of 25 households in a locality**

| Daily expenditure(Rs) | 100–150 | 150–200 | 200–250 | 250–300 | 300–350 |
|---|---|---|---|---|---|
| No.of households | 4 | 5 | 12 | 2 | 2 |

**Find the mean daily expenditure on food by a suitable method.**

a = 225 , h = 50

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 150$ | $u_i = \frac{x_i - 75.5}{3}$ | $f_i d_i$ | $f_i u_i$ |
|---|---|---|---|---|---|---|
| 100–150 | 4 | 125 | –100 | –2 | –400 | –8 |
| 150–200 | 5 | 175 | –50 | –1 | –250 | –5 |
| 200–250 | 12 | 225 | 0 | 0 | 0 | 0 |
| 250–300 | 2 | 275 | 50 | 1 | 100 | 2 |
| 300–350 | 2 | 325 | 100 | 2 | 200 | 4 |
| | $\sum f_i = 25$ | | | | –350 | –7 |

Average from assumed Mean Method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$

$= 225 + \frac{-350}{25} = 225 - 14 = 211$

Average from step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 225 + \frac{-7}{25} \times 50 = 225 - 14 = 211$

For this problem step deviation method is more suitable

7. **To find out the concentration of $SO_2$ in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below**
**Find the mean concentration of $SO_2$ in the air**

| Concentration of $SO_2$ | Freequency |
|---|---|
| 0.00 – 0.04 | 4 |
| 0.04 – 0.08 | 9 |
| 0.08 – 0.12 | 9 |
| 0.12 – 0.16 | 2 |
| 0.16 – 0.20 | 4 |
| 0.20 – 0.24 | 2 |

| C.I. | $f_i$ | $x_i$ | $f_i x_i$ |
|---|---|---|---|
| 0.00 – 0.04 | 4 | 0.02 | 0.08 |
| 0.04 – 0.08 | 9 | 0.06 | 0.54 |
| 0.08 – 0.12 | 9 | 0.10 | 0.90 |
| 0.12 – 0.16 | 2 | 0.14 | 0.28 |
| 0.16 – 0.20 | 4 | 0.18 | 0.72 |
| 0.20 – 0.24 | 2 | 0.22 | 0.44 |
| | $\sum f_i = 30$ | | 2.96 |

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2.96}{30} = 0.099$ppm

**The mean concentration of $SO_2$ in the air = 0.099ppm**

8. **A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent**

| No.of days | No.of students |
|---|---|
| 0–6 | 11 |
| 6–10 | 10 |
| 10–14 | 7 |
| 14–20 | 4 |
| 20–28 | 4 |
| 28–38 | 3 |
| 38–40 | 1 |

| C.I. | $f_i$ | $x_i$ | $f_i x_i$ |
|---|---|---|---|
| 0–6 | 11 | 3 | 33 |
| 6–10 | 10 | 8 | 80 |
| 10–14 | 7 | 12 | 84 |
| 14–20 | 4 | 17 | 68 |
| 20–28 | 4 | 24 | 96 |
| 28–38 | 3 | 33 | 99 |
| 38–40 | 1 | 39 | 39 |
| | $\sum f_i = 40$ | | 499 |

From the above table $\sum f_i = 40$, $\sum f_i x_i = 499$,

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{499}{40} = 12.475$

9. **The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.**

| Literacy rate( %) | No.of cities |
|---|---|
| 45–55 | 3 |
| 55–65 | 10 |
| 65–75 | 11 |
| 75–85 | 8 |
| 85–95 | 3 |

| C.I. | $f_i$ | $x_i$ | $f_i x_i$ | $d_i = x_i - 70$ | $f_i d_i$ |
|---|---|---|---|---|---|
| 45–55 | 3 | 50 | 150 | −20 | −60 |
| 55–65 | 10 | 60 | 600 | −10 | −100 |
| 65–75 | 11 | 70 | 770 | 0 | 0 |
| 75–85 | 8 | 80 | 640 | 10 | 80 |
| 85–95 | 3 | 90 | 270 | 20 | 60 |
| | $\sum f_i$ =35 | | 2430 | 0 | −20 |

From the above table $\sum f_i = 35$, $\sum f_i x_i = 2430$.

Average from Direct Method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2430}{35} = 69.43$

Average from Assumed mean method $\bar{x} = a + \frac{\sum d_i x_i}{\sum f_i} = 70 + \frac{-20}{35} = 60.43$

## 13.3 Mode of Grouped Data

**A mode is that value among the observations which occurs most often, that is, the value of the observation having the maximum frequency**

**Example: 4 The wickets taken by a bowler in 10 cricket matches are as follows:**

**2 6 4 5 0 2 1 3 2 3**

**Find the mode of the data**

| No.of wickets | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| No.of matches | 1 | 1 | 3 | 2 | 1 | 1 | 1 |

Clearly, 2 is the number of wickets taken by the bowler in the maximum number (i.e., 3) of matches. So, the mode of this data is 2

**Mode** $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$L$ = **lower limit of the modal class**
$h$ = **size of the class interval (assuming all class sizes to be equal),**
$f_1$ = **frequency of the modal class,**
$f_0$ = **frequency of the class preceding the modal class,**
$f_2$ = **frequency of the class succeeding the modal class**

**Example 5 : A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household**

| Family size | 1 - 3 | 3 - 5 | 5 - 7 | 7 - 9 | 9 - 11 |
|---|---|---|---|---|---|
| No.of families | 7 | 8 | 2 | 2 | 1 |

**Find the mode of this data**

Here the maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5

modal class = 3 – 5, lower limit (1) of modal class = 3, class size (h) = 2

frequency ( $f_1$ ) of the modal class = 8,

frequency ( $f_0$ ) of class preceding the modal class $f_0 = 7$

frequency ( $f_2$ ) of class succeeding the modal class $f_2 = 2$

Now substitute the values in the formula:

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h = 3 + \left[\frac{8-7}{2(8)-7-2}\right] \times 2$

$= 3 + \left[\frac{1}{16-9}\right] \times 2 = 3 + \frac{2}{7} = 3.286$

∴ **Therefore, the mode of the data above is 3.286.**

**Example 6 : The marks distribution of 30 students in a mathematics examination are given in Table 13.3 of Example 1. Find the mode of this data. Also compare and interpret the mode and the mean.**

| Class Intervals | 10–25 | 25–40 | 40–55 | 55–70 | 70–85 | 85–100 |
|---|---|---|---|---|---|---|
| No.of students | 2 | 3 | 7 | 6 | 6 | 6 |

Refer the table 13.3 of example . Maximum students are in the class interval 40-45, it is the modal class,

$\therefore$ $l = 40$, h = 15, $f_1 = 7$ , $f_0 = 3$, $f_2 = 6$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $40 + \left[\frac{7-3}{2(7)-3-6}\right] \times 15$ = $40 + \left[\frac{4}{14-9}\right] \times 15$

= $40 + \frac{4}{5} \times 15$ = 40 + 12

$\therefore$ The mode of the given data is 52

## Exercise 13.2

1. **The following table shows the ages of the patients admitted in a hospital during a year:**

| Age(in years) | 5 – 15 | 15–25 | 25–35 | 35–45 | 45–55 | 55–65 |
|---|---|---|---|---|---|---|
| No.of patients | 6 | 11 | 21 | 23 | 14 | 5 |

**Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.**

Maximum number of patients =23

Therefore 35-45 is the modal class interval

$\therefore$ $l = 35$, h = 10, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode = $35 + \left[\frac{23-21}{2(23)-21-14}\right] \times 10$= $35 + \left[\frac{2}{46-35}\right] \times 10$ = $35 + \frac{2}{11} \times 10$ = 35 + 1.81

$\therefore$ The mode of the above data is 36.81

(a = 30 , h = 10)

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 70$ | $u_i = \frac{x_i - 30}{10}$ | $f_i u_i$ |
|---|---|---|---|---|---|
| 5–15 | 6 | 10 | –20 | –2 | –12 |
| 15–25 | 11 | 20 | –10 | –1 | –11 |
| 25–35 | 21 | 30 | 0 | 0 | 0 |
| 35–45 | 23 | 40 | 10 | 1 | 23 |
| 45–55 | 14 | 50 | 20 | 2 | 28 |
| 55–65 | 5 | 60 | 30 | 3 | 15 |
| | $\sum f_i$ =80 | | | | 43 |

By step deviation method $\bar{x}$ = a + $\frac{\sum f_i u_i}{\sum f_i} \times$ h = 30 + $\frac{43}{80} \times 10$ = 30 + 5.375 = 35.375

So, we conclude that maximum number of patients admitted in the hospital are of the age 36.81 years(Approx) whereas the average age of the patient admitted in the hospital is 35.375years

2. **The following data gives the information on the observed lifetimes (in hours) of 225 electrical components**

| Life time(in hours) | 0 – 20 | 20–40 | 40–60 | 60–80 | 80–100 | 100–120 |
|---|---|---|---|---|---|---|
| Freequency | 10 | 35 | 52 | 61 | 38 | 29 |

**Determine the modal lifetimes of the components**

Maximum frequency $=61$

It is in the class interval $60-80$. So, 60 - 80 is the modal class interval.

$\therefore l=60$, $h=20$, $f_1=61$, $f_0=52$, $f_2=38$

Mode $= l+\left[\frac{f_1-f_0}{2f_1-f_0-f_2}\right] \times h$

Mode $= 60+\left[\frac{61-52}{2(61)-52-38}\right] \times 20 = 60+\left[\frac{9}{122-90}\right] \times 20$

$= 60+\frac{9}{32} \times 20 = 60+5.625 = 65.625$

$\therefore$ The mode of the above given data = 65.625

**3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.**

| Expenditure (in Rs) | No.of families |
|---|---|
| 1000 – 1500 | 24 |
| 1500 – 2000 | 40 |
| 2000 – 2500 | 33 |
| 2500 – 3000 | 28 |
| 3000 – 3500 | 30 |
| 3500 – 4000 | 22 |
| 4000 – 4500 | 16 |
| 4500 – 5000 | 7 |

Maximum frequency $= 40$

Therefore the modal class interval is (1500 – 2000)

$\therefore l=1500$, $h=500$, $f_1=40$, $f_0=24$, $f_2=33$

Mode $= l+\left[\frac{f_1-f_0}{2f_1-f_0-f_2}\right] \times h$

Mode $= 1500+\left[\frac{40-24}{2(40)-24-33}\right] \times 500 = 1500+\left[\frac{16}{80-57}\right] \times 500$

$= 1500+\frac{16}{23} \times 500 = 1500+347.83 = 1847.83$

$\therefore$ The mode of the given data = 1847.83

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 2750$ | $u_i = \frac{x_i - 2750}{500}$ | $f_i u_i$ |
|---|---|---|---|---|---|
| 1000 – 1500 | 24 | 1250 | –1500 | –3 | –72 |
| 1500 – 2000 | 40 | 1750 | –1000 | –2 | –80 |
| 2000 – 2500 | 33 | 2250 | –500 | –1 | –33 |
| 2500 – 3000 | 28 | 2750 | 0 | 0 | 0 |
| 3000 – 3500 | 30 | 3250 | 500 | 1 | 30 |
| 3500 – 4000 | 22 | 3750 | 1000 | 2 | 44 |
| 4000 – 4500 | 16 | 4250 | 1500 | 3 | 48 |
| 4500 – 5000 | 7 | 4750 | 2000 | 4 | 28 |
| | $\sum f_i = 200$ | | | | -35 |

By step deviation method $\bar{x} = a+\frac{\sum f_i u_i}{\sum f_i} \times h$

$= 2750+\frac{-35}{200} \times 500 = 2750-87.5 = 2662.5$

**4. The following distribution gives the state-wise teacher - student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures**

| No.of students per teacher | No.of state/U.Ts |
|---|---|
| 15 – 20 | 3 |
| 20 – 25 | 8 |
| 25 – 30 | 9 |
| 30 – 35 | 10 |
| 35 – 40 | 3 |
| 40 – 45 | 0 |
| 45 – 50 | 0 |
| 50 – 55 | 2 |

Maximum freeqency = 10, of the class interval 30 - 35

Therefore 30 – 35 is the modal class interval

$\therefore l=30$, $h=5$, $f_1=10$, $f_0=9$, $f_2=3$

Mode $= l+\left[\frac{f_1-f_0}{2f_1-f_0-f_2}\right] \times h$

Mode $= 30+\left[\frac{10-9}{2(10)-9-3}\right] \times 5$

$= 30+\left[\frac{1}{20-12}\right] \times 5$

$= 30+\frac{1}{8} \times 5 = 30+0.625 = 30.625$

$\therefore$ The mode of the above data is 30.625

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 32.5$ | $u_i = \frac{x_i - 32.5}{5}$ | $f_i u_i$ |
|---|---|---|---|---|---|
| 15 – 20 | 3 | 17.5 | –15 | –3 | –9 |
| 20 – 25 | 8 | 22.5 | –10 | –2 | –16 |
| 25 – 30 | 9 | 27.5 | –5 | –1 | –9 |
| 30 – 35 | 10 | 32.5 | 0 | 0 | 0 |
| 35 – 40 | 3 | 37.5 | 5 | 1 | 3 |
| 40 – 45 | 0 | 42.5 | 10 | 2 | 0 |
| 45 – 50 | 0 | 47.5 | 15 | 3 | 0 |
| 50 – 55 | 2 | 52.5 | 20 | 4 | 8 |
| | $\sum f_i = 35$ | | | | –23 |

By step deviation Method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 32.5 + \frac{-23}{35} \times 5 = 32.5 - 3.29 = 29.21$

The students – teacher ratio is 30.625 and average ratio is 29.21

5. **The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches**
**Find the mode of the data.**

| Runs scored | No.of Batsman |
|---|---|
| 3000 - 4000 | 4 |
| 4000 - 5000 | 18 |
| 5000 - 6000 | 9 |
| 6000 - 7000 | 7 |
| 7000 - 8000 | 6 |
| 8000 - 9000 | 3 |
| 9000 - 10000 | 1 |
| 10000 - 11000 | 1 |

Therefore 4000 -5000 is the modal class interval

$\therefore l = 4000,\ h = 1000,\ f_1 = 18, f_0 = 4, f_2 = 9$

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode $= 4000 + \left[\frac{18-4}{2(18)-4-9}\right] \times 1000 =$

$4000 + \left[\frac{14}{36-13}\right] \times 1000$

$= 4000 + \frac{14}{23} \times 1000 = 4000 + 608.7 = 4608.7$

$\therefore$ The mode of the above data is 4608.7

6. **A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data**

| No.of cars | 0 – 10 | 10–20 | 20–30 | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 |
|---|---|---|---|---|---|---|---|---|
| Freequency | 7 | 14 | 13 | 12 | 20 | 11 | 15 | 8 |

Maximum freequency = 20. It is in the class interval 40 - 50

Therefore 40 – 50 is the modal class interval

$\therefore l = 40,\ h = 10,\ f_1 = 20,\ f_0 = 12,\ f_2 = 11$

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

Mode $= 40 + \left[\frac{20-12}{2(20)-12-11}\right] \times 10 = 40 + \left[\frac{8}{40-23}\right] \times 10$

$= 40 + \frac{8}{17} \times 10 = 40 + 4.71 = 44.71$

$\therefore$ Mode of the given data 44.71

## 13.4 Median of Grouped Data

the median is a measure of central tendency which gives the value of the middle-most observation in the data. Recall that for finding the median of ungrouped data, we first arrange the data values of the observations in ascending order, then, if n is odd, then the meadian is $\left(\frac{n+1}{2}\right)$th observation and if n is an even, then the dedian is the average of $\left(\frac{n}{2}\right)$ and $\left(\frac{n}{2} + 1\right)$ th observation.

After finding the median class, we use the following formula for calculating the median.

## Median of Grouped Data

**Median** = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$l$ = lower limit of median class,
**n** = number of observations
**cf** = cumulative frequency of class preceding the median class,.
$f$ = frequency of median class
**h** = class size (assuming class size to be equal).

**Example 7 : A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained**
**Find the median height.**

| Heights (in cm) | No.of Girls |
|---|---|
| Less than 140 | 4 |
| Less than 145 | 11 |
| Less than 150 | 29 |
| Less than 155 | 40 |
| Less than 160 | 46 |
| Less than 165 | 51 |

| C.I. | f | cf |
|---|---|---|
| Less than 140 | 4 | 4 |
| 140 – 145 | 7 | 11 |
| 145 – 150 | 18 | 29 |
| 150 – 155 | 11 | 40 |
| 155 – 160 | 6 | 46 |
| 160 – 165 | 5 | 51 |

Now, n = 51, $\therefore \frac{n}{2} = 25.5$ It is in the class interval 145 – 150

$\therefore l$ (lower limit) = 145, cf = 11. $f$ = 18, h = 5

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

Median = $145 + \left[\frac{25.5 - 11}{18}\right] \times 5 = 145 + \left[\frac{72.5}{18}\right] = 149.03$

Therefore median of the given data is 149.03

# Exercise 13.3

1. **The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.**

| ಮಾಸಿಕ ಬಳಕೆ (ಯೂನಿಟ್‌ಗಳಲ್ಲಿ) | ಗ್ರಾಹಕರ ಸಂಖ್ಯೆ | ಸಂಚಿತ ಆವೃತ್ತಿ |
|---|---|---|
| 65 – 85 | 4 | 4 |
| 85 – 105 | 5 | 9 |
| 105 – 125 | 13 | 22 |
| 125 – 145 | 20 | 42 |
| 145 – 165 | 14 | 56 |
| 165 – 185 | 8 | 64 |
| 185 – 205 | 4 | 68 |

Now, n = 68, $\therefore \frac{n}{2} = 34$ It is in the class interval 125 - 145.

$\therefore l = 125$, cf = 22, $f = 20$, h = 20

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

median = $125 + \left[\frac{34 - 22}{20}\right] \times 20$

$= 125 + \left[\frac{12}{20}\right] \times 20 = 125 + 12 = 137$units

Therefore median is 137units

**Average:**

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 135$ | $u_i = \frac{x_i - 135}{20}$ | $f_i d_i$ |
|---|---|---|---|---|---|
| 65 – 85 | 4 | 75 | −60 | −3 | −12 |
| 85 – 105 | 5 | 95 | −40 | −2 | −10 |
| 105 – 125 | 13 | 115 | -20 | -1 | -13 |
| 125 – 145 | 20 | 135 | 0 | 0 | 0 |
| 145 – 165 | 14 | 155 | 20 | 1 | 14 |
| 165 – 185 | 8 | 175 | 40 | 2 | 16 |
| 185 – 205 | 4 | 195 | 60 | 3 | 12 |
| | $\sum f_i = 68$ | | | | 7 |

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 135 + \frac{7}{68} \times 20 = 135 + 2.1 = 137.05$

**Mode:** maximum freequency = 20. which lies in the class interval 125 – 145.

Therefore 125-145 is the modal class interval

$\therefore l = 125$, h = 20, $f_1 = 20$, $f_0 = 13$, $f_2 = 14$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 125 + \left[\frac{20 - 13}{2(20) - 13 - 14}\right] \times 20 = 125 + \left[\frac{7}{40 - 27}\right] \times 20 = 125 + \frac{7}{13} \times 20 = 125 + 10.77 = 135.77$

$\therefore$ Therefore mode of the given data is 135.77

**So, we conclude that three measures are approximately same.**

2. **If the median of the distribution given below is 28.5, find the values of x and y**

| Class interval | Freequency | cf |
|---|---|---|
| 0 - 10 | 5 | 5 |
| 10 - 20 | x | 5+x |
| 20 - 30 | 20 | 25+x |
| 30 - 40 | 15 | 40+x |
| 40 - 50 | y | 40+x+y |
| 50 - 60 | 5 | 45+x+y |
| Total | 60 | |

Total freequency = 45 + x + y $\Rightarrow 60 = 45 + x + y$

$\Rightarrow x + y = 15$ --------------------------(1)

Now, n = 60,

$\therefore \frac{n}{2} = 30$ this is in the class interval 20 – 30

$\therefore l = 20$ .$cf = 5 + x$, $f = 20$. h = 10

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$28.5 = 20 + \left[\frac{30 - (5 + x)}{20}\right] \times 10$

$8.5 \times 20 = (30 - 5 - x)10 \Rightarrow 170 = 250 - 10x \Rightarrow 10x = 80 \Rightarrow x = 8$

Substitute x = 8 in equation (1),

$\Rightarrow 8 + y = 15 \Rightarrow y = 7$

Therefore x = 8 and y = 7

**3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.**

| Age(in years) | Cumulative freequency |
|---|---|
| Below 20 | 2 |
| Below 25 | 6 |
| Below 30 | 24 |
| Below 35 | 45 |
| Below 40 | 78 |
| Below 45 | 89 |
| Below 50 | 92 |
| Below 55 | 98 |
| Below 60 | 100 |

| C.I. | f | cf |
|---|---|---|
| 15–20 | 2 | 2 |
| 20–25 | 4 | 6 |
| 25–30 | 18 | 24 |
| 30–35 | 21 | 45 |
| 35–40 | 33 | 78 |
| 40–45 | 11 | 89 |
| 45–50 | 3 | 92 |
| 50–55 | 6 | 98 |
| 55–60 | 2 | 100 |

Totoa frequency = 100

Now, n = 100, $\therefore \frac{n}{2} = 50$ This is in the class interval 35 - 40

So, $l = 35$, cf = 45, $f$ = 33, h = 5

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$= 35 + \left[\frac{50 - 45}{33}\right] \times 5 = 35 + \left[\frac{5}{33}\right] \times 5 = 35 + \frac{25}{33} = 35 + 0.76$$

Median = 35.76

**4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table . Find the median length of the leaves.**

(Hint : The data needs to be converted to continuous classes for finding the median, since th e formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, ...., 171.5 - 180.5.]

| Length(in mm) | No.of Leaves |
|---|---|
| 118 - 126 | 3 |
| 127 - 135 | 5 |
| 136 - 144 | 9 |
| 145 - 153 | 12 |
| 154 - 162 | 5 |
| 163 - 171 | 4 |
| 172 - 180 | 2 |

| C.I. | $f$ | $cf$ |
|---|---|---|
| 117.5 – 126.5 | 3 | 3 |
| 126.5 – 135.5 | 5 | 8 |
| 135.5 – 144.5 | 9 | 17 |
| 144.5 – 153.5 | 12 | 29 |
| 153.5 – 162.5 | 5 | 34 |
| 162.5 – 171.5 | 4 | 38 |
| 171.5 – 180.5 | 2 | 40 |

Now, n = 40, $\therefore \frac{n}{2} = 20$ This is in the class interval 144.5 – 153.5

So, $l$ = 144.5, cf = 17, $f$ = 12, h = 9

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$= 144.5 + \left[\frac{20 - 17}{12}\right] \times 9 = 144.5 + \left[\frac{3}{12}\right] \times 9 = 144.5 + \frac{27}{12} = 144.5 + 2.25 = 146.75\text{mm}$$

**5. The following table gives the distribution of the life time of 400 neon lamps . Find the median life time of a lamp.**

| Life time in hours | No.of Lamps |
|---|---|
| 1500-2000 | 14 |
| 2000-2500 | 56 |
| 2500-3000 | 60 |
| 3000-3500 | 86 |
| 3500-4000 | 74 |
| 4000-4500 | 62 |
| 4500-5000 | 48 |

| C.I. | $f$ | $cf$ |
|---|---|---|
| 1500-2000 | 14 | 14 |
| 2000-2500 | 56 | 70 |
| 2500-3000 | 60 | 130 |
| 3000-3500 | 86 | 216 |
| 3500-4000 | 74 | 290 |
| 4000-4500 | 62 | 352 |
| 4500-5000 | 48 | 400 |

Total freequencies = 400

Now, n = 400, $\therefore \frac{n}{2} = 200$ this is in the class interval 3000 – 3500

Now, $l$ = 3000, cf = 130, $f$ = 86, h = 500

Median= $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h = 3000 + \left[\frac{200 - 130}{86}\right] \times 500 = 3000 + \left[\frac{70}{86}\right] \times 500$

= 3000 + 406.98 = 3406.98

**6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows**

| No.of Letters | 1 - 4 | 4 - 7 | 7 - 10 | 10 - 13 | 13 - 16 | 16 - 19 |
|---|---|---|---|---|---|---|
| No.of surnames | 6 | 30 | 40 | 16 | 4 | 4 |

**Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames**

| C.I. | $f$ | $cf$ |
|---|---|---|
| 1-4 | 6 | 6 |
| 4-7 | 30 | 36 |
| 7-10 | 40 | 76 |
| 10-13 | 16 | 92 |
| 13-16 | 4 | 96 |
| 16-19 | 4 | 100 |

Total freequencies = 100

Now, n = 100, $\therefore \frac{n}{2} = 50$ this is in the class interval 7 – 10

So, $l$ = 7, cf = 36, $f$ = 40, h = 3

Median = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$7 + \left[\frac{50 - 36}{40}\right] \times 3 = 7 + \left[\frac{14}{40}\right] \times 3 = 7 + 1.05 = 8.05$

**To find the average:**

[a = 8.5, h = 3]

| C.I. | $f_i$ | $x_i$ | $d_i = x_i - 135$ | $u_i = \frac{x_i - 135}{20}$ | $f_i d_i$ |
|---|---|---|---|---|---|
| 1-4 | 6 | 2.5 | -6 | -2 | -12 |
| 4-7 | 30 | 5.5 | -3 | -1 | -30 |
| 7-10 | 40 | 8.5 | 0 | 0 | 0 |
| 10-13 | 16 | 11.5 | 3 | 1 | 16 |
| 13-16 | 4 | 14.5 | 6 | 2 | 8 |
| 16-19 | 4 | 17.5 | 9 | 3 | 12 |
| | $\sum f_i$ =100 | | | | -6 |

By step deviation method $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 8.5 + \frac{-6}{100} \times 3 = 8.5 - 0.18 = 8.32$

**To find the mode:**

Maximum freequency = 40. Which is in the class interval 7 - 10

Therefore the modal class interval is 7 - 10

$\therefore l = 7$, h = 3, $f_1 = 30$, $f_0 = 30$, $f_2 = 16$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 7 + \left[\frac{40 - 30}{2(40) - 30 - 16}\right] \times 3 = 7 + \left[\frac{10}{80 - 46}\right] \times 3 = 7 + \frac{10}{34} \times 3 = 7 + 0.88 = 7.88$

$\therefore$ The mode of the given data is 7.88

7. **The distribution below gives the weights of 30 students of a class. Find the median weight of the students**

| Weight in Kgs | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|---|---|---|---|---|---|---|---|
| No.of students | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

| C.I. | f | cf |
|---|---|---|
| 40-45 | 2 | 2 |
| 45-50 | 3 | 5 |
| 50-55 | 8 | 13 |
| 55-60 | 6 | 19 |
| 60-65 | 6 | 25 |
| 65-70 | 3 | 28 |
| 70-75 | 2 | 30 |

Total freequencies = 30

Now, n = 30, $\therefore \frac{n}{2} = 15$

which is in the class interval 55 – 60

So, $l = 55$, cf = 13, $f = 6$, h = 5

Meadian $= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

$= 55 + \left[\frac{15 - 13}{6}\right] \times 5 = 7 + \left[\frac{2}{6}\right] \times 5$

Median= 55 + 1.67 = 56.67kg

## 13.5 Graphical Representation of Cumulative Frequency Distribution

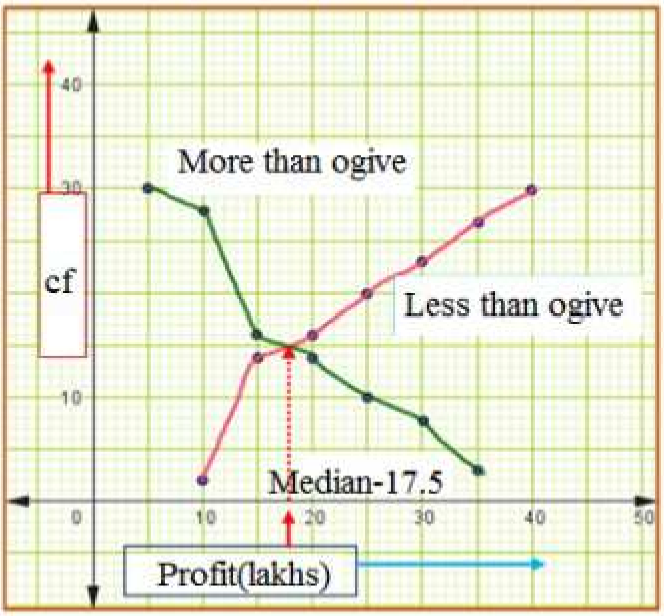
**Example 9 : The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution. draw its ogive. Hence obtain the median profit.**

| Profit (in lakhs) | No.of shopes(f) |
|---|---|
| More than or equal to 5 | 30 |
| More than or equal to 10 | 28 |
| More than or equal to 15 | 16 |
| More than or equal to 20 | 14 |
| More than or equal to 25 | 10 |
| More than or equal to 30 | 7 |
| More than or equal to 35 | 3 |

We first draw the coordinate axes, with lower limits of the profit along the horizontal axis, and the cumulative frequency along the vertical axes. Then, we plot the points (5, 30), (10, 28), (15, 16), (20, 14), (25, 10), (30, 7) and (35, 3). We join these points with a smooth curve to get the 'more than' ogive, as shown in Fig.

Now, let us obtain the classes, their frequencies and the cumulative frequency from the table above

| C.I. | 5-10 | 10-15 | 15-20 | 20-25 | 25-30 | 30-35 | 35-40 |
|---|---|---|---|---|---|---|---|
| $f$ | 2 | 12 | 2 | 4 | 3 | 4 | 3 |
| $cf$ | 2 | 14 | 16 | 20 | 23 | 27 | 30 |



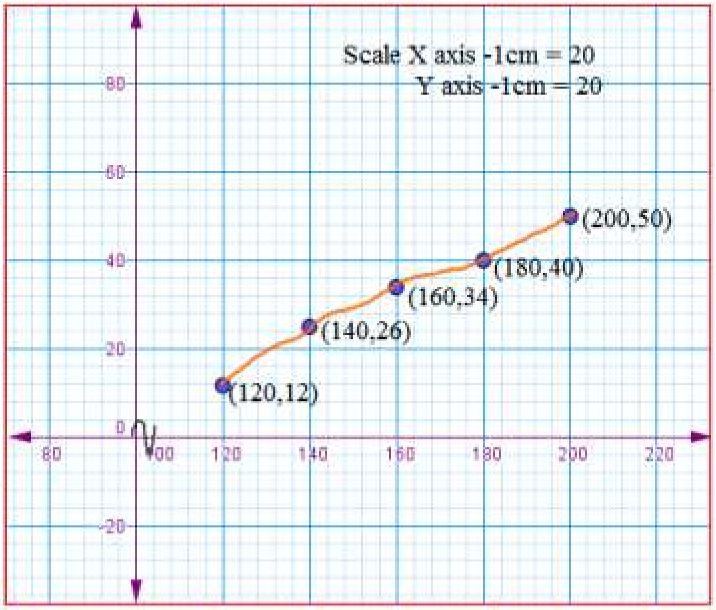
# Exercise 13.4

1. The following table gives the distribution of the life time of 400 neon lamps :

| Daily income(Rs) | 100-120 | 120-140 | 140-160 | 160-180 | 180-200 |
|---|---|---|---|---|---|
| No.of workers | 12 | 14 | 8 | 6 | 10 |

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

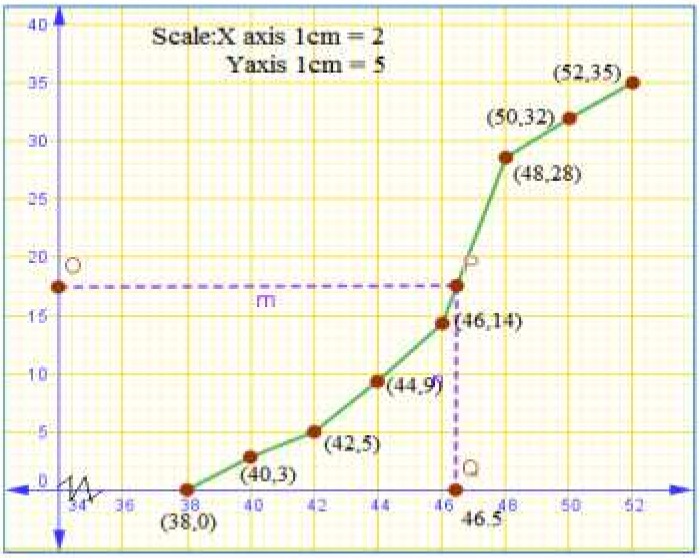
| Daily income (Rs) | No.of workers | *cf* |
|---|---|---|
| 100–120 | 12 | 12 |
| 120–140 | 14 | 26 |
| 140–160 | 8 | 34 |
| 160–180 | 6 | 40 |
| 180–200 | 10 | 50 |




2. During the medical check-up of 35 students of a class, their weights were recorded as follows:
Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula

| Weight (in kgs) | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 |
|---|---|---|---|---|---|---|---|---|
| No of students | 0 | 3 | 5 | 9 | 14 | 28 | 32 | 35 |

| Weight (in kgs) | No of students |
|---|---|
| Less than 38 | 0 |
| Less than 40 | 3 |
| Less than 42 | 5 |
| Less than 44 | 9 |
| Less than 46 | 14 |
| Less than 48 | 28 |
| Less than 50 | 32 |
| Less than 52 | 35 |

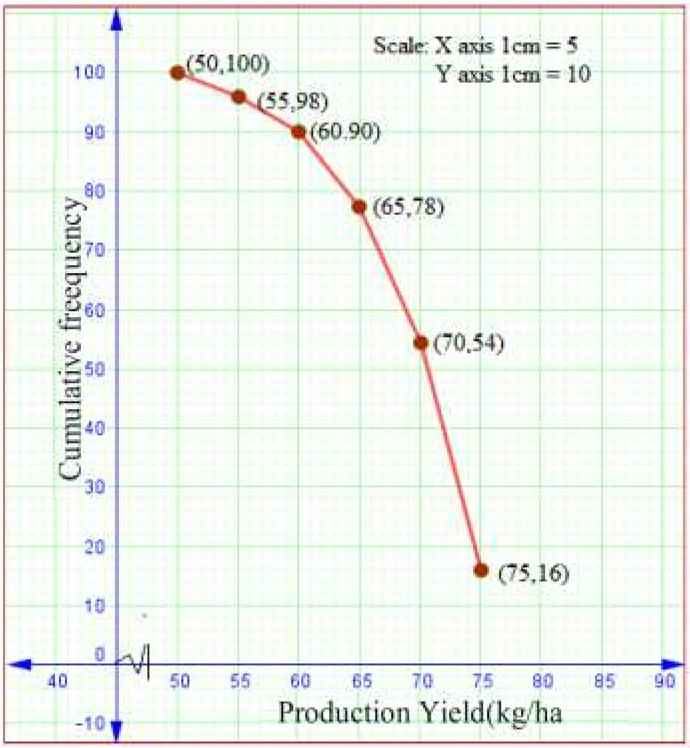


3. **The following table gives production yield per hectare of wheat of 100 farms of a village.**

| Production Yield(kg/ha) | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 | 75-80 |
|---|---|---|---|---|---|---|
| No of farms | 2 | 8 | 12 | 24 | 38 | 16 |

**Change the distribution to a more than type distribution, and draw its ogive.**

| Production Yield(kg/ha) | $f$ | $cf$ |
|---|---|---|
| 50 | 2 | 100 |
| 55 | 8 | 98 |
| 60 | 12 | 90 |
| 65 | 24 | 78 |
| 70 | 38 | 54 |
| 75 | 16 | 16 |




## Summary:

1. The mean for grouped data can be found by :

   Direct Method $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

   Assumed mean method: $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

   Step deviation method: $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

   with the assumption that the frequency of a class is centred at its mid-point, called its class mark

2. The mode for grouped data can be found by using the formula:

   Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

   Where symbols have the meanings

3. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class

4. The median for grouped data is formed by using the formula:.

   meadian = $l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$

   Where symbols have the meanings

5. Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.

6. The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

# 14 Probabilty

## 14.2 Probability — A Theoretical Approach

Suppose a coin is tossed at random the coin can only land in one of two possible ways — either head up or tail up. suppose we throw a die once. For us, a die will always mean a fair die. They are 1, 2, 3, 4, 5, 6. Each number has the same possibility of showing up.

When we speak of a coin, we assume it to be 'fair', that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being 'unbiased'. By the phrase 'random toss', we mean that the coin is allowed to fall freely without any bias or interference

**The experimental or empirical probability P(E) of an event E as**

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trails}}$$

**The theoretical probability (also called classical probability) of an event E, written as P(E), is defined as**

$$P(E) = \frac{\text{No of outcomes favarable to E}}{\text{No.of all possible outcomes of the experiment}}$$

**Example 1 : Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.**

Random experiment: Tossing a coin once

S - { Tossing a coin once};

S - {H, T} [Here, H – Head T – Tail] – n(S) = 2

A - { Getting Head }- n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$

B - { Getting Tail }- n(B) = 1

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$

**Example 2 : A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the**

**(i) Yellow ball (ii) Red ball (iii) Blue ball**

S – {Total balls in a bag } ⇒ n(S) = 3

A – { Krthika picking up yellow ball} – n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{3}$

B – { Krthika picking up red ball} – n(B) = 1

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{3}$

C – { Krthika picking up blue ball} – n(C) = 1

$P(C) = \frac{n(C)}{n(S)} = \frac{1}{3}$

**Observe that the sum of the probabilities of all the elementary events of an experiment is 1**

**Example 3 : Suppose we throw a die once. (i) What is the probability of getting a number greater than 4 ? (ii) What is the probability of getting a number less than or equal to 4 ?**

S – {Throwing a dice once } –{ 1,2,3,4,5,6}$\Rightarrow$ n(S) = 6

A – { Getting number more than 4 } – {5,6} - n(A) = 2

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

B – { Getting a number equal or less than 4} – {1,2,3,4} - n(B) = 4

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

**$P(A) = 1 - P(\bar{A})$ : where A is an event and $\bar{A}$ is complement of an event A**

That is, the probability of an event which is impossible to occur is 0. Such an event is called an **impossible** event

**Example:** We know that there are only six possible outcomes in a single throw of a die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 8, so there is no outcome favourable to 8, i.e., the number of such outcomes is zero. In other words, getting 8 in a single throw of a die, is impossible

So, the probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a certain event.

**Example:**Since every face of a die is marked with a number less than 7, it is sure that we will always get a number less than 7 when it is thrown once. So, the number of favourable outcomes is the same as the number of all possible outcomes, which is 6.

$0 \leq P(E) \leq 1$

**Example 4 : One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will**
**(i) Be an aace (ii) Not be an ace**

(i) S – {Picking a card from a deck of 52}

n(S) = 52

E – { The picked card is an ace}

P(E) = 4 [There are 4 aces in a deck of 52]

$P(A) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

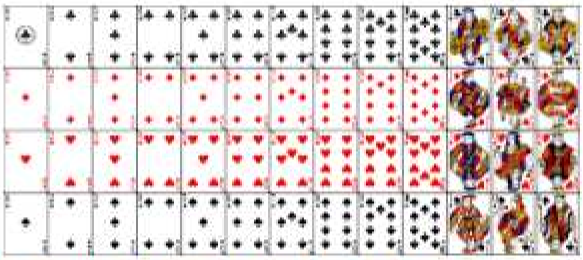
(ii) F – {The card picked is not an ace}

n(F) = 48

$P(F) = \frac{n(F)}{n(S)} = \frac{48}{52} = \frac{11}{13}$

or $P(F) = P(\bar{E}) = 1 - p(E) = 1 - \frac{1}{13} = \frac{11}{13}$

Now, let us take an example related to playing cards. Have you seen a deck of playing cards? It consists of 52 cards which are divided into 4 suits of 13 cards each— spades, hearts, diamonds and clubs. Clubs and spades are of black colour, while hearts and diamonds are of red colour. The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, queens and jacks are called face cards



**Example 5 : Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?**

The probability that Savith wins the match = P(A) = 0.62

The probability that Reshma wins the match $P(\bar{A}) = 1 - P(A) = 1 - 0.62 = 0.38$

**Example 6 : Savita and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year)**

(i) Favarable days that Savitha and Hamida have different birthdays 365–1 = 364

Probabilty of having different birthdays P(A) $= \frac{364}{365}$

Probabilty of having same birthdayP($\bar{A}$) $= \frac{1}{365}$ [P($\bar{A}$)= 1 – P(A)]

**Example 7 : There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate card, the cards being identical. Then she puts cards in a bag and stirs them thoroughly. She then draws one card from the bag. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?**

Total number of students: n(S) = 40

Number of Girls – n(A) = 25
Number of boys – n(B) = 15

The probability of drawn card with the name of a Girl P(A) $= \frac{n(A)}{n(S)} = \frac{25}{40} = \frac{5}{8}$

The probability of drawn card with the name of a BoyP(B) $= \frac{n(B)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

OR P(B) = 1 - P(A) = 1 - $\frac{5}{8} = \frac{3}{8}$

**Example 8 : A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be (i) white (ii) blue (iii) red**

The number of marbles in a box = n(S) = 9

The probability of getting white marble P(W) $= \frac{2}{9}$

The probability of getting white blue P(B) $= \frac{3}{9}$

The probability of getting white red P(B) $= \frac{4}{9}$

**Example 9 : Harpreet tosses two different coins simultaneously (say, one is of ` 1 and other of ` 2). What is the probability that she gets at least one head?**

The two different coins are tossed, the outcomes are S = { HH, HT, TH, TT }⇒ n(S) = 4

The favorable outcomes to get atleast one head – {HT, TH, TT}

Therefore the probability of getting atleast one head – $\frac{3}{4}$

**[Example 10 and 11 are not solved because they are optional]**

**Example 12 : A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that**

**(i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?**

Total number of shirts = n(S) = 100
The number of good shirts = 88

(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88

Therefore, P (shirt is acceptable to Jimmy) $= \frac{88}{100} = 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96

So, P (shirt is acceptable to Sujatha) = $\frac{96}{100}$ = 0.96

**Example 13 : Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12**

The total number of outcomes when two dice are thrown at the same time

(1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

n(S) = 6x6 = 36

(i) A – The sum of two numbers be 8

A – { (2,6), (3,5), (4,4), (5,3), (6,2)} - n(A) = 5

∴ The probability of getting the sum of two numbers be 8 = $\frac{5}{36}$

(ii) B - The sum of two numbers be 13 - n(B) = 0

∴ The probability of getting the sum of two numbers be 13 = $\frac{0}{36} = 0$

(iii) C - B - The sum of two numbers be equal or less than 12

∴ The probability of getting the sum of two numbers be equal or less than = $\frac{36}{36} = 1$

## Exercise 14.1

1. **Complete the following statements**

   **(i) Probability of an event E + Probability of the event 'not E' = ______________**

   **(ii) The probability of an event that cannot happen is _______Such an event is called _______ .**

   **(iii)The probability of an event that is certain to happen is ______ Such an event is called ______.**

   **(iv)The sum of the probabilities of all the elementary events of an experiment is ______.**

   **(v) The probability of an event is greater than or equal to _______ and less than or equal to ______**

   **Ansewers:**

   (i) 1 (ii) 0 , impossible event (iii) 1, Sure (iv) 1 (v) 0 , 1

2. **Which of the following experiments have equally likely outcomes? Explain.**

   **(i) A driver attempts to start a car. The car starts or does not start**

   **(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.**

   **(iii) A trial is made to answer a true-false question. The answer is right or wrong**

   **(iv) A baby is born. It is a boy or a girl.**

   Answer

   (i) It does not have equally likely outcomes as it depends on various reasons like mechanical problems, fuels etc.

   (ii) It does not have equally likely outcomes as it depends on the player how he/she shoots.

   (iii) It has equally likely outcomes.

   (iv)It has equally likely outcomes.

3. **Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?**

   Yes, tossing of a coin is a fair way of deciding which team should get the ball at the beginning of a football game because it has only two outcomes either head or tail. A coin is always unbiased

4. **Which of the following cannot be the probability of an event?**

   **A) $\frac{2}{3}$ B) -1.5 C) 15% D) 0.7 3**

   The probability of an event is always greater than or equal to 0 and less than or equal to 1. Thus, (B) -1.5 cannot be the probability of an event.

5. **If P(E) = 0.05, what is the probability of 'not E'?**
The probability of 'not E'=1 - P(E) = 1 − 0.05 = 0.95

6. **A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out**
**(i) an orange flavoured candy? (ii) a lemon flavoured candy?**
Answer
(i) Since the bag contains only lemon flavoured.
Therefore, No. of orange flavoured candies = 0
Probability of taking out orange flavoured candies $= \frac{0}{1} = 0$
(ii) The bag only have lemon flavoured candies.
Probability of taking out lemon flavoured candies $= \frac{1}{1} = 1$

7. **It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?**
Answer
Let E be the event of having the same birthday. P(E) = 0.992
$\Rightarrow P(E) + P(\text{not } E) = 1 \quad \Rightarrow P(\text{not } E) = 1 - P(E) \quad \Rightarrow 1 - 0.992 = 0.008$
The probability that the 2 students have the same birthday is 0.008

8. **A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?**
Total number of balls in a bag = n(S) = 3 + 5 = 8
(i) Number of red balls = n(A) = 3
Probabilty of drawing red balls $P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$
(ii) Probabilty of drawing 'not red ball' $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$

9. **A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?**
Total number of marbles in abox = n(S) = 5 + 8 + 4 = 17
(i) Number of red marbles = n(A) = 5
Probability of taking out red marbles $P(A) = \frac{n(A)}{n(S)} = \frac{5}{17}$
(ii) Number of white marbles = n(B) = 8
Probabilty of taking out white marbles $P(B) = \frac{n(B)}{n(S)} = \frac{8}{17}$
(iii) Number of green marbles = n(C) = 4
Probabilty of taking out green marbles $P(C) = \frac{n(C)}{n(S)} = \frac{4}{17}$
∴ Probabilty of 'not green' marbles $P(C^1) = 1 - \frac{n(C)}{n(S)} = 1 - \frac{4}{17} = \frac{13}{17}$

10. **A piggy bank contains hundred 50p coins, fifty Rs1 coins, twenty Rs2 coins and ten Rs5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ?(ii) will not be a Rs 5 coin?**

Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180
Total number of 50 p coins = $n(A) = 100$
Number of Rs 5 coins = $n(B) = 10$
(i) Probabilty of getting Rs 5 coins $P(A) = \frac{n(A)}{n(S)} = \frac{100}{180} = \frac{5}{9}$
(ii)Probabilty of it will not be a Rs 5 coin $1 - P(B) = 1 - \frac{n(B)}{n(S)} = 1 - \frac{10}{180} = \frac{17}{18}$

**11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see Fig. 15.4). What is the probability that the fish taken out is a male fish?**

Fig. 15.4

Total number of fish in the tank = n(S) = 5+8 = 13

Number of male fish in the tank = n(A) = 5

The probability of taking out the male fish

$= P(A) = \frac{n(A)}{n(S)} = \frac{5}{13}$

**12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see Fig. 15.5), and these are equally likely outcomes. What is the probability that it will point at (i) 8 (ii) an odd number (iii) A number greater than 2 (iv) A number less than 9**

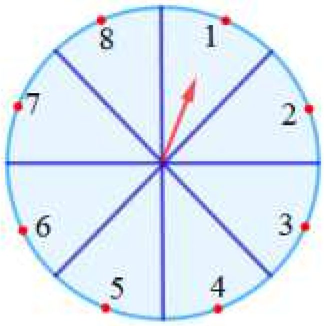



Fig 14.5

Possible number of events = 8

(i) Possible chances that an arrow pointing number 8 = 1

Probabilty of pointing 8 $= \frac{1}{8}$

(ii) Chances of pointing an odd number (1, 3, 5 ಮತ್ತು 7) = 4

Probabilty of pointing an odd number $= \frac{4}{8} = \frac{1}{2}$

(iii) Chances of pointing a number greater than 2 (i,e.3, 4, 5, 6, 7 and 8) = 6

Probablity of pointing a number greater than 2 $= \frac{6}{8} = \frac{4}{4}$

(iv) Chances of pointing less than 9 (i.e, 1,2,3,4,5,6,7,8) = 8

Probability of pointing a number less than 9 $= \frac{8}{8} = 1$

**13. A die is thrown once. Find the probability of getting (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.**

Possible numbers of events on throwing a dice = 6

Numbers on dice = 1,2,3,4,5 and 6

(i) Prime numbers = 2, 3 and 5

Favourable number of events = 3

Probability that it will be a prime number $= \frac{3}{6} = \frac{1}{2}$

(ii) Numbers lying between 2 and 6 = 3, 4 and 5

Favourable number of events = 3

Probability that a number between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

(iii) Odd numbers = 1, 3 and 5

Favourable number of events = 3

Probability that it will be an odd number $= \frac{3}{6} = \frac{1}{2}$

**14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds**

Possible numbers of events = 52

(i) Numbers of king of red colour = 2

Probability of getting a king of red colour = $= \frac{2}{52} = \frac{1}{26}$

(ii) Numbers of face cards = 12

Probability of getting a face card = $\frac{12}{52} = \frac{3}{13}$

(iii) Numbers of red face cards = 6

Probability of getting a king of red colour = $\frac{6}{52} = \frac{3}{26}$

(iv) Numbers of jack of hearts =1

Probability of getting a king of red colour = $\frac{1}{52}$

(v) Numbers of king of spade = 13

Probability of getting a king of red colour = $\frac{13}{52} = \frac{1}{4}$

(vi) Numbers of queen of diamonds = 1

Probability of getting a king of red colour = $\frac{1}{52}$

**15. Five cards the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.**
**(i) What is the probability that the card is the queen?**
**(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?**

Total numbers of cards = 5

(i) Numbers of queen = 1

Probability of picking a queen = $\frac{1}{5}$

(ii) When queen is drawn and put aside then total numbers of cards left is 4

(a) Numbers of ace = 1

Probability of picking an ace = $\frac{1}{4}$

(a) Numbers of queen = 0

Probability of picking a queen = $\frac{0}{4} = 0$

**16. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.**

Numbers of defective pens = 12

Numbers of good pens = 132

Total numbers of pen = 132 + 12 = 144 pens

Favourable number of events = 132

Probability of getting a good pen $= \frac{132}{144} = \frac{11}{12}$

**17. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?**
**(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?**

(i) Total numbers of bulbs = 20

Numbers of defective bulbs = 4

Probability of getting a defective bulb = $\frac{4}{20} = \frac{1}{5}$

(ii) One non defective bulb is drawn in (i) then the total numbers of bulb left is 19

Total numbers of events = 19

Favourable numbers of events = 19 - 4 = 15

Probability that the bulb is not defective = $\frac{15}{19}$

**18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5**

Total numbers of discs = 50

(i) Total numbers of favourable events = 81

Probability that it bears a two-digit number = $\frac{81}{90} = \frac{9}{10}$

(ii) Perfect square numbers = 1, 4, 9, 16, 25, 36, 49, 64 and 81

Favourable numbers of events = 9; Probability of getting a perfect square number = $\frac{9}{90} = \frac{1}{10}$

(iii) Numbers which are divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90

Favourable numbers of events = 18; Probability of getting a number divisible by 5 = $\frac{18}{90} = \frac{1}{5}$

**19. A child has a die whose six faces show the letters as given below:**

| | | | | | |
|---|---|---|---|---|---|
| **A** | **B** | **C** | **D** | **E** | **A** |

**The die is thrown once. What is the probability of getting (i) A? (ii) D?**

Total numbers of events = 6

(i) Total numbers of faces having A on it = 2; Probability of getting A = $\frac{2}{6} = \frac{1}{3}$

(ii) Total numbers of faces having D on it = 1 ; Probability of getting A = $\frac{1}{6}$

**20. Suppose you drop a die at random on the rectangular region shown in Fig. 15.6. What is the probability that it will land inside the circle with diameter 1m?[Not for examination]**

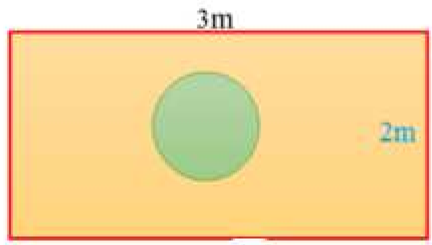



Fig 15.6

Area of the rectangle = $(3 \times 2)\ m^2 = 6m^2$

Area of the circle = $\pi r^2 = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}\ m^2$

Probability that die will land inside the circle = $\frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$

**21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that**
**(i) She will buy it? (ii) She will not buy it?**

Total numbers of pens = 144

Numbers of defective pens = 20

Numbers of non defective pens = 144 - 20 = 124

(i) Numbers of favourable events = 124 ; Probability that she will buy it = $\frac{124}{144} = \frac{31}{36}$

(ii) Numbers of favourable events = 20; Probability that she will not buy it = $\frac{20}{144} = \frac{5}{36}$

**22. Refer to Example 13. (i) Complete the following table**

| **Event Sum on two dice** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Probability** | $\frac{1}{36}$ | | | | | | $\frac{5}{36}$ | | | | $\frac{1}{36}$ |

**(ii)A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.**

Events that can happen on throwing two dices are (1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

$\Rightarrow$ n(S) = 6x6 = 36

(i) To get sum as 2, possible outcomes = (1,1)

To get sum as 3, possible outcomes = (1,2) and (2,1)

To get sum as 4, possible outcomes = (1,3); (3,1); and (2,2)

To get sum as 5, possible outcomes = (1,4); (4,1); (2,3); and (3,2)

To get sum as 6, possible outcomes = (1,5); (5,1); (2,4); (4,2); and (3,3)

To get sum as 7, possible outcomes = (1,6); (6,1); (5,2); (2,5); (4,3); and (3,4)

To get sum as 8, possible outcomes = (2,6); (6,2); (3,5); (5,3); and (4,4)

To get sum as 9, possible outcomes = (3,6); (6,3); (4,5); and (5,4)

To get sum as 10, possible outcomes = (4,6); (6,4) and (5,5)

To get sum as 11, possible outcomes = (5,6) and (6,5)

To get sum as 12, possible outcomes = (6,6)

| **Event Sum on two dice** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Probability** | $\frac{1}{36}$ | $\frac{2}{36}$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

(ii) No, i don't agree with the argument. It is already justified in (i).

**23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.**

Events that can happen in tossing 3 coins

= HHH, HHT, HTH, THH, TTH, HTT, THT, TTT

Total number of events = 8

Hinif will lose the game if he gets HHT, HTH, THH, TTH, HTT, THT

Favourable number of elementary events = 6

Probability of losing the game = = $\frac{6}{8}$ = $\frac{3}{4}$

**24. A die is thrown twice. What is the probability that**

**(i) 5 will not come up either time? (ii) 5 will come up at least once?**

**[Hint : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment]**

(i) Total number of possibilities = 6x6 =36

Possible outcomes: (1,1), (1,2), (1,3), (1,4),(1,6), (2,1), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,4), (3,6), (4,1), (4,2), (4,3), (4,4),(4,6), (6,1), (6,2), (6,3), (6,4), (6,6)

The possibility of 5 will not come either time = 25

Required probability = = $\frac{25}{36}$

(ii) Number of events when 5 comes at least once = 11

Probability = $\frac{11}{36}$ [ another way 1 - $\frac{25}{36}$ = $\frac{11}{36}$ ]

**25. Which of the following arguments are correct and which are not correct? Give reasons for your answer.**

**(i) If two coins are tossed simultaneously there are three possible outcomes—two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$**

**(ii) If a die is thrown, there are two possible outcomes—an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$**

(i) The statement is incorrect

Possible events = (H,H); (H,T); (T,H) and (T,T)

Probability of getting two heads = $\frac{1}{4}$

Probability of getting one of the each = $\frac{2}{4} = \frac{1}{2}$

(ii) Correct. The two outcomes considered are equally likely.

## Summary:

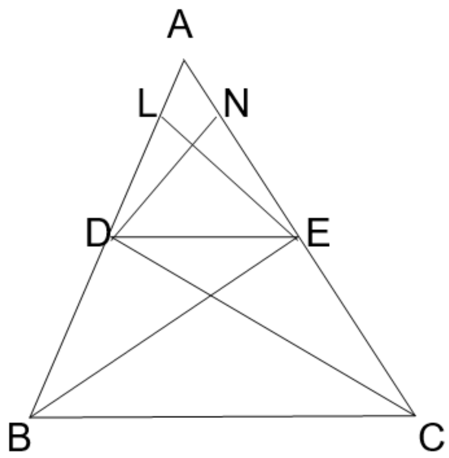
1. The difference between experimental probability and theoretical probability.
2. The theoretical (classical) probability of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

   where we assume that the outcomes of the experiment are equally likely.
3. The probability of a sure event (or certain event) is 1.
4. The probability of an impossible event is 0
5. The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is
7. For any event E, $P(E) + P(\overline{E}) = 1$ where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.

## 1. Thales Theorem (Basic proportionally theorem)

**"If a straight line is drawn parallel to a side of a triangle, then it devides the other two sides proportionally".**




Data:- In $\triangle$ ABC , DE ∥ BC

To prove :– $\frac{AD}{DB} = \frac{AE}{EC}$

Constrution :- Jain D,C and E,B draw EL $\perp$ AB and DN $\perp$ AC

Proof :– In $\triangle$ ADE and $\triangle$ BDE

$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ BDE} = \frac{\frac{1}{2}\times AD\times EL}{\frac{1}{2}\times DB\times EL} \qquad (A = \frac{1}{2} \times b \times h)$$

$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ BDE} = \frac{AD}{DB} \quad - (1)$$

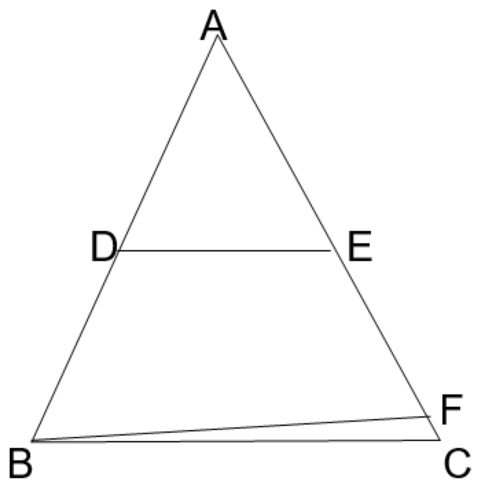
$$\frac{Area\ of\ \Delta\ ADE}{Area\ of\ \Delta\ CDE} = \frac{\frac{1}{2}\times AE\times DM}{\frac{1}{2}\times EC\times DM} \qquad (A = \frac{1}{2} \times b \times h)$$

$$\frac{Area\ of\ \ \Delta\ ADE}{Area\ of\ \ \Delta\ CDE} = \frac{AE}{EC} \quad - (2)$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC} \qquad (\Delta BDE\ \cong\ \Delta CDE) \qquad (\text{Axiom -1})$$

## 2. Converse of Thales Theorem -2

**"If a straight line divides two sides of a triangle proportionally, then the straight line is parallel to the third side".**




Data:- In $\triangle$ ABC , $\frac{AD}{DB} = \frac{AE}{EC}$

To prove :– **DE ∥ BC**

Constrution :- DE ∥ BF

Proof :– In $\triangle$ABC, DE ∥ BF (Constrution)

$\frac{AD}{DB} = \frac{AE}{EF}$ (Thales Theorem)

$\frac{AD}{DB} = \frac{AE}{EC}$ (Data)

$\frac{AE}{EC} = \frac{AE}{EF}$ (Axiom -1)

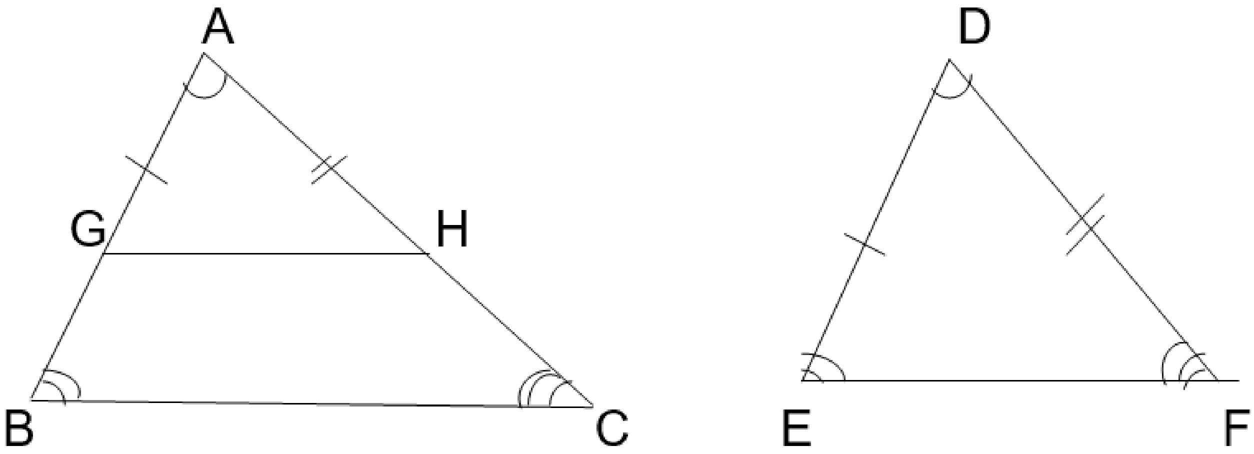
⇒ EC = EF

⇒ Point F consides point C Hence

DE ∥ BC

## 3. Theorem -3 (A.A Similarity Criterion)

**"If two triangles are equiangular, then their corresponding sides are in proportion".**




Data:- In $\triangle$ ABC and $\triangle$ DEF

$\angle BAC = \angle EDF$

$\angle ABC = \angle DEF$

To prove :– $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Constrution :- Mark point G and H on AB and AC such that AG = DE and AH = DF jain G and H .

Proof :– In $\triangle$ AGH and $\triangle$ DEF

AG = DE (Constrution)

$\angle GAH = \angle EDF$ (Data) AH = DF (Constrution)

$\triangle$ AGH $\cong$ $\triangle$ DEF (SAS poslulate)

$\therefore$ $\angle AGH = \angle DEF$ (CPCT)

$\angle ABC = \angle DEF$ (Data)

$\Rightarrow$ $\angle AGH = \angle ABC$ (Axiom -1)

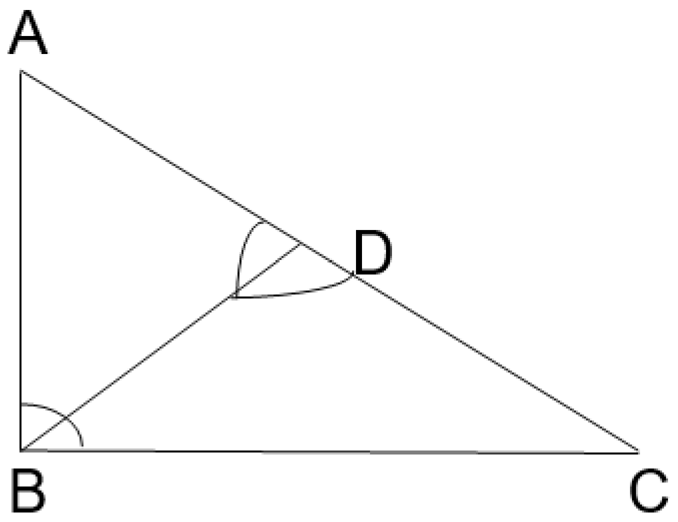
$\therefore$ GH $\parallel$ BC (If corresponding angles are equal-lines are parallel)

In $\triangle$ ABC $\frac{AB}{AG} = \frac{BC}{GH} = \frac{CA}{HA}$ (Corolary of Thales theorem)

Hence $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ( $\triangle$ AGH $\cong$ $\triangle DEF$)

## 4. Theorem -4

**" In a right angled traiangle, the perpendicular to the hypotenuse from the right angled vertex, divides the original triangle into two right angled triangles, each of which is similar to the orginal triangle"**




Data:- ⊾ ABC , $\angle ABC = 90^\circ$ , BD $\perp$ AC

To prove :– ⊾ ABD ~ ⊾ BDC ~ ⊾ ABC

Proof :– ⊾ ADB and ⊾ ABC

$\angle ADB = \angle ABC = 90^\circ$ (Data)

$\angle BAD = \angle BAC$ (comman angle)

$\therefore$ ⊾ ADB ~ ⊾ ABC .......(i) (equiangular triangle)

In ⊾ BDC and ⊾ ABC

$\angle BDC = \angle ABC = 90^\circ$ (Data)

$\angle BCD = \angle BCA$ (comman angle)

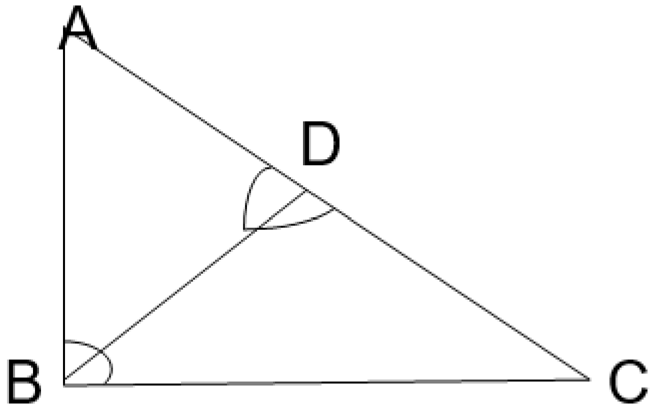
$\therefore$ ⊾ BDC ~ ⊾ ABC .......(ii) (equiangular triangle)

Now we have

⊾ ADB ~ ⊾ BDC ~ ⊾ ABC (Axiom -1 and 2)

## 6. Theorem -6 Pythagoras Thearem

**" In a right angled triangle, the square on the hypotenuse of in equal to the sum of the squares on the other two sides".**




Data:- ⊿ ABC , ∠ABC = 90°

To prove :– $(AC)^2 = AB^2 + BC^2$

Constrution :- Draw BD ⊥ AC

Proof :– In ⊿ ABC and ⊿ ADB

∠ABC = ∠ADB = 90° (Data and Constrution)

∠BAC = ∠BAD (comman angle)

⊿ ABC ~ ⊿ ADB (equiangular triangle)

$\frac{AB}{AD} = \frac{AC}{AB}$ (A,A Criteria)

$AB^2 = (AC)(AD)$ – (1)

In ⊿ ABC and ⊿ BDC

∠ABC = ∠BDC = 90° (Data and Constrution)

∠BCA = ∠BCD (comman angle)

⊿ ABC ~ ⊿ BDC (equiangular triangle)

$\frac{BC}{DC} = \frac{AC}{BC}$ (A,A Similarity Criteria)

$(BC)^2 = (AC)(DC)$ – (2)

$by\ adding\ 1\ and\ 2\ (AB)^2 + (BC)^2 = (AC)(AD) + (AC)(DC)$

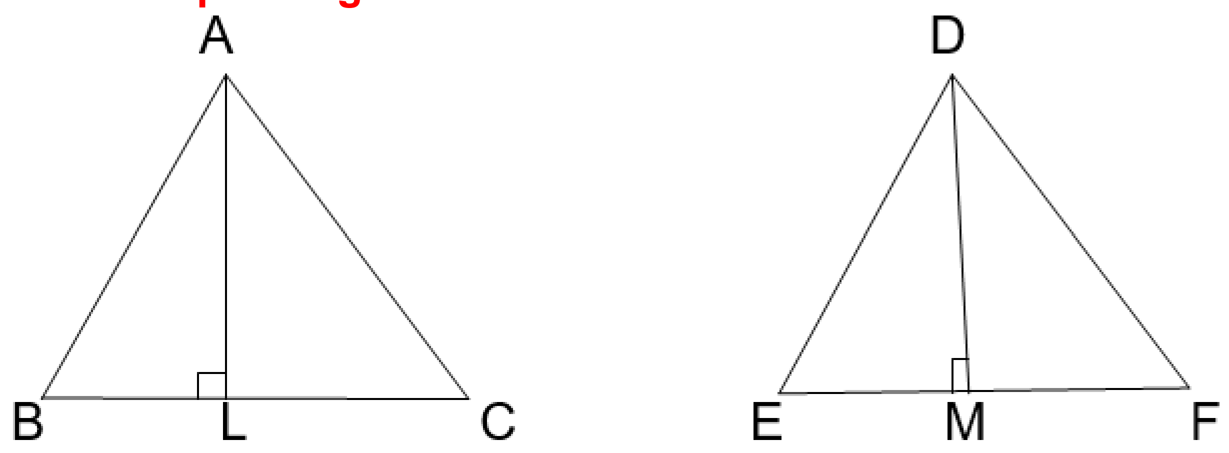
$(AB)^2 + (BC)^2 = (AC)(AD + DC)$ $(AC = AD + DC)$

$(AB)^2 + (BC)^2 = (AC)(AC)$

$(AB)^2 + (BC)^2 = (AC)^2$

## 5. Theorem -5 (Area of similar triangle)

**" The areas of similar triangles are proportional to the squares of the carrespanding sides".**




Data:- In $\triangle$ ABC and $\triangle$ DEF ಗಳಲ್ಲಿ

$\triangle$ ABC $\sim$ $\triangle$ DEF

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

To prove :– $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{(BC)^2}{(EF)^2}$

Constrution :- Draw AL $\perp$ BC and DM $\perp$ EF

Proof :– Compare $\triangle$ ALB and $\triangle$ DME

$\angle ABL = \angle DEM$ (Data)

$\angle ALB = \angle DME = 90°$ (Constrution)

$\therefore$ $\triangle$ ALB $\sim$ $\triangle$ DME (equiangular triangle)

$\frac{AL}{DM} = \frac{AB}{DE}$ (A,A Criteria)

$\frac{AB}{DE} = \frac{BC}{EF}$ Data

$\frac{AL}{DM} = \frac{BC}{EF}$ – 1 (Axiom -1)

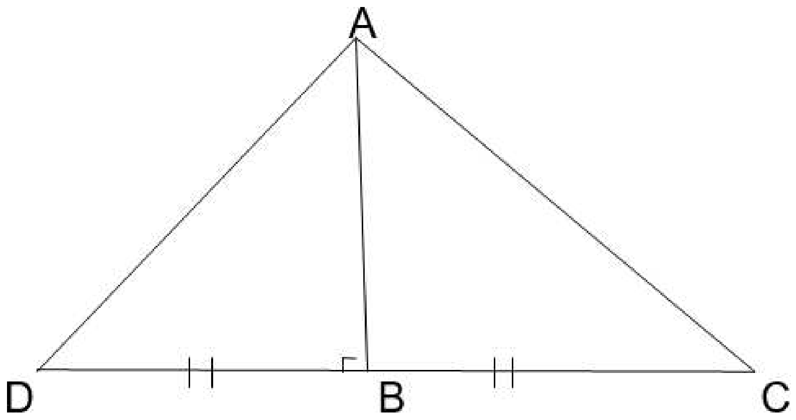
$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times EF\times DM} \quad (A = \tfrac{1}{2} \times b \times h)$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \left(\frac{BC}{EF}\right) . \left(\frac{BC}{EF}\right) \quad – \quad \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \frac{(BC)^2}{(EF)^2}$$

## 7. Converse of Pythagaras theorem – 7

**"If the square on the longer side of a triangle is equal to the sum of the squares on the other two sides, then those two sides contain a right angle".**




Data:- In △ ABC, $AC^2 = AB^2 + BC^2$

**To prove :–** $\angle$**ABC = 90°**

Constrution :- Draw a perpendicular on AB at B select a point D one it such that, DB = BC jain A and D

Proof :– In ⊿ ABD

$\angle$ABD = 90° (Constrution)

$AD^2 = AB^2 + BD^2$ (Pythagaras theorem)

In ⊿ ABC $AC^2 = AB^2 + BC^2$ Data

⇒ $AD^2 = AC^2$

AD = AC

In ⊿ ABD and ⊿ ABC

AD = AC BD = BC (Constrution)

AB = AB (common side)

△ ABD ≅ △ ABC (SSS Pastulate)

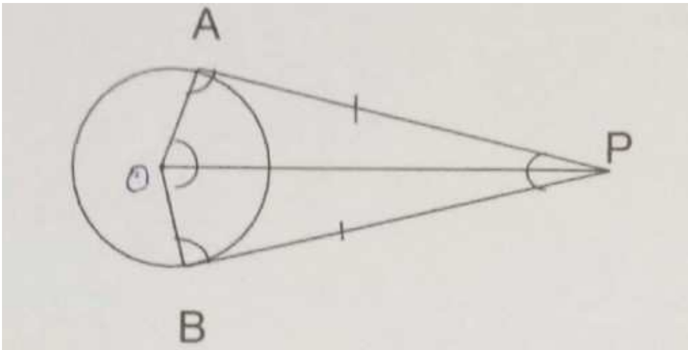
⇒ $\angle$ABD = $\angle$ABC = **90°** (CPCT)

$\angle$**ABC = 90°**

## 8. theorem

**The tangents drawn from an external point to a circle.**

a) are equal
b) subtend equal angles at the centre
c) are equally indined to the line joinging the centre and
the external point.




Data:- The circle having the 'O' and centre 'P' is an external point
'PA' and 'PB' are the tangents from external point 'P'
To the given circle join OA, OB and OP

To prove :– **a) PA = PB**

**b) $\angle AOP = \angle BOP$**

**c) $\angle APO = \angle BPO$**

Proof :– In a $\triangle$ AOP and $\triangle$ BOP ಗಳಲ್ಲಿ

OP = OB ( Radi of same circle )

$\angle OAP = \angle OBP = 90°$ (Radius drawn point of contact is prependiacular to tangent ) (right angle)

OP = OP (common side)

$\therefore \triangle AOP \cong \triangle BOP$ (RHS postulate)

$\therefore$ a) PA = PB ( C.P.C.T)

**b) $\angle AOP = \angle BOP$**

**c) $\angle APO = \angle BPO$**