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## First Semester B.E. Degree Examination, Dec.2017/Jan.2018 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

**Note: Answer FIVE full questions, choosing one full question from each module.**

### Module-1

- 1 a. Find the  $n^{\text{th}}$  derivative of  $y = e^{-x} \sin x \cos 2x$ . (06 Marks)
- b. Show that the curves  $r = a(1 + \cos \theta)$  and  $r = b(1 - \cos \theta)$  cut each other orthogonally. (05 Marks)
- c. Find the radius of curvature of the curve  $x^2 y = a(x^2 + y^2)$  at the point  $(-2a, 2a)$ . (05 Marks)

**OR**

- 2 a. If  $y = \sin(m \sin^{-1} x)$ , then prove that  $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$  (06 Marks)
- b. Find the pedal equation of  $r = 2(1 + \cos \theta)$ . (05 Marks)
- c. Find the radius of curvature of  $r^n = a^n \sin n\theta$ . (05 Marks)

### Module-2

- 3 a. Expand  $\tan^{-1} x$  in powers of  $(x-1)$  upto the fourth degree term. (06 Marks)
- b. Evaluate  $\lim_{x \rightarrow 0} \left[ \frac{1}{x} - \frac{\log(1+x)}{x^2} \right]$  (05 Marks)
- c. If  $z = f(x+ct) + g(x-ct)$ , prove that  $\frac{\partial^2 z}{\partial t^2} = C^2 \frac{\partial^2 z}{\partial x^2}$ . (05 Marks)

**OR**

- 4 a. Obtain the Maclaurin's series expansion of  $e^{\sin x}$  upto the form containing  $x^4$ . (06 Marks)
- b. If  $z = \log \left( \frac{x^4 + y^4}{x+y} \right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ . (05 Marks)
- c. If  $u = x^2 + y^2 + z^2$ ,  $v = xy + yz + zx$ ,  $w = x + y + z$ , show that  $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$ . (05 Marks)

### Module-3

- 5 a. A particle moves along the curve whose parametric equations are  $x = t^3 + 1$ ,  $y = t^2$  and  $z = 2t + 5$ . Find the components of its velocity and acceleration at time  $t = 1$  in the direction of  $i + j + 3k$ . (06 Marks)
- b. If  $\phi = 2x^3 y^2 z^4$ , find  $\text{Div}(\text{Grad } \phi)$ . (05 Marks)
- c. Show that  $\vec{F} = (y+z)i + (z+x)j + (x+y)k$  is irrotational. Also find a scalar function  $\phi$ , such that  $\vec{F} = \nabla \phi$ . (05 Marks)

OR

- 6 a. Find the directional derivative of  $\phi = x^2yz + 4xz^2$  at  $P(1, -2, -1)$  in the direction of  $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ . (06 Marks)
- b. If  $\vec{F} = (x + y + 1)\mathbf{i} + \mathbf{j} - (x + y)\mathbf{k}$ . Show that  $\vec{F} \cdot \text{curl } \vec{F} = 0$ . (05 Marks)
- c. If  $\vec{F} = \nabla(xy^3z^2)$ , find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$  at the point  $(1, -1, 1)$ . (05 Marks)

Module-4

- 7 a. Obtain the reduction formula for  $\int \cos^n x dx$ . (06 Marks)
- b. Solve  $ye^{xy} dx + (xe^{xy} + 2y)dy = 0$ . (05 Marks)
- c. Find the orthogonal trajectories of the family of curves  $y^2 = Cx^3$ . (05 Marks)

OR

- 8 a. Evaluate  $\int_0^1 x^{3/2} (1-x)^{3/2} dx$ . (06 Marks)
- b. Solve  $\frac{dy}{dx} - \frac{2}{x}y = \frac{y^2}{x^3}$ . (05 Marks)
- c. A body is heated to  $110^\circ\text{C}$  and placed in air at  $10^\circ\text{C}$ . After one hour its temperature becomes  $60^\circ\text{C}$ . How much additional time is required for it to cool to  $30^\circ\text{C}$ ? (05 Marks)

Module-5

- 9 a. Find the rank of the matrix  $A = \begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$ . (06 Marks)
- b. Solve the following system of equations by Gauss Jordan method:  
 $x + 2y + z = 3$ ,  $2x + 3y + 3z = 10$ ,  $3x - y + 2z = 13$  (05 Marks)
- c. Reduce the matrix  $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$  to the diagonal form. (05 Marks)

OR

- 10 a. Solve the following system of equations by Gauss-Seidal method:  
 $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$ . Perform three iterations. (06 Marks)
- b. Show that the transformation,  $y_1 = 2x_1 - 2x_2 - x_3$ ,  $y_2 = -4x_1 + 5x_2 + 3x_3$ ,  $y_3 = x_1 - x_2 - x_3$  is regular and find the inverse transformation. (05 Marks)
- c. Reduce the quadratic form,  $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$  into the canonical form. (05 Marks)

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