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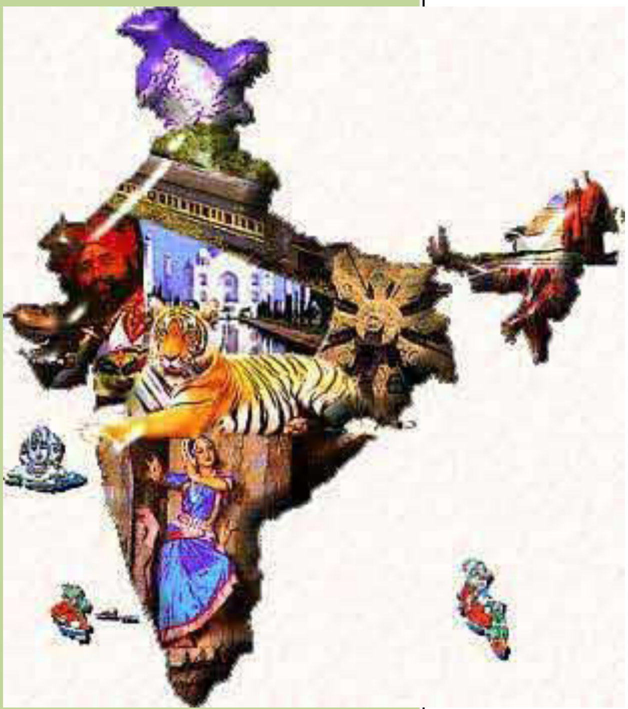
10th Mathematics



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Theorem Proof

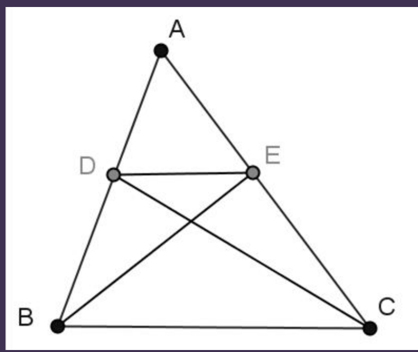
E – Book for 10th Std

(As Per KSEEB Model Paper 2018 – 19 Get 7 Marks By Practice it)

10^{th} Mathematics THEOREM PRROFS

BASIC PROPORTIONALITY THEOREM (Thales Theorem)

If a straight line is drawn parallel to one side of a triangle then it divides the other two sides proportionally



Data : In ΔABC

$DE \parallel BC$

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

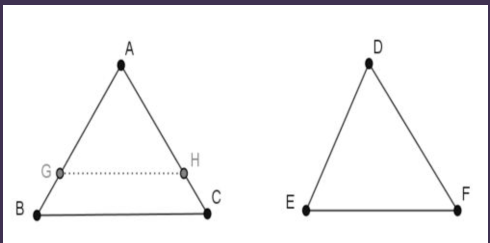
Construction : Join D, C and E, B

Proof:

STATEMENT	REASONS
$\frac{\Delta ADE}{\Delta BDE} = \frac{\frac{1}{2} X\ AD\ X\ EL}{\frac{1}{2} X\ DB\ X\ EL}$	A = ½ x b x h
$\frac{\Delta ADE}{\Delta BDE} = \frac{AD}{DB}$	
$\frac{\Delta ADE}{\Delta CDE} = \frac{\frac{1}{2} X\ AE\ X\ DM}{\frac{1}{2} X\ EC\ X\ DM}$	
$\frac{\Delta ADE}{\Delta CDE} = \frac{AE}{EC}$	
$\frac{AD}{DB} = \frac{AE}{EC}$	$\Delta BDE = \Delta CDE$ and Axiom - 1

Theorem (AA Similarity Criterion)

Statement: **If two triangles are equiangular then their corresponding sides are Proportional**



Data : In ΔABC and ΔDEF

$\angle BAC = \angle EDF$

$\angle ABC = \angle DEF$

To Prove : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

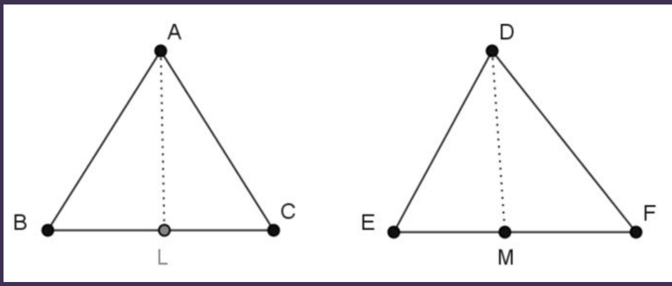
Construction : Mark Points G and H on AB and AC such that AG = DE and AH = DF, Join G and H

Proof:

STATEMENT	**REASONS**
Compare ΔAGH and ΔDEF	
AG = DE	Construction
$\angle GAH = \angle EDF$	Data
AH = DF	Construction
$\Delta AGH \cong \Delta DEF$	SAS Postulate
$\angle AGH = \angle DEF$	CPCT
But $\angle ABC = \angle DEF$	Data
$\Longrightarrow \angle AGH = \angle ABC$	Axiom 1
GH $\parallel BC$	If corresponding angles are equal then lines are $\parallel$
In ΔABC $\frac{AB}{AG} = \frac{BC}{GH} = \frac{CA}{HA}$	Third corollary to Thales Theorem
Hence $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$	$\Delta AGH \equiv \Delta DEF$

Theorem (Area of Similar Triangles)

Statement: **The areas of similar triangles are proportional to the squares of the corresponding sides**



Data: $\Delta ABC \sim \Delta DEF$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To Prove: $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{BC^2}{EF^2}$

Construction: Draw AL $\perp BC\ and\ DM\ \perp EF$

Proof:

STATEMENT

Compare $\Delta ALB\ and\ \Delta DME$

$\angle ABL = \angle DEM$

$\angle ALB = DME = 90^0$

$\Delta ALB \sim \Delta DME$

$\dfrac{AL}{DM} = \dfrac{AB}{DE}$ $\qquad \dfrac{AL}{DM} = \dfrac{AB}{DE}$

But $\dfrac{BC}{EF} = \dfrac{AB}{DE}$

$\dfrac{AL}{DM} = \dfrac{BC}{EF}$

Area of ABC = ½ x BC x AL

Area of DEF = ½ x EF x DM

$$\frac{Area\ of\ ABC}{Area\ of\ DEF} = \frac{\frac{1}{2} X\ BC\ X\ AL}{\frac{1}{2} X\ EF\ X\ DM} = \frac{BC\ X\ AL}{EF\ X\ DM}$$

Now $\dfrac{ar(\Delta ABC)}{ar\ (\Delta DEF)} = \dfrac{BC\ X\ AL}{EF\ X\ DM}$

$= \left(\dfrac{BC}{EF}\right) X \left(\dfrac{AL}{DM}\right)$

$= \dfrac{BC}{EF} X\ \dfrac{BC}{EF}$

$= \dfrac{BC^2}{EF^2}$

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{BC^2}{EF^2}$$

From data $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

REASONS

Data

Construction

AA Criteria

Data

Transitive property

$\dfrac{AL}{DM} = \dfrac{BC}{EF}$ is proved

PYTHAGORS THEOREM

Statement: **In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides**

Data: ΔABC $\angle BAC = 90°$

To Prove: $AB^2 + BC^2 = CA^2$

Construction: Draw BD $\perp AC$

Proof:

STATEMENT	REASONS
Compare ΔABC and ΔADB	
$\angle ABC = \angle ADB = 90°$	Data and Construction
$\angle BAD$ is common	
$\Delta ABC \sim \Delta ADB$	Equiangular triangles
$\frac{AB}{AD} = \frac{AC}{AB}$	AA Similarity Criteria
$AB^2 = AC . AD$ --------- (1)	
Compare ΔABC and ΔBDC	
$\angle ABC = ADB = 90°$	Data and Construction
$\angle ACB$ is common	
$\Delta ABC \sim \Delta BDC$	Equiangular triangles
$\frac{BC}{DC} = \frac{AC}{BC}$	AA Similarity criteria
$BC^2 = AC . DC$ --------- (2)	
By adding (1) and (2) we get	
$AB^2 + BC^2 = (AC.AD) + (AC.DC)$	
$AB^2 + BC^2 = AC (AD + DC)$	
$AB^2 + BC^2 = AC. AC$	(AD + DC = AC)
$AB^2 + BC^2 = AC^2$	

TANGENTS FROM EXTERNAL POINT

Statement: The tangents drawn from an external point to a circle

1. Are equal
2. Subtend equal angles at the centers
3. Are equally inclined to the line joining the centre and the external point

Data: A is the centre of the circle, B is an external point, BP and BQ are the tangents, AP, AQ and AB are joined

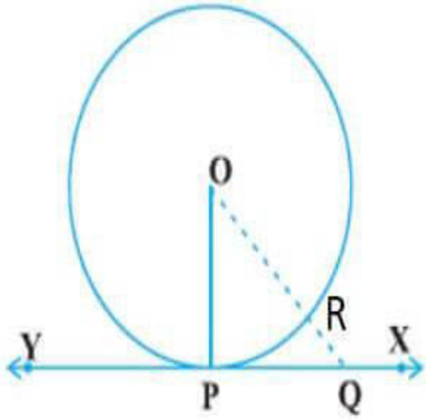
To Prove:

1. BP = BQ
2. $\angle PAB = \angle QAB$
3. $\angle PBA = QBA$

Proof:

STATEMENT	REASONS
In ΔAPB and ΔAQB	
AP = AQ	Radii of same circle
$\angle APB = AQB = 90°$	Radius drawn at the point of contact is perpendicular to the tangent
Hyp AB = Hyp AB	Common side
$\Delta APB \cong \Delta AQB$	RHS Postulate
1. BP = BQ	CPCT
2. $\angle PAB = \angle QAB$	
3. $\angle PBA = QBA$	

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given: A circle with center O.

With tangent XY at point of contact P.

To prove: $OP \perp XY$

Proof: Let Q be point on XY

Connect OQ

Suppose it touches the circle at R

Hence,

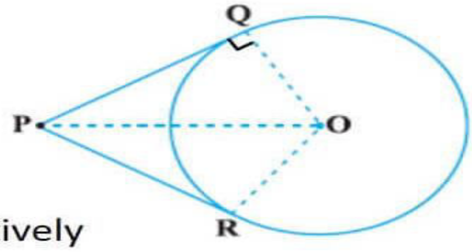
$$OQ > OR$$

$$OQ > OP \qquad (\text{as } OP = OR \text{ radius})$$

Same will be the case with all other points on circle

Hence, OP is the smallest line that connects XY

The lengths of tangents drawn from an external point to a circle are equal.



Given: Let circle be with centre O
and P be a point outside circle
PQ and PR are two tangents to circle intersecting at point Q and R respectively

To prove: Lengths of tangents are equal
i.e. PQ = PR

Construction: Join OQ , OR and OP

Proof: As PQ is a tangent

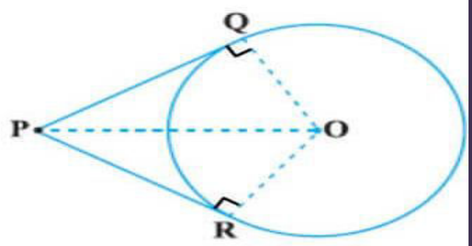
OQ ⊥ PQ (Tangent at any point of circle is perpendicular to the radius through point of contact)

So, ∠ OQP = 90°

Hence Δ OQP is right triangle

Similarly,
PR is a tangent
& OR ⊥ PR (Tangent at any point of circle is perpendicular to the radius through point of contact)

So, ∠ ORP = 90°



In Δ OQP and Δ ORP

∠ OQP = ∠ ORP *(Both 90°)*

OP = OP *(Common)*

OQ = OR *(Both radius)*

∴ Δ OQP ≅ Δ ORP *(R.H.S congruency)*

Hence, PQ = PR *(CPCT)*

Hence both tangents from external point are equal in length

Also, ∠ OPQ = ∠ OPR *(CPCT)*

Hence, OP is the angle bisector of ∠ QPR