

CBCS Scheme

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15MAT11

First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 80

Note: Answer FIVE full questions, choosing one full question from each module.

Module-1

- 1 a. Obtain the n^{th} derivative of $\frac{x}{(x-1)^2(x+2)}$. (06 Marks)
- b. Find the angle of intersection of the curves $r = a(1+\sin \theta)$ and $r = a(1-\sin \theta)$. (05 Marks)
- c. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (05 Marks)

OR

- 2 a. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. (06 Marks)
- b. Obtain the pedal equation of the curve $r^n = a^n \cos n\theta$. (05 Marks)
- c. Find the derivative of arc length of $x = a(\cos t + \log \tan(\frac{t}{2}))$ and $y = a \sin t$. (05 Marks)

Module-2

- 3 a. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e(1.1)$, correct to four decimal places. (06 Marks)
- b. If $z = \sin(ax+y) + \cos(ax-y)$, prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$. (05 Marks)
- c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (05 Marks)

OR

- 4 a. If $u(x+y) = x^2 + y^2$, then prove that $\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)^2 = 4\left(1 - \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right)$. (06 Marks)
- b. Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{\frac{1}{x}}$. (05 Marks)
- c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $xu_x + yu_y + zu_z = 0$. (05 Marks)

Module-3

- 5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. (06 Marks)
- b. If $\vec{f} = (x+y+az)\hat{i} + (bx+2y-z)\hat{j} + (x+cy+2z)\hat{k}$, find a, b, c such that \vec{f} is irrotational. (05 Marks)
- c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $P(2, -1, 2)$. (05 Marks)

OR

- 6 a. Find the directional derivative of $xy^3 + yz^3$ at $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. (06 Marks)
- b. If $\vec{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\vec{v} = yz\hat{i} + zx\hat{j} + xy\hat{k}$, show that $\vec{u} \times \vec{v}$ is a solenoidal vector. (05 Marks)
- c. For any scalar field ϕ and any vector field \vec{f} , prove that $\text{curl}(\phi \vec{f}) = \phi \text{curl} \vec{f} + (\text{grad} \phi) \times \vec{f}$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \cos^n x \, dx$, where n is a positive integer, hence evaluate $\int_0^{\pi/2} \cos^n x \, dx$. (06 Marks)
- b. Solve: $(x^2 + y^2 + x) \, dx + xy \, dy = 0$. (05 Marks)
- c. Find the orthogonal trajectories of the family of circles $r = 2a \cos \theta$, where 'a' is a parameter. (05 Marks)

OR

- 8 a. Evaluate $\int_0^\infty \frac{x^6}{(1+x^2)^{9/2}} \, dx$. (06 Marks)
- b. Solve $xy(1 + xy^2) \frac{dy}{dx} = 1$. (05 Marks)
- c. Water at temperature 10°C takes 5 minutes to warm upto 20°C in a room temperature 40°C . Find the temperature after 20 minutes. (05 Marks)

Module-5

- 9 a. Solve the following system of equations by Gauss Elimination Method. (06 Marks)
 $x + 2y + z = 3$, $2x + 3y + 2z = 5$, $3x - 5y + 5z = 2$.
- b. Find the dominant eigen value and the corresponding eigen vector by power method
 $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, perform 5 iterations, taking initial eigen vector as $[1 \ 1 \ 1]^T$. (05 Marks)
- c. Show that the transformation $y_1 = 2x + y + z$, $y_2 = x + y + 2z$, $y_3 = x - 2z$ is regular. Write down the inverse transformation. (05 Marks)

OR

- 10 a. Solve the following system of equations by Gauss – Seidel method. (06 Marks)
 $10x + 2y + z = 9$, $x + 10y - z = -22$, $-2x + 3y + 10z = 22$.
- b. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. (05 Marks)
- c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. (05 Marks)

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