MATHEMATICS

CLASS 12

Study material & notes

INDEX

Topic wise Analysis of Examples and Questions

NCERT TEXT BOOK

Chapters	Concepts	Number of Questions for revision		Total
		Questions From Solved Examples	Questions From Exercise	
01	**Relations & Functions**	**15**	**25**	**40**
02	**Inverse Trigonometric Functions**	**05**	**09**	**14**
03	**Matrices & Determinants**	**06**	**25**	**31**
04	**Continuity& Differentiability**	**08**	**21**	**29**
05	**Application of Derivative**	**06**	**14**	**20**
06	**Indefinite Integrals**	**17**	**45**	**62**
07	**Applications of Integration**	**05**	**09**	**14**
08	**Differential Equations**	**07**	**19**	**26**
09	**Vector Algebra**	**07**	**18**	**25**
10	**Three Dimensional Geometry**	**07**	**12**	**19**
11	**Linear Programming**	**09**	**12**	**21**
12	**Probability**	**19**	**27**	**46**
	TOTAL	**111**	**236**	**347**

Detail of the concepts to be mastered by every child of class XII with exercises and examples of NCERT Text Book.

SYMBOLS USED

*** : Important Questions, ** :Very Important Questions,**

***** : Very-Very Important Questions**

S.No	Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
1	***Relations & Functions***	(i) .Domain , Co-domain & Range of a relation	*	(Previous Knowledge)
		(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
		(iii).One-one , onto & inverse ofa function	***	Ex 1.2 Q.No- 7,9
		(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
		(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11
2	***Inverse Trigonometric Functions***	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
		(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 QNo- 7,13, 15 Misc Ex Q.No.9,10,11,12
3	***Matrices & Determinants***	(i) Order, Addition, Multiplication and transpose of matrices	***	Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
		(ii) Cofactors &Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
		(iii)Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4,Q.No4,5,8,12,15
		(iv)To find difference between $\|A\|$, $\|adj\ A\|$, $\|kA\|$, $\|A.adjA\|$	*	Ex 4.1 –Q.No 3,4,7,8
		(v) Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18
4	***Continuity& Differentiability***	(i).Limit of a function	*	
		(ii).Continuity	***	Ex 5.1 Q.No- 21, 26,30
		(iii).Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
		(iv).Logrithmic Differentiation	***	Ex 5.5 Q.No- 6,9,10,15
		(v) Parametric Differentiation	***	Ex 5.6 Q.No- 7,8,10,11
		(vi). Second order derivatives	***	Ex 5.7 Q.No- 14,16,17

		(vii). M. V.Th	**	Ex 5.8 Q.No- 3,4
5	***Application of Derivative.***	(i).Rate of change	*	Example 5Ex 6.1 Q.No- 9,11
		(ii).Increasing & decreasing functions	***	Ex 6.2 ,Q.No- 6 Example 12,13
		(iii).Tangents & normal	**	Ex 6.3 ,Q.No- 5,8,13,15,23
		(iv).Approximations	*	Ex 6.4,Q.No- 1,3
		(v) Maxima & Minima	***	Ex 6.5, Q.No- 8,22,23,25 Example 35,36,37
6	***Indefinite Integrals***	(i) Integration by substitution	*	Exp 5&6 Page301,303
		(ii) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
		(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	***	Edition Exp 8, 9, 10 Page 311,312Exercise 7.4 Q 3,4,8,9,13&23
		(iv) Integration using Partial Fraction	***	EditionExp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
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11	***Linear Programming***	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
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		(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
		(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
		(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

TOPIC 1
RELATIONS & FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XII Ed. 2007
Relations & Functions	(i).Domain , Co domain & Range of a relation	*	(Previous Knowledge)
	(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
	(iii).One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
	(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
	(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

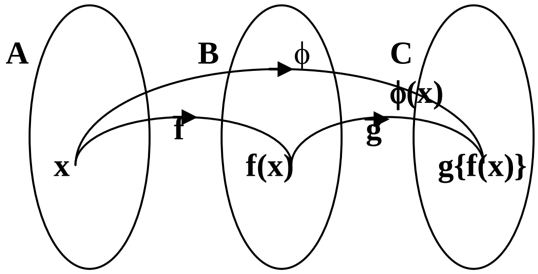
SOME IMPORTANT RESULTS/CONCEPTS

** A **relation** R in a set A is called
(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,
(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.
(iii)*transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.
** Equivalence Relation : R is equivalence if it is reflexive, symmetric and transitive.

** **Function** :A relation $f : A \to B$ is said to be a function if every element of A is correlatedto unique element in B.
* A is domain
* B is codomain
* For any x element $x \in A$, function f correlates it to an element in B, which is denoted by f(x)and is called image of x under f . Again if y= f(x), then x is called as pre-image of y.
* Range = $\{f(x) \mid x \in A\}$. Range $\subseteq$ Codomain
* The largest possible domain of a function is called domain of definition.

****Composite function** :

Let two functions be defined as $f : A \to B$ and $g : B \to C$. Then we can define a function $\phi : A \to C$ by setting $\phi(x) = g\{f(x)\}$ where $x \in A$, $f(x) \in B$, $g\{f(x)\} \in C$. This function $\phi : A \to C$ is called the composite function of f and g in that order and we write. $\phi = gof$.



**** Different type of functions** : Let ***f*** : A →B be a function.

* ***f*** is **one to one (injective) mapping**, if any two different elements in A is always correlated to different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$or, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

* ***f*** is **many one mapping,** if ∃ at least two elements in A such that their images are same.

* ***f*** is **onto mapping** (subjective), if each element in B is having at least one preimage.

****f*** is **into mapping** if range ⊆ codomain.

* ***f*** is **bijective mapping** if it is both *one to one and onto*.

**** Binary operation :** A binary operation * on a set A is a function * : A × A → A. We denote *(*a*, *b*) by *a* **b*.

* A binary operation '*' on A is a rule that associates with every ordered pair (a, b) of A x A a unique element a *b.

* An operation '*' on a is said to be commutative iff a * b = b * a ∀ a, b ∈ A.

* An operation '*' on a is said to be associative iff (a * b) * c = a * (b * c) ∀ a, b, c ∈ A.

* Given a binary operation * : A × A → A, an element *e* ∈ A, if it exists, is called ***identity*** for the operation *, if *a* **e* = *a* = *e* **a*, ∀*a* ∈ A.

* Given a binary operation * : A × A → A with the identity element *e* in A, an element *a* ∈ A is said to be ***invertible*** with respect to the operation*, if there exists an element *b* in A such that $a * b = e = b * a$ and *b* is called the *inverse of a* and is denoted by a^{-1}.

ASSIGNMENTS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b € A. Write the domain, range & co-domain.
2. Define a relation R on the set **N** of natural numbers by

   R={(*x* , *y*) : *y* = *x* +7, *x* is a natural number lesst han 4 ; *x*, *y* ∈ **N**}.

   Write down the domain and the range.

2. *Types of relations*

LEVEL II

1. Let R be the relation in the set N given by R = {(a , b)| a = b – 2 , b > 6}
   Whether the relation is reflexive or not ?justify your answer.
2. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b ∈ N} is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) ∈R iff 1+ ab> 0 ∀ a,b∈R.
   (a) Show that R is symmetric.
   (b) Show that R is reflexive.
   (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x , y)| x – 3y = 0} on A ={1, 2, 3……….13, 14}.

LEVEL III

1. Show that the relation R on A ,A = { x| x $\in Z$, $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

2.Let N be the set of all natural numbers & R be the relation on N × N defined by { (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.

3. Show that the relation R in the set A of all polygons as:
   R ={(P_1,P_2), P_1& P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?

4. Show that the relation R on A ,A = { x| x $\in Z$, $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

5. Let N be the set of all natural numbers & R be the relation on N × N defined by { (a , b) R (c ,d) iff a + d = b + c}. Show that R is an equivalence relation. [**CBSE 2010]**

6. Let A = Set of all triangles in a plane and R is defined by R={(T_1,T_2) : T_1,$T_2 \in$ A & T_1~T_2 }
   Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5; T_2 with size 5,12,13; T_3 with side 6,8,10; Which of the pairs are related?

(iii)One-one , onto & inverse of a function

LEVEL I

1. If f(x) = $x^2 - x^{-2}$, then find f(1/x).
2 Show that the function f: R→R defined by f(x)=x^2 is neither one-one nor onto.

3 Show that the function f: N→N given by f(x)=2x is one-one but not onto.

4 Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

5 Let A = {-1,0,1} and B = {0,1}. State whether the function f : A → B defined by f(x) = x^2 isbijective .

6. Let f(x) = $\frac{x-1}{x+1}$, x≠ -1,then find $f^{-1}(x)$

LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4),(2,5), (3,6)} be a function from A to B. State whether f is one-one or not. [**CBSE2011]**

2. If f : R→R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function . Find $f^{-1}(x)$.

3. Write the number of all one-one functions on the set A={a, b, c} to itself.

4. Show that function f :R→R defined by f(x)= $7 - 2x^3$ for all x $\in$ R is bijective.

5. If f: R→R is defined by f(x)= $\frac{3x+5}{2}$. Find f^{-1}.

LEVEL III

1. Show that the function f: R→R defined by $f(x) = \frac{2x-1}{3}$.$x \in$ R is one- one & onto function. Also find the f^{-1}.

2. Consider a function f :R_+→[-5, ∞) defined $f(x) = 9x^2 + 6x - 5$. Show that f is invertible &

$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$, where $R_+ = (0,\infty)$.

3. Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y)= \frac{y-3}{4}$.

4. Show that f: R→R defined by $f(x)= x^3+4$ is one-one, onto. Show that $f^{-1}(x)=(x-4)^{1/3}$.

5. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \rightarrow B$ defined by

$f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one one onto and hence find f^{-1}. **[CBSE2012]**

6. Show that $f : N \rightarrow N$ defined by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one one onto.

[CBSE2012]

(iv) *Composition of functions*

LEVEL I

1. If $f(x) = e^{2x}$ and $g(x) = \log \sqrt{x}$, x > 0, find
   (a) (f + g)(x) (b) (f .g)(x) (c) f o g (x) (d) g o f (x).

2. If $f(x) = \frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = - f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

LEVEL II

1. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [x] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).

2. Let $f(x) = \frac{x-1}{x+1}$. Then find f(f(x))

3. If $y = f(x) = \frac{3x+4}{5x-3}$, then find (fof)(x) i.e. f(y)

4. Let f : **R** →**R** be defined as f(x) = 10x +7.Find the function g : **R**→**R**such that $g \circ f(x)= f \circ g(x) = I_R$ **[CBSE2011]**

5. If f : **R** →**R** be defined as $f(x) = \left(3-x^3\right)^{\frac{1}{3}}$, then find $f \circ f(x)$.

[CBSE2010]

6. Let f :R→R& g : R→R be defined as $f(x) = x^2$, g(x) = 2x – 3 . Find fog(x).

(v)Binary Operations

LEVEL I

1. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
2. Let *be the binary on N given by a*b =HCF of {a ,b} , a,b$\in$N. Find 20*16.
3. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.

   Write the identity of *, if any.
4. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$
   Then find the value of 2 * 4.

LEVEL 2

1. Let A= N×N & * be the binary operation on A defined by (a ,b) * (c ,d) = (a+c, b+d)
   Show that * is (a) Commutative (b) Associative (c) Find identity for * on A, if any.

2. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).
   Find: (i) the identity element of A (ii) the invertible element of A.
3. Examine which of the following is a binary operation

   (i) $a * b = \frac{a+b}{2}$; a, b$\in$ N (ii) $a*b = \frac{a+b}{2}$ a, b$\in$ Q

   For binary operation check commutative & associative law.

LEVEL 3

1.Let A= N×N & * be a binary operation on A defined by (a , b) × (c , d) = (ac , bd)
∀ (a , b),(c , d) ∈N×N (i) Find (2,3) * (4,1)
(ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal
(iii) Show that * is commutative & associative on A.

2. Define a binary operation * on the set {0,1,2,3,4,5} as $a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & a+b \geq 6 \end{cases}$

Show that zero in the identity for this operation & each element of the set is invertible with 6 – a being the inverse of a. **[CBSE2011]**

3. Consider the binary operations $*$:**R** × **R** → **R**and o : **R** × **R** → **R** defined as a $*$b = |a – b| and a o b = a, ∀a, b ∈**R**. Show that $*$is commutative but not associative, o is associative but not commutative. **[CBSE2012]**

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

2. Show that each of the relation R in the set A = {x $\in$**Z** : 0 ≤ x ≤ 12}, given by R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

3. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

4. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\rightarrow$B defined by f (x) = $\left(\frac{x-2}{x-3}\right)$.

Is f one-one and onto? Justify your answer.

6. Consider f :**R+**$\rightarrow$[– 5, ∞) given by f (x) = $9x^2 + 6x - 5$. Show that f is invertible and findf^{-1}.

7. On R – {1} a binary operation '$*$' is defined as a $*$ b = a + b – ab. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'.Also prove that every element of R – {1} is invertible.

8. If A = $Q \times Q$ and '$*$' be a binary operation defined by (a, b) $*$ (c, d) = (ac, b + ad), for (a, b), (c, d)$\in A$.Then with respect to '$*$' on A

(i) examine whether '$*$' is commutative & associative

(i) find the identity element in A,

(ii) find the invertible elements of A.

TOPIC 2
INVERSE TRIGONOMETRIC FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Inverse Trigonometric Functions	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
	(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 Q No- 7,13, 15 Misc Ex Q.No. 9,10,11,12

SOME IMPORTANT RESULTS/CONCEPTS

* Domain & Range of the Inverse Trigonometric Function :

	Functions	Domain	Range (Principal value Branch)
i.	$\sin^{-1}$:	$[-1,1]$	$[-\pi/2,\pi/2]$
ii.	$\cos^{-1}$:	$[-1,1]$	$[0,\pi]$
iii.	$\operatorname{cosec}^{-1}$:	$R-(-1,1)$	$[-\pi/2,\pi/2]-\{0\}$
iv.	$\sec^{-1}$:	$R-(-1,1)$	$[0,\pi]-\{\pi/2\}$
v.	$\tan^{-1}$:	R	$(-\pi/2,\pi/2)$
vi.	$\cot^{-1}$:	R	$(0,\pi)$

* Properties of Inverse Trigonometric Function

1. (i) $\sin^{-1}(\sin x)=x$ & $\sin(\sin^{-1}x)=x$ ii. $\cos^{-1}(\cos x)=x$ & $\cos(\cos^{-1}x)=x$

   iii. $\tan^{-1}(\tan x)=x$ & $\tan(\tan^{-1}x)=x$ iv. $\cot^{-1}(\cot x)=x$ & $\cot(\cot^{-1}x)=x$

   v. $\sec^{-1}(\sec x)=x$ & $\sec(\sec^{-1}x)=x$ vi. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$ & $\operatorname{cosec}(\operatorname{cosec}^{-1}x)=x$

2. i. $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ & $\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ ii. $\cos^{-1}x=\sec^{-1}\frac{1}{x}$ & $\sec^{-1}x=\cos^{-1}\frac{1}{x}$

   iii. $\tan^{-1}x=\cot^{-1}\frac{1}{x}$ & $\cot^{-1}x=\tan^{-1}\frac{1}{x}$

3. i $\sin^{-1}(-x)=-\sin^{-1}x$ iv $\cos^{-1}(-x)=\pi-\cos^{-1}x$

   ii. $\tan^{-1}(-x)=-\tan^{-1}x$ v $\sec^{-1}(-x)=\pi-\sec^{-1}x$

   iii. $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}x$ vi $\cot^{-1}(-x)=\pi-\cot^{-1}x$

4. i $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ ii. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$

   iii. $\operatorname{cosec}^{-1}x+\sec^{-1}x=\frac{\pi}{2}$

5. $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

6. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $xy > -1$

ASSIGNMENTS

(i). Principal value branch Table

LEVEL I

Write the principal value of the following :

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(-\sqrt{3}\right)$

4. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

LEVEL II

Write the principal value of the following :

1. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[CBSE 2011]**

2. $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii). Properties of Inverse Trigonometric Functions

LEVEL I

1. Evaluate $\cot[\tan^{-1}a + \cot^{-1}a]$

2. Prove $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$

3. Find x if $\sec^{-1}\left(\sqrt{2}\right) + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

LEVEL II

1. Write the following in simplest form : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$

2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

3. Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

4. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$ **[CBSE 2011]**

5. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$ **[CBSE 2012]**

LEVEL III

1. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ **[CBSE 2011]**

3. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

4. Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

5.Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ **[CBSE 2012]**

Questions for self evaluation

1. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\quad x\in\left[-\frac{1}{\sqrt{2}},1\right]$

3. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

4. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

6. Write in the simplest form $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

7. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

8. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

TOPIC 3
MATRICES & DETERMINANTS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Matrices & Determinants	(i) Order, Addition, Multiplication and transpose of matrices	***	Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
	(ii) Cofactors &Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
	(iii)Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4–Q.No 4,5,8,12,15
	(iv)To find difference between $\|A\|$, $\|adj\ A\|$, $\|kA\|$, $\|A.adjA\|$	*	Ex 4.1 –Q.No 3,4,7,8
	(v) Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18

SOME IMPORTANT RESULTS/CONCEPTS

A matrix is a rectangular array of $m \times n$ numbers arranged in m rows and n columns.

$$A = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots a_{2n} \\ & \\ a_{m1} & a_{m2} \ldots\ldots\ldots a_{mn} \end{bmatrix}_{m\times n}$$ OR $A = [a_{ij}]_{m\times n}$, where **i** = 1, 2,…., m ; **j** = 1, 2,….,n.

*** Row Matrix**: A matrix which has one row is called row matrix. $A = [a_{ij}]_{1\times n}$

*** Column Matrix** : A matrix which has one column is called column matrix. $A = [a_{ij}]_{m\times 1}$.

*** Square Matrix**: A matrix in which number of rows are equal to number of columns, is called a square matrix $A = [a_{ij}]_{m\times m}$

*** Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all the elements, except the diagonal elements are zero. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, **i**≠ j.

$a_{ij} \neq 0$, **i** = j.

*** Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are same non-zero quantity.

$A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, **i**≠ j.

$a_{ij} \neq \alpha$, **i** = j.

*** Identity or Unit Matrix** : A square matrix in which all the non diagonal elements are zero and diagonal elements are unity is called identity or unit matrix.

*** Null Matrices** : A matrices in which all element are zero.

*** Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.

* **Transpose of matrix** : If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.\

* **Properties of Transpose** :

If A & B are matrices such that their sum & product are defined, then

(i). $\left(A^T\right)^T = A$ (ii). $(A+B)^T = A^T + B^T$ (iii). $\left(KA^T\right) = K.A^T$ where K is a scalar.

(iv). $(AB)^T = B^T A^T$ (v). $(ABC)^T = C^T B^T A^T$.

* **Symmetric Matrix** : A square matrix is said to be symmetric if A = A^T i.e. If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = a_{ji}$ for all i, j. Also elements of the symmetric matrix are symmetric about the main diagonal

* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if A^T = -A.

If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = -a_{ji}$ for all i, j.

***Singular matrix**:A square matrix 'A' of order 'n' is said to be singular, if | A| = 0.

*** Non -Singular matrix** : A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.

***Product of matrices:**

(i) If A & B are two matrices, then product AB is defined, if

Number of column of A = number of rows of B.

i.e. $A = [a_{ij}]_{m\times n}$, $B = [b_{jk}]_{n\times p}$ then AB = $AB = [C_{ik}]_{m\times p}$.

(ii) Product of matrices is not commutative. i.e. $AB \neq BA$.

(iii) Product of matrices is associative. i.e A(BC) = (AB)C

(iv) Product of matrices is distributive over addition.

***Adjoint of matrix** :

If $A = [a_{ij}]$ be a n-square matrix then transpose of a matrix $[A_{ij}]$,

where A_{ij} is the cofactor of A_{ij} element of matrix A, is called the adjoint of A.

Adjoint of A = Adj. A = $[A_{ij}]^T$.

A(Adj.A) = (Adj. A)A = | A| I.

***Inverse of a matrix** :Inverse of a square matrix A exists, if A is non-singular or square matrix

A is said to be invertible and $A^{-1} = \frac{1}{|A|}$ Adj.A

***System of Linear Equations** :

$a_1x + b_1y + c_1z = d_1$.

$a_2x + b_2y + c_2z = d_2$.

$a_3x + b_3y + c_3z = d_3$.

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow A\,X = B \;\; \Rightarrow X = A^{-1}B \quad ; \{\, |A| \neq 0\}.$$

***Criteria of Consistency**.

(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.

(ii) If $|A| = 0$ and (adj. A)B = 0, then the system of equations is consistent and has infinitely many solutions.

(iii) If $|A| = 0$ and (adj. A)B $\neq$ 0, then the system of equations is inconsistent and has no solution.

*** Determinant** :

To every square matrix we can assign a number called determinant

If $A = [a_{11}]$, det. $A = |A| = a_{11}$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $|A| = a_{11}a_{22} - a_{21}a_{12}$.

*** Properties** :

(i) The determinant of the square matrix A is unchanged when its rows and columns are interchanged.

(ii) The determinant of a square matrix obtained by interchanging two rows(or two columns) is negative of given determinant.

(iii) If two rows or two columns of a determinant are identical, value of the determinant is zero.

(iv) If all the elements of a row or column of a square matrix A are multiplied by a non-zero number k, then determinant of the new matrix is k times the determinant of A.

If elements of any one column(or row) are expressed as sum of two elements each, then determinant can be written as sum of two determinants.

Any two or more rows(or column) can be added or subtracted proportionally.

If A & B are square matrices of same order, then $|AB| = |A|\,|B|$

ASSIGNMENTS

(i). Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. If a matrix has 5 elements, what are the possible orders it can have? **[CBSE 2011]**

2. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find A –2 B.

4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA.

LEVEL II

1. For the following matrices A and B, verify $(AB)^T = B^T A^T$,

   where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

2. Give example of matrices A & B such that AB = O, but BA ≠ O, where O is a zero matrix and

A, B are both non zero matrices.

3. If B is skew symmetric matrix, write whether the matrix (ABA^T) is Symmetric or skew symmetric.
4. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$

LEVEL III

1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$

2. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where:

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

3. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \in N$

(ii) Cofactors &Adjoint of a matrix

LEVEL I

1. Find the co-factor of a_{12} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
2. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

LEVEL II

Verify $A(\text{adj}A) = (\text{adj}A)\,A = |A|I$ if

1. $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A **CBSE 2011**
2. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A ?
3. For what value of k , the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible ?

LEVEL II

1. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}

2. If A, B, C are three non zero square matrices of same order, find the condition

on A such that $AB = AC \Rightarrow B = C$.

3. Find the number of all possible matrices A of order 3 × 3 with each entry 0 or 1 and for which A $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions.

LEVEL III

1 If A = $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

2. Using matrices, solve the following system of equations:

a. $x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$ [**CBSE 2011]**

b. $4x + 3y + 2z = 60$
$x + 2y + 3z = 45$
$6x + 2y + 3z = 70$ [**CBSE 2011]**

3. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

4. Using matrices, solve the following system of equations:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$

$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

5. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(iv)To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. Evaluate $\begin{vmatrix} cos15^{\circ} & sin15^{\circ} \\ sin75^{\circ} & cos75^{\circ} \end{vmatrix}$ [**CBSE 2011]**
2. What is the value of $|3I|$, where I is identity matrix of order 3?
3. If A is non singular matrix of order 3 and $|A| = 3$, then find $|2A|$
4. For what valve of a, $\begin{bmatrix} 2a & -1 \\ -8 & 3 \end{bmatrix}$ is a singular matrix?

LEVEL II

1. If A is a square matrix of order 3 such that $|adjA| = 64$, find $|A|$
2. If A is a non singular matrix of order 3 and $|A| = 7$, then find $|adjA|$

LEVEL III

1. If $A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$ and $|A|^3 = 125$, then find a.
2. A square matrix A, of order 3, has $|A| = 5$, find $|A.adjA|$

(v).Properties of Determinants

LEVEL I

1. Find positive valve of x if $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$

2. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

LEVEL II

1. Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

[CBSE 2012]

2. $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

3. $$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

4. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$ **[CBSE 2012]**

LEVEL III

1. Using properties of determinants, solve the following for x :

a. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ **[CBSE 2011]**

b. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **[CBSE 2011]**

c. $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$ **[CBSE 2011]**

2. If a, b, c, are positive and unequal, show that the following determinant is negative:

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

3. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$ **[CBSE 2012]**

5. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

6. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

7. $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

8. If p, q, r are not in G.P and $\begin{vmatrix} 1 & \frac{q}{p} & \alpha+\frac{q}{p} \\ 1 & \frac{r}{q} & \alpha+\frac{r}{q} \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, show that $p\alpha^2+2p\alpha+r=0$.

9. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

Show that either a + b +c = 0 or a = b = c.

QUESTIONS FOR SELF EVALUTION

1. Using properties of determinants, prove that : $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

2. Using properties of determinants, prove that : $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

3. Using properties of determinants, prove that : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. .Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

5. Let $A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove by mathematical induction that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

6. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

7. Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

8. Solve the following system of equations : $x + 2y + z = 7$, $x + 3z = 11$, $2x - 3y = 1$.

9. Find the product AB, where $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

10. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

TOPIC 4
CONTINUITY AND DIFFRENTIABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Continuity& Differentiability	1.Limit of a function		
	2.Continuity	***	Ex 5.1 Q.No- 21, 26,30
	3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
	4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
	5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
	6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
	7. Mean Value Theorem	**	Ex 5.8 QNo- 3,4

SOME IMPORTANT RESULTS/CONCEPTS

* A function f is said to be continuous at x = a if Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

i.e. $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a-h) = f(a)$.

* A function is said to be differentiable at x = a if $Lf'(a) = Rf'(a)$ i.e

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

(i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$.

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) = 0, \forall\, c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1$.

(v) $\frac{d}{dx}(e^x) = e^x$.

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x\log a}\ a > 0, a \neq 1, x$

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x}, x > 0$

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x, \forall\, x \in R$.

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x, \forall\, x \in R$.

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x, \forall\, x \in R$.

(xvi) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}, \forall\, x \in R$

(xix) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}, \forall\, x \in R$.

(xx) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$,.

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$.

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(viii) $\frac{d}{dx}(\log_a \lvert x \rvert) = \frac{1}{x \log a}$, a > 0, a ≠1, x ≠ 0	(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$
(ix) $\frac{d}{dx}(\log \lvert x \rvert) = \frac{1}{x}$, x ≠ 0	(xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
(x) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R.$	
(xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R.$	
(xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R.$	

2.Continuity

LEVEL-I

1.Examine the continuity of the function $f(x)=x^2 + 5$ at x=-1.

2. Examine the continuity of the function $f(x)=\frac{1}{x+3}$, $x \in R$.

3. Show that f(x)=4x is a continuous for all $x \in R$.

LEVEL-II

1. Give an example of a function which is continuous at x=1,but not differentiable at x=1.

2. For what value of k,the function $\begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at x=2.

3.Find the relationship between "a" and "b" so that the function 'f' defined by:

[CBSE 2011]

$f(x)=\begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

4. If $f(x)=\begin{cases} \frac{\sin 3x}{x}, & \text{when } x \ne 0 \\ 1, & \text{when } x = 0 \end{cases}$. Find whether f(x) is continuous at x=0.

LEVEL-III

1.For what value of k, the function $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & x \ne 0 \\ k, & x = 0 \end{cases}$ is continuous at x=0?

2. If function $f(x)=\frac{2x+3\sin x}{3x+2\sin x}$, for $x \ne 0$ is continuous at x=0, then Find f(0).

3.Let $f(x)\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ a & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x>\dfrac{\pi}{2}\end{cases}$ = If f(x) be a continuous function at x= $\dfrac{\pi}{2}$, find a and b.

4.For what value of k,is the function f(x) = $\begin{cases}\dfrac{\sin x + x\cos x}{x} & \text{, when } x\neq 0\\ k & \text{, when } x=0\end{cases}$ continuous at x= 0?

3.Differentiation

LEVEL-I

1. Discuss the differentiability of the function $f(x)=(x-1)^{2/3}$ at x=1.

2.Differentiate $y=\tan^{-1}\dfrac{2x}{1-x^2}$.

3. If $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\dfrac{dy}{dx}$.

LEVEL-II

1. Find $\dfrac{dy}{dx}$, $y = \cos(\log x)^2$.

2. Find $\dfrac{dy}{dx}$ of $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$

3.If $y=e^{ax}\sin bx$, then prove that $\dfrac{d^2y}{dx^2}-2a\dfrac{dy}{dx}+(a^2+b^2)y=0$.

4.Find $\dfrac{d^2y}{dx^2}$, if $y=\dfrac{3at}{1+t}$, $x=\dfrac{2at^2}{1+t}$.

LEVEL-III

1.Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

2.Find $\dfrac{dy}{dx}$ $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0<x<\dfrac{\pi}{2}$.

3. If $y=\sin^{-1}\left(\dfrac{a+b\cos x}{b+a\cos x}\right)$, show that $\dfrac{dy}{dx}=\dfrac{-\sqrt{b^2-a^2}}{b+a\cos x}$.

4.Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{1-x^2}\right)\right]=\frac{1}{1+x^4}$.

4.Logrithmic Differentiation

LEVEL-I

1.Differentiate $y=\log_7(\log x)$.

2. Differentiate , sin(log x),with respect to x.

3.Differentiate $y=\tan^{-1}(\log x)$

LEVEL-II

1. If $y.\sqrt{x^2+1}=\log[\sqrt{x^2+1}-x]$,show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.
2. Find $\frac{dy}{dx}$, $y=\cos(\log x)^2$.
3. Find $\frac{dy}{dx}$ if $(\cos x)^y=(\cos y)^x$ **[CBSE 2012]**

LEVEL-III

1.If $x^p.y^q=(x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$

2. $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$

3. If $x^y=e^{x-y}$, Show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$ **[CBSE 2011]**

4. Find $\frac{dy}{dx}$ when $y=x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$ **[CBSE 2012]**

5 Parametric Differentiation

LEVEL-II

1.If $y=\tan x$, prove that $\frac{d^2y}{dx^2}=2y\frac{dy}{dx}$

2. .If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{d^2y}{dx^2}$at $\theta=\frac{\pi}{4}$.

3. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)=0$**[CBSE 2011]**

6. Second order derivatives

LEVEL-II

1. If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

2.If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$

3.If $(x-a)^2+(x-b)^2=c^2$ for some c>0.Prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant, independent

7. Mean Value Theorem

LEVEL-II

1.It is given that for the function $f(x)=x^3-6x^2+px+q$ on[1,3] , Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find the values p and q.

2. Verify Rolle's theorem for the function f(x) = sinx, in $[0,\pi]$.Find c, if verified

3.VeifyLagrange's mean Value Theorem $f(x)=\sqrt{x^2-4}$ in the interval [2,4]

Questions for self evaluation

1.For what value of k is the following function continuous at x = 2 ?

$$f(x)=\begin{cases}2x+1\,;\,x<2\\ k\,;\,x=2\\ 3x-1\,;\,x>2\end{cases}$$

2.If $f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11 & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$, continuous at x = 1, find the values of a and b.**[CBSE 2012 Comptt.]**

3. Discuss the continuity of $f(x)=|x-1|+|x-2|$ at x = 1 & x = 2.

4. If f(x), defined by the following is continuous at x = 0, find the values of a, b, c

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & ,\,x<0\\ c & ,\,x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & ,\,x>0\end{cases}$$

5.If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$.

6. If $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$.

7. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

8. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

9.If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

10. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

11.If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

12.If $x^p.y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

TOPIC 5
APPLICATIONS OF DERIVATIVES
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Application of Derivative.	1.Rate of change	*	Example 5 Ex 6.1 Q.No- 9,11
	2.Increasing & decreasing functions	***	Ex 6.2 Q.No- 6 Example 12,13
	3.Tangents & normals	**	Ex 6.3 Q.No- 5,8,13,15,23
	4.Approximations	*	Ex 6.4 QNo- 1,3
	5 Maxima & Minima	***	Ex 6.5Q.No- 8,22,23,25 Example 35,36,37,

SOME IMPORTANT RESULTS/CONCEPTS

** Whenever one quantity y varies with another quantity x, satisfying some rule y = f (x) , then $\frac{dy}{dx}$ (or f ′(x)) represents the rate of change of y with respect to x and $\left[\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x = x_0$.

** Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \le f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(iii) decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in$ I.

(iv) strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I.

** (i) f is strictly increasing in (a, b) if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is strictly decreasing in (a, b) if $f'(x) < 0$ for each $x \in (a, b)$

(iii) A function will be increasing (decreasing) in **R** if it is so in every interval of **R**.

** Slope of the tangent to the curve y = f (x) at the point (x_0, y_0) is given by $\left[\frac{dy}{dx}\right]_{(x_0, y_0)}$ $\left(= f'(x_0)\right)$.

** The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** Slope of the normal to the curve y = f (x) at (x_0, y_0) is $-\frac{1}{f'(x_0)}$.

** The equation of the normal at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

** If slope of the tangent line is zero, then tan θ = 0 and so θ = 0 which means the tangent line is parallel to the x-axis. In this case, the equation of the tangent at the point (x0, y0) is given by $y = y_0$.

** If $\theta \to \frac{\pi}{2}$, then tan $\theta \to \infty$, which means the tangent line is perpendicular to the x-axis, i.e., parallel to the y-axis. In this case, the equation of the tangent at (x_0, y_0) is given by $x = x_0$.

** Increment Δy in the function y = f(x) corresponding to increment Δ x in x is given by $\Delta y = \frac{dy}{dx}\Delta x$.

** Relative error in y = $\frac{\Delta y}{y}$.

** Percentage error in y = $\frac{\Delta y}{y} \times 100$.

** Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$.
The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) \leq f(x)$, for all $x \in I$.
The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

(c) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.
The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

* * Absolute maxima and minima

Let f be a function defined on the interval I and $c \in I$. Then

(a) f(c) is absolute minimum if f(x)□$\geq$ f(c) for all $x \in I$.

(b) f(c) is absolute maximum if $f(x) \leq f(c)$ for all $x \in I$.

(c) $c \in I$ is called the critical point off if $f'(c) = 0$

(d) Absolute maximum or minimum value of a continuous function f on [a, b] occurs at a or b or at critical points off (i.e. at the points where $f'$is zero)

If c_1 ,c_2, … , c_n are the critical points lying in [a , b], then

absolute maximum value of f = max{f(a), $f(c_1)$, $f(c_2)$, … , $f(c_n)$, f(b)}

and absolute minimum value of f = min{f(a), $f(c_1)$, $f(c_2)$, … , $f(c_n)$, f(b)}.

** Local maxima and minima

(a)A function f is said to have a local maxima or simply a maximum vaJue at x a if $f(a \pm h) \leq f(a)$ for sufficiently small h

(b)A function f is said to have a local minima or simply a minimum value at x = a if $f(a \pm h) \geq f(a)$.

** First derivative test : A function f has a maximum at a point x = a if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from + ve to –ve in the neighbourhood of ‘a’ (points taken from left to right).

However, f has a minimum at x = a, if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from –ve to +ve in the neighbourhood of ‘a’.

If $f'(a) = 0$ and f’(x) does not change sign, then f(x) has neither maximum nor minimum and the point ‘a’ is called point of inflation.

The points where $f'(x) = 0$ are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.

** Second derivative test

(i) a function has a maxima at x a if $f'(x)$ 0 and $f''(a) < 0$

(ii) a function has a minima at x = a if $f'(x) = 0$ and $f''(a) > 0$.

ASSIGNMENTS

1.Rate of change

LEVEL -I

1. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

2 .The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long ?

3. The radius of a circle is increasing at the rate of 0.7 cm/sec. what is the rate of increase of its circumference ?

LEVEL –II

1. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate?

2. A man 2 metre high walks at a uniform speed of 6km /h away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.

3. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm

LEVEL III

1. A particle moves along the curve $6y = x^3 + 2$., Find the points on the curve at which y-coordinate is changing 8 times as fast as the x-coordinate.

2. Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and altitude is 20 cm, Find the rate at which water level is dropping when it is 5 cm from top.

3. From a cylinder drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 ml/sec. If the radius of the drum is 10cm and height 50cm, find the rate at which the level of the petrol is changing when petrol level is 20 cm

2.Increasing & decreasing functions

LEVEL I

1. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$.

2. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0,1)

3. Find the intervals in which the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$ isincreasing or

decreasing.

LEVEL II

1. Indicate the interval in which the function $f(x) = \cos x$, $0 \le x \le 2\pi$ is decreasing.

2.Show that the function $f(x) = \frac{\sin x}{x}$ is strictly decreasing on $(0, \pi/2)$

3. Find the intervals in which the function $f(x) = \frac{\log x}{x}$ increasing or decreasing.

LEVEL III

1. Find the interval of monotonocity of the function $f(x) = 2x^2 - \log x$, $x \neq 0$

2. Prove that the function $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $[0, \pi/2]$

[CBSE 2011]

3.Tangents &Normals

LEVEL-I

1.Find the equations of the normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

2. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the x-coordinate of the point.

3. At what points on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x axis ?

LEVEL-II

1. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3)

2. For the curve $y = 2x^2 + 3x + 18$, find all the points at which the tangent passes through the origin.

3. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

4. Show that the equation of tangent at (x_1, y_1) to the parabola $yy_1 = 2a(x + x_1)$. **[CBSE 2012Comptt.]**

LEVEL- III

1 .Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 2$ which is parallel to the line $4x - 2y + 3 = 0$

2. Show that the curve $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0,0)

3. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.

4.Approximations

LEVEL-I

Q.1 Evaluate $\sqrt{25.3}$

Q.2 Use differentials to approximate the cube root of 66

Q.3 Evaluate $\sqrt{0.082}$

Q.4 Evaluate $\sqrt{49.5}$ **[CBSE 2012]**

LEVEL-II

1. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area

5 Maxima & Minima

LEVEL I

1. Find the maximum and minimum value of the function $f(x) = 3 – 2 \sin x$

2. Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value

3. Find two positive numbers whose sum is 24 and whose product is maximum

LEVEL II

1. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

2.A piece of wire 28(units) long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible.

3. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

LEVEL III

1 .Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one extremity of major axis.

2.An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.**[CBSE 2012 Comptt.]**

3.A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

[CBSE 2011]

Questions for self evaluation

1.Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

2. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

3. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = -2x^3 - 9x^2 - 12x + 1$

4. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$

5. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$.

7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

8. Using differentials, find the approximate value of each of the following up to 3places of decimal :

(i) $(26)^{\frac{1}{3}}$ (ii) $(32.15)^{\frac{1}{5}}$

9. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

10. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

TOPIC 6
INDEFINITE & DEFINITE INTEGRALS
SCHEMATIC DIAGRAM

Topics	Concept	Degree of Importance	References Text book of NCERT, Vol. II 2007 Edition
Indefinite Integrals	(i) Integration by substitution	*	Exp 5&6 Page301,303
	(ii)) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
	(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{1}{\sqrt{a^2 - x^2}} dx$, $\int \frac{dx}{ax^2 + bx + c}$, $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$, $\int \frac{(px+q)dx}{ax^2 + bx + c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	***	Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23
	(iv) Integration using Partial Fraction	***	Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
	(v) Integration by Parts	**	Exp 18,19&20 Page 325
	(vi)Some Special Integrals $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$	***	Exp 23 &24 Page 329
	(vii) Miscellaneous Questions	***	Solved Ex.41
Definite Integrals	(i) Definite Integrals based upon types of indefinite integrals	*	Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9
	(ii) Definite integrals as a limit of sum	**	Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
	(iii) Properties of definite Integrals	***	Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
	(iv) Integration of modulus function	**	Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

SOME IMPORTANT RESULTS/CONCEPTS

* $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
* $\int 1.dx = x + c$
* $\int \frac{1}{x^n} dx = -\frac{1}{x^n} + c$
* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$
* $\int \frac{1}{x} dx + c$
* $\int e^x dx = e^x + c$
* $\int a^x dx = \frac{a^x}{\log a} + c$
* $\int \sin x dx = -\cos x + c$
* $\int \sin x dx = -\cos x + c$
* $\int \cos x dx = \sin x + c$
* $\int \sec^2 x\, dx = \tan x + c$
* $\int \operatorname{cosec}^2 x\, dx = -\cot x + c$
* $\int \sec x . \tan x\, dx = \sec x + c$
* $\int \operatorname{cosec} x . \cot x\, dx = -\operatorname{cosec} x + c$
* $\int \tan x\, dx = -\log|\cos x| + c = \log|\sec x| + c$
* $\int \cot x\, dx = \log|\sin x| + C$
* $\int \sec x\, dx = \log|\sec x + \tan x| + C$

  $= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$
* $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C$

  $= -\log|\operatorname{cosec} x + \cot x| + C$

  $= \log\left|\tan\frac{x}{2}\right| + C$
* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$, if $x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$, if $x > a$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$, if $x > a$
* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + C$`
* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log|x + \sqrt{x^2 + a^2}| + C$
* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$
* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$
* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$
* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
* $\int \{f_1(x) \pm f_2(x) \pm \ldots\ldots\ldots f_n(x)\} dx$

  $= \int f_1(x) dx \pm \int f_2(x) dx \pm \ldots\ldots\ldots \pm \int f_n(x) dx$
* $\int \lambda f(x) dx = \lambda \int f(x) dx + C$
* $\int u.v\, dx = u.\int v.dx - \int \left[\int v.dx\right]\frac{du}{dx}.dx$
* $\int_a^b f(x)\, dx = F(b) - F(a)$, where $F(x) = \int f(x)\, dx$

* **General Properties of Definite Integrals.**

* $\int_a^b f(x)\, dx = \int_a^b f(t)\, dx$
* $\int_a^b f(x)\, dx = -\int_a^b f(x)\, dx$
* $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$
* $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$
* $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$
* $\int_{-a}^a f(x)\, dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of } x. \\ 0 & \text{if } f(x) \text{ is an odd function of } x \end{cases}$

* $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C`$	* $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$.

Assignments

(i) Integration by substitution

LEVEL I

1. $\int \frac{\sec^2(\log x)}{x}dx$ 2. $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx$ 3. $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}dx$

LEVEL II

1. $\int \frac{1}{\sqrt{x}+x}dx$ 2. $\int \frac{1}{x\sqrt{x^6-1}}dx$ 3. $\int \frac{1}{e^x-1}dx$

LEVEL III

1. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}dx$ 2. $\int \frac{\tan x}{\sec x+\cos x}dx$ 3. $\int \frac{1}{\sin x.\cos^3 x}dx$

(ii) Application of trigonometric function in integrals

LEVEL I

1. $\int \sin^3 x.dx$ 2. $\int \cos^2 3x.dx$ 3. $\int \cos x.\cos 2x.\cos 3x.dx$

LEVEL II

1. $\int \sec^4 x.\tan x.dx$ 2. $\int \frac{\sin 4x}{\sin x}dx$

LEVEL III

1. $\int \cos^5 x.dx$ 2. $\int \sin^2 x.\cos^3 x.dx$

(iii) Integration using standard results

LEVEL I

1. $\int \frac{dx}{\sqrt{4x^2-9}}$ 2. $\int \frac{1}{x^2+2x+10}dx$ 3. $\int \frac{dx}{9x^2+12x+13}$

LEVEL II

1. $\int \frac{x}{x^4+x^2+1}dx$ 2. $\int \frac{\cos x}{\sin^2 x+4\sin x+5}dx$ 3. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

LEVEL III

1. $\int \frac{2x}{\sqrt{1-x^2-x^4}} dx$
2. $\int \frac{x^2+x+1}{x^2-x+1} dx$
3. $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$
4. $\int \sqrt{\frac{1-x}{1+x}}\, dx$
5. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}$ **[CBSE 2011]**

(iv) Integration using Partial Fraction

LEVEL I

1. $\int \frac{2x+1}{(x+1)(x-1)} dx$
2. $\int \frac{x^2}{(x-1)(x-2)(x-3)} dx$
3. $\int \frac{3x-2}{(x+1)^2(x+3)} dx$

LEVEL II

1. $\int \frac{x^2+2x+8}{(x-1)(x-2)} dx$
2. $\int \frac{x^2+x+1}{x^2(x+2)} dx$
3. $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$

LEVEL III

1. $\int \frac{8}{(x+2)(x^2+4)} dx$
2. $\int \frac{dx}{\sin x + \sin 2x}$
3. $\int \frac{1}{1+x^3} dx$

(v) Integration by Parts

LEVEL I

1. $\int x.\sec^2 x.dx$
2. $\int \log x.dx$
3. $\int e^x (\tan x + \log \sec x) dx$

LEVEL II

1. $\int \sin^{-1} x.dx$
2. $\int x^2.\sin^{-1} x.dx$
3. $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}} dx$
4. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).dx$
5. $\int \sec^3 x.dx$

LEVEL III

1. $\int \cos(\log x) dx$
2. $\int \frac{e^x(1+x)}{(2+x)^2 dx}$
3. $\int \frac{\log x}{(1+\log x)^2} dx$
4. $\int \frac{2+\sin x}{1+\cos 2x} e^x.dx$
5. $\int e^{2x}.\cos 3x.dx$

(vi) Some Special Integrals

LEVEL I

1. $\int \sqrt{4+x^2}.dx$
2. $\int \sqrt{1-4x^2}.dx$

LEVEL II

1. $\int \sqrt{x^2+4x+6}.dx$
2. $\int \sqrt{1-4x-x^2}.dx$

LEVEL III

1. $\int (x+1)\sqrt{1-x-x^2}.dx$ 2. $\int (x-5)\sqrt{x^2+x}\ dx$

(vii) Miscellaneous Questions

LEVEL II

1. $\int \frac{1}{2-3\cos 2x}dx$ 2. $\int \frac{1}{3+\sin 2x}dx$ 3. $\int \frac{dx}{4\sin^2 x+5\cos^2 x}$

4. $\int \frac{dx}{1+3\sin^2 x+8\cos^2 x}$ 5. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ 6. $\int \frac{\sec x}{5\sec x+4\tan x}dx$

LEVEL III

1. $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$ 2. $\int \frac{dx}{1-\tan x}$ 3. $\int \frac{x^4}{x^4-1}dx$

4. $\int \frac{x^2+1}{x^4+x^2+1}dx$ 5. $\int \frac{x^2-1}{x^4+1}dx$ 6. $\int \sqrt{\tan x}.dx$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\int_0^1 \frac{2x+3}{5x^2+1}$ 2. $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$ 3. $\int_0^2 x\sqrt{x+2}\,dx$

LEVEL II

1. $\int_1^2 \frac{5x^2}{x^2+4x+3}dx$ 2. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}\ dx$

(ii) Definite integrals as a limit of sum

LEVEL I

1. Evaluate $\int_0^2 (x+2)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^4 (1+x)\,dx$ definite integral as the limit of a sum.

LEVEL II

1. Evaluate $\int_1^2 (3x^2 - 1)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2 + 1)\,dx$ as the limit of a sum.

LEVEL III

1. Evaluate $\int_1^2 (x^2 + x + 2)dx$ as the limit of a sum.

2. Evaluate $\int_2^4 \left(e^{2x} + x^2\right) dx$ as the limit of a sum.

(iii) Properties of definite Integrals

LEVEL I

1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$

2. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}dx$

3. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$

LEVEL II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$

2. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx$

3. $\int_0^{\pi} \frac{x \tan x}{\sec x . \operatorname{cosec} x}dx$

4. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **[CBSE 2011]**

LEVEL III

1. $\int_0^{\pi} \frac{x + \sin x}{1+\cos x}dx$ **[CBSE 2011]**

2. $\int_0^{\pi/2} \log \sin x \; dx$

3. $\int_0^{\pi/4} \log(1+\tan x)dx$

[CBSE 2011]

(iv) Integration of modulus function

LEVEL III

1. $\int_2^5 (|x-2| + |x-3| + |x-4|)dx$

2. $\int_{-1}^2 |x^3 - x|dx$

3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|]dx$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2 - 3x - 18}$

2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x.dx$

4. Evaluate $\int \frac{dx}{3+2\sin x+\cos x}$

5. Evaluate $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$

6. Evaluate $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}} dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$

8. Evaluate $\int_{-1}^{3/2} |x\sin \pi x| dx$

9. Evaluate $\int_0^{\pi/2} \log\sin x\, dx$

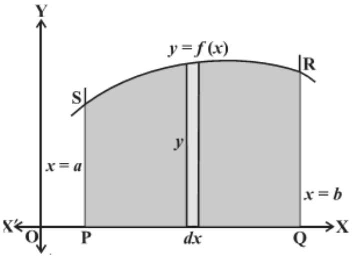
10. Evaluate $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

TOPIC 7
APPLICATIONS OF INTEGRATION
SCHEMATIC DIAGRAM

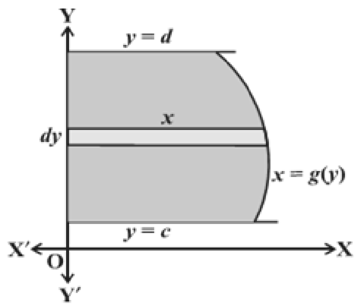
Topic	Concepts	Degree of Importance	Reference NCERT Text BookEdition 2007
Applications of Integration	(i)Area under ***Simple Curves***	*	Ex.8.1 Q.1,2,5
	(ii) Area of the region enclosed between ***Parabola and line***	***	Ex. 8.1 Q 10,11 Misc.Ex.Q 7
	(iii) Area of the region enclosed between ***Ellipse and line***	***	Example 8, page 369 Misc.Ex. 8
	(iv) Area of the region enclosed between***Circle and line***	***	Ex. 8.1 Q 6
	(v) Area of the region enclosed between ***Circle and parabola***	***	Ex 8.2 Q1, Misc.Ex.Q 15
	(vi) Area of the region enclosed between ***Two Circles***	***	Example 10, page370 Ex 8.2 Q2
	(vii) Area of the region enclosed between ***Two parabolas***	***	Example 6, page368
	(viii) Area of triangle ***when vertices are given***	***	Example 9, page370 Ex 8.2 Q4
	(ix) Area of triangle ***when sides are given***	***	Ex 8.2 Q5 ,Misc.Ex. Q 14
	(x) ***Miscellaneous Questions***	***	Example 10, page374 Misc.Ex.Q 4, 12

SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y\,dx = \int_a^b f(x)\,dx$.



** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by A= $\int_c^d x\,dy = \int_c^d g(y)\,dy$



ASSIGNMENTS

(i) Area under ***Simple Curves***

LEVEL I

1. Sketch the region of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and find its area, using integration,

.

2. Sketch the region {(x, y) : $4x^2 + 9y^2 = 36$} and find its area, using integration.

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II

1. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.
2. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

LEVEL III

1. Find the area of the region : $\{(x, y): y \le x^2 + 1,\ y \le x + 1, 0 \le x \le 2\}$

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II

1. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ and the straight line $\frac{x}{4}+\frac{y}{5}=1$.

(iv) Area of the region enclosed between ***Circle and line***

LEVEL II

1. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

LEVEL III

1. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$

(v) Area of the region enclosed between ***Circle and parabola***

LEVEL III

1. Draw the rough sketch of the region {(x, y): $x^2 \le 6y$, $x^2 + y^2 \le 16$} an find the area enclosed by the region using the method of integration.
2. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III

1. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II

1. Draw the rough sketch and find the area of the region bounded by two parabolas

$4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

(viii) Area of triangle ***when vertices are given***

LEVEL III

1. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).

2. Using integration compute the area of the region bounded by the triangle whose vertices are (–1 , 1), (0 , 5), and (3 , 2).

(ix) Area of triangle ***when sides are given***

LEVEL III

1. Using integration find the area of the region bounded by the triangle whose sides are $y = 2x + 1$, $y = 3x + 1$, $x = 4$.

2. Using integration compute the area of the region bounded by the linesx + 2y = 2, $y - x = 1$, and $2x + y = 7$.

(x) Miscellaneous Questions

LEVEL III

1. Find the area of the region bounded by the curves $y = |x-1|$ and $y = -|x-1| + 1$.

2. Find the area bounded by the curve y = x and $y = x^3$.

3. Draw a rough sketch of the curve y = sinx and y = cosx as x varies from x = 0 to $x = \frac{\pi}{2}$ and find the area of the region enclosed by them and x-axis

4. Sketch the graph of $y = |x+1|$.Evaluate $\int_{-3}^{1} |x+1|\, dx$ .What does this value represent on the graph.

5. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

6. Sketch the graph of $y = |x+3|$ and evaluate the area under the curve $y = |x+3|$ above x-axis and between $x = -6$ to $x = 0$. **[CBSE 2011]**

Questions for self evaluation

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

2. Find the area bounded by the parabola $y = x^2$ and $y = |x|$.

3. Find the area of the region : $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$

4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

5. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1, \le x + y\}$

6. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

7. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

8. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).

9. Using integration compute the area of the region bounded by the lines $2x + y = 4$,
$3x - 2y = 6$, and $x - 3y + 5 = 0$.

10. Sketch the graph of : $f(x) = \begin{cases} |x-2|+2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$.

Evaluate $\int_0^4 f(x)dx$. What does the value of this integral represent on the graph ?

TOPIC 8
DIFFERENTIAL EQUATIONS
SCHEMATIC DIAGRAM

		(ii).General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		(iii).Formation of differential equation whose general solution is given	*	Q. 7,8,10 pg 391
		(iv).Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		(vi).Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406
		(vii)Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414

SOME IMPORTANT RESULTS/CONCEPTS

** Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.

** Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives are of the given differential equation is called the degree of the differential equatin

** Homogeneous Differential Equation : $\frac{dy}{dx} = \frac{f_1(x, y)}{f_2(x, y)}$, where $f_1(x, y)$ & $f_2(x, y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

i. $\frac{dy}{dx} + py = q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx} = \int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

ii. $\frac{dx}{dy} + px = q$, where p & q be the function of y or constant.

Solution of the equation is : $x.e^{\int p\,dy} = \int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

ASSIGNMENTS

1. *Order and degree of a differential equation*

LEVEL I

1. Write the order and degree of the following differential equations

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

2. *General and particular solutions of a differential equation*

LEVEL I

1. Show that $y = e^{-x} + ax + b$ is the solution of $e^x \frac{d^2y}{dx^2} = 1$

3. *Formation of differential equation*

LEVEL II

1. Obtain the differential equation by eliminating a and b from the equation y = e^x(acosx + bsinx)

LEVEL III

1. Find the differential equation of the family of circles $(x - a)^2 - (y - b)^2 = r^2$

2. Obtain the differential equation representing the family of parabola having vertex at the origin and axis along the positive direction of x-axis

4. *Solution of differential equation by the method of separation of variables*

LEVEL II

1. Solve $\frac{dy}{dx} = 1 + x + y + xy$

2. Solve $\frac{dy}{dx} = e^{-y}\cos x$ given that y(0)=0.

3. Solve $\left(1 + x^2\right)\frac{dy}{dx} - x = \tan^{-1} x$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II

1. Solve $(x^2 + xy)dy = (x^2 + y^2)dx$

LEVEL III

Show that the given differential equation is homogenous and solve it.

1. $(x - y)\frac{dy}{dx} = x + 2y$

2. $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$

3.Solve $x dy - y dx = \sqrt{x^2 - y^2}\, dx$ 4.Solve $x^2 y dx - (x^3 + y^3) dy = 0$

5.Solve $xdy - ydx = \sqrt{(x^2 + y^2)}dx$ **CBSE2011** 6.Solve $\left(y + 3x^2\right)\frac{dx}{dy} = x$

7. Solve $x\ dy + (y - x^3)dx = 0$ **CBSE2011** 8.Solve $x\ dy + (y + 2x^2)dx = 0$

6. ***Linear Differential Equations***

LEVEL I

1.Find the integrating factor of the differential $x\frac{dy}{dx} - y = 2x^2$

LEVEL II

1.Solve $\frac{dy}{dx} + 2y\tan x = \sin x$ 2. Solve $(1 + x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

3. Solve $x\frac{dy}{dx} + y = x\log x$

LEVEL III

1. Solve $\frac{dy}{dx} = \cos(x + y)$ 2.Solve $ye^y dx = (y^3 + 2xe^y)dy$

3. Solve $x^2\frac{dy}{dx} = y(x + y)$ 4. Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = -\frac{1}{(x^2+1)^3}$

5. Solve the differential equation $\left(x + 2y^2\right)\frac{dy}{dx} = y$;given that when x=2,y=1

Questions for self evaluation

1. Write the order and degree of the differential equation $\left(\frac{d^3y}{dy^3}\right)^2 + \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$
2. Form the differential equation representing the family of ellipses having foci on x –axis and centre at origin .
3. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that y = 0 when x = 0.
4. Solve the differential equation :$xdy - y\,dx = \sqrt{x^2 + y^2}\,dx$
5. Solve the differential equation : $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$.
6. Solve the differential equation : $x^2 dy + (y^2 + xy)\, dx = 0$, $y(1) = 1$

7. Show that the differential equation $2y.e^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ is homogeneous andfind its particular solution given that y(0) = 1 .

8. Find the particular solution of differential equation

$$\frac{dy}{dx}+y\cot x=2x+x^{2}\cot x,\ \text{given that}\ y\left(\frac{\pi}{2}\right)=0\ .$$

TOPIC 9
VECTOR ALGEBRA
SCHEMATIC DIAGRAM

Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
Vector algebra	(i)Vector and scalars	*	Q2 pg428
	(ii)Direction ratio and direction cosines	*	Q 12,13 pg 440
	(iii)Unit vector	* *	Ex 6,8 Pg 436
	(iv)Position vector of a point and collinear vectors	* *	Q 15 Pg 440 , Q 11Pg440 , Q 16 Pg448
	(v)Dot product of two vectors	**	Q6 ,13 Pg445
	(vi)Projection of a vector	* * *	Ex 16 Pg 445
	(vii)Cross product of two vectors	* *	Q 12 Pg458
	(viii)Area of a triangle	*	Q 9 Pg 454
	(ix)Area of a parallelogram	*	Q 10 Pg 455

SOME IMPORTANT RESULTS/CONCEPTS

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

*Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

* Scalar Product (dot product) between two vectors: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$; θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}.\vec{b} = 0$

* $\vec{a}.\vec{a} = |\vec{a}|^2$

* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

* Vector product between two vectors :

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\ \hat{n}$; $\hat{n}$ is the normal unit vector which is perpendicular to both $\vec{a}$ & $\vec{b}$

* $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}\times\vec{b} = 0$

* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \frac{1}{2}|\vec{a}\times\vec{b}|$

* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= |\vec{a}\times\vec{b}|$

* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \frac{1}{2}|\vec{a}\times\vec{b}|$

ASSIGNMENTS

(i) Vector and scalars, Direction ratio and direction cosines&Unit vector

LEVEL I

1. If $\vec{a} = \hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 3\hat{k}$ find a unit vector parallel to $\vec{a} + \vec{b}$
2. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
3. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$; $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$ find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$
4. Find a unit vector in the direction of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ **[CBSE 2011]**
5. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$, whose magnitude is 7

LEVEL II

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a} + \vec{b})$,$(\vec{a} - \vec{b})$ where

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

2. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

LEVEL – III

1. If a line make α,β, γ with the X - axis , Y– axis and Z – axis respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
2. For what value of p, is $(\hat{i} + \hat{j} + \hat{k})\,p$ a unit vector?
3. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis
4. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

(ii)Position vector of a point and collinear vectors

LEVEL – I

1. Find the position vector of the midpoint of the line segment joining the points A($5\hat{i} + 3\hat{j}$) and B($3\hat{i} - \hat{j}$).

2. In a triangle ABC, the sides AB and BC are represents by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

3. Show that the points (1,0), (6,0) ,(0,0) are collinear.

LEVEL – II

1.Write the position vector of a point R which divides the line joining the points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2 : 1 externally.

2.Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ

(iii) Dot product of two vectors

LEVEL – I

1.Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$.

2.If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

3.Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$ **[CBSE 2011]**

LEVEL – II

1. The dot products of a vector with the vectors $\hat{i} - 3\hat{j}$, $\hat{i} - 2\hat{j}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vectors.

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$.

3. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ.

LEVEL – III

1. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$.

2. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

3. For what values of λ, vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} = \lambda\hat{i} - 4\hat{j} + 8\hat{k}$ are
   (i) Orthogonal (ii) Parallel `

4..Find$|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})=15$.

5. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \mu\hat{k}$, find μ, such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

6. Show that the vector $2\hat{i} - \hat{j} + \hat{k}$, $-3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form sides of a right angled triangle.

7.Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

8. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

9. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 4$, $\left|\vec{c}\right| = 5$ and each of them being perpendicular

to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

(iv) Projection of a vector

LEVEL – I

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\,.\,\vec{b}$ = 8 and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ **[CBSE 2011]**

3.Find the angle between the vectors $\hat{i}$ $-2\hat{j} + 3\hat{k}$ and $3\hat{i} -2\hat{j} + \hat{k}$

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

LEVEL – II

1.Three vertices of a triangle are A(0, -1, -2), B(3,1,4) and C(5,7,1). Show that it is a right angled triangle. Also find the other two angles.

2.Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

3.If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non - zero and non – coplanar vectors, prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are also coplanar

LEVEL – III

1.If a unit vector $\vec{a}$ makes angles $\pi/4$, with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$,and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

4.Show that the points A, B, C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

5. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that

$$\text{(i) } \sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}| \quad \text{(ii) } \tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$$

(vii)Cross product of two vectors

LEVEL – I

1. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$. Find $|\vec{a} \times \vec{b}|$

2. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

3. Find $|\vec{x}|$, if $\vec{p}$ is a unit vector and , $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$.

4. Find p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

LEVEL – II

1. Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

2. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.

4. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{a} \times \vec{b})$.

LEVEL – III

1. Find the value of the following: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{i}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$

2. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $(\vec{a} - \vec{d})$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

5. Express $2\hat{i}-\hat{j}+3\hat{k}$ as the sum of a vector parellal and perpendicular to $2\hat{i}+4\hat{j}-2\hat{k}$.

(viii)Area of a triangle & Area of a parallelogram

LEVEL – I

1.Find the area of Parallelogram whose adjacent sides are represented by the vectors

$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

2.If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a Parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

3. Find the area of triangle having the points A(1,1,1) , B(1,2,3) and C(2,3,1) as its vertices.

LEVEL – II

1.Show that the area of the Parallelogram having diagonals ($3\hat{i} + \hat{j} - 2\hat{k}$) and ($\hat{i} - 3\hat{j} + 4\hat{k}$) is $5\sqrt{3}$ Sq units.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a ΔABC, show that the area of the ΔABC is

$$\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|.$$

3.Using Vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5)

[CBSE 2011]

Questions for self evaluation

1.The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

3. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. Dot product of a vector with $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$, and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, 8 respectively. Find the vector.

5. Find the components of a vector which is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

TOPIC 10
THREE DIMENSIONAL GEOMETRY
SCHEMATIC DIAGRAM

Topic	**Concept**	**Degree of importance**	**Refrence** NCERT Text Book Edition 2007
Three Dimensional Geometry	(i) Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
	(ii)Cartesian and Vector equation of a line in space & conversion of one into another form	**	Ex No 8 Pg -470 Q N. 6, 7, - Pg 477 QN 9 – Pg 478
	(iii)Co-planer and skew lines	*	Ex No 29 Pg -496
	(iv) Shortest distance between two lines	***	Ex No 12 Pg -476 Q N. 16, 17 - Pg 478
	(v) Cartesian and Vector equation of a plane in space & conversion of one into another form	**	Ex No 17 Pg -482 Ex No 18 Pg – 484 Ex No 19 Pg – 485 Ex No 27 Pg – 495 Q N. 19, 20 - Pg 499
	(vi) Angle Between (iv) Two lines (v) Two planes (vi) Line & plane	 * * **	Ex No 9 Pg -472 Q N. 11 - Pg 478 Ex No 26 Pg – 494 Q N. 12 - Pg 494 Ex No 25 Pg - 492
	(vii) Distance of a point from a plane	**	Q No 18 Pg -499 Q No 14 Pg – 494
	(viii)Distance measures parallel to plane and parallel to line	**	
	(ix)Equation of a plane through the intersection of two planes	***	Q No 10 Pg -493
	(x) Foot of perpendicular and image with respect to a line and plane	**	Ex. N 16 Pg 481

SOME IMPORTANT RESULTS/CONCEPTS

** Direction cosines and direction ratios :

If a line makes angles α, β and γ with x, y and z axes respectively the $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines denoted by l, m, n respectively and $l^2 + m^2 + n^2 = 1$

Any three numbers proportional to direction cosines are direction ratios denoted by a, b, c

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c} \qquad l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}},$$

* Direction ratios of a line segment joining $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ may be taken as $x_2-x_1,\ y_2-y_1,\ z_2-z_1$
* Angle between two lines whose direction cosines are l_1,m_1,n_1 and l_2,m_2,n_2 is given by

$$\cos\theta=l_1l_2+m_1m_2+n_1n_2=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\left(a_1^2+b_1^2+c_1^2\right)\left(a_2^2+b_2^2+c_2^2\right)}}$$

* For parallel lines $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and

  for perpendicular lines $a_1a_2+b_1b_2+c_1c_2=0$ or $l_1l_2+m_1m_2+n_1n_2=0$

** STRAIGHTLINE:

* Equation of line passing through a point (x_1,y_1,z_1) with direction cosines a, b, c: $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$
* Equation of line passing through a point (x_1,y_1,z_1) and parallel to the line: $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

* Equation of line passing through two point (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$
* Equation of line (Vector form)

  Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$
* Equation of line passing through two points $\vec{a}$ & $\vec{b}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$
* Shortest distance between two skew lines : if lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$

then Shortest distance $=\frac{\left(\vec{a_2}-\vec{a_1}\right)\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}$; $\vec{b_1}\times\vec{b_2}\neq 0$

$$\frac{\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b_1}\right|}{\left|\vec{b_1}\right|} \quad ;\ \vec{b_1}\times\vec{b_2}=0$$

** PLANE:

* Equation of plane is $ax+by+cz+d=0$ where a, b & c are direction ratios of normal to the plane
* Equation of plane passing through a point (x_1,y_1,z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
* Equation of plane in intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where a, b, c are intercepts on the axes
* Equation of plane in normal form $lx+my+nz=p$ where l, m, n are direction cosines of normal to the plane p is length of perpendicular form origin to the plane

* Equation of plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and $(x_3, y_3, z\ )$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

* Equation of plane passing through two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and perpendicular to the plane

$a_1x + b_1y + c_1z + d_1 = 0$ or parralal to the line $\frac{x-\alpha_1}{a_1} = \frac{y-\beta_1}{b_1} = \frac{z-\gamma_1}{c_1}$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$

* Equation of plane passing through the point (x_1, y_1, z_1) and perpendicular to the

planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z\ + d_2 = 0$ or parralal to the lines $\frac{x-\alpha_1}{a_1} = \frac{y-\beta_1}{b_1} = \frac{z-\gamma_1}{c_1}$

and $\frac{x-\alpha_2}{a_2} = \frac{y-\beta_2}{b_2} = \frac{z-\gamma_2}{c_2}$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane contaning the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and passing through the point (x_2, y_2, z_2) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

* Condition for coplaner lines : $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplaner if

$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ and equation of common plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane passing through the intersection of two planes $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$ is $(a_1x + b_1y + c_1z) + \lambda(a_2x + b_2y + c_2z) = 0$

* Perpendicular distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

* Distance between two parallel planes $ax + by + cz + d_1 = 0$, $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

ASSIGNMENTS

(i)Direction Ratios and Direction Cosines

LEVEL-I

1. Write the direction-cosines of the line joining the points (1,0,0) and (0,1,1) **[CBSE 2011]**

2.Find the direction cosines of the line passing through the following points (-2,4,-5), (1,2,3).

3.Write the direction cosines of a line equally inclined to the three coordinate axes

LEVEL-II

1.Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$.

2.Write the direction ratios of a line parallel to the line $\frac{5-x}{3} = \frac{y+7}{-2} = \frac{z+2}{6}$.

3. If the equation of a line AB $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+6}{3}$ Find the direction cosine.

4. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axis.

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL-I

1.Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$. **[CBSE 2011]**

2. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3).

3.Express the equation of the plane $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ in the Cartesian form.

4.Express the equation of the plane $\vec{r} \cdot (2\hat{\imath} - 3\hat{\jmath} + \hat{k}) + 4 = 0$ in the Cartesian form.

(iii) Co-planer and skew lines

LEVEL-II

1.Find whether the lines $\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ intersect or not. If intersecting , find their point of intersection.

2.Show that the four points (0,-1,-1) , (4,5,1) , (3,9,4) and (-4,4,4,) are coplanar. Also, find the equation of the plane containing them.

3.Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

LEVEL-III

1. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

2. The points A(4,5,10) , B(2,3,4) and C(1,2,-1) are three vertices of a parallelogram ABCD. Find

the vector equation of the sides AB and BC and also find the coordinates

3.Find the equations of the line which intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and passes through the point (1,1,1).

4. Show that The four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

(iv) Shortest distance between two lines

LEVEL-II

1. Find the shortest distance between the lines l_1 and l_2 given by the following:

(a) $l_1 : \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{1}$ $\qquad l_2 : \frac{x-2}{2}=\frac{y+1}{1}=\frac{z+1}{2}$

(b) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$

$\vec{r}=(4\hat{\imath} + 2\mu)\hat{\imath} + (5 + 3\mu)\hat{\jmath} + (6 + \mu)\hat{k}$.

2. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Find their point of intersection.

3.. Find the shortest distance between the lines

$\vec{r} = (\hat{i}+\hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

4.Find the shortest distance between the lines

$\vec{r} = (1 - t)\hat{i} + (t-2)\hat{\jmath} + (3-t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} + (2s+1)\hat{k}$ [**CBSE 2011]**

5. Find the distance between the parallel planes x + y – z = -4 and 2x + 2y – 2z + 10 = 0.

6. Find the vector equation of the line parallel to the line $\frac{x-1}{5}=\frac{3-y}{2}=\frac{z+1}{4}$ and passing through (3,0,-4). Also, find the distance between these two lines.

(v) Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1.Find the equation of a plane passing through the origin and perpendicular to x-axis

2.Find the equation of plane with intercepts 2, 3, 4 on the x ,y, z –axis respectively.

3.Find the direction cosines of the unit vector perpendicular to the plane

$\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$ passing through the origin.

4.Find the Cartesian equation of the following planes:

(a) $r \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ $\qquad$ (b) $r \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

LEVEL II

1. Find the vector and cartesian equations of the plane which passes through the point (5, 2, – 4) and perpendicular to the line with direction ratios 2, 3, – 1.
2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector 3 iˆ + 5 ˆj - 6 kˆ.
3.Find the vector and cartesian equations of the planes that passes through the point (1, 0, – 2)and the normal to the plane is iˆ + ˆj - kˆ.

(vi) Angle Between(i)Two lines (ii)Two planes (iii)Line & plane

LEVEL-I

1. Find the angle between the lines whose direction ratios are (1, 1, 2) and ($\sqrt{3}$–1 , –$\sqrt{3}$ –1 , 4).

2.Find the angle between line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0.

3.Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane 3x – y – 2z = 7.

4.Find the angle between the planes whose vector equations are
r· (2 iˆ + 2 ˆj - 3 kˆ) = 5 and r · (3 iˆ - 3 ˆj + 5 kˆ) = 3

5.Find the angle between the line$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$and the plane 10 x + 2y – 11 z = 3.

LEVEL-II

1.Find the value of p, such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2p}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.
2. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

(vii) Distance of a point from a plane

LEVELI

1.Write the distance of plane 2x– y + 2z + 1 = 0 from the origins.
2.Find the point through which the line 2x = 3y = 4z passes.
3. Find the distance of a point $(2, 5, -3)$from the plane $r \cdot (6\,\hat{i} - 3\,\hat{j} + 2\,\hat{k}) = 4$
4. Find the distance of the following plane from origin: 2x – y + 2z + 1 = 0

5.Find the distance of the point (a,b,c) from x-axis

LEVELII

1..Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

2.Find the distance of the point (3,4,5) from the plane x + y + z =2 measured parallel to the line 2x = y = z.

3. Find the distance between the point P(6, 5, 9) and the plane determinedby the points A (3, – 1, 2), B (5, 2, 4) and C(– 1, – 1, 6).

4.Find the distance of the point $(-1, -5, -10)$from the point of intersection of theline

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right) \text{ and the plane } \vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$$

[**CBSE2011**]

LEVEL III

1.Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane 2x – y + z + 3 = 0. Find also, the image of the point in the plane.

2.Find the distance of the point P(6,5,9) from the plane determined by the points A(3,-1,2) , B(5,2,4) and C(-1,-1,6).

3.Find the equation of the plane containing the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \mu(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$.Find the distance of this plane from origin and also from the point (1,1,1).

(viii) Equation of a plane through the intersection of two planes

LEVELII

1.Find the equation of plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the perpendicular distance of the plane from the origin.

2.Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z +8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

3.Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of the planes 2x + 3y -2 z = 5 and x + 2y – 3z = 8.

LEVEL-III

1.Find the equation of the plane passing through the point (1,1,1) and containing the line $\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} - 5\hat{k})$. Also, show that the plane contains the line $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

2.Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

3.Find the Cartesian equation of the plane passing through the points A(0,0,0) and B(3,-1,2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ `

4. Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

(ix)Foot of perpendicular and image with respect to a line and plane

LEVEL II

1. Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A(1,2,3) , B(2,2,1) and C(-1,3,6).

2. Find the foot of the perpendicular from P(1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point (1,2,3).

3.Prove that the image of the point (3,-2,1) in the plane 3x – y + 4z =2 lies on the plane, x + y + z + 4 = 0.

LEVEL-III

1.Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).

2. Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.

3. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane

4. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane 2x – y+z+1=0. Find also, the image of the point in the plane.

Questions for self evaluation

1. Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

2. Find the vector equation of a line joining the points with position vectors $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$, and $2\hat{i} + \hat{j} + 2\hat{k}$. Also find the cartesian equivalent of this equation.

3. Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).

4. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

5.Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.

6. Show that the four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

7. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane.

8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

9. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

TOPIC 11
LINEAR PROGRAMMING
<u>SCHEMATIC DIAGRAM</u>

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Linear Programming	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
	(ii) Graphical method of solving LPP (bounded and unbounded solutions)	**	**Article 12.2.2** Solved Ex. 1 to 5 Q. Nos 5 to 8 Ex.12.1
	(iii) Diet Problem	***	Q. Nos 1, 2 and 9 Ex. 12.2 Solved Ex. 9 Q. Nos 2 and 3 Misc. Ex. Solved Ex. 8 Q. Nos 3,4,5,6,7 of Ex.
	(iv) Manufacturing Problem	***	12.2 Solved Ex.10 Q. Nos 4 & 10 Misc. Ex.
	(v) Allocation Problem	**	Solved Example 7Q. No 10 Ex.12.2, Q. No 5 & 8 Misc. Ex.
	(vi) Transportation Problem	*	Solved Ex.11 Q. Nos 6 & 7 Misc. Ex.
	(vii) Miscellaneous Problems	**	Q. No 8 Ex. 12.2

<u>SOME IMPORTANT RESULTS/CONCEPTS</u>

** Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = $ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is **bounded**, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is **unbounded**, we have:
4. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

ASSIGNMENTS

(i) LPP and its Mathematical Formulation

LEVEL I

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem.

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

LEVEL I

Solve the following Linear Programming Problems graphically:

1.Minimise $Z = -3x + 4y$ subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

2.Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

3.Minimise $Z = 3x + 5y$ suchthat $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

(iii) Diet Problem

LEVEL II

1.A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost? Also find the least cost.

2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost ?

(iv) Manufacturing Problem

LEVEL II

1.A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

2. A company sells two different produces A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs. 20 and that on B is Rs. 15. How many units of A and B should be produced to maximize the profit? Solve it graphically

LEVEL III

1.A manufacture makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below:

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

(v) Allocation Problem

LEVEL II

1. Ramesh wants to invest at most Rs. 70,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10,000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8 % per annum and the rate of interest on bond B is 10 % per annum , how much money should he invest to earn maximum yearly income ? Find also his maximum yearly income.

2. An oil company requires 12,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

LEVEL III

1. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

(vi) Transportation Problem

LEVEL III

1. A medicine company has factories at two places A and B . From these places, supply is to be made to each of its three agencies P, Q and R. The monthly requirement of these agencies are respectively 40, 40 and 50 packets of the medicines, While the production capacity of the factories at A and B are 60 and 70 packets are respectively. The transportation cost per packet from these factories to the agencies are given:

Transportation cost per packet (in Rs.)		
From / To	**A**	**B**
P	**5**	**4**
Q	**4**	**2**
R	**3**	**5**

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

Questions for self evaluation

1. Solve the following linear programming problem graphically : Maximize $z = x – 7y +190$ subject to the constraints $x + y \leq 8$, $x \leq 5$, $y \leq 5$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following linear programming problem graphically : Maximize $z = 3x + 5y$ subject to the constraints $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.

3. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains, 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kilogram and rice costs Rs. 4 per kilogram.

4. A shopkeeper deals only in two items — tables and chairs. He has Rs. 6,000 to invest and a space to store at most 20 pieces. A table costs him Rs. 400 and a chair Rs. 250. He can sell a table at a profit of Rs. 25 and a chair at a profit of Rs. 40. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

5. A small firm manufactures items A and B. The total number of items A and B it can manufacture a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit ? Solve the problem graphically.

6. A chemist requires 10, 12 and 12 units of chemicals A, B and C respectively for his analysis. A liquid product contains 5, 2, and 1 units of A, B and C respectively and it costs Rs. 3 per jar. A dry product contains 1, 2, and 4 units of A, B and C per carton and costs Rs. 2 per carton. How many of each should he purchase in order to minimize the cost and meet the requirement ?

7. A person wants to invest at most Rs. 18,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 4,000 in Bond A and at least Rs. 5,000 in Bond B. If the rate of interest on bond A is 9 % per annum and the rate of interest on bond B is 11 % per annum , how much money should he invest to earn maximum yearly income ?

8. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labourcost.

TOPIC 12
PROBABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Probability	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1
	(ii)Multiplication theorem on probability	**	**Article 13.3** SolvedExamples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2
	(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
	(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
	(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
	(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

SOME IMPORTANT RESULTS/CONCEPTS

** **Sample Space and Events** :

The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.

ϕ ($\subset$ S) is called an impossible event and

S($\subset$ S) is called a sure event.

** **Probability of an Event.**

(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is

defined as $P(E) \frac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$

it being assumed that the outcomes of the experiment in reference are equally likely.

(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = P(ϕ) = 0.

(iii) If $E_1, E_2, E_3, \ldots, E_k$ are mutually exclusive and exhaustive events associated with an experiment (i.e. if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \ldots, k\}$ $i \neq j$), then

$$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_k) = 1.$$

(iv) $P(E) + P(E^C) = 1$

** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \text{ provided } P(F) \neq 0$$

** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$

$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

** Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \cdot P(F)$.

** Bayes' Theorem : If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ andA is any event of nonzero probability, then

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \ldots, n$$

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^nC_x\, q^{n-x} p^x$, $x = 0, 1, \ldots, n$. $(q = 1 - p)$

ASSIGNMENTS

(i) Conditional Probability

LEVEL I

1. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events.
2. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced.

LEVEL II

1.A dice is thrown twice and sum of numbers appearing is observed to be 6. what is the conditional probability that the number 4 has appeared at least once.

LEVEL III

1.If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{2}$, find $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$

(ii)Multiplication theorem on probability

LEVEL II

1.A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red.

2. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

LEVEL III

1.A class consists of 80 students; 25 of them are girls and 55 are boys, 10 of them are rich and the remaining poor; 20 of them are fair complexioned. what is the probability of selecting a fair complexioned rich girl.

2.Two integers are selected from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd.

(iii) Independent Events

LEVEL I

1. A coin is tossed thrice and all 8 outcomes are equally likely.

E : "The first throw results in head" F : "The last throw results in tail"

Are the events independent ?

2. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent ?

3. If A and B are independent events, Find P(B) if $P(A \cup B) = 0.60$ and $P(A) = 0.35$.

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I

1. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag . Find the probability that the ball drawn is blue in colour.

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart.

3. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle.

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin.

5. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

LEVEL II

1. Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one-rupee coins and other contains 2 fifty-paise coins and 3 one-rupee coins.

2. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.

3. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 % . Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

LEVEL III

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from
(i) LONDON (ii) CLIFTON ?

2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease.

3. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III , there is one gold and one silver coin.
A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? **[CBSE 2011]**

(v) Random variables & probability distribution Mean & variance of random variables

LEVEL I

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades

2. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples.

3. A random variable X is specified by the following distribution

X	2	3	4
P(X)	0.3	0.4	0.3

Find the variance of the distribution.

LEVEL III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice.Find the probability distribution of the number of tails.

2.The sum of mean and variance of a binomial distribution for 5 trials be 1.8. Find the probability distribution.

3. The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

(vi) Bernoulli,s trials and Binomial Distribution

LEVEL II

1. If a die is thrown 5 times, what is the chance that an even number will come up exactly 3 times.
2. An experiment succeeds twice as often it fails. Find the probability that in the next six trials, there will be at least 4 success.

3. A pair of dice is thrown 200 times. If getting a sum 9 is considered a success, find the mean and variance of the number of success.

Questions for self evaluation

1. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

2. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of an experiment are performed. Find the probability that the event happens at least once.

3. A football match is either won, draw or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

4. A candidate has to reach the examination center in time. Probability of him going by bus ore scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. But he reaches in time if he uses other mode of transport. He reached late at the centre. Find the probability that he traveled by bus.

5. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0, \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \end{cases}, \text{ k is a + ve constant.}$$

Find the mean and variance of the probability distribution. 1

6. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

7. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

8. Two cards are drawn simultaneously (or successively) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

ANSWERS

TOPIC 1 RELATIONS& FUNCTIONS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. R = { (3,5),(4,4),(5,3) },Domain = {3, 4, 5}, Range = {3, 4, 5}

2. Domain = {1, 2, 3,}, Range = {8, 9, 10}

(iii).One-one , onto & inverse of a function

LEVEL I

1. - f(x)   6. $\frac{1+x}{1-x}$

LEVEL II

2. $f^{-1}(x) = \frac{(4x+7)}{2}$ **3.**6   **5.**$f^{-1}(x) = \frac{(2x-5)}{3}$

(iv).Composition of function

LEVEL II

5.$f \circ f(x) = x$ 6 . $4x^2 - 12x + 9$

(v)Binary Operations

LEVEL I

5. 15   2. 4   3. e = 5   4.50

Questions for self evaluation

2. {1, 5, 9}   3. T_1 is related to T_3 6. $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

7. $e = 0, \ a^{-1} = \frac{a}{a-1}$ 8. Identity element (1 , 0), Inverse of (a , b) is $\left(\frac{1}{a}, \frac{-b}{a}\right)$

TOPIC 2 INVERSE TRIGONOMETRIC FUNCTION

1. Principal value branch Table

LEVEL I

1.$\frac{\pi}{6}$ 2.$-\frac{\pi}{6}$ 3.$-\frac{\pi}{3}$ 4.$\frac{3\pi}{4}$

LEVEL II

1. π 2. $\frac{\pi}{5}$ 3.$\frac{5\pi}{6}$

2. Properties of Inverse Trigonometric Functions

LEVEL I

1. 0   3.$\sqrt{2}$

LEVEL II

1. $\frac{1}{2}\tan^{-1} x$

LEVEL III

3. $\frac{1}{6}$ 4. $\frac{1}{4}$ 5. $\pm\frac{1}{\sqrt{2}}$

Questions for self evaluation

6. x   7. $\pm\frac{1}{\sqrt{2}}$ 8. $\frac{1}{6}$

TOPIC 3 MATRICES & DETERMINANTS

1. Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. $1\times5,\ 5\times1$ **2**. $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$ **3**. $\begin{bmatrix} -3 & -4 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ 4**.** $2\times2,\ 3\times3$

LEVEL II

3.skew symmetric   **4.** a = 8, b = 8

LEVEL III.

1. $\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$ **2.** $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

(ii). Cofactors &Adjoint of a matrix

LEVEL I

1. 46   2. $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. $A^{-1} = -A$ **2.** $A^{-1} = A$ **3**. k = 17

LEVEL II

1. $\begin{bmatrix} -2/14 & -5/14 \\ -4/14 & -3/14 \end{bmatrix}$ **3. 0**

LEVEL III

1.x = 1, y = 2, z = 3.   **2**. x = 3, ,y = -2, z = 1.   **3.** AB = 6I, x = 2, y = - 1, z = 4

4.x = ½, y = -1, z = 1.   **5.** $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

(iv). To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. $\frac{1}{2}$ **2.** 27 **3.** 24 **4.** $\frac{4}{3}$

LEVEL II

1. 8 **2.** 49

LEVEL III

1. a = 3 **2.** 125

(v). Properties of Determinants

LEVEL I

1. x = 4 **2.** $a^2 + b^2 + c^2 + d^2$

LEVEL II

2. [Hint: Apply $C_1 \rightarrow -bC_3$ and $C_2 \rightarrow aC_3$]

LEVEL III

1a. 4 **1b.** $0, 0, 3a$ **1c.** $-\frac{a}{3}$

2. HINT $\Delta = \frac{-1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (a - b)^2]$

3.[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively]

4.[Hint : Apply $R_1 \rightarrow R_1 + R_3$ and take common a + b + c]

5.Hint : Apply $R_1 \rightarrow aR_1$, $R_2 \rightarrow bR_2$, and $R_3 \rightarrow cR_3$]

6.[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively and then apply $R_1 \rightarrow R_1 + R_2 + R_3$]

Questions for self evaluation

4. $\begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & 1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$

6. x = 8, y = 8 and $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

8. $A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$, x = 2, y = 1, z = 3

9. $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, x = 3, y = – 2, z = –1

10. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

TOPIC 4 CONTINUITY AND DIFFRENTIABILITY

2.Continuity

LEVEL-I

1.Continuous 2.Not Continuous

LEVEL-II

2.3/4 3.$3a - 3b = 2$ 4. Not Continuous

LEVEL-III

1. 1 [Hint: Use 1-cos $2\theta = 2sin^2\theta$]2. 1 [Hint: Use $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]
3.a = 1/2, b = 4 4. K = 2

3.Differentiation

LEVEL- I

1.Not Differentiable 2. $\frac{2}{1+x^2}$

3. $\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left(\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{(6x+4)}{3x^2+4x+5}\right)$

LEVEL-II

1. $2\log x \sin(\log x)^2/x$ 2.$\frac{1}{2(1+x^2)}$ 4.$-\frac{3}{2a}\left(\frac{1+t}{1-t}\right)^3$

LEVEL-III

1 .$\frac{x}{\sqrt{1-x^4}}$ [hint: Put $x^2 = \cos 2\theta$] 2. $\frac{1}{2}$[Hint: use $1 \pm sinx = (\cos\frac{x}{2} \pm \sin\frac{x}{2})^2$]

4.Logrithmic Differentiation

LEVEL-I

1.y′ = 1/(xlogxlog7) 2. $\frac{\cos(\log x)}{x}$

3.$\frac{dy}{dx} = \frac{1}{x(1+(\log x)^2)}$ [Hint: Use log(ex) = loge+logx=1+logx]

LEVEL-II

2. $2\log x \sin(\log x)^2/x$ 3. $\frac{y\tan x + \log\cos y}{x\tan y + \log\cos x}$

LEVEL-III

$2\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$ 4. $x^{\cot x}(\cot x + x\log\sin x) + \frac{2x^2+14x+3}{(x^2+x+2)^2}$

(v) Parametric differentiation

2. $\frac{2\sqrt{2}}{a}$

Questions for self evaluation

1. k = 5 2.a = 3, b = 2, 4. $a = -\frac{3}{2}, c = \frac{1}{2}, b \in R$

5. $\left[\frac{dy}{dx}\right]_{\theta=\pi/4} = 1$ 6. $(\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x.\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$

7. $\frac{y-\sec^2 x}{1-x-2y}$ 9. [Hint: Put $x = \sin\theta$; $y = \sin\varphi$] 10. $\frac{y\tan x + \log\cos y}{x\tan y + \log\cos x}$

TOPIC 5 APPLICATIONS OF DERIVATIVES

1. Rate of change

LEVEL I 1. $\frac{27\pi}{8}(2x+1)^2$ 2.64 cm^2/min 3. 4.4 cm/sec

LEVEL II 1.(2 , 4) 2. 9 km/h 3.8 cm^2/sec

LEVEL III 1. (4 , 11) and $\left(-4, \frac{-31}{3}\right)$ 2. $\frac{4}{45}\pi$cm/sec 3. $\frac{1}{10}\pi$cm/sec

2. Increasing & decreasing functions

LEVEL I 3.(0, 3π/4) U (7π/4, 2π) and (3π/4, 7π/4)

LEVEL II 1. (0 ,π) 3. (0 , e) and (e, ∞)

LEVEL III 1. (- ½ , 0) U (½ , ∞) &(-∞ , -1/2) U (0,1/2)

3. Tangents &normals

LEVEL I 1. x + 3y - 8 = 0 & x + 3y + 8 =0 2. (0 , 0)

3. (1 , 0) & (1 , 4)

LEVEL II 1. $2x + 3my - 3am^4 - 2am^2 = 0$ 2. (3 , 45) & (–3 , 27)

3. x + 14y – 254 = 0 & x + 14y + 86 = 0

LEVEL III 1.80x – 40y – 103 = 0 3. $a^2 = b^2$[Hint: Use $m_1m_2 = -1$]

4. Approximations

LEVEL I 1. 5.03 2. 4.042 3.0.2867 4. 7.036

LEVEL II 1. 2.16 π cm

5 Maxima & Minima

LEVEL I 1.1 & 5 3. 12, 12

LEVEL II 2. $\frac{112}{\pi+4}$ cm , $\frac{28\pi}{\pi+4}$ cm. 3. Length $=\frac{20}{\pi+4}$ m , bredth $\frac{10}{\pi+4}$ m.

LEVEL III 1. $\frac{3\sqrt{3}}{4}ab$ 3. $\frac{4\left(6+\sqrt{3}\right)}{11}$ m , $\frac{30-6\sqrt{3}}{11}$

Questions for self evaluation

1. $\frac{1}{48\pi}$ cm/s
2. $b\sqrt{3}cm^2/s$
3. $\uparrow$ in (– 2 , – 1) and $\downarrow$ in $(-\infty,-2)\cup(-1,\infty)$
4. $\uparrow$ in $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$ and $\downarrow$ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$
5. (0 , 0) , (1 , 2), (– 1, – 2)
6. (a) y – 2x – 3 = 0 , (b) 36y + 12x – 227 = 0

8.(i) 2.962 (ii) 2.962

10. $\frac{200}{7}$ m

TOPIC 6 INDEFINITE& DEFINITE INTEGRALS

(i) Integration by substitution

LEVEL I 1. $\tan(\log_e x)+C$ 2. $\frac{1}{m}e^{m\tan^{-1}x}+C$ 3. $e^{\sin^{-1}x}+C$

LEVEL II 1. $2\log_e\left|1+\sqrt{x}\right|+C$ 2. $\frac{1}{3}\sec^{-1}x^3+C$ 3. $\log_e\left|1-e^x\right|+C$

LEVEL III 1. $2\sqrt{\tan x}+C$ 2. $-\tan^{-1}(\cos x)+C$ 3. $\frac{\tan^2 x}{2}+\log_e|\tan x|+C$

(ii)) Application of trigonometric function in integrals

LEVEL I 1. $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x+C$ 2. $\frac{1}{2}\left[x+\frac{\sin 6x}{6}\right]+C$

3. $\frac{x}{4}+\frac{1}{4}\sin 6x+\frac{1}{16}\sin 4x+\frac{1}{8}\sin 2x+C$

LEVEL II 1. $\frac{1}{4}\sec^4 x+C$ OR $\frac{\tan^2 x}{2}+\frac{\tan^4 x}{4}+C$ 2. $\frac{2}{3}\sin 3x+2\sin x+C$

LEVEL III 1. $\sin x-\frac{2}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C$ 2. $\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$

(iii) Integration using Standard results

LEVEL I 1. $\frac{1}{2}\log_e\left|x+\frac{1}{2}\sqrt{4x^2-9}\right|+C$ 2. $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+C$ 3. $\frac{1}{9}\tan^{-1}\left(\frac{3x+2}{3}\right)+C$

LEVEL II 1. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+C$ 2. $\tan^{-1}(\sin x+2)+C$ 3. $\sin^{-1}\left(\frac{2x-1}{5}\right)+C$

LEVEL III 1. $\sin^{-1}\left(\frac{2x^2-1}{5}\right)+C$ 2. $x+\log\left|x^2-x+1\right|+\frac{2}{\sqrt{3}}\log\left|\frac{2x-1}{\sqrt{3}}\right|+C$

3. $\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

4. $\sin^{-1}x+\sqrt{1-x^2}+C$ [Hint: Put x=cos2θ]

5. $6\sqrt{x^2-9x+20}+34\log\left|\left(\frac{2x-9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$

(iv) Integration using Partial Fraction

LEVEL I 1. $\frac{1}{3}\log(x+1)+\frac{5}{3}\log(x-2)+C$ 2. $\frac{1}{2}\log(x-1)-2\log(x-2)+\frac{3}{2}\log(x-3)+C$

3. $\frac{11}{4}\log\left(\frac{x+1}{x+3}\right)+\frac{5}{2(x+1)}+C$

LEVEL II 1. $x - 11\log(x-1) + 16\log(x-2) + C$ 2. $\frac{1}{4}\log x-\frac{1}{2x}+\frac{3}{4}\log(x+2)+C$

3. $\frac{3}{8}\log(x-1)-\frac{1}{2(x-1)}+\frac{5}{8}\log(x+3)+C$

LEVEL III 1. $\log(x+2)-\frac{1}{2}\log(x^2+4)+\tan^{-1}\frac{x}{2}$ 2. $\frac{\log(1-\cos x)}{6}+\frac{\log(1+\cos x)}{2}-\frac{2\log(1+2\cos x)}{3}+C$

3. $\frac{1}{3}\log(1+x)-\frac{1}{6}\log(1-x+x^2)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+C$ [Hint: Partial fractions]

(v) Integration by Parts

LEVEL I 1. $x.\tan x + \log\cos x + C$ 2. $x\log x - x + C$ 3. $e^x.\log\sec x + C$

LEVEL II 1. $x\sin^{-1}x+\sqrt{1-x^2}+C$ 2. $\frac{x^3}{3}\sin^{-1}x+\frac{(x^2+2)\sqrt{1-x^2}}{9}+C$

3. $-\sqrt{1-x^2}\sin^{-1}x+x+C$ 4. $2x\tan^{-1}x-\log(1+x^2)+C$

5. $\frac{1}{2}(\sec x.\tan x+\log(\sec x+\tan x))+C$

LEVEL III 1. $\frac{x}{2}[\cos(\log x)+\sin(\log x)]+C$ 2. $\frac{e^x}{2+x}+C$ [Hint: $\int[e^x f(x)+f'(x)]dx= e^x f(x)+c$]

3. $\frac{x}{1+\log x}+C$

4. $e^x.\tan x + C$ 5. $\frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+C$

(vi) Some Special Integrals

LEVEL I 1. $\frac{x\sqrt{4+x^2}}{2}+2\log\left|x+\sqrt{4+x^2}\right|+C$ 2. $\frac{x\sqrt{1-4x^2}}{2}+\frac{1}{4}\sin^{-1}2x+C$

LEVEL II

1. $\frac{(x+2)\sqrt{x^2+4x+6}}{2}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

2. $\frac{(x+2)\sqrt{1-4x-x^2}}{2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

LEVEL III

1. $-\frac{1}{3}\left(1-x-x^2\right)^{3/2}+\frac{1}{8}(2x-1)\sqrt{1-x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)+C$

2. $\frac{1}{3}\left(x^2+x\right)^{3/2}-\frac{11}{8}(2x+1)\sqrt{x^2+x}+\frac{11}{16}\log\left[(2x+1)+2\sqrt{x^2+x}\right]+C$

(vii) Miscellaneous Questions

LEVEL II

1. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+C$

2. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x+1}{2\sqrt{2}}\right)+C$

3. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+C$

4. $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

5. $\tan^{-1}\left(\tan^2 x\right)+C$ [Hint: divide Nr. and Dr. by x^2]

6. $\frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}+4}{3}\right)+C$

LEVEL III

1. $-\frac{12}{13}x-\frac{5}{13}\log\left|3\cos x+2\sin x\right|+C$

2. $\frac{x}{2}-\frac{1}{2}\log\left|\cos x-\sin x\right|+C$

3. $x+\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$

4. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$

5. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$

6. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+C$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\frac{1}{5}\log\ 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

2. $\frac{64}{231}$

3. $\left[\log\frac{3}{2}-9\log\frac{5}{4}\right]$

LEVEL II

1. $5+\frac{5}{2}\left[\log\frac{3}{2}-9\log\frac{5}{4}\right]$

2. $\frac{e^2}{4}\left(e^2-2\right)$

(ii) Definite integrals as a limit of sum

LEVEL I

1. 6

2. 12

(iii) Properties of definite Integrals

LEVEL I

1. $\frac{\pi}{4}$

2. 1

3. $\frac{\pi}{4}$

LEVEL II 1. $\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 2. $\frac{\pi^2}{4}$ 3. $\frac{\pi^2}{4}$

4. $\frac{\pi}{12}$

LEVEL III 1. $\frac{\pi}{2}$ 2. $-\frac{\pi}{2}\log 2$ 3. $\frac{\pi}{8}\log 2$

(iv) Integration of modulus function

LEVEL III 1. $\frac{19}{2}$ 2. $\frac{11}{4}$ 3. 4

Questions for self evaluation

1. $\log\left|x^2+3x-18\right|-\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|+c$

2. $-3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

3. $\left[\frac{1}{8}\left(3x+2\sin 2x+\frac{\sin 4x}{4}\right)+c\right]$

4. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+c$

5. $\frac{18}{15}x+\frac{1}{25}\log\left|3\sin x+4\cos x\right|+c$

6. $x-\sqrt{1-x^2}\sin^{-1}x+c$

7. $\frac{64}{231}$

8. $\frac{3}{\pi}+\frac{1}{\pi^2}$

9. $-\frac{\pi}{2}\log 2$

10. 19/2

TOPIC 7 APPLICATIONS OF INTEGRATION

(i)Area under ***Simple Curves***

LEVEL I 1. 20π Sq. units 2. 6π Sq. units

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units 2. $\frac{32-8\sqrt{2}}{3}$ Sq. units

LEVEL III 1. $\frac{23}{6}$ Sq. units

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II 1. $5(\pi-2)$ Sq. units

(iv) Area of the region enclosed between***Circle and line***

LEVEL II 1. 4π Sq. units

LEVEL III 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units

(v) Area of the region enclosed between ***Circle and parabola***

LEVEL III 1. $2\left(4\pi-\frac{4\sqrt{3}}{3}\right)$ Sq. units 2. $\frac{4}{3}(8+3\pi)$ Sq. units

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III 1. $\left(\frac{8\pi}{3}-2\sqrt{3}\right)$ Sq. units

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II 1. 4 Sq. units

(viii) Area of triangle ***when vertices are given***

LEVEL III 1.4 Sq. units 2. $\frac{15}{2}$ Sq. units

(ix) Area of triangle ***when sides are given***

LEVEL III 1.8 Sq. units 2. 6 Sq. units

(x) ***Miscellaneous Questions***

LEVEL III 1. $\frac{1}{2}$ Sq. units 2. $\frac{1}{2}$ Sq. units

3. $\left(2-\sqrt{2}\right)$ Sq. units 4. 2 Sq. units

5. $\frac{64}{3}$ Sq. units 6. 9 Sq. units

Questions for self evaluation

1. $\frac{9}{8}$ sq. units 2. $\frac{1}{3}$ sq. units 3. $\frac{23}{6}$ sq. units

4. $\frac{3}{4}(\pi-2)$ sq. units 5. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units 6. $\frac{4}{3}(8+3\pi)$ sq. units

7. $\left(\frac{8\pi}{3}-2\sqrt{3}\right)$ sq. units 8. 4 sq. units

TOPIC 8 DIFFERENTIAL EQUATIONS

1. *Order and degree of a differential equation*

LEVEL I 1.order 2 degree 2

3. *Formation of differential equation*

LEVEL II 1. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

LEVEL III 1. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=r^2\left(\frac{d^2y}{dx^2}\right)^2$ 2. $y^2-2xy\frac{dy}{dx}=0$ [Hint: $y^2=4ax$]

4. ***Solution of differential equation by the method of separation of variables***

LEVEL II 1. $\log|1+y| = x + \frac{1}{2}x^2 + c$ 2. $e^y = \sin x + 1$

3. $y = \frac{1}{2}\log\left|1+x^2\right| + (\tan^{-1} x)^2 + c$

5. ***Homogeneous differential equation of first order and first degree***

LEVEL II 1. $\log|x| - \log|x-y| - \frac{y}{x} + c = 0$

LEVEL III 1. $cy = \log\frac{y}{x} - 1$ 2. $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$ 3. $y = ce^{\frac{x^3}{3y^3}}$

4. $y + \sqrt{x^2+y^2} = cx^2$ 5. $y = 3x^2 + cx$ 6. $y = \frac{x^3}{4} + \frac{c}{x}$

7. $y = -\frac{2}{3}x^2 + \frac{c}{x}$

6. ***Linear Differential Equations***

LEVEL I 1. 1/x

LEVEL II 1. y=cosx+c cos2x **2.** $\frac{y}{x+1} = \frac{1}{3}e^{3x} + c$ 3. xy = $\frac{x^2}{4}$ (2logx -1)+c

LEVEL III 1 $\tan\left(\frac{x+y}{2}\right) = x + c$ 2. $x = -y^2e^{-y} + cy^2$ 3. $-\frac{x}{y} = \log|x| + c$

4. $(x^2+1)^2$ = -$\tan^{-1}$x + c [Hint: Use $\frac{dy}{dx} + Py = Q$] 5. $x = 2y^2$

Questions for self evaluation

1. Order 2, Degree not defined 2. $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

3. $x = \left(\tan^{-1} y - 1\right) + Ce^{-\tan^{-1} y}$ 4. $y + \sqrt{x^2+y^2} = Cx^2$

5. $y\log x = \frac{-2}{x}\left(1+\log|x|\right)+C$ 6. $y + 2x = 3x^2y$ [Hint: use $v = \frac{x}{y}$]

7. $2e^{\frac{x}{y}} + \log|y| = 2$

8. $y = x^2 - \frac{\pi^2}{4\sin x}$

TOPIC 9
VECTOR ALGEBRA

(i)Vector and scalars, Direction ratio and direction cosines &Unit vector

LEVEL I

1. $\frac{2}{\sqrt{17}}\hat{i} - \frac{3}{\sqrt{17}}\hat{j} - \frac{2}{\sqrt{17}}\hat{k}$ 2. $5\hat{i} - 10\hat{j} + 10\hat{k}$ 3. $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

4. $\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$ 5. $7\left(\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}\right)$

LEVEL II

1. $5\left(\frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}\right)$ 2. $\sqrt{3}$ 3. $2\hat{i} - \hat{j} + 4\hat{k}$

LEVEL III

1. 2 2. $P = \pm\frac{1}{\sqrt{3}}$ 3. Cosine of the angle with y –axis is $\frac{1}{2}$ 4. $P = \frac{2}{3}$

(ii)Position vector of a point and collinear vectors

LEVEL I

1. $4\hat{i} + \hat{j}$ 2 $\overrightarrow{CA} = -(3\hat{i} + 2\hat{j} + 7\hat{k})$

LEVEL II

1. $-3\hat{i} + 3\hat{k}$

(iii). Dot product of two vectors

LEVEL I

1. 1. $\vec{a} \cdot \vec{b} = 9$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{4}$

LEVEL II

1. $\vec{a} = 15\hat{i} - 27\hat{j} + 5\hat{k}$ 2. $\theta = \frac{\pi}{4}$ 3. $\lambda=8$

LEVEL III

2. $\frac{\pi}{2}$ 3. (i) $\lambda = \frac{-40}{3}$ (ii) $\lambda = 6$ 4. $|\vec{x}| = 4$ 5. [Hint: Use $(\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = 0$]

7. $\vec{d} = 64\hat{\imath} - 2\hat{\jmath} - 28\hat{k}$ 9. $5\sqrt{2}$

(iv)Projection of a vector

LEVEL I

1. $\frac{8}{7}$ [Hint: Use projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$] 2. 0 3. $\cos^{-1}\frac{5}{7}$

4. $\frac{60}{\sqrt{114}}$

LEVEL III

1. [$\frac{1}{\sqrt{2}}\hat{i}, \frac{1}{2}\hat{j}, \frac{1}{2}\hat{k}$, $\theta = \pi/3$] 3. $\vec{\beta}_1 = \frac{1}{2}(3\hat{\imath} - \hat{\jmath})$, $\vec{\beta} = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$

(vii)Cross product of two vectors

LEVEL I

1. 12 2. $19\sqrt{2}$ 3. $|\vec{x}| = 9$ 4. $p = \frac{27}{2}$

LEVEL II

1. $\lambda = -3$ 3. $\theta = \frac{\pi}{6}$

LEVEL III

1. **1 [Hint: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$]** **2.** $\theta = \frac{\pi}{3}$ 3. $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

5. $\left(-\frac{\hat{i}}{2} - \hat{j} + \frac{\hat{k}}{2}\right) + \frac{5}{2}(\hat{i} + \hat{j})$

(viii)Area of a triangle & Area of a parallelogram

LEVEL I

1. $10\sqrt{3}$ Sq. units 2. $|\vec{a} \times \vec{b}|$ 3. $\frac{\sqrt{21}}{2}$ Squnits [Hint : Use area$\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$]

LEVEL II

3. $\frac{\sqrt{61}}{2}$.

Questions for self evaluation

1. $\lambda=1$ 2. $5\sqrt{2}$ 3. $\frac{\pi}{2}$

4. $\hat{i} + 2\hat{j} + \hat{k}$ 5. $\pm\frac{3}{\sqrt{83}}\vec{i}, \mp\frac{5}{\sqrt{83}}\vec{j}, \mp\frac{7}{\sqrt{83}}\vec{k}$

TOPIC 10THREE DIMENSIONAL GEOMETRY

(i)Direction Ratios and Direction Cosines

LEVEL I

1. $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},$ **2.** $\left[\text{Ans.} \frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right]$

3. $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}},$

LEVEL II

1. $-\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}$ 2. < -3,-2,6> 3. $\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ 4. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL I

1. $\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$ 2. $\left[\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{6}\right]$

3. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2} = \lambda$ 4. $2x - 3y + z + 4 = 0$

(iii)plane and skew lines

LEVEL II

1. Lines are intersecting & point of intersection is (3,0,-1).

[Hint: For Coplanarity use $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ & a_1 a_2 & a_3 \\ & b_1 b_2 & b_3 \end{vmatrix}$

LEVEL III

2. Equation of AB is $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 10\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$.
3. Equation of BC is $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$. Coordinates of D are (3,4,5).

(iv) Shortest distance between two lines

LEVEL II

1(a) $\frac{3\sqrt{2}}{2}$ *units*, 4. $\frac{8}{\sqrt{29}}$

1(b) $\frac{3}{\sqrt{19}}$ units

5. $\frac{1}{\sqrt{3}}$

3. 0

6. Vector equation $\vec{r} = (3\hat{\imath} - 4\hat{k}) + \lambda(5\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$ and distance = 7.75 units

(v)Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1.x = 0   2. 12x + 4y + 3z = 12   3. $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$ 4. (a) $x + y - z = 2$ (b) $2x + 3y - 4z = 1$

LEVEL II

1. $2x + 3y - z = 20$ 2. $\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7$

3. $[r - (\hat{\imath} - 2\hat{k})] \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0$; $x + y - z = 3$

(vi) Angle Between (i)Two lines(ii)Two planes(iii)Line & plane

LEVEL-I

1. 60^0 2. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ 3. $\lambda = -3$ 4. $\cos^{-1}\frac{15}{\sqrt{731}}$ 5. $\sin^{-1}\frac{8}{21}$

LEVEL-II

1. p = –3

(vii) *Distance of a point from a plane*

LEVEL-I

1. 1/3   2. (0, 0, 0)   3. $\frac{13}{7}$

4. $\frac{1}{3}$ 5. [$\sqrt{b^2+c^2}$]

LEVEL-II

1. (4 , 3, 7)   2. 6 units   3. $\frac{3\sqrt{34}}{17}$ 4. 13

LEVEL-III

1. Foot of perpendicular (-1,4,3), Image (-3,5,2), Distance = $\sqrt{6}$ units

2. 3x – 4y + 3z -19 = 0

3. x + y – z – 2 = 0 , $\frac{2}{\sqrt{3}}$ *units* , $\frac{1}{\sqrt{3}}$ units.

(viii).Equation of a plane through the intersection of two planes

LEVEL-II

1. x – y + 3z – 2 = 0 , $\frac{2\sqrt{11}}{11}$ 2. Ans. 51x + 15y – 50z + 173 = 0

3. 5x – 4y –z = 7

LEVEL-III

1. x – 2y + z = 0   3. x– 19y – 11z = 0   4. $\frac{x-2}{-6} = \frac{y-4}{-3} = \frac{z+1}{-2}$

(ix) Foot of perpendicular and image with respect to a line and plane

LEVEL-II

1. (1, -2, 7)   2. (3, 5, 9)   3. Image of the point = (0,-1,-3)

LEVEL-III

1. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$ 2. $\left(\frac{39}{7},\frac{-6}{7},\frac{-37}{7}\right)$ 3. 12x - 4y + 3z = 169   4. (-1,4,-1)

Questions for self evaluation

1. 17x + 2y – 7z = 12

2. $\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$,

3. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

4.ZERO

5. $\left(\frac{39}{7}, \frac{-6}{7}, \frac{-37}{7}\right)$

8. [Hint: second line can also be written as $\frac{(x-4)}{5} = \frac{(y-1)}{2} = \frac{(z-0)}{1}$]

TOPIC 11LINEAR PROGRAMMING

(i) LPP and its Mathematical Formulation

LEVEL I

1. Z = 50x + 70y, 2x + $y \geq$ 8, x + 2$y \geq$ 10, $x, y \geq$ 0

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

1. Minimum Z = – 12 at (4, 0),

2. Maximum Z = $\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

3. Minimum Z = 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

(iii) Diet Problem

LEVEL II

1. Least cost = Rs.110 at x = 5 and y = 30

2. Minimum cost = Rs.6 at x = 400 and y = 200

(iv) Manufacturing Problem

LEVEL II

1.Maximum profit is Rs. 120 when 12 units of A and 6 units of B are produced

2. For maximum profit, 25 units of product A and 125 units of product B are produced and sold.

(v) Allocation Problem

LEVEL II

1.Maximum annual income = Rs. 6,200 on investment of Rs. 40,000 on Bond A and

Rs. 30,000 on Bond B.

2. A should run for 60 days and B for 30 days.

LEVEL III

1.For maximum profit, 62 executive class tickets and 188 economy class ticket should be sold.

(vi) Transportation Problem

LEVEL III

1. Minimum transportation cost is Rs. 400 when 10, 0 and 50 packets are transported from factory at A and 30, 40 and 0 packets are transported from factory at B to the agencies at P, Q and R respectively.

Questions for self evaluation

1. Minimum 155 at (0 , 5).
2. Minimum value is 5 at $\left(\frac{3}{2}, \frac{1}{2}\right)$
3. Maximum is Rs 4.60 at (0.6 , 0.4)
4. Maximum is Rs.800 at (0 , 20)
5. 8 items of type A and 16 items of type B
6. 1 jar of liquid and 5 cartons of dry product.
7. Rs.4,000 in Bond A and Rs.14,000 in Bond B
8. Minimum cost Rs.1350 at (5 , 3)

TOPIC 12PROBABILITY

(i) Conditional Probability

LEVEL I 1. 0 2. $\frac{5}{14}$

LEVEL II 1. $\frac{2}{5}$

LEVEL III 1. $\frac{3}{4}$ and $\frac{3}{5}$

(ii)Multiplication theorem on probability

LEVEL II 1. $\frac{8}{65}$ 2.**(i)** $\frac{13}{21}$ (ii) $\frac{10}{21}$ [Hint : p(x>=1) = 1 – P(x<0)

LEVEL III 1. $\frac{5}{512}$ 2. $\frac{3}{5}$

(iii) Independent Events

LEVEL I 1.Yes 2.Yes [check: $P(A\cap B) = P(A).P(B)$] 3. $\frac{5}{13}$

–

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I 1. $\frac{93}{154}$ 2. $\frac{11}{50}$ 3. $\frac{3}{4}$ 4. $\frac{19}{42}$ 5. 0.27

LEVEL II 1. $\frac{1}{2}$ 2. $\frac{2}{3}$ 3. $\frac{3}{7}$

LEVEL III 1.(i) $\frac{12}{17}$ (ii) $\frac{5}{17}$ 2.0.15 3. $\frac{2}{3}$

(v) Random variables & probability distribution , Mean & variance of random variables

LEVEL I 1.

X	0	1	2
P(X)	9/16	6/16	1/16

2.

X	0	1	2	3
P(X)	28/57	24/57	24/285	1/285

3. 0.6

LEVEL III 1.

X	0	1	2
P(X)	9/16	6/16	1/16

2.. $\left(\frac{4}{5}+\frac{1}{5}\right)^5$ 3. $\frac{65}{81}$

(vi) Bernoulli's trials and Binomial Distribution

LEVEL II 1. $\frac{5}{16}$ 2. $\frac{496}{729}$ 3. $\frac{200}{9}$, $\frac{1600}{81}$ [Hint: mean =np, variance=npq]

Questions for self evaluation

1. $\frac{1}{4}$ 2.0.784 3. $\frac{1}{9}$

4. $\frac{9}{13}$

5. $\frac{19}{8}, \frac{47}{64}$

6. $\frac{625}{23328}$

7. $\frac{11}{243}$

8. 1 and 1.47