## MODEL QUESTION PAPER FOR
## 15MAT21 – Engineering Mathematics – II

| Time: 3 hrs. | | Max. Marks: 80 |
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**Note: Answer any FIVE full questions, choosing one full question from each module.**

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| | | **MODULE 1** | |
| 1) | a. | Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 2^x$ by inverse differential operator method | (6 Marks) |
| | b. | Solve $(D^2 - 2D + 5)y = e^{2x}\sin x$ by inverse differential operator method | (5 Marks) |
| | c. | Solve $D^2y + y = \tan x$ by the method of variation of parameters | (5 Marks) |
| | | **OR** | |
| 2) | a. | Solve $\frac{d^2y}{dx^2} + 16y = x\sin 3x$ by inverse differential operator method | (6 Marks) |
| | b. | Solve $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = e^x + 1$ by inverse differential operator method | (5 Marks) |
| | c. | Solve $y'' - y' - 2y = x + \sin x$ by the method of undetermined coefficient | (5 Marks) |
| | | **MODULE 2** | |
| 3) | a. | Solve $x^2y'' + xy' + y = 2\cos^2(\log x)$ | (6 Marks) |
| | b. | Solve $y\left(\frac{dy}{dx}\right)^2 + (x-y)\frac{dy}{dx} - x = 0$ | (5 Marks) |
| | c. | Solve the equation $(px - y)(py + x) = 2p$ by reducing into Clairaut's form , taking the substitution $X = x^2, Y = y^2$ | (5 Marks) |
| | | **OR** | |
| 4) | a. | Solve $(2x-1)^2y'' + (2x-1)y' - 2y = 8x^2 - 2x + 3$ | (6 Marks) |
| | b. | Solve $y = 2px + p^2y$by solving for x | (5 Marks) |
| | c | Find the general and singular solution of the equation p=log(px-y) | (5 Marks) |
| | | **MODULE 3** | |
| 5) | a. | Obtain the partial differential equation by eliminating the arbitrary function given $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ | (5 Marks) |
| | b. | Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when $y$ an odd multiple of $\frac{\pi}{2}$ | (5 Marks) |

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| | c. | Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ | (6 Marks) |
| | | **OR** | |
| 6) | a. | Obtain the partial differential equation $f(xy - z^2, x + y + z) = 0$ | (5 Marks) |
| | b. | Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$ | (5 Marks) |
| | c. | Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables for constant k=0. | (6 Marks) |
| | | **MODULE 4** | |
| 7) | a. | Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$ | (5 Marks) |
| | b. | Evaluate by changing the order of integration $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy\, dy\, dx$ | (5 Marks) |
| | c. | Evaluate $\int_0^2 (4 - x^2)^{3/2} dx$ by using Beta and Gamma functions | (6 Marks) |
| | | **OR** | |
| 8) | a. | Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates | (5 Marks) |
| | b. | Find the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$ by double integration. | (5 Marks) |
| | c. | Obtain the relation between beta and gamma function in the form $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ | (6 Marks) |
| | | **MODULE 5** | |
| 9) | a. | Find i) $L\{t^2 e^{-2t} \sin t\}$ ii) $L\left\{\frac{\sin^2 t}{t}\right\}$ | (5 Marks) |
| | b. | Given $f(t) = \begin{cases} E & 0 < t < a/2 \\ -E & a/2 < t < a \end{cases}$ where $f(t+a) = f(a)$. Show that $L\{f(t)\} = \frac{E}{S} \tanh\left(\frac{as}{4}\right)$ | (5 Marks) |
| | c. | Employ Laplace Transforms to solve the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-x}$ with the initial condition $y(0) = 0, y'(0) = 0$ | (6 Marks) |
| | | **OR** | |
| 10) | a. | find $L^{-1}\left[\frac{s+5}{s^2-4s+13}\right]$ | (5 Marks) |
| | b. | Find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ by using convolution theorem | (5 Marks) |
| | c. | Express $f(t) = \begin{cases} 1 & 0 < t \le 1 \\ t & 1 < t \le 2 \\ t^2 & t > 2 \end{cases}$ in terms of unit step function and hence find its Laplace transforms | (6 Marks) |