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First Semester B.E. Degree Examination, June/July 2016 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions,
choosing ONE full question from each module.

Module-1

- 1 a. Find the n^{th} derivative of $y = e^{-3x} \cos^3 x$. (06 Marks)
- b. Find the angle of intersection between the curves $r = a(1 + \sin \theta)$ and $r = a(1 - \cos \theta)$. (05 Marks)
- c. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (05 Marks)

OR

- 2 a. If $y = \sin(\log(x^2 + 2x + 1))$, prove that $(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$. (06 Marks)
- b. Find the pedal equation for the curve $r^m \cos m\theta = a^m$. (05 Marks)
- c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1, 1)$. (05 Marks)

Module-2

- 3 a. Expand $\sin x$ in powers of $x - \frac{\pi}{2}$ upto 4th degree terms using Taylor's series. (05 Marks)
- b. Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$. (05 Marks)
- c. If $u = \tan^{-1} \left(\frac{x^2 + y^2}{x + y} \right)$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$. (06 Marks)

OR

- 4 a. Expand $\log(1 + e^x)$ using Maclaurin's series upto 3rd degree terms. (06 Marks)
- b. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$. (05 Marks)
- c. If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, find $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$. (05 Marks)

Module-3

- 5 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time, find the component of its velocity and acceleration in the direction of the vector $i - 3j + 2k$ at $t = 1$. (06 Marks)
- b. Show that $\vec{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational, find ϕ such that $F = \nabla \phi$. (05 Marks)
- c. Prove that $\text{div}(\text{curl } u) = 0$. (05 Marks)



OR

- 6 a. If $\vec{r} = x_i + y_j + z_k$, then prove that : i) $\nabla \times \vec{r} = 0$ ii) $\nabla^2 r^n = n(n+1)r^{n-2}$. (06 Marks)
 b. Prove with usual notations $\text{Curl}(\text{grad } \phi) = 0$ (05 Marks)
 c. Find $\text{div } \vec{f}$ and $\text{curl } \vec{f}$ of $\vec{f} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$. (05 Marks)

Module-4

- 7 a. Obtain the reduction formula of $\int \sin^m x \cos^n x \, dx$. (06 Marks)
 b. Solve $(x^2 + y^3 + 6x) \, dx + y^2 x \, dy = 0$. (05 Marks)
 c. Find the orthogonal trajectory of $r^n = a^n \cos n\theta$, where a is the parameter. (05 Marks)

OR

- 8 a. Obtain the reduction formula of $\int \cos^n x \, dx$ and hence evaluate : $\int_0^{\pi/2} \cos^n x \, dx$. (06 Marks)
 b. Solve $\frac{dy}{dx} = xy^3 - xy$. (05 Marks)
 c. If the temperature of the air is 30°C and the substance cools from 100°C to 70°C in 15 minutes, find when the temperature reaches at 40°C . (Use Newton's law of cooling). (05 Marks)

Module-5

- 9 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$
 (06 Marks)
 b. Find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$
 by power method, use $[1, 0, 0]^T$ as initial vector, take five iterations. (05 Marks)
 c. Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to the diagonal form. (05 Marks)

OR

- 10 a. Use Gauss – Siedel iteration method upto 3 iterations to solve with $(0, 0, 0)$ as initial values
 $10x + y + z = 12$
 $x + 10y + z = 12$
 $x + y + 10z = 12$. (06 Marks)
 b. Show that the transformation :
 $y_1 = 2x_1 + x_2 + x_3$
 $y_2 = x_1 + x_2 + 2x_3$
 $y_3 = x_1 - 2x_3$
 is regular. Write down the inverse transformation. (05 Marks)
 c. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. (05 Marks)
