CBCS Scheme

5

USN

17MAT11

First Semester B.E. Degree Examination, Dec.2017/Jan.2018

Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Find the n^{th} derivative of $\cos x \cos 2x$. (06 Marks)

b. Find the angle between the curves $r = a\log\theta$, $r = \dfrac{a}{\log\theta}$. (07 Marks)

c. Find the radius of curvature of the curve $r = a(1+\cos\theta)$. (07 Marks)

OR

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. (06 Marks)

b. With usual notations prove that the pedal equation in the form $\dfrac{1}{p^2} = \dfrac{1}{r^2} + \dfrac{1}{r^4}\left(\dfrac{dr}{d\theta}\right)^2$. (07 Marks)

c. Find the radius of curvature of the curve $y^2 = \dfrac{a^2(a-x)}{x}$ at the point (a, 0). (07 Marks)

Module-2

3 a. Find the Taylor's series of $\log x$ in powers of $(x-1)$ upto fourth degree terms. (06 Marks)

b. If $U = \tan^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$, prove that $x\dfrac{\partial U}{\partial x} + y\dfrac{\partial U}{\partial y} = \sin 2U$ by using Euler's theorem. (07 Marks)

c. If $U = x+3y^2$, $V = 4x^2yz$, $W = 2z^2 - xy$, evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at the point (1, -1, 0). (07 Marks)

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{1/x}$. (06 Marks)

b. Find the Maclaurin's expansion of $\log(\sec x)$ upto x^4 terms. (07 Marks)

c. If $z = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$, prove that $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2$. (07 Marks)

Module-3

5 a. A particle moves along the curve $\bar{r} = (t^3-4t)\hat{i} + (t^2+4t)\hat{j} + (8t^2-3t^3)\hat{k}$. Find the velocity and acceleration vectors at time t and their magnitudes at t = 2. (06 Marks)

b. If $\bar{f} = (x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}$, prove that $\bar{f}.\text{curl}\,\bar{f} = 0$. (07 Marks)

c. Prove that $\text{div}(\text{curl}\,\bar{A}) = 0$. (07 Marks)

OR

6 a. A particle moves along the curve $\bar{r} = 2t^2\bar{i} + (t^2 - 4t)\hat{j} + (3t - 5)\vec{k}$. Find the components of velocity and acceleration along $\bar{i} - 3\bar{j} + 2\bar{k}$ at t = 2. **(06 Marks)**

b. If $\bar{f} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find $\text{div}\,\bar{f}$ and $\text{curl}\,\bar{f}$. **(07 Marks)**

c. Prove that $\text{curl}(\text{grad}\,\phi) = 0$. **(07 Marks)**

Module-4

7 a. Evaluate $\int_0^{2a} \frac{x^2}{\sqrt{2ax - x^2}} dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\tan x = y^3 \sec x$. **(07 Marks)**

c. Find the orthogonal trajectories of $r^n = a^n \cos n\theta$. **(07 Marks)**

OR

8 a. Find the reduction formula for $\int \cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} \cos^n x\, dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

c. A body originally at 80°C cools down to 60°C in 20 minutes in the surroundings of temperature 40°C. Find the temperature of the body after 40 minutes from the original instant. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$A = \begin{pmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{pmatrix}$$

by reducing it to echelon form. **(06 Marks)**

b. Using the power method find the largest eigenvalue and the corresponding eigenvector of matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ taking $(1, 1, 1)^T$ as the initial eigenvector. Perform five iterations. **(07 Marks)**

c. Show that the transformation $y_1 = x_1 + 2x_2 + 5x_3$, $y_2 = 2x_1 + 4x_2 + 11x_3$, $y_3 = -x_2 + 2x_3$ is regular. Also, find the inverse transformation. **(07 Marks)**

OR

10 a. Solve the following system of equations by using Gauss-Jordan method:
$x + y + z = 9$, $x - 2y + 3z = 8$, $2x + y - z = 3$ **(06 Marks)**

b. Diagnolize the matrix $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$. **(07 Marks)**

c. Obtain the canonical form of $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ using orthogonal transformation. **(07 Marks)**

* * * * *