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USN

MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2017/Jan.2018

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions selecting atleast TWO questions from each part.***

PART – A

1 a. Find the direction cosines ℓ, m, n of the line :
$x + y + z + 1 = 0$
$4x + y - 2z + 2 = 0.$ **(06 Marks)**

b. Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. **(07 Marks)**

c. Find the angle between the line $\frac{x+4}{4} + \frac{y-3}{-3} = \frac{z+2}{1}$ and the plane $2x + 2y - z + 15 = 0$. **(07 Marks)**

2 a. Find the equation of the plane which passes through the points A(0, 1, 1), B(1, 1, 2), C(–1, 2, –2). **(06 Marks)**

b. Find the equation of the plane which passes through the point(3, –3, 1) and normal to the line joining the points (3, 2, –1) and (2, –1, 5). **(07 Marks)**

c. Find the equations to the two planes which bisects the angle between the planes :
$3x - 4y + 5z = 3$
$5x + 3y - 4z = 9.$ **(07 Marks)**

3 a. Find the sides and the angle A of the triangle whose vertices are $\overline{OA} = I - 2J + 2K$, $\overline{OB} = 2I + J - K, \overline{OC} = 3I - J + 2K$. **(06 Marks)**

b. Show that the points $-6I + 3J + 2K$, $3I - 2J + 4K$, $5I + 7J + 3K$ and $-13I + 17J - k$ are coplanar. **(07 Marks)**

c. Prove that : $[\overline{B}\times\overline{C}, \overline{C}\times\overline{A}, \overline{A}\times\overline{B}] = [\overline{A}\,\overline{B}\,\overline{C}]^2$. **(07 Marks)**

4 a. A particle moves along the curve $x = t^2 + 1$, $y = t^2$, $z = 2t + 3 + \sin(\pi t)$ where t is the time. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. If $\overline{A} = (\cos t)I + (\sin t)J + (4t)K$ and $\overline{B} = (t^3 + 1)I + J + (8t^2 - 3t^3)K$ then find :
i) $\frac{d}{dt}(\overline{A} + \overline{B})$ ii) $\frac{d}{dt}(\overline{A}\cdot\overline{B})$. **(07 Marks)**

c. If $\phi = 3x^2y - y^3z^2$, find grad ϕ at (1, –2, 1). Also find a unit normal vector to the surface $3x^2y - y^3z^2 = 6$ at (1, –2, 1). **(07 Marks)**

PART – B

5 a. If $\overline{A} = xyz\,I + 3x^2y\,J + (xz^2 - y^2z)K$ then find curl $\overline{A}$ at (1, 2, 3). **(06 Marks)**

b. Find the directional derivative of the scalar function $f(x, y, z) = x^2 + xy + z^2$ at the point A(1, –1, –1) in the direction of $2\hat{i} + 3\hat{j} + 2\hat{k}$. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$ and $\overline{r} = xI + yJ + zK$ then find div $(u\overline{r})$ in terms of u. if $\vec{f} = \nabla(x^3 + y^3 + 2^3 - 3xyz)$ find $\nabla\cdot\vec{f}$ and $\nabla\times\vec{f}$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Find the Laplace transform of f(t) defined as :

$$f(t)=\begin{cases}\frac{t}{6}, & \text{when} \quad 0<t<6 \\ 1, & \text{when} \quad t<6\end{cases}.$$ (05 Marks)

b. Find : i) $L(\cos^2 t)$ ii) $L(t \sin h\, at)$ iii) $L\left(\frac{1}{t}\sin 2t\right)$. (15 Marks)

7 a. Find : $L(e^{2t}\cos 3t)$. (06 Marks)

b. Find : $L^{-1}\left(\dfrac{2h-5}{9s^2-25}\right)$. (07 Marks)

c. Find : $L^{-1}\left(\dfrac{s^2+4}{x^2+9}\right)$. (07 Marks)

8 a. Using Laplace transforms, find the solution of the initial value problem $y''-4y'+4y=64\sin 2t$, $y(0)=0$, $y'(0)=1$. (10 Marks)

b. Using Laplace transforms, solve $y''+9y=\cos 2t$, $y(0)=1$, $y'(0)=\frac{12}{5}$. (10 Marks)

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