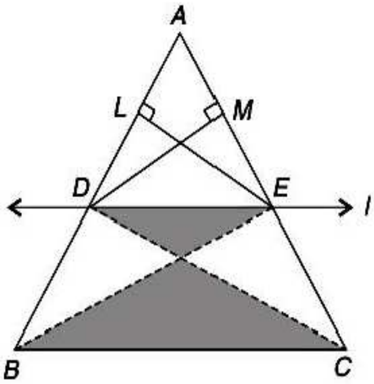
IMPORTANT THEOREMS

BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

GIVEN: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



TO PROVE : $$\frac{AD}{DB} = \frac{AE}{EC}$$

CONSTRUCTION : Join BE and CD. Draw $EL \perp$ to AB and $DM \perp AC$.

PROOF: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area } (\Delta BDE) = \text{area } (\Delta CDE) \qquad \text{...(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \qquad \text{...(ii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad \text{...(iii)}$$

Put value form (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \qquad \text{...(iv)}$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

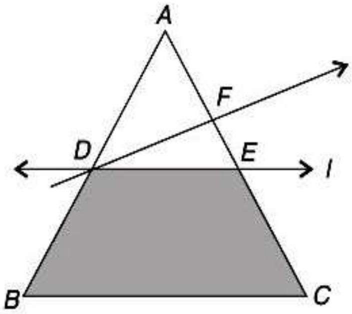
Hence Proved.

CONVERSE OF BASIC PROPORTIONALITY THEOREM (CONVERSE OF THALES THEOREM)

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

GIVEN : A Δ ABC and line 'l' intersecting the sides AB at D and AC at E such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



TO PROVE : $l \parallel BC$.

PROOF : Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{...(i) (Basic Proportionality Theorem)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{...(ii) (Given)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \quad \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow \quad FC = EC$$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$.

Hence, there is one and only line, $DE \parallel BC$, *i.e.*

$$\boxed{l \parallel BC}$$

Hence Proved.

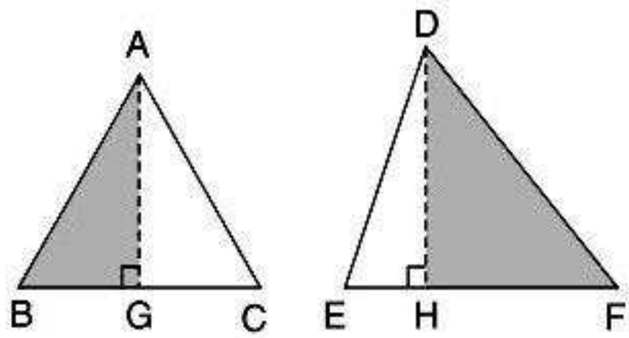
AREAS OF SIMILAR TRIANGLES THEOREM

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

GIVEN : $\Delta ABC \sim \Delta DEF$

TO PROVE : $\dfrac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \dfrac{BC^2}{EF^2} = \dfrac{AB^2}{DE^2} = \dfrac{AC^2}{DF^2}$

CONSTRUCTION : Draw $AG \perp BC$ and $DH \perp EF$.



PROOF :

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{\frac{1}{2}\times BC\times AG}{\frac{1}{2}\times EF\times DH} = \frac{BC}{EF}\times\frac{AG}{DH} \qquad ...(i)$$

(area of Δ = ½ × base × height)

Now in triangle ABG and DEH, we have

$\angle B = \angle E$ (since, $\Delta ABC \sim \Delta DEF$)

$\angle AGB = \angle DHE$ (each 90°)

Therefore, $\Delta ABG \sim \Delta DEH$ (by AA criterion)

Hence, $\frac{AB}{DE} = \frac{AG}{DH}$ ...(ii) (Using property of similar triangles)

$\Rightarrow$ $\frac{AB}{DE} = \frac{BC}{EF}$ ...(iii) (since, $\Delta ABC \sim \Delta DEF$)

Comparing (ii) and (iii), we get

$$\frac{AG}{DH} = \frac{BC}{EF} \qquad ...(iv)$$

Using (i) and (iv), we get

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC}{EF}\times\frac{BC}{EF} = \frac{BC^2}{EF^2} \qquad ...(v)$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} \qquad ...(vi) \text{ (since, } \Delta ABC \sim \Delta DEF)$$

Using (v) and (vi), we get

$$\boxed{\frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}}$$

Hence Proved.

PYTHAGORAS THEOREM

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

GIVEN : ΔABC is right angled at B.

TO PROVE : $AC^2 = AB^2 + BC^2$.

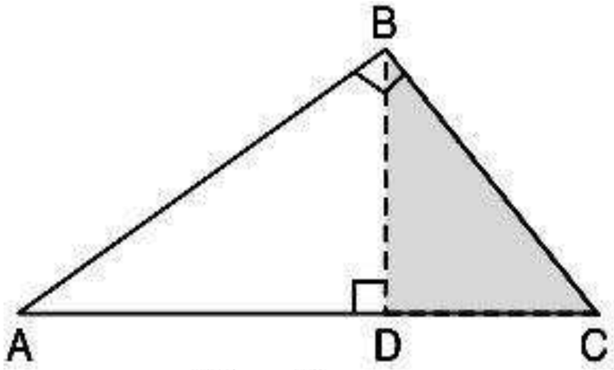
CONSTRUCTION : Draw $BD \perp AC$.

PROOF : Taking ΔADB and ΔABC

$\angle B = \angle ADB$ (each 90°)

$\angle A = \angle A$ (common)

Therefore, $\Delta ADB \sim \Delta ABC$ (by AA criterion)



Hence, $\frac{AD}{AB} = \frac{AB}{AC}$

$\Rightarrow$ $AB^2 = AD \times AC$ ...(i)

Now taking ΔCDB and ΔCBA

$\angle B = \angle BDC$ (each 90°)

$\angle C = \angle C$ (common)

Therefore, $\Delta CDB \sim \Delta CBA$ (by AA criterion)

Hence, $\frac{CD}{BC} = \frac{BC}{AC}$

$\Rightarrow$ $BC^2 = CD \times AC$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow$ $AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC \times (AD + CD)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$\boxed{AC^2 = AB^2 + BC^2}$

Hence Proved.

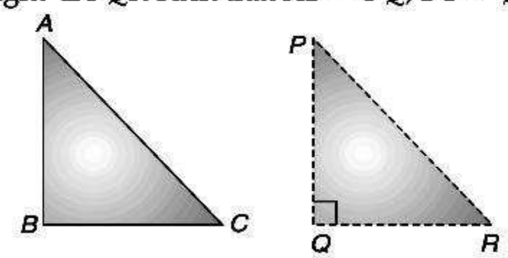
Converse of Pythagoras theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

GIVEN : $AC^2 = AB^2 + BC^2$. ...(i)

TO PROVE : ΔABC is right angled at B.

CONSTRUCTION : Draw right ΔPQR such that $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.



PROOF : Using Pythagoras theorem in ΔPQR, we get

$PR^2 = PQ^2 + QR^2$ ...(ii)

By construction, $AB = PQ$

$BC = QR$, substituting these values in (ii), we get

$PR^2 = AB^2 + BC^2$ ...(iii)

Comparing (i) and (iii), we get

$AC^2 = PR^2$

$\Rightarrow$ $AC = PR$ ...(iv)

In ΔABC and ΔPQR

$AB = PQ$ (by construction)

$BC = QR$ (by construction)

$AC = PR$ (proved above in (iv))

$\Rightarrow$ $\Delta ABC = \Delta PQR$ (by SSS congruence rule)

$\Rightarrow$ $\angle B = \angle Q$ (by cpct)

But $\angle Q = 90°$ (by construction)

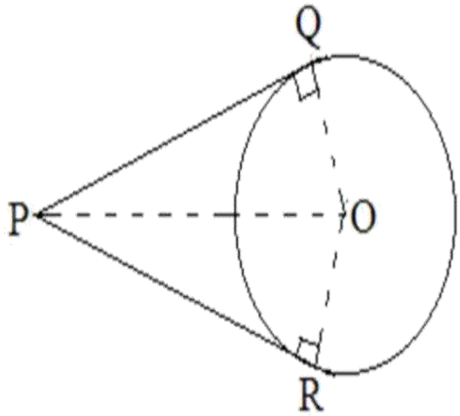
Hence, $\angle B = 90°$

$\boxed{\Delta ABC \text{ is right angled at } B.}$

Hence Proved.

CIRCLE THEORM FOR 3 MARKS

Statement: The lengths of tangents drawn from an external point to a circle are equal.



Given: Two tangents PQ and PR are drawn from a point P to a circle with centre O.

To prove: PQ = PR

Construction : Join OP, OQ and OR

Proof:

OQ⊥PQ (Tangent & radius are perpendicular)

Similarly OQ⊥PQ (Tangent & radius are perpendicular)

In right angled ΔOQP and ΔORP

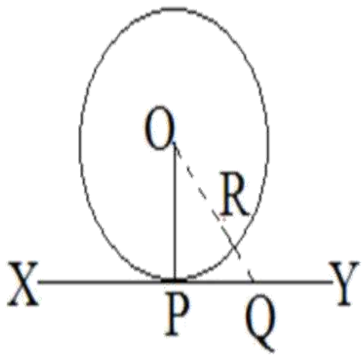
OQ=QR (Radii of the same circle)

OP = OP(Common side)

ΔOQP ≅ ΔORP (RHS Postulate)

Hence PQ = PR (CPCT)

Statement: The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given: A circle with centre O and a tangent XY at a point P of the circle.

To prove: OP⊥XY

Construction: Take a point Q, other than

Proof:

Q is point on the tangent XY, other than the point of contact P.

Q lies outside the circle.

Let OQ intersect the circle at R

OR <OQ (A part is less than the whole).......(1)

OP = OR (radii of the same circle)................(2)

OP <OQ (from 1 and 2)

Op is shorter than any other line segment joining O to any point of XY, other than P.

OP is the shortest distance between the point O and the line XY.

The shortest distance between a point and a line is the perpendicular distance

Therefore OP⊥XY

IMPORTANT QUESTIONS

Prove that $\sqrt{5}$ is an irrational number. Solution:

Let $\sqrt{5}$ is a rational number then we have

$\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow P = \sqrt{5}q$

Squaring both sides, we get

$p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5

$\Rightarrow$ P is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2$

$\Rightarrow q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5

$\Rightarrow$ q is also divisible by 5

From above we conclude that p and q has one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

QUESTION FOR PRACTICE:

1. Prove that √2 is an irrational number.
2. Prove that √3 is an irrational number.
3. Prove that 2 + 5√3 is an irrational number.
4. Prove that √2 + √3 is an irrational number.
5. Prove that 5 - √3 is an irrational number.
6. Prove that 6 + √2 is an irrational number.
7. Prove that 5 + √3 is an irrational number.

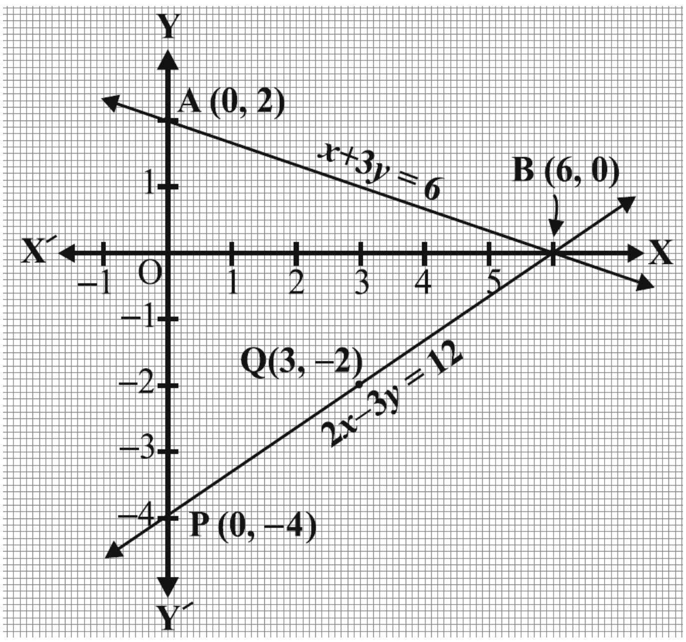
Solve the equation graphically: x + 3y = 6 and 2x – 3y = 12.

$X + 3y = 6 \Rightarrow 3y = 6 - x \Rightarrow y = \frac{6 - x}{3}$

x	0	3	6
y	2	1	0

And

$2x - 3y = 12 \Rightarrow 3y = 2x - 12 \Rightarrow y = \frac{2x - 12}{3}$



x	0	3	6
y	-4	-2	0

Now plot the points and join the points to form the lines AB and PQ as shown in graph since point B (6, 0) common to both the lines AB and PQ. Therefore, the solution of the pair of linear equations is x = 6 and y = 0

Questions for Practice

1. Solve graphically: $3x – 2y – 1 = 0$; $2x – 3y + 6 = 0$.
2. Solve graphically: $x + 4y = 10$, $y – 2 = 0$.
3. Solve graphically: $2x – 3y = 6$, $x – 6 = 0$.
4. Solve graphically: $x – 2y = 0$, $3x + 4y = 20$.
5. Solve graphically: $2x – y = 2$, $4x – y = 4$.
6. Solve graphically: $2x + y – 6 = 0$, $4x – 2y – 4 = 0$.
7. Solve graphically: $x + y – 6 = 0$, $2x – y – 3 = 0$.
8. Solve graphically: $x – y + 4 = 0$, $3x + y + 4 = 0$.

Solve the given pair of linear equations.

$X + Y = 14$

$X – Y = 4$

Solution:

$X + Y = 14 \longrightarrow (1)$

$X – Y = 4 \longrightarrow (2)$

$2X = 18$ [Adding eq. (1) and eq. (2)]

$$x = \frac{18}{2}$$

$\therefore X = 9$

Substituting the value of X in eq. (1)

$9 + Y = 14$

$Y = 14 – 9$

$\therefore Y = 5$

Solution is X= 9 and Y = 5

Question for practice:

Solve the following pairs of linear equations.

i. $2x + 3y = 11$, $2x – 4y = -24$
ii. $x – y = 26$, $x – 3y = 0$
iii. $x + y = 180$, $x – y = 18$
iv. $3x – 2y – 1 = 0$; $2x – 3y + 6 = 0$.
v. $2x – 3y = 6$, $x – 6 = 0$.
vi. $x + 4y = 10$, $y – 2 = 0$.
vii. $x + y – 6 = 0$, $2x – y – 3 = 0$.

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

Solution:

$$
\begin{array}{r|l}
 & \quad x - 2 \\ \hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & +\quad -\quad + \\ \hline
 & \quad 2x^2 - 2x + 5 \\
 & \quad 2x^2 - 2x + 2 \\
 & \quad -\quad +\quad - \\ \hline
 & \qquad\quad 3
\end{array}
$$

So, quotient = $x - 2$, remainder = 3.

Now, Divisor × Quotient + Remainder

$= (-x^2 + x - 1)(x - 2) + 3$

$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$

$= -x^3 + 3x^2 - 3x + 5$

= Dividend

Hence, the division algorithm is verified.

Questions for Practice

1. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

I. $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

II. $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

III. $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

IV. $p(x) = 3x^3 + x^2 + 2x + 5$, $g(x) = 1 + 2x + x$

CONSTRUCTION OF SIMILAR TRIANGLE

Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding Sides of the triangle ABC (i.e., of scale factor$\frac{3}{4}$)

Steps of Construction:

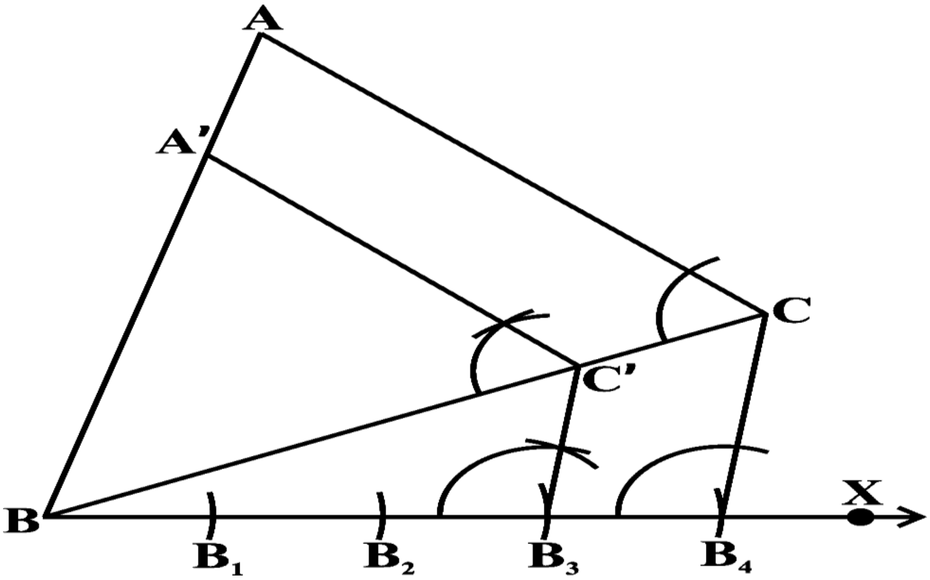
☞ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

☞ Locate 4 (the greatest of 3 and 4 in 3/4) points B_1, B_2, B_3 and B_4 on BX So that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

☞ Join B_4C and draw a line through B3 (the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C to intersect BC at C′.

☞ Draw a line through C′ parallel to the line CA to intersect BA at A′ (see below figure).

Then, Δ A′BC′ is the required triangle.



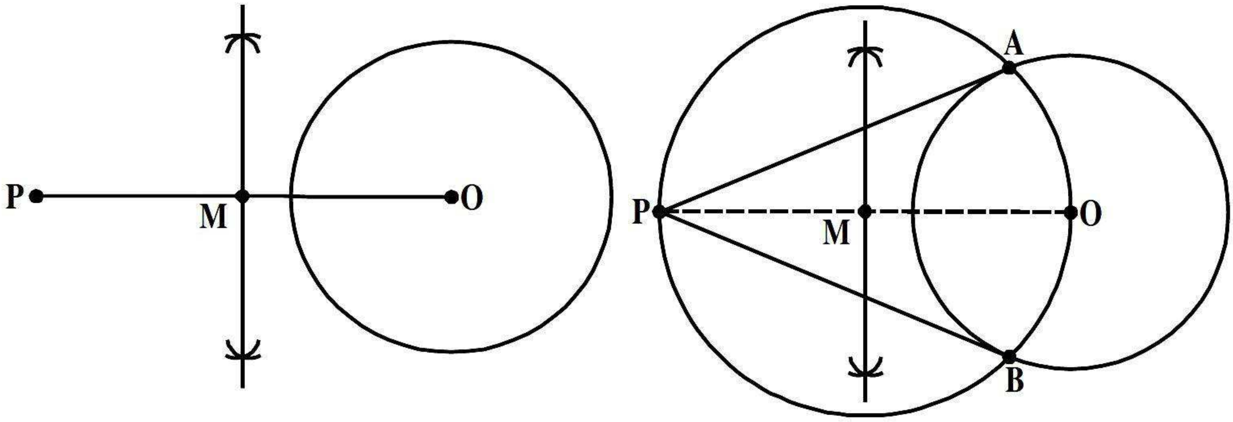
To construct the tangents to a circle from a point outside it.

Given: We are given a circle with centre 'O' and a point P outside it. We have to construct two tangents from P to the circle.

Steps of construction:

- Join PO and draw a perpendicular bisector of it. Let M be the midpoint of PO.
- Taking M as centre and PM or MO as radius, draw a circle. Let it intersect the given circle at the points A and B.
- Join PA and PB.

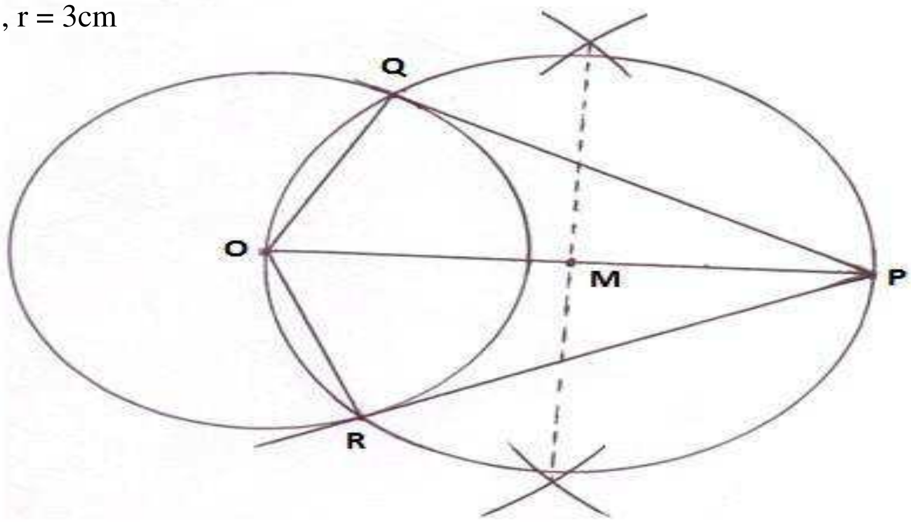
Then PA and PB are the required two tangents.



Draw a circle of radius 3 cm from a point 7cm away from its centre. Construct a pair of tangents to the circle and measure their lengths.

Solution: radius, r = 3cm

OP = 7cm

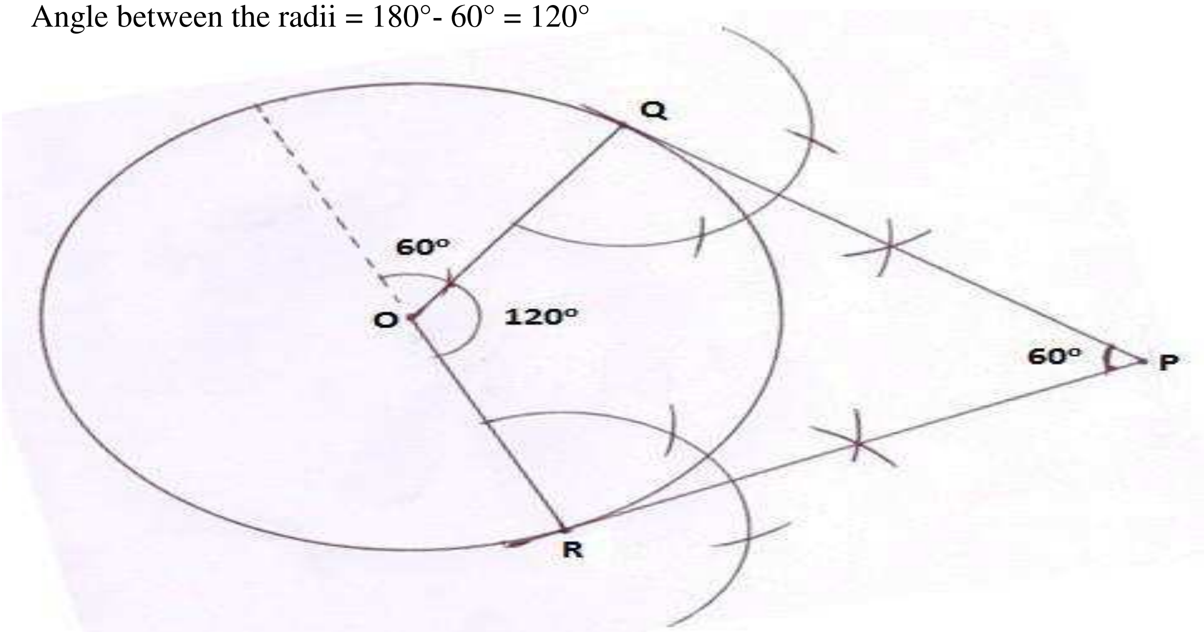


Draw a pair of tangents to a circle of radius 3cm which are inclined to each other at an angle of 60°

Solution: radius r = 3cm

Angle between the tangents = 60°

Angle between the radii = 180°- 60° = 120°

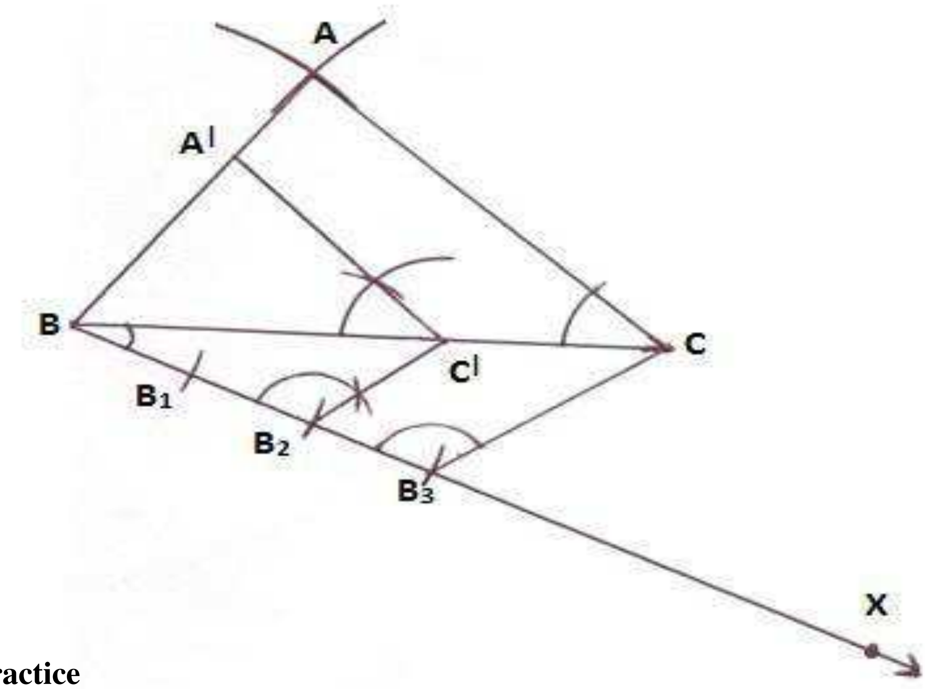


Draw a triangle ABC with sides AB = 4cm, AC = 5cm and BC = 6cm, then construct a triangle whose sides are $\frac{2}{3}$ of the corresponding sides of the triangle ABC.

AB = 4 cm

AC = 5 cm

BC = 6 cm



Questions for practice

1. Construct an isosceles triangle whose base is 7cm and altitude 4cm and construct another similar triangle whose sides are 3/2 times the corresponding sides of the isosceles triangle.

2. Draw a triangle ABC with side BC = 6cm, AB = 5cm, and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding triangle ABC
3. Draw a circle of radius 6cm. from a point 10cm away from its Centre. Construct the pair of tangents to the circle and measure their lengths
4. Draw a pair of tangents to a circle of radius 5cm which are inclined to each other at an angle of 60°
5. Draw a pair of tangents to a circle of diameter 6cm which are inclined to each other at an angle of 120°
6. Draw a pair of tangents to a circle of radius 3cm which are inclined to each other at an angle of 60°
7. Draw a pair of tangents to a circle of radius 4cm which are inclined to each other at an angle of 90°
8. Draw a triangle ABC with side BC = 6cm, AB = 5cm, and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{4}{5}$ of the corresponding triangle ABC
9. Draw a triangle ABC with side BC = 7cm, ∠A = 45°., and ∠B= 125°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding triangle ABC

Find the zeroes of the polynomial $6x^2$ - 3 -7x.

Solution: $6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (3x + 1)(2x - 3)$

$6 \times 3 = 18$

$9 \times 2 = 18$

$-9 + 2 = -7$

The zeroes, x = -1/3 and x = 3/2

Questions for practice

Find the zeroes of the polynomial

I. $4s^2 - 4s + 1$
II. $3x^2 - x - 4$
III. $4u^2 - 8u$

Find the roots of the equation $2x^2 - x + \frac{1}{8} = 0$ by factorization method

Solution: $2x^2 - x + \frac{1}{8} = 0$

$16x^2 - 8x + 1 = 0$ (multiplying by 8)

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x-1) - 1(4x-1)$

$(4x-1)(4x-1) = 0$

4x=1,

Roots are $x=\frac{1}{4}$, $x = \frac{1}{4}$

Questions for practice

Find the roots of the quadratic equation by factorization method

I. $16x^2 - 3x - 10 = 0$
II. $2x^2 + x - 6 = 0$
III. $100x^2 - 20x + 1 = 0$

Problem: Solve $x^2 - 5x + 4 = 0$ by completing its square.

Solution: I will show my work as follows;

$$x^2 - 5x + 4 = 0$$

Add and subtract $(-5/2)^2$
Which is half the coefficient of "x" squared.

$$\underline{x^2 - 5x + \left(\frac{-5}{2}\right)^2} - \left(\frac{-5}{2}\right)^2 + 4 = 0$$

Now underline the first three terms which is a perfect square.

$$\left(x - \frac{5}{2}\right)^2 - \left(\frac{-5}{2}\right)^2 + 4 = 0$$

Now $\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$; show it in the next step

$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 4 = 0$$

I will solve $-\frac{25}{4} + 4$ as follows
$-\frac{25}{4} + \frac{16}{4} = \frac{-9}{4}$

$$\left(x - \frac{5}{2}\right)^2 - \frac{9}{4} = 0$$

Add $\frac{9}{4}$ on both sides

$$\left(x - \frac{5}{2}\right)^2 = \frac{9}{4}$$

Take square root on both sides

$$x - \frac{5}{2} = \pm\sqrt{\frac{9}{4}}$$

We know $\pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$

Questions for practice

Solve the following equations by completing the square method:

i. $2x^2 - 5x + 2 = 0$
ii. $5x^2 - 6x - 2 = 0$
iii. $9x^2 - 15x + 6 = 0$
iv. $2x^2 - 5x + 3 = 0$

Solve $4x^2 + 4\sqrt{3}x + 3 = 0$ by using formula.

Solution: $4x^2 + 4\sqrt{3}x + 3 = 0$ this is in the form of $ax^2 + bx + c = 0$

$a = 4$, $b = 4\sqrt{3}$, $c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4\sqrt{3}) \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{48 - 16(3)}}{8}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$x = \frac{-4\sqrt{3}}{8} \quad or\ x = \frac{-4\sqrt{3}}{8}$$

$$x = \frac{-\sqrt{3}}{2} \quad or\ x = \frac{-\sqrt{3}}{2}$$

Questions for practice

Solve the following equations by formula method:

i. $2x^2 + x - 4 = 0$
ii. $2x^2 - 7x + 3 = 0$
iii. $2x^2 - 5x + 3 = 0$
iv. $3x^2 - 5x + 2 = 0$
v. $5x^2 - 6x - 2 = 0$
vi. $2x^2 + x = 528$
vii. $x^2 + 2x = 143$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

Where D = $b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D.

When D is greater than 0 i.e. $D < 0$, then that quadratic equation has two distinct roots.

When D is equal to 0 i.e. $D = 0$, then that quadratic equation has two equal roots

When D is lesser than 0 i.e. $D > 0$, then that quadratic equation has no real roots

Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its root.

Solution: the given equation is in the form of $ax^2 + bx + c = 0$, where a = 2, b = -4, c = 3 therefore, the discriminant, D = $\boldsymbol{b^2 - 4ac}$

D = $\boldsymbol{b^2 - 4ac}$

$= (-4)^2 - (4\times2\times3)$

$= 16 - 24$

$= -8$

$-8 > 0$ so the given equations has no real roots

Questions for Practice

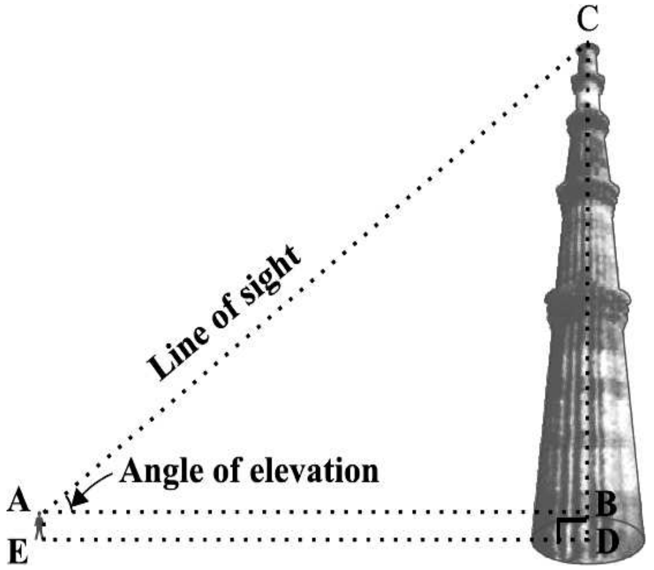
1. Find the discriminant and the nature of the roots of quadratic equation: $3\quad 3x^2 + 10x$
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 2x2 + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\quad 3x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

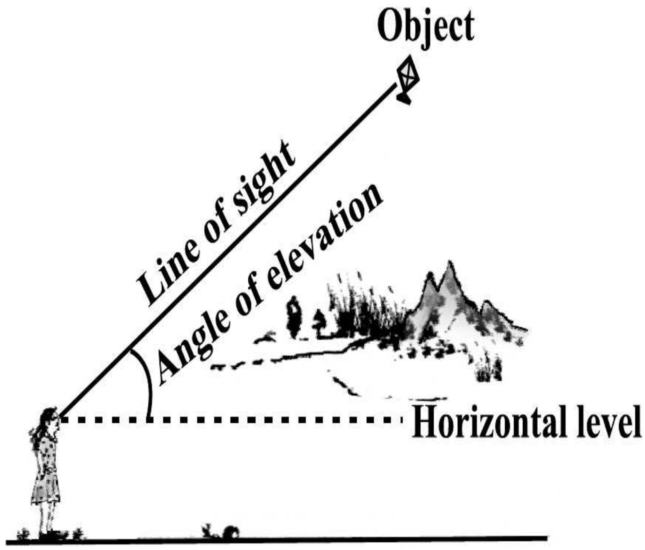
HEIGHTS AND DISTANCE:

$\angle A$	0°	30°	45°	60°	90°
Sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
Cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
Cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

ANGLE OF ELEVATION

In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

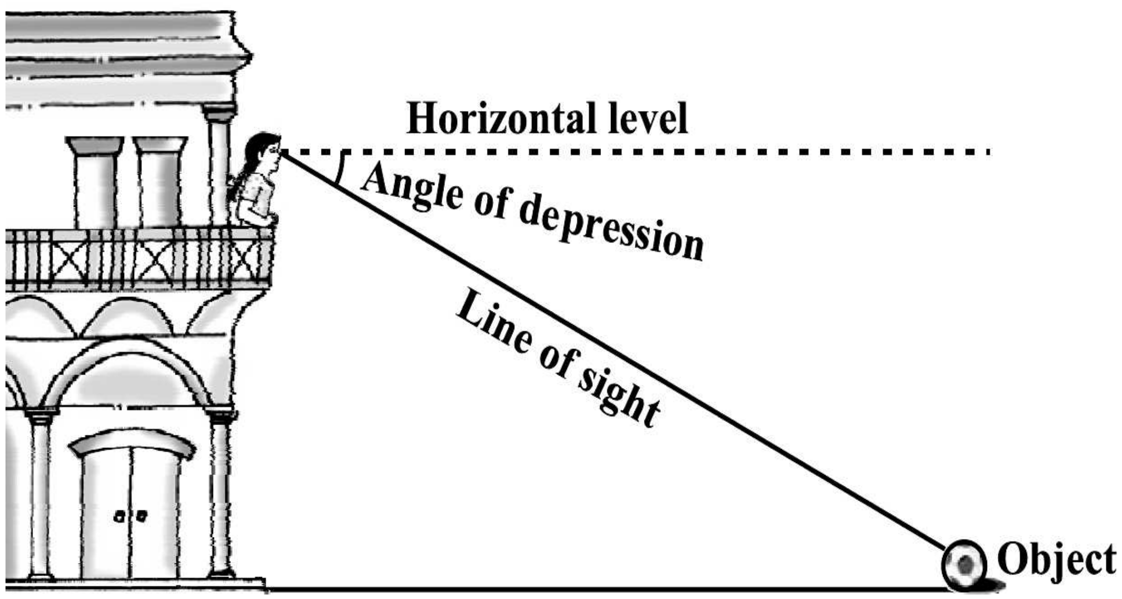




The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



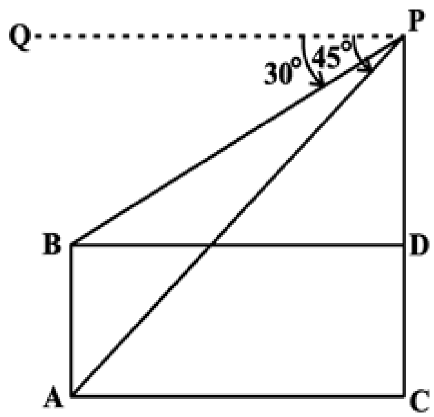
The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution: Let PC = h m be the height of multi-storyed building and AB denotes the 8 m tall building. BD = AC = x m, PC = h = PD + DC = PD + AB = PD + 8 m

So, PD = h – 8 m

Now, ∠ QPB = ∠ PBD = 30°

Similarly, ∠ QPA = ∠ PAC = 45°.



In right Δ PBD, $\tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

⇒ x = (h – 8) √3 m …………………. (1)

Also in right $\Delta PAC, \tan 45° = \frac{PC}{AC} \Rightarrow 1 = \frac{h}{x}$

From equation (1) and (2), we get h = (h – 8) √3

⇒h= h√3 - 8√3

⇒ h√3 – h = 8√3

⇒h (√3 – 1) = 8√3

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow h = \frac{8\sqrt{3}(\sqrt{3}+1)}{3-1}$

$\Rightarrow h = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})$ m

Hence, the height of the multi-storeyed building is $4(3+\sqrt{3})$ m and the distance between the two buildings is also $4(3+\sqrt{3})$ m

Questions for Practice

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.
2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°

respectively. Find the height of the tower.

5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.
6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.
7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.
8. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.
9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.
10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.
11. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.
12. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.
13. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

OGIVE:

The following distribution gives the daily income of 50 workers of a factory.

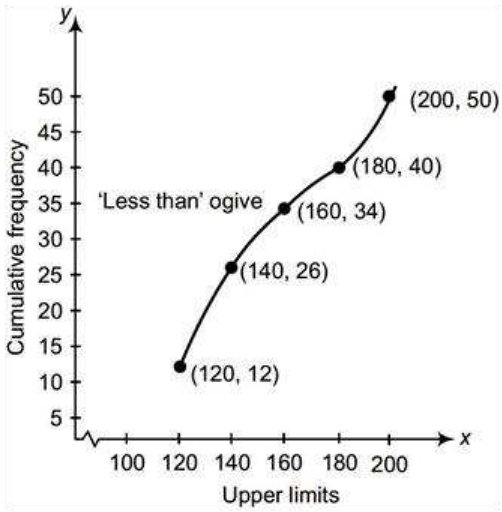
Daily income (in Rs)	100 – 120	120 – 140	140 –160	160 –180	180 –200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Solution:

Cumulative frequency less than type

Daily income (in Rs)	Less than type cf
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50



Questions for Practice

1. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

2. For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

3. Draw less than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	5	8	6	10	6	6

Also find the median from the graph and verify that by using the formula.

4. The table given below shows the frequency distribution of the cores obtained by 200 candidates in a BCA examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) less than type and (ii) more than type. Hence find median

5. Draw less than and more than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	8	5	10	6	6	6

Also find the median from the graph and verify that by using the formula

PROBABILITY:

The theoretical probability (also called classical probability) of an event A, written as p(A), is defined as

$$P(A) = \frac{\text{Number of outcomes favorable to A}}{\text{Number of all possible outcomes of the experiment}}$$

Two dice are thrown together. Find the probability that the sum of the numbers on the top of the dice is (i) 9 (ii) 10

Solution:

Here, total number of outcomes n(s) = 36

(i) Let A be the event of getting the sum of the numbers on the top of the dice is 9 then we have n(A) = 4 i.e. (3, 6), (4, 5), (5, 4), (6, 3)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 9, P (A) = $\frac{n(A)}{n(S)}$

P (A) = $\frac{4}{36} = \frac{1}{9}$

Let B be the event of getting the sum of the numbers on the top of the dice is 10

Then we have, n (B) = 3 i.e. (4, 6), (5, 5). (6, 4).

Therefore, probability of getting the sum of the numbers on the top of the dice is 10,

P (B) = $\frac{n(B)}{n(S)}$

P (B) = $\frac{3}{36} = \frac{1}{12}$

Questions for Practice

1. Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12 (iii) 7

2. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?

3. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it? (ii) She will not buy it?

4. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

5. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

7. A piggy bank contains hundred 50p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin? (ii) Will not be an Rs 5 coin?

(ii) A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) White? (iii) Not green?

8. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

9. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number

(ii) a perfect square number (iii) a number divisible by 5.

10. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that (i) it is acceptable to Jimmy? (ii) It is acceptable to Sujatha?

11. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) Consecutive days? (iii) Different days?

MEAN OF GROUPED DATA

Direct method

Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

Assume mean method or Short-cut method

Mean, $\bar{x} = A + \dfrac{\sum f_i d_i}{\sum f_i}$ where $d_i = x_i - A$

Step Deviation method

Mean, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times h$ where $u = \dfrac{x_i - A}{h}$

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Solution:

Literacy rate (in %)	Number of Cities 'f'	Class mark 'x'	$u = \dfrac{x-A}{h}$	fu
45 – 55	3	50	–2	–6
55 – 65	10	60	–1	–10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	35			–2

Here, $\sum fu = -2, \sum f = 35$, A = 70, h = 10

Mean, $\bar{x} = A + \dfrac{\sum fu}{\sum f} \times h \Rightarrow \bar{x} = 70 + \dfrac{-2}{35} \times 10 = 70 - \dfrac{20}{35} = 70 - \dfrac{4}{7} = 70 - 0.57 \Rightarrow \bar{x} = 69.43$

Questions for Practice

1. Find the mean of the following data:

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

2. Find the mean percentage of female teachers of the following data:

Percentage of female teachers	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85
Number of States/U.T	6	11	7	4	4	2	1

3. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

4. Find the mean daily wages of the workers of the factory by using an appropriate method for the following data:

Daily wages (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

MODE OF GROUPED DATA

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l = lower limit of the modal class,
h = size of the class interval (assuming all class sizes to be equal),
f_1 = frequency of the modal class,
f_0 = frequency of the class preceding the modal class,
f_2 = frequency of the class succeeding the modal class.

Find the mode for the following frequency distribution

Solution:

Here, highest frequency is 36 which belongs to class 20 – 30. So, modal class is 20 – 30,
$l = 20,\ f_0 = 16,\ f_1 = 36,\ f_2 = 34,\ h = 10$

We know that $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$\Rightarrow Mode = 20 + \frac{36 - 16}{2(36) - 16 - 34} \times 10$$

$$\Rightarrow Mode = 20 + \frac{20}{72 - 50} \times 10 = 20 + \frac{200}{22} = 20 + 9.09 = 29.09$$

Questions for Practice

1. The frequency distribution table of agriculture holdings in a village is given below:

Area of land(in ha)	1-3	3-5	5-7	79	9-11	11-13
No. of families	20	45	80	55	40	12

Find the modal agriculture holdings of the village.

2. Find the mode age of the patients from the following distribution:

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

3. Find the mode of the following frequency distribution:

Class	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

4. Find the modal height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

5. A survey regarding the heights (in cms) of 50 girls of a class was conducted and the following data was obtained.

Height(in cm)	120-130	130-140	140-150	150-160	160-170	Total
No. of girls	2	8	12	20	8	**50**

Find the mode of the above data.

MEDIAN OF GROUPED DATA

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower limit of median class,
n = number of observations,
cf = cumulative frequency of class preceding the median class,
f = frequency of median class,
h = class size (assuming class size to be equal).

Find the median of the following frequency distribution:

Class	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

Solution:

Class	True Class limits	Frequency	cf
75-84	74.5 – 84.5	8	8
85-94	84.5 – 94.5	11	19
95-104	94.5 – 104.5	26	45
105-114	104.5 – 114.5	31	76
115-124	114.5 – 124.5	18	94
125-134	124.5 – 134.5	4	98
135-144	134.5 – 144.5	2	100
Total		**100**	

Here, n = 100 $\Rightarrow \frac{n}{2} = 50$ which belongs to 104.5 – 114.5

So, l = 104.5, cf = 45, f = 31, h = 10

We know that $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

$$\Rightarrow Median = 104.5 + \frac{50-45}{31} \times 10 \;\; \Rightarrow Median = 104.5 + \frac{50}{31} = 104.5 + 1.61 = 106.11$$

Questions for Practice

1. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
No. of Students	14	16	18	23	18	8	3

Determine the median percentage of marks.

2. Weekly income of 600 families is as under:

Income (in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Compute the median income.

3. Find the median of the following frequency distribution:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	8	12	20	12	18	13	10	7

4. The following table gives the distribution of the life time of 500 neon lamps:

Life time (in hrs)	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Number of Lamps	24	86	90	115	95	72	18

AREA AND PERIMETER OF CIRCLE, QUADRANT, SEMICIRCLE

Area of Circle = πr^2, Perimeter of Circle = Circumference = $2\pi r$

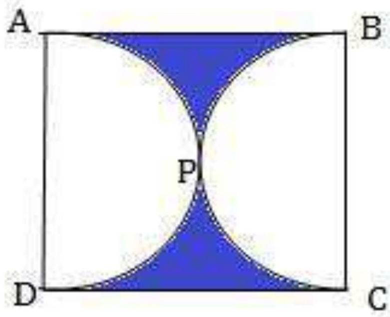
Area of Semicircle = $\frac{1}{2}\pi r^2$, Perimeter of Semicircle = $\pi r + 2r$

Area of Quadrant = $\frac{1}{4}\pi r^2$, Perimeter of Quadrant = $\frac{1}{2}\pi r + 2r$

Find the area of the shaded region in figure, if ABCD is a square of side 14 cm APD and BPC are semicircles.

Solution:- Length of side of square = 14 cm

Area of square $= 14 \times 14 = 196\ \text{cm}^2$ --------> (1)



Radius $r = \frac{14}{2} = 7$ cm

Area of semicircle = $\frac{1}{2} \times \pi r^2$

Area of semicircle = $\frac{1}{2} \times \frac{22}{7} \times 7^2$

Area of semicircle = $11 \times 7 = 77\text{cm}^2$ -----> (2)

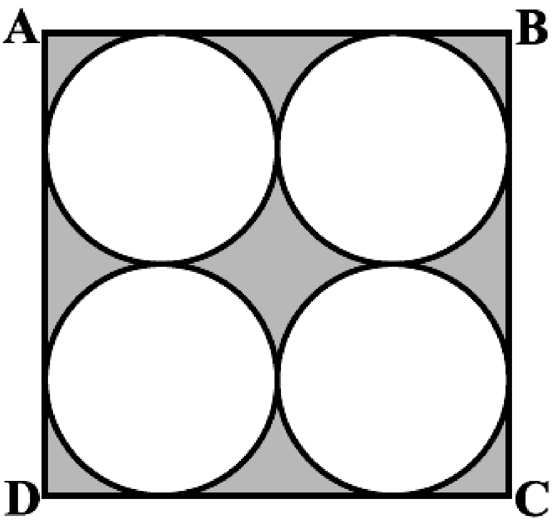
Area of shaded portion = Area of square– 2 ×Area of semicircle

$= 196 - 2 \times 77$

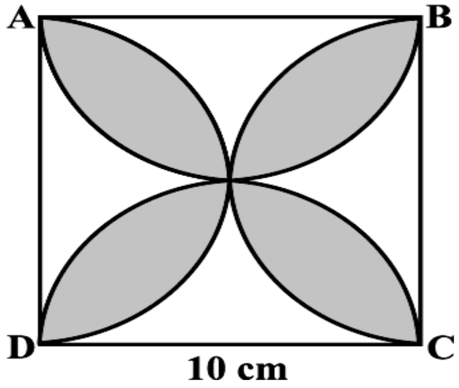
$= 196 - 154 = 42\text{cm}^2$

Questions for Practice

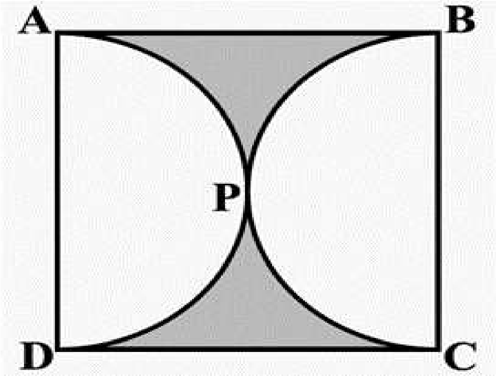
1. Find the area of the shaded region in below figure, where ABCD is a square of side 14 cm.



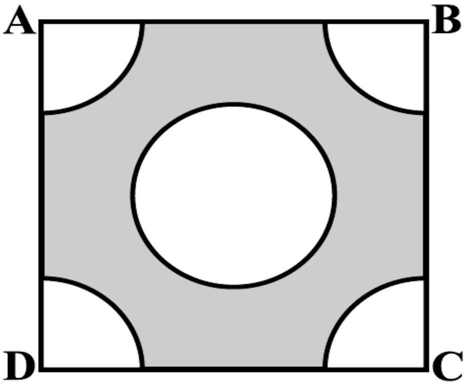
2. Find the area of the shaded design in above right figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use $\pi = 3.14$)



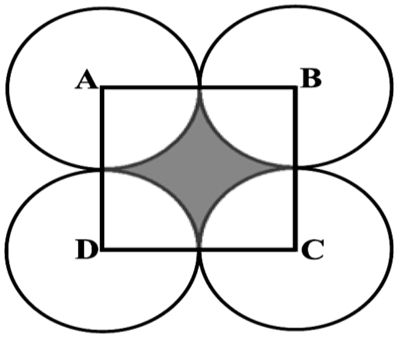
3. Find the area of the shaded region in below left figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



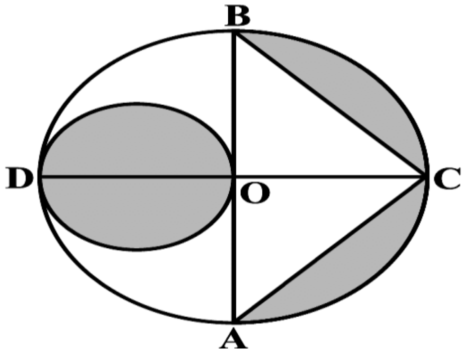
4. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in above right sided figure. Find the area of the remaining portion of the square.



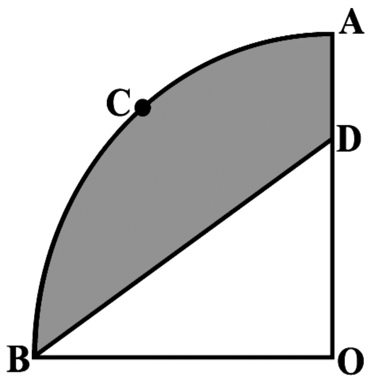
5. In the below figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region



6. In the below figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



7. In the below figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



DISTANCE FORMULA

The distance between any two points A(x_1, y_1) and B(x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \quad AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Mid-point formula

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points A(x_1, y_1) and B(x_2, y_2), are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Section formula

The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Find the distance between the points $(0, 0)$ and $(36, 15)$.

Solution: $(x_1, y_1) = (0, 0)(x_2, y_2) = (36, 15)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(36 - 0)^2 + (15 - 0)^2}$$

$$d = \sqrt{(36)^2 + (15)^2}$$

$$d = \sqrt{1296 + 225}$$

$$d = \sqrt{1521}$$

$$d = 39 \text{ units}$$

Questions for practice:

Find the distance between the following pairs:

I. (-5, -7) and (-1, 3)
II. (6, 4) and (3, 1)
III. (8, 6) and (3, 1)
IV. (6, 4) and (3, 1)
V. (1, 7) and (4, 2)
VI. (-1, -1) and (-4, 4)

Find the co-ordinates of the midpoint of the line segment joining the points $(8, 5)$ and $(6, 3)$.

Solution: $(x_1, y_1) = (8,5)(x_2, y_2) = (6,3)$

The co-ordinates of midpoint $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$(x, y) = \left(\frac{8+6}{2}, \frac{5+3}{2}\right)$$

$$(x, y) = \left(\frac{14}{2}, \frac{8}{2}\right)$$

$$(x, y) = (7,4)$$

Questions for practice:

Find the co-ordinates of the midpoint of the line segment joining by the following pairs of points:

I. (-3, 10) and (6, -8)
II. (4, -5) and (6, 3)
III. (-2, 8) and (-6, -4)
IV. (8, 3) and (8, -7)
V. (6, 5) and (4, 4)
VI. (2, 0) and (0, 3)
VII. (2, 8)and (6, 8)
VIII. (4, 6) and (6, -3)

Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$x=\frac{3(8)+1(4)}{3+1}=7, y=\frac{3(5)+1(-3)}{3+1}=3$$

Therefore, (7, 3) is the required point.

In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$y=\frac{k(-8)+1(10)}{k+1}=6$$

$$\Rightarrow -8k+10=6k+6 \Rightarrow -8k-6k=6-10$$

$$\Rightarrow -14k=-4 \Rightarrow k=\frac{4}{14}=\frac{2}{7}$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

Questions for practice:

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2: 3.
2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).
3. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A (2, – 2) and B (– 7, 4).
4. Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.
5. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).
6. Find the ratio in which the line segment joining A (1, – 5) and B (– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
7. Find the coordinates of the points which divide the line segment joining A (– 2, 2) and B (2, 8) into four equal parts