PHYSICS

PART - 2

CLASS XI

CONTENTS

6. Oscillations

Any motion that repeats itself after regular intervals of time is known as a *periodic motion***.** The examples of periodic motion are the motion of planets around the Sun, motion of hands of a clock, motion of the balance wheel of a watch, motion of Halley's comet around the Sun observable on the Earth once in 76 years.

If a body moves back and forth repeatedly about a mean position, it is said to possess *oscillatory motion*. Vibrations of guitar strings, motion of a pendulum bob, vibrations of a tuning fork, oscillations of mass suspended from a spring, vibrations of diaphragm in telephones and speaker system and freely suspended springs are few examples of oscillatory motion. In all the above cases of vibrations of bodies, the path of vibration is always directed towards the mean or equilibrium position.

The oscillations can be expressed in terms of simple harmonic functions like sine or cosine function. A harmonic oscillation of constant amplitude and single frequency is called *simple harmonic motion (SHM).*

6.1 Simple harmonic motion

A particle is said to execute simple harmonic motion if its acceleration is directly proportional to the displacement from a fixed point and is always directed towards that point.

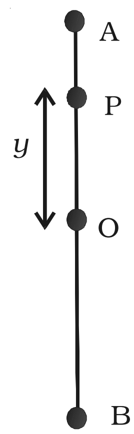


Fig. 6.1 Simple harmonic motion of a particle

Consider a particle P executing SHM along a straight line between A and B about the mean position O (Fig. 6.1). The acceleration of the particle is always directed towards a fixed point on the line and its magnitude is proportional to the displacement of the particle from this point.

(i.e) $a \propto y$

By definition $a = -\omega^2 y$

where ω is a constant known as angular frequency of the simple harmonic motion. The negative sign indicates that the acceleration is opposite to the direction of displacement. If m is the mass of the particle, restoring force that tends to bring back the particle to the mean position is given by

$$F = -m\,\omega^2\, y$$

or $$F = -k\, y$$

The constant $k = m\,\omega^2$, is called force constant or spring constant. Its unit is $N\ m^{-1}$. The restoring force is directed towards the mean position.

Thus, *simple harmonic motion is defined as oscillatory motion about a fixed point in which the restoring force is always proportional to the displacement and directed always towards that fixed point.*

6.1.1 The projection of uniform circular motion on a diameter is SHM

Consider a particle moving along the circumference of a circle of radius a and centre O, with uniform speed v, in anticlockwise direction as shown in Fig. 6.2. Let XX′ and YY′ be the two perpendicular diameters.

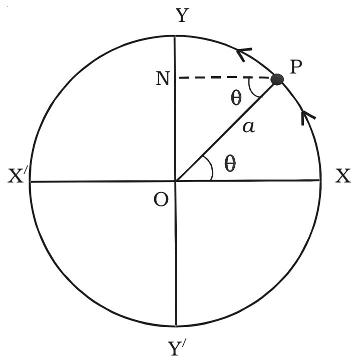


Fig. 6.2 Projection of uniform circular motion

Suppose the particle is at P after a time t. If ω is the angular velocity, then the angular displacement θ in time t is given by $\theta = \omega t$. From P draw PN perpendicular to YY'. As the particle moves from X to Y, foot of the perpendicular N moves from O to Y. As it moves further from Y to X', then from X' to Y' and back again to X, the point N moves from Y to O, from O to Y' and back again to O. When the particle completes one revolution along the circumference, the point N completes one vibration about the mean position O. The motion of the point N along the diameter YY' is simple harmonic.

Hence, the projection of a uniform circular motion on a diameter of a circle is simple harmonic motion.

Displacement in SHM

The distance travelled by the vibrating particle at any instant of time t from its mean position is known as displacement. When the particle is at P, the displacement of the particle along Y axis is y (Fig. 6.3).

Then, in Δ OPN, $sin\ \theta = \frac{ON}{OP}$

$ON = y = OP\ sin\ \theta$

$y = OP\ sin\ \omega t \quad (\because \theta = \omega t)$

since OP = a, the radius of the circle, the displacement of the vibrating particle is

$y = a\ sin\ \omega t$ ...(1)

The amplitude of the vibrating particle is defined as its maximum displacement from the mean position.

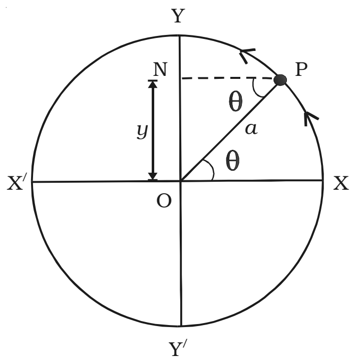


Fig. 6.3 Displacement in SHM

Velocity in SHM

The rate of change of displacement is the velocity of the vibrating particle.

Differentiating eqn. (1) with respect to time t

$$\frac{dy}{dt} = \frac{d}{dt}(a\ sin\ \omega t)$$

$\therefore\ v = a\ \omega\ cos\ \omega t$ ...(2)

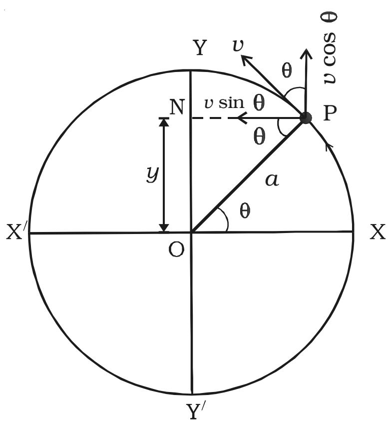


Fig. 6.4 Velocity in SHM

The velocity v of the particle moving along the circle can also be obtained by resolving it into two components as shown in Fig. 6.4.

(i) $v\ cos\ \theta$ in a direction parallel to OY

(ii) $v\ sin\ \theta$ in a direction perpendicular to *OY*

The component $v\ sin\ \theta$ has no effect along *YOY′* since it is perpendicular to *OY*.

$\therefore$ Velocity $= v \cos \theta$

$= v \cos \omega t$

We know that, linear velocity = radius × angular velocity

$\therefore\ v = a\omega$

$\therefore$ Velocity $= a\omega\ cos\ \omega t$

$\therefore$ Velocity $= a\omega \sqrt{1 - sin^2 \omega t}$

$$\text{Velocity} = a\omega \sqrt{1-\left(\frac{y}{a}\right)^2} \quad \left[\because \ \sin\theta = \frac{y}{a}\right]$$

$$\text{Velocity} = \omega \sqrt{a^2 - y^2} \qquad \ldots(3)$$

Special cases

(i) When the particle is at mean position, (i.e) $y = 0$. Velocity is $a\omega$ and is maximum. $v = \pm\, a\omega$ is called *velocity amplitude.*

(ii) When the particle is in the extreme position, (i.e) $y = \pm$ a, the velocity is zero.

Acceleration in SHM

The rate of change of velocity is the acceleration of the vibrating particle.

$$\frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}(a\omega \cos \omega t) = -\omega^2\, a \sin \omega t.$$

$$\therefore \text{ acceleration } = \frac{d^2y}{dt^2} = -\omega^2\, y \qquad \ldots(4)$$

The acceleration of the particle can also be obtained by component method.

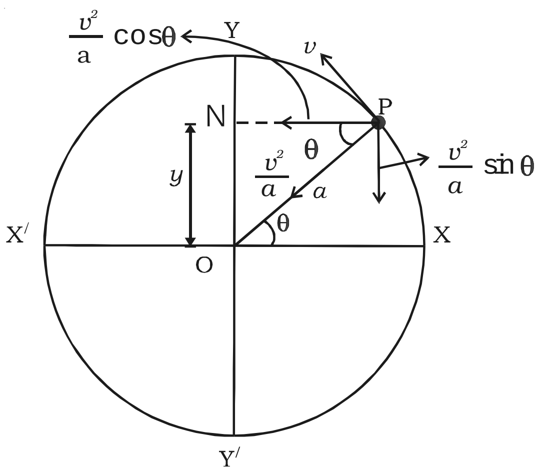


Fig. 6.5 Acceleration in SHM

The centripetal acceleration of the particle P acting along PO is $\frac{v^2}{a}$. This acceleration is resolved into two components as shown in Fig. 6.5.

(i) $\frac{v^2}{a}\cos\theta$ along PN perpendicular to OY

(ii) $\frac{v^2}{a}\sin\theta$ in a direction parallal to YO

The component $\frac{v^2}{a}$ *cos* θ has no effect along *YOY'* since it is perpendicular to *OY*.

$$\text{Hence acceleration} = -\frac{v^2}{a} \sin\theta$$

$$= -a\,\omega^2 \sin \omega t \qquad (\because v = a\,\omega)$$

$$= -\omega^2 y \qquad (\because y = a \sin \omega t)$$

$$\therefore \qquad \text{acceleation} = -\omega^2 y$$

The negative sign indicates that the acceleration is always opposite to the direction of displacement and is directed towards the centre.

Special Cases

(i) When the particle is at the mean position (i.e) $y = 0$, the acceleration is zero.

(ii) When the particle is at the extreme position (i.e) $y = \pm a$, acceleration is $\mp\, a\,\omega^2$ which is called as *acceleration amplitude.*

The differential equation of simple harmonic motion from eqn. (4) is $\frac{d^2y}{dt^2} + \omega^2 y = 0$ ...(5)

Using the above equations, the values of displacement, velocity and acceleration for the SHM are given in the Table 6.1.

It will be clear from the above, that at the mean position $y = 0$, velocity of the particle is maximum but acceleration is zero. At extreme

Table 6.1 - Displacement, Velocity and Acceleration

Time	ωt	Displacement $a \sin \omega t$	Velocity $a\omega \cos \omega t$	Acceleration $-\omega^2 a \sin \omega t$
$t = 0$	0	0	$a\omega$	0
$t = \frac{T}{4}$	$\frac{\pi}{2}$	$+a$	0	$-a\omega^2$
$t = \frac{T}{2}$	π	0	$-a\omega$	0
$t = \frac{3T}{4}$	$\frac{3\pi}{2}$	$-a$	0	$+a\omega^2$
$t = T$	2π	0	$+a\omega$	0

position $y = \pm a$, the velocity is zero but the acceleration is maximum $\mp a\omega^2$ acting in the opposite direction.

Graphical representation of SHM

Graphical representation of displacement, velocity and acceleration of a particle vibrating simple harmonically with respect to time t is shown in Fig. 6.6.

(i) Displacement graph is a sine curve. Maximum displacement of the particle is $y = \pm a$.

(ii) The velocity of the vibrating particle is maximum at the mean position i.e $v = \pm a\omega$ and it is zero at the extreme position.

(iii) The acceleration of the vibrating particle is zero at the mean position and maximum at the extreme position (i.e) $\mp a\omega^2$.

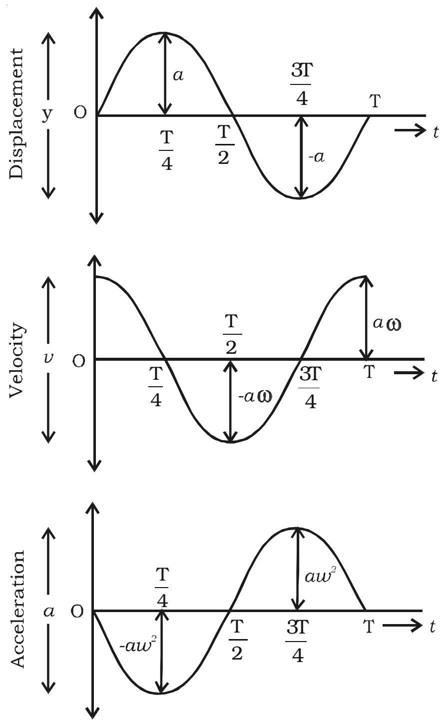


Fig. 6.6 Graphical representation

The velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$. The acceleration is ahead of the velocity by a phase angle $\frac{\pi}{2}$ or by a phase π ahead of displacement. (i.e) when the displacement has its greatest positive value, acceleration has its negative maximum value or vice versa.

6.2 Important terms in simple harmonic motion

(i) Time period

The time taken by a particle to complete one oscillation is called the time period T.

In the Fig. 6.2, as the particle P completes one revolution with angular velocity ω, the foot of the perpendicular N drawn to the vertical diameter completes one vibration. Hence T is the time period.

Then $\omega = \frac{2\pi}{T}$ or $T = \frac{2\pi}{\omega}$

The displacement of a particle executing simple harmonic motion may be expressed as

$$y(t) = a \sin \frac{2\pi}{T} t \qquad \text{...(1)}$$

and $$y(t) = a \cos \frac{2\pi}{T} t \qquad \text{...(2)}$$

where T represents the time period, a represents maximum displacement (amplitude).

These functions repeat when t is replaced by $(t + T)$.

$$y(t + T) = a \sin \left[\frac{2\pi}{T} (t + T)\right] \qquad \text{...(3)}$$

$$= a \sin \left[2\pi \frac{t}{T} + 2\pi\right]$$

$$= a \sin 2\pi \frac{t}{T} = y(t)$$

In general $y(t + nT) = y(t)$

Above functions are examples of periodic function with time period T. It is clear that the motion repeats after a time $T = \frac{2\pi}{\omega}$ where ω is the angular frequency of the motion. In one revolution, the angle covered by a particle is 2π in time T.

(ii) Frequency and angular frequency

The number of oscillations produced by the body in one second is known as frequency. It is represented by n. The time period to complete one oscillation is $\frac{1}{n}$.

$T = \frac{1}{n}$ shows the time period is the reciprocal of the frequency. Its unit is hertz. $\omega = 2\pi n$, is called as angular frequency. It is expressed in rad s^{-1}.

(iii) Phase

The phase of a particle vibrating in SHM is the state of the particle as regards to its direction of motion and position at any instant of time. In the equation $y = a \sin (\omega t + \phi_o)$ the term $(\omega t + \phi_o) = \phi$, is known as the phase of the vibrating particle.

Epoch

It is the initial phase of the vibrating particle (i.e) phase at t = 0.

$\therefore\ \phi = \phi_o \quad (\because\ \phi = \omega t + \phi_o)$

The phase of a vibrating particle changes with time but the epoch is phase constant.

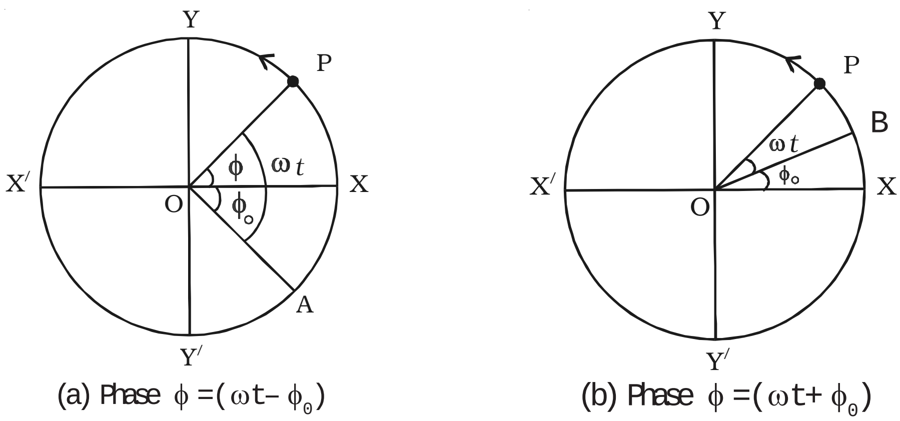


Fig. 6.7 Phase

(i) If the particle *P* starts from the position *X*, the phase of the particle is Zero.

(ii) Instead of counting the time from the instant the particle is at *X*, it is counted from the instant when the reference particle is at *A* (Fig. 6.7a) . Then $\angle XOP = (\omega t - \phi_o)$.

Here $(\omega t - \phi_o) = \phi$ is called the phase of the vibrating particle. $(-\phi_o)$ is initial phase or e*poch.*

(iii) If the time is counted from the instant the particle *P* is above *X* (i.e) at *B*, [Fig. 6.7b] then $(\omega t + \phi_o) = \phi$. Here $(+\phi_o)$ is the *initial phase.*

Phase difference

If two vibrating particles executing SHM with same time period, cross their respective mean positions at the same time in the same direction, they are said to be in phase.

If the two vibrating particles cross their respective mean position at the same time but in the *opposite direction*, they are said to be out of phase (i.e they have a phase difference of π).

If the vibrating motions are represented by equations

$y_1 = a \sin \omega t$ and

$y_2 = a \sin (\omega t - \phi)$

then the phase difference between their phase angles is equal to the phase difference between the two motions.

$\therefore$ phase difference $= \omega t - \phi - \omega t = -\phi$ negative sign indicates that the second motion lags behind the first.

If $y_2 = a \sin (\omega t + \phi)$,

phase difference $= \omega t + \phi - \omega t = \phi$

Here the second motion leads the first motion.

We have discussed the SHM without taking into account the cause of the motion which can be a force (linear SHM) or a torque (angular SHM).

Some examples of SHM

(i) Horizontal and vertical oscillations of a loaded spring.

(ii) Vertical oscillation of water in a U–tube

(iii) Oscillations of a floating cylinder

(iv) Oscillations of a simple pendulum

(v) Vibrations of the prongs of a tuning fork.

6.3 Dynamics of harmonic oscillations

The oscillations of a physical system results from two basic properties namely elasticity and inertia. Let us consider a body displaced from a mean position. The restoring force brings the body to the mean position.

(i) At extreme position when the displacement is maximum, velocity is zero. The acceleration becomes maximum and directed towards the mean position.

(ii) Under the influence of restoring force, the body comes back to the mean position and overshoots because of negative velocity gained at the mean position.

(iii) When the displacement is negative maximum, the velocity becomes zero and the acceleration is maximum in the positive direction. Hence the body moves towards the mean position. Again when the displacement is zero in the mean position velocity becomes positive.

(iv) Due to inertia the body overshoots the mean position once again. This process repeats itself periodically. Hence the system oscillates.

The restoring force is directly proportional to the displacement and directed towards the mean position.

(i.e) $F \;\alpha\; y$

$$F = -ky \qquad \text{... (1)}$$

where k is the force constant. It is the force required to give unit displacement. It is expressed in N m^{-1}.

From Newton's second law, $F = ma$ ...(2)

$$\therefore \quad -k\,y = ma$$

$$\text{or} \quad a = -\frac{k}{m}\,y \qquad \text{...(3)}$$

From definition of SHM acceleration $a = -\omega^2 y$

The acceleration is directly proportional to the negative of the displacement.

Comparing the above equations we get,

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{...(4)}$$

Therefore the period of SHM is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{\textit{inertial factor}}{\textit{spring factor}}} \qquad \text{...(5)}$$

6.4 Angular harmonic oscillator

Simple harmonic motion can also be angular. In this case, the restoring torque required for producing SHM is directly proportional to the angular displacement and is directed towards the mean position.

Consider a wire suspended vertically from a rigid support. Let some weight be suspended from the lower end of the wire. When the wire is twisted through an angle θ from the mean position, a restoring torque acts on it tending to return it to the mean position. Here restoring torque is proportional to angular displacement θ.

Hence $\tau = -C\,\theta$ ...(1)

where C is called torque constant.

It is equal to the moment of the couple required to produce unit angular displacement. Its unit is N m rad^{-1}.

The negative sign shows that torque is acting in the opposite direction to the angular displacement. This is the case of angular simple harmonic motion.

Examples : Torsional pendulum, balance wheel of a watch.

But $\tau = I\,\alpha$ ...(2)

where τ is torque, I is the moment of inertia and α is angular acceleration

∴ Angular acceleration, $\alpha = \frac{\tau}{I} = -\frac{C\theta}{I}$ ...(3)

This is similar to $a = -\omega^2 y$

Replacing y by θ, and a by α we get

$$\alpha = -\omega^2\theta = -\frac{C}{I}\theta$$

$$\therefore \quad \omega = \sqrt{\frac{C}{I}}$$

∴ Period of SHM $T = 2\pi\sqrt{\frac{I}{C}}$

∴ Frequency $n = \frac{1}{T} = \frac{1}{2\pi\sqrt{I/C}} = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$

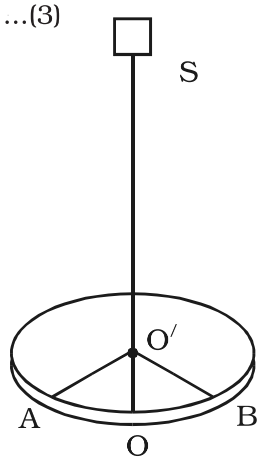


Fig. 6.8 Torsional Pendulum

6.5 Linear simple harmonic oscillator

The block – spring system is a linear simple harmonic oscillator. All oscillating systems like diving board, violin string have some element of springiness, k (spring constant) and some element of inertia, m.

6.5.1 Horizontal oscillations of spring

Consider a mass (m) attached to an end of a spiral spring (which obeys Hooke's law) whose other end is fixed to a support as shown in Fig. 6.9. The body is placed on a smooth horizontal surface. Let the body be displaced through a distance x towards right and released. It will oscillate about its mean position. The restoring force acts in the opposite direction and is proportional to the displacement.

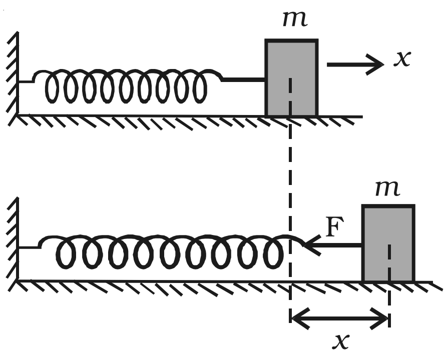


Fig. 6.9 Linear harmonic oscillator

$\therefore$ Restoring force $F = -kx$.

From Newton's second law, we know that $F = ma$

$$\therefore \quad ma = -kx$$

$$a = \frac{-k}{m}x$$

Comparing with the equation of SHM $a = -\omega^2 x$, we get

$$\omega^2 = \frac{k}{m}$$

$$\text{or} \quad \omega = \sqrt{\frac{k}{m}}$$

$$\text{But} \quad T = \frac{2\pi}{\omega}$$

$$\text{Time period} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore \text{ Frequency} \quad n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

6.5.2 Vertical oscillations of a spring

Fig 6.10a shows a light, elastic spiral spring suspended vertically from a rigid support in a relaxed position. When a mass '*m*' is attached to the spring as in Fig. 6.10b, the spring is extended by a small length dl such that the upward force *F* exerted by the spring is equal to the weight *mg*.

The restoring force $F = k\ \mathrm{d}l$; $k\ dl = mg$ ...(1)

where *k* is spring constant. If we further extend the given spring by a small distance by applying a small force by our finger, the spring oscillates up and down about its mean position. Now suppose the body is at a distance y above the equilibrium position as in Fig. 6.10c. The extension of the spring is $(dl - y)$. The upward force exerted on the body is $k\ (dl - y)$ and the resultant force *F* on the body is

$$F = k\ (dl - y) - mg = -ky \qquad ...(2)$$

The resultant force is proportional to the displacement of the body from its equilibrium position and the motion is simple harmonic.

If the total extension produced is $(dl + y)$ as in Fig. 6.10d the restoring force on the body is $k\ (dl + y)$ which acts upwards.

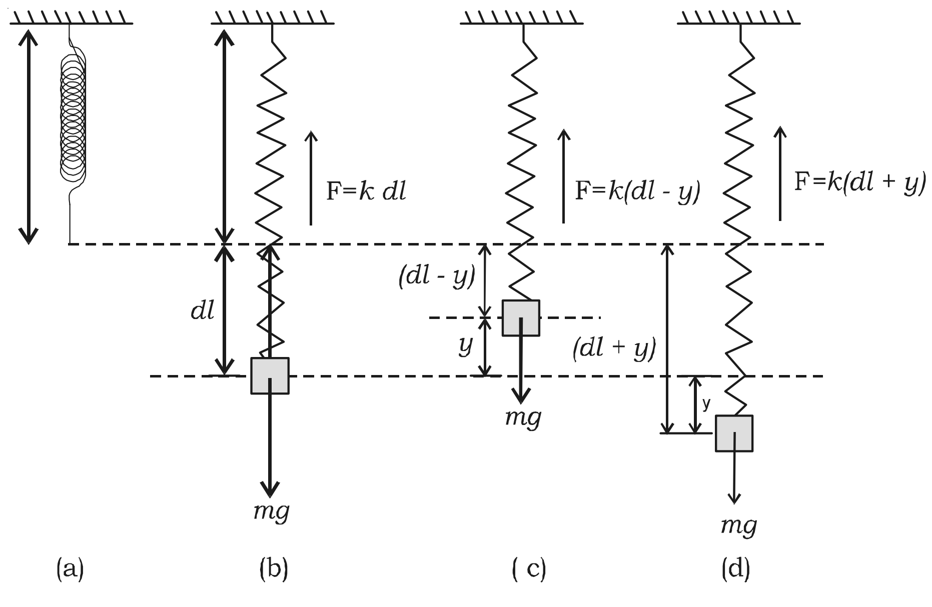


Fig. 6.10 Vertical oscillations of loaded spring

So, the increase in the upward force on the spring is

$$k\,(dl + y) - mg = ky$$

Therefore if we produce an extension downward then the restoring force in the spring increases by ky in the upward direction. As the force acts in the opposite direction to that of displacement, the restoring force is $-ky$ and the motion is SHM.

$$F = -ky$$

$$ma = -ky$$

$$a = -\frac{k}{m}y \quad \text{...(3)}$$

$$a = -\omega^2 y \quad \text{(expression for SHM)}$$

Comparing the above equations, $\omega = \sqrt{\frac{k}{m}}$ ...(4)

But $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ ...(5)

From equation (1) $mg = k\,dl$

$$\frac{m}{k} = \frac{dl}{g}$$

Therefore time period $T = 2\pi\sqrt{\frac{dl}{g}}$ ...(6)

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{g}{dl}}$

Case 1 : *When two springs are connected in parallel*

Two springs of spring factors k_1 and k_2 are suspended from a rigid support as shown in Fig. 6.11. A load m is attached to the combination.

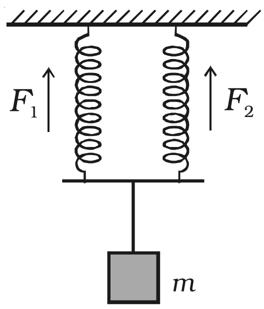


Fig. 6.11 Springs in parallel

Let the load be pulled downwards through a distance y from its equilibrium position. The increase in length is y for both the springs but their restoring forces are different.

If F_1 and F_2 are the restoring forces

$$F_1 = -k_1 y, \qquad F_2 = -k_2 y$$

$\therefore$ Total restoring force = $(F_1 + F_2) = -(k_1 + k_2)\ y$

So, time period of the body is given by

$$T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$$

If $k_1 = k_2 = k$

Then, $$T = 2\pi \sqrt{\frac{m}{2k}}$$

$\therefore$ frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}}$

Case 2 : *When two springs are connected in series.*

Two springs are connected in series in two different ways. This arrangement is shown in Fig. 6.12a and 6.12b.

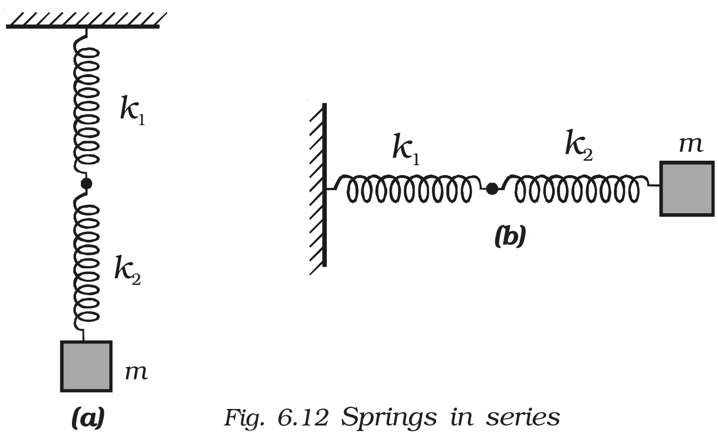


Fig. 6.12 Springs in series

In this system when the combination of two springs is displaced to a distance y, it produces extension y_1 and y_2 in two springs of force constants k_1 and k_2.

$$F = -k_1\ y_1 \quad ; \quad F = -k_2\ y_2$$

where F is the restoring force.

Total extension, $y = y_1 + y_2 = -F\left[\frac{1}{k_1}+\frac{1}{k_2}\right]$

We know that $F = -ky$

$$\therefore\ y = -\frac{F}{k}$$

From the above equations,

$$-\frac{F}{k} = -F\left[\frac{1}{k_1}+\frac{1}{k_2}\right]$$

or $$k = \frac{k_1k_2}{k_1+k_2}$$

$$\therefore \text{ Time period} = T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

$$\text{frequency } n = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)m}}$$

If both the springs have the same spring constant,

$$k_1 = k_2 = k.$$

$$\therefore\ n = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}$$

6.5.3 Oscillation of liquid column in a U - tube

Consider a non viscous liquid column of length l of uniform cross-sectional area A (Fig. 6.13). Initially the level of liquid in the limbs is the same. If the liquid on one side of the tube is depressed by blowing gently the levels of the liquid oscillates for a short time about their initial positions O and C, before coming to rest.

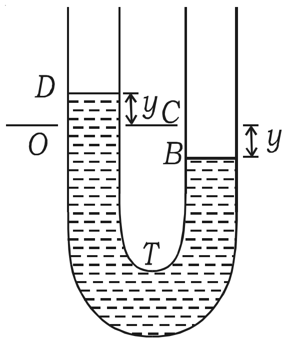


Fig. 6.13 Oscillation of a liquid column in U - tube

If the liquid in one of the limbs is depressed by y , there will be a difference of 2 y in the liquid levels in the two limbs. At some instant, suppose the level of the liquid on the left side of the tube is at D, at a height y above its original position O, the level B of the liquid on the other side is then at a depth y below its original position C. So the excess pressure P on the liquid due to the restoring force is excess height × density × g

(i.e) pressure = $2\ y\ \rho\ g$

$\therefore$ Force on the liquid = pressure × area of the cross-section of the tube

$$= -2\ y\ \rho\ g \times A \qquad \text{..... (1)}$$

The negative sign indicates that the force towards O is opposite to the displacement measured from O at that instant.

The mass of the liquid column of length l is volume × density

(i.e) $m = l\ A\ \rho$

$$\therefore F = l\ A\ \rho\ a \qquad \text{.... (2)}$$

From equations (1) and (2) $l\ A\ \rho\ a = -\ 2\ y\ A\ \rho\ g$

$$\therefore\ a = -\frac{2g}{l}y \qquad \text{..... (3)}$$

We know that $a = -\omega^2 y$

(i.e) $a = -\dfrac{2g}{l}y = -\omega^2 y$ where $\omega = \sqrt{\dfrac{2g}{l}}$

Here, the acceleration is proportional to the displacement, so the motion is simple harmonic and the period T is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{2g}}$$

6.5.4 Oscillations of a simple pendulum

A simple pendulum consists of massless and inelastic thread whose one end is fixed to a rigid support and a small bob of mass m is suspended from the other end of the thread. Let l be the length of the pendulum. When the bob is slightly displaced and released, it oscillates about its equilibrium position. Fig.6.14 shows the displaced position of the pendulum.

Suppose the thread makes an angle θ with the vertical. The distance of the bob from the equilibrium position *A* is *AB*. At *B*, the weight *mg* acts vertically downwards. This force is resolved into two components.

(i) The component *mg cos* θ is balanced by the tension in the thread acting along the length towards the fixed point O.

(ii) *mg sin* θ which is unbalanced, acts perpendicular to the length of thread. This force tends to restore the bob to the mean position. If the amplitude of oscillation is small, then the path of the bob is a straight line.

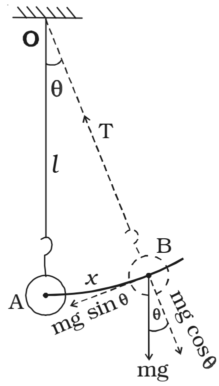


Fig. 6.14 Simple Pendulum – Linear SHM

$\therefore \quad F = -mg \sin \theta$ ...(1)

If the angular displacement is small $\sin \theta \approx \theta$

$\therefore \; F = -mg\,\theta$ ...(2)

But $\theta = \dfrac{x}{l}$

$\therefore \; F = -\,mg\,\dfrac{x}{l}$

Comparing this equation with Newton's second law, $F = ma$ we get, acceleration $a = -\dfrac{gx}{l}$ ...(3)

(negative sign indicates that the direction of acceleration is opposite to the displacement) Hence the motion of simple pendulum is SHM.

We know that $a = -\omega^2 x$

Comparing this with (3)

$$\omega^2 = \frac{g}{l} \text{ or } \omega = \sqrt{\frac{g}{l}} \quad ...(4)$$

$\therefore \quad$ Time period $T = \dfrac{2\pi}{\omega}$

$$T = 2\pi\sqrt{\frac{l}{g}} \quad ...(5)$$

$\therefore \quad$ frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$ ...(6)

Laws of pendulum

From the expression for the time period of oscillations of a pendulum the following laws are enunciated.

(i) The law of length

The period of a simple pendulum varies directly as the square root of the length of the pendulum.

$$\text{(i.e)} \quad T \propto \sqrt{l}$$

(ii) The law of acceleration

The period of a simple pendulum varies inversely as the square root of the acceleration due to gravity.

$$\text{(i.e)} \quad T \propto \frac{1}{\sqrt{g}}$$

(iii) The law of mass

The time period of a simple pendulum is independent of the mass and material of the bob.

(iv) The law of amplitude

The period of a simple pendulum is independent of the amplitude provided the amplitude is small.

Note : The length of a seconds pendulum is 0.99 m whose period is two seconds.

$$2 = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore\ l = \frac{9.81 \times 4}{4\pi^2} = 0.99 \text{ m}$$

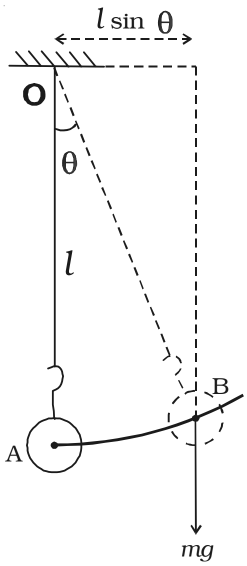


Fig. 6.15 Simple pendulum - Angular SHM

Oscillations of simple pendulum can also be regarded as a case of *angular SHM.*

Let θ be the angular displacement of the bob B at an instant of time. The bob makes rotation about the horizontal line which is perpendicular to the plane of motion as shown in Fig. 6.15.

Restoring torque about O is $\tau = -\,mg\, l \sin\theta$

$$\tau = -m\,g\,l\,\theta \quad [\because \theta \text{ is small}] \qquad ...(1)$$

Moment of inertia
about the axis = $m\,l^2$ ...(2)

If the amplitude is small, motion of the bob is angular simple harmonic. Therefore angular acceleration of the system about the axis of rotation is

$$\alpha = \frac{\tau}{I} = \frac{-mgl\theta}{ml^2}$$

$$\alpha = -\frac{g}{l}\theta \qquad ...(3)$$

We know that $\alpha = -\omega^2\,\theta$ ...(4)

Comparing (3) and (4)

$$-\omega^2\theta = -\frac{g}{l}\,\theta$$

angular frequency $\omega = \sqrt{\frac{g}{l}}$

Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$ ...(5)

Frequency $n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$ ...(6)

6.6 Energy in simple harmonic motion

The total energy (E) of an oscillating particle is equal to the sum of its kinetic energy and potential energy if conservative force acts on it.

The velocity of a particle executing SHM at a position where its displacement is y from its mean position is $v = \omega\sqrt{a^2 - y^2}$

Kinetic energy

Kinetic energy of the particle of mass m is

$$K = \frac{1}{2}m\left[\omega\sqrt{a^2 - y^2}\right]^2$$

$$K = \frac{1}{2}\,m\,\omega^2\,(a^2 - y^2) \qquad ...(1)$$

Potential energy

From definition of SHM $F = -ky$ the work done by the force during the small displacement dy is $dW = -F.dy = -(-ky)\ dy = ky\ dy$

$\therefore$ Total work done for the displacement y is,

$$W = \int dW = \int_0^y ky\ dy$$

$$W = \int_0^y m\omega^2 y\, dy \quad [\because k = m\omega^2]$$

$$\therefore W = \frac{1}{2} m\,\omega^2\, y^2$$

This work done is stored in the body as potential energy

$$U = \frac{1}{2} m\,\omega^2\, y^2 \qquad \ldots(2)$$

Total energy $E = K + U$

$$= \frac{1}{2} m\omega^2\ (a^2 - y^2) + \frac{1}{2} m\,\omega^2\, y^2$$

$$= \frac{1}{2} m\,\omega^2\, a^2$$

Thus we find that the total energy of a particle executing simple harmonic motion is $\frac{1}{2} m\,\omega^2\, a^2$.

Special cases

(i) When the particle is at the mean position $y = 0$, from eqn (1) it is known that kinetic energy is maximum and from eqn. (2) it is known that potential energy is zero. Hence the total energy is wholly kinetic

$$E = K_{max} = \frac{1}{2} m\omega^2 a^2$$

(ii) When the particle is at the extreme position $y = \pm a$, from eqn. (1) it is known that kinetic energy is zero and from eqn. (2) it is known that Potential energy is maximum. Hence the total energy is wholly potential.

$$E = U_{max} = \frac{1}{2}\ m\ \omega^2\ a^2$$

(iii) when $y = \frac{a}{2}$,

$$K = \frac{1}{2}\ m\ \omega^2 \left[a^2 - \frac{a^2}{4}\right]$$

$$\therefore K = \frac{3}{4}\left(\frac{1}{2} m\omega^2 a^2\right)$$

$$K = \frac{3}{4}E$$

$$U = \frac{1}{2} m\ \omega^2 \left(\frac{a}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2} m\ \omega^2 a^2\right)$$

$$\therefore \quad U = \frac{1}{4}\ E$$

If the displacement is half of the amplitude, $K = \frac{3}{4}\ E$ and $U = \frac{1}{4}\ E$. K and U are in the ratio 3 : 1,

$$E = K + U = \frac{1}{2} m\ \omega^2\ a^2$$

At any other position the energy is partly kinetic and partly potential.

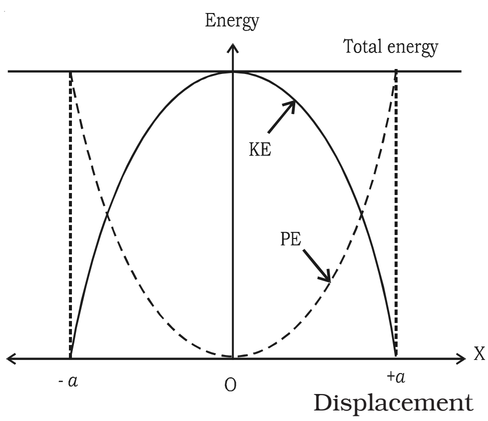


Fig. 6.16 Energy – displacement graph

This shows that the particle executing SHM obeys the law of conservation of energy.

Graphical representation of energy

The values of K and U in terms of E for different values of y are given in the Table 6.2. The variation of energy of an oscillating particle with the displacement can be represented in a graph as shown in the Fig. 6.16.

Table 6.2 *Energy of SHM*

y	0	$\frac{a}{2}$	a	$-\frac{a}{2}$	$-a$
Kinetic energy	E	$\frac{3}{4}$ E	0	$\frac{3}{4}$ E	0
Potential energy	0	$\frac{1}{4}$ E	E	$\frac{1}{4}$ E	E

6.7 Types of oscillations

There are three main types of oscillations.

(i) Free oscillations

When a body vibrates with its own natural frequency, it is said to execute free oscillations. The frequency of oscillations depends on the inertial factor and spring factor, which is given by,

$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Examples

(i) Vibrations of tuning fork

(ii) Vibrations in a stretched string

(iii) Oscillations of simple pendulum

(iv) Air blown gently across the mouth of a bottle.

(ii) Damped oscillations

Most of the oscillations in air or in any medium are damped. When an oscillation occurs, some kind of damping force may arise due to friction or air resistance offered by the medium. So, a part of the energy is dissipated in overcoming the resistive force. Consequently, the amplitude of oscillation decreases with time and finally becomes zero. Such oscillations are called damped oscillations (Fig. 6.17).

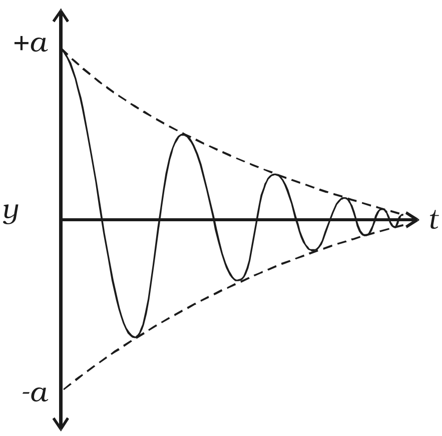


Fig. 6.17 Damped oscillations

Examples :

(i) The oscillations of a pendulum

(ii) Electromagnetic damping in galvanometer (oscillations of a coil in galvanometer)

(iii) Electromagnetic oscillations in tank circuit

(iii) Maintained oscillations

The amplitude of an oscillating system can be made constant by feeding some energy to the system. If an energy is fed to the system to compensate the energy it has lost, the amplitude will be a constant. Such oscillations are called *maintained oscillations* (Fig. 6.18).

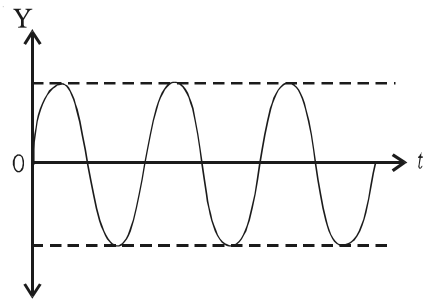


Fig. 6.18 Maintained oscillations

Example :

A swing to which energy is fed continuously to maintain amplitude of oscillation.

(iv) Forced oscillations

When a vibrating body is maintained in the state of vibration by a periodic force of frequency (n) other than its natural frequency of the body, the vibrations are called forced vibrations. The external force is *driver* and body is *driven*.

The body is forced to vibrate with an external periodic force. The amplitude of forced vibration is determined by the difference between the frequencies of the driver and the driven. The larger the frequency difference, smaller will be the amplitude of the forced oscillations.

Examples :

(i) Sound boards of stringed instruments execute forced vibration,

(ii) Press the stem of vibrating tuning fork, against tabla. The tabla suffers forced vibration.

(v) Resonance

In the case of forced vibration, if the frequency difference is small, the amplitude will be large (Fig. 6.19). Ultimately when the two frequencies are same, amplitude becomes maximum.This is a *special case of forced vibration.*

If the frequency of the external periodic force is equal to the natural frequency of oscillation of the system, then the amplitude of oscillation will be large and this is known as resonance.

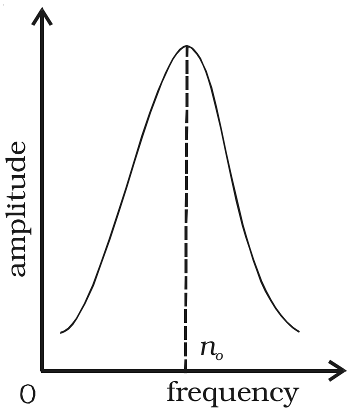


Fig. 6.19 Resonance

Advantages

(i) Using resonance, frequency of a given tuning fork is determined with a sonometer.

(ii) In radio and television, using tank circuit, required frequency can be obtained.

Disadvantages

(i) Resonance can cause disaster in an earthquake, if the natural frequency of the building matches the frequency of the periodic oscillations present in the Earth. The building begins to oscillate with large amplitude thus leading to a collapse.

(ii) A singer maintaining a note at a resonant frequency of a glass, can cause it to shatter into pieces.

Solved problems

6.1 Obtain an equation for the SHM of a particle whose amplitude is 0.05 m and frequency 25 Hz. The initial phase is $\pi/3$.

Data : $a = 0.05\ m$, $n = 25\ Hz$, $\phi_o = \pi/3$.

Solution : $\omega = 2\pi n = 2\pi \times 25 = 50\pi$

The equation of SHM is $y = a \sin(\omega t + \phi_o)$

The displacement equation of SHM is : $y = 0.05 \sin(50\pi t + \pi/3)$

6.2 The equation of a particle executing SHM is $y = 5 \sin\left(\pi t + \frac{\pi}{3}\right)$.

Calculate (i) amplitude (ii) period (iii) maximum velocity and (iv) velocity after 1 second (y is in metre).

Data : $y = 5 \sin\left(\pi t + \frac{\pi}{3}\right)$

Solution : *The equation of SHM is* $y = a \sin(\omega t + \phi_o)$

Comparing the equations

(i) *Amplitude* $a = 5\ m$

(ii) *Period,* $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\ s$

(iii) $v_{max} = a\omega = 5 \times \pi = 15.7\ m\ s^{-1}$

(iv) *Velocity after 1 s* $= a\omega \cos(\omega t + \phi_o)$

$$= 15.7\left[\cos\left(\pi \times 1 + \frac{\pi}{3}\right)\right]$$

$$= 15.7 \times \frac{1}{2} = 7.85\ m\ s^{-1}$$

$$\therefore v = 7.85\ m\ s^{-1}$$

6.3 A particle executes a simple harmonic motion of time period T. Find the time taken by the particle to have a displacement from mean position equal to one half of the amplitude.

Solution : *The displacement is given by* $y = a \sin \omega t$

When the displacement $y = \frac{a}{2}$,

$$\text{we get} \quad \frac{a}{2} = a \sin \omega t$$

$$\text{or} \quad \sin \omega t = \frac{1}{2}$$

$$\omega t = \frac{\pi}{6}$$

$$\therefore \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6.\frac{2\pi}{T}}$$

The time taken is $t = \frac{T}{12}$ *s*

6.4 The velocities of a particle executing SHM are 4 cm s^{-1} and 3 cm s^{-1}, when its distance from the mean position is 2 cm and 3 cm respectively. Calculate its amplitude and time period.

Data : $v_1 = 4\ cm\ s^{-1} = 4 \times 10^{-2}\ m\ s^{-1}$; $v_2 = 3\ cm\ s^{-1} = 3 \times 10^{-2}\ m\ s^{-1}$

$y_1 = 2\ cm = 2 \times 10^{-2}\ m$; $y_2 = 3\ cm = 3 \times 10^{-2}\ m$

Solution : $v_1 = \omega \sqrt{a^2 - y_1^2}$ ... (1)

$v_2 = \omega \sqrt{a^2 - y_2^2}$ ... (2)

Squaring and dividing the equations

$$\left(\frac{v_1}{v_2}\right)^2 = \frac{a^2 - y_1^2}{a^2 - y_2^2}$$

$$\left(\frac{4 \times 10^{-2}}{3 \times 10^{-2}}\right)^2 = \frac{a^2 - 4 \times 10^{-4}}{a^2 - 9 \times 10^{-4}}$$

$$9a^2 - 36 \times 10^{-4} = 16a^2 - 144 \times 10^{-4}$$

$$7a^2 = 108 \times 10^{-4}$$

$$\therefore a = \sqrt{15.42} \times 10^{-2} = 0.03928\ m$$

Substituting the value of a^2 in equation (1)

we have

$$4 \times 10^{-2} = \omega \sqrt{\frac{108 \times 10^{-4}}{7} - 4 \times 10^{-4}}$$

$$\therefore \quad \omega = \sqrt{\frac{7}{5}} \ rad\ s^{-1}$$

$$\therefore \quad \text{Time period } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{5}{7}}$$

$$T = 5.31\ s$$

6.5 A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5 s. The radius of the disc is 15 cm. Calculate the torsional spring constant.

Data : *$m = 10\ kg$, $T = 1.5\ s$, $r = 15\ cm = 15 \times 10^{-2}\ m$ $\quad C = ?$*

Solution : *MI of the disc about an axis through the centre is*

$$I = \frac{1}{2} MR^2$$

The time period of angular SHM is $T = 2\pi\sqrt{\frac{I}{C}}$

Squaring the equation, $T^2 = 4\pi^2 \frac{I}{C}$

$$\therefore \quad C = \frac{4\pi^2 I}{T^2}$$

$$C = \frac{4\pi^2 \times \frac{1}{2} MR^2}{T^2}$$

$$= \frac{2 \times (3.14)^2 \times 10 \times 0.15^2}{(1.5)^2}$$

$$C = 2.0\ N\ m\ rad^{-1}$$

6.6 A body of mass 2 kg executing SHM has a displacement $y = 3 \sin\left(100\,t + \frac{\pi}{4}\right)$ cm. Calculate the maximum kinetic energy of the body.

Solution : *Comparing with equation of SHM*

$$y = a \sin(\omega t + \phi_o)$$

$a = 3\ cm = 3 \times 10^{-2}\ m$, $\omega = 100\ rad\ s^{-1}$, $m = 2\ kg$

$$y = 3 \sin\left(100\ t + \frac{\pi}{4}\right)$$

$$\text{Maximum kinetic energy} = \frac{1}{2} ma^2 \omega^2$$

$$= \frac{1}{2} \times 2 \times (0.03^2 \times 100^2)$$

Maximum kinetic energy = 9 joule

6.7 A block of mass 15 kg executes SHM under the restoring force of a spring. The amplitude and the time period of the motion are 0.1 m and 3.14 s respectively. Find the maximum force exerted by the spring on the block.

Data : *$m = 15\ kg$, $a = 0.1\ m$ and $T = 3.14\ s$*

Solution : *The maximum force exerted on the block is ka, when the block is at the extreme position, where k is the spring constant.*

$$\text{The angular frequency} = \omega = \frac{2\pi}{T} = 2\ s^{-1}$$

$$\text{The spring constant } k = m\omega^2$$

$$= 15 \times 4 = 60\ N\ m^{-1}$$

The maximum force exerted on the block is $ka = 60 \times 0.1 = 6\ N$

6.8 A block of mass 680 g is attached to a horizontal spring whose spring constant is 65 Nm^{-1}. The block is pulled to a distance of 11 cm from the mean position and released from rest. Calculate : (i) angular frequency, frequency and time period (ii) displacement of the system (iii) maximum speed and acceleration of the system

Data : $m = 680\ g = 0.68\ kg$, $k = 65\ N\ m^{-1}$, $a = 11\ cm = 0.11\ m$

Solution : *The angular frequency* $\omega = \sqrt{\frac{k}{m}}$

$$\omega = \sqrt{\frac{65}{0.68}} = 9.78\ rad\ s^{-1}$$

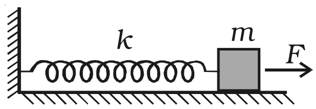
The frequency $n = \frac{\omega}{2\pi} = \frac{9.78}{2\pi} = 1.56\ Hz$

The time period $T = \frac{1}{n} = \frac{1}{1.56} = 0.64\ s$

maximum speed $= a\,\omega$

$= 0.11 \times 9.78$

$= 1.075\ m\,s^{-1}$



Acceleration of the block $= a\,\omega^2 = a\omega \times \omega$

$= 1.075 \times (9.78)$

$= 10.52\ m\,s^{-2}$

Displacement $y(t) = a \sin \omega t$

$\therefore\ y(t) = 0.11 \sin 9.78\,t$ metre

6.9 A mass of 10 kg is suspended by a spring of length 60 cm and force constant $4 \times 10^3\ N\,m^{-1}$. If it is set into vertical oscillations, calculate the (i) frequency of oscillation of the spring and (ii) the length of the stretched string.

Data : $k = 4 \times 10^3\ N\,m^{-1}$, $F = 10 \times 9.8\ N$, $l = 60 \times 10^{-2}\ m$, $m = 10\ kg$

Solution : (i) $n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{4 \times 10^3}{10}} = \frac{20}{2\pi}$

Frequency $= 3.184\ Hz$

(ii) $T = 2\pi\sqrt{\frac{dl}{g}}$ or $T^2 = 4\pi^2 \frac{dl}{g}$

length $(dl) = \frac{T^2 g}{4\pi^2} = \frac{1}{n^2} \times \frac{g}{4\pi^2}$

$\therefore\ dl = \frac{9.8}{(3.184)^2 \times 4 \times (3.14)^2}$

$dl = 0.0245\ m$

$\therefore$ The length of the stretched string = 0.6 + 0.0245 = 0.6245 m

6.10 A mass m attached to a spring oscillates every 4 seconds. If the mass is increased by 4 kg, the period increases by 1 s. Find its initial mass m.

Data : *Mass m oscillates with a period of 4 s*

When the mass is increased by 4 kg period is 5 s

Solution : *Period of oscillation* $T = 2\pi\sqrt{\frac{m}{k}}$

$$4 = 2\pi\sqrt{m/k} \quad \text{... (1)}$$

$$5 = 2\pi\sqrt{\frac{m+4}{k}} \quad \text{... (2)}$$

Squaring and dividing the equations

$$\frac{25}{16} = \frac{m+4}{m}$$

$$25\,m = 16\,m + 64$$

$$9m = 64$$

$$\therefore \quad m = \frac{64}{9} = 7.1\ kg$$

6.11 The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of the moon, if its period on the Earth is 3.5 s ?

Data : g on moon = 1.7 m s^{-2}

g on the Earth = 9.8 ms^{-2}

Time period on the Earth = 3.5 s

Solution : $T = 2\pi\sqrt{\frac{l}{g}}$

Let T_m represent the time period on moon

$$T_m = 2\pi\sqrt{\frac{l}{1.7}} \quad \text{... (1)}$$

On the Earth, $$3.5 = 2\pi\sqrt{\frac{l}{9.8}} \quad \text{... (2)}$$

Dividing the equation (2) by (1) and squaring

$$\left(\frac{3.5}{T_m}\right)^2 = \frac{1.7}{9.8}$$

$$T_m^{\ 2} \times 1.7 = (3.5)^2 \times 9.8$$

$$T_m^{\ 2} = \frac{(3.5)^2 \times 9.8}{1.7} = \frac{12.25 \times 9.8}{1.7}$$

$$\therefore \quad T_m = \sqrt{\frac{120.05}{1.7}} = 8.40\ s$$

6.12 A simple pendulum has a period 4.2 s. When the pendulum is shortened by 1 m the period is 3.7 s. Calculate its (i) acceleration due to gravity (ii) original length of the pendulum.

Data : *T = 4.2 s ; when length is shortened by 1m the period is 3.7 s.*

Solution : $T = 2\pi\sqrt{\frac{l}{g}}$

Squaring and rearranging $g = 4\pi^2 \frac{l}{T^2}$

$$g = 4\pi^2 \frac{l}{(4.2)^2} \qquad ...(1)$$

When the length is shortened by 1 m

$$g = \frac{4\pi^2(l\text{-}1)}{(3.7)^2} \qquad ...(2)$$

From the above equations

$$\frac{l}{(4.2)^2} = \frac{l-1}{(3.7)^2}$$

$$(7.9 \times 0.5)\ l = 17.64$$

$$l = \frac{17.64}{7.9 \times 0.5} = 4.46\ m$$

Substituting in equation (1)

$$g = 4\pi^2 \frac{4.46}{(4.2)^2} = \frac{175.89}{17.64}$$

$$g = 9.97\ m\ s^{-2}$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

6.1 *Which of the following is the necessary condition for SHM?*

(a) constant period

(b) constant acceleration

(c) displacement and acceleration are proportional

(d) displacement and torque are proportional

6.2 *The displacement of a particle executing SHM is given by $x = 0.01 \sin (100 \pi t + 0.05)$. Its time period is*

(a) 0.01 s	*(b) 0.02 s*
(c) 0.1 s	*(d) 0.2 s*

6.3 *If the displacement of a particle executing SHM is given by $y = 0.05 \sin (100\, t + \frac{\pi}{2})$ cm. The maximum velocity of the particle is*

(a) 0.5 cm s^{-1}	*(b) 0.05 m s^{-1}*
(c) 100 m s^{-1}	*(d) 50 m s^{-1}*

6.4 *If the magnitude of displacement is equal to acceleration, then the time period is,*

(a) 1 s	*(b) π s*
(c) 2π s	*(d) 4π s*

6.5 *A body of mass 2 g is executing SHM about a mean position with an amplitude 10 cm. If the maximum velocity is 100 cm s^{-1} its velocity is 50 cm s^{-1} at a distance of (in cm).*

(a) $5\sqrt{2}$	*(b)* $50\sqrt{3}$
(c) $5\sqrt{3}$	*(d)* $10\sqrt{3}$

6.6 *A linear harmonic oscillator has a total energy of 160 J. Its*

(a) maximum potential energy is 100 J

(b) maximum kinetic energy is 160 J

(c) minimum potential energy is 100 J

(d) maximum kinetic energy is 100 J

6.7 *A force of 6.4 N stretches a vertical spring by 0.1 m. The mass that must be suspended from the spring so that it oscillates with a period of $\frac{\pi}{4}$ s is*

(a) $\frac{\pi}{4}$ kg (b) 1 kg

(c) $\frac{1}{4}$ kg (d) 10 kg

6.8 *The length of seconds pendulum at a place where $g = 9.8$ m s^{-2} is*

(a) 0.25 m (b) 1 m

(c) 0.99 m (d) 0.50 m

6.9 *A particle executes SHM with an amplitude 4 cm. At what displacement from the mean position its energy is half kinetic and half potential?*

(a) $2\sqrt{2}$ cm (b) $\sqrt{2}$ cm

(c) 2 cm (d) 1 cm

6.10 *A particle executes SHM along a straight line with an amplitude 'a'. PE is maximum when the displacement is*

(a) $\pm a$ (b) zero

(c) $+\frac{a}{2}$ (d) $\frac{a}{\sqrt{2}}$

6.11 *Define simple harmonic motion. What are the conditions of SHM?*

6.12 *Every SHM is periodic motion but every periodic motion need not be SHM. Why? Support your answer with an example.*

6.13 *Show that the projection of uniform circular motion on the diameter of a circle is simple harmonic motion.*

6.14 *Explain : (i) displacement (ii) velocity and (iii) acceleration in SHM using component method.*

6.15 *Show graphically the variation of displacement, velocity and acceleration of a particle executing SHM.*

6.16 *What is the phase difference between (i) velocity and acceleration (ii) acceleration and displacement of a particle executing SHM?*

6.17 *Derive the differential formula for SHM.*

6.18 *Define the terms (i) time period (ii) frequency and (iii) angular frequency.*

6.19 *Define force constant. Give its unit and dimensional formula.*

6.20 *What is phase of SHM? Explain the term phase difference.*

6.21 *Derive an expression for the time period of a body when it executes angular SHM.*

6.22 *What is an epoch? Give its unit.*

6.23 *Explain the oscillations of a mass attached to a horizontal spring. Hence deduce an expression for its time period.*

6.24 *Obtain an expression for the frequency of vertical oscillations of a loaded spring.*

6.25 *Distinguish between linear and angular harmonic oscillator?*

6.26 *What is a spring factor?*

6.27 *Show that the oscillations of a simple pendulum are simple harmonic. Hence deduce the expression for the time period.*

6.28 *The bob of a simple pendulum is a hollow sphere filled with water. How does the period of oscillation change if the water begins to drain out of the sphere?*

6.29 *Why does the oscillation of a simple pendulum eventually stop?*

6.30 *What will happen to the time period of a simple pendulum if its length is doubled?*

6.31 *Derive an expression for the total energy of a particle executing SHM.*

6.32 *On what factors the natural frequency of a body depend on?*

6.33 *What is forced vibration? Give an example.*

6.34 *What forces keep the simple pendulum in SHM?*

6.35 *Illustrate an example to show that resonance is disastrous sometimes.*

6.36 *If two springs are connected in parallel, what is its equivalent spring constant?*

6.37 *If two springs are connected in series, what is its equivalent spring constant?*

Problems

6.38 *Obtain an equation for the SHM of a particle of amplitude 0.5 m, frequency 50 Hz. The initial phase is* $\frac{\pi}{2}$. *Find the displacement at t = 0.*

6.39 *The equation of SHM is represented by y = 0.25 sin (3014 t + 0.35), where y and t are in mm and s respectively. Deduce (i) amplitude (ii) frequency (iii) angular frequency (iv) period and (v) initial phase.*

6.40 *A particle executing SHM is represented by* $y = 2 \sin\left(2\pi \frac{t}{T} + \phi_o\right)$.

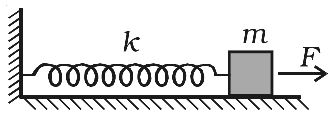
At t = 0, the displacement is $\sqrt{3}$ *cm. Find the initial phase.*

6.41 *A particle executing SHM has angular frequency of* π *rad* s^{-1} *and amplitude of 5 m. Deduce (i) time period (ii) maximum velocity (iii) maximum acceleration (iv) velocity when the displacement is 3 m.*

6.42 *A body executes SHM with an amplitude 10 cm and period 2 s. Calculate the velocity and acceleration of the body when the displacement is i) zero and ii) 6 cm.*

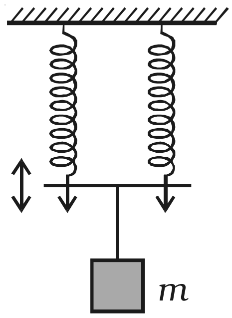
6.43 *A disc suspended by a wire, makes angular oscillations. When it is displaced through* 30^o *from the mean position, it produces a restoring torque of 4.6 N m. If the moment of inertia of the disc is 0.082 kg* m^2, *calculate the frequency of angular oscillations.*

6.44 *A spring of force constant 1200 N* m^{-1} *is mounted on a horizontal table as shown in figure. A mass of 3 kg is attached to its free end and pulled side ways to a distance of 2 cm and released. Calculate (i) the frequency of oscillation (ii) the maximum velocity and (iii) maximum acceleration of the mass.*

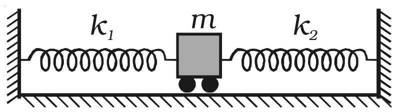


6.45 *A mass of 0.2 kg attached to one end of a spring produces an extension of 15 mm. The mass is pulled 10 mm downwards and set into vertical oscillations of amplitude 10 mm. calculate (i) the period of oscillation (ii) maximum kinetic energy.*

6.46 *A 5 kg mass is suspended by a system of two identical springs of spring constant 250 N m^{-1} as shown in figure. Determine the period of oscillation the system.*



6.47 *A trolley of mass 2 kg is connected between two identical springs of spring constant 400 N m^{-1}. If the trolley is displaced from its mean position by 3 cm and released, calculate its (i) time period (ii) maximum velocity (iii) maximum kinetic energy.*



6.48 *A vertical U tube of uniform cross section contains water to a height of 0.3 m. Show that, if water in one of the limbs is depressed and then released, the oscillations of the water column in the tube are SHM. Calculate its time period also.*

6.49 *A bob of a simple pendulum oscillates with an amplitude of 4 cm and time period 1 s. Find (i) length of the pendulum and (ii) velocity of the bob in the mean position.*

6.50 *Compare the acceleration due to gravity at two places if the time for 100 oscillations of a simple pendulum are 8 minutes 2 seconds and 8 minutes 20 seconds respectively of the two places.*

6.51 *A particle of mass 0.2 kg executes SHM of amplitude 2 cm and time period 6 s. Calculate (i) the total energy (ii) kinetic and potential energy when displacement is 1 cm from the mean position.*

6.52 *The length of a seconds pendulum in a clock is increased by 2%. How many seconds will it lose or gain in a day?*

Answers

6.1 *(c)*	***6.2*** *(b)*	***6.3*** *(b)*	***6.4*** *(c)*
6.5 *(c)*	***6.6*** *(b)*	***6.7*** *(b)*	***6.8*** *(c)*
6.9 *(a)*	***6.10*** *(a)*		

6.38 0.5 m

6.39 0.25×10^{-3} m, 480 Hz, 3014 rad s^{-1}, 0.0021 s, 0.35 rad

6.40 60^o

6.41 2 s, 15.7 m s^{-1}, 49.3 m s^{-2}, 12.56 m s^{-1}

6.42 0.314 m s^{-1}, zero; 0.2512 m s^{-1}, 0.5915 m s^{-2}

6.43 1.64 Hz

6.44 3.2 Hz, 0.40 m s^{-1}, 8.07 m s^{-2}

6.45 0.25 s, 6.533×10^{-3} J

6.46 0.628 s

6.47 0.314 s, 0.6 m s^{-1}, 0.36 J

6.48 1.0098 s

6.49 0.25 m, 0.2512 m s^{-1}

6.50 1.076

6.51 4.386×10^{-5} J, 3.286×10^{-5} J, 1.1×10^{-5} J

6.52 *loss of time is 864 s*

7. Wave Motion

Wave motion is a mode of transmission of energy through a medium in the form of a disturbance. It is due to the repeated periodic motion of the particles of the medium about an equilibrium position transferring the energy from one particle to another.

The waves are of three types - mechanical, electromagnetic and matter waves. Mechanical waves can be produced only in media which possess elasticity and inertia. Water waves, sound waves and seismic waves are common examples of this type. Electromagnetic waves do not require any material medium for propagation. Radio waves, microwaves, infrared rays, visible light, the ultraviolet rays, X rays and γ rays are electromagnetic waves. The waves associated with particles like electrons, protons and fundamental particles in motion are matter waves.

Waves on surface of water

In order to understand the concept of wave motion, let us drop a stone in a trough of water. We find that small circular waves seem to originate from the point where the stone touches the surface of water. These waves spread out in all directions. It appears as if water moves away from that point. If a piece of paper is placed on the water surface, it will be observed that the piece of paper moves up and down, when the waves pass through it. This shows that the waves are formed due to the vibratory motion of the water particles, about their mean position.

Wave motion is a form of disturbance which travels through a medium due to the repeated periodic motion of the particles of the medium about their mean position. The motion is transferred continuously from one particle to its neighbouring particle.

7.1 Characteristics of wave motion

(i) Wave motion is a form of disturbance travelling in the medium due to the periodic motion of the particles about their mean position.

(ii) It is necessary that the medium should possess elasticity and inertia.

(iii) All the particles of the medium do not receive the disturbance at the same instant (i.e) each particle begins to vibrate a little later than its predecessor.

(iv) The wave velocity is different from the particle velocity. The velocity of a wave is constant for a given medium, whereas the velocity of the particles goes on changing and it becomes maximum in their mean position and zero in their extreme positions.

(v) During the propagation of wave motion, there is transfer of energy from one particle to another without any actual transfer of the particles of the medium.

(vi) The waves undergo reflection, refraction, diffraction and interference.

7.1.1 Mechanical wave motion

The two types of mechanical wave motion are (i) transverse wave motion and (ii) longitudinal wave motion

(i) Transverse wave motion

Transverse wave motion is that wave motion in which particles of the medium execute SHM about their mean positions in a direction perpendicular to the direction of propagation of the wave. Such waves are called transverse waves. Examples of transverse waves are waves produced by plucked strings of veena, sitar or violin and electromagnetic waves. Transverse waves travel in the form of crests and troughs. *The maximum displacement of the particle in the positive direction i.e. above its mean position is called crest and maximum displacement of the particle in the negative direction i.e below its mean position is called trough.*

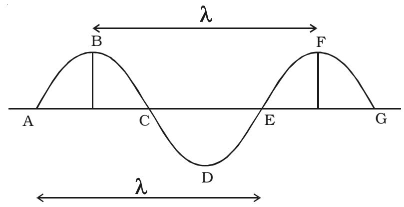


Fig. 7.1 Transverse wave

Thus if ABCDEFG is a transverse wave, the points B and F are crests while D is trough (Fig. 7.1).

For the propagation of transverse waves, the medium must possess force of cohesion and volume elasticity. Since gases and liquids do not have rigidity (cohesion), transverse waves cannot be produced in gases and liquids. Transverse waves can be produced in solids and surfaces of liquids only.

(ii) Longitudinal wave motion

'*Longitudinal wave motion is that wave motion in which each particle of the medium executes simple harmonic motion about its mean position along the direction of propagation of the wave.*'

Sound waves in fluids (liquids and gases) are examples of longitudinal wave. When a longitudinal wave travels through a medium, it produces compressions and rarefactions.

In the case of a spiral spring, whose one end is tied to a hook of a wall and the other end is moved forward and backward, the coils of the spring vibrate about their original position along the length of the spring and longitudinal waves propagate through the spring (Fig.7.2).

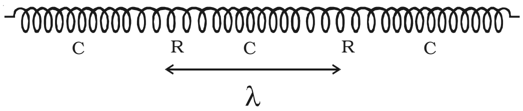


Fig. 7.2 Compression and rarefaction in spring

The regions where the coils are closer are said to be in the state of compression, while the regions where the coils are farther are said to be in the state of rarefaction.

When we strike a tuning fork on a rubber pad, the prongs of the tuning fork begin to vibrate to and fro about their mean positions. When the prong A moves outwards to A_1, it compresses the layer of air in its neighbourhood. As the compressed layer moves forward it compresses the next layer and a wave of compression passes through air. But when the prong moves inwards to A_2, the particles of the medium which moved to the right, now move backward to the left due to elasticity of air. This gives rise to rarefaction.

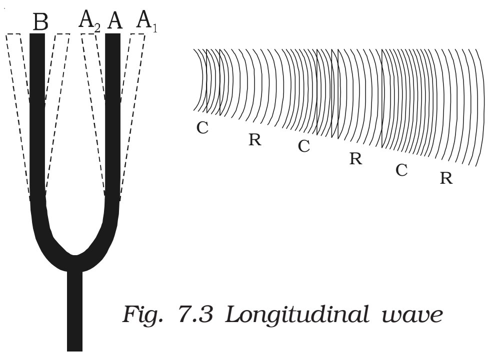


Fig. 7.3 Longitudinal wave

Thus a longitudinal wave is characterised by the formation of compressions and rarefactions following each other.

Longitudinal waves can be produced in all types of material medium, solids, liquids and gases. The density and pressure of the medium in the region of compression are more than that in the region of rarefaction.

7.1.2 Important terms used in wave motion

(i) Wavelength (λ)

The distance travelled by a wave during which a particle of the medium completes one vibration is called wavelength. It is also defined as the distance between any two nearest particles on the wave having same phase.

Wavelength may also be defined as the distance between two successive crests or troughs in transverse waves, or the distance between two successive compressions or rarefactions in longitudinal waves.

(ii) Time period (T)

The time period of a wave is the time taken by the wave to travel a distance equal to its wavelength.

(iii) Frequency (n)

This is defined as the number of waves produced in one second. If T represents the time required by a particle to complete one vibration, then it makes $\frac{1}{T}$ waves in one second.

Therefore frequency is the reciprocal of the time period (i.e) $n = \frac{1}{T}$.

Relationship between velocity, frequency and wavelength of a wave

The distance travelled by a wave in a medium in one second is called the velocity of propagation of the wave in that medium. If v represents the velocity of propagation of the wave, it is given by

$$Velocity = \frac{Distance\ travelled}{Time\ taken}$$

$$v = \frac{\lambda}{T} = n\lambda \qquad \left[\because n = \frac{1}{T}\right]$$

The velocity of a wave (v) is given by the product of the frequency and wavelength.

7.2 Velocity of wave in different media

The velocity of mechanical wave depends on elasticity and inertia of the medium.

7.2.1 Velocity of a transverse wave along a stretched string

Let us consider a string fixed at one of its ends and tension be applied at the other end. When the string is plucked at a point, it begins to vibrate.

Consider a transverse wave proceeding from left to right in the form of a pulse when the string is plucked at a point as shown in Fig. 7.4. *EF* is the displaced position of the string at an instant of time. It forms an arc of a circle with *O* as centre and R as radius. The arc *EF* subtends an angle 2θ at *O*.

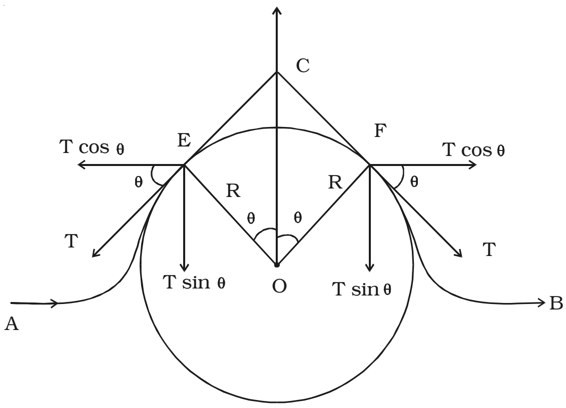


Fig. 7.4 Transverse vibration of a string

If *m* is the mass per unit length of the string and *dx* is the length of the arc *EF*, then the mass of the portion of the string is *m dx*.

$$\therefore \text{ Centripetal force } = \frac{m.dx.v^2}{R} \quad ...(1)$$

This force is along CO. To find the resultant of the tension T at the points E and F, we resolve T into two components Tcos θ and T sinθ.

T cosθ components acting perpendi- cular to CO are of equal in magnitude but opposite in direction, they cancel each other.

T sin θ components act parallel to CO. Therefore the resultant of the tensions acting at E and F is 2T sin θ. It is directed along CO. If θ is small, sinθ = θ and the resultant force due to tension is 2Tθ.

resultant force = 2Tθ

$$= 2T.\frac{dx}{2R} \qquad \left(\because 2\theta = \frac{dx}{R}\right)$$

$$= T.\frac{dx}{R} \qquad \ldots (2)$$

For the arc EF to be in equilibrium,

$$\frac{m.dx\, v^2}{R} = \frac{T.dx}{R}$$

$$v^2 = \frac{T}{m}$$

$$\text{or} \quad v = \sqrt{\frac{T}{m}} \qquad \ldots (3)$$

7.2.2 Velocity of longitudinal waves in an elastic medium

Velocity of longitudinal waves in an elastic medium is

$$v = \sqrt{\frac{E}{\rho}} \qquad \ldots(1)$$

where E is the modulus of elasticity, ρ is the density of the medium.

(i) In the case of a solid rod

$$v = \sqrt{\frac{q}{\rho}} \qquad \ldots(2)$$

where q is the Young's modulus of the material of the rod and ρ is the density of the rod.

(ii) In liquids, $v = \sqrt{\frac{k}{\rho}}$ ...(3)

where k is the Bulk modulus and ρ is the density of the liquid.

7.2.3 Newton's formula for the velocity of sound waves in air

Newton assumed that sound waves travel through air under isothermal conditions (i.e) temperature of the medium remains constant.

The change in pressure and volume obeys Boyle's law.

$\therefore$ PV = constant

Differentiating, $P \,.\, dV + V\,.dP = 0$

$$P.\ dV = -V\ dP$$

$$\therefore\ P = \frac{-dP}{\left(\frac{dV}{V}\right)} = \frac{\textit{change in pressure}}{\textit{volume strain}}$$

P = k (Volume Elasticity)

Therefore under isothermal condition, $P = k$

$$v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{P}{\rho}}$$

where P is the pressure of air and ρ is the density of air. The above equation is known as Newton's formula for the velocity of sound waves in a gas.

At NTP, P = 76 cm of mercury

= $(0.76 \times 13.6 \times 10^3 \times 9.8)\ N\ m^{-2}$

$\rho = 1.293$ kg m^{-3}.

∴ Velocity of sound in air at NTP is

$$v = \sqrt{\frac{0.76 \times 13.6 \times 10^3 \times 9.8}{1.293}} = 280\ \text{m s}^{-1}$$

The experimental value for the velocity of sound in air is 332 m s^{-1}. But the theoretical value of 280 m s^{-1} is 15% less than the experimental value. This discrepancy could not be explained by Newton's formula.

7.2.4 Laplace's correction

The above discrepancy between the observed and calculated values was explained by Laplace in 1816. Sound travels in air as a longitudinal wave. The wave motion is therefore, accompanied by compressions and rarefactions. At compressions the temperature of air rises and at rarefactions, due to expansion, the temperature decreases.

Air is a very poor conductor of heat. Hence at a compression, air cannot lose heat due to radiation and conduction. At a rarefaction it cannot gain heat, during the small interval of time. As a result, the temperature throughout the medium does not remain constant.

Laplace suggested that sound waves travel in air under adiabatic condition and not under isothermal condition.

For an adiabatic change, the relation between pressure and volume is given by

$$P\,V^{\gamma} = \text{constant}$$

where $\gamma = \left(\frac{C_P}{C_V}\right)$ is the ratio of two specific heat capacities of the gas.

Differentiating

$$P\gamma\,V^{\gamma-1}\,.\,dV + V^{\gamma}\,dP = 0$$

$$P\gamma = \frac{-V^{\gamma}dP}{V^{\gamma-1}dV}$$

$$P\gamma = -V.\frac{dP}{dV}$$

$$P\gamma = \frac{-dP}{\left(\frac{dV}{V}\right)} = k$$

$\therefore$ $P\gamma = k$ (Volume elasticity)

Therefore under adiabatic condition

velocity of sound $v = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

This is Laplace's corrected formula.

For air at NTP

$\gamma = 1.41$, $\rho = 1.293$ kg m^{-3}

$\therefore$ $v = \sqrt{\gamma}\,\sqrt{\frac{P}{\rho}} = \sqrt{1.41} \times 280 = 331.3$ ms^{-1}

This result agrees with the experimental value of 332 ms^{-1}.

7.2.5 Factors affecting velocity of sound in gases

(i) Effect of pressure

If the temperature of the gas remains constant, then by Boyle's law PV = constant

i.e $P\,.\,\frac{m}{\rho} = \text{constant}$

$\frac{P}{\rho}$ is a constant, when mass (m) of a gas is constant. If the pressure changes from P to P' then the corresponding density also will change from ρ to ρ' such that $\frac{P}{\rho}$ is a constant.

In Laplace's formula $\sqrt{\frac{\gamma P}{\rho}}$ is also a constant. Therefore the *velocity of sound in a gas is independent of the change in pressure provided the temperature remains constant.*

(ii) Effect of temperature

For a gas, $PV = RT$

$$P.\frac{m}{\rho} = RT$$

$$\text{or} \quad \frac{P}{\rho} = \frac{RT}{m}$$

where m is the mass of the gas, T is the absolute temperature and R is the gas constant.

Therefore $v = \sqrt{\frac{\gamma RT}{m}}$

It is clear that the velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

Let v_o and v_t be the velocity of sound at $O^o C$ and $t^o C$ respectively. Then, from the above equation,

$$v_o = \sqrt{\frac{\gamma R}{m}} \times \sqrt{273}$$

$$v_t = \sqrt{\frac{\gamma R}{m}} \times \sqrt{273+t}$$

$$\therefore \frac{v_t}{v_o} = \sqrt{\frac{273+t}{273}}$$

$$\therefore v_t = v_o \left(1+\frac{t}{273}\right)^{1/2}$$

Using binomial expansion and neglecting higher powers we get,

$$v_t = v_o \left(1+\frac{1}{2}.\frac{t}{273}\right)$$

$$v_t = v_o \left(1+\frac{t}{546}\right)$$

Since $\quad v_o = 331\, m\, s^{-1}\, at\, 0^o C$

$$v_t = 331 + 0.61 \ m\, s^{-1}$$

Thus the velocity of sound in air increases by 0.61 m s^{-1} per degree centigrade rise in temperature.

(iii) Effect of density

Consider two different gases at the same temperature and pressure with different densities. The velocity of sound in two gases are given by

$$v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}} \quad \text{and} \quad v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \cdot \frac{\rho_2}{\rho_1}}$$

For gases having same value of γ, $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

The velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

(iv) Effect of humidity

When the humidity of air increases, the amount of water vapour present in it also increases and hence its density decreases, because the density of water vapour is less than that of dry air. Since velocity of sound is inversely proportional to the square root of density, the sound travels faster in moist air than in dry air. Due to this reason it can be observed that on a rainy day sound travels faster.

(v) Effect of wind

The velocity of sound in air is affected by wind. If the wind blows with the velocity w along the direction of sound, then the velocity of sound increases to $v + w$. If the wind blows in the opposite direction to the direction of sound, then the velocity of sound decreases to $v - w$. If the wind blows at an angle θ with the direction of sound, the effective velocity of sound will be $(v + w \cos \theta)$.

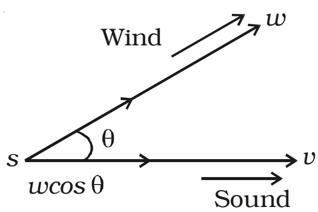


Fig. 7.5 Effect of wind

Note: In a medium, sound waves of different frequencies or wavelengths travel with the same velocity. Hence there is no effect of frequency on the velocity of sound.

Table 7.1 Velocity of sound in various media
(NOT FOR EXAMINATION)

	Medium	Velocity (ms^{-1})
Gases	Air 0º C	331
	Air 20º C	343
	Helium	965
	Hydrogen	1284
Liquids	Water 0º C	1402
	Water at 20º C	1482
	Sea water	1522
Solids	Aluminum	6420
	Steel	5921
	Granite	6000

7.3 Progressive wave

A progressive wave is defined as the onward transmission of the vibratory motion of a body in an elastic medium from one particle to the successive particle.

7.3.1 Equation of a plane progressive wave

An equation can be formed to represent generally the displacement of a vibrating particle in a medium through which a wave passes. Thus each particle of a progressive wave executes simple harmonic motion of the same period and amplitude differing in phase from each other.

Let us assume that a progressive wave travels from the origin O along the positive direction of X axis, from left to right (Fig. 7.6). The displacement of a particle at a given instant is

$$y = a \sin \omega t \qquad \dots (1)$$

where a is the amplitude of the vibration of the particle and $\omega = 2\pi n$.

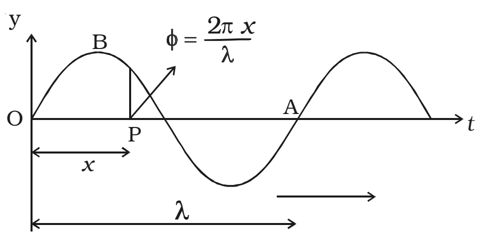


Fig. 7.6 Plane Progressive wave

The displacement of the particle P at a distance x from O at a given instant is given by,

$$y = a \sin(\omega t - \phi) \quad \text{... (2)}$$

If the two particles are separated by a distance λ, they will differ by a phase of 2π. Therefore, the phase ϕ of the particle P at a distance x is $\phi = \frac{2\pi}{\lambda}.x$

$$y = a \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \quad \text{...(3)}$$

Since $\omega = 2\pi n = 2\pi\frac{v}{\lambda}$, the equation is given by

$$y = a \sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)$$

$$y = a \sin\frac{2\pi}{\lambda}(vt - x) \quad \text{...(4)}$$

Since $\omega = \frac{2\pi}{T}$, the eqn. (3) can also be written as

$$y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad \text{...(5)}$$

If the wave travels in opposite direction, the equation becomes.

$$y = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right) \quad \text{...(6)}$$

(i) Variation of phase with time

The phase changes continuously with time at a constant distance.

At a given distance x from O let ϕ_1 and ϕ_2 be the phase of a particle at time t_1 and t_2 respectively.

$$\phi_1 = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right)$$

$$\phi_2 = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right)$$

$$\therefore \phi_2 - \phi_1 = 2\pi\left(\frac{t_2}{T} - \frac{t_1}{T}\right) = \frac{2\pi}{T}(t_2 - t_1)$$

$$\Delta\phi = \frac{2\pi}{T}\Delta t$$

This is the phase change $\Delta\phi$ of a particle in time interval Δt. If $\Delta t = T$, $\Delta\phi = 2\pi$. This shows that after a time period T, the phase of a particle becomes the same.

(ii) Variation of phase with distance

At a given time t phase changes periodically with distance x. Let ϕ_1 and ϕ_2 be the phase of two particles at distance x_1 and x_2 respectively from the origin at a time t.

Then $$\phi_1 = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)$$

$$\phi_2 = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)$$

$$\therefore \quad \phi_2 - \phi_1 = -\frac{2\pi}{\lambda}(x_2 - x_1)$$

$$\therefore \quad \Delta\phi = -\frac{2\pi}{\lambda}\Delta x$$

The negative sign indicates that the forward points lag in phase when the wave travels from left to right.

When $\Delta x = \lambda$, $\Delta\phi = 2\pi$, the phase difference between two particles having a path difference λ is 2π.

7.3.2 Characteristics of progressive wave

1. Each particle of the medium executes vibration about its mean position. The disturbance progresses onward from one particle to another.

2. The particles of the medium vibrate with same amplitude about their mean positions.

3. Each successive particle of the medium performs a motion similar to that of its predecessor along the propagation of the wave, but later in time.

4. The phase of every particle changes from 0 to 2π.

5. No particle remains permanently at rest. Twice during each

vibration, the particles are momentarily at rest at extreme positions, different particles attain the position at different time.

6. Transverse progressive waves are characterised by crests and troughs. Longitudinal waves are characterised by compressions and rarefactions.

7. There is a transfer of energy across the medium in the direction of propagation of progressive wave.

8. All the particles have the same maximum velocity when they pass through the mean position.

9. The displacement, velocity and acceleration of the particle separated by mλ are the same, where m is an integer.

7.3.3 Intensity and sound level

If we hear the sound produced by violin, flute or harmonium, we get a pleasing sensation in the ear, whereas the sound produced by a gun, horn of a motor car etc. produce unpleasant sensation in the ear.

The loudness of a sound depends on intensity of sound wave and sensitivity of the ear.

The intensity is defined as the amount of energy crossing per unit area per unit time perpendicular to the direction of propagation of the wave.

Intensity is measured in W m^{-2}.

The intensity of sound depends on (i) Amplitude of the source ($I \propto a^2$), (ii) Surface area of the source ($I \propto A$), (iii) Density of the medium ($I \propto \rho$), (iv) Frequency of the source ($I \propto n^2$) and (v) Distance of the observer from the source ($I \propto \frac{1}{r^2}$).

The lowest intensity of sound that can be perceived by the human ear is called *threshold of hearing*. It is denoted by I_o.

For sound of frequency 1 KHz, $I_o = 10^{-12}$ W m^{-2}. The level of sound intensity is measured in decibel. According to Weber-Fechner law,

$$\text{decibel level } (\beta) = 10 \log_{10} \left[\frac{I}{I_o}\right]$$

where I_o is taken as 10^{-12} W m^{-2} which corresponds to the lowest sound intensity that can be heard. Its level is 0 *dB*. I is the maximum intensity that an ear can tolerate which is 1W m^{-2} equal to 120 *dB*.

$$\beta = 10 \log_{10} \left(\frac{1}{10^{-12}}\right)$$

$$\beta = 10 \log_{10} (10^{12})$$

$$\beta = 120 \text{ dB.}$$

Table 7.2 gives the decibel value and power density (intensity) for various sources.

Table 7.2 Intensity of sound sources
(NOT FOR EXAMINATION)

Source of sound	Sound intensity(dB)	Intensity ($W m^{-2}$)
Threshold of pain	120	1
Busy traffic	70	10^{-5}
Conversation	65	3.2×10^{-6}
Quiet car	50	10^{-7}
Quiet Radio	40	10^{-8}
Whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing	0	10^{-12}

7.4. Reflection of sound

Take two metal tubes A and B. Keep one end of each tube on a metal plate as shown in Fig. 7.7. Place a wrist watch at the open end of the tube A and interpose a cardboard between A and B. Now at a particular inclination of the tube B with the cardboard, ticking of the watch is clearly heard. The angle of reflection made by the tube B with the cardboard is equal to the angle of incidence made by the tube A with the cardboard.

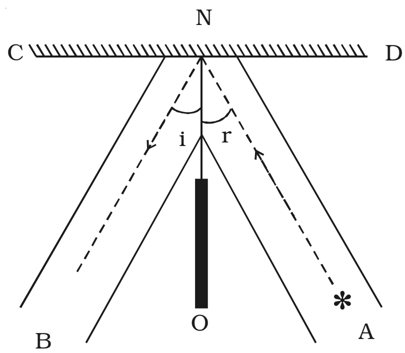


Fig. 7.7 Reflection of sound

7.4.1 Applications of reflection of sound waves

(i) Whispering gallery **:** The famous whispering gallery at

St. Paul's Cathedral is a circular shaped chamber whose walls repeatedly reflect sound waves round the gallery, so that a person talking quietly at one end can be heard distinctly at the other end. This is due to multiple reflections of sound waves from the curved walls (Fig. 7.8).

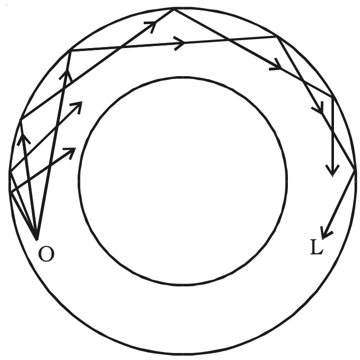


Fig. 7.8 Multiple reflections in the whispering gallery

(ii) Stethoscope : Stethoscope is an instrument used by physicians to listen to the sounds produced by various parts of the body. It consists of a long tube made of rubber or metal. When sound pulses pass through one end of the tube, the pulses get concentrated to the other end due to several reflections on the inner surface of the tube. Using this doctors hear the patients' heart beat as concentrated rays.

(iii) Echo : Echoes are sound waves reflected from a reflecting surface at a distance from the listener. Due to persistence of hearing, we keep hearing the sound for $\frac{1}{10}$th of a second, even after the sounding source has stopped vibrating. Assuming the velocity of sound as 340 ms^{-1}, if the sound reaches the obstacle and returns after 0.1 second, the total distance covered is 34 m. No echo is heard if the reflecting obstacle is less than 17 m away from the source.

7.5 Refraction of sound

This is explained with a rubber bag filled with carbon-di-oxide as shown in Fig. 7.9. The velocity of sound in carbon-di-oxide is less than that in air and hence the bag acts as a lens. If a whistle is used as a source S, the sound passes through the lens and converges at O which is located with the help of flame. The flame will be disturbed only at the point O.

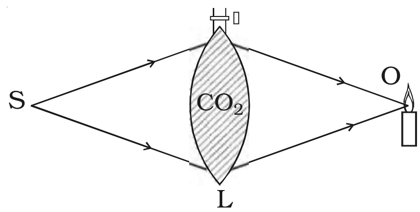


Fig. 7.9 Refraction of sound

When sound travels from one medium to another, it undergoes refraction.

7.5.1 Applications of refraction of sound

It is easier to hear the sound during night than during day-time.

During day time, the upper layers of air are cooler than the layers of air near the surface of the Earth. During night, the layers of air near the Earth are cooler than the upper layers of air. As sound travels faster in hot air, during day-time, the sound waves will be refracted upwards and travel a short distance on the surface of the Earth. On the other hand, during night the sound waves are refracted downwards to the Earth and will travel a long distance.

7.6 Superposition principle

When two waves travel in a medium simultaneously in such a way that each wave represents its separate motion, then the resultant displacement at any point at any time is equal to the vector sum of the individual displacements of the waves.

This principle is illustrated by means of a slinky in the Fig. 7.10(a).

1. In the figure, (i) shows that the two pulses pass each other,
2. In the figure, (ii) shows that they are at some distance apart
3. In the figure, (iii) shows that they overlap partly
4. In the figure, (iv) shows that resultant is maximum

Fig. 7.10 b illustrates the same events but with pulses that are equal and opposite.

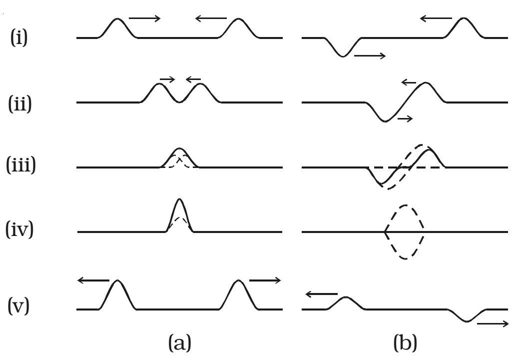


Fig.7.10 Superposition of waves

If $\vec{Y_1}$ and $\vec{Y_2}$ are the displacements at a point, then the resultant displacement is given by $\vec{Y} = \vec{Y_1} + \vec{Y_2}$.

If $|\vec{Y_1}| = |\vec{Y_2}| = a$, and if the two waves have their displacements in the same direction, then $|\vec{Y}| = a + a = 2a$

If the two waves have their displacements in the opposite direction, then $|\vec{Y}| = a + (-a) = 0$

The principle of superposition of waves is applied in wave phenomena such as interference, beats and stationary waves.

7.6.1 Interference of waves

When two waves of same frequency travelling in the same direction in a medium superpose with each other, their resultant intensity is maximum at some points and minimum at some other points. This phenomenon of superposition is called interference.

Let us consider two simple harmonic waves of same frequency travelling in the same direction. If a_1 and a_2 are the amplitudes of the waves and ϕ is the phase difference between them, then their instantaneous displacements are

$$y_1 = a_1 \sin \omega t \qquad ...(1)$$

$$y_2 = a_2 \sin (\omega t + \phi) \qquad ...(2)$$

According to the principle of superposition, the resultant displacement is represented by

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t.\ \cos \phi + \cos \omega t.\sin \phi)$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + a_2 \sin \phi \cos \omega t \qquad ...(3)$$

Put $a_1 + a_2 \cos \phi = A \cos \theta$ ...(4)

$a_2 \sin \phi = A \sin \theta$ ...(5)

where A and θ are constants, then

$$y = A \sin \omega t.\ \cos \theta + A \cos \omega t.\ \sin \theta$$

or $y = A \sin (\omega t + \theta)$ ...(6)

This equation gives the resultant displacement with amplitude A. From eqn. (4) and (5)

$$A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

$$= (a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2$$

$$\therefore A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$

$$\therefore A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi} \qquad ...(7)$$

Also $\tan \theta = \dfrac{a_2 \sin \phi}{a_1 + a_2 \cos \phi}$ ...(8)

We know that intensity is directly proportional to the square of the amplitude

(i.e) $I \propto A^2$

$\therefore \quad I \propto (a_1^2 + a_2^2 + 2a_1a_2 \cos\phi)$ ... (9)

Special cases

The resultant amplitude A is maximum, when $\cos\phi = 1$ or $\phi = 2m\pi$ where m is an integer (i.e) $I_{max} \propto (a_1 + a_2)^2$

The resultant amplitude A is minimum when

$\cos\phi = -1$ or $\phi = (2m + 1)\pi$

$I_{min} \propto (a_1 - a_2)^2$

The points at which interfering waves meet in the same phase $\phi = 2m\pi$ i.e $0, 2\pi, 4\pi, ...$ are points of maximum intensity, where constructive interference takes place. The points at which two interfering waves meet out of phase $\phi = (2m + 1)\pi$ i.e $\pi, 3\pi, ...$ are called points of minimum intensity, where destructive interference takes place.

7.6.2 Experimental demonstration of interference of sound

The phenomenon of interference between two longitudinal waves in air can be demonstrated by Quincke's tube shown in Fig. 7.11.

Quincke's tube consists of U shaped glass tubes *A* and *B*. The tube *SAR* has two openings at *S* and R. The other tube *B* can slide over the tube *A*. *A* sound wave from *S* travels along both the paths *SAR* and *SBR* in opposite directions and meet at *R*.

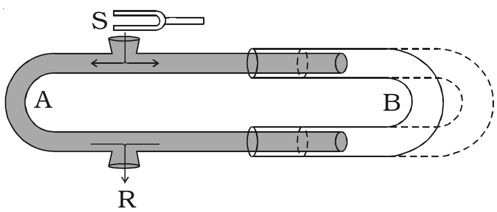


Fig. 7.11 Quincke's Tube

If the path difference between the two waves (i.e) $SAR \sim SBR$ is an integral multiple of wavelength, intensity of sound will be maximum due to constructive interference.

i.e $SAR \sim SBR = m\lambda$

The corresponding phase difference ϕ between the two waves is even multiples of π. (i.e) $\phi = m\,2\pi$ where m = 0, 1, 2, 3 ....

If the tube B is gradually slided over A, a stage is reached when the intensity of sound is zero at R due to destructive interference. Then no sound will be heard at *R*.

If the path difference between the waves is odd multiples of $\frac{\lambda}{2}$, intensity of sound will be minimum.

i.e $\quad SAR \sim SBR = (2m + 1)\frac{\lambda}{2}$

The corresponding phase difference ϕ between the two waves is odd multiples of π. (i.e) $\phi = (2m + 1)\pi$ where m = 0, 1, 2, 3 .....

7.6.3 Beats

When two waves of nearly equal frequencies travelling in a medium along the same direction superimpose upon each other, beats are produced. The amptitude of the resultant sound at a point rises and falls regularly.

The intensity of the resultant sound at a point rises and falls regularly with time. When the intensity rises to maximum we call it as waxing of sound, when it falls to minimum we call it as waning of sound.

The phenomenon of waxing and waning of sound due to interference of two sound waves of nearly equal frequencies are called beats. The number of beats produced per second is called beat frequency, which is equal to the difference in frequencies of two waves.

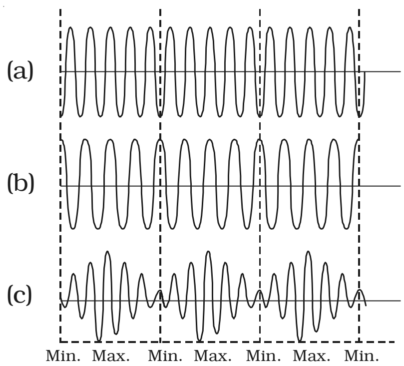


Fig. 7.12 Graphical representation of beats

Analytical method

Let us consider two waves of slightly different frequencies n_1 and n_2 ($n_1 \sim n_2 < 10$) having equal amplitude travelling in a medium in the same direction.

At time t = 0, both waves travel in same phase.

The equations of the two waves are

$$y_1 = a \sin \omega_1 t$$

$y_1 \;= a \sin (2\pi\; n_1)t$ ...(1)

$y_2 \;= a \sin \omega_2\; t$

$\;\;\;\;= a \sin (2\pi\; n_2)t$ ...(2)

When the two waves superimpose, the resultant displacement is given by

$y \;=\; y_1 + y_2$

$y \;=\; a \sin (2\pi\; n_1)\; t + a \sin (2\pi\; n_2)\; t$ ...(3)

Therefore

$$y = 2a \sin 2\pi \left(\frac{n_1+n_2}{2}\right) t \cos 2\pi \left(\frac{n_1-n_2}{2}\right) t \quad ...(4)$$

Substitute $A = 2\, a \cos 2\pi \left(\frac{n_1-n_2}{2}\right) t$ and $n = \frac{n_1+n_2}{2}$ in equation (4)

$\therefore\; y = A \sin 2\pi nt$

This represents a simple harmonic wave of frequency $n = \frac{n_1+n_2}{2}$ and amplitude A which changes with time.

(i) The resultant amplitude is maximum (i.e) $\pm 2a$, if

$$\cos 2\pi \left[\frac{n_1-n_2}{2}\right] t = \pm 1$$

$$\therefore\; 2\pi \left[\frac{n_1-n_2}{2}\right] t = m\pi$$

(where m = 0, 1, 2 ...) or $(n_1 - n_2)\; t = m$

The first maximum is obtained at $t_1 = 0$

The second maximum is obtained at

$$t_2 = \frac{1}{n_1-n_2}$$

The third maximum at $t_3 = \frac{2}{n_1-n_2}$ and so on.

The time interval between two successive maxima is

$$t_2 - t_1 = t_3 - t_2 = \frac{1}{n_1-n_2}$$

Hence the number of beats produced per second is equal to the reciprocal of the time interval between two successive maxima.

(ii) The resultant amplitude is minimum (i.e) equal to zero, if

$$\cos 2\pi \left(\frac{n_1 - n_2}{2}\right) t = 0$$

(i.e) $2\pi \left(\frac{n_1 - n_2}{2}\right) t = \frac{\pi}{2} + m\pi = (2m + 1)\frac{\pi}{2}$ or $(n_1 - n_2)t = \frac{(2m+1)}{2}$

where m = 0, 1, 2 ...

The first minimum is obtained at

$$t_1' = \frac{1}{2(n_1 - n_2)}$$

The second minimum is obtained at

$$t_2' = \frac{3}{2(n_1 - n_2)}$$

The third minimum is obtained at

$$t_3' = \frac{5}{2(n_1 - n_2)} \text{ and so on}$$

Time interval between two successive minima is

$$t_2' - t_1' = t_3' - t_2' = \frac{1}{n_1 - n_2}$$

Hence, the number of beats produced per second is equal to the reciprocal of time interval between two successive minima.

7.6.4 Uses of beats

(i) The phenomenon of beats is useful in tuning two vibrating bodies in unison. For example, a sonometer wire can be tuned in unison with a tuning fork by observing the beats. When an excited tuning fork is kept on the sonometer and if the sonometer wire is also excited, beats are heard, when the frequencies are nearly equal. If the length of the wire is adjusted carefully so that the number of beats gradually decreases to zero, then the two are said to be in unison. Most of the musical instruments are made to be in unison based on this method.

(ii) The frequency of a tuning fork can be found using beats. A standard tuning fork of frequency N is excited along with the experimental fork. If the number of beats per second is n, then the frequency of experimental tuning fork is $N \pm n$. The experimental tuning fork is then loaded with a little bees' wax, thereby decreasing its frequency. Now the observations are repeated. If the number of beats increases, then the frequency of the experimental tuning fork is N-n, and if the number of beats decreases its frequency is N + n.

7.6.5 Stationary waves

When two progressive waves of same amplitude and wavelength travelling along a straight line in opposite directions superimpose on each other, stationary waves are formed.

Analytical method

Let us consider a progressive wave of amplitude a and wavelength λ travelling in the direction of X axis.

$$y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad .....(1)$$

This wave is reflected from a free end and it travels in the negative direction of X axis, then

$$y_2 = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right) \quad .....(2)$$

According to principle of superposition, the resultant displacement is

$$y = y_1 + y_2$$

$$= a \left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

$$= a \left[2 \sin \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda}\right]$$

$$\therefore\ y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T} \quad ...(3)$$

This is the equation of a stationary wave.

(i) At points where $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}$, the values of $\cos \frac{2\pi x}{\lambda} = \pm 1$

$\therefore$ $A = \pm 2a$. At these points the resultant amplitude is maximum. They are called *antinodes* (Fig. 7.13).

(ii) At points where $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... the values of $\cos \frac{2\pi x}{\lambda} = 0$.

$\therefore$ A = 0. The resultant amplitude is zero at these points. They are called *nodes* (Fig. 7.16).

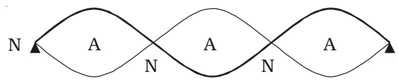


Fig. 7.13 Stationary waves

The distance between any two successive antinodes or nodes is equal to $\frac{\lambda}{2}$ and the distance between an antinode and a node is $\frac{\lambda}{4}$.

(iii) When t = 0, $\frac{T}{2}$, T, $\frac{3T}{2}$, 2T, ... then $\sin\frac{2\pi t}{T}$ = 0, the displacement is zero.

(iv) When t = $\frac{T}{4}$, $\frac{3T}{4}$, $\frac{5T}{4}$ etc, ... $\sin\frac{2\pi t}{T} = \pm 1$, the displacement is maximum.

7.6.6 Characteristics of stationary waves

1. The waveform remains stationary.

2. Nodes and antinodes are formed alternately.

3. The points where displacement is zero are called nodes and the points where the displacement is maximum are called antinodes.

4. Pressure changes are maximum at nodes and minimum at antinodes.

5. All the particles except those at the nodes, execute simple harmonic motions of same period.

6. Amplitude of each particle is not the same, it is maximum at antinodes decreases gradually and is zero at the nodes.

7. The velocity of the particles at the nodes is zero. It increases gradually and is maximum at the antinodes.

8. Distance between any two consecutive nodes or antinodes is equal to $\frac{\lambda}{2}$, whereas the distance between a node and its adjacent antinode is equal to $\frac{\lambda}{4}$.

9. There is no transfer of energy. All the particles of the medium pass through their mean position simultaneously twice during each vibration.

10. Particles in the same segment vibrate in the same phase and

between the neighbouring segments, the particles vibrate in opposite phase.

7.7 Standing waves in strings

In musical instruments like sitar, violin, etc. sound is produced due to the vibrations of the stretched strings. Here, we shall discuss the different modes of vibrations of a string which is rigidly fixed at both ends.

When a string under tension is set into vibration, a transverse progressive wave moves towards the end of the wire and gets reflected. Thus stationary waves are formed.

7.7.1 Sonometer

The sonometer consists of a hollow sounding box about a metre long. One end of a thin metallic wire of uniform cross-section is fixed to a hook and the other end is passed over a pulley and attached to a weight hanger as shown in Fig. 7.14. The wire is stretched over two knife edges P and Q by adding sufficient weights on the hanger. The distance between the two knife edges can be adjusted to change the vibrating length of the wire.

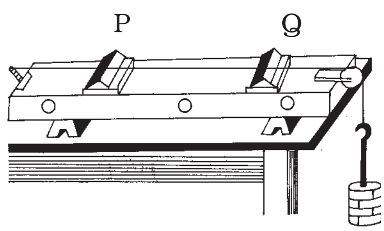


Fig. 7.14 Sonometer

A transverse stationary wave is set up in the wire. Since the ends are fixed, nodes are formed at P and Q and antinode is formed in the middle.

The length of the vibrating segment is $l = \lambda/2$

$\therefore\ \lambda = 2l$. If n is the frequency of vibrating segment, then

$$n = \frac{v}{\lambda} = \frac{v}{2l} \qquad \text{...(1)}$$

We know that $v = \sqrt{\frac{T}{m}}$ where T is the tension and m is the mass per unit length of the wire.

$$\therefore\ n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \text{...(2)}$$

Modes of vibration of stretched string

(i) Fundamental frequency

If a wire is stretched between two points, a transverse wave travels along the wire and is reflected at the fixed end. A transverse stationary wave is thus formed as shown in Fig. 7.15.

When a wire AB of length l is made to vibrate in one segment then $l = \frac{\lambda_1}{2}$.

$\therefore \lambda_1 = 2l$. This gives the lowest frequency called fundamental frequency $n_1 = \frac{v}{\lambda_1}$

$$\therefore n_1 = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad ...(3)$$

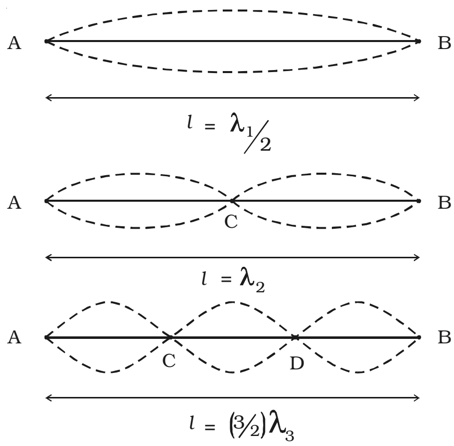


Fig. 7.15 Fundamental and overtones in stretched string

(ii) Overtones in stretched string

If the wire AB is made to vibrate in two segments then $l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$

$\therefore \quad \lambda_2 = l$.

$$\text{But, } n_2 = \frac{v}{\lambda_2} \qquad \therefore n_2 = \frac{1}{l}\sqrt{\frac{T}{m}} = 2n_1 \quad ...(4)$$

n_2 is the frequency of the first overtone.

Since the frequency is equal to twice the fundamental, it is also known as second harmonic.

Similarly, higher overtones are produced, if the wire vibrates with more segments. If there are P segments, the length of each segment is

$$\frac{l}{P} = \frac{\lambda_P}{2} \quad \text{or} \quad \lambda_p = \frac{2l}{P}$$

$$\therefore \text{ Frequency } n_P = \frac{P}{2l}\sqrt{\frac{T}{m}} = P\, n_1 \quad ...(5)$$

(i.e) P^{th} harmonic corresponds to $(P-1)^{th}$ overtone.

7.7.2 Laws of transverse vibrations of stretched strings

The laws of transverse vibrations of stretched strings are (i) the law of length (ii) law of tension and (iii) the law of mass.

(i) For a given wire (m is constant), when T is constant, the fundamental frequency of vibration is inversely proportional to the vibrating length (i.e)

$$n \ \alpha \ \frac{1}{l} \text{ or } nl = \text{constant.}$$

(ii) For constant l and m, the fundamental frequency is directly proportional to the square root of the tension (i.e) $n \ \alpha \ \sqrt{T}$.

(iii) For constant l and T, the fundamental frequency varies inversely as the square root of the mass per unit length of the wire (i.e) $n \ \alpha \ \frac{1}{\sqrt{m}}$.

7.8 Vibrations of air column in pipes

Musical wind instruments like flute, clarinet etc. are based on the principle of vibrations of air columns. Due to the superposition of the incident wave and the reflected wave, longitudinal stationary waves are formed in the pipe.

7.8.1 Organ pipes

Organ pipes are musical instruments which are used to produce musical sound by blowing air into the pipe. Organ pipes are two types (i) closed organ pipes, closed at one end (ii) open organ pipe, open at both ends.

(i) Closed organ pipe : If the air is blown lightly at the open end of the closed organ pipe, then the air column vibrates (Fig. 7.16a) in the fundamental mode. There is a node at the closed end and an antinode at the open end. If l is the length of the tube,

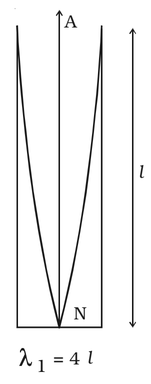


Fig. 7.16a Statinary waves in a closed pipe (Fundamental mode)

$$l = \frac{\lambda_1}{4} \text{ or } \lambda_1 = 4l \qquad \ldots (1)$$

If n_1 is the fundamental frequency of the vibrations and v is the velocity of sound in air, then

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{4l} \quad \text{... (2)}$$

If air is blown strongly at the open end, frequencies higher than fundamental frequency can be produced. They are called overtones. Fig.7.16b & Fig.7.16c shows the mode of vibration with two or more nodes and antinodes.

$$l = \frac{3\lambda_3}{4} \text{ or } \lambda_3 = \frac{4l}{3} \quad \text{...(3)}$$

$$\therefore\ n_3 = \frac{v}{\lambda_3} = \frac{3v}{4l} = 3n_1 \quad \text{...(4)}$$

This is the first overtone or third harmonic.

Similarly $n_5 = \frac{5v}{4l} = 5n_1$ ...(5)

This is called as second overtone or fifth harmonic.

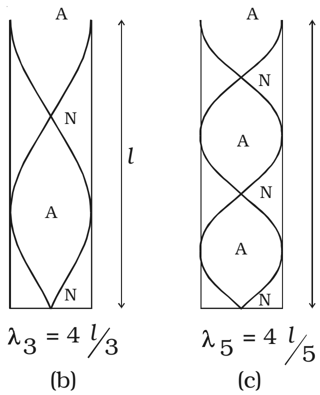


Fig. 7.16b & c Overtones in closed pipe

Therefore the frequency of pth overtone is $(2p + 1)\ n_1$ where n_1 is the fundamental frequency. In a closed pipe only odd harmonics are produced. The frequencies of harmonics are in the ratio of 1 : 3 : 5.....

(ii) Open organ pipe - When air is blown into the open organ pipe, the air column vibrates in the fundamental mode Fig. 7.17a. Antinodes are formed at the ends and a node is formed in the middle of the pipe. If l is the length of the pipe, then

$$l = \frac{\lambda_1}{2} \text{ or } \lambda_1 = 2l \quad \text{...(1)}$$

$v = n_1\lambda_1 = n_1 2l$

The fundamental frequency

$$n_1 = \frac{v}{2l} \quad \text{...(2)}$$

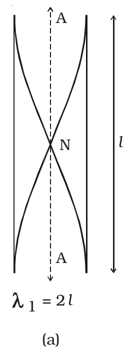


Fig. 7.17a Stationary waves in an open pipe (Fundamental mode)

In the next mode of vibration additional nodes and antinodes are formed as shown in

Fig. 7.17b and Fig.7.17c.

$$l = \lambda_2 \text{ or } v = n_2\lambda_2 = n_2 \cdot l.$$

$$\therefore n_2 = \left(\frac{v}{l}\right) = 2n_1 \qquad \text{...(3)}$$

This is the first overtone or second harmonic.

Similarly, $$n_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3n_1 \qquad \text{...(4)}$$

This is the second overtone or third harmonic.

Therefore the frequency of Pth overtone is $(P + 1)\, n_1$ where n_1 is the fundamental frequency. The frequencies of harmonics are in the ratio of 1 : 2 : 3 ....

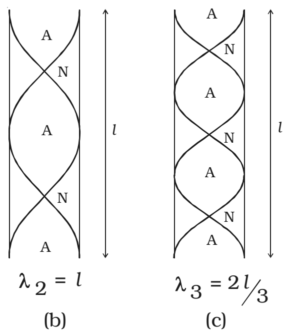


Fig. 7.17b & c Overtones in an open pipe

7.9 Resonance air column apparatus

The resonance air column apparatus consists of a glass tube G about one metre in length (Fig. 7.18) whose lower end is connected to a reservoir R by a rubber tube.

The glass tube is mounted on a vertical stand with a scale attached to it. The glass tube is partly filled with water. The level of water in the tube can be adjusted by raising or lowering the reservoir.

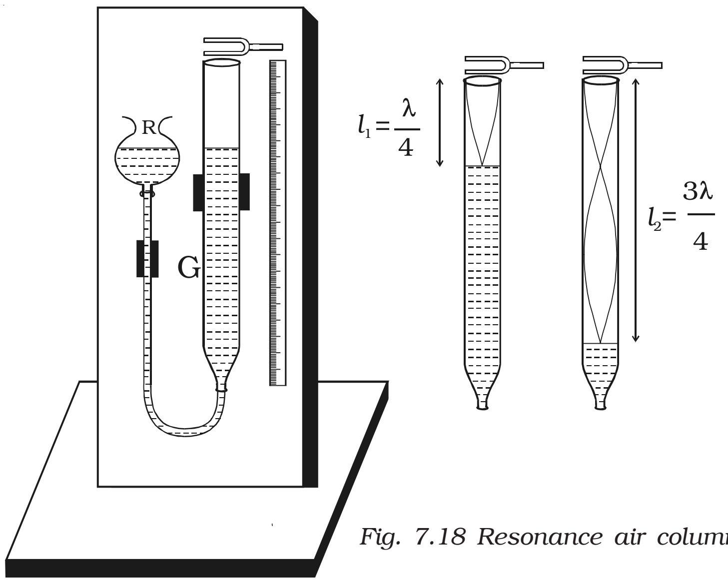


Fig. 7.18 Resonance air column apparatus

A vibrating tuning fork of frequency n is held near the open end of the tube. The length of the air column is adjusted by changing the water level. The air column of the tube acts like a closed organ pipe. When this air column resonates with the frequency of the fork the intensity of sound is maximum.

Here longitudinal stationary wave is formed with node at the water surface and an antinode near the open end. If l_1 is the length of the resonating air column

$$\frac{\lambda}{4} = l_1 + e \qquad ..(1)$$

where e is the end correction.

The length of air column is increased until it resonates again with the tuning fork. If l_2 is the length of the air column.

$$\frac{3\lambda}{4} = l_2 + e \qquad ...(2)$$

From equations (1) and (2)

$$\frac{\lambda}{2} = (l_2 - l_1) \qquad ...(3)$$

The velocity of sound in air at room temperature

$$v = n\lambda = 2n\ (l_2 - l_1) \qquad ...(4)$$

End correction

The antinode is not exactly formed at the open end, but at a small distance above the open end. This is called the end correction.

As $l_1 + e = \frac{\lambda}{4}$ and $l_2 + e = \frac{3\lambda}{4}$

$$e = \frac{(l_2 - 3l_1)}{2}$$

It is found that $e = 0.61r$, where r is the radius of the glass tube.

7.10 Doppler effect

The whistle of a fast moving train appears to increase in pitch as it approaches a stationary observer and it appears to decrease as the train moves away from the observer. This apparent change in frequency was first observed and explained by Doppler in 1845.

The phenomenon of the apparent change in the frequency of sound due to the relative motion between the source of sound and the observer is called Doppler effect.

The apparent frequency due to Doppler effect for different cases can be deduced as follows.

(i) Both source and observer at rest

Suppose S and O are the positions of the source and the observer respectively. Let n be the frequency of the sound and v be the velocity of sound. In one second, n waves produced by the source travel a distance SO = v (Fig. 7.19a).

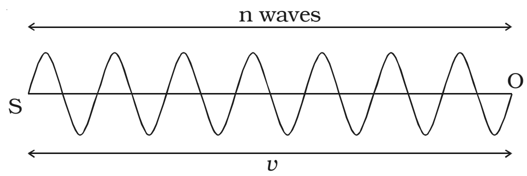


Fig. 7.19a Both source and observer at rest

The wavelength is $\lambda = \frac{v}{n}$.

(ii) When the source moves towards the stationary observer

If the source moves with a velocity v_s towards the stationary observer, then after one second, the source will reach S', such that $SS' = v_s$. Now n waves emitted by the source will occupy a distance of $(v-v_s)$ only as shown in Fig. 7.19b.

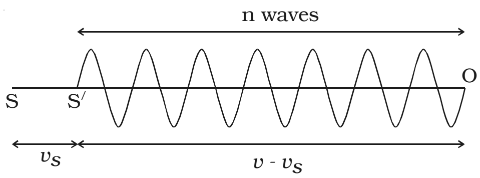


Fig. 7.19b Source moves towards observer at rest

Therefore the apparent wavelength of the sound is

$$\lambda' = \frac{v - v_s}{n}$$

The apparent frequency

$$n' = \frac{v}{\lambda'} = \left(\frac{v}{v - v_s}\right)n \qquad \ldots(1)$$

As $n' > n$, the pitch of the sound appears to increase.

When the source moves away from the stationary observer

If the source moves away from the stationary observer with velocity v_s, the apparent frequency will be given by

$$n' = \left(\frac{v}{v - (-v_s)}\right)n = \left(\frac{v}{v+v_s}\right)n \qquad \ldots(2)$$

As $n' < n$, the pitch of the sound appears to decrease.

(iii) Source is at rest and observer in motion

S and O represent the positions of source and observer respectively. The source S emits n waves per second having a wavelength $\lambda = \frac{v}{n}$. Consider a point A such that OA contains n waves which crosses the ear of the observer in one second (Fig. 7.20a). (i.e) when the first wave is at the point A, the n^{th} wave will be at O, where the observer is situated.

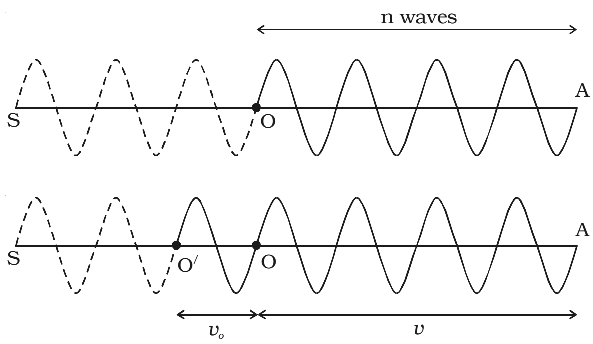


Fig. 7.20a & 7.20b Observer is moving towards a source at rest

When the observer moves towards the stationary source

Suppose the observer is moving towards the stationary source with velocity v_o. After one second the observer will reach the point O′ such that OO′ = v_o. The number of waves crossing the observer will be n waves in the distance OA in addition to the number of waves in the distance OO′ which is equal to $\frac{v_o}{\lambda}$ as shown in Fig. 7.20b.

Therefore, the apparent frequency of sound is

$$n' = n + \frac{v_o}{\lambda} = n + \left(\frac{v_o}{v}\right) n$$

$$\therefore\ n' = \left(\frac{v + v_o}{v}\right) n \qquad \ldots(3)$$

As $n' > n$, the pitch of the sound appears to increase.

When the observer moves away from the stationary source

$$n' = \left[\frac{v + (-v_o)}{v}\right] n$$

$$n' = \left(\frac{v - v_o}{v}\right)n \qquad ...(4)$$

As $n' < n$, the pitch of sound appears to decrease.

Note : If the source and the observer move along the same direction, the equation for apparent frequency is

$$n' = \left(\frac{v - v_o}{v - v_S}\right)n \qquad ...(5)$$

Suppose the wind is moving with a velocity W in the direction of propagation of sound, the apparent frequency is

$$n' = \left(\frac{v + W - v_o}{v + W - v_s}\right)n \qquad ...(6)$$

Applications of Doppler effect

(i) To measure the speed of an automobile

An electromagnetic wave is emitted by a source attached to a police car. The wave is reflected by a moving vechicle, which acts as a moving source. There is a shift in the frequency of the reflected wave. From the frequency shift using beats, the speeding vehicles are trapped by the police.

(ii) Tracking a satellite

The frequency of radio waves emitted by a satellite decreases as the satellite passes away from the Earth. The frequency received by the Earth station, combined with a constant frequency generated in the station gives the beat frequency. Using this, a satellite is tracked.

(iii) RADAR (RADIO DETECTION AND RANGING)

A RADAR sends high frequency radiowaves towards an aeroplane. The reflected waves are detected by the receiver of the radar station. The difference in frequency is used to determine the speed of an aeroplane.

(iv) SONAR (SOUND NAVIGATION AND RANGING)

Sound waves generated from a ship fitted with SONAR are transmitted in water towards an approaching submarine. The frequency of the reflected waves is measured and hence the speed of the submarine is calculated.

Solved Problems

7.1 What is the distance travelled by sound in air when a tuning fork of frequency 256 Hz completes 25 vibrations? The speed of sound in air is 343 m s^{-1}.

Data : v = 343 m s^{-1}, n = 256 Hz, d = ?

Solution : $v = n\lambda$

$$\therefore \lambda = \frac{343}{256} = 1.3398 \text{ m}$$

Wavelength is the distance travelled by the wave in one complete vibration of the tuning fork.

$\therefore$ *Distance travelled by sound wave in 25 vibrations = 25 x 1.3398*

Distance travelled by sound wave is = 33.49 m

7.2 Ultrasonic sound of frequency 100 kHz emitted by a bat is incident on a water surface. Calculate the wavelength of reflected sound and transmitted sound? (speed of sound in air *340 m* s^{-1} and in water *1486 m* s^{-1})

Data : n = 100 kHz = 10^5 Hz, v_a = 340 m s^{-1}, v_w = 1486 m s^{-1};

λ_a = ?, λ_w = ?

Wavelength of reflected sound $\lambda_a = \frac{v_a}{n}$

$$\lambda_a = \frac{340}{10^5} = 3.4 \times 10^{-3} \text{ m}$$

Wavelength of transmitted sound $\lambda_w = \frac{v_w}{n}$

$$\lambda_w = \frac{1486}{10^5} = 1.486 \times 10^{-2} \text{ m}$$

7.3 A string of mass 0.5 kg and length 50 m is stretched under a tension of 400 N. A transverse wave of frequency 10 Hz travels through the wire. (i) Calculate the wave velocity and wavelength. (ii) How long does the disturbance take to reach the other end?

Data : *m = 0.5 kg, length of the wire = 50m; T = 400 N; n=10 Hz*

v = ? ; λ = ? ; t = ?

Solution : *mass per unit length* $m = \frac{\text{mass of the wire}}{\text{length of the wire}}$

$$m = \frac{0.5}{50} = 0.01 \text{ kg m}^{-1}$$

Velocity in the stretched string $v = \sqrt{\frac{T}{m}}$

$$v = \sqrt{\frac{400}{0.01}} = 200\ m\ s^{-1}$$

$$v = n\lambda$$

$$200 = 10\lambda$$

$$\therefore \quad \lambda = 20\ m$$

The length of the wire = 50 m

$\therefore$ *Time taken for the transverse wave to travel*

a distance 50 m $= \frac{50}{200} = 0.25\ s$

7.4 Determine the velocity and wavelength of sound of frequency 256 Hz travelling in water of Bulk modulus 0.022×10^{11} Pa

Data : $k = 0.022 \times 10^{11}$ *Pa*, $\rho = 1000\ kg\ m^{-3}$, $n = 256\ Hz$

Solution : *Velocity of sound in water* $v = \sqrt{\frac{k}{\rho}}$

$$v = \sqrt{\frac{0.022 \times 10^{11}}{1000}} = 1483\ ms^{-1}$$

$$\therefore \lambda = \frac{v}{n} = \frac{1483}{256} = 5.79\ m$$

7.5 Calculate the speed of longitudinal wave in air at 27^{o} C (The molecular mass of air is 28.8 g mol^{-1}. γ for air is 1.4, R = 8.314 J mol^{-1} K^{-1})

Data : $m = 28.8 \times 10^{-3}\ kg\ mol^{-1}$, $\gamma = 1.4$,

$R = 8.314\ J\ mol^{-1}\ K^{-1}$, $T = 27^{o}C = 300\ K$

Solution : $v = \sqrt{\frac{\gamma RT}{m}} = \sqrt{\frac{1.4 \times 8.314 \times 300}{28.8 \times 10^{-3}}}$

$$v = 348.2\ m\ s^{-1}$$

7.6 For air at NTP, the density is 0.001293 g cm^{-3}. Calculate the velocity of longitudinal wave (i) using Newton's formula (ii) Laplace's correction

Data : γ = 1.4, P = 1.013 × 10^5 N m^{-2},

ρ = 0.001293 × 10^3 kg m^{-3}.

Solution : *By Newton's formula the velocity of longitudinal wave*

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.013\times10^5}{0.001293\times10^3}}$$

$$v = 279.9 \text{ m s}^{-1}$$

By Laplace's formula

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4\times1.013\times10^5}{0.001293\times10^3}}$$

$$v = 331.18 \text{ m s}^{-1}$$

7.7 The velocity of sound at 27°C is 347 m s^{-1}. Calculate the velocity of sound in air at 627° C.

Data : v_{27} = 347 m s^{-1}, v_{627} = ?

Solution : $v \; \alpha \; \sqrt{T}$

$$\frac{v_{27}}{v_{627}} = \sqrt{\frac{273+27}{273+627}} = \sqrt{\frac{300}{900}}$$

$$\frac{v_{27}}{v_{627}} = \sqrt{\frac{1}{3}}$$

$$\therefore \; v_{627} = v_{27} \times \sqrt{3} = 347 \times \sqrt{3}$$

$$= 347 \times 1.732 = 601 \text{ m s}^{-1}$$

Velocity of sound in air at 627°C is 601 m s^{-1}

7.8 The equation of a progressive wave is y = 0.50 sin (500 t - 0.025x), where y, t and x are in cm, second and metre. Calculate (i) amplitude (ii) angular frequency (iii) period (iv) wavelength and (v) speed of propagation of wave.

Solution : *The general equation of a progressive wave is given by*

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$$

given y = 0.50 sin (500 t - 0.025x)

comparing the two equations,

(i) *amplitude* $a = 0.50 \times 10^{-2}\ m$

(ii) *angular frequency* $\omega = 500\ rad\ s^{-1}$

(iii) *time period* $T = \frac{2\pi}{\omega} = \frac{2\pi}{500} = \frac{\pi}{250} s$

(iv) *wavelength* $\lambda = \frac{2\pi}{0.025} m$

$$\lambda = 80\pi = 251.2\ m$$

(v) *wave velocity* $v = n\lambda$

$$= \frac{250}{\pi} \times 80\pi$$

$$v = 2 \times 10^{4}\ m\ s^{-1}$$

7.9 A source of sound radiates energy uniformly in all directions at a rate of 2 watt. Find the intensity (i) in $W\ m^{-2}$ and (ii) in decibels, at a point 20 m from the source.

Data : *Power = 2 watt, r = 20 m*

Solution : *Intensity of sound* $I = \frac{\text{Power}}{\text{area}}$

$$I = \frac{2}{4\pi(20)^2}$$

(A spherical surface of radius 20 m with source of sound as centre is imagined)

$$I = 4 \times 10^{-4}\ W\ m^{-2}$$

$$\text{Intensity} = 10 \log_{10}\left(\frac{I}{I_o}\right)$$

$$= 10 \log_{10}\left(\frac{4\times10^{-4}}{10^{-12}}\right) \qquad (\because I_o = 10^{-12})$$

$$= 10 \log_{10}(4 \times 10^{8})$$

$$\text{Intensity} = 86\ dB$$

7.10 Two tuning forks A and B when sounded together produce 4 beats. If A is in unison with the 0.96 m length of a sonometer wire under a tension, B is in unison with 0.97 m length of the same wire under same tension. Calculate the frequencies of the forks.

Data : $l_1 = 0.96\ m;\ l_2 = 0.97\ m;\ n_1 = ?;\ n_2 = ?$

$l_1 < l_2 \qquad \therefore n_1 > n_2$

Solution : *Let* $n_1 = n$ *and* $n_2 = n - 4$

According to 1st law of transverse vibrations

$n_1 l_1 = n_2 l_2$

$n \times 0.96 = (n-4) \times 0.97$

$n(0.97 - 0.96) = 3.88$

$\therefore n = \frac{3.88}{0.01} = 388 \text{ Hz}$

$\therefore n_2 = 388 - 4 = 384 \text{ Hz}$

The frequency of the fork A is $n_1 = 388$ *Hz,*

The frequency of the fork B is $n_2 = 384$ *Hz.*

7.11 A string of length 1 m and mass 5×10^{-4} kg fixed at both ends is under a tension of 20 N. If it vibrates in two segments, determine the frequency of vibration of the string.

Data : *The string vibrates with 2 segments.*

$P = 2$ *loops,* $l = 1$ *m,* $m = 5 \times 10^{-4}$ *kg* m^{-1}*,* $T = 20$ *N*

Solution : *Frequency of vibration* $n = \frac{P}{2l}\sqrt{\frac{T}{m}}$

$$\therefore n = \frac{2}{2 \times 1}\sqrt{\frac{20}{5 \times 10^{-4}}}$$

$n = 200$ *Hz*

7.12 A stretched string made of aluminium is vibrating at its fundamental frequency of 512 Hz. What is the fundamental frequency of a second string made from the same material which has a diameter and length twice that of the original and which is subjected to three times the force of the original?

Data : $n = 512$ *Hz, In the second case, tension = 3T, length = 2 l, radius = 2r*

Solution : *Let l be the length, T be the tension and r be the radius of the wire, then*

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Mass per unit length can be written as the product of cross-sectional area of the wire and density (i.e) $m = \pi r^2 d$

$$512 = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}} \qquad ....(1)$$

In the second case

$$n = \frac{1}{2 \times 2l}\sqrt{\frac{3T}{\pi (2r)^2 d}} \qquad ....(2)$$

Dividing the second equation by first equation

$$\frac{n}{512} = \frac{1}{2}\sqrt{\frac{3}{(2)^2}} \qquad (i.e)\ n = \frac{512}{4}\sqrt{3} = 222\ Hz$$

7.13 The third overtone of a closed pipe is found to be in unison with the first overtone of an open pipe. Determine the ratio of the lengths of the pipes.

Solution : *Let l_1 and l_2 be the lengths of the closed pipe and open pipe respectively. n_1 and n_2 are their fundamental frequencies.*

For closed pipe $n_1 = \frac{v}{4l_1}$

For open pipe $n_2 = \frac{v}{2l_2}$

Third overtone of closed pipe $= (2P + 1)\, n_1 = (2 \times 3 + 1)\, n_1 = 7n_1$

First overtone of open pipe $= (P + 1)\, n_2 = (1 + 1)\, n_2 = 2n_2$

$$\therefore\ 7n_1 = 2n_2$$

$$7 \times \frac{v}{4l_1} = 2 \times \frac{v}{2l_2}$$

$$\therefore\ \frac{l_1}{l_2} = \frac{7}{4}$$

7.14 The shortest length of air in a resonance tube which resonates with a tuning fork of frequency 256 Hz is 32 cm. The corresponding length for the fork of frequency 384 Hz is 20.8 cm. Calculate the end correction and velocity of sound in air .

Data : $n_1 = 256\ Hz,\ l_1 = 32 \times 10^{-2}\ m$

$n_2 = 384\ Hz,\ l_2 = 20.8 \times 10^{-2}\ m$

Solution : *In a closed pipe* $n = \frac{v}{4(l+e)}$

For the first tuning fork, $256 = \frac{v}{4(32+e)\times 10^{-2}}$ *and* *for the second tuning fork,* $384 = \dfrac{v}{4(20.8+e)\times10^{-2}}$

Dividing the first equation by second equation,

$$\frac{256}{384} = \frac{20.8+e}{32+e}$$

$\therefore \quad e = 1.6$ *cm.*

$v = 256 \times 4\ (32 + 1.6) \times 10^{-2}$

Velocity of sound in air $v = 344\ m\ s^{-1}$

7.15 A railway engine and a car are moving parallel but in opposite direction with velocities 144 km/hr and 72 km/hr respectively. The frequency of engine's whistle is 500 Hz and the velocity of sound is 340 m s^{-1}. Calculate the frequency of sound heard in the car when (i) the car and engine are approaching each other (ii) both are moving away from each other.

Data : *The velocity of source* $v_S = 144$ *km/hr and*

the velocity of observer $v_o = 72$ *km/hr*

$v = 340\ m\ s^{-1},\ n = 500\ Hz$

Solution : *(i) When the car and engine approaches each other*

$$n' = \left(\frac{v+v_o}{v-v_S}\right)n$$

$$v_S = \frac{144\times10^3}{60\times60} = 40\ m\ s^{-1}$$

$$v_o = \frac{72\times10^3}{60\times60} = 20\ m\ s^{-1}$$

$$\therefore\ n' = \frac{340+20}{340-40} \times 500$$

The frequency of sound heard is = 600 Hz

(ii) When the car and engine are moving away from each other

$$n'' = \left(\frac{v-v_o}{v+v_S}\right)n$$

$$= \frac{340-20}{340+40} \times 500$$

The frequency of sound heard is = 421 Hz

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

7.1 *In a longitudinal wave there is state of maximum compression at a point at an instant. The frequency of wave is 50 Hz. After what time will the same point be in the state of maximum rarefaction.*

(a) 0.01 s *(b) 0.002 s*

(c) 25 s *(d) 50 s*

7.2 *Sound of frequency 256 Hz passes through a medium. The maximum displacement is 0.1 m. The maximum velocity is equal to*

(a) $60\pi \text{ m s}^{-1}$ *(b)* $51.2\pi \text{ m s}^{-1}$

(c) 256 m s^{-1} *(d)* 512 m s^{-1}

7.3 *Which of the following does not affect the velocity of sound?*

(a) temperature of the gas *(b) pressure of the gas*

(c) mass of the gas

(d) specific heat capacities of the gas

7.4 *When a wave passes from one medium to another, there is change of*

(a) frequency and velocity

(b) frequency and wavelength

(c) wavelength and velocity

(d) frequency, wavelength and velocity

7.5 *Sound waves from a point source are propagating in all directions. What will be the ratio of amplitude at a distance 9 m and 25 m from the source?*

(a) 25:9 *(b) 9: 25*

(c) 3 : 5 *(d) 81 : 625*

7.6 *The intensity level of two sounds are 100 dB and 50 dB. Their ratio of intensities are*

(a) 10^1 *(b)* 10^5

(c) 10^3 *(d)* 10^{10}

7.7 *Number of beats produced by two waves of* $y_1 = a \sin 2000 \pi t$, $y_2 = a \sin 2008 \pi t$ *is*

(a) 0 (b) 1

(c) 4 (d) 8

7.8 *In order to increase the fundamental frequency of a stretched string from 100 Hz to 400 Hz, the tension must be increased by*

(a) 2 times (b) 4 times

(c) 8 times (d) 16 times

7.9 *The second overtone of an open pipe has the same frequency as the first overtone of a closed pipe of 2 m long. The length of the open pipe is,*

(a) 2 m (b) 4 m

(c) 0.5 m (d) 0.75 m

7.10 *A source of sound of frequency 150 Hz is moving in a direction towards an observer with a velocity 110 m* s^{-1}*. If the velocity of sound is 330 m* s^{-1}*, the frequency of sound heard by the person is*

(a) 225 Hz (b) 200 Hz

(c) 150 Hz (d) 100 Hz

7.11 *Define wave motion. Mention the properties of the medium in which a wave propagates.*

7.12 *What are the important characteristics of wave motion?*

7.13 *Distinguish between transverse and longitudinal waves.*

7.14 *In solids both longitudinal and transverse waves are possible, but transverse waves are not produced in gases. Why?*

7.15 *Define the terms wavelength and frequency in wave motion. Prove that* $v = n\lambda$.

7.16 *Obtain an expression for the velocity of transverse wave in a stretched string, when it is vibrating in fundamental mode.*

7.17 *Derive Newton - Laplace formula for the velocity of sound in gases.*

7.18 *Show that the velocity of sound increases by 0.61 m* s^{-1} *for every degree rise of temperature.*

7.19 *Sound travels faster on rainy days. Why?*

7.20 *Obtain the equation for plane progressive wave.*

7.21 *Distinguish between intensity and loudness of sound.*

7.22 *What do you understand by decibel?*

7.23 *On what factors does the intensity of sound depend?*

7.24 *What is an echo? Why an echo cannot be heard in a small room?*

7.25 *Write a short note on whispering gallery.*

7.26 *State the principle of superposition.*

7.27 *What are the essential conditions for the formation of beats?*

7.28 *What are beats? Show that the number of beats produced per second is equal to the difference in frequencies.*

7.29 *What is interference of sound waves? Describe an experiment to explain the phenomenon of interference of waves.*

7.30 *How are stationary waves formed?*

7.31 *Derive the equation of stationary wave and deduce the condition for nodes and antinodes.*

7.32 *What are the properties of stationary waves?*

7.33 *State the laws of transverse vibrations in stretched strings.*

7.34 *List out the differences between a progressive wave and a stationary wave.*

7.35 *What are overtones and harmonics?*

7.36 *Why open organ pipes are preferred for making flute?*

7.37 *Prove that in a pipe closed at one end, frequency of harmonics are in the ratio 1:3:5.*

7.38 *Explain how overtones are produced in an open pipe. Show that all harmonics are present in the open pipe.*

7.39 *What is meant by end correction?*

7.40 *What is doppler effect? Derive the formula for the change in frequency (i) when the source is approaching and receding from* *the observer and (ii) when the source is stationary and observer is moving towards and away from the source.*

Problems

7.41 *A wave of length 0.60 cm is produced in air and travels with a velocity of 340 m s^{-1}. Will it be audiable to human ear?*

7.42 *The velocity of sound in water is 1480 m s^{-1}. Find the frequency of sound wave such that its wavelength in water is the same as the wavelength in air of a sound wave of frequency 1000 Hz. (The velocity of sound in air is 340 m s^{-1}).*

7.43 *Calculate the ratio of velocity of sound in hydrogen* $\left(\gamma = \frac{7}{5}\right)$ *to that in helium* $\left(\gamma = \frac{5}{3}\right)$ *at the same temperature.*

7.44 *The equation of a progressive wave travelling along the x axis is given by $y = 10 \sin \pi (2t - 0.01x)$ where y and x are in m and t in s. Calculate (i) amplitude (ii) frequency and wavelength (iii) wave velocity.*

7.45 *If the intensity is increased by a factor 60, by how many decibels the sound level is increased.*

7.46 *Two sound waves, originating from the same source, travel along different paths in air and then meet at a point. If the source vibrates at a frequency of 1.0 kHz and one path is 83 cm longer than the other, what will be the nature of interference? The speed of sound in air is 332 m s^{-1}.*

7.47 *In an experiment, the tuning fork and sonometer give 5 beats per second, when their lengths are 1m and 1.05m respectively. Calculate the frequency of the fork.*

7.48 *A steel wire of length 1.2 m with a tension of 9.8 N is found to resonate in five segments at a frequency of 240 Hz. Find the mass of the string.*

7.49 *How can a stretched string of length 114 cm be divided into three segments so that the fundamental frequency of the three segments be in the ratio of 1 : 3 : 4.*

7.50 *An open organ pipe has a fundamental frequency of 240 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? Velocity of sound at room temperature is 350 ms^{-1}.*

7.51 *A tuning fork of frequency 800 Hz produces resonance in a resonance column apparatus. If successive resonances are produced at lengths 9.75 cm and 31.25 cm, calculate the velocity of sound in air.*

7.52 *A train standing at a signal of a railway station blows a whistle of frequency 256 Hz in air. Calculate the frequency of the sound as heard by a person standing on the platform when the train (i) approches the platform with a speed of 40 m s^{-1} (ii) recedes from the platform with the same speed.*

7.53 *A whistle of frequency 480 Hz rotates in a circle of radius 1.25m at an angular speed of 16.0 rad s^{-1}. What is the lowest and highest frequency heard by a listener a long distance away at rest with respect to the centre of the circle. The velocity of sound is 340 m s^{-1}.*

7.54 *Two tuning forks A and B when sounded together give 4 beats per second. The fork A is in resonance with a closed column of air of length 15 cm, while the second is in resonance with an open column of length 30.5 cm. Calculate their frequencies.*

Answers

7.1	*(a)*	**7.2**	*(b)*	**7.3**	*(b)*
7.4	*(c)*	**7.5**	*(a)*	**7.6**	*(b)*
7.7	*(c)*	**7.8**	*(d)*	**7.9**	*(b)*
7.10	*(a)*				

7.41 5.666×10^4 Hz, not audible

7.42 4353 Hz

7.43 1.833

7.44 10 m, 1 Hz, 200 m, 200 ms^{-1}

7.45 18 dB

7.46 destructive interference, as this is odd multiple of π

7.47 205 Hz

7.48 7.38×10^{-4} kg

7.49 72×10^{-2} m, 24×10^{-2} m, 18×10^{-2} m

7.50 54.7×10^{-2} m, 72.9×10^{-2} m

7.51 344 m s^{-1}

7.52 290 Hz, 229 Hz

7.53 510 Hz, 453 Hz

7.54 240 Hz, 244 Hz

8. Heat and Thermodynamics

In early days, according to caloric theory of heat, heat was regarded as an invisible and weightless fluid called "caloric". The two bodies at different temperatures placed in contact attain thermal equilibrium by the exchange of caloric. The caloric flows from the hot body to the cold body, till their temperature becomes equal. However, this theory failed to explain the production of heat due to friction in the experiments conducted by Court Rumford. Rubbing our hands against each other produces heat. Joule's paddle wheel experiment led to the production of heat by friction. These observations led to the dynamic theory of heat, according to which *heat is a form of energy called thermal energy.*

Every body is made up of molecules. Depending on its nature and temperature, the molecules may possess translatory motion, vibratory motion and rotatory motion about its axis. Each type of motion provides some kinetic energy to the molecules. Heat possessed by a body is the total thermal energy of the body, which is the sum of kinetic energies of all the individual molecules of the body.

Temperature of a body is the degree of hotness or coldness of the body. Heat flows from a body at high temperature to a body at low temperature when they are in contact with each other. Modern concept of temperature follows from zeroth law of thermodynamics. *Temperature is the thermal state of the body, that decides the direction of flow of heat.* Temperature is now regarded as one of the fundamental quantities.

8.1 Kinetic theory of gases

The founder of modern kinetic theory of heat by common consent is Daniel Bernoulli. But the credit for having established it on a firm mathematical basis is due to Clausius and Maxwell in whose hands it attained the present form.

8.1.1 Postulates of Kinetic theory of gases

(1) A gas consists of a very large number of molecules. Each one is a perfectly identical elastic sphere.

(2) The molecules of a gas are in a state of continuous and random motion. They move in all directions with all possible velocities.

(3) The size of each molecule is very small as compared to the distance between them. Hence, the volume occupied by the molecule is negligible in comparison to the volume of the gas.

(4) There is no force of attraction or repulsion between the molecules and the walls of the container.

(5) The collisions of the molecules among themselves and with the walls of the container are perfectly elastic. Therefore, momentum and kinetic energy of the molecules are conserved during collisions.

(6) A molecule moves along a straight line between two successive collisions and the average distance travelled between two successive collisions is called the mean free path of the molecules.

(7) The collisions are almost instantaneous (i.e) the time of collision of two molecules is negligible as compared to the time interval between two successive collisions.

Avogadro number

Avogadro number is defined as the number of molecules present in one mole of a substance. It is constant for all the substances. Its value is 6.023×10^{23}.

8.1.2 Pressure exerted by a gas

The molecules of a gas are in a state of random motion. They continuously collide against the walls of the container. During each collision, momentum is transfered to the walls of the container. The pressure exerted by the gas is due to the continuous collision of the molecules against the walls of the container. Due to this continuous collision, the walls experience a continuous force which is equal to the total momentum imparted to the walls per second. The force experienced per unit area of the walls of the container determines the pressure exerted by the gas.

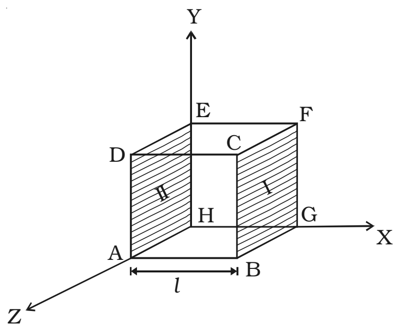


Fig. 8.1 Pressure exerted by a gas

Consider a cubic container of side l containing n molecules of perfect gas moving with velocities C_1, C_2, C_3 ... C_n *(Fig. 8.1).* A molecule moving with a velocity C_1, will have velocities u_1, v_1 and w_1 as components along the x, y and z axes respectively. Similarly u_2, v_2 and w_2 are the velocity components of the second molecule and so on. Let a molecule P (Fig. 8.2) having velocity C_1 collide against the wall marked I (BCFG) perpendicular to the x-axis. Only the x-component of the velocity of the molecule is relevant for the wall I. Hence momentum of the molecule before collision is mu_1 where m is the mass of the molecule. Since the collision is elastic, the molecule will rebound with the velocity u_1 in the opposite direction. Hence momentum of the molecule after collision is $-mu_1$.

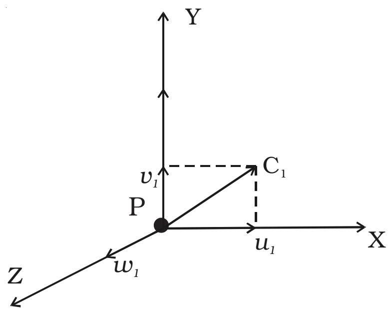


Fig. 8.2 Components of velocity

Change in the momentum of the molecule

$$= \text{Final momentum - Initial momentum}$$

$$= -mu_1 - mu_1 = -2mu_1$$

During each successive collision on face I the molecule must travel a distance $2l$ from face I to face II and back to face I.

Time taken between two successive collisions is $= \dfrac{2l}{u_1}$

$\therefore$ Rate of change of momentum

$$= \frac{\text{Change in the momentum}}{\text{Time taken}}$$

$$= \frac{-2mu_1}{\frac{2l}{u_1}} = \frac{-2mu_1^2}{2\,l} = \frac{-mu_1^2}{l}$$

(i.e) Force exerted on the molecule $= \dfrac{-mu_1^2}{l}$

$\therefore$ According to Newton's third law of motion, the force exerted by the molecule

$$= -\frac{\left(-mu_1^2\right)}{l} = \frac{mu_1^2}{l}$$

Force exerted by all the *n* molecules is

$$F_x = \frac{mu_1^2}{l} + \frac{mu_2^2}{l} + ... + \frac{mu_n^2}{l}$$

Pressure exerted by the molecules

$$P_x = \frac{F_x}{A}$$

$$= \frac{1}{l^2}\left(\frac{mu_1^2}{l} + \frac{mu_2^2}{l} + ...+ \frac{mu_n^2}{l}\right)$$

$$= \frac{m}{l^3}\left(u_1^2 + u_2^2 + ... + u_n^2\right)$$

Similarly, pressure exerted by the molecules along Y and Z axes are

$$P_y = \frac{m}{l^3}\left(v_1^2 + v_2^2 + ... + v_n^2\right)$$

$$P_z = \frac{m}{l^3}\left(w_1^2 + w_2^2 + ...+w_n^2\right)$$

Since the gas exerts the same pressure on all the walls of the container

$$P_x = P_y = P_z = P$$

$$P = \frac{P_x + P_y + p_z}{3}$$

$$P = \frac{1}{3}\frac{m}{l^3}[(u_1^2 + u_2^2 + ... + u_n^2) + (v_1^2 + v_2^2 + ... + v_n^2) + (w_1^2 + w_2^2 + ... + w_n^2)]$$

$$P = \frac{1}{3}\frac{m}{l^3}[(u_1^2 + v_1^2 + w_1^2) + (u_2^2 + v_2^2 + w_2^2) + ... + (u_n^2 + v_n^2 + w_n^2)]$$

$$P = \frac{1}{3}\frac{m}{l^3}[C_1^2 + C_2^2 + ... + C_n^2]$$

where $C_1^2 = (u_1^2 + v_1^2 + w_1^2)$

$$P = \frac{1}{3}\frac{mn}{l^3}\left[\frac{C_1^2 + C_2^2 + ... + C_n^2}{n}\right]$$

$$P = \frac{1}{3}\frac{mn}{V} . C^2$$

where C is called the root mean square (RMS) velocity, which is defined as the square root of the mean value of the squares of velocities of individual molecules.

(i.e.) $C = \sqrt{\dfrac{C_1^{\,2} + C_2^{\,2} + ... + C_n^{\,2}}{n}}$

8.1.3 Relation between the pressure exerted by a gas and the mean kinetic energy of translation per unit volume of the gas

Pressure exerted by unit volume of a gas, $P = \frac{1}{3} mnC^2$

$P = \frac{1}{3} \rho C^2$ ($\because$ mn = mass per unit volume of the gas ; mn = ρ, density of the gas)

Mean kinetic energy of translation per unit volume of the gas

$$E = \frac{1}{2} \rho C^2$$

$$\frac{P}{E} = \frac{\frac{1}{3} \rho C^2}{\frac{1}{2} \rho C^2} = \frac{2}{3}$$

$$P = \frac{2}{3} E$$

8.1.4 Average kinetic energy per molecule of the gas

Let us consider one mole of gas of mass M and volume V.

$$P = \frac{1}{3} \rho C^2$$

$$P = \frac{1}{3} \frac{M}{V} C^2$$

$$PV = \frac{1}{3} MC^2$$

From gas equation

$$PV = RT$$

$$\therefore \quad RT = \frac{1}{3} MC^2$$

$$\frac{3}{2} RT = \frac{1}{2} MC^2$$

(i.e) Average kinetic energy of one mole of the gas is equal to $\frac{3}{2}$RT

Since one mole of the gas contains N number of atoms where N is the Avogadro number we have $M = Nm$

$$\therefore \frac{1}{2} mNC^2 = \frac{3}{2} RT$$

$$\frac{1}{2} mC^2 = \frac{3}{2} \frac{R}{N} T$$

$$= \frac{3}{2} kT \text{ where } k = \frac{R}{N}, \text{ is the Boltzmann constant}$$

Its value is 1.38×10^{-23} J K^{-1}

$\therefore$ *Average kinetic energy per molecule of the gas is equal to* $\frac{3}{2} kT$

Hence, it is clear that the temperature of a gas is the measure of the mean translational kinetic energy per molecule of the gas.

8.2 Degrees of freedom

The number of degrees of freedom of a dynamical system is defined as the total number of co-ordinates or independent variables required to describe the position and configuration of the system.

For translatory motion

(i) A particle moving in a straight line along any one of the axes has one degree of freedom (e.g) Bob of an oscillating simple pendulum.

(ii) A particle moving in a plane (X and Y axes) has two degrees of freedom. (eg) An ant that moves on a floor.

(iii) A particle moving in space (X, Y and Z axes) has three degrees of freedom. (eg) a bird that flies.

A point mass cannot undergo rotation, but only translatory motion. A rigid body with finite mass has both rotatory and translatory motion. The rotatory motion also can have three co-ordinates in space, like translatory motion ; Therefore a rigid body will have six degrees of freedom ; three due to translatory motion and three due to rotatory motion.

8.2.1 Monoatomic molecule

Since a monoatomic molecule consists of only a single atom of point mass it has three degrees of freedom of translatory motion along the three co-ordinate axes as shown in Fig. 8.3.

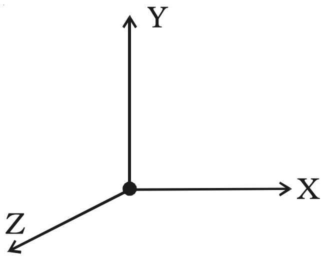


Fig. 8.3 Monoatomic molecule

Examples : molecules of rare gases like helium, argon, etc.

8.2.2 Diatomic molecule

The diatomic molecule can rotate about any axis at right angles to its own axis. Hence it has two degrees of freedom of rotational motion in addition to three degrees of freedom of translational motion along the three axes. So, a diatomic molecule has five degrees of freedom (Fig. 8.4). Examples : molecules of O_2, N_2, CO, Cl_2, etc.

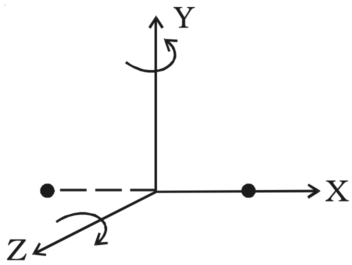


Fig. 8.4 Diatomic molecule

8.2.3 Triatomic molecule (Linear type)

In the case of triatomic molecule of linear type, the centre of mass lies at the central atom. It, therefore, behaves like a diamotic moelcule with three degrees of freedom of translation and two degrees of freedom of rotation, totally it has five degrees of freedom (Fig. 8.5). Examples : molecules of CO_2, CS_2, etc.

Fig. 8.5 Triatomic molecules (linear type)

8.2.4 Triatomic molecule (Non-linear type)

A triatomic non-linear molecule may rotate, about the three mutually perpendicular axes, as shown in Fig.8.6. Therefore, it possesses three degrees of freedom of rotation in addition to three degrees of freedom of translation along the three co-ordinate axes. Hence it has six degrees of freedom. Examples : molecules of H_2O, SO_2, etc.

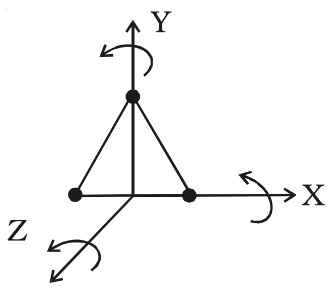


Fig. 8.6 Triatomic molecule

In all the above cases, only the translatory and rotatory motion of the molecules have been considered. The vibratory motion of the molecules has not been taken into consideration.

8.3 Law of equipartition of energy

Law of equipartition of energy states that for a dynamical system in thermal equilibrium the total energy of the system is shared equally by all the degrees of freedom. The energy associated with each degree of freedom per moelcule is $\frac{1}{2}$ kT, where k is the Boltzmann's constant.

Let us consider one mole of a monoatomic gas in thermal equilibrium at temperature T. Each molecule has 3 degrees of freedom due to translatory motion. According to kinetic theory of gases, the mean kinetic energy of a molecule is $\frac{3}{2}$ kT.

Let C_x, C_y and C_z be the components of RMS velocity of a molecule along the three axes. Then the average energy of a gas molecule is given by

$$\frac{1}{2}\ mC^2 = \frac{1}{2}mC_x{}^2 + \frac{1}{2}mC_y{}^2 + \frac{1}{2}mC_z{}^2$$

$$\therefore\ \frac{1}{2}mC_x{}^2 + \frac{1}{2}mC_y{}^2 + \frac{1}{2}mC_z{}^2 = \frac{3}{2}kT$$

Since molecules move at random, the average kinetic energy corresponding to each degree of freedom is the same.

$$\therefore\ \frac{1}{2}mC_x{}^2 = \frac{1}{2}mC_y{}^2 = \frac{1}{2}mC_z{}^2$$

$$\text{(i.e)} \qquad \frac{1}{2}mC_x{}^2 = \frac{1}{2}mC_y{}^2 = \frac{1}{2}mC_z{}^2 = \frac{1}{2}kT$$

$\therefore$ Mean kinetic energy per molecule per degree of freedom is $\frac{1}{2}$ kT.

8.4. Thermal equilibrium

Let us consider a system requiring a pair of independent co-ordinates X and Y for their complete description. If the values of X and Y remain unchanged so long as the external factors like temperature also remains the same, then the system is said to be in a state of thermal equilibrium.

Two systems A and B having their thermodynamic co-ordinates X and Y and X_1 and Y_1 respectively separated from each other, for example, by a wall, will have new and common co-ordinates X' and Y'

spontaneously, if the wall is removed. Now the two systems are said to be in thermal equilibrium with each other.

8.4.1 Zeroth law of thermodynamics

If two systems A and B are separately in thermal equilibrium with a third system C, then the three systems are in thermal equilibrium with each other. *Zeroth law of thermodynamics states that two systems which are individually in thermal equilibrium with a third one, are also in thermal equilibrium with each other.*

This Zeroth law was stated by Flower much later than both first and second laws of thermodynamics.

This law helps us to define temperature in a more rigorous manner.

8.4.2 Temperature

If we have a number of gaseous systems, whose different states are represented by their volumes and pressures V_1, V_2, V_3 ... and P_1, P_2, P_3... etc., in thermal equilibrium with one another, we will have $\phi_1 (P_1, V_1) = \phi_2 (P_2, V_2) = \phi_3 (P_3, V_3)$ and so on, where ϕ is a function of P and V. Hence, despite their different parameters of P and V, the numerical value of the these functions or the temperature of these systems is same.

Temperature may be defined as the particular property which determines whether a system is in thermal equilibrium or not with its neighbouring system when they are brought into contact.

8.5 Specific heat capacity

Specific heat capacity of a substance is defined as the quantity of heat required to raise the temperature of 1 kg of the substance through 1K. Its unit is J $kg^{-1}K^{-1}$.

Molar specific heat capacity of a gas

Molar specific heat capacity of a gas is defined as the quantity of heat required to raise the temperature of 1 mole of the gas through 1K. Its unit is J mol^{-1} K^{-1}.

Specific heat capacity of a gas may have any value between $-\infty$ and $+\infty$ depending upon the way in which heat energy is given.

Let m be the mass of a gas and C its specific heat capacity. Then $\Delta Q = m \times C \times \Delta T$ where ΔQ is the amount of heat absorbed and ΔT is the corresponding rise in temperature.

$$\text{(i.e)}\ C = \frac{\Delta Q}{m\,\Delta T}$$

Case (i)

If the gas is insulated from its surroundings and is suddenly compressed, it will be heated up and there is rise in temperature, even though no heat is supplied from outside

$$\text{(i.e)} \quad \Delta Q = 0$$

$$\therefore \quad C = 0$$

Case (ii)

If the gas is allowed to expand slowly, in order to keep the temperature constant, an amount of heat ΔQ is supplied from outside,

$$\text{then } C = \frac{\Delta Q}{m \times \Delta T} = \frac{\Delta Q}{0} = +\infty$$

($\because$ ΔQ is +ve as heat is supplied from outside)

Case (iii)

If the gas is compressed gradually and the heat generated ΔQ is conducted away so that temperature remains constant, then

$$C = \frac{\Delta Q}{m \times \Delta T} = \frac{-\Delta Q}{0} = -\infty$$

($\because$ ΔQ is -ve as heat is supplied by the system)

Thus we find that if the external conditions are not controlled, the value of the specific heat capacity of a gas may vary from $+\infty$ to $-\infty$

Hence, *in order to find the value of specific heat capacity of a gas, either the pressure or the volume of the gas should be kept constant.* Consequently a gas has two specific heat capacities (i) Specific heat capacity at constant volume (ii) Specific heat capacity at constant pressure.

Molar specific heat capacity of a gas at constant volume

Molar specific heat capacity of a gas at constant volume C_V is defined as the quantity of heat required to raise the temperature of one mole of a gas through 1 K, keeping its volume constant.

Molar specific heat capacity of a gas at constant pressure

Molar specific heat capacity of a gas at constant pressure C_p is defined as the quantity of heat to raise the temperature of one mole of a gas through 1 K keeping its pressure constant.

Specific heat capacity of monoatomic, diatomic and triatomic gases

Monoatomic gases like argon, helium etc. have three degrees of freedom.

We know, kinetic energy per molecule, per degree of freedom is $\frac{1}{2}$ kT.

$\therefore$ Kinetic energy per molecule with three degrees of freedom is $\frac{3}{2}$ kT.

Total kinetic energy of one mole of the monoatomic gas is given by $E = \frac{3}{2}\ kT \times N = \frac{3}{2}\ RT$, where N is the Avogadro number.

$$\therefore \quad \frac{dE}{dT} = \frac{3}{2}R$$

If dE is a small amount of heat required to raise the temperature of 1 mole of the gas at constant volume, through a temperature dT,

$$dE = 1 \times C_V \times dT$$

$$C_V = \frac{dE}{dT} = \frac{3}{2}R$$

As $R = 8.31\ \text{J mol}^{-1}\ \text{K}^{-1}$

$$C_V = \frac{3}{2} \times 8.31 = 12.465\ \text{J mol}^{-1}\ \text{K}^{-1}$$

Then $C_P - C_V = R$

$$C_P = C_V + R$$

$$= \frac{3}{2}\text{R} + \text{R} \quad = \frac{5}{2}\ \text{R} \ = \frac{5}{2} \times 8.31$$

$$\therefore \qquad C_p = 20.775\ \text{J mol}^{-1}\ \text{K}^{-1}$$

In diatomic gases like hydrogen, oxygen, nitrogen etc., a molecule has five degrees of freedom. Hence the total energy associated with one mole of diatomic gas is

$$E = 5 \times \frac{1}{2} kT \times N = \frac{5}{2} RT$$

$$\text{Also, } C_v = \frac{dE}{dT} = \frac{d}{dT}\left(\frac{5}{2}\text{RT}\right) = \frac{5}{2} R$$

$$C_v = \frac{5}{2} \times 8.31 = 20.775 \text{ J mol}^{-1} \text{ K}^{-1}$$

$$\text{But } C_p = C_v + R$$

$$= \frac{5}{2} R + R = \frac{7}{2} R$$

$$C_p = \frac{7}{2} \times 8.31$$

$$= 29.085 \text{ J mol}^{-1} \text{ K}^{-1}$$

similarly, Cp and Cv can be calculated for triatomic gases.

Internal energy

Internal energy U of a system is the energy possessed by the system due to molecular motion and molecular configuration. The internal kinetic energy U_K of the molecules is due to the molecular motion and the internal potential energy U_P is due to molecular configuration. Thus

$$U = U_K + U_P$$

It depends only on the initial and final states of the system. In case of an ideal gas, it is assumed that the intermolecular forces are zero. Therefore, no work is done, although there is change in the intermolecular distance. Thus $U_P = O$. Hence, internal energy of an ideal gas has only internal kinetic energy, which depends only on the temperature of the gas.

In a real gas, intermolecular forces are not zero. Therefore, a definite amount of work has to be done in changing the distance between the molecules. Thus the internal energy of a real gas is the sum of internal kinetic energy and internal potential energy. Hence, it would depend upon both the temperature and the volume of the gas.

8.6 First law of thermodynamics

Let us consider a gas inside a cylinder fitted with a movable frictionless piston. The walls of the cylinder are made up of non-conducting material and the bottom is made up of conducting material (Fig. 8.7).

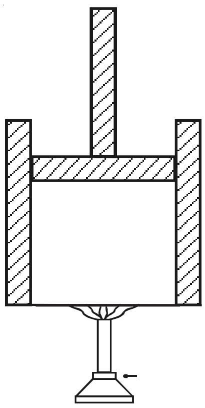


Fig. 8.7 First Law of thermodynamics

Let the bottom of the cylinder be brought in contact with a hot body like burner. The entire heat energy given to the gas is not converted into work. A part of the heat energy is used up in increasing the temperature of the gas (i.e) in increasing its internal energy and the remaining energy is used up in pushing the piston upwards (i.e.) in doing work.

If ΔQ is the heat energy supplied to the gas, U_1 and U_2 are initial and final internal energies and ΔW is the work done by the system, then

$$\Delta Q = \Delta W + (U_2 - U_1)$$

$$\Delta Q = \Delta W + \Delta U$$

where ΔU is the change in the internal energy of the system.

Hence, *the first law of thermodynamics states that the amount of heat energy supplied to a system is equal to the sum of the change in internal energy of the system and the work done by the system.* This law is in accordance with the law of conservation of energy.

8.7 Relation between C_p and C_v (Meyer's relation)

Let us consider one mole of an ideal gas enclosed in a cylinder provided with a frictionless piston of area *A*. Let *P*, *V* and *T* be the pressure, volume and absolute temperature of gas respectively (Fig. 8.8).

A quantity of heat *dQ* is supplied to the gas. To keep the volume of the gas constant, a small weight is placed over the piston. The pressure and the temperature of the gas increase to $P + dP$ and $T + dT$ respectively. This heat energy *dQ* is used to increase the internal energy *dU* of the gas. But the gas does not do any work ($dW = 0$).

$$\therefore\ dQ = dU = 1 \times C_v \times dT \qquad \dots (1)$$

The additional weight is now removed from the piston. The piston now moves upwards through a distance *dx*, such that the pressure of

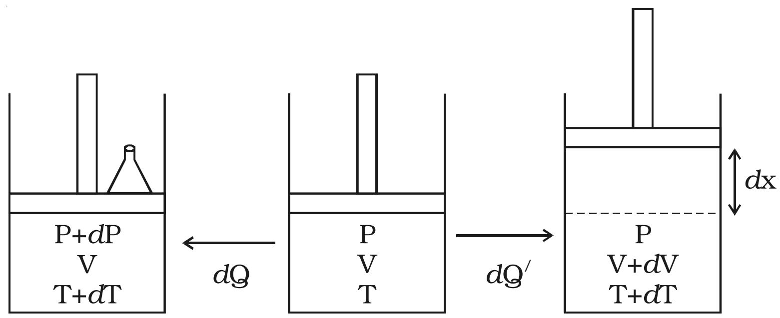


Fig. 8.8 Meyer's relation

the enclosed gas is equal to the atmospheric pressure P. The temperature of the gas decreases due to the expansion of the gas.

Now a quantity of heat dQ' is supplied to the gas till its temperature becomes $T + dT$. This heat energy is not only used to increase the internal energy dU of the gas but also to do external work dW in moving the piston upwards.

$\therefore\ dQ' = dU + dW$

Since the expansion takes place at constant pressure,

$dQ' = C_p dT$

$\therefore\ C_p dT = C_v dT + dW$ ... (2)

Work done, dW = force × distance

$= P \times A \times dx$

$dW = P\ dV$ *(since $A \times dx = dV$, change in volume)*

$\therefore\ C_p dT = C_v dT + P\ dV$ ... (3)

The equation of state of an ideal gas is

$PV = RT$

Differentiating both the sides

$PdV = RdT$ ... (4)

Substituting equation (4) in (3),

$C_p dT = C_v dT + RdT$

$C_p = C_v + R$

$\therefore \quad C_p - C_v = R$

This equation is known as Meyer's relation

8.8 Indicator diagram (P-V diagram)

A curve showing variation of volume of a substance taken along the X-axis and the variation of pressure taken along Y-axis is called an indicator diagram or P-V diagram. The shape of the indicator diagram shall depend on the nature of the thermodynamical process the system undergoes.

Let us consider one mole of an ideal gas enclosed in a cylinder fitted with a perfectly frictionless piston. Let P_1, V_1 and T be the initial state of the gas. If dV is an infinitesimally small increase in volume of the gas during which the pressure P is assumed to be constant, then small amount of workdone by the gas is $dW = PdV$

In the indicator diagram dW = area $a_1b_1c_1d_1$

$\therefore$ The total workdone by the gas during expansion from V_1 to V_2 is

$$W = \int_{V_1}^{V_2} PdV = \text{Area ABCD, in the}$$

indicator diagram.

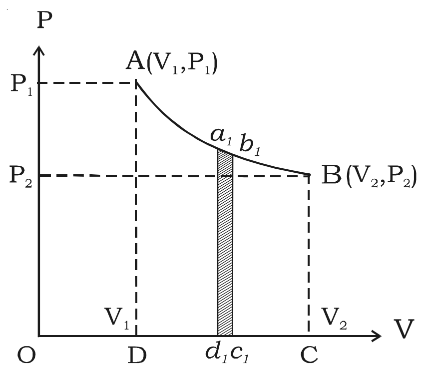


Fig. 8.9 Indicator diagram

Hence, in an indicator diagram the area under the curve represents the work done (Fig. 8.9).

8.8.1 Isothermal process

When a gas undergoes expansion or compression at constant temperature, the process is called isothermal process.

Let us consider a gas in a cylinder provided with a frictionless piston. The cylinder and the piston are made up of conducting material. If the piston is pushed down slowly, the heat energy produced will be quickly transmitted to the surroundings. Hence, the temperature remains constant but the pressure of the gas increases and its volume decreases.

The equation for an isothermal process is PV = constant.

If a graph is drawn between P and V, keeping temperature constant, we get a curve called an isothermal curve. Isotherms for three different

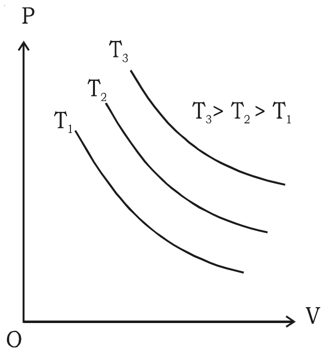


Fig. 8.10 Isothermal process

temperatures T_1, T_2 and T_3 are shown in the Fig. 8.10. The curve moves away from the origin at higher temperatures.

During an isothermal change, the specific heat capacity of the gas is infinite.

(i.e) $C = \dfrac{\Delta Q}{m\Delta T} = \infty \qquad (\because \Delta T = 0)$

(e.g) Melting of ice at its melting point and vapourisation of water at its boiling point.

8.8.2 Workdone in an isothermal expansion

Consider one mole of an ideal gas enclosed in a cylinder with perfectly conducting walls and fitted with a perfectly frictionless and conducting piston. Let P_1, V_1 and T be the initial pressure, volume and temperature of the gas. Let the gas expand to a volume V_2 when pressure reduces to P_2, at constant temperature T. At any instant during expansion let the pressure of the gas be P. If A is the area of cross section of the piston, then force $F = P \times A$.

Let us assume that the pressure of the gas remains constant during an infinitesimally small outward displacement dx of the piston. Work done

$$dW = Fdx = PAdx = PdV$$

Total work done by the gas in expansion from initial volume V_1 to final volume V_2 is

$$W = \int_{V_1}^{V_2} P\, dV$$

We know, $PV = RT$, $P = \dfrac{RT}{V}$

$$\therefore\ W = \int_{V_1}^{V_2} \frac{RT}{V}\, dV = RT \int_{V_1}^{V_2} \frac{1}{V} dV$$

$$W = RT \left[\log_e V\right]_{V_1}^{V_2}$$

$$W = RT\left[\log_e V_2 - \log_e V_1\right]$$

$$= RT \log_e \frac{V_2}{V_1}$$

$$W = 2.3026\ RT \log_{10} \frac{V_2}{V_1}$$

This is the equation for the workdone during an isothermal process.

8.8.3 Adiabatic process

In Greek, adiabatic means "nothing passes through". *The process in which pressure, volume and temperature of a system change in such a manner that during the change no heat enters or leaves the system is called adiabatic process.* Thus in adiabatic process, the total heat of the system remains constant.

Let us consider a gas in a perfectly thermally insulated cylinder fitted with a piston. If the gas is compressed suddenly by moving the piston downward, heat is produced and hence the temperature of the gas will increase. Such a process is adiabatic compression.

If the gas is suddenly expanded by moving the piston outward, energy required to drive the piston is drawn from the internal energy of the gas, causing fall in temperature. This fall in temperature is not compensated by drawing heat from the surroundings. This is adiabatic expansion.

Both the compression and expansion should be sudden, so that there is no time for the exchange of heat. Hence, *in an adiabatic process always there is change in temperature.*

Expansion of steam in the cylinder of a steam engine, expansion of hot gases in internal combustion engine, bursting of a cycle tube or car tube, propagation of sound waves in a gas are adiabatic processes.

The adiabatic relation between P and V for a gas, is

$PV^{\gamma} = k$, a constant ... (1)

where $\gamma = \dfrac{\text{specific heat capacity of the gas at constant pressure}}{\text{specific heat capacity of the gas at constant volume}}$

From standard gas equation,

$$PV = RT$$

$$P = \frac{RT}{V}$$

substituting the value P in (1)

$$\frac{RT}{V} V^{\gamma} = \text{constant}$$

$$T.V^{\gamma-1} = \text{constant}$$

In an adiabatic process Q =constant

$$\therefore \Delta Q = 0$$

$\therefore$ specific heat capacity $C = \dfrac{\Delta Q}{m\Delta T}$

$$\therefore C = 0$$

8.8.4 Work done in an adiabatic expansion

Consider one mole of an ideal gas enclosed in a cylinder with perfectly non conducting walls and fitted with a perfectly frictionless, non conducting piston.

Let P_1, V_1 *and* T_1 be the initial pressure, volume and temperature of the gas. If A is the area of cross section of the piston, then force exerted by the gas on the piston is

$F = P \times A$, where P is pressure of the gas at any instant during expansion. If we assume that pressure of the gas remains constant during an infinitesimally small outward displacement dx of the piston,

then work done $dW = F \times dx = P \times A\ dx$

$$dW = P\ dV$$

Total work done by the gas in adiabatic expansion from volume V_1 to V_2 is

$$W = \int_{V_1}^{V_2} P\ dV$$

But PV^{γ} = constant (k) for adiabatic process

where $\gamma = \dfrac{C_P}{C_V}$

$$\therefore \quad W = \int_{V_1}^{V_2} k.V^{-\gamma}\ dV = k\left[\frac{V^{1-\gamma}}{1-\gamma}\right]_{V_1}^{V_2} \quad \left(\because P = \frac{k}{V^{\gamma}}\right)$$

$$W = \frac{k}{1-\gamma}\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[kV_2^{1-\gamma} - kV_1^{1-\gamma}\right] \qquad \text{... (1)}$$

but, $P_2V_2^{\gamma} = P_1V_1^{\gamma} = k$ ... (2)

Substituting the value of k in (1)

$$\therefore\ W = \frac{1}{1-\gamma}\left[P_2V_2^{\gamma} \cdot V_2^{1-\gamma} - P_1 V_1^{\gamma} V_1^{1-\gamma}\right]$$

$$W = \frac{1}{1-\gamma}\left[P_2 V_2 - P_1V_1\right] \qquad \text{... (3)}$$

If T_2 is the final temperature of the gas in adiabatic expansion, then

$P_1V_1 = RT_1$, $P_2V_2 = RT_2$

Substituting in (3)

$$W = \frac{1}{1-\gamma}\left[RT_2 - RT_1\right]$$

$$W = \frac{R}{1-\gamma}\left[T_2 - T_1\right] \qquad \text{... (4)}$$

This is the equation for the work done during adiabatic process.

8.9 Reversible and irreversible processes

8.9.1 Reversible process

A thermodynamic process is said to be reversible when (i) the various stages of an operation to which it is subjected can be reversed in the opposite direction and in the reverse order and (ii) in every part of the process, the amount of energy transferred in the form of heat or work is the same in magnitude in either direction. At every stage of the process there is no loss of energy due to friction, inelasticity, resistance, viscosity etc. The heat losses to the surroundings by conduction, convection or radiation are also zero.

Condition for reversible process

(i) The process must be infinitely slow.

(ii) The system should remain in thermal equilibrium (i.e) system and surrounding should remain at the same temperature.

Examples

(a) Let a gas be compressed isothermally so that heat generated is conducted away to the surrounding. When it is allowed to expand in the same small equal steps, the temperature falls but the system takes up the heat from the surrounding and maintains its temperature.

(b) Electrolysis can be regarded as reversible process, provided there is no internal resistance.

8.9.2 Irreversible process

An irreversible process is one which cannot be reversed back. Examples : diffusion of gases and liquids, passage of electric current through a wire, and heat energy lost due to friction. As an irreversible process is generally a very rapid one, temperature adjustments are not possible. Most of the chemical reactions are irreversible.

8.10 Second law of thermodynamics

The first law of thermodynamics is a general statement of equivalence between work and heat. The second law of thermodynamics enables us to know whether a process which is allowed by first law of thermodynamics can actually occur or not. The second law of thermodynamics tells about the extent and direction of energy transformation.

Different scientists have stated this law in different ways to bring out its salient features.

(i) Kelvin's statement

Kelvin's statement of second law is based on his experience about the performance of heat engine.

It is impossible to obtain a continuous supply of work from a body by cooling it to a temperature below the coldest of its surroundings.

(ii) Clausius statement

It is impossible for a self acting machine unaided by any external *agency to transfer heat from a body at a lower temperature to another body at a higher temperature.*

(iii) Kelvin - Planck's statement

It is impossible to construct a heat engine operating in a cycle, that will extract heat from a reservoir and perform an equivalent amount of work.

8.11 Carnot engine

Heat engine is a device which converts heat energy into mechanical energy.

In the year 1824, Carnot devised an ideal cycle of operation for a heat engine. The machine used for realising this ideal cycle of operation is called an ideal heat engine or carnot heat engine.

The essential parts of a Carnot engine are shown in Fig. 8.11

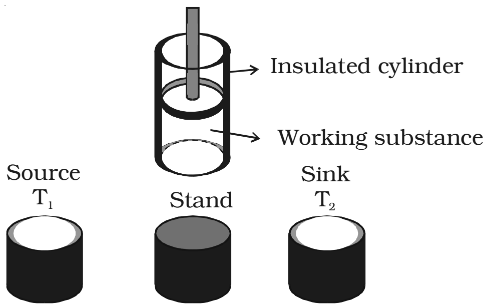


Fig. 8.11 Carnot engine

(i) Source

It is a hot body which is kept at a constant temperature T_1. It has infinite thermal capacity. Any amount of heat can be drawn from it at a constant temperature T_1 (i.e) its temperature will remain the same even after drawing any amount of heat from it.

(ii) Sink

It is a cold body which is kept at a constant lower temperature T_2. Its thermal capacity is also infinite that any amount of heat added to it will not increase its temperature.

(iii) Cylinder

Cylinder is made up of non-conducting walls and conducting bottom. A perfect gas is used as a working substance. The cylinder is fitted with a perfectly non-conducting and frcitionless piston.

(iv) Insulating stand

It is made up of non conducting material so as to perform adiabatic operations.

Working : The Carnot engine has the following four stages of operations.

1. Isothermal expansion 2. Adiabatic expansion 3. Isothermal compression 4. Adiabatic compression.

Isothermal expansion

Let us consider one mole of an ideal gas enclosed in the cylinder. Let V_1, P_1 be the initial volume and pressure of the gas respectively. The initial state of the gas is represented by the point *A* on the *P–V* diagram. The cylinder is placed over the source which is at the temperature T_1.

The piston is allowed to move slowly outwards, so that the gas expands. Heat is gained from the source and the process is isothermal at constant temperature T_1. In this process the volume of the gas changes from V_1 to V_2 and the pressure changes from P_1 to P_2. This process is represented by *AB* in the indicator diagram (Fig. 8.12). During this process, the quantity of heat absorbed from the source is Q_1 and W_1 is the corresponding amount of work done by the gas.

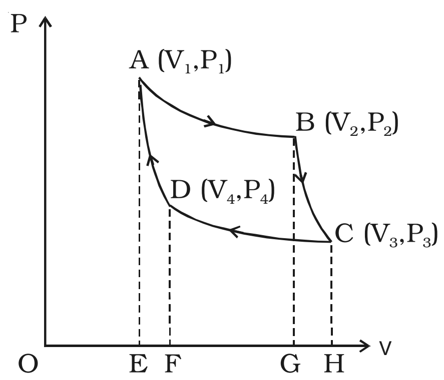


Fig. 8.12 Carnot cycle

$$\therefore Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT_1 \log_e \left(\frac{V_2}{V_1}\right)$$

= area ABGEA ...(1)

Adiabatic expansion

The cylinder is taken from the source and is placed on the insulting stand and the piston is moved further so that the volume of the gas changes from V_2 to V_3 and the pressure changes from P_2 to P_3. This adiabatic expansion is represented by BC. Since the gas is thermally insulated from all sides no heat can be gained from the surroundings. The temperature of the gas falls from T_1 to T_2.

Let W_2 be the work done by the gas in expanding adiabatically.

$$\therefore\ W_2 = \int_{V_2}^{V_3} PdV = \frac{R}{\gamma - 1}(T_1 - T_2) = \text{Area BCHGB} \qquad \ldots(2)$$

Isothermal compression

The cylinder is now placed on the sink at a temperature T_2. The piston is moved slowly downward to compress the gas isothermally. This is represented by CD. Let (V_4, P_4) be the volume and pressure corresponding to the point D. Since the base of the cylinder is conducting the heat produced during compression will pass to the sink so that, the temperature of the gas remains constant at T_2. Let Q_2 be the amount of heat rejected to the sink and W_3 be the amount of work done on the gas in compressing it isothermally.

$$Q_2 = W_3 = \int_{V_3}^{V_4} -P\ dV = -RT_2 \log_e\left(\frac{V_4}{V_3}\right) = -\text{ area CDFHC} \qquad \ldots(3)$$

The negative sign indicates that work is done on the working substance.

$$\therefore\ Q_2 = RT_2 \log_e \left(\frac{V_3}{V_4}\right)$$

Adiabatic compression

The cylinder is now placed on the insulating stand and the piston is further moved down in such a way that the gas is compressed adiabatically to its initial volume V_1 and pressure P_1. As the gas is insulated from all sides heat produced raises the temperature of the gas to T_1. This change is adiabatic and is represented by DA. Let W_4 be the work done on the gas in compressing it adiabatically from a state $D\ (V_4, P_4)$ to the initial state $A\ (V_1, P_1)$.

$$\therefore W_4 = \int_{V_4}^{V_1} -P\,dV = \frac{-R}{\gamma - 1}(T_2 - T_1)$$

The negative sign indicates that work is done on the working substance.

$$\therefore W_4 = \frac{R}{\gamma - 1}(T_1 - T_2) = \text{Area DAEFD} \quad \ldots(4)$$

Work done by the engine per cycle

Total work done by the gas during one cycle of operation is $(W_1 + W_2)$.

Total work done on the gas during one cycle of operation is $(W_3 + W_4)$.

$\therefore$ Net work done by the gas in a complete cycle

$$W = W_1 + W_2 - (W_3 + W_4)$$

But $W_2 = W_4$

$$\therefore W = W_1 - W_3$$

$$W = Q_1 - Q_2$$

Also, W = Area ABGEA + Area BCHGB - Area CDFHC - Area DAEFD

(i.e) W = Area ABCDA

Hence in Carnot heat engine, *net work done by the gas per cycle is numerically equal to the area of the loop representing the cycle.*

Efficiency of Carnot's engine

$$\eta = \frac{\text{Heat converted into work}}{\text{Heat drawn from the source}} = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

But $$\frac{Q_1}{Q_2} = \frac{W_1}{W_3} = \frac{RT_1 \log\left(V_2/V_1\right)}{RT_2 \log\left(V_3/V_4\right)}$$

$$= \frac{T_1 \log\left(V_2/V_1\right)}{T_2 \log\left(V_3/V_4\right)} \quad \ldots(5)$$

Since B and C lie on the same adiabatic curve BC

$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$ ($\because$ $TV^{\gamma-1}$ = constant) where $\gamma = \dfrac{C_p}{C_v}$

$$\therefore \frac{T_1}{T_2} = \frac{V_3^{\gamma-1}}{V_2^{\gamma-1}} \qquad \text{...(6)}$$

Similarly D & A lie on the same adiabatic curve DA

$$\therefore \; T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$$

$$\frac{T_1}{T_2} = \frac{V_4^{\gamma-1}}{V_1^{\gamma-1}} \qquad \text{...(7)}$$

From (6) & (7) $\dfrac{V_3^{\gamma-1}}{V_2^{\gamma-1}} = \dfrac{V_4^{\gamma-1}}{V_1^{\gamma-1}}$

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ (or) } \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{...(8)}$$

substituting equation (8) in equation (5)

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \; \frac{\log\left(V_3/V_4\right)}{\log\left(V_3/V_4\right)}$$

(i.e) $\dfrac{Q_2}{Q_1} = \dfrac{T_2}{T_1}$

$\therefore$ We have $\eta = 1 - \dfrac{Q_2}{Q_1} = 1 - \dfrac{T_2}{T_1}$

$$\text{or} \quad \eta = \frac{T_1 - T_2}{T_1} \qquad \text{...(9)}$$

Inferences

Efficiency of Carnot's cycle is independent of the working substance, but depends upon the temperatures of heat source and sink.

Efficiency of Carnot's cycle will be 100% if $T_1 = \infty$ *or* $T_2 = 0$ K. As neither the temperature of heat source can be made infinite, nor the temperature of the sink can be made 0 K, the inference is that the

Carnot heat engine working on the reversible cycle cannot have 100% efficiency.

8.12 Refrigerator

A refrigerator is a cooling device. *An ideal refrigerator can be regarded as Carnot's heat engine working in the reverse direction. Therefore, it is also called a heat pump.* In a refrigerator the working substance would absorb certain quantity of heat from the sink at lower temperature and reject a large amount of heat to the source at a higher temperature with the help of an external agency like an electric motor (Fig. 8.13).

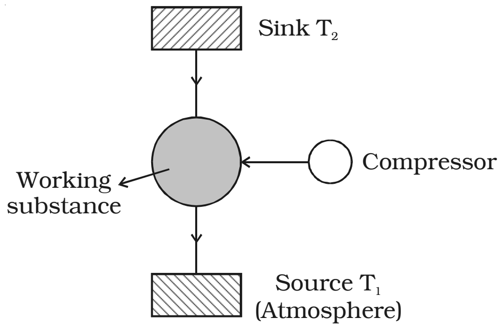


Fig. 8.13 Refrigerator

In an actual refrigerator vapours of freon (dichloro difluoro methane CCl_2F_2) act as the working substance. Things kept inside the refrigerator act as a sink at a lower temperature T_2. A certain amount of work W is performed by the compressor (operated by an electric motor) on the working substance. Therefore, it absorbs heat energy Q_2 from the sink and rejects Q_1 amount of heat energy to the source (atmosphere) at a temperature T_1.

Since this is a reversible cyclic process, the change in the internal energy of the working substance is zero (i.e) $dU = 0$

According to the first law of thermodynamics,

$$dQ = dU + dW$$

But $$dQ = Q_2 - Q_1$$

$$dW = -W \quad \text{and} \quad dU = 0$$

$$\therefore \quad dQ = Q_2 - Q_1 = -W$$

Negative sign with W represents work is done on the system

(i.e) $W = Q_1 - Q_2$

Coefficient of performance

Coefficient of performance (COP) is defined as the ratio of quantity of heat Q_2 removed per cycle from the contents of the refrigerator to the energy spent per cycle W to remove this heat.

$$\text{(i.e) COP} = \frac{Q_2}{W}$$

$$= \frac{Q_2}{Q_1 - Q_2}$$

$$\text{(i.e) COP} = \frac{T_2}{T_1 - T_2} \qquad \text{... (1)}$$

The efficiency of the heat engine is

$$\eta = 1 - \frac{T_2}{T_1}; \qquad 1 - \eta = \frac{T_2}{T_1}$$

$$\frac{1-\eta}{\eta} = \frac{T_2}{T_1} \times \frac{T_1}{T_1 - T_2}$$

$$\text{(i.e)} \frac{1-\eta}{\eta} = \frac{T_2}{T_1 - T_2} \qquad \text{... (2)}$$

From equations (1) and (2)

$$\text{COP} = \frac{1-\eta}{\eta} \qquad \text{...(3)}$$

Inferences

(i) Equation (1) shows that smaller the value of $(T_1 - T_2)$ greater is the value of COP. (i.e.) smaller is the difference in temperature between atmosphere and the things to be cooled, higher is the COP.

(ii) As the refrigerator works, T_2 goes on decreasing due to the formation of ice. T_1 is almost steady. Hence COP decreases. When the refrigerator is defrosted, T_2 increases.

Therefore defrosting is essential for better working of the refrigerator.

8.13 Transfer of heat

There are three ways in which heat energy may get transferred from one place to another. These are conduction, convection and radiation.

8.13.1 Conduction

Heat is transmitted through the solids by the process of conduction only. When one end of the solid is heated, the atoms or molecules of the solid at the hotter end becomes more strongly agitated and start vibrating with greater amplitude. The disturbance is transferred to the neighbouring molecules.

Applications

(i) The houses of Eskimos are made up of double walled blocks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the coldest surroundings.

(ii) Birds often swell their feathers in winter to enclose air between their body and the feathers. Air prevents the loss of heat from the body of the bird to the cold surroundings.

(iii) Ice is packed in gunny bags or sawdust because, air trapped in the saw dust prevents the transfer of heat from the surroundings to the ice. Hence ice does not melt.

Coefficient of thermal conductivity

Let us consider a metallic bar of uniform cross section A whose one end is heated. After sometime each section of the bar attains constant temperature but it is different at different sections. This is called steady state. In this state there is no further absorption of heat.

If Δx is the distance between the two sections with a difference in temperature of ΔT and ΔQ is the amount of heat conducted in a time Δt, then it is found that the rate of conduction of heat $\frac{\Delta Q}{\Delta t}$ is

(i) directly proportional to the area of cross section (A)

(ii) directly proportional to the temperature difference between the two sections (ΔT)

(iii) inversely proportional to the distance between the two sections (Δx).

(i.e) $$\frac{\Delta Q}{\Delta t} \propto A \frac{\Delta T}{\Delta x}$$

$$\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{\Delta x}$$

where K is a constant of proportionality called co-efficient of thermal conductivity of the metal.

$\frac{\Delta T}{\Delta x}$ is called temperature gradient

If $A = 1\ m^2$, and $\frac{\Delta T}{\Delta x}$ = unit temperature gradient

then, $\frac{\Delta Q}{\Delta t} = K \times 1 \times 1$

or $K = \frac{\Delta Q}{\Delta t}$

Coefficient of thermal conductivity of the material of a solid is equal to the rate of flow of heat per unit area per unit temperature gradient across the solid. Its unit is $W\ m^{-1}\ K^{-1}$.

8.13.2 Convection

It is a phenomenon of transfer of heat in a fluid with the actual movement of the particles of the fluid.

When a fluid is heated, the hot part expands and becomes less dense. It rises and upper colder part replaces it. This again gets heated, rises up replaced by the colder part of the fluid. This process goes on. This mode of heat transfer is different from conduction where energy transfer takes place without the actual movement of the molecules.

Application

It plays an important role in ventilation and in heating and cooling system of the houses.

8.13.3 Radiation

It is the phenomenon of transfer of heat without any material medium. Such a process of heat transfer in which no material medium takes part is known as radiation.

Thermal radiation

The energy emitted by a body in the form of radiation on account of its temperature is called thermal radiation.

It depends on,

(i) temperature of the body,

(ii) nature of the radiating body

The wavelength of thermal radiation ranges from 8×10^{-7} *m* to 4×10^{-4} *m*. They belong to infra-red region of the electromagnetic spectrum.

Properties of thermal radiations

1. Thermal radiations can travel through vacuum.
2. They travel along straight lines with the speed of light.
3. They can be reflected and refracted. They exhibit the phenomenon of interference and diffraction.
4. They do not heat the intervening medium through which they pass.
5. They obey inverse square law.

Absorptive and Emissive power

Absorptive power

Absorptive power of a body for a given wavelength and temperature is defined *as the ratio of the radiant energy absorbed per unit area per unit time to the total energy incident on it per unit area per unit time.*

It is denoted by a_λ.

Emissive power

Emissive power of a body at a given temperature is *the amount of energy emitted per unit time per unit area of the surface for a given wavelength.* It is denoted by e_λ. Its unit is $W\ m^{-2}$.

8.14 Perfect black body

A perfect black body is the one which absorbs completely heat radiations of all wavelengths which fall on it and emits heat radiations of all wavelengths when heated. Since a perfect black body neither reflects nor transmits any radiation, the absorptive power of a perfectly black body is unity.

8.14.1 Fery's black body

Fery's black body consists of a double walled hollow sphere having a small opening O on one side and a conical projection P just opposite to it (Fig. 8.14). Its inner surface is coated with lamp black. Any radiation entering the body through the opening O suffers multiple reflections at its innerwall and about 97% of it is absorbed by lamp black at each reflection. Therefore, after a few reflections almost entire radiation is absorbed. The projection helps in avoiding any direct reflections which even otherwise is not possible because of the small opening O. When this body is placed in a bath at fixed temperature, the heat radiations come out of the hole. The opening O thus acts as a black body radiator.

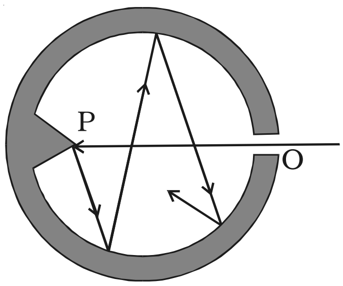


Fig. 8.14 Fery's black body

8.14.2 Prevost's theory of heat exchanges

Prevost applied the idea of thermal equilibrium to radiation. According to him the rate at which a body radiates or absorbs heat depends on the nature of its surface, its temperature and the temperature of the surroundings. The total amount of heat radiated by a body increases as its temperature rises. A body at a higher temperature radiates more heat energy to the surroundings than it receives from the surroundings. That is why we feel warm when we stand before the furnace.

Similarly a body at a lower temperature receives more heat energy than it loses to the surroundings. That is why we feel cold when we stand before an ice block.

Thus the rise or fall of temperature is due to the exchange of heat radiation. When the temperature of the body is the same as that of surroundings, the exchanges of heat do not stop. In such a case, the amount of heat energy radiated by the body is equal to the amount of heat energy absorbed by it.

A body will stop emitting radiation only when it is at absolute zero. (i.e) 0 K or -273° C. At this temperature the kinetic energy of the molecule is zero.

Therefore, Prevost theory states that all bodies emit thermal radiation at all temperatures above absolute zero, irrespective of the nature of the surroundings.

8.14.3 Kirchoff's Law

According to this law, *the ratio of emissive power to the absorptive power corresponding to a particular wavelength and at a given temperature is always a constant for all bodies.* This constant is equal to the emissive power of a perfectly black body at the same temperature and the same wavelength. Thus, if e_λ is the emissive power of a body corresponding to a wavelength λ at any given temperature, a_λ is the absorptive power of the body corresponding to the same wavelength at the same temperature and E_λ is the emissive power of a perfectly black body corresponding to the same wavelength and the same temperature, then according to Kirchoff's law

$$\frac{e_\lambda}{a_\lambda} = \text{constant} = E_\lambda$$

From the above equation it is evident that if a_λ is large, then e_λ will also be large (i.e) if a body absorbs radiation of certain wavelength strongly then it will also strongly emit the radiation of same wavelength. In other words, good absorbers of heat are good emitters also.

Applications of Kirchoff's law

(i) The silvered surface of a thermos flask is a bad absorber as well as a bad radiator. Hence, ice inside the flask does not melt quickly and hot liquids inside the flask do not cool quickly.

(ii) Sodium vapours on heating, emit two bright yellow lines. These are called D_1 and D_2 lines of sodium. When continuous white light from carbon arc passes through sodium vapour at low temperature, the continuous spectrum is absorbed at two places corresponding to the wavelengths of D_1 and D_2 lines and appear as dark lines. This is in accordance with Kirchoff's law.

8.14.4 Wien's displacement law

Wien's displacement law *states that the wavelength of the radiation corresponding to the maximum energy (λ_m) decreases as the temperature T of the body increases.*

(i.e) $\lambda_m T = b$ where b is called Wien's constant.

Its value is 2.898×10^{-3} m K

8.14.5 Stefan's law

Stefan's law states that *the total amount of heat energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of its absolute temperature.*

(i.e) $E \alpha T^4$ or $E = \sigma T^4$

where σ is called the Stefan's constant. Its value is 5.67×10^{-8} W m^{-2} K^{-4}.

It is also called Stefan - Boltzmann law, as Boltzmann gave a theoretical proof of the result given by Stefan.

8.14.6 Newton's law of cooling

Newton's law of cooling states that *the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings.*

The law holds good only for a small difference of temperature. Loss of heat by radiation depends on the nature of the surface and the area of the exposed surface.

Experimental verification of Newton's law of cooling

Let us consider a spherical calorimeter of mass m whose outer surface is blackened. It is filled with hot water of mass m_1. The calorimeter with a thermometer is suspended from a stand (Fig. 8.15).

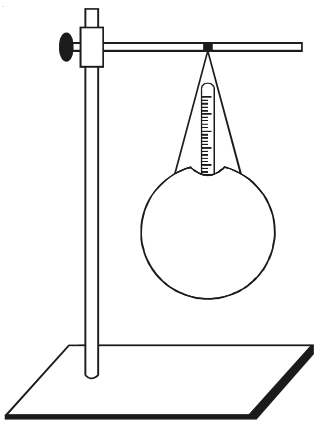


Fig. 8.15 Newton's law of cooling

The calorimeter and the hot water radiate heat energy to the surroundings. Using a stop clock, the temperature is noted for every 30 seconds interval of time till the temperature falls by about 20° C. The readings are entered in a tabular column.

If the temperature of the calorimeter and the water falls from T_1 to T_2 in t seconds, the quantity of heat energy lost by radiation $Q = (ms + m_1s_1)(T_1 - T_2)$, where s is the specific heat capacity of the material of the calorimeter and s_1 is the specific heat capacity of water.

Rate of cooling $= \dfrac{\textit{Heat energy lost}}{\textit{time taken}}$

$$\therefore \quad \frac{Q}{t} = \frac{(ms + m_1 s_1)(T_1 - T_2)}{t}$$

If the room temperature is T_o, the average excess temperature of the calorimeter over that of the surroundings is $\left(\frac{T_1 + T_2}{2} - T_o\right)$

According to Newton's Law of cooling, $\frac{Q}{t} \,\alpha \left(\frac{T_1 + T_2}{2} - T_o\right)$

$$\frac{(ms + m_1 s_1)(T_1 - T_2)}{t} \,\alpha \left(\frac{T_1 + T_2}{2} - T_o\right)$$

$$\therefore \quad \frac{(ms + m_1 s_1)(T_1 - T_2)}{t\left(\frac{T_1 + T_2}{2} - T_o\right)} = \text{constant}$$

The time for every 4° fall in temperature is noted. The last column in the tabular column is found to be the same. This proves Newton's Law of cooling.

Table 8.1 Newton's law of cooling

Temperature range	Time t for every 4° fall of temperature	Average excess of temperature $\left(\frac{T_1 + T_2}{2} - T_o\right)$	$\left(\frac{T_1 + T_2}{2} - T_o\right) t$

A cooling curve is drawn by taking time along X-axis and temperature along Y-axis (Fig. 8.16).

From the cooling curve, the rate of fall of temperature at T is $\dfrac{dT}{dt} = \dfrac{AB}{BC}$

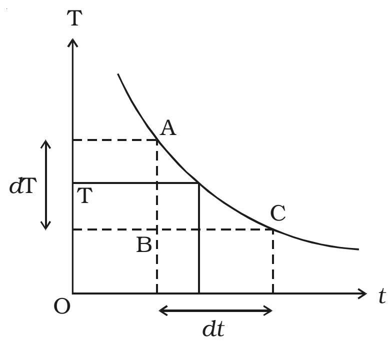


Fig. 8.16 Cooling curve

The rate of cooling $\frac{dT}{dt}$ is found to be directly proportional to $(T - T_o)$. Hence Newton's law of cooling is verified.

8.15 Solar constant

The solar constant is the amount of radiant energy received per second per unit area by a perfect black body on the Earth with its surface perpendicular to the direction of radiation from the sun in the absence of atmosphere. It is denoted by S and its value is 1.388×10^3 $W\ m^{-2}$. Surface temperature of the Sun can be calculated from solar constant.

Surface temperature of the Sun

The Sun is a perfect black body of radius r and surface temperature T. According to Stefan's law, the energy radiated by the Sun per second per unit area is equal to σT^4.

Where σ is Stefan's Constant.

Hence, the total energy radiated per second by the Sun will be given by

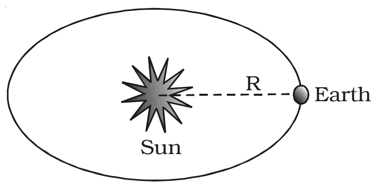


Fig. 8.17 Surface temperature of the Sun

E = surface area of the Sun $\times\ \sigma T^4$

$$E = 4\pi r^2\ \sigma T^4 \qquad ...(1)$$

Let us imagine a sphere with Sun at the centre and the distance between the Sun and Earth R as radius (Fig. 8.17). The heat energy from the Sun will necessarily pass through this surface of the sphere.

If S is the solar constant, the amount of heat energy that falls on this sphere per unit time is $E = 4\pi R^2 S$ ...(2)

By definition, equations (1) & (2) are equal.

$$\therefore \quad 4\pi r^2 \sigma T^4. = 4\pi R^2 S$$

$$T^4 = \frac{R^2 S}{r^2 \sigma}$$

$$T = \left(\frac{R^2 S}{r^2 \sigma}\right)^{\frac{1}{4}}; \quad (i.e) \quad T = \left(\frac{R}{r}\right)^{\frac{1}{2}} \left(\frac{S}{\sigma}\right)^{\frac{1}{4}}$$

Knowing the values of R, r, S and σ the surface temperature of the Sun can be calculated.

8.15.1 Angstrom pyrheliometer

Pyrheliometer is an instrument used to measure the quantity of heat radiation and solar constant.

Pyrheliometer designed by Angstrom is the simplest and most accurate.

Angstrom's pyrheliometer consists of two identical strips S_1 and S_2 of area A. One junction of a thermocouple is connected to S_1 and the other junction is connected to S_2. A sensitive galvanometer is connected to the thermo couple.

Strip S_2 is connected to an external electrical circuit as shown in Fig.8.18. When both the strips S_1 and S_2 are shielded from the solar radiation, galvanometer shows no deflection as both the junctions are at the same temperature. Now strip S_1 is exposed to the solar radiation and S_2 is shielded with a cover M. As strip S_1 receives heat radiations from the sun, its temperature rises and hence the galvanometer shows deflection. Now current is allowed to pass through the strip S_2 and it is adjusted so that galvanometer shows no deflection. Now, the strips S_1 and S_2 are again at the same temperature.

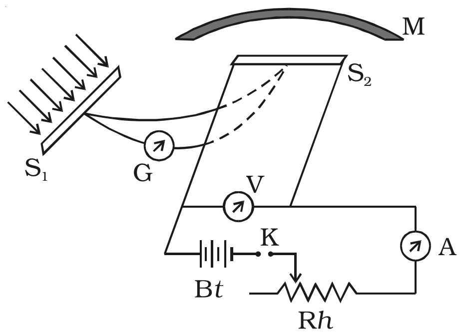


Fig. 8.18 Angstrom pyrheliometer

If the quantity of heat radiation that is incident on unit area in unit time on strip S_1 is Q and a its absorption co-efficient, then the amount of heat radiations absorbed by the strip S_1 in unit time is QAa.

Also, heat produced in unit time in the strip S_2 is given by VI, where V is the potential difference and I is the current flowing through it.

As heat absorbed = heat produced

$$QAa = VI \quad (or) \quad Q = \frac{VI}{Aa}$$

Knowing the values of V, I, A and a, Q can be calculated.

Solved Problems

8.1 At what temperature will the RMS velocity of a gas be tripled its value at NTP?

Solution : At NTP, $T_o = 273$ K

RMS velocity, $C = \sqrt{\frac{3RT_o}{M}}$

$$C = \sqrt{\frac{3R \times 273}{M}} \quad \ldots (1)$$

Suppose at the temperature T, the RMS velocity is tripled, then

$$3C = \sqrt{\frac{3RT}{M}} \quad \ldots (2)$$

Divide (2) by (1)

$$\frac{3C}{C} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{3R \times 273}{M}}}$$

$$3 = \sqrt{\frac{T}{273}}$$

$$T = 273 \times 9 \quad = 2457 \text{ K}$$

8.2 Calculate the number of degrees of freedom in 15 cm^3 of nitrogen at NTP.

Solution : *We know 22400 cm^3 of a gas at NTP contains 6.02×10^{23} molecules.*

$\therefore$ *The number of molecules in 15 cm^3 of N_2 at NTP*

$$n = \frac{15}{22400} \times 6.023 \times 10^{23} = 4.033 \times 10^{20}$$

The number degrees of freedom of a diatomic gas molecule at 273 K, is $f = 5$

$\therefore$ *Total degrees of freedom of 15 cm^3 of the gas = nf*

$\therefore$ *Total degrees of freedom = $4.033 \times 10^{20} \times 5 = 2.016 \times 10^{21}$*

8.3 A gas is a mixture of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting vibrational modes, show that the energy of the system is 11 RT where R is the universal gas constant.

Solution : *Since oxygen is a diatomic moleucle with 5 degrees of freedom, degrees of freedom of molecules in 2 moles of oxygen* $= f_1 = 2\,N \times 5 = 10\,N$

Since argon is a monatomic molecules degrees of freedom of molecules in 4 moles of argon $= f_2 = 4\,N \times 3 = 12\,N$

$\therefore$ *Total degrees of freedom of the mixture* $= f = f_1 + f_2 = 22\,N$

As per the principle of law of equipartition of energy, energy associated with each degree of freedom of a molecule $= \frac{1}{2}\,kT$

$\therefore$ *Total energy of the system* $= \frac{1}{2}\,kT \times 22\,N = 11\,RT$

8.4 Two carnot engines A and B are operating in series. The first one A receives heat at 600 K and rejects to a reservoir at temperature T. The second engine B receives the heat rejected by A and in turn rejects heat to a reservior at 150 K. Calculate the temperature T when (i) The work output of both the engines are equal, (ii) The efficiency of both the engines are equal.

Solution : *(i) When the work outputs are equal :*

For the first engine $W_1 = Q_1 - Q_2$

For the second engine $W_2 = Q_2 - Q_3$

Given (i.e) $W_1 = W_2$

$$Q_1 - Q_2 = Q_2 - Q_3$$

Divide by Q_2 *on both sides*

$$\frac{Q_1}{Q_2} - 1 = 1 - \frac{Q_3}{Q_2}$$

Also $\frac{Q_1}{Q_2} = \frac{600}{T}$

and $\frac{Q_2}{Q_3} = \frac{T}{150}$ $\qquad \left[\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right]$

$$\therefore \frac{600}{T} - 1 = 1 - \frac{150}{T}$$

$$\frac{600-T}{T} = \frac{T-150}{T}$$

$$\therefore T = 375\ K$$

(ii) When efficiencies are equal

$$\eta_1 = 1 - \frac{Q_2}{Q_1} \quad and \quad \eta_2 = 1 - \frac{Q_3}{Q_2}$$

As $\eta_1 = \eta_2$

$$1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_3}{Q_2}$$

$$1 - \frac{T}{600} = 1 - \frac{150}{T}$$

$$\frac{600-T}{600} = \frac{T-150}{T}$$

$$\frac{T}{600} = \frac{150}{T}$$

$$T^2 = 600 \times 150$$

$$\therefore \quad T = 300\ K$$

8.5 A carnot engine whose low temperature reservoir is at 7° C has an efficiency of 50 %. It is desired to increase the efficiency to 70 %. By how many degrees should the temperature of the high temperature reservoir be increased?

Data : $\eta_1 = 50\ \% = 0.5$; $T_2 = 7 + 273 = 280K$; $\eta_2 = 70\% = 0.7$

Solution : $\eta_1 = 1 - \frac{T_2}{T_1}$; $0.5 = 1 - \frac{280}{T_1}$; $\therefore T_1 = 560\ K$

Let the temperature of the high temperature reservoir be T_1'

$$\eta_2 = 1 - \frac{T_2}{T_1'} \; ; 0.7 = 1 - \frac{280}{T_1'} \quad ; \quad \therefore T_1' = 933.3\ K$$

$\therefore$ *The temperature of the reservoir should be increased by* $933.3\ K - 560\ K = 373.3\ K$

8.6 A carnot engine is operated between two reservoirs at temperature 177° C and 77° C. If the engine receives 4200 J of heat energy from the source in each cycle, calculate the amount of heat rejected to the sink in each cycle. Calculate the efficiency and work done by the engine.

Data : $T_1 = 177^o\ C = 177 + 273 = 450\ K.$

$T_2 = 77^o\ C = 77 + 273 = 350\ K$

$Q_1 = 4200\ J \quad Q_2 = ?$

Solution : $$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\therefore \quad Q_2 = Q_1 \frac{T_2}{T_1} = 4200 \times \frac{350}{450}$$

$Q_2 = 3266.67\ J$

Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

$$\eta = 1 - \frac{350}{450} = 0.2222 = 22.22\%$$

Work done

$W = Q_1 - Q_2 = 4200 - 3266.67$

$W = 933.33\ J$

8.7 A Carnot engine has the same efficiency, when operated

(i) between 100 K and 500 K

(ii) between T K and 900 K

Find the value of T

Solution : *(i) Here* $T_1 = 500\ K; T_2 = 100\ K$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{500} = 1 - 0.2 = 0.8$$

(ii) Now, $T_1 = 900\ K; T_2 = T$ *and* $\eta = 0.8$

Again, $\eta = 1 - \frac{T_2}{T_1}$

$0.8 = 1 - \frac{T}{900}$ *or* $\frac{T}{900} = 1 - 0.8 = 0.2$

$\therefore T = 180\ K$

8.8 In a refrigerator heat from inside at 277 K is transfered to a room at 300 K. How many joule of heat will be delivered to the room for each joule of electric energy consumed ideally?

Data : $T_1 = 300\ K$; $T_2 = 277\ K$
Solution : *COP of a refrigerator*

$$= \frac{T_2}{T_1 - T_2} = \frac{277}{300 - 277} = 12.04 \qquad ...(1)$$

Suppose for each joule of electric energy consumed an amount of heat Q_2 *is extracted from the inside of refrigerator. The amount of heat delivered to the room for each joule of electrical energy consumed is given by*

$$Q_1 = Q_2 + W = Q_2 + 1 \qquad (\because W = Q_1 - Q_2)$$

$$\therefore \qquad Q_1 - Q_2 = 1$$

Also for a refrigerator, $COP = \frac{Q_2}{Q_1 - Q_2} = Q_2 \qquad ...(2)$

From equations (1) and (2)

(i.e) $Q_2 = 12.04$

$\therefore Q_1 = Q_2 + 1 = 12.04 + 1 = 13.04\ J$

8.9 Two rods A and B of different material have equal length and equal temperature gradient. Each rod has its ends at temperatures T_1 and T_2. Find the condition under which rate of flow of heat through the rods A and B is same.

Solution : *Suppose the two rods A and B have the same temperature difference* $T_1 - T_2$ *across their ends and the length of each rod is l.*

When the two rods have the same rate of heat conduction,

$$\frac{K_1 A_1 (T_1 - T_2)}{l} = \frac{K_2 A_2 (T_1 - T_2)}{l}$$

$$K_1 A_1 = K_2 A_2 \text{ or } \frac{A_1}{A_2} = \frac{K_2}{K_1}$$

(i.e) for the same rate of heat conduction, the areas of cross - section of the two rods should be inversely proportional to their coefficients of thermal conductivity.

8.10 A metal cube takes 5 minutes to cool from 60^o C to 52^o C. How much time will it take to cool to 44^o C, if the temperature of the surroundings is 32^o C?

Solution : *While cooling from 60^o C to 52^o C*

Rate of cooling $= \frac{60-52}{5} = 1.6^o$ *C/minute* $= \frac{1.6^o\ C}{60}$ *per second*

$\therefore$ *Average temperature while cooling* $= \frac{60+52}{2} = 56^o$ C

$\therefore$ *Average temperature excess* $= 56 - 32 = 24^o$ *C*

According to Newton's law of cooling,

Rate of cooling α *Temperature excess*

$\therefore$ *Rate of cooling* $= K \times$ *temperature excess*

$$\frac{1.6}{60} = K \times 24 \qquad \ldots(1)$$

Suppose that the cube takes t seconds to cool from 52^o C to 44^o C

$\therefore$ *Rate of cooling* $= \frac{52-44}{t} = \frac{8}{t}$

Average temperature while cooling $= \frac{52+44}{2} = 48^o$ *C*

$\therefore$ *Average temperature excess* $= 48 - 32 = 16^o$ *C*

According to Newton's law, Rate of cooling $= K \times$ *(Temperature excess)* $\frac{8}{t} = K \times 16$

Dividing equation (1) by equation (2)

$$\frac{1.6}{60} \times \frac{t}{8} = \frac{24}{16} \quad = 450\ s$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

8.1 Avogadro number is the number of molecules in

(a) one litre of a gas at NTP

(b) one mole of a gas

(c) one gram of a gas

(d) 1 kg of a gas

8.2 First law of thermodynamics is a consequence of the conservation of

(a) momentum *(b) charge*

(c) mass *(d) energy*

8.3 At a given temperature, the ratio of the RMS velocity of hydrogen to the RMS velocity of oxygen is

(a) 4 *(b)* $\frac{1}{4}$

(c) 16 *(d) 8*

8.4 The property of the system that does not change during an adiabatic change is

(a) temperature *(b) volume*

(c) pressure *(d) heat*

8.5 For an ant moving on the horizontal surface, the number of degrees of freedom of the ant will be:

(a) 1 *(b) 2*

(c) 3 *(d) 6*

8.6 *The translational kinetic energy of gas molecules for one mole of the gas is equal to :*

(a) $\frac{3}{2}RT$ *(b)* $\frac{2}{3}kT$

(c) $\frac{1}{2}RT$ *(d)* $\frac{3}{2}kT$

8.7 *The internal energy of a perfect gas is*

(a) partly kinetic and partly potential

(b) wholly potential

(c) wholly kinetic

(d) depends on the ratio of two specific heats

8.8 *A refrigerator with its power on, is kept in a closed room. The temperature of the room will*

(a) rise *(b) fall*

(c) remains the same *(d) depend on the area of the room*

8.9 *A beaker full of hot water is kept in a room. If it cools from 80°C to 75°C in* t_1 *minutes, from 75°C to 70°C in* t_2 *minutes and from 70°C to 65°C in* t_3 *minutes then*

(a) $t_1 = t_2 = t_3$ *(b)* $t_1 < t_2 = t_3$

(c) $t_1 < t_2 < t_3$ *(d)* $t_1 > t_2 > t_3$

8.10 *Which of the following will radiate heat to the large extent*

(a) white polished surface *(b) white rough surface*

(c) black polished surface *(d) black rough surface*

8.11 *A block of ice in a room at normal temperature*

(a) does not radiate

(b) radiates less but absorbs more

(c) radiates more than it absorbs

(d) radiates as much as it absorbs

8.12 *What are the postulates of Kinetic theory of gases?*

8.13 *Derive an expression for the average kinetic energy of the molecule of gas.*

8.14 *Two different gases have exactly the same temperature. Do the molecules have the same RMS speed?*

8.15 *Explain internal energy. What is its value in one complete cyclic process?*

8.16 *What are degrees of freedom?*

8.17 *State the law of equipartition of energy and prove that for a diatomic gas, the ratio of the two specific heats at room temperature is* $\frac{7}{5}$.

8.18 *Distinguish between isothermal and adiabatic process*

8.19 *Define isothermal process. Derive an expression for the work done during the process.*

8.20 *A gas has two specific heats, whereas liquid and solid have only one. Why?*

8.21 *Derive an expression for the work done in one cycle during an adiabatic process*

8.22 *Define molar specific heat at constant pressure.*

8.23 *Derive Meyer's relation.*

8.24 *What is an indicator diagram?*

8.25 *Distinguish between reversible process and irreversible process with examples.*

8.26 *Is it possible to increase the temperature of a gas without the addition of heat? Explain.*

8.27 *On driving a scooter for a long time the air pressure in the tyre slightly increases why?*

8.28 *How is second law of thermodynamics different from first law of thermodynamics?*

8.29 *Define Clausius statement.*

8.30 *Describe the working of Carnot engine and derive its efficiency.*

8.31 *Give an example for a heat pump.*

8.32 *A heat engine with 100% efficiency is only a theoretical possibility. Explain.*

8.33 *What is Coefficient of Performance? Derive the relation between COP and efficiency.*

8.34 *Why are ventilators provided in our houses?*

8.35 *Define temperature gradient.*

8.36 *Define steady state in thermal conduction of heat.*

8.37 *What are the factors upon which coefficient of thermal conductivity depends?*

8.38 *Write the applications of Kirchoff's law.*

8.39 *Define absorptive power.*

8.40 *Define Stefan's law.*

8.41 *Explain Fery's concept of a perfect black body.*

8.42 *State Wien's displacement law.*

8.43 *State Newton's law of cooling. Explain the experimental verification of Newton's law of cooling.*

8.44 *Why does a piece of red glass when heated and taken out glow with green light?*

8.45 *Define solar constant.*

8.46 *Describe the working of pyrheliometer.*

Problems

8.47 *Calculate the kinetic energy of translational motion of a molecule of a diatomic gas at 320 K.*

8.48 *Calculate the rms velocity of hydrogen molecules at NTP (One mole of hydrogen occupies 22.4 litres at NTP).*

8.49 *The RMS speed of dust particles in air at NTP is 2.2×10^{-2} ms^{-1}. Find the average mass of the particles.*

8.50 *Find the number of molecules in 10×10^{-6} m^3 of a gas at NTP, if the mass of each molecule is 4×10^{-26} kg and the RMS velocity is 400 m s^{-1}.*

8.51 *Calculate the molecular kinetic energy of translation of one mole of hydrogen at NTP. ($R = 8.31$ J mol^{-1} K^{-1}).*

8.52 *Find the work done by 1 mole of perfect gas when it expands isothermally to double its volume. The initial temperature of the gas is 0^oC ($R=8.31$ J mol^{-1} K^{-1}).*

8.53 *A tyre pumped to a pressure of 3 atmosphere suddenly bursts. Calculate the fall in temperature if the temperature of air before expansion is 27^oC and $\gamma = 1.4$.*

8.54 *A certain volume of dry air at NTP is expanded into three times its volume, under (i) isothermal condition (ii) adiabatic condition. Calculate in each case, the final pressure and final temperature, (γ for air = 1.4).*

8.55 *A gas is suddenly compressed to $\frac{1}{2}$ of its original volume. If the original temperature is 300 K, find the increase in temperature (Assume $\gamma = 1.5$).*

8.56 *A system absorbs 8.4 k J of heat and at the same time does 500 J of work. Calculate the change in internal energy of the system.*

8.57 *How many metres can a man weighing 60 kg, climb by using the energy from a slice of bread which produces a useful work of 4.2×10^5 J. Efficiency of human body is 28 %.*

8.58 *The wavelength with maximum energy emitted from a certain star in our galaxy is 1.449×10^{-5}cm. Calculate the temperature of star.*

8.59 *The surface temperature of a spherical hot body is 1000 K. Calculate the rate at which energy is radiated.*

(Given $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4})

8.60 *The opposite faces of the top of an electric oven are at a difference of temperature of 100^oC and the area of the top surface and its* *thickness are 300 cm^2 and 0.2 cm respectively. Find the quantity of heat that will flow through the top surface in one minute.*

($K = 0.2\ W\ m^{-1}\ K^{-1}$)

8.61 *Compare the rate of loss of heat from a black metal sphere at 227°C with the rate of loss of heat from the same sphere at 127°C. The temperature of the surroundings is 27°C.*

8.62 *The ratio of radiant energies radiated per unit surface area by two bodies is 16 : 1. The temperature of hotter body is 1000 K. Calculate the temperature of the other body. Hint: $E \alpha (T^4 - T_0^4)$*

8.63 *Calculate the surface temperature of the Sun (λ_m = 4753 Å).*

8.64 *A hot solid takes 10 minutes to cool from 60° C to 50° C. How much further time will it take to cool to 40° C, if the room temperature is 20° C?*

8.65 *An object is heated and then allowed to cool when its temperature is 70°C, its rate of cooling is 3°C per minute and when the temperature is 60°C, the rate of cooling is 2.5°C per minute. Determine the temperature of the surroundings.*

Answers

8.1	*(b)*	**8.2**	*(d)*	**8.3**	*(a)*
8.4	*(d)*	**8.5**	*(b)*	**8.6**	*(a)*
8.7	*(c)*	**8.8**	*(a)*	**8.9**	*(c)*
8.10	*(d)*	**8.11**	*(b)*		

8.47 6.624×10^{-21} J **8.48** 1845 m s^{-1}

8.49 2.335×10^{-17} kg **8.50** 4.748×10^{20}

8.51 3.403×10^{3} J **8.52** 1572.6 J

8.53 80.8 K

8.54 3.376×10^{4} N m^{-2} ; 273 K ; 2.171×10^{4} N m^{-2} ; 176 K

8.55 124.2 K **8.56** 7900 J

8.57 200 m **8.58** 20000 K

8.59 5.67×10^{4} W m^{-2} **8.60** 18 K J

8.61 31 : 10 **8.62** 500 K

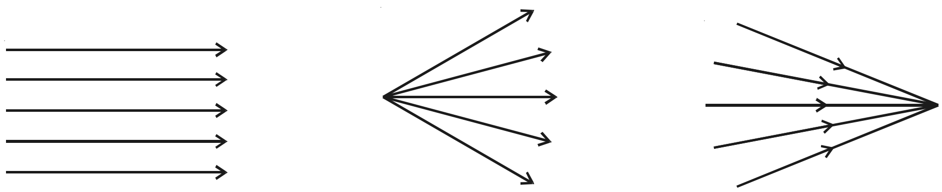
8.63 6097 K **8.64** 840 seconds

8.65 10^{o} C

9. Ray Optics

Light rays and beams

A ray of light is the direction along which the light energy travels. In practice a ray has a finite width and is represented in diagrams as straight lines. A beam of light is a collection of rays. A search light emits a parallel beam of light (Fig. 9.1a). Light from a lamp travels in all directions which is a divergent beam. (Fig. 9.1b). A convex lens produces a convergent beam of light, when a parallel beam falls on it (Fig. 9.1c).



(a) Parallel beam *(b) Divergent Beam* *(c) Convergent Beam*

Fig. 9.1 Beam of light

9.1 Reflection of light

Highly polished metal surfaces reflect about 80% to 90% of the light incident on them. Mirrors in everyday use are therefore usually made of depositing silver on the backside of the glass. The largest reflector in the world is a curved mirror nearly 5 metres across, whose front surface is coated with aluminium. It is the hale Telescope on the top of Mount Palomar, California, U.S.A. Glass by itself, will also reflect light, but the percentage is small when compared with the case of silvered surface. It is about 5% for an air-glass surface.

9.1.1 Laws of reflection

Consider a ray of light, AO, incident on a plane mirror XY at O. It is reflected along OB. Let the normal ON is drawn at the point of incidence. The angle AON between the incident ray and the normal is called angle of incidence, i (Fig. 9.2) the angle BON between the reflected ray and the normal is called angle of reflection, r. Experiments show that : (i) *The incident ray, the reflected ray and the normal drawn to the reflecting surface at the point of incidence, all lie in the same plane.*

(ii) *The angle of incidence is equal to the angle of reflection.* (i.e) $i = r$.

These are called the laws of reflection.

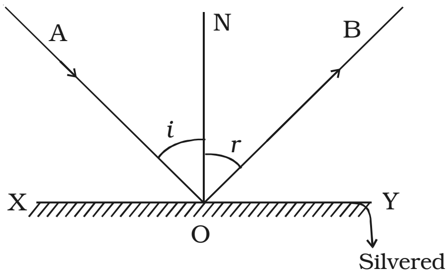


Fig. 9.2 Reflection at a plane mirror

9.1.2 Deviation of light by plane mirror

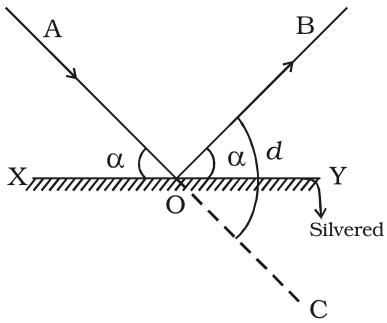


Fig. 9.3 Deviation of light by a plane mirror

Consider a ray of light, AO, incident on a plane mirror XY (Fig. 9.3) at O. It is reflected along OB. The angle AOX made by AO with XY is known as the glancing angle α with the mirror. Since the angle of reflection is equal to the angle of incidence, the glancing angle BOY made by the reflected ray OB with the mirror is also equal to α.

The light has been deviated from a direction AO to a direction OB. Since angle COY = angle AOX, it follows that angle of deviation, $d = 2\alpha$

So, in general, *the angle of deviation of a ray by a plane mirror or a plane surface is twice the glancing angle.*

9.1.3 Deviation of light due to rotation of a mirror

Let us consider a ray of light AO incident on a plane mirror XY at O. It is reflected along OB. Let α be the glancing angle with XY (Fig. 9.4). We know that the angle of deviation COB = 2α.

Suppose the mirror is rotated through an angle θ to a position X′Y′.

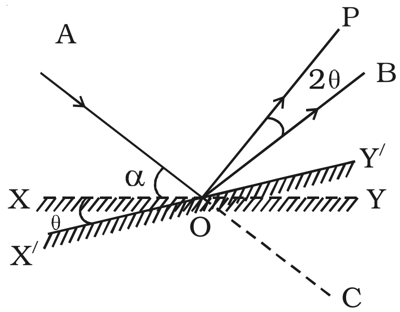


Fig. 9.4 Deviation of light due to rotation of a mirror

The same incident ray AO is now reflected along OP. Here the glancing angle with X′Y′ is ($\alpha + \theta$). Hence the new angle of deviation COP = 2 ($\alpha + \theta$). The reflected ray has thus been rotated through an angle BOP when the mirror is rotated through an angle θ.

$$\angle BOP = \angle COP - \angle COB$$

$$\angle BOP = 2(\alpha + \theta) - 2\alpha = 2\theta$$

For the same incident ray, when the mirror is rotated through an angle, the reflected ray is rotated through twice the angle.

9.2 Image in a plane mirror

Let us consider a point object A placed in front of a plane mirror M as shown in the Fig. 9.5. Consider a ray of light AO from the point object incident on the mirror and reflected along OB. Draw the normal ON to the mirror at O.

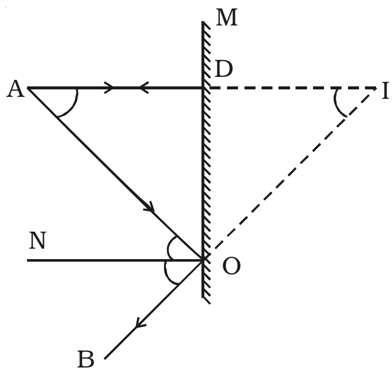


Fig. 9.5 Image in a plane mirror

The angle of incidence AON = angle of reflection BON

Another ray AD incident normally on the mirror at D is reflected back along DA. When BO and AD are produced backwards, they meet at I. Thus the rays reflected from M appear to come from a point I behind the mirror.

From the figure

$\angle AON = \angle DAO$, alternate angles and $\angle BON = \angle DIO$, corresponding angles it follows that $\angle DAO = \angle DIO$.

The triangles ODA and ODI are congruent

$\therefore$ AD = ID

For a given position of the object, A and D are fixed points. Since AD = ID, the point I is also fixed. It should be noted that AO = OI. So the object and its image in a plane mirror are at equal perpendicular distances from the mirror.

9.2.1 Virtual and real images

An object placed in front of a plane mirror has an image behind the mirror. The rays reflected from the mirror do not actually meet through I, but only appear to meet and the image cannot be received on the screen, because the image is behind the mirror. This type of image is called an unreal or virtual image (Fig. 9.6a).

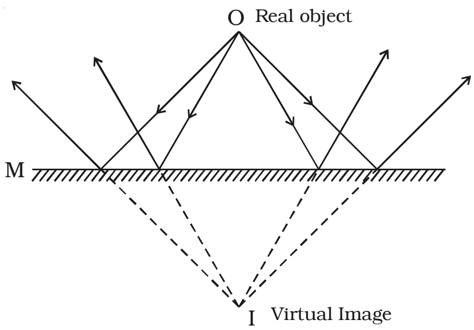


Fig. 9.6a Virtual image in a plane mirror

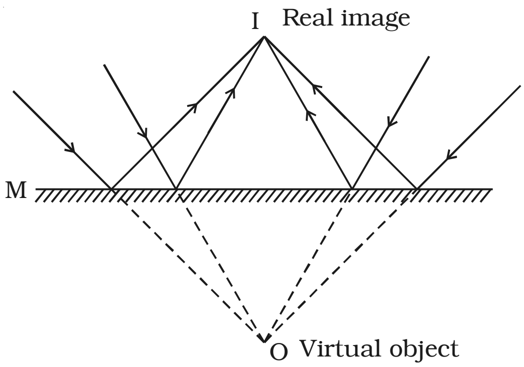


Fig. 9.6b Real image in a plane mirror

If a convergent beam is incident on a plane mirror, the reflected rays pass through a point I in front of M, as shown in the Fig. 9.6b. In the Fig. 9.6a, a real object (divergent beam) gives rise to a virtual image. In the Fig. 9.6b, a virtual object (convergent beam) gives a real image. Hence plane mirrors not only produce virtual images for real objects but also produce real images for virtual objects.

9.2.2 Characteristics of the image formed by a plane mirror

(i) Image formed by a plane mirror is as far behind the mirror as the object is in front of it and it is always virtual.

(ii) The image produced is laterally inverted.

(iii) The minimum size of the mirror required to see the complete image of the object is half the size of the object.

(iv) If the mirror turns by an angle θ, the reflected ray turns through an angle 2θ.

(v) If an object is placed between two plane mirrors inclined at an angle θ, then the number of images formed is $n = \frac{360^o}{\theta} - 1$

9.3 Reflection at curved surfaces

In optics we are mainly concerned with curved mirrors which are the part of a hollow sphere (Fig. 9.7). One surface of the mirror is silvered. Reflection takes place at the other surface. If the reflection takes place at the concave surface, (which is towards the centre of the sphere) it is called concave mirror. If the reflection takes place at the convex surface, (which is away from the centre of the sphere) it is called convex mirror. The laws of reflection at a plane mirror are equally true for spherical mirrors also.

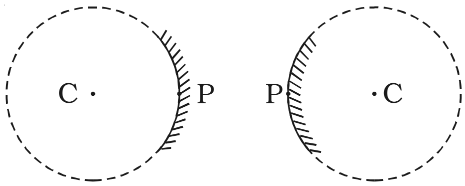


Fig.9.7 Concave and convex mirror

The centre of the sphere, of which the mirror is a part is called the *centre of curvature* (C).

The geometrical centre of the mirror is called its *pole* (P).

The line joining the pole of the mirror and its centre of curvature is called the *principal axis*.

The distance between the pole and the centre of curvature of the spherical mirror is called the *radius of curvature* of the mirror and is also equal to the radius of the sphere of which the mirror forms a part.

When a parallel beam of light is incident on a spherical mirror, the point where the reflected rays converge (concave mirror) or appear to

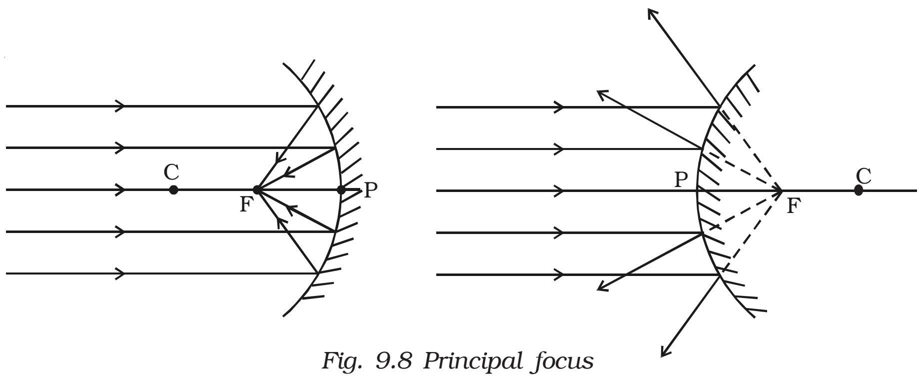


Fig. 9.8 Principal focus

diverge from the point (convex mirror) on the principal axis is called the *principal focus* (F) of the mirror. The distance between the pole and the principal focus is called the *focal length* (f) of the mirror (Fig. 9.8).

9.3.1 Images formed by a spherical mirror

The images produced by spherical mirrors may be either real or virtual and may be either larger or smaller than the object. The image can be located by graphical construction as shown in Fig. 9.9 by adopting any two of the following rules.

(i) A ray parallel to the principal axis after reflection by a concave mirror passes through the principal focus of the concave mirror and appear to come from the principal focus in a convex mirror.

(ii) A ray passing through the centre of curvature retraces its path after reflection.

(iii) A ray passing through the principal focus, after reflection is rendered parallel to the principal axis.

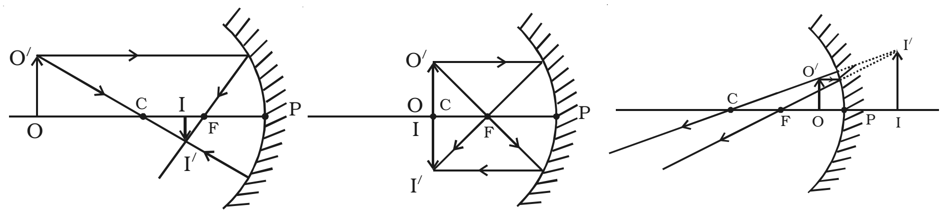


Fig. 9.9 Formation of images in concave mirror

(iv) A ray striking the pole at an angle of incidence i is reflected at the same angle i to the axis.

9.3.2 Image formed by a convex mirror

In a convex mirror irrespective of the position of the object, the image formed is always virtual, erect but diminished in size. The image lies between the pole and the focus (Fig. 9.10).

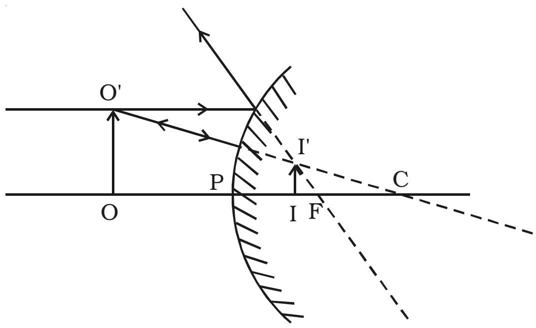


Fig. 9.10 Image formed by convex mirror

In general, real images are located in front of a mirror while virtual images behind the mirror.

9.3.3 Cartesian sign convention

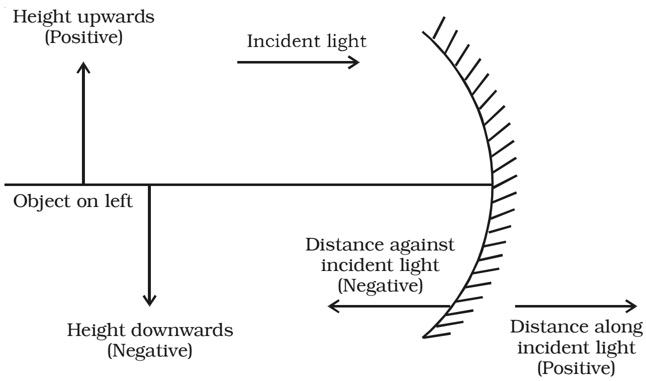


Fig. 9.11 Sign convention

The following sign conventions are used.

(1) All distances are measured from the pole of the mirror (in the case of lens from the optic centre).

(2) The distances measured in the same direction as the incident light, are taken as positive.

(3) The distances measured in the direction opposite to the direction of incident light are taken as negative.

(4) Heights measured perpendicular to the principal axis, in the upward direction are taken as positive.

(5) Heights measured perpendicular to the principal axis, in the downward direction are taken as negative.

(6) The size of the object is always taken as positive, but image size is positive for erect image and negative for an inverted image.

(7) The magnification is positive for erect (and virtual) image, and negative for an inverted (and real) image.

9.3.4 Relation between u, v and f for spherical mirrors

A mathematical relation between object distance u, the image distance v and the focal length f of a spherical mirror is known as mirror formula.

(i) Concave mirror - real image

Let us consider an object OO′ on the principal axis of a concave mirror beyond C. The incident and the reflected rays are shown in the Fig 9.12. A ray O′A parallel to principal axis is incident on the concave mirror at A, close to P. After reflections the ray passes through the focus F. Another ray O′C passing through centre of curvature C, falls normally on the mirror and reflected back along the same path. A third ray O′P incident at the pole P is reflected along PI′. The three reflected rays intersect at the point I′. Draw perpendicular I′I to the principal axis. II′ is the real, inverted image of the object OO′.

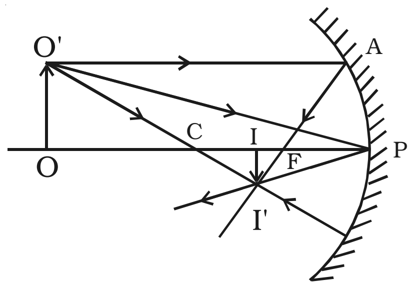


Fig. 9.12 Concave mirror-real image

Right angled triangles, *II′P* and *OO′P* are similar.

$$\therefore \frac{II'}{OO'} = \frac{PI}{PO} \qquad \dots (1)$$

Right angled triangles *II′F* and *APF* are also similar (A is close to P ; hence AP is a vertical line)

$$\therefore \frac{II'}{AP} = \frac{IF}{PF}$$

$AP = OO'$. Therefore the above equation becomes,

$$\frac{II'}{OO'} = \frac{IF}{PF} \qquad \dots (2)$$

Comparing the equations (1) and (2)

$$\frac{PI}{PO} = \frac{IF}{PF} \qquad \dots (3)$$

But, $IF = PI - PF$

Therefore equation (3) becomes,

$$\frac{PI}{PO} = \frac{PI - PF}{PF} \qquad \dots (4)$$

Using sign conventions, we have $PO = -u$,

$PI = -v$ and $PF = -f$

Substituting the values in the above equation, we get

$$\frac{-v}{-u} = \frac{-v-(-f)}{-f} \quad \text{(or)}$$

$$\frac{v}{u} = \frac{v-f}{f} = \frac{v}{f} - 1$$

Dividing by v and rearranging, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

This is called *mirror equation*. The same equation can be obtained for virtual image also.

(ii) Convex mirror - virtual image

Let us consider an object OO′ anywhere on the principal axis of a convex mirror. The incident and the reflected rays are shown in the Fig. 9.13. A ray O′A parallel to the principal axis incident on the convex mirror at A close to P. After reflection the ray appears to diverge from the focus F. Another ray O′C passing through centre of curvature C, falls normally on the mirror and is reflected back along the same path. A third ray $O'P$ incident at the pole P is reflected along PQ. The three reflected rays when produced appear to meet at the point I'. Draw perpendicular II' to the principal axis. II′ is the virtual image of the object OO′.

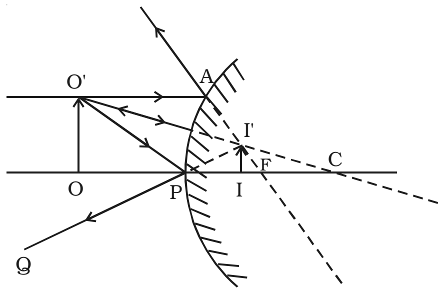


Fig. 9.13 Convex mirror – Virtual image

Right angled triangles, $II'P$ and $OO'P$ are similar.

$$\therefore \frac{II'}{OO'} = \frac{PI}{PO} \qquad \text{... (1)}$$

Right angled triangles $II'F$ and APF are also similar (A is close to P; hence AP is a vertical line)

$$\frac{II'}{AP} = \frac{IF}{PF}$$

$AP = OO'$. Therefore the above equation becomes,

$$\frac{II'}{OO'} = \frac{IF}{PF} \qquad \ldots (2)$$

Comparing the equations (1) and (2)

$$\frac{PI}{PO} = \frac{IF}{PF} \qquad \ldots (3)$$

But, $IF = PF - PI$. Therefore equation (3) becomes,

$$\frac{PI}{PO} = \frac{PF - PI}{PF}$$

Using sign conventions, we have $PO = -u$, $PI = +v$ and $PF = +f$.

Substituting the values in the above equation, we get

$$\frac{+v}{-u} = \frac{+f - (+v)}{+f} \quad \text{(or)} \quad -\frac{v}{u} = \frac{f - v}{f} = 1 - \frac{v}{f}$$

Dividing by v and rearranging we get, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

This is called *mirror equation for convex mirror producing virtual image.*

9.3.5 Magnification

The linear or transverse magnification is defined as the ratio of the size of the image to that of the object.

$$\therefore \text{Magnification} = \frac{\textit{size of the image}}{\textit{size of the object}} = \frac{h_2}{h_1}$$

where h_1 and h_2 represent the size of the object and image respectively.

From Fig. 9.12 it is known that $\frac{II'}{OO'} = \frac{PI}{PO}$

Applying the sign conventions,

$II' = -h_2$ (height of the image measured downwards)

$OO' = +h_1$ (height of the object measured upwards)

$PI = -v$ (image distance against the incident light)

$PO = -u$ (object distance against the incident light)

Substituting the values in the above equation, we get

magnification $m = \frac{-h_2}{+h_1} = \frac{-v}{-u}$ (or) $m = \frac{h_2}{h_1} = \frac{-v}{u}$

For an erect image m is positive and for an inverted image m is negative. This can be checked by substituting values for convex mirror also.

Using mirror formula, the equation for magnification can also be obtained as

$$m = \frac{h_2}{h_1} = \frac{-v}{u} = \frac{f-v}{f} = \frac{f}{f-u}$$

This equation is valid for both convex and concave mirrors.

9.4 Total internal reflection

When a ray of light AO passes from an optically denser medium to a rarer medium, at the interface XY, it is partly reflected back into the same medium along OB and partly refracted into the rarer medium along OC (Fig. 9.14).

If the angle of incidence is gradually increased, the angle of refraction r will also gradually increase and at a certain stage r becomes 90^o. Now the refracted ray OC is bent so much away from the normal and it grazes the surface of separation of two media. *The angle of incidence in the denser medium at which the refracted ray just grazes the surface of separation is called the critical angle c of the denser medium.*

If i is increased further, refraction is not possible and the incident

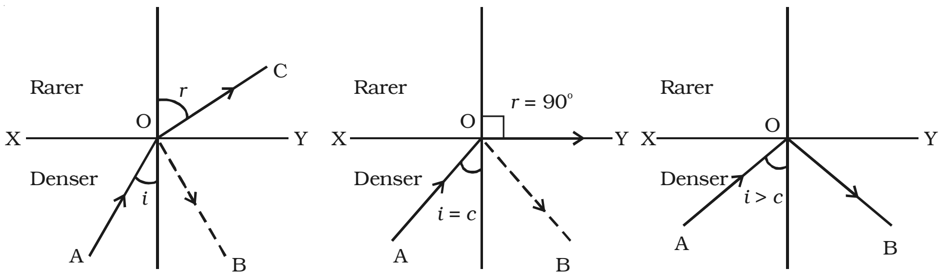


Fig. 9.14 Total internal reflection

ray is totally reflected into the same medium itself. This is called *total internal reflection.*

If μ_d is the refractive index of the denser medium then, from Snell's Law, the refractive index of air with respect to the denser medium is given by,

$$_d\mu_a = \frac{\sin i}{\sin r}$$

$$\frac{\mu_a}{\mu_d} = \frac{\sin i}{\sin r}$$

$$\frac{1}{\mu_d} = \frac{\sin i}{\sin r} \quad (\because \mu_a = 1 \text{ for air})$$

If $r = 90^o$, $i = c$

$$\frac{\sin c}{\sin 90^o} = \frac{1}{\mu_d} \quad \text{(or)} \quad \sin c = \frac{1}{\mu_d} \text{ or } c = \sin^{-1}\left(\frac{1}{\mu_d}\right)$$

If the denser medium is glass, $c = \sin^{-1}\left(\frac{1}{\mu_g}\right)$

Hence for total internal reflection to take place (i) light must travel from a denser medium to a rarer medium and (ii) the angle of incidence inside the denser medium must be greater than the critical angle i.e. $i > c$.

Table 9.1 Critical angle for some media

(NOT FOR EXAMINATION)

Medium	Refractive index	Critical angle
Water	1.33	48.75^o
Crown glass	1.52	41.14^o
Dense flint glass	1.62	37.31^o
Diamond	2.42	24.41^o

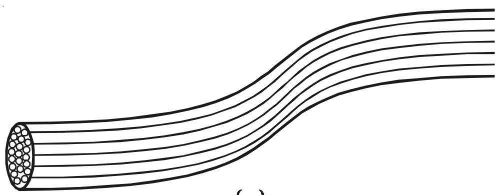
9.4.1 Applications

(i) Diamond

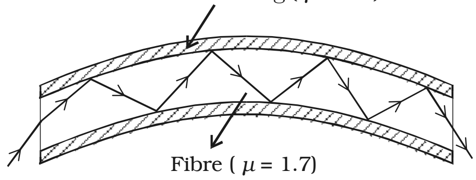
Total internal reflection is the main cause of the brilliance of diamonds. The refractive index of diamond with respect to air is 2.42. Its critical angle is 24.41^o. When light enters diamond from any face at an angle greater than 24.41^o it undergoes total internal reflection. By cutting the diamond suitably, multiple internal reflections can be made to occur.

(ii) Optical fibres

The total internal reflection is the basic principle of optical fibre. An optical fibre is a very thin fibre made of glass or quartz having radius of the order of micrometer (10^{-6}m). A bundle, of such thin fibres forms a 'light pipe' (Fig. 9.15a).



(a)



(b)

Fig.9.15 An optical fibre

Fig. 9.15b shows the principle of light transmission inside an optical fibre. The refractive index of the material of the core is higher than that of the cladding. When the light is incident at one end of the fibre at a small angle, the light passes inside, undergoes repeated total internal reflections along the fibre and finally comes out. The angle of incidence is always larger than the critical angle of the core material with respect to its cladding. Even if the fibre is bent or twisted, the light can easily travel through the fibre.

Light pipes are used in medical and optical examination. They are also used to transmit communication signals.

9.5 Michelson's method

A.A. Michelson, an American physicist, spent many years of his life in measuring the velocity of light and he devised a method in the year 1926 which is considered as accurate.

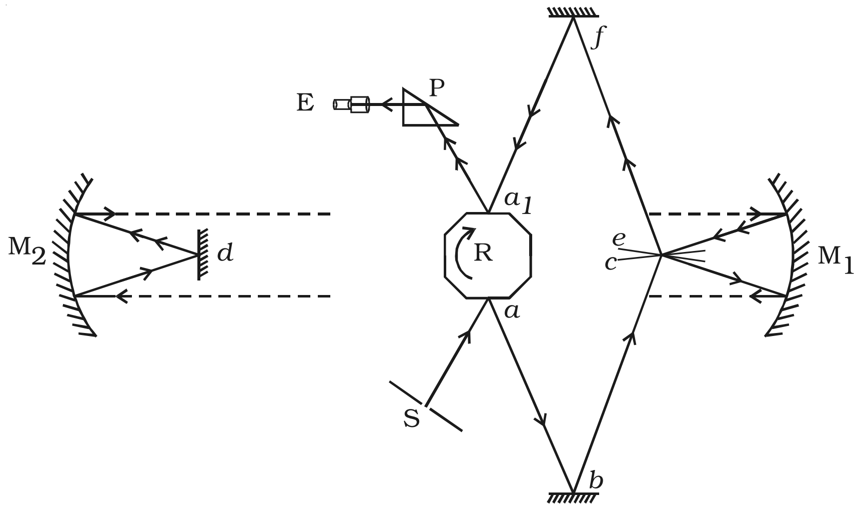


Fig. 9.16 Michelson's method

The experimental set up is shown in Fig. 9.16. Light from an arc source after passing through a narrow slit S is reflected from one face a of an octagonal mirror R. The ray after reflections at small fixed mirrors b and c is then rendered parallel by a concave mirror M_1 placed in the observing station on Mt. Wilson. This parallel beam of light travels a distance of 35 km and falls on another concave mirror M_2 placed at Mt. St Antonio, and it is reflected to a plane mirror d placed at the focus of the concave mirror M_2. The ray of light from d is rendered parallel after getting reflected by M_2 and travels back to the concave mirror M_1.

After reflections at M_1 and the plane mirrors e and f, the ray falls on the opposite face a_1 of the octagonal mirror. The final image which is totally reflected by a total reflecting prism P, is viewed through an eye piece E.

When the octagonal mirror is stationary, the image of the slit is seen through the eye piece. When it is rotated the image disappears. The speed of rotation of R is suitably adjusted so that the image is seen again clearly as when R is stationary. The speed of revolution is measured by stroboscope.

Let D be the distance travelled by light from face a to face a_1 and n be the number of rotations made by R per second.

The time taken by R to rotate through 45^{o} or $\frac{1}{8}$ of a rotation = $\frac{1}{8n}$

During this time interval, the distance travelled by the light = D

$\therefore$ The velocity of light c = $\frac{\text{Distance travelled}}{\text{Time taken}} = \frac{D}{\frac{1}{8n}}$ = $8nD$.

In general, if the number of faces in the rotating mirror is N, the velocity of light = NnD.

The velocity of light determined by him is 2.99797×10^{8} m s^{-1}.

Importance of velocity of light

The value of velocity of light in vacuum is of great importance in science. The following are some of the important fields where the value of velocity of light is used.

(1) Frequency - wavelength relation : From the relation $c = \nu\lambda$, the frequency of electromagnetic radiations can be calculated if the wavelength is known and vice versa.

(2) Relativistic mass variation with velocity : Theory of relativity has shown that the mass m of a moving particle varies with its velocity v according to the relation $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

Here m_o is the rest mass of the particle.

(3) Mass - Energy relation : E = mc^2 represents conversion of mass into energy and energy into mass. The energy released in nuclear fission and fusion is calculated using this relation.

(4) Measurement of large distance in Astronomy : Light year is a unit of distance used in astronomy. A light year is the distance travelled by light in one year. It is equal to 9.46×10^{15} metre.

(5) Refractive index : The refractive index μ of a medium is given by

$$\mu = \frac{\text{velocity of light in vacuum}}{\text{velocity of light in medium}} = \frac{c}{v}$$

9.6 Refraction of light

When a ray of light travels from one transparent medium into another medium, it bends while crossing the interface, separating the two media. This phenomenon is called refraction.

Image formation by spherical lenses is due to the phenomenon of refraction. The laws of refraction at a plane surface are equally true for refraction at curved surfaces also. While deriving the expressions for refraction at spherical surfaces, we make the following assumptions.

(i) The incident light is assumed to be monochromatic and

(ii) the incident pencil of light rays is very narrow and close to the principal axis.

9.6.1 Cartesian sign convention

The sign convention followed in the spherical mirror is also applicable to refraction at spherical surface. In addition to this two more sign conventions to be introduced which are:

(i) The power of a converging lens is positive and that of a diverging lens is negative.

(ii) The refractive index of a medium is always said to be positive. If two refractions are involved, the difference in their refractive index is also taken as positive.

9.6.2 Refraction at a spherical surface

Let us consider a portion of a spherical surface AB separating two media having refracting indices μ_1 and μ_2 (Fig. 9.17). This is symmetrical about an axis passing through the centre *C* and cuts the surface at *P*. The point *P* is called the pole of the surface. Let *R* be the radius of curvature of the surface.

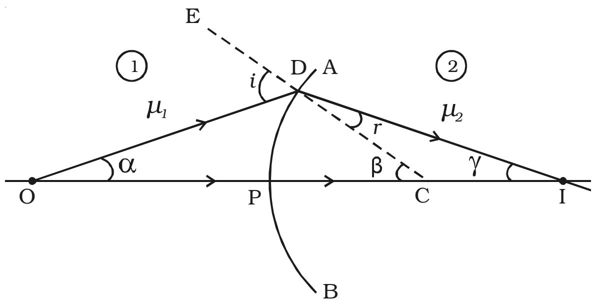


Fig. 9.17 Refraction at a spherical surface

Consider a point object *O* on the axis in the first medium. Consider two rays *OP* and *OD* originating from *O*. The ray *OP* falls normally on *AB* and goes into the second medium, undeviated. The ray *OD* falls at *D* very close to *P*. After refraction, it meets at the point *I* on the axis, where the image is formed. *CE* is the normal drawn to the point *D*. Let *i* and *r* be the angle of incidence and refraction respectively.

Let $\underline{|DOP} = \alpha, \underline{|DCP} = \beta, \ \underline{|DIC} = \gamma$

Since *D* is close to *P*, the angles α, β and γ are all small. From the Fig. 9.17.

$$\tan\alpha = \frac{DP}{PO}, \ \tan\beta = \frac{DP}{PC} \text{ and } \tan\gamma = \frac{DP}{PI}$$

$$\therefore \alpha = \frac{DP}{PO}, \ \beta = \frac{DP}{PC} \text{ and } \gamma = \frac{DP}{PI}$$

From the ΔODC, $i = \alpha + \beta$ ...(1)

From the ΔDCI, $\beta = r + \gamma$ or $r = \beta - \gamma$ ...(2)

From Snell's Law, $\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$ and for small angles of *i* and *r*, we can write, $\mu_1 i = \mu_2 r$ ...(3)

From equations (1), (2) and (3)

we get $\mu_1 (\alpha + \beta) = \mu_2 (\beta - \gamma)$ or $\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$ ... (4)

Substituting the values of α, β and γ in equation (4)

$$\mu_1\left(\frac{DP}{PO}\right) + \mu_2\left(\frac{DP}{PI}\right) = \left(\mu_2 - \mu_1\right)\frac{DP}{PC}$$

$$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \left(\frac{\mu_2 - \mu_1}{PC}\right) \quad \text{...(5)}$$

As the incident ray comes from left to right, we choose this direction as the positive direction of the axis. Therefore *u* is negative, whereas *v* and *R* are positive substitute $PO = -u$ $PI = +v$ and $PC = +R$ in equation (5),

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{... (6)}$$

Equation (6) represents the general equation for refraction at a spherical surface.

If the first medium is air and the second medium is of refractive index μ, then

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R} \qquad ...(7)$$

9.6.3 Refraction through thin lenses

A lens is one of the most familiar optical devices. A lens is made of a transparent material bounded by two spherical surfaces. If the distance between the surfaces of a lens is very small, then it is a thin lens.

As there are two spherical surfaces, there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2. The line joining C_1 and C_2 is called the *principal axis* of the lens. The centre P of the thin lens which lies on the principal aixs is called the optic centre.

9.6.4 Lens maker's formula and lens formula

Let us consider a thin lens made up of a medium of refractive index μ_2 placed in a medium of refractive index μ_1. Let R_1 and R_2 be the radii of curvature of two spherical surfaces *ACB* and *ADB* respectively and *P* be the optic centre.

Consider a point object O on the principal axis. The ray OP falls normally on the spherical surface and goes through the lens undeviated. The ray OA falls at A very close to P. After refraction at the surface ACB the image is formed at I′. Before it does so, it is again refracted by the surface ADB. Therefore the final image is formed at I as shown in Fig. 9.18.

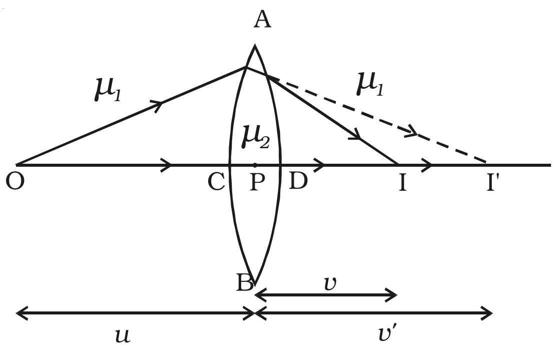


Fig. 9.18 Refraction through a lens

The general equation for the refraction at a spherical surface is given by

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{... (1)}$$

For the refracting surface ACB, from equation (1) we write

$$\frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{... (2)}$$

The image I′ acts as a virtual object for the surface ADB and the final image is formed at I. The second refraction takes place when light travels from the medium of refractive index μ_2 to μ_1.

For the refracting surface ADB, from equation (1) and applying sign conventions, we have

$$\frac{\mu_1}{v} - \frac{\mu_2}{v'} = \left(\frac{\mu_2 - \mu_1}{-R_2}\right) \quad \text{... (3)}$$

Adding equations (2) and (3) $\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Dividing the above equation by μ_1

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(4)}$$

If the object is at infinity, the image is formed at the focus of the lens.

Thus, for $u = \infty$, $v = f$. Then the equation (4) becomes.

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(5)}$$

If the refractive index of the lens is μ and it is placed in air, $\mu_2 = \mu$ and $\mu_1 = 1$. So the equation (5) becomes

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(6)}$$

This is called the *lens maker's formula*, because it tells what curvature will be needed to make a lens of desired focal length. This formula is true for concave lens also.

Comparing equation (4) and (5)

we get $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ... (7)

which is known as the *lens formula*.

9.6.5 Magnification

Let us consider an object OO' placed on the principal axis with its height perpendicular to the principal axis as shown in Fig. 9.19. The ray *OP* passing through the optic centre will go undeviated. The ray $O'A$ parallel to the principal axis must pass through the focus F_2. The image is formed where $O'PI'$ and AF_2I' intersect. Draw a perpendicular from I' to the principal axis. This perpendicular II' is the image of OO'.

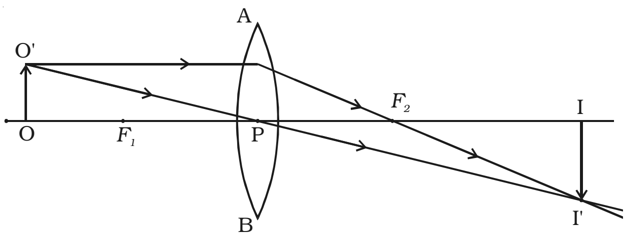


Fig. 9.19 Magnification

The linear or transverse magnification is defined as the ratio of the size of the image to that of the object.

$$\therefore \text{ Magnification } m = \frac{\textit{Size of the image}}{\textit{Size of the object}} = \frac{II'}{OO'} = \frac{h_2}{h_1}$$

where h_1 is the height of the object and h_2 is the height of the image.

From the similar right angled triangles $OO'P$ and $II'P$, we have

$$\frac{II'}{OO'} = \frac{PI}{PO}$$

Applying sign convention,

$II' = -h_2$; $OO' = +h_1$;

$PI = +v$; $PO = -u$;

Substituting this in the above equation, we get magnification

$$m = \frac{-h_2}{+h_1} = \frac{+v}{-u}$$

$$\therefore m = +\frac{v}{u}$$

The magnification is negative for real image and positive for virtual image. In the case of a concave lens, it is always positive.

Using lens formula the equation for magnification can also be obtained as $m = \frac{h_2}{h_1} = \frac{v}{u} = \frac{f-v}{f} = \frac{f}{f+u}$

This equation is valid for both convex and concave lenses and for real and virtual images.

9.6.6 Power of a lens

Power of a lens is a measure of the degree of convergence or divergence of light falling on it. *The power of a lens (P) is defined as the reciprocal of its focal length.*

$$P = \frac{1}{f}$$

The unit of power is dioptre (*D*) : *1 D* = 1 m^{-1}. *The power of the lens is said to be 1 dioptre if the focal length of the lens is 1 metre.* P is positive for converging lens and negative for diverging lens. Thus, when an optician prescribes a corrective lens of power + 0.5 D, the required lens is a convex lens of focal length + 2 m. A power of -2.0 D means a concave lens of focal length -0.5 m.

9.6.7 Combination of thin lenses in contact

Let us consider two lenses *A* and *B* of focal length f_1 and f_2 placed in contact with each other. An object is placed at *O* beyond the focus of the first lens *A* on the common principal axis. The lens *A* produces an image at I_1. This image I_1 acts as the object for the second lens *B*. The final image is produced at *I* as shown in Fig. 9.20. Since the lenses are thin, a common optical centre *P* is chosen.

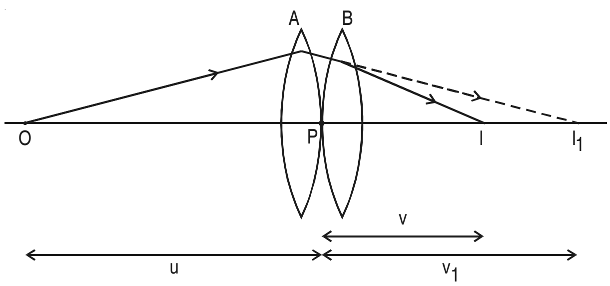


Fig. 9.20 Image formation by two thin lenses

Let *PO* = *u*, object distance for the first lens (A), *PI* = *v*, final image distance and $PI_1 = v_1$, image distance for the first lens (A) and also object distance for second lens (B).

For the image I_1 produced by the first lens A,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \qquad ...(1)$$

For the final image I, produced by the second lens B,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \qquad ...(2)$$

Adding equations (1) and (2),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \qquad ...(3)$$

If the combination is replaced by a single lens of focal length F such that it forms the image of O at the same position I, then

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F} \qquad ...(4)$$

From equations (3) and (4)

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad ...(5)$$

This F is the focal length of the equivalent lens for the combination.

The derivation can be extended for several thin lenses of focal lengths f_1, f_2, f_3 ... in contact. The effective focal length of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ ... \qquad ...(6)$$

In terms of power, equation (6) can be written as

$$P = P_1 + P_2 + P_3 + ... \qquad ...(7)$$

Equation (7) may be stated as follows :

The power of a combination of lenses in contact is the algebraic sum of the powers of individual lenses.

The combination of lenses is generally used in the design of objectives of microscopes, cameras, telescopes and other optical instruments.

9.7 Prism

A prism is a transparent medium bounded by the three plane faces. Out of the three faces, one is grounded and the other two are polished. The polished faces are called refracting faces. The angle between the refracting faces is called angle of prism, or the refracting angle. The third face is called base of the prism.

Refraction of light through a prism

Fig. 9.21 shows the cross section of a triangular prism *ABC*, placed in air. Let '*A*' be the refracting angle of the prism. *A* ray of light *PQ* incident on the refracting face *AB*, gets refracted along *QR* and emerges along *RS*. The angle of incidence and refraction at the two faces are i_1, r_1, r_2 and i_2 respectively. The angle between the incident ray *PQ* and the emergent ray *RS* is called angle of deviation, *d*.

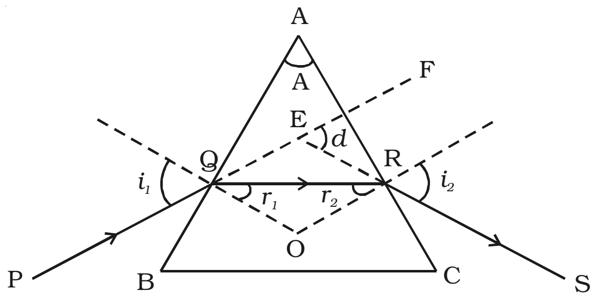


Fig. 9.21 Refraction through a prism

In the ΔQER, the exterior angle $\angle FER = \angle EQR + \angle ERQ$

$$d = (i_1 - r_1) + (i_2 - r_2)$$

$\therefore$ $d = (i_1 + i_2) - (r_1 + r_2)$ ...(1)

In the quadrilateral *AQOR*, the angles at *Q* and *R* are right angles

$$\angle Q + \angle R = 180^\circ$$

$\therefore$ $A + \angle QOR = 180^\circ$ ...(2)

Also, from the ΔQOR

$r_1 + r_2 + \angle QOR = 180^\circ$ ...(3)

From equation (2) and (3)

$r_1 + r_2 = A$ ...(4)

Substituting in (1),

$$d = i_1 + i_2 - A$$

or $A + d = i_1 + i_2$ ...(5)

For a given prism and for a light of given wavelength, the angle of deviation depends upon the angle of incidence.

As the angle of incidence i gradually increases, the angle of deviation d decreases, reaches a minimum value D and then increases. D is called the angle of minimum deviation. It will be seen from the graph (Fig. 9.22) that there is only one angle of incidence for which the deviation is a minimum.

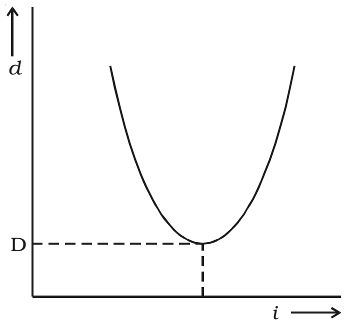


Fig. 9.22 i-d graph

At minimum deviation position the incident ray and emergent ray are symmetric with respect to the base of the prism. (i.e) the refracted ray QR is parallel to the base of the prism.

At the minimum deviation $i_1 = i_2 = i$ *and* $r_1 = r_2 = r$

$\therefore$ from equation (4) $2r = A$ or $r = \dfrac{A}{2}$

and from equation (5) $2i = A + D$ or $i = \dfrac{A+D}{2}$

The refractive index is $\mu = \dfrac{\sin i}{\sin r}$

$$\therefore \quad \mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

9.8 Dispersion of light

Dispersion is the splitting of white light into its constituent colours. This band of colours of light is called its spectrum.

In the visible region of spectrum, the spectral lines are seen in the order from violet to red. The colours are given by the word VIBGYOR (Violet, Indigo, Blue, Green, Yellow, Orange and Red) (Fig. 9.23)

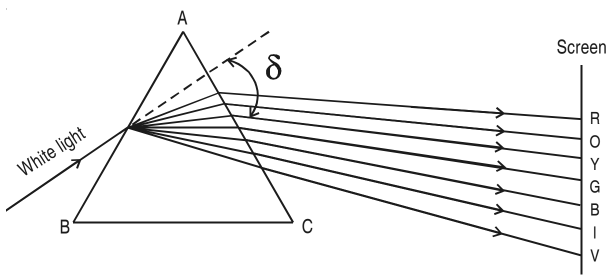


Fig. 9.23 Dispersion of light

The origin of colour after passing through a prism was a matter of much debate in physics. Does the prism itself create colour in some way or does it only separate the colours already present in white light?

Sir Isaac Newton gave an explanation for this. He placed another similar prism in an inverted position. The emergent beam from the first prism was made to fall on the second prism (Fig. 9.24). The resulting emergent beam was found to be white light. The first prism separated the white light into its constituent colours, which were then recombined by the inverted prism to give white light. Thus it can be concluded that the prism does not create any colour but it only separates the white light into its constituent colours.

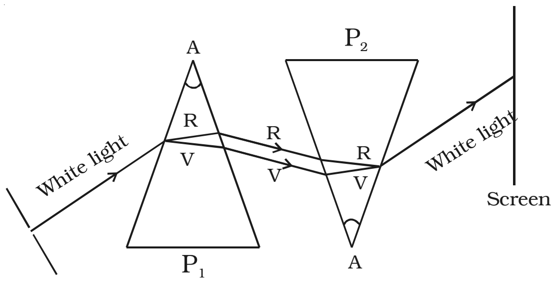


Fig. 9.24 Newton's experiment on dispersion

Dispersion takes place because the refractive index of the material of the prism is different for different colours (wavelengths). The deviation and hence the refractive index is more for violet rays of light than the corresponding values for red rays of light. Therefore the violet ray travels with a smaller velocity in glass prism than red ray. The deviation and the refractive index of the yellow ray are taken as the mean values. Table 9.2 gives the refractive indices for different wavelength for crown glass and flint glass.

Table 9.2 Refractive indices for different wavelengths (NOT FOR EXAMINATION)

Colour	Wave length (nm)	Crown glass	Flint glass
Violet	396.9	1.533	1.663
Blue	486.1	1.523	1.639
Yellow	589.3	1.517	1.627
Red	656.3	1.515	1.622

The speed of light is independent of wavelength in vacuum. Therefore vacuum is a non-dispersive medium in which all colours travel with the same speed.

9.8.1 Dispersive power

The refractive index of the material of a prism is given by the relation $\mu = \dfrac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}$

Here A is the angle of the prism and D is the angle of minimum deviation.

If the angle of prism is small of the order of 10^{o}, the prism is said to be small angled prism. When rays of light pass through such prisms the angle of deviation also becomes small.

If *A* be the refracting angle of a small angled prism and δ the angle of deviation, then the prism formula becomes $\mu = \dfrac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\frac{A}{2}}$

For small angles *A* and δ, $\sin\frac{A+\delta}{2} = \frac{A+\delta}{2}$ and $\sin\frac{A}{2} = \frac{A}{2}$

$$\therefore \quad \mu = \frac{\left(\frac{A+\delta}{2}\right)}{\frac{A}{2}}$$

$$\mu A = A + \delta$$

$$\delta = (\mu - 1)A \qquad \ldots (1)$$

If δ_v and δ_r are the deviations produced for the violet and red rays and μ_v and μ_r are the corresponding refractive indices of the material of the small angled prism then,

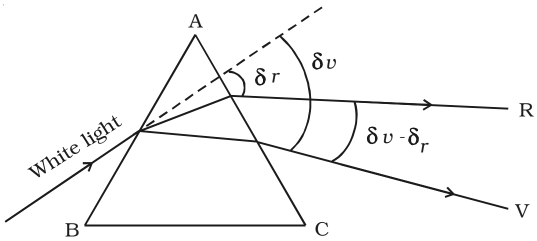


Fig. 9.25 Dispersive power

for violet light,

$$\delta_v = (\mu_v - 1)A \qquad \ldots(2)$$

for red light, $\delta_r = (\mu_r - 1)A$ ...(3)

From equations (2) and (3)

$$\delta_v - \delta_r = (\mu_v - \mu_r)A \qquad \ldots(4)$$

$\delta_v - \delta_r$ is called the angular dispersion which is the difference in deviation between the extreme colours (Fig. 9.25).

If δ_y and μ_y are the deviation and refractive index respectively for yellow ray (mean wavelength) then,

for yellow light, $\delta_y = (\mu_y - 1)\,A$ ... (5)

Dividing equation (4) by (5) we get $\dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{(\mu_v - \mu_r)A}{(\mu_y - 1)A}$

$$\frac{\delta_v - \delta_r}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

The expression $\dfrac{\delta_v - \delta_r}{\delta_y}$ is known as the dispersive power of the material of the prism and is denoted by ω.

$$\therefore \ \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

The dispersive power of the material of a prism is defined as the ratio of angular dispersion for any two wavelengths (colours) to the deviation of mean wavelength.

9.9 Spectrometer

The spectrometer is an optical instrument used to study the spectra of different sources of light and to measure the refractive indices of materials (Fig. 9.26). It consists of basically three parts. They are collimator, prism table and Telescope.

Fig. 9.26 Spectrometer (NEED NOT DRAW IN THE EXAMINATION)

Collimator

The collimator is an arrangement to produce a parallel beam of light. It consists of a long cylindrical tube with a convex lens at the inner end and a vertical slit at the outer end of the tube. The distance between the slit and the lens can be adjusted such that the slit is at the focus of the lens. The slit is kept facing the source of light. The width of the slit can be adjusted. The collimator is rigidly fixed to the base of the instrument.

Prism table

The prism table is used for mounting the prism, grating etc. It consists of two circular metal discs provided with three levelling screws. It can be rotated about a vertical axis passing through its centre and its position can be read with verniers V_1 and V_2. The prism table can be raised or lowered and can be fixed at any desired height.

Telescope

The telescope is an astronomical type. It consists of an eyepiece provided with cross wires at one end of the tube and an objective lens at its other end co-axially. The distance between the objective lens and the eyepiece can be adjusted so that the telescope forms a clear image at the cross wires, when a parallel beam from the collimator is incident on it.

The telescope is attached to an arm which is capable of rotation about the same vertical axis as the prism table. A circular scale graduated in half degree is attached to it.

Both the telescope and prism table are provided with radial screws for fixing them in a desired position and tangential screws for fine adjustments.

9.9.1 Adjustments of the spectrometer

The following adjustments must be made before doing the experiment with spectrometer.

(i) Adjustment of the eyepiece

The telescope is turned towards an illuminated surface and the eyepiece is moved to and fro until the cross wires are clearly seen.

(ii) Adjustment of the telescope

The telescope is adjusted to receive parallel rays by turning it towards a distant object and adjusting the distance between the objective lens and the eyepiece to get a clear image on the cross wire.

(iii) Adjustment of the collimator

The telescope is brought along the axial line with the collimator. The slit of the collimator is illuminated by a source of light. The distance between the slit and the lens of the collimator is adjusted until a clear image of the slit is seen at the cross wires of the telescope. Since the telescope is already adjusted for parallel rays, a well defined image of the slit can be formed, only when the light rays emerging from the collimator are parallel.

(iv) Levelling the prism table

The prism table is adjusted or levelled to be in horizontal position by means of levelling screws and a spirit level.

9.9.2 Determination of the refractive index of the material of the prism

The preliminary adjustments of the telescope, collimator and the prism table of the spectrometer are made. The refractive index of the prism can be determined by knowing the angle of the prism and the angle of minimum deviation.

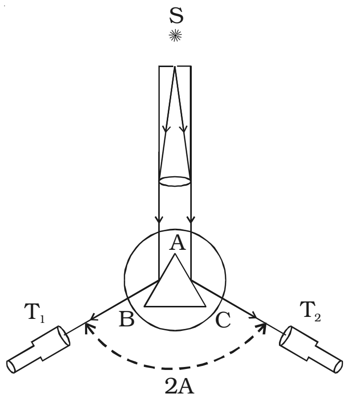


Fig. 9.27 Angle of the prism

(i) Angle of the prism (A)

The prism is placed on the prism table with its refracting edge facing the collimator as shown in Fig 9.27. The slit is illuminated by a sodium vapour lamp.

The parallel rays coming from the collimator fall on the two faces AB and AC.

The telescope is rotated to the position T_1 until the image of the slit, formed by the reflection at the face AB is made to coincide with the vertical cross wire of the telescope. The readings of the verniers are noted. The telescope is then rotated to the position T_2 where the image of the slit formed by the reflection at the face AC coincides with the vertical cross wire. The readings are again noted.

The difference between these two readings gives the angle rotated by the telescope. This angle is equal to twice the angle of the prism. Half of this value gives the angle of the prism A.

(ii) Angle of minimum deviation (D)

The prism is placed on the prism table so that the light from the collimator falls on a refracting face, and the refracted image is observed through the telescope (Fig. 9.28). The prism table is now rotated so that the angle of deviation decreases. A stage comes when the image stops for a moment and if we rotate the prism table further in the same direction, the image is seen to recede and the angle of deviation increases. The vertical cross wire of the telescope is made to coincide with the image of the slit where it turns back. This gives the minimum deviation position. The readings of the verniers are noted. Now the prism is removed and the telescope is turned to receive the direct ray and the vertical cross wire is made to coincide with the image. The readings of the verniers are noted. The difference between the two readings gives the angle of minimum deviation D.

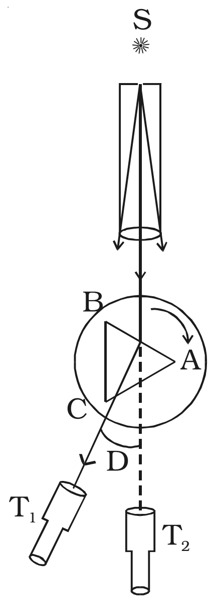


Fig. 9.28 Angle of minimum deviation

The refractive index of the material of the prism μ is calculated using the formula $\mu = \dfrac{\sin\left(\dfrac{A+D}{2}\right)}{\sin\dfrac{A}{2}}$.

The refractive index of a liquid may be determined in the same way using a hollow glass prism filled with the given liquid.

9.10 Rainbow

One of the spectacular atmospheric phenomena is the formation of rainbow during rainy days. The rainbow is also an example of dispersion of sunlight by the water drops in the atmosphere.

When sunlight falls on small water drops suspended in air during or after a rain, it suffers refraction, internal reflection and dispersion.

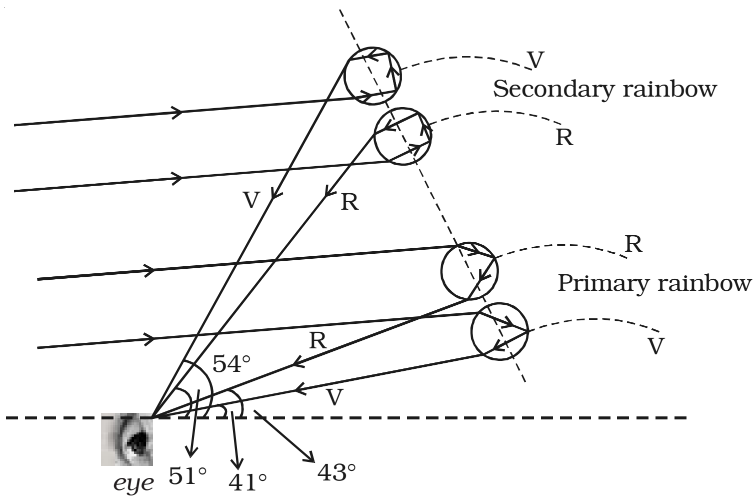


Fig. 9.29 Formation of rainbows

If the Sun is behind an observer and the water drops infront, the observer may observe two rainbows, one inside the other. The inner one is called primary rainbow having red on the outer side and violet on the inner side and the outer rainbow is called secondary rainbow, for which violet on the outer side and red on the inner side.

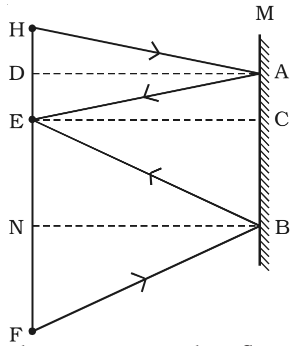
Fig. 9.29 shows the formation of primary rainbow. It is formed by the light from the Sun undergoing one internal reflection and two refractions and emerging at minimum deviation. It is however, found that the intensity of the red light is maximum at an angle of 43^o and that of the violet rays at 41^o. The other coloured arcs occur in between violet and red (due to other rain drops).

The formation of secondary rainbow is also shown in Fig. 9.31. It is formed by the light from the Sun undergoing two internal reflections and two refractions and also emerging at minimum deviation. In this case the inner red edge subtends an angle of 51^o and the outer violet edge subtends an angle of 54^o. This rainbow is less brighter and narrower than the primary rainbow. Both primary and secondary rainbows exhibit all the colours of the solar spectrum.

From the ground level an arc of the rainbow is usually visible. A complete circular rainbow may be seen from an elevated position such as from an aeroplane.

Solved Problems

9.1 A man 2 m tall standing in front of a plane mirror whose eye is 1.90 m above the ground. What is the minimum size of the mirror required to see complete image?



Solution :

M	–	*Mirror*
FH	–	*Man*
H	–	*Head*
E	–	*Eye*
F	–	*Feet*

A ray HA from the head, falls at A on the mirror and reflected to E along AE. AD is the perpendicular bisector of HE.

$$\therefore AC = \frac{1}{2} HE = \frac{1}{2} \times 0.10 = 0.05 \text{ m.}$$

A ray FB from the feet, falls at B and reflected to E along BE. BN is the perpendicular bisector of EF.

$$\therefore CB = \frac{1}{2} EF = \frac{1}{2} \times 1.90 = 0.95 \text{ m.}$$

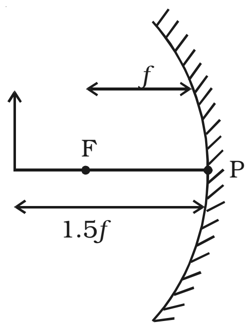
$\therefore$ *The size of the mirror* $= AC + CB$

$= 0.05 \text{ m} + 0.95 \text{ m}$

Size of the mirror $= 1 \text{ m}$

9.2 An object of length 2.5 cm is placed at a distance of 1.5 times the focal length *(f)* from a concave mirror. Find the length of the image. Is the image is erect or inverted?

Data : $f = -f$; $u = -1.5\, f$; $h_1 = 2.5$ cm; $h_2 = ?$



Solution :

We know, $$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-f} - \frac{1}{-1.5f}$$

$$\frac{1}{v} = \frac{1}{1.5f} - \frac{1}{f}$$

$v = -\ 3f$

magnification, $m = -\frac{v}{u} = (-)\ \frac{-3f}{-1.5f} = -2$

But $\frac{h_2}{h_1} = m = -2$

$\therefore \quad h_2 = -5\ cm$

The length of the image is 5.0 cm. The –ve sign indicates that the image is inverted.

9.3 In Michelson's method to determine the velocity of light in air, the distance travelled by light between reflections from the opposite faces of the octagonal mirror is 150 km. The image appears stationary when the minimum speed of rotation of the octagonal mirror is 250 rotations per second. Calculate the velocity of light.

Data :

$D = 150\ km = 150 \times 10^3\ m;\quad n = 250\ rps;\quad N = 8;\quad C = ?$

Solution :

In Michelson's method, the velocity of light is

$C = NnD$

$C = 8 \times 250 \times 150 \times 10^3$

$C = 3 \times 10^8\ ms^{-1}$

9.4 The radii of curvature of two surfaces of a double convex lens are 10 cm each. Calculate its focal length and power of the lens in air and liquid. Refractive indices of glass and liquid are 1.5 and 1.8 respectively.

Data : $R_1 = 10\ cm,\ R_2 = -10\ cm\ ;\quad \mu_g = 1.5$ *and* $\mu_l = 1.8$

Solution : In air

$$\frac{1}{f_a} = \left({}_a\mu_g - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5 - 1)\left[\frac{1}{10} + \frac{1}{10}\right]$$

$f_a \quad = 10\ cm$

$$P_a = \frac{1}{f_a} = \frac{1}{10 \times 10^{-2}}$$

$P_a \quad = 10$ *dioptres*

In liquid

$$\frac{1}{f_l} = \left({}_l\mu_g - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{\mu_g}{\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1.5}{1.8} - 1\right)\left[\frac{1}{10} + \frac{1}{10}\right] = -\frac{1}{6} \times \frac{2}{10}$$

$f_l \quad = -30\ cm$

$$P_l = \frac{1}{f_l} = -\frac{1}{30 \times 10^{-2}}$$

$P_l \quad = -3.33$ *dioptres*

9.5 A needle of size 5 cm is placed 45 cm from a lens produced an image on a screen placed 90 cm away from the lens. Identify the type of the lens and calculate its focal length and size of the image.

Data : $h_1 = 5\ cm, \quad u = -45\ cm, \quad v = 90\ cm, \quad f = ? \quad h_2 = ?$

Solution : *We know that* $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{90} - \frac{1}{-45}$

$\therefore\ f = 30\ cm$

Since f is positive, the lens is converging

Since $\frac{h_2}{h_1} = \frac{v}{u} \quad \frac{h_2}{5} = \frac{90}{-45} = -2$

$\therefore\ h_2 = -10\ cm$

(The –ve sign indicates that the image is real and inverted)

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

9.1 *The number of images of an object held between two parallel plane mirrors.*

(a) infinity *(b) 1*

(c) 3 *(d) 0*

9.2 *Radius of curvature of concave mirror is 40 cm and the size of image is twice as that of object, then the object distance is*

(a) 20 cm *(b) 10 cm*

(c) 30 cm *(d) 60 cm*

9.3 *A ray of light passes from a denser medium strikes a rarer medium at an angle of incidence i. The reflected and refracted rays are perpendicular to each other. The angle of reflection and refraction are r and r′. The critical angle is*

(a) $\tan^{-1} (\sin i)$ (b) $\sin^{-1} (\tan i)$

(c) $\tan^{-1} (\sin r)$ (d) $\sin^{-1} (\tan r')$

9.4 *Light passes through a closed tube which contains a gas. If the gas inside the tube is gradually pumped out, the speed of light inside the tube*

(a) increases *(b) decreases*

(c) remains constant *(d) first increases and then decreases*

9.5 *In Michelson's experiment, when the number of faces of rotating mirror increases, the velocity of light*

(a) decreases *(b) increases*

(c) does not change *(d) varies according to the rotation*

9.6 *If the velocity of light in a medium is (2/3) times of the velocity of light in vacuum, then the refractive index of that medium is.*

(a) 3/2c *(b) 2c/3*

(c) 2/3 *(d) 1.5*

9.7 *Two lenses of power +12 and –2 dioptre are placed in contact. The focal length of the combination is given by*

(a) 8.33 cm *(b) 12.5 cm*

(c) 16.6 cm *(d) 10 cm*

9.8 *A converging lens is used to form an image on a screen. When the lower half of the lens is covered by an opaque screen then,*

(a) half of the image will disappear

(b) complete image will be formed

(c) no image is formed

(d) intensity of the image is high

9.9 *Two small angled prism of refractive indices 1.6 and 1.8 produced same deviation, for an incident ray of light, the ratio of angle of prism*

(a) 0.88 *(b) 1.33*

(c) 0.56 *(d) 1.12*

9.10 *Rainbow is formed due to the phenomenon of*

(a) refraction and absorption

(b) dispersion and focussing

(c) refraction and scattering

(d) dispersion and total internal reflection

9.11 *State the laws of reflection.*

9.12 *Show that the reflected ray turns by 2θ when mirror turns by θ.*

9.13 *Explain the image formation in plane mirrors.*

9.14 *Draw graphically the image formation in spherical mirrors with different positions of the object and state the nature of the image.*

9.15 *What is the difference between the virtual images produced by (i) plane mirror (ii) concave mirror (iii) convex mirror*

9.16 *The surfaces of the sun glasses are curved, yet their power may be zero. Why?*

9.17 *Prove the mirror formula for reflection of light from a concave mirror producing (i) real image (ii) virtual image.*

9.18 *With the help of ray diagram explain the phenomenon of total internal reflection. Give the relation between critical angle and refractive index.*

9.19 *Write a note on optical fibre.*

9.20 *Explain Michelson's method of determining velocity of light.*

9.21 *Give the importance of velocity of light.*

9.22 *Derive lens maker's formula for a thin biconvex lens.*

9.23 *Define power of a lens. What is one dioptre?*

9.24 *Establish the relation* $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ *of thin lenses in contact.*

9.25 *Derive the relation* $\mu = \dfrac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}$.

9.26 *Does a beam of white light disperse through a hollow prism?*

9.27 *Derive an equation for dispersive power of a prism.*

9.28 *Describe a spectrometer.*

9.29 *Explain how will you determine the angle of the minimum deviation of a prism using spectrometer.*

9.30 *Write a note on formation of rainbows.*

Problems

9.31 *Light of wavelength 5000* Å *falls on a plane reflecting surface. Calculate the wavelength and frequency of reflected light. For what angle of incidence, the reflected ray is normal to the incident ray?*

9.32 *At what distance from a convex mirror of focal length 2.5 m should a boy stand, so that his image has a height equal to half the original height?*

9.33 *In a Michelson's experiment the distance travelled by the light between two reflections from the octagon rotating mirror is 4.8 km. Calculate the minimum speed of the mirror so that the image is formed at the non–rotating position.*

9.34 *If the refractive index of diamond be 2.5 and glass 1.5, then how faster does light travel in glass than in diamond?*

9.35 *An object of size 3 cm is kept at a distance of 14 cm from a concave lens of focal length 21 cm. Find the position of the image produced by the lens?*

9.36 *What is the focal length of a thin lens if the lens is in contact with 2.0 dioptre lens to form a combination lens which has a focal length of –80 cm?*

9.37 *A ray passes through an equilateral prism such that the angle of incidence is equal to the angle of emergence and the later is equal to 3/4 of the angle of prism. Find the angle of deviation.*

9.38 *The refractive indices of flint glass of equilateral prism for 400 nm and 700 nm are 1.66 and 1.61 respectively. Calculate the difference in angle of minimum deviation.*

9.39 *White light is incident on a small angled prism of angle 5^o. Calculate the angular dispersion if the refractive indices of red and violet rays are 1.642 and 1.656 respectively.*

9.40 *A thin prism of refractive index 1.5 deviates a ray by a minimum angle of 5^o. When it is kept immersed in oil of refractive index 1.25, what is the angle of minimum deviation?*

Answers

9.1	*(a)*	***9.2***	*(b)*	***9.3***	*(b)*
9.4	*(a)*	***9.5***	*(c)*	***9.6***	*(d)*
9.7	*(d)*	***9.8***	*(b)*	***9.9***	*(b)*
9.10	*(d)*				

9.31 5000 Å ; 6×10^{14} Hz ; 45°

9.32 *2.5 m*

9.33 7.8×10^{3} rps

9.34 *1.66 times*

9.35 *– 8.4 cm*

9.36 *–30.8 cm*

9.37 *30°*

9.38 *4°*

9.39 *0.07°*

9.40 *2°*

10. Magnetism

The word magnetism is derived from iron ore magnetite (Fe_3O_4), which was found in the island of magnesia in Greece. It is believed that the Chinese had known the property of the magnet even in 2000 B.C. and they used magnetic compass needle for navigation in 1100 AD. But it was Gilbert who laid the foundation for magnetism and had suggested that Earth itself behaves as a giant bar magnet. The field at the surface of the Earth is approximately 10^{-4} T and the field extends upto a height of nearly five times the radius of the Earth.

10.1 Earth's magnetic field and magnetic elements

A freely suspended magnetic needle at a point on Earth comes to rest approximately along the geographical north - south direction. This shows that the Earth behaves like a huge magnetic dipole with its magnetic poles near its geographical poles. Since the north pole of the magnetic needle approximately points towards geographic north (N_G) it is appropriate to call the magnetic pole near N_G as the magnetic south pole of Earth S_m. Also, the pole near S_G is the magnetic north pole of the Earth (N_m). (Fig.10.1)

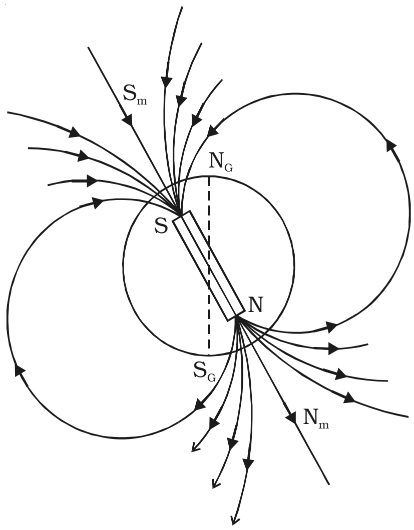


Fig. 10.1 Magnetic field of Earth

The Earth's magnetic field at any point on the Earth can be completely defined in terms of certain quantities called magnetic elements of the Earth, namely

(i) Declination or the magnetic variation θ.

(ii) Dip or inclination δ and

(iii) The horizontal component of the Earth's magnetic field B_h

Causes of the Earth's magnetism

The exact cause of the Earth's magnetism is not known even today. However, some important factors which may be the cause of Earth's magnetism are:

(i) Magnetic masses in the Earth.

(ii) Electric currents in the Earth.

(iii) Electric currents in the upper regions of the atmosphere.

(iv) Radiations from the Sun.

(v) Action of moon etc.

However, it is believed that the Earth's magnetic field is due to the molten charged metallic fluid inside the Earth's surface with a core of radius about 3500 km compared to the Earth's radius of 6400 km.

10.1.1 Bar magnet

The iron ore magnetite which attracts small pieces of iron, cobalt, nickel etc. is a natural magnet. The natural magnets have irregular shape and they are weak. A piece of iron or steel acquires magnetic properties when it is rubbed with a magnet. Such magnets made out of iron or steel are artificial magnets. Artificial magnets can have desired shape and desired strength. If the artificial magnet is in the form of a rectangular or cylindrical bar, it is called a bar magnet.

10.1.2 Basic properties of magnets

(i) When the magnet is dipped in iron filings, they cling to the ends of the magnet. The attraction is maximum at the two ends of the magnet. These ends are called poles of the magnet.

(ii) When a magnet is freely suspended, it always points along north-south direction. The pole pointing towards geographic north is called north pole *N* and the pole which points towards geographic south is called south pole *S*.

(iii) Magnetic poles always exist in pairs. (i.e) isolated magnetic pole does not exist.

(iv) The magnetic length of a magnet is always less than its geometric length, because the poles are situated a little inwards from the free ends of the magnet. (But for the purpose of calculation the geometric length is always taken as magnetic length.)

(v) Like poles repel each other and unlike poles attract each other. North pole of a magnet when brought near north pole of another magnet, we can observe repulsion, but when the north pole of one magnet is brought near south pole of another magnet, we observe attraction.

(vi) The force of attraction or repulsion between two magnetic poles is given by Coulomb's inverse square law.

Note : In recent days, the concept of magnetic poles has been completely changed. The origin of magnetism is traced only due to the flow of current. But anyhow, we have retained the conventional idea of magnetic poles in this chapter. Pole strength is denoted by m and its unit is ampere metre.

Magnetic moment

Since any magnet has two poles, it is also called a magnetic dipole.

The magnetic moment of a magnet is defined as the product of the pole strength and the distance between the two poles.

If m is the pole strength of each pole and $2l$ is the distance between the poles, the magnetic moment

$$\vec{M} = m\,(2\vec{l})$$

Magnetic moment is a vector quantity. It is denoted by M. Its unit is A m^2. Its direction is from south pole to north pole.

Magnetic field

Magnetic field is the space in which a magnetic pole experiences a force or it is the space around a magnet in which the influence of the magnet is felt.

Magnetic induction

Magnetic induction is the fundamental character of a magnetic field at a point.

Magnetic induction at a point in a magnetic field is the force experienced by unit north pole placed at that point. It is denoted by B. Its unit is $\frac{N}{Am}$. It is a vector quantity. It is also called as magnetic flux density.

If a magnetic pole of strength m placed at a point in a magnetic field experiences a force F, the magnetic induction at that point is

$$\vec{B} = \frac{\vec{F}}{m}$$

Magnetic lines of force

A magnetic field is better studied by drawing as many number of magnetic lines of force as possible.

A magnetic line of force is a line along which a free isolated north pole would travel when it is placed in the magnetic field.

Properties of magnetic lines of force

(i) Magnetic lines of forces are closed continuous curves, extending through the body of the magnet.

(ii) The direction of line of force is from north pole to south pole outside the magnet while it is from south pole to north pole inside the magnet.

(iii) The tangent to the magnetic line of force at any point gives the direction of magnetic field at that point. (i.e) it gives the direction of magnetic induction ($\vec{B}$) at that point.

(iv) They never intersect each other.

(v) They crowd where the magnetic field is strong and thin out where the field is weak.

Magnetic flux and magnetic flux density

The number of magnetic lines of force passing through an area A is called magnetic flux. It is denoted by ϕ. Its unit is weber. It is a scalar quantity.

The number of magnetic lines of force crossing unit area kept normal to the direction of line of force is magnetic flux density. Its unit is Wb m^{-2} or tesla or N $A^{-1}m^{-1}$.

$\therefore$ Magnetic flux $\phi = \vec{B} \cdot \vec{A}$

Uniform and non-uniform magnetic field

Fig. 10.2 Uniform Magneticfield

Magnetic field is said to be uniform if the magnetic induction has the same magnitude and the same direction at all the points in the region. It is represented by drawing parallel lines (Fig. 10.2).

An example of uniform magnetic field over a wide area is the Earth's magnetic field.

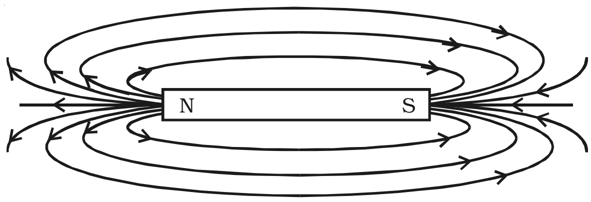


Fig. 10.3 Non-uniform magnetic field

If the magnetic induction varies in magnitude and direction at different points in a region, the magnetic field is said to be non-uniform. The magnetic field due to a bar magnet is non-uniform. It is represented by convergent or divergent lines (Fig. 10.3).

10.2 Force between two magnetic poles

In 1785, Coulomb made use of his torsion balance and discovered the law governing the force between the two magnetic poles.

Coulomb's inverse square law

Coulomb's inverse square law states that the *force of attraction or repulsion between the two magnetic poles is directly proportional to the product of their pole strengths and inversely proportional to the square of the distance between them.*

If m_1 and m_2 are the pole strengths of two magnetic poles separated by a distance of d in a medium, then

$$F \propto m_1 m_2 \text{ and } F \propto \frac{1}{d^2}$$

$$\therefore F \propto \frac{m_1 m_2}{d^2}$$

$$F = k \frac{m_1 m_2}{d^2}$$

where k is the constant of proportionality and $k = \frac{\mu}{4\pi}$ where μ is the permeability of the medium.

But $\mu = \mu_o \times \mu_r$

$\therefore \ \mu_r = \dfrac{\mu}{\mu_o}$

where μ_r - relative permeability of the medium

μ_o - permeability of free space or vacuum.

Let $m_1 = m_2 = 1$

and $d = 1\ m$

$$k = \frac{\mu_o}{4\pi}$$

In free space, $\mu_o = 4\pi \times 10^{-7}$ H m^{-1}

$$\therefore \quad F = \frac{10^{-7} \times m_1 \times m_2}{d^2}$$

$$F = \frac{10^{-7} \times 1 \times 1}{1^2}$$

$$F = 10^{-7} \text{ N}$$

Therefore, *unit pole is defined as that pole which when placed at a distance of 1 metre in free space or air from an equal and similar pole, repels it with a force of 10^{-7} N.*

10.3 Magnetic induction at a point along the axial line due to a magnetic dipole (Bar magnet)

NS is the bar magnet of length 2*l* and of pole strength m. *P* is a point on the axial line at a distance d from its mid point *O* (Fig. 10.4).

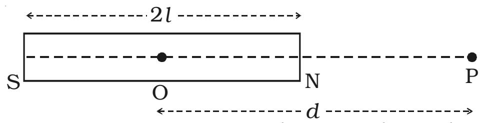


Fig. 10.4 Magnetic induction along the axial line

According to inverse square law, $F = \dfrac{\mu_o}{4\pi}\dfrac{m_1 m_2}{d^2}$

$\therefore$ Magnetic induction (B_1) at P due to north pole of the magnet,

$$B_1 = \frac{\mu_o}{4\pi}\frac{m}{NP^2} \text{ along NP} \quad \left(\because B = \frac{F}{m}\right)$$

$$= \frac{\mu_o}{4\pi}\frac{m}{(d-l)^2} \text{ along NP}$$

Magnetic induction (B_2) at P due to south pole of the magnet,

$$B_2 = \frac{\mu_o}{4\pi}\frac{m}{SP^2} \text{ along } PS$$

$$B_2 = \frac{\mu_o}{4\pi} \frac{m}{(d+l)^2} \text{ along } PS$$

$\therefore$ Magnetic induction at P due to the bar magnet,

$$B = B_1 - B_2$$

$$B = \frac{\mu_o}{4\pi} \frac{m}{(d-l)^2} - \frac{\mu_o}{4\pi} \frac{m}{(d+l)^2} \quad \text{along NP}$$

$$B = \frac{\mu_o m}{4\pi} \left(\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2} \right)$$

$$B = \frac{\mu_o m}{4\pi} \left(\frac{(d+l)^2 - (d-l)^2}{(d^2 - l^2)^2} \right)$$

$$B = \frac{\mu_o m}{4\pi} \left(\frac{4ld}{(d^2 - l^2)^2} \right)$$

$$B = \frac{\mu_o m}{4\pi} \frac{2l \times 2d}{(d^2 - l^2)^2}$$

$$B = \frac{\mu_o}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$$

where M = $2ml$ (magnetic dipole moment).

For a short bar magnet, l is very small compared to d, hence l^2 is neglected.

$$\therefore B = \frac{\mu_o}{4\pi} \frac{2M}{d^3}$$

The direction of B is along the axial line away from the north pole.

10.4 Magnetic induction at a point along the equatorial line of a bar magnet

NS is the bar magnet of length $2l$ and pole strength m. P is a point on the equatorial line at a distance d from its mid point O (Fig. 10.5).

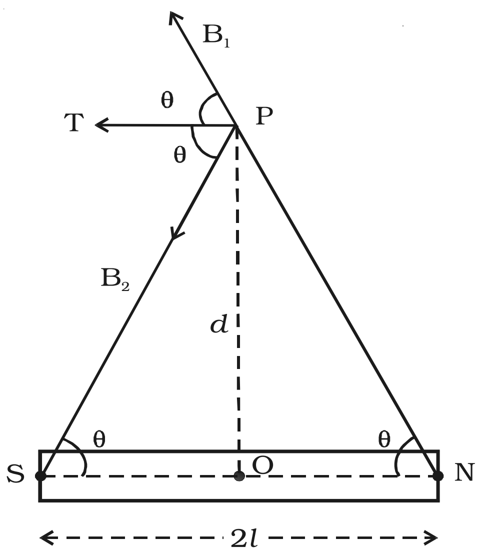


Fig. 10.5 Magnetic induction along the equatorial line

Magnetic induction (B_1) at P due to north pole of the magnet,

$$B_1 = \frac{\mu_o}{4\pi}\frac{m}{NP^2} \text{ along } NP$$

$$= \frac{\mu_o}{4\pi}\frac{m}{\left(d^2 + l^2\right)} \text{ along NP}$$

$$(\because NP^2 = NO^2 + OP^2)$$

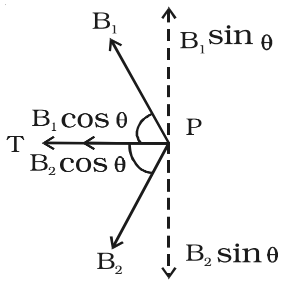


Fig. 10.6 Components of magnetic fields

Magnetic induction (B_2) at P due to south pole of the magnet,

$$B_2 = \frac{\mu_o}{4\pi}\frac{m}{PS^2} \text{ along } PS$$

$$= \frac{\mu_o}{4\pi}\frac{m}{\left(d^2 + l^2\right)} \text{ along PS}$$

Resolving B_1 and B_2 into their horizontal and vertical components.

Vertical components B_1 sin θ and B_2 *sin* θ are equal and opposite and therefore cancel each other (Fig. 10.6).

The horizontal components B_1 cos θ and B_2 cos θ will get added along PT.

Resultant magnetic induction at P due to the bar magnet is

$B = B_1 \cos\theta + B_2 \cos\theta$. (along PT)

$$B = \frac{\mu_o}{4\pi}\frac{m}{d^2 + l^2} \cdot \frac{l}{\sqrt{d^2 + l^2}} + \frac{\mu_o}{4\pi}\frac{m}{(d^2 + l^2)} \cdot \frac{l}{\sqrt{d^2 + l^2}}$$

$$\left(\because \cos\theta = \frac{SO}{PS} = \frac{NO}{NP}\right)$$

$$= \frac{\mu_o}{4\pi}\frac{2ml}{(d^2 + l^2)^{3/2}}$$

$$B = \frac{\mu_o}{4\pi}\frac{M}{(d^2 + l^2)^{3/2}}, \quad \text{(where } M = 2ml\text{)}$$

For a short bar magnet, l^2 is neglected.

$$\therefore\ B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$$

The direction of 'B' is along PT parallel to NS.

10.5 Mapping of magnetic field due to a bar magnet

A bar magnet is placed on a plane sheet of a paper. A compass needle is placed near the north pole of the magnet. The north and south poles of the compass are marked by pencil dots. The compass needle is shifted and placed so that its south pole touches the pencil dot marked for north pole. The process is repeated and a series of dots are obtained. The dots are joined as a smooth curve. This curve is a magnetic line of force. Even though few lines are drawn around a bar magnet the magnetic lines exists in all space around the magnet.

(i) Magnet placed with its north pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on the magnetic meridian such that its north pole points towards geographic north. Using a compass needle, magnetic lines of force are drawn around the magnet. (Fig. 10.7)

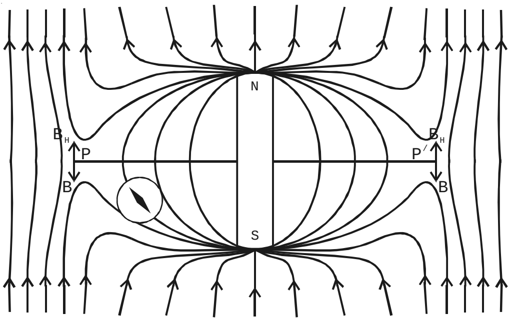


Fig. 10.7 Neutral points - equatorial line

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P ′ along the equatorial line of the magnet, the compass shows no deflection. They are called "neutral points." At these points the magnetic field due to the magnet along its equatorial line (B) is exactly balanced by the horizontal component of the Earth's magnetic field. (B_h)

Hence, *neutral points are defined as the points where the resultant magnetic field due to the magnet and Earth is zero.*

Hence, at neutral points

$$B = B_h$$

$$\frac{\mu_o}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = B_h$$

(ii) Magnet placed with its south pole facing geographic north

A sheet of paper is fixed on a drawing board. Using a compass needle, the magnetic meridian is drawn on it. A bar magnet is placed on a magnetic meridian such that its south pole facing geographic north. Using a compass needle, the magnetic lines of force are drawn around the magnet as shown in Fig. 10.8.

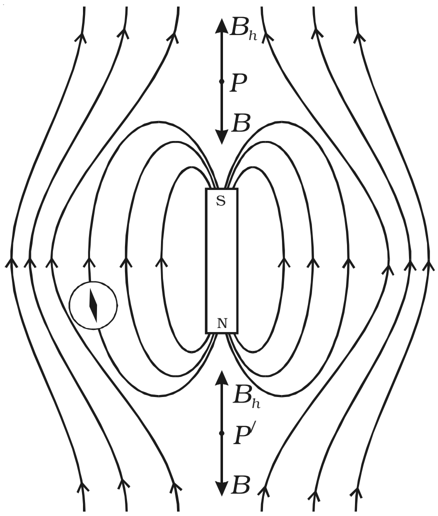


Fig. 10.8 Neutral points - axial line

The magnetic lines of force is due to the combined effect of the magnetic field due to the bar magnet and Earth. It is found that when the compass is placed at points P and P' along the axial line of the magnet, the compass shows no deflection. They are called neutral points. At these points the magnetic field (B) due to the magnet along its axial line is exactly balanced by the horizontal component of the Earth's magnetic field (B_h).

Hence at neutral points, $B = B_h$

$$\therefore \frac{\mu_o}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = B_h$$

10.6 Torque on a bar magnet placed in a uniform magnetic field

Consider a bar magnet NS of length $2l$ and pole strength m placed in a uniform magnetic field of induction B at an angle θ with the direction of the field (Fig. 10.9).

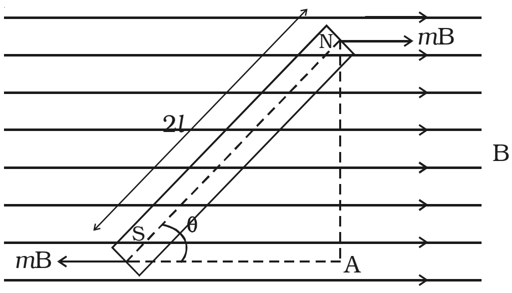


Fig. 10.9 Torque on a bar magnet

Due to the magnetic field B, a force mB acts on the north pole along the direction of the field and a force mB acts on the south pole along the direction opposite to the magnetic field.

These two forces are equal and opposite, hence constitute a couple. The torque τ due to the couple is

τ = one of the forces × perpendicular distance between them

$$\tau = F \times NA$$

$$= mB \times NA \quad ...(1)$$

$$= mB \times 2l \sin\theta$$

$$\therefore \tau = MB \sin\theta \quad ...(2)$$

Vectorially,

$$\vec{\tau} = \vec{M} \times \vec{B}$$

The direction of τ is perpendicular to the plane containing $\vec{M}$ and $\vec{B}$.

If $B = 1$ and $\theta = 90°$

Then from equation (2), $\tau = M$

Hence, *moment of the magnet M is equal to the torque necessary to keep the magnet at right angles to a magnetic field of unit magnetic induction.*

10.7 Tangent law

A magnetic needle suspended, at a point where there are two crossed magnetic fields acting at right angles to each other, will come to rest in the direction of the resultant of the two fields.

B_1 and B_2 are two uniform magnetic fields acting at right angles to each other. A magnetic needle placed in these two fields will be subjected to two torques tending to rotate the magnet in opposite directions. The torque τ_1 due to the two equal and opposite parallel forces mB_1 and mB_1 tend to set the magnet parallel to B_1. Similarly the torque τ_2 due to the two equal and opposite parallel forces mB_2 and mB_2 tends to set the magnet parallel to B_2. In a position where the torques balance each other, the

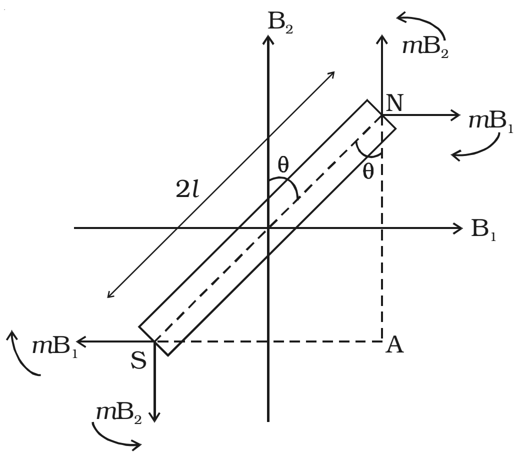


Fig. 10.10 Tangent law

magnet comes to rest. Now the magnet makes an angle θ with B_2 as shown in the Fig. 10.10.

The deflecting torque due to the forces mB_1 and mB_1

$$\tau_1 = mB_1 \times NA$$

$$= mB_1 \times NS \cos\theta$$

$$= mB_1 \times 2l \cos\theta$$

$$= 2l\, mB_1 \cos\theta$$

$$\therefore \tau_1 = MB_1 \cos\theta$$

Similarly the restoring torque due to the forces mB_2 and mB_2

$$\tau_2 = mB_2 \times SA$$

$$= mB_2 \times 2l \sin\theta$$

$$= 2lm \times B_2 \sin\theta$$

$$\tau_2 = MB_2 \sin\theta$$

At equillibrium,

$$\tau_1 = \tau_2$$

$$\therefore MB_1 \cos\theta = MB_2 \sin\theta$$

$$\therefore B_1 = B_2 \tan\theta$$

This is called Tangent law

Invariably, in the applications of tangent law, the restoring magnetic field B_2 is the horizontal component of Earth's magnetic field B_h.

10.8 Deflection magnetometer

Deflection magnetometer consists of a small magnetic needle pivoted on a sharp support such that it is free to rotate in a horizontal plane. A light, thin, long aluminium pointer is fixed perpendicular to the magnetic needle. The pointer also rotates along with the needle (Fig. 10.11).

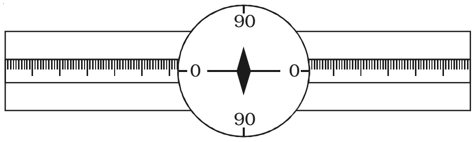


Fig. 10.11 Deflection magnetometer

There is a circular scale divided into four quadrants and each quadrant is graduated from 0° to 90°. A plane mirror fixed below the scale ensures, reading without parallax error, as the image of the pointer is made to coincide exactly with pointer itself. The needle, aluminium pointer and the scale are enclosed in a box with a glass top. There are two arms graduated in centimetre and their zeroes coincide at the centre of the magnetic needle.

10.8.1 End-on (or) Tan A position

The magnetic field at a point along the axial line of a bar magnet is perpendicular to the horizontal component of Earth's magnetic field. If a magnetometer and a bar magnet are placed in such way that this condition is satisfied, then this arrangement is called Tan A position.

To achieve this, the arms of the deflection magnetometer are placed along East-West direction (i.e) perpendicular to the magnetic meridian. The bar magnet is placed along East - West direction (i.e) parallel to the arms, as shown in the Fig. 10.12.

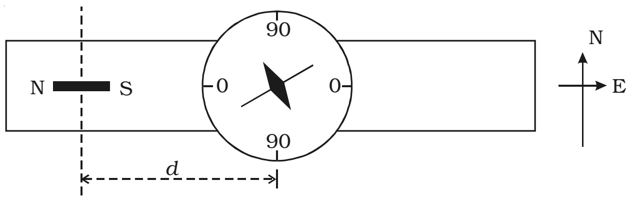


Fig. 10.12 End-on (or) Tan A position

When a bar magnet of magnetic moment M and length $2l$ is placed at a distance d from the centre of the magnetic needle, the needle gets deflected through an angle θ due to the action of two magnetic fields.

(i) the field B due to the bar magnet acting along its axis and

(ii) the horizontal component of Earth's magnetic field B_h.

The magnetic field at a distance d acting along the axial line of the bar magnet,

$$B = \frac{\mu_o}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2}$$

According to Tangent law,

$$B = B_h \tan \theta$$

$$\frac{\mu_o}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2} = B_h \tan \theta$$

Comparison of magnetic moments of two bar magnets

(i) Deflection method

The deflection magnetometer is placed in Tan A position (Fig. 10.13). A bar magnet of magnetic moment M_1 and length $2l_1$ is placed at a distance

d_1 from the centre of the magnetic needle, on one side of the compass box. Since, the sensitivity of the magnetometer is more at 45°, the distance of the bar magnet should be chosen such that the deflection lies between 30° and 60°. The readings corresponding to the ends of the aluminium pointer are noted as θ_1 and θ_2. The magnet is reversed pole to pole and kept at the same distance. Two more readings θ_3 and θ_4 are noted. By placing the magnet on the other side of the compass box at the same distance, four more readings θ_5, θ_6, θ_7 and θ_8 are noted as above. The mean of the eight readings gives a value θ_I.

The experiment is repeated as above for the second bar magnet of magnetic moment M_2 and length $2l_2$ by placing at a distance d_2. Now the mean of the eight readings gives a value of θ_{II}.

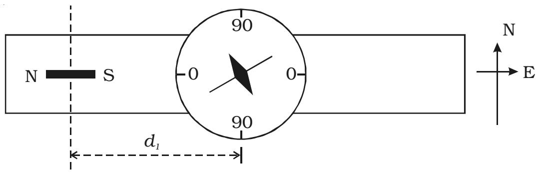


Fig. 10.13 Deflection method

Applying tangent law, for the first magnet,

$$\frac{\mu_o}{4\pi}\ \frac{2M_1d_1}{(d_1^{\,2}-l_1^{\,2})^2} = B_h \tan\theta_I \qquad \text{...(1)}$$

and for the second magnet.

$$\frac{\mu_o}{4\pi}\ \frac{2M_2d_2}{(d_2^{\,2}-l_2^{\,2})^2} = B_h \tan\theta_{II} \qquad \text{...(2)}$$

From the above equations (1) and (2), we get

$$\frac{M_1}{M_2} = \frac{\left(d_1^2 - l_1^2\right)^2}{\left(d_2^{\,2} - l_2^{\,2}\right)^2}\ \frac{d_2}{d_1}\ \frac{\tan\theta_I}{\tan\theta_{II}} \qquad \text{...(3)}$$

Special case

If the magnets are placed at the same distance, then $d_1 = d_2 = d$

$$\therefore\ \frac{M_1}{M_2} = \frac{\left(d^2 - l_1^2\right)^2}{\left(d^2 - l_2^{\,2}\right)^2}\quad \frac{\tan\theta_I}{\tan\theta_{II}}$$

In addition, if l_1 and l_2 are small compared to the distance d

then $$\frac{M_1}{M_2} = \frac{\tan\theta_I}{\tan\theta_{II}}$$

(ii) Null deflection method

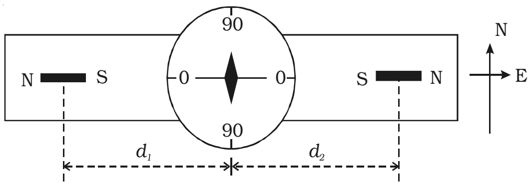


Fig. 10.14 Null deflection method

The deflection magnetometer is placed in Tan A position (Fig. 10.14). A bar magnet of magnetic moment M_1 and length $2l_1$ is placed on one side of the compass box at a distance d_1 from the centre of the magnetic needle. The second bar magnet of magnetic moment M_2 and length $2l_2$ is placed on the other side of the compass box such that like poles of the magnets face each other. The second magnet is adjusted so that the deflection due to the first magnet is nullified and the aluminium pointer reads 0° - 0°. The distance of the second magnet is x_1. The first magnet is reversed pole to pole and placed at the same distance d_1. The second magnet is also reversed and adjusted such that the aluminium pointer reads 0° - 0°. The distance of the second magnet is x_2.

The experiment is repeated by interchanging the magnets. Two more distances x_3 and x_4 are noted. The mean of x_1, x_2, x_3 and x_4 is taken as d_2.

As the magnetic fields due to the two bar magnets at the centre of the magnetic needle are equal in magnitude but opposite in direction,

(i.e) $\quad B_1 = B_2$

$$\frac{\mu_o}{4\pi}\frac{2M_1d_1}{(d_1{}^2-l_1{}^2)^2} = \frac{\mu_o}{4\pi}\frac{2M_2d_2}{(d_2{}^2-l_2{}^2)^2}$$

$$\therefore \frac{M_1}{M_2} = \frac{\left(d_1^2-l_1^2\right)^2}{\left(d_2{}^2-l_2{}^2\right)^2}\frac{d_2}{d_1}$$

If the bar magnets are short, l_1 and l_2 are negligible compared to the distance d_1 and d_2

$$\therefore \frac{M_1}{M_2} = \frac{d_1{}^3}{d_2{}^3}$$

10.8.2 Broad–side on (or) Tan B position

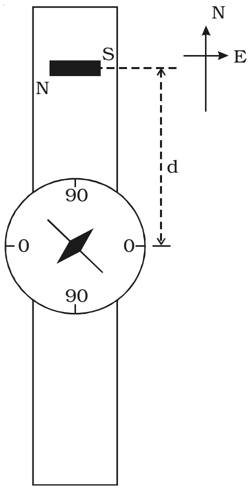


Fig. 10.15 Broad-side on or Tan B position

The magnetic field at a point along the equatorial line of a bar magnet is perpendicular to the horizontal component of Earth's magnetic field. If the magnetometer and a bar magnet are placed in such way that this condition is satisfied, then this arrangement is called Tan B position.

To achieve this, the arms of the deflection magnetometer are placed along the North - South direction (i.e) along the magnetic meridian. The magnet is placed along East - West direction (i.e) parallel to the aluminium pointer as shown in the Fig. 10.15.

When a bar magnet of magnetic moment M and length $2l$ is placed at a distance d from the centre of the magnetic needle, the needle gets deflected through an angle θ due to the action of the following two magnetic fields.

(i) The field B due to the bar magnet along its equatorial line
(ii) The horizontal component of Earth's magnetic field B_h.

The magnetic field at a distance d along the equatorial line of the bar magnet,

$$B = \frac{\mu_o}{4\pi}\frac{M}{(d^2+l^2)^{3/2}}$$

According to tangent law

$$B = B_h \tan\theta$$

$$\text{(i.e)} \quad \frac{\mu_o}{4\pi}\frac{M}{(d^2+l^2)^{3/2}} = B_h \tan\theta$$

If the magnet is short, l is small compared to d and hence l^2 is neglected.

$$\frac{\mu_o}{4\pi}\frac{M}{d^3} = B_h \tan\theta$$

Comparison of magnetic moments of two bar magnets

(i) Deflection method

The deflection magnetometer is placed in Tan B position. A bar magnet of magnetic moment M_1 and length $2l_1$ is placed at a distance d_1 from the centre of the magnetic needle, on one side of the compass box (Fig. 10.16). Since, the sensitivity of the magnetometer is more at 45°, the distance of the bar magnet should be chosen such that the deflection lies between 30° and 60°. The readings corresponding to the ends of the aluminium pointer are noted as θ_1 and θ_2. The magnet is reversed pole to pole and kept at the same distance. Two more readings θ_3 and θ_4 are noted. By placing the magnet on the other side of the compass box at the same distance, four more readings θ_5, θ_6, θ_7 and θ_8 are noted as above. The mean of the eight readings gives a value θ_I.

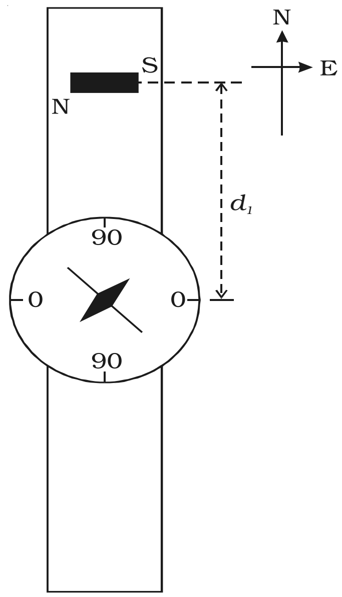


Fig.10.16 Deflection method

The experiment is repeated as above for the second bar magnet of magnetic moment M_2 and length $2l_2$ by placing at a distance d_2. Now the mean of the eight readings gives a value of θ_{II}.

Applying tangent law, for the first magnet,

$$\frac{\mu_o}{4\pi}\frac{M_1}{(d_1{}^2+l_1{}^2)^{3/2}} = B_h \tan\theta_I \qquad ...(1)$$

and for the second magnet

$$\frac{\mu_o}{4\pi}\frac{M_2}{(d_2{}^2+l_2{}^2)^{3/2}} = B_h \tan\theta_{II} \qquad ...(2)$$

From the above equations (1) and (2), we get

$$\frac{M_1}{M_2} = \frac{\left(d_1^2+l_1^2\right)^{3/2}}{\left(d_2{}^2+l_2{}^2\right)^{3/2}} \quad \frac{\tan\theta_I}{\tan\theta_{II}} \qquad ...(3)$$

Special case

If the magnets are placed at the same distance, then $d_1 = d_2 = d$

$$\frac{M_1}{M_2} = \frac{\left(d^2 + l_1^2\right)^{3/2}}{\left(d^2 + l_2^2\right)^{3/2}} \cdot \frac{\tan\theta_I}{\tan\theta_{II}}$$

In addition, if l_1 and l_2 are small compared to the distance d,

$$\frac{M_1}{M_2} = \frac{\tan\theta_I}{\tan\theta_{II}}$$

(ii) Null deflection method

The deflection magnetometer is placed in Tan B position (Fig. 10.17). A bar magnet of magnetic moment M_1 and length $2l_1$ is placed on one side of the compass box at a distance d_1 from the centre of the magnetic needle. The second bar magnet of magnetic moment M_2 and length $2l_2$ is placed on the other side of the compass box such that like poles of the magnets face in the opposite direction. The second magnet is adjusted so that the deflection due to the first magnet is nullified and the aluminium pointer reads 0° - 0°. The distance of the second magnet is x_1. The first magnet is reversed pole to pole and placed at the same distance d_1. The second magnet is also reversed and adjusted such that the aluminium pointer reads 0° - 0°. The distance of the second magnet is x_2.

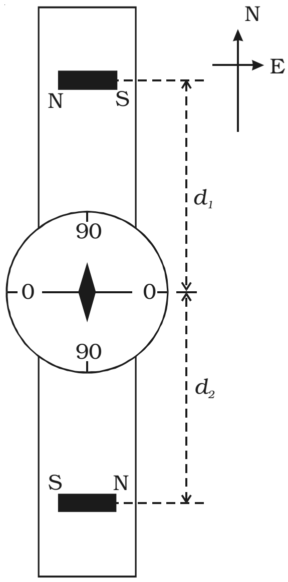


Fig. 10.17 Null deflection method

The experiment is repeated by interchanging the magnets. Two more distances x_3 and x_4 are noted. The mean of x_1, x_2, x_3 and x_4 is taken as d_2.

Since the magnetic fields due to the two bar magnets at the centre of the magnetic needle are equal in magnitude but opposite in direction.

$$\therefore \quad B_1 = B_2$$

$$\frac{\mu_o}{4\pi} \frac{M_1}{(d_1{}^2 + l_1{}^2)^{3/2}} = \frac{\mu_o}{4\pi} \frac{M_2}{(d_2{}^2 + l_2{}^2)^{3/2}}$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{\left(d_1^2 + l_1^2\right)^{3/2}}{\left(d_2{}^2 + l_2{}^2\right)^{3/2}}$$

If the bar magnets are short, l_1 and l_2 are negligible compared to the distance d_1 and d_2

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$$

10.9 Magnetic properties of materials

The study of magnetic properties of materials assumes significance since these properties decide whether the material is suitable for permanent magnets or electromagnets or cores of transformers etc. Before classifying the materials depending on their magnetic behaviour, the following important terms are defined.

(i) Magnetising field or magnetic intensity

The magnetic field used to magnetise a material is called the magnetising field. It is denoted by H and its unit is A m^{-1}.

(Note : Since the origin of magnetism is linked to the current, the magnetising field is usually defined in terms of ampere turn which is out of our purview here.)

(ii) Magnetic permeability

Magnetic permeability is the ability of the material to allow the passage of magnetic lines of force through it.

Relative permeability μ_r of a material is defined as the ratio of number of magnetic lines of force per unit area B inside the material to the number of lines of force per unit area in vacuum B_o produced by the same magnetising field.

$$\therefore \text{ Relative permeability } \mu_r = \frac{B}{B_o}$$

$$\mu_r = \frac{\mu H}{\mu_o H} = \frac{\mu}{\mu_o}$$

(since μ_r is the ratio of two identical quantities, it has no unit.)

$\therefore$ *The magnetic permeability of the medium $\mu = \mu_o \mu_r$ where μ_o is the permeability of free space.*

Magnetic permeability μ of a medium is also defined as the ratio of magnetic induction B inside the medium to the magnetising field H inside the same medium.

$$\therefore \mu = \frac{B}{H}$$

(iii) Intensity of magnetisation

Intensity of magnetisation represents the extent to which a material has been magnetised under the influence of magnetising field H.

Intensity of magnetisation of a magnetic material is defined as the magnetic moment per unit volume of the material.

$$I = \frac{M}{V}$$

Its unit is A m^{-1}.

For a specimen of length $2l$, area A and pole strength m,

$$I = \frac{2lm}{2lA}$$

$$\therefore I = \frac{m}{A}$$

Hence, *intensity of magnetisation is also defined as the pole strength per unit area of the cross section of the material.*

(iv) Magnetic induction

When a soft iron bar is placed in a uniform magnetising field H, the magnetic induction inside the specimen B is equal to the sum of the magnetic induction B_o produced in vacuum due to the magnetising field and the magnetic induction B_m due to the induced magnetisation of the specimen.

$$B = B_o + B_m$$

But $B_o = \mu_o H$ and $B_m = \mu_o I$

$$B = \mu_o H + \mu_o I$$

$$\therefore B = \mu_o (H + I)$$

(v) Magnetic susceptibility

Magnetic susceptibility χ_m is a property which determines how easily and how strongly a specimen can be magnetised.

Susceptibility of a magnetic material is defined as the ratio of intensity of magnetisation I induced in the material to the magnetising field H in which the material is placed.

Thus $\chi_m = \dfrac{I}{H}$

Since I and H are of the same dimensions, χ_m has no unit and is dimensionless.

Relation between χ_m and μ_r

$$\chi_m = \frac{I}{H}$$

$$\therefore\ I = \chi_m H$$

We know $B = \mu_o (H + I)$

$$B = \mu_o (H + \chi_m H)$$

$$B = \mu_o H (1 + \chi_m)$$

If μ is the permeability, we know that $B = \mu H$.

$$\therefore\ \mu H = \mu_o H (1 + \chi_m)$$

$$\frac{\mu}{\mu_o} = (1 + \chi_m)$$

$$\therefore\ \mu_r = 1 + \chi_m$$

10.10 Classification of magnetic materials

On the basis of the behaviour of materials in a magnetising field, the materials are generally classified into three categories namely, (i) Diamagnetic, (ii) Paramagnetic and (iii) Ferromagnetic

(i) Properties of diamagnetic substances

Diamagnetic substances are those in which the net magnetic moment of atoms is zero.

1. The susceptibility has a low negative value. (For example, for bismuth $\chi_m = -\ 0.00017$).

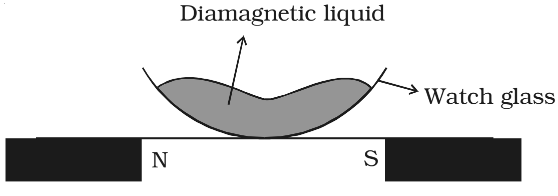


Fig. 10.18 Diamagnetic liquid

2. Susceptibility is independent of temperature.

3. The relative permeability is slightly less than one.

4. When placed in a non uniform magnetic field they have a tendency to move

away from the field. (i.e) from the stronger part to the weaker part of the field. They get magnetised in a direction opposite to the field as shown in the Fig. 10.18.

5. When suspended freely in a uniform magnetic field, they set themselves perpendicular to the direction of the magnetic field (Fig. 10.19).

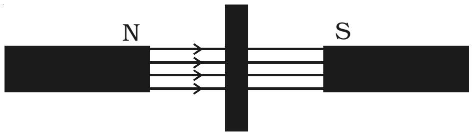


Fig. 10.19 Diamagnetic material perpendicular to the field

Examples : Bi, Sb, Cu, Au, Hg, H_2O, H_2 etc.

(ii) Properties of paramagnetic substances

Paramagnetic substances are those in which each atom or molecule has a net non-zero magnetic moment of its own.

1. Susceptibility has a low positive value.

(For example : χ_m for aluminium is +0.00002).

2. Susceptibiltity is inversely proportional to absolute temperature (i.e) $\chi_m \alpha \frac{1}{T}$. As the temperature increases susceptibility decreases.

3. The relative permeability is greater than one.

4. When placed in a non uniform magnetic field, they have a tendency to move from weaker part to the stronger part of the field. They get magnetised in the direction of the field as shown in Fig. 10.20.

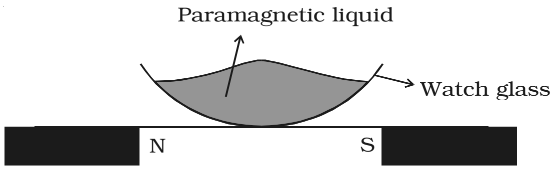


Fig. 10.20 Paramagnetic liquid

5. When suspended freely in a uniform magnetic field, they set themselves parallel to the direction of magnetic field (Fig. 10.21).

Fig. 10.21 Paramagnetic material parallel to the field

Examples : Al, Pt, Cr, O_2, Mn, $CuSO_4$ etc.

(iii) Properties of ferromagnetic substances

Ferromagnetic substances are those in which each atom or molecule has a strong spontaneous net magnetic moment. These substances exhibit strong paramagnetic properties.

1. The susceptibility and relative permeability are very large.

(For example : μ_r for iron = 200,000)

2. Susceptibility is inversely proportional to the absolute temperature.

(i.e) $\chi_m \; \alpha \; \frac{1}{T}$. As the temperature increases the value of susceptibility decreases. *At a particular temperature, ferro magnetics become para magnetics. This transition temperature is called curie temperature.* For example curie temperature of iron is about 1000 K.

3. When suspended freely in uniform magnetic field, they set themselves parallel to the direction of magnetic field.

4. When placed in a non uniform magnetic field, they have a tendency to move from the weaker part to the stronger part of the field. They get strongly magnetised in the direction of the field.

Examples : Fe, Ni, Co and a number of their alloys.

10.11 Hysteresis

Consider an iron bar being magnetised slowly by a magnetising field H whose strength can be changed. It is found that the magnetic induction B inside the material increases with the strength of the magnetising field and then attains a saturated level. This is depicted by the path OP in the Fig. 10.22.

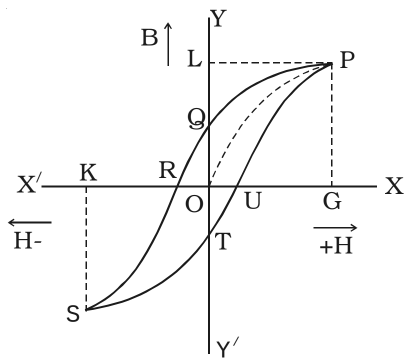


Fig. 10.22 Hysteresis loop

If the magnetising field is now decreased slowly, then magnetic induction also decreases but it does not follow the path PO. Instead, when H = 0, B has non zero value equal to OQ. This implies that some magnetism is left in the specimen. *The value of magnetic induction of a substance, when the magnetising field is reduced to zero, is called remanance or residual magnetic induction of the material.* OQ represents the residual magnetism of the material. Now, if we apply the magnetising field in the reverse direction, the magnetic induction decreases along QR till it becomes zero at R. Thus to reduce the residual magnetism (remanent magnetism) to zero, we have to apply a magnetising field OR in the opposite direction.

The value of the magnetising field H which has to be applied to the magnetic material in the reverse direction so as to reduce its residual magnetism to zero is called its coercivity.

When the strength of the magnetising field H is further increased in the reverse direction, the magnetic induction increases along RS till it acquires saturation at a point S (points P and S are symmetrical). If we now again change the direction of the field, the magnetic induction follows the path STUP. *This closed curve PQRSTUP is called the 'hysteresis loop' and it represents a cycle of magnetisation.* The word 'hysteresis' literally means lagging behind. We have seen that magnetic induction B lags behind the magnetising field H in a cycle of magnetisation. *This phenomenon of lagging of magnetic induction behind the magnetising field is called hysteresis.*

Hysteresis loss

In the process of magnetisation of a ferromagnetic substance through a cycle, there is expenditure of energy. The energy spent in magnetising a specimen is not recoverable and there occurs a loss of energy in the form of heat. This is so because, during a cycle of magnetisation, the molecular magnets in the specimen are oriented and reoriented a number of times. This molecular motion results in the production of heat. It has been found that *loss of heat energy per unit volume of the specimen in each cycle of magnetisation is equal to the area of the hysteresis loop.*

The shape and size of the hysteresis loop is characteristic of each material because of the differences in their retentivity, coercivity, permeability, susceptibility and energy losses etc. By studying hysteresis loops of various materials, one can select suitable materials for different purposes.

10.11.1 *Uses of ferromagnetic materials*

(i) Permanent magnets

The ideal material for making permanent magnets should possess high retentivity (residual magnetism) and high coercivity so that the magnetisation lasts for a longer time. Examples of such substances are steel and alnico (an alloy of Al, Ni and Co).

(ii) Electromagnets

Material used for making an electromagnet has to undergo cyclic changes. Therefore, the ideal material for making an electromagnet has to be one which has the least hysteresis loss. Moreover, the material should attain high values of magnetic induction B at low values of magnetising field H. Soft iron is preferred for making electromagnets as it has a thin hysteresis loop (Fig. 10.23) [small area, therefore less hysteresis loss] and low retentivity. It attains high values of B at low values of magnetising field H.

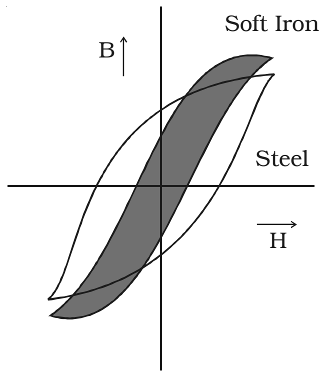


Fig. 10.23 Hysteresis loop for steel and soft iron

(iii) Core of the transformer

A material used for making transformer core and choke is subjected to cyclic changes very rapidly. Also, the material must have a large value of magnetic induction B. Therefore, soft iron that has thin and tall hysteresis loop is preferred. Some alloys with low hysteresis loss are: radio-metals, pern-alloy and mumetal.

(iv) Magnetic tapes and memory store

Magnetisation of a magnet depends not only on the magnetising field but also on the cycle of magnetisation it has undergone. Thus, the value of magnetisation of the specimen is a record of the cycles of magnetisation it has undergone. Therefore, such a system can act as a device for storing memory.

Ferro magnetic materials are used for coating magnetic tapes in a cassette player and for building a memory store in a modern computer. Examples : Ferrites (Fe, Fe_2O, $MnFe_2O_4$ etc.).

Solved Problems

10.1 A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If B = 4 × 10^{-5} T, calculate the magnetic moment of the magnet.

Data : *$d = 10 \times 10^{-2}$ m; $B = 4 \times 10^{-5}$ T; M = ?*

Solution : *When the north pole of a bar magnet points north, the neutral points will lie on its equatorial line.*

∴ The field at the neutral point on the equatorial line of a short bar magnet is, $B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$

$\therefore \quad M = B \times d^3 \times 10^7 = 4 \times 10^{-5} (10 \times 10^{-2})^3 \times 10^7$

$M = 0.4 \text{ A m}^2$

10.2 A bar magnet is suspended horizontally by a torsionless wire in magnetic meridian. In order to deflect the magnet through 30° from the magnetic meridian, the upper end of the wire has to be rotated by 270°. Now this magnet is replaced by another magnet. In order to deflect the second magnet through the same angle from the magnetic meridian, the upper end of the wire has to be rotated by 180°. What is the ratio of the magnetic moments of the two bar magnets. (Hint : $\tau = C\theta$)

Solution : *Let C be the deflecting torque per unit twist and M_1 and M_2 be the magnetic moments of the two magnets.*

The deflecting torque is $\tau = C\theta$

The restoring torque is $\tau = MB \sin \theta$

In equilibrium

deflecting torque = restoring torque

For the Magnet – I

$$C\,(270^o - 30^o) = M_1 B_h \sin \theta \qquad \dots (1)$$

For the magnet – II

$$C\,(180^o - 30^o) = M_2 B_h \sin \theta \qquad \dots (2)$$

Dividing (1) by (2)

$$\frac{M_1}{M_2} = \frac{240^o}{150^o} = \frac{8}{5}$$

10.3 A short bar magnet of magnetic moment 5.25×10^{-2} A m^2 is placed with its axis perpendicular to the Earth's field direction. At what distance from the centre of the magnet on (i) its equatorial line and (ii) its axial line, is the resultant field inclined at 45^o with the Earth's field. Magnitude of the Earth's field at the place is 0.42×10^{-4} T.

Data : $M = 5.25 \times 10^{-2}$ $A\ m^{-2}$

$\theta = 45^o$

$B_h = 0.42 \times 10^{-4}\ T$

$d = ?$

Solution : *From Tangent Law*

$$\frac{B}{B_h} = Tan\ \theta$$

$$B = B_h \tan\theta = 0.42 \times 10^{-4} \times \tan 45^o$$

$$B = 0.42 \times 10^{-4}\ T$$

(i) *For the point on the equatorial line*

$$B = \frac{\mu_o}{4\pi}\frac{M}{d^3}$$

$$d^3 = \frac{\mu_o}{4\pi}\frac{M}{B}$$

$$d^3 = \frac{4\pi \times 10^{-7} \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}}$$

$$= 12.5 \times 10^{-5} = 125 \times 10^{-6}$$

$$\therefore d = 5 \times 10^{-2}\ m$$

(ii) *For the point on the axial line*

$$B = \frac{\mu_o}{4\pi}\frac{2M}{d^3} \quad (or) \quad d^3 = \frac{\mu_o}{4\pi}\frac{2M}{B}$$

$$d^3 = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 5.25 \times 10^{-2}}{0.42 \times 10^{-4}}$$

$$d^3 = 250 \times 10^{-6} = 2 \times 125 \times 10^{-6}$$

$d \quad = 2^{1/3} \cdot (5 \times 10^{-2})$

$d \quad = 6.3 \times 10^{-2}$ m.

10.4 A bar magnet of mass 90 g has magnetic moment 3 A m^2. If the intensity of magnetisation of the magnet is 2.7×10^5 A m^{-1}, find the density of the material of the magnet.

Data : $m = 90 \times 10^{-3}$ kg; $M = 3$ A m^2

$I = 2.7 \times 10^5$ A m^{-1}; $\rho = ?$

Solution : *Intensity of magnetisation,* $I = \frac{M}{V}$

But, volume $V = \frac{m}{\rho}$

$\therefore I = \frac{M\rho}{m}$

$$\rho = \frac{Im}{M} = \frac{2.7 \times 10^5 \times 90 \times 10^{-3}}{3} = 8100$$

$\rho = 8100$ kg m^{-3}

10.5 A magnetising field of 50 A m^{-1} produces a magnetic field of induction 0.024 T in a bar of length 8 cm and area of cross section 1.5 cm^2. Calculate (i) the magnetic permeability (ii) the magnetic susceptibility.

Data : $H = 50$ A m^{-1}, $B = 0.024$ T $= 2.4 \times 10^{-2}$ T,

$2l = 8 \times 10^{-2}$ m, $\quad A = 1.5 \times 10^{-4}$ m^2 $\mu = ?$; $\chi_m = ?$

Solution : *Permeability* $\mu = \frac{B}{H} = \frac{2.4 \times 10^{-2}}{50} = 4.8 \times 10^{-4} H\ m^{-1}$

Susceptibility, $\chi_m = \mu_r - 1 = \frac{\mu}{\mu_o} - 1$

$$\chi_m = \frac{4.8 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 381.16$$

Self evaluation

(The questions and problems given in this self evaluation are only samples. In the same way any question and problem could be framed from the text matter. Students must be prepared to answer any question and problem from the text matter, not only from the self evaluation.)

10.1 *Two magnetic poles kept separated by a distance d in vacuum experience a force of 10 N. The force they would experience when kept inside a medium of relative permeability 2, separated by the same distance is*

(a) 20 N (b) 10 N

(c) 5 N (d) 40 N

10.2 *The magnetic moment of a magnet is 5 A m^2. If the pole strength is 25 A m, what is the length of the magnet?*

(a) 10 cm (b) 20 cm

(c) 25 cm (d) 1.25 cm

10.3 *A long magnetic needle of length 2l, magnetic moment M and pole strength m is broken into two at the middle. The magnetic moment and pole strength of each piece will be*

(a) M, m (b) $\frac{M}{2}, \frac{m}{2}$

(c) M, $\frac{m}{2}$ (d) $\frac{M}{2}$, m

10.4 *Two short magnets have equal pole strengths but one is twice as long as the other. The shorter magnet is placed 20 cm in tan A position from the compass needle. The longer magnet must be placed on the other side of the magnetometer for zero deflection at a distance*

(a) 20 cm (b) 20 $(2)^{1/3}$ cm

(c) 20 $(2)^{2/3}$ cm (d) 20 (2) cm

10.5 *The direction of a magnet in tan B position of a deflection magnetometer is*

(a) North – South *(b) East – West*

(c) North – West *(d) South – West*

10.6 *The relative permeability of a specimen is 10001 and magnetising field strength is 2500 A m^{-1}. The intensity of magnetisation is*

(a) 0.5×10^{-7} A m^{-1} *(b) 2.5×10^{-7} A m^{-1}*

(c) $2.5 \times 1.0^{+7}$ A m^{-1} *(d) 2.5×10^{-1} A m^{-1}*

10.7 *For which of the following substances, the magnetic susceptibility is independent of temperature?*

(a) diamagnetic

(b) paramagnetic

(c) ferromagnetic

(d) diamagnetic and paramagnetic

10.8 *At curie point, a ferromagnetic material becomes*

(a) non–magnetic *(b) diamagnetic*

(c) paramagnetic *(d) strongly ferromagnetic*

10.9 *Electromagnets are made of soft iron because soft iron has*

(a) low susceptibility and low retentivity

(b) high susceptibility and low retentivity

(c) high susceptibility and high retentivity

(d) low permeability and high retentivity

10.10 *State Coulomb's inverse square law.*

10.11 *Obtain the expressions for the magnetic induction at a point on the (i) axial line and (ii) equatorial line of a bar magnet.*

10.12 *Find the torque experienced by a magnetic needle in a uniform magnetic field.*

10.13 *State and prove tangent law.*

10.14 *What is tan A position? How will you set up the deflection magnetometer in tan A position?*

10.15 *Explain the theory of tan A position. Explain how will you compare the magnetic moments of two bar magnets in this position.*

10.16 *What is tan B position? How will you set up the deflection magnetometer in tan B position?*

10.17 *Explain the theory of tan B position. Explain how will you compare the magnetic moments of two bar magnets in this position.*

10.18 *Define the terms (i) magnetic permeability (ii) intensity of magnetisation and (iii) magnetic susceptibility.*

10.19 *Distinguish between dia, para and ferro magnetic substances. Give one example for each.*

10.20 *Explain the hysteresis cycle.*

Problems

10.21 *The force acting on each pole of a magnet placed in a uniform magnetic induction of 5×10^{-4} T is 6×10^{-3} N. If the length of the magnet is 8 cm, calculate the magnetic moment of the magnet.*

10.22 *Two magnetic poles, one of which is twice stronger than the other, repel one another with a force of 2×10^{-5} N, when kept seperated at a distance of 20 cm in air. Calculate the strength of each pole.*

10.23 *Two like poles of unequal pole strength are placed 1 m apart. If a pole of strength 4 A m is in equilibrium at a distance 0.2 m from one of the poles, calculate the ratio of the pole strengths of the two poles.*

10.24 *A magnet of pole strength 24.6×10^{-2} A m and length 10 cm is placed at 30^o with a magnetic field of 0.01 T. Find the torque acting on the magnet.*

10.25 *The magnetic moment of a bar magnet of length 10 cm is 9.8×10^{-1} A m^2. Calculate the magnetic field at a point on its axis at a distance of 20 cm from its midpoint.*

10.26 *Two mutually perpendicular lines are drawn on a table. Two small magnets of magnetic moments 0.108 and 0.192 A* m^2 *respectively are placed on these lines. If the distance of the point of intersection of these lines is 30 cm and 40 cm respectively from these magnets, find the resultant magnetic field at the point of intersection.*

10.27 *The intensity of magnetisation of an iron bar of mass 72 g, density 7200 kg* m^{-3} *is 0.72 A* m^{-1}. *Calculate the magnetic moment.*

10.28 *A magnet of volume 25* cm^3 *has a magnetic moment of* 12.5×10^{-4} *A* m^2. *Calculate the intensity of magnetisation.*

10.29 *A magnetic intensity of* 2×10^3 *A/m produces a magnetic induction of* 4π *Wb/*m^2 *in a bar of iron. Calculate the relative permeability and susceptibility.*

Answers

10.1	*(a)*	***10.2***	*(b)*	***10.3***	*(d)*
10.4	*(b)*	***10.5***	*(b)*	***10.6***	*(c)*
10.7	*(a)*	***10.8***	*(c)*	***10.9***	*(b)*

10.21 $0.96\ A\ m^2$

10.22 $2\ A\ m,\ 4\ A\ m$

10.23 *1 :16*

10.24 $1.23 \times 10^{-4}\ N\ m$

10.25 $2.787 \times 10^{-5}\ T$

10.26 $10^{-6}\ T$

10.27 $7.2 \times 10^{-6}\ A\ m^2$

10.28 $50\ A\ m^{-1}$

10.29 *5000, 4999*

ANNEXURE

(NOT FOR EXAMINATION)

1 Declination

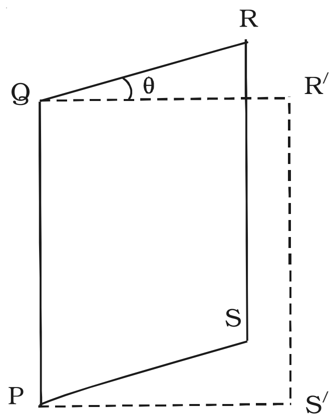


Fig. 1 Declination

A vertical plane passing through the axis of a freely suspended magnetic needle is called magnetic meridian and the vertical plane passing through the geographic north - south direction (axis of rotation of Earth) is called geographic meridian (Fig.).

In the Fig. 1 the plane *PQRS* represents the magnetic meridian and the plane *PQR′S′* represents the geographic meridian.

Declination at a place is defined as the angle between magnetic meridian and the geographic meridian at that place.

Determination of declination

A simple method of determining the geographical meridian at a place is to erect a pole of 1 to 1.5 m height on the ground and a circle is drawn with the pole O as centre and its height as radius as shown in the Fig. 2.

Mark a point P_1 on the circle before noon, when the tip of the shadow of the pole just touches the circle.

Again mark a point P_2 when the tip of the shadow touches the circle in the afternoon. The line *POQ* drawn bisecting the angle P_1OP_2 is the geographical meridian at that place.

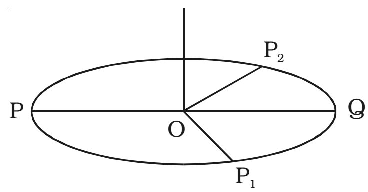


Fig. 2 Geographic Meridian

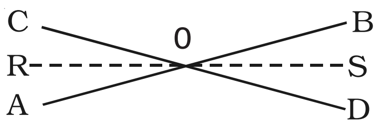


Fig. 3 Magnetic meridian

Magnetic meridian is drawn by freely suspending a magnetic needle provided with two pins fixed vertically at its ends.

When the needle is at rest, draw a line AB joining the tips of the two pins. The magnetic needle is reversed upside down. Pins are fixed at the ends of the needle. When the magnet is at rest, draw a line *CD* joining the tips of pins. *O* is the point of intersection of *AB* and *CD*. The line RS bisecting the angle *BOD* is the magnetic meridian at that place (Fig. 3).

Now the angle between geographic meridian *PQ* and the magnetic meridian RS is the angle of declination θ (Fig. 4).

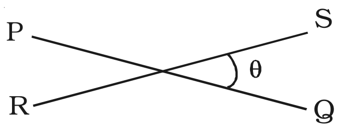


Fig. 4 Declination

2 Dip

Dip is defined as the angle between the direction of Earth's magnetic field and the direction of horizontal component of earths magentic field. It is the angle by which the total Earth's magnetic field dips or comes out of the horizontal plane. It is denoted by δ. The value of dip varies from place to place. It is O^o along the equator and 90^o at the poles. At Chennai the value of dip is about 9^{o}7'.

At a place the value of dip is measured by an instrument called dip circle.

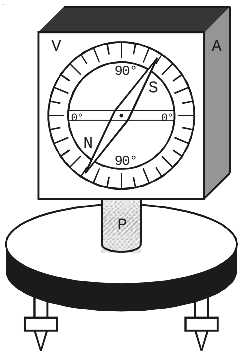


Fig. 5 Dip Circle

Dip circle

A magnetic needle *NS* is pivoted at the centre of a circular vertical scale V by means of a horizontal rod. The needle is free to move over this circular scale. The scale has four segments and each segment is graduated from 0^o to 90^o such that it reads 0^o - 0^o along the horizontal and 90^o - 90^o along the vertical. The needle and the scale are enclosed in a rectangular box A with glass window. The box is mounted on a vertical pillar *P* on a horizontal base, which is provided with levelling screws. The base has a circular scale graduated from 0^o to 360^o (FIg. 5). The box can be rotated about a vertical axis and its position can be read on the circular scale with the help of a vernier (not shown in the figure).

The levelling screws are adjusted such that the base is horizontal and the scale inside the box is vertical. The box is rotated so that the ends of the magnetic needle *NS* read 90^o - 90^o on the vertical scale.

The needle, in this position is along the vertical component of the Earth's field. The horizontal component of Earth's field being perpendicular to the plane, does not affect the needle. This shows that the vertical scale and the needle are in a plane at right angles to the magnetic meridian. Now the box is rotated through an angle of 90^o with the help of the horizontal circular scale. The magnetic needle comes to rest exactly in the magnetic meridian. The reading of the magnetic needle gives the angle of dip at that place.