USN | | | | | | | | | | 15MAT11

First/Second Semester B.E. Degree(CBCS)Examination

Engineering Mathematics-I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the nth derivative of $\frac{x^2+4x+1}{x^3+2x^2-x-2}$ (04 Marks)

b. Find the angle between the radius vector and the tangent for the curve $r = a(1+cos\theta)$ and also find the slope of the tangent at $\theta = \frac{\pi}{3}$ (04 Marks)

c. Find the angle of intersection between the curves $r^2 sin2\theta = 4$ and $r^2=16\ sin2\theta$ (04 Marks)

d. Obtain the Pedal equation of the curve $\frac{2a}{r} = 1 + cos\theta$ (04 Marks)

OR

2 a. If $y = e^{msin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (m^2+n^2)y_n = 0$ (06 Marks)

b. Find the pedal equation , $r^n=a^n\ cosn\theta$ (05 Marks)

c. Show that the radius of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ $is\ -\frac{3a}{8\sqrt{2}}$ (05 Marks)

Module-2

3 a. Expand $\log_e x$ in powers of $(x-1)$ up to fourth degree term. (04 Marks)

b. Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$ (04 Marks)

c. If $z = f(x+ct) + g(x-ct)$ prove that $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$ (04 Marks)

d. If $u = \cos^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2cotu$ (04 Marks)

OR

4 a. Prove that $\sqrt{1+sin2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{24} + \cdots\cdots\cdots$ (06 Marks)

b. If u= f(x-y, y-z, z-x) then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ (05 Marks)

c. If $u = x+y+z, v = x^2+y^2+z^2, w = xy+yz+zx, then\ find \frac{\partial(u,v,w)}{\partial(x,y,z)}$ (05 Marks)

Module-3

5 a. A particle moves along the curve $x=1-t^3$, $y=1+t^2$ and z= 2t-5, find the velocity and acceleration. Also find the components of velocity and acceleration at t=1 in the direction 2i+j+2k. (08 Marks)

b. Find the constants a, b, c so that the vector field $\vec{F} = (x+2y+az)\hat{\imath} + (bx-3y-z)\hat{\jmath} + (4x+cy+2z)\hat{k}$ is irrotational. (08 Marks)

OR

6 a. If $\hat{u} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and $\hat{v} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ then prove that $\vec{u}\times\vec{v}$ is a solenoidal vector. (08 Marks)

b. Find $div\vec{F}$ and $curl\vec{F}$ if $\vec{F} = grad(x^3+y^3+z^3-3xyz)$. (08 Marks)

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} sin^n x\, dx$. (06 Marks)

b. Solve $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$. (05 Marks)

c. Find the Orthogonal trajectories of the family of cardioids $r = a(1 + cos\theta)$ where a is the parameter. (05 Marks)

OR

8 a. Evaluate $\int_0^{\pi/6} cos^4 3\theta\, sin^3 6\theta\, d\theta$. (06 Marks)

b. Solve $x\frac{dy}{dx} + y = x^3 y^6$ (05 Marks)

c. If a substance cools from 370k to 330k in 10minutes, when the temperature of the surrounding air is 290k. Find the temperature of the substance after 40 minutes. (05 Marks)

Module-5

9 a. Solve the following system of equations by Gauss elimination method

$$x + 2y + z = 3$$

$$2x + 3y + 3z = 10$$

$$3x - y + 3z = 13$$

(06 Marks)

b. Use power method to find the largest eigen value and the corresponding eigen vector of the matrix taking $[0 \quad 1]'$ as initial eigen vector

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(05 Marks)

c. Show that the transformation $y_1 = x - y + z$

$$y_2 = 3x - y + 2z$$

$$y_3 = 2x - 2y + 3z$$

is non-singular. Find the inverse transformation (05 Marks)

OR

10 a. Solve the following system of equations by Gauss-Seidel method

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

(05 Marks)

b. Reduce the following matrix to the diagonal form

$$\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$$

(05 Marks)

c. Reduce the quadratic form $2xy + 2xz - 2yz$ to the canonical form by orthogonal transformation. (06 Marks)

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