

First Semester B.E. Degree Examination, Dec.2015/Jan.2016 Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1.
 - a. Find the n^{th} derivative of $\frac{x^2}{2x^2 + 7x + 6}$. (06 Marks)
 - b. Find the angle between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$. (05 Marks)
 - c. Find the radius of curvature of the curve represented by $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$. (05 Marks)

OR

2.
 - a. If $y = (x - \sqrt{x^2 - 1})^m$ then prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. (06 Marks)
 - b. Find the pedal equation of $r^n = a(1 - \cos n\theta)$. (05 Marks)
 - c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. (05 Marks)

Module-2

3.
 - a. Expand $\sin x$ in powers of $(x - \frac{\pi}{2})$ upto fourth degree term. (06 Marks)
 - b. Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log(1+x)}{x^2}$. (05 Marks)
 - c. If $u = x + y + z$, $uv = y + z$, $uvw = z$ then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (05 Marks)

OR

4.
 - a. Find the Maclaurin's series expansion of $\sec x$ upto x^4 term. (06 Marks)
 - b. If $V(x, y) = (1 - 2xy + y^2)^{-1/2}$ and $x \frac{\partial v}{\partial x} - y \frac{\partial v}{\partial y} = y^2 V^K$, then find K . (05 Marks)
 - c. If $u = \sin^{-1} \left\{ \frac{x + 2y + 3z}{x^2 + y^2 + z^2} \right\}$ then find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$. (05 Marks)

Module-3

5.
 - a. A particle moves along the curve whose parametric equations are $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$ where t is the time. Find the component of its velocity at $t = 1$ in the direction of $\vec{i} + \vec{j} + 3\vec{k}$. Find also the component of its acceleration at $t = 1$ along the normal to $\vec{i} + \vec{j} + 3\vec{k}$. (06 Marks)
 - b. Verify whether $\vec{A} = (2x + yz)\vec{i} + (4y + zx)\vec{j} - (6z - xy)\vec{k}$ is irrotational or not. And find the scalar potential of \vec{A} . (05 Marks)
 - c. If \vec{A} is a vector point function and ϕ is a scalar point function then prove that $\text{div}(\phi \vec{A}) = \phi \text{div} \vec{A} + (\text{grad} \phi) \cdot \vec{A}$. (05 Marks)

OR

- 6 a. If $\vec{f} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$ and $\vec{g} = y^2 \vec{i} + x^2 \vec{j} + xy \vec{k}$, then verify whether $\vec{f} \times \vec{g}$ is solenoidal or not. (06 Marks)
- b. Find the directional derivative of $\phi = x^2 + y^2 + 2z^2$ at $P(1, 2, 3)$ in the direction of line $\overrightarrow{PQ} = 4\vec{i} - 2\vec{j} + \vec{k}$. (05 Marks)
- c. Prove that $\text{curl}(\text{grad } \phi) = \vec{0}$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x \, dx$. Hence evaluate $\int_0^{\pi/2} \sin^n x \, dx$. (06 Marks)
- b. Solve $(4xy + 3y^2 - x) \, dx + x(x - 2y) \, dy = 0$. (05 Marks)
- c. Find the Orthogonal trajectories of the family $r^n = a^n \sin n\theta$, where a is the parameter. (05 Marks)

OR

- 8 a. Evaluate $\int_0^{\infty} \frac{x^6 \, dx}{(4 + x^2)^{15/2}}$. (06 Marks)
- b. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (05 Marks)
- c. A body is heated to 110°C and placed in air at 10°C . After one hour its temperature become 60°C . How much additional time is required for it to cool to 30°C ? (05 Marks)

Module-5

- 9 a. Solve the following system of equations by Gauss – Jordan method :
 $x - y + z = 8$; $-x - y + 2z = -4$; $3x + 5y - 7z = 14$. (06 Marks)
- b. Verify the transformation $y_1 = 19x_1 - 9x_2 + 2x_3$; $y_2 = -4x_1 + 2x_2 - x_3$; $y_3 = -2x_1 + x_2$ is regular or not and find the inverse transformation if possible. (05 Marks)
- c. Reduce the matrix to the diagonal form

$$A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$$

(05 Marks)

OR

- 10 a. Solve the following system by Gauss – Seidal method :
 $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. Perform three iterations. (06 Marks)
- b. Determine the largest eigen value and the corresponding eigen vector of

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

using Power method.

(05 Marks)

Take $(1, 0, 0)^T$ as the initial eigen vector and perform four iterations.

- c. Reduce the quadratic form :

$$8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz \text{ into canonical form.}$$

(05 Marks)
