

VTU QUESTION PAPERS

MODULE - 1 DIFFERENTIAL EQUATIONS – I

- 1) Solve $4\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 23\frac{d^2y}{dx^2} - 12\frac{dy}{dx} - 36 = 0$ (July 2015)
- 2) Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = e^x - 1$ **July 2015**
- 3) Solve $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$ **July 2015**
- 4) Solve $y'' - 4y' + 12y = e^{2x} + 3\sin 2x$ Jan 2016
- 5) By the method of undetermined coefficients solve $y'' - y' = 2\cos x$ (Jan 2016)
- 6) By the method of variation of parameters solve $y'' - 4y' + \tan 2x$ (Jan 2016)
- 7) Solve $(D^4 - m^4)y = 0$ (Jan 2016)
- 8) Solve $(D^4 - m^4)y = 0$ (Jan 2016)
- 9) Solve $(D^2 - 7D + 12)y = \cosh x$ Jan 2016
- 10) By the method of variation of parameters solve $y'' - x \sin x = 9$ Jan 2016
- 11) Solve $(4D^4 - 8D^3 + 7D^2 + 11D - 6)y = 0$ (June 2015)
- 12) Solve $(D^2 - 4)y = x^2 e^{-x}$ (June 2015)
- 13) Solve $(D^2 - 2D - 2)y = e^x \tan x$ using method of variation of parameters. (June 2015)
- 14) Solve $(D^3 - D)y = 2e^x + 4\cos x$ (Jan 2015)

15) Solve: $(D^2 - 2)y = x^2 e^{3x} - e^x \cos 2x$

(Jan 2015)

16) Solve the simultaneous equation $(D+5)x - 2y = t$ and $(D+1)y + 2x = 0$

(Jan 2015)

17) Solve $(D - 2)^2 y = 8 e^{2x} \sin 2x$

(June 2014)

18) Solve: $y'' - 2y' + y = x \cos x$

(June 2014)

19) Solve $\frac{dx}{dt} - 7x - y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$

(June 2014)

20) Solve $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} - 2x = \sin 2t$, given that $x = 1, y = 0$ at $t = 0$

(Dec 2013)

21) Using the method of variation of parameters solve $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$.

(June 2014, Dec 2013)

22) Solve: $x^2 y'' - xy' + y = 2 \cos^2(\log x)$.

(Dec 2013)

MODULE-2

DIFFERENTIAL EQUATIONS - II

- 1) Solve the simultaneous equations $\frac{dx}{dt} = 2y - \sin t$, $\frac{dy}{dt} = 2x - \cos t$ given that $x = 0$, $y = 1$ when $t = 0$ (Jan 2016)
- 2) Solve $x^2y - xy - 2y - x \sin \log x$ (Jan 2016)
- 3) Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ (Jan 2016, June 2015)
- 4) Solve $x - a^2y - 4x - ay - 6y - x$ (July 2015)
- 5) Solve $p \tan x = \frac{p}{1 - p^2}$ (Jan 2016)
- 6) Find the general and singular solution of the equation $y = px - p^3$ (Jan 2016)
- 7) Solve $(px - y)(py - x) = a^2p$ by reducing to Clairaut's equation (June 2015)
- 8) Solve $(1 - x)^2y - (1 - x)y - y = 2 \sin \log x$ (June 2015)
- 9) Solve $y = 2px - y^2p^3$ by solving for x . (June 2015)
- 10) Solve $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} - 2y = e^x$ (June 2015)
- 11) Solve $\frac{dx}{dt} = 7x - y$, $\frac{dy}{dt} = 2x - 5y$ (June 2015)
- 12) Solve $p^2 - 4x^5p - 12x^4y = 0$, obtain the singular solution also.. (Jan 2015)
- 13) Solve $px - y = py - x = 2p$, by reducing into Clairaut's form, taking the substitution $X = x^2, Y = y^2$. (Jan 2015)
- 14) Solve $p^3 - 4xyp - 8y^2 = 0$ by solving for x .. (Jan 2015)
- 15) Solve $p(p - y) = x(x - y)$ (June 2014)
- 16) Obtain the general solution and singular solution of the equation $y = 2px - p^2y$. (June 2014)

17) Obtain the general solution and singular solution of the Clairaut's equation $x p^3 - y p^2 - 1 = 0$.
(Dec 2013)

18) Solve $p^2 - 2py \cot x = y^2$.
(Dec 2013)

MODULE 3

PARTIAL DIFFERENTIAL EQUATION

- 1) Form the partial differential equation of $Z = y f(x) + xg(y)$ where f and g are arbitrary functions. (Jan 2016)
- 2) Derive one dimensional heat equation as $\frac{u}{t} = c^2 \frac{\partial^2 u}{\partial x^2}$. (Jan 2016, July 2015)
- 3) From the function $f(x^2 + y^2, z - xy) = 0$ form the partial differential equation. (July 2015)
- 4) Derive one dimensional wave equation as $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (July 2015)
- 5) Solve $z_{xy} = \sin x \sin y$ for which $z = 2 \sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. (Jan 2015)
- 6) Solve: $x^2 \frac{\partial^2 z}{\partial y^2} + y^2 \frac{\partial^2 z}{\partial x^2} = 2xyq - 2xz$ (Jan 2015)
- 7) Solve by the method of variables $3u_x - 2u_y = 0$, given that $u(x, 0) = 4e^{-x}$ (Jan 2015)
- 8) Form the partial differential equation by eliminating the arbitrary functions from $z = f(y-2x) + g(2y-x)$ (June 2014)
- 9) Solve: $x^2 \frac{\partial^2 z}{\partial y^2} + yz \frac{\partial^2 z}{\partial x^2} = y^2 \frac{\partial^2 z}{\partial x \partial y} + zx \frac{\partial^2 z}{\partial y^2} - xy \frac{\partial^2 z}{\partial x^2}$ (June 2014, Dec 2013)
- 10) Solve by the method of variables $4 \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 3u$ given that $u(0, y) = 2e^{5y}$ (Jan 2015, June 2014)
- 11) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$ (Dec 2013)

MODULE-4

INTEGRAL CALCULUS

1) Evaluate $\int_a^c \int_b^c \int_a^c x^2 y^2 z^2 dx dy dz$ (Jan 2016)

2) Solve $\frac{z^2}{xy} \sin x \sin y$, for which $\frac{z}{y} = 2 \sin y$ when $x = 0$ and $z = 0$

when y is an odd multiple of $\frac{\pi}{2}$ Jan 2016, July 2015

3) Evaluate $\int_R xy dx dy$, where R is the region bounded by x -axis, the ordinate $x=2a$ and the parabola $x^2=4ay$. (Jan 2016)

4) Evaluate $\int_0^1 \int_0^1 e^{(x^2+y^2)} dx dy$ by changing into polar coordinates. (Jan 2016)

5) Evaluate $\int_0^1 \int_{x^2}^x xy dy dx$ changing the order of integration. (July 2015)

6) Evaluate $\int_0^1 \int_0^x \int_0^z x y z dy dx dz$. (July 2015)

7) Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$ (July 2015)

8) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$ using beta and gamma function (June 2015)

9) Show that $\int_0^{\frac{\pi}{2}} \frac{d}{\sqrt{\sin d}} = \int_0^{\frac{\pi}{2}} \sqrt{\sin d} d$ (June 2015)

10) Evaluate $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy dx dy$ by changing the order of integration (Jan 2015, June 2014)

Solution:

11) Evaluate $\int_0^1 \int_0^x \int_0^z (x - y - z) dy dx dz$ (Jan 2015)

12) Show that $\int_1^2 \int_1^x x^{p-1} \int_1^x x^{q-1} dx = 2^{p+q-1} (m, n)$ (Jan 2015, Dec 2013)

13) Prove that $B(m, n) = \int_0^1 \frac{x^{m-1} (1-x)^{n-1}}{1-x^{m+n}} dx$ (June 2014)

14) Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dx dy$ and hence evaluate the same. (June 2014)

15) Evaluate $\int_0^a \int_0^x \int_0^{x-y} e^{x+y+z} dz dy dx$. (June 2014)

16) Change the order of integration in $\int_0^1 \int_{x^2}^x xy dx dy$ and evaluate the same (Dec 2013)

17) Evaluate $\int_0^1 \int_0^1 e^{(x^2-y^2)} dx dy$ by changing to polar coordinates (Dec 2013)

MODULE-5

LAPLACE TRANSFORMS

1) Find $L e^{-2t} \sin 3t - e^t t \cos t$. (Jan 2016)

2) Find the inverse Laplace transform of $\frac{4s-5}{s^2-1} - \frac{5}{s^2-2}$. (Jan 2016)

3) Solve $y'' - 6y' + 9y = 12t^2 e^{3t}$ by Laplace transform method with $y(0) = 0 = y'(0)$. (Jan 2016)

4) Express $f(t) = \begin{cases} \cos t, & 0 \leq t < 2 \\ 1 - t, & 2 \leq t < 4 \\ \sin t, & t \geq 4 \end{cases}$ in terms of unit step function and hence find its Laplace transform. (Jan 2016)

5) Solve $y'' - 6y' + 9y = 12t^2 e^{3t}$ by Laplace transform method with $y(0) = 0 = y'(0)$. (Jan 2016)

6) Find $L \frac{\cos at - \cos bt}{t}$ (Jan 2016)

7) Find the Laplace transform of $t e^{4t} \sin 3t$ and $\frac{e^{at} - e^{bt}}{t}$ (July 2015)

8) Express $f(t)$ in terms of unit step function and find its Laplace transform given that

$$f(t) = \begin{cases} t^2, & 0 \leq t < 2 \\ t - 4t, & 2 \leq t < 4 \\ 8, & t \geq 4 \end{cases} \quad \text{(July 2015)}$$

9) Find $L^{-1} \frac{1}{s^2 - 1} - \frac{1}{s^2 - 9}$ using convolution theorem. (July 2015)

10) A periodic function $f(t)$ with period 2 is defined by $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 2 - t, & 1 \leq t < 2 \end{cases}$ find $L\{f(t)\}$

(July 2015)

11) Find $L^{-1} \left\{ \frac{5s^2}{3s^2 - 4s - 8} \log \frac{1}{s^2} \right\}$ (July 2015)

12) Solve using Laplace transform method $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - y = te^{-t}$ with $y(0) = 1, y'(0) = 2$ (July 2015)

13) Find $L \{ t (\sin^3 t - \cos^3 t) \}$ (Jan 2015)

14) Express $f(t)$ in terms of unit step function and hence find the Laplace transform (Jan 2015)

$$f(t) = \begin{cases} t^2 & 0 \leq t < 2 \\ 4t - 2 & 2 \leq t < 4 \\ 8 - t & 4 \leq t < 6 \end{cases}$$

15) Find the value of $\int_0^{\infty} t^3 e^{-t} \sin t dt$ using Laplace transforms (Jan 2015)

16) Find Laplace transform of a periodic function $f(t) = \begin{cases} t, & 0 \leq t < 2 \\ 0, & 2 \leq t < 4 \end{cases}$ (Jan 2015)

17) Prove that $L \{ (t - a)^n e^{-as} \} = \frac{n!}{s^{n+1}}$ (June 2014)

18) If $f(t) = \begin{cases} t, & 0 \leq t < a \\ 2a - t, & a \leq t < 2a \end{cases}$, where $f(t + 2a) = f(t)$, show that $L \{ f(t) \} = \frac{E}{s} \tanh \frac{as}{2}$

Laplace transform (June 2014)

19) Find the inverse Laplace transform of $\tan^{-1}(2/s^2)$ (June 2014)

20) Find $L^{-1} \left\{ \frac{S}{(s-1)(s^2-4)} \right\}$ using convolution theorem (June 2014)

21) Solve the following initial value problem by using Laplace transforms:

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 4y = e^{-t}; y(0) = 0, y'(0) = 0$$
 (June 2014)

22) Find $L^{-1} \left\{ \frac{5s^3}{s^4 - 1} \right\}$ (Dec 2013)

23) Using convolution theorem evaluate $L^{-1} \left[\frac{s^2}{s^2 + a^2} \right]$

(Dec 2013)

24) Solve $y''' - 2y'' + y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using

Laplace transform method

(Dec 2013)

25) Solve the initial value problem $(D^3 - 3D^2 - 3D - 1)y = 2t^2e^t, y(0) = 1, y'(0) = 0, y''(0) = 2$

Dec 2013