

CBCS Scheme

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15CS36

Third Semester B.E. Degree Examination, Dec.2016/Jan.2017

Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1. a. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound proposition
 i) $(p \wedge q) \rightarrow r$ ii) $p \rightarrow (q \wedge r)$ iii) $p \wedge (r \rightarrow q)$ iv) $p \rightarrow (q \rightarrow (\neg r))$ (04 Marks)
- b. Define tautology. Prove that for any propositions p, q, r the compound proposition $[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is tautology. (04 Marks)
- c. Establish the validity of the following argument
 $\forall x, [p(x) \vee q(x)]$
 $\exists x, \neg p(x)$
 $\forall x, [\neg q(x) \vee r(x)]$
 $\forall x, [s(x) \rightarrow \neg r(x)]$
 $\therefore \exists x \neg s(x)$ (04 Marks)
- d. Give i) direct proof and ii) proof by contradiction for the following statement. "If 'n' is an odd integer, then n+9 is an even integer". (04 Marks)

OR

2. a. Define dual of a logical statement. Verify the principle of duality for the following logical equivalence $[\sim (p \wedge q) \rightarrow \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$. (04 Marks)
- b. Prove the following by using laws of logic
 i) $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$
 ii) $[\sim p \wedge (\sim q \vee r)] \vee [(q \wedge r) \vee (p \wedge q)] \Leftrightarrow r$. (04 Marks)
- c. Establish the validity of the following argument using the rules of inference:
 $[p \wedge (p \rightarrow q) \wedge (s \vee t) \wedge (r \rightarrow \sim q)] \rightarrow (s \vee t)$ (04 Marks)
- d. Define i) open sentence ii) quantifiers. For the following statements, the universe comprises all non-zero integers. Determine the truth values of each statement :
 i) $\exists x, \exists y (xy = 1)$ ii) $\exists x, \forall y (xy = 1)$ iii) $\forall x, \exists y (xy = 1)$. (04 Marks)

Module-2

3. a. By mathematical induction, prove that
 $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$. (05 Marks)
- b. For the Fibonacci sequence show that

$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$
 (05 Marks)
- c. A women has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations : i) There is no restriction on the choice ii) Two particular persons will not attend separately iii) Two particular persons will not attend together. (06 Marks)

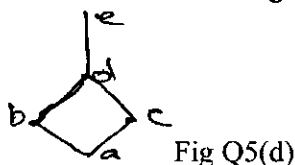
Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

OR

- 4 a. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's. (04 Marks)
 b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. (04 Marks)
 c. i) How many arrangements are there for all letters in the word SOCIOLOGICAL?
 ii) In how many of these arrangements A and G are adjacent? In how many of these arrangements all the vowels are adjacent? (04 Marks)
 d. Find the coefficient of i) $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$ ii) $a^2 b^3 c^2 d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. (04 Marks)

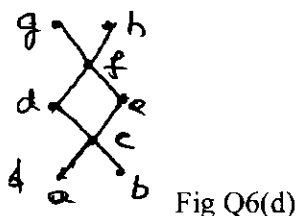
Module-3

- 5 a. Let a function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Find the images of $A_1 = \{2, 3\}$, $A_2 = \{-2, 0, 3\}$, $A_3 = (0, 1)$ and $A_4 = [-6, 3]$. (04 Marks)
 b. ABC is an equilateral triangle whose sides are of length one cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than $\frac{1}{2}$ cm. (04 Marks)
 c. Let f, g, h be functions from \mathbb{Z} to \mathbb{Z} defined by $f(x) = x - 1$, $g(x) = 3x$ and $h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$. Determine $(fo(goh))(x)$ and $((fo g)oh)(x)$ and verify that $fo(goh) = (fog)oh$. (04 Marks)
 d. For $A = \{a, b, c, d, e\}$ the Hasse diagram for the Poset (A, R) is as shown in Fig Q5(d). Determine the relation matrix for R and Construct the digraph for R . (04 Marks)



OR

- 6 a. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$. Determine the
 i) Number of binary relations on A .
 ii) Number of relations from A to B that contain $(1, 2)$ and $(1, 5)$
 iii) Number of relations from A, B that contain exactly five ordered pairs
 iv) Number of binary relations on A that contains at least seven ordered pairs. (04 Marks)
 b. Let $A = B = \mathbb{R}$ be the set of the real numbers, the functions $f : A \rightarrow B$ and $g : B \rightarrow A$ be defined by $f(x) = 2x^3 - 1$, $\forall x \in A$; $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}$ $\forall y \in B$. Show that each of f and g is the inverse of the other. (04 Marks)
 c. Define a relation R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$, where $A = \{1, 2, 3, 4, 5\}$.
 i) Verify that R is an equivalence relation on $A \times A$.
 ii) Determine the equivalence classes $[(1, 3)]$ and $[(2, 4)]$. (04 Marks)
 d. Consider the Hasse diagram of a POSET (A, R) given in Fig Q6(d). If $B = \{c, d, e\}$ find all upper bounds, lower bounds, the least upper bound and the greatest lower bound of B . (04 Marks)



Module-4

- 7 a. Determine the number of positive integers n such that $1 \leq n \leq 100$ and n is not divisible by 2, 3, or 5. (04 Marks)
- b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? (04 Marks)
- c. A girl student has Sarees of 5 different colors, blue, green red, white and yellow. On Monday she does not wear green, on Tuesdays blue or red, on Wednesday blue or green, on Thursday red or yellow; on Friday red. In how many ways can she dress without repeating a color during a week (from Monday to Friday)? (04 Marks)
- d. The number of affected files in a system 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. (04 Marks)

OR

- 8 a. In how many ways can one arrange the letters in the word CORRESPONDENTS so that
- There is no pair of consecutive identical letters?
 - There are exactly two pairs of consecutive identical letters? (06 Marks)
- b. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 , and B_4 . The boys B_1 and B_2 do not wish to have apple, the boy, B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? (05 Marks)
- c. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$ given that $a_1 = 5$ and $a_2 = 3$. (05 Marks)

Module-5

- 9 a. Define :
- Bipartite graph
 - Complete bipartite graph
 - Regular graph
 - Connected graph with an example. (04 Marks)
- b. Define isomorphism. Verify the two graphs are isomorphic (04 Marks)

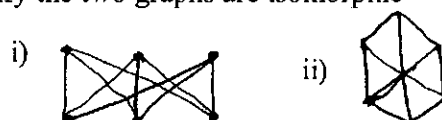


Fig Q9(b)

- c. Show that a tree with n vertices has $n-1$ edges. (04 Marks)
- d. Obtain an optimal prefix code for the message ROAD IS GOOD. Indicate the code. (04 Marks)

OR

- 10 a. Determine the order $|V|$ of the graph $G = (V, E)$ in
- G is a cubic graph with 9 edges
 - G is regular with 15 edges
 - G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. (04 Marks)
- b. Prove that in a graph
- The sum of the degrees of all the vertices is an even number and is equal to twice the number of edges in the graph.
 - The number of vertices of odd degrees is even. (04 Marks)
- c. Discuss the solution of Konigsberg bridge problem. (04 Marks)
- d. Define optimal tree and construct an optimal tree for a given set of weights $\{4, 15, 25, 5, 8, 16\}$. Hence find the weight of the optimal tree. (04 Marks)